

Conceptual design of a Doppler Backscattering diagnostic for the EXL-50U spherical tokamak

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Abstract

The EXL-50U spherical tokamak was built by Energy iNNovation to develop technologies for proton-boron fusion in spherical tokamaks (Liu et al., Phys. Plasmas 2024). We present a conceptual design of the Doppler backscattering (DBS) diagnostic for the EXL-50U spherical tokamak. DBS is a diagnostic capable of measuring plasma turbulence, which is especially important for transport in tokamaks. Starting from a set of physical design constraints, such as port window availability and in-vessel space, we used SCOTTY (Hall-Chen et al., PPCF 2022), an in-house beam tracing code, to predict the location of the cutoffs and the corresponding scattering wavenumbers for several EXL-50U plasma scenarios. We find that we are able to measure scattering locations of $0.15 < \rho < 1$, with corresponding turbulent wavenumbers of $2.47 \text{ cm}^{-1} < k_{\perp} < 9.49 \text{ cm}^{-1}$. Here, ρ is the normalised radial coordinate of the scattering location, and k_{\perp} is the corresponding turbulent wavenumber.

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ber. We then determine the optimal toroidal launch angles to ensure that the probe beam's wavevector is perpendicular to the magnetic field at the cutoff location, thereby maximising the backscattered signal. This matching is crucial due to the EXL-50U's high magnetic pitch angle, $\sim 35^\circ$ at the outboard midplane. Given our results, we propose the use of toroidal steering and tunable frequency channels to ensure beams are well-matched with the magnetic pitch angle. We propose a quasioptical system that covers the U-band range (40–60 GHz).

Keywords: Doppler backscattering, EXL-50U, Beam tracing, Diagnostics

1. Introduction

When a plasma confined in a tokamak is heated to high temperatures, it is susceptible to anomalous transport of heat, particles, and momentum [1–4], which are caused by various small-scale instabilities driven by temperature and density gradients [2–5]. This is responsible for degrading the confinement of plasmas, which increases the capital costs of the fusion device [6]. Hence, it is important to understand turbulent transport in any fusion device.

The Energy iNNovation (ENN) XuanLong-50U (EXL-50U) spherical tokamak, an upgraded version of the ENN XuanLong-50 (EXL-50), was built by the ENN Energy Research Institute in China to advance vital technologies for proton-boron fusion in spherical tokamaks [7], with its main parameters shown in Table 1 [8]. The EXL-50U tokamak was designed with a small aspect ratio to improve plasma confinement [7]. Two reasons explain this improvement. First, a smaller aspect ratio results in greater toroidicity [9, 10] — defined by the inverse aspect ratio [10] — which reduces areas with bad

curvature in the tokamak [9]. Secondly, the smaller aspect ratio in spherical tokamaks results in larger $E \times B$ shearing compared to that present in conventional tokamaks [9]. These two effects suppress electrostatic drift wave instabilities at the ion and electron scales [9], improving confinement. However, simulation results predicting ion and electron-scale transport do not always correspond to experimental observations [11]. Cross-scale simulations that resolve both ion and electron contributions have shown that ion-scale turbulence can be indirectly amplified by electron-scale turbulence through sub-ion-scale structures [12, 13], significantly reducing confinement. Hence, it is crucial to understand cross-scale turbulent interactions in spherical tokamaks like the EXL-50U via well-resolved experimental measurements of electron and ion-scale turbulence. Furthermore, EXL-50U uses proton-boron fusion, which requires ion temperatures to be much higher than electron temperatures [7]. This regime is expected to yield novel and potentially rich turbulence physics that warrants careful experimental investigation. One such technique is Doppler backscattering (DBS), a diagnostic used to measure turbulence in many tokamaks and stellarators worldwide [14–25]. DBS is able to measure flows [26] and density fluctuations of intermediate length scales, usually $1 \lesssim k_{\perp} \rho_s \lesssim 10$, where ρ_s is the deuterium ion sound gyroradius. Furthermore, it can measure turbulent fluctuations in the core of the plasma, which is challenging.

DBS involves launching a microwave probe beam into the plasma, (Fig. 1). The beam is then scattered by electron density fluctuations, and the backscattered electric field is picked up by effectively the same antenna that was used to transmit the microwave beam. This backscattered signal is then used to

Parameters	Values
Plasma current	0.5–1 MA
Major radius	0.6–0.8 m
Toroidal magnetic field ($R = 0.6$ m)	1.0–1.2 T
Aspect ratio	1.4–1.85
Elongation	1.4–2

Table 1: Main parameters of the EXL-50U [8]

determine the locations of the turbulent fluctuations and the corresponding turbulence wavenumbers k_{\perp} associated with these turbulent fluctuations. We will assume that most of the backscattered signal comes from the nominal cutoff location [27], which we define as the point where the probe beam wavenumber K is minimised. We assume that the measured turbulent fluctuations are located at this point. The exact mechanism of spatial localisation is complicated and beyond the scope of the paper. The measured turbulence wavenumber is determined by the Bragg condition, given by

$$k_{\perp} = -2K. \quad (1)$$

Here, k_{\perp} is the turbulence wavenumber and K is the wavenumber of the probe beam. Hence, to measure a particular turbulence wavenumber k_{\perp} , a beam with a corresponding wavenumber K can be launched, allowing us to receive a measurable backscattered signal from the turbulent fluctuation with that associated wavenumber, k_{\perp} . Furthermore, varying the poloidal launch angles and frequencies of the beam varies the probe beam's wavenumber at cutoff

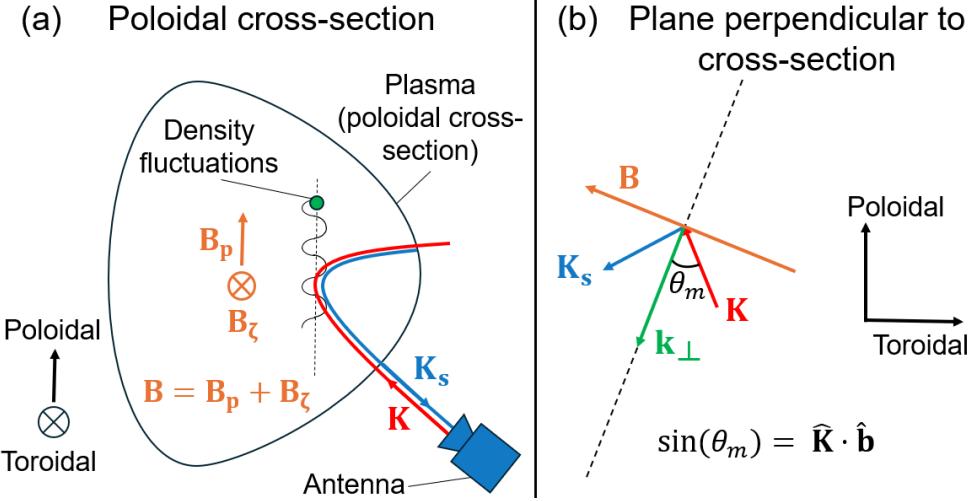


Fig. 1. An antenna emits a microwave probe beam into the plasma and receives the backscattered signal from density fluctuations. (a) Poloidal cross-section. (b) Plane tangent to the cut-off flux surface. We define the mismatch angle, θ_m , between the normal of the magnetic field and the probe beam's wavevector such that $\sin(\theta_m) = \hat{\mathbf{K}} \cdot \hat{\mathbf{b}}$, where $\hat{\mathbf{K}}$ and $\hat{\mathbf{b}}$ are unit vectors of the probe beam's wavevector and magnetic field \mathbf{B} , respectively. According to the Bragg condition, the turbulence wavenumber is double that of the probe beam's wavenumber, $k_{\perp} = -2K$. k_{\perp} , K and K_s are the wavenumbers of the turbulence, probe beam and scattered beam, respectively. \mathbf{B}_p and \mathbf{B}_{ζ} are poloidal and toroidal components of the magnetic field, respectively.

K_c , allowing us to probe a range of cutoff locations ρ_c , where ρ_c is the normalised radial coordinate of the cutoff location, and turbulent wavenumbers k_{\perp} .

In addition to the turbulent density fluctuations, the amplitude of the backscattered signal also depends strongly on the angle between the probe beam's wavevector \mathbf{K} and the plane perpendicular to the magnetic field, \mathbf{B} .

This is quantified by the mismatch angle θ_m , given by

$$\sin(\theta_m) = \hat{\mathbf{K}} \cdot \hat{\mathbf{b}}, \quad (2)$$

where $\hat{\mathbf{K}}$ is the unit vector of the wavevector of the probe beam \mathbf{K} , and $\hat{\mathbf{b}}$ is the unit vector of the magnetic field \mathbf{B} .

The backscattered power, P , is reduced when θ_m is non-zero [28],

$$P \propto \exp\left(-\frac{2\theta_m^2}{\Delta\theta_m^2}\right). \quad (3)$$

Here, $\Delta\theta_m$ is the width of the mismatch attenuation. It depends on the beam width, beam curvature and the beam's wavenumber. In this paper, the beam width is defined as the radius measured from the beam axis such that the electric field drops to $1/e$ of its on-axis value. The beam curvature is defined as the inverse of the radius of curvature of the wavefront of the beam. The explicit formula for $\Delta\theta_m$ is given in [28] and experimental validation in [29].

This results in mismatch attenuation, which is a decrease in the backscattered power because the turbulence wavevector, which is perpendicular to the magnetic field vector, is not aligned with the probe beam's wavevector. A larger mismatch attenuation means there will be no signal, which is a bigger problem at larger K values [28]. This is important to note in this work because we want to measure turbulent wavenumbers k_\perp that are high enough to be considered electron scale. Furthermore, a decrease in backscattered power could either mean a decrease in turbulent fluctuations or an increase in the mismatch angle between the normal to the magnetic field and the wavevector of the probe beam, and these causes are experimentally indistinguishable just from looking at the signal alone.

In cases where the poloidal magnetic field is significant compared to the toroidal magnetic field, such as in tokamaks with high magnetic pitch angles, the incident beam's wavevector may not be perpendicular to the magnetic field. Hence, understanding mismatch attenuation is important in this work. The EXL-50U tokamak has a high magnetic pitch angle (Fig. 3), meaning that the mismatch can be large if the toroidal launch angles are not carefully chosen. However, it is possible to minimise the mismatch by using toroidal steering to vary the toroidal launch angle [30, 31]. Hence, we will need to determine the optimal toroidal launch angle that yields zero mismatch for a given beam frequency. For $\Delta\theta_m$, the beam width and curvature values are needed to calculate it, and these depend on the design of the DBS system used to propagate a beam of a particular beam width and curvature.

Before designing and installing a DBS system to make these measurements, it is useful to use a synthetic DBS diagnostic to understand how a DBS system would measure turbulent fluctuations in a particular fusion device and to determine the optimal DBS configuration for such measurements. Research has been done using ray tracing codes to perform synthetic DBS diagnostics and to design DBS systems [24]. In ray tracing, the electric field is calculated by propagating a bundle of rays. However, the rays intersect one another near the cutoff point [32]. The points where the rays meet one another are known as caustics [33, 34], and they are problematic because the amplitude of the electric field becomes infinitely large [32] at these points. Consequently, analysis using ray tracing near the cutoff becomes impractical [35] as DBS requires one to analyse the properties of the probe beam near the cutoff [36, 37]. An alternative is beam tracing, where a single ray

is traced first, before performing an expansion around that ray [28]. This method evolves the trajectory of a Gaussian beam, with the position of the Gaussian envelope’s amplitude traced out by the central ray [28]. The theory of beam tracing has been extensively researched [38–44], and various phenomena, such as electron cyclotron resonance heating (ECRH) [45], have been simulated using beam tracing. Furthermore, this method can be used near the cutoff [46, 47]. This paper aims to use SCOTTY [48] to conduct synthetic DBS diagnostics on the EXL-50U tokamak and to measure the locations of the turbulent fluctuations in the plasma in the EXL-50U and the corresponding wavevectors of these turbulent fluctuations, while keeping in mind physical design constraints like the space available to propagate the microwave probe beam. Then, we design a conceptual design for a DBS, which includes the configuration of the lens that can conduct the simulated observations generated from SCOTTY for the EXL-50U tokamak.

The rest of the paper is structured as follows: we present the plasma scenarios used in this study in Section 2. In Section 3, we determine the launch angles and frequencies to use to probe density fluctuations, while meeting the various physical constraints. We also present the results of the scattering locations simulated by SCOTTY, and show the dependence of the mismatch attenuation on the toroidal launch angle. Section 4 discusses the quasioptical design of a DBS system for making the desired measurements put forward in the previous section. We finally conclude our paper in Section 5.

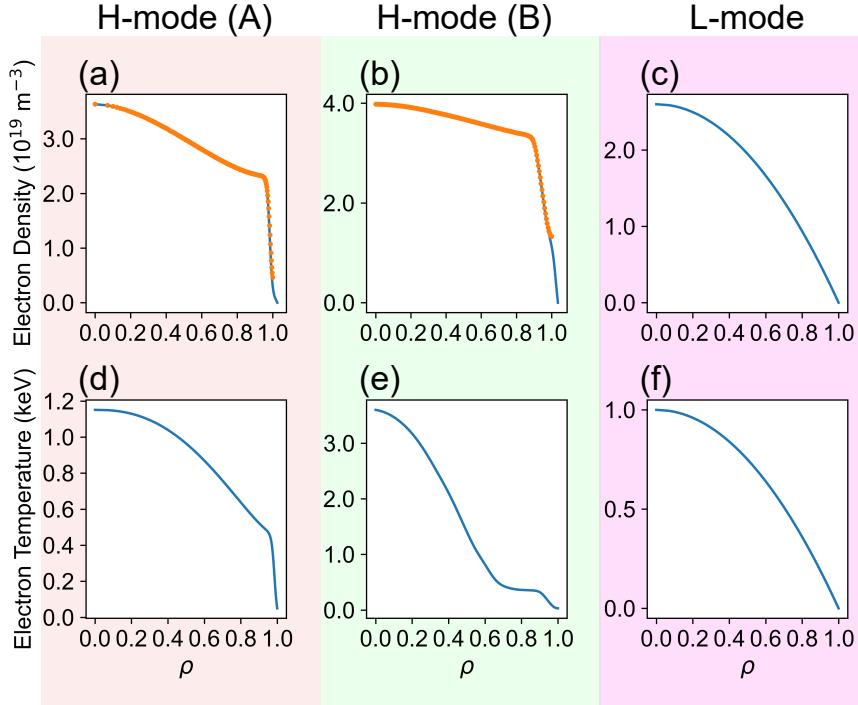


Fig. 2. (a)–(c): Electron density as a function of the normalised radial coordinate ρ for (a): H-mode (A), (b): H-mode (B), and (c): L-mode plasma scenarios. The orange dots represent the simulated data, and the blue curve is a curve fit of the data. (d)–(f): Electron temperature as a function of the normalised radial coordinate ρ for (d): H-mode (A), (e): H-mode (B), and (f): L-mode plasma scenarios. For (e), NBIs were simulated, resulting in the presence of an ITB in its temperature profile.

2. Plasma scenarios

We consider three plasma scenarios for our DBS design, covering a range of EXL-50U plasmas: two high-confinement mode (H-mode) scenarios and one low-confinement mode (L-mode) scenario were used. The electron density and temperature profiles of these scenarios are shown in Fig. 2. Profiles of the two H-mode scenarios, H-mode (A) and H-mode (B), were both simulated

using the workflow shown in Figure 1 of previous work [49]. This workflow uses codes such as EFIT [50] for the equilibrium, TGYRO [51] for the core profile, and NUBEAM [52], TORAY [53], and GENRAY [54] for heating and current drive [49]. Neutral Beam Injections (NBIs) were used in the H-mode (B) scenario, resulting in an internal transport barrier (ITB) in its temperature profile, see Fig. 2(e). The electron density profiles for both H-mode scenarios are further extrapolated at the edge to the point where the electron density reaches zero. The L-mode scenario was described with a quadratic function of the electron density against the normalised radial coordinate ρ , with the electron density at $\rho = 0$ set at $2.6 \times 10^{19} \text{ m}^{-3}$. Its temperature profile was also described with a quadratic function of the temperature with respect to ρ , with the temperature at $\rho = 0$ set at 1 keV. The electron temperature and density of the magnetic axis were chosen to be similar to MAST [55], a comparable spherical tokamak.

Temperature profiles are used to calculate $k_{\perp}\rho_s$ in the plot used to determine the spatial and poloidal flux range in Section 3, where ρ_s is the deuterium ion sound gyroradius. Relativistic corrections to the electron mass were neglected as T_e is low, lower than 5 keV [56, 57]. The magnetic field profile was also obtained from the same integrated modelling in [49]. See Fig. 3 for the components of the magnetic field. We used the same magnetic field profile for all three scenarios as an estimate. The magnetic pitch angle is also shown in Fig. 3(d); we note that the EXL-50U has a large magnetic pitch angle, $\sim 35^\circ$ at the LCFS. This is expected considering that the EXL-50U is a spherical tokamak.

We seek to determine the ranges of X and O-mode frequencies suitable for

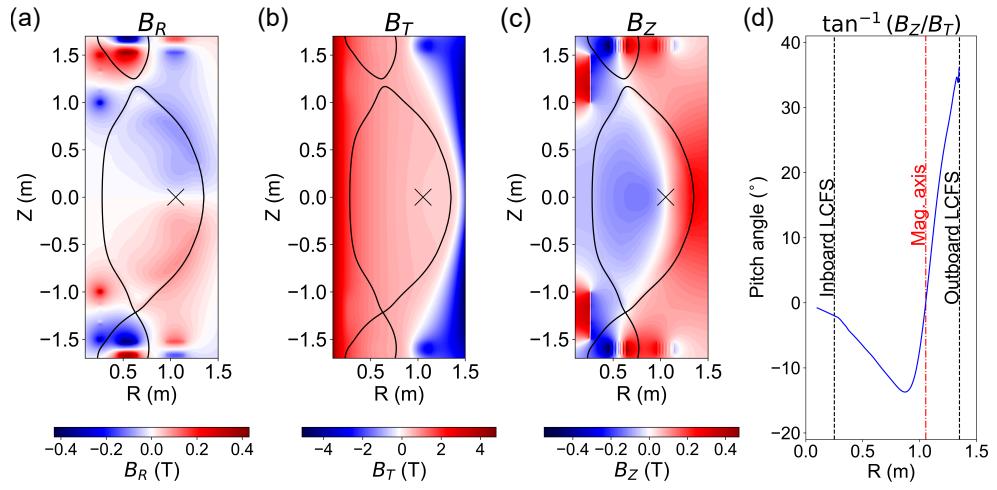


Fig. 3. Magnetic field profile in the EXL-50U. (a) B_R : Radial component of magnetic field. (b) B_T : Toroidal component of magnetic field. (c) B_Z : Z -component of magnetic field. The black contour lines and the black cross in (a)–(c) represent the last closed flux surface (LCFS) and the magnetic axis, respectively. (d) Magnetic pitch angle plot on the midplane of the EXL-50U.

probing density fluctuations, so the cutoffs along the midplane (Fig. 4) are plotted using the magnetic field and electron density profiles, giving us an estimate of the cutoff locations. From these plots, we note that frequencies between 40 GHz and 60 GHz, which is the U-band range, can probe turbulent fluctuations located at both the pedestal and the core of the plasma for the H-mode profiles, so we will use the U-band for these profiles at 2 GHz intervals. Our simulations will thus span these frequencies. For the L-mode profile, frequencies between 30 GHz and 50 GHz can probe turbulent fluctuations located at both the edge and the core, so these frequencies will be used for the L-mode profile at 2 GHz intervals. The exact range of frequencies for an actual DBS would depend on the experimental profiles achieved in EXL-50U scenario development, which has not begun at the point of writing.

3. Beam tracing results

3.1. Initial run of SCOTTY

An initial run of beam tracing is done to determine an appropriate range of launch angles. The initial launch position corresponds to an available below-midplane port at the time of this work, and is fixed at $R = 1.895$ m, $Z = -1.0$ m, as shown in Fig. 5. While the exact location may change, we will use this launch position for this study. Due to the off-normal incidence of the beams and the fact that the launch position is below the midplane, the cutoffs will not be the same as those shown in Fig. 4. In addition, enough clearance should be given to ensure no part of the beam is incident on poloidal field coil PF14, see Fig. 5. Based on an empirical guideline used in previous work [58], the beams must be launched such that the minimum

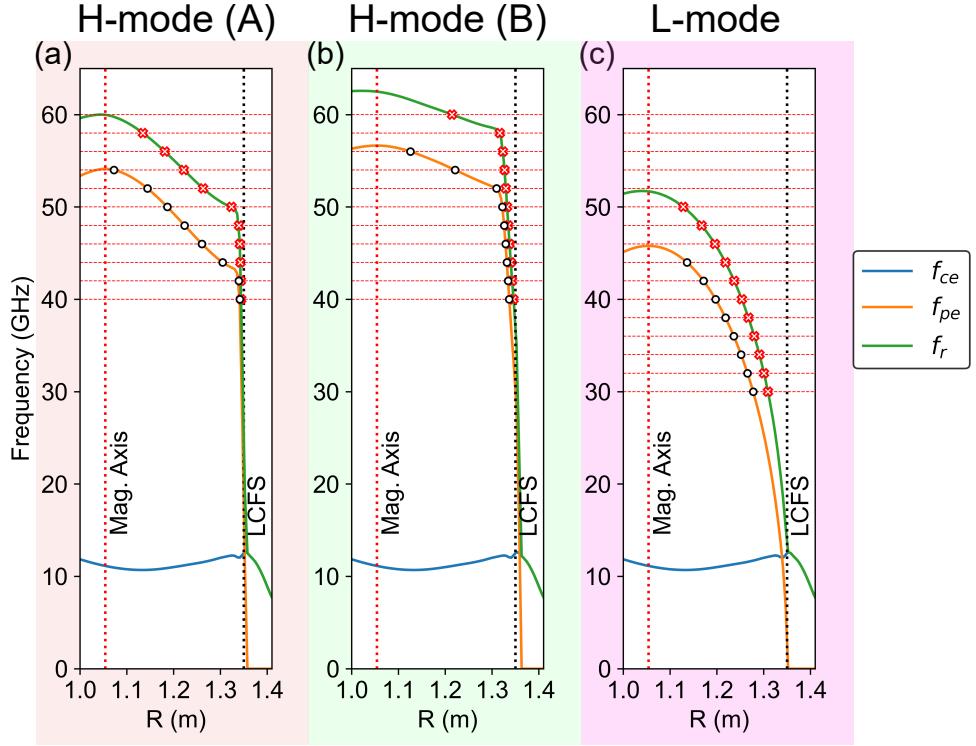


Fig. 4. Cutoff frequencies along the midplane in the EXL-50U for (a): H-mode (A), (b): H-mode (B), and (c): L-mode. f_{ce} is the fundamental harmonic electron cyclotron frequency, f_{pe} is the plasma frequency, and f_r is the X-mode cutoff frequency. The red dashed lines are the frequencies used, with the black circles and red crosses representing the O-mode cutoffs and the X-mode cutoffs, respectively. For (a), f_r is slightly lower than 60 GHz, so a point is not plotted for 60 GHz. A 20 GHz frequency range allows for core to edge coverage.

distance between PF14 and the beam is at least three times the beam width at the beam trajectory's closest point. This imposes a range of initial poloidal launch angles, which will be discussed later. There will be a lower bound on the range of poloidal launch angles because any beam launched at a poloidal angle lower than this lower bound would be too close to PF14. There will

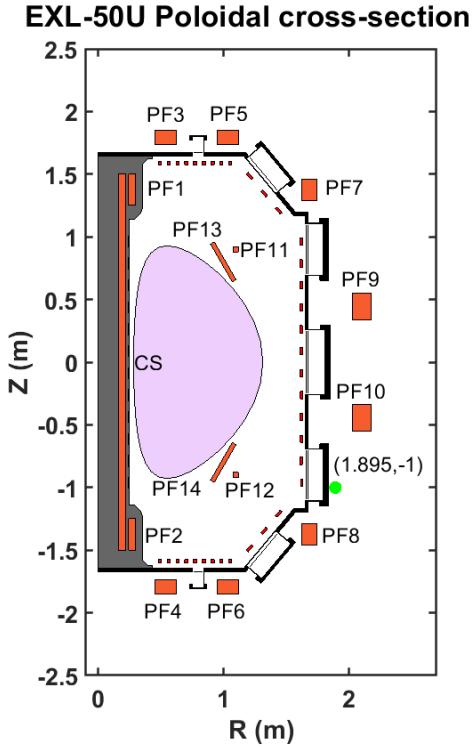


Fig. 5. Poloidal section of the EXL-50U containing the plasma (depicted in pink). The launch position is set at $R = 1.895$ m, $Z = -1.0$ m. Microwave probe beam must be launched at appropriate angles to avoid it from being incident on PF14, a coil that goes toroidally around EXL-50U. Note that the plasma shape here differs from that shown in Fig. 3 as scenario development has yet to be completed. Moreover, the vessel wall is not toroidally symmetric, so we will not consider the vessel wall in our analysis.

also be an upper bound because the beam will miss the plasma if the poloidal launch angle is sufficiently large.

Before finding the range of poloidal launch angles, we first define the plasma frame of reference: the poloidal launch angle corresponding to the normal incidence of the beam on the LCFS. The poloidal launch angle that results in normal incidence on the LCFS is -28.6° , which is indicated by the black dotted lines on the subplots of Fig. 7. We now determine the range of poloidal launch angles. The toroidal launch angle is set to 0° . Here, the sign convention for the poloidal angle is defined such that a 0° poloidal launch angle corresponds to the case where the beam is launched parallel to the midplane, and a positive angle corresponds to pointing downwards. Then, the trajectories are inspected to ensure that the minimum distance between the beam and PF14 is not too small and that the beam enters the plasma. Given these considerations, we find that the poloidal launch angle range is narrowed to $-65^\circ \leq \varphi_p \leq -36^\circ$. This range is narrowed down further by considering the ratio of the wavenumber at cutoff, K_c , to the vacuum wavenumber, K_0 , shown in Fig. 6. When K_c/K_0 is very small, which we take to be approximately less than 0.1, DBS approaches the conventional reflectometry regime, which we will not consider in this paper. When the beam is weakly refracted, the DBS filter function, which specifies the scattering intensity due to every point along the beam path, becomes broader [47]. Consequently, we expect to receive significant backscattered signals from scattering away from the cutoff [47], which is complicated to analyse. As a guideline, we take that case to be $K_c/K_0 \gtrsim 0.4$, so we will only consider cases less than 0.4. We will be propagating at the following poloidal launch angles for the following density

profiles, at 0.5° intervals in the ranges shown in Table 2.

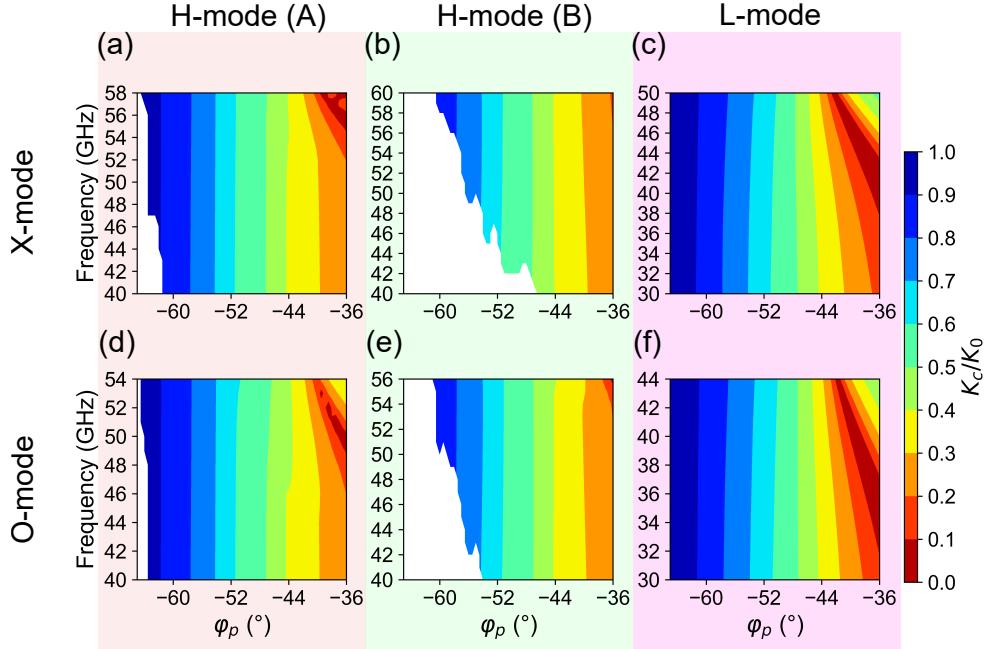


Fig. 6. (a)–(c): Ratio of the wavenumber at cutoff to the vacuum wavenumber K_c/K_0 heatmaps for X-mode beams with varying frequencies and poloidal launch angles φ_p : (a): H-mode (A), (b): H-mode (B), and (c): L-mode. (d)–(f): Same for O-mode: (d): H-mode (A), (e): H-mode (B), and (f): L-mode. The white regions indicate where the cutoffs are located outside the LCFS. Poloidal launch angles that correspond to $K_c/K_0 \lesssim 0.1$ and $K_c/K_0 \gtrsim 0.4$ and cases where the cutoffs are located outside the LCFS are omitted.

We find the scattering locations of the turbulent fluctuations and their associated wavevectors \mathbf{k}_\perp for these poloidal launch angles. These will be discussed in the next few subsections.

3.2. Cutoff locations

From Fig. 7, there are different spatial distributions of cut-off locations over the poloidal cross-section of the tokamak for every possible frequency,

Profiles	φ_p range
H-mode (A)	$-43.0^\circ \leq \varphi_p \leq -40.0^\circ$
H-mode (B)	$-43.5^\circ \leq \varphi_p \leq -36.5^\circ$
L-mode	$-44.0^\circ \leq \varphi_p \leq -40.0^\circ$

Table 2: Poloidal launch angle ranges for different profiles. Note that $\varphi_p = -28.6^\circ$ corresponds to normal incidence on the LCFS.

polarization and initial poloidal launch angle, with the toroidal launch angle set to 0° . Fig. 8 and Fig. 9 depict the poloidal range of the cut-off locations for every possible frequency, polarization and poloidal launch angle, with the toroidal launch angle set to 0° . We will be using these three figures for our analysis of each scenario.

3.2.1. H-mode (A) cutoff locations

Both the X and O mode beams can provide sufficient coverage of both the edge and the core of the plasma, covering a poloidal flux range between $\rho = 0.15$ and $\rho = 1$. We notice that at lower frequencies between 40 GHz and around 50 GHz, the cutoff locations cluster between $\rho = 0.95$ and $\rho = 1$, see Fig. 8(a). From 52 GHz onwards, the beam's penetration into the core increases significantly with increasing frequency. This is similar to the case for O-mode in Fig. 9(a), except that the cutoffs tend to occur between $\rho = 0.95$ and $\rho = 1$ when beams are launched with frequencies between 40 GHz and 44 GHz. Above 44 GHz, the radial coordinates of the cutoffs decrease drastically with increasing frequency. These two observations can be explained using Fig. 4. We notice that 40–50 GHz X-mode beams and

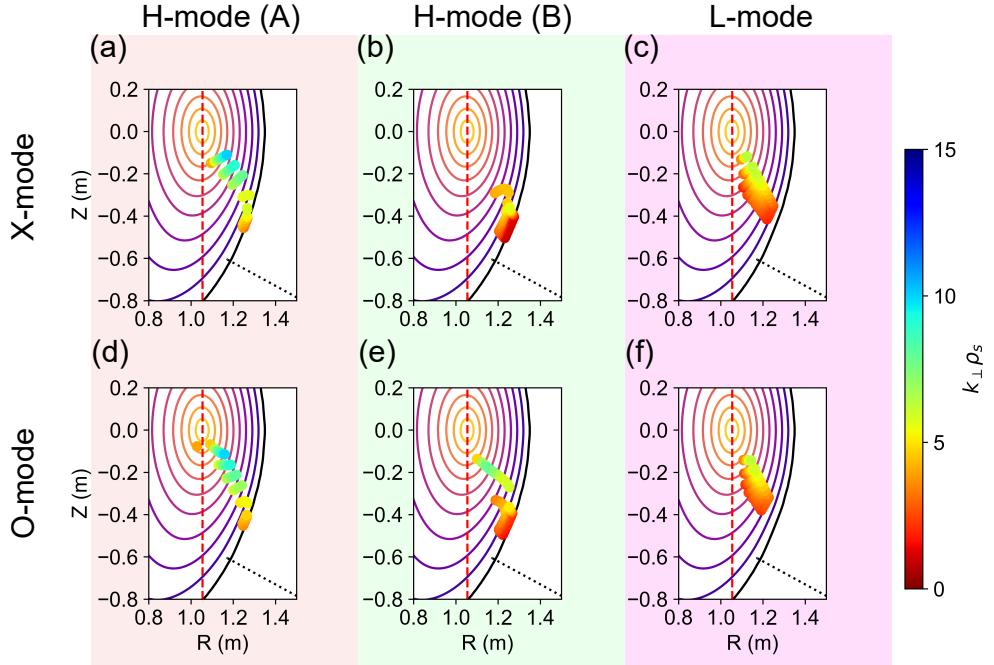


Fig. 7. (a)–(c): Cut-off positions and their corresponding normalised turbulence wavenumbers $k_{\perp}\rho_s$ for X-mode beam trajectories: (a): H-mode (A), (b): H-mode (B), and (c): L-mode. (d)–(f): Same for O-mode: (d): H-mode (A), (e): H-mode (B), and (f): L-mode. The solid black line represents the LCFS ($\rho = 1$), and flux surfaces are plotted in intervals of 0.1. The red dashed line represents the R -coordinate of the magnetic axis. The dotted black line represents the poloidal launch angle that results in the beam being normally incident on the LCFS for the magnetic geometry considered in this paper. For all profiles, there is good spatial coverage of the plasma's edge, core or both.

40–42 GHz O-mode beams measure density fluctuations in the steep pedestal region, so the cutoffs occur at similar radial coordinates. However, once the beams are launched at high enough frequencies, they will probe fluctuations in the core. Furthermore, the positions of the scattering locations in the core region are sensitive to the beam frequency because f_{pe} and f_r change slowly with position in the core, so a small change in frequency will lead to

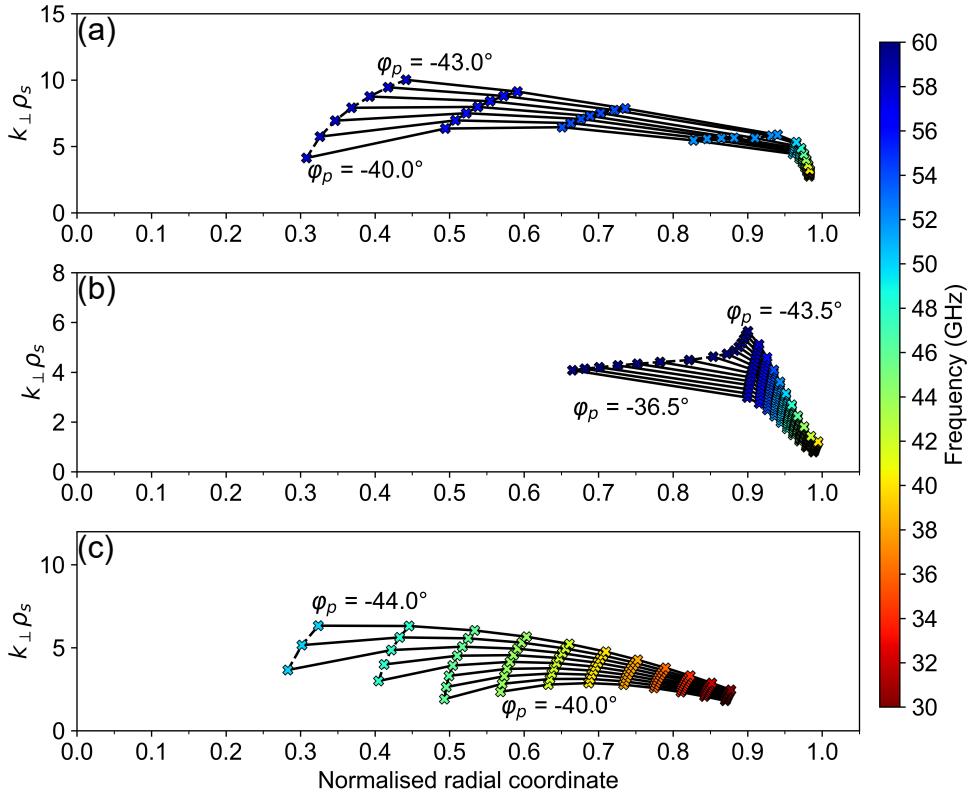


Fig. 8. Map of the localization and normalized turbulence wavenumbers $k_{\perp}\rho_s$ for X-mode beams launched at a fixed toroidal angle of 0° for (a): H-mode (A), (b): H-mode (B), and (c): L-mode. $k_{\perp}\rho_s$ vary across frequencies and poloidal launch angles φ_p , as shown by the colour of the points and the labels of the solid black lines, respectively. For all profiles, there is good poloidal flux coverage of the plasma's edge, core or both.

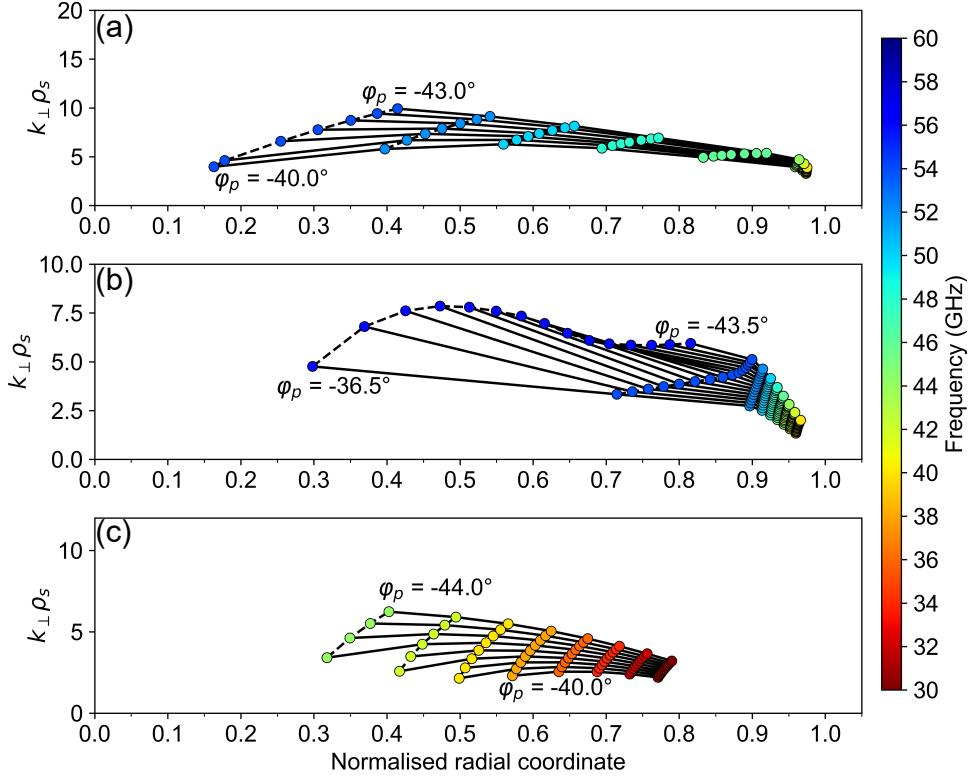


Fig. 9. Map of the localization and normalized turbulence wavenumbers $k_{\perp}\rho_s$ for O-mode beams launched at a fixed toroidal angle of 0° for (a): H-mode (A), (b): H-mode (B), and (c): L-mode. $k_{\perp}\rho_s$ vary across frequencies and poloidal launch angles φ_p , as shown by the colour of the points and the labels of the solid black lines, respectively. For all profiles, there is good poloidal flux coverage of the plasma's edge, core or both.

a significant change in the position of the scattering location.

For X-mode beams with frequencies greater than or equal to 52 GHz and for O-mode beams with frequencies greater than or equal to 42 GHz, the cutoff locations corresponding to different beam frequencies at a given poloidal launch angle tend to be more dispersed, compared to the cutoff points at lower frequencies, where they tend to be clustered very closely together, see Fig. 7(a) and Fig. 8(a). This observation can be explained using Fig. 4 as well. We see that for right-hand cutoff frequencies above 50 GHz, the cutoff frequency changes slowly with position, so a slight change in frequency results in a considerable change in the location of the X-mode cutoffs. Hence, the X-mode cutoff points are spread out for frequencies above 50 GHz at a given poloidal angle. The results for the O-mode beams in Fig. 7(d) and Fig. 9(a) can be explained in a similar way. For plasma cutoff frequencies past 42 GHz, the cutoff frequency also changes slowly with position, so a slight change in frequency also results in a considerable change in the location of the O-mode cutoffs. Hence, to probe for density fluctuations in the spaces between cutoff points past the pedestal, measurements can be sampled at smaller frequency intervals, increasing the spatial and poloidal flux coverage. This can be done using a tunable frequency channel.

3.2.2. *H-mode (B) cutoff locations*

The X-mode and O-mode beams can fully cover the plasma's edge and some parts of the core, generally covering a poloidal flux range roughly between $\rho = 0.30$ and $\rho = 1$ as seen in Fig. 8(b) and Fig. 9(b). Some O-mode beams have cutoffs localising at a lower radial coordinate. However, this current set of frequencies and poloidal launch angles cannot sufficiently cover

the plasma's core. This can be explained using Fig. 4. For f_r , we see that the beams with frequencies lower than or equal to 58 GHz will localise at the pedestal of the plasma. Only beams with a frequency of 60 GHz can penetrate the pedestal and localise in the plasma core. Similarly, for f_{pe} , only beams with frequencies above 52 GHz can penetrate the pedestal and localise deep in the plasma. This implies that a very small subset of the frequencies we are using can probe density fluctuations past the pedestal of the plasma. Furthermore, like the H-mode (A) profile, we also see that the cutoff points past the pedestal region for the H-mode (B) profile are slightly more dispersed compared to those near the edge because past the pedestal region, f_{pe} and f_r change slowly with position. Hence, higher frequency X-mode beams sampled at smaller frequency intervals can be used to probe for more density fluctuations near the core, increasing the spatial and poloidal flux coverage.

3.2.3. L-mode cutoff locations

In L-mode, the X-mode and O-mode beams can provide extensive coverage of the plasma core, with the X-mode and O-mode beams covering a poloidal flux range roughly between $\rho = 0.3$ and $\rho = 0.9$ as seen in Fig. 8(c) and Fig. 9(c), respectively. However, this set of frequencies is unable to probe the plasma's edge. This observation can be explained using Fig. 4, where we notice that when an X-mode beam of frequency 30 GHz is launched into the plasma, it does not localise near the plasma's edge. Furthermore, any X-mode beam launched with a higher frequency and any O-mode beam with a frequency greater than or equal to 30 GHz will have cutoffs localising at a point in a plasma that is deeper than the cutoff for the X-mode beam launched at

30 GHz. To probe for fluctuations at the plasma's edge, frequencies lower than 30 GHz must be used.

3.3. Spectral range

Profiles	k_{\perp} (cm $^{-1}$)	$k_{\perp}\rho_s$
H-mode (A)	$3.39 < k_{\perp} < 8.56$	$2.75 < k_{\perp}\rho_s < 10.03$
H-mode (B)	$4.00 < k_{\perp} < 9.49$	$0.79 < k_{\perp}\rho_s < 5.65$
L-mode	$1.94 < k_{\perp} < 6.27$	$1.80 < k_{\perp}\rho_s < 6.34$

Table 3: Spectral range for X-mode beams for different plasma profiles

Profiles	k_{\perp} (cm $^{-1}$)	$k_{\perp}\rho_s$
H-mode (A)	$3.34 < k_{\perp} < 8.31$	$3.28 < k_{\perp}\rho_s < 9.93$
H-mode (B)	$2.47 < k_{\perp} < 9.23$	$1.34 < k_{\perp}\rho_s < 7.86$
L-mode	$2.18 < k_{\perp} < 5.88$	$2.14 < k_{\perp}\rho_s < 6.25$

Table 4: Spectral range for O-mode beams for different plasma profiles

Turbulence measurements are on the scale of $k_{\perp}\rho_s \lesssim 10$ for all density profiles, see Table 3 and Table 4. Now, electron scale turbulence has a length scale of $10 \lesssim k_{\perp}\rho_s \lesssim 30$, which means that the DBS can measure both ion scale and the lower end of electron scale turbulence. Turbulence of these length scales is important to measure in spherical tokamaks because electron temperature gradients (ETGs) are mainly responsible for driving electron-scale turbulence, which then causes anomalous electron heat transport to

arise in spherical tokamaks [59, 60]. Furthermore, ETGs have been predicted to suppress microtearing modes (MTMs) in spherical tokamaks [61, 62], so measuring both ion and electron scale turbulence is important to understand both anomalous electron heat transport and cross-scale turbulent interactions in spherical tokamaks. Now that the scattering locations of the turbulent fluctuations and their associated wavevectors \mathbf{k}_\perp have been determined, the next subsection will discuss how to find the optimal toroidal launch angle for a given frequency to ensure zero mismatch.

3.4. Pitch angle matching and mismatch attenuation

As the EXL-50U has a large magnetic pitch angle, beams may be severely mismatched if the launch angles are not chosen carefully. Hence, to show the effect of toroidal launch angle on the mismatch attenuation of the beams, we sweep the toroidal launch angles for all frequencies for both X and O modes while keeping other quantities constant. Since the toroidal response depends strongly on beam properties [30, 31, 55], the beam parameters of the Gaussian beams at launch must be determined. The dependence of the backscattered power on the mismatch angle follows a Gaussian because there will be a collection of wavevectors within the Gaussian envelope. The implication is that even if the central wavevector does not satisfy the Bragg condition, another wavevector in this Gaussian envelope will. The properties of this Gaussian envelope are thus important, and they depend on the beam width and curvature. Since the exact DBS quasioptics will be determined at a later time, we will be using reasonable estimates of the beam parameters for the synthetic DBS. In this study, beams are launched with an initial beam width given by,

$$w_0 = 2\sqrt{\frac{z_r \lambda}{\pi}}, \quad (4)$$

and an initial curvature of -0.25 m^{-1} . Here, w_0 is the initial beam width, z_r is the Rayleigh length, which is set to be equal to the minor radius of the tokamak, and λ is the vacuum wavelength of the beam. These parameters were chosen such that the beam does not focus too tightly, and thus diverges rapidly away from the beam waist. w_0 has a frequency dependence to estimate the beam widths more realistically. The inclusion of the factor of 2 is valid because the Rayleigh range z_r is still on the order of the characteristic length scale L , giving us reasonable estimates for the beam widths. We use a converging beam because it improves performance [63]. For this study, the curvature is arbitrarily set to -0.25 m^{-1} . This will be optimised in the future based on the actual scenario.

With the beam parameters defined, beams are propagated at a toroidal launch angle range of $-5^\circ \leq \varphi_t \leq 15^\circ$, where φ_t is the toroidal launch angle. Fixed poloidal launch angles of -40.0° , -36.5° and -43.0° were used for the H-mode (A), H-mode (B) and L-mode profiles, respectively. The following subsections will discuss how to determine the optimal toroidal launch angle for a given frequency for each of the plasma profiles. Here, we also define the mismatch attenuation as $\exp(-2\theta_m^2/\Delta\theta_m^2)$ [28], which shows how much the backscattered power, expressed as a fraction of the maximum power, is reduced due to mismatch.

3.4.1. H-mode (A)

The optimal toroidal launch angle for the X-mode beams does not vary much from 40 GHz to 52 GHz. However, from 52 GHz to 58 GHz, the

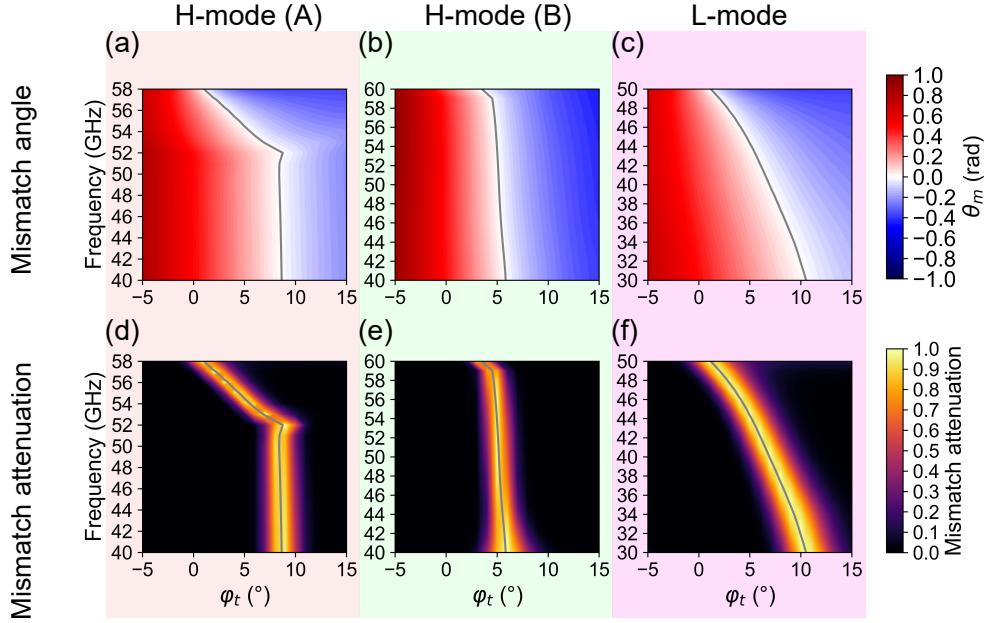


Fig. 10. (a)–(c): Heatmaps of the mismatch angle θ_m of X-mode beams across varying frequencies and toroidal launch angles φ_t : (a): H-mode (A), (b): H-mode (B), and (c): L-mode. (d)–(f): Corresponding mismatch attenuation, defined as $\exp(-2\theta_m^2/\Delta\theta_m^2)$ [28]: (d): H-mode (A), (e): H-mode (B), and (f): L-mode. The grey line represents zero mismatch. Toroidal steering is needed because it is generally impossible to achieve pitch angle matching for both the core and the edge using the same toroidal launch angle.

toroidal angle for zero mismatch decreases monotonically from around 8° to 1° , see Fig. 10(a) and Fig. 10(d). This is similar to the O-mode case, where the toroidal angle for zero mismatch does not vary much from 40 GHz to 48 GHz. However, from 48 GHz to 54 GHz, the toroidal angle for zero mismatch starts decreasing monotonically from around 8° to -1° , see Fig. 11(a) and Fig. 11(d). This is because X-mode beams launched with frequencies from 40 GHz to 52 GHz and O-mode beams launched with frequencies from 40 GHz to 48 GHz localise at roughly the same location in the pedestal

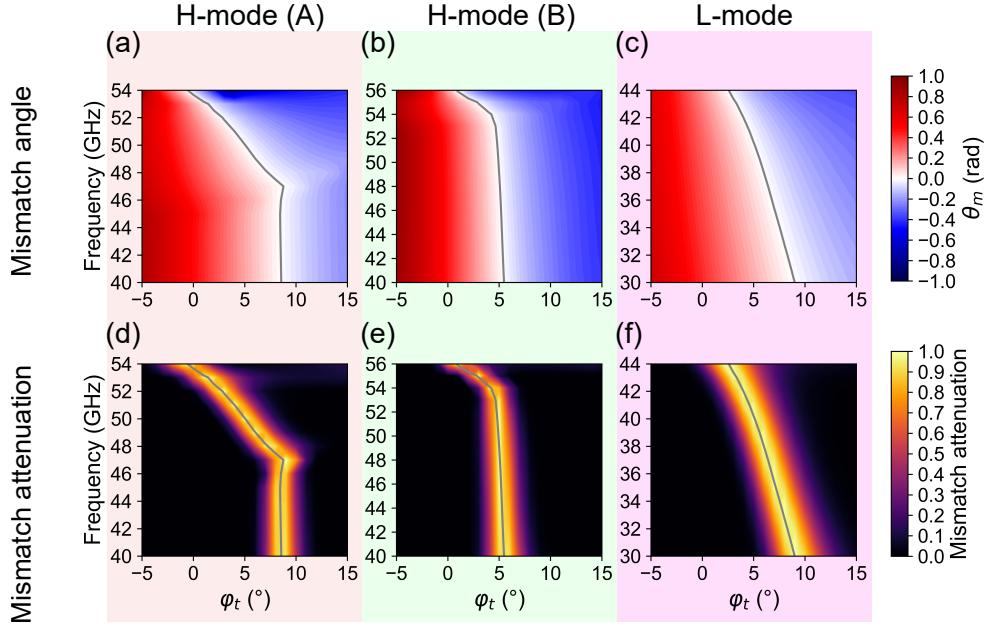


Fig. 11. (a)–(c): Heatmaps of the mismatch angle θ_m of O-mode beams across varying frequencies and toroidal launch angles φ_t : (a): H-mode (A), (b): H-mode (B), and (c): L-mode. (d)–(f): Corresponding mismatch attenuation, defined as $\exp(-2\theta_m^2/\Delta\theta_m^2)$ [28]: (d): H-mode (A), (e): H-mode (B), and (f): L-mode. The grey line represents zero mismatch. Toroidal steering is needed because it is generally impossible to achieve pitch angle matching for both the core and the edge using the same toroidal launch angle.

region of the plasma. However, when an X-mode beam is launched at a frequency greater than 52 GHz or an O-mode beam is launched at a frequency greater than 48 GHz, the beam will reach cutoff at a drastically different location, explaining the relatively large change in toroidal angle that yields zero mismatch. This closely corroborates with Fig. 4, where X-mode beams launched with frequencies from 40 GHz to 50 GHz will localise at a similar radial coordinate in the pedestal of the plasma. O-mode beams launched with frequencies from 40 GHz to 42 GHz will localise at a similar radial coordi-

nate in the pedestal. In comparison, X-mode beams with frequencies greater than 50 GHz and O-mode beams with frequencies greater than 42 GHz can penetrate this pedestal region.

This analysis shows that for a given toroidal launch angle, some frequencies will be matched, but other frequencies will be mismatched, and thus, the backscattered power received would be significantly reduced if the wrong toroidal launch angle is used. Furthermore, the range of measurable frequencies for each toroidal angle is different and generally does not span the entire 20 GHz range. Hence, toroidal steering with a tunable frequency channel should be used to ensure that beams are not severely mismatched. To probe turbulent fluctuations in the pedestal region, lower beam frequencies with a broader range (40–52 GHz for X-mode and 40–48 GHz for O-mode) should be used with a toroidal launch angle of around 8° to optimise beams of these frequencies because the optimal toroidal launch angle for zero mismatch is the same. On the other hand, to probe a particular location in the core, a narrow range of beam frequencies should be used together with an appropriate toroidal launch angle, because a single toroidal launch angle cannot be used to optimise higher frequency beams probing different locations in the core. For this specific profile, to probe fluctuations located at around $\rho = 0.30$ using a poloidal launch angle of -40° with X-mode beams, a toroidal launch angle of 1° should be used together with a narrow frequency range of 56–58 GHz. Either way, there is no single compromise angle where one is able to obtain significant backscattered from both the edge and the core channels.

3.4.2. H-mode (B)

For the X-mode beams, the optimal toroidal launch angle does not vary significantly with frequency, suggesting that the same toroidal launch angle of approximately 5° can be used to ensure beams are not severely mismatched, see Fig. 10(b) and Fig. 10(e). The toroidal launch angle for zero mismatch does not change much here, because from Fig. 4, we see that when the beam is propagated at any frequency from 40 GHz to 60 GHz, it generally localises at roughly the same location in the pedestal of the plasma. The O-mode case is similar, except for a slight bend from 54 GHz onwards, see Fig. 11(b) and Fig. 11(e). This is because, from Fig. 4, when an O-mode beam is propagated with frequencies above 54 GHz, it is able to penetrate through the pedestal and localise at a drastically different location.

Similar to the H-mode (A) profile, toroidal steering with a tunable frequency channel should be used to ensure that beams are not severely mismatched. To probe fluctuations in the pedestal, a broad range of beam frequencies (40–60 GHz for X-mode and 40–54 GHz for O-mode) can be used together with a toroidal angle of around 5° to optimise beams of lower frequencies because the optimal toroidal launch angle for zero mismatch is the same for these beams.

On the other hand, to probe a particular location in the core, a narrow frequency range should be used together with an appropriate toroidal launch angle. For this specific profile, to probe fluctuations located at $\rho = 0.30$ using a poloidal launch angle of -36.5° with O-mode beams, a frequency range of 55–56 GHz should be used together with a toroidal launch angle of around 1° to optimise these beams.

3.4.3. L-mode

For the L-mode case, the optimal toroidal launch angle decreases monotonically as the frequency increases for both the X-mode and O-mode beams, see Fig. 10(c) and Fig. 10(f) for the X-mode, and Fig. 11(c) and Fig. 11(f) for the O-mode. This is expected because, from Fig. 4, we see that whenever a beam with a given polarisation is launched at a different frequency, the beam will always probe density fluctuations at a different location in the plasma. This shows that the optimal toroidal launch angle for zero mismatch changes at every frequency, so we cannot simultaneously optimise for all locations. This implies that when using the tunable frequency channel to probe a particular location in the plasma, a narrow range of frequencies should be used together with an appropriate toroidal launch angle. For this specific profile, to probe fluctuations located at around $\rho = 0.85$ using a poloidal launch angle of -43.0° with X-mode beams, a frequency range of 30–34 GHz should be used with a toroidal launch angle of 10° . However, to probe fluctuations located at $\rho = 0.30$ using the same poloidal launch angle with X-mode beams, a frequency range of 48–50 GHz should be used together with a toroidal launch angle of around 2° . Similar to the H-mode (A) case, there is no single compromise toroidal angle that allows one to obtain a significant signal for both core and edge channels.

Now that we have determined the scattering locations of the turbulent fluctuations, their associated wavevectors \mathbf{k}_\perp , and the optimal toroidal launch angle for a given frequency, the next section will discuss how to develop a simple conceptual DBS design.

4. Conceptual DBS design

4.1. DBS design

With an understanding of the scattering locations of the turbulent fluctuations and their corresponding turbulence wavevectors, we will discuss how a DBS can be designed for the EXL-50U to make these desired measurements while keeping in mind physical design constraints like the space available to propagate the microwave beam. The configuration of the DBS we are considering consists of several components shown in Fig. 12. Here, a High-Density Polyethylene (HDPE) lens focuses the beams to an appropriate beam width and curvature because the DBS will be outside the vacuum chamber. Determining the appropriate beam widths and curvatures depends on the physical constraints, which will be mentioned in the next subsection and, in turn, affect how we design the DBS.

4.2. Constraints for DBS system design

The main constraints to consider when designing the DBS for the EXL-50U are the following:

1. At the beam's entry point, the beam width cannot exceed $1/3$ the distance from the entry point to the top of PF14, or else the beam will be incident on PF14. We take the entry point to correspond to the case where the beam is launched at a poloidal angle of -36.5° .
2. The lens must not focus the beam too much to the point where the beam width is smaller than three times the wavelength of the lowest frequency wave generated by the antenna. If the beam width becomes

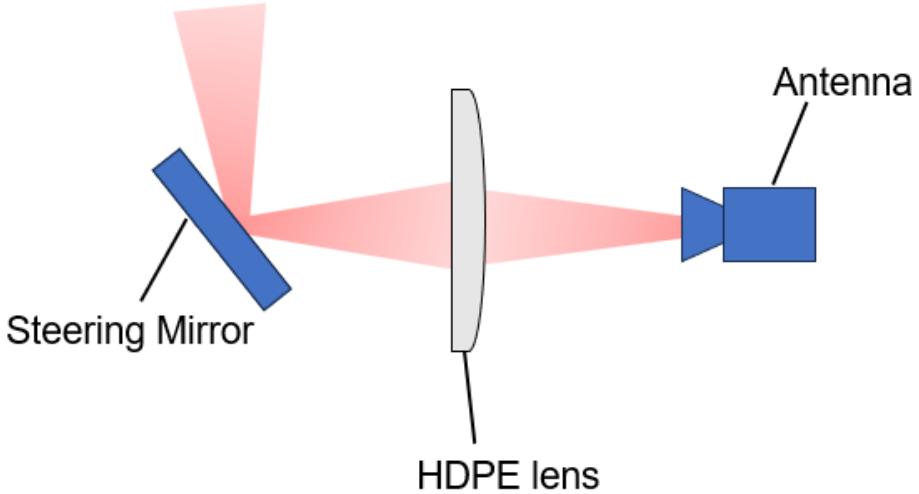


Fig. 12. Overview of the DBS used to send microwaves into the plasma. The antenna emits microwaves at various frequencies, which are then focused by a HDPE lens before being reflected into the plasma by a rotatable flat mirror that sets the initial launch angles of the beam.

too small, diffraction effects will be dominant, causing the beam to not focus effectively.

3. Beam width at the lens is no more than $1/3$ the diameter of the lens.
4. Radius of curvature of the lens cannot be less than the radius of the lens, otherwise it is impossible to manufacture the lens. We will be setting the radius of the lens to be equal to 1.5 times the length of the maximum beam width that will be attained at the lens, since that is the minimum radius the lens can have to not violate the previous constraint.
5. Diameter of the port window must be three times the beam width to ensure all beams can pass through the window.

The last constraint will be addressed in subsection 4.4. Ideally, the beams should focus at the cutoff point, which means they are not focused before the cutoff point, making it very difficult to fulfil the above constraints. Hence, the DBS is designed to focus all the beams when they enter the plasma, allowing the above constraints to be fulfilled.

To design a system that ensures the aforementioned design criteria are met, the following design parameters must be chosen wisely:

1. Distance from the horn to the lens
2. Distance from the lens to the mirror
3. Focal length of the lens
4. Type of horn and its Full Width, Half Maximum (FWHM) angle

These parameters were determined by back-propagating 40 GHz and 60 GHz beams with zero curvature and with the maximum possible beam width from the plasma's entry point to the beam waist's location to estimate the location of the horn. Since two beams of different frequencies are used and they have different widths at the beam waist, the mean of the two widths w_m is used to determine the horn aperture radius a and the FWHM angle of the horn θ_{FWHM} :

$$a = \frac{w_m}{0.644}, \quad (5)$$

and

$$\theta_{FWHM} = \sqrt{2 \ln 2} \tan^{-1} \left(\frac{c}{\pi w_m f_{mid}} \right). \quad (6)$$

Here, c is the speed of light and f_{mid} is the mid-band frequency taken as 50 GHz.

Using the lensmaker's formula, the radii of curvature of the lens, R_1 and R_2 , are determined,

$$\frac{1}{f} = (n - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right), \quad (7)$$

where f is the focal length of the lens and n is the refractive index of HDPE. However, one side of the lens is flat, so the radius of curvature on that side is infinite. This means that $\frac{1}{R_2}$ is zero and can be ignored, giving us the following simplified formula,

$$R_1 = (n - 1) f. \quad (8)$$

These quantities are then used to verify whether the design constraints are met by propagating the beam from the horn to the entry point to calculate the beam widths for 40–60 GHz beams as they vary with the propagation distance, as shown in Fig. 13.

4.3. Proposed DBS system design

Our suggested design involves a DBS system with the following parameters listed in Table 5. Using these parameters, we can determine launch widths and curvatures for each beam frequency as plotted in Fig. 14, which may be used again in SCOTTY to simulate beam trajectories.

Furthermore, the upper and lower limits of each system's possible launch widths and curvatures can also be determined. In Fig. 14, the widths and curvatures simulated in Section 3 are plotted as well, and they are similar to the beam parameters calculated using the design parameters from the DBS design. This shows that the widths and curvatures used in SCOTTY are within the range of those achievable by a physical system.

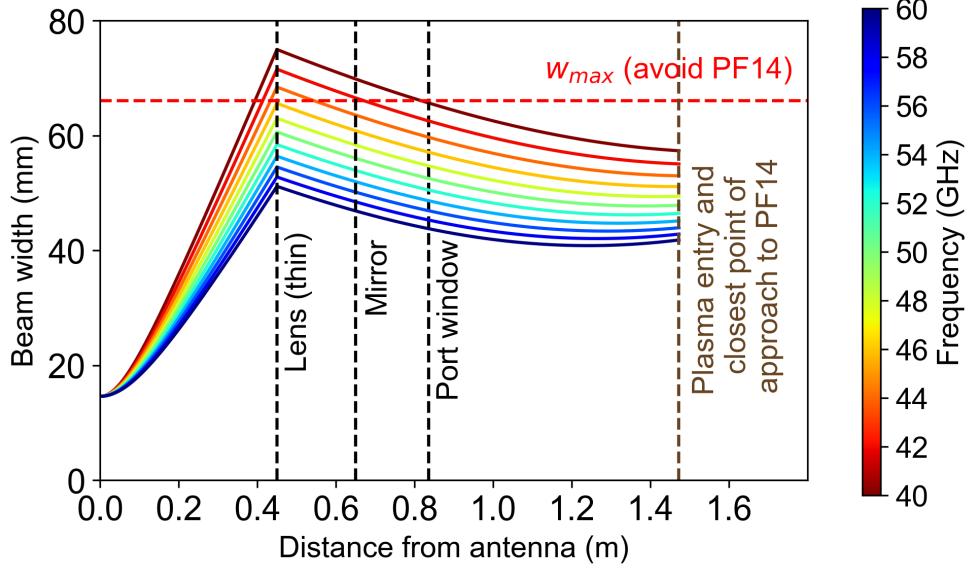


Fig. 13. Evolution of the beam width for the 40–60 GHz system as the beam propagates through the quasi-optical system and enters the plasma. Beams are realistically focused, and their widths remain below the maximum value (red dashed line) before incidence on PF14 at the plasma entry (brown dashed line), which is also the closest approach to PF14. This plot shows that we can design a DBS system that meets the physical constraints.

4.4. Port window design

As mentioned in subsection 4.2, we will be addressing the final constraint in this subsection, which is that the beam width cannot exceed more than 1/3 the diameter of the port window. The size of the port window must be large enough to account for not only the beam widths, but also the maximum separation between two beams as different launch angles are used. We will compute the beam widths at the port window first.

We suppose that the horizontal distance between the launch position and the port window is 150 mm, see Fig. 15(a). We also assume that the centre of

Horn type	Scalar horn
Aperture radius	22.67 mm
FWHM beamwidth	8.77°
Focal length	400.00 mm
Distance between the horn and the lens	450.00 mm
Distance between lens and mirror	200.00 mm
Radius of curvature of the lens	212.00 mm
Radius of the lens	112.47 mm

Table 5: Values of the design parameters used to design the DBS system.

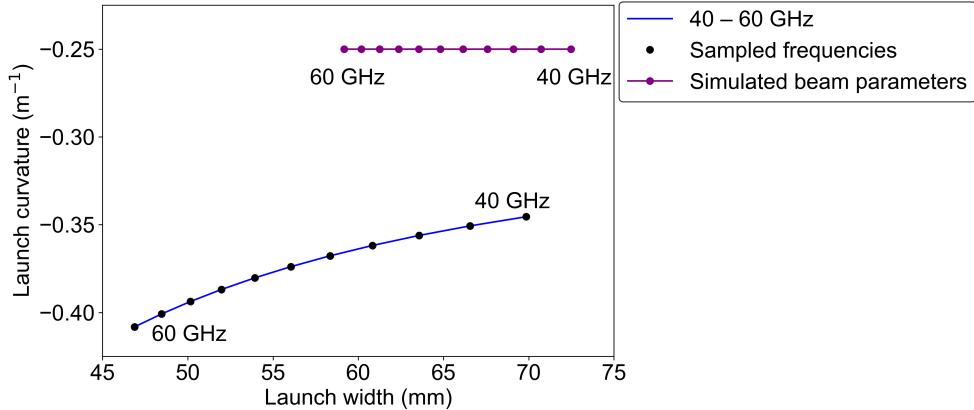


Fig. 14. Plot showing the possible launch widths and curvatures produced by each system. Widths and curvatures used in SCOTTY are within the range of those achievable by a physical system.

the port window coincides with the centre of the beam launched at a poloidal angle of -36.5° . However, the centre of the port window is not at the same vertical height as the centre of the steering mirror. Hence, we need to account

for this offset in height h_o . We calculate it to be:

$$\begin{aligned} h_o &= 150 \tan(36.5^\circ) \\ &= 110.99 \text{ mm}. \end{aligned} \tag{9}$$

Then, the distances travelled by the beams launched at poloidal angles of -36.5° and -43.5° in millimeters are calculated using:

$$l_{\varphi_p} = \frac{150}{\cos(\varphi_p)}, \tag{10}$$

where φ_p is the poloidal launch angle and l_{φ_p} is the distance travelled by a beam launched at φ_p in millimeters. Together with the parameters in Table 5, these distances are used to calculate the beam widths for 40 GHz beams at the port window:

$$w_{-36.5^\circ, \text{port}} = 65.65 \text{ mm}, \tag{11}$$

$$w_{-43.5^\circ, \text{port}} = 65.24 \text{ mm}. \tag{12}$$

Here, $w_{-36.5^\circ, \text{port}}$ corresponds to the beam width at the port window for a 40 GHz beam launched at $\varphi_p = -36.5^\circ$, and $w_{-43.5^\circ, \text{port}}$ corresponds to the beam width at the port window for a 40 GHz beam launched at $\varphi_p = -43.5^\circ$. 40 GHz beams are used because this frequency corresponds to beams with the largest beam width, see Fig. 13. Finally, the diameter of the port window D_{port} is calculated by taking the larger of the two beam width values and multiplying it by three:

$$\begin{aligned} D_{\text{port}} &= 3w_{-36.5^\circ, \text{port}} \\ &= 196.95 \text{ mm}. \end{aligned} \tag{13}$$

We see that a reasonably sized port window can be designed for this system. Now, we verify whether this port window can account for the separation between the beam centres launched at these two different angles h by calculating it:

$$\begin{aligned} h &= 150(\tan(43.5^\circ) - \tan(36.5^\circ)) \\ &= 31.35 \text{ mm.} \end{aligned} \tag{14}$$

h is much smaller than D_{port} , so the port window is large enough to allow beams to pass through, see Fig. 15(b).

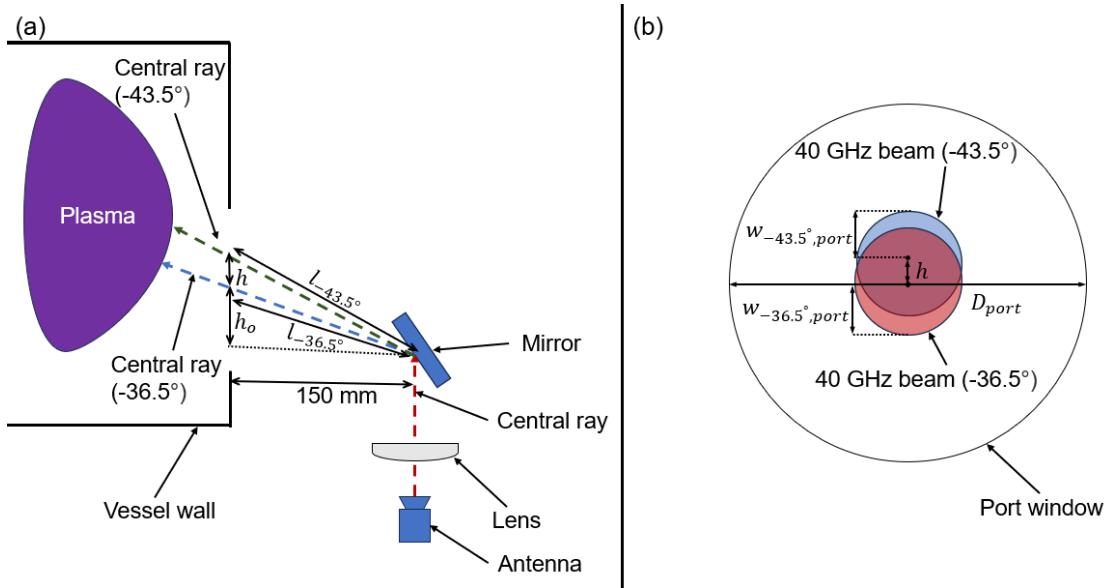


Fig. 15. (a) Diagram showing the quantities used to calculate the beam widths at the port window and the port window diameter. (b) The port window is large enough to account for not only the beam widths but also the separation between the beams.

4.5. Potential further improvements

An alternative design uses two mirrors, shown in Fig. 16, allowing better access to the plasma. At the time of writing, the volume available in the vessel for installing the DBS is indeterminate. Having sufficient space available would allow one to install the second mirror, raising the launch position to a higher Z position. This design would result in the beam probing the cutoff at lower angles, allowing for lower wavenumbers to be obtained. However, this is beyond the scope of this work.

Moreover, the exact widths and curvatures needed to optimise the mismatch tolerance depend on the actual plasma properties of the EXL-50U and the exact quasi-optics that will be used, which are currently unknown. Hence, optimising these beam parameters to optimise the mismatch tolerance will not be done in this paper, but should be considered in the future.

5. Conclusion

We have completed a conceptual DBS design for the EXL-50U, launching from below the midplane. Using SCOTTY beam tracing simulations, we show that depending on the density profile, the system can extensively probe for turbulent fluctuations in the plasma's edge, core, or both, probing a maximum k_{\perp} value of 9.49 cm^{-1} . Furthermore, it can measure turbulence on the scale of $k_{\perp}\rho_s \lesssim 10$ with a maximum range of $0.79 < k_{\perp}\rho_s < 10.03$, making it suitable for measuring ion scale and the lower end of electron scale turbulence ($10 \lesssim k_{\perp}\rho_s \lesssim 30$), which is relevant to the EXL-50U as it is a spherical tokamak.

Since EXL-50U is a spherical tokamak with a large magnetic pitch angle,

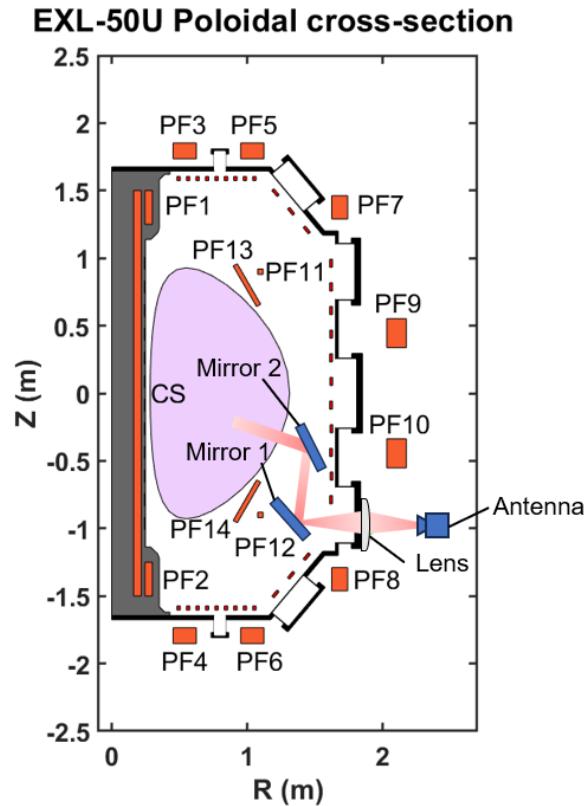


Fig. 16. A possible two-mirror DBS design that can be implemented for the EXL-50U. These mirrors can be adjusted to ensure the beams are not incident on PF14, providing easier access to the plasma and increasing the maximum poloidal launch angle range. The mirrors are concave, providing a focusing effect for the beams.

the mismatch angle can be large if launch angles are not carefully chosen. In this study, we found how the beam's mismatch attenuation varies with toroidal launch angles and launch frequencies, allowing us to find the optimal toroidal launch angle for a given launch frequency and density profile. Furthermore, we find that to achieve matching at the core and the edge with the same, fixed poloidal launch angle, toroidal steering is crucial. Therefore, toroidal steering, complemented with a tunable frequency channel, should be used during actual operation to minimise the mismatch angle and, in turn, the mismatch attenuation.

We successfully designed a conceptual quasioptical design for a DBS with a maximum poloidal launch angle range of $-43.5^\circ \leq \varphi_p \leq -36.5^\circ$ and is capable of measuring turbulent fluctuations in the edge and the core of the plasma. In this work, a quasioptical system which met various physical design constraints was designed for 40–60 GHz.

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