Theorem 1. For real numbers $a, b \in \mathbb{R}$ we have

$$a^2 + b^2 \geq 2ab$$

Proof. For $a, b \in \mathbb{R}$, we have $(a - b)^2 \geq 0$ and therefore

$$a^2 + b^2 \geq 2ab$$

□