



# លេខីទៅនៃស្ថាគម្ពស់

ថ្ងៃទី ១២

ស្រី ស៊ុអុន





## ហាតិវា

### ពិនិត្យណិត

១	បែវិទ្ទេសនុសម៉ោង	៣
១.១	និយមនីយ	៥
១.១.១	ការកំណត់សេសរ	៥
១.២	តាមមានដែរធម៌	៥
១.៣	លក្ខណៈនៃដែរធម៌	១០
១.៤	ដែរធម៌នូវគម្ពីសហគ្ភារ	១៥
១.៥	ដែរធម៌នូវគម្ពីក្រឹកការមាត្រិ	១៥
១.៦	ដែរធម៌នូវគម្ពីចល្លួយជាតិ	១៥
១.៧	ដែរធម៌នូវគម្ពីការការពិត	១៥
១.៨	ដែរធម៌នូវគម្ពីនៅក្នុងផែន	១៥
១.៩	ដែរធម៌នូវគម្ពី Arc Sine និង Arc Tangent	១៥
១.១០	រូបមន្ត្រដែរធម៌	១៥
១.១១	ចំហាត់ និង ដំឡាសក្រាយ	១៥



# ពិភ័សជាតិ

## ១ លេខិតផលនូចបញ្ជី ..... ៧

- ១.១ និយមនីយ
- ១.២ រាមានដើរីន់
- ១.៣ លក្ខណវង់នឹងដើរីន់
- ១.៤ ដើរីនឹងអនុគមន៍បណ្តាក់
- ១.៥ ដើរីនឹងអនុគមន៍ត្រីកាលាយាត្រា
- ១.៦ ដើរីនឹងអនុគមន៍អិចស្សែរុញដែលស្រួល
- ១.៧ ដើរីនឹងអនុគមន៍ការកើត
- ១.៨ ដើរីនឹងអនុគមន៍ការកើតនៅទំនើត
- ១.៩ ដើរីនឹងអនុគមន៍ Arc Sine និង Arc Tangent
- ១.១០ រូបមន្ទីដើរីន់
- ១.១១ លំហាត់ និង ដំណោះស្រាយ
- ១.១២ លំហាត់មេដ្ឋាន





## ១. លើកបង្កើតនិងលក្ខណៈ

### ១.១ លិមិនីតុលិមិនី

លិមិនីតុលិមិនី ១.១.១ ដើរីនៅអនុគមន៍  $y = f(x)$  ត្រង់  $a$  កំណត់ដោយ

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} \quad (1.1)$$

- អនុគមន៍  $f$  មានដើរីលិមិនីច្បាប់ចំណុច  $a \in (b, c)$  ។
- អនុគមន៍  $f$  មានដើរីលិមិនីបិទ  $[b, c]$  កាលណា  $f$  មានដើរីលិមិនី  $(b, c)$  ហើយ  $f$  មានដើរី ខាងឆ្វែងត្រង់  $x = b$  និងខាងស្តាំត្រង់  $x = c$  ។

■ ឧទាហរណ៍ ១.១ វក្សាដើរីនៅអនុគមន៍  $y = f(x) = 2x^2 + 3$  ត្រង់ ២ ។ ■

### ជំន៉ាមេគ្រោះ

$$f'(2) = \lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2} \text{ ដើម } f(2) = 2(2)^2 + 3 = 11$$

$$\begin{aligned} f'(2) &= \lim_{x \rightarrow 2} \frac{2x^2 + 3x - 11}{x - 2} \\ &= \lim_{x \rightarrow 2} \frac{2x^2 - 8}{x - 2} \\ &= \lim_{x \rightarrow 2} \frac{2(x+2)(x-2)}{x - 2}, x \neq 2 \\ &= 2 \times 4 = 8 \end{aligned}$$

### ១.១.១ គ្មានអនុសម្រេច

តាត  $h = x - a \implies x = h + a$  បើ  $h \rightarrow 0$  នោះ  $x \rightarrow a$  នៅសមីការ (១.១) គឺជា

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(h+a) - f(a)}{h} \quad (1.1)$$

■ ចំណាំ ១.១ គឺជាសម្រេចដោយ  $y'$ ,  $f'(x)$  ឬ  $\frac{dy}{dx}$  ។

■ ឧបាទាង ១.២ ត្រូវយថាបើ  $y = x$  នោះ  $y' = 1$  ។

### ស្រួលយកល្អាក់

តាមនិយមន៍យ

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(h+x) - f(x)}{h} \text{ ដែល } y = f(x) = x, f(h+x) = h+x \\ &= \lim_{h \rightarrow 0} \frac{h+x-x}{h} = \lim_{h \rightarrow 0} 1 = 1 \\ \therefore \quad \frac{dy}{dx} &= 1 \end{aligned}$$

**ទីផ្សារ ១.១.១** បើអនុគមន៍  $f$  មានដំឡើងត្រង់  $x_0$  នោះ  $f$  ជាប់ត្រង់  $x_0$  ។

### ស្រួលយកល្អាក់

គឺជាបង្ហាញថា  $f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$  នោះ  $\lim_{x \rightarrow a} f(x) = f(a)$

$$\begin{aligned} \text{គឺជា} \quad \lim_{x \rightarrow a} f(x) &= \lim_{x \rightarrow a} (f(x) - f(a) + f(a)) \\ &= \lim_{x \rightarrow a} \left( \frac{f(x) - f(a)}{x - a} \times (x - a) + f(a) \right) \\ &= \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} \times \lim_{x \rightarrow a} (x - a) + f(a) \\ &= f'(a) \times 0 + f(a) \\ &= f(a) \end{aligned}$$

■ ចំណាំ ១.២ បើអនុគមន៍  $f$  ជាប់ត្រង់  $x_0$  នោះ  $f$  អាចមានដំឡើងត្រង់  $x_0$  ឬ ត្រូវដំឡើងត្រង់  $x_0$  ។

### ១.២ តាមរូបរឹង

**សិល្ងមទី២ ១.២.១** អនុគមន៍  $f$  មានដំឡើងត្រង់  $x$  លូបត្រាតែ

- អនុគមន៍  $f$  ជាប់ត្រង់  $x$  ។

- ដំឡើងស្ថិតិយោគស្ថិតិក្រោចចំណុច  $x$  តើ  $f'_-(x) = f'_+(x)$  ដឹងបាន

$$f'_-(x) = \lim_{h \rightarrow 0^-} \frac{f(x+h) - f(x)}{h} \text{ និង } f'_+(x) = \lim_{h \rightarrow 0^+} \frac{f(x+h) - f(x)}{h}$$

■ **ឧទាហរណ៍ ១.៣** គឺអនុគមន៍  $f$  កំណត់ដោយ  $f(x) = \begin{cases} \cos x & \text{បើ } x \leq \frac{\pi}{4} \\ a+bx & \text{បើ } x > \frac{\pi}{4} \end{cases}$

កំណត់តម្លៃ  $a$  និង  $b$  ដើម្បីធ្វើអនុគមន៍  $f$  មានដំឡើងត្រង់  $x = \frac{\pi}{4}$  ។ ■

**សម្រាយបញ្ជាក់**

- បើអនុគមន៍  $f$  ជាប់ត្រង់  $x = \frac{\pi}{4}$  នៅ:  $\lim_{x \rightarrow \frac{\pi}{4}^-} f(x) = \lim_{x \rightarrow \frac{\pi}{4}^+} f(x) = f\left(\frac{\pi}{4}\right)$

$$\lim_{x \rightarrow \frac{\pi}{4}^-} \cos x = \lim_{x \rightarrow \frac{\pi}{4}^+} (a+bx) = \cos \frac{\pi}{4} \iff \frac{\sqrt{2}}{2} = a+b \cdot \frac{\pi}{4} = \frac{\sqrt{2}}{2} \implies a = \frac{\sqrt{2}}{2} - \frac{\pi}{4} \cdot b$$

- ដំឡើង  $f'_-(x)$

$$\begin{aligned} f'_-\left(\frac{\pi}{4}\right) &= \lim_{h \rightarrow 0^-} \frac{f\left(\frac{\pi}{4}+h\right) - f\left(\frac{\pi}{4}\right)}{h}, f(x) = \cos x \\ &= \lim_{h \rightarrow 0^-} \frac{\cos\left(\frac{\pi}{4}+h\right) - \cos\frac{\pi}{4}}{h} \\ &= \lim_{h \rightarrow 0^-} \frac{\cos\frac{\pi}{4} \cos h - \sin\frac{\pi}{4} \sin h - \cos\frac{\pi}{4}}{h} \\ &= \lim_{h \rightarrow 0^-} \frac{-\cos\frac{\pi}{4}(1 - \cos h) - \sin\frac{\pi}{4} \sin h}{h} \\ &= \frac{\sqrt{2}}{2} \lim_{h \rightarrow 0^-} \left( -\frac{1 - \cos h}{h} - \frac{\sin h}{h} \right), \lim_{h \rightarrow 0^-} \frac{1 - \cos h}{h} = 0, \lim_{h \rightarrow 0^-} \frac{\sin h}{h} = 1 \\ &= \frac{\sqrt{2}}{2}(0 - 1) = -\frac{\sqrt{2}}{2} \end{aligned}$$

- ដំឡើង  $f'_+(x)$

$$\begin{aligned} f'_+\left(\frac{\pi}{4}\right) &= \lim_{h \rightarrow 0^+} \frac{f\left(\frac{\pi}{4}+h\right) - f\left(\frac{\pi}{4}\right)}{h}, f(x) = a+bx \\ &= \lim_{h \rightarrow 0^+} \frac{a+b\left(\frac{\pi}{4}+h\right) - (a+b \cdot \frac{\pi}{4})}{h} \\ &= \lim_{h \rightarrow 0^+} \frac{a+b \cdot \frac{\pi}{4} + bh - a - b \cdot \frac{\pi}{4}}{h} \\ &= \lim_{h \rightarrow 0^+} \frac{bh}{h} = b \end{aligned}$$

$$\text{ដោយ } f \text{ មានដំឡើងត្រង់ } x = \frac{\pi}{4} \text{ នៅ: } f'_-\left(\frac{\pi}{4}\right) = f'_+\left(\frac{\pi}{4}\right) \iff b = -\frac{\sqrt{2}}{2} \implies a = \frac{\sqrt{2}}{2} \left(1 + \frac{\pi}{4}\right)$$

### ១.៣ លក្ខណៈនៃដៅវិទ្យា

**លក្ខណៈ ១** ចំណោះ  $u, v$  ជាអនុគមន៍នៃ  $x$  និង  $k$  ជាបំនុំនៃបែងបាន៖

$$១. (ku)' = ku'$$

$$\text{២. } (u - v)' = u' - v'$$

$$\text{៤. } \left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$$

$$\text{៥. } (u + v)' = u' + v'$$

$$\text{៦. } (uv)' = u'v + v'u$$

$$\text{៧. } \left(\frac{1}{v}\right)' = -\frac{v'}{v^2}$$

### ស្របតាមបញ្ជាក់

១. តាង  $f(x) = k \cdot u(x)$  ដើម្បី  $u = u(x)$  និង  $k$  ជាបំនុំនៃបែងបាន៖

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{ku(x+h) - k \cdot u(x)}{h} \\ &= k \cdot \lim_{h \rightarrow 0} \frac{u(x+h) - u(x)}{h} \\ &= k \cdot u'(x) \end{aligned}$$

$$\therefore (k \cdot u)' = k \cdot u'$$

២. តាង  $f(x) = u(x) + v(x)$  ដើម្បី  $u = u(x)$  និង  $v = v(x)$  តាមនិយមន័យ

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{u(x+h) + v(x+h) - (u(x) + v(x))}{h} \\ &= \lim_{h \rightarrow 0} \frac{u(x+h) - u(x)}{h} + \lim_{h \rightarrow 0} \frac{v(x+h) - v(x)}{h} \\ &= u'(x) + v'(x) \end{aligned}$$

$$\therefore (u + v)' = u' + v'$$

៣. ស្រាយដោចឡើង

៤. តាង  $f(x) = uv$  ដើម្បី  $u = u(x)$  និង  $v = v(x)$  តាមនិយមន៍យោគបាន

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{u(x+h).v(x+h) - u(x).v(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{u(x+h).v(x+h) - u(x).v(x+h) + u(x).v(x+h) + u(x).v(x)}{h} \\
 &= \lim_{h \rightarrow 0} \left[ \frac{u(x+h).v(x+h) - u(x).v(x+h)}{h} + \frac{u(x).v(x+h) + u(x).v(x)}{h} \right] \\
 &= \lim_{h \rightarrow 0} \left[ v(x+h) \cdot \frac{u(x+h) - u(x)}{h} + u(x) \cdot \frac{v(x+h) + v(x)}{h} \right] \\
 &= v(x) \cdot \frac{d}{dx}(u(x)) + u(x) \cdot \frac{d}{dx}(v(x))
 \end{aligned}$$

$$\therefore (uv)' = u'v + v'u \quad (9.3)$$

៥. យក  $u = u(x)$  និង  $v = v(x)$  តាង  $f(x) = \frac{u}{v} \Leftrightarrow f(x).v = u$  ធ្វើដោរអង្គទាំងពីរជូនិង  $x$

នៅពេល  $[f(x).v]' = u'$  ប្រើតាមសមីការ (9.3) គឺបាន

$$f'(x).v + v'f(x) = u', \quad f(x) = \frac{u}{v}$$

$$f'(x).v + v' \cdot \frac{u}{v} = u'$$

$$\frac{f'(x).v^2}{v} + \frac{v'u}{v} = u'$$

$$f'(x).v^2 + v'u = u'v$$

$$f'(x) = \frac{u'v - v'u}{v^2}$$

$$\therefore \left( \frac{u}{v} \right)' = \frac{u'v - v'u}{v^2} \quad (9.4)$$

៦. យក  $v = v(x)$  តាង  $f(x) = \frac{1}{v}$  ប្រើសមីការ (9.4) គឺបាន

$$\begin{aligned}
 f'(x) &= \frac{(1)' \cdot v - v' \cdot (1)}{v^2} \\
 &= \frac{0 - v'}{v^2} \\
 &= -\frac{v'}{v^2} \\
 \therefore \left( \frac{1}{v} \right)' &= -\frac{v'}{v^2}
 \end{aligned}$$

## ១.៤ ដែនិកលេខាលុកសម្រាប់បង្កើត

**ចាត់ទេស ១** បើ  $y = f(u)$  និង  $u = g(x)$  នោះ  $\frac{d}{dx}(f \circ g) = \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

### សម្រាយបញ្ជាក់

តាម  $F(x) = f \circ g = f(g(x))$  តាមនិយមន៍យករាល់មានដឹងថ្វាន់  $x = a$  នោះគឺបាន

$$\begin{aligned} F'(a) &= \lim_{x \rightarrow a} \frac{F(x) - F(a)}{x - a} \\ &= \lim_{x \rightarrow a} \frac{f(g(x)) - f(g(a))}{x - a} \\ &= \lim_{x \rightarrow a} \left( \frac{f(g(x)) - f(g(a))}{g(x) - g(a)} \times \frac{g(x) - g(a)}{x - a} \right) \\ &= f'(g(a)) \times g'(a), u = g(a), y = f(a) \\ \therefore \frac{d}{dx}(f \circ g) &= \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} \end{aligned}$$

**ចាត់ទេស ២** បើ  $y = c$  ដែល  $c$  ជាប័ណ្ណនៅរី នោះ  $y' = 0$

### សម្រាយបញ្ជាក់

គឺមាន  $y = f(x_0) = c$  នោះ  $f(x_0 + h) = c, c \in \mathbb{R}$  តាមនិយមន៍យកគឺបាន

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{c - c}{h} \\ &= \lim_{h \rightarrow 0} \frac{0}{h} \\ \therefore \frac{d}{dx}(c) &= 0 \end{aligned}$$

■ **ឧបាទេស ១.៤** តណាន  $y'$  ដែល  $y = (\ln x \cdot \log_a(\sqrt{3}))$

### ចំណែកស្តាយ

គឺមាន  $y = (\ln x \cdot \log_a(\sqrt{3})) \Rightarrow y' = (\ln x \cdot \log_a(\sqrt{3}))' = 0$

■ **ឧបាទេស ១.៥** ស្រាយបញ្ហាក់ថា បើ  $y = x^n$  នោះ  $y' = nx^{n-1}$

### ស្រោចយោបល់

គោលន៍  $f(x) = x^n$  នាំ $f(x+h) = (x+h)^n$  តាមនិយមន័យ

$$\begin{aligned}
 y' = f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(x+h)^n - x^n}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(x+h-x)(x^{n-1} + x^{n-2} \cdot x + \dots + x \cdot x^{n-2} \cdot x + x^{n-1})}{h} \\
 &= \lim_{h \rightarrow 0} (x^{n-1} + x^{n-1} + \dots + x^{n-1} + x^{n+1}) \\
 &= x^{n-1} \left( \underbrace{1 + 1 + \dots + 1 + 1}_{n \text{ ពីរលីម}} \right) \\
 &= n \cdot x^{n-1} \\
 \therefore \quad \frac{d}{dx}(x^n) &= n \cdot x^{n-1}
 \end{aligned}$$

#### ■ ឧបាទាន់ ១.៦ គណន៍ $f'(x)$

៩.  $f(x) = x^3$

ឬ.  $f(x) = \sqrt{x}$

៣.  $f(x) = \sqrt[3]{x^2}$

### ចំណោម: ស្រោចយោបល់

៩.  $f(x) = x^3 \Rightarrow f'(x) = (x^3)' = 3x^{3-1} = 3x^2$

ឬ.  $f(x) = \sqrt{x} \Rightarrow f'(x) = (\sqrt{x})' = (x^{\frac{1}{2}})' = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

៣.  $f(x) = \sqrt[3]{x^2} \Rightarrow f'(x) = (\sqrt[3]{x^2})' = (x^{\frac{2}{3}})' = \frac{2}{3}x^{\frac{2}{3}-1} = \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{x}}$

**ចាប់ដៅ ៣** បើ  $y = u^n$  ដែល  $u$  ជាអនុគមន៍នៃ  $x$  នោះ  $y' = nu'u^{n-1}$  ។

### ស្រោចយោបល់

គោលន៍  $y = u^n$  គោល  $y' = \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(u^n) \times u' = nu'u^{n-1}$

#### ■ ឧបាទាន់ ១.៧ គណន៍ $y'$

៩.  $y = (2x + \ln 2)^4$

១០.  $y = \sqrt{u}$  ដើម្បី  $u$  ជាអនុគមន៍នៃ  $x$  ។

### ចំណែក ១. ដែនិទ្ទេនៃអនុគមន៍

៩.  $y = (2x + \ln 2)^4 \Rightarrow y' = 4(2x + \ln 2)'(2x + \ln 2)^{4-1} = 4(2+0)(2x + \ln 2)^3$

$$\therefore y' = 8(2x + \ln 2)^3$$

១០.  $y = \sqrt{u} = u^{\frac{1}{2}} \Rightarrow y' = (u^{\frac{1}{2}})' = \frac{1}{2}u'u^{\frac{1}{2}-1} = \frac{1}{2}u'u^{-\frac{1}{2}} = \frac{u'}{2\sqrt{u}}$

### ១.៥ ដែនិទ្ទេនៃអនុគមន៍ស្រីកោណាតាមវត្ថុ

**ឧទ្ធផល:** ២ ដែនិទ្ទេនៃអនុគមន៍ត្រីកោណាតាមវត្ថុ

៩. បើ  $y = \sin x$  នៅ៖  $y' = \cos x$

១០. បើ  $y = \cos x$  នៅ៖  $y' = -\sin x$

១១. បើ  $y = \tan x$  នៅ៖  $y' = \frac{1}{\cos^2 x} = 1 + \tan^2 x$

១២. បើ  $y = \cot x$  នៅ៖  $y' = -\frac{1}{\sin^2 x} = -(1 + \cot^2 x)$

### សម្រាយបញ្ជាក់

៩. តើមាន  $y = f(x) = \sin x$  នៅ៖  $f(x+h) = \sin(x+h)$  តាមនិយមន៍យ

$$\begin{aligned} y' = f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin x \cos h + \sin h \cos x - \sin x}{h} \\ &= \lim_{h \rightarrow 0} \left( \cos x \cdot \frac{\sin h}{h} - \sin x \cdot \frac{1 - \cos h}{h} \right) \\ &= \cos x \quad , \lim_{h \rightarrow 0} \frac{\sin h}{h} = 1, \lim_{h \rightarrow 0} \frac{1 - \cos h}{h} = 0 \end{aligned}$$

$$\therefore \frac{d}{dx}(\sin x) = \cos x$$

២. គេបាន  $y = f(x) = \cos x$  នៅ:  $f(x+h) = \cos(x+h)$  តាមនិយមន៍យោ

$$\begin{aligned} y' = f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos x}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cos x \cdot \cos h - \sin x \cdot \sin h - \cos x}{h} \\ &= \lim_{h \rightarrow 0} \left( -\frac{\sin h}{h} \cdot \sin x - \cos x \cdot \frac{1 - \cos h}{h} \right) \\ &= -\sin x, \lim_{h \rightarrow 0} \frac{1 - \cos h}{h} = 0, \lim_{h \rightarrow 0} \frac{\sin h}{h} = 1 \\ \therefore \quad \frac{d}{dx}(\cos x) &= -\sin x \end{aligned}$$

៣. តាង  $y = \tan x = \frac{\sin x}{\cos x}$  តាមសមីការ (១.៥) គេបាន

$$\begin{aligned} y' &= \left( \frac{\sin x}{\cos x} \right)' = \frac{(\sin x)' \cos x - (\cos x)' \cdot \sin x}{(\cos x)^2} \\ &= \frac{\cos x \cdot \cos x - (-\sin x) \cdot \sin x}{\cos^2 x} \\ &= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \\ &= 1 + \tan^2 x \\ &= \frac{1}{\cos^2 x}, \sin^2 x + \cos^2 x = 1 \\ \therefore \quad (\tan x)' &= \frac{1}{\cos^2 x} = 1 + \tan^2 x \end{aligned}$$

៤. តាង  $y = \cot x = \frac{\cos x}{\sin x}$  តាមសមីការ (១.៥) គេបាន

$$\begin{aligned} y' &= \left( \frac{\cos x}{\sin x} \right)' = \frac{(\cos x)' \cdot \sin x - (\sin x)' \cdot \cos x}{(\sin^2 x)^2} \\ &= \frac{-\sin x \cdot \sin x - \cos x \cdot \cos x}{\sin^2 x} \\ &= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} \\ &= -\frac{\sin^2 x + \cos^2 x}{\sin^2 x}, \sin^2 x + \cos^2 x = 1 \\ \therefore \quad (\cot x)' &= -\frac{1}{\sin^2 x} = -(1 + \cot^2 x) \end{aligned}$$

**ចាយទេន ៤** បើ  $u$  ជាអនុគមន៍នៃ  $x$  គឺបាន

១. បើ  $y = \sin u$  នោះ  $y' = u' \cos u$

២. បើ  $y = \cos u$  នោះ  $y' = -u' \sin u$

៣. បើ  $y = \tan u$  នោះ  $y' = \frac{u'}{\cos^2 u} = u(1 + \tan^2 u)$

៤. បើ  $y = \cot u$  នោះ  $y' = -\frac{u'}{\sin^2 u} = -u'(1 + \cot^2 u)$

### សម្រាប់បញ្ជូន

១. បើ  $u$  ជាអនុគមន៍នៃ  $x$  នោះ  $y = \sin u$  គឺបាន

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(\sin u) \times \frac{du}{dx} = \cos u \times u' = u' \cos u \\ \therefore \quad \frac{d}{dx}(\sin u) &= u' \cos u\end{aligned}$$

២. បើ  $u$  ជាអនុគមន៍នៃ  $x$  នោះ  $y = \cos u$  គឺបាន

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(\cos u) \times \frac{du}{dx} = -\sin u \times u' = -u' \sin u \\ \therefore \quad \frac{d}{dx}(\cos u) &= -u' \sin u\end{aligned}$$

៣. បើ  $u$  ជាអនុគមន៍នៃ  $x$  នោះ  $y = \tan u$  គឺបាន

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(\tan u) \times \frac{du}{dx} = \frac{1}{\cos^2 u} \times u' = (1 + \tan^2 u) \times u' \\ \therefore \quad (\tan u)' &= \frac{u'}{\cos^2 u} = u'(1 + \tan^2 u)\end{aligned}$$

៤. បើ  $u$  ជាអនុគមន៍នៃ  $x$  នោះ  $y = \cot u$  គឺបាន

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(\cot u) \times \frac{du}{dx} = -\frac{1}{\sin^2 u} \times u' = -(1 + \cot^2 u) \times u' \\ \therefore \quad (\cot u)' &= -\frac{u'}{\sin^2 u} = -u'(1 + \cot^2 u)\end{aligned}$$

#### ឧត្តមាឌ ១.៤ គណនោះនៃអនុគមន៍ខាងក្រោម៖

១.  $y = \sin(2x + 1)$

៣.  $y = \tan(2x + 1)$

៥.  $y = \cos(2x + 1)$

៤.  $y = \cot(2x + 1)$

### ចំណោម: ស្ថាប័ន

$$9. \ y = \sin(2x+1) \Rightarrow y' = (2x+1)' \cos(2x+1) = 2 \cos(2x+1)$$

$$10. \ y = \cos(2x+1) \Rightarrow y' = -(2x+1)' \sin(2x+1) = -2 \sin(2x+1)$$

$$11. \ y = \tan(2x+1) \Rightarrow y' = \frac{(2x+1)'}{\cos^2(2x+1)} = \frac{2}{\cos^2(2x+1)} = 2[1 + \tan^2(2x+1)]$$

$$12. \ y = \cot(2x+1) \Rightarrow y' = -\frac{(2x+1)'}{\sin^2(2x+1)} = -\frac{2}{\sin^2(2x+1)} = -2[1 + \cot^2(2x+1)]$$

## ១.៦ លេរិនេអនុគមន៍ដិចស្សូរនៃតម្លៃ

ប្រាក់យើងបើ  $y = a^x$  នៅ:  $y' = a^x \cdot \ln a$

### ស្ថាប័នបញ្ជាក់

តើមាន  $y = a^x$  តាមនីយមនឹមួយ តើបាន

$$\begin{aligned} y' &= f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{a^{x+h} - a^x}{h} \\ &= \lim_{h \rightarrow 0} \frac{a^x(a^h - 1)}{h} \\ &= a^x \lim_{h \rightarrow 0} \frac{a^h - 1}{h} \text{ ដើម្បី } \lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \ln a \\ \therefore \quad (a^x)' &= a^x \cdot \ln a \end{aligned}$$

**ចាត់ទេន ៥** បើ  $u$  ជាអនុគមន៍នៃ  $x$  នៅ:  $(a^u)' = u' a^u \cdot \ln a$

### ស្ថាប័នបញ្ជាក់

បើ  $u$  ជាអនុគមន៍នៃ  $x$  នៅ: តើបាន

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(a^u) \times \frac{du}{dx} = a^u \cdot \ln a \times u' \\ \therefore \quad (a^u)' &= u' \cdot a^u \cdot \ln a \end{aligned}$$

■ **ឧបាទោន៍ ១.៦** តិណនា  $y'$  ចំពោះ  $u$  ជាអនុគមន៍នៃ  $x$  នៃអនុគមន៍ខាងក្រោម៖

៩.  $y = e^x$

១៩.  $y = a^{x^2 - 1}$

២៩.  $y = e^u$

### ចំណែក ១. ដែនីទេសអនុសម្ព័ន្ធ

៩.  $y = e^x$  នៅ:  $y' = (e^x)' = e^x \cdot \ln e = e^x$ ,  $\ln e = 1$

១៩.  $y = a^{x^2 - 1}$  នៅ:  $y' = (x^{x^2 - 1})' a^{x^2 - 1} \ln a = 2x \cdot a^{x^2 - 1} \ln a$

២៩.  $y = e^u$  នៅ:  $y' = (e^u)' = u' e^u \cdot \ln e = u' e^u$ ,  $\ln e = 1$

## ១.៧ ដែនីទេសអនុសម្ព័ន្ធតាមគីឡូ

ស្រាយបញ្ជាក់ថា បើ  $y = \log_a x$ ,  $a > 0, a \neq 1$  នៅ:  $y' = \frac{1}{x \ln a}$

### សម្រាយបញ្ជាក់

គឺមាន  $y = \log_a x$  តាមនិយមនីយ៍ គួលនា

$$\begin{aligned} y' &= f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\log_a(x+h) - \log_a(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \log_a \left( \frac{x+h}{x} \right) \\ &= \lim_{h \rightarrow 0} \log_a \left( 1 + \frac{h}{x} \right)^{\frac{1}{h}} \\ &= \log_a \left( \lim_{h \rightarrow 0} \left( 1 + \frac{1}{\frac{x}{h}} \right)^{\frac{x}{h}} \right)^{\frac{1}{x}} \text{ ដោយ } \lim_{x \rightarrow 0} \left( 1 + \frac{1}{x} \right)^x = e \\ &= \log_a e^{\frac{1}{x}} = \frac{1}{x} \ln a \\ \therefore \quad (\log_a x)' &= \frac{1}{x \ln a}, a > 0, a \neq 1 \end{aligned}$$

**បញ្ជាផែន ៦** បើ  $u$  ជាអនុគមន៍នៃ  $x$  នៅ:  $(\log_a u)' = \frac{u'}{u \ln a}$ ,  $a > 0, a \neq 1$

### សម្រាយបញ្ជាក់

បើ  $u$  ជាអនុគមន៍នៃ  $x$  នៅទៅ គេបាន

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(\log_a u) \times \frac{du}{dx} = \frac{1}{u \ln a} \times u'$$

$$\therefore (\log_a u)' = \frac{u'}{u \ln a}, a > 0, a \neq 1$$

## ១.៥ ដែរីទៅលើអនុគមន៍លេខការិតឡេង

ស្រាយបញ្ជាក់ថា បើ  $y = \ln x$  នៅទៅ  $y' = \frac{1}{x}$

ស្រាយបញ្ជាក់

គេមាន  $y = \ln x$  តាមនិយមន៍យ៉ា គេបាន

$$\begin{aligned} y' = f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\ln(x+h) - \ln(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\ln\left(\frac{x+h}{x}\right)}{h} \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \ln\left(1 + \frac{h}{x}\right) \\ &= \lim_{h \rightarrow 0} \ln\left(1 + \frac{h}{x}\right)^{\frac{1}{h}} \\ &= \ln \left[ \lim_{h \rightarrow 0} \left(1 + \frac{1}{\frac{x}{h}}\right)^{\frac{x}{h} \times \frac{1}{x}} \right], \lim_{h \rightarrow 0} \left(1 + \frac{1}{\frac{x}{h}}\right)^{\frac{x}{h}} = e \\ &= \ln e^x, \ln e = 1 \end{aligned}$$

$$\therefore (\ln x)' = \frac{1}{x}$$

■ ឧបាទេណ្ឌ ១.៩០ វិក  $f'(x)$  នៃអនុគមន៍ខាងក្រោម៖

៩.  $f(x) = x^2 \cdot \log_a x, a > 0, a \neq 1$

៦.  $f(x) = \log(x^2 \sqrt{x^3 - 1})$

៩.  $f(x) = \sin(2x) + \log_2(x^2 + 1)$

៥.  $f(x) = (\sin x)^{\log x}$

៣.  $f(x) = \frac{e^{2x} + \log_3 x}{x^2}$

៦.  $f(x) = (\log_a x)^{\ln(2x)}, a > 0, a \neq 1$

ស្រាយបញ្ជាក់

៩.  $f(x) = x^2 \cdot \log_a x, a > 0, a \neq 1 \implies f'(x) = (x^2)' \log_a x + (\log_a x)' x^2$

$$= 2x \log_a x + \frac{1}{x \ln a} x^2$$

$$\therefore f'(x) = 2x \log_a x + \frac{x}{\ln a}, a > 0, a \neq 1$$

១០.  $f(x) = \sin(2x) + \log_2(x^2 + 1) \implies f'(x) = -(2x)' \cos(2x) + \frac{(x^2 + 1)'}{(x^2 + 1) \ln 2}$

$$\therefore f'(x) = -2 \cos(2x) + \frac{2x}{(x^2 + 1) \ln 2}$$

១១.  $f(x) = \frac{e^{2x} + \log_3 x}{x^2} \implies f'(x) = \frac{(e^{2x} + \log_3 x)' x^2 - (x^2)' (e^{2x} + \log_3 x)}{x^4}$

$$= \frac{(2e^{2x} + \frac{1}{x \ln 3}) x^2 - 2x(e^{2x} + \log_3 x)}{x^4}$$

$$= \frac{2xe^{2x} + \frac{1}{\ln 3} - 2e^{2x} - 2\log_3 x}{x^3}$$

$$\therefore f'(x) = \frac{2e^{2x}(x-1) + \frac{1}{\ln 3} - \log_3 x^2}{x^3}$$

១២.  $f(x) = \log(x^2 \sqrt{x^3 - 1}) = \log x^2 + \log(x^3 - 1)^{\frac{1}{2}} = 2 \log x + \frac{1}{2} \log(x^3 - 1)$

$$\therefore f'(x) = \frac{2}{x \ln 10} + \frac{(x^3 - 1)'}{2(x^3 - 1) \ln 10} = \frac{2}{x \ln 10} + \frac{3x^2}{2(x^3 - 1) \ln 10}$$

១៣.  $f(x) = (\sin x)^{\log x} \iff \ln f(x) = \ln(\sin x)^{\log x}$

$$(\ln f(x))' = (\log x \cdot \ln(\sin x))'$$

$$\frac{f'(x)}{f(x)} = (\log x)' \ln(\sin x) + (\ln(\sin x))' \log x$$

$$f'(x) = f(x) \left( \frac{1}{x \ln 10} \ln(\sin x) + \frac{(\sin x)'}{\sin x} \cdot \log x \right)$$

$$\therefore f'(x) = (\sin x)^{\log x} \left( \frac{\ln(\sin x)}{x \ln 10} + \cot x \cdot \log x \right)$$

១៤.  $f(x) = (\log_a x)^{\ln(2x)}, a > 0, a \neq 1 \iff \ln f(x) = \ln(\log_a x)^{\ln(2x)}$

$$(\ln f(x))' = (\ln(2x) \cdot \ln(\log_a x))'$$

$$\frac{f'(x)}{f(x)} = (\ln(2x))' \ln(\log_a x) + (\ln(\log_a x))' \ln(2x)$$

$$f'(x) = f(x) \left( \frac{(2x)'}{2x} \ln(\log_a x) + \frac{(\log_a x)'}{\log_a x} \ln(2x) \right)$$

$$\therefore f'(x) = (\log_a x)^{\ln(2x)} \left( \frac{\ln(\log_a x)}{x} + \frac{\ln(2x)}{x \ln a \log_a x} \right), a > 0, a \neq 1$$

**ចាប់ដោយ** បើ  $u$  ជាអនុគមន៍នៃ  $x$  នៅ:  $(\ln u)' = \frac{u'}{u}$

### សម្រាយបញ្ជាក់

បើ  $u$  ជាអនុគមន៍នៃ  $x$  នៅ: គោល

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(\ln u) \times \frac{du}{dx} = \frac{1}{u} \times u' \\ \therefore (\ln u)' &= \frac{u'}{u} \end{aligned}$$

■ ឧបាទាន់ ១.១១ វិក  $f'(x)$  នៃអនុគមន៍ខាងក្រោម៖

- |                                     |                                     |
|-------------------------------------|-------------------------------------|
| ៩. $f(x) = x \cdot \ln x$           | ៤. $f(x) = \ln(x^2 \sqrt{x^3 - 1})$ |
| ៥. $f(x) = x^2 + \ln(x^2 + 1)$      | ៥. $f(x) = x^x$                     |
| ៦. $f(x) = \frac{e^x + \ln x}{x^2}$ | ៦. $f(x) = (\sin x)^{\cos x}$       |

### សម្រាយបញ្ជាក់

$$\begin{aligned} ៩. f(x) = x \cdot \ln x \implies f'(x) &= x' \ln x + (\ln x)'x = \ln x + \frac{1}{x} \cdot x = \ln x + 1 \\ ៥. f(x) = x^2 + \ln(x^2 + 1) \implies f'(x) &= (x^2)' + \frac{(x^2 + 1)'}{x^2 + 1} = 2x + \frac{2x}{x^2 + 1} \\ ៦. f(x) = \frac{e^x + \ln x}{x^2} \implies f'(x) &= \frac{(e^x + \ln x)'x^2 - (x^2)'(e^x + \ln x)}{(x^2)^2} \\ &= \frac{\left(e^x + \frac{1}{x}\right)x^2 - 2x(e^x + \ln x)}{x^4} \\ \therefore f'(x) &= \frac{xe^x + 1 - 2e^x - 2\ln x}{x^3} \end{aligned}$$

$$៤. f(x) = \ln(x^2 \sqrt{x^3 - 1}) = \ln x^2 + \ln \sqrt{x^3 - 1} = 2 \ln x + \ln(x^3 - 1)^{\frac{1}{2}}$$

$$\begin{aligned} f'(x) &= 2(\ln x)' + \frac{1}{2}[\ln(x^3 - 1)]' \\ &= 2 \cdot \frac{1}{x} + \frac{1}{2} \cdot \frac{(x^3 - 1)'}{x^3 - 1} \\ \therefore f'(x) &= \frac{2}{x} + \frac{3}{2} \cdot \frac{x^2}{x^3 - 1} \end{aligned}$$

៤.  $f(x) = x^x \iff \ln f(x) = \ln x^x = x \ln x$

$$\begin{aligned} (\ln f(x))' &= (x \ln x)' \\ \frac{f'(x)}{f(x)} &= x' \ln x + (\ln x)' x \\ f'(x) &= f(x)(\ln x + \frac{1}{x} \cdot x) \\ \therefore f'(x) &= x^x(\ln x + 1) \end{aligned}$$

៥.  $f(x) = (\sin x)^{\cos x} \iff \ln f(x) = \ln(\sin x)^{\cos x}$

$$\begin{aligned} (\ln f(x))' &= (\cos x \ln \sin x)' \\ \frac{f'(x)}{f(x)} &= (\cos x)' \ln \sin x + (\ln \sin x)' \cos x \\ f'(x) &= f(x) \left( -\sin x \ln \sin x + \frac{(\sin x)'}{\sin x} \cdot \cos x \right) \\ \therefore f'(x) &= (\sin x)^{\cos x} (\cos x \cot x - \sin x \ln \sin x) \end{aligned}$$

### ១.៦ ទេរីនៃលេខល្អតម្លៃ Arc Sine និង Arc Tangent

$$\begin{aligned} y = \arcsin x &\iff x = \sin y \text{ និង } -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}, \\ y = \arctan x &\iff x = \tan y \text{ និង } -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}, \end{aligned}$$

■ ឧបាទាន់ ១.៩២ ស្រាយថា បើ  $y = \arcsin x$  នោះ  $y' = \frac{1}{\sqrt{1-x^2}}$

#### ស្រាយបញ្ជាក់

បើ  $y = \arcsin x$  នោះ  $x = \sin y$  នូវដៅអង្គសងខាងពួរបន្តិច  $x$  គេបាន

$$\begin{aligned} (x)' &= (\sin y)' \iff 1 = y' \cos y \\ y' &= \frac{1}{\cos y} \text{ ដើម្បី } \sin^2 y + \cos^2 y = 1 \\ \implies \cos y &= \pm \sqrt{1 - \sin^2 x} \end{aligned}$$

$$\text{ដើម្បី } -\frac{\pi}{2} \leq y \leq \frac{\pi}{2} \implies \cos y \geq 0 \implies \cos y = \sqrt{1 - x^2}$$

$$\therefore (\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

### ១.៤ ដើរីនឹងអនុគមន៍ Arc Sine និង Arc Tangent

- ឧទាហរណ៍ ១.៩៣ ក្រុាយថា បើ  $y = \arctan x$  នោះ  $y' = \frac{1}{1+x^2}$

#### ស្រាយបញ្ជាក់

បើ  $y = \arctan x$  នោះ  $x = \tan y$  ដូចជាអង់រៃត្តិសមិទ្ធភាពរវាង  $x$  តើបាន

$$(x)' = (\sin y)' \iff 1 = y'(1 + \tan^2 y)$$

$$y' = \frac{1}{1 + \tan^2 y}$$

$$\therefore (\arctan x)' = \frac{1}{1 + x^2}$$

**ចាប់ផ្តែង ៤** បើ  $u$  ជាអនុគមន៍នៃ  $x$  នោះ  $(\arcsin u)' = \frac{u'}{\sqrt{1 - u^2}}$

#### ស្រាយបញ្ជាក់

បើ  $u$  ជាអនុគមន៍នៃ  $x$  នោះ តើបាន

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(\arcsin u) \times \frac{du}{dx} = \frac{1}{\sqrt{1-u^2}} \times u' \\ \therefore (\arcsin u)' &= \frac{u'}{\sqrt{1-u^2}} \end{aligned}$$

**ចាប់ផ្តែង ៥** បើ  $u$  ជាអនុគមន៍នៃ  $x$  នោះ  $(\arctan u)' = \frac{u'}{1+u^2}$

#### ស្រាយបញ្ជាក់

បើ  $u$  ជាអនុគមន៍នៃ  $x$  នោះ តើបាន

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(\arctan u) \times \frac{du}{dx} = \frac{1}{1-u^2} \times u' \\ \therefore (\arctan u)' &= \frac{u'}{1+u^2} \end{aligned}$$

- ឧទាហរណ៍ ១.៩៤ តណានាផើននៃអនុគមន៍ខាងក្រោម៖

៩.  $f(x) = \arcsin x \cdot \sin x$

ឱ.  $f(x) = \sin(\arcsin x)$

៩.  $f(x) = \arctan x \cos x$

៤.  $f(x) = \arctan(\tan x)$

### សម្រាយរបាយការ

៩.  $f(x) = \arcsin x \cdot \sin x \implies f'(x) = (\arcsin x)' \sin x + (\sin x)' \arcsin x$

$$\therefore f'(x) = \frac{\sin x}{\sqrt{1-x^2}} + \cos x \cdot \arcsin x$$

ឱ.  $f(x) = \arctan x \cos x \implies f'(x) = (\arctan x)' \cos x + (\cos x)' \arctan x$

$$\therefore f'(x) = \frac{\cos x}{1+x^2} - \sin x \cdot \arctan x$$

ឱ.  $f(x) = \sin(\arcsin x) \implies f'(x) = (\arcsin x)' \cos(\arcsin x)$

$$\therefore f'(x) = \frac{\cos(\arcsin x)}{\sqrt{1-x^2}}$$

៤.  $f(x) = \arctan(\tan x) \implies f'(x) = \frac{(\tan x)'}{1+(\tan x)^2} = \frac{1+\tan^2 x}{1+\tan^2 x}$

$$\therefore f'(x) = 1$$

## ១.១០ ប្រចាំឆ្នាំនៃដែរធម្ម

បើ  $C, a, b, c$  ជាប័ណ្ណនមួយ និង  $u$  ជាអនុគមន៍នៃ  $x$  ដើម្បី  $n \in \mathbb{N}$  តែបាន៖

- |  |   |
|--|---|
| ៩. $(C)' = 0$  | ២០. $(e^u)' = u'e^u$  |
| ១១. $(x)' = 1$   | ២១. $(\sin x)' = \cos x$                                      |
| ១២. $(ax+b)' = a$  | ២២. $(\sin u)' = u' \cos u$                                   |
| ១៣. $(ax^2+bx+c)' = 2ax+b$                               | ២៣. $(\cos x)' = -\sin x$                                     |
| ១៤. $(x^n)' = nx^{n-1}$                                  | ២៤. $(\cos u)' = -u' \sin u$                                  |
| ១៥. $(u^n)' = n.u'.u^{n-1}$                              | ២៥. $(\tan x)' = \frac{1}{\cos^2 x} = 1 + \tan^2 x$           |
| ១៦. $(x)^{-n} = -\frac{n}{x^{n+1}}$                      | ២៦. $(\tan u)' = \frac{u'}{\cos^2 u} = u'(1 + \tan^2 u)$      |
| ១៧. $\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$         | ២៧. $(\cot x)' = -\frac{1}{\sin^2 x} = -(1 + \cot^2 x)$       |
| ១៨. $\left(\frac{1}{u}\right)' = -\frac{u'}{u^2}$        | ២៨. $(\cot u)' = -\frac{u'}{\sin^2 u} = -(1 + \cot^2 u)$      |
| ១៩. $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$                  | ២៩. $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$                   |
| ១២០. $(\sqrt{u})' = \frac{u'}{2\sqrt{u}}$                | ៣០. $(\arcsin u)' = \frac{u'}{\sqrt{1-u^2}}$                  |
| ១២១. $(\sqrt[n]{x})' = \frac{1}{n\sqrt[n]{x^{n-1}}}$     | ៣១. $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$                  |
| ១២២. $(\ln x)' = \frac{1}{x}$                            | ៣២. $(\arccos u)' = -\frac{u'}{\sqrt{1-u^2}}$                 |
| ១២៣. $(\ln u)' = \frac{u'}{u}$                           | ៣៣. $(\arctan x)' = \frac{1}{1+x^2}$                          |
| ១២៤. $(\log_a x)' = \frac{1}{x \ln a}, a > 0, a \neq 1$  | ៣៤. $(\arctan u)' = \frac{u'}{1+u^2}$                         |
| ១២៥. $(\log_a u)' = \frac{u'}{u \ln a}, a > 0, a \neq 1$ | ៣៥. $(\text{arccot } x)' = -\frac{1}{1+x^2}$                  |
| ១២៦. $(a^x)' = a^x \ln a, a > 0, a \neq 1$               | ៣៦. $(\text{arccot } u)' = -\frac{u'}{1+u^2}$                 |
| ១២៧. $(a^u)' = u'a^u \ln a, a > 0, a \neq 1$             | ៣៧. $(u^\nu)' = \left(v'.\ln u + \frac{v.u'}{u}\right).u^\nu$ |
| ១២៨. $(e^x)' = e^x$                                      |   |

**១.៩៩ សំហាត់ និង សំដោះស្រាយ**

**សំហាត់ ៩** គណនា  $f'(x)$  នៃអនុគមន៍ខាងក្រោម៖

- |  |   |
|--|---|
| ៩. $f(x) = x^5 - x^4 + x^3 - x^2 + x - 1$  | ៩. $\sqrt[4]{x^3 - 2x}$                   |
| ១០. $f(x) = 2x^2 - \sqrt{x} + \frac{2}{x}$ | ១១. $f(x) = (x+1)(2x-1)^2$                |
| ១២. $f(x) = (x^4 - 7x^2 + \sin a)^7$       | ១៣. $f(x) = (x^2 + 2x + 3)(x^3 - 3x - 1)$ |
| ១៤. $f(x) = (x^2 - \sqrt{x})^{2019}$       | ១៥. $f(x) = \frac{1}{x-1}$                |
| ១៦. $f(x) = \sqrt{x^3 - x^2 + 3}$          | ១៧. $f(x) = \frac{x\sqrt{x}}{x+1}$        |

**សម្រាយបញ្ជាក់**

៩.  $f(x) = x^5 - x^4 + x^3 - x^2 + x - 1 \Rightarrow f'(x) = 5x^4 - 4x^3 + 3x^2 - 2x + 1$
១០.  $f(x) = 2x^2 - \sqrt{x} - \frac{2}{x} \Rightarrow f'(x) = 4x^2 + \frac{1}{2\sqrt{x}} - \frac{2}{x^2}$
១១.  $f'(x) = 7(x^4 - 7x^2 + \sin a)'(x^4 - 7x^2 + \sin a)^{7-1} = 7(4x^3 - 14x)(x^4 - 7x^2 + \sin a)^6$
១២.  $f(x) = (x^4 - 7x^2 + \sin a)^7$
១៣.  $f(x) = (x^2 - \sqrt{x})^{2019} \Rightarrow f'(x) = 2019(x^2 - \sqrt{x})'(x^2 - \sqrt{x})^{2019-1}$   
 $= 2019 \left( 2x - \frac{1}{2\sqrt{x}} \right) (x^2 - \sqrt{x})^{2018}$
១៤.  $f(x) = \sqrt{x^3 - x^2 + 3} \Rightarrow f'(x) = \frac{(x^3 - x^2 + 3)'}{2\sqrt{x^3 - x^2 + 3}} = \frac{3x - 2}{2\sqrt{x^3 - x^2 + 3}}$
១៥.  $\sqrt[4]{x^3 - 2x} \iff f(x) = (x^3 - 2x)^{\frac{1}{4}}$   
 $f'(x) = \frac{1}{4}(x^3 - 2x)'(x^3 - 2x)^{\frac{1}{4}-1}$   
 $= \frac{1}{4}(3x^2 - 2)(x^3 - 2x)^{-\frac{3}{4}}$   
 $\therefore f'(x) = \frac{3x^2 - 2}{4\sqrt[4]{(x^3 - 2x)^3}}$
១៦.  $f(x) = (x+1)(2x-1)^2$   
 $f'(x) = (x+1)'(2x-1)^2 + [(2x-1)^2]'(x+1)$   
 $= (2x-1)^2 + 2(2x-1)'(2x-1)(x+1)$   
 $= (2x-1)(2x-1 + 4x+4)$   
 $\therefore f'(x) = (2x-1)(6x+3)$

៤.  $f(x) = (x^2 + 2x + 3)(x^3 - 3x - 1)$

$$\begin{aligned} f'(x) &= (x^2 + 2x + 3)'(x^3 - 3x - 1) + (x^3 - 3x - 1)'(x^2 + 2x + 3) \\ &= (2x+2)(x^3 - 3x - 1) + (2x-3)(x^2 + 2x + 3) \\ &= 2x^3 - 6x^2 - 2x + 2x^2 - 6x - 2 + 2x^3 + 4x^2 + 6x - 3x^2 - 6x - 9 \\ \therefore f'(x) &= 4x^3 - 3x^2 - 8x - 11 \end{aligned}$$

៥.  $f(x) = \frac{1}{x-1} \implies f'(x) = -\frac{(x-1)'}{(x-1)^2} = -\frac{1}{(x-1)^2}$

៩០.  $f(x) = \frac{x\sqrt{x}}{x+1}$

$$\begin{aligned} f'(x) &= \frac{(x\sqrt{x})'(x+1) - (x+1)'x\sqrt{x}}{(x+1)^2} \\ &= \frac{[x'\sqrt{x} + (\sqrt{x})'x](x+1) - x\sqrt{x}}{(x+1)^2} \\ &= \frac{\left(x + \frac{x}{2\sqrt{x}}\right)(x+1) - x\sqrt{x}}{(x+1)^2} \\ &= \frac{x\sqrt{x} + \sqrt{x} + \frac{x}{2\sqrt{x}}(x+1) - x\sqrt{x}}{(x+1)^2} \end{aligned}$$

$\therefore f'(x) = \frac{x^2 + 3x}{2\sqrt{x}(x+1)^2}$

### លំហាត់ ២ គណនាដើម្បីនឹងអនុគមន៍ខាងក្រោម៖

៩.  $f(x) = x \cdot \sin x + \cos x$

៥.  $f(x) = \sin^2 \sqrt{x} + \cos^2(3x)$

៩.  $f(x) = \sin^3 x - x \cdot \cos x$

៥.  $f(x) = \cos(3x+4) + 3 \cos x \cdot \sin x$

៩០.  $f(x) = \cos(x^2 + 1) + 2 \sin(x^2 - 1)$

៥.  $f(x) = \sin(\sin \sqrt{x}) + \cos^3 x$

### ចំណែកស្រាយ

៩.  $f(x) = x \cdot \sin x + \cos x$

$$f'(x) = x' \sin x + (\sin x)' \cdot x - \sin x$$

$$= \sin x + x \cdot \cos x - \sin x$$

$\therefore f'(x) = x \cdot \cos x$

$$\text{Q. } f(x) = \sin^3 x - x \cdot \cos x$$

$$f'(x) = 3(\sin x)' \sin^{3-1} x - [x' \cdot \cos x + (\cos x)' \cdot x]$$

$$= 3 \cos x \cdot \sin^2 x - (\cos x - x \cdot \sin x)$$

$$\therefore f'(x) = 3\cos x \cdot \sin^2 x - \cos x + x \sin x$$

$$\text{m. } f(x) = \cos(x^2 + 1) + 2 \sin(x^2 - 1)$$

$$f'(x) = -(x^2 + 1)' \sin(x^2 + 1) + 2(x^2 - 1)' \cos(x^2 - 1)$$

$$\therefore f'(x) = -2x \sin(x^2 + 1) + 4x \cos(x^2 - 1)$$

$$\text{Q. } f(x) = \sin^2 \sqrt{x} + \cos^2(3x)$$

$$f'(x) = 2(\sin \sqrt{x})' \cos^{2-1} \sqrt{x} + 2(\cos(3x))' \sin(3x)$$

$$= 2(\sqrt{x})' \cdot \cos \sqrt{x} \cdot \cos \sqrt{x} - 2(3x)' \sin(3x) \cdot \sin(3x)$$

$$\therefore f'(x) = \frac{1}{\sqrt{x}} \cdot \cos^2 \sqrt{x} - 6 \sin^2(3x)$$

$$\text{Q. } f(x) = \cos(3x+4) + 3\cos x \cdot \sin x$$

$$f'(x) = -(3x+4)' \cdot \sin(3x+4) + 3[(\cos x)' \cdot \sin x + (\sin x)' \cdot \cos x]$$

$$= -3 \sin(3x + 4) + 3[-\sin x \cdot \sin x + \cos x \cdot \cos x]$$

$$\therefore f'(x) = -3[\sin(3x+4) + \sin^2 x - \cos^2 x]$$

$$5. \quad f(x) = \sin(\sin \sqrt{x}) + \cos^3 x$$

$$f'(x) = (\sin \sqrt{x})' \cdot \cos(\sin \sqrt{x}) + 3(\cos x) \cos^{3-1} x$$

$$= (\sqrt{x})' \cdot \cos \sqrt{x} \cdot \cos(\sin \sqrt{x}) - 3 \sin x \cos^2 x$$

$$\therefore f'(x) = \frac{1}{2\sqrt{x}} \cos \sqrt{x} \cdot \cos(\sin \sqrt{x}) - 3 \sin x \cdot \cos^2 x$$

ជំនាញ ៣ គណនាថៃនៃអនុគមន៍ខាងក្រោម៖

$$9. \quad f(x) = (1 + \tan x)^4$$

$$\text{म. } f(x) = x \cdot \tan(x^2 - 1) + x \cot(2x^2)$$

$$\text{Q. } f(x) = x^2 \tan x + (1 + \cot x)^2$$

$$\text{Q. } f(x) = \frac{\tan(2x)}{1 - \cos x}$$

### ចំណោះស្រាយ

$$9. \quad f(x) = (1 + \tan x)^4$$

$$f'(x) = 4(1 + \tan x)'(1 + \tan^2 x)^{4-1}$$

$$\therefore f'(x) = 4(1 + \tan^2 x)(1 + \tan x)^3$$

$$10. \quad f(x) = x^2 \tan x + (1 + \cot x)^2$$

$$f'(x) = (x^2)' \tan x + (\tan x)' x^2 + 2(1 + \cot x)' (1 + \cot x)^{2-1}$$

$$\therefore f'(x) = 2x \tan x + x^2(1 + \tan^2 x) - 2(1 + \cot^2 x)(1 + \cot x)$$

$$11. \quad f(x) = x \cdot \tan(x^2 - 1) + x \cot(2x^2)$$

$$\begin{aligned} f'(x) &= x' \tan(x^2 - 1) + [\tan(x^2 - 1)]' x + x' \cot(2x^2) + [\cot(2x^2)]' x \\ &= \tan(x^2 - 1) + (x^2 - 1)' [1 + \tan^2(x^2 - 1)] x - (2x^2)' [1 + \cot^2(2x^2)] x \\ \therefore f'(x) &= \tan(x^2 - 1) + 2x^2[1 + \tan^2(x^2 - 1)] - 4x^2[1 + \cot^2(2x^2)] \end{aligned}$$

**សំបាត់ & គណនាគេវនៃអនុគមនីខាងក្រោម៖**

$$9. \quad f(x) = \frac{1 - x - 2x^2}{x^3 - \ln 3}$$

$$10. \quad f(x) = \frac{2x^2 + 3x + 4}{\sqrt{1 + 2x - x^2}}$$

$$11. \quad f(x) = \sin x^2 \cdot \tan(2x + 3)$$

$$12. \quad f(x) = \sin(x^2 + 5) + \cos(\sin x)$$

$$13. \quad f(x) = \frac{\sin(\tan \sqrt{x})}{\sin(\sqrt{x})}$$

### ចំណោះស្រាយ

$$9. \quad f(x) = \frac{1 - x - 2x^2}{x^3 - \ln 3}$$

$$\begin{aligned} f'(x) &= \frac{(1 - x - 2x^2)'(x^3 - \ln 3) - (x^3 - \ln 3)'(1 - x - 2x^2)}{(x^3 - \ln 3)^2} \\ &= \frac{(-1 - 4x)(x^3 - \ln 3) - 3x^2(1 - x - 2x^2)}{(x^3 - \ln 3)^2} \\ &= \frac{-x^3 + \ln 3 - 4x^4 + 4x \ln 3 - 3x^2 + 3x^3 + 6x^4}{(x^3 - \ln 3)^2} \end{aligned}$$

$$\therefore f'(x) = \frac{2x^4 + 2x^3 - 3x^2 + 4x \ln 3 + \ln 3}{(x^3 - \ln 3)^2}$$

ឧ.  $f(x) = \frac{2x^2 + 3x + 4}{\sqrt{1 + 2x - x^2}} \iff f(x) \cdot \sqrt{1 + 2x - x^2} = 2x^2 + 3x + 4$  ត្រូវដើរអង្គសងខាង គេបាន

$$\begin{aligned} [f(x)\sqrt{1 + 2x - x^2}]' &= (2x^2 + 3x + 4)' \\ f'(x)\sqrt{1 + 2x - x^2} + (\sqrt{1 + 2x - x^2})'f(x) &= 4x + 3 \\ f'(x)\sqrt{1 + 2x - x^2} + \frac{(1 + 2x - x^2)'}{2\sqrt{1 + 2x - x^2}}f(x) &= 4x + 3 \\ f'(x)\sqrt{1 + 2x - x^2} &= 4x + 3 - \frac{1 - x}{\sqrt{1 + 2x - x^2}} \cdot f(x) \\ \therefore f'(x) &= \frac{4x + 3}{\sqrt{1 + 2x - x^2}} + \frac{(x - 1)(2x^2 + 3x + 4)}{(1 + 2x - x^2)\sqrt{1 + 2x - x^2}} \end{aligned}$$

ມ.  $f(x) = \sin x^2 \cdot \tan(2x + 3)$

$$\begin{aligned} f'(x) &= (\sin x^2)' \tan(2x + 3) + (\tan(2x + 3))' \sin x^2 \\ &= (x^2)' \cdot \sin x^2 \cdot \tan(2x + 3) + (2x + 3)' [1 + \tan^2(2x + 3)] \sin x^2 \\ &= 2x \sin x^2 \cdot \tan(2x + 3) + 2 \sin x^2 [1 + \tan^2(2x + 3)] \\ \therefore f'(x) &= 2 \sin x^2 [\tan^2(2x + 3) + x \tan(2x + 3) + 1] \end{aligned}$$

៥.  $f(x) = \sin(x^2 + 5) + \cos(\sin x)$

$$\begin{aligned} f'(x) &= (x^2 + 5)' \cos(x^2 + 5) - (\sin x)' \sin(\sin x) \\ \therefore f'(x) &= 2x \cos(x^2 + 5) - \cos x \sin(\sin x) \end{aligned}$$

៥.  $f(x) = \frac{\sin(\tan \sqrt{x})}{\sin(\sqrt{x})} \iff f(x) \cdot \sin \sqrt{x} = \sin(\tan \sqrt{x})$

$$\begin{aligned} (f(x) \cdot \sin \sqrt{x})' &= (\sin(\tan \sqrt{x}))' \\ f'(x) \cdot \sin \sqrt{x} + (\sin \sqrt{x})' f(x) &= (\tan \sqrt{x})' \cos(\tan \sqrt{x}) \\ f'(x) \cdot \sin \sqrt{x} + (\sqrt{x})' \cos \sqrt{x} \cdot f(x) &= (\sqrt{x})' (1 + \tan^2 \sqrt{x}) \cos(\tan \sqrt{x}) \\ f'(x) \sin \sqrt{x} + \frac{1}{2\sqrt{x}} \cos \sqrt{x} \cdot f(x) &= \frac{1}{2\sqrt{x}} (1 + \tan^2 \sqrt{x}) \cos(\tan \sqrt{x}) \\ f'(x) \sin \sqrt{x} &= \frac{1}{2\sqrt{x}} [(1 + \tan^2 \sqrt{x}) \cos(\tan \sqrt{x}) - \cos \sqrt{x} \cdot f(x)] \end{aligned}$$

$$\therefore f'(x) = \frac{(1 + \tan^2 \sqrt{x}) \cos(\tan \sqrt{x}) - \cos \sqrt{x} \cdot f(x)}{2\sqrt{x} \cdot \sin \sqrt{x}}$$

**សំបាត់ ៥ គណនាគេវនៃអនុគមន៍ខាងក្រោម៖**

៩.  $f(x) = xe^x + \frac{1}{2}x^2$

១០.  $f(x) = e^{x^2+2x+1} + (x^2 - 3)e^x$

១១.  $f(x) = \frac{\sqrt{x}}{e^x}$

១២.  $f(x) = x^3e^{-3x}$

១៣.  $f(x) = e^{2x}3^{x^2+1}$

១៤.  $f(x) = e^{\sin x \cos x}$

### ចំណោះស្រាយ

៩.  $f(x) = xe^x + \frac{1}{2}x^2$

$$f'(x) = x'e^x + (e^x)'x + \frac{1}{2} \cdot 2x = e^x + e^x x + x = e^x(1+x) + x$$

១០.  $f(x) = e^{x^2+2x+1} + (x^2 - 3)e^x$

$$f'(x) = (x^2 + 2x + 1)'e^{x^2+2x+1} + (x^2 - 3)'e^x + (e^x)'(x^2 - 3)$$

$$= (2x + 2)e^{x^2+2x+1} + 2xe^x + e^x(x^2 - 3)$$

$$\therefore f'(x) = 2(x + 1)e^{x^2+2x+1} + e^x(2x + x^2 - 3)$$

១១.  $f(x) = \frac{\sqrt{x}}{e^x}$

$$f'(x) = \frac{(\sqrt{x})'e^x + (e^x)'\sqrt{x}}{(e^x)^2} = \frac{\frac{1}{2\sqrt{x}}e^x + e^x\sqrt{x}}{e^{2x}} = \frac{1+2x}{2\sqrt{x}e^x}$$

១២.  $f(x) = x^3e^{-3x}$

$$f'(x) = (x^3)'e^{-3x} + (e^{-3x})'x^3$$

$$= 3x^2e^{-3x} + (-3x)'e^{-3x}x^3$$

$$= 3x^2e^{-3x} - 3e^3e^{-3x}$$

$$\therefore f'(x) = 3x^2e^{-3x}(1-x)$$

១៣.  $f(x) = e^{2x}3^{x^2+1}$

$$f'(x) = (e^{2x})'3^{x^2+1} + (3^{x^2+1})'.e^{2x}$$

$$= (2x)'e^{2x}.3^{x^2+1} + (x^2+1)'3^{x^2+1}\ln 3.e^{2x}$$

$$= 2.e^{2x}3^{x^2+1} + 2x3^{x^2+1}\ln 3.e^{2x}$$

$$\therefore f'(x) = 2e^{2x}3^{x^2+1}(1+x\ln 3)$$

៩.  $f(x) = e^{\sin x \cos x}$

$$\begin{aligned} f'(x) &= (\sin x \cos x)' e^{\sin x \cos x} \\ &= [(\sin x)' \cos x + (\cos x)' \sin x] e^{\sin x \cos x} \\ \therefore f'(x) &= (\cos^2 x - \sin^2 x) e^{\sin x \cos x} \end{aligned}$$

**សំបាត់ ៦** តណានាយកើនអនុគមន៍ខាងក្រោម៖

៩.  $f(x) = (x^2 - 1) \ln(x^2 - 1)$

១០.  $f(x) = \ln\left(\frac{x^2 - 2}{\sqrt[3]{x^2 - 2}}\right)$

១១.  $f(x) = \ln(\sin x \cdot \cos(2x))$

១២.  $f(x) = \ln\left(\sqrt{\frac{1 + \sin x}{1 - \sin x}}\right)$

### ចំណែក: ស្ថិតិយោង

៩.  $f(x) = (x^2 - 1) \ln(x^2 - 1) \implies f'(x) = (x^2 - 1)' \ln(x^2 - 1) + \ln(x^2 - 1)'(x^2 - 1)$

$$\begin{aligned} &= 2x \ln(x^2 - 1) + \frac{(x^2 - 1)'}{x^2 - 1} \cdot (x^2 - 1) \\ &= 2x \ln(x^2 - 1) + 2x \end{aligned}$$

$\therefore f'(x) = 2x[\ln(x^2 - 1) + 1]$

១០.  $f(x) = \ln\left(\frac{x^2 - 2}{\sqrt[3]{x^2 - 2}}\right) = \ln(x^2 - 2) - \ln(x^2 - 2)^{\frac{1}{3}}$

$$\begin{aligned} f'(x) &= \frac{(x^2 - 2)'}{x^2 - 2} - \frac{1}{3} \cdot \frac{(x^2 - 2)'}{x^2 - 2} \\ &= \frac{3(2x) - 2x}{3(x^2 - 2)} \end{aligned}$$

$\therefore f'(x) = \frac{4x}{3(x^2 - 2)}$

១១.  $f(x) = \ln(\sin x \cdot \cos(2x)) = \ln(\sin x) + \ln(\cos(2x))$

$$\begin{aligned} f'(x) &= \frac{(\sin x)'}{\sin x} + \frac{(\cos(2x))'}{\cos(2x)} \\ &= \frac{\cos x}{\sin x} - \frac{2 \sin(2x)}{\cos(2x)} \end{aligned}$$

$\therefore f'(x) = \cot x - 2 \tan(2x)$

$$\text{Q. } f(x) = \ln \left( \sqrt{\frac{1+\sin x}{1-\sin x}} \right) = \ln \left( \frac{1+\sin x}{1-\sin x} \right)^{\frac{1}{2}} = \frac{1}{2} (\ln(1+\sin x) - \ln(1-\sin x))$$

$$\begin{aligned} f'(x) &= \frac{1}{2} \left( \frac{(1+\sin x)'}{1+\sin x} - \frac{(1-\sin x)'}{1-\sin x} \right) \\ &= \frac{1}{2} \left( \frac{\cos x}{1+\sin x} + \frac{\cos x}{1-\sin x} \right) \\ &= \frac{1}{2} \cdot \frac{(\cos x(1-\sin x) + 1+\sin x)}{1-\sin^2 x} \\ \therefore f'(x) &= \frac{2\cos x}{2\cos^2 x} = \frac{1}{\cos x} \end{aligned}$$

**លំហាត់ ៤** គណនាដឹងនៃអនុគមន៍ខាងក្រោម៖

៩.  $f(x) = \cos(\arcsin x)$

៤.  $f(x) = \arcsin \sqrt{x}$

៥.  $f(x) = \cot(\arctan x)$

៦.  $f(x) = \arctan(\sin x)$

៧.  $f(x) = \tan(\arctan x)$

៨.  $f(x) = \frac{\arctan x}{\arcsin x}$

៩.  $f(x) = \arcsin(2x)$

### ចំណែកស្រាយ

៩.  $f(x) = \cos(\arcsin x) \implies f'(x) = -(\arcsin x)' \sin(\arcsin x)$

$$\therefore f'(x) = -\frac{\sin(\arcsin x)}{\sqrt{1-x^2}}$$

៥.  $f(x) = \cot(\arctan x) \implies f'(x) = -(\arctan x)' [1 + \cot^2(\arctan x)]$

$$\therefore f'(x) = -\frac{1 + \cot^2(\arctan x)}{1+x^2}$$

៧.  $f(x) = \tan(\arctan x) \implies f'(x) = (\arctan x)' [1 + \tan^2(\arctan x)]$

$$\therefore f'(x) = \frac{1 + \tan^2(\arctan x)}{1+x^2}$$

៩.  $f(x) = \arcsin(2x) \implies f'(x) = \frac{(2x)'}{\sqrt{1-(2x)^2}}$

$$\therefore f'(x) = \frac{2}{\sqrt{1-4x^2}}$$

៤.  $f(x) = \arcsin \sqrt{x} \implies f'(x) = \frac{(\sqrt{x})'}{\sqrt{1-(\sqrt{x})^2}} = \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}\sqrt{1-x^2}}$

$$\therefore f'(x) = \frac{1}{2\sqrt{x-x^2}}$$

៩.  $f(x) = \arctan(\sin x) \implies f'(x) = \frac{(\sin x)'}{1 + (\sin x)^2}, \sin^2 x + \cos^2 x = 1$

$$\therefore f'(x) = \frac{\cos x}{2 - \cos^2 x}$$

១០.  $f(x) = \frac{\arctan x}{\arcsin x} \implies f'(x) = \frac{(\arctan x)' \arcsin x - (\arcsin x)' \arctan x}{(\arcsin x)^2}$

$$\therefore f'(x) = \frac{\arcsin x}{1+x^2} - \frac{\arctan x}{\sqrt{1-x^2}} \quad (\arcsin x)^2$$

## ១.១២ សំលាក់ឡើង

១. គណនាដីវិនអនុគមន៍ខាងក្រោម៖

(១)  $y = x^3 + 2x^2$

(២)  $y = x^3 - 4x^2$

(៣)  $y = x^4 - 27x$

(៤)  $y = x^4 - 5x^2 + 4$

(៥)  $y = x^5 - 16x$

(៦)  $y = \frac{x}{x+1}$

(៧)  $y = \frac{x^2}{1+x^2}$

(៨)  $y = x - \frac{1}{x}$

(៩)  $y = x^3 + 2x^2 - x$

(១០)  $y = x^4 - 2x^3 + 2x$

(១១)  $y = \sqrt{1+x^2}$

(១២)  $y = \sqrt[4]{1+x^2}$

២. រក  $f'(x)$  នៃអនុគមន៍ខាងក្រោម៖

(១)  $f(x) = \sin x + \cos x$

(២)  $f(x) = 2 \sin x - 3 \cos x$

(៣)  $f(x) = 3 \sin x + 2 \cos x$

(៤)  $f(x) = x \sin x + \cos x$

(៥)  $f(x) = x \cos x - \sin x$

(៦)  $f(x) = \cos(2x)$

(៧)  $f(x) = \frac{1 - \sin(2x)}{1 - \sin x}$

(៨)  $f(x) = 1 + \sin x^2$

(៩)  $f(x) = \cot x - \cos x$

(១០)  $f(x) = \sin(2x) - \cos(3x)$

(១១)  $f(x) = \sin(\cos(3x))$

(១២)  $f(x) = \frac{\sin x^2}{x^2}$

(១៣)  $f(x) = \tan(1 + x^2)$

(១៤)  $f(x) = \cos 2x - \cos x^2$

(១៥)  $f(x) = (1 + \sqrt{1+x})^3$

៣. រក  $y'$  នៃអនុគមន៍ខាងក្រោម៖

## ១.១ ធនធានីមេដ្ឋាន

- |                           |  |
|---------------------------|--|
| (ក) $xy = \frac{\pi}{6}$  | (ល) $(y^2 - 1)^2 + x = 0$                  |
| (ខ) $\sin(xy) = 1$        | (ព) $(y^2 + 1)^2 - x = 0$                  |
| (គ) $xy = \frac{1}{x+y}$  | (ឃ) $x^3 + xy + y^3 = 3$                   |
| (ឃ) $x + y = xy$          | (ឌ) $\sin x + \sin y = 1$                  |
| (ឃ) $(y-1)^2 + x = 0$     | (ឧ) $\sin x + xy + y^5 = \pi$              |
| (ឃ) $(y+1)^2 + y - x = 0$ | (ឈ) $\tan x + \tan y = 1$                  |
| (ឃ) $(y-x)^2 + x = 0$     | (ឈ) $x \ln y = e^{\ln \sin x}$             |
| (ឃ) $(y+x) + 2y - x = 0$  | (ឈ) $(\sin x)^{\ln y} = (\tan y)^{e^{3x}}$ |

## ៤. គណនាដឹងនៃអនុគមន៍ខាងក្រោម៖

- |                              |   |
|------------------------------|---|
| (ក) $f(x) = \sqrt{1-x}$      | (ឃ) $y = \sqrt[3]{\sqrt{2x+1}} - x^2$                           |
| (ខ) $f(x) = \sqrt[4]{x+x^2}$ | (ឧ) $y = \sqrt[4]{x+x^2}x + x^2$                                |
| (គ) $y = \sqrt{1-\sqrt{x}}$  | (ឈ) $y = \sqrt[3]{x-\sqrt{2x+1}}$                               |
| (ឃ) $y = \sqrt{x-\sqrt{x}}$  | (ឃ) $y = \sqrt[4]{\sqrt[3]{x}} + \sqrt[3]{\sqrt{x}} + \sqrt{x}$ |

## ៥. គណនាដឹងនៃអនុគមន៍ខាងក្រោម៖

- |                                       |  |
|---------------------------------------|--|
| (ក) $f(x) = e^x + e^{-x}$             | (ឃ) $f(x) = \sqrt{x}e^{-\frac{x}{4}} + x^2e^{x+2}$                   |
| (ខ) $f(x) = e^{3x} + 4e^x$            | (ឧ) $f(x) = x^{-\frac{1}{2}x} + \ln \sqrt{x}$                        |
| (គ) $f(x) = \frac{e^x}{1+e^x}$        | (ឈ) $f(x) = (\ln x)^2 + \ln x + 1$                                   |
| (ឃ) $f(x) = \frac{2e^{2x}}{1+e^{2x}}$ | (ឈ) $f(x) = \frac{\ln x}{x} + \ln \frac{1}{x}$                       |
| (ឃ) $f(x) = xe^{-x} + x \ln x$        | (ឈ) $f(x) = \ln \left( \sqrt{\frac{\sqrt{x}+1}{\sqrt{x}-1}} \right)$ |

## ៦. គណនាដឹងនៃអនុគមន៍ខាងក្រោម៖

- |                              |  |
|------------------------------|--|
| (ក) $f(x) = \tan(\arctan x)$ | (ឃ) $f(x) = (\arcsin x)^2$             |
| (ខ) $f(x) = \arcsin(\sin x)$ | (ឧ) $f(x) = \frac{1}{1+(\arctan x)^2}$ |
| (គ) $f(x) = \sin(\arctan x)$ | (ឈ) $f(x) = \sqrt{1-(\arcsin x)^2}$    |

## ៧. គណនាដឹងនៃអនុគមន៍ខាងក្រោម៖

- |  |  |
|--|--|
| (៦) $y = (x+1)(x-1)$   | (៧) $y = x^4 \cos x + x \cos x$                              |
| (៨) $y = (x^2 + 1)(x^2 - 1)$                                 | (៩) $y = \frac{1}{2}x^2 \sin x - x \cos x + \sin x$          |
| (៩) $y = \frac{1}{x+1} + \frac{1}{1+\sin x}$                 | (៩) $y = \sqrt{x}(\sqrt{x}+1)(\sqrt{x}+2)$                   |
| (១០) $y = \frac{1}{1+x^2} + \frac{1}{1-\sin x}$              | (១១) $y = (x-6)^{10} + \sin^{10} x$                          |
| (១២) $y = (x-1)(x-2)(x-3)$                                   | (១៣) $y = (\sin x \cos x)^3 + \sin(2x)$                      |
| (១៤) $y = x^2 \cos x + 2x \sin x$                            | (១៥) $y = x^{\frac{1}{2}} \sin(2x) + (\sin x)^{\frac{1}{2}}$ |
| (១៦) $y = x^{\frac{1}{2}}(x + \sin x)$                       | (១៧) $y = \frac{\sin x - \cos x}{\sin x + \cos x}$           |
| (១៧) $y = x^{\frac{1}{2}} \sin^2 x + (\sin x)^{\frac{1}{2}}$ | (១៨) $y = \frac{1}{\tan x} - \frac{1}{\cot x}$               |