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KEY CONCEPTS
on Integrals and its Applications

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10 GREAT PROBLEMS

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(XI & XII)

MATH ARCHIVES

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CONCEPT MAP
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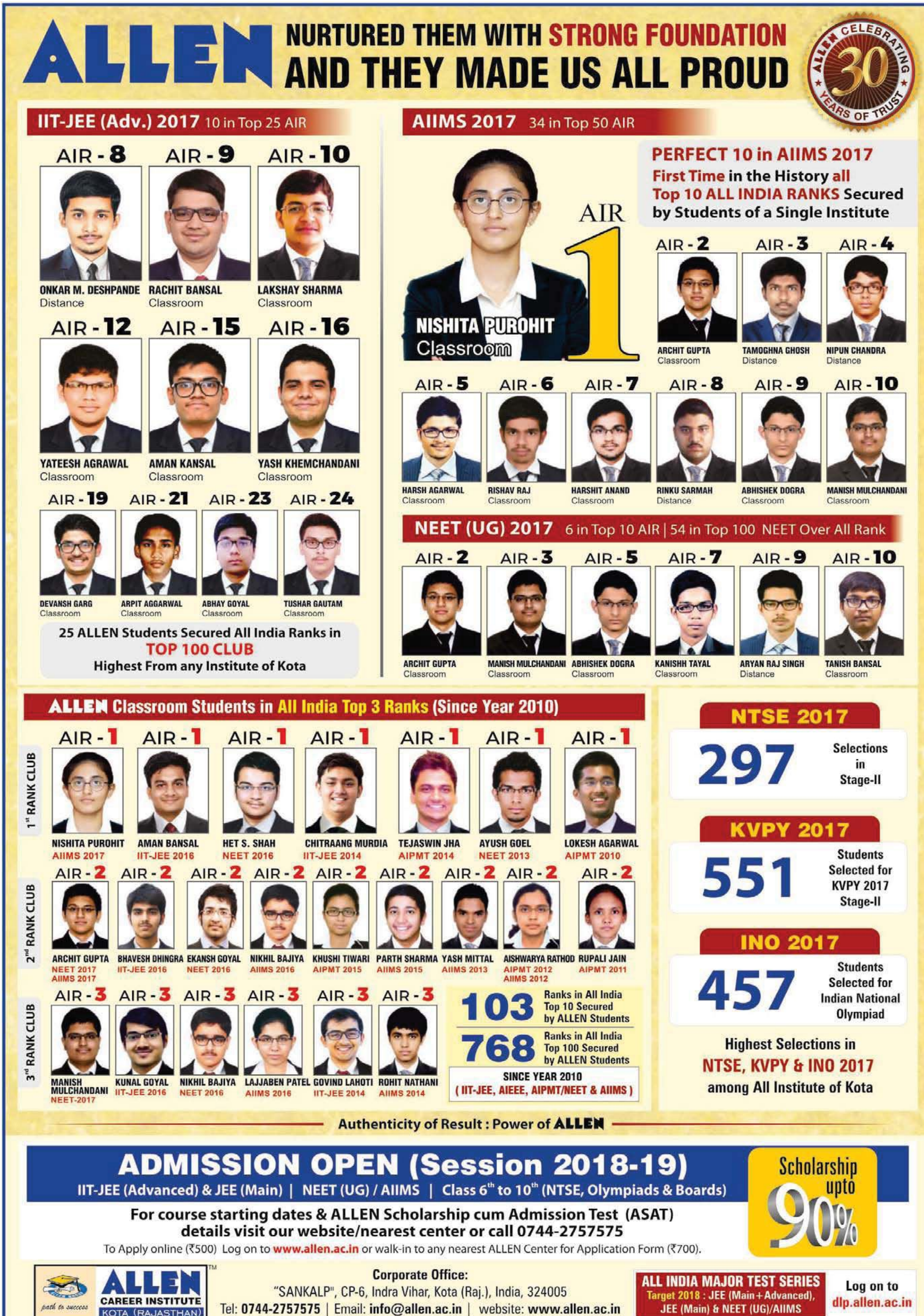

ALLEN
NURTURED THEM WITH STRONG FOUNDATION
AND THEY MADE US ALL PROUD
ALLEN CELEBRATING 30 YEARS OF TRUST
IIT-JEE (Adv.) 2017 10 in Top 25 AIR
AIR - 8 ONKAR M. DESHPANDE Distance
AIR - 9 RACHIT BANSAL Classroom
AIR - 10 LAKSHAY SHARMA Classroom
AIR - 12 YATEESH AGRAWAL Classroom
AIR - 15 AMAN KANSAL Classroom
AIR - 16 YASH KHEMCHANDANI Classroom
AIR - 19 DEVANSH GARG Classroom
AIR - 21 ARPIT AGGARWAL Classroom
AIR - 23 ABHAY GOYAL Classroom
AIR - 24 TUSHAR GAUTAM Classroom
25 ALLEN Students Secured All India Ranks in
TOP 100 CLUB
Highest From any Institute of Kota
AIIMS 2017 34 in Top 50 AIR
AIR 1 NISHITA PUROHIT Classroom
PERFECT 10 in AIIMS 2017
First Time in the History all
Top 10 ALL INDIA RANKS Secured
by Students of a Single Institute
AIR - 2 ARCHIT GUPTA Classroom
AIR - 3 TAMOGHNA GHOSH Distance
AIR - 4 NIPUN CHANDRA Distance
AIR - 5 HARSH AGARWAL Classroom
AIR - 6 RISHAV RAJ Classroom
AIR - 7 HARSHIT ANAND Classroom
AIR - 8 RINKU SARMAH Distance
AIR - 9 ABHISHEK DOGRA Classroom
AIR - 10 MANISH MULCHANDANI Classroom
NEET (UG) 2017 6 in Top 10 AIR | 54 in Top 100 NEET Over All Rank
AIR - 2 ARCHIT GUPTA Classroom
AIR - 3 MANISH MULCHANDANI Classroom
AIR - 5 ABHISHEK DOGRA Classroom
AIR - 7 KANISHH TAYAL Classroom
AIR - 9 ARYAN RAJ SINGH Distance
AIR - 10 TANISH BANSAL Classroom
ALLEN Classroom Students in All India Top 3 Ranks (Since Year 2010)
1st RANK CLUB
AIR - 1 NISHITA PUROHIT AIIMS 2017
AIR - 1 AMAN BANSAL IIT-JEE 2016
AIR - 1 HET S. SHAH NEET 2016
AIR - 1 CHITRAANG MURDIA IIT-JEE 2014
AIR - 1 TEJASWIN JHA AIPMT 2014
AIR - 1 AYUSH GOEL NEET 2013
AIR - 1 LOKESH AGARWAL AIPMT 2010
2nd RANK CLUB
AIR - 2 ARCHIT GUPTA NEET 2017 AIIMS 2017
AIR - 2 BHAVESH DHINGRA IIT-JEE 2016
AIR - 2 EKANSH GOYAL NEET 2016
AIR - 2 NIKHIL BAJIYA AIIMS 2016
AIR - 2 KHUSHI TIWARI AIPMT 2015
AIR - 2 PARTH SHARMA AIIMS 2015
AIR - 2 YASH MITTAL AIIMS 2013
AIR - 2 AISHWARYA RATHOD AIPMT 2012 AIIMS 2012
AIR - 2 RUPALI JAIN AIPMT 2011
3rd RANK CLUB
AIR - 3 MANISH MULCHANDANI NEET-2017
AIR - 3 KUNAL GOYAL IIT-JEE 2016
AIR - 3 NIKHIL BAJIYA NEET 2016
AIR - 3 LAJJABEN PATEL AIIMS 2016
AIR - 3 GOVIND LAHOTI IIT-JEE 2014
AIR - 3 ROHIT NATHANI AIIMS 2014
103 Ranks in All India Top 10 Secured by ALLEN Students
768 Ranks in All India Top 100 Secured by ALLEN Students
SINCE YEAR 2010
(IIT-JEE, AIEEE, AIPMT/NEET & AIIMS)
NTSE 2017
297 Selections in Stage-II
KVPY 2017
551 Students Selected for KVPY 2017 Stage-II
INO 2017
457 Students Selected for Indian National Olympiad
Highest Selections in
NTSE, KVPY & INO 2017
among All Institute of Kota
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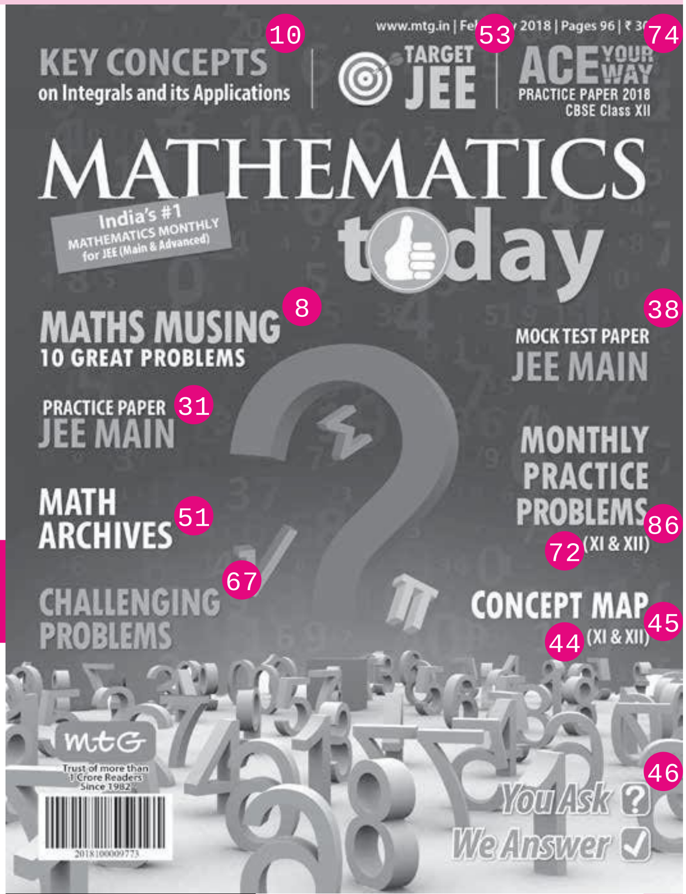

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MATHS MUSING

Maths Musing was started in January 2003 issue of Mathematics Today. The aim of Maths Musing is to augment the chances of bright students seeking admission into IITs with additional study material.
During the last 10 years there have been several changes in JEE pattern. To suit these changes Maths Musing also adopted the new pattern by changing the style of problems. Some of the Maths Musing problems have been adapted in JEE benefitting thousand of our readers. It is heartening that we receive solutions of Maths Musing problems from all over India.
Maths Musing has been receiving tremendous response from candidates preparing for JEE and teachers coaching them. We do hope that students will continue to use Maths Musing to boost up their ranks in JEE Main and Advanced.

PROBLEM Set 182

JEE MAIN

1. If $z_n = cis\dfrac{\pi}{n(n+1)(n+2)}$ for n = 1, 2, 3, ... and principle argument of $\lim\limits_{n\to\infty}(z_1z_2z_3....z_n)$ is θ. Then value of θ is
(a) $\pi/2$ (b) $\pi/3$ (c) $\pi/6$ (d) $\pi/4$

2. The coefficient of x^n in the expansion of
$$\left(1-3x+6x^2-10x^3+...+(-1)^r\frac{r(r+1)(r+2)...(r+(r-1))x^r}{r!}+...\infty\right)^{-n}$$
is
(a) $\dfrac{3n!}{(n!)^2}$ (b) $\dfrac{3n!}{n!\,2n!}$ (c) $\dfrac{3n!}{(2n!)^2}$ (d) None of these

3. At the point $P(a, a^n)$ on the graph of $y = x^n$ $(n \in N)$ in the first quadrant, a normal is drawn. The normal intersects the y-axis at the point $(0, b)$. If $\lim\limits_{a\to 0} b = \dfrac{1}{2}$, then n equals
(a) 1 (b) 3 (c) 2 (d) 4

4. In triangle ABC, $\sum\limits_{r=0}^{n}\binom{n}{r}a^{n-r}b^r\cos(rA-(n-r)B) =$
(a) $(a-b)^n$ (b) $(a+b)^n$ (c) c^n (d) c^{2n}

5. The value of $\displaystyle\int_0^1 \frac{8\log(1+x)}{1+x^2}dx$ is
(a) $\dfrac{\pi}{2}\log 2$ (b) $\log 2$ (c) $\pi\log 2$ (d) $\dfrac{\pi}{8}\log 2$

JEE ADVANCED

6. $2\cot^2\theta+2\sqrt{3}\cot\theta+4\operatorname{cosec}\theta+8=0$ if $\theta =$
(a) $n\pi+\dfrac{\pi}{6}$ (b) $n\pi-\dfrac{\pi}{6}$ (c) $2n\pi+\dfrac{\pi}{6}$ (d) $2n\pi-\dfrac{\pi}{6}$

COMPREHENSION

Let α, β be the roots of the equation $ax^2 + bx + c = 0$. Also, let $S_n = \alpha^n + \beta^n$ for $n \ge 1$ and
$$\Delta = \begin{vmatrix} 3 & 1+S_1 & 1+S_2 \\ 1+S_1 & 1+S_2 & 1+S_3 \\ 1+S_2 & 1+S_3 & 1+S_4 \end{vmatrix}$$

7. If $\Delta < 0$ then the equation $ax^2 + bx + c = 0$ has
(a) positive real roots (b) negative real roots
(c) equal roots (d) imaginary roots

8. If a, b, c are rational and one of the roots of the equation is $1+\sqrt{2}$, then the value of Δ is
(a) 8 (b) 12 (c) 30 (d) 32

INTEGER TYPE

9. Perpendiculars are drawn from the angles A, B, C of an acute-angled triangle on opposite sides and produced to meet the circumscribing circle. If these produced parts are α, β, γ respectively, then the value of $\dfrac{(a/\alpha)+(b/\beta)+(c/\gamma)}{\tan A+\tan B+\tan C}$ is

MATRIX MATCH

10. Match the following.

	List-I		List-II
P.	Number of ordered pairs which satisfy the equation $x^2 + 2x\sin(xy) + 1 = 0$ (where $y \in [0, 2\pi]$) is	1.	1
Q.	If maximum $\{5\sin\theta + 3\sin(\theta - \alpha)\} = 7$ $(\theta \in R)$ then number of solutions of α in $[0, \pi]$ are	2.	2
R.	The total number of solutions of $\cos x = \sqrt{1-\sin 2x}$ in $[0, 2\pi]$ is equal to	3.	0
S.	The number of solutions of the equation $\sin\left(\frac{1}{3}\cos^{-1}x\right)=1$	4.	3

	P	Q	R	S
(a)	1	2	3	4
(b)	2	1	4	3
(c)	4	3	2	1
(d)	3	4	1	2

See Solution Set of Maths Musing 181 on page no 70

KEY CONCEPTS on Integrals and its Applications

*ALOK KUMAR, B.Tech, IIT Kanpur

INDEFINITE INTEGRALS

If f and g are functions of x such that $g'(x) = f(x)$ then,

$$\int f(x)dx = g(x)+c \text{ or } \frac{d}{dx}\{g(x)+c\} = f(x),$$

where c is called the constant of integration.

Some Standard Formulae

- $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c, n \neq -1$
- $\int \frac{dx}{ax+b} = \frac{1}{a}\ln(ax+b)+c$
- $\int e^{ax+b}\, dx = \frac{1}{a}e^{ax+b}+c$
- $\int a^{px+q}\, dx = \frac{1}{p}\frac{a^{px+q}}{\ln a}+c;\ a>0$
- $\int \sin(ax+b)\, dx = -\frac{1}{a}\cos(ax+b)+c$
- $\int \cos(ax+b)\, dx = \frac{1}{a}\sin(ax+ab)+c$
- $\int \tan(ax+b)\, dx = \frac{1}{a}\ln\sec(ax+b)+c$
- $\int \cot(ax+b)\, dx = \frac{1}{a}\ln\sin(ax+b)+c$
- $\int \sec^2(ax+b)\, dx = \frac{1}{a}\tan(ax+b)+c$
- $\int \text{cosec}^2(ax+b)\, dx = -\frac{1}{a}\cot(ax+b)+c$
- $\int \sec(ax+b)\cdot\tan(ax+b)dx = \frac{1}{a}\sec(ax+b)+c$
- $\int \text{cosec}(ax+b)\cdot\cot(ax+b)dx = -\frac{1}{a}\text{cosec}(ax+b)+c$
- $\int \sec x\, dx = \ln(\sec x+\tan x)+c$
- $\int \text{cosec}\, x\, dx = \ln(\text{cosec}\, x-\cot x)+c$
- $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}+c$
- $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}+c$
- $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a}+c$
- $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left[x+\sqrt{x^2+a^2}\right]+c$
- $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left[x+\sqrt{x^2-a^2}\right]+c$
- $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right|+c$
- $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|+c$
- $\int \sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+c$
- $\int \sqrt{x^2+a^2}\, dx = \frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\ln\left(x+\sqrt{x^2+a^2}\right)+c$
- $\int \sqrt{x^2-a^2}dx = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\ln\left(x+\sqrt{x^2-a^2}\right)+c$
- $\int e^{ax}\cdot\sin bx\, dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx-b\cos bx)+c$

* Alok Kumar is a winner of INDIAN NATIONAL MATHEMATICS OLYMPIAD (INMO-91). He trains IIT and Olympiad aspirants.

- $\int e^{ax} \cdot \cos bx\, dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx) + c$
- $\int c\, f(x)\, dx = c\int f(x) dx$
- $\int (f(x) \pm g(x)) dx = \int f(x) dx \pm \int g(x) dx$
- $\int f(x) dx = g(x) + c \Rightarrow \int f(ax+b) dx = \frac{g(ax+b)}{a} + c$
- $\int [f(x)]^n f'(x)\, dx = \frac{(f(x))^{n+1}}{n+1} + c, n \neq -1$
- $\int \frac{f'(x)}{[f(x)]^n} dx = \begin{cases} \log|f(x)| + c, \text{ if } n = 1 \\ \int t^{-n} dt, \text{ where } t = f(x) \end{cases}$

Integration by Parts

- $\int \underset{\text{I}}{f(x)}\, \underset{\text{II}}{g(x)} dx = f(x)\int g(x) dx - \int\left(\frac{d}{dx}(f(x))\int (g(x)) dx\right) dx$

Some Particular Substitutions

- $\int \frac{dx}{ax^2+bx+c}, \int \frac{dx}{\sqrt{ax^2+bx+c}}, \int \sqrt{ax^2+bx+c}\, dx,$

  express $ax^2 + bx + c$ in the form of perfect square and then apply the standard results.
- $\int \frac{px+q}{ax^2+bx+c} dx, \int \frac{px+q}{\sqrt{ax^2+bx+c}} dx,$

  $\int (px+q)\sqrt{ax^2+bx+c}\, dx,$

  express $px + q = A$ (differential coefficient of denominator) $+ B$.
- $\int \frac{a\cdot\cos x + b\cdot\sin x + c}{l\cdot\cos x + m\cdot\sin x + n} dx$

  Express $\text{Num.} \equiv A(\text{Den.}) + B\frac{d}{dx}(\text{Den.}) + C$ and proceed.
- $\int \frac{x^2 \pm 1}{x^4 + Kx^2 + 1} dx$ where 'K' is any constant.

  Divide Num. and Den. by x^2 and put $x \mp \frac{1}{x} = t$.
- $\int \frac{dx}{(ax^2+bx+c)\sqrt{px+q}}$; put $px + q = t^2$
- $\int \frac{dx}{(ax+b)\sqrt{px^2+qx+r}}$; put $ax + b = \frac{1}{t}$
- $\int \frac{dx}{(ax^2+b)\sqrt{px^2+q}}$; put $x = \frac{1}{t}$
- $\int \sqrt{\frac{x-\alpha}{\beta - x}} dx$ or $\int \sqrt{(x-\alpha)(\beta - x)}$;

  put $x = \alpha\cos^2\theta + \beta\sin^2\theta$
- $\int \frac{dx}{\sqrt{(x-\alpha)(x-\beta)}}$; put $x - \alpha = t^2$ or $x - \beta = t^2$
- Integrals of the form $\int \tan^n x\, dx$, $\int \cot^n x\, dx$, $\int \sec^n x\, dx$, $\int \text{cosec}^n x\, dx$ is solved as :

  Let $I_n = \int \tan^n x\, dx = \int \tan^2 x \tan^{n-2} x dx$

  $= \int (\sec^2 x - 1)\tan^{n-2} x\, dx$ for

  $\Rightarrow I_n = \int \sec^2 x\ \tan^{n-2} x dx - I_{n-2}$

  $\Rightarrow I_n = \frac{\tan^{n-1} x}{n-1} - I_{n-2}$

Similarly, we can obtain integral for other trigonometric functions.

DEFINITE INTEGRALS & ITS APPLICATIONS

Let $f(x)$ be a continuous function defined on $[a, b]$, $\int f(x)\, dx = F(x) + c$. Then $\int_a^b f(x)\, dx = F(b) - F(a)$ is called definite integral. This formula is known as Newton-Leibnitz formula.

Properties of Definite Integral

- $\int_a^b f(x)\, dx = \int_a^b f(t)\, dt = -\int_b^a f(x)\, dx$
- $\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx$, where $c \in (a, b)$
- $\int_{-a}^{a} f(x)\, dx = \int_0^a (f(x) + f(-x))\, dx$

  $= \begin{cases} 2\int_0^a f(x) dx, \text{ if } f(-x) = f(x) \text{ i.e. } f(x) \text{ is even} \\ 0 \qquad\qquad\quad, \text{ if } f(-x) = -f(x) \text{ i.e. } f(x) \text{ is odd} \end{cases}$
- $\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$

  Further $\int_0^a f(x)\, dx = \int_0^a f(a-x) dx$

- $\displaystyle\int_0^{2a} f(x)\,dx = \int_0^a (f(x)+f(2a-x))\,dx$

$$= \begin{cases} 2\displaystyle\int_0^a f(x)dx, & \text{if } f(2a-x)=f(x) \\ 0 & \text{, if } f(2a-x)=-f(x) \end{cases}$$

If $f(x)$ is a periodic function with period T, then

- $\displaystyle\int_0^{nT} f(x)\,dx = n\int_0^T f(x)\,dx, n \in Z$
- $\displaystyle\int_a^{a+nT} f(x)\,dx = n\int_0^T f(x)\,dx, n \in Z, a \in R$
- $\displaystyle\int_{mT}^{nT} f(x)\,dx = (n-m)\int_0^T f(x)\,dx, m, n \in Z$
- $\displaystyle\int_{a+nT}^{b+nT} f(x)\,dx = \int_a^b f(x)\,dx, n \in z, a, b \in R$
- If $\psi(x) \le f(x) \le \phi(x)$ for $a \le x \le b$, then
  $\displaystyle\int_a^b \psi(x)\,dx \le \int_a^b f(x)\,dx \le \int_a^b \phi(x)\,dx$
- $\displaystyle\left|\int_a^b f(x)\,dx\right| \le \int_a^b |f(x)|\,dx$

Leibnitz Theorem

If $F(x) = \displaystyle\int_{g(x)}^{h(x)} f(t)\,dt$,

then $\dfrac{dF(x)}{dx} = h'(x)\,f(h(x)) - g'(x)\,f(g(x))$

Walli's formula

- $\displaystyle\int_0^{\frac{\pi}{2}} \sin^m x \cos^n x\,dx =$

$$\begin{cases} \dfrac{[(m-1)(m-3)...5\cdot3\cdot1][(n-1)(n-3)(n-5)...5\cdot3\cdot1]}{(m+n)(m+n-2)...6\cdot4\cdot2} \times \dfrac{\pi}{2} \\ \quad m, n \text{ both are even positive integers} \\ \dfrac{[(m-1)(m-3)...5\cdot3\cdot1][(n-1)(n-3)...6\cdot4\cdot2]}{(m+n)(m+n-2)...5\cdot3\cdot1} \\ \quad \text{if } m \text{ is even } \& \, n \text{ is odd positive integer} \\ \dfrac{[(m-1)(m-3)...6\cdot4\cdot2][(n-1)(n-3)...5\cdot3\cdot1]}{(m+n)(m+n-2)...5\cdot3\cdot1} \\ \quad \text{if } m \text{ is odd and } n \text{ is even positive integer.} \end{cases}$$

where m, n is positive integers.

Note : (i) If all the powers of 'x' and 'y' in the equation are even, the curve is symmetrical about the axis of 'x' as well as 'y' such as $x^2 + y^2 = a^2$.

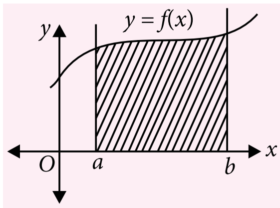
(ii) If the equation of the curve remains unchanged on interchanging 'x' and 'y', then the curve is symmetrical about the line $y = x$ such as $x^3 + y^3 = 3axy$

(iii) If all the powers of 'x' in the equation are even then the curve is symmetrical about the y-axis.

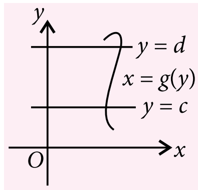
(iv) If all the powers of 'y' in the equation are even then the curve is symmetrical about the x-axis.

Area Under the Curve

- If $f(x) \ge 0$ for $x \in [a, b]$, then area bounded by curve $y = f(x)$ and x-axis between $x = a$ and $x = b$ is $\displaystyle\int_a^b f(x)\,dx$.

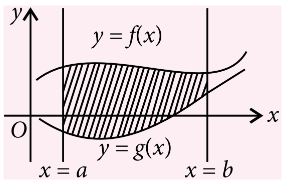


- If $g(y) \ge 0$ for $y \in [c,d]$ then area bounded by curve $x = g(y)$ and y-axis between abscissa $y = c$ and $y = d$ is $\displaystyle\int_c^d g(y)\,dy$

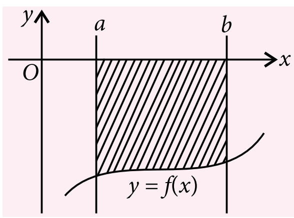


- If $f(x) > 0$ for $x \in [a, c]$ and $f(x) \le 0$ for $x \in [c, b]$ $(a < c < b)$ then area bounded by curve $y = f(x)$ and x-axis between $x = a$ and $x = b$ is $\displaystyle\int_a^c f(x)\,dx - \int_c^b f(x)\,dx$

- If $f(x) \ge g(x)$ for $x \in [a, b]$ then area bounded by curves $y = f(x)$ and $y = g(x)$ between $x = a$ and $x = b$ is $\displaystyle\int_a^b (f(x) - g(x))\,dx$



- If $f(x) \le 0$ for $x \in [a, b]$, then area bounded by curve $y = f(x)$ and x-axis between $x = a$ and $x = b$ is $-\displaystyle\int_a^b f(x)\,dx$.



PROBLEMS

Single Correct Answer Type

1. $\int x^{51}(\tan^{-1}x+\cot^{-1}x)\,dx=$

(a) $\frac{x^{52}}{52}(\tan^{-1}x+\cot^{-1}x)+c$

(b) $\frac{x^{52}}{52}(\tan^{-1}x-\cot^{-1}x)+c$

(c) $\frac{\pi x^{52}}{104}+\frac{\pi}{2}+c$ (d) $\frac{x^{52}}{52}+\frac{\pi}{2}+c$

2. $\int\frac{dx}{\sin x+\cos x}=$

(a) $\log\tan\left(\frac{\pi}{8}+\frac{x}{2}\right)+c$ (b) $\log\tan\left(\frac{\pi}{8}-\frac{x}{2}\right)+c$

(c) $\frac{1}{\sqrt{2}}\log\tan\left(\frac{\pi}{8}+\frac{x}{2}\right)+c$ (d) None of these

3. $\int\frac{\sin^3x+\cos^3x}{\sin^2x\cos^2x}dx=$

(a) $\tan x+\cot x+c$ (b) $\tan x-\cot x+c$
(c) $\operatorname{cosec}x-\cot x+c$ (d) $\sec x-\operatorname{cosec}x+c$

4. $\int\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}dx=$

(a) $2[\sin x+x\cos\alpha]+c$ (b) $2[\sin x+\sin\alpha]+c$
(c) $2[-\sin x+x\cos\alpha]+c$ (d) $-2[\sin x+\sin\alpha]+c$

5. $\int\frac{x^4+x^2+1}{x^2-x+1}dx=$

(a) $\frac{1}{3}x^3+\frac{1}{2}x^2+x+c$ (b) $\frac{1}{3}x^3-\frac{1}{2}x^2+x+c$

(c) $\frac{1}{3}x^3+\frac{1}{2}x^2-x+c$ (d) None of these

6. $\int\frac{1}{\sqrt{1+\cos x}}dx=$

(a) $\sqrt{2}\log\left(\sec\frac{x}{2}+\tan\frac{x}{2}\right)+c$

(b) $\frac{1}{\sqrt{2}}\log\left(\sec\frac{x}{2}+\tan\frac{x}{2}\right)+c$

(c) $\log\left(\sec\frac{x}{2}+\tan\frac{x}{2}\right)+c$ (d) None of these

7. $\int\frac{dx}{x+x\log x}=$

(a) $\log(1+\log x)$ (b) $\log\log(1+\log x)$
(c) $\log x+\log(\log x)$ (d) None of these

8. $\int\sec^p x\tan x\,dx=$

(a) $\frac{\sec^{p+1}x}{p+1}+c$ (b) $\frac{\sec^p x}{p}+c$

(c) $\frac{\tan^{p+1}x}{p+1}+c$ (d) $\frac{\tan^p x}{p}+c$

9. $\int\frac{e^x(x+1)}{\cos^2(xe^x)}dx=$

(a) $\tan(xe^x)+c$ (b) $\sec(xe^x)\tan(xe^x)+c$
(c) $-\tan(xe^x)+c$ (d) None of these

10. $\int\frac{\sin x\cos x}{a\cos^2x+b\sin^2x}dx=$

(a) $\frac{1}{2(b-a)}\log(a\cos^2x+b\sin^2x)+c$

(b) $\frac{1}{b-a}\log(a\cos^2x+b\sin^2x)+c$

(c) $\frac{1}{2}\log(a\cos^2x+b\sin^2x)+c$

(d) None of these

11. $\int\frac{e^{2x}-1}{e^{2x}+1}dx=$

(a) $\frac{e^{2x}-1}{e^{2x}+1}+c$ (b) $\log(e^{2x}+1)-x+c$

(c) $\log(e^{2x}+1)+c$ (d) None of these

12. $\int\frac{1}{\cos^{-1}x\cdot\sqrt{1-x^2}}dx=$

(a) $\log(\cos^{-1}x)+c$ (b) $-\log(\cos^{-1}x)+c$

(c) $-\frac{1}{2(\cos^{-1}x)^2}+c$ (d) None of these

13. $\int\frac{1}{x^2\sqrt{1+x^2}}dx=$

(a) $-\frac{\sqrt{1+x^2}}{x}+c$ (b) $\frac{\sqrt{1+x^2}}{x}+c$

(c) $-\frac{\sqrt{1-x^2}}{x}+c$ (d) $-\frac{\sqrt{x^2-1}}{x}+c$

14. $\int\sin^3x\,dx$ is equal to

(a) $\sin^2x+1$ (b) $\sin x^2+x^2+1$

(c) $\frac{\cos^3x}{3}-\cos x$ (d) $\frac{1}{4}\sin^4x-\frac{3}{4}\sin^2x$

15. $\int \frac{dx}{(1+e^x)(1+e^{-x})} =$

(a) $\frac{-1}{1+e^x}$ (b) $\frac{e^x}{1+e^x}$

(c) $\frac{1}{1+e^x}$ (d) None of these

16. $\int \frac{1+\tan^2 x}{1-\tan^2 x} dx$ equals to

(a) $\log\left(\frac{1-\tan x}{1+\tan x}\right)+c$ (b) $\log\left(\frac{1+\tan x}{1-\tan x}\right)+c$

(c) $\frac{1}{2}\log\left(\frac{1-\tan x}{1+\tan x}\right)+c$ (d) $\frac{1}{2}\log\left(\frac{1+\tan x}{1-\tan x}\right)+c$

17. $\int \operatorname{cosec}^4 x\, dx =$

(a) $\cot x+\frac{\cot^3 x}{3}+c$ (b) $\tan x+\frac{\tan^3 x}{3}+c$

(c) $-\cot x-\frac{\cot^3 x}{3}+c$ (d) $-\tan x-\frac{\tan^3 x}{3}+c$

18. $\int \sqrt{\frac{1-x}{1+x}} dx =$

(a) $\sin^{-1} x-\frac{1}{2}\sqrt{1-x^2}+c$ (b) $\sin^{-1} x+\frac{1}{2}\sqrt{1-x^2}+c$

(c) $\sin^{-1} x-\sqrt{1-x^2}+c$ (d) $\sin^{-1} x+\sqrt{1-x^2}+c$

19. $\int \tan^{-1} x\, dx =$

(a) $x\tan^{-1} x+\frac{1}{2}\log(1+x^2)$

(b) $x\tan^{-1} x-\frac{1}{2}\log(1+x^2)$

(c) $(x-1)\tan^{-1} x$

(d) $x\tan^{-1} x-\log(1+x^2)$

20. $\int \left[\log(\log x)+\frac{1}{(\log x)^2}\right] dx =$

(a) $x\log(\log x)+\frac{x}{\log x}+c$

(b) $x\log(\log x)-\frac{x}{\log x}+c$

(c) $x\log(\log x)+\frac{\log x}{x}+c$

(d) $x\log(\log x)-\frac{\log x}{x}+c$

21. $\int \frac{x+\sin x}{1+\cos x} dx$ is equal to

(a) $-x\tan\frac{x}{2}+c$ (b) $x\tan\frac{x}{2}+c$

(c) $x\tan x+c$ (d) $\frac{1}{2}x\tan x+c$

22. If $\int \frac{e^x(1+\sin x)dx}{1+\cos x} = e^x f(x)+c$, then $f(x) =$

(a) $\sin\frac{x}{2}$ (b) $\cos\frac{x}{2}$ (c) $\tan\frac{x}{2}$ (d) $\log\frac{x}{2}$

23. $\int \cos\sqrt{x}\, dx =$

(a) $2[\sqrt{x}\sin\sqrt{x}+\cos\sqrt{x}]+c$

(b) $2[\sqrt{x}\sin\sqrt{x}-\cos\sqrt{x}]+c$

(c) $2[\cos\sqrt{x}-\sqrt{x}\sin\sqrt{x}]+c$

(d) $-2[\sqrt{x}\sin\sqrt{x}+\cos\sqrt{x}]+c$

24. $\int \tan^{-1}\frac{2x}{1-x^2} dx =$

(a) $x\tan^{-1} x+c$

(b) $x\tan^{-1}x-\log(1+x^2)+c$

(c) $2x\tan^{-1}x+\log(1+x^2)+c$

(d) $2x\tan^{-1}x-\log(1+x^2)+c$

25. $\int \frac{x\sin^{-1} x}{\sqrt{1-x^2}} dx =$

(a) $x-\sqrt{1-x^2}\sin^{-1} x+c$ (b) $x+\sqrt{1-x^2}\sin^{-1} x+c$

(c) $\sqrt{1-x^2}\sin^{-1} x-x+c$ (d) None of these

26. $\int \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx =$

(a) $\frac{x}{\sqrt{1-x^2}}\sin^{-1} x+\frac{1}{2}\log(1-x^2)+c$

(b) $\frac{x}{\sqrt{1-x^2}}\sin^{-1} x-\frac{1}{2}\log(1-x^2)+c$

(c) $\frac{1}{\sqrt{1-x^2}}\sin^{-1} x-\frac{1}{2}\log(1-x^2)+c$

(d) $\frac{1}{\sqrt{1-x^2}}\sin^{-1} x+\frac{1}{2}\log(1-x^2)+c$

27. $\int e^{\tan^{-1} x}\left(\frac{1+x+x^2}{1+x^2}\right) dx$ is equal to

(a) $xe^{\tan^{-1} x}+c$ (b) $x^2e^{\tan^{-1} x}+c$

(c) $\frac{1}{x}e^{\tan^{-1} x}+c$ (d) None of these

28. $I_1 = \int \sin^{-1} x\, dx$ and $I_2 = \int \sin^{-1} \sqrt{1-x^2}\, dx$ then

(a) $I_1 = I_2$ (b) $I_2 = (\pi/2)I_1$

(c) $I_1 + I_2 = (\pi/2)x + K$ (d) $I_1 + I_2 = \pi/2$

29. $\int \dfrac{x^2}{(x^2+2)(x^2+3)} dx =$

(a) $-\sqrt{2}\tan^{-1} x + \sqrt{3}\tan^{-1} x + c$

(b) $-\sqrt{2}\tan^{-1} \dfrac{x}{\sqrt{2}} + \sqrt{3}\tan^{-1} \dfrac{x}{\sqrt{3}} + c$

(c) $\sqrt{2}\tan^{-1} \dfrac{x}{\sqrt{2}} + \sqrt{3}\tan^{-1} \dfrac{x}{\sqrt{3}} + c$

(d) None of these

30. $\int \sin^3 x \cos^2 x\, dx =$

(a) $\dfrac{\cos^5 x}{5} - \dfrac{\cos^3 x}{3} + c$ (b) $\dfrac{\cos^5 x}{5} + \dfrac{\cos^3 x}{3} + c$

(c) $\dfrac{\sin^5 x}{5} - \dfrac{\sin^3 x}{3} + c$ (d) $\dfrac{\sin^5 x}{5} + \dfrac{\sin^3 x}{3} + c$

31. $\int \dfrac{\cos x}{(1+\sin x)(2+\sin x)} dx =$

(a) $\log[(1 + \sin x)(2 + \sin x)] + c$

(b) $\log \dfrac{2+\sin x}{1+\sin x} + c$ (c) $\log \dfrac{1+\sin x}{2+\sin x} + c$

(d) None of these

32. $\int \dfrac{dx}{e^x + 1 - 2e^{-x}} =$

(a) $\log(e^x - 1) - \log(e^x - 2)$

(b) $\dfrac{1}{2}\log(e^x - 1) - \dfrac{1}{3}\log(e^x + 2) + c$

(c) $\dfrac{1}{3}\log(e^x - 1) - \dfrac{1}{3}\log(e^x + 2) + c$

(d) $\dfrac{1}{3}\log(e^x - 1) + \dfrac{1}{3}\log(e^x + 2) + c$

33. $\int \sin^5 x \cos^4 x\, dx =$

(a) $-\dfrac{1}{5}\cos^5 x + \dfrac{2}{7}\cos^7 x - \dfrac{1}{9}\cos^9 x + c$

(b) $\dfrac{1}{5}\cos^5 x + \dfrac{2}{7}\cos^7 x - \dfrac{1}{9}\cos^9 x + c$

(c) $\dfrac{1}{5}\cos^5 x + \dfrac{2}{7}\cos^7 x + \dfrac{1}{9}\cos^9 x + c$

(d) None of these

34. $\int \dfrac{dx}{5 + 4\cos x} =$

(a) $\dfrac{2}{3}\tan^{-1}\left(\dfrac{1}{3}\tan x\right) + c$ (b) $\dfrac{1}{3}\tan^{-1}\left(\dfrac{1}{3}\tan x\right) + c$

(c) $\dfrac{2}{3}\tan^{-1}\left(\dfrac{1}{3}\tan \dfrac{x}{2}\right) + c$ (d) $\dfrac{1}{3}\tan^{-1}\left(\dfrac{1}{3}\tan \dfrac{x}{2}\right) + c$

35. $\int_0^1 \dfrac{dx}{[ax + b(1-x)]^2} =$

(a) $\dfrac{a}{b}$ (b) $\dfrac{b}{a}$ (c) ab (d) $\dfrac{1}{ab}$

36. $\int_0^{1/\sqrt{2}} \dfrac{\sin^{-1} x}{(1-x^2)^{3/2}} dx =$

(a) $\dfrac{\pi}{4} + \dfrac{1}{2}\log 2$ (b) $\dfrac{\pi}{4} - \dfrac{1}{2}\log 2$

(c) $\dfrac{\pi}{2} + \log 2$ (d) $\dfrac{\pi}{2} - \log 2$

37. $\int_0^{\pi} \dfrac{dx}{1 + \sin x} =$

(a) 0 (b) $\dfrac{1}{2}$ (c) 2 (d) $\dfrac{3}{2}$

38. $\int_0^{\pi/2} \dfrac{\sin x \cos x}{1 + \sin^4 x} dx =$

(a) $\dfrac{\pi}{2}$ (b) $\dfrac{\pi}{4}$ (c) $\dfrac{\pi}{6}$ (d) $\dfrac{\pi}{8}$

39. If $\int_0^1 x \log\left(1 + \dfrac{x}{2}\right) dx = a + b \log \dfrac{2}{3}$, then

(a) $a = \dfrac{3}{2},\ b = \dfrac{3}{2}$ (b) $a = \dfrac{3}{4},\ b = -\dfrac{3}{4}$

(c) $a = \dfrac{3}{4},\ b = \dfrac{3}{2}$ (d) $a = b$

40. $\int_0^{\pi/4} \dfrac{4\sin 2\theta\, d\theta}{\sin^4\theta + \cos^4\theta} =$

(a) $\pi/4$ (b) $\pi/2$

(c) π (d) None of these

41. If $x(x^4 + 1)\,\phi(x) = 1$, then $\int_1^2 \phi(x)\, dx =$

(a) $\dfrac{1}{4}\log\dfrac{32}{17}$ (b) $\dfrac{1}{2}\log\dfrac{32}{17}$

(c) $\dfrac{1}{4}\log\dfrac{16}{17}$ (d) None of these

42. $\int_0^{\pi/4} \dfrac{\sec x}{1 + 2\sin^2 x}$ is equal to

(a) $\frac{1}{3}\left[\log(\sqrt{2}+1)+\frac{\pi}{2\sqrt{2}}\right]$ (b) $\frac{1}{3}\left[\log(\sqrt{2}+1)-\frac{\pi}{2\sqrt{2}}\right]$

(c) $3\left[\log(\sqrt{2}+1)-\frac{\pi}{2\sqrt{2}}\right]$ (d) $3\left[\log(\sqrt{2}+1)+\frac{\pi}{2\sqrt{2}}\right]$

43. The value of $\int_0^{\sin^2 x}\sin^{-1}\sqrt{t}\,dt+\int_0^{\cos^2 x}\cos^{-1}\sqrt{t}\,dt$ is

(a) $\frac{\pi}{2}$ (b) 1

(c) $\frac{\pi}{4}$ (d) None of these

44. If for non-zero x, $af(x)+bf\left(\frac{1}{x}\right)=\frac{1}{x}-5$, where $a\neq b$, then $\int_1^2 f(x)\,dx=$

(a) $\frac{1}{(a^2+b^2)}\left[a\log 2-5a+\frac{7}{2}b\right]$

(b) $\frac{1}{(a^2-b^2)}\left[a\log 2-5a+\frac{7}{2}b\right]$

(c) $\frac{1}{(a^2-b^2)}\left[a\log 2-5a-\frac{7}{2}b\right]$

(d) $\frac{1}{(a^2+b^2)}\left[a\log 2-5a-\frac{7}{2}b\right]$

45. $\int_0^{\pi/4}\sec x\log(\sec x+\tan x)\,dx=$

(a) $\frac{1}{2}[\log(1+\sqrt{2})]^2$ (b) $[\log(1+\sqrt{2})]^2$

(c) $\frac{1}{2}[\log(\sqrt{2}-1)]^2$ (d) $[\log(\sqrt{2}-1)]^2$

46. $\int_0^{\pi/4}\frac{\sec^2 x}{(1+\tan x)(2+\tan x)}dx=$

(a) $\log_e\left(\frac{2}{3}\right)$ (b) $\log_e 3$

(c) $\frac{1}{2}\log_e\left(\frac{4}{3}\right)$ (d) $\log_e\left(\frac{4}{3}\right)$

47. $\int_0^{\pi/4}(\cos x-\sin x)dx+\int_{\pi/4}^{5\pi/4}(\sin x-\cos x)dx$ $+\int_{2\pi}^{\pi/4}(\cos x-\sin x)\,dx$ is equal to

(a) $\sqrt{2}-2$ (b) $2\sqrt{2}-2$

(c) $3\sqrt{2}-2$ (d) $4\sqrt{2}-2$

48. $\int_8^{15}\frac{dx}{(x-3)\sqrt{x+1}}=$

(a) $\frac{1}{2}\log\frac{5}{3}$ (b) $\frac{1}{3}\log\frac{5}{3}$

(c) $\frac{1}{2}\log\frac{3}{5}$ (d) $\frac{1}{5}\log\frac{3}{5}$

49. $\int_0^1\sin\left(2\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right)dx=$

(a) $\pi/6$ (b) $\pi/4$ (c) $\pi/2$ (d) π

50. Let $I_1=\int_1^2\frac{dx}{\sqrt{1+x^2}}$ and $I_2=\int_1^2\frac{dx}{x}$ then

(a) $I_1>I_2$ (b) $I_2>I_1$

(c) $I_1=I_2$ (d) $I_2>2I_2$

51. The value of $\int_{1/e}^{\tan x}\frac{t\,dt}{1+t^2}+\int_{1/e}^{\cot x}\frac{dt}{t(1+t^2)}=$

(a) -1 (b) 1

(c) 0 (d) None of these

52. The value of $\int_1^{e^2}\frac{dx}{x(1+\ln x)^2}$ is

(a) 2/3 (b) 1/3 (c) 3/2 (d) ln 2

53. $\int_0^{\pi}x\log\sin x\,dx=$

(a) $\frac{\pi}{2}\log\frac{1}{2}$ (b) $\frac{\pi^2}{2}\log\frac{1}{2}$

(c) $\pi\log\frac{1}{2}$ (d) $\pi^2\log\frac{1}{2}$

54. $\int_0^{\pi/2}\log\tan x\,dx=$

(a) $\frac{\pi}{2}\log_e 2$ (b) $-\frac{\pi}{2}\log_e 2$

(c) $\pi\log_e 2$ (d) 0

55. $\int_{-\pi/2}^{\pi/2}\log\left(\frac{2-\sin\theta}{2+\sin\theta}\right)d\theta=$

(a) 0 (b) 1

(c) 2 (d) None of these

56. Area under the curve $y=\sqrt{3x+4}$ between $x=0$ and $x=4$ is

(a) $\frac{56}{9}$ sq. unit (b) $\frac{64}{9}$ sq. unit

(c) 8 sq. unit (d) None of these

57. The ratio of the areas bounded by the curves $y=\cos x$ and $y=\cos 2x$ between $x=0$, $x=\pi/3$ and x-axis, is

(a) $\sqrt{2}:1$ (b) 1 : 1

(c) 1 : 2 (d) 2 : 1

58. The area bounded by the curve $y = x^3$, x-axis and two ordinates $x = 1$ to $x = 2$ equal to

(a) $\frac{15}{2}$ sq. unit (b) $\frac{15}{4}$ sq. unit

(c) $\frac{17}{2}$ sq. unit (d) $\frac{17}{4}$ sq. unit

59. For $0 \le x \le \pi$, the area bounded by $y = x$ and $y = x + \sin x$, is

(a) 2 (b) 4 (c) 2π (d) 4π

60. If a curve $y = a\sqrt{x} + bx$ passes through the point (1, 2) and the area bounded by the curve, line $x = 4$ and x-axis is 8 sq. unit, then

(a) $a = 3, b = -1$ (b) $a = 3, b = 1$

(c) $a = -3, b = 1$ (d) $a = -3, b = -1$

61. The area of the region (in the square unit) bounded by the curve $x^2 = 4y$, line $x = 2$ and x-axis is

(a) 1 (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) $\frac{8}{3}$

62. Area bounded by the curve $xy - 3x - 2y - 10 = 0$, x-axis and the lines $x = 3$, $x = 4$ is

(a) 16 log 2 – 13 (b) 16 log 2 – 3

(c) 16 log 2 + 3 (d) None of these

63. Area bounded by the parabola $y^2 = 2x$ and the ordinates $x = 1$, $x = 4$ is

(a) $\frac{4\sqrt{2}}{3}$ sq. unit (b) $\frac{28\sqrt{2}}{3}$ sq. unit

(c) $\frac{56}{3}$ sq. unit (d) None of these

64. The area bounded by the x-axis, the curve $y = f(x)$ and the lines $x = 1$, $x = b$ is equal to $\sqrt{b^2+1} - \sqrt{2}$ for all $b > 1$, then $f(x)$ is

(a) $\sqrt{x-1}$ (b) $\sqrt{x+1}$

(c) $\sqrt{x^2+1}$ (d) $\frac{x}{\sqrt{1+x^2}}$

65. The area of smaller part between the circle $x^2 + y^2 = 4$ and the line $x = 1$ is

(a) $\frac{4\pi}{3} - \sqrt{3}$ (b) $\frac{8\pi}{3} - \sqrt{3}$

(c) $\frac{4\pi}{3} + \sqrt{3}$ (d) $\frac{5\pi}{3} + \sqrt{3}$

66. Area of the region bounded by the curve $y = \tan x$, tangent drawn to the curve at $x = \frac{\pi}{4}$ and the x-axis is

(a) $\frac{1}{4}$ (b) $\log\sqrt{2} + \frac{1}{4}$

(c) $\log\sqrt{2} - \frac{1}{4}$ (d) None of these

67. Area enclosed by the parabola $ay = 3(a^2 - x^2)$ and x-axis is

(a) $4a^2$ sq. unit (b) $12a^2$ sq. unit

(c) $4a^3$ sq. unit (d) None of these

Assertion & Reason Type

Directions : *In the following questions, Statement-1 is followed by Statement-2. Mark the correct choice as :*

(a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1.

(c) Statement-1 is true, Statement-2 is false.

(d) Statement-1 is false, Statement-2 is true.

68. Statement-1 : The inequality $\int_0^1 \frac{dx}{1+x^n} > 1 - \frac{1}{n}$ is true for $n \in N$.

Statement-2 : $\int_0^1 \frac{x^{n-1} - x^n + x^{2n-1}}{1+x^n} dx > 0, \ \forall \ n \in N$.

69. Statement-1: If $F(x) = \int_1^x \frac{\ln t}{1+t+t^2} dt$, then $F(x) = -F(1/x)$

Statement-2: If $F(x) = \int_1^x \frac{\ln t}{t+1} dt$, then $F(x) + F(1/x) = (1/2)(\ln x)^2$.

70. Statement-1 : $\int e^{3x}(3\sin x + \cos x)dx = e^{3x}\sin x + c$

Statement-2 : Antiderivative of a periodic function is a periodic function.

71. Statement-1: $\int \frac{1}{4e^{-x} - 9e^x} dx = \frac{1}{6}\log\left|\frac{2+3e^x}{2-3e^x}\right| + C$

Statement - 2: $\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$

72. Statement-1: $\int \frac{e^x(2-x^2)}{(1-x)\sqrt{1-x^2}} dx = e^x\sqrt{\frac{1-x}{1+x}} + c$

Statement-2 : $\int e^x(f(x) + f'(x))dx = e^x f(x) + c$

73. **Statement-1:** $\sum_{r=0}^{12}\int \cot^{2r} x\,dx = -\sum_{r=1}^{6}\frac{\cot^{4r-1} x}{4r-1}+c$

Statement-2: $\int \cot^n x\,dx + \int \cot^{n-2} x\,dx = -\frac{\cot^{n-1} x}{n-1}+c$

74. **Statement-1:** $\int \frac{e^{\cot x}}{\sin^2 x}(2\ln \operatorname{cosec} x + \sin 2x)dx$

$= 2e^{\cot x}\ln\sin x + c$

Statement-2: $\int e^x(f(x)+f'(x))dx = e^x f(x)+c$

75. **Statement-1 :** $\int_1^{\infty}\frac{x^2-2}{x^3\sqrt{x^2-1}}dx = 0$

Statement-2 : $f(x)=\frac{x^2-2}{x^3\sqrt{x^2-1}}$ is an odd function

76. Let $T>0$ be a fixed real number. Suppose $f(x)$ is a continuous function for all $x \in R$, $f(x+T)=f(x)$ then

Statement-1 : $\int_6^{6+6T} f(4x)dx = 6\int_0^T f(x)dx$

Statement-2 : $\int_0^{24T} f(x)dx = 24\int_0^T f(x)dx$

77. **Statement-1 :** $\int_0^{\pi/2}(\sin^6 x+\cos^6 x)\,dx$ lies in the interval $\left(\frac{\pi}{8},\frac{\pi}{2}\right)$.

Statement-2 : $\sin^6 x + \cos^6 x$ is periodic with period $\pi/2$.

78. **Statement-1 :** If root of $ax^2+bx+c=0$ are non real and $c=\int_0^{\sin^2 x}\sin^{-1}\sqrt{x}dx+\int_0^{\cos^2 x}\cos^{-1}\sqrt{x}dx$ then 'a' must be positive.

Statement-2 : If roots of quadratic equation $ax^2+bx+c=0$ are non-real then 'a' and 'c' have same sign.

79. Let $f(x)$ and $g(x)$ be continuous functions of x in (a, b) and $f(x) < g(x) < 0\ \forall\ x \in (a, b)$ then

Statement-1 : Area of the region bounded by $y=f(x)$, x-axis, $x=a$ and $x=b$ is less than the area of the region bounded by $y=g(x)$, x-axis, $x=a$ and $x=b$.

Statement-2 : $|f(x)| > |g(x)|\ \forall\ x \in (a, b)$

80. If $f(x)=\frac{e^x}{1+e^x}$

Statement-1 : $I_1=\int_{f(-a)}^{f(a)} xg(x(1-x))dx$ and $I_2=\int_{f(-a)}^{f(a)} g(x(1-x))dx$ then the value of $\frac{I_2}{I_1}$ is 2.

Statement-2 : Here $f(a)+f(-a)=1$ and also by the property $\int_p^q f(x)\,dx=\int_p^q f(p+q-x)\,dx$

81. Let $\int \frac{x+(\cos^{-1}3x)^2}{\sqrt{1-9x^2}}dx = A\sqrt{1-9x^2} - B(\cos^{-1}3x)^3 + c$, where c is a constant of integration, then

Statement-1 : $A+B=0$

Statement-2 : $A=\frac{2}{9}$ and $B=-\frac{2}{9}$

Comprehension Type

Paragraph for Q. No. 82 – 84

If a curve is given by its parametric equation in the form $x=f(t)$, $y=g(t)$ and suppose the derivatives $f'(t)$ and $g'(t)$ are continuous functions on the interval $[t_1, t_2]$. If t_1 and t_2 are the values of parameter 't' corresponding respectively to the initial and final position in which the curve can be described as a contour in the positive direction (*i.e.*, figure remains left), then the area described by the curve is given by

$$A=-\int_{t_1}^{t_2} g(t)f'(t)dt = -\int_{t_1}^{t_2} f(t)g\,f'(t)dt$$

$$=\frac{1}{2}\int_{t_1}^{t_2}(xg'(t)dt - yf'(t))dt$$

82. Area enclosed by the curve $x=a\cos^3 t$, $y=a\sin^3 t$, $0\le t\le 2\pi$

(a) $\frac{3}{2}a^2\pi$ (b) $\frac{3}{4}a^2\pi$ (c) $\frac{3}{8}a^2\pi$ (d) $\frac{3}{16}a^2\pi$

83. Area of the loop of the curve $x=a(1-t^2)$, $y=at(1-t^2)$, $-1\le t\le 1$ must be

(a) $\frac{2}{15}a^2$ (b) $\frac{4}{15}a^2$ (c) $\frac{8}{15}a^2$ (d) $\frac{a^2}{5}$

84. The area of the curve $x=2\cos t$, $y=2\sin t$ must be

(a) 4π (b) 2π (c) 8π (d) 16π

Paragraph for Q. No. 85 – 87

Let n be a non-negative integer and let $I_n=\int x^n\sqrt{a^2-x^2}dx$, $(a>0)$ then we can find relation among I_n, I_{n-1}, I_{n-2}. It can be observed that I_1 is elementary integration whose value is $-\frac{1}{3}(a^2-x^2)^{3/2}$.

If $I_n = -\dfrac{x^{n-1}(a^2-x^2)^{3/2}}{A} + a^2 B I_{n-2}$ where A and B are constants. Then

85. A must be equal to

(a) $n+1$ (b) $n-1$ (c) $n+2$ (d) n

86. B must be equal to

(a) $\dfrac{n+1}{n+2}$ (b) $\dfrac{n+2}{n+1}$ (c) $\dfrac{n}{n+2}$ (d) $\dfrac{n-1}{n+2}$

87. The value of $\displaystyle\int_0^a x^4\sqrt{a^2-x^2}\,dx$ must be

(a) $\dfrac{\pi a^6}{32}$ (b) $\dfrac{\pi a^4}{16}$ (c) $\dfrac{\pi a^4}{64}$ (d) $\dfrac{\pi a^6}{16}$

Paragraph for Q. No. 88 – 90

A curve passing through origin is such that slope of tangent at any point is reciprocal of sum of co-ordinate of point of tangency.

88. Slope of tangent at ordinate ln3.

(a) 1 (b) $\dfrac{1}{3}$ (c) $\dfrac{1}{2}$ (d) -2

89. Area bounded by curve and the abscissa $y = 0$ and $y = 1$ is

(a) $e-\dfrac{1}{2}$ (b) $e-\dfrac{3}{2}$ (c) $e-\dfrac{5}{2}$ (d) $e+1$

90 $\displaystyle\int_{-1}^{[|\sin\alpha|+|\cos\alpha|]} xe^{-y}\,d(e^y)$ where $[\cdot]$ represents integer function, is equal to

(a) $e-e^{-1}-\dfrac{1}{3}$ (b) $e-e^{-1}-2$

(c) $e-\dfrac{1}{e}$ (d) $e+e^{-1}+\dfrac{1}{3}$

Paragraph for Q. No. 91 – 93

Differentiate I w.r.t. the parameter within the sign of integrals taking variable of the integrand as constant. Now, evaluate the integral so obtained as usual as a function of the parameter and then integrate the result to get I. Constant of integration can be computed by giving some arbitrary values to the parameter and the corresponding value of I.

91. The value of $\displaystyle\int_0^1 \frac{x^a-1}{\log x}\,dx$ is

(a) $\log(a-1)$ (b) $\log(a+1)$

(c) $a\log(a+1)$ (d) None of these

92. The value of $\displaystyle\int_0^{\pi/2} \log(\sin^2\theta + k^2\cos^2\theta)\,d\theta$, where $k \ge 0$, is

(a) $\pi\log(1+k)+\pi\log 2$ (b) $\pi\log(1+k)$

(c) $\pi\log(1+k)-\pi\log 2$ (d) $\pi\log(1+k)-\log 2$

93. If $\displaystyle\int_0^{\pi}\frac{dx}{(a-\cos x)} = \frac{\pi}{\sqrt{a^2-1}}$, then the value of $\displaystyle\int_0^{\pi}\frac{dx}{(\sqrt{10}-\cos x)^3}$ is

(a) $\dfrac{\pi}{81}$ (b) $\dfrac{7\pi}{81}$ (c) $\dfrac{7\pi}{162}$ (d) $\dfrac{7\pi}{8}$

Matrix–Match Type

94. Match the following.

Column-I		Column-II	
(A)	If $\int x^2 d(\tan^{-1}x)dx = x - f(x) + c$ then $f(1)$ is equal to	(p)	$\dfrac{1}{8}$
(B)	If $\int x^2 e^{2x}dx = e^{2x}f(x)+c$, then the minimum value of $f(x)$ is	(q)	0
(C)	If $\displaystyle\int\frac{(x^4+1)}{x(x^2+1)^2}dx = a\log\lvert x\rvert + \frac{b}{x^2+1}+c$ then $a-b$ is equal to	(r)	$\dfrac{\pi}{4}$

95. Match the following.

$[x]$ denotes greatest integer less than or equal to 'x'.

Column I		Column II	
(A)	$\displaystyle 25\int_0^{\pi/4}(\tan^6(x-[x]) + \tan^4(x-[x]))\,dx =$	(p)	4
(B)	$\displaystyle I_1 = \int_{-2}^{2}\frac{x^6+3x^5+7x^4}{x^4+2}dx$, $\displaystyle I_2 = \int_{-3}^{1}\frac{2(x+1)^2+11(x+1)+14}{(x+1)^4+2}dx$ then the $I_1+I_2=$	(q)	2
(C)	$\displaystyle \frac{2}{\pi}\int_{-1}^{3}\left(\tan^{-1}\frac{x}{x^2+1}+\tan^{-1}\frac{x^2+1}{x}\right)dx =$	(r)	$\dfrac{100}{3}$
		(s)	5

Integer Answer Type

96. If $\lim_{n\to\infty}\dfrac{\sum_{r=1}^{n}\sqrt{r}\sum_{r=1}^{n}\dfrac{1}{\sqrt{r}}}{\sum_{r=1}^{n}r}=\dfrac{k}{3}$ then the value of k is

97. $\lim_{n\to\infty}\sum_{k=1}^{n}\dfrac{k^{1/a}\left\{n^{a-\frac{1}{a}}+k^{a-\frac{1}{a}}\right\}}{n^{a+1}}$ is equal to

98. The value of

$$\int_{-\pi/4}^{\pi/4}[(x^9-3x^5+7x^3-x+1)/\cos^2 x]dx \text{ is}$$

99. $f:[0,5]\to R$, $y=f(x)$ such that $f''(x)=f''(5-x)$ $\forall\ x\in[0,5]$ $f'(0)=1$ and $f'(5)=7$, then value of $\int_1^4 f'(x)dx-4$ is

100. Let α,β be roots of the quadratic equation $18x^2-9\pi x+\pi^2=0$, where $\alpha<\beta$. Also $f(x)=x^2$ and $g(x)=\cos x$. If the area bounded by the curve $y=(fog)(x)$, the vertical lines $x=\alpha$, $x=\beta$ and x-axis is $\dfrac{\pi}{\lambda}$, then the sum of the digit of λ is

101. Let $I_n=\int_0^{\pi/2}(\sin x+\cos x)^n\,dx\ (n\geq 2)$. Then the value of $nI_n-2(n-1)I_{n-2}$ is

102. If $\int_0^{\pi/2}\ln\sin x\,dx=\dfrac{\pi}{2}\ln\dfrac{1}{2}$

and $\int_0^{\pi/2}\left(\dfrac{x}{\sin x}\right)^2 dx=\dfrac{k\cdot\pi}{2}\ln 2$, then value of k is

103. Find the value of $\dfrac{29\int_0^1(1-x^4)^7\,dx}{4\int_0^1(1-x^4)^6\,dx}$.

104. Evaluate $\left[\int_0^{\pi}\dfrac{dx}{1+2\sin^2 x}\right]$ where [.] denotes greatest integer function.

105. The function $f(x)=\int_1^x\{2(t-1)(t-2)^3+3(t-1)^2(t-2)^2\}dt$ attains its maximum at x is equal to

SOLUTIONS

Single Correct Answer Type

1. (a) : $\int x^{51}(\tan^{-1}x+\cot^{-1}x)dx=\int x^{51}.\dfrac{\pi}{2}dx$

$$\left\{\because \tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}\right\}$$

$$=\frac{\pi x^{52}}{104}+c=\frac{x^{52}}{52}(\tan^{-1}x+\cot^{-1}x)+c$$

2. (c) : $\int\dfrac{dx}{\sin x+\cos x}=\dfrac{1}{\sqrt{2}}\int\dfrac{dx}{\sin x\cos\frac{\pi}{4}+\cos x\sin\frac{\pi}{4}}$

$$=\frac{1}{\sqrt{2}}\int\operatorname{cosec}\left(x+\frac{\pi}{4}\right)dx=\frac{1}{\sqrt{2}}\log\tan\left(\frac{\pi}{8}+\frac{x}{2}\right)+c$$

3. (d) : $\int\dfrac{\sin^3 x+\cos^3 x}{\sin^2 x\cos^2 x}dx=\int\left(\dfrac{\sin x}{\cos^2 x}+\dfrac{\cos x}{\sin^2 x}\right)dx$

$$=\int(\sec x\tan x+\operatorname{cosec}x\cot x)dx=\sec x-\operatorname{cosec}x+c$$

4. (a) : $\int\dfrac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}dx=\int\dfrac{2(\cos^2 x-\cos^2\alpha)}{\cos x-\cos\alpha}dx$

$$=2\int(\cos x+\cos\alpha)dx=2(\sin x+x\cos\alpha)+c$$

5. (a) : $\int\dfrac{x^4+x^2+1}{x^2-x+1}dx=\int(x^2+x+1)dx$

$$=\frac{x^3}{3}+\frac{x^2}{2}+x+c$$

6. (a) : $\int\dfrac{1}{\sqrt{1+\cos x}}dx=\int\dfrac{dx}{\sqrt{2\cos^2(x/2)}}$

$$=\frac{1}{\sqrt{2}}\int\sec\frac{x}{2}dx$$

$$=\frac{1}{\sqrt{2}}\left\{\log\left(\sec\frac{x}{2}+\tan\frac{x}{2}\right)\right\}.\frac{1}{1/2}$$

$$=\sqrt{2}\log\left(\sec\frac{x}{2}+\tan\frac{x}{2}\right)+c$$

7. (a) : Let $I=\int\dfrac{dx}{x+x\log x}=\int\dfrac{dx}{x(1+\log x)}$

Put $1+\log x=t\Rightarrow\dfrac{1}{x}dx=dt$

$\therefore\ I=\int\dfrac{dt}{t}=\log(t)=\log(1+\log x)$

8. (b) : Put $\sec x=t\Rightarrow\sec x\tan x\,dx=dt$

$\therefore\ \int\sec^p x\tan x\,dx=\int t^{p-1}dt=\dfrac{t^p}{p}+c=\dfrac{\sec^p x}{p}+c$

9. (a) : $\int \frac{e^x(x+1)}{\cos^2(xe^x)}dx = \int e^x(x+1)\sec^2(xe^x)dx$

Putting $xe^x = t \Rightarrow (x+1)e^x dx = dt$, we get

$\int \sec^2 t\,dt = \tan t + c = \tan(xe^x)+c$

10. (a) : Put $a\cos^2 x + b\sin^2 x = t$

$\Rightarrow \quad 2(b-a)\sin x\cos x = dt,$

then $\int \frac{\sin x\cos x\,dx}{a\cos^2 x + b\sin^2 x} = \frac{1}{2(b-a)}\int \frac{1}{t}dt$

$$= \frac{1}{2(b-a)}\log(a\cos^2 x + b\sin^2 x)+c$$

11. (b) : Let $I = \int \frac{e^{2x}-1}{e^{2x}+1}dx = \int \frac{e^x - e^{-x}}{e^x + e^{-x}}dx$

Now put $e^x + e^{-x} = t \Rightarrow (e^x - e^{-x})dx = dt,$

$\therefore \quad I = \int \frac{dt}{t} = \log t = \log(e^x + e^{-x}) = \log(e^{2x}+1) - x + c$

12. (b)

13. (a) : Put $x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$, then

$\int \frac{1}{x^2\sqrt{1+x^2}}dx = \int \frac{\sec^2\theta\, d\theta}{\tan^2\theta\sec\theta} = \int \operatorname{cosec}\theta\cot\theta\, d\theta$

$$= -\operatorname{cosec}\theta + c = \frac{-\sqrt{x^2+1}}{x}+c$$

14. (c) : $\int \sin^3 x\,dx = \int \sin^2 x\cdot \sin x\,dx$

$= \int \sin x(1-\cos^2 x)dx = \int \sin x\,dx - \int \cos^2 x\cdot\sin x\,dx$

$= -\cos x + \frac{\cos^3 x}{3}+c$

15. (a) : Let $I = \int \frac{dx}{(1+e^x)\left(1+\frac{1}{e^x}\right)} = \int \frac{e^x dx}{(1+e^x)^2}$

Let $1 + e^x = t$, $\therefore$ $e^x\,dx = dt$

$\therefore \quad I = \int \frac{dt}{t^2} = \int t^{-2}dt = \frac{t^{-1}}{-1} = \frac{(1+e^x)^{-1}}{-1} = \frac{-1}{1+e^x}$

16. (d) : $I = \int \frac{1+\tan^2 x}{1-\tan^2 x}dx = \int \frac{\sec^2 x}{1-\tan^2 x}dx$

Put $\tan x = t \Rightarrow \sec^2 x.dx = dt \quad \Rightarrow \quad I = \int \frac{dt}{1-t^2}$

$= \frac{1}{2\times 1}\log\left[\frac{1+t}{1-t}\right]+c = \frac{1}{2}\log\left|\frac{1+\tan x}{1-\tan x}\right|+c$

17. (c) : $\int \operatorname{cosec}^4 x\,dx = \int \operatorname{cosec}^2 x.\operatorname{cosec}^2 x\,dx$

$= \int \operatorname{cosec}^2 x(1+\cot^2 x)dx$

$= \int \operatorname{cosec}^2 x\,dx + \int \cot^2 x.\operatorname{cosec}^2 x\,dx$

$= -\cot x - \frac{\cot^3 x}{3}+c$

18. (d) : $\int \sqrt{\frac{1-x}{1+x}}dx = \int \frac{1-x}{\sqrt{1-x^2}}dx$

$= \int \frac{1}{\sqrt{1-x^2}}dx - \int \frac{x\,dx}{\sqrt{1-x^2}} = \sin^{-1}x + \sqrt{1-x^2}+c$

19. (b) : $\int \tan^{-1}x\,dx = x\tan^{-1}x - \int \frac{x}{1+x^2}dx + c$

$= x\tan^{-1}x - \frac{1}{2}\log(1+x^2)+c.$

20. (b) : $\int \left[\log(\log x) + \frac{1}{(\log x)^2}\right] =$

$$\int \log(\log x)dx + \int \frac{1}{(\log x)^2}$$

$= x\log(\log x) - \int \frac{x}{x\log x}dx + \int \frac{1}{(\log x)^2}dx$

$= x\log(\log x) - \frac{x}{\log x} - \int \frac{1}{(\log x)^2}dx + \int \frac{1}{(\log x)^2}dx$

$= x\log(\log x) - \frac{x}{\log x}+c.$

21. (b) : $\int \frac{x+\sin x}{1+\cos x}dx = \frac{1}{2}\int x\sec^2\frac{x}{2}dx + \int \tan\frac{x}{2}dx$

$= \frac{1}{2}\frac{x\tan\frac{x}{2}}{\frac{1}{2}} - \int \tan\frac{x}{2}dx + \int \tan\frac{x}{2}dx = x\tan\frac{x}{2}+c$

22. (c) : Let $I = \int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx$

$= \int e^x\left[\frac{1+2\sin(x/2)\cos(x/2)}{2\cos^2(x/2)}\right]dx$

$I = \int e^x\left[\frac{1}{2}\sec^2(x/2) + \tan(x/2)\right]dx = e^x\tan(x/2)+c$

$\{\because \int e^x[f(x)+f'(x)]dx = e^x\cdot f(x)+c\}$

23. (a) : Put $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}}dx = dt \Rightarrow dx = 2t\,dt$, then

$$\int 2t.\cos t\,dt = 2\left[t\sin t - \int \sin t\,dt\right]$$
$$= 2t\sin t + 2\cos t + c = 2[\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x}] + c$$

24. (d) : Put $x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$, then

$$\int \tan^{-1}\frac{2x}{1-x^2}dx = \int \tan^{-1}\frac{2\tan\theta}{1-\tan^2\theta}\sec^2\theta\, d\theta$$
$$= \int \tan^{-1}(\tan 2\theta)\sec^2\theta\, d\theta = \int 2\theta\sec^2\theta\, d\theta$$
$$= 2\left[\theta\tan\theta - \int\tan\theta\, d\theta\right] = 2x\tan^{-1}x - \log(x^2+1) + c$$

25. (a) : Putting $\sin^{-1}x = t \Rightarrow \frac{1}{\sqrt{1-x^2}}dx = dt$, we get

$$\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx = \int t\sin t\,dt = -t\cos t + \sin t + c$$
$$= -\sin^{-1}x\cos(\sin^{-1}x) + \sin(\sin^{-1}x) + c$$
$$= x - \sqrt{1-x^2}\sin^{-1}x + c$$

26. (a) : Put $t = \sin^{-1}x$

$\Rightarrow$ $\sin t = x \Rightarrow \cos t\, dt = dx$, then

$$\int \frac{\sin^{-1}x}{(1-x^2)^{3/2}}dx = t\sec^2 t\,dt = t\tan t + \log\cos t + c$$
$$= \sin^{-1}x\tan(\sin^{-1}x) + \log\cos(\sin^{-1}x) + c$$
$$= \frac{x}{\sqrt{1-x^2}}\sin^{-1}x + \frac{1}{2}\log(1-x^2) + c.$$

27. (a) : Putting $\tan^{-1}x = t$ and $\frac{dx}{1+x^2} = dt$, we get

$$\int e^{\tan^{-1}x}\left(\frac{1+x+x^2}{1+x^2}\right)dx = \int e^t(\tan t + \sec^2 t)dt$$
$$= e^t\tan t + c = x e^{\tan^{-1}x} + c$$
$$\left[\text{Using } \int e^x\{f(x) + f'(x)\}dx = e^x f(x) + C\right]$$

28. (c)

29. (b) : $$\int \frac{x^2}{(x^2+2)(x^2+3)}dx = \int\left[\frac{3}{x^2+3} - \frac{2}{x^2+2}\right]dx$$
$$= \frac{3}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} - \frac{2}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + c$$
$$= \sqrt{3}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) - \sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + c$$

30. (a) : Let $I = \int \sin^3 x\cos^2 x\,dx$
$$= \int (1-\cos^2 x)\cos^2 x\cdot\sin x\,dx$$

Put $\cos x = t \Rightarrow -\sin x\,dx = dt$

$$\therefore\ I = -\int (t^2 - t^4)dt = \frac{t^5}{5} - \frac{t^3}{3} + c = \frac{(\cos x)^5}{5} - \frac{(\cos x)^3}{3} + c$$

31. (c) : Put $\sin x = t \Rightarrow \cos x\,dx = dt$, then

$$\int \frac{\cos x}{(1+\sin x)(2+\sin x)}dx = \int \frac{dt}{(t+1)(t+2)}$$
$$= \int \frac{1}{t+1}dt - \int \frac{1}{t+2}dt = \log\left(\frac{t+1}{t+2}\right) + c$$
$$= \log\left(\frac{\sin x + 1}{\sin x + 2}\right) + c$$

32. (c) : $$\int \frac{e^x dx}{e^{2x} + e^x - 2} = \int \frac{dt}{t^2 + t - 2},\ t = e^x$$
$$= \int \frac{dt}{(t+2)(t-1)} = \int \frac{1}{3}\left[\frac{1}{t-1} - \frac{1}{t+2}\right]dt$$
$$= \frac{1}{3}\log(e^x - 1) - \frac{1}{3}\log(e^x + 2) + c$$

33. (a) : Put $\cos x = t \Rightarrow -\sin x\,dx = dt$, then

$$\int (1-\cos^2 x)^2\cdot\cos^4 x\cdot\sin x\,dx = -\int (1-t^2)^2\cdot t^4 dt$$
$$= -\frac{t^5}{5} + \frac{2}{7}t^7 - \frac{1}{9}t^9 + c$$
$$= -\frac{\cos^5 x}{5} + \frac{2}{7}\cos^7 x - \frac{1}{9}\cos^9 x + c$$

34. (c) : $$\int \frac{dx}{5 + 4\cos x}$$
$$= \int \frac{dx}{5 + 4\left[\dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right]} = \int \frac{\sec^2\frac{x}{2}}{9 + \tan^2\frac{x}{2}}dx$$

Put $\tan\frac{x}{2} = t \Rightarrow \sec^2\frac{x}{2}\cdot\frac{1}{2}dx = dt$,

$$2\int \frac{dt}{3^2 + t^2} = \frac{2}{3}\tan^{-1}\left[\frac{1}{3}\tan\frac{x}{2}\right] + c$$

35. (d) : Let $I = \int_0^1 \frac{dx}{[(a-b)x + b]^2}$

Put $t = (a - b)x + b \Rightarrow dt = (a - b)dx$

As $x = 1 \Rightarrow t = a$ and $x = 0 \Rightarrow t = b$, then

$$I = \frac{1}{a-b}\int_b^a \frac{1}{t^2}dt = \frac{1}{(a-b)}\left[-\frac{1}{t}\right]_b^a = \frac{1}{(a-b)}\left(\frac{a-b}{ab}\right) = \frac{1}{ab}$$

36. (b) : $I=\int_0^{1/\sqrt{2}}\frac{\sin^{-1}x}{(1-x^2)^{3/2}}dx$

Put $\sin^{-1}x=t\Rightarrow\frac{1}{\sqrt{1-x^2}}dx=dt$ and $x=\sin t$

Also $t=0$ to $\frac{\pi}{4}$ as $x=0$ to $\frac{1}{\sqrt{2}}$

$$\Rightarrow I=\int_0^{\pi/4}t\cdot\sec^2t\,dt=\frac{\pi}{4}-\frac{1}{2}\log 2$$

37. (c) : $\int_0^{\pi}\frac{dx}{1+\sin x}=\int_0^{\pi}\frac{1-\sin x}{\cos^2 x}dx$

$=\int_0^{\pi}(\sec^2x-\sec x\tan x)dx$

$=[\tan x-\sec x]_0^{\pi}=[\tan\pi-\sec\pi+1]=[0+1+1]=2$

38. (d) : Put $\sin^2x=t\Rightarrow dt=2\sin x\cos x\,dx$

Now $\int_0^{\pi/2}\frac{\sin x\cos x}{1+\sin^4x}dx=\frac{1}{2}\int_0^1\frac{1}{1+t^2}dt=\frac{1}{2}[\tan^{-1}t]_0^1=\frac{\pi}{8}$

39. (c) : Let $I=\int_0^1 x\log\left(1+\frac{x}{2}\right)dx$

$$=\left[\log\left(1+\frac{x}{2}\right)\frac{x^2}{2}\right]_0^1-\int_0^1\frac{1}{1+\frac{x}{2}}\cdot\frac{1}{2}\cdot\frac{x^2}{2}dx$$

$$=\frac{1}{2}\log\frac{3}{2}-\frac{1}{2}\int_0^1\frac{x^2}{x+2}dx$$

$$=\frac{1}{2}\log\frac{3}{2}-\frac{1}{2}\left[\frac{1}{2}-2+4\log 3-4\log 2\right]=\frac{3}{4}+\frac{3}{2}\log\frac{2}{3}$$

On comparing with the given value $a=3/4$, $b=3/2$.

40. (c) : $4\int_0^{\pi/4}\frac{\sin 2\theta\,d\theta}{\sin^4\theta+\cos^4\theta}=4\int_0^{\pi/4}\frac{2\sin\theta\cos\theta\,d\theta}{\sin^4\theta+\cos^4\theta}$

$$=4\int_0^{\pi/4}\frac{2\tan\theta\sec^2\theta\,d\theta}{\tan^4\theta+1}$$

Now put $\tan^2\theta=t\Rightarrow 2\tan\theta\sec^2\theta\,d\theta=dt$, then the reduced form is

$$4\int_0^1\frac{dt}{t^2+1}=4[\tan^{-1}t]_0^1=4\left[\frac{1}{4}\pi-0\right]=\pi$$

41. (a) : Here $\phi(x)=\frac{1}{x(x^4+1)}=\frac{1}{x}-\frac{x^3}{x^4+1}$

$$\Rightarrow\int_1^2\phi(x)dx=\int_1^2\left(\frac{1}{x}-\frac{x^3}{x^4+1}\right)dx$$

$$=|\log x|_1^2-\left|\frac{1}{4}\log(x^4+1)\right|_1^2=\frac{1}{4}\log\frac{32}{17}$$

42. (a)

43. (c) : Let $I=\int_0^{\sin^2x}\sin^{-1}\sqrt{t}\,dt+\int_0^{\cos^2x}\cos^{-1}\sqrt{t}\,dt$

Putting $t=\sin^2u$ in the first integral and $t=\cos^2v$ in the second integral, we have

$$I=\int_0^x u\sin 2u\,du-\int_{\pi/2}^x v\sin 2v\,dv$$

$$=\int_0^{\pi/2}u\sin 2u\,du+\int_{\pi/2}^x u\sin 2u\,du-\int_{\pi/2}^x v\sin 2v\,dv$$

$$I=\int_0^{\pi/2}u\sin 2u\,du=\left(\frac{-u\cos 2u}{2}\right)_0^{\pi/2}+\frac{1}{2}\int_0^{\pi/2}\cos 2u\,du$$

$$=\left(\frac{-u\cos 2u}{2}\right)_0^{\pi/2}+\frac{1}{4}(\sin 2u)_0^{\pi/2}=\frac{\pi}{4}$$

44. (b) : $af(x)+bf\left(\frac{1}{x}\right)=\frac{1}{x}-5$ (for each $x\neq 0$) ...(i)

Replacing x by $\frac{1}{x}$ in (i), we get

$af\left(\frac{1}{x}\right)+bf(x)=x-5$ ...(ii)

Eliminating $f\left(\frac{1}{x}\right)$ from (i) and (ii), we get

$$(a^2-b^2)f(x)=\frac{a}{x}-bx-5a+5b$$

$$\Rightarrow (a^2-b^2)\int_1^2 f(x)dx=\left[\left(a\log|x|-\frac{b}{2}x^2-5(a-b)x\right)\right]_1^2$$

$$=a\log 2-2b-10(a-b)-a\log 1+\frac{b}{2}+5(a-b)$$

$$=a\log 2-5a+\frac{7}{2}b$$

$$\Rightarrow\int_1^2 f(x)dx=\frac{1}{a^2-b^2}\left[a\log 2-5a+\frac{7}{2}b\right]$$

45. (a) : Let $I=\int_0^{\pi/4}\sec x\log(\sec x+\tan x)dx$

Put $\log(\sec x+\tan x)=t\Rightarrow\sec x\,dx=dt$

$$\Rightarrow I=\int_0^{\log(\sqrt{2}+1)}t\,dt=\left[\frac{t^2}{2}\right]_0^{\log(\sqrt{2}+1)}=\frac{[\log(\sqrt{2}+1)]^2}{2}$$

46. (d) : Put $1+\tan x=t\Rightarrow\sec^2x\,dx=dt$

$$\therefore\int_0^{\pi/4}\frac{\sec^2x}{(1+\tan x)(2+\tan x)}dx$$

$$=\int_1^2\frac{dt}{t(1+t)}=\int_1^2\frac{dt}{t}-\int_1^2\frac{dt}{1+t}=[\log t-\log(1+t)]_1^2$$

$$=\log_e 2-\log_e 3+\log_e 2=\log_e\frac{4}{3}$$

47. (d) : Let $I=\int_0^{\pi/4}(\cos x-\sin x)\,dx$

$$+\int_{\pi/4}^{5\pi/4}(\sin x-\cos x)\,dx+\int_{2\pi}^{\pi/4}(\cos x-\sin x)\,dx$$

$$=[\sin x+\cos x]_0^{\frac{\pi}{4}}-[\sin x+\cos x]_{\frac{\pi}{4}}^{\frac{5\pi}{4}}+[\sin x+\cos x]_{2\pi}^{\frac{\pi}{4}}$$

$$=\left[\frac{1}{\sqrt2}+\frac{1}{\sqrt2}-1\right]-\left[-\frac{1}{\sqrt2}-\frac{1}{\sqrt2}-\left(\frac{1}{\sqrt2}+\frac{1}{\sqrt2}\right)\right]$$

$$+\left[\frac{1}{\sqrt2}+\frac{1}{\sqrt2}-1\right]$$

$$=[\sqrt2-1]-[-\sqrt2-\sqrt2]+[\sqrt2-1]=4\sqrt2-2$$

48. (a) : $I=\int_8^{15}\frac{dx}{(x-3)\sqrt{x+1}}$

Put $x=\tan^2\theta$ and $\theta=\tan^{-1}\sqrt{x}$

$\Rightarrow\ dx=2\tan\theta\sec^2\theta\,d\theta$

$$\therefore\ I=\int_{\tan^{-1}\sqrt8}^{\tan^{-1}\sqrt{15}}\frac{2\tan\theta\sec^2\theta}{(\tan^2\theta-3)\sqrt{\tan^2\theta+1}}\,d\theta$$

$$=\int_{\tan^{-1}\sqrt8}^{\tan^{-1}\sqrt{15}}\frac{2\tan\theta\sec^2\theta}{(\sec^2\theta-4)\sec\theta}\,d\theta$$

$$=\left[\frac12\log\frac{(\sec\theta-2)}{(\sec\theta+2)}\right]_{\tan^{-1}\sqrt8}^{\tan^{-1}\sqrt{15}}$$

$$=\frac12\left[\log\frac26-\log\frac15\right]=\frac12\log\frac53$$

49. (b) **50. (b)**

51. (b) : On integrating both functions, we get

$$=\frac12\left|\log(1+t^2)\right|_{1/e}^{\tan x}+\left|\left\{\log t-\frac12\log(1+t^2)\right\}\right|_{1/e}^{\cot x}$$

$$=\frac12\left[\log\sec^2x-\log\left(1+\frac{1}{e^2}\right)\right]+\log\cot x-\log\left(\frac1e\right)$$

$$=-\frac12\left\{\log(\operatorname{cosec}^2x)-\log\left(1+\frac{1}{e^2}\right)\right\}=-\log\left(\frac1e\right)=\log e=1$$

52. (a)

53. (b) : Let $I=\int_0^{\pi}x\log\sin x\,dx$ ...(i)

$$=\int_0^{\pi}(\pi-x)\log\sin(\pi-x)\,dx \quad ...(ii)$$

By adding (i) and (ii), we get

$$2I=\int_0^{\pi}\pi\log\sin x\,dx\Rightarrow I=\frac{2\pi}{2}\int_0^{\pi/2}\log\sin x\,dx$$

$$=\pi\left(\frac{\pi}{2}\log\frac12\right)=\frac{\pi^2}{2}\log\frac12$$

54. (d) : $\int_0^{\pi/2}\log\tan x\,dx=\int_0^{\pi/2}\log\left(\frac{\sin x}{\cos x}\right)dx$

$$=\int_0^{\pi/2}\log\sin x\,dx-\int_0^{\pi/2}\log\cos x\,dx=0$$

$$\left\{\because\int_0^a f(x)dx=\int_0^a f(a-x)dx\right\}$$

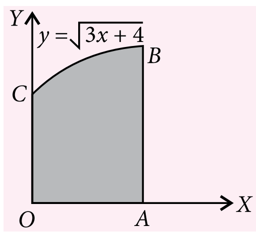
55. (a) : Since $f(-\theta)=\log\left(\frac{2-\sin\theta}{2+\sin\theta}\right)^{-1}$

$$=-\log\left(\frac{2-\sin\theta}{2+\sin\theta}\right)=-f(\theta)$$

$\therefore$ $f(x)$ is an odd function of x.

Therefore, $2\int_0^{\pi/2}\log\left(\frac{2-\sin\theta}{2+\sin\theta}\right)d\theta=0.$

56. (d) : Area $=\int_0^4\sqrt{3x+4}\,dx$



$$=\left|\frac{(3x+4)^{3/2}}{3\cdot(3/2)}\right|_0^4$$

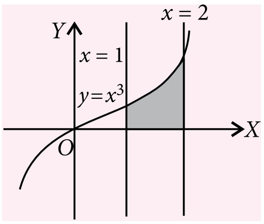
$$=\frac{112}{9}\text{ sq. unit}$$

57. (d)

EXAM CORNER 2018

Exam	Date
VITEEE	4th to 15th April
JEE Main	8th April (Offline), 15th & 16th April (Online)
SRMJEEE	16th to 30th April
Karnataka CET	18th & 19th April
WBJEE	22nd April
Kerala PET	23rd & 24th April
MHT CET	10th May
COMEDK (Engg.)	13th May
AMU (Engg.)	13th May (Revised)
BITSAT	16th to 31st May
JEE Advanced	20th May

58. (b) : Required area



$= \int_1^2 y\,dx = \int_1^2 x^3 dx = \frac{1}{4}[x^4]_1^2$

$= \frac{1}{4}[16-1] = \frac{15}{4}$ sq. unit

59. (a) : The curves $y = x$ and $y = x + \sin x$ intersect at (0, 0) and (π, π). Hence area bounded by the two curves

$= \int_0^{\pi} (x+\sin x)dx - \int_0^{\pi} x\,dx = \int_0^{\pi} \sin x\,dx$

$= [-\cos x]_0^{\pi} = -\cos\pi + \cos 0 = -(-1)+(1) = 2$

60. (a) : Given curve $y = a\sqrt{x} + bx$.

This curve passes through (1, 2), $\therefore\ 2 = a + b$ ...(i)

and area bounded by this curve and line $x = 4$ and x-axis is 8 sq. unit, then

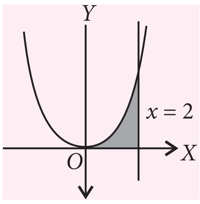
$\int_0^4 (a\sqrt{x}+bx)dx = 8$

$\Rightarrow \frac{2a}{3}[x^{3/2}]_0^4 + \frac{b}{2}[x^2]_0^4 = 8 \Rightarrow \frac{2a}{3}(8)+8b = 8$

$\Rightarrow\ 2a + 3b = 3$ ...(ii)

From equation (i) and (ii), we get $a = 3$, $b = -1$.

61. (b) : Given $x^2 = 4y$



$\therefore$ Required area $= \int_0^2 \frac{x^2}{4} dx$

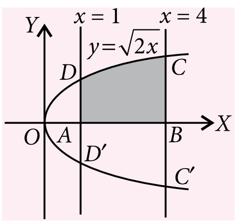
$= \frac{1}{12}[x^3]_0^2 = \frac{8}{12} = \frac{2}{3}$

62. (c) : Given curve is $y(x-2) = 3x+10 \Rightarrow y = \frac{3x+10}{x-2}$

$\therefore$ Required area is $\int_3^4 y\,dx = \int_3^4 \frac{3x+10}{x-2} dx$

$= [3x + 16\log(x-2)]_3^4 = (3+16\log 2)$ sq. unit

63. (b) : Required area = $CDD'C'$



$= 2 \times ABCD$

$= 2\int_1^4 \sqrt{2}x^{1/2}\,dx$

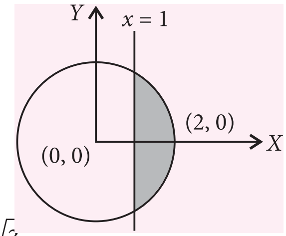
$= \frac{28\sqrt{2}}{3}$ sq. unit

64. (d) : $\int_1^b f(x)dx = \sqrt{b^2+1} - \sqrt{2} = \sqrt{b^2+1} - \sqrt{1+1}$

$= \left[\sqrt{x^2+1}\right]_1^b$

$\therefore\ f(x) = \frac{d}{dx}\sqrt{x^2+1} = \frac{2x}{2\sqrt{x^2+1}} = \frac{x}{\sqrt{x^2+1}}$

65. (a) : Area of smaller part $= 2\int_1^2 \sqrt{4-x^2}\,dx$



$= 2\left[\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2}\right]_1^2$

$= 2\left[2\cdot\frac{\pi}{2} - \left[\frac{\sqrt{3}}{2} + 2\cdot\frac{\pi}{6}\right]\right]$

$= 2\left[\pi - \left[\frac{\sqrt{3}}{2} + \frac{\pi}{3}\right]\right] = \frac{4\pi}{3} - \sqrt{3}$

66. (d) : Required area $= \int_0^{\pi/4} \tan x\,dx = [\log\sec x]_0^{\pi/4}$

$= \log\sec(\pi/4) - \log\sec 0 = \log\sqrt{2} - \log 1 = \log\sqrt{2}$

67. (a) : The parabola meets x-axis at the points, where

$\frac{3}{a}(a^2-x^2) = 0 \Rightarrow x = \pm a.$

So the required area

$= \int_{-a}^{a} \frac{3}{a}(a^2-x^2)dx = \frac{6}{a}\int_0^a (a^2-x^2)dx = (4a^2)$ sq. unit

68. (a) : $\because\ \int_0^1 \frac{x^{n-1} - x^n + x^{2n-1}}{1+x^n} dx > 0\ \forall\ n \in N$

$\Rightarrow \int_0^1 \left\{\frac{1}{1+x^n} - (1-x^{n-1})\right\} dx > 0$

$\Rightarrow \int_0^1 \frac{dx}{1+x^n} > \int_0^1 (1-x^{n-1})\,dx$

$\Rightarrow \int_0^1 \frac{dx}{1+x^n} > 1 - \frac{1}{n};\ \forall\ n \in N$

69. (d)

70. (c) : Put $3x = t \Rightarrow\ dx = \frac{dt}{3}$

$\frac{1}{3}\int e^t\left(3\sin\frac{t}{3} + \cos\frac{t}{3}\right)dt = e^t\sin\frac{t}{3} + c = e^{3x}\sin x + c$

Statement-2 is false.

71. (d) : $\int \frac{e^x}{2^2 - (3e^x)^2} dx = \frac{1}{4}\log\left|\frac{2+3e^x}{2-3e^x}\right| + C$

72. (a) : $\int \frac{e^x(2-x^2)}{(1-x)\sqrt{1-x^2}} dx = \int \frac{e^x(1+(1-x^2))}{(1-x)\sqrt{1-x^2}} dx$

$$= \int e^x \left(\frac{1}{(1-x)\sqrt{1-x^2}} + \frac{1+x}{\sqrt{1-x^2}} \right) dx$$

$$= \int e^x \left[\frac{1}{(1-x)^{3/2}(1+x)^{1/2}} + \sqrt{\frac{1+x}{1-x}} \right] dx$$

$$= e^x \sqrt{\frac{1+x}{1-x}} + c \; dx$$

73. (d) : $\int \cot^n x dx = \int \cot^{n-2} x \cdot \cot^2 x \, dx$

$$= \int \cot^{n-2} x \cdot \operatorname{cosec}^2 x \, dx - \int \cot^{n-2} x \, dx$$

$$\therefore \int \cot^n x = \frac{-\cot^{n-1} x}{n-1} - \int \cot^{n-2} x \, dx$$

$$I = \int (\cot^{24} x + \cot^{22} x + \ldots\ldots\ldots + 1) dx$$

$= (I_{24} + I_{22}) + (I_{20} + I_{18}) + \ldots.. + (I_4 + I_2) + \int 1 dx$

$$= \frac{-\cot^{23} x}{23} - \frac{\cot^{19} x}{19} - \ldots\ldots - \frac{\cot^3 x}{3} + x + c$$

74. (a) : We have, $\int \frac{e^{\cot x}}{\sin^2 x} \left(\ln \operatorname{cosec}^2 x + \frac{2\tan x}{1+\tan^2 x} \right) dx$

Let $t = \cot x \Rightarrow dt = -\operatorname{cosec}^2 x \, dx$

$$= -\int e^t \left(\ln(1+t^2) + \frac{\frac{2}{t}}{1+\frac{1}{t^2}} \right) dt$$

$-e^t \ln|1 + t^2| + c = -2e^{\cot x} \ln|\operatorname{cosec} x| + c$

$= 2e^{\cot x} \ln \sin x + c$

75. (b) : Let $I = \int_1^\infty \frac{x^2 - 2}{x^3\sqrt{x^2-1}} dx$

$$= \int_1^\infty \frac{x^2}{x^3\sqrt{x^2-1}} dx - 2\int_1^\infty \frac{dx}{x^3\sqrt{x^2-1}}$$

Put $x = \sec\theta \Rightarrow dx = \sec\theta \tan\theta \, d\theta$

$$\therefore I = \int_0^{\frac{\pi}{2}} \frac{\sec\theta \cdot \tan\theta d\theta}{\sec\theta \cdot \tan\theta} - 2\int_0^{\frac{\pi}{2}} \frac{\sec\theta \cdot \tan\theta d\theta}{\sec^3\theta \cdot \tan\theta}$$

$$= [\theta]_0^{\frac{\pi}{2}} - 2\int_0^{\pi/2} \cos^2\theta d\theta = \frac{\pi}{2} - 2\int_0^{\pi/2} \left(\frac{1+\cos 2\theta}{2} \right) d\theta$$

$$= \frac{\pi}{2} - \frac{\pi}{2} = 0.$$

76. (a) : $\int_a^{a+nT} f(x) dx = \int_0^{nT} f(x) dx = n\int_0^T f(x) dx$

77. (b)

78. (a) : Let $f(x) = \int_0^{\sin^2 x} \sin^{-1}\sqrt{x} dx + \int_0^{\cos^2 x} \cos^{-1}\sqrt{x} dx$

$\therefore \; f'(x) = 0 \;\; \Rightarrow \;\; f(x) = \text{constant}.$

and $f\left(\frac{\pi}{4}\right) = \int_0^{1/2} \frac{\pi}{2} dx = \frac{\pi}{4} \;\; \Rightarrow c = \frac{\pi}{4}$ ...(i)

As roots of $g(x) = ax^2 + bx + c = 0$ are non real

$\Rightarrow \; D < 0$ and $g(0) = c > 0$ [Using (i)]

$\Rightarrow \; a > 0$

79. (d)

80. (a) : $f(x) = \frac{e^x}{1+e^x} = \frac{1}{1+e^{-x}}$

$f(a) = \frac{1}{1+e^{-a}}, \; f(-a) = \frac{1}{1+e^a}$

$f(a) + f(-a) = 1$

81. (c) : Put $3x = \cos\theta \Rightarrow 3dx = -\sin\theta \, d\theta$

$$\therefore \; -\frac{1}{3}\int \frac{\frac{\cos\theta}{3} + \theta^2}{\sin\theta} \cdot \sin\theta \, d\theta = -\frac{1}{3}\int \left(\frac{1}{3}\cos\theta + \theta^2 \right) d\theta$$

$$-\frac{1}{9}\int \cos\theta d\theta - \frac{1}{3}\int \theta^2 d\theta = -\frac{1}{9}\sin\theta - \frac{1}{9}\theta^3 + c$$

$$= -\frac{1}{9}\sqrt{1-9x^2} - \frac{1}{9}(\cos^{-1} 3x)^3 + c$$

Clearly, $A = -\frac{1}{9}$ and $B = \frac{1}{9}$

Hence, $A + B = 0$.

82. (c) : $x = a\cos^3 t, \; y = a\sin^3 t, \; 0 \le t \le 2\pi$

$(xg'(t) - yf'(t)) = a^2(\cos^3 t \cdot 3\sin^2 t \cos t + \sin^3 t . 3\cos^2 t \sin t)$

$= 3a^2 \cos^2 t \sin^2 t$

$\Rightarrow$ Area of the curve $= \frac{1}{2}\int_0^{2\pi} (xg'(t) - yf'(t)) dt$

$$= \frac{1}{2}\int_0^{2\pi} 3a^2 \cos^2 t \sin^2 t \, dt = \frac{3}{8}a^2\pi$$

83. (c) : The curve is described when t varies from -1 to 1. Indeed when $t = -1$, both x and y are zero. When $t = 0$, $x = a$, $y = 0$. Thus is the adjoining figure upper portion is described when $0 \le t \le 1$ and the lower portion is described when $-1 < t < 0$

Required area

$$= -\int_{-1}^{1} g(t) f'(t)dt = -\int_{-1}^{1} at(1-t^2)(-2at)\,dt = \frac{8a^2}{15}$$

84. (a) : This is a circle of radius 2.

85. (c) **86. (d)** **87. (a)**

$$I_n = \int x^n \sqrt{a^2 - x^2}\,dx = \int x^{n-1}\left(x\sqrt{a^2-x^2}\right)dx$$

Apply integration by parts

$$I_n = -\frac{1}{3}(a^2-x^2)^{3/2}x^{n-1} + \frac{n-1}{3}\int x^{n-2}(a^2-x^2)^{3/2}dx$$

$$= -\frac{1}{3}(a^2-x^2)^{3/2}x^{n-1} + \frac{n-1}{3}\int\left(a^2x^{n-2}\sqrt{a^2-x^2} - x^n\sqrt{a^2-x^2}\right)dx$$

$$I_n = -\frac{1}{3}(a^2-x^2)^{3/2}x^{n-1} + \frac{n-1}{3}a^2 I_{n-2} - \frac{n-1}{3}I_n$$

$$\Rightarrow I_n = \frac{-(a^2-x^2)^{3/2}x^{n-1}}{n+2} + \left(\frac{n-1}{n+2}\right)a^2 I_{n-2}$$

$$\Rightarrow A = n+2,\ B = \frac{n-1}{n+2} \quad \text{If } I_n = \int_0^a x^n\sqrt{a^2-x^2}\,dx$$

$$\Rightarrow I_n = \left(\frac{n-1}{n+2}\right)a^2 I_{n-2} \Rightarrow I_4 = \frac{3}{6}a^2 I_2$$

and $I_2 = \frac{1}{4}a^2 I_0$. Also, $I_0 = \frac{\lambda a^2}{4}$

$$\therefore\ I_4 = \frac{3}{6}\times\frac{1}{4}\times\frac{\pi}{4}a^6 = \frac{\pi a^6}{32}$$

88. (c) : $\frac{dy}{dx} = \frac{1}{x+y}$ ...(i) [Given]

Put $y + x = v$ ...(ii)

$\Rightarrow \frac{dy}{dx} + 1 = \frac{dv}{dx} \Rightarrow \frac{dv}{dx} - 1 = \frac{1}{v}$ [Using (i) and (ii)]

$$\Rightarrow \frac{dv}{dx} = \frac{v+1}{v} \Rightarrow dv\left(\frac{v+1-1}{v+1}\right) = dx$$

Integrating both sides, we get

$\Rightarrow\ v - \ln(v+1) = x + c$

$\Rightarrow\ y + x - \ln(x+y+1) = x + c$

Also $x = 0, y = 0 \Rightarrow c = 0$

$\therefore\ y = \ln(x+y+1) \Rightarrow x = e^y - y - 1$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{e^y - 1} \Rightarrow \left.\frac{dy}{dx}\right|_{\ln 3} = \frac{1}{e^{\ln 3} - 1} = \frac{1}{2}$$

89. (c) : $\text{Area} = \int_0^1 (e^y - y - 1)dy = \left[e - \frac{1}{2} - 1\right] - 1 = e - \frac{5}{2}$

90. (b) : $\int_{-1}^{[|\sin\alpha|+|\cos\alpha|]} xe^{-y}d\left(e^y\right)$

Let $e^y = z \Rightarrow e^y\frac{dy}{dz} = 1 \Rightarrow dy = e^{-y}dz$

Since $1 \le |\sin\alpha| + |\cos\alpha| \le \sqrt{2}\ \therefore\ [|\sin\alpha| + |\cos\alpha|] = 1$

$$\Rightarrow \int_{-1}^{1}(e^y - y - 1)dy = e - e^{-1} - 2$$

91. (b) : Let $I(a) = \int_0^1 \frac{x^a - 1}{\log x}dx$

Diff w.r.t. a keeping x as constant

$$\frac{d}{da}I(a) = \int_0^1 x^a dx \Rightarrow \frac{d}{d(a)}I(a) = \frac{1}{a+1}$$

Integrating both sides w.r.t a, we get

$I(a) = \log(a+1) + C$

$I(0) = 0 \Rightarrow C = 0 \Rightarrow I(a) = \log(a+1)$

92. (c)

93. (c) : $\int_0^\pi \frac{dx}{a - \cos x} = \frac{\pi}{\sqrt{a^2 - 1}}$

Diff w.r.t. a on both sides treating x as constant, we get

$$-\int_0^\pi \frac{dx}{(a-\cos x)^2} = \frac{-\pi a}{(a^2-1)^{\frac{3}{2}}}$$

Again diff. w.r.t. a treating x as a constant, we get

$$2\int_0^\pi \frac{dx}{(a-\cos x)^3} = \frac{\pi(1+2a^2)}{(a^2-1)^{\frac{5}{2}}}$$

$$\therefore\ \int_0^\pi \frac{dx}{(\sqrt{10}-\cos x)^3} = \frac{\pi(1+2a^2)}{2(9)^{5/2}} = \frac{7\pi}{162}$$

94. (A) → (r), (B) → (p), (C) → (q)

(A) $\int x^2 \cdot \frac{1}{1+x^2}dx = x - \tan^{-1}x + c$

$\Rightarrow f(x) = \tan^{-1}x \Rightarrow f(1) = \tan^{-1}1 = \pi/4$

(B) $\int x^2 e^{2x}dx = \frac{1}{2}x^2 \cdot e^{2x} - \int 2x \cdot \frac{1}{2}e^{2x}dx$

$$= \frac{1}{2}x^2 e^{2x} - \left[x \cdot \frac{1}{2}e^{2x} - \frac{1}{4}e^{2x}\right] = \left(\frac{x^2}{2} - \frac{x}{2} + \frac{1}{4}\right)e^{2x} + c$$

(C) $\int \frac{(x^4+1)}{x(x^2+1)^2}dx = \int \frac{\left((x^2+1)^2-2x^2\right)dx}{x(x^2+1)^2}$

$$= \int \frac{1}{x}dx - 2\int \frac{xdx}{(x^2+1)^2} = \log|x| + \frac{1}{x^2+1} + c$$

95. (A) → (s), (B) → (r), (C) → (q), (D) → (q)

(A) Let $I = 25\int_0^{\pi/4}(\tan^6 x + \tan^4 x)dx$

$$= 25\int_0^{\pi/4}(\tan^4 x \sec^2 x)dx = 5$$

(B) Let $x + 1 = t$ in I_2 then $I_1 + I_2 = \int_{-2}^{2}(x^2+7)dx = \frac{100}{3}$

(C) Let $I = \int_{-1}^{0}\frac{-\pi}{2}dx + \int_0^3 \frac{\pi}{2}dx = \pi$

96. (8) : $\lim_{n\to\infty}\frac{\sum_{r=1}^{n}\sqrt{r}\sum_{r=1}^{n}\frac{1}{\sqrt{r}}}{\frac{n(n+1)}{2}} = \lim_{n\to\infty}\frac{\sum_{r=1}^{n}\sqrt{\frac{r}{n}}\sum_{r=1}^{n}\frac{1}{\sqrt{\frac{r}{n}}}}{\frac{n^2}{2}\left(1+\frac{1}{n}\right)}$

$$= 2\times\int_0^1\sqrt{x}\,dx\int_0^1\frac{1}{\sqrt{x}}dx$$

$$= 2\times\left[\frac{x^{3/2}}{\frac{3}{2}}\right]_0^1\left[\frac{x^{1/2}}{\frac{1}{2}}\right]_0^1 = 2\times\frac{2}{3}\times 2 = \frac{8}{3}$$

$\therefore \quad k = 8$

97. (1): $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k^{\frac{1}{a}}\left\{n^{a-\frac{1}{a}}+k^{a-\frac{1}{a}}\right\}}{n^{a+1}}$

$$= \lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}.\left\{\left(\frac{k}{n}\right)^{1/a}+\left(\frac{k}{n}\right)^{a}\right\} = \int_0^1(x^{1/a}+x^a)dx$$

$$= \left\{\frac{x^{(1/a)+1}}{\frac{1}{a}+1}+\frac{x^{a+1}}{a+1}\right\}_0^1 = \frac{a}{a+1}+\frac{1}{a+1} = 1$$

98. (2) : $f(x) = \frac{x^9-3x^5+7x^3-x}{\cos^2 x}+\sec^2 x$

$$= \sec^2 x(x^9-3x^5+7x^3-x)+\sec^2 x$$

$$\Rightarrow \int_{-\pi/4}^{\pi/4}f(x)dx = \int_{-\pi/4}^{\pi/4}\sec^2 x\,dx$$

$$= 2\int_0^{\pi/4}\sec^2 x\,dx = 2\tan x\,|_0^{\pi/4} = 2$$

99. (8) : $\int_1^4 f'(x)dx = [xf'(x)]_1^4 - \int_1^4 xf''(x)dx$

Let $I = \int_1^4 x f''(x)dx = \int_1^4(5-x)f''(5-x)dx$

$\Rightarrow I = 5\int_1^4 f''(x)dx - I \quad \therefore \quad I = \frac{5}{2}[f'(4)-f'(1)]$

So, $\int_1^4 f'(x)\,dx = \frac{3}{2}[f'(4)+f'(1)]$

Now, $f''(x) = f''(5-x) \Rightarrow f'(x) = -f'(5-x) + c$

$f'(0) + f'(5) = c \Rightarrow c = 8$

So, $f'(x) + f'(5-x) = 8 \Rightarrow f'(4) + f'(1) = 8$

100. (3): Let α, β be roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$

$\therefore \quad \alpha = \frac{\pi}{6}, \beta = \frac{\pi}{3}$

And, $(fog)(x) = f(\cos x) = \cos 2x$

$$\text{Area} = \int_{\pi/6}^{\pi/3}\cos^2 x\,dx = \frac{\pi}{12}$$

101. (2) : Let $I_n = \int_0^{\pi/2}(\sin x+\cos x)^{n-1}\cdot(\sin x+\cos x)dx$

$$I_n = \int_0^{\pi/2}(\sin x+\cos x)^{n-1}(\sin x-\cos x)'dx$$

$$= [\sin x + \cos x]^{n-1}\cdot(\sin x - \cos x)]_0^{\pi/2}$$

$$-\int_0^{\pi/2}(n-1)(\sin x+\cos x)^{n-2}\cdot(\cos x-\sin x)(\sin x-\cos x)dx$$

$$= 2+(n-1)\int_0^{\pi/2}(\sin x+\cos x)^{n-2}(\cos x-\sin x)^2dx$$

$$= 2+(n-1)\int_0^{\pi/2}(\sin x+\cos x)^{n-2}\left[2-(\sin x+\cos x)^2\right]dx$$

$= 2 + 2(n-1)I_{n-2} - (n-1)I_n$

$\Rightarrow \quad nI_n - 2(n-1)\,I_{n-2} = 2$

102. (2) : $I=\int_0^{\pi/2} x^2 \operatorname{cosec}^2 x\,dx$

$=\left[-x^2\cot x\right]_0^{\pi/2}+\int_0^{\pi/2}2x\cot x\,dx$

$=2\left[x\ln\sin x\right]_0^{\pi/2}-2\int_0^{\pi/2}\ln\sin x\,dx$

103. (7) : $I=\int_0^1(1-x^4)^7\cdot 1dx$

$=\left[x(1-x^4)^7\right]_0^1+7\times 4\int_0^1 x(1-x^4)^6x^3dx$

$=0-28\int_0^1(1-x^4)^6[(1-x^4)-1]dx$

$=-28\int_0^1(1-x^4)^6(1-x^4)dx+28\int_0^1(1-x^4)^6dx$

$=-28I+28\int_0^1(1-x^4)^6dx$

$\Rightarrow\ 29I=28\int_0^1(1-x^4)^6dx$

$\Rightarrow\ \dfrac{29\int_0^1(1-x^4)^7dx}{4\int_0^1(1-x^4)^6dx}=\dfrac{28\int_0^1(1-x^4)^6dx}{4\int_0^1(1-x^4)^6dx}=7$

104. (1) : $\int_0^{\pi}\dfrac{dx}{1+2\sin^2x}=2\int_0^{\pi/2}\dfrac{dx}{1+2\sin^2x}$

$=2\int_0^{\pi/2}\dfrac{\sec^2x\,dx}{1+3\tan^2x}=2\int_0^{\infty}\dfrac{dt}{1+\left(\sqrt{3}t\right)^2}$

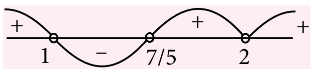
Put $t=\tan x\ \Rightarrow\ dt=\sec^2x\,dx$

$=\dfrac{2}{\sqrt3}\left[\tan^{-1}\sqrt{3}t\right]_0^{\infty}$

105. (1): $f'(x)=2(x-1)(x-2)^3+3(x-1)^2(x-2)^2$

$=(x-1)(x-2)^2\{2(x-2)+3(x-1)\}$

$=(x-1)(x-2)^2(5x-7)$



sign change of $f'(x)$ from +ve to –ve at $x=1$

$\therefore$ maximum at $x=1$.

PRACTICE PAPER 2018 JEE MAIN

Exam Dates
OFFLINE : 8th April
ONLINE : 15th & 16th April

1. Number of 4 digit numbers of the form $N = abcd$ which satisfy following three conditions:

(i) $4000 \le N < 6000$ (ii) N is a multiple of 5

(iii) $3 \le b < c \le 6$, is equal to

(a) 12 (b) 18 (c) 24 (d) 48

2. The angle between pair of tangents drawn to the curve $7x^2 - 12y^2 = 84$ from $M(1, 2)$ is

(a) $2\tan^{-1}\frac{1}{2}$ (b) $2\tan^{-1}2$

(c) $2\left(\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{2}\right)$

(d) $2\tan^{-1}3$

3. Let $\binom{n}{k}$ represents the combination of 'n' things taken 'k' at a time, then the value of the sum $\binom{99}{97}+\binom{98}{96}+\binom{97}{95}+\ldots+\binom{3}{1}+\binom{2}{0}$ equals

(a) $\binom{99}{97}$ (b) $\binom{100}{98}$ (c) $\binom{99}{98}$ (d) $\binom{100}{97}$

4. For any natural number m,
$\int (x^{7m}+x^{2m}+x^m)(2x^{6m}+7x^m+14)^{1/m}\,dx$
(where $x > 0$), equals

(a) $\dfrac{(7x^{7m}+2x^{2m}+14x^m)^{\frac{m+1}{m}}}{14(m+1)}+C$

(b) $\dfrac{(2x^{7m}+14x^{2m}+7x^m)^{\frac{m+1}{m}}}{14(m+1)}+C$

(c) $\dfrac{(2x^{7m}+7x^{2m}+14x^m)^{\frac{m+1}{m}}}{14(m+1)}+C$

(d) $\dfrac{(7x^{7m}+2x^{2m}+x^m)^{\frac{m+1}{m}}}{14(m+1)}+C$

where C is constant of integration.

5. Let $A=\begin{bmatrix} l & m & n \\ p & q & r \\ 1 & 1 & 1 \end{bmatrix}$ and $B = A^2$.

If $(l - m)^2 + (p - q)^2 = 9$, $(m - n)^2 + (q - r)^2 = 16$, $(n - l)^2 + (r - p)^2 = 25$, then the value of det. B equals

(a) 100 (b) 125 (c) 144 (d) 169

6. Let $\alpha, \beta \in R$. If α, β^2 be the roots of quadratic equation $x^2 - px + 1 = 0$ and α^2, β be the roots of quadratic equation $x^2 - qx + 8 = 0$, then the value of 'r' if $\frac{r}{8}$ be arithmetic mean of p and q, is

(a) $\frac{83}{8}$ (b) $\frac{83}{4}$ (c) $\frac{83}{2}$ (d) 83

7. The range of k for which the inequality $k\cos^2x - k\cos x + 1 \ge 0 \;\forall\; x$, is

(a) $k<-\frac{1}{2}$ (b) $k > 4$

(c) $-\frac{1}{2}\le k\le 4$ (d) $-\frac{1}{2}\le k\le 2$

8. Let $g(x) = ax + b$, where $a < 0$ and g is defined from $[1, 3]$ onto $[0, 2]$ then the value of
$\cot(\cos^{-1}(|\sin x|+|\cos x|)+\sin^{-1}(-|\cos x|-|\sin x|))$
is equal to

(a) $g(1)$ (b) $g(2)$

(c) $g(3)$ (d) $g(1) + g(3)$

9. $\int_0^{\theta}\ln(1+\tan\theta\cdot\tan x)\,dx$ is equal to

(a) $\theta\ln(\sec\theta)$ (b) $\theta\ln(\mathrm{cosec}\theta)$

(c) $\frac{\theta\cdot\ln 2}{2}$ (d) $2\theta\ln\sec\theta$

10. $\frac{\sin x}{\cos 3x}+\frac{\sin 3x}{\cos 9x}+\frac{\sin 9x}{\cos 27x}$ equals

(a) $\tan 3x - \tan x$ (b) $\frac{1}{2}(\tan 9x - \tan 3x)$

(c) $\frac{1}{2}(\tan 27x - \tan x)$ (d) $\frac{1}{2}(\tan 27x + \tan 9x)$

11. Let a, b, c, d are non-zero real numbers such that $6a + 4b + 3c + 3d = 0$, then the equation $ax^3 + bx^2 + cx + d = 0$ has

(a) atleast one root in $[-2, 0]$
(b) atleast one root in $[0, 2]$
(c) atleast two roots in $[-2, 2]$
(d) no root in $[-2, 2]$

12. Let $f : (0, \infty) \to R$ and $F(x) = \int_0^x t\, f(t)dt$

If $F(x^2) = x^4 + x^5$, then $\sum_{r=1}^{12} f(r^2)$ is equal to

(a) 216 (b) 219 (c) 222 (d) 225

13. If $\vec{a}, \vec{b}$ and $\vec{c}$ are non-zero vectors then the value of the scalar $\left((\vec{a}\times\vec{b})\times\vec{a}\right)\cdot\left((\vec{b}\times\vec{a})\times\vec{b}\right)$ equals

(a) $-(\vec{a}\cdot\vec{b})\,|\vec{a}\times\vec{b}|^2$ (b) $\vec{a}^2\,|\vec{a}\times\vec{b}|^2$
(c) $|\vec{b}|^2\,|\vec{a}\times\vec{b}|^2$ (d) $(\vec{a}\cdot\vec{b})\,|\vec{a}\times\vec{b}|^2$

14. If Z is a non real complex number then the minimum value of $\frac{\operatorname{Im} Z^5}{\operatorname{Im}^5 Z}$, is

(a) -1 (b) -2 (c) -4 (d) -5

15. Suppose that three points on the parabola $y = x^2$ have the property that their normal lines intersect at a common point (a, b). The sum of their x-coordinates is

(a) 0 (b) $\frac{2b-1}{2}$ (c) $\frac{a}{2}$ (d) $a + b$

16. If the circle $x^2 + y^2 + (3 + \sin\beta)x + (2\cos\alpha)y = 0$ and $x^2 + y^2 + (2\cos\alpha)x + 2cy = 0$ touches each other, then the maximum value of 'c' is

(a) 1/2 (b) 1 (c) 3/2 (d) 2

17. If range of the function $f(x) = \sin^{-1}x + 2\tan^{-1}x + x^2 + 4x + 1$ is $[p, q]$ then $p + q$ equals

(a) $-\pi + 4$ (b) $\frac{3\pi}{4}+2$ (c) 4 (d) 8

18. The complete set of values of the parameter α so that the point $P(\alpha, (1 + \alpha^2)^{-1})$ does not lie outside the triangle formed by the lines $L_1 : 15y = x + 1$, $L_2 : 78y = 118 - 23x$ and $L_3 : y + 2 = 0$, is

(a) (0. 5) (b) [2, 5] (c) [1, 5] (d) [0, 2]

19. If the variable line $y = kx + 2h$ is tangent to an ellipse $2x^2 + 3y^2 = 6$, then locus of $P(h, k)$ is a conic C whose eccentricity equals

(a) $\frac{\sqrt{5}}{2}$ (b) $\frac{\sqrt{7}}{3}$ (c) $\frac{\sqrt{7}}{2}$ (d) $\sqrt{\frac{7}{3}}$

20. An ellipse has semi major axis of length 2 and semi minor axis of length 1. The distance between its foci (units) is

(a) $2\sqrt{3}$ (b) 3 (c) $2\sqrt{2}$ (d) $\sqrt{3}$

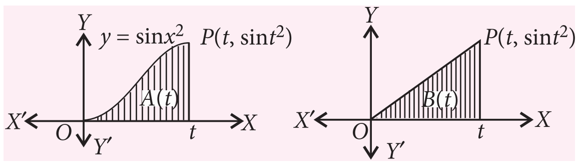
21. A coin that comes up head with probability $p > 0$ and tails with probability $1 - p > 0$ independently on each flip, is flipped eight times. Suppose the probability of three heads and five tails is equal to 1/25 of the probability of five heads and three tails. Let $p = \frac{m}{n}$, where m and n are relative prime positive integers. The value of $m + n$ equals

(a) 9 (b) 11 (c) 13 (d) 15

22. Equation of the circle which cuts the circle $x^2 + y^2 - 2x - 4y + 4 = 0$ and the lines $xy - 2x - y + 2 = 0$ orthogonally, is

(a) $x^2 + y^2 - 2x - 4y + 12 = 0$
(b) $x^2 + y^2 - 2x - 4y + 6 = 0$
(c) $x^2 + y^2 - 2x - 4y - 6 = 0$
(d) none of these

23. The figure shows two regions in the first quadrant. $A(t)$ is the area under the curve $y = \sin x^2$ from 0 to t and $B(t)$ is the area of the triangle with vertices O, P and $M(t, 0)$. Then, $\lim_{t\to 0}\frac{A(t)}{B(t)}$ equals



(a) 3/5 (b) 2/3 (c) 1/3 (d) 1/2

24. Let $S(t)$ be the area of the ΔOAB with $O(0, 0, 0)$, $A(2, 2, 1)$ and $B(t, 1, t + 1)$. The value of the definite integral $\int_1^e (S(t))^2 \ln t\, dt$, is equal to

(a) $\frac{2e^3+5}{2}$ (b) $\frac{e^3+5}{2}$

(c) $\frac{2e^3+15}{2}$ (d) $\frac{e^3+15}{2}$

25. The number of straight lines equidistant from three non-collinear points in the plane of the points is equal to

(a) 0 (b) 1 (c) 2 (d) 3

26. Two fair dice are rolled simultaneously. It is found that one of them shows odd prime numbers. The probability that remaining dice also shows an odd prime number, is equal to

(a) 1/5 (b) 2/5 (c) 3/5 (d) 4/5

27. The straight lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$, will intersect provided

(a) $k = \{3, -3\}$ (b) $k = \{0, -1\}$
(c) $k = \{-1, 1\}$ (d) $k = \{0, -3\}$

28. The length of projection, of the line segment joining the points $(1, -1, 0)$ and $(-1, 0, 1)$, to the plane $2x + y + 6z = 1$, is equal to

(a) $\sqrt{\frac{255}{61}}$ (b) $\sqrt{\frac{237}{41}}$ (c) $\sqrt{\frac{137}{41}}$ (d) $\sqrt{\frac{155}{61}}$

29. The sum $\frac{1}{2}\,{}^{10}C_0-{}^{10}C_1+2\cdot{}^{10}C_2-2^2\cdot{}^{10}C_3+...+2^9\cdot{}^{10}C_{10}$ is equal to

(a) $\frac{1}{2}$ (b) 0 (c) $\frac{1}{2}\cdot 3^{10}$ (d) 3^3

30. Let the r^{th} term, t_r, of a series is given by $t_r=\frac{r}{1+r^2+r^4}$. Then $\lim_{n\to\infty}\sum_{r=1}^{n} t_r$ is

(a) 1/4 (b) 1
(c) 1/2 (d) none of these

31. Let $f : [0, 1] \to [0, 1]$ be a continuous function. Then

(a) $f(x) = x$ for at least one $0 \le x \le 1$
(b) $f(x)$ will be differentiable in $[0, 1]$
(c) $f(x) + x = 0$ for at least one x such that $0 \le x \le 1$
(d) none of these

32. Let $f(x)$ be a polynomial function of degree 2 and $f(x) > 0$ for all $x \in R$. If $g(x) = f(x) + f'(x) + f''(x)$ then for any x

(a) $g(x) < 0$ (b) $g(x) > 0$
(c) $g(x) = 0$ (d) $g(x) \ge 0$

33. Let $f(x) = \tan^{-1}(\phi(x))$, where $\phi(x)$ is increasing for $0 < x < \pi/2$. Then $f(x)$ is

(a) increasing in $(0, \pi/2)$
(b) decreasing in $(0, \pi/2)$
(c) increasing in $(0, \pi/4)$ and decreasing in $(\pi/4, \pi/2)$
(d) none of these

34. If $f(x) = a\log_e|x| + bx^2 + x$ has extremums at $x = 1$ and $x = 3$ then

(a) $a=-\frac{3}{4},\ b=-\frac{1}{8}$ (b) $a=\frac{3}{4},\ b=-\frac{1}{8}$

(c) $a=-\frac{3}{4},\ b=\frac{1}{8}$ (d) none of these

SOLUTIONS

1. (c) : We have, $N=\boxed{a\,|\,b\,|\,c\,|\,d}$

First place a can be filled in 2 ways *i.e.* 4, 5 $(\because\ 4000 \le N < 6000)$

For b and c, total possibilities are '6'

i.e. 34, 35, 36, 45, 46, 56 $(\because\ 3 \le b < c \le 6)$

Last place d can be filled in 2 ways *i.e.* 0, 5 $(\because\ N$ is a multiple of 5)

Hence total numbers = $2 \times 6 \times 2 = 24$

2. (c) : The director circle of given hyperbola $\frac{x^2}{12}-\frac{y^2}{7}=1$, is $x^2 + y^2 = 5$ and given point $M(1, 2)$ lies on it.

$\Rightarrow$ The angle between pair of tangents = $\pi/2$

As, $\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{2}=\tan^{-1}1=\frac{\pi}{4}$

$$\Rightarrow\ 2\left(\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{2}\right)\right)=\frac{\pi}{2}$$

3. (d) : $S = {}^2C_0 + {}^3C_1 + {}^4C_2 + {}^5C_3 + ... + {}^{99}C_{97}$

$= {}^4C_1 + {}^4C_2 + {}^5C_3 + ... + {}^{99}C_{97}$ $(\because {}^2C_0 + {}^3C_1 = {}^4C_1)$

$= {}^5C_2 + {}^5C_3 + ... + {}^{99}C_{97}$

$\therefore\ S = {}^{100}C_{97}$

4. (c) : Let $I=\int(x^{7m-1}+x^{2m-1}+x^{m-1})(2x^{7m}+7x^{2m}+14x^m)^{1/m}\,dx$

Now, put $2x^{7m} + 7x^{2m} + 14x^m = t^m$

$14m(x^{7m-1} + x^{2m-1} + x^{m-1})dx = m\,t^{m-1}\,dt$

$$\therefore\ I=\frac{1}{14}\int t^{m-1}(t^m)^{1/m}\,dt$$

$$=\frac{t^{m+1}}{14(m+1)}+C=\frac{(2x^{7m}+7x^{2m}+14x^m)^{\frac{m+1}{m}}}{14(m+1)}+C$$

5. (c) : Det.A is twice the area of the triangle with vertices (l, p), (m, q), (n, r) with sides 3, 4, 5.

$\Delta^2 = s(s - a)(s - b)(s - c)$

$\Delta^2 = 6(6 - 3)(6 - 4)(6 - 5)$

$\Delta^2 = 36 \ \Rightarrow\ \Delta = 6$

Now, det$A$ = 2Δ = 12

$\Rightarrow$ det.B = $(\det A)^2 = 4\Delta^2 = 144$

6. **(d) :** For the equation $x^2 - px + 1 = 0$,
the product of roots, $\alpha\beta^2 = 1$
and for the equation $x^2 - qx + 8 = 0$,
the product of roots, $\alpha^2\beta = 8$
Hence, $(\alpha\beta^2)(\alpha^2\beta) = 8 \Rightarrow \alpha^3\beta^3 = 8 \Rightarrow \alpha\beta = 2$
$\therefore$ From $\alpha\beta^2 = 1$, we have $\beta = 1/2$ and from $\alpha^2\beta = 8$, we have $\alpha = 4$
Hence, from sum of roots $= -\dfrac{b}{a}$ relation, we have

$$p = \alpha + \beta^2 = 4 + \frac{1}{4} = \frac{17}{4} \text{ and } q = \alpha^2 + \beta = 16 + \frac{1}{2} = \frac{33}{2}$$

$\because \dfrac{r}{8}$ is arithmetic mean of p and q

$$\therefore \frac{r}{8} = \frac{p+q}{2} \Rightarrow r = 4(p+q) = 4\left(\frac{17}{4} + \frac{33}{2}\right) = 83$$

7. **(c) :** We have, $k\cos^2 x - k\cos x + 1 \geq 0 \forall x$
$\Rightarrow k(\cos^2 x - \cos x) + 1 \geq 0$

Now, $\cos^2 x - \cos x = \left(\cos x - \dfrac{1}{2}\right)^2 - \dfrac{1}{4}$

$$\because -\frac{1}{4} \leq \cos^2 x - \cos x \leq 2$$

$$\therefore 2k + 1 \geq 0 \text{ and } -\frac{k}{4} + 1 \geq 0 \Rightarrow -\frac{1}{2} \leq k \leq 4$$

8. **(c) :** Consider $F(x) = \cot(\cos^{-1}(|\sin x| + |\cos x|) + \sin^{-1}(-|\cos x| - |\sin x|))$

But $|\sin x| + |\cos x| \in [1, \sqrt{2}]$
$\therefore F(x) = \cot(\cos^{-1}(1) + \sin^{-1}(-1))$

$= \cot\left(0 - \dfrac{\pi}{2}\right) = 0 = g(3)$ (As $F(x) = 0$, $\forall\, x \in D_F$)

9. **(a) :** Let $I = \displaystyle\int_0^\theta \ln(1 + \tan\theta \cdot \tan(\theta - x))\, dx$

$$= \int_0^\theta \ln\left(1 + \frac{\tan\theta \cdot (\tan\theta - \tan x)}{1 + \tan\theta \cdot \tan x}\right) dx$$

$$= \int_0^\theta \ln\left(\frac{1 + \tan^2\theta}{1 + \tan\theta \cdot \tan x}\right) dx$$

$$= \int_0^\theta \ln(1 + \tan^2\theta)\, dx - \int_0^\theta \ln(1 + \tan\theta \cdot \tan x)\, dx$$

$I = 2\theta \ln \sec\theta - I$
$2I = 2\theta \ln \sec\theta \Rightarrow I = \theta \ln(\sec\theta)$

10. **(c) :** $\dfrac{\sin x}{\cos 3x} = \dfrac{\sin 2x}{2\cos x \cos 3x} = \dfrac{\sin(3x - x)}{2\cos x \cos 3x}$

$$= \frac{1}{2}(\tan 3x - \tan x) \text{ etc.}$$

11. **(b) :** We have $ax^3 + bx^2 + cx + d = 0$

Let $f(x) = \dfrac{ax^4}{4} + \dfrac{bx^3}{3} + \dfrac{cx^2}{2} + dx + e$

$\therefore f(0) = e$

$$f(2) = 4a + \frac{8b}{3} + 2c + 2d + e = \frac{(12a + 8b + 6c + 6d)}{3} + e$$

$$= \frac{2}{3}(6a + 4b + 3c + 3d) + e = 0$$

$\Rightarrow f(2) = e$
$\therefore$ By Rolle's theorem, there exist atleast one value of $x \in (0, 2)$ such that $f'(x) = 0$
$\Rightarrow$ The equation $ax^3 + bx^2 + cx + d = 0$ has atleast one real root in $[0, 2]$

12. **(b) :** We have $F(x^2) = \displaystyle\int_0^{x^2} t\, f(t)\, dt = x^4 + x^5$

$\therefore$ On differentiating w.r.t. x, we get

$$\Rightarrow f(x)^2 = 2 + \frac{5}{2}x \quad \therefore \sum_{r=1}^{12} f(r^2) = 219$$

13. **(a) :** $\left((\vec{a} \times \vec{b}) \times \vec{a}\right) \cdot \left((\vec{b} \times \vec{a}) \times \vec{b}\right)$

$= \left((\vec{a} \cdot \vec{a})\vec{b} - (\vec{b} \cdot \vec{a})\vec{a}\right) \cdot \left((\vec{b} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{b})\vec{b}\right)$

$= (\vec{a}^2\, \vec{b}^2)\, \vec{a} \cdot \vec{b} - \vec{a}^2\, \vec{b}^2\, (\vec{a} \cdot \vec{b}) - (\vec{a} \cdot \vec{b})\, \vec{a}^2\, \vec{b}^2 + (\vec{a} \cdot \vec{b})^3$

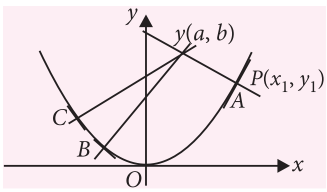
$= -(\vec{a} \cdot \vec{b})\, |\vec{a} \times \vec{b}\,|^2$

14. **(c) :** Let $Z = a + ib$, $b \neq 0$ where Im $Z = b$
$\therefore Z^5 = (a + ib)^5 = a^5 + {}^5C_1\, a^4bi + {}^5C_2\, a^3b^2i^2 + {}^5C_3\, a^2b^3i^3 + {}^5C_4\, ab^4i^4 + i^5b^5$
$\Rightarrow$ Im $Z^5 = 5a^4b - 10a^2b^3 + b^5$

Now, $y = \dfrac{\text{Im}\, Z^5}{\text{Im}^5\, Z} = 5\left(\dfrac{a}{b}\right)^4 - 10\left(\dfrac{a}{b}\right)^2 + 1$

Let $\left(\dfrac{a}{b}\right)^2 = x$ (say), $x \in R^+$

$y = 5x^2 - 10x + 1 = 5[x^2 - 2x] + 1 = 5[(x - 1)^2] - 4$
Hence $y_{min} = -4$



15. **(a) :** $y = x^2$

$$\left.\frac{dy}{dx}\right]_{x_1} = 2x_1$$

m (slope of normal) $= -\dfrac{1}{2x_1}$

Equation of normal at (x, x_1^2) is

$$y - x_1^2 = -\frac{1}{2x_1}(x - x_1) \qquad \text{....(i)}$$

(i) passes through (a, b)

$$b - x_1^2 = -\frac{1}{2x_1}(a - x_1) \Rightarrow 2x_1(b - x_1^2) = x_1 - a$$

$\Rightarrow \; 2x_1^3 + x_1(1 - 2b) - a = 0$

$\therefore$ There is no coefficient of x_1^2

Hence, sum of all the x coordinates = 0

16. (b) : Equation of tangent at (0, 0) will be same

$(3 + \sin\beta)x + (2\cos\alpha)y = 0$ ....(i)

$2\cos\alpha x + 2cy = 0$ ....(ii)

$\therefore$ (i) and (ii) must be identical

$\Rightarrow \; c = \dfrac{2\cos^2\alpha}{3 + \sin\beta} \Rightarrow c_{max} = 1$ when $\sin\beta = -1$ and $\alpha = 0$

17. (c) : $f(x) = \sin^{-1}x + 2\tan^{-1}x + x^2 + 4x + 1$

Domain of $f(x)$ is $[-1, 1]$

$f(x)$ is increasing in the domain

$$\therefore \; f(x)/\min = -\frac{\pi}{2} + 2\left(\frac{-\pi}{4}\right) + 1 - 4 + 1$$

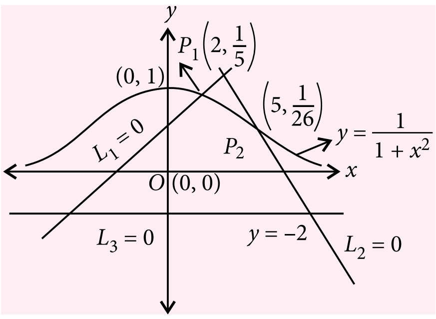
$= -\pi - 2$ at $x = -1$

$f(x)/\max = \dfrac{\pi}{2} + 2\cdot\dfrac{\pi}{4} + 1 + 4 + 1 = \pi + 6$ at $x = 1$

$\therefore$ Range of $f(x)$ is $[-\pi - 2, \pi + 6]$

$\Rightarrow \; p + q = 4$

18. (b) : As $P(\alpha, (1 + \alpha^2)^{-1})$ lie on $y = \dfrac{1}{1 + x^2}$



$\therefore$ On solving $y = \dfrac{1}{1 + x^2}$ with L_1,

we get $P_1\left(2, \dfrac{1}{5}\right)$ ....(i)

and with L_2, we get $P_2\left(5, \dfrac{1}{26}\right)$ ....(ii)

$\therefore$ From (i) and (ii), we get $2 \le \alpha \le 5$

19. (d) : By using condition of tangency, we get

$4h^2 = 3k^2 + 2$

$\therefore$ Locus of $P(h, k)$ is $4x^2 - 3y^2 = 2$ (which is hyperbola.)

Hence, $e^2 = 1 + \dfrac{4}{3} \Rightarrow e = \sqrt{\dfrac{7}{3}}$

20. (a) : We have $d = F_1F_2 = 2ae$

$d^2 = 4a^2e^2 = 4(a^2 - b^2) = 4(4 - 1) = 12 \Rightarrow d = 2\sqrt{3}$ units

21. (b) : We have ${}^8C_3\, p^3(1 - p)^5 = \dfrac{1}{25}\, {}^8C_3\, p^5(1 - p)^3$

or $1 - p = \dfrac{(1)(p)}{5}$

So, we get $p = \dfrac{5}{6} \Rightarrow m + n = 11$

22. (b) : We have, Line $(x - 1)(y - 2) = 0$

$\Rightarrow \; x = 1$ and $y = 2$

Centre (1, 2) *i.e.* $-g = 1$ and $-f = 2$

Let the equation of the required circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$ ....(i)

As (i) is orthogonal to $x^2 + y^2 - 2x - 4y + 4 = 0$

$\therefore \; 2 + 8 = 4 + c \Rightarrow c = 6$

Hence, $x^2 + y^2 - 2x - 4y + 6 = 0$

23. (b) : We have $A(t) = \displaystyle\int_0^t \sin x^2\, dx$; $B(t) = \dfrac{t \sin t^2}{2}$

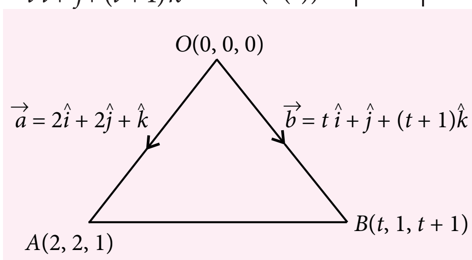
$$\therefore \; \lim_{t \to 0} \frac{A(t)}{B(t)} = \lim_{t \to 0} \frac{2\int_0^t \sin x^2\, dx}{t \sin t^2}$$

$$= \lim_{t \to 0} \frac{2\int_0^t \sin x^2\, dx}{t^3 \dfrac{\sin t^2}{t^2}} = \lim_{t \to 0} \frac{2\int_0^t \sin x^2\, dx}{t^3}$$

Hence $\displaystyle\lim_{t \to 0} \frac{A(t)}{B(t)} = \lim_{t \to 0} \frac{2\sin t^2}{3t^2} = \frac{2}{3}$

24. (b) : We have $S(t) = \dfrac{1}{2}|\vec{a} \times \vec{b}|$ where $\vec{a} = 2\hat{i} + 2\hat{j} + \hat{k}$

and $\vec{b} = t\hat{i} + \hat{j} + (t + 1)\hat{k} \Rightarrow 4(S(t))^2 = |\vec{a} \times \vec{b}|^2$



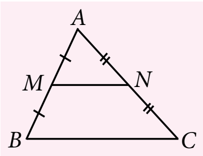
But $4(S(t))^2 = |\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$

$= 9(t^2 + 1 + (t + 1)^2) - (2t + 2 + t + 1)^2 = 9(t^2 + 1)$

$\therefore \; (S(t))^2 = \dfrac{9}{4}(t^2 + 1)$

Now $I = \displaystyle\int_1^e \frac{9}{4}(t^2 + 1) \ln t\, dt$, we get $I = \dfrac{1}{2}(e^3 + 5)$

25. (d) : Three non collinear points form a triangle and the line joining the mid points of any two sides is equidistant from all the three vertices.



26. (a) : 3 and 5 are the only odd prime numbers, among the possible outcomes. The following are the outcomes when one of them show odd prime numbers.

(3, 1), (1, 3), (3, 2), (2, 3), (3, 3), (3, 4), (4, 3), (3, 5), (5, 3), (3, 6), (6, 3), (5, 1), (1, 5), (5, 2), (2, 5), (5, 4), (4, 5), (5, 5), (5, 6), (6, 5).

Out of these 20 equally likely outcomes exactly 4 favour the presence of odd prime numbers on both dice.

Thus, required probability = $\frac{4}{20}=\frac{1}{5}$

27. (d) : Any point on the first line can be taken as

$P_1 \equiv (r_1 + 2, r_1 + 3, -kr_1 + 4)$

Similarly any point on second line can be taken as

$P_2 \equiv (kr_2 + 1, 2r_2 + 4, r_2 + 5)$.

These lines will intersect if for some r_1 and r_2 we have

$r_1 + 2 = kr_2 + 1, r_1 + 3 = 2r_2 + 4, -kr_1 + 4 = r_2 + 5$.

$\therefore \quad r_1 - kr_2 + 1 = 0, r_1 = 2r_2 + 1$

$\Rightarrow \quad r_2 = \frac{2}{k-2}, r_1 = \frac{k+2}{k-2}$

Putting these values in the last condition, we get

$k^2 + 3k = 0 \quad \Rightarrow k = \{-3, 0\}$

28. (b) : Let $A \equiv (1, -1, 0)$, $B \equiv (-1, 0, 1)$

Direction ratios of segment AB are $2, -1, -1$

If 'θ' be the acute angle between segment AB and normal to plane,

$$\cos\theta = \frac{|2\cdot 2-1\cdot 1-1\cdot 6|}{\sqrt{4+1+36}\cdot\sqrt{4+1+1}} = \frac{3}{\sqrt{246}}$$

Length of projection = $|AB| \sin\theta$

$$= \sqrt{6}\cdot\sqrt{1-\frac{9}{246}} = \sqrt{\frac{237}{41}} \text{ units}$$

29. (a) : We have,

$$\frac{1}{2}({}^{10}C_0 - 2\,{}^{10}C_1 + 2^2\,{}^{10}C_2 - .....+ 2^{10}\,{}^{10}C_{10})$$

$$= \frac{1}{2}(2-1)^{10} = \frac{1}{2}.$$

30. (c) : $t_r = \frac{1}{2}\cdot\frac{2r}{(r^2+1)^2-r^2} = \frac{1}{2}\left\{\frac{1}{r^2-r+1} - \frac{1}{r^2+r+1}\right\}$

$$= \frac{1}{2}\left\{\frac{1}{r(r-1)+1} - \frac{1}{(r+1)r+1}\right\}$$

$$\therefore \sum_{r=1}^{n} t_r = \sum_{r=1}^{n}\frac{1}{2}\{f(r)-f(r+1)\}, \text{ where } f(r) = \frac{1}{r(r-1)+1}$$

$$= \frac{1}{2}\{f(1)-f(n+1)\}$$

$$= \frac{1}{2}\left\{1-\frac{1}{(n+1)n+1}\right\} \to \frac{1}{2} \text{ as } n \to \infty$$

31. (a) : Clearly, $0 \le f(0) \le 1$ and $0 \le f(1) \le 1$. As $f(x)$ is continuous, $f(x)$ attains all values between $f(0)$ to $f(1)$, and the graph will have no breaks. So, the graph will cut the line $y = x$ at one point x at least where $0 \le x \le 1$. So, $f(x) = x$ at that point.

32. (b) : Let $f(x) = ax^2 + bx + c$.

Then $g(x) = ax^2 + bx + c + 2ax + b + 2a$

$= ax^2 + (2a + b)x + 2a + b + c$

As $f(x) > 0$ for all x, $a > 0$ and $D = b^2 - 4ac < 0$.

$g(x) > 0$ for all x if $a > 0$

and $D = (2a + b)^2 - 4a(2a + b + c) < 0$

Now, $(2a + b)^2 - 4a(2a + b + c)$

$= -4a^2 + b^2 - 4ac < 0$

Hence, $g(x) > 0$.

33. (a) : $f'(x) = \frac{\phi'(x)}{1+\{\phi(x)\}^2} > 0$

$\because 0 < x < \frac{\pi}{2}, \phi'(x) > 0$

34. (a) : Around $x = 1, 3$ we have $|x| = x$.

$\therefore \quad f(x) = a\log_e x + bx^2 + x$

$\therefore \quad f'(x) = \frac{a}{x} + 2bx + 1$

According to question, $f'(1) = 0, f'(3) = 0$

$\therefore \quad a + 2b + 1 = 0, \frac{a}{3} + 6b + 1 = 0$

On solving, we get $a = \frac{-3}{4}$, $b = \frac{-1}{8}$

Solution Sender of Maths Musing

SET-181

- Khokon Kumar Nandi (West Bengal)
- Gouri Sankar Adhikary (West Bengal)
- N. Jayanthi (Hyderabad)
- Sannibha Pande (West Bengal)

SET-180

- Gouri Sankar Adhikary (West Bengal)
- Sannibha Pande (West Bengal)

Exam Dates
OFFLINE : 8th April
ONLINE : 15th & 16th April

JEE Main 2018
MOCK TEST PAPER

Series-8

Time: 1 hr 15 min.

The entire syllabus of Mathematics of JEE MAIN is being divided in to eight units, on each unit there will be a Mock Test Paper (MTP) which will be published in the subsequent issues. The syllabus for module break-up is given below:

Unit No. 8	Topic	Syllabus In Detail
	Integral calculus	Integral as limit of a sum. Fundamental theorem of calculus. Properties of definite integrals. Evaluation of definite integrals, determining areas of the regions bounded by simple curves in standard form.
	Differential equation	Ordinary differential equations, their order and degree. Formation of differential equations. Solution of differential equations by the method of separation of variables, solution of homogeneous and linear differential equation
	Probability	Baye's theorem, Probability distribution of a random variate, Bernoulli trials and Binomial distribution

1. $\displaystyle\int_{-1}^{1}\frac{x^3+|x|+1}{x^2+2|x|+1}dx=$

(a) $\ln 3$ (b) $2\ln 3$
(c) $\frac{1}{2}\ln 3$ (d) none of these

2. Let $f(x)=\max\{2-x,2,1+x\}$. Then $\displaystyle\int_{-1}^{1}f(x)dx=$

(a) 0 (b) 2 (c) 9/2 (d) 3

3. $\displaystyle\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{2n}\frac{r}{\sqrt{n^2+r^2}}=$

(a) $1+\sqrt{5}$ (b) $-1+\sqrt{5}$
(c) $-1+\sqrt{2}$ (d) $1+\sqrt{2}$

4. $\displaystyle\int_{\log\sqrt{\frac{\pi}{2}}}^{\log\sqrt{\pi}}e^{2x}\sec^2\left(\frac{1}{3}e^{2x}\right)dx=$

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{3\sqrt{3}}{2}$ (d) $\frac{1}{2\sqrt{3}}$

5. If $[x]$ stands for the greatest integer function, the value of $\displaystyle\int_{4}^{10}\frac{[x^2]dx}{[x^2-28x+196]+[x^2]}$ is

(a) 0 (b) 1
(c) 3 (d) none of these

6. The area bounded by the curve $x^2+2x+y-3=0$, the x-axis and the tangent at the point where it meets the y-axis is

(a) $\frac{7}{12}$ sq. units (b) $\frac{12}{7}$ sq. units
(c) $\frac{7}{6}$ sq. units (d) $\frac{6}{7}$ sq. units.

7. If $f(x)=\displaystyle\int_{0}^{x}\frac{dt}{(f(t))^2}$ and $\displaystyle\int_{0}^{2}\frac{dt}{(f(t))^2}=\sqrt[3]{6}$, $f(9)=$

By : Sankar Ghosh, S.G.M.C, Mob : 09831244397.

(a) 2 (b) 0
(c) 3 (d) none of these

8. Let $\frac{d}{dx}(f(x))=\frac{e^{\sin x}}{x}, x>0$. If $\int_1^4 \frac{2e^{\sin x^2}}{x}dx$ $=f(k)-f(1)$, then one of the possible value of k is
(a) -4 (b) 0 (c) 2 (d) 16

9. The area between the curves $y = xe^x$ and $y = xe^{-x}$ and the line $x = 1$ is
(a) $2e$ (b) e (c) $2/e$ (d) $1/e$

10. The area bounded by the curve, $y = f(x) = x^4 - 2x^3 + x^2 + 3$, the x-axis and the ordinates corresponding to the minimum of function $f(x)$ is
(a) 1 (b) $\frac{91}{30}$ (c) $\frac{30}{9}$ (d) 4

11. The order and degree, respectively of the differential equation whose solution is $ax^2 + by^2 = 1$, a and b being arbitrary constants are
(a) 1, 1 (b) 2, 1 (c) 2, 2 (d) 1, 2

12. If $x^3dy+xydx=x^2dy+2ydx$, $y(2)=e$, then $y(-1)=$
(a) $\frac{2}{e}$ (b) $\frac{4}{e}$ (c) $\frac{2}{e^2}$ (d) $\frac{4}{e^2}$

13. If $xdy - ydx + x\cos\ln x\, dx = 0$, $y(1) = 1$, then $y(e) =$
(a) $e(1 - \cos 1)$ (b) $e(1 - \sin 1)$
(c) $e(1 + \cos 1)$ (d) $e(1 + \sin 1)$

14. If $(x^2 + y^2)dy = xydx$, $y(1) = 1$, and $y(x_0) = e$ then $x_0 =$
(a) $\sqrt{2(e^2-1)}$ (b) $\sqrt{2(e^2+1)}$
(c) $\sqrt{3}\cdot e$ (d) $\sqrt{\frac{e^2+1}{2}}$

15. The solution of $1+y^2+\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx}=0$
(a) $x-2=ce^{-\tan^{-1}y}$ (b) $2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+c$
(c) $xe^{\tan^{-1}y}=c+\tan^{-1}y$
(d) $xe^{2\tan^{-1}y}=c+\tan^{-1}y$

16. If $y-\cos x\frac{dy}{dx}=y^2(1-\sin x)\cos x$, $y(0)=1$, then $y\left(\frac{\pi}{3}\right)=$
(a) 1 (b) 2 (c) 1/2 (d) $\sqrt{3}$

17. If $(2x - 2y + 5)dy = (x - y + 3)dx$, then $\ln(x - y + 2) =$
(a) $x - 2y + c$ (b) $2x - y + c$
(c) $2y - x + c$ (d) $y - x + c$

18. A curve passes through (2, 0) and slope of the tangent at the point $(x, y)=\frac{(x+1)^2+y-3}{x+1}$. The area bounded by the curve and the x-axis is
(a) 1 (b) 2/3 (c) 4/3 (d) 2

19. A right circular cone with radius R and height H contains a liquid which evaporates at a rate proportional to the surface area in contact with the area ($\lambda > 0$ is proportional constant). Then the time at which liquid evaporates completely is
(a) $\frac{H}{\lambda}$ (b) $\frac{2H}{\lambda}$ (c) $\frac{H}{2\lambda}$ (d) $\frac{R}{2\lambda}$

20. The differential equation which represent the family of curves $y = ae^{bx}$ where a and b are arbitrary constants, is
(a) $y' = y^2$ (b) $y'' = yy'$
(c) $yy'' = y'$ (d) $yy'' = (y')^2$

21. A letter is known to have come either from TATANAGAR or CALCUTTA. On the envelope just two consecutive letters are visible. The probability that the letter came from CALCUTTA is
(a) $\frac{4}{11}$ (b) $\frac{7}{11}$
(c) $\frac{5}{11}$ (d) none of these

22. A coffee connoisseur claims that he can distinguish between a cup of instant coffee and a cup of percolator coffee 75% of time. It is agreed that his claim will be accepted if he correctly identifies at least 5 of the 6 cups. His chances of having the claim accepted is
(a) 0.534 (b) 0.466
(c) 0.763 (d) none of these

23. If on an average one vessel in every 10 is wrecked, the probability that out of 5 vessels atleast 4 will arrive safely is
(a) 0.92 (b) 0.72
(c) 0.82 (d) none of these

24. The probability distribution of a random variable is $P(X)=\begin{cases}\frac{X}{15}; X=1,2,3,4,5\\ 0, \text{ otherwise}\end{cases}$. Then

$$P\left\{\frac{1}{2}<X<\frac{5}{2} \mid X>1\right\}=$$

(a) 1/5 (b) 1/7 (c) 2/15 (d) 1/15

25. A perfect die is thrown twice. The expected value of the product of the number of points obtained in two throws is

(a) 7/2 (b) 7
(c) 49/4 (d) none of these

26. For a fixed value of n, the maximum value of variance of binomial distribution

(a) $n/4$ (b) $n/2$
(c) $n/8$ (d) none of these

27. The variance of a symmetric binomial distribution with mean 5 is

(a) 2/5 (b) 5/2
(c) 3/2 (d) none of these

28. If a random variable X follows binomial distribution with mean 2 and $E(X^2)$= 28/5, then $\langle n, p\rangle =$

(a) $\left\langle 10, \frac{1}{5}\right\rangle$ (b) $\left\langle 5, \frac{1}{10}\right\rangle$

(c) $\left\langle 10, \frac{4}{5}\right\rangle$ (d) none of these

29. There are 4 white and 3 black balls in a box. In another box there are 3 white and 4 black balls. An unbiased dice is rolled. If it shows a number less than or equal to 3 then a ball is drawn from the first box but if it shows a number more than 3 then a ball is drawn from the second box. If the ball drawn is black then the probability that the ball was drawn from the first box is

(a) 1/2 (b) 6/7 (c) 4/7 (d) 3/7

30. A coin is tossed n times. The probability of getting at least one head is greater than that of getting at least two tails by 5/32. Then n is

(a) 5 (b) 10
(c) 15 (d) none of these

SOLUTIONS

1. (d) : $\displaystyle\int_{-1}^{1}\frac{x^3+|x|+1}{x^2+2|x|+1}dx$

$$=\int_{-1}^{1}\frac{x^3dx}{x^2+2|x|+1}+\int_{-1}^{1}\frac{|x|+1}{x^2+2|x|+1}dx$$

$$=0+2\int_{0}^{1}\frac{|x|+1}{x^2+2|x|+1}=2\int_{0}^{1}\frac{dx}{1+x}=2\ln 2$$

2. (c) : In (–1, 0), 2 – x is maximum and in (0, 1), 2 is maximum.

$$\therefore \int_{-1}^{1}f(x)dx=\int_{-1}^{0}(2-x)dx+\int_{0}^{1}2dx$$

$$=\left[2x-\frac{x^2}{2}\right]_{-1}^{0}+2=\frac{5}{2}+2=\frac{9}{2}$$

3. (b) : $$\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{2n}\frac{r}{\sqrt{n^2+r^2}}=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{2n}\frac{\frac{r}{n}}{\sqrt{1+\left(\frac{r}{n}\right)^2}}$$

$$=\int_{0}^{2}\frac{x}{\sqrt{1+x^2}}dx=\frac{1}{2}\cdot\left(2\sqrt{1+x^2}\right)_{0}^{2}=\sqrt{5}-1$$

4. (a) : Let $$I=\int_{\log\sqrt{\frac{\pi}{2}}}^{\log\sqrt{\pi}}e^{2x}\sec^2\left(\frac{1}{3}e^{2x}\right)dx$$

$$=\frac{3}{2}\tan\left(\frac{1}{3}e^{2x}\right)\Bigg|_{\ln\sqrt{\frac{\pi}{2}}}^{\ln\sqrt{\pi}}=\frac{3}{2}\left[\tan\frac{\pi}{3}-\tan\frac{\pi}{6}\right]=\sqrt{3}$$

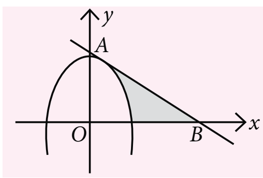
5. (c): $$I=\int_{4}^{10}\frac{[x^2]dx}{[x^2-28x+196]+[x^2]}$$

$$=\int_{4}^{10}\frac{[x^2]dx}{[(14-x)^2]+[x^2]}dx=\int_{4}^{10}\frac{\left[(14-x)^2\right]dx}{[x^2]+\left[(14-x)^2\right]}$$

$$\left[\because \int_{a}^{b}f(x)dx=\int_{a}^{b}f(a+b-x)dx\right]$$

$$\therefore\quad 2I=\int_{4}^{10}dx=6\ \Rightarrow I=3$$

6. (a) : The equation of the curve is y – 4= –(x + 1)^2. The curve meets y-axis at(0, 3) and x-axis at (–3, 0)and (1, 0). The tangent at (0, 3) is y – 3 = –2x



The required area $=$ Area of $\Delta OAB-\displaystyle\int_{0}^{1}\{4-(x+1)^2\}dx$

$$=\frac{9}{4}-\left\{4x-\frac{(x+1)^3}{3}\right\}_{0}^{1}=\frac{9}{4}-\left\{\left(4-\frac{8}{3}\right)+\frac{1}{3}\right\}=\frac{7}{12}$$

7. (c) : Here, $f(x)=\int_0^x \frac{dt}{(f(t))^2} \Rightarrow f'(x)=\frac{1}{(f(x))^2}$

$\Rightarrow \quad f'(x)(f(x))^2=1 \quad \Rightarrow \quad \frac{(f(x))^3}{3}=x+c$

Since $f(2)=\sqrt[3]{6} \quad \Rightarrow \quad \frac{6}{3}=2+c \Rightarrow \quad c=0$

$\therefore \quad f(x)=\sqrt[3]{3x} \quad \Rightarrow \quad f(9)=3$

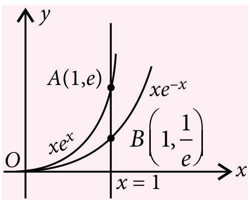
8. (d) : Given, $\frac{d}{dx}(f(x))=\frac{e^{\sin x}}{x}, x>0$

Now, $I=\int_1^4 \frac{2e^{\sin x^2}}{x}dx$

$=\int_1^4 \frac{2xe^{\sin x^2}}{x^2}dx \quad$ [Put $t=x^2 \Rightarrow dt=2xdx$]

$=\int_1^{16} \frac{e^{\sin t}dt}{t}=[f(t)]_1^{16}=f(16)-f(1) \quad \therefore \quad k=16$

9. (c) : The line $x = 1$ meets the curves in $A(1, e)$ and $B\left(1,\frac{1}{e}\right)$. Both the curves pass through origin.



The required area

$=\int_0^1 (xe^x-xe^{-x})dx=\frac{2}{e}$ sq.units

10. (b) : We have, $f(x)=x^4-2x^3+x^2+3$

$\therefore \quad f'(x)=4x^3-6x^2+2x$

$f'(x)=0 \quad \Rightarrow \quad 4x^3-6x^2+2x=0 \quad \Rightarrow \quad x=0,1,\frac{1}{2}$

$f''(x)=12x^2-12x+2=2(6x^2-6x+1)$

$f''(x)_{x=0}>0 \Rightarrow$ Minimum exists at $x=0$ and $x=1$

$f''(x)_{x=0}<0 \Rightarrow$ Maximum exists at $x=\frac{1}{2}$

$\Rightarrow$ The curve is bounded by the ordinates $x = 0$ and $x = 1$

$\therefore$ Required area

$=\int_0^1 (x^4-2x^3+x^2+3)dx=\left(\frac{x^5}{5}-\frac{x^4}{2}+\frac{x^3}{3}+3x\right)_0^1$

$=\frac{91}{30}$ sq.units

11. (b) : Differentiating, $ax^2 + by^2 = 1$ ...(i)

we get, $ax + byy_1 = 0$ ...(ii)

$a+b(y_1^2+yy_2)=0$ ...(iii)

Eliminating a and b we get,

$$\begin{vmatrix} x^2 & y^2 & -1 \\ x & yy_1 & 0 \\ 1 & y_1^2+yy_2 & 0 \end{vmatrix}=0 \Rightarrow xyy_2+xy_1^2-yy_1=0$$

$\therefore$ Required order is 2 and degree is 1

12. (d) : We have, $(x^3 - x^2)dy = (2y - xy)dx$

$\Rightarrow \quad \frac{dy}{y}=\frac{2-x}{x^2(x-1)}dx=\left(\frac{1}{x-1}-\frac{1}{x}-\frac{2}{x^2}\right)dx$

$\Rightarrow \int\frac{dy}{y}=\int\left(\frac{1}{x-1}-\frac{1}{x}-\frac{2}{x^2}\right)dx \Rightarrow \ln y=\ln\frac{x-1}{x}+\frac{2}{x}+c$

Now, $y(2)=e \quad \Rightarrow 1=-\ln 2+1+c \quad \Rightarrow c=\ln 2$

$\therefore \quad \ln y=\ln\frac{x-1}{x}+\frac{2}{x}+\ln 2$

Now, $x=-1 \Rightarrow \ln y=\ln 4-2=\ln\frac{4}{e^2} \quad \Rightarrow \quad y=\frac{4}{e^2}$

13. (b) : $\frac{xdy-ydx}{x^2}+\frac{\cos\ln x}{x}dx=0$

$\Rightarrow \quad \frac{d}{dx}\left(\frac{y}{x}\right)+\frac{d}{dx}(\sin\ln x)=0 \quad \Rightarrow \quad \frac{y}{x}+\sin\ln x=c$

Now, $x = 1, y = 1 \Rightarrow c = 1$

and $x = e \Rightarrow y = e(1 - \sin 1)$

14. (c): We have, $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$ [put $y=vx$]

$v+x\frac{dv}{dx}=\frac{v}{1+v^2} \quad \Rightarrow \quad x\frac{dv}{dx}=\frac{v}{1+v^2}-v=-\frac{v^3}{1+v^2}$

$\Rightarrow \quad \frac{1+v^2}{v^3}dv=-\frac{dx}{x} \quad \Rightarrow \quad \int\frac{1+v^2}{v^3}dv=-\int\frac{dx}{x}$

$\Rightarrow \quad \ln x+\ln v-\frac{1}{2v^2}=c \quad \Rightarrow \ln y=\frac{x^2}{2y^2}+c$

Now $x=1, y=1 \quad \Rightarrow \quad c=-\frac{1}{2}$

Again $y=e \quad \Rightarrow \quad 1=\frac{x_0^2}{2e^2}-\frac{1}{2} \Rightarrow x_0^2=3e^2, x_0=\sqrt{3}\cdot e.$

15. (b) : The given equation is

$1+y^2+(x-e^{\tan^{-1}y})\frac{dx}{dy}=0 \Rightarrow \frac{dx}{dy}+\frac{1}{1+y^2}\cdot x=\frac{e^{\tan^{-1}y}}{1+y^2}$

General solution is $xe^{\tan^{-1}y} = \int \frac{e^{2\tan^{-1}y}}{1+y^2}dy$

$$= \frac{e^{2\tan^{-1}y}}{2} + c_1$$

$\Rightarrow \quad 2xe^{\tan^{-1}y} = e^{2\tan^{-1}y} + c$

16. (b) : Given, $y - \cos x \frac{dy}{dx} = y^2(1-\sin x)\cos x$

Dividing the above differential equation by $y^2 \cos x$, we get

$$-\frac{1}{y^2}\frac{dy}{dx} + \sec x \cdot \frac{1}{y} = 1 - \sin x$$

$$\Rightarrow \quad \frac{d}{dx}\left(\frac{1}{y}\right) + \sec x \cdot \left(\frac{1}{y}\right) = 1 - \sin x$$

Here, I.F. $= e^{\int \sec x dx} = \sec x + \tan x$

General solution is

$$\frac{\sec x + \tan x}{y} = \int \frac{(1-\sin x)(1+\sin x)dx}{\cos x}$$

$$= \int \cos x dx = \sin x + c$$

$\therefore \quad x = 0, y = 1 \quad \Rightarrow \quad c = 1 \quad \Rightarrow \quad y\left(\frac{\pi}{3}\right) = 2$

17. (c): The given equation is

$(2x - 2y + 5)dy = (x - y + 3)dx$

$\Rightarrow \frac{dy}{dx} = \frac{(x-y+3)}{(2x-2y+5)} \quad \left[\text{put } x - y = z \Rightarrow 1 - \frac{dy}{dx} = \frac{dz}{dx}\right]$

$\Rightarrow 1 - \frac{dz}{dx} = \frac{z+3}{2z+5} \quad \Rightarrow \int dx = \int\left(\frac{2z+5}{z+2}\right)dz$

$\Rightarrow \quad x + c = 2z + \ln(z + 2) = 2x - 2y + \ln(x - y + 2)$

$\Rightarrow \quad \ln(x - y + 2) = 2y - x + c$

18. (c) : The given differential equation is

$$\frac{dy}{dx} = \frac{(x+1)^2 + y - 3}{x+1} = (x+1) + \frac{y-3}{x+1}$$

[Let $x + 1 = X$ and $y - 3 = Y$], $\frac{dY}{dX} - \frac{Y}{X} = X$, I.F. $= \frac{1}{X}$

General solution is $\frac{Y}{X} = X + c$

$\Rightarrow \quad \frac{y-3}{x+1} = x + 1 + c$ ...(i)

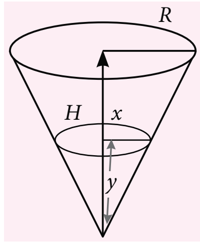
When $x = 2$, $y = 0$, then $c = -4$

Now(i) becomes $y - 3 = (x+1)^2 - 4(x+1) = x^2 - 2x - 3$

$\Rightarrow \quad y = x^2 - 2x$ meets x-axis at $x = 0, 2$

$\therefore \quad \text{Area} = \int_0^2 (2x - x^2)dx = 4 - \frac{8}{3} = \frac{4}{3}$

19. (a) : Let y be the height at time t.



$\frac{x}{y} = \frac{R}{H}$, $v = \frac{1}{3}\pi x^2 y = \frac{\pi}{3} \cdot \frac{Hx^3}{R}$

$\Rightarrow \quad \frac{dv}{dt} = \frac{\pi H x^2}{R}\frac{dx}{dt}$

But $\frac{dv}{dt} = -\lambda\pi x^2 \quad \Rightarrow \quad \frac{dx}{dt} = \frac{-\lambda R}{H}$

$\Rightarrow \quad x = \frac{-\lambda R t}{H} + c$, $t = 0, x = R \quad \Rightarrow \quad c = R$

$\Rightarrow \quad x = R\left(1 - \frac{\lambda t}{H}\right)$, $x = 0 \quad \Rightarrow \quad t = \frac{H}{\lambda}$.

20. (d) : We have, $\ln y = \ln a + bx$

Differentiating twice, we get $\frac{1}{y}y' = b$,

$$\frac{y''}{y} - \frac{1}{y^2}(y')^2 = 0 \quad \Rightarrow \quad yy'' = (y')^2$$

21. (a) : Let E_1 and E_2 denote the event that letters came from TATANAGAR and CALCUTTA respectively. Let A denotes the event that two consecutive visible letters on the envelop are 'TA'. We have

$P(E_1) = P(E_2) = \frac{1}{2}$, $P(A \mid E_1) = \frac{2}{8}$ and $P(A \mid E_2) = \frac{1}{7}$.

Using the Bayes' theorem, we get

$$P(E_2 \mid A) = \frac{P(E_2)P(A \mid E_2)}{P(E_1)P(A \mid E_1) + P(E_2)P(A \mid E_2)}$$

$$= \frac{\frac{1}{2}\cdot\frac{1}{7}}{\frac{1}{2}\cdot\frac{2}{8} + \frac{1}{2}\cdot\frac{1}{7}} = \frac{4}{11}$$

22. (a) : Let p denotes the probability of a correct distinction between a cup of instant coffee and a cup of percolator coffee.

Then, $p = \frac{75}{100} = \frac{3}{4} \quad \Rightarrow q = 1 - p = \frac{1}{4}$ and $n = 6$

Let random variable X denotes the number of correct distinctions.

Then, $P(X = x) = p(x) = {}^6C_x\left(\frac{3}{4}\right)^x\left(\frac{1}{4}\right)^{6-x}$, $x = 0, 1, ..., 6$

The probability of the claim being accepted is

$$P(X \geq 5) = p(5) + p(6) = {}^6C_5\left(\frac{3}{4}\right)^5\left(\frac{1}{4}\right)^{6-5} + {}^6C_6\left(\frac{3}{4}\right)^6$$

$= 0.534$ (approx.)

23. (a) : Probability of a vessel arriving safe = 9/10

Required probability $= {}^5C_4\left(\frac{9}{10}\right)^4 \times \frac{1}{10} + \left(\frac{9}{10}\right)^5$

$$= \frac{9^4 \times 14}{10^5} = 0.92 \text{ (approx.)}$$

24. (b) : $P(X) = \begin{cases} \frac{X}{15}, & X = 1, 2, 3, 4, 5 \\ 0, & \text{otherwise} \end{cases}$

$\therefore \; P(X = 1 \text{ or } 2) = P(X = 1) + P(X = 2) = \frac{1}{15} + \frac{2}{15} = \frac{1}{5}$

Now, $P\left(\frac{1}{2} < X < \frac{5}{2} \,|\, X > 1\right) = \frac{P\left\{\left(\frac{1}{2} < X < \frac{5}{2}\right) \cap X > 1\right\}}{P(X > 1)}$

$$= \frac{P\{(X = 1 \text{ or } 2) \cap (X > 1)\}}{P(X > 1)} = \frac{P(X = 2)}{1 - P(X = 1)} = \frac{\frac{2}{15}}{1 - \frac{1}{15}} = \frac{1}{7}.$$

25. (c) : Let X and Y denote respectively the number of points obtained in the first and the second throws. Then both of them take the values 1, 2, 3, 4, 5 and 6 with each probability 1/6.

$\therefore \; E(X) = (1 + 2 + 3 + 4 + 5 + 6) \times \frac{1}{6} = \frac{7}{2}.$

Similarly, $E(Y) = \frac{7}{2}.$

Thus $E(XY) = E(X)E(Y) = \frac{7}{2} \cdot \frac{7}{2} = \frac{49}{4}$

26. (a) : We know, variance of Binomial distribution is

$$\text{Var}(X) = npq = n\left[\left(\frac{p+q}{2}\right)^2 - \left(\frac{p-q}{2}\right)^2\right]$$

$$= n\left[\frac{1}{4} - \left(\frac{p-q}{2}\right)^2\right] \le \frac{n}{4} \qquad \left[\because \left(\frac{p-q}{2}\right)^2 \ge 0\right]$$

Hence, for a fixed value of n, the variance of the binomial distribution can never exceed $n/4$. So, the maximum value of var(X) is $n/4$.

27. (b) : Let the parameters of the distribution be n and p. Since the distribution is symmetric

$\therefore \quad p = 1/2.$

Again, $np = 5$ (Given)

So, variance $= np(1 - p) = \frac{5}{2}.$

28. (a) : Let a random variable X follows a binomial distribution with parameter n and p.

Given that mean $= E(X) = np = 2$ and $E(X^2) = \frac{28}{5}.$

$\therefore \quad npq = E(X^2) - [E(X)]^2 = \frac{28}{5} - 2^2 = \frac{8}{5}.$

Now, $\frac{npq}{np} = \frac{8}{5} \div 2 = \frac{4}{5} \;\Rightarrow\; q = \frac{4}{5}$

$\therefore \quad p = \frac{1}{5}$ and hence $n = 10$

29. (d) : Let E_1: The ball drawn from the first box

E_2: The ball drawn from the second box

A: The drawn ball is black.

$\therefore \quad P(E_1) = \frac{3}{6} = \frac{1}{2}$ and $P(E_2) = \frac{3}{6} = \frac{1}{2}$

Now, $P(A \,|\, E_1) = \frac{3}{7}$ and $P(A \,|\, E_2) = \frac{4}{7}.$

$\therefore \quad P(E_1 \,|\, A) = \frac{P(E_1) \cdot P(A \,|\, E_1)}{P(E_1) \cdot P(A \,|\, E_1) + P(E_2) \cdot P(A \,|\, E_2)}$

$$= \frac{\frac{1}{2} \cdot \frac{3}{7}}{\frac{1}{2} \cdot \frac{3}{7} + \frac{1}{2} \cdot \frac{4}{7}} = \frac{3}{7}$$

30. (a) : Let X be the random variable denoting number of heads obtain. Clearly X follows Binomial distribution.

$$P(X = x) = {}^nC_x p^x q^{n-x} = {}^nC_x \left(\frac{1}{2}\right)^n \quad \left[\because \; p = q = \frac{1}{2}\right]$$

Now $P(X \ge 1) = 1 - P(X = 0) = 1 - {}^nC_0\left(\frac{1}{2}\right)^n = 1 - \frac{1}{2^n}$

Again, let Y be the random variable denoting the number of tails obtain.

$\therefore \; P(Y = y) = {}^nC_y p^y q^{n-y}$

$$= {}^nC_y\left(\frac{1}{2}\right)^n \quad \left[\because \; p = q = \frac{1}{2}\right]$$

Now, $P(Y \ge 2) = 1 - P(Y = 0) - P(Y = 1)$

$$= 1 - {}^nC_0\left(\frac{1}{2}\right)^n - {}^nC_1\left(\frac{1}{2}\right)^n = 1 - \frac{1}{2^n} - n\frac{1}{2^n}$$

According to the question, we get

$$\left(1 - \frac{1}{2^n}\right) - \left(1 - \frac{1+n}{2^n}\right) = \frac{5}{32} \;\Rightarrow\; \frac{n}{2^n} = \frac{5}{32} \;\Rightarrow\; n = 5$$

CONCEPT MAP

BINOMIAL THEOREM

Class XI

Expansion

For positive integral index $n \in N$

- $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + \ldots + {}^nC_nb^n = \sum_{r=0}^{n} {}^nC_r a^{n-r}b^r$

For negative integral index

- $(a+b)^{-n} = \sum_{r=0}^{n} (-1)^n \, {}^nC_r a^{n-r}b^r$

General Term

In the binomial expansion for $(a+b)^n$, general term

- $T_{r+1} = {}^nC_r a^{n-r} b^r$
- T_{r+1} from the end $= T_{n-r+1}$ from the beginning

Middle Term

In the expansion $(a+b)^n$,

- If n is even, then $\left(\frac{n}{2}+1\right)^{\text{th}}$ term is the middle term.
- If n is odd, then $\left(\frac{n+1}{2}\right)^{\text{th}}, \left(\frac{n+3}{2}\right)^{\text{th}}$ are two middle terms.

Binomial Theorem for Fractional Index

For $r \geq 0$, $|x| < 1$ and $n \in Q$

- $(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \ldots + \frac{n(n-1)\ldots(n-r+1)x^r}{r!} + \ldots + \infty$
- $T_{r+1} = \frac{n(n-1)(n-2)\ldots.(n-r+1)}{r!}x^r$
- T_{r+1} in $(1+x)^{-n} = \frac{(-1)^r n(n+1)(n+2)\ldots(n+r-1)x^r}{r!}$
- T_{r+1} in $(1-x)^{-n} = \frac{n(n+1)(n+2)\ldots.(n+r-1)x^r}{r!}$

Binomial Coefficients

Greatest Binomial Coefficient are given by

- ${}^nC_{n/2}$, if n is even,
- ${}^nC_{n-1/2} = \frac{{}^nC_{n+1}}{2}$, if n is odd

Coefficient of

- $(r+1)^{\text{th}}$ term in $(1+x)^n$ is ${}^nC_r$.
- x^r in $(1+x)^n$ is ${}^nC_r$
- x^r in $(1-x)^n$ is $(-1)^r\,{}^nC_r$.
- $(r+1)^{\text{th}}$ term in $(1-x)^n$ is $(-1)^r\,{}^nC_r$

Some Important Results

- ${}^nC_r = {}^nC_{n-r}$
- ${}^nC_r = \frac{n}{r}\,{}^{n-1}C_{r-1} = \frac{n}{r}\left(\frac{n-1}{r-1}\right){}^{n-1}C_{r-2}$
- ${}^nC_x = {}^nC_y \Rightarrow$ Either $x = y$ or $x + y = n$
- ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
- $\sum_{r=0}^{n} {}^nC_r = \sum_{r=0}^{n} C_r = 2^n$
- $C_0 + C_2 + C_4 + \ldots = C_1 + C_3 + C_5 + \ldots = 2^{n-1}$
- $\sum_{r=0}^{n} r \cdot C_r = n2^{n-1}$
- $C_0^2 - C_1^2 + C_2^2 - C_3^2 + \ldots. + (-1)^n C_n^2 = \begin{cases} 0, n \text{ is odd} \\ (-1)^{\frac{n}{2}}\,{}^nC_{\frac{n}{2}}, \; n \text{ is even} \end{cases}$
- $C_0 - C_2 + C_4 - C_6 + \ldots = 2^{\frac{n}{2}} \cos\frac{n\pi}{4}$
- $C_1 - C_3 + C_5 - C_7 + \ldots = 2^{\frac{n}{2}} \sin\frac{n\pi}{4}$
- If $n = p_1^{\alpha_1}.p_2^{\alpha_2}.\,p_3^{\alpha_3}\ldots.\,p_r^{\alpha_r}$, where $p_1, p_2, \ldots. p_r$ are prime numbers and $\alpha_1, \alpha_2, \ldots \alpha_r$ are positive integers. Then sum of positive divisors of $n = (1 + p_1 + p_1^2 + \ldots + p_1^{\alpha_1})(1 + p_2 + p_2^2 + \ldots + p_2^{\alpha_2})\ldots.(1 + p_r + p_r^2 + \ldots. + p_r^{\alpha_r})$.

INDEFINITE INTEGRALS

CONCEPT MAP

Class XII

INDEFINITE INTEGRALS

Integration is the inverse process of differentiation $\frac{d}{dx}F(x) = f(x)$, $F(x)$ is an indefinite integral of $f(x)$ or, $\int f(x)dx = F(x) + c$, where c is an arbitrary constant of integration

Properties

- $\int f'(x)dx = f(x) + c$
- $\int [f(x) \pm g(x)]dx = \int f(x)\,dx \pm \int g(x)\,dx$
- $\int k \cdot f(x)\,dx = k\int f(x)\,dx$, k being any real number.

Some Standard Integrals

- $\int dx = x + c$
- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$, where $n \neq -1$
- $\int e^x dx = e^x + c$
- $\int a^x dx = \frac{a^x}{\log_e a} + c$, where $a > 0, a \neq 1$
- $\int \frac{1}{x}dx = \log_e |x| + c$, where $x \neq 0$
- $\int \sin x\,dx = -\cos x + c$
- $\int \cos x\,dx = \sin x + c$
- $\int \tan x\,dx = \log|\sec x| + c$
- $\int \cot x\,dx = \log|\sin x| + c$
- $\int \sec x\,dx = \log|\sec x + \tan x| + c$
- $\int \operatorname{cosec} x\,dx = \log|\operatorname{cosec} x - \cot x| + c$

where 'c' is the constant of integration.

Methods

Using Substitution

The given integral $\int f(x)\,dx$ can be transformed into another form by changing the independent variable x to t by substituting $x = g(t)$.

Using by Parts

If u and v are two differentiable functions of x, then

$$\int (uv)\,dx = \left[u \cdot \int v dx\right] - \int \left\{\frac{du}{dx} \cdot \int v dx\right\} dx.$$

In order to choose 1st function, we take the letter which comes first in the word ILATE.

I – Inverse Trigonometric Function

L – Logarithmic Function, A – Algebraic Function

T – Trigonometric Function, E – Exponential Function

Using Partial Fractions

- If $f(x)$ and $g(x)$ are two polynomials such that $\deg f(x) \geq \deg g(x)$, then we divide $f(x)$ by $g(x)$.
  $\therefore \frac{f(x)}{g(x)} = \text{Quotient} + \frac{\text{Remainder}}{g(x)}$
- If $f(x)$ and $g(x)$ are two polynomials such that the degree of $f(x)$ is less than the degree of $g(x)$, then we can evaluate $\int \frac{f(x)}{g(x)}dx$ by decomposing $\frac{f(x)}{g(x)}$ into partial fraction.

Integrals of Some Particular Functions

- $\int \frac{1}{\sqrt{x^2 - a^2}}dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c$
- $\int \frac{1}{\sqrt{x^2 + a^2}}dx = \log\left|x + \sqrt{x^2 + a^2}\right| + c$
- $\int \frac{1}{x^2 - a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$
- $\int \frac{1}{a^2 - x^2}dx = \frac{1}{2a}\log\left|\left(\frac{a+x}{a-x}\right)\right| + c, a > x$
- $\int \frac{1}{x^2 + a^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$
- $\int \frac{1}{x\sqrt{x^2 - 1}}dx = \sec^{-1} x + c = -\operatorname{cosec}^{-1} x + c$, where $|x| > 1$
- $\int \sqrt{x^2 + a^2}dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + c$
- $\int \sqrt{x^2 - a^2}dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + c$
- $\int \sqrt{a^2 - x^2}dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$
  $\int \frac{1}{\sqrt{a^2 - x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\int e^x(f(x) + f'(x))dx = e^x f(x) + c$

YOU ASK WE ANSWER

Do you have a question that you just can't get answered?
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. Prove that :

$$\begin{vmatrix} a & a+b & a+2b & .... & a+nb \\ a+nb & a & a+b & .... & a+(n-1)b \\ a+(n-1)b & a+nb & a & .... & a+(n-2)b \\ .... & .... & .... & .... & .... \\ a+b & a+2b & a+3b & .... & a \end{vmatrix} = (-b)^n (n+1)^n \left(a+\frac{nb}{2}\right).$$

(Mandeep, Delhi)

Ans. We have,

$$\text{L.H.S.} = \begin{vmatrix} a & a+b & a+2b & .... & a+nb \\ a+nb & a & a+b & .... & a+(n-1)b \\ a+(n-1)b & a+nb & a & .... & a+(n-2)b \\ .... & .... & .... & .... & .... \\ a+b & a+2b & a+3b & .... & a \end{vmatrix}$$

$$= (n+1)\left(a+\frac{nb}{2}\right)\begin{vmatrix} 1 & a+b & a+2b & .... & a+nb \\ 1 & a & a+b & .... & a+(n-1)b \\ 1 & a+nb & a & .... & a+(n-2)b \\ .... & .... & .... & .... & .... \\ 1 & a+2b & a+3b & .... & a \end{vmatrix}$$

$$\left[C_1 \to C_1 + C_2 + ... + C_{n+1} \text{ and taking out } (n+1)\left(a+\frac{nb}{2}\right)\right]$$

$$= (n+1)\left(a+\frac{nb}{2}\right)\begin{vmatrix} 1 & a+b & a+2b & .... & a+nb \\ 0 & -b & -b & .... & a+nb \\ 0 & (n-1)b & -2b & .... & -2b \\ .... & .... & .... & .... & .... \\ 1 & b & b & .... & -nb \end{vmatrix}$$

$[R_2 \to R_2 - R_1, R_3 \to R_3 - R_1, ..., R_{n+1} \to R_{n+1} - R_1]$

$$= (-b)^n (n+1)\left(a+\frac{nb}{2}\right)\begin{vmatrix} 1 & a+b & a+2b & .... & a+nb \\ 0 & 1 & 1 & .... & 1 \\ 0 & (1-n) & 2 & .... & 1 \\ .... & .... & .... & .... & .... \\ 0 & -1 & -1 & .... & n \end{vmatrix}$$

[taking $(-b)$ common from $R_2, R_3, ..., R_{n+1}$]

$$= (-b)^n (n+1)\left(a+\frac{nb}{2}\right)\begin{vmatrix} 1 & 1 & 1 & .... & 1 \\ (1-n) & 2 & 2 & .... & 2 \\ (2-n) & (2-n) & 3 & .... & 3 \\ .... & .... & .... & .... & .... \\ -1 & -1 & -1 & .... & n \end{vmatrix}$$

[expanding along C_1]

$$= (-b)^n (n+1)\left(a+\frac{nb}{2}\right)\begin{vmatrix} 1 & 0 & 0 & .... & 1 \\ (1-n) & (n+1) & (n+1) & .... & (n+1) \\ (2-n) & 0 & (n+1) & .... & (n+1) \\ .... & .... & .... & .... & .... \\ -1 & 0 & 0 & .... & (n+1) \end{vmatrix}$$

$[C_2 \to C_2 - C_1, C_3 \to C_3 - C_1, ..., C_n \to C_n - C_1]$

$$= (-b)^n (n+1)^n \left(a+\frac{nb}{2}\right)\begin{vmatrix} 1 & 0 & 0 & .... & 0 \\ (1-n) & 1 & 1 & .... & 1 \\ (2-n) & 0 & 1 & .... & 1 \\ .... & .... & .... & .... & .... \\ -1 & 0 & 0 & .... & 1 \end{vmatrix}$$

[taking common $(n + 1)$ from $C_2, C_3, ..., C_n$]

$$= (-b)^n (n+1)^n \left(a+\frac{nb}{2}\right)\begin{vmatrix} 1 & 1 & .... & 1 \\ 0 & 1 & .... & 1 \\ .... & .... & .... & .... \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

[expanding along R_1]

$$= (-b)^n (n+1)^n \left(a+\frac{nb}{2}\right) = \text{R.H.S.}$$

2. $\bar{b}z + b\bar{z} = c$ is the equation of a straight line. If z_1 and z_2 be mirror image of each other in this line, then prove that $\bar{b}z_2 + b\bar{z}_1 = c$.

(Rituraj, Chandigarh)

Ans. If z_1 and z_2 are mirror image of each other in the line

$\bar{b}z + b\bar{z} = c$ ...(i)

then the mid-point of z_1, z_2 must lie on (i) ...(ii)

and the line joining z_1, z_2 must be perpendicular to (i) ...(iii)

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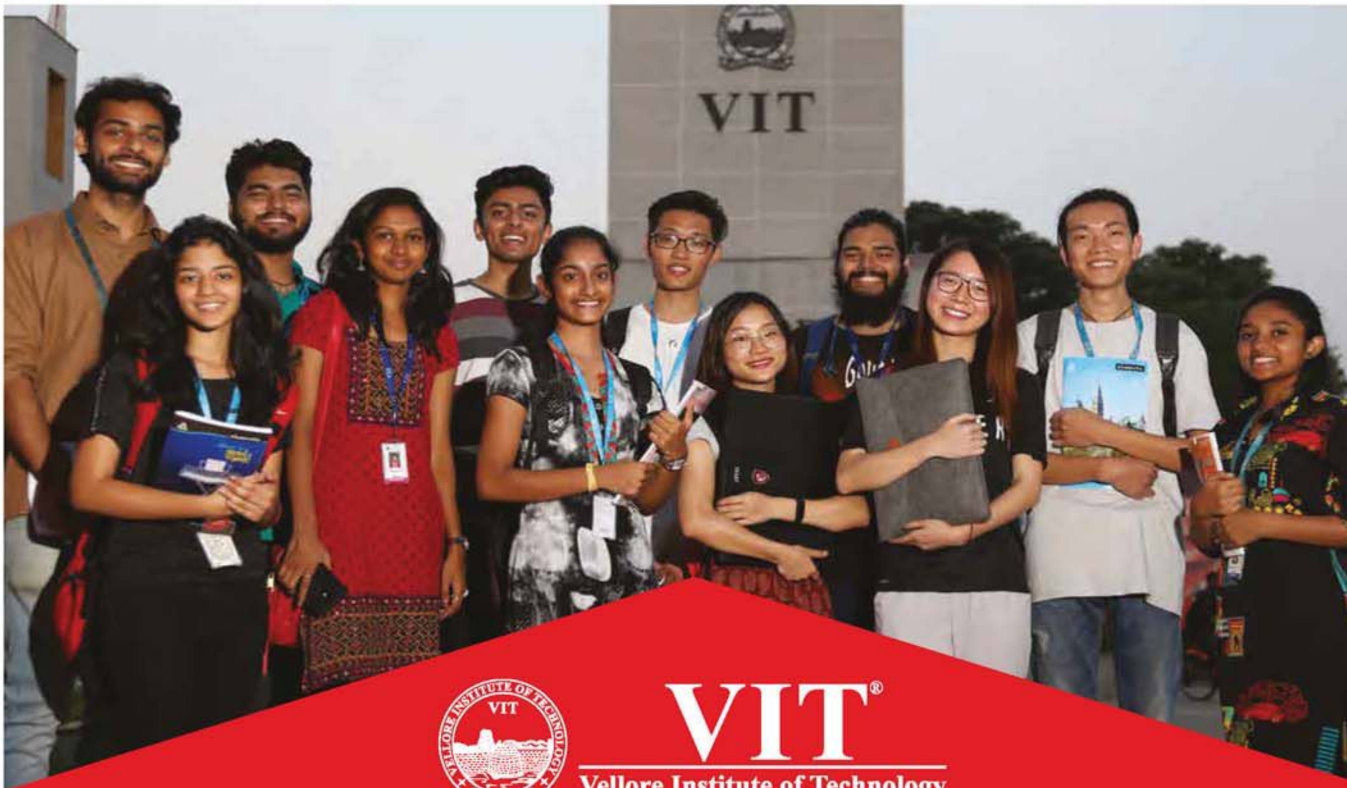
VIT

MATH archives

Math Archives, as the title itself suggests, is a collection of various challenging problems related to the topics of JEE Main & Advanced Syllabus. This section is basically aimed at providing an extra insight and knowledge to the candidates preparing for JEE Main & Advanced. In every issue of MT, challenging problems are offered with detailed solution. The readers' & comments and suggestions regarding the problems and solutions offered are always welcome.

1. Equation of a common tangent to the curves $y^2 = 8x$ and $xy = -1$ is
(a) $3y = 9x + 2$ (b) $y = 2x + 1$
(c) $2y = x + 8$ (d) $y = x + 2$

2. Integrating factor of the differential equation $2y\cos y^2 \dfrac{dy}{dx} - \dfrac{2}{x+1}\sin y^2 = (x+1)^3$ is
(a) $(x+1)^2$ (b) $\log_e(x+1)$
(c) $e^{\log_e(x+1)}$ (d) $\dfrac{1}{(x+1)^2}$

3. The radius of a right circular cylinder of maximum volume which can be inscribed in a sphere of radius R is
(a) R (b) $\dfrac{R}{2}$ (c) $\sqrt{\dfrac{2}{3}}R$ (d) $\sqrt{\dfrac{3}{2}}R$

4. The distance between the line $\dfrac{x-2}{1} = \dfrac{y+2}{-1} = \dfrac{z-3}{4}$ and the plane $x + 5y + z = 5$ is
(a) $\dfrac{10}{3\sqrt{3}}$ units (b) $\dfrac{10}{3}$ units
(c) $\dfrac{10}{9}$ units (d) $\dfrac{10}{7}$ units

5. The number of ways in which 13 identical gold coins can be distributed among three persons such that each person can get at least two gold coins is
(a) 36 (b) 24 (c) 12 (d) 6

6. If A, B, C are the angles of a triangle, the system of equations $(\sin A)x + y + z = \cos A$, $x + (\sin B)y + z = \cos B$, $x + y + (\sin C)z = 1 - \cos C$ has
(a) no solution
(b) unique solution
(c) infinitely many solutions
(d) finitely many solutions

7. Let $f(x) = 2x^n + \lambda$, $\lambda \in R$, $f(4) = 133$ and $f(5) = 255$ then sum of the factors of $(f(3) - f(2))$ is
(a) 20 (b) 60 (c) 21 (d) 59

8. If $|z^2 - 1| = |z|^2 + 1$, then z lies on
(a) the real axis (b) an ellipse
(c) a circle (d) the imaginary axis

9. If $x^2 - 4x + 5 - \sin y = 0$, $y \in [0, 2\pi]$, then
(a) $x = 1, y = 0$ (b) $x = 1, y = \pi/2$
(c) $x = 2, y = 0$ (d) $x = 2, y = \pi/2$

10. A solution of the equation
$$\tan^{-1}(1+x) - \tan^{-1}(x-1) = \frac{\pi}{2} \text{ is}$$
(a) $x = 1$ (b) $x = -1$ (c) $x = 0$ (d) $x = \pi$

SOLUTIONS

1. (d): Any tangent to $y^2 = 8x$ is $y = mx + \dfrac{2}{m}$ ...(i)
For (i) be tangent to $xy = -1$

$\Rightarrow x\left(mx + \dfrac{2}{m}\right) = -1 \Rightarrow m^2x^2 + 2x + m = 0$

Now, $D = 0 \Rightarrow 4 - 4m^3 = 0$
$\Rightarrow 4m^3 = 1 \therefore m = 1$
Hence, tangent is $y = x + 2$.

2. (d): $\text{I.F.} = e^{-\int \frac{2}{x+1}dx} = \dfrac{1}{(x+1)^2}$

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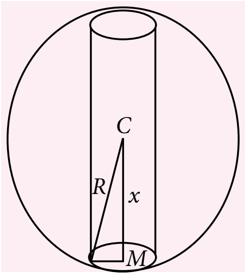
3. **(c) :** Let r be radius of cylinder and R be radius of sphere.



Let $CM = x$

$v = \pi(R^2 - x^2)2x$

For Maximum volume $\frac{dv}{dx} = 0$

$\Rightarrow \quad x = \frac{R}{\sqrt{3}};\ r = \sqrt{R^2 - x^2};$

Also, $\frac{d^2v}{dx^2} < 0$

$\therefore$ For maximum volume,

$$r = \sqrt{R^2 - \frac{R^2}{3}};\ r = \sqrt{\frac{2}{3}R^2} = \sqrt{\frac{2}{3}}R$$

$\therefore$ For maximum volume

4. **(a) :** Distance of line from the plane

$$= \frac{|(2\hat{i} - 2\hat{j} + 3\hat{k})\cdot(\hat{i} + 5\hat{j} + \hat{k}) - 5|}{\sqrt{(1)^2 + (5)^2 + (1)^2}} = \frac{10}{3\sqrt{3}}$$

5. **(a) :** $x + y + z = 13 - 6 = 7$

$\therefore$ No.of non negative solutions $= {}^{n+r-1}C_{r-1}$

$= {}^{7+3-1}C_{3-1} = {}^{9}C_{2} = 36.$

6. **(b) :** Let $\Delta = \begin{vmatrix} \sin A & 1 & 1 \\ 1 & \sin B & 1 \\ 1 & 1 & \sin C \end{vmatrix}$

Apply $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

Then expand along C_1, we get

$\Delta = \sin A(1 - \sin B)(1 - \sin C) + (1 - \sin A)(1 - \sin C) + (1 - \sin A)(1 - \sin B)$

Since A, B, C are angles of a triangle,

$0 < \sin A, \sin B, \sin C \le 1 \ \Rightarrow\ \Delta > 0$

(or) Put $A = B = C = 60° \Rightarrow \Delta \ne 0$

7. **(b) :** $f(x) = 2x^n + \lambda,\ f(4) = 133,\ f(5) = 255$

$\therefore \quad f(4) = 2\cdot 4^n + \lambda = 133$ ...(i)

$\quad f(5) = 2\cdot 5^n + \lambda = 255$ ...(ii)

$\therefore \quad f(5) - f(4) = 2(5^n - 4^n) = 122$

$\Rightarrow \quad 5^n - 4^n = 61 = 5^3 - 4^3 \quad \therefore \quad n = 3$

Now, $\quad f(4) = 133 = 2\times 4^3 + \lambda \ \Rightarrow\ \lambda = 5$

$\therefore \quad f(x) = 2x^3 + 5$

$\therefore \quad f(3) = 59$ and $f(2) = 21$

$\therefore \quad f(3) - f(2) = 38 = 2\times 19 = 2^a 19^b,\ (a = 1, b = 1)$

Sum of factors of $f(3) - f(2)$

$$= \frac{2^{a+1} - 1}{2 - 1}\cdot\frac{19^{b+1} - 1}{19 - 1} = \frac{3\times 360}{18} = 60$$

8. **(d) :** Let $z = x + iy$

Given, $|z^2 - 1| = |z|^2 + 1$

$\Rightarrow \quad (x^2 - y^2 - 1)^2 + 4x^2y^2 = (x^2 + y^2 + 1)^2 \ \Rightarrow\ x = 0$

9. **(d) :** $x^2 - 4x + 5 = \sin y \Rightarrow (x - 2)^2 + 1 = \sin y \le 1$

$\Rightarrow \quad \sin y = 1,\ x = 2 \ \Rightarrow x = 2,\ y = \pi/2$

10. **(c):** $\tan^{-1}(1 + x) - \tan^{-1}(x - 1) = \frac{\pi}{2}$

$\Rightarrow \quad \tan^{-1}\frac{2}{x^2} = \frac{\pi}{2} \quad \Rightarrow \quad x = 0$

YOU ASK WE ANSWER

Contd. from page no. 46

Satisfying condition (ii), we have

$$\bar{b}\left(\frac{z_1 + z_2}{2}\right) + b\left(\overline{\frac{z_1 + z_2}{2}}\right) = c$$

i.e., $\bar{b}(z_1 + z_2) + b(\bar{z}_1 + \bar{z}_2) = 2c$

i.e., $(\bar{b}z_1 + b\bar{z}_2) + (\bar{b}z_2 + b\bar{z}_1) = 2c$ ...(iv)

Putting $z = x + iy$ in equation (i), it reduces to

$\bar{b}(x + iy) + b(x - iy) - c = 0$

i.e., $(\bar{b} + b)x + i(\bar{b} - b)y - c = 0$

whose slope $= \frac{(b + \bar{b})}{i(b - \bar{b})}$

Equation of the line joining z_1, z_2 is

$$\begin{vmatrix} z & \bar{z} & 1 \\ z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \end{vmatrix} = 0$$

i.e., $z\bar{a} - \bar{z}a + d = 0$

where $a = z_1 - z_2$ and $d = z_1\bar{z}_2 - \bar{z}_1, z_2$

Putting $z = x + iy$, the above equation reduces to

$\bar{a}(x + iy) - a(x - iy) + d = 0$

i.e., $(\bar{a} - a)x + i(\bar{a} + a)y + d = 0$

whose slope $= \frac{(a - \bar{a})}{i(a + \bar{a})}$

Satisfying equation (iii), we have

$$\frac{(b + \bar{b})}{i(b - \bar{b})}\times\frac{(a - \bar{a})}{i(a + \bar{a})} = -1$$

$\Rightarrow \quad (b + \bar{b})(a - \bar{a}) = (b - \bar{b})(a + \bar{a}) \ \Rightarrow\ \bar{b}a = b\bar{a}$

$\Rightarrow \quad \bar{b}(z_1 - z_2) = b(\bar{z}_1 - \bar{z}_2)$

$\Rightarrow \quad \bar{b}z_1 + b\bar{z}_2 = \bar{b}z_2 + b\bar{z}_1$ ...(v)

From equations (iv) and (v), we have

$\bar{b}z_2 + b\bar{z}_1 = c$

which is the desired result.

TARGET JEE

Probability

DEFINITIONS OF VARIOUS TERMS

- **Random experiment :** An experiment is called random if all possible outcomes of the experiment are known in advance but the exact result of any specific performance cannot be predicted in advance.
- **Sample space :** The set of all possible outcomes of an experiment is known as sample space of the experiment and is usually denoted by S. The sample space of rolling a die is $S = \{1, 2, 3, 4, 5, 6\}$ and $n(S)$ denote the number of numbers in S. When a die is rolled twice, then $n(S) = 6^2 = 36$. Each element of the sample space is called **sample point**. There are two types of sample spaces namely discrete and continuous. The sample space whose elements are countable is discrete and for non-countable it is said to be continuous sample space.
- **Trial and event :** To perform an experiment is known as trial and possible outcomes of the experiment is called event. For example, tossing a coin or rolling a dice (any number of times) is trial and turning up head or tail or occurrence of numbers 1, 2, 3, 4, 5 or 6 are events. In fact, every subset of a sample space is an event.

TYPES OF EVENT

- **Simple event :** An event is said to be simple if it contains only one sample point of the sample space.
- **Compound event :** An event is said to be a compound event if it contains more than one sample point of the sample space.
- **Equally likely event or equally probable event:** Events are said to be equally likely if there is no reason for an event to occur in preference to any other event. It means events are said to be equally likely when each event is equally likely to occur as any other.
- **Mutually exclusive events (Disjoint events) :** Two or more than two events are said to be mutually exclusive if the occurrence any one of them exclude/ruled out the occurrence of the others. The events $A_1, A_2, ..., A_n$ are mutually exhaustive if $A_i \cap A_j = \phi$ or $P(A_i \cap A_j) = 0\ \forall\ i \neq j$ and $1 \leq i, j \leq n$.
  (Note : Mutually exclusive events are also known as incompatible events).
- **Exhaustive events :** The events $A_1, A_2, ..., A_n$ are said to be exhaustive events, if their union form the sample space of the experiment. Thus, events A_i $(i = 1, 2, ..., n)$ are said to be exhaustive if $\bigcup_{i=1}^{n} A_i = S$ (sample space).
- **Mutually exclusive and exhaustive events:** Let S be the sample space associated with a random experiment.
  Then the events $A_1, A_2, ..., A_m$ form a mutually exclusive and exhaustive system of events if
  (i) $P(A_i \cap A_j) = 0\ (\forall\ i \neq j)$
  (ii) $P(A_i \cup A_j) = P(A_1 \cup A_2 \cup .... \cup A_m) = P(S) = 1$ $(i \neq j)$
- **Independent events :** Events A and B are said to be independent if happening or non-happening of A does not effect the happening or non-happening of B. In such type of events, we have the following results.
- $P(A \cap B) = P(A) \cdot P(B)$ and
- $P(A \cup B) = P(A) + P(B) - P(A)\, P(B)$

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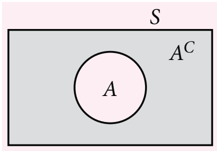
Note : If the events A, B, C are mutually independent,
$\Rightarrow\ P(A \cap B) = P(A)\,P(B),\ P(B \cap C) = P(B)\,P(C)$
and $P(C \cap A) = P(A)\,P(C)$ and
$P(A \cap B \cap C) = P(A)\,P(B)\,P(C)$

- **Dependent events :** Two or more than two events are said to be dependent if happening of one effects the happening of other. For example, drawing two card from a deck of card without replacement. If A and B are dependent events, then
$P(A \cap B) = P(A)\,P(B/A) = P(B)\,P(A/B)$
- **Complement of an event A :**
The set of all outcomes which are in S but not in A is called complement of A which is denoted by $\bar{A}$ or A^C.



$\therefore\quad A^C = S - A$
- **Impossible event :** Event which do not consist any point of the sample space is called impossible event. Throwing a die and hope to obtain tail or head is impossible or null event.
- **Some important points**
On a coin, a die, a chess board and deck of playing cards
(1) Number of rectangles formed on a normal chess board $= {}^9C_2 \times {}^9C_2$.
(2) Number of non-congruent squares of sides $1 \times 1, 2 \times 2, ..., 8 \times 8$ is 1 in each.
$\therefore$ Total number of non-congruent squares = 8
(3) The number of non-congruent rectangles that can be found on a normal chess board are equal to the sum of the non-congruent squares and non-congruent rectangles which are not square $= {}^{8\,+\,8}C_2 = 36$
(4) Number of non-congruent rectangle which are not square $= {}^8C_2 = 28$

MATHEMATICAL OR CLASSICAL DEFINITION OF PROBABILITY

- If a random experiment having n mutually exclusive, equally likely and exhaustive events (outcomes) out of which m events (outcomes) are favourable to the occurrence of an event, then probability of occurrence of A is given by

$$P(A) = \frac{\text{No. of favourable outcomes to } A}{\text{No. of total outcomes}} = \frac{m}{n}$$

$$P(\bar{A}) = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(A)$$

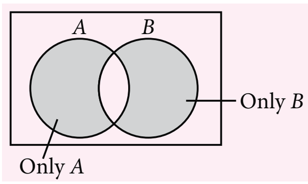
Note : (i) $\quad 0 \le P(A) \le 1$
(ii) $\quad P(\bar{A}) + P(A) = 1$

- Odd in favour of the event E

$$= \frac{\text{No. of favourable cases (outcomes)}}{\text{No. of non-favourable cases (outcomes)}}$$

$$\therefore\ \frac{P(E)}{P(\bar{E})} = \frac{a}{b}\ \ (n(S) = a + b)$$

- Similarly, odd against of the event, $E = \dfrac{P(\bar{E})}{P(E)}$
or $\dfrac{n(\bar{E})}{n(E)}$
- Let A and B are two events then probability that at least one occur is denoted by $P(A$ or $B)$ *i.e.* $P(A \cup B)$ and is given by
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$\qquad = P(A \cap \bar{B}) + P(\bar{A} \cap B) + P(A \cap B)$
- Probability that exactly one of the events A, B occurs
$= P(\text{only } A) + P(\text{only } B) = P(A - B) + P(B - A)$
$= P(A \cap B^C) + P(B \cap A^C)$
$= P(A) - P(A \cap B) + P(B) - P(A \cap B)$



$= P(A) + P(B) - P(A \cap B) - P(A \cap B)$
$= P(A \cup B) - P(A \cap B)$
$= P(A^C \cup B^C) - P(A \cup B)^C$
$= P(A^C \cup B^C) - P(A^C \cap B^C)$
- For any three events, A, B, C
P (at least one of three events occurs)
$= P(A) + P(B) + P(C) - [P(A \cap B) + P(B \cap C)$
$\qquad + P(C \cap A) - P(A \cap B \cap C)]$
The above result is called addition theorem on probability for three events which can be generalized for n events.
- P (at least two of A, B, C occurs) $= P(A \cap B)$
$\qquad + P(B \cap C) + P(C \cap A) - 2P(A \cap B \cap C)$
- P(exactly two of A, B, C occurs) $= P(A \cap B)$
$+ P(B \cap C) + P(C \cap A) - 3P(A \cap B \cap C)$
- P(exactly one of A, B, C occurs) $= P(A) + P(B)$
$+ P(C) - 2P(A \cap B) - 2P(B \cap C) - 2P(C \cap A)$
$\qquad + 3P(A \cap B \cap C)$
- $P(A$ and B but not C occurs$) = P(A \cap B \cap \bar{C})$
$\qquad = P(A \cap B) - P(A \cap B \cap C)$
- $P(\text{only } A) = P(A \cap B' \cap C') = P(A \cap (B \cup C)')$
$= P(A) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C)$

The results of P(only B) and P(only C) are similar.

- P(none of the event A, B, C) $= P(A \cup B \cup C)'$
  $= 1 - P(A \cup B \cup C)$
  $= 1 - P$ (At least one of A, B, C occurs)
- If three events A, B, C are pair wise disjoint *i.e.* mutually exclusive then
  $P(A \cap B) = P(B \cap C) = P(C \cap A) = P(A \cap B \cap C) = 0$
  and $P(A$ or B or $C) = P(A) + P(B) + P(C)$
- Addition theorem for n events : Let $A_1, A_2, A_3, \ldots, A_n$ are n events associated with a random experiment, then

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i) - \sum_{i, i \neq j, j=1}^{n} P(A_i \cap A_j) + \sum_{\substack{i,j,k=1 \\ i \neq j \neq k}}^{n} P(A_i \cap A_j \cap A_k + \ldots + (-1)^{n-1} P(A_1 \cap A_2 \cap \ldots \cap A_n) \quad \ldots (*)$$

If the events $A_1, A_2, \ldots, A_n$ form a pairwise disjoint family of events then (*) reduces the following result.

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i) = P(A_1 + A_2 + \ldots + A_n)$$
$$= P(A_1) + P(A_2) + P(A_3) + \ldots + P(A_n)$$

CONDITIONAL PROBABILITY (MULTIPLICATION THEOREM ON PROBABILITY)

Let A and B are two events associated with a random experiment. Then, the probability of occurrence (happening) of A when it is known that the event B has already happened *i.e.* $P(B) \neq 0$, is called the conditional probability, denoted by $P(A|B)$ and defined as

$$P(A \mid B) = \frac{n(A \cap B)}{n(B)} = \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}} = \frac{P(A \cap B)}{P(B)}$$

$\Rightarrow \; P(A \cap B) = P(B)\, P(A|B),\; P(B) \neq 0$

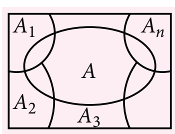
Similarly, we have $P(A \cap B) = P(A)\, P(B|A),\; P(A) \neq 0$

- **Extension of multiplication theorem**
  (1) If A, B, C are three events of a sample space, then
  $P(A \cap B \cap C)$ or $P(ABC)$
  $= P(A)\, P(B|A)\, P(C|AB)$
  (2) If A_i $(i = 1, 2, \ldots, n)$ are n events associated with a sample space then
  $P(A_1 \cap A_2 \cap \ldots . \cap A_n) = P(A_1)\, P(A_2|A_1)$
  $P(A_3|A_1A_2) \ldots .\, P(A_n|A_1A_2 \ldots A_{n-1})$

- **Multiplication theorem for independent events**
  If the events A and B are independent associated with a random experiment then probability of their simultaneously occurrence is equal to the product of their individual occurrence.
  i.e. $P(A \cap B) = P(A)\, P(B)$
- **Extension of multiplication theorem for independent events**
  (i) If A, B, C are three events associated with a sample space then
  $P(A \cap B \cap C) = P(A)\, P(B)\, P(C)$, events are pair wise independents.
  (ii) If A_i $(i = 1, 2, \ldots, n)$ are n independent events associated with a sample space of a random experiment, then
  $P(A_1 \cap A_2 \cap A_3 \cap \ldots \cap A_n) = P(A_1)\, P(A_2) \ldots P(A_n)$
- **Probability of atleast one event among the two, three or more independent events**
  (1) For two events: probability of happening at least one is given by $P(A \cup B)$
  $= 1 - P(\bar{A})\, P(\bar{B})$
  (2) For three events A, B, C
  $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B)$
  $- P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$
  $= 1 - P(\bar{A})\, P(\bar{B})\, P(\bar{C})$
  (3) For n events $A_1, A_2, A_3, \ldots, A_n$
  $P(A_1 \cup A_2 \cup \ldots \cup A_n)$
  $= 1 - P(\bar{A}_1)\, P(\bar{A}_2)\, P(\bar{A}_3) \ldots .\, P(\bar{A}_n)$
  - Probability of happening of none of them
  $= P(\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3 \cap \ldots . \cap \bar{A}_n)$
  $= P(\bar{A}_1)\, P(\bar{A}_2)\, P(\bar{A}_3) \ldots .\, P(\bar{A}_n)$
  $= (1 - p_1)(1 - p_2) \ldots (1 - p_n)$ where $P(A_i) = p_i$

LAW OF TOTAL PROBABILITY AND BAYES' THEOREM

- Partition of a sample space
  A set of events A_i $(i = 1, 2, \ldots, n)$ is said to represent a partition of the sample space S is
  (i) $A_i \cap A_j = \phi,\; i \neq j,$
  $j = 1, 2, \ldots, n$
  (ii) $A_1 \cup A_2 \cup \ldots . \cup A_n = S$ and
  (iii) $P(A_i) > 0 \;\forall\; i = 1, 2, 3, \ldots .., n$



  In otherwords, partitions $\{A_1, A_2, \ldots, A_n\}$ of a sample space S represent a set of mutually exclusive and exhaustive events having non-zero probabilities.

Law of total probability

- If $A_1, A_2, \ldots ., A_n$ are mutually exclusive and exhaustive and A be any event which occurs with A_1 or A_2 or .... or A_n, then

$$P(A) = P(A_1)\,P(A|A_1) + P(A_2)\,P(A|A_2) + \ldots + P(A_n)\,P(A|A_n)$$

Bayes' theorem

- If $A_1, A_2, \ldots., A_n$ are n mutually exclusive and exhaustive events, then

$$P(A_i\,|\,A) = \frac{P(A_i)P(A\,|\,A_i)}{\sum_{i=1}^{n} P(A_i)P(A\,|\,A_i)}$$

RANDOM VARIABLES

- A random variable is often described as a variable whose values are determined by outcome of a random experiment.
- If to each point of sample space we assign a real number, we have a function defined on the sample space. This function is called random function. Thus, a random variable is a real function whose domain is the sample space of a random experiment and whose range is a real line. For example, the sample space (S) of a simultaneous throw of two coins is S = {HH, TH, HT, TT}.

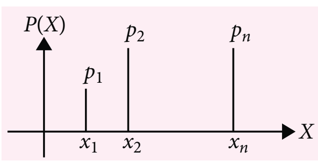
  Let X denote the number of heads. Then X is a random variable. It can have values X(TT) = 0, X(TH) = 1, X(HH) = 2 and X(HT) = 1. Thus, the domain of X is S and range is {0, 1, 2}. On the other hand if we denote Y as the number of tails then Y is also a random variable, having domain S and range is {0, 1, 2}.

PROBABILITY DISTRIBUTION

- Let $\{X = x_i\}$ is an event and $P(X = x_i) = p_i\ (1 \le i \le n)$ then the system of numbers

X_i	x_1	x_2	...	...	x_n
$P(x = x_i)$	p_1	p_2	...	...	p_n

is said to be probability distribution of the random variable X whose graphical representation is



Note : (i) $\sum_{i=1}^{n} P(X = x_i) = \sum_{i=1}^{n} p_i = 1$

(ii) $P(X = x_i)$ lies between 0 and 1 *i.e.* $0 \le P(X = x_i) \le 1$

BERNOULLI TRIALS

The set of n trials is said to be Bernoulli trial if

- The value of n is finite *i.e.* number of trials is finite.
- Each and every trial is independent.
- Trial (experiment) consists of only two outcomes namely success and failure.
- Probability of success and failure for each trial is fixed (same).

BINOMIAL DISTRIBUTION(B.D.)

- Let X be a random variable which assume the values $0, 1, 2, \ldots, n$ is said to be in binomial distribution if its distribution function is given by
  $P(X = r) = {}^nC_r\,p^r\,q^{n-r}$, $r = 0, 1, 2, \ldots., n$ and
  $p, q > 0$ and $p + q = 1$
- For the random variable X, the notation $X \sim B(n, p)$ is used for binomial distribution with parameters n and p, where n is the number of trials and p is the probability of success for each trial.

Mean and variance of binomial distribution

The binomial distribution is

X	0	1	2	....	n
$P(X)$	${}^nC_rq^n$	${}^nC_1q^{n-1}p$	${}^nC_2q^{n-2}p^2$	...	${}^nC_nq^0p^n$

- Mean

$$\bar{X} = E(X) = \sum_{i=1}^{n} x_i p_i = \sum_{r=1}^{n} r\,{}^nC_r q^{n-r} p^r = np$$

- Variance : The variance of the binomial distribution is denoted by var(X) or σ^2 and given by

$$\text{var}(X) = \sum_{r=0}^{n} r^2 P(r) - \sum_{r=0}^{n} rP(r) = npq$$

  Note : (Mean – variance) = $np - npq$
  $= np(1 - q) = np^2 > 0,\ 0 \le q \le 1$
  $\therefore$ mean – variance > 0 $\Rightarrow$ mean > variance
- Recurrence relation for binomial distribution recurrence relation is given by

$$\frac{P(r+1)}{P(r)} = \frac{{}^nC_{r+1}}{{}^nC_r}\left(\frac{p}{q}\right) \Rightarrow P(r+1) = \frac{n-r}{r+1}\frac{p}{q}P(r)$$

$$r \in \{0, 1, 2, \ldots, n-1\}$$

- Use of multi-nominal expansion : If a die has m faces marked with numbers 1, 2, ... m and if such n dice are rolled up, then the probability that sum of the numbers appears (exhibited) on the upper faces equal to λ is the coefficient of x^λ in the expansion of

$$\frac{(x^1 + x^2 + x^3 + \ldots. + x^m)^n}{m^n}$$

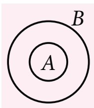
GEOMETRICAL METHOD FOR PROBABILITY

When the number of points in the sample space is infinite it is difficult to use the mathematical definition of probability. For instance, if we need to find the probability that a circle of radius 2 unit lies in a square of side 4 units. We cannot apply the mathematical definition. In such cases we define the probability as follows:

$$P\{y \in A\} = \frac{\text{Measure of region } A}{\text{Measure of sample space}}$$

SOME IMPORTANT RESULTS

- **Letters and envelopes :** Let n letters corresponding to n envelopes are placed in the envelopes randomly, then
  (1) Probability that all letters are placed in the right envelopes $= \frac{1}{n!}$
  (2) Probability that all letters are not placed in the right envelopes $= 1 - \frac{1}{n!}$
  (3) P(no letter goes to right envelopes)
  $= \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - .... + (-1)^n \frac{1}{n!}$
  (4) P(exactly m letters are goes to right envelopes)
  $= \frac{1}{m!}\left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - .... + (-1)^{n-m}\frac{1}{(n-m)!}\right]$
- If $P(A \cup B) = P(A \cap B) \Rightarrow P(A) = P(B)$
- If $A \subseteq B \Rightarrow n(A) \le n(B)$ then
  (i) $P(A) \le P(B)$
  (ii) $P(B - A) = P(B) - P(A)$



- If A, B are such that $P(A) > 0$, $P(B) \neq 1$ then $P(\bar{A} \mid \bar{B}) = \frac{1 - P(A \cup B)}{P(\bar{B})}$
- If A and B are events such that $P(A|B) = P(B|A)$ then $P(A) = P(B)$

PROBLEMS

Single Correct Answer Type

1. A square is inscribed in a circle. If p_1 is the probability that a randomly selected point of the circle lies within the square and p_2 is the probability that point lies outside the square then which of the following is true?

(a) $p_1 < p_2$ (b) $p_1 = p_2$
(c) $p_1 > p_2$ and $p_1^2 - p_2^2 < 1/3$
(d) none of these

2. A natural number x is chosen randomly from the first 150 natural numbers, the probability that $\frac{x^2 - 48x + 540}{x - 25} < 0$ is

(a) $\frac{17}{50}$ (b) $\frac{7}{50}$ (c) $\frac{4}{50}$ (d) $\frac{13}{50}$

3. A coin is tossed n times. If the probability of getting at least two heads is less than that of getting at least one tail by $\frac{3}{32}$, then the value of n equals

(a) 3 (b) 4 (c) 5 (d) 6

4. In a 3×3 matrix, entries a_{ij} are selected randomly from the digits 0, 1, 2, ... 9 with replacement. The probability that 3 digit number in each row will be divisible by 15 is

(a) $\frac{67}{10^3}$ (b) $\frac{77}{10^3}$ (c) $\frac{66}{10^3}$ (d) $\frac{48}{10^3}$

5. An ellipse whose eccentricity is $\frac{2\sqrt{6}}{5}$ is inscribed in a circle and a point that lies within the circle is chosen randomly, the probability that chosen point lies outside the ellipse is

(a) 2/3 (b) 3/5 (c) 4/5 (d) 1/5

6. Two players A and B throw a die one after the other for a lottery whose prize worth is ₹ 22. The player who first throws five will win the game. If A starts the game, the respective expectations of A and B are (in ₹) is

(a) ₹ 12, ₹ 10 (b) ₹ 10, ₹ 12
(c) ₹ 9, ₹ 13 (d) ₹ 11 each

7. Two numbers are selected at random from a set of first 120 natural numbers. Two numbers are selected randomly, the probability that product of selected number is divisible by 3.

(a) $\frac{13}{357}$ (b) $\frac{199}{357}$ (c) $\frac{158}{357}$ (d) $\frac{160}{357}$

8. A man throws a fair coin a number of times and obtained 2 points for each head he throws and for each tail he gets only a point. The probability that he gets a total of 6 points is

(a) $\frac{39}{64}$ (b) $\frac{43}{64}$ (c) $\frac{41}{64}$ (d) $\frac{29}{64}$

9. Let E and F are independent events such that $P(E \cap F^c) = 0.12$ and $P(E^c \cap F) = 0.32$ and $P(E) > 0.5$ then the value of $P(E) + P(F)$ equals

(a) 1.5 (b) 1.6 (c) 1.8 (d) 0.6

10. Bag A consists of 3 red and 4 blue balls and the bag B consists of 5 red and 8 blue balls. One ball is drawn randomly from one of the bags and is found to be blue, the probability that ball was drawn from bag B is

(a) $\frac{14}{27}$ (b) $\frac{13}{27}$ (c) $\frac{39}{74}$ (d) $\frac{35}{74}$

More than One Correct Answer Type

11. Bag A contains 2 white and 3 red balls and bag B contains 4 white and 7 red balls, one bag is selected randomly and a ball is drawn from the bag then

(a) If drawn ball is found to be red, the probability that it was drawn from bag B is $\frac{35}{68}$

(b) If drawn ball is found to be white, the probability that it was drawn from bag A is $\frac{11}{21}$

(c) If drawn ball is found to be red, the probability that it was drawn from bag A is $\frac{33}{68}$

(d) If drawn ball is found to be white, the probability that it was drawn from bag B is $\frac{10}{21}$

12. The independent probabilities that A, B, C can solve the problem are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively then

(a) Probability that the problem will be solved is $\frac{3}{4}$

(b) Probability that problem can not be solved is $\frac{1}{4}$

(c) Probability that only two of them solve the problem is $\frac{1}{4}$

(d) Probability exactly one of then solve the problem is $\frac{11}{24}$

13. Let $0 < P(A) < 1$, $0 < P(B) < 1$ and $P(A \cup B) = P(A) + P(B) - P(A)\,P(B)$ then which of the following is incorrect?

(a) $P(\overline{A \cup B}) = P(\bar{A})\,P(\bar{B})$

(b) $P(A|B) = P(A)$

(c) $P(\bar{A} \cup \bar{B}) = P(\bar{A}) + P(\bar{B})$

(d) $P(B|A) = P(B) - P(A)$

14. For two events A and B, the probability that exactly one of them occurs is given by

(a) $P(A) + P(B) - 2P(A \cap B)$

(b) $P(A \cup B) - P(A \cap B)$

(c) $P(A') + P(B') - 2P(A' \cap B')$

(d) $P(A' \cap B) + P(A \cap B')$

15. A cubical die is thrown 11 times and the numbers obtained are formed as a number whose digits are equal in numbers as many times the die is thrown then probability that number

(a) begins with 531 is $\frac{1}{6^3}$

(b) end with 642 is $\frac{1}{6^3}$

(c) begins with 642 and ends with 531 is $\frac{1}{6^6}$

(d) begins with 531 and ends with 642 is $\frac{431}{216 \times 216}$

16. An electric component manufactured by 'RAJU' Electronics is tested for its defectiveness by a sophisticated testing device. Let 'A' denote the event " the device is defective" and 'B' denote the event "the testing device reveals the component to be defective' If $P(A) = \alpha$ and $P\left(\frac{B}{A}\right) = P\left(\frac{B'}{A'}\right) = 1 - \alpha$ where $0 < \alpha < 1$

then which of the following is true?

(a) $P(B) = 2\alpha(1 - \alpha)$ (b) $P\left(\frac{A'}{B}\right) = \frac{1}{2}$

(c) $P(B') = \alpha^2 + (1 - \alpha)^2$

(d) $P\left(\frac{A'}{B'}\right) = \left(\frac{\alpha}{1-\alpha}\right)^2$

17. Three numbers are selected at random without replacement from $\{1, 2, 3, ..., 11\}$. Let A_1 be the event that minimum of selected numbers is 4 and A_2 be the event their maximum is 7 then

(a) $P(A_1) = \frac{7}{55}$ (b) $P(A_2) = \frac{1}{11}$

(c) $P\left(\frac{A_2}{A_1}\right) = \frac{2}{21}$ (d) $P\left(\frac{A_1}{A_2}\right) = \frac{2}{15}$

Comprehension Type

Paragraph for Q.No. 18-20

A bag contains 9 black, 7 white and 4 green balls. If 3 balls are drawn randomly then,

18. Probability of getting 3 balls of different colours

(a) $\frac{7}{95}$ (b) $\frac{14}{95}$ (c) $\frac{21}{95}$ (d) $\frac{7}{228}$

19. Probability of getting balls in order of colour green, white, black respectively.

(a) $\frac{7}{190}$ (b) $\frac{11}{190}$ (c) $\frac{17}{190}$ (d) $\frac{29}{190}$

20. Probability of getting 2 black and one green ball

(a) $\frac{7}{95}$ (b) $\frac{12}{95}$ (c) $\frac{13}{95}$ (d) $\frac{20}{95}$

Paragraph for Q. No. 21-23

The contents of urns I, II, III are as follows:
Urn I : 1 White, 2 Black, 3 Red balls and
Urn II : 2 White, 1 Black, 1 Red ball and
Urn III : 4 White, 5 Black and 3 Red balls

One urn is chosen randomly and two balls are drawn. If E_1, E_2, E_3 denote the event that urn I, II and III are chosen respectively and E be the event of selection of two balls from the selecting urn as white and red. Then

21. $P(E)$ equals

(a) $\frac{118}{495}$ (b) $\frac{131}{495}$ (c) $\frac{137}{495}$ (d) $\frac{132}{495}$

22. $P\left(\frac{E_1}{E}\right)=$

(a) $\frac{118}{495}$ (b) $\frac{33}{118}$ (c) $\frac{55}{118}$ (d) $\frac{15}{59}$

23. $P\left(\frac{E_2}{E}\right)+P\left(\frac{E_3}{E}\right)=$

(a) $\frac{55}{118}$ (b) $\frac{30}{118}$ (c) $\frac{85}{118}$ (d) $\frac{33}{118}$

Paragraph for Q.No. 24-26

An urn contains 4 white and 3 red balls. Three balls are drawn one by one with replacement. Then,

24. Probability of getting 2 white and1 red ball is

(a) $\frac{144}{343}$ (b) $\frac{199}{343}$ (c) $\frac{216}{343}$ (d) $\frac{197}{343}$

25. Probability that at most two balls are red is

(a) $\frac{199}{343}$ (b) $\frac{316}{343}$ (c) $\frac{180}{343}$ (d) $\frac{319}{343}$

26. Probability of getting at least 2 white balls is

(a) $\frac{172}{343}$ (b) $\frac{171}{343}$ (c) $\frac{208}{343}$ (d) $\frac{216}{343}$

Matrix Match Type

27. Match the following.

	Column - I		Column - II
A.	Three number are selected from the set {1, 2, 3, ..., 10}, the the probability that they form a G. P.	p.	$\frac{5}{18}$
B.	If x and y are selected from the set of numbers {1, 2, 3, ..., 10} the probability that $x^3 + y^3$ is divisible by 3 is	q.	$\frac{1}{4}$
C.	A card is lost from a pack of 52 cards. If a card is drawn from the remaining cards, then the probability that it is a heart is	r.	$\frac{1}{3}$
D.	Two balls are drawn from an urn containing 2 white, 3 red and 4 black balls one by one without replacement. The probability that they are of same colour is	s.	$\frac{1}{15}$

28. Match the following.

	Column - I		Column - II
A.	A and B are two independent events such that $P(A)=\frac{1}{2}$ and $P(B)=\frac{1}{5}$	p.	$P(A\cup B)\geq\frac{3}{5}$
B.	A and B are two events such that $P(A)=\frac{1}{2}$, $P(B)=\frac{2}{3}$	q.	$P\left(\frac{A}{B}\right)=\frac{1}{2}$
C.	A and B are two independent events such that $P(B)=\frac{1}{5}$ and $P(A\cup B)=\frac{3}{5}$	r.	$P(A\cap B)\leq\frac{1}{2}$
D.	A and B are two independent events such that $P(A)=\frac{3}{7}$ and $P(B)=\frac{1}{2}$	s.	P(Exactly one of A, B) $=\frac{1}{2}$

Integer Type

29. A bag contains 4 balls, two are drawn at random and found to be white and the probability that all balls are white is λ then the value of 5λ is

30. If X and Y are independent binomial variates $X\left(5,\frac{1}{2}\right)$ and $Y\left(7,\frac{1}{2}\right)$ and the value of $P(X + Y = 4)$ is m then, value of $\frac{4096}{99}m$ equals

31. If the papers of 5 students can be checked by any one of the 5 teachers. If the probability that all the 5 papers are checked by exactly 2 teachers is m then the value of $\frac{125m}{2}$ equals

32. Two numbers x and y are randomly selected from the set $A = \{1, 2, 3, 4, 5, ..., 25\}$ and the probability that $x^2 - y^2$ is divisible by 3 is λ then the value of $\frac{75\lambda}{41}$ is

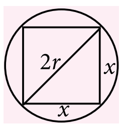
33. A pair of dice is rolled once and a total of 8 has appeared. The chance that even number appear on each dice is $\frac{A}{B}$ and $B^2 - A^2 = k^2$, $k > 0$ then k equals

34. If A and B be any two events such that $P(A) = \frac{2}{5}$ and $P(A \cap B) = \frac{3}{20}$ and conditional probability $P(A|A' \cup B') = \frac{l}{m}$ then l equals

35. If the mean and variance of a binomial variate x are 2 and 1 respectively and the probability that x takes the value greater than or equal to 1 is λ, then value of $\frac{16\lambda}{5}$ is

SOLUTIONS

1. (c) : Let the radius of the circle be r and side of square be x.



So, side of square = $\sqrt{2}r$

Area of square = $2r^2$

$$\therefore\ p_1 = \frac{\text{Area of square}}{\text{Area of circle}} = \frac{2r^2}{\pi r^2} = \frac{2}{\pi}$$

$$\Rightarrow\ p_2 = 1 - p_1 = \frac{\pi - 2}{\pi} \qquad \therefore\ p_1 > p_2$$

Now, $p_1^2 - p_2^2 = (p_1 + p_2)(p_1 - p_2) = (1)\left(\frac{4-\pi}{\pi}\right)$

$$p_1^2 - p_2^2 < \frac{1}{3} \quad \text{as } 3 < \pi < 4$$

2. (b) : As $\frac{x^2 - 48x + 540}{x - 25} < 0$

$$\Rightarrow\ \frac{(x-18)(x-30)}{x-25} < 0$$

$$\Rightarrow\ x \in \{1, 2, 3, ..., 17\} \cup \{26, 27, 28, 29\}$$

$$\therefore\ \text{Required probability} = \frac{21}{150} = \frac{7}{50}$$

3. (d) : We have, $P(X = r) = {}^nC_r\, p^r q^{n-r}$, $p = q = 1/2$

$\therefore$ Probability of getting at least 2 heads

$$= 1 - P(0) - P(1) = 1 - \frac{1}{2^n} - n \cdot \frac{1}{2^n}$$

Probability of getting at least 1 tail

$$= 1 - P(0) = 1 - \frac{1}{2^n}$$

Now, as per question we have

$$\left(1 - \frac{1}{2^n}\right) - \left(1 - \frac{1}{2^n} - \frac{n}{2^n}\right) = \frac{3}{32}$$

$$\Rightarrow\ \frac{n}{2^n} = \frac{3}{32} = \frac{6}{64} = \frac{6}{2^6} \ \Rightarrow\ n = 6$$

4. (a) : Here $n(S)$ = total three digit number with repetition of digit = 10^3

Now numbers from 000 to 999 which are divisible by 15 is 67 given by rows

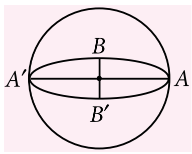
$$\begin{array}{llllll}
000, & 015, & 030, & ... & 090 & = 7 \\
105, & 120, & ... & ... & 195 & = 7 \\
210, & 225, & ... & ... & 300 & = 7 \\
... & ... & ... & ... & ... & ... \\
... & ... & ... & ... & ... & ... \\
915, & 930, & 945, & ... & 990 & = 6
\end{array}$$

$\Rightarrow$ Total favourable numbers = $7 \times 7 + 6 \times 3 = 67$

$$\therefore\ \text{Required probability} = \frac{67}{10^3}$$

5. (c) : Let the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, b < a$

We know that, if length of major and minor axes are a and b units respectively, then the area of ellipse is given by πab where $b^2 = a^2(1 - e^2)$



$$\therefore\ \text{Area of ellipse} = \pi a(a\sqrt{1-e^2})$$

$$= \pi a^2 \left(\sqrt{1 - \frac{24}{25}}\right) = \frac{\pi a^2}{5}$$

Also, area of circle = πa^2

$\therefore$ Favourable region = area of circle – area of ellipse

$$= \pi a^2 - \frac{\pi a^2}{5} = \frac{4\pi a^2}{5}$$

$$\therefore\ \text{Required probability} = \frac{4\pi a^2}{5} \times \frac{1}{\pi a^2} = \frac{4}{5}$$

6. (a) : Probability of throwing the number 5 = 1/6

Now, it is given that A starts the game, which means A gets the chance in first, third, fifth, ... so on throws.

$$\therefore\ P(A) = \frac{1}{6} \ \Rightarrow\ P(\bar{A}) = 1 - \frac{1}{6} = \frac{5}{6}$$

Similarly $P(B) = \frac{1}{6} \ \Rightarrow\ P(\bar{B}) = \frac{5}{6}$

Probability of winning of A

$$= P(A) + (P(\bar{A}))^2 P(A) + (P(\bar{A}))^4 P(A) + ...$$

$$= \frac{1}{6} + \left(\frac{5}{6}\right)^2 \left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^4 \frac{1}{6} + ...$$

$$= \frac{1}{6}\left[1 + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^4 + \left(\frac{5}{6}\right)^6 + ... + \infty\right]$$

$$= \frac{1}{6}\left(\frac{1}{1-\frac{25}{36}}\right) = \frac{6}{11}$$

Probability of winning of $B = 1 - \frac{6}{11} = \frac{5}{11}$

∴ Expectation of A and B are

$$E(A) = \frac{6}{11} \times 22 = 12 \text{ (in ₹)}$$

$$E(B) = \frac{5}{11} \times 22 = 10 \text{ (in ₹)}$$

7. (b) : Given set of numbers is {1, 2, 3, 4, ..., 120}

Now, {1, 2, 3, 4, 5, ..., 120}

$$= \underbrace{\{1, 2, 4, 5, ..., 119\}}_{\text{80 numbers}} \cup \underbrace{\{3, 6, 9, ..., 120\}}_{\text{40 numbers}}$$

The product of two numbers will be divisible by 3 if both the numbers are from the set {3, 6, 9, ..., 120} (*i.e.* from 40 numbers) or one from the set {1, 2, 4, 5, .., 119} and other from the set {3, 6, 9, ..., 120}.

Let A = event that product of number is divisible by 3

∴ $n(A) = {}^{40}C_2 + {}^{80}C_1 \times {}^{40}C_1$ and $n(S) = {}^{120}C_2$

$$\therefore \ P(A) = \frac{{}^{40}C_2 + {}^{80}C_1 \times {}^{40}C_1}{{}^{120}C_2} = \frac{199}{357}$$

8. (b) : Given, the number of points on throwing head = 2 and for tail = 1 point.

∴ Cases to get a total of 6 points
HHH, HHTT, HTTTT and TTTTTT and
$P(\text{T}) = 1/2 = P(\text{H})$

∴ Required probability

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + {}^4C_2\left(\frac{1}{2} \times \frac{1}{2}\right)^2 + {}^5C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^4 + {}^6C_6\left(\frac{1}{2}\right)^6$$

$$= \frac{8 + 24 + 10 + 1}{64} = \frac{43}{64}$$

9. (d) : Given, $P(E \cap F^c) = 0.12$ and $P(E^c \cap F) = 0.32$, $P(E) > 0.5$

Let $P(E) = m$ and $P(F) = n$

Since, $P(E \cap F^c) = P(E) - P(E \cap F)$

$\Rightarrow$ $0.12 = P(E)\,(1 - P(F))$ ($\because$ E and F are independent)

$$\Rightarrow \ 0.12 = m(1-n) \ \therefore \ m = \frac{0.12}{1-n} \quad ....(i)$$

Also $P(F \cap E^c) = P(F) - P(E \cap F) = P(F)\,(1 - P(E)) = n(1 - m)$

$$\Rightarrow \ 0.32 = n(1-m) \ \Rightarrow \ m = 1 - \frac{0.32}{n} \quad ....(ii)$$

From (i) and (ii), we get

$n^2 - 1.2n + 0.32 = 0 \ \Rightarrow (n - 0.8)(n - 0.4) = 0$

$\Rightarrow$ $n = 0.8$ or $n = 0.4$

When $n = 0.8 \Rightarrow m = 0.6$ and

when $n = 0.4 \Rightarrow m = 0.2$

∴ $m + n = 1.4$ and $m + n = 0.6$

$\Rightarrow$ $P(E) + P(F) = 1.4$ or $P(E) + P(F) = 0.6$

10. (a) : Let E_1 and E_2 be the event of selecting the bags A and B respectively.

$$\therefore \ P(E_1) = P(E_2) = \frac{1}{2}$$

and $P\left(\frac{\text{Blue}}{E_1}\right) = \frac{4}{7}$ and $P\left(\frac{\text{Blue}}{E_2}\right) = \frac{8}{13}$

Using Baye's theorem

$$P\left(\frac{E_2}{\text{Blue}}\right) = \frac{P(E_2)P\left(\frac{\text{Blue}}{E_2}\right)}{P(E_1)P\left(\frac{\text{Blue}}{E_1}\right) + P(E_2)P\left(\frac{\text{Blue}}{E_2}\right)}$$

$$= \frac{\frac{1}{2}\cdot\frac{8}{13}}{\frac{1}{2}\cdot\frac{4}{7} + \frac{1}{2}\cdot\frac{8}{13}} = \frac{\frac{8}{13}}{\frac{4}{7} + \frac{8}{13}} = \frac{14}{27}$$

11. (a, b, c, d) : Let E_1 and E_2 be the events of selecting bags then,

$$P(E_1) = P(E_2) = \frac{1}{2}$$

$$\therefore \ P\left(\frac{\text{red}}{E_1}\right) = \frac{3}{5}, P\left(\frac{\text{red}}{E_2}\right) = \frac{7}{11}, P\left(\frac{\text{white}}{E_1}\right) = \frac{2}{5}$$

and $P\left(\frac{\text{white}}{E_2}\right) = \frac{4}{11}$

$$\therefore \ P\left(\frac{E_2}{\text{red}}\right) = \frac{P(E_2)P\left(\frac{\text{red}}{E_2}\right)}{P(E_1)P\left(\frac{\text{red}}{E_1}\right) + P(E_2)P\left(\frac{\text{red}}{E_2}\right)}$$

$$=\frac{\frac{1}{2}\cdot\frac{7}{11}}{\frac{1}{2}\cdot\frac{3}{5}+\frac{1}{2}\cdot\frac{7}{11}}=\frac{35}{68}$$

$$\therefore\ P\left(\frac{E_1}{\text{red}}\right)=1-P\left(\frac{E_2}{\text{red}}\right)=1-\frac{35}{68}=\frac{33}{68}$$

$$\text{Now, } P\left(\frac{E_1}{\text{white}}\right)=\frac{P(E_1)P\left(\frac{\text{white}}{E_1}\right)}{P(E_1)P\left(\frac{\text{white}}{E_1}\right)+P(E_2)P\left(\frac{\text{white}}{E_2}\right)}$$

$$=\frac{\frac{2}{5}}{\frac{2}{5}+\frac{4}{11}}=\frac{11}{21}$$

$$\therefore\ P\left(\frac{E_2}{\text{white}}\right)=1-P\left(\frac{E_1}{\text{white}}\right)=1-\frac{11}{21}=\frac{10}{21}$$

12. (a, b, c, d) : Given, $P(A)=\frac{1}{2}\ \therefore\ P(\bar{A})=\frac{1}{2}$

$P(B)=\frac{1}{3}\ \therefore\ P(\bar{B})=\frac{2}{3}$; $P(C)=\frac{1}{4}\ \therefore\ P(\bar{C})=\frac{3}{4}$

Now, P(none of A, B, C solve the problem) $=P\left(\bar{A}\,\bar{B}\,\bar{C}\right)$

$$=\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}=\frac{1}{4}$$

$\therefore$ P(Problem will be solved) = 1 – P (none of them can solve the problem) $=1-\frac{1}{4}=\frac{3}{4}$

P(only two of them solve the problem)

$$=P(AB\bar{C})+P(A\bar{B}C)+P(\bar{A}BC)$$

$$=\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{3}{4}+\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{1}{4}+\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{4}=\frac{6}{24}=\frac{1}{4}$$

P(exactly one of them solve the problem)

$$=P(A\overline{BC})+P(\overline{AC}B)+P(\overline{AB}C)$$

$$=\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{3}+\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{1}{4}=\frac{11}{24}$$

13. (c, d) : We know, $P(A\cup B)$

$= P(A) + P(B) - P(A\cap B)$

But it is given that $P(A\cup B)=P(A)+P(B)-P(A)\,P(B)$

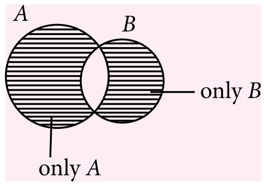
$\therefore$ $P(A\cap B) = P(A)\,P(B)$ *i.e.* A, B are independent events.

$\therefore$ $A, \bar{B}$; $\bar{A}, B$ and $\bar{A}, \bar{B}$ are also pairwise independent events.

$$\therefore\ P(\overline{A\cup B})=P(\bar{A}\cap\bar{B})=P(\bar{A})\,P(\bar{B})$$

$$P(A\,|\,B)=\frac{P(A\cap B)}{P(B)}=\frac{P(A)P(B)}{P(B)}=P(A)$$

14. (a, b, c, d) : Probability that exactly one of them occurs



$= P(\text{only } A \text{ or only } B)$
$= P(\text{only } A) + P(\text{only } B)$
$= P(A\cap B') + P(A'\cap B)$ (option (d))
$= P(A) - P(A\cap B) + P(B) - P(A\cap B)$
$= P(A) + P(B) - 2P(A\cap B)$ (option (a))
$= P(A) + P(B) - P(A\cap B) - P(A\cap B)$
$= P(A\cup B) - P(A\cap B)$ (option (b))
$= 1 - P(A\cup B)' - P(A\cap B)$
$= 1 - P(A'\cap B') - P(A\cap B)$
$= 1 - P(A\cap B) - P(A'\cap B')$
$= P(A'\cup B') - P(A'\cap B')$
$= P(A') + P(B') - P(A'\cap B') - P(A'\cap B')$
$= P(A') + P(B') - 2P(A'\cap B')$ (option (c))

15. (a, b, c, d) : Here $n(s) = 6^{11}$

Now fixing 5, 3, 1 in the first three places $\underbrace{\overset{531}{\bullet\bullet\bullet\bullet}}_{\text{fixed}}\ \underbrace{\bullet\bullet\bullet\bullet\bullet\bullet\bullet\bullet\bullet}$

and remaining 8 places can be fill by 6^8 way as digit can be used as many times

$\therefore$ Required probability the number begins with 531 is $\frac{6^8}{6^{11}}=\frac{1}{6^3}$

Similarly Probability when number ends with 642 is $\frac{1}{216}=\frac{1}{6^3}$

Again the probability that number begins with 642 $\underset{632}{\bullet\bullet\bullet}\ \underbrace{\qquad}\ \underset{531}{\bullet\bullet\bullet}$ and ending with 531 is $\frac{6^5}{6^{11}}=\frac{1}{6^6}$

$\therefore$ P (number begins with 531 or ends with 642)
$= P$(begin with 531) + P(end with 642) – P(begin with 531 and end with 642) $=\frac{1}{216}+\frac{1}{216}-\frac{1}{216\times216}=\frac{431}{216\times216}$

16. (a, b, c) : $P(B)=P(A)P\left(\frac{B}{A}\right)+P(A')P\left(\frac{B}{A'}\right)$

$$=P(A)P\left(\frac{B}{A}\right)+P(A')\left[1-P\left(\frac{B'}{A'}\right)\right]$$

$= \alpha(1 - \alpha) + (1 - \alpha)\,[1 - (1 - \alpha)]$
$= \alpha(1 - \alpha) + (1 - \alpha)(\alpha) = 2\alpha(1 - \alpha)$

$$\text{Again, } P\left(\frac{A'}{B}\right)=\frac{P(B)-P(A\cap B)}{P(B)}$$

$$\therefore\ P\left(\frac{A'}{B}\right)=\frac{2\alpha(1-\alpha)-\alpha(1-\alpha)}{2\alpha(1-\alpha)}=\frac{1}{2}$$

Again $P(B') = 1 - P(B) = 1 - (2\alpha - 2\alpha^2)$
$= \alpha^2 + (\alpha^2 - 2\alpha + 1) = \alpha^2 + (\alpha - 1)^2$

Also, $P\left(\frac{A'}{B'}\right) = \frac{P(A' \cap B')}{P(B')} = \frac{1 - P(A \cup B)}{1 - P(B)}$

$$= \frac{1-(2\alpha-\alpha^2)}{1-(2\alpha-2\alpha^2)}$$

$$\Rightarrow \quad P\left(\frac{A'}{B'}\right) = \frac{(\alpha-1)^2}{2\alpha^2-2\alpha+1} \neq \left(\frac{\alpha}{1-\alpha}\right)^2$$

17. (a, b, c, d) : $\therefore$ $P(A_1) = P$ (Selecting 4 and two from the numbers 5 to 11) $= \frac{{}^1C_1 \times {}^7C_2}{{}^{11}C_3} = \frac{7}{55}$

$P(A_2)$ = P(Selecting number 7 and two from the numbers 1 to 6) $= \frac{1 \times {}^6C_2}{{}^{11}C_3} = \frac{1}{11}$

and $P(A_1 \cap A_2) = P$(Selecting 4 and 7 and one between the number 4 and 7)

$$= \frac{{}^1C_1 \times {}^1C_1 \times {}^2C_1}{{}^{11}C_3} = \frac{2 \times 6}{11 \times 10 \times 9} = \frac{2}{165}$$

$$\therefore \quad P\left(\frac{A_1}{A_2}\right) = \frac{P(A_1 \cap A_2)}{P(A_2)} = \frac{2}{165} \times \frac{11}{1} = \frac{2}{15}$$

and $P\left(\frac{A_2}{A_1}\right) = \frac{P(A_1 \cap A_2)}{P(A_1)} = \frac{2}{165} \times \frac{55}{7} = \frac{2}{21}$

18. (c) : Total balls in the bag = 9 + 7 + 4 = 20
P(balls are of different colour)

$$= \frac{{}^9C_1 \times {}^7C_1 \times {}^4C_1}{{}^{20}C_3} = \frac{9 \cdot 7 \cdot 4 \cdot 6}{20 \cdot 19 \cdot 18} = \frac{21}{95}$$

19. (a) : P(orderly green, white, black ball)

$$= P(\text{green,white,black}) = \frac{4}{20} \cdot \frac{7}{19} \cdot \frac{9}{18} = \frac{7}{190}$$

20. (b) : P(two black and one green ball)

$$= \frac{{}^9C_2 \times {}^4C_1}{{}^{20}C_3} = \frac{12}{95}$$

(21-23) :

From given we have $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$ and

$$P\left(\frac{E}{E_1}\right) = \frac{1 \times 3}{{}^6C_2} = \frac{1}{5}, \; P\left(\frac{E}{E_2}\right) = \frac{2 \times 1}{{}^4C_2} = \frac{1}{3},$$

$$P\left(\frac{E}{E_3}\right) = \frac{4 \times 3}{{}^{12}C_2} = \frac{2}{11}$$

21. (a) : $P(E) = \sum_{i=1}^{3} P(E_i)P\left(\frac{E}{E_i}\right) = \frac{1}{3}\left[\frac{1}{5} + \frac{1}{3} + \frac{2}{11}\right] = \frac{118}{495}$

22. (b) : $P\left(\frac{E_1}{E}\right) = \frac{P(E_1)P\left(\frac{E}{E_1}\right)}{P(E)} = \frac{1}{3} \cdot \frac{1}{5} \times \frac{495}{118} = \frac{33}{118}$

23. (c) : We have, $P\left(\frac{E_2}{E}\right) = \frac{P(E_2 \cap E)}{P(E)} = \frac{55}{118}$

and $P\left(\frac{E_3}{E}\right) = \frac{P(E_3 \cap E)}{P(E)} = \frac{P(E_3) \cdot P\left(\frac{E}{E_3}\right)}{P(E)}$

$$= \frac{1}{3} \times \frac{2}{11} \times \frac{495}{118} = \frac{30}{118}$$

$$\therefore \quad P\left(\frac{E_2}{E}\right) + P\left(\frac{E_3}{E}\right) = \frac{55}{118} + \frac{30}{118} = \frac{85}{118}$$

24. (a) : Total balls = 4 + 3 = 7

$\therefore$ $P(\text{red ball}) = \frac{3}{7}$ and $P(\text{white ball}) = \frac{4}{7}$

$\therefore$ $P(\text{2 white and one red ball}) = 3\left(\frac{4}{7} \times \frac{4}{7} \times \frac{3}{7}\right) = \frac{144}{343}$

25. (b) : P(getting at most 2 red balls) = P (getting 0 red or 1 red or 2 red balls) = 1 – P(getting 3 red balls)

$$1 - \left(\frac{3}{7}\right)^3 = \frac{316}{343}$$

26. (c) : P(getting at least 2 white balls)

$$= {}^3C_2\left(\frac{4}{7}\right) \times \frac{4}{7} \times \frac{3}{7} + {}^3C_3\left(\frac{4}{7}\right)^3$$

$$= 3 \times \left(\frac{4}{7}\right)^2 \left(\frac{3}{7}\right) + \left(\frac{4}{7}\right)^3 = \left(\frac{4}{7}\right)^2 \left[\frac{3 \times 3}{7} + \frac{4}{7}\right]$$

$$= \frac{16}{49} \times \frac{13}{7} = \frac{208}{343}$$

27. A $\to$ s, B $\to$ r, C $\to$ q, D $\to$ p

A. Let $A = \{1, 2, 3, 4, ..., 10\}$.

So, possible G.P.'s are (1, 2, 4), (4, 6, 9), (1, 3, 9), (2, 4, 8)
$n(S) = {}^{10}C_3$ and $n(A) = {}^4C_1 \times 2$

$$\therefore \text{ Required probability} = \frac{{}^4C_1 \times 2}{{}^{10}C_3} = \frac{4 \times 6 \times 2}{10 \cdot 9 \cdot 8} = \frac{1}{15}$$

B. Let set $A = \{1, 2, 3, ..., 10\}$

$$\begin{cases} 1 & 4 & 7 & 10 \\ 2 & 5 & 8 & \\ 3 & 6 & 9 & \end{cases} \begin{matrix} \to R_1 \\ \to R_2 \\ \to R_3 \end{matrix}$$

Now $x^3 + y^3 = 3M \Rightarrow x + y = 3M$

Required case are

1 Selecting x from R_1 and y from R_2

2. Selecting x from R_2 and y from R_1

3. Selecting x and y both from R_3.

$\therefore$ Required probability $= \dfrac{^4C_1 \times {}^3C_1 + {}^3C_2}{^{10}C_2} = \dfrac{1}{3}$

C. Let HH stand for Heart lost and heart drawn, NH denote non heart lost and Heart draw.

$\therefore$ Required probability $= P(\text{HH}) + P(\text{NH})$

$$= \frac{13}{52}\cdot\frac{12}{51} + \frac{39}{52}\cdot\frac{13}{51} = \frac{1}{4}\left(\frac{12}{51} + \frac{39}{51}\right) = \frac{1}{4}$$

D. Number of white balls = 2, Red balls = 3, Black balls = 4

P(both are of same colour) = P(2 white or 2 red or 2 black balls)

$$= \frac{^2C_2}{^9C_2} + \frac{^3C_2}{^9C_2} + \frac{^4C_2}{^9C_2} = \frac{5}{18}$$

28. A $\to$ p, q, r, s, B $\to$ p, r, C $\to$ p, q, r, s, D $\to$ r, s

A. $P(A \cup B) = 1 - P(A' \cap B') = 1 - \left(\dfrac{1}{2} \times \dfrac{4}{5}\right) = \dfrac{3}{5}$

and $P\left(\dfrac{A}{B}\right) = P(A) = \dfrac{1}{2}$

($\because$ A, B are independent events)

Now, $P(A \cap B) \le$ minimum $\{P(A), P(B)\}$

$= \text{minimum}\left\{\dfrac{1}{2}, \dfrac{1}{5}\right\} = \dfrac{1}{5} \le \dfrac{1}{2}$

P(exactly A, B) = P(only A) + P(only B)

$= P(A \cap B') + P(A' \cap B) = P(A)P(B') + P(A')P(B)$

$= \dfrac{1}{2} \times \dfrac{4}{5} + \dfrac{1}{2}\cdot\dfrac{1}{5} = \dfrac{1}{2}$

B. $P(A \cup B) \ge$ maximum $\{P(A), P(B)\}$

$= \text{maximum}\left\{\dfrac{2}{3}, \dfrac{1}{2}\right\} = \dfrac{2}{3} \ge \dfrac{3}{5}$

and $P(A \cap B) \le \text{minimum}\{P(A), P(B)\} = \dfrac{1}{2}$

C. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \dfrac{3}{5} = P(A) + \dfrac{1}{5} - P(A)\left(\dfrac{1}{5}\right)$

($\because$ A, B are independent events)

$\Rightarrow \dfrac{2}{5} = \dfrac{4}{5}P(A) \Rightarrow P(A) = \dfrac{1}{2}$

$\therefore P\left(\dfrac{A}{B}\right) = P(A)$

Also, $P(A \cap B) = P(A)\,P(B) = \dfrac{1}{2}\cdot\dfrac{1}{5} = \dfrac{1}{10} < \dfrac{1}{2}$

So, P(exactly one of A, B) = P(only A) + P(only B)

$= P(A \cap B') + P(A' \cap B)$

$= P(A)\,P(B') + P(A')\,P(B)$

$= \dfrac{1}{2}\cdot\dfrac{4}{5} + \dfrac{1}{2}\cdot\dfrac{1}{5} = \dfrac{1}{2}$

D. $P(A \cup B) \ge \dfrac{1}{2}$, $P(A \cap B) \le \dfrac{3}{7} \le \dfrac{1}{2}$

$\therefore$ $P(A|B) = P(A) = 3/7$ (given)

and $P(B) = 1/2$. $\therefore$ $P(B') = 1/2$

$\therefore$ P(exactly one of A, B) = P(only A) + P(only B)

$= P(A \cap B') + P(A' \cap B)$

$= P(A)\,P(B') + P(A')\,P(B) = \dfrac{3}{7}\cdot\dfrac{1}{2} + \dfrac{4}{7}\cdot\dfrac{1}{2} = \dfrac{1}{2}$

29. (3) : Let E_1 = the event that bag contains 2 white balls and 2 balls of other colour.

E_2 = the event that bag contains 3 white balls and 1 ball of other colour

E_3 = the event that bag contains all white balls and

A = event of drawing 2 white balls from the bag.

$\therefore$ $P(E_1) = P(E_2) = P(E_3) = \dfrac{1}{3}$ and

$$P(A\,|\,E_1) = \frac{^2C_2}{^4C_2} = \frac{1}{6},\ P(A\,|\,E_2) = \frac{^3C_2}{^4C_2} = \frac{1}{2} \text{ and}$$

$P(A\,|\,E_3) = 1$ so $P(A) = \dfrac{1}{3}\left(\dfrac{1}{6} + \dfrac{1}{2} + 1\right) = \dfrac{10}{18}$

$\therefore$ Required probability $P(E_3\,|\,A) = \lambda = \dfrac{P(E_3 \cap A)}{P(A)}$

$\Rightarrow \dfrac{P(E_3)\cdot P(A\,|\,E_3)}{P(A)} = \lambda \Rightarrow \dfrac{1}{3} \times \dfrac{18}{10} = \lambda$

$\Rightarrow \lambda = \dfrac{3}{5} \Rightarrow 5\lambda = 3$

30. (5) : $P(X + Y = 4) = P(X = 0, Y = 4)$
$+ P(X = 1, Y = 3) + P(X = 2, Y = 2)$
$+ P(X = 3, Y = 1) + P(X = 4, Y = 0)$

$= P(X = 0)\,P(Y = 4) + P(X = 1)\,P(Y = 3) + P(X = 2)$
$P(Y = 2) + P(X = 3)\,P(Y = 1) + P(X = 4)\,P(Y = 0)$

where $P(X = r) = {}^nC_r\left(\dfrac{1}{2}\right)^n$; $n = 5, r = 0, 1, 2, 3, 4$ and

$$P(Y=r)={}^nC_r\left(\frac{1}{2}\right)^n, n=7,\ r=0,1,2,3,4$$

$$\therefore\ P(X+Y=4)={}^5C_0\left(\frac{1}{2}\right)^5\cdot{}^7C_4\left(\frac{1}{2}\right)^7$$
$$+{}^5C_1\left(\frac{1}{2}\right)^5\cdot{}^7C_3\left(\frac{1}{2}\right)^7+{}^5C_2\left(\frac{1}{2}\right)^5\ {}^7C_2\left(\frac{1}{2}\right)^7$$
$$+{}^5C_3\left(\frac{1}{2}\right)^5\cdot{}^7C_1\left(\frac{1}{2}\right)^7+{}^5C_4\left(\frac{1}{2}\right)^5\ {}^7C_0\left(\frac{1}{2}\right)^7$$
$$=\left(\frac{1}{2}\right)^{12}\left[{}^5C_0\cdot{}^7C_4+{}^5C_1\cdot{}^7C_3+{}^5C_2\cdot{}^7C_2+{}^5C_3\cdot{}^7C_1+{}^5C_4\cdot{}^7C_0\right]$$
$$=\left(\frac{1}{2}\right)^{12}[1(35)+5(35)+10(21)+10(7)+5(1)]$$
$$\therefore\ m=\frac{1}{4096}[35+175+210+70+5]=\frac{495}{4096}$$
$$\Rightarrow\ \frac{4096}{99}m=5$$

31. (6) : $n(S)$ = The number of ways in which papers of 5 students can be checked by the 5 teachers = 5^5

$n(A)$ = choosing two teachers out of 5 × the number of ways in which 5 papers can be checked by exactly 2 teachers

$= {}^5C_2\times(2^5-2) = 300$

$$\therefore\ \text{Required probability}=\frac{n(A)}{n(S)}=\frac{300}{5^5}=\frac{12}{125}=m$$

(given)

Hence, $\frac{125m}{2}=6$

32.(1) : $A = \{1, 2, 3, 4, ..., 25\}$

Let x = subset of A in which each number is multiple of 3

y = subset of A in which numbers are not multiple of 3

$\therefore\ x = \{3, 6, 9, 12, 18, 21, 24\}.\ \therefore\ n(x) = 8$ and

$y = A - x = \{1, 2, 4, 5, 7, ..., 25\}.\ \therefore\ n(y) = 17$

Now, the number $x^2 - y^2$ will be divisible by 3 if both number are selected either from x or y.

$\therefore$ Number of favourable cases $= {}^8C_2 + {}^{17}C_2 = 164$

and $n(S)={}^{25}C_2=300$

$$\therefore\ \text{Required probability}=\frac{\text{favourable cases}}{\text{total cases}}=\frac{164}{300}$$

$$\therefore\ \lambda=\frac{164}{300}=\frac{41}{75}\ \Rightarrow\ \frac{75\lambda}{41}=1$$

33. (4) : Let E_1 = 'total of 8'

i.e., $E_1 = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$

$\therefore\ P(E_1)=\frac{5}{36}$

E_2 = even number on each dice

i.e., $E_2 = \{(2, 6), (4, 4), (6, 2)\}.\ \therefore\ P(E_2)=\frac{3}{36}$

$\therefore\ E_1\cap E_2=E_2$

Now, $P(E_2\,|\,E_1)=\frac{P(E_2\cap E_1)}{P(E_1)}=\frac{3}{5}=\frac{A}{B}$

$\therefore\ B^2 - A^2 = k^2 = 25 - 9 = 16\ \Rightarrow\ k = \pm 4$

$\therefore\ k = 4$ (as $k > 0$)

34. (5) : Given, $P(A)=\frac{2}{5},\ P(A\cap B)=\frac{3}{20}$

Now, $P(A'\cup B') = P((A\cap B)') = 1 - P(A\cap B)$

$$=1-\frac{3}{20}=\frac{17}{20}$$

Again, $A\cap(A'\cup B') = A\cap(A\cap B)'$

$= A - ((A\cap B)')' = A - (A\cap B)$

$$\therefore\ P(A-(A\cap B))=\frac{2}{5}-\frac{3}{20}=\frac{1}{4}=P(\text{only }A)$$

$$\therefore\ \frac{l}{m}=P(A\,|\,(A'\cup B'))=\frac{P(A-(A\cap B))}{P(A'\cup B')}=\frac{\frac{1}{4}}{\frac{17}{20}}=\frac{5}{17}$$

$\therefore\ l=5$

35. (3) : Given, mean of B.D. $= np = 2$ and variance $= npq = 1$

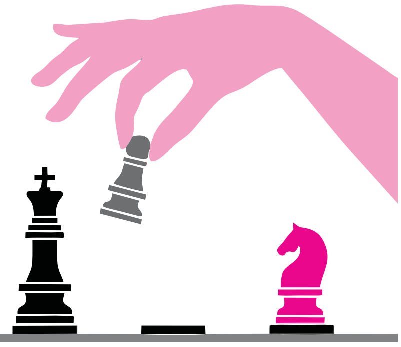
$$\Rightarrow\ p=q=\frac{1}{2}\ \text{ and }\ n=4$$

Now, $\lambda = P(x\geq 1) = P(1) + P(2) + P(3) + P(4)$

$$=1-P(0)=1-{}^4C_0\left(\frac{1}{2}\right)^4=1-\frac{1}{16}$$

$$\lambda=\frac{15}{16}\ \therefore\ \frac{16\lambda}{5}=3$$

Challenging PROBLEMS ON JEE Main



1. Given that $x_1 + x_2 + x_3 = 0$, $y_1 + y_2 + y_3 = 0$ and $x_1y_1 + x_2y_2 + x_3y_3 = 0$. Then
$$\frac{x_1^2}{x_1^2+x_2^2+x_3^2}+\frac{y_1^2}{y_1^2+y_2^2+y_3^2}=$$
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) $\frac{4}{3}$

2. $\sum_{r=1}^{n} r(r+1)(r+2)...(r+p)=$ (where n and p are positive integers)
(a) $\frac{n(n+1)(n+2)....(n+p+1)}{(p+2)}-(p+1)!$
(b) $\frac{n(n+1)(n+2)....(n+p+1)}{(n+p+2)}$
(c) $\frac{n(n+1)(n+2)....(n+p)}{(p+2)}$
(d) $\frac{n(n+1)(n+2)....(n+p)}{(n+p+2)}$

3. Let $z = (18 + 26i)$ and $z_0 = a + ib$ is the cube root of z having least positive argument then $a + b =$
(a) 1 (b) 2 (c) 3 (d) 4

4. Consider the two lines L_1, L_2 and a circle C
$L_1 : 2x + 3y + p - 3 = 0$, $L_2 : 2x + 3y + p + 3 = 0$
$C : x^2 + y^2 + 6x + 10y + 30 = 0$, $(p \in I)$
It is given that at least one of the lines L_1, L_2 is a chord of C then the probability that both are chords of C is
(a) $\frac{2}{7}$ (b) $\frac{3}{7}$ (c) $\frac{4}{7}$ (d) $\frac{5}{7}$

5. Let $f(x)=\log\left(\frac{x^2+e}{x^2+1}\right)$ and $g(x)=\sqrt{\sin f(x)}+\sqrt{\cos f(x)}$, then range of $g(x)$ is
(a) $(1, 2^{3/4}]$ (b) $(2^{1/2}, 2^{3/4}]$
(c) $(\pi^{1/2}, \pi^{3/4})$ (d) $(e^{1/2}, \pi^{1/2}]$

6. If the parabola $y = ax^2 + bx + c$ has vertex at (4, 2) and $a \in [1, 3]$ then maximum value of product $(a\,b\,c)$ is
(a) 144 (b) 12 (c) −12 (d) −144

7. For $p \geq 2$, the equation
$\sqrt{2p+1-x^2}+\sqrt{3x+p+4}=\sqrt{x^2+9x+3p+9}$ has
(a) exactly 1 real root
(b) exactly 2 distinct real roots
(c) exactly 3 distinct real roots
(d) no real roots

8. If $f : R \to R$, $g : R \to R$ and $f(x) + f''(x) = -xg(x)f'(x)$ and $g(x) > 0\ \forall\ x \in R$ then $f^2(x) + (f'(x))^2$ has
(a) a minima at $x = 0$
(b) a maxima at $x = 0$
(c) a point of inflexion at $x = 0$
(d) data insufficient

9. All bases of logarithms in which a real positive number can be equal to its logarithm is/ are
(a) $(0, 1) \cup (1, e^{1/e}]$ (b) $(1, e)$
(c) $(1, e^{1/2})$ (d) $[e^{1/e}, e^e)$

10. Let $f(x)=\lim_{m\to 0}\frac{1}{m^4}\cdot\int_0^m \frac{(e^{x+t}-e^x)(\log(1+t))^2}{3+2t^3}dt$ then $f(\log 3) =$
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{12}$

11. The non-negative real numbers $x_r (r = 1$ to $5)$ satisfy the following relations :
(1) $\sum_{r=1}^{5} r\cdot x_r = a$ (2) $\sum_{r=1}^{5} r^3 x_r = a^2$ and

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(3) $\sum_{r=1}^{5} r^5 x_r = a^3$, then number of possible values of a is/are

(a) 0 (b) 1 (c) 6 (d) 12

12. Let $\sum_{k=1}^{\infty} \cot^{-1}\left(\frac{k^2}{8}\right) = \frac{-\pi}{n}$, $n \in I$ then $n =$

(a) 1 (b) 2 (c) 4 (d) 8

13. The natural number n for which $2^8 + 2^{11} + 2^n$ is a perfect square is

(a) 11 (b) 12 (c) 14 (d) 15

14. If $f(x)$ is a differentiable function defined $\forall\, x \in R$ such that $(f(x))^3 = x - f(x)$ then $\int_0^{\sqrt{2}} f^{-1}(x)dx =$

(a) 1 (b) 2 (c) $\sqrt{2}$ (d) $2\sqrt{2}$

15. Let $f(x) = \begin{cases} e^{\{x^2\}} - 1, & x > 0 \\ 0, & x = 0 \\ \dfrac{\sin x - \tan x + \cos x - 1}{2x^2 + \tan x + \log(x+2)}, & x < 0 \end{cases}$

Lines L_1 and L_2 represent tangent and normal to curve $y = f(x)$ at $x = 0$. Consider the family of circles touching both lines L_1 and L_2. Then the ratio of the radii of two orthogonally circles of this family is

(a) $2+\sqrt{2}$ (b) $2+\sqrt{3}$ (c) $2-\sqrt{2}$ (d) $2-\sqrt{3}$

16. Suppose a and b are single digit positive integers chosen independently and at random. The probability that the point (a, b) lies above the parabola $y = ax^2 - bx$ is

(a) $\frac{17}{81}$ (b) $\frac{19}{81}$ (c) $\frac{21}{81}$ (d) $\frac{23}{81}$

17. Let $f(x) = ax^2 + bx + c$, $a, b, c \in I$. Let $f(1) = 0$, $f(7) \in (50, 60)$, $f(8) \in (70, 80)$ then $f(2) \in$

(a) $(-2, 0)$ (b) $(0, 10)$ (c) $(1, 12)$ (d) $(20, 30)$

18. In ΔABC, H is the orthocentre and $AH \cdot BH \cdot CH = 3$ and $AH^2 + BH^2 + CH^2 = 7$ then sum of the possible circumradius (R) of the ΔABC is

(a) $\frac{2}{5}$ (b) $\frac{5}{2}$ (c) $\frac{3}{2}$ (d) $\frac{2}{3}$

SOLUTIONS

1. (b) : Consider three vectors $\vec{n}_1 = (x_1, x_2, x_3)$, $\vec{n}_2 = (y_1, y_2, y_3)$ and $\vec{n}_3 = (1, 1, 1)$.

From the given data,

$$\vec{n}_1 \cdot \vec{n}_2 = 0, \vec{n}_2 \cdot \vec{n}_3 = 0 \text{ and } \vec{n}_1 \cdot \vec{n}_3 = 0$$

i.e., $\vec{n}_1, \vec{n}_2$ and $\vec{n}_3$ are mutually $\perp^r$ vectors.

Now, $\frac{x_1^2}{x_1^2+x_2^2+x_3^2}, \frac{y_1^2}{y_1^2+y_2^2+y_3^2}$ and $\frac{1}{3}$ are the squares of the projections of the vector (1, 0, 0) onto the direction of $\vec{n}_1, \vec{n}_2, \vec{n}_3$ respectively and hence their sum = 1

i.e., $\frac{x_1^2}{x_1^2+x_2^2+x_3^2} + \frac{y_1^2}{y_1^2+y_2^2+y_3^2} + \frac{1}{3} = 1$

2. (a) : Since $r(r+1) \ldots\ldots (r+p) = (p+1)!\,{}^{r+p}C_{p+1}$

$= (p+1)!\,[{}^{r+p+1}C_{p+2} - {}^{r+p}C_{p+2}]$

and required sum $= (p+1)!\,[{}^{n+p+1}C_{p+2-1}]$

$= \frac{n(n+1)\ldots\ldots(n+p+1)}{p+2} - (p+1)!$

3. (d) : $z = 18 + 26i = 10\sqrt{10}\,[\cos\theta + i\sin\theta]$

where $\tan\theta = \frac{13}{9} \Rightarrow \tan\left(\frac{\theta}{3}\right) = \frac{1}{3}$

and $z_0 = z^{1/3} = \sqrt{10}\,[\cos(\theta/3) + i\sin(\theta/3)]$

$$z_0 = \sqrt{10}\left[\frac{3}{\sqrt{10}} + \frac{i}{\sqrt{10}}\right] = 3 + i$$

4. (b) : For L_1 to be a chord of the circle, possible integral value of p are {17, 18, ..., 31}. Similarly, for L_2 to be a chord, possible integral value of p are {11, 12, ..., 25}.

So, in total possible $p = 21$ and common values are 9.

Hence, probability $= \frac{9}{21} = \frac{3}{7}$

5. (a) : Notice that $\frac{x^2+e}{x^2+1} \in [1, e]$

Hence, $f(x) \in (0, 1]$

So, $g(\alpha) = \sqrt{\sin\alpha} + \sqrt{\cos\alpha}$, $\alpha \in (0, 1]$

$g'(\alpha) = 0$ gives $\alpha = \pi/4$

So, $g(x) \in (1, 2^{3/4}]$

6. (d) : From given data, $\frac{-b}{2a} = 4$ and $\frac{-D}{4a} = 2$

So, $c = 2 + 16a$

and $E = abc = -16(a^2 + 8a^3)$

So, $\frac{dE}{da} = -16(2a + 24a^2) < 0$, for $a \in [1, 3]$

Hence, $E_{\text{max.}} = -16(1^2 + 8 \cdot 1^3) = -144$

7. (b) : Let $h = x^2 + x - p$, then given equation becomes

$$\sqrt{(x+1)^2 - 2h} + \sqrt{(x+2)^2 - h} = \sqrt{(2x+3)^2 - 3h}$$

Simplifying further, $h[2h - 2(x+2)^2 - (x+1)^2] = 0$

If $2h - 2(x+2)^2 - (x+1)^2 = 0$ then above square root equation is invalid. Hence, only $h = 0$ possible. So, now above equation becomes

$$|x+1| + |x+2| = |2x+3| \Rightarrow x \notin (-2, -1)$$

Hence, the number of real solutions of $h = x^2 + x - p = 0$ which are not in $(-2, -1)$ is zero if $p < -\frac{1}{4}$, one if $p = -\frac{1}{4}$ or $p \in (0, 2)$ and two otherwise. Hence, exactly 2 real roots for $p \in \left(\frac{-1}{4}, 0\right] \cup [2, \infty)$.

8. (b) : Let $h(x) = (f(x))^2 + (f'(x))^2$

So, $h'(x) = -2x\, g(x) \cdot (f'(x))^2$

9. (a) : We require $a \in R - \{1\}$ such that $x = \log_a x$

i.e. $f(x) = \frac{\log x}{x} = \log a$

$f'(x) = 0 \Rightarrow x = e$

So, max. $f(x) = f(e) = 1/e$. So, $a_{\text{max.}} = e^{1/e}$

Hence, $a \in (0, 1) \cup (1, e^{1/e}]$

10. (c) : $f(x) = e^x \cdot \lim\limits_{m\to 0} \dfrac{\displaystyle\int_0^m \frac{(e^t-1)\cdot(\log(1+t))^2}{2t^3+3}\,dt}{m^4} \quad \left(\frac{0}{0}\right)$

$$\Rightarrow f(x) = e^x \cdot \lim_{m\to 0} \frac{(e^m-1)\cdot(\log(1+m))^2}{(3+2m^3)\cdot 4m^3}$$

$$\Rightarrow f(x) = e^x \cdot \lim_{m\to 0} \frac{e^m-1}{m} \cdot \left(\frac{\log(1+m)}{m}\right)^2 \cdot \frac{1}{4} \cdot \frac{1}{3+2m^3}$$

$$\Rightarrow f(x) = \frac{e^x}{12}, \text{ so, } f(\log 3) = \frac{3}{12} = \frac{1}{4}$$

11. (c) : Notice that

$$\sum_{r=1}^{5} r(a-r^2)^2 x_r = a^2 \sum_{r=1}^{5} r \cdot x_r - 2a \sum_{r=1}^{5} r^3 x_r + \sum_{r=1}^{5} r^5 x_r$$

$= a^2(a) - 2a(a^2) + (a^3) = 0$

Since, L.H.S. terms are non-negative.

Hence, each term in L.H.S. is zero.

So, possible values of a are $\{0, 1, 4, 9, 16, 25\}$

12. (c) : $T_k = \cot^{-1}\left(\frac{k^2}{8}\right)$

$$= \tan^{-1}\left(\frac{8}{k^2}\right) = \tan^{-1}\left[\frac{2}{1+\frac{k^2}{4}-1}\right]$$

$$= \tan^{-1}\left[\frac{k}{2}+1\right] - \tan^{-1}\left(\frac{k}{2}-1\right)$$

13. (b) : $2^8 + 2^{11} + 2^n = 2^8(9 + 2^{n-8})$

Hence, $9 + 2^{n-8}$ should be a perfect square.

So, $9 + 2^{n-8} = k^2$ (say) i.e. $2^{n-8} = (k+3)(k-3)$

So, $(k+3)$ and $(k-3)$ are both powers of 2.

$\Rightarrow$ $k = 5$ being the only possibility. Hence, $n = 12$

14. (b) : $(f(x))^3 + f(x) = x$. Hence, $f^{-1}(x) = x^3 + x$

15. (b) : $\text{L.H.D.} = \lim\limits_{h\to 0}\left[\dfrac{-\sin h + \tan h + \cos h - 1}{2h^2 - \tanh + \log(2-h)}\right] \times \dfrac{1}{h} = 0$

and $\text{R.H.D.} = \lim\limits_{h\to 0} \dfrac{e^{h^2} - 1 - 0}{h} = 0$

Hence, $L_1 = y = 0$ and $L_2 = x = 0$.

16. (b) : If (a, b) lies above the curve then $b > y(a)$

i.e., $b > a^3 - ba \Rightarrow b > \dfrac{a^3}{a+1}$

The only possibilities are $a = 1, 2, 3$

For $a = 1$, $b = 1, 2, 3, \ldots.9$

For $a = 2$, $b = 3, 4, \ldots.\ 9$

For $a = 3$, $b = 7, 8, 9$. So, in total there are 19 points out of $9 \times 9 = 81$ points.

Hence, required probability $= \dfrac{19}{81}$

17. (a) : $f(1) = 0 \Rightarrow a + b + c = 0$

$f(7) \in (50, 60) \Rightarrow 50 < 49a + 7b + c < 60$

or $50 < 48a + 6b < 60$ i.e., $8a + b \in \left(\dfrac{25}{3}, 10\right)$

i.e., $8a + b = 9$

Similarly, $9a + b = 11$

Hence, $a = 2$, $b = -7$, $c = 5$

i.e., $f(x) = 2x^2 - 7x + 5 \Rightarrow f(2) = 8 - 14 + 5 = -1$

18. (b) : If ΔABC is acute, then Cosine rule gives,

$AB^2 = AH^2 + BH^2 - 2AH \cdot BH\cos(\pi - C)$ ...(i)

and $AB = 2R\sin C$, $CH = 2R\cos C$

$\Rightarrow AB^2 + CH^2 = 4R^2$ ...(ii)

From (i) and (ii),

$$4R^2 = AH^2 + BH^2 + CH^2 + \frac{AH \cdot BH \cdot CH}{R}$$

Now, A.T.Q., $4R^2 = 7 + \dfrac{3}{R}$ i.e., $4R^3 - 7R - 3 = 0$

i.e., $(R+1)(2R+1)(2R-3) = 0$

Since, $3 = AH \cdot BH \cdot CH < (2R)^3 \Rightarrow R = 1$

Similarly when ΔABC is obtuse, we have $R = \dfrac{3}{2}$

So, sum of possibilities of $R = \dfrac{3}{2} + 1 = \dfrac{5}{2}$

MATHS MUSING

SOLUTION SET-181

1. (c) : Let $x = (1)^{1/n} \Rightarrow x^n - 1 = 0$

or $x^n - 1 = (x-1)(x-\omega)(x-\omega^2)\ ....\ (x-\omega^{n-1})$

$$\Rightarrow \frac{x^n-1}{x-1} = (x-\omega)(x-\omega^2)\ ....\ (x-\omega^{n-1})$$

Putting $x = 9$ in both sides, we have

$$(9-\omega)(9-\omega^2)(9-\omega^3)....(9-\omega^{n-1}) = \frac{9^n-1}{8}.$$

2. (d) : Since, $3^{2\sin 2x-1}, 14, 3^{4-2\sin 2x}$ are in A.P.

So, $28 = 3^{2\sin 2x}\cdot\frac{1}{3} + 3^4\cdot\frac{1}{3^{2\sin 2x}}$

Put $3^{2\sin 2x} = t$, we get

$$28 = \frac{t}{3} + \frac{81}{t} \Rightarrow 84t = t^2 + 243$$

$\Rightarrow t^2 - 84t + 243 = 0$

$\Rightarrow (t-81)(t-3) = 0 \Rightarrow t = 3, 81$

$\Rightarrow 3^{2\sin 2x} = 3^1, 3^4$

$\Rightarrow 2\sin 2x = 1$ or $2\sin 2x = 4$

$\Rightarrow \sin 2x = \frac{1}{2}\ (\because \sin 2x \neq 2)$

$\therefore$ First term $= 3^{1-1} = 1$, second term $= 14$

and third term $= 27$

Here, common difference $= 13$

$\therefore$ Fifth term $= 1 + 4 \times 13 = 53$.

3. (a) : $\sum_{i=0}^{n}\sum_{j=1}^{n} {}^nC_j\,{}^jC_i = {}^nC_1({}^1C_0 + {}^1C_1) +$

${}^nC_2({}^2C_0 + {}^2C_1 + {}^2C_2) + {}^nC_3({}^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3)$
$+ {}^nC_4({}^4C_0 + {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4)$
$+ ... + {}^nC_n({}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n)$

$= {}^nC_1(2) + {}^nC_2(2)^2 + {}^nC_3(2)^3 + {}^nC_4(2)^4 + ... + {}^nC_n(2)^n$

$= (1+2)^n - 1 = 3^n - 1$

4. (b) : Putting $t = 0$ on both sides, we get $E = -39$

5. (a) : Let $A = \begin{pmatrix} -1 & 2 & 5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{pmatrix}$

Applying $R_1 \to (-1)R_1$, we get

$$A = \begin{pmatrix} 1 & -2 & -5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{pmatrix}$$

Applying $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - R_1$, we get

$$A = \begin{pmatrix} 1 & -2 & -5 \\ 0 & 0 & a+6 \\ 0 & 0 & a+6 \end{pmatrix}$$

For $a = -6$

$$A = \begin{pmatrix} 1 & -2 & -5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \therefore \quad \text{Rank}(A) = 1.$$

6. (d) : If f is injective and g is injective

$\Rightarrow fog$ is injective $\therefore$ fof is injective.

7. (d) : $\lim_{n\to\infty}\left\{\frac{1}{n} + \frac{n^2}{(n+1)^3} + \frac{n^2}{(n+2)^3} + ... + \frac{1}{8n}\right\}$

$$= \lim_{n\to\infty}\left\{\frac{n^2}{(n+0)^3} + \frac{n^2}{(n+1)^3} + \frac{n^2}{(n+2)^3} + ... + \frac{n^2}{(n+n)^3}\right\}$$

$$= \lim_{n\to\infty}\sum_{r=0}^{n}\frac{n^2}{(n+r)^3} = \lim_{n\to\infty}\sum_{r=0}^{n}\frac{1}{\left(1+\frac{r}{n}\right)^3 n}$$

$$= \int_0^1\frac{dx}{(1+x)^3} = -\left[\frac{1}{2(1+x)^2}\right]_0^1 = -\left\{\frac{1}{8} - \frac{1}{2}\right\} = \frac{1}{2} - \frac{1}{8} = \frac{3}{8}$$

8. (a) : Let $P = \lim_{n\to\infty}\left\{\prod_{r=1}^{n}\tan\left(\frac{r\pi}{2n}\right)\right\}^{1/n}$

$$\therefore \ln P = \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\ln\tan\left(\frac{r\pi}{2n}\right) = \int_0^1\ln\tan\left(\frac{\pi x}{2}\right)dx$$

$$\Rightarrow \ln P = \frac{2}{\pi}\int_0^{\pi/2}\ln\tan x\,dx \qquad ...(i)$$

and $\ln P = \frac{2}{\pi}\int_0^{\pi/2}\ln\tan\left(\frac{\pi}{2}-x\right)dx$ (by property)

$$= \frac{2}{\pi}\int_0^{\pi/2}\ln\cot x\,dx \qquad ...(ii)$$

Adding (i) and (ii), we get

$$2\ln P = \frac{2}{\pi}\int_0^{\pi/2}(\ln\tan x + \ln\cot x)dx = \frac{2}{\pi}\int_0^{\pi/2}\ln 1 dx = 0$$

$\Rightarrow 2\ln P = 0 \Rightarrow \ln P = 0 \quad \therefore \quad P = e^0 = 1$

MPP-10 CLASS XI ANSWER KEY

1.	(d)	**2.**	(c)	**3.**	(a)	**4.**	(a)	**5.**	(c)
6.	(c)	**7.**	(c)	**8.**	(a, b)	**9.**	(a, b)	**10.**	(c)
11.	(a, c)	**12.**	(a, c)	**13.**	(a, d)	**14.**	(d)	**15.**	(b)
16.	(b)	**17.**	(9)	**18.**	(5)	**19.**	(2)	**20.**	(5)

9. (2) : We have, $\lim\limits_{x\to 0}\frac{\sin(\sin x)-\sin x}{ax^3+bx^5+c}=-\frac{1}{12}$

$$\Rightarrow \lim_{x\to 0}\frac{2\cos\left(\frac{\sin x+x}{2}\right)\sin\left(\frac{\sin x-x}{2}\right)}{(ax^3+bx^5+c)}=-\frac{1}{12}$$

$$\Rightarrow \lim_{x\to 0}\frac{2\cos\left(\frac{x+\sin x}{2}\right)\frac{\sin\left(\frac{x-\sin x}{2}\right)}{\left(\frac{x-\sin x}{2}\right)}\left(\frac{x-\sin x}{2}\right)}{\left(a+bx^2+\frac{c}{x^3}\right)x^3}=\frac{1}{12}$$

$$\Rightarrow \lim_{x\to 0}\frac{2\cdot 1\cdot\frac{1}{2}\cdot\frac{1}{6}}{\left(a+bx+\frac{c}{x^3}\right)}=\frac{1}{12}\left(\because \lim_{x\to 0}\frac{x-\sin x}{x^3}=\frac{1}{6}\right)$$

$$\Rightarrow \lim_{x\to 0}\frac{1}{\left(a+bx+\frac{c}{x^3}\right)}=\frac{1}{2}$$

$\because$ R.H.S. is finite quantity.

$\therefore$ $c=0$, $b\in R$ and $\frac{1}{a}=\frac{1}{2}\Rightarrow a=2$

10. (a) : (P) → 4, (Q) → 3, (R) → 1, (S) → 2

P. $\because y=\sin^{-1}(3x-4x^3)$

$$=\begin{cases}-\pi-3\sin^{-1}x, & -1\le x<-\frac{1}{2}\\ 3\sin^{-1}x, & -\frac{1}{2}\le x\le\frac{1}{2}\\ \pi-3\sin^{-1}x, & \frac{1}{2}<x\le 1\end{cases}$$

$$\therefore \frac{dy}{dx}=\begin{cases}-\frac{3}{\sqrt{1-x^2}}, & -1<x<-\frac{1}{2}\\ \frac{3}{\sqrt{1-x^2}}, & -\frac{1}{2}<x<\frac{1}{2}\\ -\frac{3}{\sqrt{(1-x^2)}}, & \frac{1}{2}<x<1\end{cases}$$

Q. $\because y=\cos^{-1}(4x^3-3x)$

$$=\begin{cases}3\cos^{-1}x-2\pi, & -1<x<-\frac{1}{2}\\ 2\pi-3\cos^{-1}x, & -\frac{1}{2}\le x\le\frac{1}{2}\\ 3\cos^{-1}x, & \frac{1}{2}<x\le 1\end{cases}$$

$$\therefore \frac{2dy}{dx}=\begin{cases}\frac{-6}{\sqrt{1-x^2}}, & -1<x<-\frac{1}{2}\\ \frac{6}{\sqrt{1-x^2}}, & -\frac{1}{2}<x<\frac{1}{2}\\ \frac{-6}{\sqrt{1-x^2}}, & \frac{1}{2}<x<1\end{cases}$$

R. $\because y=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

$$=\begin{cases}3\tan^{-1}x, & -\frac{1}{\sqrt{3}}<x<\frac{1}{\sqrt{3}}\\ \pi+3\tan^{-1}x, & x<-\frac{1}{\sqrt{3}}\\ -\pi+3\tan^{-1}x, & x>\frac{1}{\sqrt{3}}\end{cases}$$

$$\therefore \frac{dy}{dx}=\begin{cases}\frac{3}{1+x^2}, & -\frac{1}{\sqrt{3}}<x<\frac{1}{\sqrt{3}}\\ \frac{3}{1+x^2}, & x<-\frac{1}{\sqrt{3}}\\ \frac{3}{1+x^2}, & x>\frac{1}{\sqrt{3}}\end{cases}$$

S. $f(x)=\left(\frac{\log\sin x}{\log\cos x}\right)^2+2\tan^{-1}x$

$f'(x)=2$

$$\left(\frac{\log\sin x}{\log\cos x}\right)\frac{(\log\cos x)\cot x+(\log\sin x)\tan x}{(\log\cos x)^2}+\frac{2}{1+x^2}$$

$$\therefore f'\left(\frac{\pi}{4}\right)=2\cdot 2\frac{1}{\log\cos\frac{\pi}{4}}+\frac{2}{1+\frac{\pi^2}{16}}=\frac{4}{\log\frac{1}{\sqrt{2}}}+\frac{32}{16+\pi^2}$$

$$=-\frac{8}{\log 2}+\frac{32}{\pi^2+16}$$

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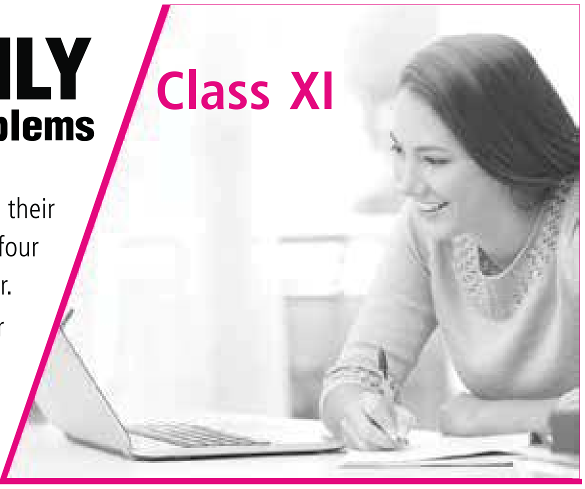
MPP-10 CLASS XII ANSWER KEY

1.	(c)	2.	(a)	3.	(a)	4.	(a)	5.	(c)
6.	(c)	7.	(a,b,c,d)			8.	(a, c)	9.	(a, c)
10.	(a,b)	11.	(c)	12.	(b, d)	13.	(b, c)	14.	(a)
15.	(b)	16.	(a)	17.	(8)	18.	(4)	19.	(4)
20.	(2)								

MPP-10 MONTHLY Practice Problems

Class XI



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Statistics and Probability

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. If $\frac{1+3p}{3}, \frac{1-p}{4}$ and $\frac{1-2p}{2}$ are the probabilities of the three mutually exclusive events, then $p \in$

(a) $[0, 1]$ (b) $\left[0, \frac{1}{2}\right]$ (c) $\left[\frac{1}{3}, 1\right]$ (d) $\left[\frac{1}{3}, \frac{1}{2}\right]$

2. Mean of 100 items is 49. It was discovered that 3 items 60, 70, 80 were wrongly read as 38, 22, 50 respectively. The correct mean is

(a) 48 (b) 78 (c) 50 (d) 80

3. The standard deviation of 25 number is 40. If each of the number is increased by 5, then the new standard deviation will be

(a) 40 (b) 25 (c) $\sqrt{40}$ (d) 1600

4. If $P(B) = \frac{3}{4}, P(A \cap B \cap \bar{C}) = \frac{1}{3}$ and $P(\bar{A} \cap B \cap \bar{C}) = \frac{1}{3}$, then $P(B \cap C) =$

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$ (c) $\frac{1}{15}$ (d) $\frac{1}{9}$

5. Two dice are thrown simultaneously to get the coordinates of a point on xy-plane. Then the probability that this point lies inside or on the region bounded by $|x| + |y| = 3$, is

(a) $\frac{3}{14}$ (b) $\frac{2}{3}$ (c) $\frac{1}{12}$ (d) $\frac{4}{14}$

6. Out of $(2n + 1)$ tokens consecutively numbered from 1 to $(2n + 1)$, 3 tokens are drawn at random. The probability that the numbers on them are in A.P. is

(a) $\frac{2n}{4n^2-1}$ (b) $\frac{4n}{4n^2-1}$ (c) $\frac{3n}{4n^2-1}$ (d) $\frac{5n}{4n^2-1}$

One or More Than One Option(s) Correct Type

7. If the mean deviation of numbers $1, 1 + d, 1 + 2d, ..., 1 + 100d$ from their mean is 255, then the d is equal to

(a) 10.0 (b) 20.0 (c) 10.1 (d) 20.2

8. The variable x takes two values x_1 and x_2 with frequencies f_1 and f_2, respectively. If σ denotes the standard deviation of x, then

(a) $\sigma^2 = \frac{f_1x_1^2 + f_2x_2^2}{f_1 + f_2} - \left(\frac{f_1x_1 + f_2x_2}{f_1 + f_2}\right)^2$

(b) $\sigma^2 = \frac{f_1 f_2}{(f_1 + f_2)^2} - (x_1 - x_2)^2$

(c) $\sigma^2 = \frac{(x_1 - x_2)^2}{(f_1 + f_2)^2}$ (d) none of these

9. Which of the following is correct statement?

(a) The sum of the deviations from arithmetic mean is zero.
(b) Computation of arithmetic mean is based on all observations.
(c) It gives no importance to extreme values.
(d) none of these.

10. The probability that the 13th day of a randomly chosen month is a second Saturday, is

(a) $\frac{1}{7}$ (b) $\frac{1}{12}$ (c) $\frac{1}{84}$ (d) $\frac{19}{84}$

11. The chance of an event happening is the square of the chance of a second event but the odds against the first are the cube of the odds against the second. The chances of the events are

(a) $p_1 = 1/9$ (b) $p_1 = 1/16$
(c) $p_2 = 1/3$ (d) $p_2 = 1/4$

12. The variance of the numbers 2, 3, 11 and x is $\frac{49}{4}$. Find the value of x.

(a) 6 (b) $\frac{14}{5}$ (c) $\frac{14}{3}$ (d) $\frac{6}{15}$

13. If runs of two players A and B in 10 cricket matches are such that players A has mean 50 and variance 36 and player B has mean 60 and variance 81 of runs, then the player

(a) A has less coeff. of variation
(b) B has less coeff. of variation
(c) Both are equally consistent
(d) A is more consistent

Comprehension Type

In an objective paper, there are two sections of 10 questions each. For 'section 1', each question has five options and only one option is correct and 'section 2' has four options with multiple answers and marks for a question in this section is awarded only if he ticks all correct answers. Marks for each question in 'section 1' is 1 and in 'section 2' is 3. (There is no negative marking).

14. If a candidate in total attempts four questions all by guessing, then the probability of scoring 10 marks is

(a) $1/15(1/15)^3$ (b) $3/5(1/15)^3$
(c) $1/15(14/15)^3$ (d) None of these

15. The probability of getting a score less than 40 by answering all the questions by guessing in this paper is

(a) $(1/75)^{10}$ (b) $1 - (1/75)^{10}$
(c) $(74/75)^{10}$ (d) None of these

Matrix Match Type

16. Match the following.

	Column I		Column II
P.	There are 5 duplicate and 10 original items in an automobile shop and 3 items are brought at random by a customer. The probability that none of the items is duplicate, is	1.	$\frac{1}{6}$
Q.	There are 2 teams with n persons in each. The probability of selecting 2 persons from one team and 1 person from the other team is $\frac{6}{7}$. Then n=	2.	$\frac{24}{91}$
R.	If the letters of the word NALGONDA are arranged in arbitrary order, the probability that the letters G, O, D appear in that order is	3.	$\frac{1}{35}$
S.	There are 8 children, 4 boys and 4 girls. They are randomly divided into 2 groups of 4 children each. The probability that all the girls are in one group is	4.	4

	P	Q	R	S
(a)	2	1	3	4
(b)	2	4	1	3
(c)	3	1	4	2
(d)	3	4	1	2

Integer Answer Type

17. A card is drawn from a pack of 52 cards and a gambler bets that it is a spade or an ace. If the odds against his winning this bet is $\frac{k}{k-5}$, then the value of k is

18. The A.M. and S.D. of 100 items was recorded as 40 and 5.1 respectively. Later on it was discovered that one observation 40 was wrongly copied down as 50. Then the correct S.D is

19. The mean and variance of 7 observations are 8 and 16 respectively. If the observations are 2, x, 4, 10, 12, y, 14. Then the positive value of $x - y$ is

20. Let $A = \{1, 3, 5, 7, 9\}$ and $B = \{2, 4, 6, 8\}$. An element (a,b) of their cartesian product $A \times B$ is chosen at random. If the probability that $\{a + b = 9\}$ is $\frac{p}{q}$ then $\frac{q}{p}$ is

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Keys are published in this issue. Search now! ☺

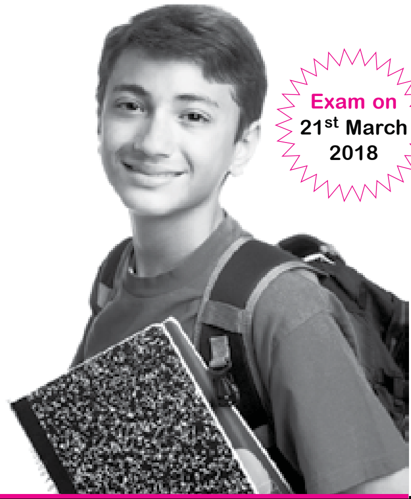
SELF CHECK

No. of questions attempted ......
No. of questions correct ......
Marks scored in percentage ......

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

ACE YOUR WAY CBSE XII



Exam on 21st March 2018

PRACTICE PAPER 2018

Time Allowed : 3 hours | Maximum Marks : 100

GENERAL INSTRUCTIONS

(i) All questions are compulsory.
(ii) This question paper contains **29** questions.
(iii) Questions **1-4** in Section-A are very short-answer type questions carrying **1** mark each.
(iv) Questions **5-12** in Section-B are short-answer type questions carrying **2** marks each.
(v) Questions **13-23** in Section-C are long-answer-**I** type questions carrying **4** marks each.
(vi) Questions **24-29** in Section-D are long-answer-**II** type questions carrying **6** marks each.

SECTION - A

1. Find the domain of $\sec^{-1}(2x + 1)$.

2. Find the distance of the point $P(2, 1, -1)$ from the plane $x - 2y + 4z = 9$.

3. If A, B, C are three non-zero square matrices of same order, then find the condition on A such that $AB = AC \Rightarrow B = C$.

4. Let $f : R \to R$ be defined by $f(x) = 3x^2 - 5$ and $g : R \to R$ be defined by $g(x) = \dfrac{x}{x^2+1}$. Find gof.

SECTION - B

5. The volume of a cube is increasing at a constant rate. Prove that the increase in surface area varies inversely as the length of the edge of the cube.

6. How many equivalence relations on the set $\{1, 2, 3\}$ containing $(1, 2)$ and $(2, 1)$ are there in all?

7. Find the cartesian and vector equations of the line which passes through the point $(-2, 4, -5)$ and parallel to the line given by
$$\frac{x+3}{3} = \frac{y-4}{5} = \frac{8-z}{-6}.$$

8. Form the differential equation of the family of curves $y = a\cos(x + b)$, where a and b are arbitrary constants.

9. If $f(x) = \begin{cases} 1 & , \text{ if } x \le 3 \\ ax+b & , \text{ if } 3 < x < 5 \\ 7 & , \text{ if } 5 \le x \end{cases}$. Determine the values of a and b so that $f(x)$ is continuous.

10. Let $l_i, m_i, n_i; i = 1, 2, 3$ be the direction cosines of three mutually perpendicular vectors in space. Show that $AA' = I_3$, where $A = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & m_3 \end{bmatrix}$.

11. Two balls are drawn at random from a bag containing 5 white and 7 black balls. Then, find the odds in favour of drawing two black balls.

12. Solve the following linear programming problem graphically.
Maximize $Z = 3x + 4y$
subject to $x + y \le 4$, $x \ge 0$ and $y \ge 0$.

SECTION - C

13. Find the inverse of the matrix, if it exists, using elementary row operations. $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$.

OR

Using properties of determinants, show that

$$\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix} = (b^2-ac)(ax^2+2bxy+cy^2)$$

14. Using Lagrange's Mean value theorem, show that $|\cos a - \cos b| \le |a-b|$.

15. Find the absolute maximum and minimum values of $f(x)=x^2\sqrt{1+x}$ in $\left[-1,\frac{1}{2}\right]$.

OR

Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$.

16. Evaluate $\int_1^3 (2x+1)dx$ as limit of sums.

17. Find the area bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$.

18. Solve the following differential equation :

$\frac{dy}{dx} - y = \cos x$, given that if $x = 0$, $y = 1$.

OR

Check whether the following differential equation is homogeneous or not.

$$x^2\frac{dy}{dx} - xy = 1+\cos\left(\frac{y}{x}\right), x \ne 0$$

Find the general solution of the differential equation using substitution $y = vx$.

19. Let $\vec{a}=\hat{i}+4\hat{j}+2\hat{k}, \vec{b}=3\hat{i}-2\hat{j}+7\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}\cdot\vec{d}=15$.

20. An urn contains 10 white and 3 black balls. Another urn contains 3 white and 5 black balls. Two balls are drawn from first urn without noticing their colour and put into the second urn and then a ball is drawn from the second urn. Find the probability that it is a white ball.

21. A random variable X has the following probability distribution :

X	0	1	2	3	4	5	6	7
$P(X)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Find each of the following :

(i) k (ii) $P(X < 6)$

(iii) $P(X \ge 6)$ (iv) $P(0 < X < 5)$

22. Find the equation of the plane passing through the points (2, 2, 1) and (9, 3, 6) and perpendicular to the line $2x + 6y + 6z = 1$.

23. Solve the following L.P.P. graphically :

Minimize $Z = 5x + 10y$

Subject to constraints $x + 2y \le 120$, $x + y \ge 60$, $x - 2y \ge 0$ and $x, y \ge 0$

SECTION - D

24. Evaluate : $\int \frac{1}{\sin x + \sec x}dx$.

OR

Evaluate : $\int_0^{\pi/2} (2\log\sin x - \log\sin 2x)dx$

25. Find the maximum and minimum values of $(\sin^{-1} x)^3 + (\cos^{-1} x)^3$, where $-1 \le x \le 1$.

OR

On the set $R - \{-1\}$ a binary operation $*$ is defined by $a * b = a + b + ab$ for all $a, b \in R - \{-1.\}$. Prove that $*$ is commutative as well as associative on $R - \{-1\}$. Find the identity element and prove that every element of $R - \{-1\}$ is invertible.

26. An amount of ₹ 5000 is put into three investments at the rate of interest of 6%, 7% and 8% per annum respectively. The total annual income is ₹ 358. If the combined income from the first two investments is ₹ 70 more than the income from the third, find the amount of each investment by matrix method.

OR

Verify : $A(\text{adj } A) = (\text{adj } A)A = |A|\ I$ for matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$

27. If $(ax + b)e^{y/x} = x$ or $y = x\log\left(\frac{x}{a+bx}\right)$, prove that $x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$.

28. Find the shortest distance between the lines

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4} \text{ and } \frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$$

29. Find the cartesian as well as vector equations of the planes through the intersection of the planes $\vec{r}\cdot(2\hat{i}+6\hat{j})+12=0$ and $\vec{r}\cdot(3\hat{i}-\hat{j}+4\hat{k})=0$ which are at a unit distance from the origin.

SOLUTIONS

1. The domain of $\sec^{-1} x$ is $(-\infty, -1] \cup [1, \infty)$.

$\therefore\ 2x+1 \geq 1$ or $2x+1 \leq -1$

$\Rightarrow\ 2x \geq 0$ or $2x \leq -2 \Rightarrow x \geq 0$ or $x \leq -1$

$\Rightarrow\ x \in (-\infty, -1] \cup [0, \infty)$.

Hence, the domain of $\sec^{-1}(2x+1)$ is $(-\infty, -1] \cup [0, \infty)$.

2. We have point $P(2, 1, -1)$ and plane $x - 2y + 4z = 9$

$\therefore$ Required distance $=\dfrac{|2-2\times1+4\times(-1)-9|}{\sqrt{1^2+(-2)^2+(4)^2}}=\dfrac{13}{\sqrt{21}}$.

3. We have $AB = AC$

To show : $B = C$

Since, $B = C$ is possible if $\exists\, A^{-1}$

i.e., $A^{-1}AB = A^{-1}AC \Rightarrow B = C$ $\quad [\because A^{-1}A = I]$

Hence, A should be invertible matrix.

4. Since, $gof(x) = g\{f(x)\} = g(3x^2-5)$

$$=\frac{3x^2-5}{(3x^2-5)^2+1}=\frac{3x^2-5}{9x^4-30x^2+26}.$$

5. Let x be the length of each edge of the cube, S be its surface area and V be its volume at any time t. Then,

$S = 6x^2$ and $V = x^3$, Given $\dfrac{dV}{dt}=k$ (constant)

Now, $V = x^3 \Rightarrow \dfrac{dV}{dt}=3x^2\dfrac{dx}{dt}$

$\Rightarrow k = 3x^2\dfrac{dx}{dt} \Rightarrow \dfrac{dx}{dt}=\dfrac{k}{3x^2}$ ...(i)

and $S = 6x^2 \Rightarrow \dfrac{dS}{dt}=12x\dfrac{dx}{dt}$

$\Rightarrow \dfrac{dS}{dt}=12x\left(\dfrac{k}{3x^2}\right)$ [Using (i)]

$\Rightarrow \dfrac{dS}{dt}=\dfrac{4k}{x} \Rightarrow \dfrac{dS}{dt}\propto\dfrac{1}{x}$.

6. Equivalence relations could be the following:

{(1,1), (2,2), (3,3), (1,2), (2,1)} and {(1,1), (2, 2), (3, 3), (1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3, 2)}

So, only two equivalence relations.

7. Given equation of line is $\dfrac{x+3}{3}=\dfrac{y-4}{5}=\dfrac{8-z}{-6}$

or $\dfrac{x+3}{3}=\dfrac{y-4}{5}=\dfrac{z-8}{6}$

Since a line through $(-2, 4, -5)$ is parallel to the given line having D.R.'s $(3, 5, 6)$

$\therefore$ Required equation of line in cartesian form is

$$\frac{x+2}{3}=\frac{y-4}{5}=\frac{z+5}{6}$$

and vector equation of the line is $\vec{r}=\vec{a}+\lambda\vec{b}$

$\Rightarrow \vec{r}=(-2\hat{i}+4\hat{j}-5\hat{k})+\lambda(3\hat{i}+5\hat{j}+6\hat{k})$

8. Here, $y = a\cos(x+b)$ ...(i)

Differentiating (i) w.r.t. x, we get

$$\frac{dy}{dx}=-a\sin(x+b)$$

Again differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2}=-a\cos(x+b)\Rightarrow\frac{d^2y}{dx^2}=-y\Rightarrow\frac{d^2y}{dx^2}+y=0.$$

9. For $f(x)$ to be continuous at $x = 3$, we must have

$\lim\limits_{x\to3^-} f(x)=\lim\limits_{x\to3^+} f(x)=f(3)$

$\Rightarrow\ 1 = 3a + b$ ...(i)

For $f(x)$ to be continuous at $x = 5$, we must have

$\lim\limits_{x\to5^-} f(x)=\lim\limits_{x\to5^+} f(x)=f(5)$

$\Rightarrow\ 5a + b = 7$ ...(ii)

Solving (i) and (ii), we get $a = 3$, $b = -8$.

10. Given, l_i, m_i, n_i, are three mutually $\perp$r vector in space.

$\therefore\ l_i^2+m_i^2+n_i^2=1$, for each $i = 1, 2, 3$ and

$l_il_j + m_im_j + n_in_j = 0\ (i \neq j)$ for each $i, j = 1, 2, 3$

Hence $AA'=\begin{bmatrix} l_1 & m_1 & n_1\\ l_2 & m_2 & n_2\\ l_3 & m_3 & n_3\end{bmatrix}\begin{bmatrix} l_1 & l_2 & l_3\\ m_1 & m_2 & m_3\\ n_1 & n_2 & n_3\end{bmatrix}$

$$=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=I_3$$

11. The probability of drawing two black balls

$$=\frac{{}^7C_2}{{}^{12}C_2}=\frac{7\cdot6}{1\cdot2}\times\frac{1\cdot2}{12\cdot11}=\frac{7}{22}$$

So, the odds in favour of drawing two black balls are

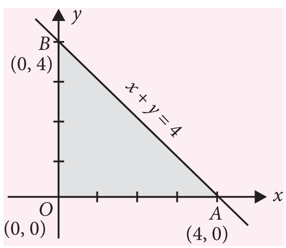
$$p:(1-p)=\frac{7}{22}:\left(1-\frac{7}{22}\right)=\frac{7}{22}:\frac{15}{22}=7:15.$$

12. We have, maximize $Z = 3x + 4y$,

Subject to the constraints

$x + y \leq 4, x \geq 0, y \geq 0$

On plotting the given constraint we get, OAB bounded region as feasible.



Hence, on evaluating Z at corner points $O(0, 0)$, $A(4, 0)$, $B(0, 4)$, we get

Corner points	$Z = 3x + 4y$
$O(0, 0)$	$3 \times 0 + 4 \times 0 = 0$
$A(4, 0)$	$3 \times 4 + 4 \times 0 = 12$
$B(0, 4)$	$3 \times 0 + 4 \times 4 = 16$ (Maximum)

Hence, the maximum value of Z is 16 at (0, 4).

13. Let $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$, then $A = I_3 A$

$$\Rightarrow \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \leftrightarrow R_2$, we get

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_3 \to R_3 - 3R_1$, we get

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A$$

Applying $R_3 \to R_3 + 5R_2$, and $R_1 \to R_1 - 2R_2$ we get

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - R_3$, we get

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ -4 & 3 & -1 \\ 5 & -3 & 1 \end{bmatrix} A$$

Applying $R_3 \to \frac{1}{2} R_3$, we get

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ -4 & 3 & -1 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix} A$$

Applying $R_1 \to R_1 + R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix} A$$

$\Rightarrow \quad I_3 = BA$

Hence, $A^{-1} = B = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$.

OR

$$\text{L.H.S.} = \begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix}$$

Applying $C_1 \to C_1 \times x$ and $C_2 \to C_2 \times y$, we get

$$\frac{1}{xy}\begin{vmatrix} ax & by & ax+by \\ bx & cy & bx+cy \\ ax^2+bxy & bxy+cy^2 & 0 \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2$, we get

$$\frac{1}{xy}\begin{vmatrix} ax+by & by & ax+by \\ bx+cy & cy & bx+cy \\ ax^2+2bxy+cy^2 & bxy+cy^2 & 0 \end{vmatrix}$$

Applying $C_1 \to C_1 - C_3$, we get

$$\frac{1}{xy}\begin{vmatrix} 0 & by & ax+by \\ 0 & cy & bx+cy \\ ax^2+2bxy+cy^2 & bxy+cy^2 & 0 \end{vmatrix}$$

Expanding the determinant along C_1, we get

$$\frac{1}{xy}\left[(ax^2 + 2bxy + cy^2)\{by(bx + cy) - cy(ax + by)\}\right]$$

$$= \frac{1}{xy}\left[(ax^2 + 2bxy + cy^2)\{b^2xy + bcy^2 - caxy - bcy^2\}\right]$$

$$= \frac{1}{xy}\left[(ax^2 + 2bxy + cy^2)(xy)(b^2 - ac)\right]$$

$= [ax^2 + 2bxy + cy^2]\,[b^2 - ac] =$ R.H.S.

14. Two cases arise:

Case I: When $a = b$

$|\cos a - \cos b| = 0 = |a - b|$.

Case II: When $a \neq b$, let $a < b$.

Consider the function $f(x) = \cos x$ ...(i)

As $f(x)$ is continuous and differentiable for all $x \in R$,

(i) $f(x)$ is continuous in $[a, b]$ and

(ii) $f(x)$ is differentiable in (a, b).

Thus, both the conditions of Lagrange's Mean value theorem are satisfied, therefore, there exists atleast one real number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a} \quad \text{...(ii)}$$

Differentiating (i) w.r.t. x, we get

$f'(x) = -\sin x \Rightarrow f'(c) = -\sin c$

From (ii), we get

$$-\sin c = \frac{\cos b - \cos a}{b - a} \Rightarrow \frac{\cos a - \cos b}{a - b} = -\sin c$$

$$\Rightarrow \quad \left|\frac{\cos a-\cos b}{a-b}\right|=|-\sin c|=|\sin c|\le 1$$

$[\because |\sin x|\le 1\ \forall\ x\in R]$

$$\Rightarrow \quad \frac{|\cos a-\cos b|}{|a-b|}\le 1\Rightarrow |\cos a-\cos b|\le |a-b|.$$

From cases I and II, we get $|\cos a - \cos b| \le |a - b|$.

15. Given $f(x)=x^2\sqrt{1+x}$ ...(i)

The given function is differentiable for all x in $\left[-1,\frac{1}{2}\right]$.
Differentiating (i) w.r.t. x, we get

$$f'(x)=x^2\cdot\frac{1}{2}(1+x)^{-1/2}+\sqrt{1+x}\cdot 2x$$

$$=\frac{x^2}{2\sqrt{1+x}}+2x\sqrt{1+x}=\frac{5x^2+4x}{2\sqrt{1+x}}.$$

Now $f'(x)=0\Rightarrow \dfrac{5x^2+4x}{2\sqrt{1+x}}=0\Rightarrow 5x^2+4x=0$

$$\Rightarrow \quad x(5x+4)=0\Rightarrow x=0,-\frac{4}{5}.$$

Also $0,-\frac{4}{5}$ both lie in $\left[-1,\frac{1}{2}\right]$, therefore, 0 and $-\frac{4}{5}$ both are stationary points.

Further, $f(0)=0, f\left(-\frac{4}{5}\right)=\frac{16}{25}\cdot\frac{1}{\sqrt{5}}=\frac{16}{25\sqrt{5}}$

$$f(-1)=0, f\left(\frac{1}{2}\right)=\frac{1}{4}\cdot\sqrt{\frac{3}{2}}=\frac{\sqrt{6}}{8}.$$

Therefore, the absolute maximum value is $\frac{\sqrt{6}}{8}$ and the absolute minimum value is 0.

The point of maxima is $\frac{1}{2}$ and points of minima are $\{-1, 0\}$.

OR

We have, equation of curve, $y = x^3 - 11x + 5$

Differentiating w.r.t. x, we get $\dfrac{dy}{dx}=3x^2-11$

Slope of the given tangent line is 1.
Thus, $3x^2 - 11 = 1$ that gives $x = \pm 2$
when $x = 2$, $y = 2 - 11 = -9$
when $x = -2$, $y = 2 - 11 = -13$
Out of the two points $(2, -9)$ and $(-2, -13)$
only the point $(2, -9)$ lies on the curve $y = x^3 - 11x + 5$.
Thus, the required point is $(2, -9)$.

16. We have

$$\int_a^b f(x)dx = \lim_{h\to 0} h[f(a) + f(a + h) + f(a + 2h) + \ldots + f(a + (n - 1)h), \text{ where } h=\frac{b-a}{n}$$

Here, $a = 1$, $b = 3$, $f(x) = 2x + 1$ and $h=\frac{3-1}{n}=\frac{2}{n}$

$$\therefore \quad I=\int_1^3 (2x+1)dx$$

$$\Rightarrow \quad I=\lim_{h\to 0} h[f(1) + f(1 + h) + f(1 + 2h) + \ldots + f(1 + (n - 1)h)]$$

$$= \lim_{h\to 0} h[(2 \times 1 + 1) + \{2(1 + h) + 1\} + \{2(1 + 2h) + 1\} + \ldots + \{2(1 + (n - 1)h)+ 1\}]$$

$$= \lim_{h\to 0} h[3 + (3 + 2h) + (3 + 2 \times 2h) + \ldots + (3 + 2(n - 1)h)]$$

$$= \lim_{h\to 0} h\{3n + 2h(1 + 2 + 3 + \ldots + (n - 1))\}$$

$$= \lim_{h\to 0} h\left\{3n+2h\frac{n(n-1)}{2}\right\}$$

$$= \lim_{n\to\infty} \frac{2}{n}\left\{3n+\frac{2}{n}\times n(n-1)\right\}$$

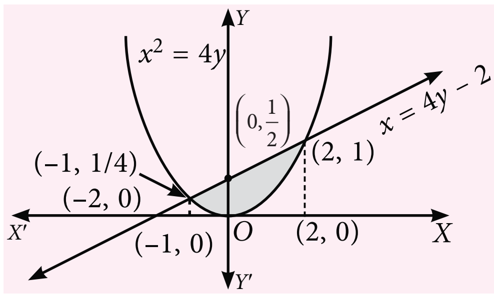
$$\left[\because\ h=\frac{2}{n} \text{ and } h\to 0\ \Rightarrow\ n\to\infty\right]$$

$$\Rightarrow \quad I=\lim_{n\to\infty}\left\{6+4\left(1-\frac{1}{n}\right)\right\} = 6 + 4\,(1 - 0) = 10$$

17. The equations of the given curves are

$x^2 = 4y$ ...(i) and, $x = 4y - 2$ ...(ii)

Equation (i) represents a parabola having vertex at the origin and axis along positive direction of y-axis. Equation (ii) represent a line which meets coordinate axes at $(-2, 0)$ and $(0, 1/2)$ respectively. Now, solving the two equations simultaneously we get $(2, 1)$ adn $(-1, 1/4)$ as their interection points.



So, required area $(A)=\int_{-1}^{2}|y_2-y_1|dx=\int_{-1}^{2}(y_2-y_1)dx$

$[\because\ y_2>y_1\ \therefore\ |y_2-y_1|=y_2-y_1]$

$$\Rightarrow \quad A=\int_{-1}^{2}\left(\frac{x+2}{4}-\frac{x^2}{4}\right)dx=\left[\frac{x^2}{8}+\frac{1}{2}x-\frac{x^3}{12}\right]_{-1}^{2}$$

$$=\left(\frac{4}{8}+\frac{2}{2}-\frac{8}{12}\right)-\left(\frac{1}{8}-\frac{1}{2}+\frac{1}{12}\right)=\frac{9}{8} \text{ sq. units}$$

18. We have, $\frac{dy}{dx} - y = \cos x$ ... (i)

This is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$, where $P = -1$, $Q = \cos x$

$\therefore$ I.F. $= e^{\int Pdx} = e^{-x}$

Hence the solution of (i) is

$y.e^{-x} = \int e^{-x}.\cos x\,dx + C$

Now, $\int e^{-x}\cos x\,dx = -e^{-x}\cos x + e^{-x}\sin x - \int e^{-x}\cos x\,dx$

$\Rightarrow \int e^{-x}\cos x dx = \frac{1}{2}e^{-x}(\sin x - \cos x)$

Thus the required solution of (i) is

$y.e^{-x} = \frac{1}{2}e^{-x}(\sin x - \cos x) + C$ ...(ii)

Putting, $x = 0$, $y = 1$ in (ii), we get

$\Rightarrow 1 = \frac{1}{2}(-1) + C \Rightarrow C = \frac{3}{2}$

From (ii), $y = \frac{1}{2}(\sin x - \cos x) + \frac{3}{2}e^{x}$

Which is the required solution.

OR

Given differential equation can be written as

$$\frac{dy}{dx} = \frac{1 + xy + \cos\left(\frac{y}{x}\right)}{x^2} = \frac{y}{x} + \left[\frac{1 + \cos\left(\frac{y}{x}\right)}{x^2}\right] \quad ..(i)$$

Let $F(x, y) = \frac{y}{x} + \left[\frac{1 + \cos\left(\frac{y}{x}\right)}{x^2}\right]$

Replacing x to λx and y to λy, we get

$$F(\lambda x, \lambda y) = \frac{\lambda y}{\lambda x} + \left[\frac{1 + \cos\left(\frac{\lambda y}{\lambda x}\right)}{(\lambda x)^2}\right]$$

$$= \frac{y}{x} + \left[\frac{1 + \cos\left(\frac{y}{x}\right)}{\lambda^2 x^2}\right] \neq F(x, y)$$

Hence, the given differential equation is not a homogeneous equation.

Putting $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ in(i), we get

$$v + x\frac{dv}{dx} = v + \frac{1 + \cos v}{x^2}$$

$$\Rightarrow \frac{dv}{1 + \cos v} = \frac{dx}{x^3} \Rightarrow \sec^2\left(\frac{v}{2}\right)dv = \frac{2}{x^3}dx$$

Integrating both sides, we get

$$2\tan\frac{v}{2} = -\frac{1}{x^2} + C \quad \text{or} \quad 2\tan\frac{y}{2x} = -\frac{1}{x^2} + C$$

19. We have, $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$

$\vec{a} \cdot \vec{b} = (\hat{i} + 4\hat{j} + 2\hat{k}) \cdot (3\hat{i} - 2\hat{j} + 7\hat{k}) = 3 - 8 + 14 = 9$

Let $\vec{d} = (d_1\hat{i} + d_2\hat{j} + d_3\hat{k})$

Now, $\vec{a} \perp \vec{d}$ if $\vec{a} \cdot \vec{d} = 0$

$\Rightarrow d_1 + 4d_2 + 2d_3 = 0$ ...(i)

Also, $\vec{b} \perp \vec{d}$ if $\vec{b} \cdot \vec{d} = 0$

$\Rightarrow 3d_1 - 2d_2 + 7d_3 = 0$ ...(ii)

and $\vec{c} \cdot \vec{d} = 15$

$\Rightarrow 2d_1 - d_2 + 4d_3 = 15$ ...(iii)

Solving (i), (ii) and (iii) for d_1, d_2, d_3, we get

$$d_1 = \frac{160}{3}, d_2 = \frac{-5}{3}, d_3 = -\frac{70}{3}$$

$$\therefore \quad \vec{d} = \frac{160}{3}\hat{i} - \frac{5}{3}\hat{j} - \frac{70}{3}\hat{k} = \frac{1}{3}[160\hat{i} - 5\hat{j} - 70\hat{k}]$$

20. Urn - I (10 W and 3 B), Urn - II (3 W and 5 B)

Let E_1, E_2 and E_3 and A be the events as defined below:

E_1 = Two black balls are drawn from the first bag.

E_2 = Two white balls are drawn from the first bag.

E_3 = One white and one black ball is drawn from first bag.

A = One ball drawn from the second bag is white.

$$P(E_1) = \frac{^3C_2}{^{13}C_2} = \frac{1}{26}, \; P(E_2) = \frac{^{10}C_2}{^{13}C_2} = \frac{15}{26},$$

$$P(E_3) = \frac{^{10}C_1 \times {^3C_1}}{^{13}C_2} = \frac{10}{26}$$

Also, $P(A/E_1) = \frac{^3C_1}{^{10}C_1} = \frac{3}{10}$; $P(A/E_2) = \frac{^5C_1}{^{10}C_1} = \frac{5}{10}$

$P(A/E_3) = \frac{^4C_1}{^{10}C_1} = \frac{4}{10} = \frac{2}{5}$

$P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3)\,P(A/E_3)$

$$= \frac{1}{26} \times \frac{3}{10} + \frac{15}{26} \times \frac{5}{10} + \frac{10}{26} \times \frac{2}{5} = \frac{59}{130}$$

21. (i) Since the sum of all the probabilities in a probability distribution is always unity.

$\therefore$ $P(X = 0) + P(X = 1) + ... + P(X = 7) = 1$

$\Rightarrow 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$\Rightarrow 10k^2 + 9k - 1 = 0 \Rightarrow (10k - 1)(k + 1) = 0$

$\Rightarrow 10k - 1 = 0$ $[\because k \geq 0 \;\therefore\; k + 1 \neq 0]$

$\Rightarrow k = \frac{1}{10}$

(ii) $P(X < 6) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)$

$\Rightarrow P(X < 6) = 0 + k + 2k + 2k + 3k + k^2$

$= k^2 + 8k = \left(\frac{1}{10}\right)^2 + \frac{8}{10}$ $\quad [\because k = 1/10]$

$\Rightarrow P(X < 6) = \frac{81}{100}$

(iii) $P(X \geq 6) = 1 - P(X < 6) = 1 - \frac{81}{100} = \frac{19}{100}$

(iv) $P(0 < X < 5) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$

$\Rightarrow P(0 < X < 5) = k + 2k + 2k + 3k = 8k$

$\Rightarrow P(0 < X < 5) = \frac{8}{10} = \frac{4}{5}$ $\quad [\because k = 1/10]$

22. Equation of a plane passing through the point (2, 2, 1) is $a(x - 2) + b(y - 2) + c(z - 1) = 0$ ...(i)

It passes through (9, 3, 6) then

$7a + b + 5c = 0$ ...(ii)

Since (i) is perpendicular to plane $2x + 6y + 6z = 1$, then

$2a + 6b + 6c = 0$ ...(iii)

Solving (ii) and (iii) by the method of cross-multiplication, we obtain

$$\frac{a}{6-30} = \frac{b}{10-42} = \frac{c}{42-2} \Rightarrow \frac{a}{-24} = \frac{b}{-32} = \frac{c}{40}$$

$\Rightarrow \frac{a}{-3} = \frac{b}{-4} = \frac{c}{5} = \lambda$ (say) $\Rightarrow a = -3\lambda, b = -4\lambda, c = 5\lambda$

Putting $a = -3\lambda$, $b = -4\lambda$ and $c = 5\lambda$ in (i), we get

$-3\lambda(x - 2) - 4\lambda(y - 2) + 5\lambda(z - 1) = 0$

$\Rightarrow -3x + 6 - 4y + 8 + 5z - 5 = 0$

$\Rightarrow -3x - 4y + 5z + 9 = 0$

23. We have, Minimize $Z = 5x + 10y$

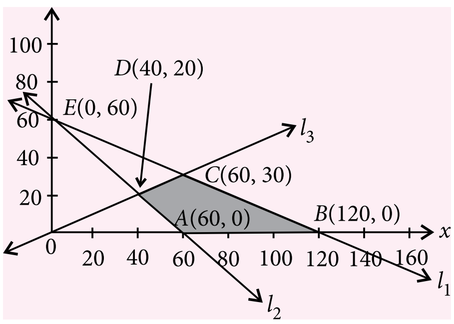
subject : $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$ and $x, y \geq 0$

To solve L.P.P graphically, we convert inequations into equations.

$l_1 : x + 2y = 120$, $l_2 : x + y = 60$, $l_3 : x - 2y = 0$ and $x = 0, y = 0$

l_1 and l_2 intersect at $E(0, 60)$, l_1 and l_3 intersect at $C(60, 30)$, l_2 and l_3 intersect at $D(40, 20)$.

The shaded region $ABCD$ is the feasible region and is bounded.



Corner points	Value of $Z = 5x + 10y$
$A(60, 0)$	300 ← (Minimum)
$B(120, 0)$	600
$C(60, 30)$	600
$D(40, 20)$	400

Hence, Z is minimum at $A(60, 0)$ *i.e.*, 300.

24. Let $I = \int \frac{1}{\sin x + \sec x} dx$. Then, $I = \int \frac{\cos x\, dx}{1 + \sin x \cos x}$

$$= \int \frac{2\cos x}{2 + 2\sin x \cos x} dx$$

$$= \int \frac{\cos x + \sin x}{2 + 2\sin x \cos x} dx + \int \frac{\cos x - \sin x}{2 + 2\sin x \cos x} dx$$

$$= \int \frac{\cos x + \sin x}{3 - (1 - 2\sin x \cos x)} dx + \int \frac{\cos x - \sin x}{1 + (1 + 2\sin x \cos x)} dx$$

$$= \int \frac{(\cos x + \sin x)}{3 - (\sin x - \cos x)^2} dx + \int \frac{\cos x - \sin x}{1 + (\sin x + \cos x)^2} dx$$

$$= \int \frac{1}{(\sqrt{3})^2 - u^2} du + \int \frac{1}{1 + v^2} dv,$$

where $u = \sin x - \cos x \Rightarrow du = (\cos x + \sin x)dx$

and $v = \sin x + \cos x \Rightarrow dv = (\cos x - \sin x)dx$

$$\Rightarrow I = \frac{1}{2\sqrt{3}} \log\left|\frac{\sqrt{3} + u}{\sqrt{3} - u}\right| + \tan^{-1} v + C$$

$$\Rightarrow I = \frac{1}{2\sqrt{3}} \log\left|\frac{\sqrt{3} + (\sin x - \cos x)}{\sqrt{3} - (\sin x - \cos x)}\right| + \tan^{-1}(\sin x + \cos x) + C$$

OR

Let $I = \int_0^{\pi/2} (2\log \sin x - \log \sin 2x) dx$

$$= \int_0^{\pi/2} \{2\log \sin x - \log(2\sin x \cos x)\} dx$$

$$= \int_0^{\pi/2} \{2\log \sin x - \log 2 - \log \sin x - \log \cos x\} dx$$

$$= \int_0^{\pi/2} \log \sin x\, dx - \int_0^{\pi/2} \log 2\, dx - \int_0^{\pi/2} \log \cos x\, dx$$

$$= \int_0^{\pi/2} \log \sin x\, dx - (\log 2) \int_0^{\pi/2} 1 \cdot dx - \int_0^{\pi/2} \log \cos\left(\frac{\pi}{2} - x\right) dx$$

$$= \int_0^{\pi/2} \log \sin x\, dx - (\log 2)[x]_0^{\pi/2} - \int_0^{\pi/2} \log \sin x\, dx$$

$$= -(\log 2)\left(\frac{\pi}{2} - 0\right) = -\frac{\pi}{2}\log 2$$

25. Let $y = (\sin^{-1} x)^3 + (\cos^{-1} x)^3$. Then,

$$y = (\sin^{-1} x + \cos^{-1} x)^3 - 3\sin^{-1} x \cos^{-1} x(\sin^{-1} x + \cos^{-1} x)$$

$$\Rightarrow y = \left(\frac{\pi}{2}\right)^3 - \frac{3\pi}{2}\sin^{-1} x\left(\frac{\pi}{2} - \sin^{-1} x\right) \quad \left[\because \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}\right]$$

$$\Rightarrow y = \frac{\pi^3}{8} - \frac{3\pi^2}{4}\sin^{-1} x + \frac{3\pi}{2}(\sin^{-1} x)^2$$

$$\Rightarrow \frac{3\pi}{2}(\sin^{-1} x)^2 - \frac{3\pi^2}{4}(\sin^{-1} x) + \left(\frac{\pi^3}{8} - y\right) = 0$$

$$\Rightarrow (\sin^{-1} x)^2 - \frac{\pi}{2}(\sin^{-1} x) + \frac{2}{3\pi}\left(\frac{\pi^3}{8} - y\right) = 0$$

$$\Rightarrow \left(\sin^{-1} x - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{48} - \frac{2y}{3\pi} = 0$$

$$\Rightarrow \left(\sin^{-1} x - \frac{\pi}{4}\right)^2 = \frac{2y}{3\pi} - \frac{\pi^2}{48} \quad \text{...(i)}$$

We know that

$$-\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2} \text{ for all } x \in [-1, 1]$$

$$\Rightarrow -\frac{3\pi}{4} < \sin^{-1} x - \frac{\pi}{4} \le \frac{\pi}{4} \Rightarrow 0 \le \left(\sin^{-1} x - \frac{\pi}{4}\right)^2 \le \frac{9\pi^2}{16} \quad \text{...(ii)}$$

From (i) and (ii), we find that

$$\Rightarrow 0 \le \frac{2y}{3\pi} - \frac{\pi^2}{48} \le \frac{9\pi^2}{16}$$

$$\Rightarrow \frac{\pi^2}{48} \le \frac{2y}{3\pi} \le \frac{9\pi^2}{16} + \frac{\pi^2}{48} \quad \Rightarrow \quad \frac{\pi^3}{32} \le y \le \frac{7\pi^3}{8}$$

Hence, the maximum and minimum values of

$(\sin^{-1} x)^3 + (\cos^{-1} x)^3$ are $\frac{7\pi^3}{8}$ and $\frac{\pi^3}{32}$.

OR

We observe the following properties of $*$ on $R - \{-1\}$.

Commutativety : For any $a, b \in R - \{-1\}$, we have

$a * b = a + b + ab$ and $b * a = b + a + ba$

$\because \; a + b + ab = b + a + ba$

$\Rightarrow a * b = b * a$

So, $*$ is commutative on $R - \{-1\}$.

Associativity : For any $a, b, c \in R - \{-1\}$, we have

$$(a * b) * c = (a + b + ab) * c$$
$$= (a + b + ab) + c + (a + b + ab)c$$
$$= a + b + c + ab + bc + ac + abc \quad \text{...(i)}$$

and, $a * (b * c) = a * (b + c + bc)$

$$= a + (b + c + bc) + a(b + c + bc)$$
$$= a + b + c + ab + bc + ac + abc \quad \text{...(ii)}$$

From (i) and (ii), we have

$(a * b) * c = a * (b * c)$ for all $a, b, c \in R - \{-1\}$.

So, $*$ is associative on $R - \{-1\}$

Existence of Identity : Let e be the identity element. Then, $a * e = a = e * a$ for all $a \in R - \{-1\}$

$\Rightarrow a + e + ae = a$ and $e + a + ea = a$ for all $a \in R - \{-1\}$

$\Rightarrow e(1 + a) = 0$ for all $a \in R - \{-1\}$

$\Rightarrow e = 0.$

Thus, 0 is the identity element for $*$ defined on $R - \{-1\}$.

Existence of Inverse : Let $a \in R - \{-1\}$ and let b be the inverse of a. Then,

$a * b = e = b * a$

$\Rightarrow a * b = e \;\Rightarrow\; a + b + ab = 0 \quad [\because e = 0]$

$$\Rightarrow b = \frac{-a}{a+1}$$

Now, $a \in R - \{-1\} \;\Rightarrow\; a \ne -1 \;\Rightarrow\; a + 1 \ne 0$

$$\Rightarrow b = \frac{-a}{a+1} \in R$$

Also, $\frac{-a}{a+1} = -1 \;\Rightarrow\; -a = -a - 1$

$\Rightarrow -1 = 0$, which is absurd.

Thus, $\frac{-a}{a+1} \in R - \{-1\}$.

Hence, every element of $R - \{-1\}$ is invertible and the inverse of an element a is $\frac{-a}{a+1}$.

26. Let ₹ x, ₹ y and ₹ z be the investments at the rate of interest of 6%, 7% and 8% per annum respectively.

Total investment = ₹ 5000

$\Rightarrow x + y + z = 5000 \quad \text{...(i)}$

Now, income from first investment of ₹ x = ₹ $\frac{6x}{100}$

Income from second investment of ₹ y = ₹ $\frac{7y}{100}$

Income from third investment of ₹ z = ₹ $\frac{8z}{100}$.

$\therefore$ Total annual income = ₹ $\left(\frac{6x}{100} + \frac{7y}{100} + \frac{8z}{100}\right)$

$$\Rightarrow \frac{6x}{100} + \frac{7y}{100} + \frac{8z}{100} = 358 \quad \text{...(ii)}$$

[$\because$ Total annual income = ₹ 358]

$\Rightarrow 6x + 7y + 8z = 35800.$

It is given that the combined income from the first two investments is ₹ 70 more than the income from the third.

$\therefore \frac{6x}{100}+\frac{7y}{100}=70+\frac{8z}{100} \Rightarrow 6x+7y-8z=7000$ ...(iii)

From (i), (ii) and (iii), we have

$$\begin{bmatrix}1&1&1\\6&7&8\\6&7&-8\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}5000\\35800\\7000\end{bmatrix}$$

or, $AX = B$, where

$$A=\begin{bmatrix}1&1&1\\6&7&8\\6&7&-8\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix} \text{ and } B=\begin{bmatrix}5000\\35800\\7000\end{bmatrix}$$

$$\text{Now, } |A|=\begin{vmatrix}1&1&1\\6&7&8\\6&7&-8\end{vmatrix}$$

$= 1(-56-56)-(-48-48)+42-42=-16\neq 0.$

So, A^{-1} exists and the solution of the given system of equations is given by $X = A^{-1}B$.

$$\text{adj } A=\begin{bmatrix}-112&96&0\\15&-14&-1\\1&-2&1\end{bmatrix}^T=\begin{bmatrix}-112&15&1\\96&-14&-2\\0&-1&1\end{bmatrix}$$

$$\text{So, } A^{-1}=\frac{1}{|A|}(\text{adj } A)=-\frac{1}{16}\begin{bmatrix}-112&15&1\\96&-14&-2\\0&-1&1\end{bmatrix}$$

Hence, the solution is given by

$$X=A^{-1}B=-\frac{1}{16}\begin{bmatrix}-112&15&1\\96&-14&-2\\0&-1&1\end{bmatrix}\begin{bmatrix}5000\\35800\\7000\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1000\\2200\\1800\end{bmatrix}$$

$\Rightarrow x = 1000, y = 2200$ and $z = 1800$

Hence, three investments are of ₹ 1000, ₹ 2200 and ₹ 1800 respectively.

OR

$$\text{Here, } |A|=\begin{vmatrix}1&-1&2\\3&0&-2\\1&0&3\end{vmatrix}=1(0+0)+1(9+2)+2(0-0)=11$$

$$\Rightarrow |A|I=\begin{bmatrix}11&0&0\\0&11&0\\0&0&11\end{bmatrix}$$

$$\text{adj}A=\begin{bmatrix}0&3&2\\-11&1&8\\0&-1&3\end{bmatrix}$$

$$A(\text{adj } A)=\begin{bmatrix}1&-1&2\\3&0&-2\\1&0&3\end{bmatrix}\begin{bmatrix}0&3&2\\-11&1&8\\0&-1&3\end{bmatrix}=\begin{bmatrix}11&0&0\\0&11&0\\0&0&11\end{bmatrix}$$

$$(\text{adj } A)A=\begin{bmatrix}0&3&2\\-11&1&8\\0&-1&3\end{bmatrix}\begin{bmatrix}1&-1&2\\3&0&-2\\1&0&3\end{bmatrix}=\begin{bmatrix}11&0&0\\0&11&0\\0&0&11\end{bmatrix}$$

Thus, it is verified that $A(\text{adj}A) = (\text{adj}A)A = |A|I$

27. We have, $(ax + b)e^{y/x} = x$

$\Rightarrow e^{y/x}=\frac{x}{ax+b} \quad \Rightarrow \quad \frac{y}{x}=\log\left(\frac{x}{ax+b}\right)$

[On taking log both sides]

$\Rightarrow \frac{y}{x}=\log x-\log(a+bx)$

On differentiating with respect to x, we get

$$\frac{x\frac{dy}{dx}-y}{x^2}=\frac{1}{x}-\frac{b}{a+bx}$$

$$\Rightarrow x\frac{dy}{dx}-y=x^2\left\{\frac{1}{x}-\frac{b}{a+bx}\right\}$$

$$\Rightarrow x\frac{dy}{dx}-y=\frac{ax}{a+bx} \quad \text{...(i)}$$

Differentiating both sides of (i) with respect to x, we get

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}-\frac{dy}{dx}=\frac{(a+bx)a-ax(0+b)}{(a+bx)^2}$$

$$\Rightarrow x\frac{d^2y}{dx^2}=\frac{a^2}{(a+bx)^2} \quad \Rightarrow \quad x^3\frac{d^2y}{dx^2}=\frac{a^2x^2}{(a+bx)^2}$$

$$\Rightarrow x^3\frac{d^2y}{dx^2}=\left(\frac{ax}{a+bx}\right)^2=\left(x\frac{dy}{dx}-y\right)^2 \quad \text{[Using (i)]}$$

28. The equations of two given lines are

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4} \quad \text{...(i)}$$

$$\text{and } \frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5} \quad \text{...(ii)}$$

Vector equation of (i) is

$$\vec{r}=\vec{a}_1+\lambda\vec{b}_1 \quad \text{...(iii)}$$

where, $\vec{a}_1=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b}_1=2\hat{i}+3\hat{j}+4\hat{k}$.

Vector equation of (ii) is

$$\vec{r}=\vec{a}_2+\mu\vec{b}_2 \quad \text{...(iv)}$$

where, $\vec{a}_2=2\hat{i}+4\hat{j}+5\hat{k}$ and $\vec{b}_2=3\hat{i}+4\hat{j}+5\hat{k}$.

Since, S.D. $= \left|\dfrac{(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$ ...(v)

Now, $\vec{a}_2-\vec{a}_1=(2\hat{i}+4\hat{j}+5\hat{k})-(\hat{i}+2\hat{j}+3\hat{k})=\hat{i}+2\hat{j}+2\hat{k}$

and, $\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 4\\ 3 & 4 & 5\end{vmatrix}=-\hat{i}+2\hat{j}-\hat{k}$

$\therefore\ |\vec{b}_1\times\vec{b}_2|=\sqrt{1+4+1}=\sqrt{6}$

and, $(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)=(\hat{i}+2\hat{j}+2\hat{k})\cdot(-\hat{i}+2\hat{j}-\hat{k})$

$= -1 + 4 - 2 = 1$

On substituting the values, we get

S.D. $=\dfrac{1}{\sqrt{6}}$

29. The equation of the planes through the intersection of the planes $\vec{r}\cdot(2\hat{i}+6\hat{j})+12=0$ and $\vec{r}\cdot(3\hat{i}-\hat{j}+4\hat{k})=0$ is

$[\vec{r}\cdot(2\hat{i}+6\hat{j})+12]+\lambda[\vec{r}\cdot(3\hat{i}-\hat{j}+4\hat{k})]=0$

$\Rightarrow\ \vec{r}\cdot\{(2+3\lambda)\hat{i}+(6-\lambda)\hat{j}+4\lambda\hat{k}\}+12=0$ ...(i)

$$\Rightarrow\ \frac{\vec{r}\cdot\{(-2-3\lambda)\hat{i}+(\lambda-6)\hat{j}+(-4\lambda)\hat{k}\}}{\sqrt{(2+3\lambda)^2+(\lambda-6)^2+(4\lambda)^2}}=\frac{12}{\sqrt{(2+3\lambda)^2+(\lambda-6)^2+(4\lambda)^2}}$$

It is given that the plane (i) is at a unit distance from the origin.

$$\therefore\ \frac{12}{\sqrt{(2+3\lambda)^2+(\lambda-6)^2+(4\lambda)^2}}=1$$

$\Rightarrow\ 144 = (2 + 3\lambda)^2 + (\lambda - 6)^2 + (4\lambda)^2$

$\Rightarrow\ 26\lambda^2 = 104\ \Rightarrow\ \lambda^2 = 4\ \Rightarrow\ \lambda = \pm 2$

Putting the values of λ in (i), we obtain

$\vec{r}\cdot(8\hat{i}+4\hat{j}+8\hat{k})+12=0$ and

$\vec{r}\cdot(-4\hat{i}+8\hat{j}-8\hat{k})+12=0$, as the equations of the required planes.

These equations can also be written as

$\vec{r}\cdot(2\hat{i}+\hat{j}+2\hat{k})+3=0$ and $\vec{r}\cdot(-\hat{i}+2\hat{j}-2\hat{k})+3=0$

or $2x + y + 2z + 3 = 0$ and $-x + 2y - 2z + 3 = 0$

MPP-10 MONTHLY Practice Problems

Class XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Probability

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. A determinant is chosen at random from the set of all determinants of order 2 with elements 1 or 0 only. The probability that the value of the determinant is positive is

(a) $\frac{1}{4}$ (b) $\frac{2}{9}$ (c) $\frac{3}{16}$ (d) $\frac{1}{10}$

2. A bag A contains 2 white and 3 red balls and bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. The probability that it was drawn from bag B is

(a) 25/52 (b) 13/27 (c) 8/17 (d) 25/51

3. If X and Y are independent binomial variates $X\left(5, \frac{1}{2}\right)$ and $Y\left(7, \frac{1}{2}\right)$, then $P(X + Y = 3)$ is

(a) $\frac{55}{1024}$ (b) $\frac{55}{4098}$

(c) $\frac{55}{2048}$ (d) None of these

4. If the integers λ and μ are chosen at random between 1 to 100, then the probability that a number of the form $7^{\lambda} + 7^{\mu}$ is divisible by 5, is

(a) $\frac{1}{4}$ (b) $\frac{1}{7}$ (c) $\frac{1}{8}$ (d) $\frac{1}{49}$

5. A natural number x is chosen at random from the first hundred natural numbers. The probability that $\frac{(x-20)(x-40)}{(x-30)} < 0$ is

(a) $\frac{1}{50}$ (b) $\frac{3}{50}$ (c) $\frac{7}{25}$ (d) $\frac{9}{50}$

6. Three natural numbers are taken at random from the set $A = \{x \mid 1 \le x \le 100, x \in N\}$. The probability that the A.M. of the numbers taken is 75, is

(a) $\frac{^{77}C_2}{^{100}C_3}$ (b) $\frac{^{25}C_2}{^{100}C_3}$ (c) $\frac{^{74}C_{72}}{^{100}C_{97}}$ (d) $\frac{^{75}C_2}{^{100}C_3}$

One or More Than One Option(s) Correct Type

7. A random variable X takes values 0, 1, 2, 3, ...with probability proportions to $(x+1)\left(\frac{1}{5}\right)^x$, then

(a) $P(X=0) = \frac{16}{25}$ (b) $P(X \le 1) = \frac{112}{125}$

(c) $P(X \ge 1) = \frac{9}{25}$ (d) $E(X) = \frac{25}{32}$

8. A fair coin is tossed 99 times. If r is the number of times tail occurs, then $P(X = r)$ is maximum when r is equal to

(a) 49 (b) 51

(c) 50 (d) None of these

9. Let X be a set containing n elements. If two subsets A and B of X are picked at random, the probability that A and B have the same number of elements is

(a) $\frac{^{2n}C_n}{2^{2n}}$ (b) $\frac{1}{^{2n}C_n}$

(c) $\frac{1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n-1)}{2^n \cdot n!}$ (d) $\frac{3^n}{4^n}$

10. Let X and Y be two events such that

$P(X/Y) = \frac{1}{2}$, $P(Y/X) = \frac{1}{3}$ and $P(X \cap Y) = \frac{1}{6}$.

Which of the following is(are) correct?

(a) $P(X \cup Y) = \frac{2}{3}$

(b) X and Y are independent

(c) X and Y are not independent

(d) $P(X^c \cap Y) = \frac{1}{3}$

11. A coin is tossed $(2n + 1)$ times, the probability that head appear odd number of times is

(a) $\frac{n}{2n+1}$ (b) $\frac{n+1}{2n+1}$ (c) $\frac{1}{2}$ (d) $\frac{n!}{2n+1}$

12. A card is selected at random from cards numbered as 00, 01, 02, ..., 99. An event is said to have occured. If product of digits of the card number is 16. If cards is selected 5 times with replacement each time, then the probability that the event occurs exactly three times is

(a) ${}^5C_3\left(\frac{3}{100}\right)^2\left(\frac{97}{100}\right)^3$ (b) ${}^5C_3\left(\frac{3}{100}\right)^3\left(\frac{97}{100}\right)^2$

(c) ${}^5C_3\left(\frac{0.3}{100}\right)^3\left(\frac{9.7}{100}\right)^3$ (d) $10(0.03)^3(0.97)^2$

13. A coin is tossed repeatedly. A and B call alternately for winning a prize of ₹ 30. One who calls correctly first wins the prize. A starts the call. Then, the expectation of

(a) A is ₹ 10 (b) B is ₹ 10

(c) A is ₹ 20 (d) B is ₹ 20

Comprehension Type

The probability of happening of an event in one trial being known, then the probability of its happening exactly x times in n trials is given by ${}^nC_x\, q^{n-x} \cdot p^x$ where

p = probability of happening the event

q = probability of not happening the event = $1 - p$

Now, ${}^nC_x\, q^{n-x}\, p^x$ is $(x + 1)^{\text{th}}$ term in the expansion of $(q + p)^n$ whose expansion gives the happening of the event 0, 1, 2, ..., n times, respectively.

14. In four throws with a pair of dice, the chance of throwing doublets atleast twice is

(a) $\frac{19}{144}$ (b) $\frac{125}{144}$ (c) $\frac{17}{144}$ (d) $\frac{18}{144}$

15. Unbiased coin is tossed 6 times. The probability of getting atmost 4 heads is

(a) $\frac{7}{64}$ (b) $\frac{57}{64}$ (c) $\frac{21}{32}$ (d) $\frac{11}{32}$

Matrix Match Type

16. Sixteen players $S_1, S_2, ..., S_{16}$ play in a tournament. They are divided into eight pairs at random. From each players a winner is decided on the basis of a game played between the two players of the pair. Assume that all the players are of equal strength.

	Column I		Column II
P.	The probability that player S_1 is among eight winners is	1.	1/3
Q.	If the probability that exactly one of S_1 and S_2 among eight winners is $\frac{3\lambda}{2}$, then λ is	2.	7/30
R.	The probability that both S_1 and S_2 are among the eight winners is	3.	1/2

	P	Q	R
(a)	3	1	2
(b)	3	2	1
(c)	1	2	3
(d)	2	1	3

Integer Answer Type

17. Let A and B be two events such that $P(A') = 0.3$, $P(B) = 0.4$ and $P(A \cap B') = 0.5$, then $P(B/A \cup B') = \frac{2}{n}$, where n is

18. If four squares are chosen at random on a chess board. If the probability that they lie on a diagonal line is $\frac{xyz}{{}^{64}C_4}$, then the unit place of λ is

19. The probability that the roots of the equation $x^2 + nx + \frac{n}{2} = 0$ are real, $n \in N$ and $n \le 5$, is $\frac{m}{n}$, then m =

20. A, B, C and D cut a pack of 52 cards successively in the order given. If the person who cuts a spade first receives ₹ 350, then the value of $\frac{E(x)}{64}$ is

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ......

No. of questions correct ......

Marks scored in percentage ......

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

DR. G. VISWANATHAN UNANIMOUSLY ELECTED INDIAN ECONOMIC ASSOCIATION CONFERENCE PRESIDENT

Founder Chancellor of Vellore Institute of Technology (VIT), Dr. G.Viswanathan has been unanimously elected as President of the Centenary Second Year Conference of the Indian Economic Association.

The First Year of the Centenary Celebrations was held at Acharya Nagarjuna University in Guntur, Andhra Pradesh between December 27 and 30, 2017. The selection was made during the General Body to be President of Centenary Second Year Conference of IEA.

Mr. Ram Nath Kovind, President of India, inaugurated the first year centenary celebrations at Acharya Nagarjuna University. E.S.L.Narasimhan, Governor of Andhra Pradesh, Chandrababu Naidu, Chief Minister, Muhammad Yunus, Nobel Laureate for Peace and Founder of Grameen Bank, former RBI Governor C.Rangarajan were among others present on the occasion in the inaugural function.

In his address, Mr. Kovind called upon emerging economies, including India, to speak up for an interconnected world with a fair and growing trade, in the wake of some countries turning protectionist. He also said imaginative policy-making was required to overcome the social and economic inequalities between different sections and regions.

During the Conference, Dr. G. Viswanathan presided over a special lecture by Prof Jomo Kwane Sundaram, a leading World Bank and United Nations economist from Malaysia. Mr. Sundaram spoke on the importance of trade for economic development. Dr G.Viswanathan stated that good politics should ensure cooperation among numerous political parties of India to have a common development agenda and clean politics free from corruption. He invited the IEA to guide the nation with good ideas for optimum use of public expenditure and allied fiscal policies.

The IEA was founded by Dr. Gilbert Slater, First Professor of Indian Economics, University of Madras along with Prof. Percy Anstey and C.J. Hamilton of Bombay and Calcutta Presidency Universities in 1917, along with Madras Economic Association. Former Presidents of IEA include former Prime Minister Dr. Manmohan Singh, Nobel Laureate Dr. Amartya Sen, Nobel Laureate, Dr. G.Patel, Director, London School of Economics and Political Science, Vice Chancellors Dr. V.K.R.V. Rao, Dr. Malcolm S Adiseshiah and Dr Yasodha Shanmugasundaram among others.

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OUTSTANDING PERFORMANCE BY AAKASHIANS IN KVPY, NSEs & NSO

Achal Kundu — KVPY | Abhinav Barnawal — NSEJS | Arjun R — NSO (Level-1)

Kishore Vaigyanik Protsahan Yojana (KVPY)

It is a scholarship program initiated by Department of Science & Technology, aimed at encouraging students to take up Research Careers in areas of Basic Sciences.

422 (379 Classroom | 43 Distance) Aakashians Qualified in **KVPY Aptitude Test 2017**

Our Top Performers from Classroom Programs

Achal Kundu (Stream-SX)	Aarushi Asrani (Stream-SA)	Chirag Singhal (Stream-SX)	Dev Chauhan (Stream-SX)	Dipanjan Das (Stream-SA)
Elamathi V. (Stream-SA)	Gaurav Vaidya (Stream-SX)	Govind Sharma (Stream-SX)	Gurbaaz Singh (Stream-SA)	Gayathri A. S. (Stream-SA)

and many more...

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This examination in Junior Science, Physics, Chemistry, Biology & Astronomy is the first step for the worldwide Olympiad. After NSEs, next step for the eligible students is the Indian National Olympiad (INO).

400 (Classroom Programs) Selections in **NSEs 2017**

Our Top Performers from Classroom Programs

Uttar Pradesh	Delhi	Uttar Pradesh	Gujarat	Uttar Pradesh
Abhinav Barnawal NSEJS	Samanyu Mahajan NSEP	Raman Kumar NSEC	Sahil Shah NSEB	Darpan Kumar Yadav NSEA

and many more...

Our No. of Selections in NSE (JS/P/C/B/A)

NSEJS[1] 2017	NSEP[2] 2017	NSEC[3] 2017	NSEB[4] 2017	NSEA[5] 2017
83	56	111	96	54

[1]NSEJS: National Standard Examination in Junior Science | [2]NSEP: National Standard Examination in Physics | [3]NSEC: National Standard Examination in Chemistry | [4]NSEB: National Standard Examination in Biology | [5]NSEA: National Standard Examination in Astronomy

National Science Olympiad (NSO): An Annual Scholarship Exam that aims at cultivating scientific reasoning and logical ability among school students.

667 (662 Classroom, 5 Distance) Aakashians Qualified in **NSO (Level-1) 2017**

Our Top Zonal Rankers in Various States from Classroom Programs

Tamil Nadu	Sooraj S.	Zonal Rank-1	Class X
Punjab	Pratham Uppal	Zonal Rank-1	Class IX
Punjab	Sushant Sondhi	Zonal Rank-1	Class XII
Kerala	Arjun R	Zonal Rank-1	Class X
Punjab	Anushka Garg	Zonal Rank-2	Class X
Punjab	Kavya Goyal	Zonal Rank-3	Class VIII
Punjab	Charbhi Gupta	Zonal Rank-3	Class XII
Karnataka	Shrey Jha	Zonal Rank-3	Class X
Delhi	Vaibhav Gupta	Zonal Rank-3	Class IX
Tamil Nadu	Bhagya Sri	Zonal Rank-3	Class IX

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