ដេរីវេនៃអនុគមន៍

ថ្នាក់ទី ១២

ស៊ុំ សំអុន

មាតិកា

I ពិជគណិត

ពិជគណិត

១. ដេរីវេនៃអនុគមន៍

១.១ និយមន័យ

និយមន័យ ១.១.១ $y=f(x)$ a

$$f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a} \qquad (.)$$

- f (b,c) f $a\in(b,c)$
- f $[b,c]$ f (b,c) f $x=b$ $x=c$

▪ **ឧទាហរណ៍ ១.១** $y=f(x)=2x^2+3$ 2 ▪

ដំណោះស្រាយ

$$f'(2)=\lim_{x\to 2}\frac{f(x)-f(2)}{x-2} \quad f(2)=2(2)^2+3=11$$

$$\begin{aligned}
f'(2)&=\lim_{x\to 2}\frac{2x^2+3x-11}{x-2}\\
&=\lim_{x\to 2}\frac{2x^2-8}{x-2}\\
&=\lim_{x\to 2}\frac{2(x+2)(x-2)}{x-2}\ ,x\neq 2\\
&=2\times 4=8
\end{aligned}$$

១.១.១ ការកំណត់សរសេរ

$h = x - a \Longrightarrow x = h + a \quad h \longrightarrow 0 \quad x \longrightarrow a \quad (.)$

$$f'(a) = \lim_{h\to 0} \frac{f(h+a) - f(a)}{h} \qquad (.)$$

■ **ចំណាំ ១.១** y' , $f'(x)$ $\dfrac{dy}{dx}$

■ **ឧទាហរណ៍ ១.២** $y = x$ $y' = 1$

សម្រាយបញ្ជាក់

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(h+x) - f(x)}{h} \quad y = f(x) = x, f(h+x) = h + x \\ &= \lim_{h=\to 0} \frac{h + x - x}{h} = \lim_{h\to 0} 1 = 1 \\ \therefore \quad \frac{dy}{dx} &= 1 \end{aligned}$$

■

> **ទ្រឹស្តីបទ ១.១.១** f x_0 f x_0

សម្រាយបញ្ជាក់

$$f'(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x - a} \quad \lim_{x\to a} f(x) = f(a)$$

$$\begin{aligned} \lim_{x\to a} f(x) &= \lim_{x\to a} (f(x) - f(a) + f(a)) \\ &= \lim_{x\to a} \left(\frac{f(x) - f(a)}{x - a} \times (x - a) + f(a) \right) \\ &= \lim_{x\to a} \frac{f(x) - f(a)}{x - a} \times \lim_{x\to a} (x - a) + f(a) \\ &= f'(a) \times 0 + f(a) \\ &= f(a) \end{aligned}$$

■ **ចំណាំ ១.២** f x_0 f x_0 x_0

១.២ ភាពមានដេរីវេ

និយមន័យ ១.២.១ f x

- f x
- x $f'_-(x) = f'_+(x)$

$$f'_-(x) = \lim_{h\to 0^-} \frac{f(x+h)-f(x)}{h} \quad f'_+(x) = \lim_{h\to 0^+} \frac{f(x+h)-f(x)}{h}$$

■ **ឧទាហរណ៍ ១.៣** f $f(x) = \begin{cases} \cos x & x \le \dfrac{\pi}{4} \\ a+bx & x > \dfrac{\pi}{4} \end{cases}$

a b f $x = \dfrac{\pi}{4}$ ■

សម្រាយបញ្ជាក់

- f $x = \dfrac{\pi}{4}$ $\displaystyle\lim_{x\to \frac{\pi}{4}^-} f(x) = \lim_{x\to \frac{\pi}{4}^+} f(x) = f\left(\frac{\pi}{4}\right)$

$$\lim_{x\to \frac{\pi}{4}^-} \cos x = \lim_{x\to \frac{\pi}{4}^+} (a+bx) = \cos\frac{\pi}{4} \iff \frac{\sqrt{2}}{2} = a + b.\frac{\pi}{4} = \frac{\sqrt{2}}{2} \implies a = \frac{\sqrt{2}}{2} - \frac{\pi}{4}.b$$

- $f'_-(x)$

$$\begin{aligned}
f'_-\left(\frac{\pi}{4}\right) &= \lim_{h\to 0^-} \frac{f(\frac{\pi}{4}+h) - f\left(\frac{\pi}{4}\right)}{h}, f(x) = \cos x \\
&= \lim_{h\to 0^-} \frac{\cos(\frac{\pi}{4}+h) - \cos\frac{\pi}{4}}{h} \\
&= \lim_{h\to 0^-} \frac{\cos\frac{\pi}{4}\cos h - \sin\frac{\pi}{4}\sin h - \cos\dfrac{\pi}{4}}{h} \\
&= \lim_{h\to 0^-} \frac{-\cos\frac{\pi}{4}(1-\cos h) - \sin\frac{\pi}{4}\sin h}{h} \\
&= \frac{\sqrt{2}}{2} \lim_{h\to 0^-} \left(-\frac{1-\cos h}{h} - \frac{\sin h}{h} \right), \lim_{h\to 0^-} \frac{1-\cos h}{h} = 0, \lim_{h\to 0^-} \frac{\sin h}{h} = 1 \\
&= \frac{\sqrt{2}}{2}(0-1) = -\frac{\sqrt{2}}{2}
\end{aligned}$$

- $f'_+(x)$

$$\begin{aligned}
f'_+\left(\frac{\pi}{4}\right) &= \lim_{h\to 0^+} \frac{f(\frac{\pi}{4}+h) - f\left(\frac{\pi}{4}\right)}{h}, f(x) = a+bx \\
&= \lim_{h\to 0^+} \frac{a+b(\frac{\pi}{4}+h) - (a+b.\frac{\pi}{4})}{h} \\
&= \lim_{h\to 0^+} \frac{a+b.\frac{\pi}{4}+bh-a-b.\frac{\pi}{4}}{h} \\
&= \lim_{h\to 0^+} \frac{bh}{h} = b
\end{aligned}$$

$$f \quad x = \frac{\pi}{4} \quad f'_-(\frac{\pi}{4}) = f'_+(\frac{\pi}{4}) \iff b = -\frac{\sqrt{2}}{2} \implies a = \frac{\sqrt{2}}{2}\left(1+\frac{\pi}{4}\right)$$

១.៣ លក្ខណៈនៃដេរីវេ

លក្ខណៈ ១ u,v x k

. $(ku)' = ku'$

. $(u+v)' = u'+v$

. $(u-v)' = u'-v'$

. $(uv)' = u'v+v'u$

. $\left(\frac{u}{v}\right)' = \frac{u'v-v'u}{v^2}$

. $\left(\frac{1}{v}\right)' = -\frac{v'}{v^2}$

សម្រាយបញ្ជាក់

. $f(x) = k.u(x)$ $u = u(x)$ k

$$\begin{aligned} f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0}\frac{ku(x+h)-k.u(x)}{h} \\ &= k.\lim_{h\to 0}\frac{u(x+h)-u(x)}{h} \\ &= k.u'(x) \end{aligned}$$

$$\therefore \quad (k.u)' = k.u'$$

. $f(x) = u(x)+v(x)$ $u = u(x)$ $v = v(x)$

$$\begin{aligned} f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0}\frac{u(x+h)+v(x+h)-(u(x)+v(x))}{h} \\ &= \lim_{h\to 0}\frac{u(x+h)-u(x)}{h}+\lim_{h\to 0}\frac{v(x+h)-v(x)}{h} \\ &= u'(x)+v'(x) \end{aligned}$$

$$\therefore \quad (u+v)' = u'+v'$$

.

. $f(x) = uv \quad u = u(x) \quad v = v(x)$

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \frac{u(x+h).v(x+h)-u(x).v(x)}{h} \\
&= \lim_{h\to 0} \frac{u(x+h).v(x+h)-u(x).v(x+h)+u(x).v(x+h)+u(x).v(x)}{h} \\
&= \lim_{h\to 0} \left[\frac{u(x+h).v(x+h)-u(x).v(x+h)}{h} + \frac{u(x).v(x+h)+u(x).v(x)}{h}\right] \\
&= \lim_{h\to 0} \left[v(x+h).\frac{u(x+h)-u(x)}{h} + u(x).\frac{v(x+h)+v(x)}{h}\right] \\
&= v(x).\frac{d}{dx}(u(x)) + u(x).\frac{d}{dx}(v(x))
\end{aligned}$$

$$\therefore \quad (uv)' = u'v + v'u \tag{.}$$

. $u = u(x) \quad v = v(x) \quad f(x) = \frac{u}{v} \Leftrightarrow f(x).v = u \quad x$

$[f(x).v]' = u' \quad (.)$

$$\begin{aligned}
f'(x).v + v'f(x) = u',\ f(x) = \frac{u}{v} \\
f'(x).v + v'.\frac{u}{v} = u' \\
\frac{f'(x).v^2}{v} + \frac{v'u}{v} = u' \\
f'(x).v^2 + v'u = u'v \\
f'(x) = \frac{u'v - v'u}{v^2}
\end{aligned}$$

$$\therefore \quad \left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2} \tag{.}$$

. $v = v(x) \quad f(x) = \frac{1}{v} \quad (.)$

$$\begin{aligned}
f'(x) &= \frac{(1)'.v - v'.(1)}{v^2} \\
&= \frac{0 - v'}{v^2} \\
&= -\frac{v'}{v^2}
\end{aligned}$$

$$\therefore \quad \left(\frac{1}{v}\right)' = -\frac{v'}{v^2}$$

១.៤ ដេរីវេនៃអនុគមន៍បណ្ដាក់

ជាទូទៅ ១ $y = f(u)$ $u = g(x)$ $\frac{d}{dx}(f \circ g) = \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

សម្រាយបញ្ជាក់

$F(x) = f \circ g = f(g(x))$ $x = a$

$$\begin{aligned} F'(a) &= \lim_{x \to a} \frac{F(x) - F(a)}{x - a} \\ &= \lim_{x \to a} \frac{f(g(x)) - f(g(a))}{x - a} \\ &= \lim_{x \to a} \left(\frac{f(g(x)) - f(g(a))}{g(x) - g(a)} \times \frac{g(x) - g(a)}{x - a} \right) \\ &= f'(g(a)) \times g'(a) \quad , u = g(a), y = f(a) \end{aligned}$$

$$\therefore \quad \frac{d}{dx}(f \circ g) = \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

ជាទូទៅ ២ $y = c$ c $y' = 0$

សម្រាយបញ្ជាក់

$y = f(x_0) = c$ $f(x_0 + h) = c \, , c \in \mathbb{R}$

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \to 0} \frac{c - c}{h} \\ &= \lim_{h \to 0} \frac{0}{h} \end{aligned}$$

$$\therefore \quad \frac{d}{dx}(c) = 0$$

■ **ឧទាហរណ៍ ១.៤** y' $y = \left(\ln x . \log_a(\sqrt{3})\right)$ ■

ដំណោះស្រាយ

$$y = \left(\ln x . \log_a(\sqrt{3})\right) \Rightarrow y' = \left(\ln x . \log_a(\sqrt{3})\right)' = 0$$

■ **ឧទាហរណ៍ ១.៥** $y = x^n$ $y' = nx^{n-1}$ ■

សម្រាយបញ្ជាក់

$f(x) = x^n \quad f(x+h) = (x+h)^n$

$$
\begin{aligned}
y' = f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{(x+h)^n - x^n}{h} \\
&= \lim_{h\to 0} \frac{(x+h-x)(x^{n-1} + x^{n-2}.x + \ldots + x.x^{n-2}x + x^{n-1})}{h} \\
&= \lim_{h\to 0} (x^{n-1} + x^{n-1} + \ldots + x^{n-1} + x^{n+1}) \\
&= x^{n-1}(\underbrace{1 + 1 + \ldots + 1 + 1}_{n \quad 1}) \\
&= n.x^{n-1}
\end{aligned}
$$

$$\therefore \quad \frac{d}{dx}(x^n) = n.x^{n-1}$$

▪ **ឧទាហរណ៍ ១.៦** $f'(x)$

. $f(x) = x^3$. $f(x) = \sqrt{x}$. $f(x) = \sqrt[3]{x^2}$

▪

ដំណោះស្រាយ

. $f(x) = x^3 \Rightarrow f'(x) = (x^3)' = 3x^{3-1} = 3x^2$

. $f(x) = \sqrt{x} \Rightarrow f'(x) = (\sqrt{x})' = (x^{\frac{1}{2}})' = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

. $f(x) = \sqrt[3]{x^2} \Rightarrow f'(x) = (\sqrt[3]{x^2})' = (x^{\frac{2}{3}})' = \frac{2}{3}x^{\frac{2}{3}-1} = \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{x}}$

ជាទូទៅ ៣ $y = u^n \quad u \quad x \quad y' = nu'u^{n-1}$

សម្រាយបញ្ជាក់

$$y = u^n \quad y' = \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(u^n) \times u' = nu'u^{n-1}$$

▪ **ឧទាហរណ៍ ១.៧** y'

. $y=(2x+\ln 2)^4$. $y=\sqrt{u}$ u x

■

ដំណោះស្រាយ

. $y=(2x+\ln 2)^4 \Longrightarrow y'=4(2x+\ln 2)'(2x+\ln 2)^{4-1}=4(2+0)(2x+\ln 2)^3$

$$\therefore \quad y'=8(2x+\ln 2)^3$$

. $y=\sqrt{u}=u^{\frac{1}{x}} \Longrightarrow y'=(u^{\frac{1}{2}})'=\frac{1}{2}u'u^{\frac{1}{2}-1}=\frac{1}{2}u'u^{-\frac{1}{2}}=\frac{u'}{2\sqrt{u}}$

១.៥ ដេរីវេនៃអនុគមន៍ត្រីកោណមាត្រ

លក្ខណៈ ២

. $y=\sin x$ $y'=\cos x$

. $y=\cos x$ $y'=-\sin x$

. $y=\tan x$ $y'=\dfrac{1}{\cos^2 x}=1+\tan^2 x$

. $y=\cot x$ $y'=-\dfrac{1}{\sin^2 x}=-(1+\cot^2 x)$

■

សម្រាយបញ្ជាក់

. $y=f(x)=\sin x$ $f(x+h)=\sin(x+h)$

$$\begin{aligned}
y'=f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&= \lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}\\
&= \lim_{h\to 0}\frac{\sin x\cos h+\sin h.\cos x-\sin x}{h}\\
&= \lim_{h\to 0}\left(\cos x.\frac{\sin h}{h}-\sin x.\frac{1-\cos h}{h}\right)\\
&= \cos x \quad , \lim_{h\to 0}\frac{\sin h}{h}=1, \lim_{h\to 0}\frac{1-\cos h}{h}=0
\end{aligned}$$

$$\therefore \quad \frac{d}{dx}(\sin x)=\cos x$$

. $y = f(x) = \cos x \quad f(x+h) = \cos(x+h)$

$$\begin{aligned}
y' = f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&= \lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h}\\
&= \lim_{h\to 0}\frac{\cos x.\cos h-\sin x.\sin h-\cos x}{h}\\
&= \lim_{h\to 0}\left(-\frac{\sin h}{h}.\sin x-\cos x.\frac{1-\cos h}{h}\right)\\
&= -\sin x \quad ,\lim_{h\to 0}\frac{1-\cos h}{h}=0, \lim_{h\to 0}\frac{\sin h}{h}=1
\end{aligned}$$

$$\therefore \quad \frac{d}{dx}(\cos x) = -\sin x$$

. $y = \tan x = \dfrac{\sin x}{\cos x}$ (.)

$$\begin{aligned}
y' = \left(\frac{\sin x}{\cos x}\right)' &= \frac{(\sin x)'\cos x-(\cos x)'.\sin x}{(\cos x)^2}\\
&= \frac{\cos x.\cos x-(-\sin x).\sin x}{\cos^2 x}\\
&= \frac{\cos^2 x+\sin^2 x}{\cos^2 x}\\
&= 1+\tan^2 x\\
&= \frac{1}{\cos^2 x}, \ \sin^2 x+\cos^2 x = 1
\end{aligned}$$

$$\therefore \quad (\tan x)' = \frac{1}{\cos^2} = 1+\tan^2 x$$

. $y = \cot x = \dfrac{\cos x}{\sin x}$ (.)

$$\begin{aligned}
y' = \left(\frac{\cos x}{\sin x}\right)' &= \frac{(\cos x)'.\sin x-(\sin x)'.\cos x}{(\sin^2 x)^2}\\
&= \frac{-\sin x.\sin x-\cos x.\cos x}{\sin^2 x}\\
&= \frac{-\sin^2 x-\cos^2 x}{\sin^2 x}\\
&= -\frac{\sin^2 x+\cos^2 x}{\sin^2 x} \ ,\sin^2 x+\cos^2 x = 1
\end{aligned}$$

$$\therefore \quad (\cot x)' = -\frac{1}{\sin^2 x} = -(1+\cot^2 x)$$

ជាទូទៅ ៤ u x

. $y=\sin u$ $y'=u'\cos u$

. $y=\cos u$ $y'=-u'\sin u$

. $y=\tan u$ $y'=\dfrac{u'}{\cos^2 u}=u(1+\tan^2 u)$

. $y=\cot u$ $y'=-\dfrac{u'}{\sin^2 u}=-u'(1+\cot^2 u)$

សម្រាយបញ្ជាក់

. u x $y=\sin u$

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}=\frac{d}{du}(\sin u)\times\frac{du}{dx}=\cos u\times u'=u'\cos u$$
$$\therefore\quad \frac{d}{dx}(\sin u)=u'\cos u$$

. u x $y=\cos u$

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}=\frac{d}{du}(\cos u)\times\frac{du}{dx}=-\sin u\times u'=-u'\sin u$$
$$\therefore\quad \frac{d}{dx}(\cos u)=-u'\sin u$$

. u x $y=\tan u$

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}=\frac{d}{du}(\tan u)\times\frac{du}{dx}=\frac{1}{\cos^2 u}\times u'=(1+\tan^2 u)\times u'$$
$$\therefore\quad (\tan u)'=\frac{u'}{\cos^2 u}=u'(1+\tan^2 u)$$

. u x $y=\cot u$

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}=\frac{d}{du}(\cot u)\times\frac{du}{dx}=-\frac{1}{\sin^2 u}\times u'=-(1+\cot^2 u)\times u'$$
$$\therefore\quad (\cot u)'=-\frac{u'}{\sin^2 u}=-u'(1+\cot^2 u)$$

■ **ឧទាហរណ៍ ១.៨**

. $y=\sin(2x+1)$

. $y=\tan(2x+1)$

. $y=\cos(2x+1)$

. $y=\cot(2x+1)$

■

ដំណោះស្រាយ

. $y=\sin(2x+1)\Rightarrow y'=(2x+1)'\cos(2x+1)=2\cos(2x+1)$

. $y=\cos(2x+1)\Rightarrow y'=-(2x+1)'\sin(2x+1)=-2\sin(2x+1)$

. $y=\tan(2x+1)\Rightarrow y'=\dfrac{(2x+1)'}{\cos^2(2x+1)}=\dfrac{2}{\cos^2(2x+1)}=2[1+\tan^2(2x+1)]$

. $y=\cot(2x+1)\Rightarrow y'=-\dfrac{(2x+1)'}{\sin^2(2x+1)}=-\dfrac{2}{\sin^2(2x+1)}=-2[1+\cot^2(2x+1)]$

១.៦ ដេរីវេអនុគមន៍អិចស្ប៉ូណង់ស្យែល

$y=a^x \quad y'=a^x.\ln a$

សម្រាយបញ្ជាក់

$y=a^x$

$$\begin{aligned} y'=f'(x)&=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}\\ &=\lim_{h\to0}\frac{a^{x+h}-a^x}{h}\\ &=\lim_{h\to0}\frac{a^x(a^h-1)}{h}\\ &=a^x\lim_{h\to0}\frac{a^h-1}{h}\quad \lim_{x\to0}\frac{a^x-1}{x}=\ln a \end{aligned}$$

$\therefore \quad (a^x)'=a^x.\ln a$

ជាទូទៅ ៥ $u \quad x \quad (a^u)'=u'a^u.\ln a$

សម្រាយបញ្ជាក់

$u \quad x$

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}=\frac{d}{du}(a^u)\times\frac{du}{dx}=a^u.\ln a\times u'$$

$\therefore \quad (a^u)'=u'.a^u.\ln a)$

▪ **ឧទាហរណ៍ ១.៩** $y' \quad u \quad x$

. $y=e^x$. $y=a^{x^2-1}$. $y=e^u$

■

ដំណោះស្រាយ

. $y=e^x$ $y'=(e^x)'=e^x.\ln e=e^x$,$\ln e=1$

. $y=a^{x^2-1}$ $y'=(x^{x^2-1})'a^{x^2-1}\ln a=2x.a^{x^2-1}\ln a$

. $y=e^u$ $y'=(e^u)'=u'e^u.\ln e=u'e^u$, $\ln e=1$

១.៧ ដេរីវេនៃអនុគមន៍លោការីត

$y=\log_a x$,$a>0,a\neq 1$ $y'=\dfrac{1}{x\ln a}$

សម្រាយបញ្ជាក់

$y=\log_a x$

ជាទូទៅ ៦ u x $(\log_a u)'=\dfrac{u'}{u\ln a}$,$a>0,a\neq 1$

សម្រាយបញ្ជាក់

u x

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}=\frac{d}{du}(\log_a u)\times\frac{du}{dx}=\frac{1}{u\ln a}\times u'$$
$$\therefore\quad (\log_a u)'=\frac{u'}{u\ln a}\ ,a>0,a\neq 1$$

១.៨ ដេរីវេនៃអនុគមន៍លោការីតនេពែ

$y=\ln x$ $y'=\dfrac{1}{x}$

សម្រាយបញ្ជាក់

$y = \ln x$

$$\begin{aligned}
y' = f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&= \lim_{h\to 0}\frac{\ln(x+h)-\ln(x)}{h}\\
&= \lim_{h\to 0}\frac{\ln\left(\dfrac{x+h}{x}\right)}{h}\\
&= \lim_{h\to 0}\frac{1}{h}\ln\left(1+\frac{h}{x}\right)\\
&= \lim_{h\to 0}\ln\left(1+\frac{h}{x}\right)^{\dfrac{1}{h}}\\
&= \ln\left[\lim_{h\to 0}\left(1+\frac{1}{\frac{x}{h}}\right)^{\frac{x}{h}\times\frac{1}{x}}\right], \lim_{h\to 0}\left(1+\frac{1}{\frac{x}{h}}\right)^{\frac{x}{h}} = e\\
&= \ln e^{\dfrac{1}{x}}, \ln e = 1
\end{aligned}$$

$$\therefore \quad (\ln x)' = \frac{1}{x}$$

■ **ឧទាហរណ៍ ១.១០** $f'(x)$

. $f(x) = x^2.\log_a x, a>0, a\neq 1$

. $f(x) = \sin(2x)+\log_2(x^2+1)$

. $f(x) = \dfrac{e^{2x}+\log_3 x}{x^2}$

. $f(x) = \log(x^2\sqrt{x^3-1})$

. $f(x) = (\sin x)^{\log x}$

. $f(x) = (\log_a x)^{\ln(2x)}, a>0, a\neq 1$

■

សម្រាយបញ្ជាក់

. $f(x) = x^2.\log_a x, a>0, a\neq 1 \Longrightarrow f'(x) = (x^2)'\log_a x + (\log_a x)'x^2$

$$= 2x\log_a x + \frac{1}{x\ln a}x^2$$

$$\therefore \quad f'(x) = 2x\log_a x + \frac{x}{\ln a}, a>0, a\neq 1$$

. $f(x) = \sin(2x)+\log_2(x^2+1) \Longrightarrow f'(x) = -(2x)'\cos(2x) + \dfrac{(x^2+1)'}{(x^2+1)\ln 2}$

$$\therefore \quad f'(x) = -2\cos(2x) + \frac{2x}{(x^2+1)\ln 2}$$

. $f(x)=\dfrac{e^{2x}+\log_3 x}{x^2} \Longrightarrow f'(x)=\dfrac{(e^{2x}+\log_3 x)'x^2-(x^2)'(e^{2x}+\log_3 x)}{x^4}$

$$=\frac{(2e^{2x}+\dfrac{1}{x\ln 3})x^2-2x(e^{2x}+\log_3 x)}{x^4}$$

$$=\frac{2xe^{2x}+\dfrac{1}{\ln 3}-2e^{2x}-2\log_3 x}{x^3}$$

$$\therefore \quad f'(x)=\frac{2e^{2x}(x-1)+\dfrac{1}{\ln 3}-\log_3 x^2}{x^3}$$

. $f(x)=\log(x^2\sqrt{x^3-1})=\log x^2+\log(x^3-1)^{\frac{1}{2}}=2\log x+\dfrac{1}{2}\log(x^3-1)$

$$\therefore \quad f'(x)=\frac{2}{x\ln 10}+\frac{(x^3-1)'}{2(x^3-1)\ln 10}=\frac{2}{x\ln 10}+\frac{3x^2}{2(x^3-1)\ln 10}$$

. $f(x)=(\sin x)^{\log x} \Longleftrightarrow \ln f(x)=\ln(\sin x)^{\log x}$

$$(\ln f(x))'=(\log x.\ln(\sin x))'$$

$$\frac{f'(x)}{f(x)}=(\log x)'\ln(\sin x)+(\ln(\sin x))'\log x$$

$$f'(x)=f(x)\left(\frac{1}{x\ln 10}\ln(\sin x)+\frac{(\sin x)'}{\sin x}.\log x\right)$$

$$\therefore \quad f'(x)=(\sin x)^{\log x}\left(\frac{\ln(\sin x)}{x\ln 10}+\cot x.\log x\right)$$

. $f(x)=(\log_a x)^{\ln(2x)}, a>0, a\neq 1 \Longleftrightarrow \ln f(x)=\ln(\log_a x)^{\ln(2x)}$

$$(\ln f(x))'=(\ln(2x).\ln(\log_a x))'$$

$$\frac{f'(x)}{f(x)}=(\ln(2x))'\ln(\log_a x)+(\ln(\log_a x))'\ln(2x)$$

$$f'(x)=f(x)\left(\frac{(2x)'}{2x}\ln(\log_a x)+\frac{(\log_a x)'}{\log_a x}\ln(2x)\right)$$

$$\therefore \quad f'(x)=(\log_a x)^{\ln(2x)}\left(\frac{\ln(\log_a x)}{x}+\frac{\ln(2x)}{x\ln a\log_a x}\right), a>0, a\neq 1$$

ឧទាហរណ៍ ៧ u x $(\ln u)'=\dfrac{u'}{u}$

សម្រាយបញ្ជាក់

u x

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}=\frac{d}{du}(\ln u)\times\frac{du}{dx}=\frac{1}{u}\times u'$$

$$\therefore \quad (\ln u)'=\frac{u'}{u}$$

▪ **ឧទាហរណ៍ ១.១១** $f'(x)$

. $f(x) = x.\ln x$

. $f(x) = x^2 + \ln(x^2+1)$

. $f(x) = \dfrac{e^x + \ln x}{x^2}$

. $f(x) = \ln(x^2\sqrt{x^3-1})$

. $f(x) = x^x$

. $f(x) = (\sin x)^{\cos x}$

▪

សម្រាយបញ្ជាក់

. $f(x) = x.\ln x \Longrightarrow f'(x) = x'\ln x + (\ln x)'x = \ln x + \dfrac{1}{x}.x = \ln x + 1$

. $f(x) = x^2 + \ln(x^2+1) \Longrightarrow f'(x) = (x^2)' + \dfrac{(x^2+1)'}{x^2+1} = 2x + \dfrac{2x}{x^2+1}$

. $f(x) = \dfrac{e^x + \ln x}{x^2} \Longrightarrow f'(x) = \dfrac{(e^x+\ln x)'x^2 - (x^2)'(e^x+\ln x)}{(x^2)^2}$

$$= \frac{\left(e^x + \frac{1}{x}\right)x^2 - 2x(e^x+\ln x)}{x^4}$$

$$\therefore \quad f'(x) = \frac{xe^x + 1 - 2e^x - 2\ln x}{x^3}$$

. $f(x) = \ln(x^2\sqrt{x^3-1}) = \ln x^2 + \ln\sqrt{x^3-1} = 2\ln x + \ln(x^3-1)^{\frac{1}{2}}$

$$f'(x) = 2(\ln x)' + \frac{1}{2}[\ln(x^3-1)]'$$

$$= 2.\frac{1}{x} + \frac{1}{2}.\frac{(x^3-1)'}{x^3-1}$$

$$\therefore \quad f'(x) = \frac{2}{x} + \frac{3}{2}.\frac{x^2}{x^3-1}$$

. $f(x) = x^x \Longleftrightarrow \ln f(x) = \ln x^x = x\ln x$

$$(\ln f(x))' = (x\ln x)'$$

$$\frac{f'(x)}{f(x)} = x'\ln x + (\ln x)'x$$

$$f'(x) = f(x)(\ln x + \frac{1}{x}.x)$$

$$\therefore \quad f'(x) = x^x(\ln x + 1)$$

. $f(x)=(\sin x)^{\cos x} \Longleftrightarrow \ln f(x)=\ln(\sin x)^{\cos x}$

$$(\ln f(x))'=(\cos x \ln \sin x)'$$
$$\frac{f'(x)}{f(x)}=(\cos x)'\ln \sin x+(\ln \sin x)'\cos x$$
$$f'(x)=f(x)\left(-\sin x \ln sinx+\frac{(\sin x)'}{\sin x}.\cos x\right)$$
$$\therefore \quad f'(x)=(\sin x)^{\cos x}(\cos x \cot x-\sin x \ln \sin x)$$

១.៩ ដេរីវេនៃអនុគមន៍ Arc Sine និង Arc Tangent

$$y=\arcsin x \Longleftrightarrow x=\sin y \quad -\frac{\pi}{2}\leq y\leq \frac{\pi}{2},$$
$$y=\arctan x \Longleftrightarrow x=\tan y \quad -\frac{\pi}{2}\leq y\leq \frac{\pi}{2},$$

■ **ឧទាហរណ៍ ១.១២** $y=\arcsin x \quad y'=\dfrac{1}{\sqrt{1-x^2}}$ ■

សម្រាយបញ្ជាក់

$y=\arcsin x \quad x=\sin y \quad x$

$$(x)'=(\sin y)' \Longleftrightarrow 1=y'\cos y$$
$$y'=\frac{1}{\cos y} \quad \sin^2 y+\cos^2 y=1$$
$$\Longrightarrow \cos y=\pm\sqrt{1-\sin^2 x}$$
$$-\frac{\pi}{2}\leq y\leq \frac{\pi}{2} \Longrightarrow \cos y\geq 0 \Longrightarrow \cos y=\sqrt{1-x^2}$$
$$\therefore \quad (\arcsin x)'=\frac{1}{\sqrt{1-x^2}}$$

■ **ឧទាហរណ៍ ១.១៣** $y=\arctan x \quad y'=\dfrac{1}{1+x^2}$ ■

សម្រាយបញ្ជាក់

$y=\arctan x \, x=\tan y \quad x$

$$(x)'=(\sin y)' \Longleftrightarrow 1=y'(1+\tan^2 y)$$
$$y'=\frac{1}{1+\tan^2 y}$$
$$\therefore \quad (\arctan x)'=\frac{1}{1+x^2}$$

ជាទូទៅ ៨ u x $(\arcsin u)' = \dfrac{u'}{\sqrt{1-x^2}}$

សម្រាយបញ្ជាក់

u x

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(\arcsin u) \times \frac{du}{dx} = \frac{1}{\sqrt{1-u^2}} \times u'$$

$$\therefore \quad (\arcsin u)' = \frac{u'}{\sqrt{1-u^2}}$$

ជាទូទៅ ៩ u x $(\arctan u)' = \dfrac{u'}{1+u^2}$

សម្រាយបញ្ជាក់

u x

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(\arctan u) \times \frac{du}{dx} = \frac{1}{1-u^2} \times u'$$

$$\therefore \quad (\arctan u)' = \frac{u'}{1+u^2}$$

■ **ឧទាហរណ៍ ១.១៩**

. $f(x) = \arcsin x . \sin x$

. $f(x) = \sin(\arcsin x)$

. $f(x) = \arctan x \cos x$

. $f(x) = \arctan(\tan x)$

■

សម្រាយបញ្ជាក់

. $f(x) = \arcsin x . \sin x \Longrightarrow f'(x) = (\arcsin x)' \sin x + (\sin x)' \arcsin x$

$$\therefore \quad f'(x) = \frac{\sin x}{\sqrt{1-x^2}} + \cos x . \arcsin x$$

. $f(x) = \arctan x \cos x \Longrightarrow f'(x) = (\arctan x)' \cos x + (\cos x)' \arctan$

$$\therefore \quad f'(x) = \frac{\cos x}{1+x^2} - \sin x . \arctan x$$

. $f(x)=\sin(\arcsin x)\Longrightarrow f'(x)=(\arcsin x)'\cos(\arcsin x)$

$$\therefore \quad f'(x)=\frac{\cos(\arcsin x)}{\sqrt{1-x^2}}$$

. $f(x)=\arctan(\tan x)\Longrightarrow f'(x)=\dfrac{(\tan x)'}{1+(\tan x)^2}=\dfrac{1+\tan^2 x}{1+\tan^2 x}$

$$\therefore \quad f'(x)=1$$

១.១០ រូបមន្តនៃដេរីវេ

$C, a, b, c \quad u \quad x \quad n \in \mathbb{N}$

. $(C)' = 0$

. $(x)' = 1$

. $(ax+b)' = a$

. $(ax^2+bx+c)' = 2ax+b$

. $(x^n)' = nx^{n-1}$

. $(u^n)' = n.u'.u^{n-1}$

. $(x)^{-n} = -\frac{n}{x^{n+1}}$

. $\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$

. $\left(\frac{1}{u}\right)' = -\frac{u}{u^2}$

. $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$

. $(\sqrt{u})' = \frac{u'}{2\sqrt{u}}$

. $(\sqrt[n]{x})' = \frac{1}{n\sqrt[n]{x^{n-1}}}$

. $(\ln x)' = \frac{1}{x}$

. $(\ln u)' = \frac{u'}{u}$

. $(\log_a x)' = \frac{1}{x \ln a}, a > 0, a \neq 1$

. $(\log_a u)' = \frac{u'}{u. \ln a}, a > 0, a \neq 1$

. $(a^x)' = a^x \ln a, a > 0, a \neq 1$

. $(a^u)' = u' a^u \ln a, a > 0, a \neq 1$

. $(e^x)' = e^x$

. $(e^u)' = u' e^u$

. $(\sin x)' = \cos x$

. $(\sin u)' = u' \cos u$

. $(\cos x)' = -\sin x$

. $(\cos u)' = -u' \sin u$

. $(\tan x)' = \frac{1}{\cos^2 x} = 1 + \tan^2 x$

. $(\tan u)' = \frac{u'}{\cos^2 u} = u'(1 + \tan^2 u)$

. $(\cot x)' = -\frac{1}{\sin^2 x} = -(1 + \cot^2 x)$

. $(\cot u)' = -\frac{u'}{\sin^2 u} = -(1 + \cot^2 u)$

. $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$

. $(\arcsin u)' = \frac{u'}{\sqrt{1-u^2}}$

. $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$

. $(\arccos u)' = -\frac{u'}{\sqrt{1-u^2}}$

. $(\arctan x)' = \frac{1}{1+x^2}$

. $(\arctan u)' = \frac{u'}{1+u^2}$

. $(\text{arccot}\, x)' = -\frac{1}{1+x^2}$

. $(\text{arccot}\, u)' = -\frac{u'}{1+u^2}$

. $(u^v)' = \left(v'. \ln u + \frac{v.u'}{u}\right).u^v$

១.១១ លំហាត់ និង ដំណោះស្រាយ

លំហាត់ ១ $f'(x)$

. $f(x)=x^5-x^4+x^3-x^2+x-1$

. $f(x)=2x^2-\sqrt{x}+\dfrac{2}{x}$

. $f(x)=(x^4-7x^2+\sin a)^7$

. $f(x)=(x^2-\sqrt{x})^{2019}$

. $f(x)=\sqrt{x^3-x^2+3}$

. $\sqrt[4]{x^3-2x}$

. $f(x)=(x+1)(2x-1)^2$

. $f(x)=(x^2+2x+3)(x^3-3x-1)$

. $f(x)=\dfrac{1}{x-1}$

. $f(x)=\dfrac{x\sqrt{x}}{x+1}$

សម្រាយបញ្ជាក់

. $f(x)=x^5-x^4+x^3-x^2+x-1 \Longrightarrow f'(x)=5x^4-4x^3+3x^2-2x+1$

. $f(x)=2x^2-\sqrt{x}-\dfrac{2}{x} \Longrightarrow f'(x)=4x^2+\dfrac{1}{2\sqrt{x}}-\dfrac{2}{x^2}$

$$f'(x)=7(x^4-7x^2+\sin a)'(x^4-7x^2+\sin a)^{7-1}=7(4x^3-14x)(x^4-7x^2+\sin a)^6$$

. $f(x)=(x^4-7x^2+\sin a)^7$

. $f(x)=(x^2-\sqrt{x})^{2019} \Longrightarrow f'(x)=2019(x^2-\sqrt{x})'(x^2-\sqrt{x})^{2019-1}$

$$=2019\left(2x-\frac{1}{2\sqrt{x}}\right)(x^2-\sqrt{x})^{2018}$$

. $f(x)=\sqrt{x^3-x^2+3} \Longrightarrow f'(x)=\dfrac{(x^3-x^2+3)'}{2\sqrt{x^3-x^2+3}}=\dfrac{3x-2}{2\sqrt{x^3-x^2+3}}$

. $\sqrt[4]{x^3-2x} \Longleftrightarrow f(x)=(x^3-2x)^{\frac{1}{4}}$

$$\begin{aligned} f'(x)&=\frac{1}{4}(x^3-2x)'(x^3-2x)^{\frac{1}{4}-1}\\ &=\frac{1}{4}(3x^2-2)(x^3-2x)^{-\frac{3}{4}} \end{aligned}$$

$$\therefore\quad f'(x)=\frac{3x^2-2}{4\sqrt[4]{(x^3-2x)^3}}$$

. $f(x)=(x+1)(2x-1)^2$

$$\begin{aligned} f'(x)&=(x+1)'(2x-1)^2+[(2x-1)^2]'(x+1)\\ &=(2x-1)^2+2(2x-1)'(2x-1)(x+1)\\ &=(2x-1)(2x-1+4x+4) \end{aligned}$$

$$\therefore\quad f'(x)=(2x-1)(6x+3)$$

. $f(x)=(x^2+2x+3)(x^3-3x-1)$

$$\begin{aligned}f'(x)&=(x^2+2x+3)'(x^2-3x-1)+(x^2-3x-1)'(x^2+2x+3)\\&=(2x+2)(x^2-3x-1)+(2x-3)(x^2+2x+3)\\&=2x^3-6x^2-2x+2x^2-6x-2+2x^3+4x^2+6x-3x^2-6x-9\end{aligned}$$

$$\therefore\quad f'(x)=4x^3-3x^2-8x-11$$

. $f(x)=\dfrac{1}{x-1}\Longrightarrow f'(x)=-\dfrac{(x-1)'}{(x-1)^2}=-\dfrac{1}{(x-1)^2}$

. $f(x)=\dfrac{x\sqrt{x}}{x+1}$

$$\begin{aligned}f'(x)&=\frac{(x\sqrt{x})'(x+1)-(x+1)'x\sqrt{x}}{(x+1)^2}\\&=\frac{[x'\sqrt{x}+(\sqrt{x})'x](x+1)-x\sqrt{x}}{(x+1)^2}\\&=\frac{\left(x+\dfrac{x}{2\sqrt{x}}\right)(x+1)-x\sqrt{x}}{(x+1)^2}\\&=\frac{x\sqrt{x}+\sqrt{x}+\dfrac{x}{2\sqrt{x}}(x+1)-x\sqrt{x}}{(x+1)^2}\end{aligned}$$

$$\therefore\quad f'(x)=\frac{x^2+3x}{2\sqrt{x}(x+1)^2}$$

លំហាត់ ២

. $f(x)=x.\sin x+\cos x$

. $f(x)=\sin^2\sqrt{x}+\cos^2(3x)$

. $f(x)=\sin^3 x-x.\cos x$

. $f(x)=\cos(3x+4)+3\cos x.\sin x$

. $f(x)=\cos(x^2+1)+2\sin(x^2-1)$

. $f(x)=\sin(\sin\sqrt{x})+\cos^3 x$

ដំណោះស្រាយ

. $f(x)=x.\sin x+\cos x$

$$\begin{aligned}f'(x)&=x'\sin x+(\sin x)'.x-\sin x\\&=\sin x+x.\cos x-\sin x\end{aligned}$$

$$\therefore\quad f'(x)=x.\cos x$$

. $f(x)=\sin^3 x - x.\cos x$

$$f'(x)=3(\sin x)'\sin^{3-1}x-[x'.\cos x+(\cos x)'.x]$$
$$=3\cos x.\sin^2 x-(\cos x-x.\sin x)$$
$$\therefore\quad f'(x)=3\cos x.\sin^2 x-\cos x+x\sin x$$

. $f(x)=\cos(x^2+1)+2\sin(x^2-1)$

$$f'(x)=-(x^2+1)'\sin(x^2+1)+2(x^2-1)'\cos(x^2-1)$$
$$\therefore\quad f'(x)=-2x\sin(x^2+1)+4x\cos(x^2-1)$$

. $f(x)=\sin^2\sqrt{x}+\cos^2(3x)$

$$f'(x)=2(\sin\sqrt{x})'\cos^{2-1}\sqrt{x}+2(\cos(3x))'\sin(3x)$$
$$=2(\sqrt{x})'.\cos\sqrt{x}.\cos\sqrt{x}-2(3x)'\sin(3x).\sin(3x)$$
$$\therefore\quad f'(x)=\frac{1}{\sqrt{x}}.\cos^2\sqrt{x}-6\sin^2(3x)$$

. $f(x)=\cos(3x+4)+3\cos x.\sin x$

$$f'(x)=-(3x+4)'.\sin(3x+4)+3[(\cos x)'.\sin x+(\sin x)'.\cos x]$$
$$=-3\sin(3x+4)+3[-\sin x.\sin x+\cos x.\cos x]$$
$$\therefore\quad f'(x)=-3[\sin(3x+4)+\sin^2 x-\cos^2 x]$$

. $f(x)=\sin(\sin\sqrt{x})+\cos^3 x$

$$f'(x)=(\sin\sqrt{x})'.\cos(\sin\sqrt{x})+3(\cos x)\cos^{3-1}x$$
$$=(\sqrt{x})'.\cos\sqrt{x}.\cos(\sin\sqrt{x})-3\sin x\cos^2 x$$
$$\therefore\quad f'(x)=\frac{1}{2\sqrt{x}}\cos\sqrt{x}.\cos(\sin\sqrt{x})-3\sin x.\cos^2 x$$

លំហាត់ ៣

. $f(x)=(1+\tan x)^4$

. $f(x)=x.\tan(x^2-1)+x\cot(2x^2)$

. $f(x)=x^2\tan x+(1+\cot x)^2$

. $f(x)=\dfrac{\tan(2x)}{1-\cos x}$

ដំណោះស្រាយ

. $f(x) = (1+\tan x)^4$

$$f'(x) = 4(1+\tan x)'(1+\tan^2 x)^{4-1}$$
$$\therefore \quad f'(x) = 4(1+\tan^2 x)(1+\tan x)^3$$

. $f(x) = x^2\tan x + (1+\cot x)^2$

$$f'(x) = (x^2)'\tan x + (\tan x)'x^2 + 2(1+\cot x)'(1+\cot x)^{2-1}$$
$$\therefore \quad f'(x) = 2x\tan x + x^2(1+\tan^2 x) - 2(1+\cot^2 x)(1+\cot x)$$

. $f(x) = x.\tan(x^2-1) + x\cot(2x^2)$

$$f'(x) = x'\tan(x^2-1) + [\tan(x^2-1)]'x + x'\cot(2x^2) + [\cot(2x^2)]'x$$
$$= \tan(x^2-1) + (x^2-1)'[1+\tan^2(x^2-1)]x - (2x^2)'[1+\cot^2(2x^2)]x$$
$$\therefore \quad f'(x) = \tan(x^2-1) + 2x^2[1+\tan^2(x^2-1)] - 4x^2[1+\cot^2(2x^2)]$$

លំហាត់ ៤

. $f(x) = \dfrac{1-x-2x^2}{x^3-\ln 3}$

. $f(x) = \dfrac{2x^2+3x+4}{\sqrt{1+2x-x^2}}$

. $f(x) = \sin x^2.\tan(2x+3)$

. $f(x) = \sin(x^2+5) + \cos(\sin x)$

. $f(x) = \dfrac{\sin(\tan\sqrt{x})}{\sin(\sqrt{x})}$

ដំណោះស្រាយ

. $f(x) = \dfrac{1-x-2x^2}{x^3-\ln 3}$

$$f'(x) = \frac{(1-x-2x^2)'(x^3-\ln 3) - (x^3-\ln 3)'(1-x-2x^2)}{(x^3-\ln 3)^2}$$
$$= \frac{(-1-4x)(x^3-\ln 3) - 3x^2(1-x-2x^2)}{(x^3-\ln 3)^2}$$
$$= \frac{-x^3+\ln 3-4x^4+4x\ln 3-3x^2+3x^3+6x^4}{(x^3-\ln 3)^2}$$
$$\therefore \quad f'(x) = \frac{2x^4+2x^3-3x^2+4x.\ln 3+\ln 3}{(x^3-\ln 3)^2}$$

. $f(x)=\dfrac{2x^2+3x+4}{\sqrt{1+2x-x^2}} \Longleftrightarrow f(x).\sqrt{1+2x-x^2}=2x^2+3x+4$

$$[f(x)\sqrt{1+2x-x^2}]'=(2x^2+3x+4)'$$
$$f'(x)\sqrt{1+2x-x^2}+(\sqrt{1+2x-x^2})'f(x)=4x+3$$
$$f'(x)\sqrt{1+2x-x^2}+\frac{(1+2x-x^2)'}{2\sqrt{1+2x-x^2}}f(x)=4x+3$$
$$f'(x)\sqrt{1+2x-x^2}=4x+3-\frac{1-x}{\sqrt{1+2x-x^2}}.f(x)$$
$$\therefore \quad f'(x)=\frac{4x+3}{\sqrt{1+2x-x^2}}+\frac{(x-1)(2x^2+3x+4)}{(1+2x-x^2)\sqrt{1+2x-x^2}}$$

. $f(x)=\sin x^2.\tan(2x+3)$

$$\begin{aligned} f'(x)&=(\sin x^2)'\tan(2x+3)+(\tan(2x+3))'\sin x^2\\ &=(x^2)'.\sin x^2.\tan(2x+3)+(2x+3)'[1+\tan^2(2x+3)]\sin x^2\\ &=2x\sin x^2.\tan(2x+3)+2\sin x^2[1+\tan^2(2x+3)]\end{aligned}$$
$$\therefore \quad f'(x)=2\sin x^2[\tan^2(2x+3)+x\tan(2x+3)+1]$$

. $f(x)=\sin(x^2+5)+\cos(\sin x)$

$$f'(x)=(x^2+5)'\cos(x^2+5)-(\sin x)'\sin(\sin x)$$
$$\therefore \quad f'(x)=2x\cos(x^2+5)-\cos x\sin(\sin x)$$

. $f(x)=\dfrac{\sin(\tan\sqrt{x})}{\sin(\sqrt{x})} \Longleftrightarrow f(x).\sin\sqrt{x}=\sin(\tan\sqrt{x})$

$$\big(f(x).\sin\sqrt{x}\big)'=\big(\sin(\tan\sqrt{x})\big)'$$
$$f'(x).\sin\sqrt{x}+(\sin\sqrt{x})'f(x)=(\tan\sqrt{x})'\cos(\tan\sqrt{x})$$
$$f'(x).\sin\sqrt{x}+(\sqrt{x})'\cos\sqrt{x}.f(x)=(\sqrt{x})'(1+\tan^2\sqrt{x})\cos(\tan\sqrt{x})$$
$$f'(x)\sin\sqrt{x}+\frac{1}{2\sqrt{x}}\cos\sqrt{x}.f(x)=\frac{1}{2\sqrt{x}}(1+\tan^2\sqrt{x})\cos(\tan\sqrt{x})$$
$$f'(x)\sin\sqrt{x}=\frac{1}{2\sqrt{x}}\left[(1+\tan^2\sqrt{x})\cos(\tan\sqrt{x})-\cos\sqrt{x}.f(x)\right]$$

$$\therefore \quad f'(x)=\frac{(1+\tan^2\sqrt{x})\cos(\tan\sqrt{x})-\cos\sqrt{x}.f(x)}{2\sqrt{x}.\sin\sqrt{x}},f(x)=\frac{\sin(\tan\sqrt{x})}{\sin(\sqrt{x})}$$

លំហាត់ ៥

. $f(x)=xe^x+\frac{1}{2}x^2$

. $f(x)=x^3e^{-3x}$

. $f(x)=e^{x^2+2x+1}+(x^2-3)e^x$

. $f(x)=e^{2x}3^{x^2+1}$

. $f(x)=\frac{\sqrt{x}}{e^x}$

. $f(x)=e^{\sin x\cos x}$

ដំណោះស្រាយ

. $f(x)=xe^x+\frac{1}{2}x^2$

$$f'(x)=x'e^x+(e^x)'x+\frac{1}{2}.2x=e^x+e^xx+x=e^x(1+x)+x$$

. $f(x)=e^{x^2+2x+1}+(x^2-3)e^x$

$$\begin{aligned} f'(x)&=(x^2+2x+1)'e^{x^2+2x+1}+(x^2-3)'e^x+(e^x)'(x^2-3)\\ &=(2x+2)e^{x^2+2x+1}+2xe^x+e^x(x^2-3)\\ \therefore\quad f'(x)&=2(x+1)e^{x^2+2x+1}+e^x(2x+x^2-3)\end{aligned}$$

. $f(x)=\frac{\sqrt{x}}{e^x}$

$$f'(x)=\frac{(\sqrt{x})'e^x+(e^x)'\sqrt{x}}{(e^x)^2}=\frac{\frac{1}{2\sqrt{x}}e^x+e^x\sqrt{x}}{e^{2x}}=\frac{1+2x}{2\sqrt{x}e^x}$$

. $f(x)=x^3e^{-3x}$

$$\begin{aligned} f'(x)&=(x^3)'e^{-3x}+(e^{-3x})'x^3\\ &=3x^2e^{-3x}+(-3x)'e^{-3x}x^3\\ &=3x^2e^{-3x}-3e^3e^{-3x}\\ \therefore\quad f'(x)&=3x^2e^{-3x}(1-x)\end{aligned}$$

. $f(x)=e^{2x}3^{x^2+1}$

$$\begin{aligned} f'(x)&=(e^{2x})'3^{x^2+1}+(3^{x^2+1})'.e^{2x}\\ &=(2x)'e^{2x}.3^{x^2+1}+(x^2+1)'3^{x^2+1}\ln 3.e^{2x}\\ &=2.e^{2x}3^{x^2+1}+2x3^{x^2+1}\ln 3.e^{2x}\\ \therefore\quad f'(x)&=2e^{2x}3^{x^2+1}(1+x\ln 3)\end{aligned}$$

. $f(x)=e^{\sin x\cos x}$

$$\begin{aligned} f'(x) &= (\sin x\cos x)' e^{\sin x\cos x} \\ &= [(\sin x)'\cos x+(\cos x)'\cos x]e^{\sin x\cos x} \\ \therefore \quad f'(x) &= (\cos^2 x-\sin^2 x)e^{\sin x\cos x} \end{aligned}$$

លំហាត់ ៦

. $f(x)=(x^2-1)\ln(x^2-1)$

. $f(x)=\ln(\sin x.\cos(2x))$

. $f(x)=\ln\left(\dfrac{x^2-2}{\sqrt[3]{x^2-2}}\right)$

. $f(x)=\ln\left(\sqrt{\dfrac{1+\sin x}{1-\sin x}}\right)$

ដំណោះស្រាយ

. $f(x)=(x^2-1)\ln(x^2-1)\Longrightarrow f'(x)=(x^2-1)'\ln(x^2-1)+\ln(x^2-1)'(x^2-1)$

$$\begin{aligned} &= 2x\ln(x^2-1)+\frac{(x^2-1)'}{x^2-1}.(x^2-1) \\ &= 2x\ln(x^2-1)+2x \\ \therefore \quad f'(x) &= 2x[\ln(x^2-1)+1] \end{aligned}$$

. $f(x)=\ln\left(\dfrac{x^2-2}{\sqrt[3]{x^2-2}}\right)=\ln(x^2-2)-\ln(x^2-2)^{\frac{1}{3}}$

$$\begin{aligned} f'(x) &= \frac{(x^2-2)'}{x^2-2}-\frac{1}{3}.\frac{(x^2-2)'}{x^2-2} \\ &= \frac{3(2x)-2x}{3(x^2-2)} \\ \therefore \quad f'(x) &= \frac{4x}{3(x^2-2)} \end{aligned}$$

. $f(x)=\ln(\sin x.\cos(2x))=\ln(\sin x)+\ln(\cos(2x))$

$$\begin{aligned} f'(x) &= \frac{(\sin x)'}{\sin x}+\frac{(\cos(2x))'}{\cos(2x)} \\ &= \frac{\cos x}{\sin x}-\frac{2\sin(2x)}{\cos(2x)} \\ \therefore \quad f'(x) &= \cot x-2\tan(2x) \end{aligned}$$

. $f(x)=\ln\left(\sqrt{\dfrac{1+\sin x}{1-\sin x}}\right)=\ln\left(\dfrac{1+\sin x}{1-\sin x}\right)^{\frac{1}{2}}=\dfrac{1}{2}(\ln(1+\sin x)-\ln(1-\sin x))$

$$\begin{aligned} f'(x) &= \frac{1}{2}\left(\frac{(1+\sin x)'}{1+\sin x}-\frac{(1-\sin x)'}{1-\sin x}\right)\\ &= \frac{1}{2}\left(\frac{\cos x}{1+\sin x}+\frac{\cos x}{1-\sin x}\right)\\ &= \frac{1}{2}.\frac{(\cos x(1-\sin x+1+\sin x))}{1-\sin^2 x}\\ \therefore\quad f'(x) &= \frac{2\cos x}{2\cos^2 x}=\frac{1}{\cos x}\end{aligned}$$

លំហាត់ ៧

. $f(x)=\cos(\arcsin x)$

. $f(x)=\cot(\arctan x)$

. $f(x)=\tan(\arctan x)$

. $f(x)=\arcsin(2x)$

. $f(x)=\arcsin\sqrt{x}$

. $f(x)=\arctan(\sin x)$

. $f(x)=\dfrac{\arctan x}{\arcsin x}$

ដំណោះស្រាយ

. $f(x)=\cos(\arcsin x)\Longrightarrow f'(x)=-(\arcsin x)'\sin(\arcsin x)$

$$\therefore\quad f'(x)=-\frac{\sin(\arcsin x)}{\sqrt{1-x^2}}$$

. $f(x)=\cot(\arctan x)\Longrightarrow f'(x)=-(\arctan x)'[1+\cot^2(\arctan x)]$

$$\therefore\quad f'(x)=-\frac{1+\cot^2(\arctan x)}{1+x^2}$$

. $f(x)=\tan(\arctan x)\Longrightarrow f'(x)=(\arctan x)'[1+\tan^2(\arctan x)]$

$$\therefore\quad f'(x)=\frac{1+\tan^2(\arctan x)}{1+x^2}$$

. $f(x)=\arcsin(2x)\Longrightarrow f'(x)=\dfrac{(2x)'}{\sqrt{1-(2x)^2}}$

$$\therefore\quad f'(x)=\frac{2}{\sqrt{1-4x^2}}$$

. $f(x)=\arcsin\sqrt{x}\Longrightarrow f'(x)=\dfrac{(\sqrt{x})'}{\sqrt{1-(\sqrt{x})^2}}=\dfrac{\frac{1}{2\sqrt{x}}}{\sqrt{1-x^2}}=\dfrac{1}{2\sqrt{x}\sqrt{1-x^2}}$

$$\therefore \quad f'(x) = \frac{1}{2\sqrt{x-x^2}}$$

. $f(x) = \arctan(\sin x) \Longrightarrow f'(x) = \frac{(\sin x)'}{1+(\sin x)^2}, \sin^2 x + \cos^2 x = 1$

$$\therefore \quad f'(x) = \frac{\cos x}{2-\cos^2 x}$$

. $f(x) = \frac{\arctan x}{\arcsin x} \Longrightarrow f'(x) = \frac{(\arctan x)' \arcsin x - (\arcsin x)' \arctan x}{(\arcsin x)^2}$

$$\therefore \quad f'(x) = \frac{\frac{\arcsin x}{1+x^2} - \frac{\arctan x}{\sqrt{1-x^2}}}{(\arcsin x)^2}$$

១.១២ លំហាត់មេរៀន

.

() $y = x^3 + 2x^2$

() $y = x^3 - 4x^2$

() $y = x^4 - 27x$

() $y = x^4 - 5x^2 + 4$

() $y = x^5 - 16x$

() $y = \frac{x}{x+1}$

() $y = \frac{x^2}{1+x^2}$

() $y = x - \frac{1}{x}$

() $y = x^3 + 2x^2 - x$

() $y = x^4 - 2x^3 + 2x$

() $y = \sqrt{1+x^2}$

() $y = \sqrt[4]{1+x^2}$

. $f'(x)$

() $f(x) = \sin x + \cos x$

() $f(x) = 2\sin x - 3\cos x$

() $f(x) = 3\sin x + 2\cos x$

() $f(x) = x\sin x + \cos x$

() $f(x) = x\cos x - \sin x$

() $f(x) = \cos(2x)$

() $f(x) = \frac{1-\sin(2x)}{1-\sin x}$

() $f(x) = 1 + \sin x^2$

() $f(x) = \cot x - \cos x$

() $f(x) = \sin(2x) - \cos(3x)$

() $f(x) = \sin(\cos(3x))$

() $f(x) = \frac{\sin x^2}{x^2}$

() $f(x) = \tan(1+x^2)$

() $f(x) = \cos 2x - \cos x^2$

() $f(x) = (1+\sqrt{1+x})^3$

. y'

() $xy = \dfrac{\pi}{6}$

() $\sin(xy) = 1$

() $xy = \dfrac{1}{x+y}$

() $x+y = xy$

() $(y-1)^2 + x = 0$

() $(y+1)^2 + y - x = 0$

() $(y-x)^2 + x = 0$

() $(y+x) + 2y - x = 0$

() $(y^2-1)^2 + x = 0$

() $(y^2+1)^2 - x = 0$

() $x^3 + xy + y^3 = 3$

() $\sin x + \sin y = 1$

() $\sin x + xy + y^5 = \pi$

() $\tan x + \tan y = 1$

() $x \ln y = e^{\ln \sin x}$

() $(\sin x)^{\ln y} = (\tan y)^{e^{3x}}$

.

() $f(x) = \sqrt{1-x}$

() $f(x) = \sqrt[4]{x+x^2}$

() $y = \sqrt{1-\sqrt{x}}$

() $y = \sqrt{x-\sqrt{x}}$

() $y = \sqrt[3]{\sqrt{2x+1}} - x^2$

() $y = \sqrt[4]{x+x^2}x + x^2$

() $y = \sqrt[3]{x - \sqrt{2x+1}}$

() $y = \sqrt[4]{\sqrt[3]{x}} + \sqrt[3]{\sqrt{x}} + \sqrt{x}$

.

() $f(x) = e^x + e^{-x}$

() $f(x) = e^{3x} + 4e^x$

() $f(x) = \dfrac{e^x}{1+e^x}$

() $f(x) = \dfrac{2e^{2x}}{1+e^{2x}}$

() $f(x) = xe^{-x} + x\ln x$

() $f(x) = \sqrt{x}e^{-\frac{x}{4}} + x^2e^{x+2}$

() $f(x) = x^{-\frac{1}{2}x} + \ln\sqrt{x}$

() $f(x) = (\ln x)^2 + \ln x + 1$

() $f(x) = \dfrac{\ln x}{x} + \ln\dfrac{1}{x}$

() $f(x) = \ln\left(\sqrt{\dfrac{\sqrt{x}+1}{\sqrt{x}-1}}\right)$

.

() $f(x) = \tan(\arctan x)$

() $f(x) = \arcsin(\sin x)$

() $f(x) = \sin(\arctan x)$

() $f(x) = (\arcsin x)^2$

() $f(x) = \dfrac{1}{1+(\arctan x)^2}$

() $f(x) = \sqrt{1-(\arcsin x)^2}$

.

() $y=(x+1)(x-1)$

() $y=(x^2+1)(x^2-1)$

() $y=\dfrac{1}{x+1}+\dfrac{1}{1+\sin x}$

() $y=\dfrac{1}{1+x^2}+\dfrac{1}{1-\sin x}$

() $y=(x-1)(x-2)(x-3)$

() $y=x^2\cos x+2x\sin x$

() $y=x^{\frac{1}{2}}(x+\sin x)$

() $y=x^{\frac{1}{2}}\sin^2 x+(\sin x)^{\frac{1}{2}}$

() $y=x^4\cos x+x\cos x$

() $y=\dfrac{1}{2}x^2\sin x-x\cos x+\sin x$

() $y=\sqrt{x}(\sqrt{x}+1)(\sqrt{x}+2)$

() $y=(x-6)^{10}+\sin^{10}x$

() $y=(\sin x\cos x)^3+\sin(2x)$

() $y=x^{\frac{1}{2}}\sin(2x)+(\sin x)^{\frac{1}{2}}$

() $y=\dfrac{\sin x-\cos x}{\sin x+\cos x}$

() $y=\dfrac{1}{\tan x}-\dfrac{1}{\cot x}$