Enhanced Gravitational Effects of Radiation and Cosmological Implications

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In the momentarily comoving frame of a cosmological fluid, the determinant of the energy-momentum tensor (EMT) is highly sensitive to its pressure. This component is significant during radiation-dominated epochs, and becomes naturally negligible as the universe transitions to the matter-dominated era. Here, we investigate the cosmological consequences of gravity sourced by the determinant of the EMT. Unlike Azri and Nasri, Phys. Lett. B 836, 137626 (2023), we consider the most general case in which the second derivative of the perfect-fluid Lagrangian does not vanish. We derive the gravitational field equations for the general power-law case and examine the cosmological implications of the scale-independent model characterized by dimensionless couplings to photons and neutrinos. We show that, unlike various theories based on the EMT, the present setup, which leads to an enhanced gravitational effects of radiation, does not alter the time evolution of the energy density of particle species. Furthermore, we confront the model with the predictions of primordial nucleosynthesis, and discuss its potential to alleviate the Hubble tension by reducing the sound horizon. The radiation-gravity couplings we propose here are expected to yield testable cosmological and astrophysical signatures, probing whether gravity distinguishes between relativistic and nonrelativistic species in the early universe.

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I. INTRODUCTORY REMARKS AND MOTIVATIONS

General Relativity (GR) provides a consistent description of the gravitational phenomena and has passed numerous observational and experimental tests [1, 2]. However, several issues suggest that GR may not be the final theory of gravity. At the theoretical level, the presence of singularities (black hole and big-bang) and the lack of a consistent quantum formulation indicate fundamental limitations of the theory [3, 4]. From the observational side, the accelerated expansion of the universe, together with the recently emerged cosmological tensions within the standard model of cosmology, point to open questions that may require modifications of the gravitational interaction [5, 6].

A wide range of extensions to GR have been developed [7, 8]. Some of these modify the geometric part of the action by introducing curvature invariants beyond the Ricci scalar [7, 9, 10]. Others modify the matter sector directly by incorporating explicit dependencies on the energy-momentum tensor (EMT) [11–14]. Another line of investigation considers determinant-based actions. Determinants of rank-two tensors define scalar densities consistent with general covariance and have appeared historically in alternative formulations of gravity, such as the Eddington action [15, 16]. More recently, determinant structures involving the Ricci tensor and combinations with the metric determinant have been studied as possible extensions [17, 18].

On the one hand, it is important to note that within the field-theoretic approach to GR, the Lagrangian contains the EMT as a source term coupled to the spin-2 field. Integrating out this gravitational degree of freedom induces effective interactions among the matter sources (EMT coupling terms) at the level of the Lagrangian [19, 20]. On the other hand, the inclusion of the determinant of the EMT arises quite naturally when constructing invariant terms. Indeed, the determinant of the spacetime metric tensor, which is essential for maintaining diffeomorphism invariance of the gravitational action, may itself be interpreted as the determinant of a "rescaled" EMT corresponding to the vacuum energy (cosmological constant) [15].

Motivated by these constructions, one of us with a collaborator proposed an extension of matter-gravity coupling in GR, in which the determinant of the EMT plays a central role [21]. It was shown that the determinant of the stress-energy tensor is highly sensitive to the pressure of the perfect fluid that describe an astrophysical object. As a consequence, significant deviations from the predictions of GR appear in compact objects such as neutron stars, where pressure is an essential component in the relativistic regime.

In this paper, we revisit this determinant-based coupling framework and investigate its implications in a cosmological context. We propose an early-universe dynamics that operates entirely within the framework of the known particle content, which interacts with gravity minimally as in GR, but is supplemented by additional generally invariant interaction terms constructed from the determinant of their EMT. We show that the determinant structure, being strongly pressure-sensitive, enhances the gravitational effect of radiation while leaving pressureless components unaffected, in contrast to trace- or quadratic-EMT couplings that generically alter both relativistic and nonrelativistic matter across all epochs [11–13].

After deriving the gravitational field equations for the most general case involving an arbitrary function of the EMT

determinant, we tackle the power-law models in a Friedmann-Lemaître-Robertson-Walker background and examine the associated continuity equations that govern deviations from the standard time evolution of radiation. We then focus on a scale-independent realization, in which the new radiation-gravity couplings are described by dimensionless parameters associated with the photon and neutrino sectors. We also derive the linear perturbation equations in the Newtonian gauge and track the deviations from standard radiation-gravity couplings. For this scale-independent scenario, we show that the redshift evolution of the radiation energy density coincides with the standard form. This result is notable, as it demonstrates that the new couplings dilute away analogously to standard cosmology, while still leading to an enhancement of the expansion rate. We show that the enhancement of the expansion rate remains consistent with the bounds from big bang nucleosynthesis. The allowed parameter space is constrained at the level of order ten percent, thus preserving the successful predictions of early-universe physics while permitting measurable deviations from the standard model during the radiation-dominated era.

We investigate the implications of the framework for the Hubble tension, focusing on its connection to the sound horizon. In particular, we provide analytic estimates showing how an enhancement of the early-time Hubble constant can be realized through a reduction of the sound horizon induced by what we refer to as the "enhanced gravitational effects of radiation". This analysis will clarify the extent to which the scenario can contribute to resolving the discrepancy between local and early-universe determinations of the Hubble constant without invoking any exotic energy.

The paper is organized as follows. In Sec. II, we introduce the theoretical framework based on the determinant of the EMT and discuss its incorporation into the gravitational action. We then derive the corresponding cosmological background equations, including the expansion rate and the evolution of the energy densities, for the power-law class of models. In Sec. III, we focus on the scale-independent scenario, where we derive the linear perturbation equations and obtain analytic estimates of the parameter space relevant for addressing the Hubble tension. Finally, Sec. IV summarizes our findings and outlines future directions.

II. ENHANCED GRAVITATIONAL EFFECT OF RADIATION

A. The determinant of the stress-energy tensor and the gravitational action

In this section, we introduce our gravitational framework which is based on the usual Einstein-Hilbert action of general relativity, minimally coupled to matter fields, and extended by the determinant of the EMT $T_{\mu\nu}$. The latter is defined as

$$\det T = \frac{1}{4!} \epsilon^{\alpha\beta\gamma\rho} \epsilon^{\bar{\alpha}\bar{\beta}\bar{\gamma}\bar{\rho}} T_{\alpha\bar{\alpha}} T_{\beta\bar{\beta}} T_{\gamma\bar{\gamma}} T_{\rho\bar{\rho}}, \tag{1}$$

where $\epsilon^{\alpha\beta\gamma\rho}$ is the anti-symmetric Levi-Civita symbol. This determinant transforms identically to $\det g$, and a physically meaningful quantity is then constructed from the ratio

$$D = \frac{|\det T|}{|\det g|}.$$
 (2)

The quantity D transforms clearly as scalar function under general coordinate transformations. The generally invariant action involving the most general couplings from the determinant of the EMT is written as [21]

$$S = \int d^4x \sqrt{|\det g|} \left\{ \frac{(R - 2\Lambda)}{16\pi G} + \mathcal{L}[g] \right\} + \int d^4x \sqrt{|\det g|} f(\mathbf{D}), \tag{3}$$

where $f(\mathbf{D})$ is an arbitrary function of \mathbf{D} . An analogous formulation could also be implemented in the Palatini approach, where the geometric part of the action is written in terms of both the metric and an independent symmetric connection. In this paper, however, we will consider the standard metric formulation. The field equations are then obtained by performing a variation of the total action with respect to the metric tensor. The variation of the quantity \mathbf{D} takes the form

$$\delta \mathbf{D} = \frac{\delta |\mathbf{det} \ T|}{|\mathbf{det} \ g|} + \mathbf{D} g_{\mu\nu} \delta g^{\mu\nu},\tag{4}$$

where the variation of the determinant of the EMT is given by

$$\delta |\det T| = |\det T| \left(T^{\text{inv}}\right)^{\mu\nu} \delta T_{\mu\nu},\tag{5}$$

where $(T^{\text{inv}})^{\mu\nu}$ is the inverse of the EMT. Now we need to evaluate the right-hand side of this expression. Using the definition of the EMT in terms of the Lagrangian, $T_{\mu\nu} = \mathcal{L}g_{\mu\nu} - 2\delta\mathcal{L}/\delta g^{\mu\nu}$, we get

$$\delta T_{\mu\nu} = \mathcal{L}\delta g_{\mu\nu} + \left\{ \frac{1}{2} g_{\alpha\beta} \left(\mathcal{L} g_{\mu\nu} - T_{\mu\nu} \right) - 2 \frac{\delta^2 \mathcal{L}}{\delta g^{\alpha\beta} g^{\mu\nu}} \right\} \delta g^{\alpha\beta}. \tag{6}$$

Finally

$$(T^{\text{inv}})^{\mu\nu} \delta T_{\mu\nu} = -\left\{ \mathcal{L} \left(T^{\text{inv}}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T^{\text{inv}} \right) + \frac{1}{2} T^{\text{inv}} T_{\mu\nu} \right\} \delta g^{\mu\nu} - 2 \left(T^{\text{inv}} \right)^{\alpha\beta} \frac{\delta^2 \mathcal{L}}{\delta g^{\alpha\beta} \delta g^{\mu\nu}} \delta g^{\mu\nu},$$
 (7)

where T^{inv} being the trace of the inverse of the EMT, and $T^{\text{inv}}_{\mu\nu} = g_{\alpha\mu}g_{\beta\nu} \left(T^{\text{inv}}\right)^{\alpha\beta}$.

All put together, the variation of the quantity D which is given by (4) takes the form

$$\delta \mathbf{D} = \mathbf{D} \left\{ g_{\mu\nu} - \mathcal{L} \left(T_{\mu\nu}^{\text{inv}} - \frac{1}{2} g_{\mu\nu} T^{\text{inv}} \right) - \frac{1}{2} T^{\text{inv}} T_{\mu\nu} - 2 \left(T^{\text{inv}} \right)^{\alpha\beta} \frac{\delta^2 \mathcal{L}}{\delta g^{\alpha\beta} \delta g^{\mu\nu}} \right\} \delta g^{\mu\nu}.$$
 (8)

Using the above variations, the principle of least action applied to (3) implies the gravitational field equations

$$G_{\mu\nu} = -\Lambda g_{\mu\nu} + \kappa T_{\mu\nu} + \kappa f(\mathbf{D})g_{\mu\nu} + 2\kappa \mathbf{D}f'(\mathbf{D})\mathcal{T}_{\mu\nu}, \tag{9}$$

where $G_{\mu\nu}$ is the standard Einstein tensor, $\kappa = 8\pi G$ (with G being Newton's constant), and $f'(\mathbf{D}) = df/d\mathbf{D}$. The tensor $\mathcal{T}_{\mu\nu}$ takes the form

$$\mathcal{T}_{\mu\nu} = -g_{\mu\nu} + \mathcal{L}\left(T_{\mu\nu}^{\text{inv}} - \frac{1}{2}g_{\mu\nu}T^{\text{inv}}\right) + \frac{1}{2}T^{\text{inv}}T_{\mu\nu} + 2(T^{\text{inv}})^{\alpha\beta}\frac{\delta^2\mathcal{L}}{\delta g^{\alpha\beta}\delta g^{\mu\nu}}.$$
 (10)

Before choosing the specific form for f(D) to be studied here, it is worth examining the effect of the quantity \mathbf{D} first. For a perfect fluid (a good approximation for a cosmological fluid) where $T_{\mu\nu} = (\rho + p)u_{\mu}u_{\nu} + pg_{\mu\nu}$ for each species, the determinant of the EMT, $\det T \equiv \det [T_{\mu\nu}]$, takes the form $\det [g_{\mu\lambda} T^{\lambda}_{\nu}] = \det g \times \det \hat{T}$ where \hat{T} is nothing but the matrix with the elements

$$\hat{T}^{\mu}_{\nu} = (\rho + p) u^{\mu} u_{\nu} + p \delta^{\mu}_{\nu}. \tag{11}$$

Hence, one gets $\mathbf{D} = |\mathbf{det} \ \hat{T}|$. Now, in the momentarily inertial frame of the fluid, the calculation of the determinant of the matrix \hat{T}^{μ}_{ν} is straightforward, and one finally gets

$$D = |\rho p^3|. \tag{12}$$

Therefore, in the comoving frame of the perfect fluid, \mathbf{D} vanishes for baryons and cold dark matter (negligible pressure), ensuring that these species decouple from the new gravitational interaction we introduced. As a result, the coupling proportional to \mathbf{D} becomes active exclusively in radiation-dominated epochs, precisely when relativistic content governs the expansion history. Additionally, the whole structure is well-defined only when $\mathbf{D} \neq 0$, a condition that is required by the appearance of $T_{\mu\nu}^{\text{inv}}$ in the field equations. Given its characteristics, we refer to this scenario as enhanced gravitational effects of radiation (EGER).

B. Power-law models and cosmological dynamics

In analogy with extended gravity theories, power-law structure is interesting on its own. One can consider models of the form \mathbf{D}^n where the exponent n is not necessarily an integer. Because the determinant itself carries a large mass dimension, making the action dimensionless requires introducing a constant with correspondingly high dimensionality. By introducing some constants M_i with the dimension of mass, the general form of power-law models can therefore be written as

$$f(\mathbf{D}) = \sum_{i} M_{i}^{4(1-4n)} \mathbf{D}_{i}^{n}. \tag{13}$$

where we considered the contributions from various species i.

The gravitational equations (9) involve the inverse of the EMT, $(T^{\text{inv}})^{\alpha\beta}$. For a perfect fluid, this is evaluated as follows. First, we write $(T^{\text{inv}})^{\mu\alpha} = g^{\alpha\nu}(T^{\text{inv}})^{\mu}_{\ \nu}$ and then determine the inverse of the matrix (11). Given a matrix of the form $A + UV^{\text{T}}$ where A is a square invertible matrix and U, V are column vectors, its inverse is given by the Sherman–Morrison formula [22]

$$(A + UV^{\mathrm{T}})^{-1} = A^{-1} - \frac{A^{-1}U \cdot V^{\mathrm{T}}A^{-1}}{1 + V^{\mathrm{T}}A^{-1}U}.$$
 (14)

For the case of a perfect fluid (11), $A = p_i I$ where I is the 4×4 identity matrix and $U = V = \sqrt{\rho_i + p_i} u$. By applying

this to the above formula, one finally gets

$$(T^{\text{inv}})^{\mu\nu} = \frac{1}{p_{\text{i}}} \left\{ g^{\mu\nu} + \frac{(\rho_{\text{i}} + p_{\text{i}})}{\rho_{\text{i}}} u^{\mu} u^{\nu} \right\}. \tag{15}$$

According to the previous discussion, this expression is not singular since it is valid only for $p_i \neq 0$ (relativistic species) whereas for $p_i = 0$ (dust), the function D_i vanishes in the first place, and the structure tends to be the standard matter coupling of GR without any modification. By considering $\mathcal{L} = p_i$ for the Lagrangian of each fluid [23, 24], its second-order variation reads

$$\frac{\delta^2 \mathcal{L}}{\delta g^{\mu\nu} \delta g^{\alpha\beta}} = \frac{1}{4} \left(\frac{1}{c_{\rm si}^2} - 1 \right) \left(\rho_{\rm i} + p_{\rm i} \right) u_{\mu} u_{\nu} u_{\alpha} u_{\beta}, \tag{16}$$

with $c_{\rm si}^2 = \delta p_{\rm i}/\delta \rho_{\rm i}$ (see [25] for its derivation). Consequently, the presence of the second derivative of the Lagrangian through the equations of motion induces the adiabatic sound speed squared even at the background level. This was unjustifiably ignored when the determinant of the EMT was originally introduced [21]. With these expressions at hand, we easily determine the tensor $\mathcal{T}_{\mu\nu}$ in (10) as

$$\mathcal{T}_{\mu\nu} = \frac{1}{2} \left(1 + \frac{p_{\rm i}}{\rho_{\rm i}} \right) \left(\frac{3\rho_{\rm i}}{p_{\rm i}} + \frac{1}{c_s^2} \right) u_{\mu} u_{\nu}. \tag{17}$$

All these put together, the gravitational field equations (9) adapted to the power-law models take the form

$$G_{\mu\nu} = -\Lambda g_{\mu\nu} + p_{\rm i} \left(1 + M_{\rm i}^{4(1-4n)} \frac{\rho_{\rm i}^n}{p_{\rm i}^{1-3n}} \right) g_{\mu\nu} + (\rho_{\rm i} + p_{\rm i}) \left\{ 1 + n M_{\rm i}^{4(1-4n)} \frac{p_{\rm i}^{3n}}{\rho_{\rm i}^{1-n}} \left(\frac{3\rho_{\rm i}}{2p_{\rm i}} + \frac{1}{2c_{s_i}^2} \right) \right\} u_{\mu} u_{\nu}$$
 (18)

where we took $\kappa = 1$.

1. Friedmann and continuity equations of the power-law cases

In what follows, the universe in its homogeneous approximation will be described by the Friedmann-Lemêtre-Robertson-Walker (FLRW) flat spacetime metric given by its line element

$$ds^2 = -dt^2 + a^2(t)d\vec{\mathbf{x}} \cdot d\vec{\mathbf{x}},\tag{19}$$

where a(t) is the scale factor. Next, we will be interested in the energy evolution of the constituents of the universe which can be described by their energy density and pressure as the only relevant properties in the smooth background.

Applying the covariant divergence on the left-hand side of (18), and taking its time component ($\nu = 0$), we derive the modified continuity equation

$$\begin{split} \dot{\rho_{\rm i}} + 3H(\rho_{\rm i} + p_{\rm i}) \\ + nM_{\rm i}^{4(1-4n)} \rho_{\rm i}^{4n-1} \left(\frac{p_{\rm i}}{\rho_{\rm i}}\right)^{3n} \left\{ \left[4n\left(3 + \frac{1}{c_{si}^2} + \frac{p_{\rm i}}{\rho_{\rm i}c_{si}^2} + \frac{3\rho_{\rm i}}{p_{\rm i}}\right) - 4 \right] \dot{\rho}_{\rm i} + 3H\left(3 + \frac{1}{c_{si}^2} + \frac{p_{\rm i}}{\rho_{\rm i}c_{si}^2} + \frac{3\rho_{\rm i}}{p_{\rm i}}\right) \rho_{\rm i} \right\} = 0, (20) \end{split}$$

where $H = \dot{a}/a$ is the Hubble parameter, and $p_i/\rho_i = \omega_i$ is the constant equation of state of the ith fluid component. Unlike the standard continuity equation, we notice here the presence of the inverse of ω_i which results from the inverse of the energy momentum-tensor of radiation as we have mentioned previously. We notice again that there are no effects from nonrelativitic matter where $\omega_i = 0$.

Assuming that the various species interact only gravitationally, the continuity equation (20) holds for each type of particles separately, namely, cold dark matter (i = dm), baryons (i = b), photons (i = γ) and neutrinos (i = ν). Now, we adapt the gravitational field equation (18) for the background metric (19) and get the expansion rate

$$3H^{2} = \Lambda + \sum_{m=b,dm} \rho_{m} + \sum_{r=\gamma,\nu} \left[\rho_{r} + M_{r}^{4(1-4n)} \left(\frac{1}{3} \right)^{3n} (16n-1) \rho_{r}^{4n} \right], \tag{21}$$

where we have used $\omega_{\rm b} = \omega_{\rm cdm} = 0$ for baryons and cold dark matter species, $\omega_{\rm r} = c_s^2 = 1/3$ for radiation, and have taken $u_{\mu} = (1,0,0,0)$ for a comoving observer. Here, it is worth to note that the constants $M_{\rm r}$ of mass dimension should not be confused with the masses of the relativistic species.

The space-space components of the field equations (18) lead to the time change of the Hubble parameter as

$$\dot{H} = -\frac{1}{2} \sum_{m=b,dm} \rho_m - \sum_{r=\gamma,\nu} \left[\frac{2}{3} \rho_r + 8n M_r^{4(1-4n)} \left(\frac{1}{3} \right)^{3n} \rho_r^{4n} \right]$$
 (22)

Returning to the continuity equation (20), since the quantity \mathbf{D} vanishes for nonrelativistic matter, the time evolution of the latter is not affected by the new interaction terms, thus $\dot{\rho}_{\rm b,cdm} + 3H\rho_{\rm b,cdm} = 0$, and in terms of the redshift z one has $\rho_{\rm b,cdm} = \rho_{\rm 0b,cdm} (1+z)^3$. For photons and (relativistic) neutrinos, it reads

$$\left(\frac{\rho_{\rm r}^{1-4n} + \Theta_1}{\rho_{\rm r}^{1-4n} + \Theta_2}\right) \frac{d\ln \rho_{\rm r}}{dt} + 4 \frac{d\ln a}{dt} = 0,\tag{23}$$

where

$$\Theta_1 = 4nM_{\rm r}^{4(1-4n)} \left(\frac{1}{3}\right)^{3n} (16n-1),$$
(24)

$$\Theta_2 = 12nM_{\rm r}^{4(1-4n)} \left(\frac{1}{3}\right)^{3n}.$$
 (25)

It is clear that the time evolution of relativistic species generally differs from that of standard cosmology $\rho_r \sim (1+z)^4$. However, it should be noted that the evolution becomes identical to the standard case, i.e. unaffected by the modification when $\Theta_1 = \Theta_2$, a condition satisfied by the scale-independent model (n = 1/4), which we examine in the next section.

III. COSMOLOGICAL IMPLICATIONS OF THE SCALE-FREE MODEL

A. Background evolution and big bang nucleosynthesis constraints

The scale-independent (or scale-free) model of the EGER arises as

$$f(\mathbf{D}) = \sum_{i} \lambda_{i} \, \mathbf{D}_{i}^{1/4}, \tag{26}$$

where $\lambda_{\rm i}$ are dimensionless constants referring to the couplings of various species. Again, the preceding analysis shows that a non-vanishing determinant implies that the modification affects only the radiation sector. This forces the non-relativistic matter to detach from these couplings. Therefore, the novel contribution targets only the radiation sector which will be described by the free parameters $\lambda_{\rm r} = \lambda_{\gamma}, \lambda_{\nu}$ for photons and relativistic neutrinos respectively. For this model, the Friedmann equations (21)-(22) take the form

$$3H^2 = \rho_{\rm m} + \sum_{r=\gamma,\nu} \left(1 + 3^{1/4} \lambda_{\rm r} \right) \rho_{\rm r} + \Lambda \tag{27}$$

$$\dot{H} = -\frac{1}{2}\rho_{\rm m} - \frac{2}{3} \sum_{\rm r=\gamma,\nu} \left(1 + 3^{1/4} \lambda_{\rm r} \right) \rho_{\rm r}. \tag{28}$$

Again, as in standard cosmology $\rho_{\rm m}$ involves both baryons and cold dark matter energy densities whilst radiation, encoded in $\rho_{\rm r}$, involves photons (and e^+e^- pairs when prior to big bang nucleosynthesis) and possibly, three flavors of left-handed neutrinos as described by the SM of particle physics. Despite the complexity of the gravitational field equations (9)-(10), the cosmological equations (27)-(28) reveal a simple but key consequence: the present setup leads to effective gravitational couplings that differ between matter and radiation. While pressureless matter (modeled as dust) continues to gravitate with the standard Newton constant G, radiation experiences a rescaled coupling of the form $(1+3^{1/4}\lambda_{\rm r})G$. The values of the coupling parameters $\lambda_{\rm r}$ assigned to each relativistic species determine their influence on key cosmological quantities, such as the Hubble parameter and the sound horizon.

On the other hand, the continuity equations (23) reduce to their standard form for this model (n = 1/4). Consequently, the solution is given by $\rho_{\rm r} = \rho_{\rm r_0}(1+z)^4$ in terms of the redshift z. This feature is central to the mechanism by which the enhanced gravitational coupling effectively tracks the radiation component and naturally dilutes as the universe transitions to the matter-dominated phase. As we shall discuss later, an interesting implication of this behavior is that the increase in H(z) prior to recombination reduces the sound horizon and raises the CMB-inferred value of H_0 , which may contribute to easing the Hubble tension.

In a broad class of scenarios beyond the standard model of cosmology or particle physics (if new particle species are involved), departure from the the standard dynamics is conveniently described in terms of an effective expansion rate H', related to the standard Hubble rate H through a dimensionless factor S as $H \to H' = SH$. It has been shown that analytic fits to big bang nucleosynthesis implies that for non-standard expansion rate SH which might arise generally from new physics must satisfy $0.85 \le S \le 1.15$ [26]. In the EGER, deviations from the standard case S = 1 arise from the dimensionless couplings λ_r as $S = \left(1 + 3^{1/4}\lambda_r\right)^{1/2}$ according to (27), and therefore $-1.1 \times 10^{-1} \le \lambda_r \le 1.1 \times 10^{-1}$.

These bounds show that the EGER remains tightly constrained by big bang nucleosynthesis, with the free parameters limited to values of order one tenth. The result ensures that the scenario preserves the successful predictions of early-universe physics while still allowing for measurable deviations from the standard model in the radiation-dominated era.

B. Linear scalar perturbations

In this section, we will derive the scalar perturbations of the scale-independent model of the EGER. We will work in conformal-Newtonian gauge and write our perturbed metric as

$$ds^{2} = a^{2}(\eta) \left[-(1 + 2\Psi(\vec{\mathbf{x}}, t))d\eta^{2} + (1 - 2\Phi(\vec{\mathbf{x}}, t))d\vec{\mathbf{x}} \cdot d\vec{\mathbf{x}} \right]. \tag{29}$$

Here η is the conformal time, Ψ is the gravitational potential from which the Newtonian gravity is recovered at scales smaller than the Hubble radius. The function Φ represents a local distribution of the scale factor. For perfect fluids, one immediately has $\Phi = \Psi$. Additionally, the speed of sound reads $c_s^2 = \bar{p}/\bar{\rho} = \delta p/\delta \rho$ where $\bar{\rho}$, \bar{p} are the background quantities whereas $\delta \rho$ and δp are the perturbation quantities. In addition, one writes the fluid velocity perturbation as $u^{\mu} = a^{-1}\delta_0^{\mu} + \delta u^{\mu}$ in which $\delta u^i = v^i$ is a small velocity. From the latter, one defines the scalar degree of freedom (velocity divergence) $\theta = \vec{\nabla} \vec{v}$. On the other hand, since the particle species are approximated by perfect fluids, then no anisotropic stresses are considered. Therefore, the perturbations are totally described by only the two degrees of freedom, δ and θ .

From the gravitational field equations (18), and for the scale-independent model (n = 1/4), one writes a total (an effective) EMT involving the EGER corrections as

$$T_{\text{tot }\nu}^{\mu} = p \left(1 + \lambda \left(\frac{\rho}{p} \right)^{1/4} \right) \delta_{\nu}^{\mu} + (\rho + p) \left\{ 1 + \frac{\lambda}{2} \left(\frac{p}{\rho} \right)^{3/4} \left(\frac{3\rho}{2p} + \frac{1}{2c_s^2} \right) \right\} u^{\mu} u_{\nu}$$
 (30)

where we neglected the cosmological constant term. Now, we consider linear perturbations for the energy density and pressure about the background as

$$\rho = \bar{\rho} + \delta \rho, \quad p = \bar{p} + \delta p \tag{31}$$

for various species, and define the dimensionless perturbation $\delta = \delta \rho / \bar{\rho}$ which describes the relative deviation of the energy density from the mean background density. For the cosmological perturbation equations, we will use almost the same notation of Ref. [27] for the main variables. To linear order in the perturbations, the components of this EMT read

$$T_{\text{tot }0}^{0} = -(\bar{\rho} + \delta \rho) + \tilde{T}_{0}^{0}$$
(32)

$$T_{\text{tot},i}^0 = (\bar{\rho} + \bar{p})v_i + \tilde{T}_i^0 \tag{33}$$

$$T_{\text{tot }j}^{i} = (\bar{p} + \delta p)\delta_{j}^{i} + \tilde{T}_{j}^{i} + \tilde{\Sigma}_{j}^{i}, \tag{34}$$

where the first contributions are the standard terms that arise in standard cosmology, and the last terms are given by

$$\delta \tilde{T}_{0}^{0} = \frac{\lambda}{16} \left(\frac{\bar{p}}{\bar{\rho}} \right)^{3/4} \left(\frac{3\bar{p}}{\bar{\rho}c_{s}^{2}} - \frac{15\bar{\rho}}{\bar{p}} + 1 - \frac{1}{c_{s}^{2}} \right) \delta \rho + \frac{\lambda}{16} \left(\frac{\bar{p}}{\bar{\rho}} \right)^{3/4} \left(\frac{3\bar{\rho}^{2}}{\bar{p}^{2}} + \frac{3\bar{\rho}}{\bar{p}} - \frac{7}{c_{s}^{2}} - \frac{3\bar{\rho}}{\bar{p}c_{s}^{2}} \right) \delta p \tag{35}$$

$$\tilde{T}_{i}^{0} = \frac{\lambda}{4} \left(\frac{\bar{p}}{\bar{\rho}} \right)^{3/4} \left(\frac{3\bar{\rho}}{\bar{p}} + \frac{\bar{p}}{\bar{\rho}c_{si}^{2}} + 3 + \frac{1}{c_{si}^{2}} \right) \bar{\rho}v_{i}$$
(36)

$$\tilde{T}^{i}_{j} = \lambda \left(\frac{\bar{p}}{\bar{\rho}}\right)^{3/4} \bar{\rho} \delta^{i}_{j} + \frac{\lambda}{4} \left(\frac{\bar{p}}{\bar{\rho}}\right)^{3/4} \left(\delta \rho + 3\frac{\bar{\rho}}{\bar{p}} \delta p\right) \delta^{i}_{j}, \tag{37}$$

where λ is the dimensionless constant characterizing the coupling to the EMT in action (3). Needless to say, the terms involving λ contribute to relativistic species (radiation) only. The tensor $\tilde{\Sigma}^i_{\ j}$ is the total anisotropic stress of the fluid, that is, $\tilde{\Sigma}^i_{\ j} = T^i_{\rm tot \ j} - \delta^i_j T^k_{\rm tot \ k}/3$. Here, the terms proportional to $\bar{\rho}/\bar{p}$, i.e. the inverse of the equation of state, are generated from varying the determinant of the EMT.

The evolution equation for the gravitational scalar potentials read

$$k^{2}\Phi + 3\mathcal{H}(\Phi' + \mathcal{H}\Psi) = 4\pi G a^{2} \delta T^{0}_{0} + 4\pi G a^{2} \delta \tilde{T}^{0}_{0}$$
(38)

$$k^{2}\left(\Phi' + \mathcal{H}\Psi\right) = 4\pi G a^{2}\left(\bar{\rho} + \bar{p}\right)\theta + \lambda\pi G a^{2}\left(\frac{\bar{p}}{\bar{\rho}}\right)^{3/4}\left(\frac{3\bar{\rho}}{\bar{p}} + \frac{\bar{p}}{\bar{\rho}c_{s}^{2}} + 3 + \frac{1}{c_{s}^{2}}\right)\bar{\rho}\theta\tag{39}$$

$$\Phi'' + \mathcal{H}(\Psi' + 2\Phi') + \frac{k^2}{3}(\Phi - \Psi) + (2\mathcal{H}' + \mathcal{H}^2)\Psi = \frac{4\pi}{3}Ga^2\delta T^i_{\ i} + \frac{4\pi}{3}Ga^2\delta \tilde{T}^i_{\ i}$$
(40)

$$k^{2}\left(\Phi - \Psi\right) = 12\pi G a^{2}\left(\bar{\rho} + \bar{p}\right)\tilde{\sigma},\tag{41}$$

where we have introduced $\delta T^0_{\ 0} = -\delta \rho, \, \delta T^i_{\ i} = 3\delta p$ and

$$(\bar{\rho} + \bar{p})\,\tilde{\sigma} \equiv -\left(1 + \lambda \left(\frac{\bar{\rho}}{\bar{p}}\right)^{1/4}\right) \left(\hat{k}_i \hat{k}^j - \frac{1}{3}\delta^j_{\ i}\right) \Sigma^i_{\ j},\tag{42}$$

with Σ^{i}_{j} being the anisotropic stress of the fluids. Applying the covariant conservation law (arising from the Bianchi identity) on the total EMT (30), and working in the Fourier k-space, we obtain the Euler and the continuity equations as

$$\delta' + \frac{b}{a} 3\mathcal{H} \left(\frac{\delta p}{\delta \rho} - \omega \right) \delta + \frac{c}{a} (1 + \omega)(\theta - 3\Phi') = 0 \tag{43}$$

$$\theta' + \mathcal{H}\left(1 - 3\omega\frac{d}{f}\right)\theta - \frac{e}{c}\frac{\delta p/\delta\rho}{(1+\omega)}k^2\delta + \frac{1}{c}k^2\tilde{\sigma} - k^2\Psi = 0 \tag{44}$$

with the following coefficients

$$a = 1 - \frac{\lambda}{16}\omega^{3/4} \left[\left(\frac{3\omega}{c_s^2} - 15\omega^{-1} + 1 - \frac{1}{c_s^2} \right) + \left(3\omega^{-2} + 3\omega^{-1} - \frac{7}{c_s^2} - \frac{3\omega^{-1}}{c_s^2} \right) \frac{\delta p}{\delta \rho} \right]$$
(45)

$$b = \frac{1 + \frac{\lambda}{16}\omega^{3/4} \left(27\omega^{-1} - \frac{3\omega}{c_s^2} - 1 + \frac{1}{c_s^2}\right) + \frac{\lambda^2}{4}\omega^{3/2} \left(3\omega^{-2} - \frac{1}{c_s^2}\right)}{1 + \frac{\lambda}{4}\omega^{3/4} \left(3\omega^{-1} + \frac{\omega}{c_s^2} - 1 + \frac{1}{c_s^2}\right)}$$
(46)

$$c = 1 + \frac{\lambda}{4}\omega^{3/4} \left(3\omega^{-1} + \frac{\omega}{c_s^2} + 3 + \frac{1}{c_s^2}\right) (1+\omega)^{-1}$$
(47)

$$d = 1 + \lambda \omega^{-1/4} \tag{48}$$

$$e = 1 + \frac{\lambda}{4}\omega^{3/4} \left(\left(\frac{\delta p}{\delta \rho} \right)^{-1} + 3\omega^{-1} \right) \tag{49}$$

$$f = 1 + \frac{\lambda}{4}\omega^{3/4} \left(3\omega^{-1} + \frac{\omega}{c_s^2} - 1 + \frac{1}{c_s^2} \right)$$
 (50)

Since we consider $\dot{\omega} = 0$ for the equation of state, that is $\delta p/\delta \rho = c_s^2 = \omega$, we get $a = c = d = e = f = 1 + \lambda \omega^{-1/4}$ which simplifies the equations for δ and θ as

$$\delta' + (1 + \omega)(\theta - 3\Phi') = 0 \tag{51}$$

$$\theta' + \mathcal{H}(1 - 3\omega)\theta - \frac{\omega}{(1 + \omega)}k^2\delta + k^2\sigma - k^2\Psi = 0,$$
(52)

where $\sigma = \tilde{\sigma}/(1 + \lambda \omega^{-1/4})$ is the same as that of standard cosmology.

C. Impact on the sound horizon and implications for the Hubble tension

The key mechanism by which EGER addresses the Hubble tension is through its impact on the sound horizon at recombination. The enhanced radiation couplings modify the Hubble parameter at early times, which in turn alters the sound horizon for acoustic waves in the photon–baryon fluid

$$r_s = \int_z^\infty \frac{c_{s\gamma,b}(z)}{H(z)} dz,\tag{53}$$

where $c_{s\gamma,b}(z)$ is the sound speed and H(z) includes the modified radiation contributions

$$H(z) = 100 \sqrt{w_{m0}(1+z)^3 + \sum_{r} (1+3^{1/4}\lambda_r) w_{r0}(1+z)^4} \quad \text{km s}^{-1} \,\text{Mpc}^{-1},$$
 (54)

with $w_i = \Omega_i h^2$ for each species. The modified radiation sector effectively increases the expansion rate before recombination, thus reducing r_s . Since the observed angular scale of the acoustic peaks $\theta_s = r_s/D_A$ is tightly constrained by the CMB, a smaller r_s implies a smaller angular diameter distance D_A . Given that $D_A \propto H_0^{-1}$, this naturally leads to a higher inferred value of the Hubble constant, thus helping to reconcile early- and late-universe measurements of H_0 .

Below we denote early- and late-universe measurements of H_0 as H_0^{CMB} and H_0^{loc} , respectively. The measurements

of each H_0 are given in the interval $67.24 \le H_0^{\rm CMB} \le 68.08$ [28] and $72 \le H_0^{\rm loc} \le 74.08$ [29]. To get the value of $H_0^{\rm loc}$, the value of $H_0^{\rm CMB}$ should increase by around $7.96 \pm 2.21\%$, which implies that D_A should decrease by $7.34 \pm 1.89\%$. Since $D_A \propto r_s, r_s$ should also decrease by $7.34 \pm 1.89\%$ to get $H_0^{\rm loc}$. The sound horizon has a value of about $r_s = 144.57 \pm 0.22$ Mpc [28] with the parameters $w_{b0}^{\rm Pl} = 0.02242 \pm 0.00014$, $w_{\gamma 0}^{\rm Pl} = 2.47282 \times 10^{-5}$, $w_{\nu 0}^{\rm Pl} = 1.71061 \times 10^{-5}$, $w_{m0}^{\rm Pl} = 0.14240 \pm 0.00087$ and $z_{ls} = 1089.80$ [28]. As we have clarified above, to get the value of $H_0^{\rm loc}$, $r_s = 144.57 \pm 0.22$ should be reduced by $7.34 \pm 1.89\%$, which yields $r_s \simeq 133.97 \pm 2.94$ Mpc. We therefore look for possible values of λ_{γ} and λ_{ν} . To obtain $r_s \simeq 133.97 \pm 2.94$ Mpc for a set of parameters $w_{m0}^{\rm Pl} = 0.1424$, $w_{b0}^{\rm Pl} = 0.02242$, $w_{\gamma 0}^{\rm Pl} = 2.47282 \times 10^{-5}$, $w_{\nu 0}^{\rm Pl} = 1.71061 \times 10^{-5}$ [28], the additional component of the radiation energy density (in Eq.(54)) should satisfy

$$\sum_{r=\gamma,\nu} 3^{1/4} \lambda_r w_{r0} \simeq (1,3775 \pm 0.4355) \times 10^{-5},\tag{55}$$

which implies

$$3.2544 \ \lambda_{\gamma} + 2.2513 \ \lambda_{\nu} \simeq 1,3775 \pm 0.4355$$
 (56)

For instance, for the values $\lambda_{\gamma} = 0.32$ and $\lambda_{\nu} = 0.1$ in the Eq.(54), one obtains $r_s = 134.616$ Mpc, which is reduced by 6.88 % compared to CMB referred value. This indicates that D_A should also be reduced by the same ratio, which results in a higher value of early-universe measurements of H_0 as 72,66 km s⁻¹ Mpc⁻¹.

It is worth to note that these estimates establish only a starting point for a systematic confrontation of the framework with cosmological observations. While the present work has focused on analytical estimates and order-of-magnitude constraints on the free parameters, a full exploration of the parameter space requires a numerical analysis. In particular, a Markov Chain Monte Carlo approach will make it possible to map the viable regions of the model in detail and to determine whether the enhanced gravitational effects of radiation can simultaneously preserve the successes of early-universe physics and alleviate present-day cosmological tensions. We plan to address this question in a near future work.

IV. CONCLUDING REMARKS AND PROSPECTS

In this work we have revisited the determinant-based extension of matter-gravity coupling, originally proposed in the astrophysical context of compact stars by one of us with a collaborator [21], and explored its consequences for cosmology. The central feature of this construction is the dependence on the determinant of the stress-energy tensor, which is particularly sensitive to pressure. As a result, the modification becomes relevant in relativistic regimes, while leaving nonrelativistic epochs largely unaffected.

We have shown that in the early universe the stress-energy-determinant coupling selectively influences the radiation-dominated era, providing a correction to the early expansion rate without invoking additional fields beyond the Standard Model particle content. This distinguishes the framework from other stress-energy-based extensions, such as couplings to its trace or its quadratic terms which generically affect both relativistic and nonrelativistic components [11–14]. The determinant structure therefore offers a natural mechanism to alter early-universe dynamics while

preserving the standard cosmological evolution at later times.

We have presented a detailed derivation of both the background evolution and the linear perturbation dynamics within the framework we refer to as enhanced gravitational effects of radiation. As a specific realization, we examined the scale-independent model and showed that, for constrained values of the dimensionless couplings to photons and neutrinos, the scenario preserves the successful predictions of primordial nucleosynthesis, thus remaining consistent with one of the most stringent early-universe probes. In addition, we addressed the Hubble tension in terms of the sound horizon, providing analytic estimates of how an increased early-time Hubble constant can be achieved through a reduction of the sound horizon induced by the enhanced gravitational effects of radiation.

The analytical work presented here, together with the constraints we have derived on the free parameters of the model, provide a well-defined setting for a more comprehensive numerical investigation. In particular, a natural next step is to identify the optimal region of parameter space by means of a full Markov Chain Monte Carlo analysis against current cosmological data sets, thus quantifying the viability of the framework in light of precision observations. Such a study will allow us to assess in detail the extent to which the enhanced gravitational effects of radiation can alleviate existing cosmological tensions while remaining consistent with established probes of the early universe. We leave this investigation for a near future work.

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