ដេរីវេនៃអនុគមន៍

ថ្នាក់ទី ១២

ស៊ុំ សំអុន

មាតិកា

I ពិជគណិត

ពិជគណិត

១. ដេរីវេនៃអនុគមន៍

១.១ និយមន័យ

និយមន័យ ១.១.១ ដេរីវេនៃអនុគមន៍ $y=f(x)$ ត្រង់ a កំណត់ដោយ

$$f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a} \text{ ។} \tag{១.១}$$

- អនុគមន៍ f មានដេរីវេលើចន្លោះបើក (b,c) កាលណា f មានដេរីវេលើគ្រប់ចំណុច $a\in(b,c)$ ។
- អនុគមន៍ f មានដេរីវេលើចន្លោះបិទ $[b,c]$ កាលណា f មានដេរីវេលើចន្លោះ (b,c) ហើយ f មានដេរីវេខាងឆ្វេងត្រង់ $x=b$ និងខាងស្តាំត្រង់ $x=c$ ។

■ **ឧទាហរណ៍ ១.១** រកដេរីវេនៃអនុគមន៍ $y=f(x)=2x^2+3$ ត្រង់ 2 ។ ■

ដំណោះស្រាយ

$$\begin{aligned}
f'(2)&=\lim_{x\to 2}\frac{f(x)-f(2)}{x-2} \text{ ដែល } f(2)=2(2)^2+3=11\\
f'(2)&=\lim_{x\to 2}\frac{2x^2+3x-11}{x-2}\\
&=\lim_{x\to 2}\frac{2x^2-8}{x-2}\\
&=\lim_{x\to 2}\frac{2(x+2)(x-2)}{x-2}\ ,x\neq 2\\
&=2\times 4=8
\end{aligned}$$

១.១.១ ការកំណត់សរសេរ

តាង $h=x-a \Longrightarrow x=h+a$ បើ $h \longrightarrow 0$ នោះ $x \longrightarrow a$ នោះសមីការ (១.១) គេបាន

$$f'(a)=\lim_{h\to 0}\frac{f(h+a)-f(a)}{h} \qquad (១.២)$$

- **ចំណាំ ១.១** គេអាចសរសេរដេរីវេដោយ y' , $f'(x)$ ឬ $\dfrac{dy}{dx}$ ។

- **ឧទាហរណ៍ ១.២** ស្រាយថាបើ $y=x$ នោះ $y'=1$ ។

សម្រាយបញ្ជាក់

តាមនិយមន័យ

$$\begin{aligned} f'(x) &= \lim_{h\to 0}\frac{f(h+x)-f(x)}{h} \text{ ដែល } y=f(x)=x, f(h+x)=h+x \\ &= \lim_{h=\to 0}\frac{h+x-x}{h}=\lim_{h\to 0}1=1 \\ \therefore \quad \frac{dy}{dx} &= 1 \end{aligned}$$

■

> **ទ្រឹស្តីបទ ១.១.១** បើអនុគមន៍ f មានដេរីវេត្រង់ x_0 នោះ f ជាប់ត្រង់ x_0 ។

សម្រាយបញ្ជាក់

គេនឹងបង្ហាញថា បើ $f'(a)=\lim_{x\to a}\dfrac{f(x)-f(a)}{x-a}$ នោះ $\lim_{x\to a}f(x)=f(a)$

គេមាន
$$\begin{aligned} \lim_{x\to a}f(x) &= \lim_{x\to a}\left(f(x)-f(a)+f(a)\right) \\ &= \lim_{x\to a}\left(\frac{f(x)-f(a)}{x-a}\times(x-a)+f(a)\right) \\ &= \lim_{x\to a}\frac{f(x)-f(a)}{x-a}\times\lim_{x\to a}(x-a)+f(a) \\ &= f'(a)\times 0+f(a) \\ &= f(a) \end{aligned}$$

- **ចំណាំ ១.២** បើអនុគមន៍ f ជាប់ត្រង់ x_0 នោះ f អាចមានដេរីវេត្រង់ x_0 ឬ គ្មានដេរីវេត្រង់ x_0 ។

១.២ ភាពមានដេរីវេ

និយមន័យ ១.២.១ អនុគមន៍ f មានដេរីវេត្រង់ x លុះត្រាតែ

- អនុគមន៍ f ជាប់ត្រង់ x ។
- ដេរីវេឆ្វេងស្មើដេរីវេស្តាំត្រង់ចំណុច x គឺ $f'_-(x) = f'_+(x)$ ដែល
$f'_-(x) = \lim\limits_{h\to 0^-}\dfrac{f(x+h)-f(x)}{h}$ និង $f'_+(x) = \lim\limits_{h\to 0^+}\dfrac{f(x+h)-f(x)}{h}$ ។

■ **ឧទាហរណ៍ ១.៣** គេឲ្យអនុគមន៍ f កំណត់ដោយ $f(x) = \begin{cases} \cos x & \text{បើ } x \le \dfrac{\pi}{4} \\ a+bx & \text{បើ } x > \dfrac{\pi}{4} \end{cases}$
កំណត់តម្លៃ a និង b ដើម្បីឲ្យអនុគមន៍ f មានដេរីវេត្រង់ $x = \dfrac{\pi}{4}$ ។ ■

សម្រាយបញ្ជាក់

- បើអនុគមន៍ f ជាប់ត្រង់ $x = \dfrac{\pi}{4}$ នោះ $\lim\limits_{x\to\frac{\pi}{4}^-} f(x) = \lim\limits_{x\to\frac{\pi}{4}^+} f(x) = f\left(\dfrac{\pi}{4}\right)$

$$\lim_{x\to\frac{\pi}{4}^-}\cos x = \lim_{x\to\frac{\pi}{4}^+}(a+bx) = \cos\frac{\pi}{4} \Longleftrightarrow \frac{\sqrt{2}}{2} = a+b.\frac{\pi}{4} = \frac{\sqrt{2}}{2} \Longrightarrow a = \frac{\sqrt{2}}{2}-\frac{\pi}{4}.b$$

- ដេរីវេឆ្វេង $f'_-(x)$

$$\begin{aligned}
f'_-\left(\frac{\pi}{4}\right) &= \lim_{h\to 0^-}\frac{f(\frac{\pi}{4}+h)-f\left(\frac{\pi}{4}\right)}{h}, f(x)=\cos x \\
&= \lim_{h\to 0^-}\frac{\cos(\frac{\pi}{4}+h)-\cos\frac{\pi}{4}}{h} \\
&= \lim_{h\to 0^-}\frac{\cos\frac{\pi}{4}\cos h-\sin\frac{\pi}{4}\sin h-\cos\dfrac{\pi}{4}}{h} \\
&= \lim_{h\to 0^-}\frac{-\cos\frac{\pi}{4}(1-\cos h)-\sin\frac{\pi}{4}\sin h}{h} \\
&= \frac{\sqrt{2}}{2}\lim_{h\to 0^-}\left(-\frac{1-\cos h}{h}-\frac{\sin h}{h}\right), \lim_{h\to 0^-}\frac{1-\cos h}{h}=0, \lim_{h\to 0^-}\frac{\sin h}{h}=1 \\
&= \frac{\sqrt{2}}{2}(0-1) = -\frac{\sqrt{2}}{2}
\end{aligned}$$

- ដេរីវេស្តាំ $f'_+(x)$

$$\begin{aligned}
f'_+\left(\frac{\pi}{4}\right) &= \lim_{h\to 0^+}\frac{f(\frac{\pi}{4}+h)-f\left(\frac{\pi}{4}\right)}{h}, f(x)=a+bx \\
&= \lim_{h\to 0^+}\frac{a+b(\frac{\pi}{4}+h)-(a+b.\frac{\pi}{4}))}{h} \\
&= \lim_{h\to 0^+}\frac{a+b.\frac{\pi}{4}+bh-a-b.\frac{\pi}{4}}{h} \\
&= \lim_{h\to 0^+}\frac{bh}{h} = b
\end{aligned}$$

ដោយ f មានដេរីវេត្រង់ $x = \dfrac{\pi}{4}$ នោះ $f'_-(\dfrac{\pi}{4}) = f'_+(\dfrac{\pi}{4}) \Longleftrightarrow b = -\dfrac{\sqrt{2}}{2} \Longrightarrow a = \dfrac{\sqrt{2}}{2}\left(1+\dfrac{\pi}{4}\right)$

១.៣ លក្ខណៈនៃដេរីវេ

លក្ខណៈ ១ ចំពោះ u, v ជាអនុគមន៍នៃ x និង k ជាចំនួនថេ នោះគេបាន៖

១. $(ku)' = ku'$

២. $(u+v)' = u' + v$

៣. $(u-v)' = u' - v'$

៤. $(uv)' = u'v + v'u$

៥. $\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$

៦. $\left(\frac{1}{v}\right)' = -\frac{v'}{v^2}$

សម្រាយបញ្ជាក់

១. តាង $f(x) = k.u(x)$ ដែល $u = u(x)$ និង k ជាចំនួនថេ តាមនិយមន័យ

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{ku(x+h) - k.u(x)}{h} \\ &= k.\lim_{h\to 0} \frac{u(x+h) - u(x)}{h} \\ &= k.u'(x) \end{aligned}$$

$$\therefore \quad (k.u)' = k.u'$$

២. តាង $f(x) = u(x) + v(x)$ ដែល $u = u(x)$ និង $v = v(x)$ តាមនិយមន័យ

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{u(x+h) + v(x+h) - (u(x) + v(x))}{h} \\ &= \lim_{h\to 0} \frac{u(x+h) - u(x)}{h} + \lim_{h\to 0} \frac{v(x+h) - v(x)}{h} \\ &= u'(x) + v'(x) \end{aligned}$$

$$\therefore \quad (u+v)' = u' + v'$$

៣. ស្រាយដូចទី២

៤. តាង $f(x)=uv$ ដែល $u=u(x)$ និង $v=v(x)$ តាមនិយមន័យគេបាន

$$\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&= \lim_{h\to 0}\frac{u(x+h).v(x+h)-u(x).v(x)}{h}\\
&= \lim_{h\to 0}\frac{u(x+h).v(x+h)-u(x).v(x+h)+u(x).v(x+h)+u(x).v(x)}{h}\\
&= \lim_{h\to 0}\left[\frac{u(x+h).v(x+h)-u(x).v(x+h)}{h}+\frac{u(x).v(x+h)+u(x).v(x)}{h}\right]\\
&= \lim_{h\to 0}\left[v(x+h).\frac{u(x+h)-u(x)}{h}+u(x).\frac{v(x+h)+v(x)}{h}\right]\\
&= v(x).\frac{d}{dx}(u(x))+u(x).\frac{d}{dx}(v(x))
\end{aligned}$$

$$\therefore\quad (uv)'=u'v+v'u \tag{១.៣}$$

៥. យក $u=u(x)$ និង $v=v(x)$ តាង $f(x)=\dfrac{u}{v}\Leftrightarrow f(x).v=u$ ធ្វើដេរីវេអង្គទាំងពីរធៀបនឹង x នោះគេបាន $[f(x).v]'=u'$ ប្រើតាមសមីការ (១.៣) គេបាន

$$\begin{aligned}
f'(x).v+v'f(x) &= u',\ f(x)=\frac{u}{v}\\
f'(x).v+v'.\frac{u}{v} &= u'\\
\frac{f'(x).v^2}{v}+\frac{v'u}{v} &= u'\\
f'(x).v^2+v'u &= u'v\\
f'(x) &= \frac{u'v-v'u}{v^2}
\end{aligned}$$

$$\therefore\quad \left(\frac{u}{v}\right)'=\frac{u'v-v'u}{v^2} \tag{១.៤}$$

៦. យក $v=v(x)$ តាង $f(x)=\dfrac{1}{v}$ ប្រើសមីការ (១.៤) គេបាន

$$\begin{aligned}
f'(x) &= \frac{(1)'.v-v'.(1)}{v^2}\\
&= \frac{0-v'}{v^2}\\
&= -\frac{v'}{v^2}\\
\therefore\quad \left(\frac{1}{v}\right)' &= -\frac{v'}{v^2}
\end{aligned}$$

១.៤ ដេរីវេនៃអនុគមន៍បណ្តាក់

ជាទូទៅ ១ បើ $y=f(u)$ និង $u=g(x)$ នោះ $\frac{d}{dx}(f\circ g)=\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$ ។

សម្រាយបញ្ជាក់

តាង $F(x)=f\circ g=f(g(x))$ តាមនិយមន័យភាពមានដេរីវេត្រង់ $x=a$ នោះគេបាន

$$\begin{aligned}
F'(a)&=\lim_{x\to a}\frac{F(x)-F(a)}{x-a}\\
&=\lim_{x\to a}\frac{f(g(x))-f(g(a))}{x-a}\\
&=\lim_{x\to a}\left(\frac{f(g(x))-f(g(a))}{g(x)-g(a)}\times\frac{g(x)-g(a)}{x-a}\right)\\
&=f'(g(a))\times g'(a)\quad ,u=g(a),y=f(a)
\end{aligned}$$

$$\therefore\quad \frac{d}{dx}(f\circ g)=\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$$

ជាទូទៅ ២ បើ $y=c$ ដែល c ជាចំនួនថេរ នោះ $y'=0$ ។

សម្រាយបញ្ជាក់

គេមាន $y=f(x_0)=c$ នោះ $f(x_0+h)=c\ ,c\in\mathbb{R}$ តាមនិយមន័យគេបាន

$$\begin{aligned}
f'(x)&=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&=\lim_{h\to 0}\frac{c-c}{h}\\
&=\lim_{h\to 0}\frac{0}{h}
\end{aligned}$$

$$\therefore\quad \frac{d}{dx}(c)=0$$

■ **ឧទាហរណ៍ ១.៤** គណនា y' ដែល $y=\left(\ln x.\log_a(\sqrt{3})\right)$ ។ ■

ដំណោះស្រាយ

គេមាន $y=\left(\ln x.\log_a(\sqrt{3})\right)\Rightarrow y'=\left(\ln x.\log_a(\sqrt{3})\right)'=0$

■ **ឧទាហរណ៍ ១.៥** ស្រាយបញ្ជាក់ថា បើ $y=x^n$ នោះ $y'=nx^{n-1}$ ។ ■

សម្រាយបញ្ជាក់

គេមាន $f(x)=x^n$ នាំឲ្យ $f(x+h)=(x+h)^n$ តាមនិយមន័យ

$$\begin{aligned}
y'=f'(x)&=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&=\lim_{h\to 0}\frac{(x+h)^n-x^n}{h}\\
&=\lim_{h\to 0}\frac{(x+h-x)(x^{n-1}+x^{n-2}.x+\ldots+x.x^{n-2}x+x^{n-1})}{h}\\
&=\lim_{h\to 0}(x^{n-1}+x^{n-1}+\ldots+x^{n-1}+x^{n+1})\\
&=x^{n-1}(\underbrace{1+1+\ldots+1+1}_{n\text{ តួលេខ }1})\\
&=n.x^{n-1}
\end{aligned}$$

$$\therefore \quad \frac{d}{dx}(x^n)=n.x^{n-1}$$

■ **ឧទាហរណ៍ ១.៦** គណនា $f'(x)$

១. $f(x)=x^3$ ២. $f(x)=\sqrt{x}$ ៣. $f(x)=\sqrt[3]{x^2}$

■

ដំណោះស្រាយ

១. $f(x)=x^3\Rightarrow f'(x)=(x^3)'=3x^{3-1}=3x^2$

២. $f(x)=\sqrt{x}\Rightarrow f'(x)=(\sqrt{x})'=(x^{\frac{1}{2}})'=\frac{1}{2}x^{\frac{1}{2}-1}=\frac{1}{2}x^{-\frac{1}{2}}=\frac{1}{2\sqrt{x}}$

៣. $f(x)=\sqrt[3]{x^2}\Rightarrow f'(x)=(\sqrt[3]{x^2})'=(x^{\frac{2}{3}})'=\frac{2}{3}x^{\frac{2}{3}-1}=\frac{2}{3}x^{-\frac{1}{3}}=\frac{2}{3\sqrt[3]{x}}$

ជាទូទៅ ៣ បើ $y=u^n$ ដែល u ជាអនុគមន៍នៃ x នោះ $y'=nu'u^{n-1}$ ។

សម្រាយបញ្ជាក់

គេមាន $y=u^n$ គេបាន $y'=\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}=\frac{d}{du}(u^n)\times u'=nu'u^{n-1}$

■ **ឧទាហរណ៍ ១.៧** គណនា y'

១. $y=(2x+\ln 2)^4$ ២. $y=\sqrt{u}$ ដែល u ជាអនុគមន៍នៃ x ។

■

ដំណោះស្រាយ

១. $y=(2x+\ln 2)^4 \Longrightarrow y'=4(2x+\ln 2)'(2x+\ln 2)^{4-1}=4(2+0)(2x+\ln 2)^3$

$$\therefore \quad y'=8(2x+\ln 2)^3$$

២. $y=\sqrt{u}=u^{\frac{1}{x}} \Longrightarrow y'=(u^{\frac{1}{2}})'=\frac{1}{2}u'u^{\frac{1}{2}-1}=\frac{1}{2}u'u^{-\frac{1}{2}}=\frac{u'}{2\sqrt{u}}$ ។

១.៥ ដេរីវេនៃអនុគមន៍ត្រីកោណមាត្រ

លក្ខណៈ ២ ដេរីវេអនុគមន៍ត្រីកោណមាត្រ

១. បើ $y=\sin x$ នោះ $y'=\cos x$

២. បើ $y=\cos x$ នោះ $y'=-\sin x$

៣. បើ $y=\tan x$ នោះ $y'=\frac{1}{\cos^2 x}=1+\tan^2 x$

៤. បើ $y=\cot x$ នោះ $y'=-\frac{1}{\sin^2 x}=-(1+\cot^2 x)$

■

សម្រាយបញ្ជាក់

១. គេមាន $y=f(x)=\sin x$ នោះ $f(x+h)=\sin(x+h)$ តាមនិយមន័យ

$$\begin{aligned}
y'=f'(x)&=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&=\lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}\\
&=\lim_{h\to 0}\frac{\sin x\cos h+\sin h.\cos x-\sin x}{h}\\
&=\lim_{h\to 0}\left(\cos x.\frac{\sin h}{h}-\sin x.\frac{1-\cos h}{h}\right)\\
&=\cos x \quad ,\lim_{h\to 0}\frac{\sin h}{h}=1,\lim_{h\to 0}\frac{1-\cos h}{h}=0
\end{aligned}$$

$$\therefore \quad \frac{d}{dx}(\sin x)=\cos x$$

២. គេមាន $y=f(x)=\cos x$ នោះ $f(x+h)=\cos(x+h)$ តាមនិយមន័យ

$$\begin{aligned}
y'=f'(x)&=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&=\lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h}\\
&=\lim_{h\to 0}\frac{\cos x.\cos h-\sin x.\sin h-\cos x}{h}\\
&=\lim_{h\to 0}\left(-\frac{\sin h}{h}.\sin x-\cos x.\frac{1-\cos h}{h}\right)\\
&=-\sin x \quad ,\lim_{h\to 0}\frac{1-\cos h}{h}=0,\lim_{h\to 0}\frac{\sin h}{h}=1
\end{aligned}$$

$$\therefore\quad \frac{d}{dx}(\cos x)=-\sin x$$

៣. តាង $y=\tan x=\dfrac{\sin x}{\cos x}$ តាមសមីការ (១.៤) គេបាន

$$\begin{aligned}
y'=\left(\frac{\sin x}{\cos x}\right)'&=\frac{(\sin x)'\cos x-(\cos x)'.\sin x}{(\cos x)^2}\\
&=\frac{\cos x.\cos x-(-\sin x).\sin x}{\cos^2 x}\\
&=\frac{\cos^2 x+\sin^2 x}{\cos^2 x}\\
&=1+\tan^2 x\\
&=\frac{1}{\cos^2 x},\ \sin^2 x+\cos^2 x=1
\end{aligned}$$

$$\therefore\quad (\tan x)'=\frac{1}{\cos^2}=1+\tan^2 x$$

៤. តាង $y=\cot x=\dfrac{\cos x}{\sin x}$ តាមសមីការ (១.៤) គេបាន

$$\begin{aligned}
y'=\left(\frac{\cos x}{\sin x}\right)'&=\frac{(\cos x)'.\sin x-(\sin x)'.\cos x}{(\sin^2 x)^2}\\
&=\frac{-\sin x.\sin x-\cos x.\cos x}{\sin^2 x}\\
&=\frac{-\sin^2 x-\cos^2 x}{\sin^2 x}\\
&=-\frac{\sin^2 x+\cos^2 x}{\sin^2 x},\sin^2 x+\cos^2 x=1
\end{aligned}$$

$$\therefore\quad (\cot x)'=-\frac{1}{\sin^2 x}=-(1+\cot^2 x)$$

ជាទូទៅ ៤ បើ u ជាអនុគមន៍នៃ x គេបាន

១. បើ $y = \sin u$ នោះ $y' = u'\cos u$

២. បើ $y = \cos u$ នោះ $y' = -u'\sin u$

៣. បើ $y = \tan u$ នោះ $y' = \dfrac{u'}{\cos^2 u} = u(1+\tan^2 u)$

៤. បើ $y = \cot u$ នោះ $y' = -\dfrac{u'}{\sin^2 u} = -u'(1+\cot^2 u)$

សម្រាយបញ្ជាក់

១. បើ u ជាអនុគមន៍នៃ x នោះ $y = \sin u$ គេបាន

$$\frac{dy}{dx} = \frac{dy}{du}\times\frac{du}{dx} = \frac{d}{du}(\sin u)\times\frac{du}{dx} = \cos u\times u' = u'\cos u$$

$$\therefore\quad \frac{d}{dx}(\sin u) = u'\cos u$$

២. បើ u ជាអនុគមន៍នៃ x នោះ $y = \cos u$ គេបាន

$$\frac{dy}{dx} = \frac{dy}{du}\times\frac{du}{dx} = \frac{d}{du}(\cos u)\times\frac{du}{dx} = -\sin u\times u' = -u'\sin u$$

$$\therefore\quad \frac{d}{dx}(\cos u) = -u'\sin u$$

៣. បើ u ជាអនុគមន៍នៃ x នោះ $y = \tan u$ គេបាន

$$\frac{dy}{dx} = \frac{dy}{du}\times\frac{du}{dx} = \frac{d}{du}(\tan u)\times\frac{du}{dx} = \frac{1}{\cos^2 u}\times u' = (1+\tan^2 u)\times u'$$

$$\therefore\quad (\tan u)' = \frac{u'}{\cos^2 u} = u'(1+\tan^2 u)$$

៤. បើ u ជាអនុគមន៍នៃ x នោះ $y = \cot u$ គេបាន

$$\frac{dy}{dx} = \frac{dy}{du}\times\frac{du}{dx} = \frac{d}{du}(\cot u)\times\frac{du}{dx} = -\frac{1}{\sin^2 u}\times u' = -(1+\cot^2 u)\times u'$$

$$\therefore\quad (\cot u)' = -\frac{u'}{\sin^2 u} = -u'(1+\cot^2 u)$$

■ **ឧទាហរណ៍ ១.៨** គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

១. $y = \sin(2x+1)$

២. $y = \cos(2x+1)$

៣. $y = \tan(2x+1)$

៤. $y = \cot(2x+1)$ ។

■

ដំណោះស្រាយ

១. $y=\sin(2x+1)\Rightarrow y'=(2x+1)'\cos(2x+1)=2\cos(2x+1)$

២. $y=\cos(2x+1)\Rightarrow y'=-(2x+1)'\sin(2x+1)=-2\sin(2x+1)$

៣. $y=\tan(2x+1)\Rightarrow y'=\dfrac{(2x+1)'}{\cos^2(2x+1)}=\dfrac{2}{\cos^2(2x+1)}=2[1+\tan^2(2x+1)]$

៤. $y=\cot(2x+1)\Rightarrow y'=-\dfrac{(2x+1)'}{\sin^2(2x+1)}=-\dfrac{2}{\sin^2(2x+1)}=-2[1+\cot^2(2x+1)]$ ។

១.៦ ដេរីវេអនុគមន៍អិចស្ប៉ូណង់ស្យែល

ស្រាយថាបើ $y=a^x$ នោះ $y'=a^x.\ln a$

សម្រាយបញ្ជាក់

គេមាន $y=a^x$ តាមនិយមន័យ គេបាន

$$\begin{aligned} y'=f'(x)&=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\ &=\lim_{h\to 0}\frac{a^{x+h}-a^x}{h}\\ &=\lim_{h\to 0}\frac{a^x(a^h-1)}{h}\\ &=a^x\lim_{h\to 0}\frac{a^h-1}{h}\ \text{ដោយ}\ \lim_{x\to 0}\frac{a^x-1}{x}=\ln a\end{aligned}$$

$$\therefore\quad (a^x)'=a^x.\ln a$$

ជាទូទៅ ៥ បើ u ជាអនុគមន៍នៃ x នោះ $(a^u)'=u'a^u.\ln a$ ។

សម្រាយបញ្ជាក់

បើ u ជាអនុគមន៍នៃ x នោះ គេបាន

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}=\frac{d}{du}(a^u)\times\frac{du}{dx}=a^u.\ln a\times u'$$

$$\therefore\quad (a^u)'=u'.a^u.\ln a)$$

■ **ឧទាហរណ៍ ១.៩** គណនា y' ចំពោះ u ជាអនុគមន៍នៃ x នៃអនុគមន៍ខាងក្រោម៖

១. $y=e^x$　　　　២. $y=a^{x^2-1}$　　　　៣. $y=e^u$

■

ដំណោះស្រាយ

១. $y=e^x$ នោះ $y'=(e^x)'=e^x.\ln e=e^x$,$\ln e=1$

២. $y=a^{x^2-1}$ នោះ $y'=(x^{x^2-1})'a^{x^2-1}\ln a=2x.a^{x^2-1}\ln a$

៣. $y=e^u$ នោះ $y'=(e^u)'=u'e^u.\ln e=u'e^u$, $\ln e=1$

១.៧ ដេរីវេនៃអនុគមន៍លោការីត

ស្រាយបញ្ជាក់ថា បើ $y=\log_a x$,$a>0,a\neq 1$ នោះ $y'=\dfrac{1}{x\ln a}$ ។

សម្រាយបញ្ជាក់

គេមាន $y=\log_a x$ តាមនិយមន័យ គេបាន

$$\begin{aligned}
y'=f'(x)&=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&=\lim_{h\to 0}\frac{\log_a(x+h)-\log_a(x)}{h}\\
&=\lim_{h\to 0}\frac{1}{h}\log_a\left(\frac{x+h}{x}\right)\\
&=\lim_{h\to 0}\log_a\left(1+\frac{h}{x}\right)^{\frac{1}{h}}\\
&=\log_a\left(\lim_{h\to 0}\left(1+\frac{1}{\frac{x}{h}}\right)^{\frac{x}{h}}\right)^{\frac{1}{x}} \quad \text{ដោយ } \lim_{x\to 0}(1+\frac{1}{x})^x=e\\
&=\log_a e^{\frac{1}{x}}=\frac{1}{x}\frac{\ln e}{\ln a}
\end{aligned}$$

$$\therefore\quad (\log_a x)'=\frac{1}{x\ln a}\ ,a>0,a\neq 1$$

ជាទូទៅ ៦ បើ u ជាអនុគមន៍នៃ x នោះ $(\log_a u)'=\dfrac{u'}{u\ln a}$,$a>0,a\neq 1$ ។

សម្រាយបញ្ជាក់

បើ u ជាអនុគមន៍នៃ x នោះ គេបាន

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}=\frac{d}{du}(\log_a u)\times\frac{du}{dx}=\frac{1}{u\ln a}\times u'$$
$$\therefore\quad (\log_a u)'=\frac{u'}{u\ln a},a>0,a\neq 1$$

១.៨ ដេរីវេនៃអនុគមន៍លោការីតនេពៃ

ស្រាយបញ្ជាក់ថា បើ $y=\ln x$ នោះ $y'=\frac{1}{x}$ ។

សម្រាយបញ្ជាក់

គេមាន $y=\ln x$ តាមនិយមន័យ គេបាន

$$\begin{aligned}
y'=f'(x)&=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&=\lim_{h\to 0}\frac{\ln(x+h)-\ln(x)}{h}\\
&=\lim_{h\to 0}\frac{\ln\left(\frac{x+h}{x}\right)}{h}\\
&=\lim_{h\to 0}\frac{1}{h}\ln\left(1+\frac{h}{x}\right)\\
&=\lim_{h\to 0}\ln\left(1+\frac{h}{x}\right)^{\frac{1}{h}}\\
&=\ln\left[\lim_{h\to 0}\left(1+\frac{1}{\frac{x}{h}}\right)^{\frac{x}{h}\times\frac{1}{x}}\right],\lim_{h\to 0}\left(1+\frac{1}{\frac{x}{h}}\right)^{\frac{x}{h}}=e\\
&=\ln e^{\frac{1}{x}},\ln e=1
\end{aligned}$$
$$\therefore\quad (\ln x)'=\frac{1}{x}$$

■ **ឧទាហរណ៍ ១.១០** រក $f'(x)$ នៃអនុគមន៍ខាងក្រោម៖

១. $f(x)=x^2.\log_a x,a>0,a\neq 1$

២. $f(x)=\sin(2x)+\log_2(x^2+1)$

៣. $f(x)=\frac{e^{2x}+\log_3 x}{x^2}$

៤. $f(x)=\log(x^2\sqrt{x^3-1})$

៥. $f(x)=(\sin x)^{\log x}$

៦. $f(x)=(\log_a x)^{\ln(2x)},a>0,a\neq 1$

■

សម្រាយបញ្ជាក់

១. $f(x)=x^2.\log_a x, a>0, a\neq 1 \Longrightarrow f'(x)=(x^2)'\log_a x+(\log_a x)'x^2$

$$=2x\log_a x+\frac{1}{x\ln a}x^2$$

$$\therefore\quad f'(x)=2x\log_a x+\frac{x}{\ln a}, a>0, a\neq 1$$

២. $f(x)=\sin(2x)+\log_2(x^2+1) \Longrightarrow f'(x)=-(2x)'\cos(2x)+\dfrac{(x^2+1)'}{(x^2+1)\ln 2}$

$$\therefore\quad f'(x)=-2\cos(2x)+\frac{2x}{(x^2+1)\ln 2}$$

៣. $f(x)=\dfrac{e^{2x}+\log_3 x}{x^2} \Longrightarrow f'(x)=\dfrac{(e^{2x}+\log_3 x)'x^2-(x^2)'(e^{2x}+\log_3 x)}{x^4}$

$$=\frac{(2e^{2x}+\frac{1}{x\ln 3})x^2-2x(e^{2x}+\log_3 x)}{x^4}$$

$$=\frac{2xe^{2x}+\frac{1}{\ln 3}-2e^{2x}-2\log_3 x}{x^3}$$

$$\therefore\quad f'(x)=\frac{2e^{2x}(x-1)+\frac{1}{\ln 3}-\log_3 x^2}{x^3}$$

៤. $f(x)=\log(x^2\sqrt{x^3-1})=\log x^2+\log(x^3-1)^{\frac{1}{2}}=2\log x+\dfrac{1}{2}\log(x^3-1)$

$$\therefore\quad f'(x)=\frac{2}{x\ln 10}+\frac{(x^3-1)'}{2(x^3-1)\ln 10}=\frac{2}{x\ln 10}+\frac{3x^2}{2(x^3-1)\ln 10}$$

៥. $f(x)=(\sin x)^{\log x} \Longleftrightarrow \ln f(x)=\ln(\sin x)^{\log x}$

$$(\ln f(x))'=(\log x.\ln(\sin x))'$$

$$\frac{f'(x)}{f(x)}=(\log x)'\ln(\sin x)+(\ln(\sin x))'\log x$$

$$f'(x)=f(x)\left(\frac{1}{x\ln 10}\ln(\sin x)+\frac{(\sin x)'}{\sin x}.\log x\right)$$

$$\therefore\quad f'(x)=(\sin x)^{\log x}\left(\frac{\ln(\sin x)}{x\ln 10}+\cot x.\log x\right)$$

៦. $f(x)=(\log_a x)^{\ln(2x)}, a>0, a\neq 1 \Longleftrightarrow \ln f(x)=\ln(\log_a x)^{\ln(2x)}$

$$(\ln f(x))'=(\ln(2x).\ln(\log_a x))'$$

$$\frac{f'(x)}{f(x)}=(\ln(2x))'\ln(\log_a x)+(\ln(\log_a x))'\ln(2x)$$

$$f'(x)=f(x)\left(\frac{(2x)'}{2x}\ln(\log_a x)+\frac{(\log_a x)'}{\log_a x}\ln(2x)\right)$$

$$\therefore\quad f'(x)=(\log_a x)^{\ln(2x)}\left(\frac{\ln(\log_a x)}{x}+\frac{\ln(2x)}{x\ln a\log_a x}\right), a>0, a\neq 1$$

ខាងនៅ ៧ បើ u ជាអនុគមន៍នៃ x នោះ $(\ln u)' = \dfrac{u'}{u}$ ។

សម្រាយបញ្ជាក់

បើ u ជាអនុគមន៍នៃ x នោះ គេបាន

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(\ln u) \times \frac{du}{dx} = \frac{1}{u} \times u'$$

$$\therefore \quad (\ln u)' = \frac{u'}{u}$$

▪ **ឧទាហរណ៍ ១.១១** រក $f'(x)$ នៃអនុគមន៍ខាងក្រោម៖

១. $f(x) = x.\ln x$

២. $f(x) = x^2 + \ln(x^2+1)$

៣. $f(x) = \dfrac{e^x + \ln x}{x^2}$

៤. $f(x) = \ln(x^2\sqrt{x^3-1})$

៥. $f(x) = x^x$

៦. $f(x) = (\sin x)^{\cos x}$

▪

សម្រាយបញ្ជាក់

១. $f(x) = x.\ln x \Longrightarrow f'(x) = x'\ln x + (\ln x)'x = \ln x + \dfrac{1}{x}.x = \ln x + 1$

២. $f(x) = x^2 + \ln(x^2+1) \Longrightarrow f'(x) = (x^2)' + \dfrac{(x^2+1)'}{x^2+1} = 2x + \dfrac{2x}{x^2+1}$

៣. $f(x) = \dfrac{e^x + \ln x}{x^2} \Longrightarrow f'(x) = \dfrac{(e^x+\ln x)'x^2 - (x^2)'(e^x+\ln x)}{(x^2)^2}$

$$= \frac{\left(e^x + \frac{1}{x}\right)x^2 - 2x(e^x+\ln x)}{x^4}$$

$$\therefore \quad f'(x) = \frac{xe^x + 1 - 2e^x - 2\ln x}{x^3}$$

៤. $f(x) = \ln(x^2\sqrt{x^3-1}) = \ln x^2 + \ln\sqrt{x^3-1} = 2\ln x + \ln(x^3-1)^{\frac{1}{2}}$

$$f'(x) = 2(\ln x)' + \frac{1}{2}[\ln(x^3-1)]'$$

$$= 2.\frac{1}{x} + \frac{1}{2}.\frac{(x^3-1)'}{x^3-1}$$

$$\therefore \quad f'(x) = \frac{2}{x} + \frac{3}{2}.\frac{x^2}{x^3-1}$$

៥. $f(x)=x^x \Longleftrightarrow \ln f(x)=\ln x^x = x\ln x$

$$\begin{aligned}
(\ln f(x))' &= (x\ln x)' \\
\frac{f'(x)}{f(x)} &= x'\ln x + (\ln x)'x \\
f'(x) &= f(x)(\ln x + \frac{1}{x}.x) \\
\therefore \quad f'(x) &= x^x(\ln x+1)
\end{aligned}$$

៦. $f(x)=(\sin x)^{\cos x} \Longleftrightarrow \ln f(x)=\ln(\sin x)^{\cos x}$

$$\begin{aligned}
(\ln f(x))' &= (\cos x\ln\sin x)' \\
\frac{f'(x)}{f(x)} &= (\cos x)'\ln\sin x + (\ln\sin x)'\cos x \\
f'(x) &= f(x)\left(-\sin x\ln sinx + \frac{(\sin x)'}{\sin x}.\cos x\right) \\
\therefore \quad f'(x) &= (\sin x)^{\cos x}(\cos x\cot x - \sin x\ln\sin x)
\end{aligned}$$

១.៩ ដេរីវេនៃអនុគមន៍ Arc Sine និង Arc Tangent

$$y=\arcsin x \Longleftrightarrow x=\sin y \text{ និង } -\frac{\pi}{2}\le y\le\frac{\pi}{2},$$
$$y=\arctan x \Longleftrightarrow x=\tan y \text{ និង } -\frac{\pi}{2}\le y\le\frac{\pi}{2},$$

▪ **ឧទាហរណ៍ ១.១២** ស្រាយថា បើ $y=\arcsin x$ នោះ $y'=\dfrac{1}{\sqrt{1-x^2}}$ ។ ▪

សម្រាយបញ្ជាក់

បើ $y=\arcsin x$ នោះ $x=\sin y$ ធ្វើដេរីវេអង្គសងខាងធៀបនឹង x គេបាន

$$(x)'=(\sin y)' \Longleftrightarrow 1=y'\cos y$$
$$y'=\frac{1}{\cos y} \text{ ដោយ } \sin^2 y+\cos^2 y=1$$
$$\Longrightarrow \cos y=\pm\sqrt{1-\sin^2 x}$$

ដោយ $-\dfrac{\pi}{2}\le y\le\dfrac{\pi}{2} \Longrightarrow \cos y\ge 0 \Longrightarrow \cos y=\sqrt{1-x^2}$

$$\therefore \quad (\arcsin x)'=\frac{1}{\sqrt{1-x^2}}$$

■ **ឧទាហរណ៍ ១.១៣** ស្រាយថា បើ $y = \arctan x$ នោះ $y' = \dfrac{1}{1+x^2}$ ។ ■

សម្រាយបញ្ជាក់

បើ $y = \arctan x$ នោះ$x = \tan y$ ធ្វើដេរីវេអង្គសងខាងធៀបនឹង x គេបាន

$$(x)' = (\sin y)' \Longleftrightarrow 1 = y'(1+\tan^2 y)$$

$$y' = \frac{1}{1+\tan^2 y}$$

$$\therefore \quad (\arctan x)' = \frac{1}{1+x^2}$$

ជាទូទៅ ៨ បើ u ជាអនុគមន៍នៃ x នោះ $(\arcsin u)' = \dfrac{u'}{\sqrt{1-x^2}}$

សម្រាយបញ្ជាក់

បើ u ជាអនុគមន៍នៃ x នោះ គេបាន

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(\arcsin u) \times \frac{du}{dx} = \frac{1}{\sqrt{1-u^2}} \times u'$$

$$\therefore \quad (\arcsin u)' = \frac{u'}{\sqrt{1-u^2}}$$

ជាទូទៅ ៩ បើ u ជាអនុគមន៍នៃ x នោះ $(\arctan u)' = \dfrac{u'}{1+u^2}$

សម្រាយបញ្ជាក់

បើ u ជាអនុគមន៍នៃ x នោះ គេបាន

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(\arctan u) \times \frac{du}{dx} = \frac{1}{1-u^2} \times u'$$

$$\therefore \quad (\arctan u)' = \frac{u'}{1+u^2}$$

■ **ឧទាហរណ៍ ១.១៤** គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

១. $f(x)=\arcsin x.\sin x$

២. $f(x)=\arctan x\cos x$

៣. $f(x)=\sin(\arcsin x)$

៤. $f(x)=\arctan(\tan x)$

■

សម្រាយបញ្ជាក់

១. $f(x)=\arcsin x.\sin x \Longrightarrow f'(x)=(\arcsin x)'\sin x+(\sin x)'\arcsin x$

$$\therefore \quad f'(x)=\frac{\sin x}{\sqrt{1-x^2}}+\cos x.\arcsin x$$

២. $f(x)=\arctan x\cos x \Longrightarrow f'(x)=(\arctan x)'\cos x+(\cos x)'\arctan$

$$\therefore \quad f'(x)=\frac{\cos x}{1+x^2}-\sin x.\arctan x$$

៣. $f(x)=\sin(\arcsin x) \Longrightarrow f'(x)=(\arcsin x)'\cos(\arcsin x)$

$$\therefore \quad f'(x)=\frac{\cos(\arcsin x)}{\sqrt{1-x^2}}$$

៤. $f(x)=\arctan(\tan x) \Longrightarrow f'(x)=\dfrac{(\tan x)'}{1+(\tan x)^2}=\dfrac{1+\tan^2 x}{1+\tan^2 x}$

$$\therefore \quad f'(x)=1$$

១.១០ រូបមន្តនៃដេរីវេ

បើ C,a,b,c ជាចំនួនថេ និង u ជាអនុគមន៍នៃ x ដែល $n\in\mathbb{N}$ គេបាន៖

១. $(C)'=0$

២. $(x)'=1$

៣. $(ax+b)'=a$

៤. $(ax^2+bx+c)'=2ax+b$

៥. $(x^n)'=nx^{n-1}$

៦. $(u^n)'=n.u'.u^{n-1}$

៧. $(x)^{-n}=-\dfrac{n}{x^{n+1}}$

៨. $\left(\dfrac{1}{x}\right)'=-\dfrac{1}{x^2}$

៩. $\left(\dfrac{1}{u}\right)'=-\dfrac{u}{u^2}$

១០. $(\sqrt{x})'=\dfrac{1}{2\sqrt{x}}$

១១. $(\sqrt{u})'=\dfrac{u'}{2\sqrt{u}}$

១២. $(\sqrt[n]{x})'=\dfrac{1}{n\sqrt[n]{x^{n-1}}}$

១៣. $(\ln x)'=\dfrac{1}{x}$

១៤. $(\ln u)'=\dfrac{u'}{u}$

១៥. $(\log_a x)'=\dfrac{1}{x\ln a},a>0,a\neq 1$

១៦. $(\log_a u)'=\dfrac{u'}{u.\ln a},a>0,a\neq 1$

១៧. $(a^x)'=a^x\ln a,a>0,a\neq 1$

១៨. $(a^u)'=u'a^u\ln a,a>0,a\neq 1$

១៩. $(e^x)'=e^x$

២០. $(e^u)'=u'e^u$

២១. $(\sin x)'=\cos x$

២២. $(\sin u)'=u'\cos u$

២៣. $(\cos x)'=-\sin x$

២៤. $(\cos u)'=-u'\sin u$

២៥. $(\tan x)'=\dfrac{1}{\cos^2 x}=1+\tan^2 x$

២៦. $(\tan u)'=\dfrac{u'}{\cos^2 u}=u'(1+\tan^2 u)$

២៧. $(\cot x)'=-\dfrac{1}{\sin^2 x}=-(1+\cot^2 x)$

២៨. $(\cot u)'=-\dfrac{u'}{\sin^2 u}=-(1+\cot^2 u)$

២៩. $(\arcsin x)'=\dfrac{1}{\sqrt{1-x^2}}$

៣០. $(\arcsin u)'=\dfrac{u'}{\sqrt{1-u^2}}$

៣១. $(\arccos x)'=-\dfrac{1}{\sqrt{1-x^2}}$

៣២. $(\arccos u)'=-\dfrac{u'}{\sqrt{1-u^2}}$

៣៣. $(\arctan x)'=\dfrac{1}{1+x^2}$

៣៤. $(\arctan u)'=\dfrac{u'}{1+u^2}$

៣៥. $(\text{arccot}\, x)'=-\dfrac{1}{1+x^2}$

៣៦. $(\text{arccot}\, u)'=-\dfrac{u'}{1+u^2}$

៣៧. $(u^v)'=\left(v'.\ln u+\dfrac{v.u'}{u}\right).u^v$

១.១១ លំហាត់ និង ដំណោះស្រាយ

លំហាត់ ១ គណនា $f'(x)$ នៃអនុគមន៍ខាងក្រោម៖

១. $f(x)=x^5-x^4+x^3-x^2+x-1$

២. $f(x)=2x^2-\sqrt{x}+\dfrac{2}{x}$

៣. $f(x)=(x^4-7x^2+\sin a)^7$

៤. $f(x)=(x^2-\sqrt{x})^{2019}$

៥. $f(x)=\sqrt{x^3-x^2+3}$

៦. $\sqrt[4]{x^3-2x}$

៧. $f(x)=(x+1)(2x-1)^2$

៨. $f(x)=(x^2+2x+3)(x^3-3x-1)$

៩. $f(x)=\dfrac{1}{x-1}$

១០. $f(x)=\dfrac{x\sqrt{x}}{x+1}$

សម្រាយបញ្ជាក់

១. $f(x)=x^5-x^4+x^3-x^2+x-1 \Longrightarrow f'(x)=5x^4-4x^3+3x^2-2x+1$

២. $f(x)=2x^2-\sqrt{x}-\dfrac{2}{x} \Longrightarrow f'(x)=4x^2+\dfrac{1}{2\sqrt{x}}-\dfrac{2}{x^2}$

$$f'(x)=7(x^4-7x^2+\sin a)'(x^4-7x^2+\sin a)^{7-1}=7(4x^3-14x)(x^4-7x^2+\sin a)^6$$

៣. $f(x)=(x^4-7x^2+\sin a)^7$

៤. $f(x)=(x^2-\sqrt{x})^{2019} \Longrightarrow f'(x)=2019(x^2-\sqrt{x})'(x^2-\sqrt{x})^{2019-1}$

$$=2019\left(2x-\frac{1}{2\sqrt{x}}\right)(x^2-\sqrt{x})^{2018}$$

៥. $f(x)=\sqrt{x^3-x^2+3} \Longrightarrow f'(x)=\dfrac{(x^3-x^2+3)'}{2\sqrt{x^3-x^2+3}}=\dfrac{3x-2}{2\sqrt{x^3-x^2+3}}$

៦. $\sqrt[4]{x^3-2x} \Longleftrightarrow f(x)=(x^3-2x)^{\frac{1}{4}}$

$$\begin{aligned} f'(x)&=\frac{1}{4}(x^3-2x)'(x^3-2x)^{\frac{1}{4}-1}\\ &=\frac{1}{4}(3x^2-2)(x^3-2x)^{-\frac{3}{4}} \end{aligned}$$

$$\therefore\quad f'(x)=\frac{3x^2-2}{4\sqrt[4]{(x^3-2x)^3}}$$

៧. $f(x)=(x+1)(2x-1)^2$

$$\begin{aligned} f'(x)&=(x+1)'(2x-1)^2+[(2x-1)^2]'(x+1)\\ &=(2x-1)^2+2(2x-1)'(2x-1)(x+1)\\ &=(2x-1)(2x-1+4x+4) \end{aligned}$$

$$\therefore\quad f'(x)=(2x-1)(6x+3)$$

៨. $f(x)=(x^2+2x+3)(x^3-3x-1)$

$$\begin{aligned}f'(x)&=(x^2+2x+3)'(x^2-3x-1)+(x^2-3x-1)'(x^2+2x+3)\\&=(2x+2)(x^2-3x-1)+(2x-3)(x^2+2x+3)\\&=2x^3-6x^2-2x+2x^2-6x-2+2x^3+4x^2+6x-3x^2-6x-9\\ \therefore\quad f'(x)&=4x^3-3x^2-8x-11\end{aligned}$$

៩. $f(x)=\dfrac{1}{x-1}\Longrightarrow f'(x)=-\dfrac{(x-1)'}{(x-1)^2}=-\dfrac{1}{(x-1)^2}$

១០. $f(x)=\dfrac{x\sqrt{x}}{x+1}$

$$\begin{aligned}f'(x)&=\frac{(x\sqrt{x})'(x+1)-(x+1)'x\sqrt{x}}{(x+1)^2}\\&=\frac{[x'\sqrt{x}+(\sqrt{x})'x](x+1)-x\sqrt{x}}{(x+1)^2}\\&=\frac{\left(x+\dfrac{x}{2\sqrt{x}}\right)(x+1)-x\sqrt{x}}{(x+1)^2}\\&=\frac{x\sqrt{x}+\sqrt{x}+\dfrac{x}{2\sqrt{x}}(x+1)-x\sqrt{x}}{(x+1)^2}\\ \therefore\quad f'(x)&=\frac{x^2+3x}{2\sqrt{x}(x+1)^2}\end{aligned}$$

លំហាត់ ២ គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

១. $f(x)=x.\sin x+\cos x$	៤. $f(x)=\sin^2\sqrt{x}+\cos^2(3x)$
២. $f(x)=\sin^3 x-x.\cos x$	៥. $f(x)=\cos(3x+4)+3\cos x.\sin x$
៣. $f(x)=\cos(x^2+1)+2\sin(x^2-1)$	៦. $f(x)=\sin(\sin\sqrt{x})+\cos^3 x$

ដំណោះស្រាយ

១. $f(x)=x.\sin x+\cos x$

$$\begin{aligned}f'(x)&=x'\sin x+(\sin x)'.x-\sin x\\&=\sin x+x.\cos x-\sin x\\ \therefore\quad f'(x)&=x.\cos x\end{aligned}$$

២. $f(x)=\sin^3 x - x.\cos x$

$$\begin{aligned} f'(x) &= 3(\sin x)'\sin^{3-1}x - [x'.\cos x + (\cos x)'.x] \\ &= 3\cos x.\sin^2 x - (\cos x - x.\sin x) \\ \therefore\quad f'(x) &= 3\cos x.\sin^2 x - \cos x + x\sin x \end{aligned}$$

៣. $f(x)=\cos(x^2+1)+2\sin(x^2-1)$

$$\begin{aligned} f'(x) &= -(x^2+1)'\sin(x^2+1) + 2(x^2-1)'\cos(x^2-1) \\ \therefore\quad f'(x) &= -2x\sin(x^2+1) + 4x\cos(x^2-1) \end{aligned}$$

៤. $f(x)=\sin^2\sqrt{x}+\cos^2(3x)$

$$\begin{aligned} f'(x) &= 2(\sin\sqrt{x})'\cos^{2-1}\sqrt{x} + 2(\cos(3x))'\sin(3x) \\ &= 2(\sqrt{x})'.\cos\sqrt{x}.\cos\sqrt{x} - 2(3x)'\sin(3x).\sin(3x) \\ \therefore\quad f'(x) &= \frac{1}{\sqrt{x}}.\cos^2\sqrt{x} - 6\sin^2(3x) \end{aligned}$$

៥. $f(x)=\cos(3x+4)+3\cos x.\sin x$

$$\begin{aligned} f'(x) &= -(3x+4)'.\sin(3x+4) + 3[(\cos x)'.\sin x + (\sin x)'.\cos x] \\ &= -3\sin(3x+4) + 3[-\sin x.\sin x + \cos x.\cos x] \\ \therefore\quad f'(x) &= -3[\sin(3x+4) + \sin^2 x - \cos^2 x] \end{aligned}$$

៦. $f(x)=\sin(\sin\sqrt{x})+\cos^3 x$

$$\begin{aligned} f'(x) &= (\sin\sqrt{x})'.\cos(\sin\sqrt{x}) + 3(\cos x)\cos^{3-1}x \\ &= (\sqrt{x})'.\cos\sqrt{x}.\cos(\sin\sqrt{x}) - 3\sin x\cos^2 x \\ \therefore\quad f'(x) &= \frac{1}{2\sqrt{x}}\cos\sqrt{x}.\cos(\sin\sqrt{x}) - 3\sin x.\cos^2 x \end{aligned}$$

លំហាត់ ៣ គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

១. $f(x)=(1+\tan x)^4$

២. $f(x)=x^2\tan x+(1+\cot x)^2$

៣. $f(x)=x.\tan(x^2-1)+x\cot(2x^2)$

៤. $f(x)=\dfrac{\tan(2x)}{1-\cos x}$

ដំណោះស្រាយ

១. $f(x)=(1+\tan x)^4$

$$f'(x)=4(1+\tan x)'(1+\tan^2 x)^{4-1}$$
$$\therefore\quad f'(x)=4(1+\tan^2 x)(1+\tan x)^3$$

២. $f(x)=x^2\tan x+(1+\cot x)^2$

$$f'(x)=(x^2)'\tan x+(\tan x)'x^2+2(1+\cot x)'(1+\cot x)^{2-1}$$
$$\therefore\quad f'(x)=2x\tan x+x^2(1+\tan^2 x)-2(1+\cot^2 x)(1+\cot x)$$

៣. $f(x)=x.\tan(x^2-1)+x\cot(2x^2)$

$$\begin{aligned} f'(x)&=x'\tan(x^2-1)+[\tan(x^2-1)]'x+x'\cot(2x^2)+[\cot(2x^2)]'x\\ &=\tan(x^2-1)+(x^2-1)'[1+\tan^2(x^2-1)]x-(2x^2)'[1+\cot^2(2x^2)]x\\ \therefore\quad f'(x)&=\tan(x^2-1)+2x^2[1+\tan^2(x^2-1)]-4x^2[1+\cot^2(2x^2)]\end{aligned}$$

លំហាត់ ៤ គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

១. $f(x)=\dfrac{1-x-2x^2}{x^3-\ln 3}$

២. $f(x)=\dfrac{2x^2+3x+4}{\sqrt{1+2x-x^2}}$

៣. $f(x)=\sin x^2.\tan(2x+3)$

៤. $f(x)=\sin(x^2+5)+\cos(\sin x)$

៥. $f(x)=\dfrac{\sin(\tan\sqrt{x})}{\sin(\sqrt{x})}$

ដំណោះស្រាយ

១. $f(x)=\dfrac{1-x-2x^2}{x^3-\ln 3}$

$$\begin{aligned} f'(x)&=\frac{(1-x-2x^2)'(x^3-\ln 3)-(x^3-\ln 3)'(1-x-2x^2)}{(x^3-\ln 3)^2}\\ &=\frac{(-1-4x)(x^3-\ln 3)-3x^2(1-x-2x^2)}{(x^3-\ln 3)^2}\\ &=\frac{-x^3+\ln 3-4x^4+4x\ln 3-3x^2+3x^3+6x^4}{(x^3-\ln 3)^2}\\ \therefore\quad f'(x)&=\frac{2x^4+2x^3-3x^2+4x.\ln 3+\ln 3}{(x^3-\ln 3)^2}\end{aligned}$$

២. $f(x)=\dfrac{2x^2+3x+4}{\sqrt{1+2x-x^2}} \Longleftrightarrow f(x).\sqrt{1+2x-x^2}=2x^2+3x+4$ ធ្វើដេរីវេអង្គសងខាង គេបាន

$$\begin{aligned}
[f(x)\sqrt{1+2x-x^2}]' &= (2x^2+3x+4)' \\
f'(x)\sqrt{1+2x-x^2}+(\sqrt{1+2x-x^2})'f(x) &= 4x+3 \\
f'(x)\sqrt{1+2x-x^2}+\frac{(1+2x-x^2)'}{2\sqrt{1+2x-x^2}}f(x) &= 4x+3 \\
f'(x)\sqrt{1+2x-x^2} &= 4x+3-\frac{1-x}{\sqrt{1+2x-x^2}}.f(x) \\
\therefore \quad f'(x) &= \frac{4x+3}{\sqrt{1+2x-x^2}}+\frac{(x-1)(2x^2+3x+4)}{(1+2x-x^2)\sqrt{1+2x-x^2}}
\end{aligned}$$

៣. $f(x)=\sin x^2.\tan(2x+3)$

$$\begin{aligned}
f'(x) &= (\sin x^2)'\tan(2x+3)+(\tan(2x+3))'\sin x^2 \\
&= (x^2)'.\sin x^2.\tan(2x+3)+(2x+3)'[1+\tan^2(2x+3)]\sin x^2 \\
&= 2x\sin x^2.\tan(2x+3)+2\sin x^2[1+\tan^2(2x+3)] \\
\therefore \quad f'(x) &= 2\sin x^2[\tan^2(2x+3)+x\tan(2x+3)+1]
\end{aligned}$$

៤. $f(x)=\sin(x^2+5)+\cos(\sin x)$

$$\begin{aligned}
f'(x) &= (x^2+5)'\cos(x^2+5)-(\sin x)'\sin(\sin x) \\
\therefore \quad f'(x) &= 2x\cos(x^2+5)-\cos x\sin(\sin x)
\end{aligned}$$

៥. $f(x)=\dfrac{\sin(\tan\sqrt{x})}{\sin(\sqrt{x})} \Longleftrightarrow f(x).\sin\sqrt{x}=\sin(\tan\sqrt{x})$

$$\begin{aligned}
\big(f(x).\sin\sqrt{x}\big)' &= \big(\sin(\tan\sqrt{x})\big)' \\
f'(x).\sin\sqrt{x}+(\sin\sqrt{x})'f(x) &= (\tan\sqrt{x})'\cos(\tan\sqrt{x}) \\
f'(x).\sin\sqrt{x}+(\sqrt{x})'\cos\sqrt{x}.f(x) &= (\sqrt{x})'(1+\tan^2\sqrt{x})\cos(\tan\sqrt{x}) \\
f'(x)\sin\sqrt{x}+\frac{1}{2\sqrt{x}}\cos\sqrt{x}.f(x) &= \frac{1}{2\sqrt{x}}(1+\tan^2\sqrt{x})\cos(\tan\sqrt{x}) \\
f'(x)\sin\sqrt{x} &= \frac{1}{2\sqrt{x}}\left[(1+\tan^2\sqrt{x})\cos(\tan\sqrt{x})-\cos\sqrt{x}.f(x)\right]
\end{aligned}$$

$$\therefore \quad f'(x)=\frac{(1+\tan^2\sqrt{x})\cos(\tan\sqrt{x})-\cos\sqrt{x}.f(x)}{2\sqrt{x}.\sin\sqrt{x}} \text{ ដែល, } f(x)=\frac{\sin(\tan\sqrt{x})}{\sin(\sqrt{x})}$$

លំហាត់ ៥ គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

១. $f(x) = xe^x + \frac{1}{2}x^2$

២. $f(x) = e^{x^2+2x+1} + (x^2-3)e^x$

៣. $f(x) = \frac{\sqrt{x}}{e^x}$

៤. $f(x) = x^3e^{-3x}$

៥. $f(x) = e^{2x}3^{x^2+1}$

៦. $f(x) = e^{\sin x\cos x}$

ដំណោះស្រាយ

១. $f(x) = xe^x + \frac{1}{2}x^2$

$$f'(x) = x'e^x + (e^x)'x + \frac{1}{2}.2x = e^x + e^x x + x = e^x(1+x) + x$$

២. $f(x) = e^{x^2+2x+1} + (x^2-3)e^x$

$$\begin{aligned} f'(x) &= (x^2+2x+1)'e^{x^2+2x+1} + (x^2-3)'e^x + (e^x)'(x^2-3) \\ &= (2x+2)e^{x^2+2x+1} + 2xe^x + e^x(x^2-3) \\ \therefore \quad f'(x) &= 2(x+1)e^{x^2+2x+1} + e^x(2x+x^2-3) \end{aligned}$$

៣. $f(x) = \frac{\sqrt{x}}{e^x}$

$$f'(x) = \frac{(\sqrt{x})'e^x + (e^x)'\sqrt{x}}{(e^x)^2} = \frac{\frac{1}{2\sqrt{x}}e^x + e^x\sqrt{x}}{e^{2x}} = \frac{1+2x}{2\sqrt{x}e^x}$$

៤. $f(x) = x^3e^{-3x}$

$$\begin{aligned} f'(x) &= (x^3)'e^{-3x} + (e^{-3x})'x^3 \\ &= 3x^2e^{-3x} + (-3x)'e^{-3x}x^3 \\ &= 3x^2e^{-3x} - 3e^3e^{-3x} \\ \therefore \quad f'(x) &= 3x^2e^{-3x}(1-x) \end{aligned}$$

៥. $f(x) = e^{2x}3^{x^2+1}$

$$\begin{aligned} f'(x) &= (e^{2x})'3^{x^2+1} + (3^{x^2+1})'.e^{2x} \\ &= (2x)'e^{2x}.3^{x^2+1} + (x^2+1)'3^{x^2+1}\ln 3.e^{2x} \\ &= 2.e^{2x}3^{x^2+1} + 2x3^{x^2+1}\ln 3.e^{2x} \\ \therefore \quad f'(x) &= 2e^{2x}3^{x^2+1}(1+x\ln 3) \end{aligned}$$

៦. $f(x) = e^{\sin x \cos x}$

$$
\begin{aligned}
f'(x) &= (\sin x \cos x)' e^{\sin x \cos x} \\
&= [(\sin x)' \cos x + (\cos x)' \cos x] e^{\sin x \cos x} \\
\therefore \quad f'(x) &= (\cos^2 x - \sin^2 x) e^{\sin x \cos x}
\end{aligned}
$$

លំហាត់ ៦ គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

១. $f(x) = (x^2-1)\ln(x^2-1)$

២. $f(x) = \ln\left(\dfrac{x^2-2}{\sqrt[3]{x^2-2}}\right)$

៣. $f(x) = \ln(\sin x . \cos(2x))$

៤. $f(x) = \ln\left(\sqrt{\dfrac{1+\sin x}{1-\sin x}}\right)$

ដំណោះស្រាយ

១. $f(x) = (x^2-1)\ln(x^2-1) \Longrightarrow f'(x) = (x^2-1)'\ln(x^2-1) + \ln(x^2-1)'(x^2-1)$

$$
\begin{aligned}
&= 2x\ln(x^2-1) + \frac{(x^2-1)'}{x^2-1}.(x^2-1) \\
&= 2x\ln(x^2-1) + 2x \\
\therefore \quad f'(x) &= 2x[\ln(x^2-1)+1]
\end{aligned}
$$

២. $f(x) = \ln\left(\dfrac{x^2-2}{\sqrt[3]{x^2-2}}\right) = \ln(x^2-2) - \ln(x^2-2)^{\frac{1}{3}}$

$$
\begin{aligned}
f'(x) &= \frac{(x^2-2)'}{x^2-2} - \frac{1}{3}.\frac{(x^2-2)'}{x^2-2} \\
&= \frac{3(2x)-2x}{3(x^2-2)} \\
\therefore \quad f'(x) &= \frac{4x}{3(x^2-2)}
\end{aligned}
$$

៣. $f(x) = \ln(\sin x . \cos(2x)) = \ln(\sin x) + \ln(\cos(2x))$

$$
\begin{aligned}
f'(x) &= \frac{(\sin x)'}{\sin x} + \frac{(\cos(2x))'}{\cos(2x)} \\
&= \frac{\cos x}{\sin x} - \frac{2\sin(2x)}{\cos(2x)} \\
\therefore \quad f'(x) &= \cot x - 2\tan(2x)
\end{aligned}
$$

៤. $f(x)=\ln\left(\sqrt{\dfrac{1+\sin x}{1-\sin x}}\right)=\ln\left(\dfrac{1+\sin x}{1-\sin x}\right)^{\frac{1}{2}}=\dfrac{1}{2}(\ln(1+\sin x)-\ln(1-\sin x))$

$$\begin{aligned} f'(x)&=\frac{1}{2}\left(\frac{(1+\sin x)'}{1+\sin x}-\frac{(1-\sin x)'}{1-\sin x}\right)\\ &=\frac{1}{2}\left(\frac{\cos x}{1+\sin x}+\frac{\cos x}{1-\sin x}\right)\\ &=\frac{1}{2}\cdot\frac{(\cos x(1-\sin x+1+\sin x))}{1-\sin^2 x}\\ \therefore\quad f'(x)&=\frac{2\cos x}{2\cos^2 x}=\frac{1}{\cos x}\end{aligned}$$

លំហាត់ ៧ គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

១. $f(x)=\cos(\arcsin x)$

២. $f(x)=\cot(\arctan x)$

៣. $f(x)=\tan(\arctan x)$

៤. $f(x)=\arcsin(2x)$

៥. $f(x)=\arcsin\sqrt{x}$

៦. $f(x)=\arctan(\sin x)$

៧. $f(x)=\dfrac{\arctan x}{\arcsin x}$

ដំណោះស្រាយ

១. $f(x)=\cos(\arcsin x)\Longrightarrow f'(x)=-(\arcsin x)'\sin(\arcsin x)$

$$\therefore\quad f'(x)=-\frac{\sin(\arcsin x)}{\sqrt{1-x^2}}$$

២. $f(x)=\cot(\arctan x)\Longrightarrow f'(x)=-(\arctan x)'[1+\cot^2(\arctan x)]$

$$\therefore\quad f'(x)=-\frac{1+\cot^2(\arctan x)}{1+x^2}$$

៣. $f(x)=\tan(\arctan x)\Longrightarrow f'(x)=(\arctan x)'[1+\tan^2(\arctan x)]$

$$\therefore\quad f'(x)=\frac{1+\tan^2(\arctan x)}{1+x^2}$$

៤. $f(x)=\arcsin(2x)\Longrightarrow f'(x)=\dfrac{(2x)'}{\sqrt{1-(2x)^2}}$

$$\therefore\quad f'(x)=\frac{2}{\sqrt{1-4x^2}}$$

៥. $f(x)=\arcsin\sqrt{x}\Longrightarrow f'(x)=\dfrac{(\sqrt{x})'}{\sqrt{1-(\sqrt{x})^2}}=\dfrac{\frac{1}{2\sqrt{x}}}{\sqrt{1-x^2}}=\dfrac{1}{2\sqrt{x}\sqrt{1-x^2}}$

$$\therefore \quad f'(x) = \frac{1}{2\sqrt{x-x^2}}$$

៦. $f(x) = \arctan(\sin x) \Longrightarrow f'(x) = \dfrac{(\sin x)'}{1+(\sin x)^2}, \sin^2 x + \cos^2 x = 1$

$$\therefore \quad f'(x) = \frac{\cos x}{2-\cos^2 x}$$

៧. $f(x) = \dfrac{\arctan x}{\arcsin x} \Longrightarrow f'(x) = \dfrac{(\arctan x)'\arcsin x - (\arcsin x)'\arctan x}{(\arcsin x)^2}$

$$\therefore \quad f'(x) = \frac{\dfrac{\arcsin x}{1+x^2} - \dfrac{\arctan x}{\sqrt{1-x^2}}}{(\arcsin x)^2}$$

១.១២ លំហាត់មេរៀន

១. គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

(ក) $y = x^3 + 2x^2$

(ខ) $y = x^3 - 4x^2$

(គ) $y = x^4 - 27x$

(ឃ) $y = x^4 - 5x^2 + 4$

(ង) $y = x^5 - 16x$

(ច) $y = \dfrac{x}{x+1}$

(ឆ) $y = \dfrac{x^2}{1+x^2}$

(ជ) $y = x - \dfrac{1}{x}$

(ឈ) $y = x^3 + 2x^2 - x$

(ញ) $y = x^4 - 2x^3 + 2x$

(ដ) $y = \sqrt{1+x^2}$

(ឋ) $y = \sqrt[4]{1+x^2}$

២. រក $f'(x)$ នៃអនុគមន៍ខាងក្រោម៖

(ក) $f(x) = \sin x + \cos x$

(ខ) $f(x) = 2\sin x - 3\cos x$

(គ) $f(x) = 3\sin x + 2\cos x$

(ឃ) $f(x) = x\sin x + \cos x$

(ង) $f(x) = x\cos x - \sin x$

(ច) $f(x) = \cos(2x)$

(ឆ) $f(x) = \dfrac{1-\sin(2x)}{1-\sin x}$

(ជ) $f(x) = 1 + \sin x^2$

(ឈ) $f(x) = \cot x - \cos x$

(ញ) $f(x) = \sin(2x) - \cos(3x)$

(ដ) $f(x) = \sin(\cos(3x))$

(ឋ) $f(x) = \dfrac{\sin x^2}{x^2}$

(ឌ) $f(x) = \tan(1+x^2)$

(ឍ) $f(x) = \cos 2x - \cos x^2$

(ណ) $f(x) = (1+\sqrt{1+x})^3$

៣. រក y' នៃអនុគមន៍ខាងក្រោម៖

(ក) $xy = \frac{\pi}{6}$

(ខ) $\sin(xy) = 1$

(គ) $xy = \frac{1}{x+y}$

(ឃ) $x + y = xy$

(ង) $(y-1)^2 + x = 0$

(ច) $(y+1)^2 + y - x = 0$

(ឆ) $(y-x)^2 + x = 0$

(ជ) $(y+x) + 2y - x = 0$

(ឈ) $(y^2-1)^2 + x = 0$

(ញ) $(y^2+1)^2 - x = 0$

(ដ) $x^3 + xy + y^3 = 3$

(ឋ) $\sin x + \sin y = 1$

(ឌ) $\sin x + xy + y^5 = \pi$

(ឍ) $\tan x + \tan y = 1$

(ណ) $x \ln y = e^{\ln \sin x}$

(ត) $(\sin x)^{\ln y} = (\tan y)^{e^{3x}}$

៤. គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

(ក) $f(x) = \sqrt{1-x}$

(ខ) $f(x) = \sqrt[4]{x+x^2}$

(គ) $y = \sqrt{1-\sqrt{x}}$

(ឃ) $y = \sqrt{x - \sqrt{x}}$

(ង) $y = \sqrt[3]{\sqrt{2x+1} - x^2}$

(ច) $y = \sqrt[4]{x + x^2}x + x^2$

(ឆ) $y = \sqrt[3]{x - \sqrt{2x+1}}$

(ជ) $y = \sqrt[4]{\sqrt[3]{x}} + \sqrt[3]{\sqrt{x}} + \sqrt{x}$

៥. គណនាដេរីវេនៃអនៃអនុគមន៍ខាងក្រោម៖

(ក) $f(x) = e^x + e^{-x}$

(ខ) $f(x) = e^{3x} + 4e^x$

(គ) $f(x) = \frac{e^x}{1+e^x}$

(ឃ) $f(x) = \frac{2e^{2x}}{1+e^{2x}}$

(ង) $f(x) = xe^{-x} + x\ln x$

(ច) $f(x) = \sqrt{x}e^{-\frac{x}{4}} + x^2e^{x+2}$

(ឆ) $f(x) = x^{-\frac{1}{2}x} + \ln\sqrt{x}$

(ជ) $f(x) = (\ln x)^2 + \ln x + 1$

(ឈ) $f(x) = \frac{\ln x}{x} + \ln\frac{1}{x}$

(ញ) $f(x) = \ln\left(\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}-1}}\right)$

៦. គណនាដេរីវេនៃអនៃអនុគមន៍ខាងក្រោម៖

(ក) $f(x) = \tan(\arctan x)$

(ខ) $f(x) = \arcsin(\sin x)$

(គ) $f(x) = \sin(\arctan x)$

(ឃ) $f(x) = (\arcsin x)^2$

(ង) $f(x) = \frac{1}{1+(\arctan x)^2}$

(ច) $f(x) = \sqrt{1-(\arcsin x)^2}$

៧. គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

(ក) $y=(x+1)(x-1)$

(ខ) $y=(x^2+1)(x^2-1)$

(គ) $y=\dfrac{1}{x+1}+\dfrac{1}{1+\sin x}$

(ឃ) $y=\dfrac{1}{1+x^2}+\dfrac{1}{1-\sin x}$

(ង) $y=(x-1)(x-2)(x-3)$

(ច) $y=x^2\cos x+2x\sin x$

(ឆ) $y=x^{\frac{1}{2}}(x+\sin x)$

(ជ) $y=x^{\frac{1}{2}}\sin^2 x+(\sin x)^{\frac{1}{2}}$

(ឈ) $y=x^4\cos x+x\cos x$

(ញ) $y=\dfrac{1}{2}x^2\sin x-x\cos x+\sin x$

(ដ) $y=\sqrt{x}(\sqrt{x}+1)(\sqrt{x}+2)$

(ឋ) $y=(x-6)^{10}+\sin^{10} x$

(ឌ) $y=(\sin x\cos x)^3+\sin(2x)$

(ឍ) $y=x^{\frac{1}{2}}\sin(2x)+(\sin x)^{\frac{1}{2}}$

(ណ) $y=\dfrac{\sin x-\cos x}{\sin x+\cos x}$

(ត) $y=\dfrac{1}{\tan x}-\dfrac{1}{\cot x}$