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2 AKSHAT CHUGH

Cracking the JEE ADVANCED

NEET Class XI-XII | JEE ESSENTIALS

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JEE Advanced 2017 SOLVED PAPER

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BRAIN Map

MONTHLY PRACTICE PROBLEMS (XI & XII)

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Result : IIT-JEE (Advanced) 2017
ALLEN PROVES AGAIN
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AIR 8
AIR 9
AIR 10
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RACHIT BANSAL
From Classroom Course
LAKSHAY SHARMA
From Classroom Course
25
STUDENTS SECURE AIR IN TOP 100 CLUB
HIGHEST FROM ANY INSTITUTE OF KOTA
10 in TOP 25 All India Ranks
54 in TOP 200 All India Ranks
130 in TOP 500 All India Ranks
239 in TOP 1000 All India Ranks
AIR-12 Yateesh Agrawal Classroom
AIR-15 Aman Kansal Classroom
AIR-16 Yash Khemchandani Classroom
AIR-19 Devansh Garg Classroom
AIR-21 Arpit Aggarwal Classroom
AIR-23 Abhay Goyal Classroom
AIR-24 Tushar Gautam Classroom
AIR-37 Piyush Tibarewal Classroom
AIR-42 Satvik Mashkaria Classroom
AIR-45 Irin Ghosh Distance
AIR-50 Shourya Aggarwal Distance
AIR-52 Chitrank Gupta Classroom
AIR-56 Kritin Garg Classroom
AIR-59 Himanshu Sheoran Classroom
AIR-66# Mayank Dubey Classroom
AIR-66# Saksham Dhull Classroom
AIR-82 Vedant Raval Classroom
AIR-86 Vardhan Jain Classroom
AIR-94 Aaron John Sabu Distance
AIR-96 Prateek Garg Classroom
AIR-98 Hrithik Maheshwari Classroom
AIR-100 Pranay Reddy Classroom
6289
Total Selections
Classroom 4383
Distance 1906
#Similar Ranks Declared by JEE
Admission Announcement KOTA CENTER (Session 2017-18)
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JEE (Advanced) | Nurture (X to XI Moving) | 19 June, 03 July, 19 July
JEE (Advanced) | Leader (XII Pass/Appeared) | 12 June, 26 June, 10 July
JEE (Main) | Nurture (X to XI Moving) | 03 July
JEE (Main) | Leader (XII Pass/Appeared) | 14 June, 28 June, 10 July
PRE-MEDICAL (NEET-UG, AIIMS) | Nurture (X to XI Moving) | 11 June, 26 June, 10 July
PRE-MEDICAL (NEET-UG, AIIMS) | Leader (XII Pass/Appeared) | 25 June, 09 July, 30 July
PRE-MEDICAL (NEET-UG, AIIMS) | Achiever (XII Pass/Repeaters) | 12 June, 02 July, 23 July, 07 Aug
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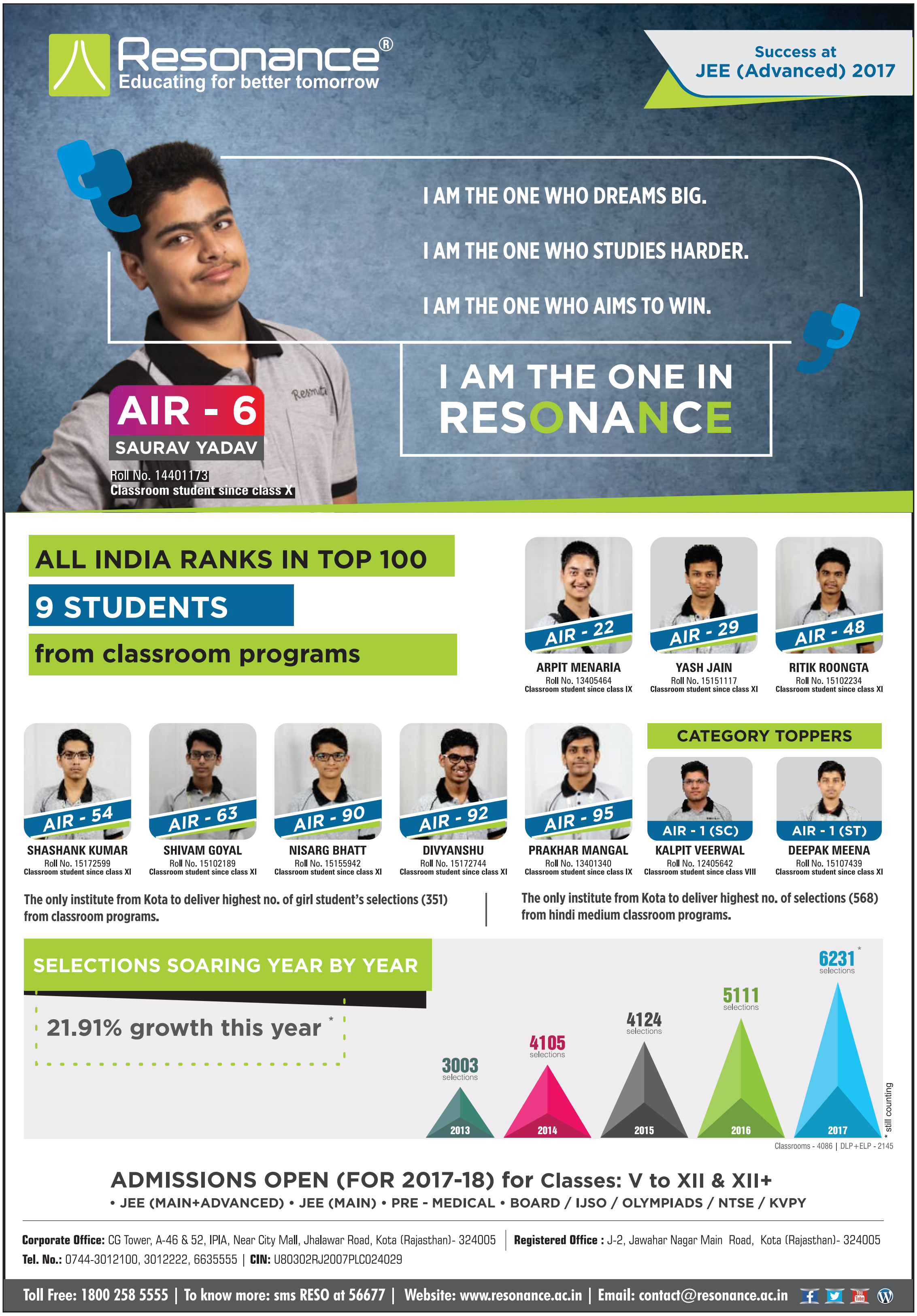
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ALL INDIA RANKS IN TOP 100
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Roll No. 13405464
Classroom student since class IX
AIR - 29
YASH JAIN
Roll No. 15151117
Classroom student since class XI
AIR - 48
RITIK ROONGTA
Roll No. 15102234
Classroom student since class XI
AIR - 54
SHASHANK KUMAR
Roll No. 15172599
Classroom student since class XI
AIR - 63
SHIVAM GOYAL
Roll No. 15102189
Classroom student since class XI
AIR - 90
NISARG BHATT
Roll No. 15155942
Classroom student since class XI
AIR - 92
DIVYANSHU
Roll No. 15172744
Classroom student since class XI
AIR - 95
PRAKHAR MANGAL
Roll No. 13401340
Classroom student since class IX
CATEGORY TOPPERS
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KALPIT VEERWAL
Roll No. 12405642
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21.91% growth this year *
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2013
4105 selections
2014
4124 selections
2015
5111 selections
2016
6231 * selections
2017
* still counting
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PHYSICS for you

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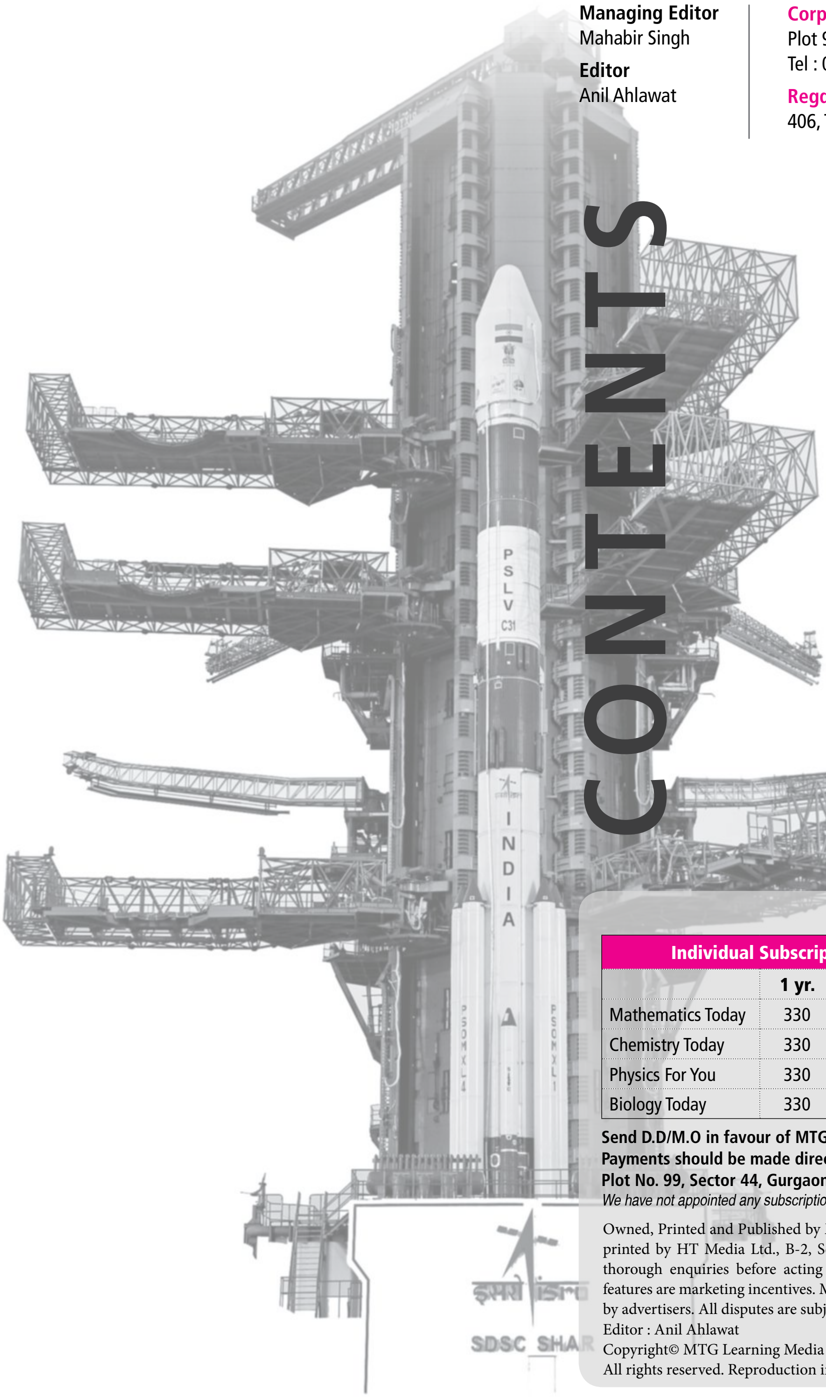

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Editor : Anil Ahlawat

Akshat Chug

AIR 2

Cracking the JEE Advanced EXAM

- **MTG : Why did you appear for Engineering Entrance?**

  Akshat : I was always interested in studying Science and Mathematics. I wanted to study in the best institutes and it's a dream come true to be able to do so.

- **MTG : What exams have you appeared for and what are your ranks in these exams?**

  Akshat : JEE Advanced - AIR 2
  JEE Main - AIR 7
  BITSAT - cleared

- **MTG : How many hours in a day did you study to prepare for the examination?**

  Akshat : I studied for 6-7 hours per day.

- **MTG : On which topics and chapters you laid more stress in each subject?**

  Akshat : I have always given equal importance to all the chapters.

- **MTG : How much time does one require for serious preparation for this exam?**

  Akshat : I started studying in class 9 and 10 itself but it was only when I was in class 11 then I decided to seriously pursue engineering as a career therefore it was important that I crack the JEE exam. As far as preparation is concerned, after school hours, I used to attend the classes at my coaching institute for three hours. Once, I returned from the coaching institute, I used to do self-study and every night I would study for at least one hour.

> "Clarity of mind, a clear strategy and then follow it strictly. This has been the secret of my success."

- **MTG : How was the preparation for JEE Advanced different from JEE Main?**

  Akshat : As far as JEE Main preparation is concerned it is more about memorizing the formulae and speed accompanied with accuracy. While one is preparing for JEE Advanced, there has to be clarity of concepts. If you have studied properly, then you need to relax before JEE Advanced.

- **MTG : Any extra coaching?**

  Akshat : I joined Bakliwal Tutorials and it was of great help to me.

- **MTG : Which Books/Magazines you read?**

  Akshat : I studied from all the standard books for JEE Advanced. There are no special text books otherwise. The


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2nd ENGG
AVISHEK DAS
WBJEE 2013 | IITJEE 115
• ENGINEERING Rankers
Rohit Bose IIT-54(sc)
Avishek Das IIT-115, WBJEE-2
Gyanesh Gupta IIT-143, WBJEE-1
Ankit Biswas IIT-199 (sc), WBJEE-14 (sc)
Aritra Bera IIT-719, WBJEE-19
Saugata Halder IIT-1326, WBJEE-35
Ratul Das IIT-1728, WBJEE-14
Soumyajit Ghosh IIT-1813, (OBC) WBJEE-32(OBC)
Arnab Bose IIT-2014, WBJEE-554
Diptangshu Sen IIT-2313, WBJEE-195
Ayush Dhanuka IIT-2804
Saurav Chowdhury IIT-2927
Debsourya De IIT-3188, WBJEE-13
Udit Surya Saha IIT-3406, WBJEE-55
Rishiraj Bose IIT-3523 WBJEE-35
Arghya Chakraborty IIT-3606, WBJEE-81
Susnata Bhowmick IIT-4206, WBJEE-375
Samarjit Karmakar IIT-4580, WBJEE-75
Subhrajit Chowdhury IIT-4905, WBJEE-196
Amartya Saha IIT-6064, WBJEE-103
Pramit Das IIT-7379, WBJEE-59
Sayantan Adhikary IIT-8892, WBJEE-182
Dibyendu Paul IIT-8989, WBJEE-1289
Chirayata Bhattacharyya IIT-9501, WBJEE-146
Suraj Mondal WBJEE-14 (OBC)
Baidik Chandra WBJEE-55
Soumik Paul WBJEE-107
Shamik Majumder WBJEE-113
Pratyasha Chakraborty WBJEE-124
Debapriyo Roy WBJEE-173
• MEDICAL Rankers
Subhashis Saha WBJEE - 1
Rima Mondal WBJEE-6 (sc)
Subhajit Paul WBJEE-14 (OBC)
Somashree Manna WBJEE-20
Anirban Ghosh WBJEE-33
Rupam Paul WBJEE-37
Debankita Das WBJEE-39 (sc)
Trishit Saha WBJEE-44
Avishek Das NEET-49
Pritam Adak WBJEE-51
Pean Basu WBJEE-55
Moumita Mallick NEET-58 (ST)
Tanmoy Paul WBJEE-65 (OBC)
Aman Shaw WBJEE-68 (OBC)
Hemabha Saha WBJEE-72
Priti Goswami WBJEE-75
Upamanyu Roy WBJEE-75
Uttiya Nath WBJEE-96 (OBC)
Vedavati Bandyopadhyay NEET-104
Sanket Mondal NEET-116
Sanjukta Dey NEET-128
Soumya Kanti Ghatak NEET-131
Rounak Roy WBJEE-137
Ritika Bhakat WBJEE-145 (sc)
Sagnik De WBJEE-169
Saikat Chandra NEET-181
Saptarshi Das WBJEE-190
Puja Jha WBJEE-197
Sourav Nandy WBJEE-199
Soumyajit Goswami WBJEE-200
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syllabus is the same; just a slight difference in the approach is needed I love to read mystery novels and I am a big fan of Sherlock Holmes.

- **MTG : In your words what are the components of an ideal preparation plan?**

  Akshat : Regular study is a must. Time Management is required, you should utilise your time properly. After studying for long hours, the mind gets exhausted so some recreational activities are also required.

- **MTG : What role did the following play in your success:**

  (a) Parents

  (b) Teachers

  (c) School

  Akshat : Parents - They have been very supportive and they have never pressurised me for anything.

  Teachers - At my coaching institute, Bakliwal Tutorials teacher's were very supportive. Whenever, I approached them with any kind of problem, they helped me a lot.

  School - I did my class 12th from DPS, Pune and all my teachers in school were very supportive and were always ready to help me out.

- **MTG : Your family background?**

  Akshat : I belong to Jaipur. My family shifted to Pune when my father got transferred here 6 years back. My father Mr. Vikas Chugh is Vice President, Western Reigon in Tata Sky and my mother is a home maker. My sister is in class 8th.

- **MTG : What mistake you think you shouldn't have made?**

  Akshat : I think I should have studied more in the beginning.

- **MTG : Was this your first attempt?**

  Akshat : Yes, it was my first attempt.

- **MTG : What do you think is the secret of your success?**

  Akshat : Clarity of mind, a clear strategy and then follow it strictly. This has been the secret of my success.

- **MTG : How did you de-stress yourself during the preparation? What are your hobbies? How often could you pursue them?**

  Akshat : Yes, it is important to take break but you should not loose your focus in doing so. My hobbies were reading and watching TV shows such as 'Arrow' and 'Flash' but was not able to do so during my JEE Advanced preparation.

- **MTG : What do you feel is lacking in our education/ examination system? Is the examination system fair to the student?**

  Akshat : Too much of mugging up is there in our educational/examination system.

- **MTG : Had you not been selected then what would have been your future plan?**

  Akshat : I would love to join BITS.

- **MTG : What advice would you like to give our readers who are JEE aspirants?**

  Akshat : Plan out your strategy. Work more on your weak points. Practice and remain focussed. Try to strike a balance between your hobbies and studies.

All the Best!☺☺

NEET | JEE ESSENTIALS

Class XI

Maximize your chance of success, and high rank in NEET, JEE (Main and Advanced) by reading this column. This specially designed column is updated year after year by a panel of highly qualified teaching experts well-tuned to the requirements of these Entrance Tests.

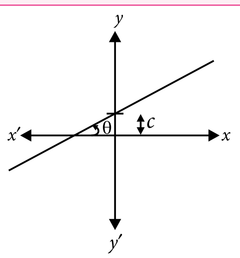
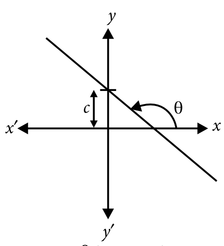
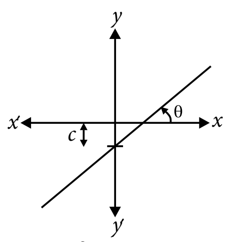
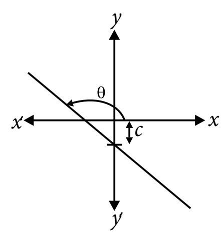
Unit 1 Mathematical Tools and Measurements

ESSENTIALS IN 2-D COORDINATE GEOMETRY

Straight Line

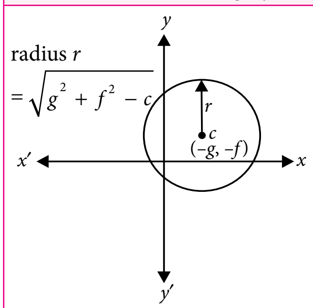
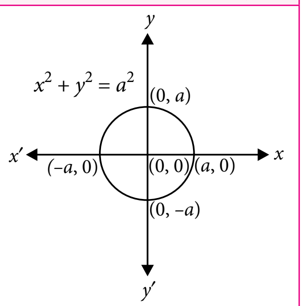
- Slope-intercept form : $y = mx + c$

Cases of Slope-Intercept Form

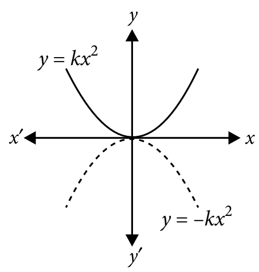
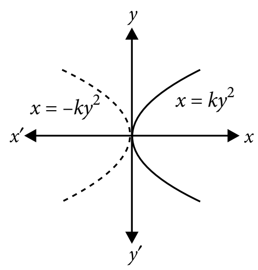
			
$m = \tan\theta$ is positive $c = y$-intercept is positive	$m = \tan\theta$ is negative $c = y$-intercept is positive	$m = \tan\theta$ is positive $c = y$-intercept is negative	$m = \tan\theta$ is negative $c = y$-intercept is negative

Conic Section

- **Circle** General Equation: $x^2 + y^2 + 2gx + 2fy + c = 0$

Circle with centre $(-g, -f)$	Circle with centre at origin
	

- **Parabola**

$y = \pm kx^2$ Passing through origin	$x = \pm ky^2$ Passing through origin
	

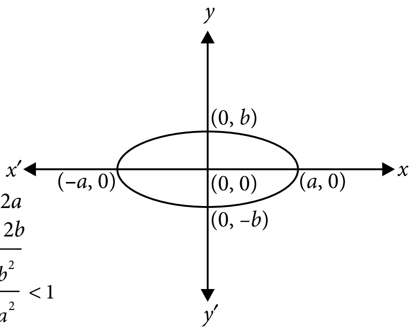
- ## Ellipse

Horizontal Ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a^2>b^2)$$

Centre: (0, 0)
Foci: ($\pm$ *ae*, 0)
Vertices: ($\pm$ *a*, 0)
Length of major axis = 2*a*
Length of minor axis = 2*b*

Eccentricity, $e=\sqrt{1-\frac{b^2}{a^2}}<1$

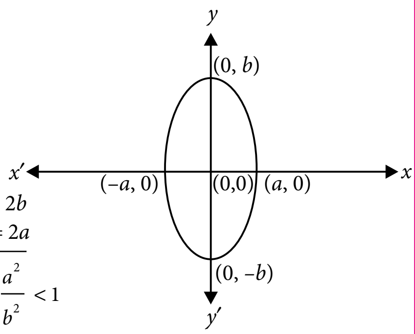


Vertical Ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a^2<b^2)$$

Centre: (0, 0)
Foci: (0, $\pm$ *be*)
Vertices: (0, $\pm$ *b*)
Length of major axis = 2*b*
Length of minor axis = 2*a*

Eccentricity, $e=\sqrt{1-\frac{a^2}{b^2}}<1$



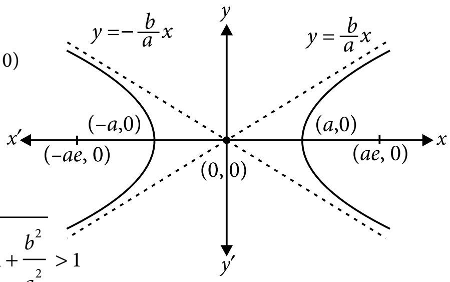
- ## Hyperbola

Transverse Hyperbola

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1\ (a,\ b>0)$$

Centre: (0, 0)
Foci: ($\pm$ *ae*, 0)
Vertices: ($\pm$ *a*, 0)

Eccentricity, $e=\sqrt{1+\frac{b^2}{a^2}}>1$

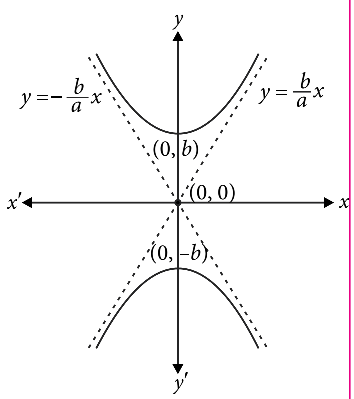


Conjugate Hyperbola

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1\ (a,\ b>0)$$

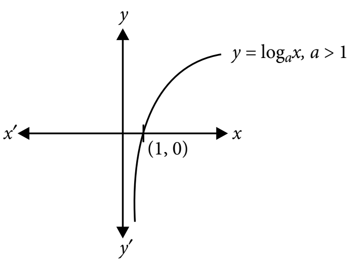
Centre: (0, 0)
Foci: (0, $\pm$ *be*)
Vertices: (0, $\pm$ *b*)
Eccentricity,

$$e=\sqrt{1+\frac{a^2}{b^2}}>1$$

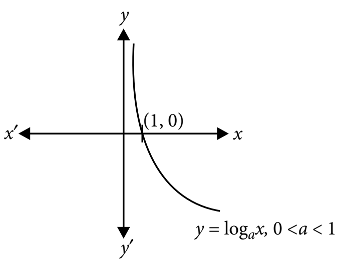


- ## Logarithmic and Exponential Functions

Logarithmic Graphs

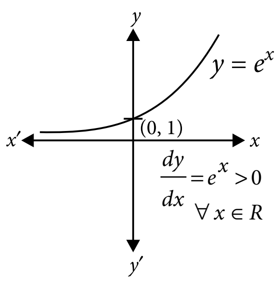


$x=a^y\ (a>0)\ a\neq 1\ (a>1)$



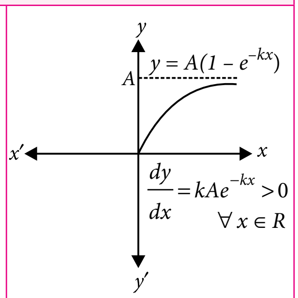
$x=a^y\ (a>0)\ a\neq 1\ (0<a<1)$

Exponential Graphs



$$\frac{dy}{dx}=e^x>0\quad \forall\, x\in R$$

Exponentially increasing

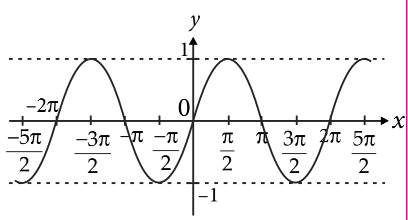


$$\frac{dy}{dx}=kAe^{-kx}>0\quad \forall\, x\in R$$

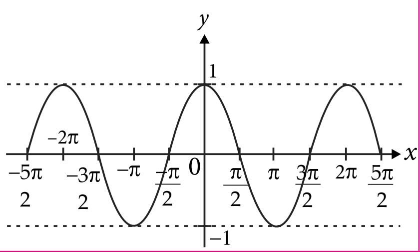
Exponentially increasing

- ## Trigonometric Functions

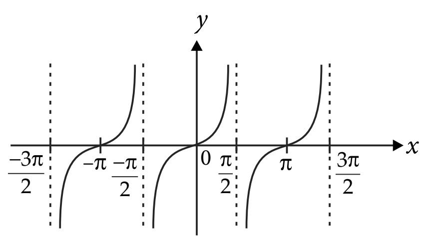
f(*x*) = sin *x*



f(*x*) = cos *x*



f(*x*) = tan *x*



ESSENTIALS IN TRIGONOMETRY

Basic Trigonometric Results

- $\sin^2\theta + \cos^2\theta = 1$
- $1 + \tan^2\theta = \sec^2\theta$
- $1 + \cot^2\theta = \text{cosec}^2\theta$
- $\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \cos^2\theta - \sin^2\theta$
- $\sin 2\theta = 2\sin\theta\cos\theta$
- $\cos(A + B) = \cos A\cos B - \sin A\sin B$
- $\cos(A - B) = \cos A\cos B + \sin A\sin B$
- $\sin(A + B) = \sin A\cos B + \cos A\sin B$
- $\sin(A - B) = \sin A\cos B - \cos A\sin B$
- $\sin C + \sin D = 2\sin\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$
- $\sin C - \sin D = 2\cos\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$
- $\cos C + \cos D = 2\cos\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$
- $\cos C - \cos D = -2\sin\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$
- $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$
- $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$
- $\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

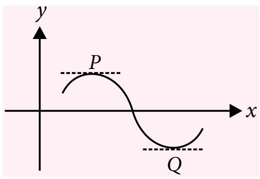
ESSENTIALS IN CALCULUS

Differentiation

- $\dfrac{d}{dx}x^n = nx^{n-1}$
- $\dfrac{d}{dx}\log_e x = \dfrac{1}{x}$
- $\dfrac{d}{dx}\sin x = \cos x$
- $\dfrac{d}{dx}\cos x = -\sin x$
- $\dfrac{d}{dx}\tan x = \sec^2 x$
- $\dfrac{d}{dx}\cot x = -\text{cosec}^2 x$
- $\dfrac{d}{dx}\sec x = \sec x\tan x$
- $\dfrac{d}{dx}\text{cosec}\, x = -\text{cosec}x\cot x$
- $\dfrac{d}{dx}e^x = e^x$
- $\dfrac{d}{dx}a^x = a^x\log_e a$
- $\dfrac{d}{dx}e^{ax} = ae^{ax}$
- $\dfrac{d}{dx}a^{bx} = ba^{bx}\log_e a$
- $\dfrac{d}{dx}\dfrac{f_1(x)}{f_2(x)} = \dfrac{f_2(x)\dfrac{d}{dx}f_1(x) - f_1(x)\dfrac{d}{dx}f_2(x)}{(f_2(x))^2}$
- $\dfrac{d}{dx}(f_1(x)f_2(x)) = f_1(x)\dfrac{d}{dx}f_2(x) + f_2(x)\dfrac{d}{dx}f_1(x)$
- $\dfrac{d}{dx}(ax+b) = a$
- $\dfrac{d}{dx}(ax+b)^n = an(ax+b)^{n-1}$

Maxima and Minima

Let $y = f(x)$ be a function. Then first draw the graph of $f(x)$ as shown.



From the graph we see that at maxima or minima, slope dy/dx of the graph at P and Q respectively is zero.

$\therefore\ \dfrac{dy}{dx} = 0$ at maximum or minimum values of y.

Put $\dfrac{dy}{dx} = 0$ and solve for x. (We may get different values of x.)

At all those values of x for which $\dfrac{d^2y}{dx^2}$ is negative, we have maximum value of y.

Similarly, at all those values of x for which $\dfrac{d^2y}{dx^2}$ is positive, we have minimum value of y.

Illustration 1. Find maximum or minimum values of the functions $y = 25x^2 + 5 - 10x$

Sol.: For maximum and minimum value, we can put

$$\frac{dy}{dx} = 0$$

or $\dfrac{dy}{dx} = 50x - 10 - 0 = 0 \quad \therefore\ x = \dfrac{1}{5}$

Further $\dfrac{d^2y}{dx^2} = 50 > 0$ at $x = \dfrac{1}{5}$. Therefore, y has minimum value at $x = \dfrac{1}{5}$. Substituting $x = \dfrac{1}{5}$ in given equation, we get

$$y_{\text{min}} = 25\left(\frac{1}{5}\right)^2 + 5 - 10\left(\frac{1}{5}\right) = 4$$

Integration

- $\int x^n dx = \dfrac{x^{n+1}}{n+1} + c \qquad (n \neq -1)$
- $\int \dfrac{dx}{x} = \log_e |x| + c$
- $\int \sin x\, dx = -\cos x + c$
- $\int \cos x\, dx = \sin x + c$
- $\int \sec x\tan x\, dx = \sec x + c$

- $\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + c$
- $\int \sec x\, dx = \log_e |\sec x + \tan x| + c$
- $\int \operatorname{cosec} x\, dx = \log_e |\operatorname{cosec} x - \cot x| + c$
- $\int (ax + b)\, dx = \dfrac{ax^2}{2} + bx + c$

- **Area bounded by a region in the graph can be done as**

  Area $= \int_{x_1}^{x_2} y\,dx$

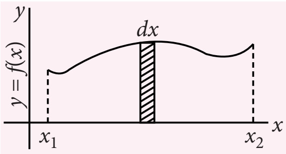


Illustration 2. Integrate the following functions

(a) $\int (5x^2 + 3x - 2)dx$ (b) $\int \left(4\sin x - \dfrac{2}{x}\right)dx$

Sol.:

(a) $\int (5x^2 + 3x - 2)dx = 5\int x^2 dx + 3\int x\, dx - 2\int dx$

$$= \frac{5x^3}{3} + \frac{3x^2}{2} - 2x + c$$

(b) $\int \left(4\sin x - \dfrac{2}{x}\right)dx = 4\int \sin x\, dx - 2\int \dfrac{dx}{x}$

$= -4 \cos x - 2 \log_e x + c$

ESSENTIALS IN VECTOR ALGEBRA

- **Triangle Law of Vector Addition**

  If two vectors are represented in both magnitude and direction by the two sides of a triangle in same order, the resultant $\vec{R}$ is represented by the third side of a triangle both in magnitude and direction but in opposite order.

- **Parallelogram Law of Vector Addition**

  For two coinitial vectors acting on a particle represented in magnitude and direction by the two adjacent sides of a parallelogram, the diagonal of the parallelogram so formed will be the resultant.

- **Resolution of a Vector**

  Consider a vector $\vec{a}$ in x-y plane making an angle θ with x-axis

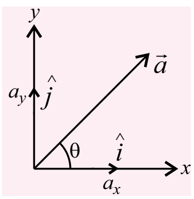
  x component of $\vec{a} = a_x = a \cos\theta$

  y component of $\vec{a} = a_y = a \sin\theta$

  and $\vec{a} = a_x\hat{i} + a_y\hat{j}$

  Magnitude of $\vec{a}$ is

  $|\vec{a}| = \sqrt{a_x^2 + a_y^2} = a$



Here $\tan\theta = \dfrac{a_y}{a_x} \Rightarrow \theta = \tan^{-1}\left(\dfrac{a_y}{a_x}\right)$

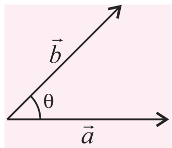
- **Dot or Scalar Product**

  Consider two vectors $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ and $\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$ and angle between $\vec{a}$ and $\vec{b}$ is θ, Their scalar product is given by $\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta$,

  where $|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$, $|\vec{b}| = \sqrt{b_x^2 + b_y^2 + b_z^2}$

  Angle θ can be determined as

  $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$



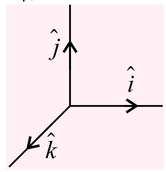
For three unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$ along three axes,

$\hat{i}\cdot\hat{i} = 1\times 1\cos 0° = 1$ $(\because |\hat{i}| = 1 = |\hat{j}| = |\hat{k}|)$

$\therefore\ \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = \hat{i}\cdot\hat{i} = 1$

and $\hat{i}\cdot\hat{j} = 1\times 1\times\cos 90° = 0$

$\therefore\ \hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$



In component form,

$\vec{a}\cdot\vec{b} = (a_x\hat{i} + a_y\hat{j} + a_z\hat{k})\cdot(b_x\hat{i} + b_y\hat{j} + b_z\hat{k})$

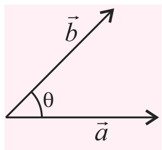
$\Rightarrow\ \vec{a}\cdot\vec{b} = (a_x b_x + a_y b_y + a_z b_z)$

- **Cross or Vector Product**

  Consider two vectors $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ and $\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$ and angle between $\vec{a}$ and $\vec{b}$ is θ. So vector product of two vectors is

  $(\vec{a}\times\vec{b}) = |a||b|\sin\theta\ \hat{n}$

  - $(\vec{a}\times\vec{b})$ is a vector quantity and $\hat{n}$ is the perpendicular to the plane carrying vectors $\vec{a}$ and $\vec{b}$



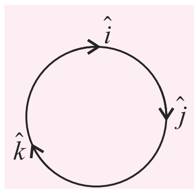
  - For three unit vectors $\hat{i}, \hat{j}, \hat{k}$ along three axes.

    $\hat{i}\times\hat{j} = 1\times 1\times\sin 90°\,\hat{k} = \hat{k}$;

    Where $|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$

    $\hat{j}\times\hat{k} = 1\times 1\times\sin 90°\,\hat{i} = \hat{i}$

    $\hat{k}\times\hat{i} = 1\times 1\times\sin 90°\,\hat{j} = \hat{j}$



and $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$

In short, for cyclic order, $\hat{i}\times\hat{j} = \hat{k}$, $\hat{j}\times\hat{k} = \hat{i}$, $\hat{k}\times\hat{i} = \hat{j}$

For anti cyclic order, $\hat{i}\times\hat{k} = -\hat{j}$, $\hat{j}\times\hat{i} = -\hat{k}$, $\hat{k}\times\hat{j} = -\hat{i}$,

  - Calculation of $(\vec{a}\times\vec{b})$ in component form

    $\vec{a}\times\vec{b} = (a_x\hat{i} + a_y\hat{j} + a_z\hat{k})\times(b_x\hat{i} + b_y\hat{j} + b_z\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = (a_y b_z - a_z b_y)\hat{i} - (a_x b_z - a_z b_x)\hat{j}$$
$$(a_x b_y - a_y b_x)\hat{k}$$

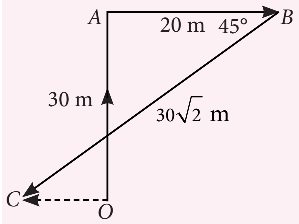
Illustration 3. A person moves 30 m north and then 20 m towards east and finally $30\sqrt{2}$ m in south-west direction. The displacement of the person from the origin will be

(a) 10 m along north (b) 10 m along south
(c) 10 m along west (d) Zero

Sol.: (c) From figure, $\overrightarrow{OA} = 0\hat{i} + 30\hat{j}, \overrightarrow{AB} = 20\hat{i} + 0\hat{j}$

$\overrightarrow{BC} = -30\sqrt{2}\cos 45°\hat{i} - 30\sqrt{2}\sin 45°\hat{j}$

$= -30\hat{i} - 30\hat{j}$



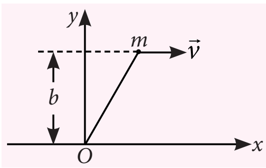
$\therefore$ Net displacement,

$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{BC}$

$= -10\hat{i} + 0\hat{j}$

$|\overrightarrow{OC}| = 10$ m along west

Illustration 4. If a particle of mass m is moving with constant velocity v parallel to x-axis in x-y plane as shown in figure. Its angular momentum $\vec{L} = \vec{r} \times \vec{p}$ with respect to origin at any time t will be



(a) $mvb\,\hat{k}$ (b) $-mvb\,\hat{k}$
(c) $mvb\,\hat{i}$ (d) $mv\,\hat{i}$

Sol.: (b) We know that angular momentum $\vec{L} = \vec{r} \times \vec{p}$ in terms of component becomes

$$\vec{L} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$

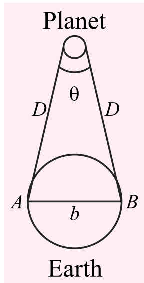
As motion is in x-y plane ($z = 0$ and $p_z = 0$), so $\vec{L} = \hat{k}(xp_y - yp_x)$

$\therefore \quad \vec{L} = \hat{k}[vt \times 0 - b\,mv] = -mvb\,\hat{k}$

MEASUREMENT OF PHYSICAL QUANTITIES

- **Fundamental Quantities :** Quantities which are independent of all other quantities and do not require any other physical quantity for their definition are called fundamental or base quantities and the units in which base quantities are measured are called fundamental units.
- **Derived Quantities :** The quantities that can be expressed as combinations of the base quantities are called derived quantities and the units for measuring them is also the combinations of fundamental units are called derived units.
- **Different Systems of Units**
  - CGS system
  - FPS system
  - MKS system
- **Measurement of Length**
  - **Large scale measurements**
    1 astronomical units (AU) = 1.496×10^{11} m
    1 light year (ly) = 9.46×10^{15} m
    1 parsec (pc) = 3.08×10^{16} m
  - **Small scale measurements**
    1 fermi (fm) = 10^{-15} m ($\approx$ size of nucleus)
    1 X-ray unit (xμ) = 10^{-13} m ($\approx$ size of an atom)
    1 angstrom (Å) = 10^{-10} m
    1 micron (μ) = 10^{-6} m
  - **Parallax Method :** To measure the distance of far away planet,

    $\therefore \theta = \dfrac{b}{D}$ or $D = \dfrac{b}{\theta}$



- **Measurement of Mass**
  - **Large scale units**
    1 Quintal = 100 kg
    1 Metric tonne = 1000 kg
    1 Chandrasekhar unit = 1.4 times solar mass $= 2.8 \times 10^{30}$ kg
  - **Small scale units**
    1 a.m.u. = 1.67×10^{-27} kg = $\dfrac{1}{12}$ of mass of carbon 12 (in kg)
    Nuclear mass = 10^{-27} kg
- **Measurement of Time**
  1 second = time interval for 9192631770 vibrations of the radiation corresponding to the transition between the two hyper fine levels of Cs^{133} (g).
  1 year = 3.156×10^7; 1 solar year = 365.25 day

ESSENTIALS IN ERRORS IN MEASUREMENT

- **Errors :** The difference in the true value and measured value of a quantity is called error of the measurement.
- **Absolute, Relative and Percentage Error**
  - **Mean absolute error,**

    $$\Delta a_{\text{mean}} = \frac{|\Delta a_1| + |\Delta a_2| + \ldots + |\Delta a_n|}{n}$$
  - **Relative error :** The ratio of mean absolute error to the mean value of observations is called relative error.

    $$\text{Relative error} = \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}}$$

- **Percentage error :** Percent representation of relative error is called percentage error.

$$\delta a = \frac{\Delta a_{mean}}{a_{mean}} \times 100\%$$

- **Combination of Errors**
  - **Error of a sum or a difference**
    If $Z = A \pm B$
    Maximum possible error can be
    $\Delta Z = \Delta A + \Delta B$
  - **Error of a product or a quotient**
    If $Z = AB$ or A/B
    Maximum relative error, $\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$
  - **Error of a measured quantity raised to a power**
    If $Z = A^n$
    then $\frac{\Delta Z}{Z} = n\frac{\Delta A}{A}$

Illustration 5. The length and breadth of a rectangle sheet are 16.2 cm and 10.1 cm, respectively. The area of the sheet in appropriate significant figures and error is

(a) 164 ± 3 cm^2 (b) 163.62 ± 2.6 cm^2
(c) 163.6 ± 2.6 cm^2 (d) 163.62 ± 3 cm^2

Sol.: (a) If Δx is error in a physical quantity, then relative error is calculated as $\frac{\Delta x}{x}$

Given, length $l = (16.2 \pm 0.10)$ cm
Breadth $b = (10.1 \pm 0.1)$ cm
Area $A = l \times b = (16.2\text{ cm}) \times (10.1\text{ cm}) = 163.62\text{ cm}^2$
Rounding off to three significant digits, area $A = 164\text{ cm}^2$

$$\frac{\Delta A}{A} = \frac{\Delta l}{l} + \frac{\Delta b}{b} = \frac{0.1}{16.2} + \frac{0.1}{10.1} = \frac{1.01+1.62}{16.2 \times 10.1} = \frac{2.63}{163.62}$$

$$\Rightarrow \quad \Delta A = A \times \frac{2.63}{163.62} = 163.62 \times \frac{2.63}{163.62} = 2.63\text{ cm}^2$$

$\Delta A = 3$ cm^2 (By rounding off to one significant figure)
Area, $A = A \pm \Delta A = (164 \pm 3)$ cm^2

Illustration 6. A physical parameter a can be determined by measuring the parameters b, c, d and e using the relation $a = \frac{b^\alpha c^\beta}{d^\gamma e^\delta}$. If the maximum errors in the measurement of b, c, d and e are b_1 %, c_1 %, d_1 % and e_1 %, then the maximum error in the value of a determined by the experiment is

(a) $(b_1 + c_1 + d_1 + e_1)\%$ (b) $(b_1 + c_1 - d_1 - e_1)\%$
(c) $(\alpha b_1 + \beta c_1 - \gamma d_1 - \delta e_1)\%$
(d) $(\alpha b_1 + \beta c_1 + \gamma d_1 + \delta e_1)\%$

Sol.: (d) $a = \frac{b^\alpha c^\beta}{d^\gamma e^\delta}$

So maximum error in a is given by

$$\left(\frac{\Delta a}{a} \times 100\right)_{max} = \alpha \cdot \frac{\Delta b}{b} \times 100 + \beta \cdot \frac{\Delta c}{c} \times 100 + \gamma \cdot \frac{\Delta d}{d} \times 100 + \delta \cdot \frac{\Delta e}{e} \times 100$$
$$= (\alpha b_1 + \beta c_1 + \gamma d_1 + \delta e_1)\%$$

SIGNIFICANT FIGURES

Rules for counting significant figures

- All non-zero digits are significant.
- A zero between two non zero digits is significant.
- Leading zeros to the left of non zero number are not significant.
- Trailing zeros to right of the number without decimal point are not significant.
- The powers of 10 are not taken as significant figure.
- All zeros to the right of the decimal point are significant.

ESSENTIALS IN DIMENSIONAL ANALYSIS

- The derived quantities can be expressed in terms of fundamental quantities as a product of different powers of the letters M,L,T etc. where M = Mass; L = Length; T = Time.
- **Uses of Dimensional Analysis**
  - **To check the correctness of a given relation**
    According to principle of homogeneity, dimensions of each term on both sides of an equation must be same.
  - **To convert a physical quantity from one to another system of units**
    $Q_1 n_1 = Q_2 n_2$; where Q_1 = unit in 1st system, Q_2 = unit in 2nd system n_1 and n_2 be constant values in 1st and 2nd system.
    $\therefore n_2 = \frac{Q_1 n_1}{Q_2}$, $n_2 = n_1\left[\frac{Q_1}{Q_2}\right]$
  - **To derive a relation among the physical quantities :**
    If we know the dependency of a physical quantity on the other quantities then using dimension analysis relation between them can be derived.
- **Limitations of Dimensional analysis**
  - Dimensional method cannot be used to derive relations other than multiplication and division. Also, it can not be used to derive trigonometric relations.

- We cannot determine the value of constants in a relation.
- The physical quantities depending on more than three quantities cannot be derived by dimensional method.

Illustration 7. Density of a liquid in CGS system is 0.625 g cm^{-3}. What is its magnitude in SI system?
(a) 0.625 (b) 0.0625
(c) 0.000625 (d) 625

Sol.: (b) Since $n_1Q_1 = n_2Q_2$

$\Rightarrow n_1[M_1L_1^{-3}] = n_2[M_2L_2^{-3}]$

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]\times\left[\frac{L_1}{L_2}\right]^{-3} = 0.625\left[\frac{1\text{ g}}{1\text{ kg}}\right]\times\left[\frac{1\text{ cm}}{1\text{ m}}\right]^{-3}$$

$= 0.625 \times 10^{-3} \times 10^{6} = 625$

Illustration 8. If the speed v of a particle of mass m as function of time t is given by $v = \omega A \sin\left[\left(\sqrt{\frac{k}{m}}\right)t\right]$, where A had dimension of length.
Which of the following statements is correct?
(a) The argument of trigonometric function must be a dimensionless quantity.
(b) Dimensional formula of ω is $[LT^{-1}]$.
(c) Dimensional formula of k is $[MLT^{-2}]$.
(d) Dimensional formula of $\sqrt{\frac{k}{m}}$ is [T].

Sol.: (a) Plane angle is dimensionless.

$[\omega A] = [v] = LT^{-1} \Rightarrow [\omega] = T^{-1}$

$$\left[\sqrt{\frac{k}{m}}\right] = \frac{1}{[t]} = [T^{-1}]$$

VERNIER CALLIPERS AND SCREW GUAGE

- **Vernier constant :** It is the difference between values of one main scale division and one vernier scale division of vernier callipers. Let n vernier scale divisions (VSD) coincide with $(n - 1)$ main scale divisions (MSD)

  $\therefore$ n VSD = $(n - 1)$ MSD

  $$1\text{ VSD} = \left(\frac{n-1}{n}\right)\text{MSD}$$

  Vernier constant, VC = 1 MSD – 1 VSD

  $$= 1\text{ MSD} - \left(\frac{n-1}{n}\right)\text{MSD} = \frac{1}{n}\text{MSD}$$

  $$\text{VC} = \frac{\text{Value of one main scale division}}{\text{Total number of divisions on vernier scale}}$$

- **Reading of Vernier callipers :** Place the body between the jaws and the zero of vernier scale lies ahead of N^{th} division of main scale. Then

  Main scale reading (MSR) = N

  If n^{th} division of vernier scale coincides with any division of main scale, then

  Vernier scale reading (VSR) = $n \times$ (VC)

  Total reading = MSR + VSR = $N + n \times$ (VC)

- Pitch of the screw $= \dfrac{\text{Distance moved on linear scale}}{\text{Number of rotations}}$

- **Least count of the screw gauge**

  $$= \frac{\text{Pitch of the screw}}{\text{Total number of divisions on the circular scale}}$$

- **Reading of a Screw Gauge :** Place a wire between A and B, the edge of the cap lies ahead of N^{th} division of linear scale. Then

  Linear scale reading (LSR) = N

  If n^{th} division of circular scale lies over reference line, then

  Circular scale reading (CSR) = $n \times$ (LC)

  Total reading = LSR + CSR = $N + n \times$ (LC)

Illustration 9. The pitch of a screw gauge is 1 mm and there are 100 divisions on circular scale. When faces A and B are just touching each other without putting anything between the studs 32nd division of the circular scale coincides with the reference line. When a glass plate is placed between the studs, the linear scale reads 4 divisions and the circular scale reads 16 divisions. Find the thickness of the glass plate. Zero, of linear scale is not hidden from circular scale when A and B touches each other.

Sol.: Least count L.C.

$$= \frac{\text{Pitch}}{\text{Number of divisions on circular scale}} = \frac{1}{100}\text{ mm} = 0.01\text{ mm}$$

As zero is not hidden from circular scale when A and B touches each other. Hence, the screw gauge has positive error.

$e = + n$ (L.C.) = 32 × 0.01 = 0.32 mm

Linear scale reading = 4 × (1 mm) = 4 mm

Circular scale reading = 16 × (0.01 mm) = 0.16 mm

$\therefore$ Measured reading = (4 + 0.16) mm = 4.16 mm

$\therefore$ Absolute reading = Measured reading – e

= (4.16 – 0.32) mm = 3.84 mm

Therefore, thickness of the glass plate is 3.84 mm.

SPEED PRACTICE

1. The fundamental unit which has same power in the dimensional formula of surface tension and coefficient of viscosity is
   (a) mass (b) length
   (c) time (d) none of these

2. Which of the following systems of units is not based on units of mass, length and time alone?
   (a) SI (b) MKS (c) FPS (d) CGS

3. The radius of a circle is 1.22 m. Area enclosed by it upto correct significant figures is
   (a) 4.6778 m^2 (b) 4.677 m^2
   (c) 4.67782 m^2 (d) 4.68 m^2

4. If voltage $V = (200 \pm 8)$ V and current $I = (20 \pm 0.5)$ A, the percentage error in resistance R is
   (a) 5.2% (b) 25% (c) 3% (d) 6.5%

5. The pitch of a screw gauge is 1 mm and there are 100 divisions on the circular scale. In measuring the diameter of a sphere there are six divisions on the linear scale and forty divisions on circular scale coincides with the reference line. The diameter of the sphere is
   (a) 6.0 mm (b) 5.0 mm (c) 5.4 mm (d) 6.4 mm

6. Three concurrent forces of the same magnitude are in equilibrium. The angle between the forces and name of the triangle formed by the forces as sides will be
   (a) 60° and equilateral triangle
   (b) 90°, 45°, 45° and right angled triangle
   (c) 120°, 30°, 30° and an isosceles triangle
   (d) 120° and an obtuse angled triangle.

7. The resultant $\vec{P}$ and $\vec{Q}$ is perpendicular to $\vec{P}$. what is the angle between $\vec{P}$ and $\vec{Q}$?
   (a) $\cos^{-1}\left(\frac{P}{Q}\right)$ (b) $\cos^{-1}\left(-\frac{P}{Q}\right)$
   (c) $\sin^{-1}\left(\frac{P}{Q}\right)$ (d) $\sin^{-1}\left(-\frac{P}{Q}\right)$

8. N-divisions on the main scale of a vernier callipers coincide with $N+1$ divisions on the vernier scale. If each division on the main scale is of a units, the least count of the instrument is
   (a) $\frac{N}{N+1}$ (b) $\frac{N}{a+1}$
   (c) $\frac{a}{N+1}$ (d) $\frac{a}{N}$

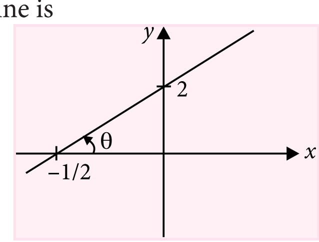
9. The charge flown through a circuit in the time internal between t and $t + dt$ is given by $dq = e^{-t/\tau}\,dt$, where τ is a constant. Find the total charge flown through the circuit between $t = 0$ to $t = \tau$.
   (a) $\tau\left(1-\frac{1}{e}\right)$ (b) $\tau(e-1)$
   (c) $\tau\left(\frac{e+1}{e}\right)$ (d) τ

10. The position vector of a particle is determined by the expression $\vec{r} = 3t^2\hat{i} + 4t^2\hat{j} + 7\hat{k}$. The distance travelled in first 10 s is
   (a) 500 m (b) 300 m
   (c) 150 m (d) 100 m

11. The height reached in time t by a particle thrown upward with a speed u is given by $h = ut - \frac{1}{2}gt^2$.
   The time taken in reaching the maximum height will be
   (a) $\frac{u}{4g}$ (b) $\frac{u}{g}$ (c) $\frac{h}{u}$ (d) $\left(\frac{1}{2}\right)\frac{h}{u}$

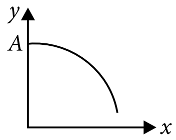
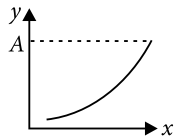
12. If $|\vec{V}_1 + \vec{V}_2| = |\vec{V}_1 - \vec{V}_2|$ and $\vec{V}_2$ is finite, then
   (a) $\vec{V}_1$ is parallel to $\vec{V}_2$
   (b) $\vec{V}_1 = \vec{V}_2$
   (c) $\vec{V}_1$ and $\vec{V}_2$ are mutually perpendicular
   (d) $|\vec{V}_1| = |\vec{V}_2|$

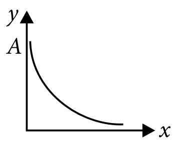
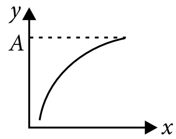
13. Choose the correct statement for the given graph.
   (i) slope of the line is 4
   (ii) Equation of the line is
   $2x + \frac{y}{2} = -1$
   (iii) $\left(\frac{dy}{dx}\right) = 4$
   (iv) none of these



(a) (i) and (ii) (b) (ii) only
(c) (iv) (d) (i) and (iii)

14. For the equation $y = Ae^{-kx}$ graphical representation will be

(a) 
(b) 

(c) 
(d) 

15. In an experiment to determine the acceleration due to gravity g, the formula used for the time period of a periodic motion is $T = 2\pi\sqrt{\frac{7(R-r)}{5g}}$. The value of R and r are measured to be (60 ± 1) mm and (10 ± 1) mm, respectively. In five successive measurements, the time period is found to be 0.52 s, 0.56 s, 0.57 s, 0.54 s and 0.59 s. The least count of the watch used for the measurement of time period is 0.01 s. Which of the following statement is true?
(a) The error in the measurement of r is 9%
(b) The error in the measurement of T is 4.57%
(c) The error in the measurement of T is 2%
(d) The error in the determined value of g is 11%

16. A physical quantity of the dimensions of length that can be formed out of c, G and $\frac{e^2}{4\pi\varepsilon_0}$ is [c is velocity of light, G is universal constant of gravitation and e is charge]

(a) $c^2\left[G\frac{e^2}{4\pi\varepsilon_0}\right]^{1/2}$ (b) $\frac{1}{c^2}\left[\frac{e^2}{G\,4\pi\varepsilon_0}\right]^{1/2}$

(c) $\frac{1}{c}G\frac{e^2}{4\pi\varepsilon_0}$ (d) $\frac{1}{c^2}\left[G\frac{e^2}{4\pi\varepsilon_0}\right]^{1/2}$

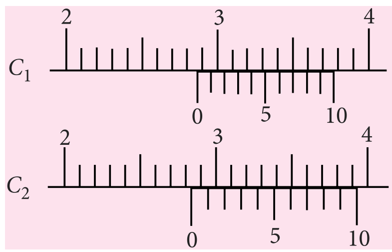
[NEET 2017]

17. Time (T), velocity (C) and angular momentum (h) are chosen as fundamental quantities instead of mass, length and time. In terms of these, the dimensions of mass would be
(a) $[M] = [TC^{-2}h]$
(b) $[M] = [T^{-1}C^{-2}h^{-1}]$
(c) $[M] = [T^{-1}C^{-2}h]$
(d) $[M] = [T^{-1}C^{2}h]$ **[JEE Main Online 2017]**

18. A physical quantity P is described by the relation $P = a^{1/2}\,b^2\,c^3\,d^{-4}$. If the relative errors in the measurement of a, b, c and d respectively, are 2%, 1%, 3% and 5%, then the relative error in P will be
(a) 25% (b) 12% (c) 8% (d) 32%
[JEE Main Online 2017]

19. The following observations were taken for determining surface tension T of water by capillary method. Diameter of capillary, $D = 1.25 \times 10^{-2}$ m and rise of water, $h = 1.45 \times 10^{-2}$ m. Using $g = 9.80$ m s^{-2} and the simplified relation, $T = \frac{rhg}{2} \times 10^3$ N m^{-1}, the possible error in surface tension is closest to
(a) 0.15% (b) 1.5% (c) 2.4% (d) 10%
[JEE Main Online 2017]

20. There are two Vernier calipers both of which have 1 cm divided into 10 equal divisions on the main scale. The Vernier scale of one of the calipers (C_1) has 10 equal divisions that correspond to 9 main scale divisions. The Vernier scale of the other caliper (C_2) has 10 equal divisions that correspond to 11 main scale divisions. The readings of the two calipers are shown in the figure. The measured values (in cm) by calipers C_1 and C_2 respectively are



(a) 2.87 and 2.86 (b) 2.85 and 2.82
(c) 2.87 and 2.87 (d) 2.87 and 2.83
[JEE Advanced 2016]

SOLUTIONS

1. **(a) :** Since [surface tension] = $[ML^0T^{-1}]$
[coefficient of viscosity] = $[ML^{-1}T^{-1}]$
Mass has the same power in both dimensional formula.

2. **(a) :** International system (SI) is not based on units of mass, length and time alone.

3. **(d):** Area $= \pi r^2 = \frac{22}{7} \times (1.22)^2 = 4.67782 \text{ m}^2$.
As per rule, the area will have three significant figures. Rounding off, we get
$A = 4.68 \text{ m}^2$

4. **(d):** $V = (200 \pm 8)$ V
$I = (20 \pm 0.5)$ A

$$R = \frac{V}{I} = \frac{200}{20} = 10\ \Omega$$

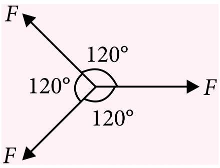
$$\frac{\Delta R}{R} = \pm\left(\frac{\Delta V}{V} + \frac{\Delta I}{I}\right) = \pm\left(\frac{8}{200} + \frac{0.5}{20}\right) = \pm\frac{13}{200}$$

$$\frac{\Delta R}{R} \times 100 = \pm\frac{13}{200} \times 100 = \pm\ 6.5\%.$$

5. **(d):** Least count, L.C. $= \frac{1}{100} = 0.01$ mm

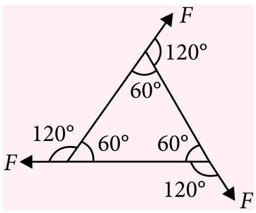
Linear scale reading = 6 (pitch) = 6 mm
Circular scale reading = n(L.C.) = $40 \times 0.01 = 0.4$ mm
$\therefore$ Total reading = (6 + 0.4) = 6.4 mm

6. **(a):** In N forces of equal magnitude works on a single point and their resultant is zero, angle between any two forces is given

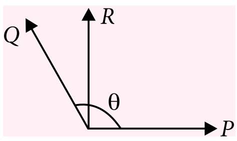


$$\theta = \frac{360°}{N} = \frac{360°}{3} = 120°$$

The angle between the forces and the triangle formed by the forces is shown in the figure.



7. **(b):**



$$\Rightarrow \tan 90° = \frac{Q\sin\theta}{P + Q\cos\theta}$$

$\Rightarrow$ $P + Q\cos\theta = 0$

$$\cos\theta = \frac{-P}{Q} \quad \therefore \quad \theta = \cos^{-1}\left(\frac{-P}{Q}\right)$$

8. **(c):** $(N + 1)$ divisions on the vermier scale
$= N$ divisions on main scale

$\therefore$ 1 division on vernier scale $= \frac{N}{N+1}$ divisions on main scale.
Given, each division on the main scale is of a units.

$\therefore$ 1 division on vernier scale $= \left(\frac{N}{N+1}\right) a$ units
$= d$ (say)

Least count = 1 main scale division
− 1 vernier scale division

$$= a - d = a - \left(\frac{N}{N+1}\right)a = \frac{a}{N+1}$$

9. **(a):** The total charge flown is the sum of all the dq for t varying from $t = 0$ to $t = \tau$. Thus, the total charge flown is

$$Q = \int_0^{\tau} e^{-t/\tau} dt = \left[\frac{e^{-t/\tau}}{-1/\tau}\right]_0^{\tau} = \tau\left(1 - \frac{1}{e}\right).$$

10. **(a):** $\vec{r} = 3t^2\hat{i} + 4t^2\hat{j} + 7\hat{k}$
at $t = 0$, $\vec{r}_1 = 7\hat{k}$

at $t = 10s$, $\vec{r}_2 = 300\hat{i} + 400\hat{j} + 7\hat{k}$,

$\overrightarrow{\Delta r} = \vec{r}_2 - \vec{r}_1 = 300\hat{i} + 400\hat{j}$

$|\overrightarrow{\Delta r}| = |\vec{r}_2 - \vec{r}_1| = \sqrt{(300)^2 + (400)^2} = 500$ m

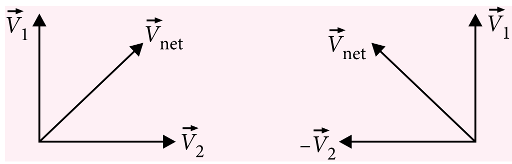
11. **(b):** Given, $h = ut - \frac{1}{2}gt^2$

or $\frac{dh}{dt} = u - gt$

For, maximum height, $\frac{dh}{dt} = 0$

or $u - gt = 0$ or $t = \frac{u}{g}$

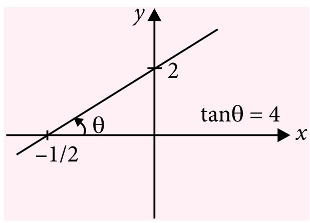
12. **(c):**



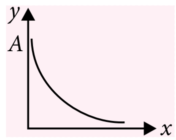
According to problem $|\vec{V}_1 + \vec{V}_2| = |\vec{V}_1 - \vec{V}_2|$
$\Rightarrow$ $|\vec{V}_{net}| = |\vec{V}_{net}|$
So, $\vec{V}_1$ and $\vec{V}_2$ will be mutually perpendicular.

13. **(d):** Here slope is +4 and y-intercept is 2. For graph as shown in the figure, equation will be $y = 4x + 2$

Slope of line $= \left(\frac{dy}{dx}\right) \quad \therefore \quad \frac{dy}{dx} = 4$,



14. (c) : $y = Ae^{-kx}$ represents exponentially decreasing graph. Value of y decreases exponentially from A to 0. The graph is shown in the figure.



From the graph and the equation, the value of $y = A$ at $x = 0$ and $y \to 0$ as $x \to \infty$.

15. (d): As $T = \sum_{i=1}^{n} \frac{T_i}{n}$

$$\therefore\ T = \frac{0.52\text{ s} + 0.56\text{ s} + 0.57\text{ s} + 0.54\text{ s} + 0.59\text{ s}}{5}$$

$$= \frac{2.78\text{ s}}{5} = 0.556\text{ s} = 0.56\text{ s}$$

($\because$ least count of watch is 0.01 s)

Percentage error in $T = \frac{\Delta T}{T} \times 100\%$

where $\Delta T = \frac{\sum_{i=1}^{n} |T_i - T|}{n}$

$$= \frac{|0.52 - 0.56| + |0.56 - 0.56| + |0.57 - 0.56| + |0.54 - 0.56| + |0.59 - 0.56|}{5}$$

$$= \frac{0.04 + 0.00 + 0.01 + 0.02 + 0.03}{5} = \frac{0.10}{5} = 0.02\text{ s}$$

$\therefore$ Percentage error in $T = \frac{0.02}{0.56} \times 100\% = 3.57\%$

Given, $r = (10 \pm 1)$ mm, $R = (60 \pm 1)$ mm

$\therefore$ % error in measurement of r

$$= \frac{1}{10} \times 100\% = 10\%$$

As $T = 2\pi\sqrt{\frac{7(R - r)}{5g}}$ (given)

$$\therefore\ g = \frac{7(R - r)}{5} \times \frac{4\pi^2}{T^2}$$

$$\Rightarrow \frac{\Delta g}{g} = \frac{\Delta(R - r)}{R - r} + \frac{2\Delta T}{T}$$

$\therefore$ Percentage error in $g = \frac{\Delta g}{g} \times 100\%$

$$= \frac{\Delta(R - r)}{R - r} \times 100\% + 2 \times \frac{\Delta T}{T} \times 100\%$$

$$= \frac{2\text{ mm}}{50\text{ mm}} \times 100\% + 2 \times 3.57\%$$

($\because \Delta(R - r) = \Delta R + \Delta r$)

$= 4\% + 7.14\% \approx 11\%$

16. (d): Dimensions of

$$\frac{e^2}{4\pi\varepsilon_o} = [F \times d^2] = [\text{ML}^3\text{T}^{-2}]$$

Dimensions of $G = [\text{M}^{-1}\text{L}^3\text{T}^{-2}]$,

Dimensions of $c = [\text{LT}^{-1}]$

$$l \propto \left(\frac{e^2}{4\pi\varepsilon_o}\right)^p G^q c^r$$

$\therefore$ $[\text{L}^1] = [\text{ML}^3\text{T}^{-2}]^p\,[\text{M}^{-1}\text{L}^3\text{T}^{-2}]^q\,[\text{LT}^{-1}]^r$

On comparing both sides and solving, we get

$$p = \frac{1}{2},\quad q = \frac{1}{2} \text{ and } r = -2$$

$$\therefore\ [l] = \frac{1}{c^2}\left[\frac{Ge^2}{4\pi\varepsilon_o}\right]^{1/2}$$

17. (c) : Let $m = kT^xC^yh^z$

where k is a dimensionless constant.

$\therefore$ $[\text{ML}^0\text{T}^0] = [\text{T}]^x\,[\text{LT}^{-1}]^y\,[\text{ML}^2\text{T}^{-1}]^z$

$[\text{ML}^0\text{T}^0] = [\text{M}^z\text{L}^{y+2z}\text{T}^{x-y-z}]$

$\Rightarrow$ $z = 1$, $y + 2z = 0$ and $x - y - z = 0$

Solving, we get, $x = -1$, $y = -2$, $z = 1$; on putting values we get

$\therefore$ $[\text{M}] = [\text{T}^{-1}\text{C}^{-2}\text{h}]$

18. (d): Here, $P = a^{1/2}\,b^2\,c^3\,d^{-4}$

$$\frac{\Delta P}{P} = \frac{1}{2}\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + 3\frac{\Delta c}{c} + 4\frac{\Delta d}{d}$$

or $\left(\frac{\Delta P}{P} \times 100\right)\%$

$$= \left(\frac{1}{2}\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + 3\frac{\Delta c}{c} + 4\frac{\Delta d}{d}\right) \times 100\%$$

$\therefore$ Relative error in P

$$= \left(\frac{1}{2} \times 2 + 2 \times 1 + 3 \times 3 + 4 \times 5\right)\% = 32\%$$

19. (b): Surface tension is given by

$$T = \frac{rhg}{2} \times 10^3\text{ N m}^{-1} = \frac{Dhg}{4} \times 10^3\text{N m}^{-1}$$

Possible error in the surface tension is

$$\frac{\Delta T}{T}\times 100=\frac{\Delta D}{D}\times 100+\frac{\Delta h}{h}\times 100+0$$

$$=\left(\frac{0.01\times 10^{-2}}{1.25\times 10^{-2}}+\frac{0.01\times 10^{-2}}{1.45\times 10^{-2}}\right)\times 100$$

(Permissible error in D and h is the place value of the last digit.)

$$\frac{\Delta T}{T}\times 100=\left(\frac{100}{125}+\frac{100}{145}\right)$$

$$\frac{\Delta T}{T}\times 100=0.8+0.689=1.489\approx 1.5\%$$

20. (d): For Vernier calipers C_1,

Smallest division on the main scale $=\frac{1\text{ cm}}{10}=1\text{ mm}$

As 10 V.S.D. = 9 M.S.D.

or $1\text{ V.S.D.}=\frac{9}{10}\text{ M.S.D.}$

$\therefore$ Vernier constant = 1 M.S.D. – 1 V.S.D.

$$=1\text{ M.S.D.}-\frac{9}{10}\text{ M.S.D.}$$

$$=\frac{1}{10}\text{ M.S.D.}=\frac{1}{10}\times 1\text{ mm}=0.1\text{ mm}=0.01\text{ cm}$$

$\therefore$ Reading = Main scale reading + Vernier divisions coinciding × Vernier constant

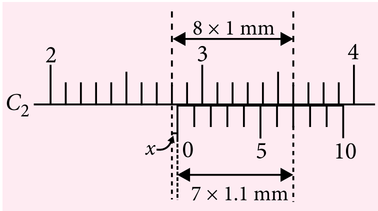
$= 2.8\text{ cm} + 7\times 0.01\text{ cm} = 2.87\text{ cm}$

For Vernier calipers C_2,

Smallest division on the main scale $=\frac{1\text{ cm}}{10}=1\text{ mm}$

Now, 10 V.S.D. = 11 M.S.D.

or $1\text{ V.S.D.}=\frac{11}{10}\text{ M.S.D}=\frac{11}{10}\times 1\text{ mm}=1.1\text{ mm}$



From figure, reading = 2.8 cm + x

where x = (8 × 1) mm – (7 × 1.1) mm

= 0.3 mm = 0.03 cm

$\therefore$ Reading = 2.8 cm + 0.03 cm = 2.83 cm

EXAM PREP 2018

CLASS XI

Useful for Medical/Engg. Entrance Exams

CHAPTERWISE MCQs FOR PRACTICE

MOTION IN A PLANE

1. What are the components of a vector $\vec{A} = 2\hat{i} + 3\hat{j}$ along the directions of $(\hat{i} + \hat{j})$ and $(\hat{i} - \hat{j})$?

(a) $\left(2, \frac{1}{2}\right)$ (b) $\left(\frac{5}{2}, \frac{-1}{2}\right)$

(c) $\left(\frac{5}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ (d) $\left(\frac{-5}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

2. A man rows a boat with a speed of 18 km h^{-1} in the north-west direction. The shoreline makes an angle of 15° south of west. The component of the velocity of the boat along the shoreline and perpendicular to the shoreline are respectively

(a) 9 km h^{-1}, 12 km h^{-1} (b) 12 km h^{-1}, 9 km h^{-1}

(c) 9 km h^{-1}, 15.5 km h^{-1} (d) 15 km h^{-1}, 9.5 km h^{-1}

3. Two projectiles A and B thrown with speeds in the ratio $1:\sqrt{2}$ acquired the same heights. If A is thrown at an angle of 45° with the horizontal, the angle of projection of B will be

(a) 0° (b) 60° (c) 30° (d) 45°

4. If the position vector of a particle is given by

$\vec{r} = (4\cos 2t)\hat{i} + (4\sin 2t)\hat{j} + (6t)\hat{k}$ m.

Find its acceleration at $t = \pi/4$.

(a) $-8\hat{j}$ m s^{-2} (b) $-16\hat{j}$ m s^{-2}

(c) $12\hat{j}$ m s^{-2} (d) $4\hat{j}$ m s^{-2}

5. For a particle in uniform circular motion, the acceleration $\vec{a}$ at a point $P(R, \theta)$ on the circle of radius R is (Here v is the speed of the particle. θ is acute angle and measured from the x-axis.)

(a) $\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$

(b) $-\frac{v^2}{R}\cos\theta\hat{i} + \frac{v^2}{R}\sin\theta\hat{j}$

(c) $-\frac{v^2}{R}\sin\theta\hat{i} + \frac{v^2}{R}\cos\theta\hat{j}$

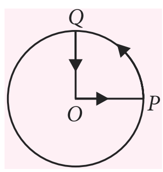
(d) $-\frac{v^2}{R}\cos\theta\hat{i} - \frac{v^2}{R}\sin\theta\hat{j}$

6. A cricket ball thrown across a field is at heights h_1 and h_2 from the point of projection at times t_1 and t_2 respectively after the throw. The ball is caught by a fielder at the same height as that of projection. The time of flight of the ball in this journey is

(a) $\frac{h_1 t_2^2 - h_2 t_1^2}{h_1 t_2 - h_2 t_1}$ (b) $\frac{h_1 t_1^2 + h_2 t_2^2}{h_2 t_1 + h_1 t_2}$

(c) $\frac{h_1 t_2^2 + h_2 t_1^2}{h_1 t_2 + h_2 t_1}$ (d) $\frac{h_1 t_1^2 - h_2 t_2^2}{h_1 t_1 - h_2 t_2}$

7. A cyclist starts from the centre O of a circular park of radius 1 km, reaches the edge P of the park, then cycles along the circumference and returns to the centre along QO as shown in the figure. If the round trip takes 10 min, the net displacement and average speed of the cyclist (in m and km h^{-1}) are



(a) 0, 1 (b) $\frac{\pi + 4}{2}, 0$

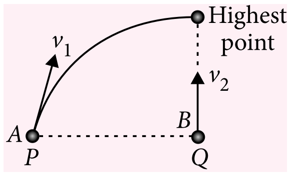
(c) $21.4, \frac{\pi + 4}{2}$ (d) 0, 21.4

8. A particle is projected from the ground with an initial speed of 15 m s^{-1} at an angle 60° with horizontal. The average velocity of the particle between its point of projection and highest point of trajectory is

(a) $15\sqrt{7}$ m s^{-1} (b) $9\sqrt{3}$ m s^{-1}

(c) $\frac{15\sqrt{7}}{4}$ m s^{-1} (d) $\frac{15\sqrt{7}}{2}$ m s^{-1}

9. A projectile A is thrown thrown at an angle of 30° to the horizontal from point P. At the same time, another projectile B is thrown with velocity v_2 upwards from the point Q vertically below the highest point.



For B to collide with A, $\frac{v_2}{v_1}$ should be

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) 4

10. The resultant of two vectors $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If the magnitude of $\vec{Q}$ is doubled, the new resultant becomes perpendicular to $\vec{P}$. Then the magnitude of $\vec{R}$ is

(a) $P + Q$ (b) Q

(c) P (d) $\frac{P+Q}{2}$

11. A ball is projected from the ground at a speed of 10 m s^{-1} making an angle of 30° with the horizontal. Another ball is simultaneously released from a point on the vertical line along the maximum height of the projectile. This ball collides with first ball at the maximum height of projectile. The initial height of the second ball is (g = 10 m s^{-2})

(a) 6.25 m (b) 2.5 m

(c) 3.75 m (d) 5 m

12. Let $\vec{r}_1(t) = 3t\,\hat{i} + 4t^2\,\hat{j}$ and $\vec{r}_2(t) = 4t^2\,\hat{i} + 3t\,\hat{j}$ represent the positions of particles 1 and 2 respectively as function of time t; $\vec{r}_1(t)$ and $\vec{r}_2(t)$ are in m and t in s. The relative speed of the two particles at the instant t = 1 s, will be

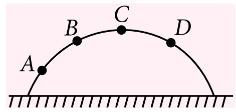
(a) 1 m s^{-1} (b) $3\sqrt{2}$ m s^{-1}

(c) $5\sqrt{2}$ m s^{-1} (d) $7\sqrt{2}$ m s^{-1}

13. The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 m s^{-1} can go without hitting the ceiling ?

(a) 108 m (b) 120 m (c) 150 m (d) 162 m

14. A stone is projected from ground. Its path is as shown in figure. At which point its speed is decreasing at fastest rate?



(a) A (b) B

(c) C (d) D

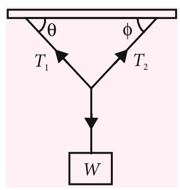
15. A particle is moving uniformly in a circular path of radius r. When it moves through an angular displacement θ, then the magnitude of the corresponding linear displacement will be

(a) $2r\cos\left(\frac{\theta}{2}\right)$ (b) $2r\cot\left(\frac{\theta}{2}\right)$

(c) $2r\tan\left(\frac{\theta}{2}\right)$ (d) $2r\sin\left(\frac{\theta}{2}\right)$

LAWS OF MOTION

16. A weight W hangs from a rope that is tied to two other ropes that are fastened to the ceiling as shown in figure. The upper ropes make angles θ and ϕ with the horizontal. Now, the values of T_1 and T_2 are



(a) $\frac{W\sin\phi}{\sin(\theta+\phi)}, \frac{W\sin\theta}{\sin(\theta+\phi)}$

(b) $\frac{W\sin\phi}{\cos(\theta+\phi)}, \frac{W\sin\theta}{\cos(\theta+\phi)}$

(c) $\frac{W\cos\phi}{\sin(\theta+\phi)}, \frac{W\cos\theta}{\sin(\theta+\phi)}$

(d) $\frac{W\cos\phi}{\tan(\theta+\phi)}, \frac{W\cos\theta}{\tan(\theta+\phi)}$

17. A mass of 5 kg is suspended in equilibrium, by two light inextensible strings S_1 and S_2 which make angle of 30° and 45° respectively with the horizontal. Then (Take g = 10 m s^{-2})

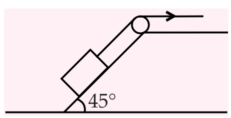
(a) tension in both the strings is same

(b) tension in S_1 is more than that in S_2

(c) tension in S_1 is less than that in S_2

(d) sum of tension in both is equal to 50 N

18. A block of mass 200 kg is being pulled up by men on an inclined plane at angle of 45° as shown in figure. The coefficient of static friction is 0.5. Each man can only apply a maximum force of 500 N. Find the minimum number of men required for the block to just start moving up the plane.

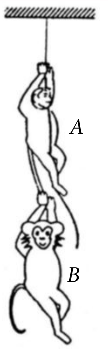


(a) 10 (b) 15 (c) 5 (d) 3

19. Which one of the following motions on a smooth plane surface does not involve force?

(a) Accelerated motion in a straight line.
(b) Retarded motion in a straight line.
(c) Motion with constant momentum along a straight line.
(d) Motion along a straight line with varying velocity.

20. The monkey B shown in figure is holding on to the tail of monkey A which is climbing up a rope. The masses of the monkeys A and B are 5 kg and 2 kg respectively. If A can tolerate a tension of 30 N in its tail, what force should it apply on the rope in order to carry the monkey B with it? (Take $g = 10\ \text{m s}^{-2}$).

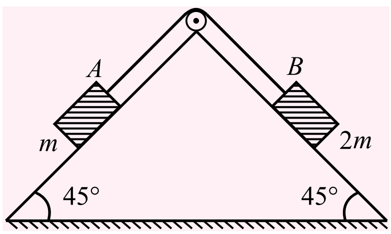


(a) Between 70 N and 105 N
(b) Between 50 N and 69 N
(c) Between 30 N and 50 N
(d) Between 30 N and 116 N

21. A gramophone record is revolving with an angular velocity ω. A coin is placed at a distance r from the centre of the record. The coefficient of static friction is μ. The coin will revolve with the record if

(a) $r = \mu g \omega^2$ (b) $r < \frac{\omega^2}{\mu g}$
(c) $r \le \frac{\mu g}{\omega^2}$ (d) $r \ge \frac{\mu g}{\omega^2}$

22. Block A of mass m and block B of mass $2m$ are placed on a fixed triangular wedge by means of a massless, inextensible string and a frictionless pulley as shown in the figure. The wedge is inclined at 45° to the horizontal on both the sides. If the coefficient of friction between the block A and the wedge is 2/3 and that between the block B and the wedge is 1/3. Both A and B are released from rest, the acceleration of A will be



(a) 0.1 (b) zero
(c) 0.2 (d) 0.6

23. A body of 100 kg is placed on a truck. The coefficient of static friction between the body and the truck is 0.2. The truck suddenly decreases its speed from 90 km h^{-1} to 36 km h^{-1} in 5 s. Then

(a) the block does not move.
(b) the block slips forward and hits the driver's cabin
(c) block shifts backward
(d) nothing can be said about the block.

24. A boy stands on a weighing machine inside a lift. When the lift is going down with acceleration $g/4$, the machine shows a reading 30 kg. When the lift goes upwards with acceleration $g/4$, the reading would be

(a) 18 kgf (b) 37.5 kgf
(c) 50 kgf (d) 67.5 kgf

25. A body of mass m is travelling with a velocity u. When a constant retarding force F is applied, it comes to rest after travelling a distance s_1. If the initial velocity is $2u$, with the same force F, the distance travelled before it comes to rest is s_2. Then

(a) $s_2 = 2s_1$ (b) $s_2 = \frac{s_1}{2}$
(c) $s_2 = s_1$ (d) $s_2 = 4s_1$

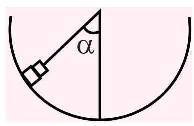
26. A person sitting in an open car moving at constant velocity throws a ball vertically up into air. The ball falls

(a) Outside the car
(b) In the car ahead of the person
(c) In the car to the side of the person
(d) Exactly in the hand which threw it up

27. The rear side of a truck is open and a box of mass 20 kg is placed on the truck 4 m away from the open end. If μ is 0.15 and $g = 10\ \text{m s}^{-2}$ and the truck starts from rest with an acceleration of 2 m s^{-2} on a straight road, then the box will fall off the truck when it is at a distance of x metre from the starting point. The value of x is

(a) 4 m (b) 8 m (c) 16 m (d) 32 m.

28. An insect crawls up a hemispherical surface very slowly as shown in the figure. The coefficient of friction between the insect and the surface is 1/3. If the line joining the centre of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is

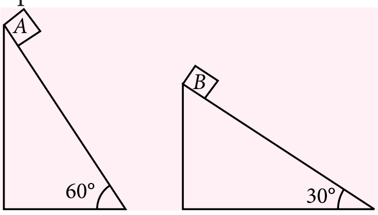


(a) cot α = 3 (b) tan α = 3
(c) sec α = 3 (d) cosec α = 3

29. A weightless thread can bear tension upto 3.7 kg wt. A stone of mass 500 g is tied to it and revolves in a circular path of radius 4 m in vertical plane. If g = 10 m s^{-2}, then what will be the maximum angular velocity of the stone?

(a) 2 rad s^{-1} (b) 4 rad s^{-1}
(c) 6 rad s^{-1} (d) 10 rad s^{-1}

30. Two fixed frictionless inclined planes making an angle 30° and 60° with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B?



(a) 4.9 m s^{-2} in vertical direction
(b) 4.9 m s^{-2} in horizontal direction
(c) 9.8 m s^{-2} in vertical direction
(d) zero

SOLUTIONS

1. **(c):** $\vec{A} = 2\hat{i} + 3\hat{j} = \lambda(\hat{i}+\hat{j}) + u(\hat{i}-\hat{j})$

$2\hat{i}+3\hat{j} = (\lambda+u)\hat{i} + (\lambda-u)\hat{j}$

$\Rightarrow \lambda + u = 2$ and $\lambda - u = 3$

$\Rightarrow \lambda = \dfrac{5}{2}$ and $u = \dfrac{-1}{2}$

Now, unit vector along $\hat{i}+\hat{j}$ is $\dfrac{\hat{i}+\hat{j}}{\sqrt{2}}$ and unit vector along $\hat{i}-\hat{j} = \dfrac{\hat{i}-\hat{j}}{\sqrt{2}}$

Thus, $2\hat{i}+3\hat{j} = \dfrac{5}{2}(\hat{i}+\hat{j}) - \dfrac{1}{2}(\hat{i}-\hat{j})$

$$= \left(\frac{5}{\sqrt{2}}\right)\cdot\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right) - \frac{1}{\sqrt{2}}\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)$$

As, the components of $2\hat{i}+3\hat{j}$ along $\hat{i}+\hat{j}$ and $\hat{i}-\hat{j}$ directions are $\left(\dfrac{5}{\sqrt{2}}, \dfrac{-1}{\sqrt{2}}\right)$.

2. **(c)**

3. **(c):** For projectile A maximum height,

$$H_A = \frac{u_A^2 \sin^2 45°}{2g}$$

For projectile B maximum height,

$$H_B = \frac{u_B^2 \sin^2\theta}{2g}$$

As per question, $H_A = H_B$

$$\frac{u_A^2 \sin^2 45°}{2g} = \frac{u_B^2 \sin^2\theta}{2g} \quad \text{or} \quad \frac{\sin^2\theta}{\sin^2 45°} = \frac{u_A^2}{u_B^2}$$

$$\sin^2\theta = \left(\frac{u_A}{u_B}\right)^2 \sin^2 45°$$

$$\sin^2\theta = \left(\frac{1}{\sqrt{2}}\right)^2\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{4} \quad \left(\because \frac{u_A}{u_B} = \frac{1}{\sqrt{2}}\right)$$

$$\sin\theta = \frac{1}{2} \quad \text{or} \quad \theta = \sin^{-1}\left(\frac{1}{2}\right) = 30°$$

4. **(b):** Position, $\vec{r} = (4\cos 2t)\hat{i} + (4\sin 2t)\hat{j} + 6t\hat{k}$

Velocity,

$$\vec{v} = \frac{d\vec{r}}{dt} = [4(-\sin 2t)(2)]\hat{i} + [4(\cos 2t)\cdot(2)]\hat{j} + 6\hat{k}$$

$$= (-8\sin 2t)\hat{i} + (8\cos 2t)\hat{j} + 6\hat{k}$$

Acceleration,

$$\vec{a} = \frac{d\vec{v}}{dt} = [-8(\cos 2t)(2)]\hat{i} + [8(-\sin 2t)(2)]\hat{j}$$

$$= (-16\cos 2t)\hat{i} + (-16\sin 2t)\hat{j}$$

When $t = \pi/4$

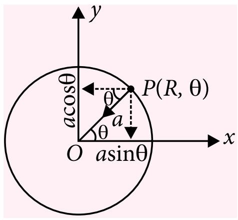
$$\vec{a} = (-16\cos\pi/2)\hat{i} + (-16\sin\pi/2)\hat{j}$$

$$= (-16\times 0)\hat{i} + (-16\times 1)\hat{j} = -16\hat{j} \text{ m s}^{-2}$$

5. **(d):** For a particle in uniform circular motion,

acceleration, $a = \dfrac{v^2}{R}$

towards the centre



From figure,

$$\vec{a} = -a\cos\theta\hat{i} - a\sin\theta\hat{j} = -\frac{v^2}{R}\cos\theta\hat{i} - \frac{v^2}{R}\sin\theta\hat{j}$$

6. **(a):** Let a cricket ball be thrown with velocity u at an angle θ with the horizontal.

As per question, $h_1 = u\sin\theta t_1 - \dfrac{1}{2}gt_1^2$

or $u\sin\theta t_1 = h_1 + \dfrac{1}{2}gt_1^2$ ... (i)

and $h_2 = u\sin\theta t_2 - \dfrac{1}{2}gt_2^2$

or $u\sin\theta t_2 = h_2 + \frac{1}{2}gt_2^2$ ... (ii)

Divide eqn. (i) by eqn. (ii), we get

$$\frac{t_1}{t_2} = \frac{h_1 + \frac{1}{2}gt_1^2}{h_2 + \frac{1}{2}gt_2^2};\ h_2t_1 + \frac{1}{2}gt_2^2 t_1 = h_1t_2 + \frac{1}{2}gt_1^2 t_2$$

$$h_1t_2 - h_2t_1 = \frac{1}{2}g(t_1t_2^2 - t_1^2t_2) \quad \text{... (iii)}$$

Time of flight, $T = \frac{2u\sin\theta}{g}$

$$T = \frac{2}{g}\left[\frac{h_1 + \frac{1}{2}gt_1^2}{t_1}\right] \quad \text{(Using (i))}$$

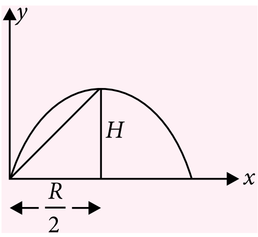
$$= \frac{2}{g}\frac{h_1}{t_1} + t_1 = \frac{h_1}{t_1}\left(\frac{t_1t_2^2 - t_1^2t_2}{h_1t_2 - h_2t_1}\right) + t_1 \quad \text{(Using (iii))}$$

$$= \frac{h_1t_2^2 - h_1t_1t_2}{h_1t_2 - h_2t_1} + t_1 = \frac{h_1t_2^2 - h_2t_1^2}{h_1t_2 - h_2t_1}$$

7. (d)

8. (c) : Average velocity $= \frac{\text{Displacement}}{\text{Time}}$

$$v_{av} = \frac{\sqrt{H^2 + \frac{R^2}{4}}}{T/2} \quad \text{...(i)}$$



Here, H = maximum height

$$= \frac{v^2\sin^2\theta}{2g}$$

$R = \text{range} = \frac{v^2\sin 2\theta}{g}$ and $T = \text{time of flight} = \frac{2v\sin\theta}{g}$

Putting these values in eqn. (i) we get

$$v_{av} = \frac{v}{2}\sqrt{1 + 3\cos^2\theta}$$

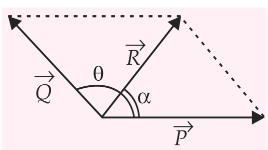
$$v_{av} = \frac{15}{2}\sqrt{1 + 3\times\cos^2 60^\circ} = \frac{15}{2}\sqrt{1 + \frac{3}{4}} = \frac{15\sqrt{7}}{4}\text{ m s}^{-1}$$

9. (c) : Both the projectiles will collide in the air if vertical component of velocity of projectile A is equal to the velocity of projectile B.

$$v_1\sin 30^\circ = v_2 \Rightarrow \frac{v_2}{v_1} = \frac{1}{2}$$

10. (b): $\vec{P} + \vec{Q} = \vec{R}$

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$

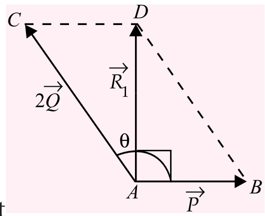


and $R^2 = P^2 + Q^2 + 2PQ\cos\theta$ ...(i)

When $\vec{Q}$ is doubled, resultant $\vec{R}_1$ is perpendicular to $\vec{P}$

$\therefore\ R_1^2 = P^2 + 4Q^2 + 4PQ\cos\theta$ ...(ii)

From right angled triangle BAD



$4Q^2 = R_1^2 + P^2$,

$R_1^2 = 4Q^2 - P^2$

Substituting in (ii) and solving, we get

$P^2 + 2PQ\cos\theta = 0$ ...(iii)

Substituting (iii) in (i), we get

$R^2 = Q^2$ or $R = Q$

11. (b): Maximum height of projectile, $H = \frac{u^2\sin^2\theta}{2g}$

$$\therefore\ H = \frac{(10)^2\times\sin^2 30^\circ}{2\times 10} = \frac{5}{4} = 1.25\text{ m}$$

Time for attaining maximum height, $t = \frac{u\sin\theta}{g}$

$$\therefore\ t = \frac{10\times\sin 30^\circ}{10} = 0.5\text{ s}$$

$\therefore$ Distance of vertical fall in 0.5 s, $h = \frac{1}{2}gt^2$

or $h = \frac{1}{2}\times 10\times(0.5)^2 = 1.25$ m

$\therefore$ Height of second ball = 1.25 + 1.25 = 2.5 m

12. (c) : Here, $\vec{r}_1(t) = 3t\,\hat{i} + 4t^2\,\hat{j}$, $\vec{r}_2(t) = 4t^2\,\hat{i} + 3t\,\hat{j}$

Velocity, $\vec{v}_1(t) = \frac{d\vec{r}_1}{dt} = \frac{d}{dt}(3t\,\hat{i} + 4t^2\,\hat{j}) = 3\hat{i} + 8t\,\hat{j}$

$$\vec{v}_2(t) = \frac{d\vec{r}_2}{dt} = \frac{d}{dt}(4t^2\,\hat{i} + 3t\,\hat{j}) = 8t\,\hat{i} + 3\hat{j}$$

The relative speed of particle 1 with respect to particle 2 is

$$\vec{v}_{12} = \vec{v}_1 - \vec{v}_2 = (3\hat{i} + 8t\,\hat{j}) - (8t\,\hat{i} + 3\hat{j})$$

$$= (3 - 8t)\hat{i} + (8t - 3)\hat{j}$$

At $t = 1$ s, $\vec{v}_{12} = (3 - 8)\hat{i} + (8 - 3)\hat{j} = -5\hat{i} + 5\hat{j}$

$$|\vec{v}_{12}| = \sqrt{(-5)^2 + (5)^2} = \sqrt{25 + 25} = 5\sqrt{2}\text{ m s}^{-1}$$

13. (c) : $h_{max} = \frac{u^2\sin^2\theta}{2g} \Rightarrow 25 = \frac{40^2\times\sin^2\theta}{2g}$

$$\sin^2\theta = \frac{50g}{40^2} = \frac{50\times 10}{40\times 40} = \frac{5}{16};\ \sin\theta = \frac{\sqrt{5}}{4}$$

$$\cos\theta = \sqrt{1 - \sin^2\theta} = \frac{\sqrt{11}}{4}$$

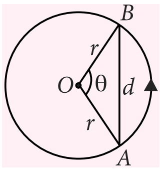
$$R = \frac{u^2\sin 2\theta}{g} = \frac{2u^2\sin\theta\cos\theta}{g} = \frac{2(40)^2\left(\frac{\sqrt{5}}{4}\right)\left(\frac{\sqrt{11}}{4}\right)}{10}$$

$$= \frac{2 \times 40^2 \times \sqrt{55}}{4 \times 4 \times 10} = 20\sqrt{55} = 148.3 \approx 150 \text{ m}$$

14. (a)

15. (d): The situation is shown in figure.

According to cosine formula



$$\cos\theta = \frac{r^2 + r^2 - d^2}{2r^2}$$

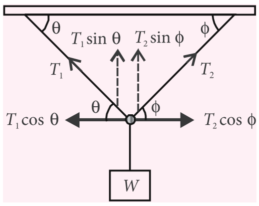
$2r^2\cos\theta = r^2 + r^2 - d^2$

$d^2 = 2r^2 - 2r^2\cos\theta = 2r^2[1 - \cos\theta]$

$$= 2r^2\left[2\sin^2\frac{\theta}{2}\right] \quad \text{or} \quad d = 2r\sin\left(\frac{\theta}{2}\right)$$

16. (c) : For equilibrium,

Along the horizontal direction



$T_1 \cos\theta = T_2 \cos\phi$ ...(i)

Along the vertical direction

$T_1 \sin\theta + T_2 \sin\phi = W$ ...(ii)

Substituting the value of T_2 from eqn. (i) in eqn. (ii).

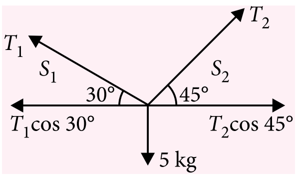
$$T_1 \sin\theta + \left(\frac{T_1\cos\theta}{\cos\phi}\right)\sin\phi = W$$

$\Rightarrow$ $T_1 (\sin\theta \cos\phi + \cos\theta \sin\phi) = W \cos\phi$

$$\Rightarrow \quad T_1 = \frac{W\cos\phi}{\sin(\theta + \phi)}$$

Similarly, $T_2 = \dfrac{W\cos\theta}{\sin(\theta + \phi)}$.

17. (c) :



From figure, we concluded

$T_1 \cos 30° = T_2 \cos 45°$

$$T_1\frac{\sqrt{3}}{2} = T_2\frac{1}{\sqrt{2}} \;\Rightarrow\; T_2 = \sqrt{\frac{3}{2}}T_1 \;\Rightarrow\; T_2 > T_1$$

18. (c) : Here,

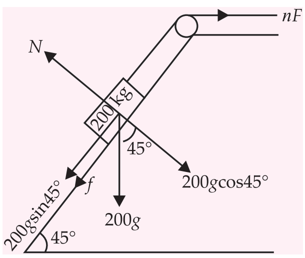
Mass of the block,

$m = 200$ kg

Coefficient of static friction, $\mu_s = 0.5 = \dfrac{1}{2}$

Angle of incline plane,

$\theta = 45°$



Maximum force that each man be apply, $F = 500$ N

Let n number of men are required for the block to just start moving up the plane. Then

$nF = mg\sin\theta + f = mg\sin\theta + \mu_s N$

$= mg\sin\theta + \mu_s mg\cos\theta = mg[\sin\theta + \mu_s\cos\theta]$

$$= 200 \times 10\left[\sin 45° + \frac{1}{2}\cos 45°\right]$$

$$= 200 \times 10\left[\frac{1}{\sqrt{2}} + \frac{1}{2\sqrt{2}}\right] = \frac{200 \times 10 \times 3}{2\sqrt{2}}$$

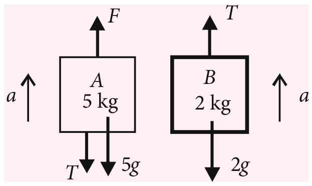
$$n = \frac{200 \times 10 \times 3}{2\sqrt{2} \times 500} \approx 5$$

19. (c) : Motion with constant momentum along a straight line implies $\vec{p} =$ constant.

So, according to Newton's 2nd law

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(\text{constant}) = 0$$

20. (a) : Let us make the free body diagram for both monkeys separately.



As the monkey A, applies a force F downward on the rope, the rope applies a force F upward on the monkey A.

For A, $F - 5g - T = 5a$ ...(i)

For B, $T - 2g = 2a$ ...(ii)

Dividing the two eqns.

$$\frac{F - 5g - T}{T - 2g} = \frac{5}{2} \Rightarrow 2(F - 5g - T) = 5(T - 2g)$$

$$2F - 10g - 2T = 5T - 10g \Rightarrow 2F = 7T \quad \text{or} \quad T = \frac{2F}{7}$$

Now from eqn. (ii), for $a = 0$

$T_{min} = 2g = 20$ N; $T_{max} = 30$ N is given.

$$\text{Hence, } 20 \text{ N} \le T \le 30 \text{ N} \Rightarrow 20 \text{ N} \le \frac{2F}{7} \le 30 \text{ N}$$

$$\frac{7}{2}(20 \text{ N}) \le \frac{7}{2}\left(\frac{2}{7}\right)F \le \frac{7}{2}(30 \text{ N})$$

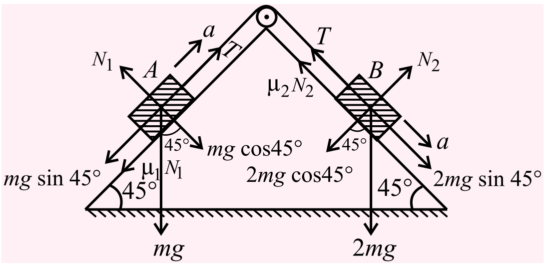
$\Rightarrow$ $70 \text{ N} \le F \le 105 \text{ N}$

21. (c) : The coin will revolve with the record, if

Centripetal force $\le$ force of friction

$$mr\omega^2 \le \mu mg \quad \text{or} \quad r \le \frac{\mu g}{\omega^2}$$

22. (b): The free body diagram of the blocks is shown in the figure



The equation of motion of block B is

$$2mg\sin 45° - \mu_2 N_2 - T = 2ma$$

or $\quad 2mg\sin 45° - \frac{1}{3}2mg\cos 45° - T = 2ma$

or $\quad 2a = \left(2 - \frac{2}{3}\right)g\frac{1}{\sqrt{2}} - \frac{T}{m} \quad$ or $\quad a = \frac{2}{3\sqrt{2}}g - \frac{T}{2m}$

But $(m_B - m_A)g\sin\theta = \frac{mg}{\sqrt{2}}$ will be less than

$(\mu_A m_A + \mu_B m_B)\, g\cos\theta = \frac{4mg}{3\sqrt{2}}$.

Therefore the masses will not move and hence acceleration of A or B will be zero.

23. (b): Force of static friction = μmg

$= 0.2 \times 100 \times 10 = 200$ N

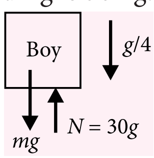
$$\text{Deceleration} = \frac{(90 \text{ km h}^{-1} - 36 \text{ km h}^{-1})}{5 \text{ s}}$$

$$= \frac{25 \text{ m s}^{-1} - 10 \text{ m s}^{-1}}{5 \text{ s}} = 3 \text{ m s}^{-2}$$

Force on the mass = 100 kg × 3 m s^{-2} = 300 N

This is greater than the force due to static friction. Therefore due to pseudo force, the block of mass m will go forward and hit the driver's cabin.

24. (c) : While going down, machine reading is 30 kg.

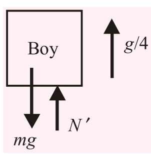


$\Rightarrow \quad mg - N = m\left(\frac{g}{4}\right);$

$mg - 30g = \frac{mg}{4}$; $3\frac{mg}{4} = 30g$

$m = \frac{30 \times 4}{3} = 40$ kg

While going up, the reaction would be



$N' - mg = m\frac{g}{4}$

$N' = 5\frac{mg}{4} = \frac{5}{4}(40 \text{ kg})g$

$\Rightarrow \quad N' = 50$ kgf.

25. (d) **26. (d)**

27. (c) : $a = 2$ m s^{-2}, $N = mg$

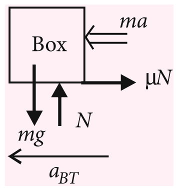
$ma - \mu N = ma_{BT} \quad \Rightarrow \quad (ma - \mu mg) = ma_{BT}$

$\Rightarrow a_{BT} = a - \mu g$

$= 2$ m s^{-2} – (0.15) (10 m s^{-2})

$= 0.5$ m s^{-2}

For the box's motion w.r.t. truck,



$s_{BT} = u_{BT}\,.\,t + \frac{1}{2}\,a_{BT}\,.\,t^2$

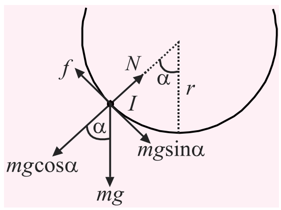
$4 = 0 + \frac{1}{2}(0.5)t^2 \Rightarrow t^2 = \frac{4}{0.25}$

$\Rightarrow \quad t = 4$ s.

After 4 s, the box will fall off the truck. During 4 s, the truck travels,

$x = \frac{1}{2}\,at^2 = (2)(4)2 = 16$ m.

28. (a) : The insect I is under the action of two forces. N denotes the normal reaction while mg denotes the weight of the insect. The weight mg can be resolved into two perpendicular components. For equilibrium



$N = mg\cos\alpha,\ f = mg\sin\alpha$

where f denotes force of friction.

But $f = \mu N \quad \therefore \quad \mu(mg\cos\alpha) = mg\sin\alpha$

or $\quad \mu = \tan\alpha \quad$ or $\quad \frac{1}{3} = \frac{1}{\cot\alpha} \quad$ or $\cot\alpha = 3$

29. (b)

30. (a) : The acceleration of the body down the smooth inclined plane is $a = g\sin\theta$

It is along the inclined plane.

where θ is the angle of inclination

$\therefore$ The vertical component of acceleration a is

$a_{(\text{along vertical})} = (g\sin\theta)\sin\theta = g\sin^2\theta$

For block A,

$a_{A(\text{along vertical})} = g\sin^2 60°$

For block B,

$a_{B(\text{along vertical})} = g\sin^2 30°$

The relative vertical acceleration of A with respect to B is

$a_{AB(\text{along vertical})} = a_{A(\text{along vertical})} - a_{B(\text{along vertical})}$

$$= g\sin^2 60° - g\sin^2 30° = g\left(\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right)$$

$$= \frac{g}{2} = 4.9 \text{ m s}^{-2} \text{ in vertical direction.}$$

ACE YOUR WAY CBSE XI

Series 1

CHAPTERWISE PRACTICE PAPER

UNITS AND MEASUREMENTS | MOTION IN A STRAIGHT LINE

Time Allowed : 3 hours — Maximum Marks : 70

GENERAL INSTRUCTIONS

(i) All questions are compulsory.
(ii) Q. no. 1 to 5 are very short answer questions and carry 1 mark each.
(iii) Q. no. 6 to 10 are short answer questions and carry 2 marks each.
(iv) Q. no. 11 to 22 are also short answer questions and carry 3 marks each.
(v) Q. no. 23 is a value based question and carries 4 marks.
(vi) Q. no. 24 to 26 are long answer questions and carry 5 marks each.
(vii) Use log tables if necessary, use of calculators is not allowed.

SECTION - A

1. A famous relation in physics relates moving mass (m) to the rest mass (m_o)of a particle in terms of its speed v and the speed of light, c. (This relation first arose as a consequence of special theory of relativity due to Albert Einstein). A boy recalls the relation almost correctly but forgets where to put the constant c. He writes : $m = \frac{m_o}{(1-v^2)^{1/2}}$.

Guess where to put the missing c.

2. Distinguish between accuracy and precision.

3. If in case of a motion, displacement is directly proportional to the square of the time elapsed, what do you think about its acceleration *i.e.*, constant or variable? Explain why.

4. Which of the two-velocity or acceleration, gives the direction of motion of the body. Justify your answer by an example.

5. When an observer is standing on earth, the trees and houses appear stationary to him. However, when he is sitting in a moving train, all these objects appear to move in backward direction. Why?

SECTION - B

6. Give reasons why is platinum iridium alloy used in making prototype metre and kilogram.

7. A ball is dropped from a building of height 45 m. Simultaneously another ball is thrown up with a speed 40 m s^{-1}. Calculate the relative speed of the balls as a function of time.

8. Distinguish between distance and displacement.

9. Suggest an indirect method for measuring the height of a tree on a sunny day.

OR

The nearest star to our solar system is 4.29 light years away. How much is this distance in terms of parsecs? How much parallax would this star (named Alpha Centauri) show when viewed from two locations of the Earth six months apart in its orbit around the Sun?

10. The unit of length convenient on the atomic scale is known as an angstrom and is denoted by Å : 1 Å = 10^{-10} m. The size of a hydrogen atom is about 0.5 Å. What is the total atomic volume in m^3 of one mole of hydrogen atoms?

SECTION - C

11. Define relative velocity. Deduce an expression for relative velocity of one object with respect to another in terms of their velocities relative to the earth.

12. What is meant by dimensional formula and dimensional equation? Give examples.

13. A car accelerates from rest at a constant rate α for sometime after which it decelerates at constant rate β to come to rest. If the total time elapsed is t second, evaluate : (a) the maximum velocity reached and (b) the total distance travelled.

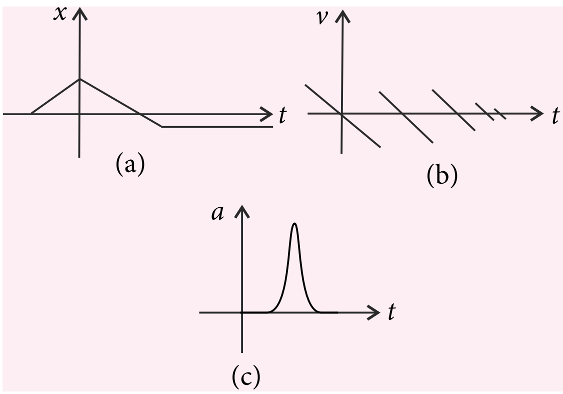
14. Which of the following is the most precise device for measuring length :

(a) a vernier callipers with 20 divisions on the sliding scale

(b) a screw gauge of pitch 1 mm and 100 divisions on the circular scale

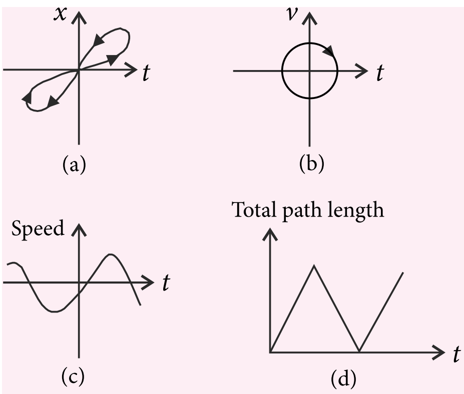
(c) an optical instrument that can measure length to within a wavelength of light?

15. Suggest a suitable physical situation for each of the following graphs shown in figure.



OR

Look at the graphs (a) to (d) carefully and state, with reasons, which of these cannot possibly represent one-dimensional motion of a particle.



16. Einstein's mass - energy relation emerging out of his famous theory of relativity relates mass (m) to energy (E) as $E = mc^2$, where c is speed of light in vacuum. At the nuclear level, the magnitudes of energy are very small. The energy at nuclear level is usually measured in MeV, where $1\ \text{MeV} = 1.6012 \times 10^{-13}$ J; the masses are measured in unified atomic mass unit (u) where $1\ \text{u} = 1.6605 \times 10^{-27}$ kg.

(a) Show that the energy equivalent of 1 u is 931.5 MeV.

(b) A student writes the relation as 1 u = 931.5 MeV. The teacher points out that the relation is dimensionally incorrect. Write the correct relation.

17. Two trains, each of length 100 m, are running on parallel tracks. One overtakes the other in 20 s and one crosses the other in 10 s. Calculate the speed of each of the trains.

18 A new system of units is proposed in which unit of mass is α kg, unit of length β m and unit of time γ s. How much will 5 J measure in this new system?

19 The displacement of a particle is zero at $t = 0$ and x at $t = t$. It starts moving in the positive x-direction with a velocity which varies as $v = k\sqrt{x}$, where k is a constant. Show that the velocity is proportional to time.

20 A point moves from rest with a uniform acceleration and $\bar{v}_1, \bar{v}_2$ and $\bar{v}_3$ denote the average velocities in the three successive intervals of time t_1, t_2 and t_3. Find the ratio $\dfrac{(\bar{v}_1 - \bar{v}_2)}{(\bar{v}_2 - \bar{v}_3)}$.

21. A car starts from rest and accelerates uniformly for 10 s to a velocity of 8 m s^{-1}. It then runs with constant velocity and is finally brought to rest in 64 m with a constant retardation. The total distance covered by the car is 584 m. Find the value of acceleration, retardation and total time taken.

22. Time for 20 oscillations of a pendulum is measured as $t_1 = 39.6$ s; $t_2 = 39.9$ s; $t_3 = 39.5$ s. What is the precision in the measurement? What is the accuracy of the measurement?

SECTION - D

23. Ramesh was travelling along with his father in a train. When the train was moving fast, Ramesh looked out of the window of the train. To his surprise, he found that the nearby trees, houses, lamp-posts, etc. seem to move rapidly in a direction opposite to the train's motion while the distant objects such as hill tops, the Moon, the stars, etc. seem to be stationary. Out of shear enthusiasm, he asked his father, who was a science teacher, to explain him the reason behind this observation. He gave the explanation of the phenomenon in detail.

(a) What are the values being displayed by Ramesh here?

(b) Name the phenomenon involved in the above observation.

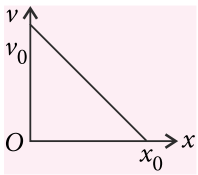
(c) Explain the above observation clearly.

SECTION - E

24. A point moving with constant acceleration from A to B in the straight line AB has velocities v_0 and v at A and B respectively. Find its velocity at C, the mid-point of AB. Also show that if the time from A to C is twice that from C to B, then $v = 7v_0$.

OR

The velocity-displacement graph of a particle is shown in figure.



(a) Write the relation between v and x.

(b) Obtain the relation between acceleration and displacement and plot it.

25. Define the principle of homogeneity of dimensions. An artificial satellite is revolving around a planet of mass M and radius R, in a circular orbit of radius r. From Kepler's third law about the period of a satellite around a common central body, square of the period of revolution T is proportional to the cube of the radius of the orbit r. Show using dimensional analysis, that $T = \frac{k}{R}\sqrt{\frac{r^3}{g}}$, where k is a dimensionless constant and g is acceleration due to gravity.

OR

(a) Deduce the general rule for evaluating the error in combind calculation.

(b) The specific resistance σ of a thin wire of radius r cm, resistance $R\ \Omega$ and length L cm is given by

$$\sigma = \frac{\pi r^2 R}{L}$$

If $r = 0.26 \pm 0.02$ cm, $R = 32 \pm 1\ \Omega$ and $L = 78 \pm 0.01$ cm, find the percentage error in σ.

26. Derive the following equations of motion for uniformly accelerated motion from velocity - time graph :

(a) $v = u + at$ (b) $s = ut + \frac{1}{2}at^2$

(c) $v^2 - u^2 = 2as$.

OR

If velocity of light c, Planck's constant h and gravitational constant G are taken as fundamental quantities then express mass, length and time in terms of dimensions of these quantities.

SOLUTIONS

1. From principle of homogeneity of dimensions both sides of given formula must have same dimensions. For this, $(1 - v^2)^{1/2}$ must be dimensionless. Therefore, instead of $(1 - v^2)^{1/2}$, it will be $(1 - v^2/c^2)^{1/2}$.

Hence relation should be $m = \dfrac{m_o}{(1 - v^2/c^2)^{1/2}}$.

2. By accuracy of a measurement we mean that the measured value of a physical quantity is as close to the true value as possible. On the other hand, a measurement is said to be precise, if same value of that quantity is obtained in each of the various measurements carried out with the given apparatus.

3. Given $x \propto t^2$

or $x = ct^2$ where c is a constant

Velocity, $v = \dfrac{dx}{dt} = c \times 2t$

Acceleration, $a = \dfrac{dv}{dt} = 2c$ = a constant

Hence the object is moving with a uniform acceleration.

4. It is the velocity and not the acceleration, which gives the direction of motion of a body. When a body is projected upward, both its direction of motion and velocity are in upward direction but the acceleration is in the downward direction.

5. For the stationary observer, the relative velocity of trees and houses is zero. For the observer sitting in the moving train, the relative velocity of houses and trees is negative. So these objects appear to move in backward direction.

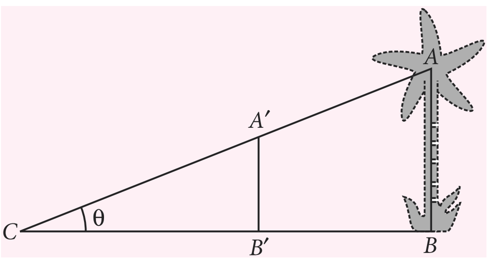
6. The reasons for making standard kilogram and metre from platinum-iridium alloy are as follows:
(i) The alloy is least affected by temperature variations.
(ii) It is non-corrosive and so does not wear out easily.
(iii) It is quite hard.
(iv) It does not change with time.

7. Velocity of dropped ball after time, t
$v_d = gt$ (downward)
Velocity of thrown ball after time, t
$v_t = 40 - gt$ (upward)
$\therefore$ Relative speed of balls $= v_{dt} = v_{td}$
$= v_d + v_t$ (As they are in opposite direction)
$= gt + (40 - gt) = 40 \text{ m s}^{-1}$

8.

	Distance	Displacement
(i)	Distance is the length of the actual path traversed by a body, irrespective of its motion.	Displacement is the shortest distance between the initial and final positions of a body in a given direction.
(ii)	Distance between two points may be same or different for different paths chosen.	Displacement between two given points is always same.
(iii)	It is a scalar quantity.	It is a vector quantity.
(iv)	Distance covered may be positive or zero.	Displacement covered may be positive, negative or zero.

9. Let AB be the height of a tree, as shown in figure and BC be its shadow cast by the sun.



Let $\angle ACB = \theta$

Clearly, $\tan\theta = \dfrac{AB}{BC}$

Take a rod $A'B'$ and fix it at such a point that the tip of its shadow coincides with the point C. Then

$\tan\theta = \dfrac{A'B'}{B'C}$.

Hence $\dfrac{AB}{BC} = \dfrac{A'B'}{B'C}$.

$\therefore$ Height of tree, $AB = \dfrac{A'B'}{B'C} \times BC$.

OR

Distance $= 4.29$ light years $= 4.29 \times 9.46 \times 10^{15}$ m

($\because$ 1 light year $= 9.46 \times 10^{15}$ m)

$= \dfrac{4.29 \times 9.46 \times 10^{15}}{3.08 \times 10^{16}}$ parsec

($\because$ 1 parsec $= 3.08 \times 10^{16}$ m)

$= 1.318$ parsec $= 1.32$ parsec

Parallax of the star,

$$\theta = \frac{\text{Arc}}{\text{Radius}} = \frac{b}{s} = \frac{2 \text{ AU}}{1.32 \text{ parsec}} = 1.515''$$

$$\left(\because 1'' = \frac{1 \text{ AU}}{1 \text{ parsec}}\right)$$

10. Here, $r = 0.5$ Å $= 0.5 \times 10^{-10}$ m
V_1 = Volume of each hydrogen atom

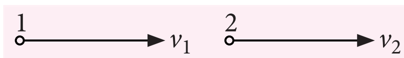
$= \dfrac{4}{3}\pi r^3 = \dfrac{4}{3} \times 3.14 \times (0.5 \times 10^{-10})^3$

$= 5.233 \times 10^{-31} \text{ m}^3$

According to Avogadro's hypothesis, one mole of hydrogen contains :
$N = 6.023 \times 10^{23}$ atoms
$\therefore$ Atomic volume of 1 mole of hydrogen atoms,
$V = NV_1$
or $V = 6.023 \times 10^{23} \times 5.233 \times 10^{-31}$
$= 3.152 \times 10^{-7} \text{ m}^3 \cong 3 \times 10^{-7} \text{ m}^3$

11. The relative velocity of an object 2 with respect to object 1, when both are in motion, is the time rate of change of position of object 2 with respect to that of object 1.

As shown in figure, consider the objects 1 and 2 moving along the same direction with constant velocities v_1 and v_2 (relative to the earth) respectively.



Suppose the position coordinates of the two objects are x_1 (0) and x_2 (0) at time $t = 0$. At time $t = t$, their position coordinates will be

$x_1(t) = x_1(0) + v_1 t$ ...(i)

$x_2(t) = x_2(0) + v_2 t$ ...(ii)

Subtracting (i) from (ii), we find that

$x_2(t) - x_1(t) = x_2(0) - x_1(0) + (v_2 - v_1)t$

or $[x_2(t) - x_2(0)] - [x_1(t) - x_1(0)] = (v_2 - v_1)t$

or Displacement of object 2 in time t
– Displacement of object 1 in time $t = (v_2 - v_1)t$

or Relative displacement of object 2 w.r.t. object 1 in time $t = (v_2 - v_1)t$

or $\dfrac{\text{Relative displacement of object 2 w.r.t.object 1}}{\text{Time } t}$
$= v_2 - v_1$

or Relative velocity of object 2 w.r.t. object 1,
$v_{21} = v_2 - v_1$

Similarly, relative velocity of object 1 w.r.t. object 2,
$v_{12} = v_1 - v_2$

12. The expression which shows how and which of the fundamental quantities represent the dimensions of a physical quantity is called the dimensional formula of the given physical quantity.
The dimensional formula of the volume is $[M^0L^3T^0]$ and that of momentum is $[MLT^{-1}]$.
The equation obtained by equating a physical quantity with its dimensional formula is called the dimensional equation of the given physical quantity.
The dimensional equation for force is
$[\text{Force}] = [MLT^{-2}]$
The dimensional equation for pressure is
$[\text{Pressure}] = [ML^{-1}T^{-2}]$
Dimensional formulae for force is $[MLT^{-2}]$ and for pressure is $[ML^{-1}T^{-2}]$.

13. Let t_1 be the time during which the car accelerates and t_2 be the time for its deceleration.
(a) Let v_m be the maximum velocity reached.
As $v = v_0 + at$,

For time t_1, $v_m = 0 + \alpha t_1$, or $t_1 = \dfrac{v_m}{\alpha}$ ...(i)

For time t_2, $0 = v_m - \beta t_2$, or $t_2 = \dfrac{v_m}{\beta}$ ...(ii)

As $t_1 + t_2 = t$,

$$\frac{v_m}{\alpha} + \frac{v_m}{\beta} = t \quad \text{or} \quad v_m\left(\frac{\alpha+\beta}{\alpha\beta}\right) = t$$

$$\text{or } v_m = \frac{\alpha\beta t}{(\alpha+\beta)} \quad \text{...(iii)}$$

(b) As $s = v_0 t + \dfrac{1}{2}at^2$,

for time t_1, $s_1 = \dfrac{1}{2}\alpha t_1^2$ (as $v_0 = 0$, $a = \alpha$, $t = t_1$)

and for time t_2, $s_2 = v_m t_2 + \dfrac{1}{2}(-\beta)t_2^2$
(as $v_0 = v_m$, $a = -\beta$, $t = t_2$)

Total distance covered, $s = s_1 + s_2$

$$\text{or} \quad s = \frac{1}{2}\alpha t_1^2 + \left(v_m t_2 - \frac{1}{2}\beta t_2^2\right)$$

$$\text{or} \quad s = \frac{1}{2}\alpha\left(\frac{v_m}{\alpha}\right)^2 + v_m \times \frac{v_m}{\beta} - \frac{1}{2}\beta\left(\frac{v_m}{\beta}\right)^2$$

(Using (i) and (ii))

$$s = \frac{v_m^2}{2\alpha} + \frac{v_m^2}{\beta} - \frac{v_m^2}{2\beta} = \frac{v_m^2}{2\alpha} + \frac{v_m^2}{2\beta} = \frac{1}{2}v_m^2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

$$\text{or} \quad s = \frac{1}{2}\left(\frac{\alpha\beta t}{\alpha+\beta}\right)^2\left(\frac{\alpha+\beta}{\alpha\beta}\right) = \frac{\alpha\beta t^2}{2(\alpha+\beta)} \quad \text{(Using (iii))}$$

14. The most precise device is that whose least count is minimum.
Now,
(a) Least count of vernier callipers

$$= 1\text{ MSD} - 1\text{ VSD} = 1\text{ MSD} - \frac{19}{20}\text{MSD} = \frac{1}{20}\text{MSD}$$

$$= \frac{1}{20}\text{ mm} = \frac{1}{200}\text{ cm} = 0.005\text{ cm}$$

(b) Least count of screw gauge

$$= \frac{\text{pitch}}{\text{number of divisions on circular scale}}$$

$$= \frac{1}{100}\text{ mm} = \frac{1}{1000}\text{ cm} = 0.001\text{ cm}$$

(c) Wavelength of light, $\lambda \approx 10^{-5}$ cm = 0.00001 cm
$\therefore$ Least count of optical instrument = 0.00001 cm
Thus, clearly the optical instrument is the most precise instrument.

15. Figure (a) : The (x-t) graph shows that initially x is equal to 0, attains a certain value of x, again x becomes zero and then x increases in opposite direction till it again attains a constant x (*i.e.* comes to rest). Therefore, it may represent a physical situation such as a ball (initially at rest) on being kicked, rebounds from the wall with reduced speed and then moves to the opposite wall and then stops.

Figure (b) : From the (v-t) graph, it follows that the velocity changes sign again and again with the passage of time and every time losing some speed. Therefore, it may represent a physical situation such as a ball falling freely (after being thrown up), on striking the ground rebounds with reduced speed after each hit against the ground.

Figure (c) : The (a-t) graph shows that the body gets accelerated for a short duration only. Therefore, it may represent a physical situation such as a ball moving with uniform speed is hit with a bat for a very small time interval.

OR

(a) A line drawn for a given time parallel to position axis will cut the graph at two points which means that at a given instant of time, the particle will have two positions which is not possible. Hence graph (a) is not possible *i.e.*, does not represent one dimensional motion.

(b) This graph does not represent one dimensional motion because at a given instant of time, the particle will have two values of velocity in positive, as well as in negative direction which is not possible in one dimensional motion.

(c) This is a graph between speed and time and does not represent one dimensional motion as this graph tells that the particle can have the negative speed but the speed can never be negative.

(d) This does not represent one dimensional motion, as this graph tells that the total path length decreases after certain time but total path length of a moving particle can never decrease with time.

16. Here, 1 u = 1.6605×10^{-27} kg, $c = 2.998 \times 10^{8}$ m s^{-1}

(a) As we know, $E = mc^2 = 1\text{ u }(c^2)$

$= (1.6605 \times 10^{-27}\text{ kg}) \times (2.998 \times 10^{8}\text{ m s}^{-1})^2$

$\Rightarrow E = 1.4924 \times 10^{-10}$ J

$$= \frac{1.4924 \times 10^{-10}}{1.6012 \times 10^{-19}} \approx 9.315 \times 10^{8}\text{ eV } = 931.5\text{ MeV}$$

(b) As $E = mc^2$, $m = E/c^2$

According to this, 1 u = 931.5 MeV/c^2

or 1 u c^2 = 931.5 MeV

Hence, 1 u = 931.5 MeV is incorrect relation.

17. Let $|v_A|$ and $|v_B|$ be the speeds of the two trains (say A and B).

Total distance to be covered in each case = sum of the lengths of the two trains = 200 m

While one train overtakes the other, relative speed between the two trains moving in the same direction, *i.e.*, $|v_{\text{same}}| = |v_A| - |v_B|$

And while crossing each other, relative speed between the two trains moving in the opposite directions, *i.e.*, $|v_{\text{opp}}| = |v_A| + |v_B|$

According to the given condition,

$$\frac{200}{|v_{\text{same}}|} = 20, \quad \text{or} \quad |v_{\text{same}}| = |v_A| - |v_B| = 10$$

$$\text{and} \quad \frac{200}{|v_{\text{opp}}|} = 10 \quad \text{or} \quad |v_{\text{opp}}| = |v_A| + |v_B| = 20$$

Clearly, $|v_A| = 15$ m s^{-1} and $|v_B| = 5$ m s^{-1}

18. In SI unit, $M_1 = 1$ kg, $L_1 = 1$ m, $T_1 = 1$ s

In other proposed unit, $M_2 = \alpha$ kg, $L_2 = \beta$ m, $T_2 = \gamma$ s, $n_1 = 5$ J, $n_2 = ?$

Also, $[U] = [\text{M}^1\text{L}^2\text{T}^{-2}]$

We know, $n_1 u_1 = n_2 u_2$

$$\Rightarrow n_2 = n_1\left(\frac{u_1}{u_2}\right) = n_1\left(\frac{M_1}{M_2}\right)^1\left(\frac{L_1}{L_2}\right)^2\left(\frac{T_1}{T_2}\right)^{-2}$$

$$\Rightarrow n_2 = n_1\left(\frac{1}{\alpha}\right)^1\left(\frac{1}{\beta}\right)^2\left(\frac{1}{\gamma}\right)^{-2} = \frac{n_1\gamma^2}{\alpha\beta^2}$$

$$\Rightarrow n_2 = \frac{5\gamma^2}{\alpha\beta^2}\text{ J which is the required value of energy.}$$

19. As $v = k\sqrt{x}$, $\dfrac{dx}{dt} = kx^{1/2}$ or $\dfrac{dx}{x^{1/2}} = k\,dt$

Integrating both sides within the given limits, we get

$$\int_0^x \frac{dx}{x^{1/2}} = k\int_0^t dt$$

$$\text{or} \quad \left|\frac{x^{1/2}}{1/2}\right|_0^x = k\,|t|_0^t \quad \text{or} \quad 2\sqrt{x} = kt \quad \text{or} \quad \sqrt{x} = \frac{kt}{2}$$

$$\text{Thus, } v = k\sqrt{x} = k\left(\frac{kt}{2}\right) = \left(\frac{k^2}{2}\right)t$$

or $v \propto t$

20. If v_1, v_2, v_3 are the velocities at t_1, t_2 and t_3 respectively, then $v_1 = at_1$, $v_2 = a(t_1 + t_2)$ and $v_3 = a(t_1 + t_2 + t_3)$

$$\text{Clearly, } \bar{v}_1 = \frac{0 + v_1}{2} = \frac{1}{2}at_1$$

$$\bar{v}_2 = \frac{v_1 + v_2}{2} = \frac{at_1 + a(t_1 + t_2)}{2} = at_1 + \frac{1}{2}at_2$$

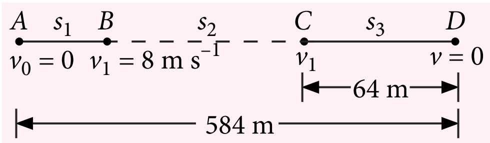
$$\bar{v}_3 = \frac{v_2 + v_3}{2} = \frac{a(t_1 + t_2) + a(t_1 + t_2 + t_3)}{2}$$

$$\bar{v}_3 = a(t_1 + t_2) + \frac{1}{2}at_3$$

Clearly, $\bar{v}_1 - \bar{v}_2 = -\frac{1}{2}a(t_1 + t_2)$

and $\bar{v}_2 - \bar{v}_3 = -\frac{1}{2}a(t_2 + t_3)$

Obviously, $\frac{\bar{v}_1 - \bar{v}_2}{\bar{v}_2 - \bar{v}_3} = \frac{t_1 + t_2}{t_2 + t_3}$

21. 

For the part *AB* of the journey.

From $v_1 = v_0 + at_1$,

$$a = \frac{v_1 - v_0}{t_1} = \frac{8 - 0}{10} = 0.8 \text{ m s}^{-2}$$

$$s_1 = \frac{1}{2}at_1^2 = \frac{1}{2}(0.8)(10)^2 = 40 \text{ m}$$

For the part *CD* of the journey.

From $v^2 - v_1^2 = 2as_3$, $0 - (8)^2 = 2a(64)$,

$a = -0.5 \text{ m s}^{-2}$

Also, from $v = v_1 + at_3$, $0 = 8 + (-0.5)t_3$

or $t_3 = 16$ s

For the part *BC* of the journey.

$s_2 = 584 - (s_1 + s_3) = 584 - (40 + 64) = 480$ m

From $s_2 = v_1 t_2$; $t_2 = \frac{s_2}{v_1} = \frac{480}{8} = 60$ s

Total time taken $= t_1 + t_2 + t_3 = 10 \text{ s} + 60 \text{ s} + 16 \text{ s} = 86$ s

The acceleration of car in first part of journey is 0.8 m s^{-2}, retardation in last part of journey is 0.5 m s^{-2} and total time taken to complete the journey is 86 s.

22. Time for 20 oscillations of a pendulum, $t_1 = 39.6$ s, $t_2 = 39.9$ s, $t_3 = 39.5$ s

According to the given data, least count of stop watch = 0.1 s

Precision for 20 oscillations is the least count of watch *i.e.*, 0.1 s

$\therefore$ Precision for 1 oscillation is $\frac{0.1}{20} = 0.005$ s

Average time for 20 oscillations of the pendulum is given by

$$t = \frac{t_1 + t_2 + t_3}{3} = \frac{39.6 + 39.9 + 39.5}{3} = 39.66 \text{ s} = 39.7 \text{ s}$$

Average time period, $t' = \frac{39.7}{20} = 1.98$ s

Maximum value of time period observed

$$= \frac{39.9}{20} \text{ s} = 1.995 \text{ s}$$

Accuracy of measurement is the maximum observed error, and is given by

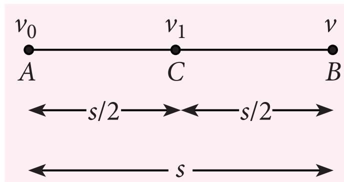
Accuracy $= |1.995 - 1.980| = 0.015$ s

23. (a) Keen observer, inquisitive and a good learner.

(b) The phenomenon involved is parallax.

(c) The line joining the object and the eye is called the line of sight. The direction of the line of sight of the nearby objects like trees changes very rapidly due to the fast motion of the train and so they appear to move in a direction opposite to the train's motion. But the direction of line of sight of a distant object like stars does not change due to large distance from the eye, so the distant objects appear stationary.

24. The journey of the particle is shown in figure, where $AB = s$.



For the path *AB*, $v^2 - v_0^2 = 2as$ ...(i)

If v_1 is the velocity at *C*,

$v_1^2 - v_0^2 = 2a(s/2) = as$ ...(ii)

From eqns. (i) and (ii),

$2v_1^2 - 2v_0^2 = v^2 - v_0^2$

or $v_1^2 = \frac{v^2 + v_0^2}{2}$ or $v_1 = \sqrt{\frac{v^2 + v_0^2}{2}}$ ...(iii)

If *t* is the time for the journey of the particle from *C* to *B*, then as per condition of the problem, time for the journey from *A* to *C* is 2*t*. Clearly, time for the journey from *A* to *B* = 3*t*.

For the path *CB*, $\frac{s}{2} = \left(\frac{v_1 + v}{2}\right)t$ ...(iv)

and for the path *AB*, $s = \left(\frac{v_0 + v}{2}\right)3t$ ...(v)

From eqns. (iv) and (v), we get

$(v_1 + v) = \frac{3}{2}(v_0 + v)$

or $3v_0 + v = 2v_1 = 2\sqrt{\frac{v^2 + v_0^2}{2}}$ (using eqn. (iii)) or $9v_0^2 + v^2 + 6vv_0 = 2v^2 + 2v_0^2$

or $v^2 - 6vv_0 - 7v_0^2 = 0$ or $(v + v_0)(v - 7v_0) = 0$

Since $(v + v_0) \neq 0$, $v - 7v_0 = 0$ or $v = 7v_0$

OR

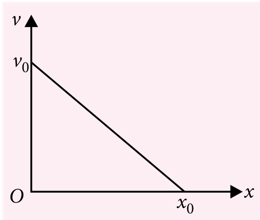
(a) Clearly slope of the graph,

$$m = -\frac{v_0}{x_0}$$

Also, at $x = 0$, $v = v_0$

$$\therefore \quad v = \left(-\frac{v_0}{x_0}\right)x + v_0$$

which is required relation



(b) As, $v = \left(-\frac{v_0}{x_0}\right)x + v_0$

$$\Rightarrow \quad a = \frac{dv}{dt} = \left(-\frac{v_0}{x_0}\right)\frac{dx}{dt} + 0 \Rightarrow \quad a = \left(-\frac{v_0}{x_0}\right)v$$

$$\Rightarrow \quad a = \left(-\frac{v_0}{x_0}\right)\left[\left(-\frac{v_0}{x_0}\right)x + v_0\right]$$

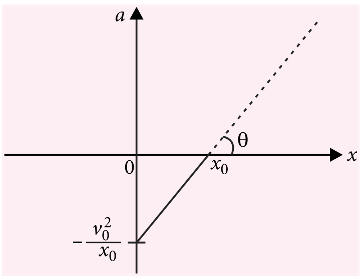
$$\Rightarrow \quad a = \left(\frac{v_0^2}{x_0^2}\right)x - \frac{v_0^2}{x_0}$$

which is required relation between acceleration and displacement.

Compare it with, $y = mx + c$

Here, $m = \frac{v_0^2}{x_0^2}$, $c = -\frac{v_0^2}{x_0}$

Slope is positive and intercept is negative.



25. According to this principle, when a physical equation consists of a number of terms, each of these terms must be of the same dimensions in each of the fundamental units.

Given that, according to Kepler's third law,

$$T^2 \propto r^3 \quad \Rightarrow \quad T \propto r^{3/2}$$

Also, T depends on g and R as given below,

$$T \propto r^{3/2} g^x R^y$$

where x and y are exponents of g and R respectively.

$$\Rightarrow \quad T = kr^{3/2} g^x R^y$$

where k is dimensionless constant of proportionality.

Writing dimensions of the physical quantities, both sides

$[M^0L^0T^1] = [L]^{3/2} [LT^{-2}]^x [L]^y$

$\Rightarrow \quad [M^0 L^0 T^1] = [M^0L^{(3/2 + x + y)}T^{-2x}]$

Using the principle of homogeneity of dimensions

$x + y + \frac{3}{2} = 0$ ... (i) $\quad -2x = 1$ ... (ii)

From eqn. (i) and (ii), $x = -\frac{1}{2}$, $y = -1$

$$\therefore \quad T = kr^{3/2} g^{-1/2} R^{-1} \quad \Rightarrow \quad T = \frac{k}{R}\sqrt{\frac{r^3}{g}}$$

which is required quantity.

OR

(a) We have, $Z = \frac{A^p B^q}{C^r}$

Taking logarithms, we get

$$\log Z = p \log A + q \log B - r \log C$$

On differentiating both sides, we get

$$\frac{dZ}{Z} = p\frac{dA}{A} + q\frac{dB}{B} - r\frac{dC}{C}$$

Writing the above equation in terms of fractional errors,

$$\pm\frac{\Delta Z}{Z} = \pm p\frac{\Delta A}{A} \pm q\frac{\Delta B}{B} \pm r\frac{\Delta C}{C}$$

The maximum permissible error in Z is given by

$$\frac{\Delta Z}{Z} = p\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C}$$

(b) The percentage error in specific resistance σ is given by

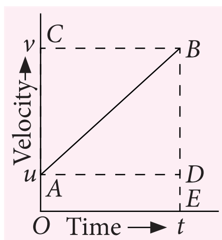
$$\frac{\Delta\sigma}{\sigma} \times 100 = \left(2\frac{\Delta r}{r} + \frac{\Delta R}{R} + \frac{\Delta L}{L}\right) \times 100$$

$$= \left[\frac{2\times 0.02}{0.26} + \frac{1}{32} + \frac{0.01}{78}\right] \times 100$$

$= [0.15 + 0.03 + 0.0001] \times 100$

$= 0.1801 \times 100 = 0.18 \times 100 = 18\%$

26. Consider an object moving along a straight line path with initial velocity u and uniform acceleration a. Suppose it travels distance s in time t. As shown in figure, its velocity-time graph is straight line.



Here $OA = ED = u$, $OC = EB = v$ and $OE = t = AD$.

(a) We know that,

Acceleration = Slope of velocity-time graph AB

or $a = \dfrac{DB}{AD} = \dfrac{EB - ED}{OE} = \dfrac{v - u}{t}$ ...(i)

or $v - u = at$ or $v = u + at$

This proves the first equation of motion.

(b) From eqn. (i), we have

$a = \dfrac{DB}{AD} = \dfrac{DB}{t}$ or $DB = at$

Distance travelled by the object in time t is

s = Area of the trapezium $OABE$

= Area of rectangle $OADE$ + Area of triangle ADB

$= OA \times OE + \dfrac{1}{2} \times DB \times AD = ut + \dfrac{1}{2} at \times t$

or $s = ut + \dfrac{1}{2}at^2$

This proves the second equation of motion.

(c) Distance travelled by object in time t is

s = Area of trapezium $OABE$.

$= \dfrac{1}{2}(EB + OA) \times OE = \dfrac{1}{2}(EB + ED) \times OE$

Acceleration,

a = Slope of velocity - time graph AB

or $a = \dfrac{DB}{AD} = \dfrac{EB - ED}{OE}$ or $OE = \dfrac{EB - ED}{a}$

$\therefore \quad s = \dfrac{1}{2}(EB + ED) \times \dfrac{(EB - ED)}{a}$

$= \dfrac{1}{2a}(EB^2 - ED^2) = \dfrac{1}{2a}(v^2 - u^2)$ or $v^2 - u^2 = 2as$

This proves the third equation of motion.

OR

Velocity of light, $[c] = [\mathrm{LT}^{-1}]$

Planck's constant, $[h] = [\mathrm{ML}^2\mathrm{T}^{-1}]$

Gravitational constant, $[G] = [\mathrm{M}^{-1}\mathrm{L}^3\mathrm{T}^{-2}]$

For mass, let $m = k_1 c^x h^y G^z$

where x, y and z are exponents of c, h and G respectively and k_1 is dimensionless constant.

$\Rightarrow \quad [\mathrm{ML}^0\mathrm{T}^0] = [\mathrm{LT}^{-1}]^x [\mathrm{ML}^2\mathrm{T}^{-1}]^y [\mathrm{M}^{-1}\mathrm{L}^3\mathrm{T}^{-2}]^z$

Using the principle of homogeneity of dimensions,

$y - z = 1$ ... (i)

$x + 2y + 3z = 0$ ... (ii)

$-x - y - 2z = 0$ ... (iii)

Adding eqns. (i), (ii) and (iii),

$2y = 1 \Rightarrow y = 1/2$

From eqn. (i), $z = y - 1 = \dfrac{1}{2} - 1 = -\dfrac{1}{2}$

From eqn. (iii), $x = -(y + 2z) = -\left(\dfrac{1}{2} - 1\right) = \dfrac{1}{2}$

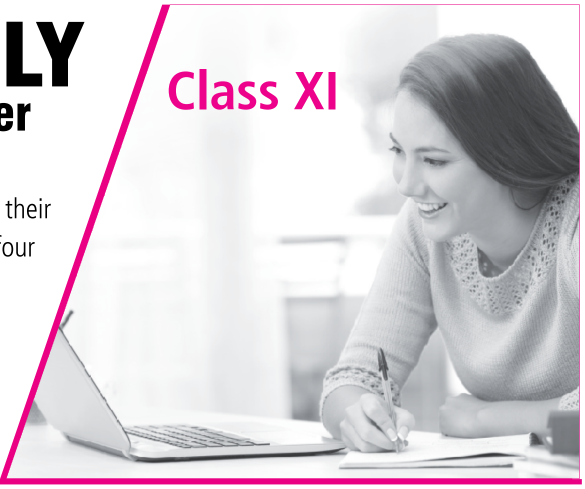
Hence, $m = k_1 c^{1/2} h^{1/2} G^{-1/2} \Rightarrow m = k_1\sqrt{\dfrac{ch}{G}}$

Similarly, for length, $L = k_2\sqrt{\dfrac{hG}{c^3}}$

and for time, $T = k_3\sqrt{\dfrac{hG}{c^5}}$

MPP-3 MONTHLY Practice paper

Class XI



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer.

Self check table given at the end will help you to check your readiness.

Laws of Motion

Total Marks : 120 **Time Taken : 60 min**

NEET / AIIMS

Only One Option Correct Type

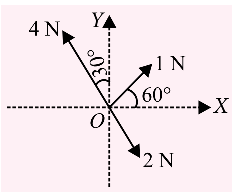
1. A rocket of mass 120 kg is moving vertically up at 600 m s^{-1} such that gas is escaping at the rate of 1 kg s^{-1}. Find out acceleration of the rocket.

(a) 2 m s^{-2} (b) 5 m s^{-2}
(c) 6 m s^{-2} (d) 10 m s^{-2}

2. With increase of temperature, the frictional force acting between two surfaces

(a) increases (b) remains the same
(c) decreases (d) becomes zero

3. Three forces acting on a body are shown in the figure. To have the resultant force only along the Y-axis, the magnitude of the minimum additional force needed along OX is

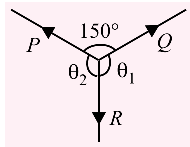


(a) $\frac{\sqrt{3}}{4}$ N (b) $\sqrt{3}$ N
(c) 0.5 N (d) 1.5 N

4. A block is released from an inclined plane of inclination 45°. Find out the acceleration of the block down the incline, if coefficient of friction between block and the incline is 0.5.

(a) $3\sqrt{2}$ m s^{-2} (b) $\sqrt{2}$ m s^{-2}
(c) $\frac{10}{\sqrt{2}}$ m s^{-2} (d) $\frac{5}{\sqrt{2}}$ m s^{-2}

5. P, Q and R are three coplanar forces acting at a point and are in equilibrium. (Given P = 1.9318 kg wt, $\sin\theta_1$ = 0.9659), the value of R (in kg wt) is



(a) 0.9659
(b) 2
(c) 1
(d) 0.5

6. A car of mass m is moving on a level circular track of radius R. If μ_s represents the static friction between the road and tyres of the car, the maximum speed of the car in circular motion is given by

(a) $\sqrt{\mu_s mRg}$ (b) $\sqrt{\frac{Rg}{\mu_s}}$
(c) $\sqrt{\frac{mRg}{\mu_s}}$ (d) $\sqrt{\mu_s Rg}$

7. A car is racing on a circular track of 180 m radius with a speed of 32 m s^{-1}. What should be the banking angle of the road to avoid skidding of the vehicle at this speed without taking into consideration the friction between the tyre and the road ?

(a) 45° (b) 60°
(c) 30° (d) 15°

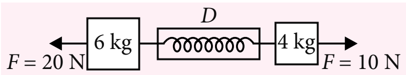
8. Sand is being dropped on a conveyer belt at the rate of M kg s^{-1}. The force necessary to keep the belt moving with a constant velocity of v m s^{-1} will be

(a) $\frac{Mv}{2}$ N (b) zero
(c) Mv N (d) $2Mv$ N

9. A weight W rests on a rough horizontal plane. If the angle of friction be θ, the least force that will move the body along the plane will be

(a) $W \cos\theta$ (b) $W \cot\theta$
(c) $W \tan\theta$ (d) $W \sin\theta$

10. A dynamometer D is attached to two blocks of masses 6 kg and 4 kg. Forces of 20 N and 10 N are applied on the blocks as shown in figure. The dynamometer reads



(a) 10 N (b) 20 N
(c) 6 N (d) 14 N

11. A particle is moving in the $x - y$ plane. At certain instant of time, the components of its velocity and acceleration are as follows : $v_x = 3$ m s^{-1}, $v_y = 4$ m s^{-1}, $a_x = 2$ m s^{-2} and $a_y = 1$ m s^{-2}. The rate of change of speed at this moment is

(a) $\sqrt{10}$ m s^{-2} (b) 4 m s^{-2}
(c) $\sqrt{5}$ m s^{-2} (d) 2 m s^{-2}

12. A particle of mass 2 kg moves with an initial velocity of $(4\hat{i} + 2\hat{j})$ m s^{-1} on the x-y plane. A force $\vec{F} = (2\hat{i} - 8\hat{j})$ N acts on the particle. The initial position of the particle is (2 m, 3 m). Then for $y = 3$ m,

(a) Possible value of x is only $x = 2$ m,
(b) Possible value of x is not only $x = 2$ m, but there exists some other value of x also.
(c) Time taken is 2 s
(d) All of these

Assertion & Reason Type

Directions : In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as :

(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false.
(d) If both assertion and reason are false.

13. Assertion : The driver of a moving car sees a wall in front of him. To avoid collision, he should apply brakes rather than taking a turn away from the wall.
Reason : Frictional force is needed to stop the car or taking a turn on a horizontal road.

14. Assertion : A concept of pseudo force is valid both for inertial as well as non-inertial frame of reference.
Reason : Pseudo force acts along the direction of inertial frame.

15. Assertion : A reference frame attached to the earth is a non-inertial frame of reference.
Reason : Newton's laws can be applied in this frame of reference.

JEE MAIN / JEE ADVANCED

Only One Option Correct Type

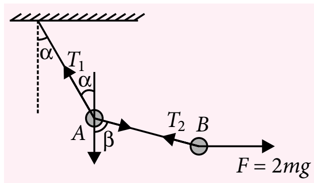
16. A man slides down a light rope whose breaking strength is η times his weight ($\eta < 1$). The maximum acceleration of the man so that the rope just breaks is

(a) $g(1 - \eta)$ (b) $g(1 + \eta)$
(c) $g\eta$ (d) $\dfrac{g}{\eta}$

17. A body placed on the surface of a smooth inclined plane reaches the bottom of the plane in time t. When it is identically placed on a rough inclined plane, it reaches the bottom in time nt where $n > 1$. In both the cases, the angle of inclination θ is same. The coefficient of friction μ is equal to

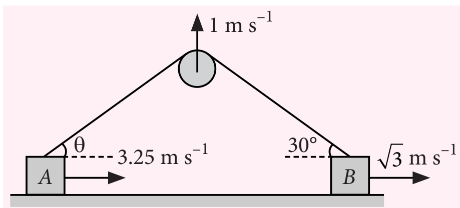
(a) $\left[1 - \dfrac{1}{n^2}\right] \sin\theta$ (b) $\left[1 - \dfrac{1}{n^2}\right] \cos\theta$
(c) $\left[1 - \dfrac{1}{n^2}\right] \tan\theta$ (d) $\tan\theta$

18. Two particles A and B, each of mass m are kept stationary by applying horizontal force $F = 2mg$ on the particle B as shown in figure, then (T_1 and T_2 are tension in the strings as shown in figure).



(a) $\tan\beta = 2$ (b) $T_1 = \sqrt{2}mg$
(c) $T_2 = \sqrt{3}mg$ (d) $\tan\beta = 2\sqrt{2}$

19. In the figure shown, find out the value of θ (assume string to be tight)



(a) $\tan^{-1}\left(\frac{3}{4}\right)$ (b) $\tan^{-1}\left(\frac{4}{3}\right)$

(c) $\tan^{-1}\left(\frac{3}{8}\right)$ (d) None of these

More than One Options Correct Type

20. A car of mass M is moving on a horizontal circular path fo radius R. At an instant its speed is v and is increasing at constant rate β.

(a) The net acceleration of the car is towards the centre of its path.

(b) The magnitude of the frictional force on the car is greater than $\frac{Mv^2}{R}$.

(c) The friction coefficient between the ground and the car is not less than $\frac{\beta}{g}$.

(d) The friction coefficient between the ground and the car is not less than $\frac{v^2}{Rg}$.

21. A particle of mass m starts moving at $t = 0$ due to a force $F = F_0 \cos \omega t$ where F_0 and ω are constant. Which of the following statements are correct ?

(a) It will stop first time at $t = \frac{\pi}{\omega}$

(b) It will travel distance $S = \frac{2F_0}{m\omega^2}$ during time $t = 0$ to $t = \frac{\pi}{\omega}$.

(c) Maximum velocity of particle is $v_{max} = \frac{F_0}{m\omega}$

(d) None of these

22. When a bicycle is in motion, the force of friction exerted by the ground on the two wheels is such that it acts

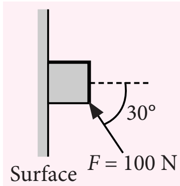
(a) In the backward direction on the front wheel and in the forward direction on the rear wheel.

(b) In the forward direction on the front wheel and in the backward direction on the rear wheel.

(c) In the backward direction on both front and the rear wheels.

(d) In the forward direction on both front and the rear wheels.

23. A force of 100 N is applied on a stationary block of mass 3 kg as shown in figure. If the coefficient of friction between the surface and the block is 0.25 then



(a) The frictional force acting on the block is 20 N downwards

(b) The normal reaction on the block is $50\sqrt{3}$ N

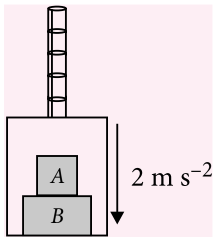
(c) The friction force (kinetic) acting on the block is $\frac{25\sqrt{3}}{2}$ N

(d) If coefficient of friction is changed to 0.35 then the friction force acting on the block is again 20 N downwards.

Integer Answer Type

24. A block is moving on an inclined plane making an angle 45° with the horizontal and the coefficient of friction is μ. The force required to just push it up the inclined plane is three times the force required to just prevent it from sliding down. If we define $N = 10\mu$, then N is

25. The elevator shown in figure is descending with an acceleration of 2 m s^{-2}. The mass of the block A = 0.5 kg. Find the force (in N) exerted by block A on block B.

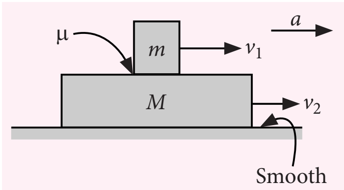


26. Two blocks m_1 = 4 kg and m_2 = 2 kg connected by a weightless rod slide down a plane having an inclination of 37°. The coefficient of dynamic friction of m_1 and m_2 with the inclined plane are

$\mu_1 = 0.75$ and $\mu_2 = 0.25$ respectively. The tension (in N) in the rod is approximately (Take sin 37° = 0.6 and cos 37° = 0.8)

Comprehension Type

A small block of mass m is placed over a long plank of mass M. The coefficient of friction between them is μ. Ground is smooth. At $t = 0$, m and M are given velocities v_1 and v_2 ($> v_1$) respectively as shown in figure. After this M is maintained at constant acceleration a ($< \mu g$).



Initially there will be some relative motion between the block and the plank, but after some time relative motion will cease and velocities of both will become same.

27. Find the time t_0 when the velocities of both block and plank become same.

(a) $\dfrac{v_2 - v_1}{\mu g + a}$ (b) $\dfrac{v_2 + v_1}{\mu g - a}$

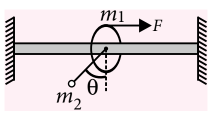
(c) $\dfrac{v_2 - v_1}{\mu g - a}$ (d) $\dfrac{v_2 + v_1}{\mu g + a}$

28. Find the forward force acting on the plank before and after t_0, respectively.

(a) Ma and $(M + m)\,a$
(b) $\mu mg + Ma$ and $(M + m)\,a$
(c) $\mu Mg + ma$ and Ma
(d) $(M + m)\,a$ and $\mu mg + Ma$

Matrix Match Type

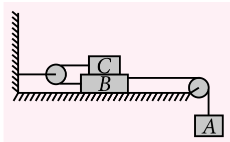
29. A horizontal force F pulls a ring of mass m_1 such that θ remains constant with time. The ring is constrained to move along a smooth rigid horizontal wire. A bob of mass m_2 hangs from m_1 by an inextensible light string. Then match the entries of column I with that of column II.



Column I	Column II
(A) F	(P) $(m_1 + m_2)g$
(B) Force acting on m_2	(Q) $m_2 g \sec\theta$
(C) Tension in the string	(R) $m_2 \dfrac{F}{m_1 + m_2}$
(D) Force acting on m_1 by the wire	(S) $(m_1 + m_2)\, g \tan\theta$

	A	B	C	D
(a)	P	R	Q	S
(b)	S	R	Q	P
(c)	S	Q	R	P
(d)	R	Q	S	P

30. When the system shown in figure is released, A accelerates downwards.



Match the entries of column I with column II.

Column I	Column II
(A) Acceleration of B	(P) Towards left
(B) Acceleration of C w.r.t B	(Q) Towards right
(C) Acceleration of A w.r.t. C	(R) At some angle θ with horizontal ($0 < \theta < 90°$)
(D) Acceleration of B w.r.t A	(S) At some angle θ with vertical ($0 < \theta < 90°$)

	A	B	C	D
(a)	Q	P	R,S	R,S
(b)	P	Q	R,S	R,S
(c)	Q	R	P	P,S
(d)	S	R	P	Q

◈◈

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ……
No. of questions correct ……
Marks scored in percentage ……

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

BRAIN MAP

WAVES

Class XI

TYPES OF WAVES

Electromagnetic Waves
Waves propagating in form of oscillating electric and magnetic fields.
Do not require medium for propagation.

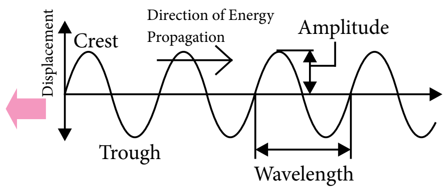
Matter Waves
Waves associative with microscopic particles such as electrons, protons etc. in motion are called matter waves.

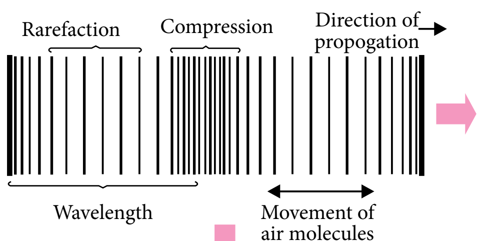
Mechanical Waves
Waves which require a material medium for their propagation are called mechanical waves.

Transverse Waves
The individual particles of the medium oscillate perpendicular to the direction of wave propagation.

Longitudinal Waves
The individual particles of medium oscillate along the direction of wave propagation.

Velocity of Transverse Wave in Solids and Strings
- In solids, $v = \sqrt{\frac{\eta}{\rho}}$
  where η is modulus of rigidity and ρ is density of solids.
- In stretched string, $v = \sqrt{\frac{T}{m}}$
  here, T is tension in string and m is mass per unit length of string.





Velocity of Longitudinal Waves
- In a solid of bulk modulus κ, modulus of rigidity η and density ρ is
  $$v = \sqrt{\frac{\kappa + \frac{4}{3}\eta}{\rho}}$$
- In a fluid of bulk modulus κ and density ρ is
  $$v = \sqrt{\frac{\kappa}{\rho}}$$
- Newton's formula for the velocity of sound in a gas is
  $$v = \sqrt{\frac{\kappa_{iso}}{\rho}} = \sqrt{\frac{P}{\rho}} \ (P = \text{pressure of the gas})$$

WAVE MOTION

Progressive Waves
- **Displacement,** $y = A\sin(\omega t - kx + \phi_0)$
  $y = A\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) = A\sin\frac{2\pi}{\lambda}(vt - x)$
- **Phase,** $\phi = 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + \phi_0$
  where ϕ_0 is the initial phase.
- **Phase change :**
  (a) with time $\Delta\phi = \frac{2\pi}{T}\Delta t$ (b) with position $\Delta\phi = \frac{2\pi}{\lambda}\Delta x$.

Superposition of Waves
Identical waves of same speed superposes in opposite direction

Waves with same speed and different frequency superposes in same direction

Stationary Waves
- Wave formed by the superposition of incident wave and reflected wave is given by $y = \pm 2a\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi t}{T}$
- Position of antinodes : $x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}$
- Position of nodes : $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}...$
- Frequency of vibration of a string fixed at both ends, $\upsilon = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{T}{m}}$
  L = length of string, n = mode of vibration

- **Open organ pipe :**
  Fundamental mode,
  $\upsilon_1 = v/2L = \upsilon$ (1st harmonic)
  n^{th} mode, $\upsilon_n = nv/2L$ (n^{th} harmonic and $(n-1)^{th}$ overtone)
- **Closed organ pipe :**
  Fundamental mode,
  $\upsilon_1 = v/4L = \upsilon$ (1st harmonic)
  n^{th} mode, $\upsilon_n = (2n-1)\upsilon$
  $[(2n-1)^{th}$ harmonic or $(n-1)^{th}$ overtone$]$

Doppler's Effect in Sound
- If v, v_0, v_s and v_m are the velocities of sound, observer, source and medium respectively, then the apparent frequency,
  $$\upsilon' = \frac{v + v_m - v_0}{v + v_m - v_s} \times \upsilon$$
- If the medium is at rest, $(v_m = 0)$ then
  $$\upsilon' = \frac{v - v_0}{v - v_s} \times \upsilon$$

Beats Formation
- **Beat frequency** = Number of beats sec^{-1}
  = Difference in frequencies of two sources.
  $\upsilon_{beat} = (\upsilon_1 - \upsilon_2)$ or $(\upsilon_2 - \upsilon_1)$
  $\therefore \quad \upsilon_2 = \upsilon_1 + \upsilon_{beat}$
- If prongs of tuning fork is filed υ increases.
- If prongs is loaded with a wax υ decreases.
- **Uses :**
  - For tuning musical instruments
  - For detection of marsh gas in mines
  - For using as a low frequency oscillator.

COMMUNICATION SYSTEMS

Class XII

BRAIN MAP

MODES OF COMMUNICATION

POINT TO POINT COMMUNICATION
Communication takes place over a link between a single transmitter and a single receiver.

BROADCAST MODE COMMUNICATION
Large number of receivers corresponding to a single transmitter.

BASIC COMMUNICATION TECHNIQUE

Transmitter
Converts the message signal suitable for transmission.

Communication Channel
A medium that connects a transmitter to a receiver.

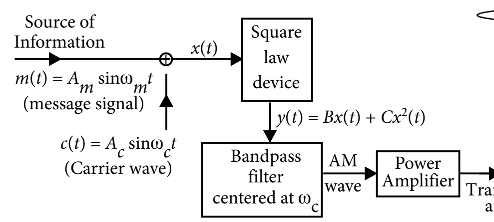
Receiver
Retrieves the message signal into original form.

Modulation
Process of variation of some characteristic of a high frequency wave in accordance with the message signal.

Phase Modulation

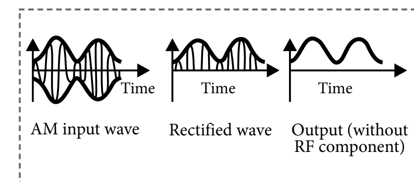
Frequency Modulation

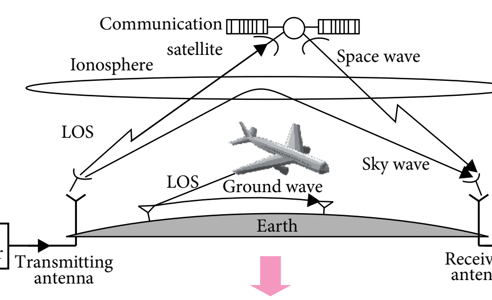
Amplitude Modulation
Amplitude of the high frequency carrier wave changes in accordance with modulating signal.

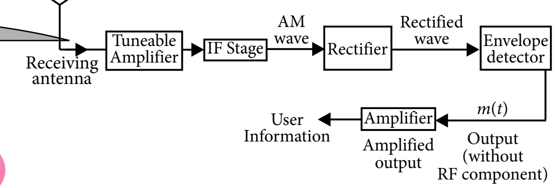


Necessity of Modulation
- *To reduce the size of antenna, need a high frequency carrier wave.*
- *We need high power transmission as $P \propto (1/\lambda)^2$*
- *To avoid the mixing up of signals a band of frequency is allotted to each user for different radio channels.*

Demodulation
Process of recovering the audio signal from the modulated wave is called demodulation or detection.

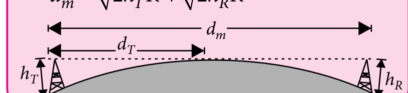






SPACE COMMUNICATION

Space Wave Propagation
A radio wave transmitted from an antenna, directly reaches the receiving antenna by LOS propagation.
- Maximum LOS distance
  $d_m = \sqrt{2h_T R} + \sqrt{2h_R R}$



Range and Application:
- VHF : 30 MHz - 300 MHz
  TV, FM radio, metrology devices
- UHF : 300 MHz - 3 GHz
  TV, aircraft landing systems

Ground Wave Propagation
Here EM wave glides over the earth surface along its curvature from transmitter to receiver placed close to the surface of earth.

Range and Application:
- LF : 30 kHz - 300 kHz
  Long wave radio communication
- MF : 300 kHz - 3 MHz
  AM radio broadcast for local areas

Sky Wave Propagation
The radio wave directed towards the sky and reflected by the ionosphere towards the desired location on the earth.
- Critical frequency $\upsilon_c = 9(N_{max})^{½}$

Range and Application:
- HF : 3 MHz - 30 MHz
  Short wave radio communication, CB radio

NEET | JEE ESSENTIALS

Class XII

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Unit 1 ELECTROSTATICS

Electrostatics is the branch of science which deals with the study of electric charges at rest. Here we study the forces, fields and potentials associated with static charges.

ELECTRIC CHARGES AND COULOMB'S LAW

Electric Charges

Charges are of two types, positive charge and negative charge. The charge developed on a glass rod when rubbed with silk is positive charge. The charge developed on a plastic rod when rubbed with wool is negative charge.

Basic Properties of Charge

- Charge is always associated with mass, *i.e.*, charge can not exist without mass though mass can exist without charge.
- Electric charge is quantised in nature
  $q = ne;\ n = 0, \pm1, \pm2, \pm3, ...$
- The basic unit of charge is the charge that an electron or proton carries . By convention the charge on electron is $-e\,(-1.6\times10^{-19}\,\text{C})$ and charge on proton is $+e\ (1.6\times10^{-19}\ \text{C})$.
- Charge of a system is additive in nature.
- Total charge of an isolated system remains conserved.

Continuous Charge Distribution

- **Linear charge density :** Charge per unit length is known as linear charge density. It is denoted by symbol λ.
  $$\lambda = \frac{\text{Charge}}{\text{Length}} = \frac{q}{l}$$
  Its SI unit is C m^{-1}.
- **Surface charge density :** Charge per unit area is known as surface charge density. It is denoted by symbol σ.
  $$\sigma = \frac{\text{Charge}}{\text{Area}} = \frac{q}{A}$$
  Its SI unit is C m^{-2}.
- **Volume charge density :** Charge per unit volume is known as volume charge density. It is denoted by symbol ρ.
  $$\rho = \frac{\text{Charge}}{\text{Volume}} = \frac{q}{V}$$
  Its SI unit is C m^{-3}.

Coulomb's Law

- The electrostatic force between two stationary charges is proportional to the product of magnitude of charges and inversely proportional to the square of the distance between them.
  i.e., $F \propto \frac{q_1q_2}{r^2}$
  $$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r^2}$$
  (Here $\frac{1}{4\pi\varepsilon_0} = k = 9\times10^9\ \text{N m}^2\text{C}^{-2}$)
  $\varepsilon_0 = 8.854\times10^{-12}\,\text{C}^2\text{N}^{-1}\,\text{m}^{-2}$ is permittivity of free space.

- Coulomb's law in vector form can be written as

$$\vec{F}_{12}=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{|\vec{r}_1-\vec{r}_2|^3}(\vec{r}_1-\vec{r}_2)$$

where the position vectors of charges q_1 and q_2 are $\vec{r}_1$ and $\vec{r}_2$.

- The Coulomb's force between two charged particles in a medium is

$$F_{\text{medium}}=\frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2}=\frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1q_2}{r^2}$$

(Here ε_r = relative permittivity of the medium)

$$F_{\text{medium}}=\frac{F_{\text{vacuum}}}{\varepsilon_r}$$

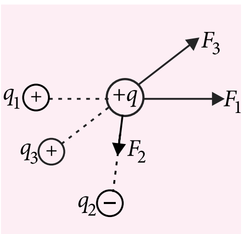
- If a dielectric slab (ε_r) of thickness t is placed between two charges kept at a distance d then the electric force decreases. This reduced force is given by

$$F=\frac{q_1q_2}{4\pi\varepsilon_0 r^2} \text{ where } r=d-t+t\sqrt{\varepsilon_r}$$

- If $F_g = F_e$ for two identical charges, then

$$\frac{q}{m}=\sqrt{4\pi\varepsilon_0 G}$$

- **Superposition principle :** The net force on any one charge is the vector sum of all the forces exerted on it due to each of the other charges interacting with it independently *i.e,*



Total force on charge q,

$$\vec{F}=\vec{F}_1+\vec{F}_2+\vec{F}_3+.......$$

Illustration 1. Two identical balls, each having a density ρ, are suspended from a common point by two insulating strings of equal length. Both the balls have equal mass and charge. In equilibrium, each string makes an angle θ with the vertical. Now, both the balls are immersed in a liquid. As a result, the angle θ does not change. The density of the liquid is σ. The dielectric constant of the liquid is

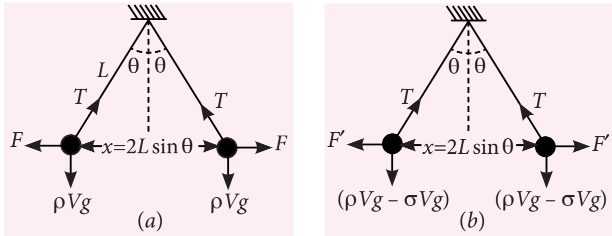
(a) $\dfrac{\rho}{\rho-\sigma}$ (b) $\dfrac{\rho-\sigma}{\sigma}$ (c) $\dfrac{\rho}{\sigma}$ (d) $\dfrac{\sigma}{\rho-\sigma}$

Sol. (a) : Let V be the volume of each ball, then the mass of each ball is $m = \rho V$. When the balls are in air,

$$T\cos\theta = \rho Vg$$

$$T\sin\theta = F$$

$$F = mg\tan\theta = \rho Vg\tan\theta \quad ...(i)$$



When the balls are suspended in liquid, the Coulombic force is reduced to $F' = F/K$ and

apparent weight = weight – upthrust

$$W' = \rho Vg - \sigma Vg$$

According to the question, angle θ is unchanged. Therefore,

$$F' = W'\tan\theta = (\rho Vg - \sigma Vg)\tan\theta \quad ...(ii)$$

From eqs. (i) and (ii), we get

$$\frac{F}{F'}=\frac{\rho Vg}{\rho Vg-\sigma Vg}=\frac{\rho}{\rho-\sigma}$$

$$\therefore\ K=\frac{\rho}{\rho-\sigma}$$

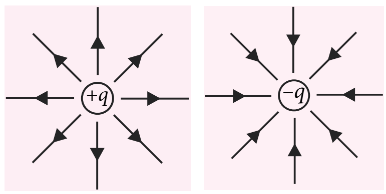
ELECTRIC FIELD

- An electric field is a region where an electric charge experiences a force. The magnitude of $\vec{E}$ is the force per unit charge and its direction is that of $\vec{F}$.

$$\vec{E}=\frac{\vec{F}}{q_0}=\frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$$

Thus, electric field is a vector quantity.
The unit of E is newton per coulomb (N C^{-1}).

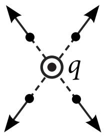
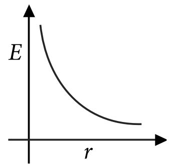
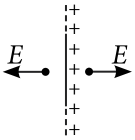
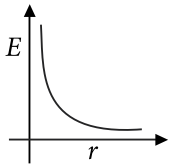
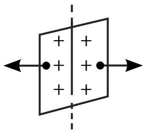
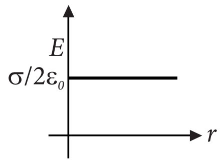
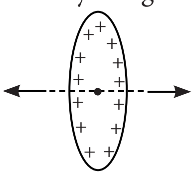
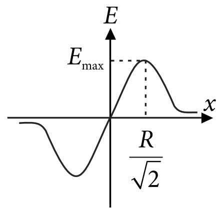
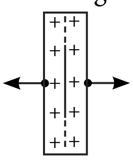
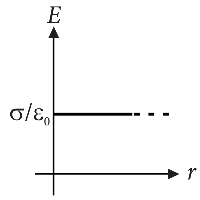
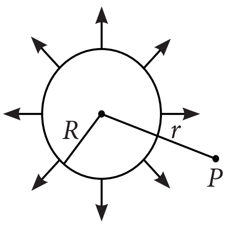
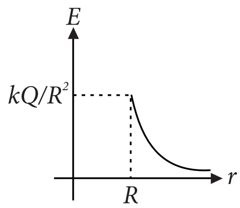
- **Electric Field Lines :** The field lines at a point, (or the tangent to it if it is curved) gives the direction of $\vec{E}$ at that point, *i.e.*, the direction in which positive charge would move and the number of lines per unit cross-sectional area is proportional to E.



(i) (ii)

- Crowded lines represent strong field while distant lines represent weak field.
- A small metal ball is suspended in a uniform $\vec{E}$ with the help of insulated thread. If X-ray beam falls on it, the ball will be deflected in the direction of $\vec{E}$.

- **Electric field intensity due to various types of charge distribution :**

Name /Type	Formula	Note	Graph
Point charge 	$\vec{E} = \frac{kq}{\lvert\vec{r}\rvert^2}\cdot\hat{r}$	▸ q is a source charge. ▸ $\vec{r}$ is vector drawn from source charge to the test point. ▸ outwards due to +ve charges and inwards due to –ve charges	
Infinitely long line charge 	$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r} = \frac{2k\lambda\hat{r}}{r}$	▸ λ is linear charge density (assumed uniform). ▸ r is perpendicular distance of point from line charge. ▸ $\hat{r}$ is radial unit vector drawn from the charge to test point.	
Infinitely non-conducting this sheet 	$\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$	▸ σ is surface charge density (assumed uniform). ▸ $\hat{n}$ is normal unit vector	
Uniformly charged ring 	$E = \frac{kQx}{(R^2+x^2)^{3/2}}$ $E_{\text{centre}} = 0$	▸ Q is total charge on the ring ▸ x = distance of point on the axis from centre of the ring. ▸ Electric field is always along the axis.	
Infinitely large charged conducting sheet 	$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$	▸ σ is the surface charge density (assumed uniform). ▸ $\hat{n}$ is the unit vector perpendicular to the surface.	
Uniformly charged hollow shell or solid conducting sphere 	(i) For $r \geq R$, $\vec{E} = \frac{kQ}{\lvert\vec{r}\rvert^2}\hat{r}$ (ii) For $r < R$, $E = 0$ (iii) For $r = R$, $\vec{E} = \frac{kQ}{R^2}\hat{r}$	▸ R is radius of the sphere. ▸ $\vec{r}$ is vector drawn from centre of sphere to the point. ▸ Sphere acts like a point charge placed at the centre for points outside the sphere. ▸ $\vec{E}$ is always along radial direction. ▸ Q is the total charge ($= \sigma 4\pi R^2$) (σ = surface charge density)	

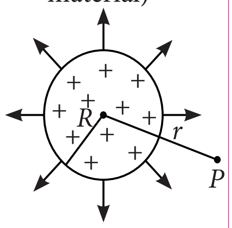
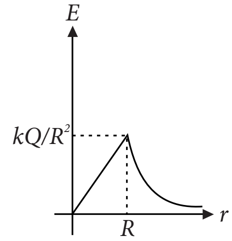
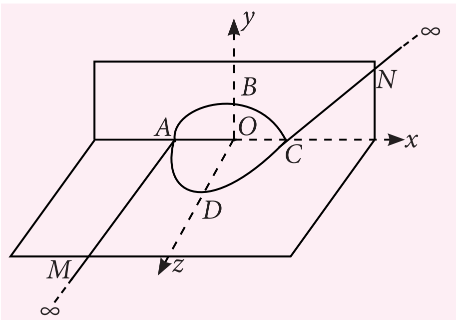
Uniformly charged solid non-conducting sphere (insulating material) 	(i) For $r \geq R$, $\vec{E} = \frac{kQ}{\lvert\vec{r}\rvert^2}\hat{r}$ (ii) For $r < R$, $\vec{E} = \frac{kQ}{R^3}\vec{r}$ (iii) For $r = R$, $\vec{E} = \frac{kQ}{R^2}\hat{r}$	▸ $\vec{r}$ is vector drawn from centre of sphere to the point. ▸ Sphere acts like a point charge placed at the centre for points outside the sphere. ▸ $\vec{E}$ is always along radial direction. ▸ Q is total charge $\left(\frac{4}{3}\pi R^3\rho\right)$, ($\rho$ = volume charge density). ▸ Inside the sphere, $E \propto r$ ▸ Outside the sphere, $E \propto \frac{1}{r^2}$	

Illustration 2. Two semicircular wires ABC and ADC, each of radius R, are lying on xy and xz planes, respectively, as shown in figure.



If the linear charge density of the semicircular parts and straight parts is λ. The electric field intensity $\vec{E}$ at the origin is

(a) $\frac{\lambda}{\varepsilon_0 R}(\hat{i}+\hat{j})$ (b) $\frac{-\lambda}{2\pi\varepsilon_0 R}(\hat{j}+\hat{k})$

(c) $\frac{\lambda}{4\pi\varepsilon_0 R}(-\hat{k})$ (d) $\frac{\lambda}{2\pi\varepsilon_0 R}(-\hat{j})$

Sol. (b) : Electric field due to MA; $\vec{E}_1 = \frac{\lambda}{4\pi\varepsilon_0 R}(\hat{i}-\hat{k})$

Electric field due to ADC; $\vec{E}_2 = \frac{\lambda}{4\pi\varepsilon_0 R}(-\hat{k})$

Electric field due to ABC; $\vec{E}_3 = \frac{\lambda}{4\pi\varepsilon_0 R}(-\hat{j})$

Electric field due to NC; $\vec{E}_4 = \frac{\lambda}{4\pi\varepsilon_0 R}(-\hat{i}-\hat{j})$

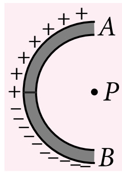
Net electric field is

$$\vec{E}_0 = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 = \frac{-\lambda}{2\pi\varepsilon_0 R}(\hat{j}+\hat{k})$$

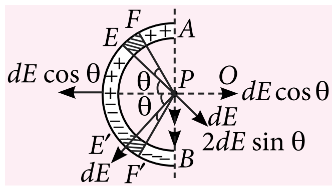
Illustration 3. A thin glass rod is bent into a semicircle of radius r. A charge $+Q$ is uniformly distributed along the upper half, a charge $-Q$ is uniformly distributed along the lower half, as shown in figure. The electric field at P, the center of the semicircle, is

(a) $\frac{Q}{\pi^2\varepsilon_0 r^2}$ (b) $\frac{Q}{\sqrt{2}\,\pi^2\varepsilon_0 r^2}$

(c) $\frac{2\sqrt{2}Q}{\pi^2\varepsilon_0 r^2}$ (d) $\frac{\sqrt{2}Q}{\pi^2\varepsilon_0 r^2}$



Sol. (a) : The PO as the x-axis and PA as the y-axis. Consider two elements EF and $E'F'$ of width $d\theta$ at angular distance θ above and below PO, respectively.



The magnitude of the field at P due to either element is

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{rd\theta \times Q/(\pi r/2)}{r^2} = \frac{Q}{2\pi^2\varepsilon_0 r^2}d\theta$$

Resolving the fields, we find that the components along PO sum up to zero, and hence the resultant field is along PB.

Therefore, field at P due to pair of elements is $2dE \sin\theta$.

$$\therefore E = \int_0^{\pi/2} 2dE\sin\theta = 2\int_0^{\pi/2}\frac{Q}{2\pi^2\varepsilon_0 r^2}\sin\theta d\theta = \frac{Q}{\pi^2\varepsilon_0 r^2}$$

ELECTRIC FLUX AND GAUSS'S LAW

Electric Flux

If the lines of force pass through a surface then the surface is said to have flux linked with it. It is given by $d\phi = \vec{E}\cdot d\vec{S}$ ($d\vec{S}$ is the area vector of the small area element)

- The total flux linked with whole of the body
  $$\phi = \oint \vec{E}\cdot d\vec{S}$$
  where $\oint$ represents closed integral done for a closed surface.
- The SI unit of electric flux is N m^2 C^{-1} and its dimensional formula is [ML^{3}T^{-3}A^{-1}].
- The value of ϕ is zero in the following circumstances :
  - If a dipole is (or many dipoles are) enclosed by a closed surface.
  - Magnitude of positive and negative charges are equal inside a closed surface.
  - If no charge is enclosed by a closed surface.
  - Incoming flux (–ve) = outgoing flux (+ve)

Gauss's Law

It states that the total electric flux through a closed surface S is $1/\varepsilon_0$ times the total charge enclosed by S.

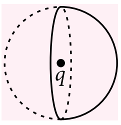
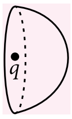
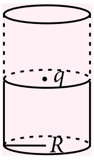
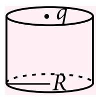
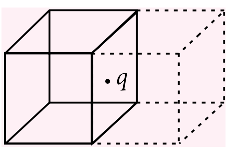
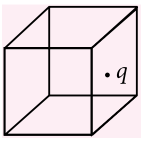
$$\oint \vec{E}\cdot d\vec{S} = \frac{q}{\varepsilon_0}$$

where q is charge enclosed by the closed surface S.

Applications of Gauss's Law

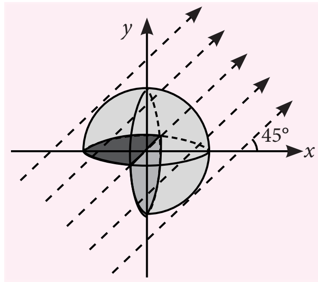
- For an imaginary cube

Position of charge	Main centre	Face centre	Centre of side	Corner
Flux from cube	$\frac{q}{\varepsilon_0}$	$\frac{q}{2\varepsilon_0}$	$\frac{q}{4\varepsilon_0}$	$\frac{q}{8\varepsilon_0}$

-  $\phi_{\text{Total}} = \frac{q}{\varepsilon_0} \Rightarrow$  $\phi_{\text{hemisphere}} = \frac{q}{2\varepsilon_0}$
-  $\phi_{\text{Total}} = \frac{q}{\varepsilon_0} \Rightarrow$  $\phi_{\text{cylinder}} = \frac{q}{2\varepsilon_0}$
-  

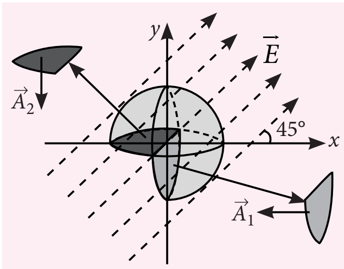
  $\phi_{\text{Total}} = \frac{q}{\varepsilon_0}$ $\qquad$ $\phi_{\text{cube}} = \frac{q}{2\varepsilon_0}$

Illustration 4.
One-fourth of a sphere of radius R is removed as shown in figure. An electric field E exists parallel to the xy plane. Find the flux through the remaining curved part.



(a) $\pi R^2 E$ (b) $\sqrt{2}\pi R^2 E$
(c) $-\pi R^2 E/\sqrt{2}$ (d) none of these

Sol. (c) : $\phi_{\text{plane}} + \phi_{\text{curve}} = 0$ or $\phi_{\text{plane}} = -\phi_{\text{curve}}$



$$\vec{A}_1 = \frac{-\pi R^2}{2}\hat{i},\ \vec{A}_2 = -\frac{\pi R^2}{2}\hat{j}$$

$$\vec{E} = E\cos 45°\,\hat{i} + E\sin 45°\,\hat{j} = \frac{E}{\sqrt{2}}\hat{i} + \frac{E}{\sqrt{2}}\hat{j}$$

$$\phi = \vec{E}\cdot(\vec{A}_1 + \vec{A}_2) = \frac{-E}{\sqrt{2}}\frac{\pi R^2}{2} - \frac{E}{\sqrt{2}}\frac{\pi R^2}{2} = \frac{-\pi R^2 E}{\sqrt{2}}$$

This is the flux entering. So required flux is $\frac{-\pi R^2 E}{\sqrt{2}}$

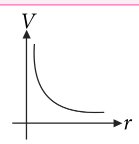
ELECTRIC POTENTIAL AND ELECTRIC POTENTIAL ENERGY

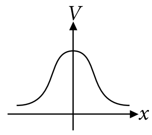
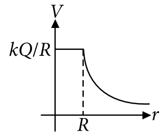
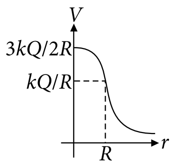
- Electric potential at a point is defined as amount of work done in bringing a unit positive charge from infinity to that point. It is denoted by symbol V.

  $$V = \frac{W}{q}$$

  Electric potential is a scalar quantity. The SI unit and its dimensional formula is [ML^{2}T^{-3}A^{-1}].

- **Electric potential due to various charge distributions :**

Name /Type	Formula	Note	Graph
Point charge	$V = \frac{kq}{r}$	▸ q is source charge. ▸ r is the distance of the point from the point charge.	

Ring (uniform/ non-uniform charge distribution)	$V = \frac{kQ}{R}$, at centre $V = \frac{kQ}{\sqrt{R^2 + x^2}}$, along the axis	▸ Q is charge on the ring. ▸ x is the distance of the point on the axis.	
Uniformly charged hollow conducting/ non-conducting or solid conducting sphere	For $r \geq R$, $V = \frac{kQ}{r}$ For $r \leq R$, $V = \frac{kQ}{R}$	▸ R is radius of sphere ▸ r is the distance from centre of sphere to the point ▸ Q is total charge $= \sigma 4\pi R^2$	
Uniformly charged solid non-conducting sphere	For $r \geq R$, $V = \frac{kQ}{R}$ For $r \leq R, V = \frac{kQ(3R^2 - r^2)}{2R^3}$ $= \frac{\rho}{6\varepsilon_0}(3R^2 - r^2)$	▸ R is radius of sphere ▸ r is distance from centre to the point ▸ $V_{\text{centre}} = \frac{3}{2} V_{\text{surface}}$ ▸ Q is total charge $= \rho \frac{4}{3}\pi R^3$ ▸ Inside sphere potential varies parabolically ▸ Outside potential varies hyperbolically.	
Infinite line charge	▸ Potential difference between two points is given by formula $V_B - V_A = -2k\lambda \ln (r_B/r_A)$	▸ Absolute potential is not defined.	
Infinite non-conducting thin sheet	▸ Potential difference between two points is given by formula $V_B - V_A = -\frac{\sigma}{2\varepsilon_0}(r_B - r_A)$	▸ Absolute potential is not defined.	
Infinite charged conducting thin sheet	▸ Potential difference between two points is given by formula $V_B - V_A = -\frac{\sigma}{\varepsilon_0}(r_B - r_A)$	▸ Absolute potential is not defined.	

Equipotential Surface

An equipotential surface is a surface with a constant value of potential at all points on the surface.

Properties of an equipotential surface

- Electric field lines are always perpendicular to an equipotential surface.
- Work done in moving an electric charge from one point to another on an equipotential surface is zero.
- Two equipotential surfaces can never intersect one another.

Relationship between $\vec{E}$ and V

$$\vec{E} = -\vec{\nabla} V$$

where $\vec{\nabla} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z} \right)$

Negative sign shows that the direction of $\vec{E}$ is the direction of decreasing potential.

Electric Potential Energy

Electric potential energy of a system of charges is

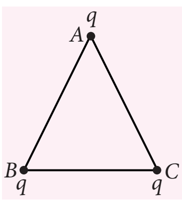
- The total amount of work done in bringing the various charges to their respective positions from infinitely large mutual separations. The SI unit of electrical potential energy is joule.
- Electric potential energy of a system of two charges is $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}}$

  where r_{12} is the distance between q_1 and q_2.
- Electric potential energy of a system of n point charges

$$U = \frac{1}{4\pi\varepsilon_0}\sum_{\text{all pairs}}\frac{q_jq_k}{r_{jk}}$$

- **Note :** In this summation, we should include only one term for each pair of charges.

Illustration 4. Three equal charges q are placed at the corners of an equilateral triangle of side a.



(i) The potential energy of charge system is $\frac{3}{4\pi\varepsilon_0}\frac{q^2}{a}$

(ii) The work required to decrease the side of triangle to $a/2$ is $\frac{3}{4\pi\varepsilon_0}\frac{q^2}{a}$.

(iii) If the charges are released from the shown position and each of them has same mass m, then the speed of each particle when they lie on triangle of side $2a$ is $\frac{1}{2}\sqrt{\frac{q^2}{\pi\varepsilon_0 am}}$.

The correct statement(s) from the above is/are

(a) (i) and (ii) (b) (ii) and (iii)

(c) (ii) only (d) (i), (ii) and (iii)

Sol. (d) : Net potential energy, $U = U_{AB} + U_{BC} + U_{CA}$

$$= 3U_{AB} = 3k\frac{q^2}{a}$$

$$= \frac{3}{4\pi\varepsilon_0}\frac{q^2}{a}$$

Work required to decrease the sides is

$$W = U_f - U_i = \frac{3kq^2}{a/2} - \frac{3kq^2}{a} = \frac{3kq^2}{a} = \frac{3}{4\pi\varepsilon_0}\frac{q^2}{a}$$

By conservation of mechanical energy

$$\Delta U + \Delta K = 0$$

or $$U_i - U_f = K_f - K_i$$

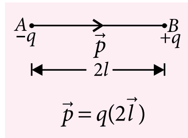
or $$\frac{3kq^2}{a} - \frac{3kq^2}{2a} = 3\left(\frac{1}{2}mv^2\right) - 0$$

or $$v = \sqrt{\frac{kq^2}{am}} = \frac{1}{2}\sqrt{\frac{q^2}{\pi\varepsilon_0 am}}$$

ELECTRIC DIPOLE

- Electric dipole moment,

  $\vec{p} = q(2\vec{l})$



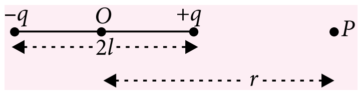
- Electric field at any point on the axial line

$$\vec{E}_{\text{axial}} = \frac{1}{4\pi\varepsilon_0}\frac{2pr}{(r^2 - l^2)^2}\hat{p} \approx \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}\hat{p} \text{ (For } r >> l)$$

- Electric field at any point on the equatorial line

$$E_{\text{eq}} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\hat{p} \qquad \text{(For } r >> l)$$

- Net electric potential at axial point



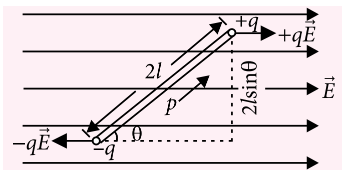
$$V = V_1 + V_2 = \frac{kp}{r^2 - l^2}$$

If $r >> l, V = \frac{kp}{r^2}$

- Net electric potential at equatorial line

$$V = V_1 + V_2 = \frac{kq}{x} - \frac{kq}{x} = 0$$

- In general, $V = \frac{kp\cos\theta}{r^2} = k\frac{\vec{p}\cdot\hat{r}}{r^2}$
- Torque, $\vec{\tau} = \vec{p}\times\vec{E} = pE\sin\theta\,\hat{n}$



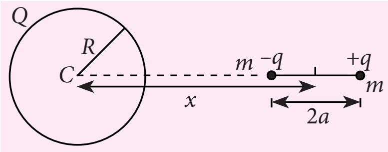
- Potential energy of a dipole is work done in rotating a dipole from a direction perpendicular to the field to the given direction

$$W = \int_{90°}^{\theta}\tau d\theta = \int_{90°}^{\theta} pE\sin\theta d\theta = -pE\cos\theta$$

This work is stored in the form of potential energy.

So, $U = W = -\vec{p}\cdot\vec{E}$

Illustration 5. In figure shown, an electric dipole lies at a distance x from the centre of the axis of a charged ring of radius R with charge Q uniformly distributed over it.



The dipole is slightly rotated about its equilibrium position by a very small angle. What is the time period of oscillation? (Assume that dipole is linearly restrained)

Sol.: Field at a distance x on the axis of a ring is given by

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Qx}{(R^2 + x^2)^{3/2}} \quad \text{...(i)}$$

Restoring torque, $\tau = -pE \sin\theta \approx -pE\theta$ (for small θ) ...(ii)

$$\because \quad \tau = I\frac{d^2\theta}{dt^2} = 2(ma^2)\frac{d^2\theta}{dt^2} \quad \text{...(iii)}$$

From eqn. (ii) and (iii)

$$\text{or} \quad \frac{d^2\theta}{dt^2} = -\frac{pE}{2ma^2}\theta \qquad (\text{using } \alpha = -\omega^2\theta)$$

$$\because \quad T = 2\pi\sqrt{\frac{2ma^2}{pE}} = 2\pi\sqrt{\frac{2ma^2}{2aq}\frac{(R^2+x^2)^{3/2}4\pi\varepsilon_0}{Qx}}$$

$$= \sqrt{\frac{16\pi^3\varepsilon_0 ma}{qQx}}(R^2+x^2)^{3/4}$$

- **Basic electrostatics properties of a conductor**
  - Inside a conductor, electric field is zero.
  - At the surface of a charged conductor, electric field must be normal to the surface at every point.
  - The interior of a conductor can have no excess charge in the static situation.
  - Electric potential is constant throughout the volume of the conductor and has the same value (as inside) on its surface.
  - Electric field at the surface of a charged conductor $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$

  where σ is the surface charge density and $\hat{n}$ is a unit vector normal to the surface in the outward direction.
- **Electrostatic shielding :** It is the phenomenon of protecting a certain region of space from external electric field.

CAPACITOR AND CAPACITANCES

Capacitor

A capacitor is a device that stores electrical energy.

Capacity of capacitor (Capacitance)

Capacitance (C) of a capacitor is the ratio of charge(Q) given and the potential (V) to which it is raised.

$$C = \frac{Q}{V}$$

The SI unit of capacitance is farad (F).

Capacitance is a scalar quantity.

The dimensional formula of capacitance is $[M^{-1}L^{-2}T^4A^2]$.

- Capacitance of a spherical conductor of radius R is $C = 4\pi\varepsilon_0 R$

Capacity of capacitor depends upon

- Total outer surface area
- Dielectric constant of the medium inside the capacitor.
- Separation of two conductors.

Types of Capacitors

- **Parallel plate capacitor :** $C = \frac{\varepsilon_0 A}{d}$

  (when air is between the plates)
- **Spherical capacitor :** $C = 4\pi\varepsilon_0\frac{ab}{b-a}$

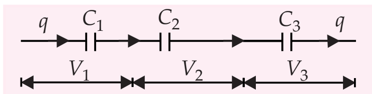
  (a and b are the radii of inner and outer coatings spherical shells).
- **Cylindrical capacitor :**

  $$C = 2\pi\varepsilon_0\frac{l}{\log_e\frac{b}{a}} = 2\pi\varepsilon_0\frac{l}{2.303\log_{10}\frac{b}{a}}$$

  (a and b are the radii of the inner and outer coatings of cylindrical shells)

Combination of Capacitors

Capacitors in series

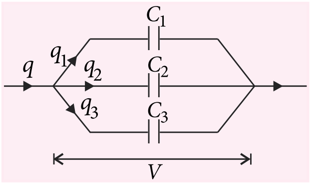


- Same charge flows through each capacitor.
- Different potential difference exists across each capacitor.

$$V = V_1 + V_2 + V_3$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Capacitors in parallel



- Across each capacitor, potential difference is same.
- Different charges flow through each capacitor.

$$q = q_1 + q_2 + q_3$$
$$C = C_1 + C_2 + C_3$$

Thus the ultimate effect is to enhance the capacitance in a circuit.

Energy Stored in a Capacitor

Work done in charging a capacitor gets stored in the capacitor in the form of its electrostatic potential energy.

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$$

Electric Energy Density (u_E)

The energy stored per unit volume in the electric field between the plates is called energy density.

$$u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{\sigma^2}{2\varepsilon_0}.$$

Sharing of Charges

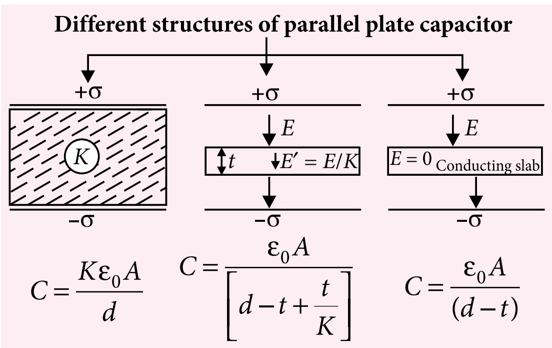
When two capacitors charged to different potentials are connected by a conducting wire. Common potential (V),

$$V = \frac{\text{Total charge}}{\text{Total capacity}} = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

Energy lost in the process which is given by

$$U_1 - U_2 = \frac{C_1C_2(V_1 - V_2)^2}{2(C_1 + C_2)}$$

Effect of Dielectric

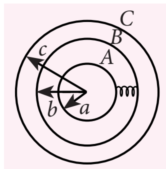


- When a dielectric slab of dielectric constant K is introduced between the plates of a charged parallel plate capacitor and if charging battery remains connected, then, $V = V_0$ (remains constant)
  - Capacitance C increases *i.e.*, $C = KC_0$
  - Charge on a capacitor increases *i.e.*, $Q = KQ_0$
  - Electric field between the plates remains unchanged *i.e.*, $E = E_0$
  - Energy stored in a capacitor increases *i.e.*, $U = KU_0$
- If in this case the charging battery is disconnected then, charge remains unchanged *i.e.*, $Q = Q_0$
  - Capacitance increases *i.e.*, $C = KC_0$
  - Potential difference between the plates decreases *i.e.*, $V = \frac{V_0}{K}$
  - Electric field between the plates decreases *i.e.*, $E = \frac{E_0}{K}$
  - Energy stored in the capacitor decreases *i.e.*, $U = \frac{U_0}{K}$

Polarisation : The dipole moment per unit volume is called polarisation and is denoted by $\vec{P}$. For linear isotropic dielectrics $\vec{P} = \chi_e \vec{E}$,

where χ_e is called the electric susceptibility of the dielectric medium.

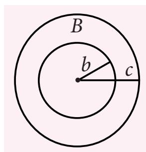
Illustration 6. Three conducting spheres A, B and C are as shown in figure. The radii of the spheres are a, b and c respectively. A and B are connected by a conducting wire. The capacity of the system is :



(a) $4\pi\varepsilon_0 (a + b + c)$ (b) $4\pi\varepsilon_0 \left[\frac{bc}{c - b}\right]$

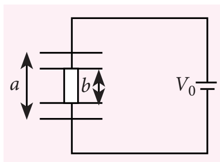
(c) $4\pi\varepsilon_0 \left[\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right]$ (d) $4\pi\varepsilon_0 \left[\frac{abc}{ab + bc + ca}\right]$

Sol. (b) : When A, B are connected then the capacitor formed in between shells A and B is short-circuited *i.e.*, all the charges is transferred to shell B.



Now capacitance $= \frac{4\pi\varepsilon_0 bc}{c - b}$

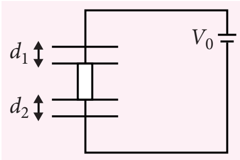
Illustration 7. Figure shows two capacitors in series. The rigid center section of length b is movable. The area of each plate is S. If the voltage difference between the plates is maintained constant at V_0. If the center section is removed the change in stored energy will be



(a) $\dfrac{S\varepsilon_0 V_0^2 a}{2b(a-b)}$ (b) $\dfrac{S\varepsilon_0 V_0^2 b}{2a(a-b)}$

(c) $\dfrac{S\varepsilon_0 V_0^2 b}{a(a-b)}$ (d) $\dfrac{S\varepsilon_0 V_0^2 a}{b(a-b)}$

Sol. (b) : Let d_1 be the distance between the two upper plates and d_2 be the distance between the two lower plates.



$\therefore \quad d_1 + d_2 = a - b$

$\therefore \quad C_1 = \dfrac{\varepsilon_0 S}{d_1},\ C_2 = \dfrac{\varepsilon_0 S}{d_2}$

$\Rightarrow \quad C_{net} = \dfrac{C_1 C_2}{C_1 + C_2} = \dfrac{S\varepsilon_0}{d_1 + d_2} = \dfrac{S\varepsilon_0}{a-b}$

As C_{net} is independent of d_1 and d_2

$\Rightarrow$ energy stored is independent of position of centre section

$$U = \frac{1}{2}CV_0^2 = \frac{S\varepsilon_0 V_0^2}{2(a-b)}$$

If centre section is removed energy stored

$$= U' = \frac{1}{2}CV_0^2 = \frac{S\varepsilon_0 V_0^2}{2a}$$

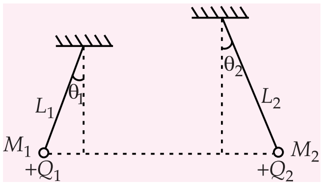
$$\Delta U = U - U' = \frac{S\varepsilon_0 V_0^2 b}{2(a-b)a}$$

SPEED PRACTICE

1. Initially, sphere A has charge of $-50e$ and sphere B has a charge of $+20e$. The spheres are made of conducting material and are identical in size. If the spheres touch each other, what is the resulting charge on sphere A?

(a) $-15e$ (b) $-30e$ (c) $+30e$ (d) $+15e$

2. Two small spheres of masses M_1 and M_2 are suspended by weightless insulating threads of lengths L_1 and L_2. The spheres carry charges Q_1 and Q_2 respectively. The spheres are suspended such that they are in level with one another and the threads are inclined to the vertical at angles of θ_1 and θ_2 as shown. Which one of the following conditions is essential, if $\theta_1 = \theta_2$?

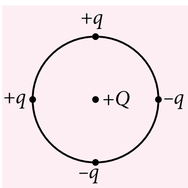


(a) $M_1 \neq M_2$, but $Q_1 = Q_2$

(b) $M_1 = M_2$

(c) $Q_1 = Q_2$ (d) $L_1 = L_2$

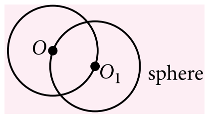
3. Four charges, $+q, +q, -q, -q$ are placed on the circumference of a circle of radius r. What is the force on the charge $+Q$ placed at the centre of the circle as shown in the figure?



(a) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{Qq}{r^2}$ (b) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{\sqrt{8}Qq}{r^2}$

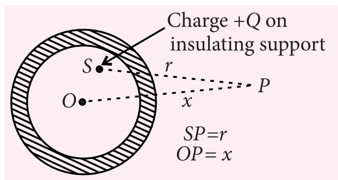
(c) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{8Qq}{r^2}$ (d) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{2Qq}{r^2}$

4. A charge Q is distributed uniformly on a ring of radius r. A sphere of equal radius r is constructed with its centre at the periphery of the ring. The flux of electric field through the sphere is



(a) $\dfrac{Q}{3\varepsilon_0}$ (b) $\dfrac{2Q}{3\varepsilon_0}$ (c) $\dfrac{Q}{2\varepsilon_0}$ (d) $\dfrac{3Q}{4\varepsilon_0}$

5. The adjacent diagram shows a charge $+Q$ held on an insulating support S and enclosed by a hollow spherical conductor. O represents the centre of the spherical conductor and P is a point such that $OP = x$ and $SP = r$. The electric field at point P will be



(a) $\dfrac{Q}{4\pi\varepsilon_0 x^2}$ (b) $\dfrac{Q}{4\pi\varepsilon_0 r^2}$

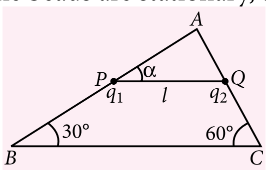
(c) 0 (d) none of the above.

6. If an electron enters into space between the plates of an parallel plate capacitor at an angle α with the plates and leaves at an angle β to the plates, the ratio of its kinetic energy while entering the capacitor to that while leaving will be

(a) $\left(\frac{\sin\beta}{\sin\alpha}\right)^2$ (b) $\left(\frac{\cos\beta}{\cos\alpha}\right)^2$

(c) $\left(\frac{\cos\alpha}{\cos\beta}\right)^2$ (d) $\left(\frac{\sin\alpha}{\sin\beta}\right)^2$

7. A rigid insulated wire frame in the form of a right angled triangle ABC, is set in a vertical plane as shown in figure. Two beads of equal masses m each and carrying charges q_1 and q_2 are connected by a cord of length l and can slide without friction on the wires. If the beads are stationary, then



(a) The angle α is 30°

(b) The tension in the cord is q_1q_2/l^2

(c) The normal reaction on the bead with charge q_1 is $\sqrt{3}\,mg$

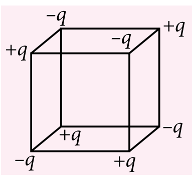
(d) The normal reaction on bead with q_2 is 2 mg

8. Two square metal plates of side 1 m are kept 0.01 m apart like a parallel plate capacitor in air in such a way that one of their edges is perpendicular to oil surface in tank filled with an insulating oil. The plates are connected to a battery of emf 500 V. The plates are then lowered vertically into the oil at a speed of 0.001 m s^{-1}. Calculate the current drawn from the battery during the process. (Dielectric constant of oil =11, $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-1})

(a) 4.425 nA (b) 5.625 nA

(c) 4.125 μA (d) 3.896 μA

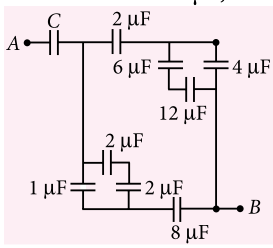
9. Charges $+q$ and $-q$ are located at the corners of a cube of side a as shown in the figure. Find the work done to separate the charges to infinite distance.



(a) $1.154\left(\frac{q^2}{4\pi\varepsilon_0 a}\right)$ (b) $5.824\left(\frac{q^2}{4\pi\varepsilon_0 a}\right)$

(c) $2.309\left(\frac{q^2}{4\pi\varepsilon_0 a}\right)$ (d) $3.535\left(\frac{q^2}{4\pi\varepsilon_0 a}\right)$

10. In the given network, the value of C, so that an equivalent capacitance between A and B is 3 μF, is



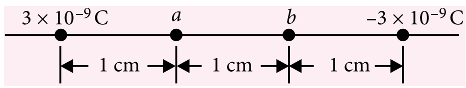
(a) $\frac{1}{5}$ μF

(b) $\frac{31}{5}$ μF

(c) 48 μF

(d) 36 μF

11. In figure, a particle having mass m = 5 g and charge $q' = 2 \times 10^{-9}$ C starts from rest at point a and moves in a straight line to point b. What is its speed v at point b?



(a) 2.65 cm s^{-1} (b) 3.65 cm s^{-1}

(c) 4.65 cm s^{-1} (d) 5.65 cm s^{-1}

12. A parallel plate capacitor of capacity C_0 is charged to a potential V_0. (i) The energy stored in the capacitor when the battery is disconnected and the plate separation is doubled is U_1. (ii) The energy stored in the capacitor when the charging battery is kept connected and the separation between the capacitor plates is doubled is U_2. Then $\frac{U_1}{U_2}$ is

(a) 4 (b) $\frac{3}{2}$ (c) 2 (d) $\frac{1}{2}$

13. A capacitor is charged by a battery. The battery is removed and another identical uncharged capacitor is connected in parallel. The total electrostatic energy of resulting system

(a) decreases by a factor of 2

(b) remains the same

(c) increases by a factor of 2

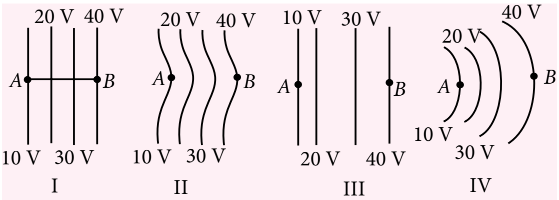
(d) increases by a factor of 4 **[NEET 2017]**

14. Suppose the charge of a proton and an electron differ slightly. One of them is $-e$, the other is $(e + \Delta e)$. If the net of electrostatic force and gravitational force between two hydrogen atoms placed at a distance d (much greater than atomic size) apart is zero, then Δe is of the order of [Given : mass of hydrogen $m_h = 1.67 \times 10^{-27}$ kg]

(a) 10^{-23} C (b) 10^{-37} C

(c) 10^{-47} C (d) 10^{-20} C **[NEET 2017]**

15. The diagrams below show regions of equipotentials.



A positive charge is moved from A to B in each diagram.

(a) In all the four cases the work done is the same.
(b) Minimum work is required to move q in figure (I).
(c) Maximum work is required to move q in figure (II).
(d) Maximum work is required to move q in figure (III). **[NEET 2017]**

16. An electric dipole has a fixed dipole moment $\vec{p}$, which makes angle θ with respect to x-axis. When subjected to an electric field $\vec{E}_1 = E\hat{i}$, it experiences a torque $\vec{T}_1 = \tau\hat{k}$. When subjected to another electric field $\vec{E}_2 = \sqrt{3}E_1\hat{j}$ it experiences a torque $\vec{T}_2 = -\vec{T}_1$. The angle θ is

(a) 30° (b) 45° (c) 60° (d) 90° **[JEE Main Offline 2017]**

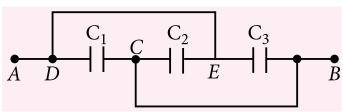
17. A capacitance of 2 μF is required in an electrical circuit across a potential difference of 1.0 kV. A large number of 1 μF capacitors are available which can withstand a potential difference of not more than 300 V. The minimum number of capacitors required to achieve this is

(a) 2 (b) 16 (c) 24 (d) 32 **[JEE Main Offline 2017]**

18. The energy stored in the electric field produced by a metal sphere is 4.5 J. If the sphere contains 4 μC charge, its radius will be

(a) 32 mm (b) 20 mm
(c) 16 mm (d) 28 mm **[JEE Main Online 2017]**

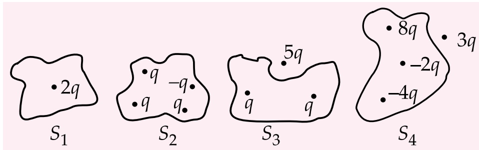
19. A combination of parallel plate capacitors is maintained at a certain potential difference.



When a 3 mm thick slab is introduced between all the plates, in order to maintain the same potential difference, the distance between the plates is increased by 2.4 mm. Find the dielectric constant of the slab.

(a) 6 (b) 5 (c) 4 (d) 3 **[JEE Main Online 2017]**

20. Four closed surfaces and corresponding charge distributions are shown below.



Let the respective electric fluxes through the surfaces be ϕ_1, ϕ_2, ϕ_3 and ϕ_4. Then

(a) $\phi_1 = \phi_2 = \phi_3 = \phi_4$ (b) $\phi_1 > \phi_3; \phi_2 < \phi_4$
(c) $\phi_1 > \phi_2 > \phi_3 > \phi_4$ (d) $\phi_1 < \phi_2 = \phi_3 > \phi_4$ **[JEE Main Online 2017]**

SOLUTIONS

1. (a) : Since, the spheres are identical in size, so when they touch each other,

Charge lost by one = charge gained by other

Let a charge q flows from sphere B to A, then

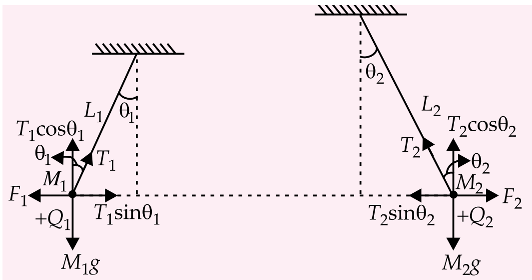
$-50e + q = 20e - q$ or $2q = 20e + 50e = 70e$

or $q = 35e$

$\therefore$ Resulting charge on sphere $A = -50e + 35e$
$= -15e$

2. (b) : The three forces acting on each sphere are:

(i) Tension (ii) Weight
(iii) Electrostatic force of repulsion.



For sphere 1, in equilibrium, from figure

$T_1\cos\theta_1 = M_1g$ and $T_1\sin\theta_1 = F_1$ $\therefore$ $\tan\theta_1 = \dfrac{F_1}{M_1g}$

For sphere 2, in equilibrium, from figure

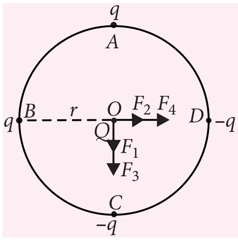
$T_2\cos\theta_2 = M_2g$ and $T_2\sin\theta_2 = F_2$ $\therefore$ $\tan\theta_2 = \dfrac{F_2}{M_2g}$

$$\theta_1 = \theta_2 \text{ only if } \frac{F_1}{M_1g} = \frac{F_2}{M_2g}$$

But $F_1 = F_2$, then $M_1 = M_2$

3. **(b) :** Force on charge Q due to charge q at A,

$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} \text{ along } AO$$



Force on charge Q due to charge q at B,

$$F_2 = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} \text{ along } BO$$

Force on charge Q due to charge $-q$ at C,

$$F_3 = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} \text{ along } OC$$

Force on charge Q due to charge $-q$ at D,

$$F_4 = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} \text{ along } OD$$

$\because \quad |F_1| = |F_2| = |F_3| = |F_4| = F$

The resultant force on charge Q is

$$F_{net} = \sqrt{(2F)^2 + (2F)^2 + 2(2F)(2F)\cos 90^\circ}$$

$$= F\sqrt{8} = \frac{1}{4\pi\varepsilon_0}\frac{\sqrt{8}Qq}{r^2}$$

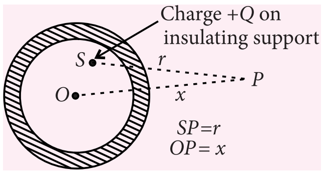
4. **(a)**

5. **(a) :** According to Gauss's theorem,

$$\oint \vec{E}\cdot d\vec{S} = \frac{1}{\varepsilon_0}Q_{enclosed}$$

$$E\cdot 4\pi x^2 = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0 x^2}$$



6. **(b) :** Suppose

u = velocity of electron while entering the field

v = velocity of electron while leaving the field

As electric field is perpendicular to the plates, component of velocity parallel to plates will remain unchanged.

$\therefore \quad u\cos\alpha = v\cos\beta$

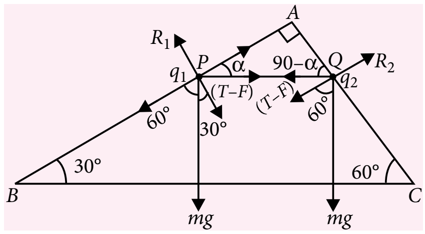
$$\frac{u}{v} = \frac{\cos\beta}{\cos\alpha}$$

$$\frac{\text{K.E. while entering}}{\text{K.E. while leaving}} = \frac{\frac{1}{2}mu^2}{\frac{1}{2}mv^2} = \left(\frac{\cos\beta}{\cos\alpha}\right)^2$$

7. **(c) :** Consider first the equilibrium of bead at P, having charge q_1. Different forces acting on it are shown in the figure. If the charges q_1 and q_2 are similar in nature, then electric force F will be repulsive and is given by

$$F = \frac{q_1 q_2}{l^2}$$

Resolving the forces mg and $(T - F)$ parallel and perpendicular to AB, we get following two equations for equilibrium



$mg\cos 60^\circ = (T - F)\cos\alpha$ ...(i)

$R_1 = mg\cos 30^\circ + (T - F)\sin\alpha$ ...(ii)

Similarly, for the equilibrium of bead at Q

$mg\sin 60^\circ = (T - F)\sin\alpha$ ...(iii)

$R_2 = mg\cos 60^\circ + (T - F)\cos\alpha$ ...(iv)

where T = Tension in the string

Dividing equation (iii) by (i), we get

$\tan 60^\circ = \tan\alpha$

i.e. $\alpha = 60^\circ$

From equation (iii) $T - F = mg$

or $\quad T = F + mg = (q_1q_2/l^2) + mg$

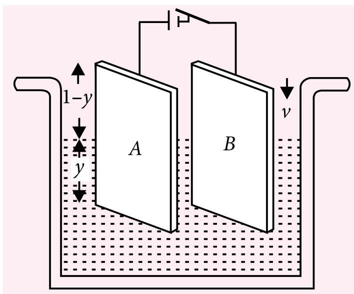
From equation (ii) $R_1 = mg\cos 30^\circ + (T - F)\sin 60^\circ$

$R_1 = mg\cos 30^\circ + mg\sin 60^\circ$

$\therefore \quad R_1 = \sqrt{3}\,mg$

From equation (iv) $R_2 = mg\cos 60^\circ + mg\cos 60^\circ = mg$

8. **(a) :** As here battery remains attached to the circuit V = constant. Now as due to lowering of plates in the oil, capacity will increase gradually and as $q = CV$ the charge on capacitor will increase.



This gradual increase in q results in current drawn from the battery.

If at any instant, y length of plate is dipped in oil, the system will be equivalent to two capacitors in parallel, one with air and the other with oil as dielectric so that

$$C = C_{\text{Air}} + C_{\text{oil}} = \frac{\varepsilon_0 \times 1 \times (1-y)}{0.01} + \frac{\varepsilon_0 \times K \times 1 \times y}{0.01}$$

$$= \frac{\varepsilon_0}{0.01}[1 + (K-1)y]$$

So, $q = CV = \frac{\varepsilon_0 V}{0.01}[1 + y(K-1)]$

And hence, $I = \frac{dq}{dt} = \frac{\varepsilon_0 V}{0.01}[K-1]\frac{dy}{dt}$

So, $I = \frac{8.85 \times 10^{-12} \times 500}{0.01}[11-1] \times 0.001$

$= 4.425 \times 10^{-9}$ A $= 4.425$ nA

9. **(b) :** Potential energy between two charges $= \frac{Q_1 Q_2}{4\pi\varepsilon_0 r}$

In the present case, the charge-pairs are ${}^8C_2$.

$${}^8C_2 = \frac{8 \times 7}{1 \times 2} = 28$$

Of these 28 pairs, we have

(i) 12 pairs of dissimilar charges placed at a mutual separation of a.

(ii) 12 pairs of similar charges placed at a mutual separation of $\sqrt{2}a$.

(iii) 4 pairs of dissimilar charges placed at a mutual separation of $\sqrt{3}a$.

For (i) above, P.E. $= \frac{12(q)(-q)}{4\pi\varepsilon_0 a} = -\frac{12q^2}{4\pi\varepsilon_0 a}$

For (ii) above, P.E. $= \frac{12(q)(q)}{4\pi\varepsilon_0\sqrt{2}\,a} = \frac{12q^2}{4\pi\varepsilon_0 a} \times 0.707$

For (iii) above, P.E. $= \frac{4(q)(-q)}{4\pi\varepsilon_0\sqrt{3}\,a} = -\frac{4q^2}{4\pi\varepsilon_0 a} \times 0.577$

Total Potential energy of system $= U$

$\therefore\ U = -5.824\left(\frac{q^2}{4\pi\varepsilon_0 a}\right)$

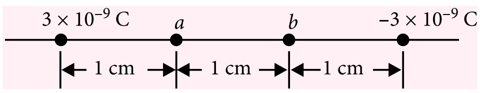
$\therefore$ Binding energy of system $= 5.824\left(\frac{q^2}{4\pi\varepsilon_0 a}\right)$

Hence work done to separate the charge is

$W = 5.824\left(\frac{q^2}{4\pi\varepsilon_0 a}\right)$.

10. **(c)**

11. **(c) :**



According to conservation of energy, we get

$K_a + U_a = K_b + U_b$

Here, $K_a = 0$ and the potential energies are

$U_a = q'V_a$ and $U_b = q'V_b$

$\therefore\ 0 + q'V_a = \frac{1}{2}mv^2 + q'V_b$ or $v = \sqrt{\frac{2q'(V_a - V_b)}{m}}$

$$V_a = (9.0 \times 10^9)\left(\frac{3 \times 10^{-9}\text{ C}}{0.01\text{ m}} + \frac{-3 \times 10^{-9}\text{ C}}{0.02\text{ m}}\right) = 1350\text{ V}$$

$$V_b = (9.0 \times 10^9)\left(\frac{3 \times 10^{-9}\text{ C}}{0.02\text{ m}} + \frac{-3 \times 10^{-9}\text{ C}}{0.01\text{ m}}\right) = -1350\text{ V}$$

$$\therefore\ v = \sqrt{\frac{2(2 \times 10^{-9}\text{ C})(2700\text{ V})}{5 \times 10^{-3}\text{ kg}}} = 4.65 \times 10^{-2}\text{ m s}^{-1}$$

$= 4.65$ cm s^{-1}

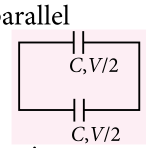
12. **(a)**

13. **(a) :** When the capacitor is charged by a battery of potential V, then energy stored in the capacitor,

$U_i = \frac{1}{2}CV^2$ ...(i)

When the battery is removed and another identical uncharged capacitor is connected in parallel

Common potential,



$V' = \frac{CV}{C+C} = \frac{V}{2}$

$\therefore$ Then the energy stored in the capacitor,

$U_f = \frac{1}{2}(2C)\left(\frac{V}{2}\right)^2 = \frac{1}{4}CV^2$ ...(ii)

$\therefore$ From eqns. (i) and (ii)

$U_f = U_i/2$

that means the total electrostatic energy of resulting system will decreases by a factor of 2.

14. **(b) :** According to questions (for two hydrogen atoms) electrostatic force = Gravitational force

$\therefore$ Charge on one hydrogen atom

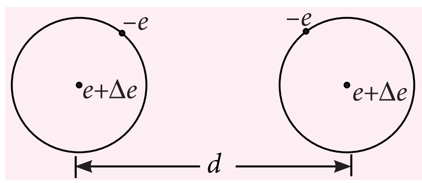
$= q_e + q_p = -e + (e + \Delta e) = \Delta e$

Since a hydrogen atom carry a net charge Δ_e,

$\therefore$ Electrostatic force,

$F_e = \frac{1}{4\pi\varepsilon_o}\frac{(\Delta e)^2}{d^2}$ ...(i)

will act between two hydrogen atoms.



The gravitational force between two hydrogen atoms is given as

$$F_g = \frac{G m_h m_h}{d^2} \quad \ldots(\text{ii})$$

Since, the net force on the system is zero, $F_e = F_g$

Using eqns. (i) and (ii), we get

$$\frac{(\Delta e)^2}{4\pi\varepsilon_o d^2} = \frac{Gm_h^2}{d^2}$$

$(\Delta e)^2 = 4\pi\varepsilon_o G m_h^2$

$= 6.67 \times 10^{-11} \times (1.67 \times 10^{-27})^2/(9 \times 10^9)$

$\Delta e \approx 10^{-37}$ C

15. (a) : Work done is given as $W = q\Delta V$

In all the four cases the potential difference from A to B is same.

$\therefore$ In all the four cases the work done is same.

16. (c) : Dipole moment of fixed dipole can be written as

$$\vec{p} = p\cos\theta\,\hat{i} + p\sin\theta\,\hat{j}$$

For electric field, $\vec{E}_1 = E\,\hat{i}$

Torque on the dipole; $\vec{T}_1 = (\vec{p}\times\vec{E}_1) = pE\sin\theta(-\hat{k})$ … (i)

Now for $\vec{E}_2 = \sqrt{3}E_1\hat{j} = \sqrt{3}E\hat{j}$

In this case, torque on the dipole

$$\vec{T}_2 = (p\cos\theta\,\hat{i} + p\sin\theta\,\hat{j})\times(\sqrt{3}E\,\hat{j})$$

$$\vec{T}_2 = \sqrt{3}pE\cos\theta(\hat{k}) \quad \ldots(\text{ii})$$

Now given, $\vec{T}_2 = -\vec{T}_1$

$$\sqrt{3}pE\cos\theta(\hat{k}) = -pE\sin\theta(-\hat{k})$$

$$\sqrt{3}\cos\theta = \sin\theta \ \text{ or } \ \frac{\sin\theta}{\cos\theta} = \sqrt{3}\,;\ \tan\theta = \sqrt{3}$$

$\Rightarrow$ $\theta = 60°$

17. (d): We have to get equivalent capacitance of 2 μF across 1000 V using 1 μF capacitor.

To obtain the desired capacitance, 8 capacitors of 1 μF should be connected in parallel with four such branches in series as shown in the figure.

$$\frac{1}{C_{eq}} = \frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8} = \frac{4}{8} = \frac{1}{2}\,;\ \therefore\ C_{eq} = 2\ \mu\text{F}$$

$\therefore$ Total number of capacitor used = 8 × 4 = 32

18. (c) : The energy stored in the electric field produced by a metal sphere = 4.5 J

$$\Rightarrow \frac{Q^2}{2C} = 4.5 \ \text{ or } \ C = \frac{Q^2}{2\times 4.5} \quad \ldots(\text{i})$$

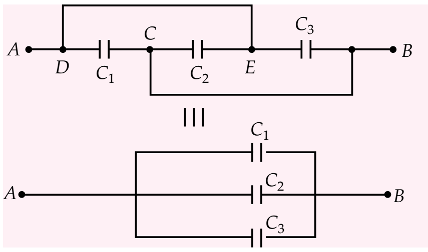
Capacitance of spherical conductor = $4\pi\varepsilon_0 R$

$$C = 4\pi\varepsilon_0 R = \frac{Q^2}{2\times 4.5} \quad [\text{from eqn. (i)}]$$

$$R = \frac{1}{4\pi\varepsilon_0}\times\frac{(4\times10^{-6})^2}{2\times4.5}$$

$$= 9\times10^9\times\frac{16}{9}\times10^{-12} = 16\times10^{-3}\text{ m} = 16\text{ mm}$$

19. (b):



As the capacitors are in parallel combination so they have equal potential differences.

$$C_{\text{before}} = \frac{\varepsilon_0 A}{3} \quad \ldots(\text{i})$$

$$C_{\text{after}} = \frac{\frac{k\varepsilon_0 A}{3}\cdot\frac{\varepsilon_0 A}{2.4}}{\frac{k\varepsilon_0 A}{3}+\frac{\varepsilon_0 A}{2.4}} \quad \ldots(\text{ii})$$

From (i) and (ii),

$$\frac{\varepsilon_0 A}{3} = \frac{\frac{k\varepsilon_0 A}{3}\cdot\frac{\varepsilon_0 A}{2.4}}{k\frac{\varepsilon_0 A}{3}+\frac{\varepsilon_0 A}{2.4}}$$

or $3k = 2.4k + 3$ or $0.6k = 3$

$\Rightarrow$ $k = \dfrac{3}{0.6}$ or $k = 5$

20. (a) : $\phi = \dfrac{q_{\text{enclosed}}}{\varepsilon_0}$

For S_1, $\phi_1 = \dfrac{2q}{\varepsilon_0}$; For S_2, $\phi_2 = \dfrac{3q-q}{\varepsilon_0} = \dfrac{2q}{\varepsilon_0}$

For S_3, $\phi_3 = \dfrac{q+q}{\varepsilon_0} = \dfrac{2q}{\varepsilon_0}$; For S_4, $\phi_4 = \dfrac{8q-6q}{\varepsilon_0} = \dfrac{2q}{\varepsilon_0}$

Hence, $\phi_1 = \phi_2 = \phi_3 = \phi_4 = \dfrac{2q}{\varepsilon_0}$

MPP-3 CLASS XI ANSWER KEY

1. (b)	2. (b)	3. (c)	4. (d)	5. (c)
6. (d)	7. (c)	8. (c)	9. (c)	10. (d)
11. (d)	12. (b)	13. (b)	14. (d)	15. (c)
16. (a)	17. (c)	18. (a)	19. (a)	20. (b, c, d)
21. (a, b, c)	22. (a, c)	23. (a, b, d)	24. (5)	25. (4)
26. (5)	27. (c)	28. (b)	29. (b)	30. (a)

ACE YOUR WAY CBSE XII

Series 2

CHAPTERWISE PRACTICE PAPER

CURRENT ELECTRICITY

Time Allowed : 3 hours

Maximum Marks : 70

GENERAL INSTRUCTIONS

(i) All questions are compulsory.
(ii) Q. no. 1 to 5 are very short answer questions and carry 1 mark each.
(iii) Q. no. 6 to 10 are short answer questions and carry 2 marks each.
(iv) Q. no. 11 to 22 are also short answer questions and carry 3 marks each.
(v) Q. no. 23 is a value based question and carries 4 marks.
(vi) Q. no. 24 to 26 are long answer questions and carry 5 marks each.
(vii) Use log tables if necessary, use of calculators is not allowed.

SECTION - A

1. Electrons are continuously in motion within a conductor but there is no current in it unless some source of potential is applied across its ends. Give reason.

2. If the current flowing in a copper wire is passed through another copper wire of the same length but of double the radius of the first one. How would the drift velocity of free electrons change?

3. A steady current is flowing in a cylindrical conductor. Does electric field exist within the conductor?

4. What is the advantage of using thick metallic strips to join wires in a potentiometer?

5. Why are alloys used for making standard resistance coils?

SECTION - B

6. A wire connected to a bulb does not glow, whereas the filament of the bulb glows when same current flows through them. Why?

7. A cell of emf ε and internal resistance r is connected across an external resistance R. Plot a graph showing the variation of potential difference (across R) versus R.

OR

First, a set of n equal resistors of R each are connected in series to a battery of emf ε and internal resistance R. A current I is observed to flow. Then the n resistors are connected in parallel to the same battery. It is observed that the current is increased 10 times. What is the value of n?

8. On what factors, does the potential gradient of the potentiometer wire depend?

9. Explain how electron mobility changes for a good conductor, when (a) the temperature of the conductor is decreased at constant potential difference and (b) applied potential difference is doubled at constant temperature.

10. Two conductors are made of the same material and have the same length. Conductor A is a solid wire of diameter 1 mm. Conductor B is a hollow tube of outer diameter 2 mm and inner diameter 1 mm. Find the ratio of resistance R_A to R_B.

SECTION - C

11. Two heating elements of resistance R_1 and R_2 when operated at a constant supply of voltage V, consume powers P_1 and P_2 respectively. Deduce the expression for the power of their combination when they are in turn, connected in (a) series and (b) parallel across the same voltage supply.

12. A storage battery of emf 8.0 V and internal resistance 0.5 Ω is being charged by a 120 V dc supply using a series resistor of 15.5 Ω. What is the terminal voltage of the battery during charging? What is the purpose of having a series resistor in the charging circuit?

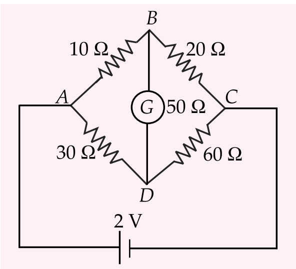
13. Two wires of equal length, one of aluminium and the other of copper have the same resistance. Which of the two wires is lighter? Hence explain why aluminium wires are preferred for overhead power cables.
($\rho_{Al} = 2.63 \times 10^{-8}$ Ω m, $\rho_{Cu} = 1.72 \times 10^{-8}$ Ω m, Relative density of Al = 2.7 and of Cu = 8.9).

14. (a) Three resistors 2 Ω, 4 Ω and 5 Ω are combined in parallel. What is the total resistance of the combination?

(b) If the combination is connected to a battery of emf 20 V and negligible internal resistance determine the current through each resistor, and the total current drawn from the battery.

OR

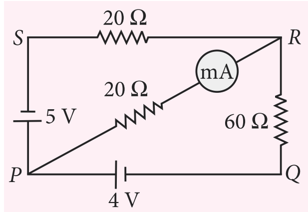
The given figure shows a network of resistances. Name the circuit so formed. What is the current flowing in the arm BD of this circuit? State the two laws used to find the current in different branches of this circuit.



15. In a discharge tube, the number of hydrogen ions (*i.e.*, protons) drifting across a cross-section per second is 1.0×10^{18}. While the number of electrons drifting in the opposite direction across another cross-section is 2.7×10^{18} per second. If the supply voltage is 230 V, what is the effective resistance of the tube?

16. A conductor of length l is connected to a dc source of potential V. If the length of the conductor is tripled by stretching it, keeping V constant, explain how do the following factors vary in the conductor :
(a) Drift speed of electrons
(b) Resistance (c) Resistivity.

17. Network $PQRS$ as shown in figure is made. PQ has a battery of 4 V and negligible resistance with positive terminal connected to P. QR has a resistance of 60 Ω. PS has a battery of 5 V negligible resistance with positive terminal connected to P. RS has resistance of 20 Ω. If a milliammeter, of 20 Ω resistance is connected between P and R, calculate the reading of the milliammeter.



18. (a) Six lead-acid type of secondary cells each of emf 2.0 V and internal resistance of 0.015 Ω are joined in series to provide a supply to a resistance of 8.5 Ω. What are the current drawn from the supply and its terminal voltage ?

(b) A secondary cell after long use has an emf of 1.9 V and a large internal resistance of 380 Ω. What maximum current can be drawn from the cell ? Could the cell drive the starting motor of a car ?

19. Three cells are connected in parallel with their like poles connected together with wires of negligible resistance. If the emfs of the cells are 2 V, 1 V and 4 V respectively and their internal resistances are 4 Ω, 3 Ω and 2 Ω respectively, find the current through each cell.

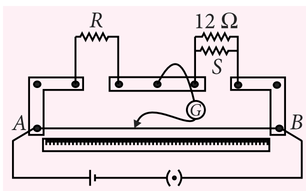
20. Explain why an electric bulb becomes dim when an electric heater in parallel circuit is made on. Why dimness decreases after some time?

OR

A battery of emf 10 V and internal resistance 3 Ω is connected to a resistor R.
(a) If the current in the circuit is 0.5 A, calculate the value of R.
(b) What is the terminal voltage of the battery?

21. Define resistivity of a conductor. Plot a graph showing the variation of resistivity with temperature for a metallic conductor. How does one explain such a behaviour, using the mathematical expression of the resistivity of a material.

22. In a meter bridge, the null point is found at a distance of 40 cm from A. If a resistance of 12 Ω is connected in parallel with S, the null point occurs at 50.0 cm from A. Determine the values of R and S.



SECTION - D

23. Manish and Rajnish lived in an unauthorised colony. They found that most people of that colony stole power from transmission lines using hooks. They had read in the newspapers about different fire accidents caused due to electric short circuits. Alongwith some of their friends and some responsible representatives of that area, they visited house to house of that colony and made people aware of the risks involved in short circuiting. They also explained the people the importance of paying electricity bills. They succeeded in changing the mindset of the people. Answer the following questions based on the above information:

(a) What according to you, are the values of displayed by Manish and Rajnish?

(b) A household circuit has a fuse of 5 A rating. Find the maximum number of bulbs of rating 60 W, – 220 V each which can be connected in this circuit.

SECTION - E

24. Using the concept of drift velocity, drive ohm's law.

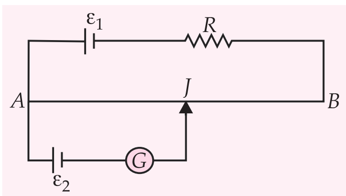
OR

Show that the current density of a metallic conductor is directly proportional to the drift speed of electrons.

25. Deduce the expression of temperature T °C of a platinum wire measured using Wheatstone's bridge.

OR

In the given circuit diagram AB is uniform wire of resistance 10 Ω and length 1 m. It is connected to a series arrangement of cell ε_1, of emf 2.0 V and negligible internal resistance and a resistor R terminal A is also connected to an electrochemical cell ε_2 of emf 100 mV and a galvanometer G. In this set-up a balancing point is obtained at 40 cm mark from A. Calculate the value of resistance R. If ε_2 were to have an emf of 300 mV, where will you expect the balancing point to be.



26. Draw the circuit diagram of potentiometer which can be used to determine the internal resistance (r) of a given cell of emf (ε). With the help of this diagram describe the method to find the internal resistance of the primary cell.

OR

Find the conditions for maximum current in the external resistor R when number of cells each of emf ε and internal resistance r are connected (i) in series (ii) in parallel and (iii) in mixed grouping.

SOLUTIONS

1. In the absence of any external voltage source the motion of electrons in a conductor is random and electrons collide continuously with the positive ions of metal. This causes a random change in direction. The average velocity of random motion of electrons in any direction is zero, hence current is zero.

2. Drift velocity, $v_d = \dfrac{I}{nAe} = \dfrac{I}{n\pi r^2 e}$

i.e., $v_d \propto \dfrac{1}{r^2}$ $\quad \therefore \dfrac{v_{d_2}}{v_{d_1}} = \dfrac{r^2}{(2r)^2} = \dfrac{1}{4}$ or $v_{d_2} = \dfrac{1}{4} v_{d_1}$

So the drift velocity becomes one fourth when radius of the conductor is doubled.

3. Yes, electric field exists within the conductor because it is the electric field which imparts acceleration to electrons for the flow of current.

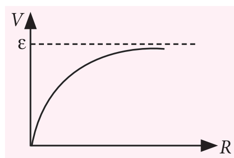
4. The thick metallic strips have very low resistances. As such there is no need to add their resistances to the resistance of the potentiometer length at balance point. Instead, if we use wires whose resistances are not negligible, these will have to be determined and taken into account.

5. Alloys are used for making standard resistance coils because they have low value of temperature coefficient of resistance and high resistivity.

6. Filament of the bulb and connecting wires are connected in series so the same current flows through them. Since the resistance of connecting wires is negligibly small as compared to the resistance of filament and heat produced due to given current is directly proportional to its resistance (from Joule's law of heating), therefore, the heat produced in the filament is very large. Hence the bulb glows, but the connecting wires remain practically unheated.

7. Potential difference across R,

i.e., $V = IR = \dfrac{\varepsilon R}{(R+r)}$



When R increases,
V increases.
In the extreme case, when
$R >> r$, $R + r \rightarrow R$ and $V \rightarrow \varepsilon$ as shown in figure.

OR

When n resistors are in series, then current in the circuit,

$$I = \frac{\varepsilon}{R+nR} = \frac{\varepsilon}{(n+1)R}$$

When n resistors are in parallel, then effective resistance is $R_{\text{eff}} = R/n$.

Current in circuit, $I' = \dfrac{\varepsilon}{R+R/n} = \dfrac{n\varepsilon}{R(n+1)}$

As per question, $I' = 10\,I$

$$\therefore \quad \frac{n\varepsilon}{R(n+1)} = \frac{10\,\varepsilon}{(n+1)R} \quad \text{or} \quad n = 10$$

8. The fall of potential across a potentiometer wire of length l is given by

$$V = IR = \frac{I\rho l}{A}$$

$\therefore$ Potential gradient, $k = \dfrac{V}{l} = \dfrac{I\rho}{A}$.

Thus potential gradient of a wire depends upon the following factors :

(i) $k \propto I$ *i.e.*, current passing through the potentiometer wire

(ii) $k \propto \rho$ *i.e.*, specific resistance of the material of the potentiometer wire

(iii) $k \propto \dfrac{1}{A}$, where A is the area of cross-section of the potentiometer wire.

9. Electron mobility in a conductor is given by

$$\mu = \frac{v_d}{E} = \frac{\frac{eE}{m}\tau}{E} = \frac{e\tau}{m} \qquad \left(\because\ v_d = \frac{eE}{m}\tau\right)$$

(a) When the temperature of the conductor decreases, the relaxation time τ of the electrons increases, so mobility μ increases.

(b) Mobility μ is independent of applied potential difference.

10. For a solid wire of resistance R_A

$l_1 = l$, $\rho_1 = \rho$, $D_1 = 1$ mm

$$A_1 = \frac{\pi D_1^2}{4} = \frac{\pi(1)^2}{4}\ \text{mm}^2$$

$$R_A = \frac{\rho_1 l_1}{A_1} = \frac{\rho l}{\pi(1)^2/4} = \frac{4\rho l}{\pi}$$

For hollow tube of resistance R_B, $l_2 = l$, $\rho_2 = \rho$,

$$A_2 = \pi\frac{(D_2^2 - D_1^2)}{4} = \frac{\pi(2^2 - 1^2)}{4} = \frac{3\pi}{4}\ \text{mm}^2$$

$$R_B = \frac{\rho_2 l_2}{A_2} = \frac{\rho l}{(3\pi/4)} = \frac{4\rho l}{3\pi}$$

$$\therefore \quad \frac{R_A}{R_B} = 3:1$$

11. Here, $P_1 = \dfrac{V^2}{R_1}$ and $P_2 = \dfrac{V^2}{R_2}$

(a) For series combination, $P_s = \dfrac{V^2}{R_s} = \dfrac{V^2}{R_1 + R_2}$

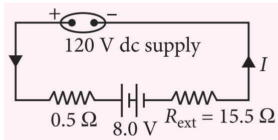
$$\text{or} \quad P_s = \frac{V^2}{\frac{V^2}{P_1} + \frac{V^2}{P_2}} = \frac{1}{\frac{1}{P_1} + \frac{1}{P_2}} = \frac{P_1 P_2}{P_1 + P_2}$$

(b) For parallel combination,

$$P_p = \frac{V^2}{R_p} = V^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\text{or} \quad P_p = \frac{V^2}{R_1} + \frac{V^2}{R_2} = P_1 + P_2$$

12. During charging, the electric current is sent into the 8.0 V battery. The current in the circuit



$$I = \frac{\varepsilon_1 - \varepsilon_2}{R_{\text{ext}} + r} = \frac{120 - 8}{15.5 + 0.5} \quad \text{or} \quad I = \frac{112}{16} = 7\ \text{A}$$

Now, terminal voltage of the battery during charging

$V = \varepsilon_2 + Ir = 8 + 7(0.5) = 11.5$ V

A series resistance is joined in the charging circuit to limit the excessive current so that charging is slow and permanent.

13. Two wires have same length and resistance. As the specific resistances are unequal, the areas are different.

For copper wire, $R_{\text{Cu}} = \rho_{\text{Cu}}\dfrac{l}{A_{\text{Cu}}}$

For aluminium wire, $R_{\text{Al}} = \rho_{\text{Al}}\dfrac{l}{A_{\text{Al}}}$

So, $\rho_{\text{Cu}}\dfrac{l}{A_{\text{Cu}}} = \rho_{\text{Al}}\dfrac{l}{A_{\text{Al}}}$

$$\frac{A_{\text{Al}}}{A_{\text{Cu}}} = \frac{\rho_{\text{Al}}}{\rho_{\text{Cu}}} = \frac{2.63\times10^{-8}}{1.72\times10^{-8}} = \frac{263}{172}$$

Ratio of masses,

$$\frac{M_{\text{Al}}}{M_{\text{Cu}}} = \frac{d_{\text{Al}}\, l\, A_{\text{Al}}}{d_{\text{Cu}}\, l\, A_{\text{Cu}}} = \frac{2.7\times263}{8.9\times172} = 0.46 < 1$$

$\therefore \quad M_{Al} < M_{Cu}$

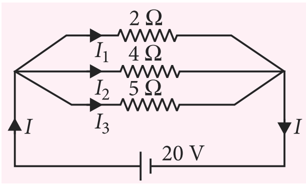
Thus, the aluminium wire for the same resistance is very light than copper and has greater thickness which increases its durability. Therefore aluminium wires are preferred for overhead power cables.

14. (a) Total resistance of the combination in parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad \text{or} \quad \frac{1}{R_{eq}} = \frac{1}{2} + \frac{1}{4} + \frac{1}{5}$$

$$R_{eq} = \frac{20}{19} = 1.05\ \Omega$$

(b) Potential of 20 V will be same across each resistance, so current



$$I_1 = \frac{V}{R_1} = \frac{20}{2} = 10\text{ A},$$

$$I_2 = \frac{V}{R_2} = \frac{20}{4} = 5\text{ A}, \; I_3 = \frac{V}{R_3} = \frac{20}{5} = 4\text{ A}$$

Total current drawn from the cell,

$I = I_1 + I_2 + I_3$ or $I = 19$ A

OR

The given circuit is called Wheatstone's bridge. Wheatstone's bridge is balanced when

$$\frac{\text{Resistance in branch } AB}{\text{Resistance in branch } BC} = \frac{\text{Resistance in branch } AD}{\text{Resistance in branch } DC}$$

Here, $\dfrac{R_{AB}}{R_{BC}} = \dfrac{10}{20} = \dfrac{1}{2}; \dfrac{R_{AD}}{R_{DC}} = \dfrac{30}{60} = \dfrac{1}{2}$

$\therefore$ Wheatstone's bridge is balanced.

$\therefore$ No current flows through the arm BD containing galvanometer, as B and D are at same potential. Two laws used to find the current in different branches of this circuit are:

(i) Kirchhoff's junction rule : It states that the algebraic sum of the currents meeting at a junction is zero.

(ii) Kirchhoff's loop rule : It states that in any closed loop in a circuit, algebraic sum of applied emf's and potential drops across the resistors is equal to zero.

15. In the discharge tube the protons move in the direction of electric current and electrons move opposite to the direction of current, the total current is sum of current due to electrons and current due to protons.

$I_{net} = I_p + I_e$

Current due to protons

$$I_p = \frac{q}{t} = \frac{10^{18} \times 1.6 \times 10^{-19}\text{ C}}{1\text{ s}} = 0.16\text{ A}$$

Current due to electrons

$$I_e = \frac{q}{t} = \frac{2.7 \times 10^{18} \times 1.6 \times 10^{-19}\text{ C}}{1\text{ s}} = 0.432\text{ A}$$

Total current (I_{net}) in discharge tube $= I_p + I_e = 0.592$ A

Effective resistance, $R = \dfrac{V}{I} = \dfrac{230}{0.592} = 388.51\ \Omega.$

16. The drift speed is given by

$$v_d = \frac{eE\tau}{m} = \frac{eV}{ml}\tau \qquad \text{...(i)}$$

(a) $v_d \propto \dfrac{1}{l}.$ (Using (i))

Thus, if length is tripled, the drift speed will become one-third.

(b) Since volume remains constant,

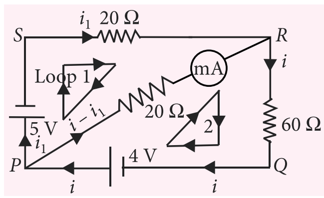
$A'l' = Al$ or $A' = \dfrac{Al}{l'} = \dfrac{A}{3}$ $[\because l' = 3l]$

$$R = \frac{\rho l}{A}, \; R' = \frac{\rho l'}{A'}$$

$$\Rightarrow \frac{R'}{R} = \left(\frac{l'}{A'}\right)\left(\frac{A}{l}\right) = \left(\frac{l'}{l}\right)\left(\frac{A}{A'}\right) = 3 \times 3 = 9$$

(c) Resistivity remains unchanged.

17. An ammeter reads the current flowing through it. Let us distribute the currents in different branches.



Loop 1 :

$20i_1 - 20(i - i_1) = -5$ or $40i_1 - 20i = -5$

$4i - 8i_1 = 1$ ...(i)

Loop 2 :

$60i + 20(i - i_1) = 4$ or $80i - 20i_1 = 4$

$20i - 5i_1 = 1$ ...(ii)

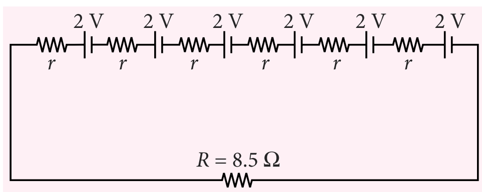
Solving (i) and (ii), $i_1 = -\dfrac{4}{35}$ A, $i = \dfrac{3}{140}$ A

So, the current through milliammeter is

$$i - i_1 = \frac{3}{140} - \left(-\frac{4}{35}\right) = \frac{19}{140} = 0.1357\text{ A}$$

So, the milliammeter reads 135.7 mA.

18. Six cells are joined in series.



Equivalent emf = 2 × 6 = 12 V
Equivalent internal resistance = 0.015 × 6 = 0.09 Ω
Current drawn from supply,

$$I = \frac{12}{8.5+0.09} = \frac{12}{8.59} \approx 1.39 \text{ A}$$

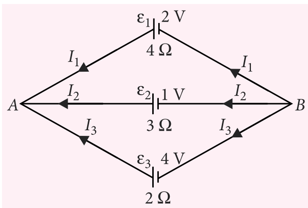
Terminal voltage of battery,
$V = \varepsilon - Ir = 12 - 1.39 \times 0.09 \approx 11.8$ V

(b) Maximum current is drawn from a battery when external resistance is treated to be zero.

$$I_{max} = \frac{1.9}{380} = 0.005 \text{ A}$$

To start a car, a higher amount of current is needed, so the battery can not drive the starting motor.

19. The scheme of connections is shown in figure. Let I_1, I_2 and I_3 be the currents flowing through the three cells ε_1, ε_2 and ε_3.



Applying Kirchhoff's junction law at the junction A, we get, $I_1 + I_2 + I_3 = 0$ or $I_3 = -(I_1 + I_2)$ ...(i)

Applying Kirchhoff's loop law to the closed loop $B\varepsilon_1A\varepsilon_2B$ and we get, $4I_1 - 2 - 3I_2 + 1 = 0$

or $4I_1 - 3I_2 = 2 - 1 = 1$ ... (ii)

Applying the Kirchhoff's loop law to the closed loop $B\varepsilon_1A\varepsilon_3B$, we get

$4I_1 - 2 - 2I_3 + 4 = 0$ or $4I_1 - 2I_3 = -2$

or $4I_1 + 2 \times (I_1 + I_2) = -2$ (Using (i))

or $6I_1 + 2I_2 = -2$ or $3I_1 + I_2 = -1$ ... (iii)

Multiplying (iii) by 3 and adding to (ii), we get

$$(9 + 4)I_1 = 1 - 3 = -2 \text{ or } I_1 = -\frac{2}{13} \text{ A}$$

From (iii),

$$I_2 = -1 - 3I_1 = -1 - 3\left(-\frac{2}{13}\right) = -1 + \frac{6}{13} = -\frac{7}{13} \text{ A}$$

From (i),

$$I_3 = -\left[-\frac{2}{13} - \frac{7}{13}\right] = \frac{9}{13} \text{ A}$$

Note : Negative sign of currents shows that the actual direction is opposite to what has been taken in figure.

20. Since the electric heater has more power than that of electric bulb and power is reciprocal to resistance for a given supply voltage *i.e.*, $P \propto \frac{1}{R}$.

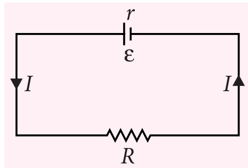
Hence, the resistance of heater coil is less than that of electric bulb. When the heater, which is connected in parallel with the illuminating bulb is made on, it draws more current from the circuit. Due to it, the bulb becomes dim.

After some time, the heater coil becomes hot. Its resistance becomes more. Due to it some current is diverted into bulb, and also the supply voltage maintains the current. As a result of which dimness of the bulb decreases.

OR

(i) Here, ε = 10 V, r = 3 Ω,

I = 0.5 A



Current in the circuit is,

$$I = \frac{\varepsilon}{R + r}$$

or $$R = \frac{\varepsilon - Ir}{I} = \frac{10 - 0.5 \times 3}{0.5} = 17\,\Omega$$

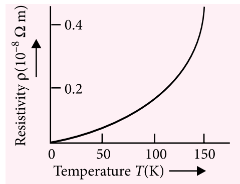
(ii) Terminal voltage, $V = \varepsilon - Ir = 10 - 0.5 \times 3$

V = 8.5 V.

21. $$R = \rho\frac{l}{A}$$

If $l = 1,\ A = 1 \Rightarrow \rho = R$

Thus, resistivity of a material is numerically equal to the resistance of the conductor having unit length and unit cross-sectional area.



Resistivity ρ of metallic conductor as a function of temperature T

The resistivity of a material is found to be dependent on the temperature. Different materials do not exhibit the same dependence on temperature. Over a limited range of temperatures, that is not too large, the resistivity of a metallic conductor is approximately given by,

$\rho_T = \rho_0\,[1 + \alpha(T - T_0)]$ ...(i)

where ρ_T is the resistivity at a temperature T and ρ_0 is the same at a reference temperature T_0. α is called the temperature coefficient of resistivity. The relation in eqn. (i) implies that a graph of ρ_T plotted against T would be a straight line. At temperatures much lower than 0°C, the graph, however, deviates considerably from a straight line.

22. When resistances R and S are connected then balance point is found at a distance 40 cm from the zero end.

$$\therefore \quad \frac{R}{S} = \frac{40}{100 - 40}$$

$$\frac{R}{S} = \frac{40}{60} \text{ or } \frac{R}{S} = \frac{2}{3} \quad \text{... (i)}$$

When a resistance of 12 Ω is connected in parallel with S then total resistance in the right gap is

$$S_1 = \frac{12S}{S+12} \qquad \text{...(ii)}$$

Since balance point is obtained at a distance of 50 cm from the zero end.

$$\therefore \quad \frac{R}{S_1} = \frac{50}{100-50} \quad \text{or} \quad \frac{R}{S_1} = \frac{50}{50}$$

$$\therefore \quad \frac{R}{S_1} = 1 \qquad \text{...(iii)}$$

Dividing (i) by (iii), we get

$$\therefore \quad \frac{\frac{R}{S}}{\frac{R}{S_1}} = \frac{\frac{2}{3}}{1} \quad \text{or} \quad \frac{R}{S} \times \frac{S_1}{R} = \frac{2}{3} \quad \text{or} \quad \frac{S_1}{S} = \frac{2}{3} \quad \text{or} \quad S_1 = \frac{2}{3}S$$

Putting the value of S_1 in (ii), we get

$$\frac{2}{3}S = \frac{12S}{S+12} \quad \text{or} \quad \frac{2}{3} = \frac{12}{S+12} \quad \text{or } 2S + 24 = 36 \quad \text{or } 2S = 12$$

$\therefore \quad S = 6$

Putting the value of S in (i), we get

$$\frac{R}{6} = \frac{2}{3} \Rightarrow R = \frac{2}{3} \times 6 = 4$$

$\therefore \quad R = 4\ \Omega$ and $S = 6\ \Omega$

23. (a) Critical thinking, decision making, team spirit and assertive communication.

(b) Number of bulbs that can be safely used with 5 A fuse $= \dfrac{5}{3/11} = \dfrac{55}{3} = 18.33$

Hence, 18 bulbs can be safely used with 5 A fuse.

24. The current I in a conductor of cross-sectional area A and having n free electrons per unit volume is given by

$I = neAv_d$ ...(i)

e being the magnitude of electronic charge and v_d, the drift velocity of electrons.

The drift velocity v_d of electrons is given by

$$v_d = \frac{eE}{m}\tau \qquad \text{...(ii)}$$

where m is the mass of an electron, E the magnitude of electric field in the conductor and τ is the relaxation time.

Substituting the value of v_d in eqn. (i), we get

$$I = neA\left(\frac{eE\tau}{m}\right)$$

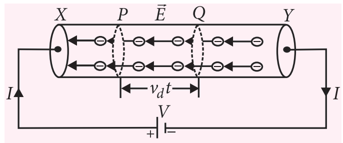
$$\text{or} \quad I = \frac{e^2nA\tau}{m}E \qquad \text{...(iii)}$$

If the electric field E is due to potential difference V applied across the ends of the conductor of length l, then $E = \dfrac{V}{l}$

Substituting this value of E in equation (iii), we get

$$I = \frac{e^2nA\tau}{ml}V \quad \text{or} \quad \frac{V}{I} = \frac{ml}{e^2nA\tau}$$

For a given conductor, m, e, n, τ, l and A are all constants,



$\therefore \quad \dfrac{V}{I} = R$, a constant

where $R = \dfrac{ml}{e^2nA\tau}$

Thus Ohm's law has been derived.

OR

Refer to point 4, 5 page no. 92 (MTG Excel in Physics)

25. Refer to point 5 page no. 100 (MTG Excel in Physics)

OR

If k is potential gradient and l_1 the balancing length for cell ε_2, then we have

$\varepsilon_2 = kl_1$

100 mV = $k \times 40$ cm

$$k = \frac{100}{40}\ \text{mV cm}^{-1} = 2.5\ \text{mV cm}^{-1}$$

$k = 2.5 \times 10^{-3}$ V cm^{-1} = 0.25 V m^{-1} ...(i)

Current in primary circuit

$$I = \frac{\varepsilon_1}{R + R_{AB}}, \text{ where } R_{AB} \text{ is resistance of wire } AB$$

$$I = \frac{2.0}{R+10}$$

Potential difference across wire AB, $V_{AB} = I \times R_{AB}$

$$V_{AB} = \frac{2.0}{R+10} \times 10 = \frac{20}{R+10}$$

$$\therefore \quad \text{Potential gradient } k = \frac{V_{AB}}{l_{AB}} = \frac{20/(R+10)}{1}$$

$$= \frac{20}{R+10} \qquad \text{...(ii)}$$

Comparing (i) and (ii), we get

$$\frac{20}{R+10} = 0.25 \quad \text{or} \quad R + 10 = \frac{20}{0.25}\ \Omega = 80\ \Omega$$

$R = 80 - 10 = 70\ \Omega$

Balancing length

when ε_2 = 300 mV = 300 × 10^{-3} V = 0.3 V

$$\varepsilon_2 = kl_2 \quad \text{or} \quad 0.3 = 0.25 \times l_2 \quad \text{or} \quad l_2 = \frac{0.3}{0.25} = 1.2\ \text{m}$$

At ε_2 = 300 mV, we do not get the balance point as it will lie outside the length of the wire.

26. Refer to point 7(f) page no. 102 (MTG Excel in Physics)

OR

Refer to point 9, 10, 11 page no. 97 (MTG Excel in Physics)

MPP-3 MONTHLY Practice Problems

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer.

Self check table given at the end will help you to check your readiness.

Current Electricity

Total Marks : 120 **Time Taken : 60 min**

NEET / AIIMS

Only One Option Correct Type

1. In a closed circuit, the current I (in A) at an instant of time t (in s) is given by $I = 4 - 0.08\,t$. The number of electrons flowing in 50 s through the cross section of the conductor is
(a) 1.25×10^{19} (b) 6.25×10^{20}
(c) 5.25×10^{19} (d) 2.55×10^{20}

2. When a current I flows through a wire, the drift velocity of the electrons is v. When current $2I$ flows through another wire of the same material having double the length and double the area of cross-section, the drift velocity of the electrons will be
(a) $v/8$ (b) $v/4$ (c) $v/2$ (d) v

3. If R_1 and R_2 be the resistances of the filaments of 200 W and 100 W electric bulbs operating at 220 V, then (R_1/R_2) is
(a) 1 (b) 2 (c) 0.5 (d) 4

4. A thin wire of resistance 4 Ω is bent to form a circle. The resistance across any diameter is
(a) 4 Ω (b) 2 Ω (c) 1 Ω (d) 8 Ω

5. In a meter bridge experiment, the ratio of the left gap resistance to right gap resistance is 2 : 3, the balance point from left is
(a) 60 cm (b) 50 cm
(c) 40 cm (d) 20 cm

6. Ten identical batteries each of emf 2 V are connected in series to a 8 Ω resistor. If the current in the circuit is 2 A, then the internal resistance of each battery is
(a) 0.2 Ω (b) 0.3 Ω (c) 0.4 Ω (d) 0.5 Ω

7. A student measures the terminal potential difference V of a cell of emf ε and internal resistance r as a function of the current I flowing through it. The slope and intercept of the graph between V and I then respectively equal to
(a) $-r$ and ε (b) r and $-\varepsilon$
(c) $-\varepsilon$ and r (d) ε and $-r$

8. One kg of copper is drawn into a wire of 1 mm diameter and a wire of 2 mm diameter. The resistance of the two wires will be in the ratio of
(a) 2 : 1 (b) 1 : 2
(c) 16 : 1 (d) 4 : 1

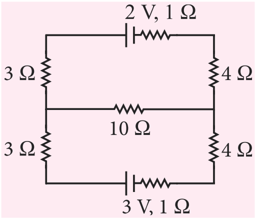
9. The charge flowing in a conductor varies with time as $Q = at - bt^2$. Then, the current
(a) increases linearly with time
(b) reaches a maximum and then decreases
(c) falls to zero at time $t = a/2b$
(d) changes at a rate $-3b$

10. The current voltage relation for a device is given by

$$I = I_0(e^{\alpha V} - 1);\ \alpha > 0$$

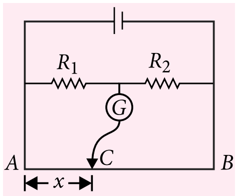
where V denotes voltage
(a) Power lost in the device is $VI_0(e^{-\alpha V} - 1)$
(b) Power lost in device is $\dfrac{I_0}{\alpha V}e^{\alpha V}$
(c) Power lost is minimum at $V = 0$
(d) Power lost is maximum at $V = 0$

11. A network of five resistances and two batteries is shown in figure. The potential drop across 10 Ω resistor is



(a) 1.79 V (b) 2.31 V (c) 3.70 V (d) zero.

12. In the metre bridge experiment shown in figure, the balance length AC corresponds to null deflection of the galvanometer is x. What would be the balance length if the radius of wire AB is doubled?



(a) $x/2$ (b) x (c) $2x$ (d) $4x$

Assertion & Reason Type

Directions : In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as :

(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false.
(d) If both assertion and reason are false.

13. Assertion : Two identical cells each of emf ε and internal resistance r are connected in series and in parallel. Maximum power transferred to the load is same in both cases.

Reason : The value of load resistance for maximum power transfer for series and parallel combinations of cells are same.

14. Assertion : In a meter bridge, if its wire is replaced by another wire having same length, made of same material but having twice the cross-sectional area, the accuracy decreases.

Reason : If its wire is replaced by another wire of same material, having same cross-sectional area but of twice the length, accuracy decreases.

15. Assertion : Kirchhoff's loop law represents conservation of energy.

Reason : If the sum of potential changes around a closed loop is not zero, unlimited energy could be gained by repeatedly carrying a charge around a loop.

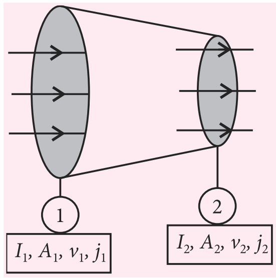
JEE MAIN / JEE ADVANCED

Only One Option Correct Type

16. The heat generated through 4 Ω and 9 Ω resistances separately, when a capacitor of 100 μF capacity charged to 200 V is discharged one by one, will be

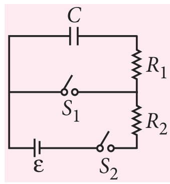
(a) 2 J and 8 J, respectively
(b) 8 J and 2 J, respectively
(c) 2 J and 4 J, respectively
(d) 2 J and 2 J, respectively

17. Current is flowing from a conductor of non uniform cross-sectional area as shown in figure. If $A_1 > A_2$, then the correct option is
(Here I is current, j is current density and v is drift velocity.)



(a) $I_1 = I_2$ (b) $j_1 > j_2$
(c) $v_1 < v_2$ (d) Both (a) and (c)

18. The capacitor shown in figure has been charged to a potential difference of V so that it carries a charge CV with both the switches S_1 and S_2 remaining open. Switch S_1 is closed at $t = 0$. At $t = R_1C$, switch S_1 is opened and S_2 is closed.

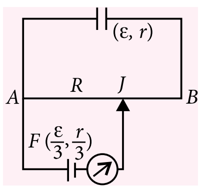


Find the charge on the capacitor at $t = 2R_1C + R_2C$.

(a) $C\varepsilon(1-e)+C\dfrac{V}{e^2}$ (b) $C\varepsilon(1-e^{-1})+C\dfrac{V}{e^2}$

(c) $C\varepsilon e+C\dfrac{V}{e^2}$ (d) $C\varepsilon(1+e)+C\dfrac{V}{e^2}$

19. A potentiometer arrangement is shown in figure. The driver cell has emf ε and internal resistance r. The resistance of potentiometer wire AB is R. F is the cell of emf $\varepsilon/3$ and internal resistance $r/3$. Balance point J can be obtained for all finite values if



(a) $R > r/2$ (b) $R < r/2$ (c) $R > r/3$ (d) $R < r/3$

More than One Options Correct Type

20. Which of the following statements are correct?

(a) Potential difference between the terminals of non-ideal battery can never be greater than its emf.

(b) If a non-ideal battery is short-circuited by a wire, heat generated in the wire is less than electrical energy developed in the battery.

(c) Emf of an ideal battery is measured by potentiometer and then by voltmeter. Both the measurements are equally correct.

(d) The current inside the cell is always due to motion of both positive and negative ions while outside it depends on the circuit elements (conductor, semi-conductor, gas or electrolytes etc.).

21. When capacitor discharges through resistance R, the time constant is τ and maximum current in the circuit is I_0. Then

(a) Initial charge on the capacitor was $I_0\tau$.

(b) Initial charge on the capacitor was $\frac{1}{2}I_0\tau$.

(c) Initial energy stored in the capacitor was $I_0^2R\tau$.

(d) Initial energy stored in the capacitor was $\frac{1}{2}I_0^2R\tau$.

22. Two bulbs consume same energy when operated at 200 V and 300 V, respectively. When these bulbs are connected in series across a dc source of 500 V, then

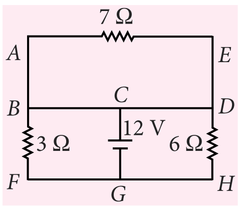
(a) ratio of potential difference across them is 3/2

(b) ratio of potential difference across them is 4/9

(c) ratio of power produced in them is 4/9

(d) ratio of power produced in them is 2/3.

23. A single battery is connected to three resistance as shown in figure.



(a) The current through 7 Ω resistance is 4 A.

(b) The current through 3 Ω resistance is 4 A.

(c) The current through 6 Ω resistance is 2 A.

(d) The current through 7 Ω resistance is 0.

Integer Answer Type

24. A potentiometer wire of length 10 m and resistance 30 Ω is connected in series with a battery of emf 2.5 V, internal resistance 5 Ω and an external resistance R. If the fall of potential along the potentiometer wire is 50 μV mm^{-1}, then the value of R is found to be $23n$ Ω. What is the value of n?

25. Three identical resistors are connected in series. When a certain potential difference is applied across the combination, the total power dissipated is 27 W. How many times the power would be dissipated if the three resistors were connected in parallel across the same potential difference?

26. The area of cross section, length and density of a piece of metal of atomic weight 60 are 10^{-6} m^2, 1 m and 5×10^3 kg m^{-3} respectively. If every atom contributes one free electron, find the drift velocity (in mm s^{-1}) of electons in the metal when a current of 16 A passes through it. Avogadro's number is $N_A = 6 \times 10^{23}$ mol^{-1} and charge on an electron is $q = 1.6 \times 10^{-19}$ C.

Comprehension Type

A three-way light bulb has three brightness settings (low, medium and high) but only two filaments. The two filaments are arranged in three settings, when connected across a 120 V line and can dissipate 60, 120 and 180 W.

27. Which of the following statements are correct?

i. High resistance filament works for 60 W
ii. Low resistance filament works for 120 W
iii. Low resistance filament works for 60 W
iv. High resistance filament works for 120 W
v. Low and high resistance filaments in parallel for 180 W.
vi. Low and high resistance filaments in series for 180 W.

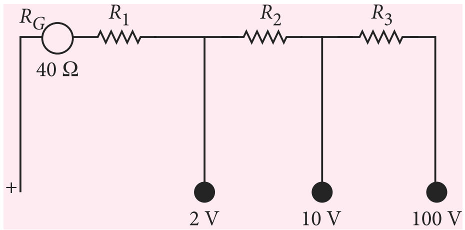
(a) i, ii and v (b) i, ii and vi
(c) iii, iv and v (d) iii, iv and vi

28. When the filament of high resistance burns out, then intensity in

(a) all three settings is 120 W
(b) all three settings is 60 W
(c) two settings is 60 W
(d) two settings is 120 W

Matrix Match Type

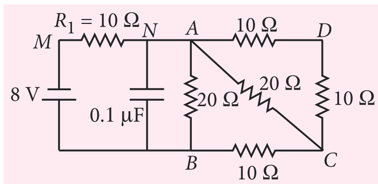
29. Figure shows the internal wiring of a three-range voltmeter whose binding posts are marked +, 2 V, 10 V and 100 V. When the volt meter is connected to the circuit being measured, one connection is made to the post marked + and the other to the post marked with the desired voltage range. The resistance of the moving coil R_G is 40 Ω and a current of 1 mA in the coil causes it to deflect full-scale. Then match the following



Column I	Column II
(A) Value of resistance R_1 in kΩ	(P) 100
(B) Value of resistance R_3 in kΩ	(Q) 2
(C) Overall resistance of the volt meter in 100 V range in kΩ	(R) 1.96
(D) Overall resistance of the volt meter in 2 V range in kΩ	(S) 90

	A	B	C	D
(a)	P	R	Q	S
(b)	S	P	R	Q
(c)	R	S	P	Q
(d)	Q	P	R	S

30. A capacitor of capacitance 0.1 μF is connected to a battery of emf 8 V as shown in figure under the steady-state condition. Then match the following



Column I	Column II
(A) Charge on the capacitor	(P) 0.4 μC
(B) Current in AC branch	(Q) 0.2 A
(C) Current in AB branch	(R) 0.1 A
(D) Current in R_1 connected between M and N	(S) 0.4 A

	A	B	C	D
(a)	P	R	Q	S
(b)	Q	S	P	R
(c)	P	Q	S	R
(d)	P	R	S	Q

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ......
No. of questions correct ......
Marks scored in percentage ......

Check your score! If your score is

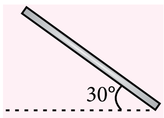
> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

PHYSICS MUSING

Physics Musing was started in August 2013 issue of Physics For You. The aim of Physics Musing is to augment the chances of bright students preparing for JEE (Main and Advanced) / NEET / AIIMS / JIPMER with additional study material.
In every issue of Physics For You, 10 challenging problems are proposed in various topics of JEE (Main and Advanced) / NEET. The detailed solutions of these problems will be published in next issue of Physics For You.
The readers who have solved five or more problems may send their detailed solutions with their names and complete address. The names of those who send atleast five correct solutions will be published in the next issue.
We hope that our readers will enrich their problem solving skills through "Physics Musing" and stand in better stead while facing the competitive exams.

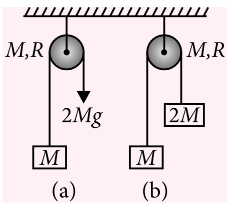
PROBLEM Set 48

1. A slender rod of mass m and length L is pivoted about a horizontal axis through one end and released from rest at an angle of 30° with the horizontal. The force exerted by the pivot on the rod at the instant when the rod passes through a horizontal position is



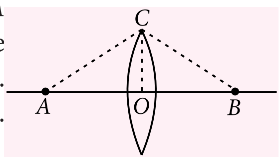
(a) $\frac{\sqrt{10}}{4}mg$ along horizontal
(b) mg along vertical
(c) $\frac{\sqrt{10}}{4}mg$ along a line making an angle of $\tan^{-1}\left(\frac{1}{3}\right)$ with the horizontal
(d) $\frac{\sqrt{10}}{4}mg$ along a line making an angle of $\tan^{-1}(3)$ with the horizontal.

2. A cord is wrapped on a pulley (disk) of mass M and radius R as shown in figure. To one end of the cord, a block of mass M is connected as shown and to other end in (a) a force of 2 Mg and in (b) a block of mass $2M$. Let angular acceleration of the disk in A and B is α_A and α_B respectively, then (cord is not slipping on the pulley)



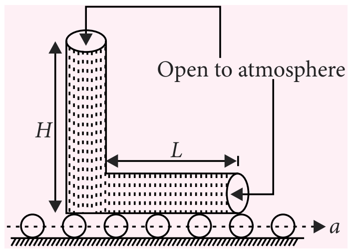
(a) $\alpha_A = \alpha_B$
(b) $\alpha_A > \alpha_B$
(c) $\alpha_A < \alpha_B$
(d) none of these

3. An object is placed at A ($OA > f$). Here f is the focal length of the lens. The image is formed at B. A perpendicular is erected at O and C is chosen such that $\angle BCA = 90°$. Let $OA = a$, $OB = b$ and $OC = c$. Then the value of f is



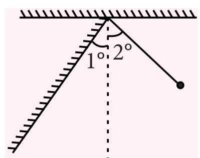
(a) $\frac{(a+b)^3}{c^2}$
(b) $\frac{(a+b)c}{(a+c)}$
(c) $\frac{c^2}{a+b}$
(d) $\frac{a^2}{a+b+c}$

4. A narrow tube completely filled with a liquid is lying on a series of cylinders as shown in the figure. Assuming pure rolling (no sliding between any surfaces), the value of acceleration a of the cylinders for which liquid will not come out of the tube from anywhere is given by



(a) $\frac{gH}{\sqrt{2}L}$
(b) $\frac{gH}{L}$
(c) $\frac{2gH}{L}$
(d) $\frac{gH}{2L}$

5. A simple pendulum of length 1 m is allowed to oscillate with amplitude 2°. If it collides elastically with a wall inclined at 1° to the vertical. Its time period will be (use $g = \pi^2$ m s^{-2})



(a) $\frac{2}{3}$s
(b) $\frac{4}{3}$s
(c) 2 s
(d) none of these

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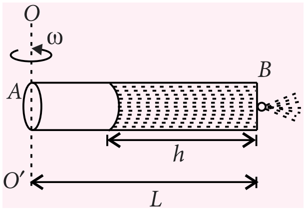
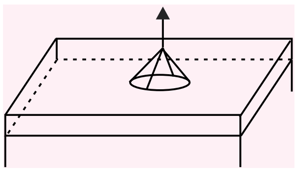
6. A horizontally oriented tube AB of length L rotates with a constant angular velocity ω about a stationary axis OO' passing through the end A as shown in the figure. The tube is filled with an ideal fluid. The end A of the tube is opened and the closed end B has a very small orifice. Find the velocity of the fluid relative to the tube as a function of length of fluid h.



(a) $\dfrac{\omega h}{2}\sqrt{\dfrac{2L}{h}-1}$ (b) $\omega h\sqrt{\dfrac{2L}{h}-1}$

(c) $\omega h\sqrt{\dfrac{L}{h}-1}$ (d) $\dfrac{\omega h}{2}\sqrt{\dfrac{2L}{h}-\dfrac{1}{2}}$

7. The surface tension of water is 75 dyn cm^{-1}. Find the minimum vertical force required to pull a thin wire ring up (refer figure) if it is initially resting on a horizontal water surface. The circumference of the ring is 20 cm and its weight is 0.1 N.



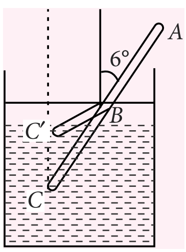
(a) 0.125 N (b) 0.225 N
(c) 0.115 N (d) 0.130 N

8. A solid homogeneous cylinder of height h and base radius r is kept vertically on a conveyer belt moving horizontally with an increasing velocity $v = a + bt^2$. If the cylinder is not allowed to slip then the time when the cylinder is about to topple, will be equal to

(a) $\dfrac{2rg}{bh}$ (b) $\dfrac{rg}{bh}$

(c) $\dfrac{2bg}{rh}$ (d) $\dfrac{rg}{2bh}$

9. A small rod ABC is put in water making an angle 6° with vertical. If it is viewed paraxially from above, it will look like bent shaped ABC', the angle of bending ($\angle CBC'$) will be

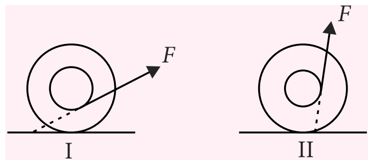


$\left(\text{refactive index of water} = \dfrac{4}{3}\right)$

(a) 2° (b) 3°

(c) 4° (d) 4.5°

10. The string of step rolling wheel is pulled by applying force F with different lines of action in two situations as shown in figure. The wheel starts rolling without slipping due to application of the force.



(a) The wheel rolls to the left in situation I and to the right in situation II
(b) The wheel rolls to the right in situation I and to the left in situation II
(c) The wheel rolls to the right in both situations
(d) The wheel rolls to the left in both situations.

PAPER-I

Section 1 (Maximum Marks : 28)

- *This section contains SEVEN questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four options is(are) correct.*
- *For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.*
- *For each question, marks will be awarded in one of the following categories :*

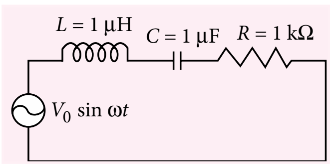
  Full Marks : +4 *If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.*

  Partial Marks : +1 *For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened.*

  Zero Marks : 0 *If none of the bubbles is darkened.*

  Negative Marks : –2 *In all other cases.*
- *For example, if (a), (c) and (d) are all the correct options for a question, darkening all these three will get +4 marks; darkening only (a) and (d) will get +2 marks; and darkening (a) and (b) will get –2 marks, as a wrong option is also darkened.*

1. In the circuit shown, $L = 1\ \mu H$, $C = 1\ \mu F$ and $R = 1\ k\Omega$. They are connected in series with an a.c. source $V = V_0 \sin \omega t$ as shown.



Which of the following options is/are correct?

(a) At $\omega \sim 0$ the current flowing through the circuit becomes nearly zero

(b) The frequency at which the current will be in phase with the voltage is independent of R

(c) The current will be in phase with the voltage if $\omega = 10^4$ rad s^{-1}.

(d) At $\omega >> 10^6$ rad s^{-1}, the circuit behaves like a capacitor.

2. For an isosceles prism of angle A and refractive index μ, it is found that the angle of minimum deviation $\delta_m = A$. Which of the following options is/are correct?

(a) For the angle of incidence $i_1 = A$, the ray inside the prism is parallel to the base of the prism

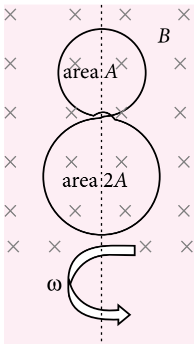
(b) At minimum deviation, the incident angle i_1 and the refracting angle r_1 at the first refracting surface are related by $r_1 = (i_1/2)$

(c) For this prism, the emergent ray at the second surface will be tangential to the surface when the angle of incidence at the first surface is

$$i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\frac{A}{2}-1}-\cos A\right]$$

(d) For this prism, the refractive index μ and the angle of prism A are related as $A = \frac{1}{2}\cos^{-1}\left(\frac{\mu}{2}\right)$

3. A circular insulated copper wire loop is twisted to form two loops of area A and $2A$ as shown in the figure. At the point of crossing the wires remain electrically insulated from each other. The entire loop lies in the plane (of the paper). A uniform magnetic field $\vec{B}$ points into the plane of the paper. At $t = 0$, the loop starts rotating about the common diameter as axis with a constant angular velocity ω in the magnetic field. Which of the following options is/are correct?



(a) The emf induced in the loop is proportional to the sum of the areas of the two loops

(b) The rate of change of the flux is maximum when the plane of the loops is perpendicular to plane of the paper

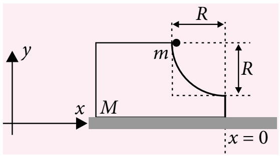
(c) The net emf induced due to both the loops is proportional to $\cos \omega t$

(d) The amplitude of the maximum net emf induced due to both the loops is equal to the amplitude of maximum emf induced in the smaller loop alone

4. A flat plate is moving normal to its plane through a gas under the action of a constant force F. The gas is kept at a very low pressure. The speed of the plate v is much less than the average speed u of the gas molecules. Which of the following options is/are true?

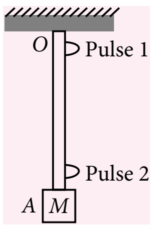
(a) At a later time the external force F balances the resistive force
(b) The plate will continue to move with constant non-zero acceleration, at all times
(c) The resistive force experienced by the plate is proportional to v
(d) The pressure difference between the leading and trailing faces of the plate is proportional to uv.

5. A block of mass M has a circular cut with a frictionless surface as shown. The block rests on the horizontal on frictionless surface of a fixed table. Initially the right edge of the block is at $x = 0$, in a co-ordinate system fixed to the table. A point mass m is released from rest at the topmost point of the path as shown and it slides down. When the mass loses contact with the block, its position is x and the velocity is v. At that instant, which of the following options is/are correct?



(a) The velocity of the point mass m is: $v = \sqrt{\dfrac{2gR}{1+\dfrac{m}{M}}}$
(b) The x component of displacement of the center of mass of the block M is : $-\dfrac{mR}{M+m}$
(c) The position of the point mass is : $x = -\sqrt{2}\dfrac{mR}{M+m}$
(d) The velocity of the block M is : $V = -\dfrac{m}{M}\sqrt{2gR}$

6. A block M hangs vertically at the bottom end of a uniform rope of constant mass per unit length. The top end of the rope is attached to a fixed rigid support at O. A transverse wave pulse (Pulse 1) of wavelength λ_0 is produced at point O on the rope. The pulse takes time T_{OA} to reach point A. If the wave pulse of wavelength λ_0 is produced at point A (Pulse 2) without disturbing the position of M it takes time T_{AO} to reach point O. Which of the following options is/are correct?



(a) The time $T_{AO} = T_{OA}$
(b) The wavelength of Pulse 1 becomes longer when it reaches point A
(c) The velocity of any pulse along the rope is independent of its frequency and wavelength
(d) The velocities of the two pulses (Pulse 1 and Pulse 2) are the same at the midpoint of rope.

7. A human body has a surface area of approximately 1 m^2. The normal body temperature is 10 K above the surrounding room temperature T_0. Take the room temperature to be T_0 = 300 K. For T_0 = 300 K, the value of σT_0^4 = 460 W m^{-2} (where σ is the Stefan Boltzmann constant). Which of the following options is/are correct?
(a) If the body temperature rises significantly then the peak in the spectrum of electromagnetic radiation emitted by the body would shift to longer wavelengths
(b) If the surrounding temperature reduces by a small amount $\Delta T_0 << T_0$, then to maintain the same body temperature the same (living) human being needs to radiate $\Delta W = 4\sigma T_0^3 \Delta T_0$ more energy per unit time
(c) The amount of energy radiated by the body in 1 second is close to 60 Joules
(d) Reducing the exposed surface area of the body (e.g. by curling up) allows humans to maintain the same body temperature while reducing the energy lost by radiation.

Section 2 (Maximum Marks : 15)

- *This section contains FIVE questions.*
- *The answer to each question is a SINGLE DIGIT INTEGER ranging from 0 to 9, both inclusive.*
- *For each question, darken the bubble corresponding to the correct integer in the ORS.*
- *For each question, marks will be awarded in one of the following categories :*

Full Marks : +3 *If only the bubble corresponding to the correct answer is darkened.*

Zero Marks : 0 *In all other cases.*

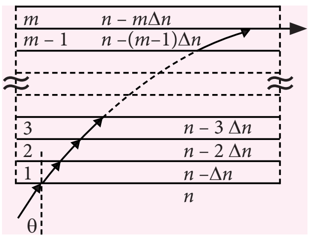
8. An electron in a hydrogen atom undergoes a transition from an orbit with quantum number n_i to another with quantum number n_f. V_i and V_f are respectively the initial and final potential energies of the electron. If $\dfrac{V_i}{V_f} = 6.25$, then the smallest possible n_f is

9. A drop of liquid of radius $R = 10^{-2}$ m having surface tension $S = \dfrac{0.1}{4\pi}$ N m^{-1} divides itself into K identical drops. In this process the total change in the surface energy $\Delta U = 10^{-3}$ J. If $K = 10^{\alpha}$, then the value of α is

10. A stationary source emits sound of frequency υ_0 = 492 Hz. The sound is reflected by a large car approaching the source with a speed of 2 m s^{-1}. The reflected signal is received by the source and superposed with the original. What will be the beat frequency of the resulting signal in Hz? (Given that the speed of sound in air is 330 m s^{-1} and the car reflects the sound at the frequency it has received).

11. ^{131}I is an isotope of Iodine that β decays to an isotope of Xenon with a half-life of 8 days. A small amount of a serum labelled with ^{131}I is injected into the blood of a person. The activity of the amount of ^{131}I injected was 2.4×10^5 Becquerel (Bq.) It is known that the injected serum will get distributed uniformly in the blood stream in less than half an hour. After 11.5 hours, 2.5 ml of blood is drawn from the person's body, and gives an activity of 115 Bq. The total volume of blood in the person's body, in litres is approximately (you may use $e^x \approx 1 + x$ for $|x| \ll 1$ and $\ln 2 \approx 0.7$).

12. A monochromatic light is travelling in a medium of refractive index $n = 1.6$. It enters a stack of glass layers from the bottom side at an angle $\theta = 30°$. The interfaces of the glass layers are parallel to each other. The refractive indices of different glass layers are monotonically decreasing as $n_m = n - m\Delta n$, where n_m is the refractive index of the m^{th} slab and $\Delta n = 0.1$ (see the figure). The ray is refracted out parallel to the interface between the $(m - 1)^{th}$ and m^{th} slabs from the right side of the stack. What is the value of m?



Section 3 (Maximum Marks : 18)

- *This section contains SIX questions of matching type.*
- *This section contains TWO tables (each having 3 columns and 4 rows).*
- *Based on each table, there are THREE questions.*
- *Each question has FOUR options (a), (b), (c), and (d). ONLY ONE of these four options is correct.*
- *For each question, darken the bubble corresponding to the correct option in the ORS.*
- *For each question, marks will be awarded in one of the following categories :*

Full Marks : +3 *If only the bubble corresponding to the correct option is darkened.*

Zero Marks : 0 *If none of the bubbles is darkened*

Negative Marks : −1 *In all other cases*

Answer Q.13, Q.14 and Q.15 by appropriately matching the information given in the three columns of the following table.

A charged particle (electron or proton) is introduced at the origin ($x = 0, y = 0, z = 0$) with a given initial velocity $\vec{v}$. A uniform electric field $\vec{E}$ and a uniform magnetic field $\vec{B}$ exist everywhere. The velocity $\vec{v}$, electric field $\vec{E}$ and magnetic field $\vec{B}$ are given in columns 1, 2 and 3, respectively. The quantities E_0, B_0 are positive in magnitude.

Column I	Column II	Column III
(I) Electron with $\vec{v} = 2\dfrac{E_0}{B_0}\hat{x}$	(i) $\vec{E} = E_0\hat{z}$	(P) $\vec{B} = -B_0\hat{x}$
(II) Electron with $\vec{v} = \dfrac{E_0}{B_0}\hat{y}$	(ii) $\vec{E} = -E_0\hat{y}$	(Q) $\vec{B} = B_0\hat{x}$
(III) Proton with $\vec{v} = 0$	(iii) $\vec{E} = -E_0\hat{x}$	(R) $\vec{B} = B_0\hat{y}$
(IV) Proton with $\vec{v} = 2\dfrac{E_0}{B_0}\hat{x}$	(iv) $\vec{E} = E_0\hat{x}$	(S) $\vec{B} = B_0\hat{z}$

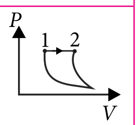
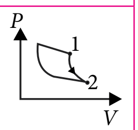
13. In which case would the particle move in a straight line along the negative direction of y-axis (*i.e.*, move along $-\hat{y}$)?
(a) (IV) (ii) (S)
(b) (II) (iii) (Q)
(c) (III) (ii) (R)
(d) (III) (ii) (P)

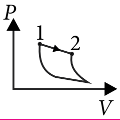
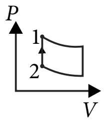
14. In which case will the particle move in a straight line with constant velocity?
(a) (II) (iii) (S)
(b) (III) (iii) (P)
(c) (IV) (i) (S)
(d) (III) (ii) (R)

15. In which case will the particle describe a helical path with axis along the positive z direction?
(a) (II) (ii) (R)
(b) (III) (iii) (P)
(c) (IV) (i) (S)
(d) (IV) (ii) (R)

Answer Q. 16, Q.17 and Q. 18 by appropriately matching the information given in the three columns of the following table.

An ideal gas is undergoing a cyclic thermodynamic process in different ways as shown in the corresponding $P - V$ diagrams in column 3 of the table. Consider only the path from state 1 to state 2. W denotes the corresponding work done on the system. The equations and plots in the table have standard notations as used in thermodynamic processes. Here γ is the ratio of heat capacities at constant pressure and constant volume. The number of moles in the gas is n.

Column I	Column II	Column III
(I) $W_{1\to2} = (P_2V_2 - P_1V_1)\dfrac{1}{\gamma-1}$	(i) Isothermal	(P) 
(II) $W_{1\to2} = -PV_2 + PV_1$	(ii) Isochoric	(Q) 

(III) $W_{1\rightarrow 2} = 0$	(iii) Isobaric	(R) 
(IV) $W_{1\rightarrow 2} = -nRT\ln\left(\frac{V_2}{V_1}\right)$	(iv) Adiabatic	(S) 

16. Which one of the following options correctly represents a thermodynamic process that is used as a correction in the determination of the speed of sound in an ideal gas?

(a) (IV) (ii) (R) (b) (I) (ii) (Q)
(c) (I) (iv) (Q) (d) (III) (iv) (R)

17. Which of the following options is the only correct representation of a process in which $\Delta U = \Delta Q - P\Delta V$?

(a) (II) (iii) (S) (b) (II) (iii) (P)
(c) (III) (iii) (P) (d) (II) (iv) (R)

18. Which one of the following options is the correct combination?

(a) (II) (iv) (P) (b) (III) (ii) (S)
(c) (II) (iv) (R) (d) (IV) (ii) (S)

PAPER-II

Section 1 (Maximum Marks : 21)

- *This section contains SEVEN questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONLY ONE of these four options is correct.*
- *For each question, darken the bubble corresponding to the correct option in the ORS.*
- *For each question, marks will be awarded in one of the following categories :*

Full Marks : +3 *If only the bubble corresponding to the correct option is darkened.*
Zero Marks : 0 *If none of the bubbles is darkened.*
Negative Marks : –1 *In all other cases.*

1. A rocket is launched normal to the surface of the Earth, away from the Sun, along the line joining the Sun and the Earth. The Sun is 3×10^5 times heavier than the Earth and is at a distance 2.5×10^4 times larger than the radius of the Earth. The escape velocity from Earth's gravitational field is $v_e = 11.2$ km s^{-1}. The minimum initial velocity (v_s) required for the rocket to be able to leave the Sun-Earth system is closest to (Ignore the rotation and revolution of the Earth and the presence of any other planet)

(a) $v_s = 62$ km s^{-1} (b) $v_s = 22$ km s^{-1}
(c) $v_s = 72$ km s^{-1} (d) $v_s = 42$ km s^{-1}

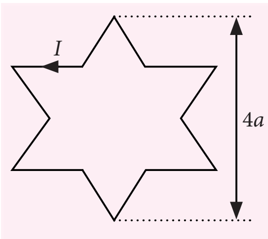
2. A person measures the depth of a well by measuring the time interval between dropping a stone and receiving the sound of impact with the bottom of the well. The error in his measurement of time is $\delta T = 0.01$ s and he measures the depth of the well to be $L = 20$ m. Take the acceleration due to gravity $g = 10$ m s^{-2} and the velocity of sound is 300 m s^{-1}. Then the fractional error in the measurement, $\delta L/L$, is closest to

(a) 5% (b) 1% (c) 3% (d) 0.2%

3. Consider an expanding sphere of instantaneous radius R whose total mass remains constant. The expansion is such that the instantaneous density ρ remains uniform throughout the volume. The rate of fractional change in density $\left(\frac{1}{\rho}\frac{d\rho}{dt}\right)$ is constant. The velocity v of any point on the surface of the expanding sphere is proportional to

(a) R (b) $\frac{1}{R}$ (c) $R^{2/3}$ (d) R^3

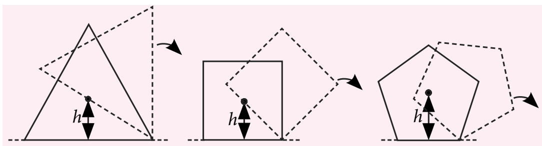
4. A symmetric star shaped conducting wire loop is carrying a steady state current I as shown in the figure. The distance between the diametrically opposite vertices of the star is $4a$. The magnitude of the magnetic field at the center of the loop is



(a) $\frac{\mu_0 I}{4\pi a}3[2-\sqrt{3}]$ (b) $\frac{\mu_0 I}{4\pi a}6[\sqrt{3}-1]$

(c) $\frac{\mu_0 I}{4\pi a}3[\sqrt{3}-1]$ (d) $\frac{\mu_0 I}{4\pi a}6[\sqrt{3}+1]$

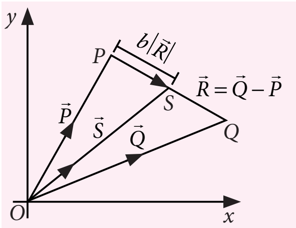
5. Consider regular polygons with number of sides $n = 3, 4, 5$ ...... as shown in the figure. The centre of mass of all the polygons is at height h from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in height of the locus of the center of mass for each polygon is Δ. Then Δ depends on n and h as



(a) $\Delta = h\sin^2\left(\frac{\pi}{n}\right)$ (b) $\Delta = h\tan^2\left(\frac{\pi}{2n}\right)$

(c) $\Delta = h\left(\frac{1}{\cos\left(\frac{\pi}{n}\right)}-1\right)$ (d) $\Delta = h\sin\left(\frac{2\pi}{n}\right)$

6. Three vectors $\vec{P}, \vec{Q}$ and $\vec{R}$ are shown in the figure. Let S be any point on the vector $\vec{R}$. The distance between the points P and S is $b|\vec{R}|$. The general relation among vectors $\vec{P}$, $\vec{Q}$ and $\vec{S}$ is



(a) $\vec{S} = (1-b)\vec{P} + b\vec{Q}$

(b) $\vec{S} = (b-1)\vec{P} + b\vec{Q}$

(c) $\vec{S} = (1-b)\vec{P} + b^2\vec{Q}$

(d) $\vec{S} = (1-b^2)\vec{P} + b\vec{Q}$

7. A photoelectric material having work-function ϕ_0 is illuminated with light of wavelength $\lambda \left(\lambda < \frac{hc}{\phi_0}\right)$. The fastest photoelectron has a de Broglie wavelength λ_d. A change in wavelength of the incident light by $\Delta\lambda$ results in a change $\Delta\lambda_d$ in λ_d. Then the ratio $\Delta\lambda_d/\Delta\lambda$ is proportional to

(a) λ_d^3/λ (b) λ_d^3/λ^2 (c) λ_d^2/λ^2 (d) λ_d/λ

Section 2 (Maximum Marks : 28)

- *This section contains SEVEN questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four options is(are) correct.*
- *For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.*
- *For each question, marks will be awarded in one of the following categories :*

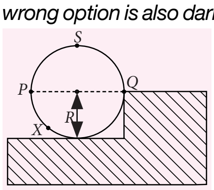
  Full Marks : +4 *If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.*

  Partial Marks : +1 *For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened.*

  Zero Marks : 0 *If none of the bubbles is darkened.*

  Negative Marks : –2 *In all other cases.*
- *For example, if (a), (c) and (d) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (a) and (d) will result in +2 marks; and darkening (a) and (b) will result in –2 marks, as a wrong option is also darkened.*

8. A wheel of radius R and mass M is placed at the bottom of a fixed step of height R as shown in the figure.



A constant force is continuously applied on the surface of the wheel so that it just climbs the step without slipping. Consider the torque τ about an axis normal to the plane of the paper passing through the point Q. Which of the following options is/are correct?

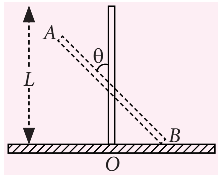
(a) If the force is applied at point P tangentially then τ decreases continuously as the wheel climbs.

(b) If the force is applied tangentially at point S then $\tau \neq 0$ but the wheel never climbs the step.

(c) If the force is applied normal to the circumference at point P then τ is zero.

(d) If the force is applied normal to the circumference at point X then τ is constant.

9. A rigid uniform bar AB of length L is slipping from its vertical position on a frictionless floor (as shown in the figure). At some instant of time, the angle made by the bar with the vertical is θ. Which of the following statements about its motion is/are correct?



(a) When the bar makes an angle θ with the vertical, the displacement of its midpoint from the initial position is proportional to $(1 - \cos\theta)$.

(b) The midpoint of the bar will fall vertically downward.

(c) Instantaneous torque about the point in contact with the floor is proportional to $\sin\theta$.

(d) The trajectory of the point A is a parabola.

10. The instantaneous voltages at three terminals marked X, Y and Z are given by $V_X = V_0 \sin \omega t$,

$$V_Y = V_0 \sin\left(\omega t + \frac{2\pi}{3}\right) \text{ and } V_Z = V_0 \sin\left(\omega t + \frac{4\pi}{3}\right)$$

An ideal voltmeter is configured to read rms value of the potential difference between its terminals. It is connected between points X and Y and then between Y and Z. The reading(s) of the voltmeter will be

(a) independent of the choice of the two terminals

(b) $V_{XY}^{\text{rms}} = V_0$

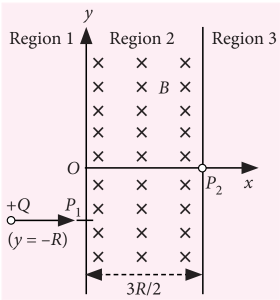
(c) $V_{YZ}^{\text{rms}} = V_0\sqrt{\frac{1}{2}}$

(d) $V_{XY}^{\text{rms}} = V_0\sqrt{\frac{3}{2}}$

MPP-3 CLASS XII ANSWER KEY

1. (b)	2. (d)	3. (c)	4. (c)	5. (c)
6. (a)	7. (a)	8. (c)	9. (c)	10. (c)
11. (a)	12. (b)	13. (c)	14. (d)	15. (a)
16. (d)	17. (d)	18. (b)	19. (a)	20. (b,c,d)
21. (a, d)	22. (b, c)	23. (b,c,d)	24. (5)	25. (9)
26. (2)	27. (a)	28. (d)	29. (c)	30. (a)

11. A uniform magnetic field B exists in the region between $x = 0$ and $x = \frac{3R}{2}$ (region 2 in the figure) pointing normally into the plane of the paper. A particle with charge $+Q$ and momentum p directed along x-axis enters region 2 from region 1 at point $P_1(y = -R)$. Which of the following option(s) is/are correct?



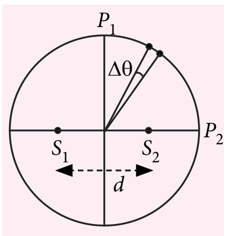
(a) For $B > \frac{2}{3}\frac{p}{QR}$, the particle will re-enter region 1.

(b) For $B = \frac{8}{13}\frac{p}{QR}$, the particle will enter region 3 through the point P_2 on x-axis.

(c) For a fixed B, particles of same charge Q and same velocity v, the distance between the point P_1 and the point of re-entry into region 1 is inversely proportional to the mass of the particle.

(d) When the particle re-enters region 1 through the longest possible path in region 2, the magnitude of the change in its linear momentum between point P_1 and the farthest point from y-axis is $p/\sqrt{2}$.

12. Two coherent monochromatic point sources S_1 and S_2 of wavelength λ = 600 nm are placed symmetrically on either side of the center of the circle as shown. The sources are separated by a distance d = 1.8 mm. This arrangement produces interference fringes visible as alternate bright and dark spots on the circumference of the circle. The angular separation between two consecutive bright spots on the circumference of the circle. The angular separation between two consecutive bright spots is $\Delta\theta$. Which of the following options is/are correct?



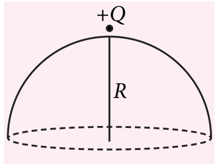
(a) The angular separation between two consecutive bright spots decreases as we move from P_1 to P_2 along the first quadrant.

(b) At P_2 the order of the fringe will be maximum.

(c) A dark spot will be formed at the point P_2.

(d) The total number of fringes produced between P_1 and P_2 in the first quadrant is close to 3000.

13. A point charge $+Q$ is placed just outside an imaginary hemispherical surface of radius R as shown in the figure. Which of the following statements is/are correct?



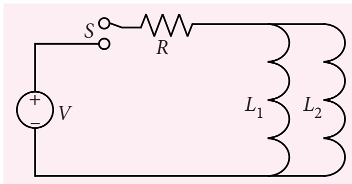
(a) The circumference of the flat surface is an equipotential.

(b) The component of the electric field normal to the flat surface is constant over the surface.

(c) Total flux through the curved and the flat surfaces is $\frac{Q}{\varepsilon_0}$.

(d) The electric flux passing through the curved surface of the hemisphere is $-\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$.

14. A source of constant voltage V is connected to a resistance R and two ideal inductors L_1 and L_2 through a switch S as shown. There is no mutual inductance between the two inductors. The switch S is initially open. At $t = 0$, the switch is closed and current begins to flow. Which of the following options is/are correct?



(a) At $t = 0$, the current through the resistance R is $\frac{V}{R}$.

(b) The ratio of the currents through L_1 and L_2 is fixed at all times ($t > 0$).

(c) After a long time, the current through L_2 will be $\frac{V}{R}\frac{L_1}{L_1 + L_2}$.

(d) After a long time, the current through L_1 will be $\frac{V}{R}\frac{L_2}{L_1 + L_2}$.

Section 3 (Maximum Marks : 12)

- *This section contains TWO paragraphs.*
- *Based on each paragraph, there are TWO questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONLY ONE of these four options is correct.*
- *For each question, darken the bubble corresponding to the correct option in the ORS.*
- *For each question, marks will be awarded in one of the following categories :*

Full Marks : +3 *If only the bubble corresponding to the correct option is darkened.*

Zero Marks : 0 *In all other cases.*

PARAGRAPH 1

Consider a simple RC circuit as shown in figure 1.

Process 1 : In the circuit the switch S is closed at $t = 0$ and the capacitor is fully charged to voltage V_0 (*i.e.*, charging continues for time $T >> RC$). In the process some dissipation (E_D) occurs across the resistance R. The amount of energy finally stored in the fully charged capacitor is E_C.

Process 2 : In a different process the voltage is first set to $\frac{V_0}{3}$ and maintained for a charging time $T >> RC$. Then the voltage is raised to $\frac{2V_0}{3}$ without discharging the capacitor and again maintained for a time $T >> RC$. The process is repeated one more time by raising the voltage to V_0 and the capacitor is charged to the same final voltage V_0 as in Process 1.

These two processes are depicted in figure 2.

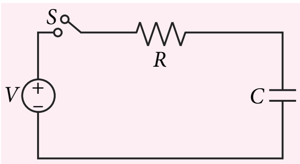


figure 1

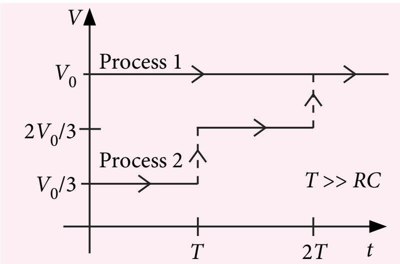


figure 2

15. In Process 2, total energy dissipated across the resistance E_D is

(a) $E_D = 3\left(\frac{1}{2}CV_0^2\right)$ (b) $E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$

(c) $E_D = 3CV_0^2$ (d) $E_D = \frac{1}{2}CV_0^2$

16. In Process 1, the energy stored in the capacitor E_C and heat dissipated across resistance E_D are related by

(a) $E_C = E_D$ (b) $E_C = E_D \ln 2$

(c) $E_C = 2E_D$ (d) $E_C = \frac{1}{2}E_D$

PARAGRAPH 2

One twirls a circular ring (of mass M and radius R) near the tip of one's finger as shown in figure 1. In the process the finger never loses contact with the inner rim of the ring. The finger traces out the surface of a cone, shown by the dotted line. The radius of the path traced out by the point where the ring and the finger is in contact is r. The finger rotates with an angular velocity ω_0. The rotating ring rolls without slipping on the outside of a smaller circle described by the point where the ring and the finger is in contact (figure 2). The coefficient of friction between the ring and the finger is μ and the acceleration due to gravity is g.

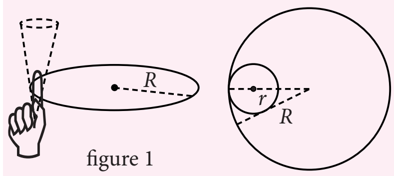


figure 1 figure 2

17. The minimum value of ω_0 below which the ring will drop down is

(a) $\sqrt{\frac{3g}{2\mu(R-r)}}$ (b) $\sqrt{\frac{g}{\mu(R-r)}}$

(c) $\sqrt{\frac{g}{2\mu(R-r)}}$ (d) $\sqrt{\frac{2g}{\mu(R-r)}}$

18. The total kinetic energy of the ring is

(a) $M\omega_0^2(R-r)^2$ (b) $\frac{1}{2}M\omega_0^2(R-r)^2$

(c) $\frac{3}{2}M\omega_0^2(R-r)^2$ (d) $M\omega_0^2R^2$

ANSWER KEYS

PAPER-I

1. (a,b)	**2.** (a,b,c)	**3.** (b,d)	**4.** (a,c,d)	**5.** (a,b)
6. (a,c)	**7.** (d)	**8.** (5)	**9.** (6)	**10.** (6)
11. (5)	**12.** (8)	**13.** (c)	**14.** (a)	**15.** (c)
16. (c)	**17.** (b)	**18.** (b)		

PAPER-II

1. (d)	**2.** (b)	**3.** (a)	**4.** (b)	**5.** (c)
6. (a)	**7.** (b)	**8.** (b or b,c)		
9. (a,b,c)	**10.** (a,d)	**11.** (a,b)	**12.** (b,d)	**13.** (a,d)
14. (b,c,d)	**15.** (b)	**16.** (a)	**17.** (b)	
18. (bonus)				

For detail solutions refer to :

MTG JEE Advanced Explorer,
MTG 40 years JEE Advanced Chapterwise Solutions.

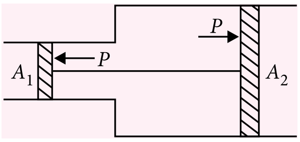
◈◈

PHYSICS MUSING

SOLUTION SET-47

1. **(c) :** Let specific heat of water be s J $g^{-1}\,°C^{-1}$
Energy released by water to come to 0°C
$= m_w s_w (T_i - T_f)_w = (10\text{ g})\, s\,(10 - 0)\,°\text{C}$
$= (10\text{ g})\, s\,(10°\text{C}) = 100$ J
Energy absorbed by ice to come to 0°C
$$= m_I s_I (T_f - T_i)_I = (10\text{ g})\left(\frac{s}{2}\right)(0-(-20))\,°\text{C}$$
$$= (10\text{ g})\,\frac{s}{2}(20°\text{C}) = (10\text{ g})\,s\,(10°\text{C}) = 100\text{ J}$$
(Here, we have used subscript w and I for water and ice respectively).
Since heat absorbed by ice is equal to the heat released by water, therefore, no ice will convert into water. Hence, option (c) is correct.

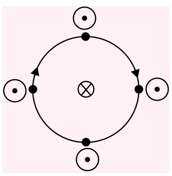
2. **(b) :** When we heat the gas, the temperature of system rises and gas starts expanding. Since, pistons are movable, the gas exerts pressure on both the piston to increase space. As the piston are joined by inextensible string, hence, both the pistons will move in the direction of net force acting on the system.



As $A_1 < A_2$ $\therefore$ $PA_1 < PA_2$ or $F_1 < F_2$ ($\because$ $F = PA$)
Net force, $F = F_2 - F_1$ (rightwards)

3. **(b) :** Current in primary circuit, $I = \dfrac{2}{(5+R)}$
Also, $6\times10^{-3} = \dfrac{2}{(5+R)}\times5\times0.6$
$\Rightarrow$ $5 + R = 1000$ $\Rightarrow$ $R = 995\ \Omega$

4. **(d) :** For a circular wire direction of magnetic field is shown



5. **(a) :** The equivalent resistance of the given circuit is $\left(\dfrac{R\times G}{R+G}\right)+1.$
Current supplied by the cell, $I = \dfrac{24}{\dfrac{RG}{R+G}+1}$
Voltmeter reading,
$V = 24$ V – Potential difference across 1 Ω
$= 24 - I \times 1$
$$V = 24 - \frac{24}{\left(\dfrac{RG}{R+G}\right)+1} = 24 - \frac{24}{\left(\dfrac{1}{\dfrac{1}{R}+\dfrac{1}{G}}\right)+1}$$
For $G \to \infty$; $V = 20$ V
$$\Rightarrow\ 20 = 24 - \frac{24}{R+1}\ \Rightarrow\ R = 5\ \Omega$$

6. **(b, c) :** Magnetic field due to loop of radius R carrying current I at a distance x is given by,
$$B = \frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}}$$
Let us consider that the distance of point O from centres of loop X and Y be x and x' respectively.
Using properties of similar triangles,
$$\frac{x}{x'} = \frac{r}{2r}\ \Rightarrow\ x' = 2x$$
Now, the current in each loop is of same magnitude and flowing in same sense, hence magnetic field at point O is in same direction,
$$\text{For loop } X : B = \frac{\mu_0 I r^2}{2(r^2+x^2)^{3/2}}$$
$$\text{For loop } Y : B' = \frac{\mu_0 I (2r)^2}{2((2r)^2+(2x)^2)^{3/2}} = \frac{B}{2}$$
$\therefore$ Total magnetic field at O is $\left(B+\dfrac{B}{2}\right)$ or $\dfrac{3B}{2}$

7. **(a, c, d) :** Reading of V_1 and V_2 are equal
$$\Rightarrow\ X_C = X_L\ \Rightarrow\ \omega = \frac{1}{\sqrt{LC}} = \frac{\pi}{4}\times10^3\text{ Hz}$$
$$2\pi\upsilon = \frac{\pi}{4}\times10^3\text{ Hz}\quad\Rightarrow\ \upsilon = 125\text{ Hz}$$
Net impedance of LCR circuit, $Z = R = 100\ \Omega$
$\therefore$ Current flowing in the circuit,
$$I_{\text{rms}} = \frac{V}{Z} = \left(\frac{200}{100}\right)\text{A} = 2\text{ A}$$
$$V_1 = I_{\text{rms}} X_L = (2)\times\frac{2}{\pi}\times\frac{\pi}{4}\times10^3 = 1000\text{ V}$$

8. **(0) :** If the volume of liquid at temperature 10 °C is 2 litre, then the volume at temperature 0°C is
$$V_{10} = \frac{2}{(1+10\gamma)}$$

Solution Senders of Physics Musing

SET-47

1. Avnish Mishra, Patna
2. Subrata Sinha, Delhi
3. Nishita Kelkar, Pune

Similarly if the volume of liquid at temperature is 20°C is 7 litre then the volume at temperature 0°C is

$$V_{20} = \frac{7}{(1+20\gamma)}$$

If the density at 0°C is d, then the masses are

$m_1 = V_{10}d$ and $m_2 = V_{20}d$

After mixing the liquids, the temperature is

$$T = \frac{m_1T_1 + m_2T_2}{m_1 + m_2} = \frac{10m_1 + 20m_2}{m_1 + m_2}$$

The volume of liquids at this temperature are

$V_{10}(1 + \gamma T)$ and $V_{20}(1 + \gamma T)$

The sum of volume of liquids after mixing is

$$V = V_{10}(1 + \gamma T) + V_{20}(1 + \gamma T)$$

$$= V_{10} + V_{20} + \gamma(V_{10} + V_{20})T$$

$$= V_{10} + V_{20} + \gamma\left(\frac{m_1 + m_2}{d}\right)\frac{10m_1 + 20m_2}{m_1 + m_2}$$

$$= V_{10} + V_{20} + 10\gamma\, V_{10} + 20\gamma\, V_{20} = V_1 + V_2$$

Hence the sum of volume of liquids is constant.

9. (3) : $U^\beta \propto V \quad \Rightarrow T^\beta \propto V$

$P^\beta V^\beta \propto V \qquad (\because\ PV = nRT)$

$P^\beta V^{\beta-1}$ = constant; $PV^{(\beta-1)/\beta}$ = constant

$$\frac{\Delta U}{\Delta Q} = \frac{nC_V\Delta T}{nC\Delta T} = \frac{1}{3};\ C = 3C_V$$

$$C_V + \frac{R}{1-x} = 3\,C_V\text{, where } x = \frac{\beta-1}{\beta}$$

For monoatomic gas, $C_V = \frac{3R}{2}$

$\Rightarrow\ \beta = 3$

10. (4) : For dc source : $R = \frac{12}{4} = 3\,\Omega$

For ac source : $Z = \frac{12}{2.4} = 5\,\Omega$

As, $Z = \sqrt{R^2 + X_L^2}$

$\therefore\ R^2 + (\omega L)^2 = 25 \qquad (\because\ X_L = \omega L)$

$(\omega L)^2 = 25 - 9 = 16$

$\omega L = 4 \quad \Rightarrow \quad L = \frac{4}{\omega}$

$$\omega = 2\pi\upsilon = 2\pi \times \frac{50}{\pi} = 100 \text{ rad s}^{-1}$$

$$L = \frac{4}{100} = 4 \times 10^{-2}\text{ H}$$

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