

Emergence of power laws in hierarchical dynamics on multi-level graphs

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Abstract

Power-law distributions are widely recognized in complex systems physics as indicative of underlying complexity in interaction networks and critical macroscopic behavior. Previous studies, notably those of Newman and others, have emphasized the importance of network structure and dynamics in understanding the emergence of such statistical patterns and predicting extreme events. In this study, we investigate the emergence of power-law behavior in delay distributions within a multi-level hierarchical network of agents governed by simple priority rules. Using railway systems as a case study, we model the dynamics of high-speed and local trains assigned distinct priority levels—operating within a simplified hierarchical network framework. By introducing Laplacian-distributed stochastic fluctuations into scheduled travel times, derived from empirical data, we observe that local trains exhibit a markedly higher incidence of higher delays than high-speed trains. To account for this phenomenon, we propose a queue-based dynamical model, calibrated using Italian railway data, and validate our findings through comparative analysis with both Italian and German datasets. The model accurately reproduces the empirically observed power-law exponent associated with the Italian local train delays. Furthermore, we analyze the influence of operational policies, such as priority assignment and delay compensation thresholds, revealing distinct cut-offs in delay distributions at 30 and 60 minutes

for high-speed and local trains, respectively—corresponding to refund eligibility criteria in Italy. Such cut-offs are absent in the German case, where no comparable priority-change policies are in effect. These results underscore the capacity of simple hierarchical structures and rule-based dynamics to generate complex statistical behaviors without necessitating intricate interaction networks.

Keywords: Network Dynamics, Hierarchical Systems, Complex Systems, Delay Dynamics

1 Introduction

The study of complex systems has revealed that macroscopic observables often display heavy-tailed statistics, notably power-law distributions, which signal the presence of long-range correlations and critical phenomena [1, 2]. In the context of interaction networks, such distributions have been linked to scale-free topologies and self-organized criticality, as exemplified by the Barabási–Albert model of network growth [3] and sandpile models on lattices [4]. New empirical analyses [5, 6] further suggest that understanding the structure and dynamics of real-world networks is indispensable for predicting extreme events, ranging from large-magnitude earthquakes to systemic failures in infrastructural systems.

Railway transportation networks constitute a paradigmatic example of engineered complex systems, where heterogeneous agents (trains) navigate a shared infrastructure under constraints imposed by scheduling, priority rules, and operational policies. Empirical investigations have documented non-Gaussian delay distributions across various national networks, with pronounced heavy tails for commuter and regional services [7]. Such findings have motivated the development of stochastic and queuing-based models aiming to reproduce observed delay statistics [8–10]. However, prevailing approaches often assume intricate interaction kernels or detailed timetable dependencies, leaving open the question of whether simple hierarchical rules suffice to generate power-law behavior.

In this work, we propose and analyze a minimal hierarchical framework in which agents are assigned to one of two priority classes, namely high-speed versus local services, and interact solely through queue-discipline rules at network bottlenecks. Travel time fluctuations are modeled via a Laplacian noise term whose parameters are inferred from empirical data. We show that such a simple scheme naturally yields power-law delay distributions for low-priority trains, without invoking complex network topologies or critical tuning of parameters.

To validate our theoretical predictions, we calibrate a queue-dynamics model using performance metrics from the Italian railway system and compare its output against delay data collected from both Italian and German networks. We further explore how operational policies, particularly delay-compensation thresholds linked to passenger-refund eligibility, imprint characteristic cut-offs in the tail distributions. Our results highlight that simple hierarchical interactions and policy-driven rules can play a central role in shaping complex statistical patterns in infrastructural networks.

ViaggiaTreno Category	Category	Priority Level
EC	<i>High Speed</i>	6
Frecciarossa	<i>High Speed</i>	6
Frecciaargento	<i>High Speed</i>	5
Frecciabianca	<i>High Speed</i>	5
IC	<i>Local</i>	3
ICN	<i>Local</i>	3
RV	<i>Local</i>	2
Regionale	<i>Local</i>	1
SFM	<i>Local</i>	0

Table 1 ViaggiaTreno train categories mapped into *high speed* and *regular* with an integer priority level.

By elucidating the mechanisms through which minimal priority-based hierarchies engender emergent heavy tails in delay statistics, this study contributes to a deeper understanding of complexity in engineered systems and offers practical insights into the design of operational policies for resilience enhancement.

2 Methods

2.1 Data

To construct a realistic model capable of reproducing delay propagation within railway systems, we collected and analyzed empirical data from the Italian and German railway networks. The Italian dataset comprises records from multiple dates in 2023—specifically, September 19, 21, 26, 27, and 28, as well as October 6, 7, 9, 10, 11, and 13—and from a continuous period in early 2025, spanning January 31 to April 3. All Italian data were retrieved from the ViaggiaTreno portal [11]. Complementary data for the German railway system, covering the months of February, March, and April 2025, were obtained from a publicly accessible online archive [12].

For the Italian railway network, the ViaggiaTreno platform provides detailed information essential to our study, including train category (which determines priority levels), station-to-station delay times, and geolocation data for all train stations. We remapped the original train categories into two aggregated groups: *regular* and *high-speed*, as detailed in Table 1. The priority levels reported are those directly used in our model and are not subject to further aggregation. *EuroCity* and *Frecciarossa* trains are assigned the highest priority, while *InterCity* services receive intermediate priority, and *SFM* (metropolitan services) are designated the lowest priority. For the German data set, trains were classified as *high-speed* only if they belong to the *ICE*, *TGV*, or *RJX* classes. The impact of freight traffic was considered negligible for the purposes of this study.

The Italian railway network topology was reconstructed by analyzing train itineraries. In total, 2073 stations were identified as active, meaning that each was visited by at least one train per day during the observation periods.

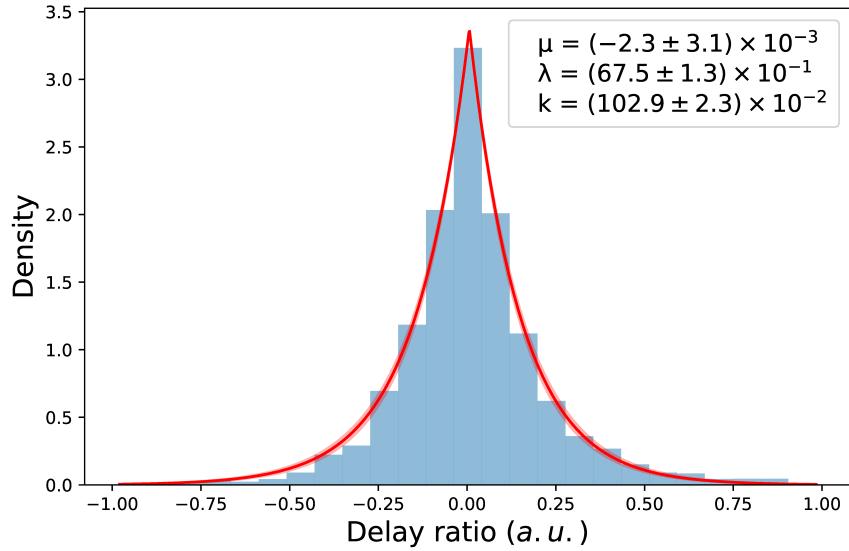


Fig. 1 Distribution of the station-to-station ratio between delay and scheduled travel time. The red line is the Asymmetric Laplace Distribution fit.

2.2 Delay distribution

Consider a train journey traversing n stations denoted by S_1, \dots, S_n , with scheduled travel times between successive stations given by T_1, \dots, T_{n-1} . In practice, scheduled timings are subject to stochastic variations, resulting in either delays or early arrivals. To account for variability in segment lengths, we define a normalized delay metric for each segment: $d_i := \frac{D_i}{T_i}$, where D_i represents the observed delay at segment i .

The Italian railway system exhibits a hierarchical operational structure, in which delays incurred by lower-priority trains are often influenced by higher-priority services. Based on empirical observations and theoretical considerations, we posit that normalized delays between stations follow an exponential distribution $\exp(\lambda_+)$. Similarly, early arrivals (negative delays) are assumed to follow another exponential distribution $\exp(\lambda_-)$ with a distinct rate parameter. Since trains are statistically more likely to experience delays than advances, we expect $\lambda_+ < \lambda_-$.

Accordingly, the overall distribution of normalized station-to-station delay is modeled as the difference between two exponentials, resulting in an *Asymmetric Laplace Distribution*, whose probability density function (PDF) is given by:

$$f(x; m, \lambda, k) = \frac{\lambda}{k + k^{-1}} e^{-(x-m)\lambda s k^s} \quad (1)$$

where $s = \text{sign}(x - m)$. A formal derivation of this result is provided in [Appendix A](#).

In [Figure 1](#) we reported the normalized station-to-station delay for *high-speed* trains. The asymmetric Laplace PDF well fits the histogram, with a mean close to

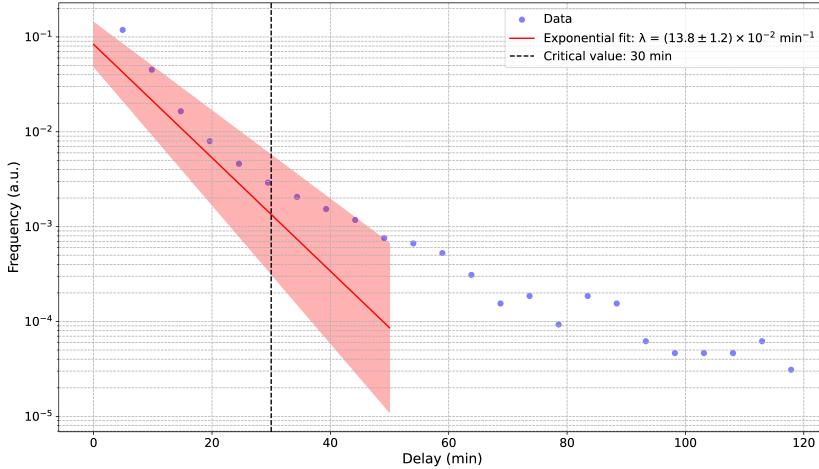


Fig. 2 Italian high-speed trains show an exponential decay until 30 minutes. The exponential fit (red) is calculated over the delay interval between 0 to 30 minutes and then it is plotted over a wider range as to show the not-exponential behavior

zero ($\mu = (10.7 \pm 8.8) \times 10^{-3}$ and an asymmetry parameter $k = (93.5 \pm 2.5) \times 10^{-2}$). Notice that $k < 1$ implies $\lambda_+ < \lambda_-$, as expected.

When analyzing the average delay over entire trips, a different behavior emerges. For *high-speed* trains, the mean delay distribution remains exponential, whereas for *regular-speed* trains, it follows a power-law trend. This distinction potentially reflects Trenitalia's operational policies. Notably, the exponential distribution for high-speed services holds only up to a threshold of approximately 30 minutes—a duration which coincides with the minimum delay required to qualify for a refund, according to the company's official guidelines. This suggests a deliberate reduction in priority for trains exceeding this threshold.

This interpretation is reinforced by comparing the delay distribution for Italian and German high-speed services, as shown in [Figure 2](#) and [Figure 3](#), respectively. In particular, one can notice how the German distribution is perfectly exponential, meaning that they do not have policies like Trenitalia's ones.

3 Model

The aim of this project is to show that in a hierarchical network, as the Italian railway is, extreme events are more frequent in the lower-priority class.

To simulate this, we create a dynamic-framework managed by a queue model. We have as infrastructure the nodes on the network—in this case, the stations—where the agents—i.e. the trains—move from one to the other. The hierarchy of the network is due to the rules for which an agent can leave a node or not. The departure of one

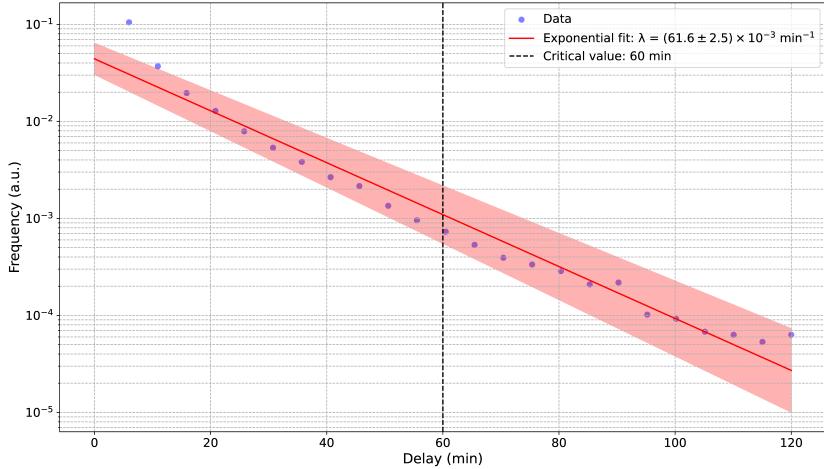


Fig. 3 German high-speed trains show an exponential decay over all the time interval. The exponential fit (red) is calculated and plotted over the same delay range.

agent must never prevent the one of another agent with higher priority. In order to simplify the dynamic, we make some assumptions:

1. Each node can always receive a new agent, i.e. a new train arrives at the station as soon as it can, without any consideration about the number of free platforms. This can be done because the agents surplus in a node is managed when agents leave the node;
2. Each node can manage the departures of a fixed number of agents at the same time (we can see it as the number of possible directions of the railway) and it is equivalent to the *degree* of the node;
3. The departure of an agent from a node must wait a well-defined time after previous agent's departure on the same direction. This time will be called *management time* (τ) of the node and it is the time between two train in the same track;
4. Each agent must not depart before its theoretical departure time or if its departure impedes to an agent with higher priority to leave the node. It is related to points (2) and (3) of this list and it is the hierarchical condition;
5. The transit of a train in a station where it does not stop does not require τ : it is immediate and it does not reserve a track. This assumption is strong but necessary for our data since we have not the train's path but just the list of station in which it stops. If there is a station between two stops, we have no information on it.

We create the network starting from scheduled train routes. In this way, we can connect the stations between them if they are consecutive stops for at least one train. Thus, we assign the number of possible directions as the degree of the station, and hence the number of trains that may depart simultaneously. This produces the bias that stations have a degree higher than the real number of possible directions. The error occurs when

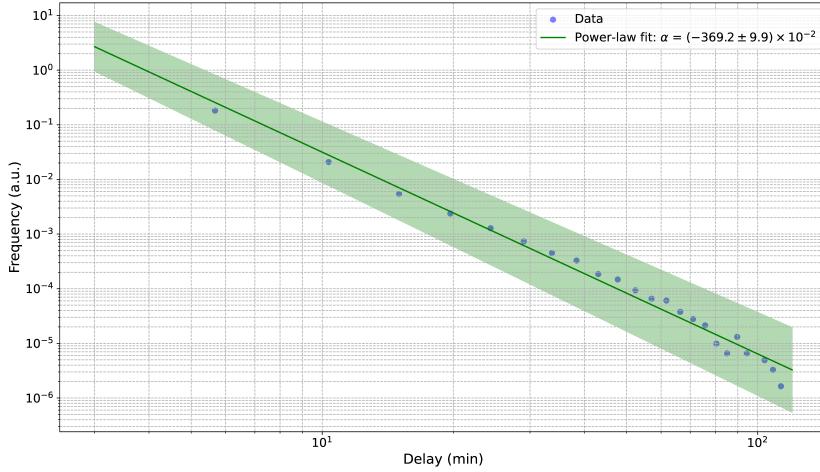


Fig. 4 Italian local trains show a nearly perfect power-law decay. The power-law fit (green) is calculated and plotted over the same delay range.

trains skip small stations between two bigger ones: the latter two result connected in our network, meanwhile there is not any direct track between them. To evaluate management time we assume it would be enough short to allow trains to respect scheduled timetable if there are not delays. We assume it is smaller for big stations, where track control shall be faster. Performing simulations, we find management times that satisfy these conditions.

3.1 Station Dynamics

When train should be depart from station, two conditions must be satisfied: there is at least one free track and its departure must not prevent the departure of higher priority trains at their scheduled time. This has to be checked only for trains that suppose to leave within the management time, because if they depart later, they are not affected by the current station condition. The former condition is a base check of any dynamic model with flow constraint. The latter introduces the hierarchy inside the model as lower priority trains should wait until departure of higher priority trains if they will overlap.

Since there are inaccuracies about degree and real directions, we do not assign a specific track to trains moving towards specific direction. This reduces information about dependencies between two trains delay and it shall be improved in future models.

3.2 Train Dynamics

We do not simulate what happens between two stations, but we just modify the scheduled travel time with fluctuations. Trains may take longer or shorter time for the route but they can not leave before scheduled time. If it is in late, its next departure

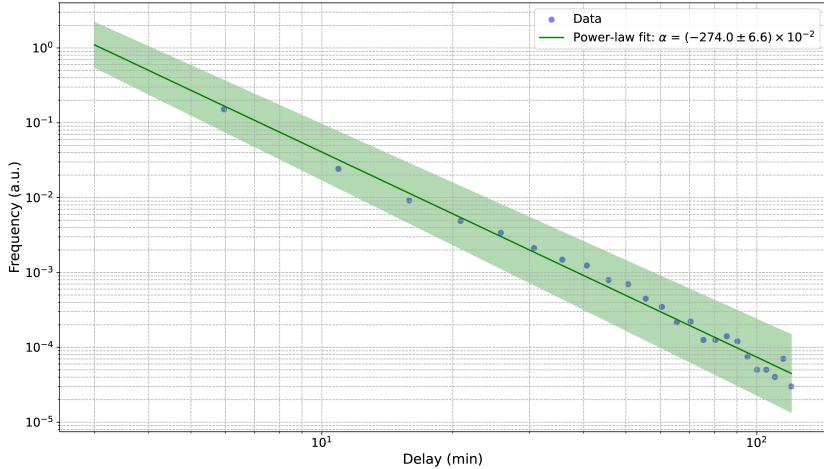


Fig. 5 German local trains show a nearly perfect power-law decay. The power-law fit (green) is calculated and plotted over the same delay range.

time is postponed; in the next route it may catch up or increase the delay. If at its departure time it can not depart (no free track or there are trains with higher priority) its departure time is postponed.

If the delay is greater than a threshold value, the train is downgraded in priority. This rule is taken from Italian railway network and it is related to refund policy and to avoid avalanche delays. Usually these trains have lower priority than train's ones with same priority but it is still higher than train's ones with lower priority (e.g. all high-speed trains will have priority equal to 4, meanwhile all the other trains will have priority equal to 0).

4 Discussion

We performed simulations of the trains dynamic to check if the delay distributions of the *High speed* and *Regular* trains follow the real ones. The stations' *degree* was evaluated from the scheduled trips or regular trains. If two stations are directly connected (i.e. without intermediate stop) their degrees increase by one. It is greater than the number of possible directions since some little stations are often skipped to low-speed trains too. The *management time* assumes the same value for all the stations. We set it at 5 minutes empirically.

We add fluctuations to the travel time of the trains in order to reproduce the effective travel time. The added noise follows the distributions of delay and early of real trains (see subsection 2.2 and Figure 1). We performed 75 independent simulations, equivalent to 75 different days as the number of days for which we have real data. The high speed trains distribution has an exponential behavior at little delay, then it has a power law trend after the loss of priority (see Figure 6). The regular trains, instead,

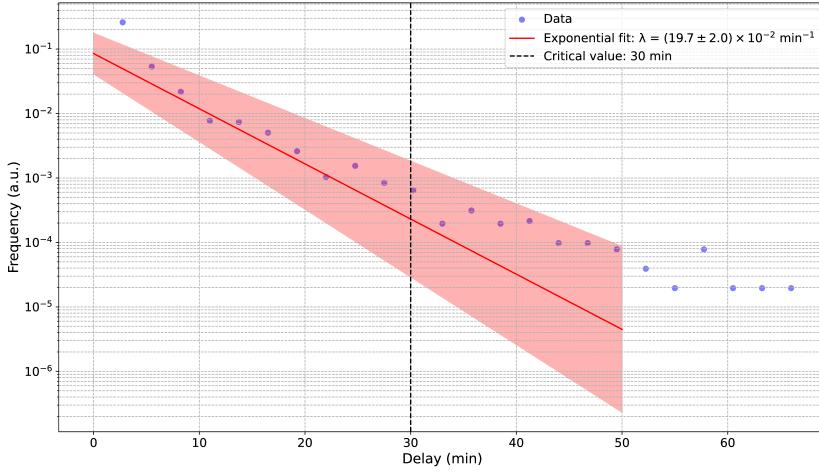


Fig. 6 Simulated high-speed trains show an exponential delay until 30 minutes delay and then a power-law decay. The exponential fit (red) is calculated until the critical value but it is plotted over a wider delay range.

follow always a power law distribution as showed in [Figure 7](#). All the coefficients are coherent with the real data distributions.

5 Conclusions

In this work, we have shown that simple hierarchical rules, implemented via priority-based queue dynamics on a multi-level network, are sufficient to reproduce the heavy-tailed delay distributions observed in real-world railway systems. By modeling high-speed and local trains as agents with distinct priority levels and introducing empirically calibrated Laplacian-distributed fluctuations in inter-station travel times, our simulations naturally yield exponential delay statistics for high-priority services and power-law tails for lower-priority services. These findings confirm that minimal rule sets, rather than elaborate interaction kernels or finely tuned network topologies, can generate complex macroscopic phenomena such as power-law delays.

A direct comparison between the Italian and German datasets further highlights how operational policies imprint themselves on delay statistics. The Italian network exhibits sharp cut-offs at 30 minutes for high-speed and 60 minutes for local trains: this thresholds are directly tied to passenger refund policies. On the other hand, the German system, which lacks analogous priority-change provisions, displays uninterrupted exponential or power-law behavior across the entire delay spectrums. This divergence is reflected in a systematically lower power-law exponent for German local trains, indicating that aggressive prioritization of high-speed services can significantly exacerbate delays in local mobility.

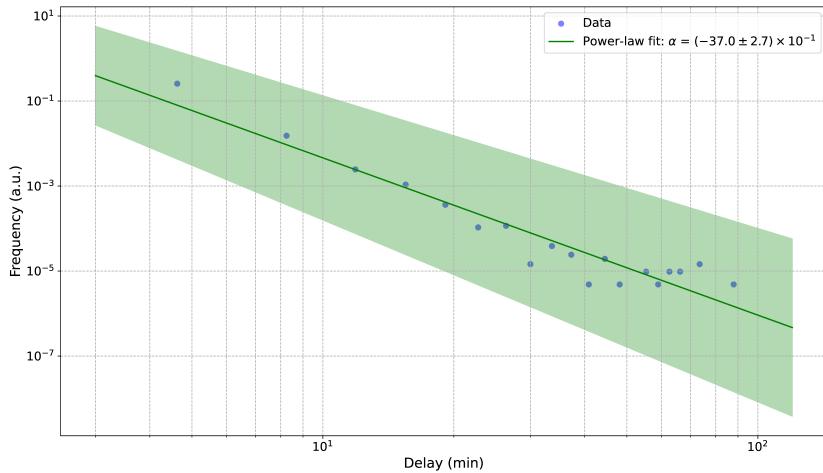


Fig. 7 Simulated local trains show a power-law decay. The power-law fit (green) is calculated and plotted over the same delay range.

Despite these successes, our model carries several simplifying assumptions that merit future investigation. First, our station-degree approximation abstracts away real track layouts and platform constraints; incorporating detailed infrastructure maps and direction-specific capacities could refine delay propagation patterns. Second, the instantaneous transit assumption at non-stop stations omits potential interactions at intermediate nodes; richer path information, including all intermediate halts, would enable more accurate modeling of congestion cascades. Finally, freight traffic, which we treated as negligible, may play a significant role in mixed-use corridors and should be explicitly accounted for.

In summary, our study advances both the theoretical understanding of how simple hierarchical dynamics produce emergent power laws and offers actionable guidance for transport operators seeking to balance service quality with operational resilience.

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Appendix A Asymmetric Laplace Distribution as difference of two exponentials

Starting from (1) we can compute the characteristic function of the Asymmetric Laplace distribution using the definition [13]. For simplicity we put $m = 0$, but the

proof is indeed general.

$$\begin{aligned}
\phi^{AL}(t; \lambda, k) &= \int_{-\infty}^{\infty} f(x; \lambda, k) e^{itx} dx = \\
&= \frac{\lambda}{k + k^{-1}} \left(\int_{-\infty}^0 e^{(it + \frac{\lambda}{k})x} dx + \int_0^{\infty} e^{-(it + \lambda k)x} dx \right) = \quad (\text{A1}) \\
&= \frac{\lambda}{k + k^{-1}} \left(\frac{1}{it + \frac{\lambda}{k}} + \frac{1}{-it + \lambda k} \right) = \frac{1}{1 + \frac{itk}{\lambda}} \frac{1}{1 - \frac{it}{\lambda k}}
\end{aligned}$$

The exponential chf is indeed

$$\phi^{EXP}(t; \lambda) = \frac{1}{1 - \frac{it}{\lambda}}$$

Given $X \sim \exp(\lambda_+)$ and $Y \sim \exp(\lambda_-)$, the chf of $X - Y$ is the product of the two chf, namely

$$\phi(t; \lambda_+, \lambda_-) = \frac{1}{1 - \frac{it}{\lambda_+}} \frac{1}{1 + \frac{it}{\lambda_-}} \quad (\text{A2})$$

By comparing (A1) and (A2) one can find that $\lambda = \sqrt{\lambda_+ \lambda_-}$ and $k = \sqrt{\frac{\lambda_+}{\lambda_-}}$.