

Dineutron clusters in ^7He and ^8He structure

E Garrido¹ and A S Jensen²

¹Instituto de Estructura de la Materia, CSIC, Serrano 123, E-28006 Madrid, Spain

²Department of Physics and Astronomy, Aarhus University, DK-8000 Aarhus C, Denmark

E-mail: e.garrido@csic.es

Abstract. The hyper-radial barrier strongly hinders formation of more than three clusters. We investigate how well the dominating cluster components in ^7He and ^8He , respectively can be described as $\alpha+n+^2n$ and $\alpha+^2n+^2n$, where 2n is the dineutron. Effective interactions compatible with ^5He and ^6He are used. We vary the lesser known $n-^2n$ and $^2n-^2n$ interactions, where very small strengths are required. We provide energies, radii, and partial wave decomposition of all computed, predicted or measured, ground and resonance states. We predict substructures within each of the three-body quantum states. We also calculate the neutron-structure sensitive invariant mass spectrum of the four-nucleon system, after fast removal of the α -particle from ^8He . We show that all available experimental information are fairly well reproduced. Very little room is left for variation of the effective interaction parameters. Thus, the dominating features of the subsequently derived reaction and structure properties are well supported.

1. Introduction

Cluster structures have been recognized in a number of nuclear systems [1]. The description of such structures by means of ab-initio calculations are exceedingly difficult, since the bare interactions and the intrinsic cluster structures would be involved, and the simple cluster picture would be lost. The necessity of few-body calculations with effective interactions is then obvious.

Concerning weakly bound many-neutron systems, which structures would characterize them? A general observation formulated as a theorem is helpful: “Two and three clusters are strongly preferred” [2, 3]. Therefore, four (and more)-body halos do not exist. The proof of this statement is that the many-body hyper-radial potential barrier prevents more than three clusters. The bulk of the wave function would be either squeezed inside the barrier and overlap with the core or be pushed outside the barrier without any chance to bind even in a resonance state. The intrinsic structures of the clusters do not have to be point-like, although they are treated as one entity. The effective interactions between clusters should be adjusted to account for this. The accuracy is reduced, if more than one three-cluster division is contributing.

1.1. Neutrons around a core

A tightly bound core surrounded by one or two weakly bound neutrons are prototypes of two- and three-body clusters as exemplified by, respectively, $^{11}\text{Be}(^{10}\text{Be}+n)$ [4], and $^6\text{He}(\alpha+n+n)$ or $^{11}\text{Li}(^{10}\text{Li}+n+n)$ [5, 6]. The neutron-neutron probability distribution for these systems has a peak at short distance, which is characterized by a relative s -wave and total spin equal to zero. The core is spatially well separated.

Although a cluster as an object making part of a bigger structure is typically understood as a bound constituent of the big system, unbound entities as constituents in clusterized systems do actually become possible when another body indirectly delivers a sufficient attraction. The unbound dineutron, 2n , can subsequently be considered as a constituent particle, often described as a spatially correlated structure of two neutrons. The structure is not unlike models of two neutrons in a shell of given orbital and total angular momentum. In shell models, they have a strong tendency to form 0^+ spin-correlated nucleon pairs arranged in shells outside a core.

Dineutron correlation has previously been proposed as structures of neutron dripline nuclei, e.g. in the halo defining the description of $^{11}\text{Li}(^9\text{Li}+^2n)$ [7], or, more recently, in three-body calculations like $^8\text{Li}(\alpha+\text{deuteron}+^2n)$ [8], $^7\text{H}(\text{triton}+^2n+^2n)$ [9], $^8\text{He}(\alpha+^2n+^2n)$ [10, 11, 12, 13, 14], and perhaps even $^{28}\text{O}(^{24}\text{O}+^2n+^2n)$ [15]. Experiments on these systems are adding information [12, 14, 16, 17] to be explained by theory. The loosely bound dineutrons are part of a cluster structure, where core and valence constituents (neutrons and dineutrons) essentially are completely decoupled.

From this, we deduce that the dominating structures of $^8\text{He}(\alpha+^2n+^2n)$ and $^7\text{He}(\alpha+n+^2n)$, are the indicated three-body clusters. This can be viewed as a continuation of the structure of the lighter Helium isotopes. The α -particle, ^4He , surrounded itself with an increasing number of neutrons. First, the different partial-wave α - n interactions must match the description of $^5\text{He}(\alpha+n)$. Then the partial-wave α - 2n interactions produce $^6\text{He}(\alpha+^2n)$, which can be viewed as an effective interaction, since it accounts for omitted intrinsic structure of the dineutron. The $^7\text{He}(\alpha+n+^2n)$ system also needs the n - 2n partial wave effective interaction. Then ^7He could get contributions from more than one intrinsic structure of 2n , where most is accounted for by the effective interaction. The $^8\text{He}(\alpha+^2n+^2n)$ system needs the 2n - 2n partial wave effective interaction, which also accounts for the intrinsic 2n structures, but not another unlikely cluster divisions like $\alpha+n+^3n$.

The purpose of this work is to investigate how the three-body structures, $\alpha+^2n+^2n$ and $\alpha+n+^2n$, permit to describe the known properties of ^8He and ^7He , respectively. We treat the dineutron, 2n , as a zero-spin particle, with a typical size, root-mean-square radius, of about 2 fm [10]. The main uncertainty comes from the choices of 2n - 2n and n - 2n interactions. For this reason particular emphasis will be put on what the main characteristics of these interactions have to be such that the ^8He and ^7He properties are fairly well reproduced.

1.2. From data to predictions

The experimental data on the weakly bound neutron dripline nuclei, ^7He and ^8He , are binding energies, spins and parities of ground and excited (resonance) states, and some of their widths. In addition, for ^8He the root-mean-square radius is also known as well as the missing mass spectrum after gentle removal of an α -particle. Two levels are known for ^7He , that is $\frac{3}{2}^-$ and $\frac{5}{2}^-$, and five levels for ^8He , that is two 0^+ , and one of each 2^+ , 1^- , and 3^- .

We shall use the three-body cluster model to describe ^7He and ^8He structures. The technique is the three-body hyperspherical expansion method combined with complex scaling for resonance calculations. First the cluster divisions with the three constituents are selected, that is $^7\text{He}(\alpha+n+^2n)$ and $^8\text{He}(\alpha+^2n+^2n)$. Then the input data are the two-body interactions between the three subsystems, where each is a nucleus in itself, that is ^5He and ^6He in addition to the nuclear combinations $n-^2n$ and ^2n-n . In this work the spatial form of the two-body potentials are sums of Gaussian functions, usually only one or two are sufficient.

The strength and range of the Gaussian potentials, together with the angular momentum and spin dependencies, are carefully chosen to reproduce the known properties of the corresponding nuclei. Both spatial and angular momentum dependencies are chosen to be consistent with the Pauli exclusion principle.

To determine the proper interactions between unstable constituents, ^2n-n and $n-^2n$, where direct two-body information is not available, is more delicate. However, as constituents of a three-body system, these interactions influence the resulting three-body structure. Information about three-body properties can then, through the calculations, provide information about the two-body input interaction. Therefore, such interactions are intricately connected with the three-body calculation.

The accuracy of the calculations are of few-body character, which means that inaccuracies arise from choices of cluster division and how well the two-body interactions can be determined from the available data. The numerical accuracy is insignificant compared to the model choice. The cluster constituents are crucial and chosen from inspection of internal and external separation energies, which strongly must suggest a small spatial overlap. A wrong choice would be catastrophic and the results would not be possible to repair by use of adjusted phenomenological interactions. The two-body interactions contain uncertainties as well, but the two-body data combined with three-body data and calculations are rather efficient in narrowing down their uncertainties. In this way, successful three-body calculations can supply information about these unknown interactions.

The results of these cluster calculations must be in fair agreement with much of the available data, otherwise the employed clusterization is not preferred by nature. Good or bad agreement is significant to assess how well the cluster division can describe the investigated structures. Predictions of so far unknown properties clearly also provide possible tests. This includes the detailed structures obtained from the partial wave

Table 1. Strength, S , and range, b , in MeV and fm, respectively, for the Gaussian potentials used for the α - 2n and α - n interactions.

	α - 2n		α - n			
	$V_c^{(s)}$	$V_c^{(d)}$	$V_c^{(s)}$	$V_c^{(p)}$	$V_c^{(d)}$	V_{so}
S (MeV)	-20.37	-63.35	48.0	-47.4	-21.93	-25.49
b (fm)	1.8	2.5	2.33	2.30	2.03	1.72

decomposition of bound state sand resonances. No matter how well the models turn out, they can never be equally accurate for all properties of all states. In case the energies are essential for example close to a threshold, we introduce the three-body potential for fine tuning. This does not change the structure, only lowering or lifting the corresponding energy. However, it is an indication of how well the model is doing for that state.

The paper is organized as follows. After the introduction and the brief explanation of the overall procedure, we describe in Section II the basic interactions related to the experimentally known subsystems $^5\text{He}(\alpha+n)$ and $^6\text{He}(\alpha+^2n)$. Section III investigates various dineutron-dineutron interactions and their effect, in comparison with measurements, on the three-body 0^+ ground state of $^8\text{He}(\alpha+^2n+^2n)$. The best interactions are used in Section IV to compute excited states of $^8\text{He}(\alpha+^2n+^2n)$. In Section V, we turn to $^7\text{He}(\alpha+n+^2n)$, where much less is known. Finally, Section VI summarizes and gives the conclusions.

2. The α - 2n and α - n interactions

The experimentally known subsystems, ^5He and ^6He , can first be used to determine the two-body interactions. These potentials are assumed to be partial-wave dependent, and they contain a central, and when needed, a spin-orbit part:

$$V^{(\ell)}(r) = V_c^{(\ell)}(r) + V_{so}^{(\ell)}(r)\boldsymbol{\ell} \cdot \boldsymbol{s}, \quad (1)$$

where ℓ is the relative orbital angular momentum between the two particles, and s is the total spin. The potential radial functions, $V(r)$, are considered to have a Gaussian shape, $V(r) = Se^{-r^2/b^2}$, whose strength, S , and range, b , are determined such that they reproduce the available experimental information of the two-body system.

The α -dineutron potential: We here use the same α - 2n potential as the one described in reference [8] and employed for the description of ^8Li as an $\alpha+t+^2n$ structure.

This potential is based on the fact that the three-body structure of the ^6He ground state corresponds to a very large extent with a relative s -wave between the two neutrons and a relative s -wave between the α -particle and the dineutron center of mass [18]. This fact permits to obtain the parameters of the s -wave α - 2n central potential in such a way that the two-neutron separation energy is reproduced (-0.975 MeV [19]). The corresponding values for the strength and the range of the Gaussian s -wave potential are given in the second column of Table 1.

For the d -wave interaction we proceed in the same way, fitting the potential parameters in order to reproduce the properties of the 2^+ resonance in ${}^6\text{He}$. More precisely, the strength and range of the d -wave α - 2n potential are adjusted to reproduce the experimental resonance energy and width, 0.83 MeV above the two-neutron threshold and 0.113 MeV, respectively [20]. The values of these parameters are given in the third column of Table 1. It is important to note that the picture employed here assumes that the 2^+ resonance wave function is fully given by the component having the two neutrons in a relative s -wave and the α -particle in a relative d -wave with respect to the dineutron center of mass. However, three-body calculations show that the component with a relative d -wave between the two neutrons and a relative s -wave between the α -particle and the dineutron, although it gives a small contribution, its weight is not negligible [21]. The effective interaction accounts for this assumption by reproducing the 2^+ energy.

As discussed in reference [8], a p -wave α - 2n interaction should in principle be considered as well. However, the absence of a known low-lying 1^- state in ${}^6\text{He}$, and the fact the p -wave α - 2n components play a very minor role in the three-body description of the 0^+ and 2^+ states in ${}^6\text{He}$ leads us to simply employ for the p -wave potential the same interaction as for s -waves. In any case, as mentioned in [8], the use of this p -wave potential, making it equal to zero, or even excluding the p -wave components from the calculations, do not produce significant changes in the description of the low-lying ${}^8\text{Li}$ states.

The α -neutron potential: For the α - n interaction we use the one specified in reference [22], and whose parameters for the s - p - and d -wave central potential as well as the ℓ -independent spin-orbit potential are given in the right part of Table 1. These parameters have been obtained in order to reproduce the s -, p -, and d -wave α -neutron phase shifts obtained from scattering experiments up to an energy of 20 MeV. Furthermore, these parameters give rise to $\frac{3}{2}^-$ and $\frac{1}{2}^-$ resonances whose energy and width agree with the experimental values [20, 22]. Note that the s -wave potential is repulsive, which guarantees that the neutron does not occupy the Pauli forbidden $s_{1/2}$ -shell filled by the two neutrons in the α -particle.

3. The ${}^8\text{He}$ ground state 0^+

Let us start by investigating the ground state, 0^+ , of ${}^8\text{He}$ assuming an α - 2n - 2n structure. The three-body calculations are made following the hyperspherical adiabatic expansion method described in detail in [23]. They have been made including all the partial waves consistent with $\ell_x, \ell_y \leq 3$, where ℓ_x and ℓ_y refer to the relative orbital angular momentum between any pair of particles and ℓ_y is the orbital angular momentum between the third particle and the center of mass of the first two. As mentioned above, and although the p - and f -waves play a minor role, for the p -wave interaction between the α and the dineutron we use the same potential as for s -waves, whereas for $\ell = 3$ we use the same

Table 2. Strength, S , and range, b , of the four $^2n\text{-}^2n$ potentials used in this work. For each of them, and for the 0^+ ground state of ^8He , we also give the three-body binding energy, B , and rms radius, $\langle r^2 \rangle^{\frac{1}{2}}$, when no three-body potential is used, the strength, S_{3b} , of the three-body potential ($b_{3b} = 4.5$ fm) needed to fit the experimental binding energy (-3.11 MeV [25]), and the rms radius, $\langle r^2 \rangle_{3b}^{\frac{1}{2}}$, obtained when the three-body potential is included. All the energies are given in MeV and the distances in fm.

	<i>s</i> -wave		<i>d</i> -wave		B	$\langle r^2 \rangle^{\frac{1}{2}}$	S_{3b}	$\langle r^2 \rangle_{3b}^{\frac{1}{2}}$
	S	b	S	b				
Potential I	0	--	0	--	-2.20	2.80	-2.48	2.58
Potential II	-8.3	1.5	-8.3	1.5	-3.11	2.54	0	2.54
Potential III	30.0	1.5	-220.0	1.5	-2.07	2.75	-3.25	2.61
Potential IV	30.0	1.5	-229.8	1.5	-3.11	0	0	2.57

potential as for *d*-waves.

The three-body calculation provides the root mean square (rms) radius given by

$$\langle r^2 \rangle = \frac{\langle \rho^2 \rangle}{A} + \sum_{i=1}^3 \frac{A_i}{A} \langle r_{ci}^2 \rangle, \quad (2)$$

where A_i is the mass number of cluster i , $A = \sum_{i=1}^3 A_i$ is the mass number of the full system, $\langle r_{ci}^2 \rangle^{1/2}$ is the rms radius of cluster i , and $\langle \rho^2 \rangle$ is the expectation value of the square of the hyperradius ρ , defined in [23] with normalization mass as that of the nucleon.

3.1. The $^2n\text{-}^2n$ interaction

In order to perform the calculations, the $^2n\text{-}^2n$ interaction still has to be specified. In reference [24] the dineutron correlation in the ground state of ^8He was investigated using a core+4*n* model. They found that the neutron pairs coupled to zero spin exhibit a strong dineutron correlation, and furthermore, they also concluded that the two dineutrons interact very weakly with each other. For this reason we start our three-body calculation simply putting the $^2n\text{-}^2n$ interaction equal to zero.

When this is done, we obtain a bound 0^+ state, with a four-neutron separation energy of 2.20 MeV, about 0.9 MeV smaller than the experimental value of 3.11 MeV [25]. A not very much attractive three-body force is sufficient to fit the computed energy to the experimental value. In particular, if the Gaussian three-body force

$$V_{3b} = S_{3b} e^{-\rho^2/b_{3b}^2} \quad (3)$$

is used, and we choose a range $b_{3b}=4.5$ fm, a strength $S_{3b} = -2.48$ MeV permits then to recover the experimental energy.

The root mean square radius is also a measurable quantity. In our case, since $\langle r_\alpha^2 \rangle^{1/2} = 1.67$ fm [26], and considering $\langle r_{2n}^2 \rangle^{1/2} \approx 2$ fm [10], we obtain from equation (2) a rms radius for the ^8He ground state of 2.80 fm and 2.58 fm without and with inclusion of the three-body force, respectively. When compared to the experimental value, $\langle r^2 \rangle^{1/2} = 2.53 \pm 0.02$ fm [27], we see that the inclusion of the three-body force,

besides the four-neutron separation energy, gives rise to a rms radius also in reasonable good agreement with the experiment. The results obtained with this potential, referred as potential I, are collected in the first line of Table 2.

The fact that a zero interaction between the two dineutrons gives rise to a separation energy of the ${}^8\text{He}$ ground state not dramatically different from the experimental value, leads us to consider the case where the ${}^2n\text{-}{}^2n$ interaction is slightly attractive, simply to reduce the role of the three-body force. Playing with the strength and the range of the Gaussian potential, it is possible to get values that permit to reproduce simultaneously the experimental separation energy and rms radius without need of any three-body force. This happens with the strength and range $S = -8.3$ MeV and $b = 1.5$ fm. This potential will be referred as potential II, and the computed results for the 0^+ ground state are given in the second row of Table 2.

In reference [28] the authors investigated the properties of the tetraneutron as a dineutron-dineutron molecule. In their work they estimated that the s -wave dineutron-dineutron potential is predominantly repulsive. For this reason we have considered as well the possibility of a repulsive s -wave ${}^2n\text{-}{}^2n$ potential and an attractive d -wave potential that permits to recover the correct three-body separation energy. These are the potentials denoted as III and IV in Table 2. They have been constructed similar to potentials I and II, respectively. For potential III some three-body attraction is needed to fit the experimental separation energy, and in potential IV the d -wave interaction is used to adjust the energy and the three-body force is not needed. As we can see, these potentials, III and IV, produce a slightly bigger ground state, although still in reasonable agreement with the experimental value.

In the very recent paper [14], the authors have investigated the structure of the two known 0^+ states in ${}^8\text{He}$. They allowed both dineutrons and shell-model $p_{3/2}$ -configurations. The separation between two-body, three-body and shell-model structure is defined through the choice of a non-orthogonal, and substantially overlapping, basis. They find $\alpha+{}^2n+{}^2n$ configurations (their two- and three-body clusters) in the 0^+ ground state with probability 0.883. Their shell model consists of two neutrons in $p_{3/2}$ -configurations, which to some degree could also be describing a dineutron. This result supports the $\alpha+{}^2n+{}^2n$ description used here. Furthermore, in [14] they obtain a rms radius of 2.49 fm, in fairly good agreement with our result of about 2.6 fm obtained with the four potentials, as shown in the last column of Table 2, and with the experimental value of 2.53 ± 0.02 fm. In any case, the large amount of $\alpha+{}^2n+{}^2n$ configuration found in [14] contrasts with the result in reference [12], where a rather small squared overlap of about 45% with the $\alpha+{}^2n+{}^2n$ configuration is assigned to the 0^+ ground state.

3.2. Evaluating the potentials

We have then constructed a set of four different ${}^2n\text{-}{}^2n$ potentials all of them reproducing the experimental ground state three-body separation energy in ${}^8\text{He}$ (using an attractive three-body force when needed), and also reproducing reasonably well the experimental

Table 3. For the 0_1^+ state in ${}^8\text{He}$, and for each of the two different Jacobi sets, contribution to the norm of the wave function of the components included in the calculation. The results with the four ${}^2n\text{-}{}^2n$ potentials in Table 2 are given. When needed the three-body potential (3) has been included in the calculation.

						0_1^+ state (ground state)							
$4n({}^2n, {}^2n) + \alpha$						${}^6\text{He}(\alpha, {}^2n) + {}^2n$							
ℓ_x	ℓ_y	L	weights				ℓ_x	ℓ_y	L	weights			
			I	II	III	IV				I	II	III	IV
0	0	0	96.9	96.7	44.3	35.3	0	0	0	96.2	96.6	37.1	33.8
2	2	0	3.1	3.3	55.7	64.7	2	2	0	1.4	2.2	51.7	53.1
							1	1	0	2.3	1.2	9.6	10.7
							3	3	0			1.6	2.3

rms radius. However, the structure of the state produced by these potentials is very different. This is shown in Table 3, where we give the contribution to the norm of the ground state wave function of the components included in the calculation. The quantum numbers ℓ_x and ℓ_y , which couple to L , refer to the relative orbital angular momenta between two of the particles, and between the third particle and the center of mass of the first two, respectively. The results in the two possible Jacobi sets, with the α -Jacobi coordinate between the two dineutrons or between the α -particle and one of the dineutrons, are given in the left and right parts of the table. As we can see, potentials I and II favor the contribution of the s -wave components in both Jacobi sets, which give about 97% of the wave function. Thus, this ${}^8\text{He}$ state is mostly the ground state of ${}^6\text{He}(0^+)$ supplemented by another s -wave dineutron, but the structure can as well in the other basis be described as an alpha-particle surrounded by two dineutrons in a relative s -wave.

On the contrary, potentials III and IV enhance significantly the role of the d -waves, which in both Jacobi sets contribute with at least 50% of the norm. Thus, now the ${}^8\text{He}$ structure is equally shared between ${}^6\text{He}(0^+)$ and excited ${}^6\text{He}^*(2^+)$ states again supplemented by another dineutron. Furthermore, in these cases, the p -wave component between the α -particle and the dineutron has a relevant contribution of about 10%. The similarity between the results for potentials I and II, and for potentials III and IV, indicate that the need or not of the three-body force is not playing an important role, at least with respect to the partial wave content in the wave function.

The different structure produced by the potentials can also be seen in figure 1, where we show the contour plot of probability function as a function of the ${}^2n\text{-}{}^2n$ distance and the distance between the α -particle and the center of mass of the two dineutrons. Only the results with potentials I and III are shown, since the plots with potential II and IV are indistinguishable by naked eye from the ones with potentials I and III, respectively. The probability distribution with potential I (and II) is elongated along the dineutron-dineutron distance with a single maximum with the α -particle about 1 fm away from the ${}^2n\text{-}{}^2n$ center of mass. This is a relatively tight structure, although the three particles still remain separated. However, for potential III (and IV) the distribution presents

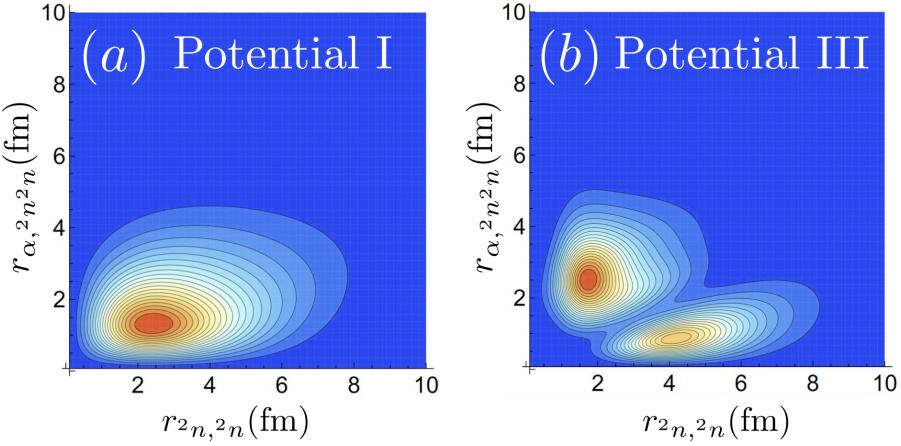


Figure 1. Contour plot of the 0^+ ground state with $^2n\text{-}^2n$ potentials I and III as a function of the $^2n\text{-}^2n$ distance and the distance between the α -particle and the center of mass of the two dineutrons.

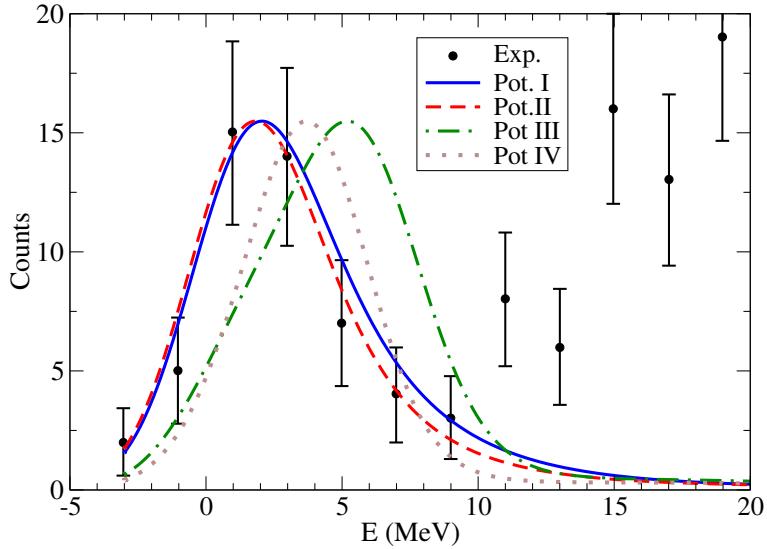


Figure 2. Missing-mass spectrum after removal of the α -particle in the $\alpha+^2n+^2n$ system obtained with the four $^2n\text{-}^2n$ potentials given in Table 2. Experimental data from [16].

two peaks, one of them also elongated along the $^2n\text{-}^2n$ distance, but the second one, which actually dominates, appears for clearly closer dineutrons to each other and the α -particle further away from the two-dineutron center of mass.

3.3. Missing-mass spectrum

Very recently, the missing-mass spectrum of the four-neutron system after the $^8\text{He}(p,p\alpha)4n$ reaction has been measured at a ^8He collision energy of 156 MeV/nucleon [16]. As a test, we can estimate this spectrum by means of the sudden approximation, which assumes that in the collision one of the three-constituents of the system (the α -

particle in our case) is suddenly removed, is such a way that the other two particles (the two dineutrons) continue flying together without being too much altered by the removal of the core. As shown in [29], the momentum distributions of the fragments is very much determined by the final state interaction between them. Taking this into account, and following [30], we have computed the invariant mass spectrum for ^8He after removal of the core. The result is shown in figure 2 for the four 2n - 2n potentials considered in this work. The computed curves have been convoluted with an experimental resolution of 2 MeV (which is assumed to be energy independent) [13] and scaled to the same maximum.

As we can see, potentials I and II both give rise to spectra reproducing reasonably well the low-energy peak of the experimental spectrum. On the contrary, the spectra produced by potentials III and IV show a peak clearly shifted with respect to the experimental data.

The fact that the spectrum with potential IV (for which there is no need of including a three-body force in the three-body calculation) gets closer to the experimental peak than with potential III (for which a three-body attractive potential is needed) could lead us to think that a more attractive d -wave 2n - 2n potential and a repulsive three-body force would reproduce better the experimental spectrum in figure 2. However, there is not much room for this, since a d -wave attractive strength of about -240 MeV binds the two dineutrons into an unphysical 2^+ four-neutron bound state. Therefore, from this perspective, the dineutron-dineutron potentials showing a very weak interaction are more appropriate than the ones containing repulsive s -wave and attractive d -wave terms.

In any case, experimentally the energy E in the figure refers to the four-neutron relative energy, whereas in the calculations E represents the dineutron-dineutron relative energy. The four-neutron relative energy equals the dineutron-dineutron relative energy added to twice the intrinsic n - n energy, which in turn should be slightly positive. A guess could be the n - n virtual energy of $\hbar^2/(ma_{sc}^2) \simeq 0.12$ MeV (where the n - n scattering length $a_{sc} = 18.7$ fm has been used). The curves should then be shifted by about 0.24 MeV to the left in figure 2, which would be hardly noticeable.

4. ^8He excited states

Experimentally, all the excited states of ^8He are known to be resonances [25]. In order to get them numerically, the calculations are made after complex rotation of the hyperspherical adiabatic expansion method mentioned above, as done for instance in [31]. In this way, the resonances appear as states with bound state asymptotic behavior, but with complex energy, $E = E_R - i\Gamma_R/2$, where E_R and Γ_R are the resonance energy and width, respectively.

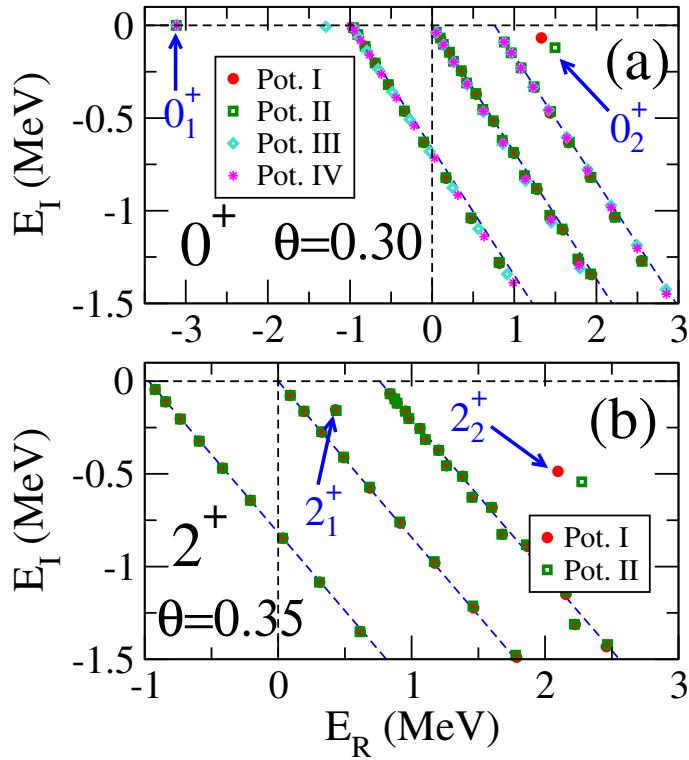


Figure 3. Complex scaled energy spectrum of the (a) 0^+ (with complex rotation angle $\theta=0.30$ rads) and (b) 2^+ states (with $\theta=0.35$ rads) obtained with the potentials given in Table 2.

4.1. 0^+ states

As mentioned above, the possibility of existence of excited 0^+ states is investigated by solving the three-body problem after a complex scaling transformation [31]. More precisely, this is done after discretization of the continuum spectrum by imposing a box boundary condition.

The result for the 0^+ states is shown in figure 3a, where a complex scaling angle $\theta=0.30$ rads has been used. The discrete three-body energies shown in the figure are distributed along three different cuts, which are rotated by an angle 2θ . The first cut starts at the ${}^6\text{He}$ two-neutron separation energy (-0.97 MeV), and the dots along this cut represent ${}^6\text{He}-{}^2n$ continuum states. The second cut, which starts at zero energy, represents genuine three-body continuum states. Finally, the third cut starts at 0.82 MeV, which is the energy of the 2^+ resonance in ${}^6\text{He}$, and the dots correspond to a sort of two-body continuum states between the 2^+ resonance in ${}^6\text{He}$ and the dineutron. The dots out of these three cuts, indicated by the arrows in the figure, are the bound and resonant 0^+ states in ${}^8\text{He}$. For each of the potentials the calculation has been made using the same three-body force as the one needed to reproduce the energy of the ground state.

As we can see, together with the bound state at -3.11 MeV, potentials I and

Table 4. Energies, E_R , and widths, Γ_R , of the computed ${}^8\text{He}$ states with the four 2n - 2n potentials given in Table 2. The energies (with respect to the three-body threshold) and the widths are given in MeV. For each state we also give the strength, in MeV, of the three-body force (3) used in the calculation. The experimental data have been taken from [25] except the one of the 0_2^+ state, which has been taken from [17].

	Pot. I			Pot. II			Pot. III			Pot. IV			Exp.	
	E_R	Γ_R	S_{3b}	E_R	Γ_R	S_{3b}	E_R	Γ_R	S_{3b}	E_R	Γ_R	S_{3b}	E_R	Γ_R
0_1^+	-3.11	—	-2.48	-3.11	—	0.0	-3.11	—	-3.25	-3.11	—	0.0	-3.11	—
0_2^+	1.33	0.13	-2.48	1.50	0.24	0.0	-1.29	—	-3.25	-1.0	—	0.0	1.43 ± 0.06	
2_1^+	0.43	0.31	5.0	0.43	0.31	7.25							0.43 ± 0.06	0.89 ± 0.11
2_2^+	2.10	0.97	5.0	2.27	1.09	7.25								
1_1^-	0.33	1.35	-6.0	0.19	1.39	-6.0							1.25 ± 0.20	1.3 ± 0.5
3_1^-	0.70	0.23	0.0	0.01	0.23	0.0							4.05 ± 0.04	0.1 ± 0.1

II show a second, resonant, 0^+ state with $E_R=1.33$ MeV and 1.50 MeV above the four-neutron threshold (therefore with excitation energies 4.44 MeV and 3.61 MeV, respectively). The computed width of the resonance is 0.13 MeV and 0.24 MeV for each of the two potentials. This resonance excitation energy agrees very well with the one recently observed in reference [17], where they have identified a 0^+ resonance in ${}^8\text{He}$ with excitation energy 4.54 ± 0.06 MeV, which corresponds to an energy of 1.43 MeV above the four-neutron threshold. In [17] this state is interpreted as an exotic $\alpha+{}^2n+{}^2n$ cluster structure. Nothing is said in [17] concerning the width of the 0_2^+ state. This 0^+ excited state has also been obtained in [11, 32] with excitation energies of 4.5 MeV and 6.3 MeV, respectively. In [32] a width of 3.2 MeV is given, clearly bigger than the values provided by potentials I and II.

However, when potentials III and IV are considered, we have found that the 0_2^+ state is actually bound, as clearly seen in figure 3a for potential III, where a second bound state is observed at -1.29 MeV below the three-body threshold. For potential IV the energy of the 0_2^+ state is -1.0 MeV, so close to the ${}^6\text{He}(0^+) + {}^2n$ threshold that the state can not be seen in the figure. In any case, these two energies clearly disagree with the experimental value of 1.43 MeV. Even more, if we try to force the energy of the 0_2^+ state to reproduce the experimental energy by means of a sufficiently repulsive three-body force, we then find that the 0_2^+ state disappears very fast in the ${}^6\text{He}(0^+) + {}^2n$ continuum, and the experimental energy can not be reached.

In the first two rows of Table 4 we collect the computed energies and widths for the 0_1^+ and 0_2^+ states using the same three-body force as the one needed to reproduce the ground state energy. It is clear that potentials I and II are more appropriate for a correct description of the 0_2^+ state, since a small retuning of the three-body force would be sufficient to recover the experimental energy.

Although the resonances are obtained after a complex scaling calculation, the real part of the squared complex rotated wave function can give a clear hint about the spatial distribution of the clusters when the resonance is populated. The contour plot of this probability function is shown in figure 4 for the 0_2^+ resonance with potential I (the result

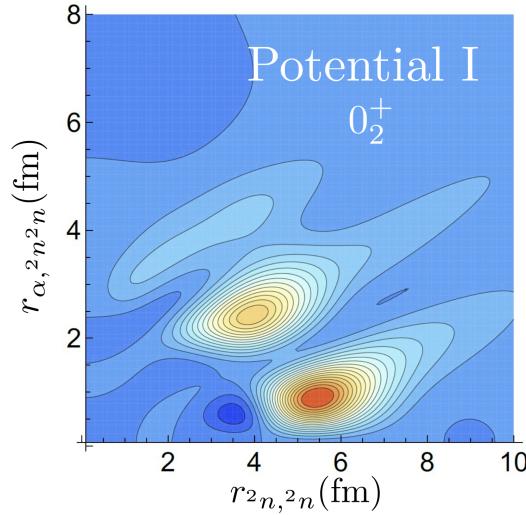


Figure 4. The same as figure 1 for the excited 0^+ resonance.

Table 5. The same as Table 3 for the 0_2^+ state.

0_2^+ state													
			$4n(^2n, ^2n)+\alpha$				${}^6\text{He}(\alpha, {}^2n)+{}^2n$						
ℓ_x	ℓ_y	L	weights				ℓ_x	ℓ_y	L	weights			
			I	II	III	IV				I	II	III	IV
0	0	0	77.3	78.5	84.6	65.3	0	0	0	10.9	11.2	79.7	75.4
2	2	0	22.7	21.5	15.4	34.7	2	2	0	72.7	73.3	10.8	8.1
							1	1	0	9.3	9.0	8.9	13.3
							3	3	0	7.1	6.5	0.6	3.2

obtained with potential II is very similar). As in figure 1, we plot it as a function of the ${}^2n-{}^2n$ distance and the distance between the α -particle and the center of mass of the two dineutrons. As we can see, the function presents two prominent peaks, where the main one is located for a large separation between the two dineutrons, about 5.5 fm, and the α -particle pretty close, about 1 fm, to the two-dineutron center of mass. When comparing to figure 1a we can see that, as expected, the 0_2^+ wave function is more extended than the one of the ground state. This is also reflected in the rms radius obtained for the 0_2^+ resonance, which is 3.1 ± 0.6 fm with both, potentials I and II. As in the case of the complex energy, the imaginary part of the computed rms radius is interpreted as the uncertainty of the observable.

In Table 5 we give the contribution to the norm of the different components in the 0_2^+ state for the energies given in Table 4. We see that for potentials I and II the waves with $\ell > 0$ start playing now a more important role than for the ground state. In fact, from the right part of the table we can see that a large contribution is coming from ${}^6\text{He}$ in the 2^+ state with the second dineutron in a relative d -wave. For potentials III and IV, which fail in reproducing the experimental energy, it is just the opposite, since the dominating component corresponds to ${}^6\text{He}$ in the ground state (0^+), and the dineutron in a relative s -wave. We can also see that, for all the cases, the components with odd

value of ℓ give a significant contribution, which means that a more detailed analysis of the p - and f -wave α -dineutron potential than the one made here is very likely necessary to be more accurate about the structure of this state.

The 0_2^+ state in ^8He was also investigated in references [10, 11, 12] with the conclusion that there is a large component of $\alpha+^2n+^2n$ structure. In [14] they obtain an overlap probability of 0.926 for this structure. They also give a computed radius of 4.67 fm for this first excited 0^+ state in ^8He . As already mentioned, in our case this state is a resonance obtained by use of complex scaling technique, where the imaginary part of the radius is interpreted as an uncertainty resulting in the value 3.1 ± 0.6 fm.

The discretization of the continuum 0^+ states allows us to compute the monopole transition strength to the ground state as:

$$\mathcal{B} = \sum_k |\langle \Psi_{\text{g.s.}} | \hat{\mathcal{O}} | \Psi_{0^+}(E_k) \rangle|^2, \quad (4)$$

where $\Psi_{\text{g.s.}}$ is the wave function of the 0_1^+ ground state, $\Psi_{0^+}(E_k)$ is the wave function of the discretized continuum state k with energy E_k , and $\hat{\mathcal{O}} = \sum_{i=1}^A r_i^2$ is the monopole transition operator [33] (where r_i is the radial coordinate of nucleon i in the nucleus, referred to the nucleus center of mass).

The strength in equation (4) can be computed on the real energy axis (without complex scaling). When the summation over the discrete states k is limited to the ones whose energy is in the interval $E_R \pm 1$ MeV (where E_R is the energy of 0_2^+ resonance) we get $\mathcal{B} = 113.4$ fm 4 . The square root of this value, 10.7 fm 2 , compares well with the experimental value, $11^{+1.8}_{-2.3}$ fm 2 , given in reference [17] for the monopole transition matrix element.

4.2. 2^+ states

Concerning the 2^+ states, in reference [34] a 2^+ resonance in ^8He was identified with excitation energy 3.57 ± 0.12 MeV (0.46 MeV above the four-neutron threshold) and a width of 0.50 ± 0.35 MeV. These values have been recently confirmed in [35], where they have observed the resonance with excitation energy 3.54 ± 0.06 MeV (0.43 MeV above the four-neutron threshold) and a width of 0.89 ± 0.11 MeV.

After the calculations, and without the inclusion of any three-body potential, we have obtained that the $^2n-^2n$ potentials I and II give rise to a 2^+ state at about 0.8 MeV and 1.7 MeV, respectively, below the four-neutron threshold. Therefore, some repulsive three-body force is necessary to push up the energy to the experimental value. Using again the Gaussian three-body potential in equation (3) with $b = 4.5$ fm, this is achieved with the repulsive strengths $S_{3b}=5.0$ MeV and $S_{3b}=7.25$ MeV, for each of the two potentials. The corresponding discretized continuum spectra are shown in figure 3b, which has been computed with the complex scaling angle $\theta = 0.35$ rads. In the figure we can see that both potentials give rise to a 2_1^+ state with a very similar width, $\Gamma_R=0.31$ MeV, which is close to a factor of 2 smaller than the experimental observation reported in [35].

Table 6. The same as Table 3 for the 2_1^+ and 2_2^+ states with potentials I and II.

			2_1^+ and 2_2^+ states							${}^6\text{He}(\alpha, {}^2n) + {}^2n$			
			$4n({}^2n, {}^2n) + \alpha$							${}^6\text{He}(\alpha, {}^2n) + {}^2n$			
ℓ_x	ℓ_y	L	2_1^+		2_2^+		ℓ_x	ℓ_y	L	2_1^+		2_2^+	
			I	II	I	II				I	II	I	II
0	2	2	63.7	67.9	24.0	25.9	0	2	2	35.7	36.1	3.7	3.1
2	0	2	34.1	31.8	27.7	22.6	2	0	2	54.8	56.9	3.7	3.4
2	2	2	2.3	0.3	48.3	51.4	2	2	2	2.6	0.6	81.0	83.1
							1	1	2	0.0	1.1	3.3	2.8
							1	3	2	4.4	3.6	0.4	0.0
							3	1	2	2.0	1.4	0.2	0.4
							3	3	2	0.5	0.3	7.7	7.2

Concerning potentials III and IV, the results for the 2^+ state are very different. In fact, they produce a lowest 2_1^+ state, which actually would be the ground state, since it appears with a rather big three-body separation energy of about 20 MeV. Even more, they also show a pretty much bound 2_2^+ excited state, about 8 MeV below the four-neutron threshold. These results are so unrealistic, and so far from the known experimental information about ${}^8\text{He}$, that we definitely can conclude that potentials III and IV must be disregarded as potentials suitable for the description of ${}^8\text{He}$ as an $\alpha + {}^2n + {}^2n$ system. This is in fact confirming what we could already suspect, from the result shown in figure 2 and the fact that the experimentally known 0_2^+ state is not predicted either when these two potentials are used.

In figure 3b we can also see that, together with the 2_1^+ state, a second 2^+ resonance is found at about 2.2 MeV above the three-body threshold (about 5.3 MeV excitation energy) with a width in the vicinity of 1 MeV. So far there is no experimental evidence for the existence of such an excited 2^+ state. In reference [36] a second 2^+ resonance is also obtained, but at a higher energy (about 4.5 MeV above the four-neutron threshold) and clearly wider (about 4.4 MeV). The precise computed energies and widths of the 2^+ states obtained with potentials I and II are also given in Table 4.

In Table 6 we give the contribution of the different components in the two different Jacobi sets for the 2_1^+ and 2_2^+ states with potentials I and II. We see that the 2_1^+ state has basically an sd structure, with two of the particles in a relative s (or d) wave, and the third particle in a d (or s) wave relative to the center of mass of the first two. In the first Jacobi set the relative s -wave between the two dineutrons dominate, giving about two thirds of the norm of the wave function. In the second Jacobi set the dominating components corresponds to ${}^6\text{He}$ in a 2^+ state. For the 2_2^+ state we find that the $\ell_x = \ell_y = 2$ components are the dominant ones, mainly in the second Jacobi set, where they give more than 80% of the norm. The difference between potentials I and II is not significant.

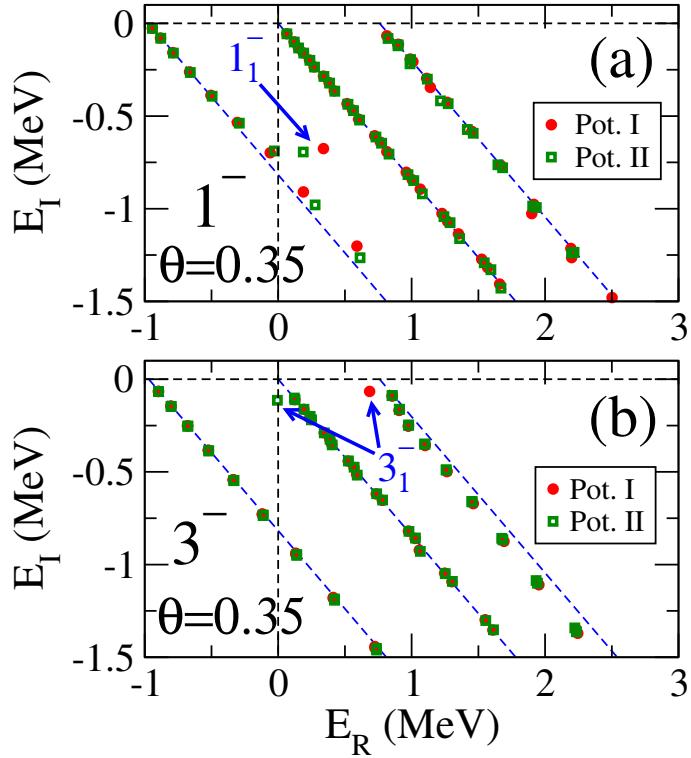


Figure 5. The same as figure 3 for the (a) 1^- and (b) 3^- states.

4.3. 1^- and 3^- states

The experimental information about ^8He available in reference [25] indicates that, although not fully confirmed, two negative parity states, 1^- and 3^- , could be present in the spectrum.

Focusing again on potentials I and II, when no three-body force is employed, our calculations do not give rise to any 1^- state, or at least they are not reachable with the complex scaling angle used in the calculations, $\theta=0.35$ rads. However, when the three-body potential is used to introduce some additional attraction, a wide resonant state starts soon to show up. In figure 5a we show the 1^- discretized continuum spectrum obtained with $S_{3b} = -6$ MeV and $\theta=0.35$ rads. We see that with both potentials, a resonance is now present at about 0.3 MeV above the three-body threshold and a width of about 1.4 MeV. The precise values are given in Table 4. Although the computed width agrees with the one of the tentative experimental 1^- resonance, 1.3 ± 0.5 MeV [25], the energy is clearly smaller than the experimental one, 1.25 ± 0.20 MeV. In principle this energy could be reached by making the three-body force less attractive, but in that case the computed width increases rapidly, and the resonance disappears in the ${}^6\text{He}(0^+) + {}^2\text{n}$ continuum far before reaching the experimental energy.

Therefore, within our model, the computed 1^- state either has an energy about 1 MeV below the experimental value and a width matching experiment, or reproducing the measured energy with a width much larger than the experimental value. In any case,

Table 7. The same as Table 3 for the 1^-_1 state with potentials I and II.

1^-_1 state										
$4n(^2n, ^2n) + \alpha$			${}^6\text{He}(\alpha, {}^2n) + {}^2n$							
ℓ_x	ℓ_y	L	weights			ℓ_x	ℓ_y	L	weights	
			I	II					I	II
0	1	1	100.0	97.3		0	1	1	45.1	49.1
2	1	1	0.0	1.2		1	0	1	35.1	33.2
2	3	1	0.0	1.5		1	2	1	1.6	1.9
						2	1	1	13.4	10.3
						2	3	1	2.2	2.5
						3	2	1	2.6	3.0

Table 8. The same as Table 3 for the 3^-_1 state with potentials I and II.

3^-_1 state										
$4n(^2n, ^2n) + \alpha$			${}^6\text{He}(\alpha, {}^2n) + {}^2n$							
ℓ_x	ℓ_y	L	weights			ℓ_x	ℓ_y	L	weights	
			I	II					I	II
0	3	3	47.2	51.9		0	3	3	10.8	7.7
2	1	3	49.5	38.1		1	2	3	9.3	11.8
2	3	3	3.3	2.2		2	1	3	74.9	74.6
						2	3	3	1.2	0.2
						3	0	3	1.3	2.0
						3	2	3	2.5	1.5

it is worth noticing that the component with a relative p -wave between the α -particle and the dineutron plays a very relevant role in this case, contributing with about 35% of the norm of the wave function. This is seen in the right part of Table 7, where we give the weight of the different components for the 1^-_1 resonance shown in figure 5a. Since the α -dineutron p -wave potential has not been adjusted to any ${}^6\text{He}$ property, but simply has been taken equal to the $\ell=0$ potential, a more careful treatment of this interaction could modify the results. For example, reproduction of the 1^-_1 state in ${}^8\text{He}$ might be achieved by adjusting the unknown 1^-_1 in ${}^6\text{He}$. These properties are connected in our model. From the left part of the table we can see that this state is fully given by a single component in the first Jacobi set, with the two dineutrons in a relative s -wave and the α in a p -wave relative to the ${}^2n-{}^2n$ center of mass.

We close this section with the results for the 3^-_1 states. The computed discretized continuum spectrum is shown in figure 5b for potentials I and II. These results have been obtained without inclusion of a three-body force. We can see that one resonance can be clearly seen for each of the two potentials, whose energies and widths are given in the last row of Table 4. The computed energies are several MeV below the tentative experimental value given in reference [25]. This means therefore that a quite repulsive three-body force should be needed in order to place the resonance at the experimental value. In any case, when we try to do this, we find that the 3^-_1 resonance disappears rather fast in the continuum.

In Table 8 we give the contribution of the components included in the calculation of the 3^- state (with no three-body force). We see that this state is dominated by ${}^6\text{He}$ in the 2^+ state, and, contrary to the 1^- case, the p -wave α -dineutron component plays a limited role (about 10% of the norm). Therefore, the uncertainty coming from the unknown p -wave α -dineutron interaction is not that crucial in this case.

4.4. Properties of excited ${}^8\text{He}$ states

The two best potentials are used to obtain excited states. Two of them, 0_2^+ and 2_1^+ , are experimentally seen. Two more states are also known experimentally with the same tentative spin and parity, 1_1^- and 3_1^- , as found in this work. The computed energies, especially of the 3_1^- state, do not fully agree with the experimental values. In any case, the computed odd parity energies are less reliable, because an uncertainty from an unknown p -wave in ${}^6\text{He}(\alpha+{}^2n)$ contributes significantly, mainly for the 1_1^- state. We predict the existence of a 2_2^+ state. Energies and radii are in fair agreement with the available data. We did not fine-tune the interactions to reproduce experiments more precisely.

In contrast to the ground state, the partial wave decomposition for the second 0^+ state reveals a content of about 73% ${}^6\text{He}(\alpha+{}^2n)$ in the 2^+ excited state. The average-distance density distribution for this excited state is delicate, since it is found as a resonance after complex scaling. The structure reveals very well separated clusters, where the dineutrons to a large extent are located on each side of the α -particle.

The monopole transition between the two 0^+ -states is less precisely defined, since 0_2^+ is a continuum structure. The spurious large-distance contribution from ${}^6\text{He}(0_1^+) + {}^2n$ has to be removed, but unfortunately difficult to differentiate for real three-body energies, from the desired short-distance contributions of 0_2^+ . Still, we give an estimate, which within the inherent uncertainties agree with the measured value.

The partial wave decompositions reveal predicted substructures. The 2_1^+ -state has about 36% and 58%, respectively in ground and first excited ${}^6\text{He}(\alpha+{}^2n)$ states, whereas 2_2^+ has about 85% in the ${}^6\text{He}$ excited 2^+ state. The 1_1^- -state has all strength in the s -wave between the two dineutrons. Translation to the other Jacobi coordinate showing ${}^6\text{He}(\alpha+{}^2n)$ with an attached dineutron is then equally sharing s and p waves between α and 2n . The 3_1^- -state is sharing almost equally the s and d -waves between dineutrons, but arising from different couplings to the α -particle. The translation to the other Jacobi coordinate gives about 75% of the ${}^6\text{He}$ 2^+ -state. The estimates of the odd parity structures are more uncertain, because they depend on the unknown p -wave of ${}^6\text{He}$.

The structure of the four valence neutrons in ${}^8\text{He}$ is tested by pushing the α -core gently away by a fast moving projectile and measuring the missing-mass spectrum. We believe this is a relatively sensitive structure quantity, which in the present calculation with dineutrons agrees with the measurement.

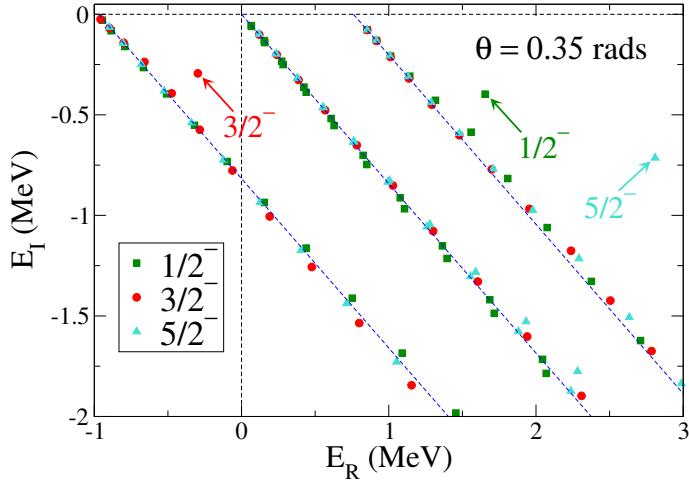


Figure 6. Complex scaled energy spectrum of the $\frac{1}{2}^-$, $\frac{3}{2}^-$, and $\frac{5}{2}^-$ states in ^7He .

Table 9. Energies, E_R , and widths, Γ_R , of the computed ^7He states. The energies (with respect to the three-body threshold) and the widths are given in MeV. S_{3b} gives the strength, in MeV, of the Gaussian three-body force in equation (3). The experimental data have been taken from [20].

J^π	E_R	Γ_R	S_{3b}	E_R	Γ_R
$\frac{3}{2}^-$	-0.52	0.35	-0.84	-0.529	0.15 ± 0.02
$\frac{1}{2}^-$	1.65	0.79	0.0		
$\frac{5}{2}^-$	2.39	0.94	-2.0	2.39 ± 0.09	1.99 ± 0.17

5. ^7He as an $\alpha+^2n+n$ system

Focusing now on the ^7He nucleus, our aim here is to investigate how the $\alpha+^2n+n$ structure is able to describe its main properties. Following the results obtained for ^8He , where we have seen that a very weak, or even zero, $^2n-^2n$ interaction is more appropriate to describe the nucleus, we assume here that the same applies for the ^2n-n interaction. We have then made the calculations for ^7He taking the ^2n-n potential equal to zero.

When this is done we obtain for ^7He the discretized continuum spectrum shown in figure 6 for the $\frac{1}{2}^-$, $\frac{3}{2}^-$, and $\frac{5}{2}^-$ states. We can immediately see that the ground state is given by a resonant $\frac{3}{2}^-$ state at -0.30 MeV with respect to the three-neutron threshold, or, in other words, 0.67 MeV above the ${}^6\text{He}(0^+)+n$ threshold. This energy is close to the experimental value, -0.529 MeV given in reference [20]. A small attractive three-body force is then sufficient to fit the experiment ($S_{3b} = -0.84$ MeV in equation (3)). The resulting energy and width are quoted in the first row of Table 9, where we can see that the width of the computed resonance is about a factor of two bigger than the experimental one.

In figure 6 we also see that the first excited state corresponds to spin and parity $\frac{1}{2}^-$, which appears at an energy 1.65 MeV above the three-neutron threshold, with a width of about 0.8 MeV. So far, there has not been experimental evidence of such an

Table 10. For the $\frac{3}{2}^-$ and $\frac{1}{2}^-$ states in ^7He , and for each of the three different Jacobi sets, contribution to the norm of the wave function of the components included in the calculation.

$3n(^2n,n)+\alpha$						$\frac{3}{2}^-$ and $\frac{1}{2}^-$ states ${}^5\text{He}(\alpha,n)+{}^2n$				${}^6\text{He}(\alpha,{}^2n)+n$				
ℓ_x	ℓ_y	L	weights		ℓ_x	ℓ_y	L	weights		ℓ_x	ℓ_y	L	weights	
			$\frac{3}{2}^-$	$\frac{1}{2}^-$				$\frac{3}{2}^-$	$\frac{1}{2}^-$				$\frac{3}{2}^-$	$\frac{1}{2}^-$
0	1	1	35.6	48.6	0	1	1	1.3	16.2	0	1	1	91.8	1.7
1	0	1	64.4	21.9	1	0	1	98.7	0.0	1	0	1	3.8	7.3
1	2	1	0.0	29.5	1	2	1	0.0	82.7	1	2	1	2.7	0.0
2	1	1	0.0	0.0	2	1	1	0.0	1.1	2	1	1	1.1	88.0
2	3	1	0.0	0.0	2	3	1	0.0	0.0	2	3	1	0.6	0.0
3	2	1	0.0	0.0	3	2	1	0.0	0.0	3	2	1	0.0	3.0

Table 11. The same as Table 10 for the $\frac{5}{2}^-$ state.

$3n(^2n,n)+\alpha$				$\frac{5}{2}^-$ state ${}^5\text{He}(\alpha,n)+{}^2n$				${}^6\text{He}(\alpha,{}^2n)+n$			
ℓ_x	ℓ_y	L	weight	ℓ_x	ℓ_y	L	weight	ℓ_x	ℓ_y	L	weight
1	2	2	68.0	1	2	2	90.4	1	2	2	0.2
2	1	2	22.1	2	1	2	8.1	2	1	2	99.8
2	3	2	8.7	2	3	2	1.5	2	3	2	0.0
3	2	2	1.2	3	2	2	0.0	3	2	2	0.0

excited state.

It is important to note here that the components entering in the description of the $\frac{3}{2}^-$ and $\frac{1}{2}^-$ states are the same, since they both result from the coupling of the total orbital angular momentum $L = 1$ and the total spin $S = \frac{1}{2}$. The only difference between the two states comes from the different weight of the components. These weights are shown in Table 10 for the two states and the three possible Jacobi sets. The different structure of the $\frac{3}{2}^-$ and $\frac{1}{2}^-$ states can be seen in the central part of the table, which indicates that both states are mainly given by ${}^5\text{He}$ in a p -state, but with the dineutron in a relative s -wave in the $\frac{3}{2}^-$ case, or in a relative d -wave in the $\frac{1}{2}^-$ case. A similar difference can be seen in the Jacobi set shown in the right part of the table, which shows that the structure of these two states can also be understood as the neutron in relative p -wave with respect to ${}^6\text{He}$, but whereas ${}^6\text{He}$ is in a 0^+ state for the $\frac{3}{2}^-$ state, it is in a 2^+ state in the $\frac{1}{2}^-$ state.

Finally, in figure 6 we see as well that the calculations give rise to an excited $\frac{5}{2}^-$ state with energy 2.81 MeV above the three-neutron threshold and a width of 1.4 MeV. In reference [20] a resonant state at 2.4 MeV above the three-neutron threshold and width of about 2.0 MeV is reported, and the tentative spin and parity of $\frac{5}{2}^-$ is assigned. It is then reasonable to assume that our computed state actually corresponds to this experimentally known resonance. When the three-body potential is used to fit the experimental value (which is done with $S_{3b}=-2.0$ MeV), we get the result given in the last row of Table 9. We see that the width of the computed state is then about 1 MeV,

roughly half the size of the experimental value.

For completeness we close this section giving in Table 11 the contribution to the $\frac{5}{2}^-$ wave function of the different components included in the calculation. We see that the wave function is to a very large extent given by ($\ell_x = 1, \ell_y = 2$) component in the second Jacobi set (${}^5\text{He}$ in a p -state and 2n in a relative d -wave), or the ($\ell_x = 2, \ell_y = 1$) component in the third Jacobi set (${}^6\text{He}$ in a d -state and n in a relative p -wave). These two are also the components giving rise to the $\frac{1}{2}^-$ state (see Table 10), but with the crucial difference that whereas in the $\frac{1}{2}^-$ case ℓ_x and ℓ_y couple to $L = 1$, in the $\frac{5}{2}^-$ case they couple to $L = 2$.

To close this section let us mention that, as done in the ${}^8\text{He}$ case, the calculations could have been done assuming a small 2n - n interaction such that three-body potential is not needed. For the $\frac{3}{2}^-$ state this is achieved with an attractive Gaussian potential with strength -6.3 MeV and range 1.5 fm. This gives rise to a resonance energy of -0.52 MeV and a width of 0.32 MeV, very much the same as quoted in Table 9. Furthermore, the partial wave content is also very similar to the one given in Table 10. The computed content of the dominating $(0, 1, 1)$ and $(1, 0, 1)$ components is now $(36.5, 63.5)$, $(1.0, 99.0)$ and $(91.8, 4.4)$ in the first, second, and third Jacobi sets, respectively. As one can see, the change is minimal. For the other two states, $\frac{1}{2}^-$ and $\frac{5}{2}^-$, the variation in the partial wave content compared to the results given in Tables 10 and 11 is also very small, not relevant either.

6. Summary and conclusions

We have investigated two structures with three and four neutrons outside an α -particle core. In the shell model, all neutrons would be in the $p_{3/2}$ state, but it may be advantageous to exploit other degrees of freedom. Since neutrons are fermions, the most obvious is to form pairs of clusters, i.e. the entities n and 2n . This attempt is also suggested by the hyper-radial prediction that only two- and three-body clusters are likely to exist. Thus, we assume the structures ${}^7\text{He}(\alpha + {}^2n + n)$ and ${}^8\text{He}(\alpha + {}^2n + {}^2n)$ and perform the corresponding three-body calculations. The dineutron, 2n , is treated as a spin-zero particle without intrinsic structure.

The known subsystems, ${}^5\text{He}$ and ${}^6\text{He}$, are used to constrain the two-body interactions, α - 2n and α - n . This incorporates effects of possible intrinsic 2n structure. The other two-body interactions, 2n - 2n and 2n - n , are required to reproduce the energies of the three-body (ground) states. They turn out to be rather weak and almost compatible with zero. The adiabatic hyperspherical expansion method supplemented by complex scaling is applied to obtain three-body solutions for different angular momenta and parities. If necessary, a three-body potential is added to match precisely the ground state energy.

The computations are in fair agreement with the available data, first of all the energy spectra, where the existence of a second 2^+ in ${}^8\text{He}$ and one $\frac{1}{2}^-$ in ${}^7\text{He}$ are predicted. In addition, the average-distance density distributions for both 0^+ states

in ^8He show non-overlapping three-body constituents. The missing mass spectrum of the four neutrons after removal of the α -particle as well as the monopole transition between the two 0^+ -states are within error bars consistent with measurements.

The calculated partial wave decompositions describe the structures of the states. The realistic potentials reveal that the 0^+ ground state of ^8He has a content of 97% of 0^+ ground state of ^6He . In contrast, the second 0^+ state has a content of about 73% of ^6He in the 2^+ excited state. Both with ^6He surrounded by a dineutron 2n . The 2_1^+ -state has about 36% and 58%, respectively in ground and first excited $^6\text{He}(\alpha+^2n)$ states, whereas 2_2^+ has about 85% in the ^6He excited 2^+ state. The 1_1^- -state has all strength in the s -wave between the two dineutrons. The 3_1^- -state has about 75% of the ^6He 2^+ -state.

The $\frac{3}{2}^-$ ground state of ^7He can be understood either as 99% ^5He surrounded by an s -wave dineutron, or as 92% ^6He surrounded by a p -wave neutron. The dominating part of the $\frac{1}{2}^-$ state can also be understood either as 83% ^5He surrounded by a d -wave dineutron, or as 88% $^6\text{He}^*(2^+)$ surrounded by a p -wave neutron. The highest found resonance, $\frac{5}{2}^-$, is described either as 90% ^5He surrounded by an d -wave dineutron, or as 100% $^6\text{He}^*(2^+)$ surrounded by a p -wave neutron. The different descriptions of the same state are the consequence of the reference to different bases.

In conclusion, the dominating components in the structures of ^7He and ^8He can be described as three-body systems consisting of an α -particle combined with one neutron and one dineutron and two dineutrons, respectively. In this work, we derived contributions of substructures for each three-body state, which expressed in different bases appears different, although it is the same state. We see connections between interactions and structures of nuclei in the isotopic chain. Specifically, a measurement of $^8\text{He}(1^-)$ would provide information about an unknown $^6\text{He}(1^-)$, and the ^8He spectrum show that the effective interactions between 2n - 2n and 2n - n are very weak. The perspective of this work is that a number of other neutron dripline nuclei can be understood as specific two or three-body clusters. The main uncertainty in such descriptions would arise from nucleons arranged in several different (coupled) clusters.

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