

**KEY CONCEPTS**  
on Integrals and its Applications



**ACE YOUR WAY**  
PRACTICE PAPER 2018  
CBSE Class XII

# MATHEMATICS

## today

India's #1  
MATHEMATICS MONTHLY  
for JEE (Main & Advanced)

**MATHS MUSING**  
10 GREAT PROBLEMS

PRACTICE PAPER  
**JEE MAIN**

**MATH ARCHIVES**

**CHALLENGING  
PROBLEMS**



MOCK TEST PAPER  
**JEE MAIN**

**MONTHLY  
PRACTICE  
PROBLEMS**  
(XI & XII)

**CONCEPT MAP**  
(XI & XII)

**mtG**

Trust of more than  
1 Crore Readers  
Since 1982



**You Ask ?**  
**We Answer** ✓

# ALLEN

NURTURED THEM WITH STRONG FOUNDATION  
AND THEY MADE US ALL PROUD



IIT-JEE (Adv.) 2017 10 in Top 25 AIR

AIR - 8



ONKAR M. DESHPANDE  
Distance

AIR - 9



RACHIT BANSAL  
Classroom

AIR - 10



LAKSHAY SHARMA  
Classroom

AIR - 12



YATEESH AGRAWAL  
Classroom

AIR - 15



AMAN KANSAL  
Classroom

AIR - 16



YASH KHEMCHANDANI  
Classroom

AIR - 19



DEVANSH GARG  
Classroom

AIR - 21



ARPIT AGGARWAL  
Classroom

AIR - 23



ABHAY GOYAL  
Classroom

AIR - 24



TUSHAR GAUTAM  
Classroom

AIIMS 2017 34 in Top 50 AIR



NISHITA PUROHIT  
Classroom

AIR  
1

PERFECT 10 in AIIMS 2017

First Time in the History all  
Top 10 ALL INDIA RANKS Secured  
by Students of a Single Institute



ARCHIT GUPTA  
Classroom



TAMOGHNA GHOSH  
Distance



NIPUN CHANDRA  
Distance

AIR - 5



HARSH AGARWAL  
Classroom

AIR - 6



RISHAV RAJ  
Classroom

AIR - 7



HARSHIT ANAND  
Classroom

AIR - 8



RINKU SARMAH  
Distance

AIR - 9



ABHISHEK DOGRA  
Classroom

AIR - 10



MANISH MULCHANDANI  
Classroom

NEET (UG) 2017

6 in Top 10 AIR | 54 in Top 100 NEET Over All Rank

AIR - 2



ARCHIT GUPTA  
Classroom

AIR - 3



MANISH MULCHANDANI  
Classroom

AIR - 5



ABHISHEK DOGRA  
Classroom

AIR - 7



KANISHH TAYAL  
Classroom

AIR - 9



ARYAN RAJ SINGH  
Distance

AIR - 10



TANISH BANSAL  
Classroom

25 ALLEN Students Secured All India Ranks in  
TOP 100 CLUB

Highest From any Institute of Kota

ALLEN Classroom Students in All India Top 3 Ranks (Since Year 2010)

AIR - 1



NISHITA PUROHIT  
AIIMS 2017

AIR - 1



AMAN BANSAL  
IIT-JEE 2016

AIR - 1



HET S. SHAH  
NEET 2016

AIR - 1



CHITRAANG MURDIA  
IIT-JEE 2014

AIR - 1



TEJASWIN JHA  
AIPMT 2014

AIR - 1



AYUSH GOEL  
NEET 2013

AIR - 1



LOKESH AGARWAL  
AIPMT 2010

1<sup>st</sup> RANK CLUB

AIR - 2



ARCHIT GUPTA  
NEET 2017  
AIPMT 2017

AIR - 2



BHAVESH DHINGRA  
IIT-JEE 2016

AIR - 2



EKANSH GOYAL  
NEET 2016

AIR - 2



NIKHL BAJIJA  
AIIMS 2016

AIR - 2



KHUSHI TIWARI  
AIPMT 2015

AIR - 2



PARTH SHARMA  
AIIMS 2015

AIR - 2



YASH MITTAL  
AIIMS 2013

NTSE 2017

297

Selections  
in  
Stage-II

KVPY 2017

551

Students  
Selected for  
KVPY 2017  
Stage-II

INO 2017

457

Students  
Selected for  
Indian National  
Olympiad

103  
768  
Ranks in All India  
Top 10 Secured  
by ALLEN Students  
Ranks in All India  
Top 100 Secured  
by ALLEN Students  
SINCE YEAR 2010  
( IIT-JEE, AIEEE, AIPMT/NEET & AIIMS )

Highest Selections in  
NTSE, KVPY & INO 2017  
among All Institute of Kota

2<sup>nd</sup> RANK CLUB

AIR - 3



MANISH  
MULTCHANDANI  
NEET-2017

AIR - 3



KUNAL GOYAL  
IIT-JEE 2016

AIR - 3



NIKHL BAJIJA  
NEET 2016

AIR - 3



LAJABEN PATEL  
AIIMS 2016

AIR - 3



GOVIND LAHOTI  
IIT-JEE 2014

AIR - 3



ROHIT NATHANI  
AIIMS 2014

Authenticity of Result : Power of ALLEN

## ADMISSION OPEN (Session 2018-19)

IIT-JEE (Advanced) & JEE (Main) | NEET (UG) / AIIMS | Class 6<sup>th</sup> to 10<sup>th</sup> (NTSE, Olympiads & Boards)

For course starting dates & ALLEN Scholarship cum Admission Test (ASAT)  
details visit our website/nearest center or call 0744-2757575

To Apply online (₹500) Log on to [www.allen.ac.in](http://www.allen.ac.in) or walk-in to any nearest ALLEN Center for Application Form (₹700).



Corporate Office:  
"SANKALP", CP-6, Indra Vihar, Kota (Raj.), India, 324005

Tel: 0744-2757575 | Email: [info@allen.ac.in](mailto:info@allen.ac.in) | website: [www.allen.ac.in](http://www.allen.ac.in)

Scholarship upto  
**90%**

ALL INDIA MAJOR TEST SERIES  
Target 2018 : JEE (Main+Advanced),  
JEE (Main) & NEET (UG/AIIMS)

Log on to  
[dip.allen.ac.in](http://dip.allen.ac.in)



# JOSHI MATHS CLASSES

[11th & 12th Math]  
(Board, CET, JEE Main + Advance)

Plot No. 78, Udyog Bhavan, Behind Latur Urban bank, Shivaji Nagar, Latur.  
Ph - 02382-256547, 9422469519, Email - sudhakarjoshi9@gmail.com

- ISO 9001/2008 Certified
- Teaching Experience 23 years since 1995-1996
- Regular Parents Meeting
- SMS Facility for Absent & Marks
- Weekly Tests for Board, CET & JEE.
- Proper Teaching to Clear Understanding.
- Personalized Counselling for Weak Students
- Library & Reading Room Facility Available
- Virtual Classroom
- Selected Batch for Topper Students.

## Toppers of our class



श्री अमृत  
3856852377  
amritsudhakarjoshi9@gmail.com



joshimathsclasses



www.joshimathclasses.com



Joshi Maths Classes



**AMRITA**  
VISHWA VIDYAPEETHAM

Knowledge enriched  
by human values...

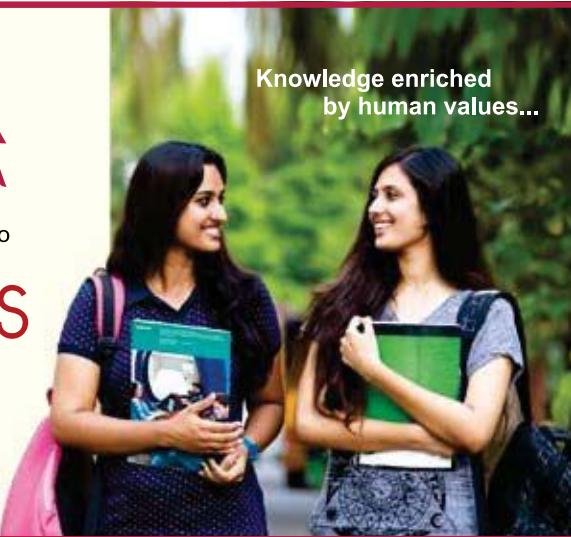
Applications are invited from eligible candidates for admission to

# B.Tech. Programmes

for the Academic Year 2018-19

offered at **Amritapuri, Bengaluru & Coimbatore** campuses

Selection is based on the performance in  
Amrita Entrance Examination - Engineering (AEEE) 2018



**Last Date for Receiving Application: 31 March, 2018**

**Success  
is here;  
Make  
it yours**

Life skills and industry-oriented technical training increases your chance of success in campus placements



200+ reputed companies visit Amrita for placement



Multiple Placement Opportunities in top-notch National & Multinational companies



Internship with stipend upto ₹1.0 Lakh p.m.



Highest Salary ₹25.16 Lakhs p.a.

**OUR RECRUITERS** ABB, Amazon, AstraZeneca, Blue Star, Bosch, Caterpillar, Cerner, Cisco, Cognizant, Dell, Directi, Ericsson, E&Y, FireEye, Ford, GE Digital, Google, Honeywell, Hyundai, IBM, Infosys, Intuit, Juniper, KLA Tencor, Larsen & Toubro, Mahindra & Mahindra, MicroFocus, Microsoft, National Instruments, Nationstar Mortgage, Oracle, Philips, Quess Corp, Reliance Power, Renault Nissan, Roadrunnr, Samsung, SAP, Seagate, TCS, Thermo Fisher, Toyota, United Technologies, Verizon, Wipro, Xome, Zoho to name a few....

## International Academic Relations

Student exchange programmes and dual degree programmes with several prestigious universities in Europe, Australia, East Asia, South America and North America



University of New Mexico (USA)



University at Buffalo, SUNY (USA)



VU Amsterdam (Netherlands)



TELECOM School of Management (France)



University of L'Aquila (Italy)



University of Louisiana, Lafayette (USA)



Tuition fee waiver ranging from 90% to 25% based on the rank scored in AEEE 2018



Entrepreneurship development opportunity through Amrita Technology Business Incubator (TBI) & Amrita Centre for Entrepreneurship (ACE)



AMRITA is RANKED 168 in Asia

#1 among Private Universities in India  
[www.topuniversities.com/university-rankings/asian-university-rankings/2018](http://www.topuniversities.com/university-rankings/asian-university-rankings/2018)

#9

RANKED 9  
in University category by  
NIRF, MHRD, Govt. of India  
[www.nirfindia.org/UniversityRanking.html](http://www.nirfindia.org/UniversityRanking.html)

#1

SWACHHTA RANKING 2017  
Ranked 'First' among the cleanest Higher Education institutions in the country in the category of Technical Institutions



How to apply ?

SCAN  
QR  
CODE



[www.amrita.edu/btech2018](http://www.amrita.edu/btech2018)

For  
Queries

Amritapuri Campus

0476 2809 400 / 402 / 405

[admissions@am.amrita.edu](mailto:admissions@am.amrita.edu)

Bengaluru Campus

080 2518 3700

[admissions@blr.amrita.edu](mailto:admissions@blr.amrita.edu)

Coimbatore Campus

0422 2685 169 / 170

[admissions@amrita.edu](mailto:admissions@amrita.edu)

Amrita Vishwa Vidyapeetham, Amrita Nagar P.O., Ettimadai, Coimbatore - 641 112. Tamil Nadu | Accredited by NAAC with 'A' grade

**RESULT 2017**

**FELICITATION**



### Achievements

Academic Year		2014	2015	2016	2017
Selections	Medical	80	98	115	139
	Engineering	60	76	94	109

### COURSES OFFERED

#### Two Years Integrated Programme

- ◆ NEET (Medical Entrance Exam.)
- ◆ IIT-JEE (Engineering Entrance Exam.)

#### One Year Integrated Programme

- ◆ NEET Repeater Batch

#### Test Series NEET/JEE (Online /Offline)

- ◆ Special Online Distance Learning / Test Series Programme.
- ◆ Tab with study material available at Lalit Tutorials.
- ◆ For Free Test Series - Download Lalit Eduapp from Google Play Store
- ◆ Well Equiped Digital Class Room
- ◆ Well Furnished Library & Laboratory



**The Best Institute in Maharashtra for Medical & Engineering Entrance Exams ...**

Kalpande Sir's



Toshniwal Lay-out, Akola (Maharashtra) Phone : 0724-2456 654, 7720 857 857

#### Opportunity For Faculties

Faculties	Min. Exp.	Min. Package / Annum
Senior	6 yr.	12 Lac
Junior	3 yr.	6 Lac

- ★ Special Incentives & Increments According to Performance
- ★ Special Reward to Faculty on Producing Best Ranks.

**For Recruitment of Senior & Junior Faculties  
send your Resume to [hrltmaharashtra@gmail.com](mailto:hrltmaharashtra@gmail.com)**



**DESIGN YOUR OWN DEGREE  
CHOOSE YOUR OWN CURRICULUM  
DETERMINE YOUR OWN CAREER**

Introducing Inter Disciplinary Experiential Active Learning (IDEAL)

**Take the SRMJEEE (B.Tech) 2018.**

Online Examination: Apr 16 - 30, 2018

## GLOBAL RECOGNITION

SRM Institute of Science and Technology is India's only multidisciplinary university with a 4-star\* QS world rating.



QS also awarded SRM Institute of Science and Technology

- 5 stars\* for Teaching, Employability and Inclusiveness

## NATIONAL RECOGNITION

- NAAC "A" grade
- MHRD "A" category
- Ranked as best engineering institution by NIRF

## WHY SRM?

75000  
Students at  
SRM Group Institutions  
from 53 countries

120  
Crores of research funding  
from government departments  
like DST, DBT etc.

105  
MoUs with  
20 countries

9500+  
Publications and  
117 patents filed

## TO APPLY

Visit: [applications.srmuniv.ac.in](http://applications.srmuniv.ac.in)  
Make e-payment for ₹1100.  
For eligibility and fee details, visit [www.srmuniv.ac.in](http://www.srmuniv.ac.in)

## FOR QUERIES

Call: +91 44 2745 5510 / +91 44 4743 7500  
Email: [admissions.india@srmuniv.ac.in](mailto:admissions.india@srmuniv.ac.in)



/SRMUniversityOfficial



/SRM\_Univ

# MATHEMATICS today



Vol. XXXVI

No. 2

February 2018

**Corporate Office:**

Plot 99, Sector 44 Institutional Area,  
Gurgaon -122 003 (HR), Tel : 0124-6601200  
e-mail : info@mtg.in website : www.mtg.in

**Regd. Office:**

406, Taj Apartment, Near Safdarjung Hospital,  
Ring Road, New Delhi - 110029.  
Managing Editor : Mahabir Singh  
Editor : Anil Ahlawat

## CONTENTS

Competition Edge  
Class XI  
Class XII

- 8 Maths Musing Problem Set - 182
- 10 Key Concepts on Integrals and its Applications
- 31 Practice Paper - JEE Main
- 38 Mock Test Paper - JEE Main 2018 (Series 8)
- 46 You Ask We Answer
- 51 Math Archives
- 53 Target JEE
- 67 Challenging Problems
- 70 Maths Musing Solutions
- 44 Concept Map
- 72 MPP-10
- 45 Concept Map
- 74 Ace Your Way Practice Paper
- 86 MPP-10

www.mtg.in | Feb 2018 | Pages 96 | ₹ 3/-

KEY CONCEPTS 10  
on Integrals and its Applications

TARGET JEE 53  
ACE YOUR WAY 74  
PRACTICE PAPER 2018  
CBSE Class XII

# MATHEMATICS today

India's #1 MATHEMATICS MONTHLY for JEE (Main & Advanced)

**MATHS MUSING** 8  
10 GREAT PROBLEMS

PRACTICE PAPER 31  
JEE MAIN

MATH ARCHIVES 51

CHALLENGING PROBLEMS 67

MONTHLY PRACTICE PROBLEMS 86  
72 (XI & XII)

CONCEPT MAP 45  
44 (XI & XII)

You Ask ?  
We Answer ✓

mtg  
Trust of more than 3 Crore Readers Since 1982  
2018100009775

Subscribe online at [www.mtg.in](http://www.mtg.in)

### Individual Subscription Rates

	1 yr.	2 yrs.	3 yrs.
Mathematics Today	330	600	775
Chemistry Today	330	600	775
Physics For You	330	600	775
Biology Today	330	600	775

### Combined Subscription Rates

	1 yr.	2 yrs.	3 yrs.
PCM	900	1500	1900
PCB	900	1500	1900
PCMB	1000	1800	2300

Send D.D/M.O in favour of MTG Learning Media (P) Ltd.

Payments should be made directly to : MTG Learning Media (P) Ltd,  
Plot 99, Sector 44 Institutional Area, Gurgaon - 122 003, Haryana.

We have not appointed any subscription agent.

Printed and Published by Mahabir Singh on behalf of MTG Learning Media Pvt. Ltd. Printed at HT Media Ltd., B-2, Sector-63, Noida, UP-201307 and published at 406, Taj Apartment, Ring Road, Near Safdarjung Hospital, New Delhi - 110029.

Editor : Anil Ahlawat

Readers are advised to make appropriate thorough enquiries before acting upon any advertisements published in this magazine. Focus/Infocus features are marketing incentives. MTG does not vouch or subscribe to the claims and representations made by advertisers. All disputes are subject to Delhi jurisdiction only.

Copyright© MTG Learning Media (P) Ltd.

All rights reserved. Reproduction in any form is prohibited.

# MATHS MUSING

**M**aths Musing was started in January 2003 issue of Mathematics Today. The aim of Maths Musing is to augment the chances of bright students seeking admission into IITs with additional study material.

During the last 10 years there have been several changes in JEE pattern. To suit these changes Maths Musing also adopted the new pattern by changing the style of problems. Some of the Maths Musing problems have been adapted in JEE benefitting thousand of our readers. It is heartening that we receive solutions of Maths Musing problems from all over India.

Maths Musing has been receiving tremendous response from candidates preparing for JEE and teachers coaching them. We do hope that students will continue to use Maths Musing to boost up their ranks in JEE Main and Advanced.

## PROBLEM Set 182

### JEE MAIN

- If  $z_n = cis \frac{\pi}{n(n+1)(n+2)}$  for  $n = 1, 2, 3, \dots$  and principle argument of  $\lim_{n \rightarrow \infty} (z_1 z_2 z_3 \dots z_n)$  is  $\theta$ . Then value of  $\theta$  is  
(a)  $\pi/2$  (b)  $\pi/3$  (c)  $\pi/6$  (d)  $\pi/4$
- The coefficient of  $x^n$  in the expansion of  $\left(1 - 3x + 6x^2 - 10x^3 + \dots + (-1)^r \frac{(r+1)(r+2)\dots(r+(r-1))x^r}{r!} + \dots\right)^{-n}$  is  
(a)  $\frac{3n!}{(n!)^2}$  (b)  $\frac{3n!}{n! 2n!}$  (c)  $\frac{3n!}{(2n!)^2}$  (d) None of these
- At the point  $P(a, a^n)$  on the graph of  $y = x^n$  ( $n \in N$ ) in the first quadrant, a normal is drawn. The normal intersects the  $y$ -axis at the point  $(0, b)$ . If  $\lim_{a \rightarrow 0} b = \frac{1}{2}$ , then  $n$  equals  
(a) 1 (b) 3 (c) 2 (d) 4
- In triangle  $ABC$ ,  $\sum_{r=0}^n \binom{n}{r} a^{n-r} b^r \cos(rA - (n-r)B) =$   
(a)  $(a-b)^n$  (b)  $(a+b)^n$  (c)  $c^n$  (d)  $c^{2n}$
- The value of  $\int_0^1 \frac{8 \log(1+x)}{1+x^2} dx$  is  
(a)  $\frac{\pi}{2} \log 2$  (b)  $\log 2$  (c)  $\pi \log 2$  (d)  $\frac{\pi}{8} \log 2$

### JEE ADVANCED

- $2 \cot^2 \theta + 2\sqrt{3} \cot \theta + 4 \operatorname{cosec} \theta + 8 = 0$  if  $\theta =$   
(a)  $n\pi + \frac{\pi}{6}$  (b)  $n\pi - \frac{\pi}{6}$  (c)  $2n\pi + \frac{\pi}{6}$  (d)  $2n\pi - \frac{\pi}{6}$

### COMPREHENSION

Let  $\alpha, \beta$  be the roots of the equation  $ax^2 + bx + c = 0$ . Also, let  $S_n = \alpha^n + \beta^n$  for  $n \geq 1$  and

$$\Delta = \begin{vmatrix} 3 & 1+S_1 & 1+S_2 \\ 1+S_1 & 1+S_2 & 1+S_3 \\ 1+S_2 & 1+S_3 & 1+S_4 \end{vmatrix}$$

- If  $\Delta < 0$  then the equation  $ax^2 + bx + c = 0$  has  
(a) positive real roots (b) negative real roots  
(c) equal roots (d) imaginary roots
- If  $a, b, c$  are rational and one of the roots of the equation is  $1 + \sqrt{2}$ , then the value of  $\Delta$  is  
(a) 8 (b) 12 (c) 30 (d) 32

### INTEGER TYPE

- Perpendiculars are drawn from the angles  $A, B, C$  of an acute-angled triangle on opposite sides and produced to meet the circumscribing circle. If these produced parts are  $\alpha, \beta, \gamma$  respectively, then the value of  $\frac{(a/\alpha)+(b/\beta)+(c/\gamma)}{\tan A + \tan B + \tan C}$  is

### MATRIX MATCH

- Match the following.

List-I	List-II
P. Number of ordered pairs which satisfy the equation $x^2 + 2x \sin(xy) + 1 = 0$ (where $y \in [0, 2\pi]$ ) is	1. 1
Q. If maximum $\{5\sin\theta + 3\sin(\theta - \alpha)\} = 7$ ( $\theta \in R$ ) then number of solutions of $\alpha$ in $[0, \pi]$ are	2. 2
R. The total number of solutions of $\cos x = \sqrt{1 - \sin 2x}$ in $[0, 2\pi]$ is equal to	3. 0
S. The number of solutions of the equation $\sin\left(\frac{1}{3}\cos^{-1} x\right) = 1$	4. 3

P	Q	R	S
(a) 1	2	3	4
(b) 2	1	4	3
(c) 4	3	2	1
(d) 3	4	1	2

See Solution Set of Maths Musing 181 on page no 70

# **KNOWLEDGE SERIES**

## **(for JEE / Olympiad Aspirants)**

### **Myth**

After an attempt, if not able to solve question, check solution to save time.

### **Reality**

Attempt question multiple times, it helps to improve your imagination.  
Suggestion - Do not check complete solution, only use hints.

Please visit youtube to watch more myth and reality discussions on KCS EDUCATE channel.

**Check your concepts & win prizes.**

### **Knowledge Quiz - 3**

Prove that, if  $m$  ends with the digit five,  
then  $1991 \mid 12^m + 9^m + 8^m + 6^m$

Please send your detailed solution before 15<sup>th</sup> February 2018 to [quiz@kcse educate.in](mailto:quiz@kcse educate.in)  
along with your name, father's name, class, school and contact details.

#### **Winner Knowledge Quiz - 2**

- Manish Ranjan Rajbongshi, (Class - IX), Delhi Public School, Assam.
- Ajit Kumar Sharma, (Class - XII), ECR Inter College School, Mughalsarai, Uttar Pradesh.
- Sahil Jain, (Class - XII), S.R.Public Sr. Sec. School, Kota, Rajasthan.

### **Stars of KCS-2017**



Successful Students With Chief Guest Dr. M.K. Verma (Vice - Chancellor, CCSVTU, C.G.)

**KCS**  
**Educate**  
...revives internal teacher

Knowledge Centre for Success Educate Pvt. Ltd.

**Team Avnish**  
for JEE | Aptitude Test  
NTSE | KVPY | Olympiad

#### **Contact us**

157, New Civic Centre, Bhilai, Dist. Durg (C.G.)

Telephone : **0788-6454505**

[www.kcse educate.in](http://www.kcse educate.in)

[info@kcse educate.in](mailto:info@kcse educate.in)

[facebook.com/kcse educate](https://facebook.com/kcse educate)

CIN No. U74140DL2011PTC227887



# KEY CONCEPTS

*on*

## Integrals and its Applications

\*ALOK KUMAR, B.Tech, IIT Kanpur

### INDEFINITE INTEGRALS

If  $f$  and  $g$  are functions of  $x$  such that  $g'(x) = f(x)$  then,

$$\int f(x)dx = g(x) + c \text{ or } \frac{d}{dx}\{g(x) + c\} = f(x),$$

where  $c$  is called the constant of integration.

#### Some Standard Formulae

- $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c, n \neq -1$
- $\int \frac{dx}{ax+b} = \frac{1}{a} \ln(ax+b) + c$
- $\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + c$
- $\int a^{px+q} dx = \frac{1}{p} \frac{a^{px+q}}{\ln a} + c; a > 0$
- $\int \sin(ax+b) dx = -\frac{1}{a} \cos(ax+b) + c$
- $\int \cos(ax+b) dx = \frac{1}{a} \sin(ax+b) + c$
- $\int \tan(ax+b) dx = \frac{1}{a} \ln \sec(ax+b) + c$
- $\int \cot(ax+b) dx = \frac{1}{a} \ln \sin(ax+b) + c$
- $\int \sec^2(ax+b) dx = \frac{1}{a} \tan(ax+b) + c$
- $\int \operatorname{cosec}^2(ax+b) dx = -\frac{1}{a} \cot(ax+b) + c$
- $\int \sec(ax+b) \cdot \tan(ax+b) dx = \frac{1}{a} \sec(ax+b) + c$
- $$\int \operatorname{cosec}(ax+b) \cdot \cot(ax+b) dx \\ = -\frac{1}{a} \operatorname{cosec}(ax+b) + c$$

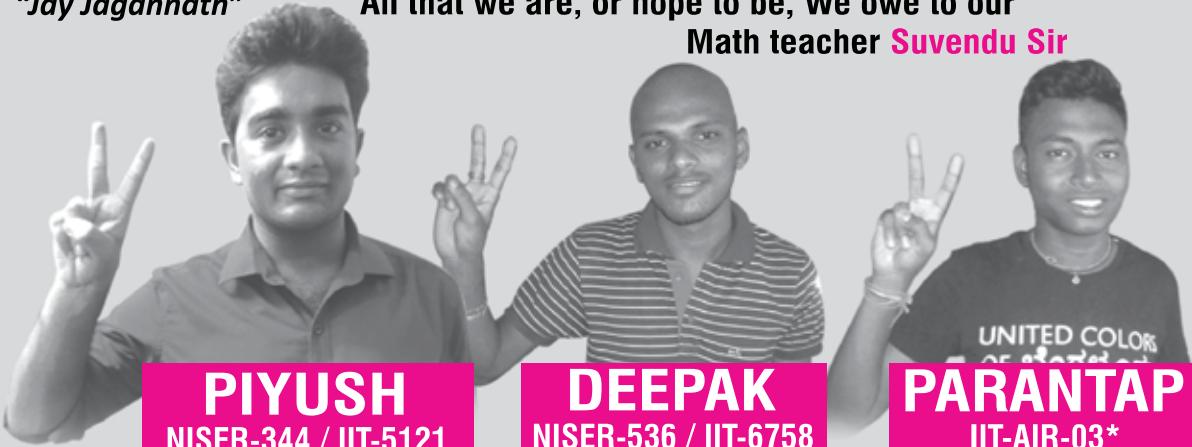
- $\int \sec x dx = \ln(\sec x + \tan x) + c$
- $\int \operatorname{cosec} x dx = \ln(\operatorname{cosec} x - \cot x) + c$
- $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + c$
- $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$
- $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{x}{a} + c$
- $\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln \left[ x + \sqrt{x^2 + a^2} \right] + c$
- $\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln \left[ x + \sqrt{x^2 - a^2} \right] + c$
- $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + c$
- $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + c$
- $\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + c$
- $$\int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \\ \ln \left( x + \sqrt{x^2 + a^2} \right) + c$$
- $$\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \\ \ln \left( x + \sqrt{x^2 - a^2} \right) + c$$
- $$\int e^{ax} \cdot \sin bx dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) + c$$

---

\* Alok Kumar is a winner of INDIAN NATIONAL MATHEMATICS OLYMPIAD (INMO-91).  
He trains IIT and Olympiad aspirants.

*"Jay Jagannath"*

All that we are, or hope to be, We owe to our  
Math teacher **Suvendu Sir**



**PIYUSH**

NISER-344 / IIT-5121

**DEEPAK**

NISER-536 / IIT-6758

**PARANTAP**

IIT-AIR-03\*



PIYUSH

98%



SOMALI

96%



RISHAB

95%



SWAPNA

95%



R.S. SUBHAM

95%



SAYOK

95%



RISHI

95%



DIPSA

95%



SWETA

95%



RICHA

95%



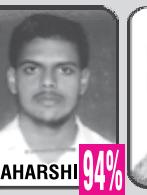
PRATIK

95%



DEBADUTTA

95%



MAHARSHI

94%



BIKASH

94%



ANUSKA

92%



MOHAN

95%

7 Selected in IITs & 56 Qualified in JEE (Main)

## Screening Test for 2 yr. regular JEE (M & A) Batch 2018

1 yr JEE (M & A) Class Starts on 2018

# S.S. GUIDELINES ■ ■ ■ MATHEMATICS

CBSE, CHSE, JEE (M & A), NISER, MCA, NDA

OMKARA Classes, A.D. Market, Cuttack / Pithapur, College Square, Cuttack  
Ph : 0671-2334577, Mob : 09861233591, 09439353862

- $\int e^{ax} \cdot \cos bx dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx) + c$
- $\int c f(x) dx = c \int f(x) dx$
- $\int (f(x) \pm g(x)) dx = \int f(x) dx \pm \int g(x) dx$
- $\int f(x) dx = g(x) + c \Rightarrow \int f(ax + b) dx = \frac{g(ax + b)}{a} + c$
- $\int [f(x)]^n f'(x) dx = \frac{(f(x))^{n+1}}{n+1} + c, n \neq -1$
- $\int \frac{f'(x)}{[f(x)]^n} dx = \begin{cases} \log |f(x)| + c, & \text{if } n = 1 \\ \int t^{-n} dt, & \text{where } t = f(x) \end{cases}$

### Integration by Parts

- $\int_I f(x) g(x) dx = f(x) \int_{II} g(x) dx - \int \left( \frac{d}{dx} (f(x)) \int (g(x)) dx \right) dx$

### Some Particular Substitutions

- $\int \frac{dx}{ax^2 + bx + c}, \int \frac{dx}{\sqrt{ax^2 + bx + c}}, \int \sqrt{ax^2 + bx + c} dx$ , express  $ax^2 + bx + c$  in the form of perfect square and then apply the standard results.
- $\int \frac{px + q}{ax^2 + bx + c} dx, \int \frac{px + q}{\sqrt{ax^2 + bx + c}} dx, \int (px + q) \sqrt{ax^2 + bx + c} dx$ , express  $px + q = A$  (differential coefficient of denominator) +  $B$ .
- $\int \frac{a \cdot \cos x + b \cdot \sin x + c}{l \cdot \cos x + m \cdot \sin x + n} dx$   
Express Num.  $\equiv A(\text{Den.}) + B \frac{d}{dx} (\text{Den.}) + C$  and proceed.

- $\int \frac{x^2 \pm 1}{x^4 + Kx^2 + 1} dx$  where 'K' is any constant.  
Divide Num. and Den. by  $x^2$  and put  $x \mp \frac{1}{x} = t$ .
- $\int \frac{dx}{(ax^2 + bx + c) \sqrt{px + q}}$ ; put  $px + q = t^2$
- $\int \frac{dx}{(ax + b) \sqrt{px^2 + qx + r}}$ ; put  $ax + b = \frac{1}{t}$

- $\int \frac{dx}{(ax^2 + b) \sqrt{px^2 + q}}$ ; put  $x = \frac{1}{t}$
- $\int \frac{\sqrt{x - \alpha}}{\beta - x} dx$  or  $\int \sqrt{(x - \alpha)(\beta - x)}$ ; put  $x = \alpha \cos^2 \theta + \beta \sin^2 \theta$
- $\int \frac{dx}{\sqrt{(x - \alpha)(x - \beta)}}$ ; put  $x - \alpha = t^2$  or  $x - \beta = t^2$
- Integrals of the form  $\int \tan^n x dx, \int \cot^n x dx, \int \sec^n x dx, \int \cosec^n x dx$  is solved as :  
Let  $I_n = \int \tan^n x dx = \int \tan^2 x \tan^{n-2} x dx$   
 $= \int (\sec^2 x - 1) \tan^{n-2} x dx$  for  
 $\Rightarrow I_n = \int \sec^2 x \tan^{n-2} x dx - I_{n-2}$   
 $\Rightarrow I_n = \frac{\tan^{n-1} x}{n-1} - I_{n-2}$

Similarly, we can obtain integral for other trigonometric functions.

### DEFINITE INTEGRALS & ITS APPLICATIONS

Let  $f(x)$  be a continuous function defined on  $[a, b]$ ,  
 $\int_a^b f(x) dx = F(x) + c$ . Then  $\int_a^b f(x) dx = F(b) - F(a)$   
is called definite integral. This formula is known as Newton-Leibnitz formula.

### Properties of Definite Integral

- $\int_a^b f(x) dx = \int_a^b f(t) dt = - \int_b^a f(x) dx$
- $\int_a^a f(x) dx = \int_a^a f(x) dx + \int_c^b f(x) dx$ , where  $c \in (a, b)$
- $\int_{-a}^a f(x) dx = \int_0^a (f(x) + f(-x)) dx$   
 $= \begin{cases} 2 \int_0^a f(x) dx, & \text{if } f(-x) = f(x) \text{ i.e. } f(x) \text{ is even} \\ 0 & , \text{if } f(-x) = -f(x) \text{ i.e. } f(x) \text{ is odd} \end{cases}$
- $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$   
Further  $\int_0^a f(x) dx = \int_0^a f(a-x) dx$

- $$\begin{aligned} \int_0^{2a} f(x) dx &= \int_0^a (f(x) + f(2a-x)) dx \\ &= \begin{cases} 2 \int_0^a f(x) dx, & \text{if } f(2a-x) = f(x) \\ 0, & \text{if } f(2a-x) = -f(x) \end{cases} \end{aligned}$$

If  $f(x)$  is a periodic function with period  $T$ , then

- $$\int_0^{nT} f(x) dx = n \int_0^T f(x) dx, n \in \mathbb{Z}$$
- $$\int_{a+nT}^{a+2nT} f(x) dx = n \int_0^T f(x) dx, n \in \mathbb{Z}, a \in \mathbb{R}$$
- $$\int_{mT}^{mT+(n-m)T} f(x) dx = (n-m) \int_0^T f(x) dx, m, n \in \mathbb{Z}$$
- $$\int_{a+nT}^{b+nT} f(x) dx = \int_a^b f(x) dx, n \in \mathbb{Z}, a, b \in \mathbb{R}$$
- $$\text{If } \psi(x) \leq f(x) \leq \phi(x) \text{ for } a \leq x \leq b, \text{ then}$$

$$\int_a^b \psi(x) dx \leq \int_a^b f(x) dx \leq \int_a^b \phi(x) dx$$
- $$\left| \int_a^b f(x) dx \right| \leq \int_a^b |f(x)| dx$$

### Leibnitz Theorem

If  $F(x) = \int_{g(x)}^{h(x)} f(t) dt,$

then  $\frac{dF(x)}{dx} = h'(x) f(h(x)) - g'(x) f(g(x))$

### Walli's formula

- $$\int_0^{\frac{\pi}{2}} \sin^m x \cos^n x dx =$$

$$\begin{cases} \frac{[(m-1)(m-3)\dots 5 \cdot 3 \cdot 1][(n-1)(n-3)(n-5)\dots 5 \cdot 3 \cdot 1]}{(m+n)(m+n-2)\dots 6 \cdot 4 \cdot 2} \times \frac{\pi}{2} & m, n \text{ both are even positive integers} \\ \frac{[(m-1)(m-3)\dots 5 \cdot 3 \cdot 1][(n-1)(n-3)\dots 6 \cdot 4 \cdot 2]}{(m+n)(m+n-2)\dots 5 \cdot 3 \cdot 1} & \text{if } m \text{ is even \& } n \text{ is odd positive integer} \\ \frac{[(m-1)(m-3)\dots 6 \cdot 4 \cdot 2][(n-1)(n-3)\dots 5 \cdot 3 \cdot 1]}{(m+n)(m+n-2)\dots 5 \cdot 3 \cdot 1} & \text{if } m \text{ is odd and } n \text{ is even positive integer.} \end{cases}$$

where  $m, n$  is positive integers.

**Note :** (i) If all the powers of 'x' and 'y' in the equation are even, the curve is symmetrical about the axis of 'x' as well as 'y' such as  $x^2 + y^2 = a^2$ .

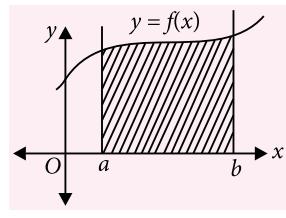
(ii) If the equation of the curve remains unchanged on interchanging 'x' and 'y', then the curve is symmetrical about the line  $y = x$  such as  $x^3 + y^3 = 3axy$

(iii) If all the powers of 'x' in the equation are even then the curve is symmetrical about the  $y$ -axis.

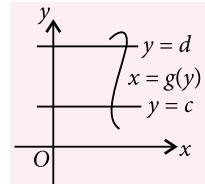
(iv) If all the powers of 'y' in the equation are even then the curve is symmetrical about the  $x$ -axis.

### Area Under the Curve

- If  $f(x) \geq 0$  for  $x \in [a, b]$ , then area bounded by curve  $y = f(x)$  and  $x$ -axis between  $x = a$  and  $x = b$  is  $\int_a^b f(x) dx$ .

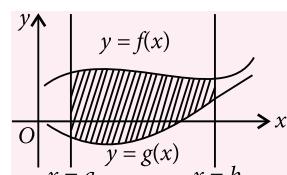


- If  $g(y) \geq 0$  for  $y \in [c, d]$  then area bounded by curve  $x = g(y)$  and  $y$ -axis between abscissa  $y = c$  and  $y = d$  is  $\int_c^d g(y) dy$

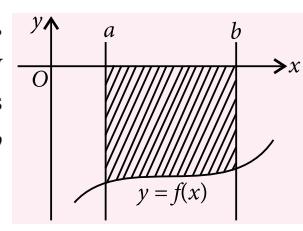


- If  $f(x) > 0$  for  $x \in [a, c]$  and  $f(x) \leq 0$  for  $x \in [c, b]$  ( $a < c < b$ ) then area bounded by curve  $y = f(x)$  and  $x$ -axis between  $x = a$  and  $x = b$  is  $\int_a^c f(x) dx - \int_c^b f(x) dx$

- If  $f(x) \geq g(x)$  for  $x \in [a, b]$  then area bounded by curves  $y = f(x)$  and  $y = g(x)$  between  $x = a$  and  $x = b$  is  $\int_a^b (f(x) - g(x)) dx$



- If  $f(x) \leq 0$  for  $x \in [a, b]$ , then area bounded by curve  $y = f(x)$  and  $x$ -axis between  $x = a$  and  $x = b$  is  $-\int_a^b f(x) dx$ .



## PROBLEMS

### Single Correct Answer Type

**1.**  $\int x^{51}(\tan^{-1}x + \cot^{-1}x)dx =$

(a)  $\frac{x^{52}}{52}(\tan^{-1}x + \cot^{-1}x) + c$

(b)  $\frac{x^{52}}{52}(\tan^{-1}x - \cot^{-1}x) + c$

(c)  $\frac{\pi x^{52}}{104} + \frac{\pi}{2} + c$       (d)  $\frac{x^{52}}{52} + \frac{\pi}{2} + c$

**2.**  $\int \frac{dx}{\sin x + \cos x} =$

(a)  $\log \tan\left(\frac{\pi}{8} + \frac{x}{2}\right) + c$     (b)  $\log \tan\left(\frac{\pi}{8} - \frac{x}{2}\right) + c$

(c)  $\frac{1}{\sqrt{2}} \log \tan\left(\frac{\pi}{8} + \frac{x}{2}\right) + c$     (d) None of these

**3.**  $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx =$

(a)  $\tan x + \cot x + c$     (b)  $\tan x - \cot x + c$   
 (c)  $\operatorname{cosec} x - \cot x + c$     (d)  $\sec x - \operatorname{cosec} x + c$

**4.**  $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx =$

(a)  $2[\sin x + x \cos \alpha] + c$     (b)  $2[\sin x + \sin \alpha] + c$   
 (c)  $2[-\sin x + x \cos \alpha] + c$     (d)  $-2[\sin x + \sin \alpha] + c$

**5.**  $\int \frac{x^4 + x^2 + 1}{x^2 - x + 1} dx =$

(a)  $\frac{1}{3}x^3 + \frac{1}{2}x^2 + x + c$     (b)  $\frac{1}{3}x^3 - \frac{1}{2}x^2 + x + c$

(c)  $\frac{1}{3}x^3 + \frac{1}{2}x^2 - x + c$     (d) None of these

**6.**  $\int \frac{1}{\sqrt{1 + \cos x}} dx =$

(a)  $\sqrt{2} \log\left(\sec \frac{x}{2} + \tan \frac{x}{2}\right) + c$

(b)  $\frac{1}{\sqrt{2}} \log\left(\sec \frac{x}{2} + \tan \frac{x}{2}\right) + c$

(c)  $\log\left(\sec \frac{x}{2} + \tan \frac{x}{2}\right) + c$     (d) None of these

**7.**  $\int \frac{dx}{x + x \log x} =$

(a)  $\log(1 + \log x)$     (b)  $\log \log(1 + \log x)$   
 (c)  $\log x + \log(\log x)$     (d) None of these

**8.**  $\int \sec^p x \tan x dx =$

(a)  $\frac{\sec^{p+1} x}{p+1} + c$     (b)  $\frac{\sec^p x}{p} + c$

(c)  $\frac{\tan^{p+1} x}{p+1} + c$     (d)  $\frac{\tan^p x}{p} + c$

**9.**  $\int \frac{e^x(x+1)}{\cos^2(xe^x)} dx =$

(a)  $\tan(xe^x) + c$     (b)  $\sec(xe^x) \tan(xe^x) + c$   
 (c)  $-\tan(xe^x) + c$     (d) None of these

**10.**  $\int \frac{\sin x \cos x}{a \cos^2 x + b \sin^2 x} dx =$

(a)  $\frac{1}{2(b-a)} \log(a \cos^2 x + b \sin^2 x) + c$

(b)  $\frac{1}{b-a} \log(a \cos^2 x + b \sin^2 x) + c$

(c)  $\frac{1}{2} \log(a \cos^2 x + b \sin^2 x) + c$

(d) None of these

**11.**  $\int \frac{e^{2x} - 1}{e^{2x} + 1} dx =$

(a)  $\frac{e^{2x} - 1}{e^{2x} + 1} + c$     (b)  $\log(e^{2x} + 1) - x + c$

(c)  $\log(e^{2x} + 1) + c$     (d) None of these

**12.**  $\int \frac{1}{\cos^{-1} x \cdot \sqrt{1-x^2}} dx =$

(a)  $\log(\cos^{-1} x) + c$     (b)  $-\log(\cos^{-1} x) + c$

(c)  $-\frac{1}{2(\cos^{-1} x)^2} + c$     (d) None of these

**13.**  $\int \frac{1}{x^2 \sqrt{1+x^2}} dx =$

(a)  $-\frac{\sqrt{1+x^2}}{x} + c$     (b)  $\frac{\sqrt{1+x^2}}{x} + c$

(c)  $-\frac{\sqrt{1-x^2}}{x} + c$     (d)  $-\frac{\sqrt{x^2-1}}{x} + c$

**14.**  $\int \sin^3 x dx$  is equal to

(a)  $\sin^2 x + 1$     (b)  $\sin x^2 + x^2 + 1$

(c)  $\frac{\cos^3 x}{3} - \cos x$     (d)  $\frac{1}{4} \sin^4 x - \frac{3}{4} \sin^2 x$

15.  $\int \frac{dx}{(1+e^x)(1+e^{-x})} =$

- (a)  $\frac{-1}{1+e^x}$  (b)  $\frac{e^x}{1+e^x}$   
 (c)  $\frac{1}{1+e^x}$  (d) None of these

16.  $\int \frac{1+\tan^2 x}{1-\tan^2 x} dx$  equals to

- (a)  $\log\left(\frac{1-\tan x}{1+\tan x}\right) + c$  (b)  $\log\left(\frac{1+\tan x}{1-\tan x}\right) + c$   
 (c)  $\frac{1}{2}\log\left(\frac{1-\tan x}{1+\tan x}\right) + c$  (d)  $\frac{1}{2}\log\left(\frac{1+\tan x}{1-\tan x}\right) + c$

17.  $\int \operatorname{cosec}^4 x dx =$

- (a)  $\cot x + \frac{\cot^3 x}{3} + c$  (b)  $\tan x + \frac{\tan^3 x}{3} + c$   
 (c)  $-\cot x - \frac{\cot^3 x}{3} + c$  (d)  $-\tan x - \frac{\tan^3 x}{3} + c$

18.  $\int \sqrt{\frac{1-x}{1+x}} dx =$

- (a)  $\sin^{-1} x - \frac{1}{2}\sqrt{1-x^2} + c$  (b)  $\sin^{-1} x + \frac{1}{2}\sqrt{1-x^2} + c$   
 (c)  $\sin^{-1} x - \sqrt{1-x^2} + c$  (d)  $\sin^{-1} x + \sqrt{1-x^2} + c$

19.  $\int \tan^{-1} x dx =$

- (a)  $x \tan^{-1} x + \frac{1}{2}\log(1+x^2)$   
 (b)  $x \tan^{-1} x - \frac{1}{2}\log(1+x^2)$   
 (c)  $(x-1)\tan^{-1} x$   
 (d)  $x \tan^{-1} x - \log(1+x^2)$

20.  $\int \left[ \log(\log x) + \frac{1}{(\log x)^2} \right] dx =$

- (a)  $x \log(\log x) + \frac{x}{\log x} + c$   
 (b)  $x \log(\log x) - \frac{x}{\log x} + c$   
 (c)  $x \log(\log x) + \frac{\log x}{x} + c$   
 (d)  $x \log(\log x) - \frac{\log x}{x} + c$

21.  $\int \frac{x+\sin x}{1+\cos x} dx$  is equal to

- (a)  $-x \tan \frac{x}{2} + c$  (b)  $x \tan \frac{x}{2} + c$   
 (c)  $x \tan x + c$  (d)  $\frac{1}{2}x \tan x + c$

22. If  $\int \frac{e^x(1+\sin x)dx}{1+\cos x} = e^x f(x) + c$ , then  $f(x) =$

- (a)  $\sin \frac{x}{2}$  (b)  $\cos \frac{x}{2}$  (c)  $\tan \frac{x}{2}$  (d)  $\log \frac{x}{2}$

23.  $\int \cos \sqrt{x} dx =$

- (a)  $2[\sqrt{x} \sin \sqrt{x} + \cos \sqrt{x}] + c$   
 (b)  $2[\sqrt{x} \sin \sqrt{x} - \cos \sqrt{x}] + c$   
 (c)  $2[\cos \sqrt{x} - \sqrt{x} \sin \sqrt{x}] + c$   
 (d)  $-2[\sqrt{x} \sin \sqrt{x} + \cos \sqrt{x}] + c$

24.  $\int \tan^{-1} \frac{2x}{1-x^2} dx =$

- (a)  $x \tan^{-1} x + c$   
 (b)  $x \tan^{-1} x - \log(1+x^2) + c$   
 (c)  $2x \tan^{-1} x + \log(1+x^2) + c$   
 (d)  $2x \tan^{-1} x - \log(1+x^2) + c$

25.  $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx =$

- (a)  $x - \sqrt{1-x^2} \sin^{-1} x + c$  (b)  $x + \sqrt{1-x^2} \sin^{-1} x + c$   
 (c)  $\sqrt{1-x^2} \sin^{-1} x - x + c$  (d) None of these

26.  $\int \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx =$

- (a)  $\frac{x}{\sqrt{1-x^2}} \sin^{-1} x + \frac{1}{2}\log(1-x^2) + c$   
 (b)  $\frac{x}{\sqrt{1-x^2}} \sin^{-1} x - \frac{1}{2}\log(1-x^2) + c$   
 (c)  $\frac{1}{\sqrt{1-x^2}} \sin^{-1} x - \frac{1}{2}\log(1-x^2) + c$   
 (d)  $\frac{1}{\sqrt{1-x^2}} \sin^{-1} x + \frac{1}{2}\log(1-x^2) + c$

27.  $\int e^{\tan^{-1} x} \left( \frac{1+x+x^2}{1+x^2} \right) dx$  is equal to

- (a)  $x e^{\tan^{-1} x} + c$  (b)  $x^2 e^{\tan^{-1} x} + c$   
 (c)  $\frac{1}{x} e^{\tan^{-1} x} + c$  (d) None of these

**28.**  $I_1 = \int \sin^{-1} x \, dx$  and  $I_2 = \int \sin^{-1} \sqrt{1-x^2} \, dx$  then

- (a)  $I_1 = I_2$  (b)  $I_2 = (\pi/2)I_1$   
 (c)  $I_1 + I_2 = (\pi/2)x + K$  (d)  $I_1 + I_2 = \pi/2$

**29.**  $\int \frac{x^2}{(x^2+2)(x^2+3)} \, dx =$

- (a)  $-\sqrt{2} \tan^{-1} x + \sqrt{3} \tan^{-1} x + c$   
 (b)  $-\sqrt{2} \tan^{-1} \frac{x}{\sqrt{2}} + \sqrt{3} \tan^{-1} \frac{x}{\sqrt{3}} + c$   
 (c)  $\sqrt{2} \tan^{-1} \frac{x}{\sqrt{2}} + \sqrt{3} \tan^{-1} \frac{x}{\sqrt{3}} + c$   
 (d) None of these

**30.**  $\int \sin^3 x \cos^2 x \, dx =$

- (a)  $\frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} + c$  (b)  $\frac{\cos^5 x}{5} + \frac{\cos^3 x}{3} + c$   
 (c)  $\frac{\sin^5 x}{5} - \frac{\sin^3 x}{3} + c$  (d)  $\frac{\sin^5 x}{5} + \frac{\sin^3 x}{3} + c$

**31.**  $\int \frac{\cos x}{(1+\sin x)(2+\sin x)} \, dx =$

- (a)  $\log[(1+\sin x)(2+\sin x)] + c$   
 (b)  $\log \frac{2+\sin x}{1+\sin x} + c$  (c)  $\log \frac{1+\sin x}{2+\sin x} + c$   
 (d) None of these

**32.**  $\int \frac{dx}{e^x + 1 - 2e^{-x}} =$

- (a)  $\log(e^x - 1) - \log(e^x - 2)$   
 (b)  $\frac{1}{2} \log(e^x - 1) - \frac{1}{3} \log(e^x + 2) + c$   
 (c)  $\frac{1}{3} \log(e^x - 1) - \frac{1}{3} \log(e^x + 2) + c$   
 (d)  $\frac{1}{3} \log(e^x - 1) + \frac{1}{3} \log(e^x + 2) + c$

**33.**  $\int \sin^5 x \cos^4 x \, dx =$

- (a)  $-\frac{1}{5} \cos^5 x + \frac{2}{7} \cos^7 x - \frac{1}{9} \cos^9 x + c$   
 (b)  $\frac{1}{5} \cos^5 x + \frac{2}{7} \cos^7 x - \frac{1}{9} \cos^9 x + c$   
 (c)  $\frac{1}{5} \cos^5 x + \frac{2}{7} \cos^7 x + \frac{1}{9} \cos^9 x + c$   
 (d) None of these

**34.**  $\int \frac{dx}{5+4 \cos x} =$

- (a)  $\frac{2}{3} \tan^{-1} \left( \frac{1}{3} \tan x \right) + c$  (b)  $\frac{1}{3} \tan^{-1} \left( \frac{1}{3} \tan x \right) + c$   
 (c)  $\frac{2}{3} \tan^{-1} \left( \frac{1}{3} \tan \frac{x}{2} \right) + c$  (d)  $\frac{1}{3} \tan^{-1} \left( \frac{1}{3} \tan \frac{x}{2} \right) + c$

**35.**  $\int_0^1 \frac{dx}{[ax+b(1-x)]^2} =$

- (a)  $\frac{a}{b}$  (b)  $\frac{b}{a}$  (c)  $ab$  (d)  $\frac{1}{ab}$   
**36.**  $\int_0^{1/\sqrt{2}} \frac{\sin^{-1} x}{(1-x^2)^{3/2}} \, dx =$
- (a)  $\frac{\pi}{4} + \frac{1}{2} \log 2$  (b)  $\frac{\pi}{4} - \frac{1}{2} \log 2$   
 (c)  $\frac{\pi}{2} + \log 2$  (d)  $\frac{\pi}{2} - \log 2$

**37.**  $\int_0^\pi \frac{dx}{1+\sin x} =$

- (a) 0 (b)  $\frac{1}{2}$  (c) 2 (d)  $\frac{3}{2}$

**38.**  $\int_0^{\pi/2} \frac{\sin x \cos x}{1+\sin^4 x} \, dx =$

- (a)  $\frac{\pi}{2}$  (b)  $\frac{\pi}{4}$  (c)  $\frac{\pi}{6}$  (d)  $\frac{\pi}{8}$

**39.** If  $\int_0^1 x \log \left( 1 + \frac{x}{2} \right) dx = a + b \log \frac{2}{3}$ , then

- (a)  $a = \frac{3}{2}$ ,  $b = \frac{3}{2}$  (b)  $a = \frac{3}{4}$ ,  $b = -\frac{3}{4}$   
 (c)  $a = \frac{3}{4}$ ,  $b = \frac{3}{2}$  (d)  $a = b$

**40.**  $\int_0^{\pi/4} \frac{4 \sin 2\theta d\theta}{\sin^4 \theta + \cos^4 \theta} =$

- (a)  $\pi/4$  (b)  $\pi/2$   
 (c)  $\pi$  (d) None of these

**41.** If  $x(x^4 + 1)\phi(x) = 1$ , then  $\int_1^2 \phi(x) \, dx =$

- (a)  $\frac{1}{4} \log \frac{32}{17}$  (b)  $\frac{1}{2} \log \frac{32}{17}$   
 (c)  $\frac{1}{4} \log \frac{16}{17}$  (d) None of these

**42.**  $\int_0^{\pi/4} \frac{\sec x}{1+2\sin^2 x}$  is equal to



(a)  $\frac{1}{3} \left[ \log(\sqrt{2}+1) + \frac{\pi}{2\sqrt{2}} \right]$  (b)  $\frac{1}{3} \left[ \log(\sqrt{2}+1) - \frac{\pi}{2\sqrt{2}} \right]$

(c)  $3 \left[ \log(\sqrt{2}+1) - \frac{\pi}{2\sqrt{2}} \right]$  (d)  $3 \left[ \log(\sqrt{2}+1) + \frac{\pi}{2\sqrt{2}} \right]$

**43.** The value of  $\int_0^{\sin^2 x} \sin^{-1} \sqrt{t} dt + \int_0^{\cos^2 x} \cos^{-1} \sqrt{t} dt$  is

(a)  $\frac{\pi}{2}$  (b) 1

(c)  $\frac{\pi}{4}$  (d) None of these

**44.** If for non-zero  $x$ ,  $af(x) + bf\left(\frac{1}{x}\right) = \frac{1}{x} - 5$ , where  $a \neq b$ , then  $\int_1^2 f(x) dx =$

(a)  $\frac{1}{(a^2 + b^2)} \left[ a \log 2 - 5a + \frac{7}{2}b \right]$

(b)  $\frac{1}{(a^2 - b^2)} \left[ a \log 2 - 5a + \frac{7}{2}b \right]$

(c)  $\frac{1}{(a^2 - b^2)} \left[ a \log 2 - 5a - \frac{7}{2}b \right]$

(d)  $\frac{1}{(a^2 + b^2)} \left[ a \log 2 - 5a - \frac{7}{2}b \right]$

**45.**  $\int_0^{\pi/4} \sec x \log(\sec x + \tan x) dx =$

(a)  $\frac{1}{2} [\log(1+\sqrt{2})]^2$  (b)  $[\log(1+\sqrt{2})]^2$

(c)  $\frac{1}{2} [\log(\sqrt{2}-1)]^2$  (d)  $[\log(\sqrt{2}-1)]^2$

**46.**  $\int_0^{\pi/4} \frac{\sec^2 x}{(1+\tan x)(2+\tan x)} dx =$

(a)  $\log_e \left( \frac{2}{3} \right)$  (b)  $\log_e 3$

(c)  $\frac{1}{2} \log_e \left( \frac{4}{3} \right)$  (d)  $\log_e \left( \frac{4}{3} \right)$

**47.**  $\int_0^{\pi/4} (\cos x - \sin x) dx + \int_{\pi/4}^{5\pi/4} (\sin x - \cos x) dx + \int_{2\pi}^{\pi/4} (\cos x - \sin x) dx$  is equal to

(a)  $\sqrt{2} - 2$  (b)  $2\sqrt{2} - 2$   
(c)  $3\sqrt{2} - 2$  (d)  $4\sqrt{2} - 2$

**48.**  $\int_8^{15} \frac{dx}{(x-3)\sqrt{x+1}} =$

(a)  $\frac{1}{2} \log \frac{5}{3}$  (b)  $\frac{1}{3} \log \frac{5}{3}$

(c)  $\frac{1}{2} \log \frac{3}{5}$  (d)  $\frac{1}{5} \log \frac{3}{5}$

**49.**  $\int_0^1 \sin \left( 2 \tan^{-1} \sqrt{\frac{1+x}{1-x}} \right) dx =$

(a)  $\pi/6$  (b)  $\pi/4$  (c)  $\pi/2$  (d)  $\pi$

**50.** Let  $I_1 = \int_1^2 \frac{dx}{\sqrt{1+x^2}}$  and  $I_2 = \int_1^2 \frac{dx}{x}$  then

(a)  $I_1 > I_2$  (b)  $I_2 > I_1$   
(c)  $I_1 = I_2$  (d)  $I_2 > 2I_1$

**51.** The value of  $\int_{1/e}^{\tan x} \frac{t dt}{1+t^2} + \int_{1/e}^{\cot x} \frac{dt}{t(1+t^2)} =$

(a) -1 (b) 1  
(c) 0 (d) None of these

**52.** The value of  $\int_1^{e^2} \frac{dx}{x(1+\ln x)^2}$  is

(a)  $2/3$  (b)  $1/3$  (c)  $3/2$  (d)  $\ln 2$

**53.**  $\int_0^\pi x \log \sin x dx =$

(a)  $\frac{\pi}{2} \log \frac{1}{2}$  (b)  $\frac{\pi^2}{2} \log \frac{1}{2}$

(c)  $\pi \log \frac{1}{2}$  (d)  $\pi^2 \log \frac{1}{2}$

**54.**  $\int_0^{\pi/2} \log \tan x dx =$

(a)  $\frac{\pi}{2} \log_e 2$  (b)  $-\frac{\pi}{2} \log_e 2$

(c)  $\pi \log_e 2$  (d) 0

**55.**  $\int_{-\pi/2}^{\pi/2} \log \left( \frac{2-\sin \theta}{2+\sin \theta} \right) d\theta =$

(a) 0 (b) 1  
(c) 2 (d) None of these

**56.** Area under the curve  $y = \sqrt{3x+4}$  between  $x=0$  and  $x=4$  is

(a)  $\frac{56}{9}$  sq. unit (b)  $\frac{64}{9}$  sq. unit

(c) 8 sq. unit (d) None of these

**57.** The ratio of the areas bounded by the curves  $y = \cos x$  and  $y = \cos 2x$  between  $x=0$ ,  $x=\pi/3$  and  $x$ -axis, is

(a)  $\sqrt{2} : 1$  (b)  $1 : 1$   
(c)  $1 : 2$  (d)  $2 : 1$

**58.** The area bounded by the curve  $y = x^3$ ,  $x$ -axis and two ordinates  $x = 1$  to  $x = 2$  equal to

- |                             |                             |
|-----------------------------|-----------------------------|
| (a) $\frac{15}{2}$ sq. unit | (b) $\frac{15}{4}$ sq. unit |
| (c) $\frac{17}{2}$ sq. unit | (d) $\frac{17}{4}$ sq. unit |

**59.** For  $0 \leq x \leq \pi$ , the area bounded by  $y = x$  and  $y = x + \sin x$ , is

- (a) 2      (b) 4      (c)  $2\pi$       (d)  $4\pi$

**60.** If a curve  $y = a\sqrt{x} + bx$  passes through the point  $(1, 2)$  and the area bounded by the curve, line  $x = 4$  and  $x$ -axis is 8 sq. unit, then

- |                     |                      |
|---------------------|----------------------|
| (a) $a = 3, b = -1$ | (b) $a = 3, b = 1$   |
| (c) $a = -3, b = 1$ | (d) $a = -3, b = -1$ |

**61.** The area of the region (in the square unit) bounded by the curve  $x^2 = 4y$ , line  $x = 2$  and  $x$ -axis is

- |       |                   |                   |                   |
|-------|-------------------|-------------------|-------------------|
| (a) 1 | (b) $\frac{2}{3}$ | (c) $\frac{4}{3}$ | (d) $\frac{8}{3}$ |
|-------|-------------------|-------------------|-------------------|

**62.** Area bounded by the curve  $xy - 3x - 2y - 10 = 0$ ,  $x$ -axis and the lines  $x = 3, x = 4$  is

- |                      |                     |
|----------------------|---------------------|
| (a) $16 \log 2 - 13$ | (b) $16 \log 2 - 3$ |
| (c) $16 \log 2 + 3$  | (d) None of these   |

**63.** Area bounded by the parabola  $y^2 = 2x$  and the ordinates  $x = 1, x = 4$  is

- |                                    |                                     |
|------------------------------------|-------------------------------------|
| (a) $\frac{4\sqrt{2}}{3}$ sq. unit | (b) $\frac{28\sqrt{2}}{3}$ sq. unit |
| (c) $\frac{56}{3}$ sq. unit        | (d) None of these                   |

**64.** The area bounded by the  $x$ -axis, the curve  $y = f(x)$  and the lines  $x = 1, x = b$  is equal to  $\sqrt{b^2 + 1} - \sqrt{2}$  for all  $b > 1$ , then  $f(x)$  is

- |                    |                              |
|--------------------|------------------------------|
| (a) $\sqrt{x-1}$   | (b) $\sqrt{x+1}$             |
| (c) $\sqrt{x^2+1}$ | (d) $\frac{x}{\sqrt{1+x^2}}$ |

**65.** The area of smaller part between the circle  $x^2 + y^2 = 4$  and the line  $x = 1$  is

- |                                 |                                 |
|---------------------------------|---------------------------------|
| (a) $\frac{4\pi}{3} - \sqrt{3}$ | (b) $\frac{8\pi}{3} - \sqrt{3}$ |
| (c) $\frac{4\pi}{3} + \sqrt{3}$ | (d) $\frac{5\pi}{3} + \sqrt{3}$ |

**66.** Area of the region bounded by the curve  $y = \tan x$ , tangent drawn to the curve at  $x = \frac{\pi}{4}$  and the  $x$ -axis is

- |                                   |                                   |
|-----------------------------------|-----------------------------------|
| (a) $\frac{1}{4}$                 | (b) $\log \sqrt{2} + \frac{1}{4}$ |
| (c) $\log \sqrt{2} - \frac{1}{4}$ | (d) None of these                 |

**67.** Area enclosed by the parabola  $ay = 3(a^2 - x^2)$  and  $x$ -axis is

- |                     |                      |
|---------------------|----------------------|
| (a) $4a^2$ sq. unit | (b) $12a^2$ sq. unit |
| (c) $4a^3$ sq. unit | (d) None of these    |

### Assertion & Reason Type

**Directions :** In the following questions, Statement-1 is followed by Statement-2. Mark the correct choice as :

- Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1.
- Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1.
- Statement-1 is true, Statement-2 is false.
- Statement-1 is false, Statement-2 is true.

**68. Statement-1 :** The inequality  $\int_0^1 \frac{dx}{1+x^n} > 1 - \frac{1}{n}$  is true for  $n \in N$ .

**Statement-2 :**  $\int_0^1 \frac{x^{n-1} - x^n + x^{2n-1}}{1+x^n} dx > 0, \forall n \in N$ .

**69. Statement-1 :** If  $F(x) = \int_1^x \frac{\ln t}{1+t+t^2} dt$ ,

then  $F(x) = -F(1/x)$

**Statement-2 :** If  $F(x) = \int_1^x \frac{\ln t}{t+1} dt$ ,

then  $F(x) + F(1/x) = (1/2)(\ln x)^2$ .

**70. Statement-1 :**  $\int e^{3x} (3 \sin x + \cos x) dx = e^{3x} \sin x + c$

**Statement-2 :** Antiderivative of a periodic function is a periodic function.

**71. Statement-1 :**  $\int \frac{1}{4e^{-x} - 9e^x} dx = \frac{1}{6} \log \left| \frac{2+3e^x}{2-3e^x} \right| + C$

**Statement-2 :**  $\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$

**72. Statement-1 :**  $\int \frac{e^x (2-x^2)}{(1-x)\sqrt{1-x^2}} dx = e^x \sqrt{\frac{1-x}{1+x}} + c$

**Statement-2 :**  $\int e^x (f(x) + f'(x)) dx = e^x f(x) + c$

**73. Statement-1:**  $\sum_{r=0}^{12} \int \cot^{2r} x dx = -\sum_{r=1}^6 \frac{\cot^{4r-1} x}{4r-1} + c$

**Statement-2:**  $\int \cot^n x dx + \int \cot^{n-2} x dx = -\frac{\cot^{n-1} x}{n-1} + c$

**74. Statement-1:**  $\int \frac{e^{\cot x}}{\sin^2 x} (2 \ln \cosec x + \sin 2x) dx = 2e^{\cot x} \ln \sin x + c$

**Statement-2:**  $\int e^x (f(x) + f'(x)) dx = e^x f(x) + c$

**75. Statement-1:**  $\int_1^\infty \frac{x^2 - 2}{x^3 \sqrt{x^2 - 1}} dx = 0$

**Statement-2:**  $f(x) = \frac{x^2 - 2}{x^3 \sqrt{x^2 - 1}}$  is an odd function

**76.** Let  $T > 0$  be a fixed real number. Suppose  $f(x)$  is a continuous function for all  $x \in R$ ,  $f(x+T) = f(x)$  then

**Statement-1:**  $\int_6^{6+6T} f(4x) dx = 6 \int_0^T f(x) dx$

**Statement-2:**  $\int_0^{24T} f(x) dx = 24 \int_0^T f(x) dx$

**77. Statement-1:**  $\int_0^{\pi/2} (\sin^6 x + \cos^6 x) dx$  lies in the interval  $\left(\frac{\pi}{8}, \frac{\pi}{2}\right)$ .

**Statement-2:**  $\sin^6 x + \cos^6 x$  is periodic with period  $\pi/2$ .

**78. Statement-1:** If root of  $ax^2 + bx + c = 0$  are non real

and  $c = \int_0^{\sin^2 x} \sin^{-1} \sqrt{x} dx + \int_0^{\cos^2 x} \cos^{-1} \sqrt{x} dx$  then

'a' must be positive.

**Statement-2:** If roots of quadratic equation  $ax^2 + bx + c = 0$  are non-real then 'a' and 'c' have same sign.

**79.** Let  $f(x)$  and  $g(x)$  be continuous functions of  $x$  in  $(a, b)$  and  $f(x) < g(x) < 0 \forall x \in (a, b)$  then

**Statement-1:** Area of the region bounded by  $y = f(x)$ ,  $x$ -axis,  $x = a$  and  $x = b$  is less than the area of the region bounded by  $y = g(x)$ ,  $x$ -axis,  $x = a$  and  $x = b$ .

**Statement-2:**  $|f(x)| > |g(x)| \forall x \in (a, b)$

**80.** If  $f(x) = \frac{e^x}{1+e^x}$

**Statement-1:**  $I_1 = \int_{f(-a)}^{f(a)} x g(x(1-x)) dx$

and  $I_2 = \int_{f(-a)}^{f(a)} g(x(1-x)) dx$  then the value of  $\frac{I_2}{I_1}$  is 2.

**Statement-2 :** Here  $f(a) + f(-a) = 1$  and also by the property  $\int_p^q f(x) dx = \int_p^q f(p+q-x) dx$

**81.** Let  $\int \frac{x + (\cos^{-1} 3x)^2}{\sqrt{1-9x^2}} dx = A \sqrt{1-9x^2} - B(\cos^{-1} 3x)^3 + c$ ,

where  $c$  is a constant of integration, then

**Statement-1 :**  $A + B = 0$

**Statement-2 :**  $A = \frac{2}{9}$  and  $B = -\frac{2}{9}$

### Comprehension Type

#### Paragraph for Q. No. 82 – 84

If a curve is given by its parametric equation in the form  $x = f(t)$ ,  $y = g(t)$  and suppose the derivatives  $f'(t)$  and  $g'(t)$  are continuous functions on the interval  $[t_1, t_2]$ . If  $t_1$  and  $t_2$  are the values of parameter 't' corresponding respectively to the initial and final position in which the curve can be described as a contour in the positive direction (i.e., figure remains left), then the area described by the curve is given by

$$A = - \int_{t_1}^{t_2} g(t) f'(t) dt = - \int_{t_1}^{t_2} f(t) g'(t) dt \\ = \frac{1}{2} \int_{t_1}^{t_2} (x g'(t) dt - y f'(t) dt)$$

**82.** Area enclosed by the curve

$$x = a \cos^3 t, y = a \sin^3 t, 0 \leq t \leq 2\pi$$

- (a)  $\frac{3}{2} a^2 \pi$  (b)  $\frac{3}{4} a^2 \pi$  (c)  $\frac{3}{8} a^2 \pi$  (d)  $\frac{3}{16} a^2 \pi$

**83.** Area of the loop of the curve  $x = a(1 - t^2)$ ,  $y = at(1 - t^2)$ ,  $-1 \leq t \leq 1$  must be

- (a)  $\frac{2}{15} a^2$  (b)  $\frac{4}{15} a^2$  (c)  $\frac{8}{15} a^2$  (d)  $\frac{a^2}{5}$

**84.** The area of the curve  $x = 2 \cos t$ ,  $y = 2 \sin t$  must be

- (a)  $4\pi$  (b)  $2\pi$  (c)  $8\pi$  (d)  $16\pi$

#### Paragraph for Q. No. 85 – 87

Let  $n$  be a non-negative integer and let

$I_n = \int x^n \sqrt{a^2 - x^2} dx$ , ( $a > 0$ ) then we can find relation among  $I_n$ ,  $I_{n-1}$ ,  $I_{n-2}$ . It can be observed that  $I_1$  is elementary integration whose value is  $-\frac{1}{3}(a^2 - x^2)^{3/2}$ .

If  $I_n = -\frac{x^{n-1}(a^2 - x^2)^{3/2}}{A} + a^2 B I_{n-2}$  where A and B are constants. Then

**85.** A must be equal to

- (a)  $n+1$  (b)  $n-1$  (c)  $n+2$  (d)  $n$

**86.** B must be equal to

- (a)  $\frac{n+1}{n+2}$  (b)  $\frac{n+2}{n+1}$  (c)  $\frac{n}{n+2}$  (d)  $\frac{n-1}{n+2}$

**87.** The value of  $\int_0^a x^4 \sqrt{a^2 - x^2} dx$  must be

- (a)  $\frac{\pi a^6}{32}$  (b)  $\frac{\pi a^4}{16}$  (c)  $\frac{\pi a^4}{64}$  (d)  $\frac{\pi a^6}{16}$

#### Paragraph for Q. No. 88 – 90

A curve passing through origin is such that slope of tangent at any point is reciprocal of sum of co-ordinate of point of tangency.

**88.** Slope of tangent at ordinate  $\ln 3$ .

- (a) 1 (b)  $\frac{1}{3}$  (c)  $\frac{1}{2}$  (d) -2

**89.** Area bounded by curve and the abscissa  $y = 0$  and  $y = 1$  is

- (a)  $e - \frac{1}{2}$  (b)  $e - \frac{3}{2}$  (c)  $e - \frac{5}{2}$  (d)  $e + 1$

**90.**  $\int_{-1}^{[\sin \alpha]} xe^{-y} d(e^y)$  where  $[.]$  represents integer function, is equal to

- (a)  $e - e^{-1} - \frac{1}{3}$  (b)  $e - e^{-1} - 2$   
 (c)  $e - \frac{1}{e}$  (d)  $e + e^{-1} + \frac{1}{3}$

#### Paragraph for Q. No. 91 – 93

Differentiate  $I$  w.r.t. the parameter within the sign of integrals taking variable of the integrand as constant. Now, evaluate the integral so obtained as usual as a function of the parameter and then integrate the result to get  $I$ . Constant of integration can be computed by giving some arbitrary values to the parameter and the corresponding value of  $I$ .

**91.** The value of  $\int_0^1 \frac{x^a - 1}{\log x} dx$  is

- (a)  $\log(a-1)$  (b)  $\log(a+1)$   
 (c)  $a \log(a+1)$  (d) None of these

**92.** The value of  $\int_0^{\pi/2} \log(\sin^2 \theta + k^2 \cos^2 \theta) d\theta$ , where  $k \geq 0$ , is

- (a)  $\pi \log(1+k) + \pi \log 2$  (b)  $\pi \log(1+k)$   
 (c)  $\pi \log(1+k) - \pi \log 2$  (d)  $\pi \log(1+k) - \log 2$

**93.** If  $\int_0^\pi \frac{dx}{(a - \cos x)} = \frac{\pi}{\sqrt{a^2 - 1}}$ , then the value of

$\int_0^\pi \frac{dx}{(\sqrt{10} - \cos x)^3}$  is

- (a)  $\frac{\pi}{81}$  (b)  $\frac{7\pi}{81}$  (c)  $\frac{7\pi}{162}$  (d)  $\frac{7\pi}{8}$

#### Matrix-Match Type

**94.** Match the following.

	Column-I	Column-II
(A)	If $\int x^2 d(\tan^{-1} x) dx = x - f(x) + c$ then $f(1)$ is equal to	(p) $\frac{1}{8}$
(B)	If $\int x^2 e^{2x} dx = e^{2x} f(x) + c$ , then the minimum value of $f(x)$ is	(q) 0
(C)	If $\int \frac{(x^4 + 1)}{x(x^2 + 1)^2} dx = a \log x  + \frac{b}{x^2 + 1} + c$ then $a - b$ is equal to	(r) $\frac{\pi}{4}$

**95.** Match the following.

[ $x$ ] denotes greatest integer less than or equal to ' $x$ '.

	Column I	Column II
(A)	$25 \int_0^{\pi/4} (\tan^6(x - [x]) + \tan^4(x - [x])) dx =$	(p) 4
(B)	$I_1 = \int_{-2}^2 \frac{x^6 + 3x^5 + 7x^4}{x^4 + 2} dx$ $I_2 = \int_{-3}^1 \frac{2(x+1)^2 + 11(x+1) + 14}{(x+1)^4 + 2} dx$ then the $I_1 + I_2 =$	(q) 2
(C)	$\frac{2}{\pi} \int_{-1}^3 \left( \tan^{-1} \frac{x}{x^2 + 1} + \tan^{-1} \frac{x^2 + 1}{x} \right) dx =$	(r) $\frac{100}{3}$ (s) 5

### Integer Answer Type

**96.** If  $\lim_{n \rightarrow \infty} \frac{\sum_{r=1}^n \sqrt{r} \sum_{r=1}^n \frac{1}{\sqrt{r}}}{\sum_{r=1}^n r} = \frac{k}{3}$  then the value of  $k$  is

$$97. \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{k^{1/a} \left\{ n^{a-\frac{1}{a}} + k^{a-\frac{1}{a}} \right\}}{n^{a+1}}$$

**98.** The value of

$$\int_{-\pi/4}^{\pi/4} [(x^9 - 3x^5 + 7x^3 - x + 1) / \cos^2 x] dx$$

**99.**  $f: [0, 5] \rightarrow R$ ,  $y = f(x)$  such that  $f''(x) = f''(5-x)$   $\forall x \in [0, 5]$   $f'(0) = 1$  and  $f'(5) = 7$ , then value of  $\int_1^4 f'(x) dx - 4$  is

**100.** Let  $\alpha, \beta$  be roots of the quadratic equation  $18x^2 - 9\pi x + \pi^2 = 0$ , where  $\alpha < \beta$ . Also  $f(x) = x^2$  and  $g(x) = \cos x$ . If the area bounded by the curve  $y = (fog)(x)$ ,

the vertical lines  $x = \alpha$ ,  $x = \beta$  and  $x$ -axis is  $\frac{\pi}{\lambda}$ , then the sum of the digit of  $\lambda$  is

**101.** Let  $I_n = \int_0^{\pi/2} (\sin x + \cos x)^n dx$  ( $n \geq 2$ ). Then the value of  $nI_n - 2(n-1)I_{n-2}$  is

**102.** If  $\int_0^{\pi/2} \ln \sin x dx = \frac{\pi}{2} \ln \frac{1}{2}$

and  $\int_0^{\pi/2} \left( \frac{x}{\sin x} \right)^2 dx = \frac{k \cdot \pi}{2} \ln 2$ , then value of  $k$  is

$$29 \int_0^1 (1-x^4)^7 dx$$

**103.** Find the value of  $\frac{0}{\int_1^1 (1-x^4)^6 dx}$ .

$$4 \int_0^1 (1-x^4)^6 dx$$

**104.** Evaluate  $\left[ \int_0^{\pi} \frac{dx}{1+2\sin^2 x} \right]$  where  $[.]$  denotes greatest integer function.

**105.** The function  $f(x) = \int_1^x \{2(t-1)(t-2)^3 + 3(t-1)^2(t-2)^2 dt$  attains its maximum at  $x$  is equal to

### SOLUTIONS

#### Single Correct Answer Type

**1. (a) :**  $\int x^{51}(\tan^{-1} x + \cot^{-1} x) dx = \int x^{51} \cdot \frac{\pi}{2} dx$   
 $\left\{ \because \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2} \right\}$

$$= \frac{\pi x^{52}}{104} + c = \frac{x^{52}}{52} (\tan^{-1} x + \cot^{-1} x) + c$$

**2. (c) :**  $\int \frac{dx}{\sin x + \cos x} = \frac{1}{\sqrt{2}} \int \frac{dx}{\sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4}}$   
 $= \frac{1}{\sqrt{2}} \int \operatorname{cosec} \left( x + \frac{\pi}{4} \right) dx = \frac{1}{\sqrt{2}} \log \tan \left( \frac{\pi}{8} + \frac{x}{2} \right) + c$

**3. (d) :**  $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = \int \left( \frac{\sin x}{\cos^2 x} + \frac{\cos x}{\sin^2 x} \right) dx$   
 $= \int (\sec x \tan x + \operatorname{cosec} x \cot x) dx = \sec x - \operatorname{cosec} x + c$

**4. (a) :**  $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx = \int \frac{2(\cos^2 x - \cos^2 \alpha)}{\cos x - \cos \alpha} dx$   
 $= 2 \int (\cos x + \cos \alpha) dx = 2(\sin x + x \cos \alpha) + c$

**5. (a) :**  $\int \frac{x^4 + x^2 + 1}{x^2 - x + 1} dx = \int (x^2 + x + 1) dx$   
 $= \frac{x^3}{3} + \frac{x^2}{2} + x + c$

**6. (a) :**  $\int \frac{1}{\sqrt{1+\cos x}} dx = \int \frac{dx}{\sqrt{2\cos^2(x/2)}}$   
 $= \frac{1}{\sqrt{2}} \int \sec \frac{x}{2} dx$   
 $= \frac{1}{\sqrt{2}} \left\{ \log \left( \sec \frac{x}{2} + \tan \frac{x}{2} \right) \right\} \cdot \frac{1}{1/2}$   
 $= \sqrt{2} \log \left( \sec \frac{x}{2} + \tan \frac{x}{2} \right) + c$

**7. (a) :** Let  $I = \int \frac{dx}{x+x \log x} = \int \frac{dx}{x(1+\log x)}$

Put  $1+\log x = t \Rightarrow \frac{1}{x} dx = dt$

$\therefore I = \int \frac{dt}{t} = \log(t) = \log(1+\log x)$

**8. (b) :** Put  $\sec x = t \Rightarrow \sec x \tan x dx = dt$

$$\therefore \int \sec^p x \tan x dx = \int t^{p-1} dt = \frac{t^p}{p} + c = \frac{\sec^p x}{p} + c$$

**9. (a) :**  $\int \frac{e^x(x+1)}{\cos^2(xe^x)} dx = \int e^x(x+1)\sec^2(xe^x)dx$

Putting  $xe^x = t \Rightarrow (x+1)e^x dx = dt$ , we get

$$\int \sec^2 t dt = \tan t + c = \tan(xe^x) + c$$

**10. (a) :** Put  $a \cos^2 x + b \sin^2 x = t$

$$\Rightarrow 2(b-a) \sin x \cos x = dt,$$

$$\text{then } \int \frac{\sin x \cos x dx}{a \cos^2 x + b \sin^2 x} = \frac{1}{2(b-a)} \int \frac{1}{t} dt$$

$$= \frac{1}{2(b-a)} \log(a \cos^2 x + b \sin^2 x) + c$$

**11. (b) :** Let  $I = \int \frac{e^{2x}-1}{e^{2x}+1} dx = \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$

Now put  $e^x + e^{-x} = t \Rightarrow (e^x - e^{-x})dx = dt$ ,

$$\therefore I = \int \frac{dt}{t} = \log t = \log(e^x + e^{-x}) = \log(e^{2x} + 1) - x + c$$

**12. (b)**

**13. (a) :** Put  $x = \tan \theta \Rightarrow dx = \sec^2 \theta d\theta$ , then

$$\begin{aligned} \int \frac{1}{x^2 \sqrt{1+x^2}} dx &= \int \frac{\sec^2 \theta d\theta}{\tan^2 \theta \sec \theta} = \int \cosec \theta \cot \theta d\theta \\ &= -\cosec \theta + c = \frac{-\sqrt{x^2+1}}{x} + c \end{aligned}$$

**14. (c) :**  $\int \sin^3 x dx = \int \sin^2 x \cdot \sin x dx$

$$\begin{aligned} &= \int \sin x (1 - \cos^2 x) dx = \int \sin x dx - \int \cos^2 x \cdot \sin x dx \\ &= -\cos x + \frac{\cos^3 x}{3} + c \end{aligned}$$

**15. (a) :** Let  $I = \int \frac{dx}{(1+e^x)(1+\frac{1}{e^x})} = \int \frac{e^x dx}{(1+e^x)^2}$

Let  $1+e^x = t$ ,  $\therefore e^x dx = dt$

$$\therefore I = \int \frac{dt}{t^2} = \int t^{-2} dt = \frac{t^{-1}}{-1} = \frac{(1+e^x)^{-1}}{-1} = \frac{-1}{1+e^x}$$

**16. (d) :**  $I = \int \frac{1+\tan^2 x}{1-\tan^2 x} dx = \int \frac{\sec^2 x}{1-\tan^2 x} dx$

Put  $\tan x = t \Rightarrow \sec^2 x dx = dt \Rightarrow I = \int \frac{dt}{1-t^2}$

$$= \frac{1}{2 \times 1} \log \left[ \frac{1+t}{1-t} \right] + c = \frac{1}{2} \log \left| \frac{1+\tan x}{1-\tan x} \right| + c$$

**17. (c) :**  $\int \cosec^4 x dx = \int \cosec^2 x \cdot \cosec^2 x dx$

$$= \int \cosec^2 x (1 + \cot^2 x) dx$$

$$= \int \cosec^2 x dx + \int \cot^2 x \cdot \cosec^2 x dx$$

$$= -\cot x - \frac{\cot^3 x}{3} + c$$

**18. (d) :**  $\int \sqrt{\frac{1-x}{1+x}} dx = \int \frac{1-x}{\sqrt{1-x^2}} dx$

$$= \int \frac{1}{\sqrt{1-x^2}} dx - \int \frac{x dx}{\sqrt{1-x^2}} = \sin^{-1} x + \sqrt{1-x^2} + c$$

**19. (b) :**  $\int \tan^{-1} x dx = x \tan^{-1} x - \int \frac{x}{1+x^2} dx + c$

$$= x \tan^{-1} x - \frac{1}{2} \log(1+x^2) + c.$$

**20. (b) :**  $\int \left[ \log(\log x) + \frac{1}{(\log x)^2} \right] = \int \log(\log x) dx + \int \frac{1}{(\log x)^2}$

$$= x \log(\log x) - \int \frac{x}{x \log x} dx + \int \frac{1}{(\log x)^2} dx$$

$$= x \log(\log x) - \frac{x}{\log x} - \int \frac{1}{(\log x)^2} dx + \int \frac{1}{(\log x)^2} dx$$

$$= x \log(\log x) - \frac{x}{\log x} + c.$$

**21. (b) :**  $\int \frac{x+\sin x}{1+\cos x} dx = \frac{1}{2} \int x \sec^2 \frac{x}{2} dx + \int \tan \frac{x}{2} dx$

$$= \frac{1}{2} \frac{x \tan \frac{x}{2}}{\frac{1}{2}} - \int \tan \frac{x}{2} dx + \int \tan \frac{x}{2} dx = x \tan \frac{x}{2} + c$$

**22. (c) :** Let  $I = \int e^x \left( \frac{1+\sin x}{1+\cos x} \right) dx$

$$= \int e^x \left[ \frac{1+2\sin(x/2)\cos(x/2)}{2\cos^2(x/2)} \right] dx$$

$$I = \int e^x \left[ \frac{1}{2} \sec^2(x/2) + \tan(x/2) \right] dx = e^x \tan(x/2) + c$$

$$\{ \because \int e^x [f(x) + f'(x)] dx = e^x \cdot f(x) + c \}$$

**23. (a) :** Put  $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt \Rightarrow dx = 2t dt$ , then

$$\begin{aligned} \int 2t \cdot \cos t dt &= 2 \left[ t \sin t - \int \sin t dt \right] \\ &= 2t \sin t + 2 \cos t + c = 2[\sqrt{x} \sin \sqrt{x} + \cos \sqrt{x}] + c \end{aligned}$$

**24. (d) :** Put  $x = \tan \theta \Rightarrow dx = \sec^2 \theta d\theta$ , then

$$\begin{aligned} \int \tan^{-1} \frac{2x}{1-x^2} dx &= \int \tan^{-1} \frac{2 \tan \theta}{1-\tan^2 \theta} \sec^2 \theta d\theta \\ &= \int \tan^{-1} (\tan 2\theta) \sec^2 \theta d\theta = \int 2\theta \sec^2 \theta d\theta \\ &= 2 \left[ \theta \tan \theta - \int \tan \theta d\theta \right] = 2x \tan^{-1} x - \log(x^2 + 1) + c \end{aligned}$$

**25. (a) :** Putting  $\sin^{-1} x = t \Rightarrow \frac{1}{\sqrt{1-x^2}} dx = dt$ , we get

$$\begin{aligned} \int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx &= \int t \sin t dt = -t \cos t + \sin t + c \\ &= -\sin^{-1} x \cos(\sin^{-1} x) + \sin(\sin^{-1} x) + c \\ &= x - \sqrt{1-x^2} \sin^{-1} x + c \end{aligned}$$

**26. (a) :** Put  $t = \sin^{-1} x$   
 $\Rightarrow \sin t = x \Rightarrow \cos t dt = dx$ , then

$$\begin{aligned} \int \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx &= t \sec^2 t dt = t \tan t + \log \cos t + c \\ &= \sin^{-1} x \tan(\sin^{-1} x) + \log \cos(\sin^{-1} x) + c \\ &= \frac{x}{\sqrt{1-x^2}} \sin^{-1} x + \frac{1}{2} \log(1-x^2) + c. \end{aligned}$$

**27. (a) :** Putting  $\tan^{-1} x = t$  and  $\frac{dx}{1+x^2} = dt$ , we get

$$\begin{aligned} \int e^{\tan^{-1} x} \left( \frac{1+x+x^2}{1+x^2} \right) dx &= \int e^t (\tan t + \sec^2 t) dt \\ &= e^t \tan t + c = x e^{\tan^{-1} x} + c \\ \left[ \text{Using } \int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C \right] \end{aligned}$$

**28. (c)**

$$\begin{aligned} \int \frac{x^2}{(x^2+2)(x^2+3)} dx &= \int \left[ \frac{3}{x^2+3} - \frac{2}{x^2+2} \right] dx \\ &= \frac{3}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} - \frac{2}{\sqrt{2}} \tan^{-1} \left( \frac{x}{\sqrt{2}} \right) + c \\ &= \sqrt{3} \tan^{-1} \left( \frac{x}{\sqrt{3}} \right) - \sqrt{2} \tan^{-1} \left( \frac{x}{\sqrt{2}} \right) + c \end{aligned}$$

**30. (a) :** Let  $I = \int \sin^3 x \cos^2 x dx$   
 $= \int (1 - \cos^2 x) \cos^2 x \cdot \sin x dx$

Put  $\cos x = t \Rightarrow -\sin x dx = dt$

$$\therefore I = - \int (t^2 - t^4) dt = \frac{t^5}{5} - \frac{t^3}{3} + c = \frac{(\cos x)^5}{5} - \frac{(\cos x)^3}{3} + c$$

**31. (c) :** Put  $\sin x = t \Rightarrow \cos x dx = dt$ , then

$$\begin{aligned} \int \frac{\cos x}{(1+\sin x)(2+\sin x)} dx &= \int \frac{dt}{(t+1)(t+2)} \\ &= \int \frac{1}{t+1} dt - \int \frac{1}{t+2} dt = \log \left( \frac{t+1}{t+2} \right) + c \\ &= \log \left( \frac{\sin x + 1}{\sin x + 2} \right) + c \end{aligned}$$

$$32. (c) : \int \frac{e^x dx}{e^{2x} + e^x - 2} = \int \frac{dt}{t^2 + t - 2}, t = e^x$$

$$\begin{aligned} &= \int \frac{dt}{(t+2)(t-1)} = \int \frac{1}{3} \left[ \frac{1}{t-1} - \frac{1}{t+2} \right] dt \\ &= \frac{1}{3} \log(e^x - 1) - \frac{1}{3} \log(e^x + 2) + c \end{aligned}$$

**33. (a) :** Put  $\cos x = t \Rightarrow -\sin x dx = dt$ , then

$$\begin{aligned} \int (1 - \cos^2 x)^2 \cdot \cos^4 x \cdot \sin x dx &= - \int (1 - t^2)^2 \cdot t^4 dt \\ &= - \frac{t^5}{5} + \frac{2}{7} t^7 - \frac{1}{9} t^9 + c \\ &= - \frac{\cos^5 x}{5} + \frac{2}{7} \cos^7 x - \frac{1}{9} \cos^9 x + c \end{aligned}$$

$$34. (c) : \int \frac{dx}{5+4 \cos x}$$

$$= \int \frac{dx}{5+4 \left[ \frac{1-\tan^2 \frac{x}{2}}{1+\tan^2 \frac{x}{2}} \right]} = \int \frac{\sec^2 \frac{x}{2}}{9+\tan^2 \frac{x}{2}} dx$$

Put  $\tan \frac{x}{2} = t \Rightarrow \sec^2 \frac{x}{2} \cdot \frac{1}{2} dx = dt$ ,

$$2 \int \frac{dt}{3^2 + t^2} = \frac{2}{3} \tan^{-1} \left[ \frac{1}{3} \tan \frac{x}{2} \right] + c$$

$$35. (d) : \text{Let } I = \int_0^1 \frac{dx}{[(a-b)x+b]^2}$$

Put  $t = (a-b)x + b \Rightarrow dt = (a-b)dx$

As  $x = 1 \Rightarrow t = a$  and  $x = 0 \Rightarrow t = b$ , then

$$I = \frac{1}{a-b} \int_b^a \frac{1}{t^2} dt = \frac{1}{(a-b)} \left[ -\frac{1}{t} \right]_b^a = \frac{1}{(a-b)} \left( \frac{a-b}{ab} \right) = \frac{1}{ab}$$

**36. (b) :**  $I = \int_0^{1/\sqrt{2}} \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx$

Put  $\sin^{-1} x = t \Rightarrow \frac{1}{\sqrt{1-x^2}} dx = dt$  and  $x = \sin t$

Also  $t = 0$  to  $\frac{\pi}{4}$  as  $x = 0$  to  $\frac{1}{\sqrt{2}}$

$$\Rightarrow I = \int_0^{\pi/4} t \cdot \sec^2 t dt = \frac{\pi}{4} - \frac{1}{2} \log 2$$

**37. (c) :**  $\int_0^\pi \frac{dx}{1+\sin x} = \int_0^\pi \frac{1-\sin x}{\cos^2 x} dx$

$$= \int_0^\pi (\sec^2 x - \sec x \tan x) dx$$

$$= [\tan x - \sec x]_0^\pi = [\tan \pi - \sec \pi + 1] = [0 + 1 + 1] = 2$$

**38. (d) :** Put  $\sin^2 x = t \Rightarrow dt = 2 \sin x \cos x dx$

$$\text{Now } \int_0^{\pi/2} \frac{\sin x \cos x}{1+\sin^4 x} dx = \frac{1}{2} \int_0^1 \frac{1}{1+t^2} dt = \frac{1}{2} [\tan^{-1} t]_0^1 = \frac{\pi}{8}$$

**39. (c) :** Let  $I = \int_0^1 x \log\left(1 + \frac{x}{2}\right) dx$

$$= \left[ \log\left(1 + \frac{x}{2}\right) \frac{x^2}{2} \right]_0^1 - \int_0^1 \frac{1}{1 + \frac{x}{2}} \cdot \frac{1}{2} \cdot \frac{x^2}{2} dx$$

$$= \frac{1}{2} \log \frac{3}{2} - \frac{1}{2} \int_0^1 \frac{x^2}{x+2} dx$$

$$= \frac{1}{2} \log \frac{3}{2} - \frac{1}{2} \left[ \frac{1}{2} - 2 + 4 \log 3 - 4 \log 2 \right] = \frac{3}{4} + \frac{3}{2} \log \frac{2}{3}$$

On comparing with the given value  $a = 3/4$ ,  $b = 3/2$ .

**40. (c) :**  $4 \int_0^{\pi/4} \frac{\sin 2\theta d\theta}{\sin^4 \theta + \cos^4 \theta} = 4 \int_0^{\pi/4} \frac{2 \sin \theta \cos \theta d\theta}{\sin^4 \theta + \cos^4 \theta}$

$$= 4 \int_0^{\pi/4} \frac{2 \tan \theta \sec^2 \theta d\theta}{\tan^4 \theta + 1}$$

Now put  $\tan^2 \theta = t \Rightarrow 2 \tan \theta \sec^2 \theta d\theta = dt$ , then the reduced form is

$$4 \int_0^1 \frac{dt}{t^2 + 1} = 4 [\tan^{-1} t]_0^1 = 4 \left[ \frac{1}{4} \pi - 0 \right] = \pi$$

**41. (a) :** Here  $\phi(x) = \frac{1}{x(x^4 + 1)} = \frac{1}{x} - \frac{x^3}{x^4 + 1}$

$$\Rightarrow \int_1^2 \phi(x) dx = \int_1^2 \left( \frac{1}{x} - \frac{x^3}{x^4 + 1} \right) dx$$

$$= [\log x]_1^2 - \left[ \frac{1}{4} \log(x^4 + 1) \right]_1^2 = \frac{1}{4} \log \frac{32}{17}$$

**42. (a)**

**43. (c) :** Let  $I = \int_0^{\sin^2 x} \sin^{-1} \sqrt{t} dt + \int_0^{\cos^2 x} \cos^{-1} \sqrt{t} dt$

Putting  $t = \sin^2 u$  in the first integral and  $t = \cos^2 v$  in the second integral, we have

$$I = \int_0^x u \sin 2u du - \int_{\pi/2}^x v \sin 2v dv$$

$$= \int_0^{\pi/2} u \sin 2u du + \int_{\pi/2}^x u \sin 2u du - \int_{\pi/2}^x v \sin 2v dv$$

$$I = \int_0^{\pi/2} u \sin 2u du = \left( \frac{-u \cos 2u}{2} \right)_0^{\pi/2} + \frac{1}{2} \int_0^{\pi/2} \cos 2u du$$

$$= \left( \frac{-u \cos 2u}{2} \right)_0^{\pi/2} + \frac{1}{4} (\sin 2u)_0^{\pi/2} = \frac{\pi}{4}$$

**44. (b) :**  $af(x) + bf\left(\frac{1}{x}\right) = \frac{1}{x} - 5$  (for each  $x \neq 0$ ) ... (i)

Replacing  $x$  by  $\frac{1}{x}$  in (i), we get

$$af\left(\frac{1}{x}\right) + bf(x) = x - 5 \quad \dots \text{(ii)}$$

Eliminating  $f\left(\frac{1}{x}\right)$  from (i) and (ii), we get

$$(a^2 - b^2) f(x) = \frac{a}{x} - bx - 5a + 5b$$

$$\Rightarrow (a^2 - b^2) \int_1^2 f(x) dx = \left[ \left( a \log|x| - \frac{b}{2} x^2 - 5(a-b)x \right) \right]_1^2$$

$$= a \log 2 - 2b - 10(a-b) - a \log 1 + \frac{b}{2} + 5(a-b)$$

$$= a \log 2 - 5a + \frac{7}{2}b$$

$$\Rightarrow \int_1^2 f(x) dx = \frac{1}{a^2 - b^2} \left[ a \log 2 - 5a + \frac{7}{2}b \right]$$

**45. (a) :** Let  $I = \int_0^{\pi/4} \sec x \log(\sec x + \tan x) dx$

Put  $\log(\sec x + \tan x) = t \Rightarrow \sec x dx = dt$

$$\Rightarrow I = \int_0^{\log(\sqrt{2}+1)} t dt = \left[ \frac{t^2}{2} \right]_0^{\log(\sqrt{2}+1)} = \frac{[\log(\sqrt{2}+1)]^2}{2}$$

**46. (d) :** Put  $1 + \tan x = t \Rightarrow \sec^2 x dx = dt$

$$\therefore \int_0^{\pi/4} \frac{\sec^2 x}{(1 + \tan x)(2 + \tan x)} dx$$

$$= \int_1^2 \frac{dt}{t(t+1)} = \int_1^2 \frac{dt}{t} - \int_1^2 \frac{dt}{1+t} = [\log t - \log(1+t)]_1^2$$

$$= \log_e 2 - \log_e 3 + \log_e 2 = \log_e \frac{4}{3}$$



**47. (d) :** Let  $I = \int_0^{\pi/4} (\cos x - \sin x) dx$

$$+ \int_{\pi/4}^{5\pi/4} (\sin x - \cos x) dx + \int_{2\pi}^{\pi/4} (\cos x - \sin x) dx$$

$$= [\sin x + \cos x]_0^{\frac{\pi}{4}} - [\sin x + \cos x]_{\frac{\pi}{4}}^{\frac{5\pi}{4}} + [\sin x + \cos x]_{2\pi}^{\frac{\pi}{4}}$$

$$= \left[ \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 \right] - \left[ -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \left( \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) \right]$$

$$+ \left[ \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 \right]$$

$$= [\sqrt{2} - 1] - [-\sqrt{2} - \sqrt{2}] + [\sqrt{2} - 1] = 4\sqrt{2} - 2$$

**48. (a) :**  $I = \int_8^{15} \frac{dx}{(x-3)\sqrt{x+1}}$

Put  $x = \tan^2 \theta$  and  $\theta = \tan^{-1} \sqrt{x}$   
 $\Rightarrow dx = 2 \tan \theta \sec^2 \theta d\theta$

$$\therefore I = \int_{\tan^{-1} \sqrt{8}}^{\tan^{-1} \sqrt{15}} \frac{2 \tan \theta \sec^2 \theta}{(\tan^2 \theta - 3)\sqrt{\tan^2 \theta + 1}} d\theta$$

$$= \int_{\tan^{-1} \sqrt{8}}^{\tan^{-1} \sqrt{15}} \frac{2 \tan \theta \sec^2 \theta}{(\sec^2 \theta - 4)\sec \theta} d\theta$$

$$= \left[ \frac{1}{2} \log \frac{(\sec \theta - 2)}{(\sec \theta + 2)} \right]_{\tan^{-1} \sqrt{8}}^{\tan^{-1} \sqrt{15}}$$

$$= \frac{1}{2} \left[ \log \frac{2}{6} - \log \frac{1}{5} \right] = \frac{1}{2} \log \frac{5}{3}$$

**49. (b)      50. (b)**

**51. (b) :** On integrating both functions, we get

$$= \frac{1}{2} \left| \log(1+t^2) \right|_{1/e}^{\tan x} + \left| \left\{ \log t - \frac{1}{2} \log(1+t^2) \right\} \right|_{1/e}^{\cot x}$$

$$= \frac{1}{2} \left[ \log \sec^2 x - \log \left( 1 + \frac{1}{e^2} \right) \right] + \log \cot x - \log \left( \frac{1}{e} \right)$$

$$= -\frac{1}{2} \left\{ \log(\cosec^2 x) - \log \left( 1 + \frac{1}{e^2} \right) \right\} = -\log \left( \frac{1}{e} \right) = \log e = 1$$

**52. (a)**

**53. (b) :** Let  $I = \int_0^\pi x \log \sin x dx$  ... (i)

$$= \int_0^\pi (\pi - x) \log \sin(\pi - x) dx$$
 ... (ii)

By adding (i) and (ii), we get

$$2I = \int_0^\pi \pi \log \sin x dx \Rightarrow I = \frac{2\pi}{2} \int_0^{\pi/2} \log \sin x dx$$

$$= \pi \left( \frac{\pi}{2} \log \frac{1}{2} \right) = \frac{\pi^2}{2} \log \frac{1}{2}$$

**54. (d) :**  $\int_0^{\pi/2} \log \tan x dx = \int_0^{\pi/2} \log \left( \frac{\sin x}{\cos x} \right) dx$

$$= \int_0^{\pi/2} \log \sin x dx - \int_0^{\pi/2} \log \cos x dx = 0$$

$$\left\{ \because \int_0^a f(x) dx = \int_0^a f(a-x) dx \right\}$$

**55. (a) :** Since  $f(-\theta) = \log \left( \frac{2-\sin \theta}{2+\sin \theta} \right)^{-1}$

$$= -\log \left( \frac{2-\sin \theta}{2+\sin \theta} \right) = -f(\theta)$$

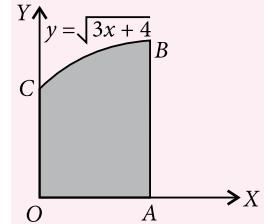
$\therefore f(x)$  is an odd function of  $x$ .

Therefore,  $2 \int_0^{\pi/2} \log \left( \frac{2-\sin \theta}{2+\sin \theta} \right) d\theta = 0$ .

**56. (d) :** Area  $= \int_0^4 \sqrt{3x+4} dx$

$$= \left| \frac{(3x+4)^{3/2}}{3 \cdot (3/2)} \right|_0^4$$

$$= \frac{112}{9}$$
 sq. unit



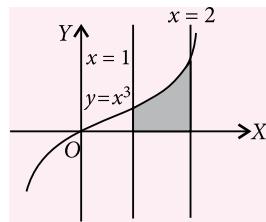
**57. (d)**

### EXAM CORNER 2018

Exam	Date
VITEEE	4 <sup>th</sup> to 15 <sup>th</sup> April
JEE Main	8 <sup>th</sup> April (Offline), 15 <sup>th</sup> & 16 <sup>th</sup> April (Online)
SRMJEEE	16 <sup>th</sup> to 30 <sup>th</sup> April
Karnataka CET	18 <sup>th</sup> & 19 <sup>th</sup> April
WBJEE	22 <sup>nd</sup> April
Kerala PET	23 <sup>rd</sup> & 24 <sup>th</sup> April
MHT CET	10 <sup>th</sup> May
COMEDK (Engg.)	13 <sup>th</sup> May
AMU (Engg.)	13 <sup>th</sup> May (Revised)
BITSAT	16 <sup>th</sup> to 31 <sup>st</sup> May
JEE Advanced	20 <sup>th</sup> May

**58. (b) :** Required area

$$\begin{aligned} &= \int_1^2 y \, dx = \int_1^2 x^3 \, dx = \frac{1}{4} [x^4]_1^2 \\ &= \frac{1}{4} [16 - 1] = \frac{15}{4} \text{ sq. unit} \end{aligned}$$



**59. (a) :** The curves  $y = x$  and  $y = x + \sin x$  intersect at  $(0, 0)$  and  $(\pi, \pi)$ . Hence area bounded by the two curves

$$\begin{aligned} &= \int_0^\pi (x + \sin x) \, dx - \int_0^\pi x \, dx = \int_0^\pi \sin x \, dx \\ &= [-\cos x]_0^\pi = -\cos \pi + \cos 0 = -(-1) + (1) = 2 \end{aligned}$$

**60. (a) :** Given curve  $y = a\sqrt{x} + bx$ .

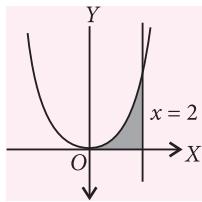
This curve passes through  $(1, 2)$ ,  $\therefore 2 = a + b$  ... (i)  
and area bounded by this curve and line  $x = 4$  and  $x$ -axis is 8 sq. unit, then

$$\begin{aligned} &\int_0^4 (a\sqrt{x} + bx) \, dx = 8 \\ &\Rightarrow \frac{2a}{3}[x^{3/2}]_0^4 + \frac{b}{2}[x^2]_0^4 = 8 \Rightarrow \frac{2a}{3}(8) + 8b = 8 \\ &\Rightarrow 2a + 3b = 3 \quad \dots \text{(ii)} \end{aligned}$$

From equation (i) and (ii), we get  $a = 3$ ,  $b = -1$ .

**61. (b) :** Given  $x^2 = 4y$

$$\begin{aligned} &\therefore \text{Required area} = \int_0^2 \frac{x^2}{4} \, dx \\ &= \frac{1}{12}[x^3]_0^2 = \frac{8}{12} = \frac{2}{3} \end{aligned}$$

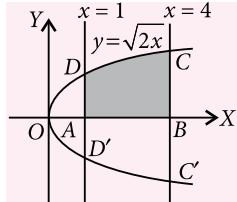


**62. (c) :** Given curve is  $y(x-2) = 3x+10 \Rightarrow y = \frac{3x+10}{x-2}$

$$\begin{aligned} &\therefore \text{Required area is } \int_3^4 y \, dx = \int_3^4 \frac{3x+10}{x-2} \, dx \\ &= [3x + 16 \log(x-2)]_3^4 = (3 + 16 \log 2) \text{ sq. unit} \end{aligned}$$

**63. (b) :** Required area =  $CDD'C'$

$$\begin{aligned} &= 2 \times ABCD \\ &= 2 \int_1^4 \sqrt{2x^{1/2}} \, dx \\ &= \frac{28\sqrt{2}}{3} \text{ sq. unit} \end{aligned}$$

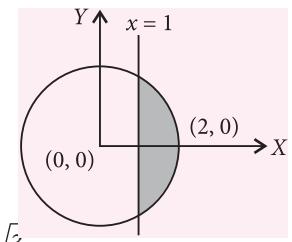


$$\begin{aligned} \text{64. (d) : } &\int_1^b f(x) \, dx = \sqrt{b^2 + 1} - \sqrt{2} = \sqrt{b^2 + 1} - \sqrt{1+1} \\ &= [\sqrt{x^2 + 1}]_1^b \end{aligned}$$

$$\therefore f(x) = \frac{d}{dx} \sqrt{x^2 + 1} = \frac{2x}{2\sqrt{x^2 + 1}} = \frac{x}{\sqrt{x^2 + 1}}$$

**65. (a) :** Area of smaller part =  $2 \int_1^2 \sqrt{4-x^2} \, dx$

$$\begin{aligned} &= 2 \left[ \frac{x}{2} \sqrt{4-x^2} + 2 \sin^{-1} \frac{x}{2} \right]_1^2 \\ &= 2 \left[ 2 \cdot \frac{\pi}{2} - \left[ \frac{\sqrt{3}}{2} + 2 \cdot \frac{\pi}{6} \right] \right] \\ &= 2 \left[ \pi - \left[ \frac{\sqrt{3}}{2} + \frac{\pi}{3} \right] \right] = \frac{4\pi}{3} - \sqrt{3} \end{aligned}$$



**66. (d) :** Required area =  $\int_0^{\pi/4} \tan x \, dx = [\log \sec x]_0^{\pi/4}$

$$= \log \sec(\pi/4) - \log \sec 0 = \log \sqrt{2} - \log 1 = \log \sqrt{2}$$

**67. (a) :** The parabola meets  $x$ -axis at the points, where

$$\frac{3}{a}(a^2 - x^2) = 0 \Rightarrow x = \pm a.$$

So the required area

$$= \int_{-a}^a \frac{3}{a}(a^2 - x^2) \, dx = \frac{6}{a} \int_0^a (a^2 - x^2) \, dx = (4a^2) \text{ sq. unit}$$

**68. (a) :**  $\because \int_0^1 \frac{x^{n-1} - x^n + x^{2n-1}}{1+x^n} \, dx > 0 \quad \forall n \in N$

$$\Rightarrow \int_0^1 \left\{ \frac{1}{1+x^n} - (1-x^{n-1}) \right\} \, dx > 0$$

$$\Rightarrow \int_0^1 \frac{dx}{1+x^n} > \int_0^1 (1-x^{n-1}) \, dx$$

$$\Rightarrow \int_0^1 \frac{dx}{1+x^n} > 1 - \frac{1}{n}; \quad \forall n \in N$$

**69. (d)**

**70. (c) :** Put  $3x = t \Rightarrow dx = \frac{dt}{3}$

$$\frac{1}{3} \int e^t \left( 3 \sin \frac{t}{3} + \cos \frac{t}{3} \right) dt = e^t \sin \frac{t}{3} + c = e^{3x} \sin x + c$$

Statement-2 is false.

$$\text{71. (d) : } \int \frac{e^x}{2^2 - (3e^x)^2} \, dx = \frac{1}{4} \log \left| \frac{2+3e^x}{2-3e^x} \right| + C$$

$$\text{72. (a) : } \int \frac{e^x (2-x^2)}{(1-x)\sqrt{1-x^2}} \, dx = \int \frac{e^x (1+(1-x^2))}{(1-x)\sqrt{1-x^2}} \, dx$$

$$\begin{aligned}
&= \int e^x \left( \frac{1}{(1-x)\sqrt{1-x^2}} + \frac{1+x}{\sqrt{1-x^2}} \right) dx \\
&= \int e^x \left[ \frac{1}{(1-x)^{3/2} (1+x)^{1/2}} + \sqrt{\frac{1+x}{1-x}} \right] dx \\
&= e^x \sqrt{\frac{1+x}{1-x}} + c \quad dx
\end{aligned}$$

$$\begin{aligned}
73. (d) : \int \cot^n x dx &= \int \cot^{n-2} x \cdot \cot^2 x dx \\
&= \int \cot^{n-2} x \cdot \operatorname{cosec}^2 x dx - \int \cot^{n-2} x dx
\end{aligned}$$

$$\therefore \int \cot^n x = \frac{-\cot^{n-1} x}{n-1} - \int \cot^{n-2} x dx$$

$$I = \int (\cot^{24} x + \cot^{22} x + \dots + 1) dx$$

$$\begin{aligned}
&= (I_{24} + I_{22}) + (I_{20} + I_{18}) + \dots + (I_4 + I_2) + \int 1 dx \\
&= \frac{-\cot^{23} x}{23} - \frac{\cot^{19} x}{19} - \dots - \frac{\cot^3 x}{3} + x + c
\end{aligned}$$

$$74. (a) : \text{We have, } \int \frac{e^{\cot x}}{\sin^2 x} \left( \ln \operatorname{cosec}^2 x + \frac{2 \tan x}{1 + \tan^2 x} \right) dx$$

Let  $t = \cot x \Rightarrow dt = -\operatorname{cosec}^2 x dx$

$$\begin{aligned}
&= - \int e^t \left( \ln(1+t^2) + \frac{\frac{2}{t}}{1+\frac{1}{t^2}} \right) dt \\
&- e^t \ln|1+t^2| + c = -2e^{\cot x} \ln|\operatorname{cosec} x| + c \\
&\quad = 2e^{\cot x} \ln \sin x + c
\end{aligned}$$

$$\begin{aligned}
75. (b) : \text{Let } I &= \int_1^\infty \frac{x^2 - 2}{x^3 \sqrt{x^2 - 1}} dx \\
&= \int_1^\infty \frac{x^2}{x^3 \sqrt{x^2 - 1}} dx - 2 \int_1^\infty \frac{dx}{x^3 \sqrt{x^2 - 1}}
\end{aligned}$$

Put  $x = \sec \theta \Rightarrow dx = \sec \theta \tan \theta d\theta$

$$\begin{aligned}
&\therefore I = \int_0^{\frac{\pi}{2}} \frac{\frac{\pi}{2} \sec \theta \cdot \tan \theta d\theta}{\sec \theta \cdot \tan \theta} - 2 \int_0^{\frac{\pi}{2}} \frac{\sec \theta \cdot \tan \theta d\theta}{\sec^3 \theta \cdot \tan \theta} \\
&= [\theta]_0^{\frac{\pi}{2}} - 2 \int_0^{\frac{\pi}{2}} \cos^2 \theta d\theta = \frac{\pi}{2} - 2 \int_0^{\frac{\pi}{2}} \left( \frac{1 + \cos 2\theta}{2} \right) d\theta \\
&= \frac{\pi}{2} - \frac{\pi}{2} = 0.
\end{aligned}$$

$$76. (a) : \int_a^{a+nT} f(x) dx = \int_0^{nT} f(x) dx = n \int_0^T f(x) dx$$

77. (b)

$$78. (a) : \text{Let } f(x) = \int_0^{\sin^2 x} \sin^{-1} \sqrt{x} dx + \int_0^{\cos^2 x} \cos^{-1} \sqrt{x} dx$$

$$\therefore f'(x) = 0 \Rightarrow f(x) = \text{constant.}$$

$$\text{and } f\left(\frac{\pi}{4}\right) = \int_0^{1/2} \frac{\pi}{2} dx = \frac{\pi}{4} \Rightarrow c = \frac{\pi}{4} \quad \dots(i)$$

As roots of  $g(x) = ax^2 + bx + c = 0$  are non real

$$\Rightarrow D < 0 \text{ and } g(0) = c > 0 \quad [\text{Using (i)}]$$

$$\Rightarrow a > 0$$

79. (d)

$$80. (a) : f(x) = \frac{e^x}{1+e^x} = \frac{1}{1+e^{-x}}$$

$$f(a) = \frac{1}{1+e^{-a}}, f(-a) = \frac{1}{1+e^a}$$

$$f(a) + f(-a) = 1$$

$$81. (c) : \text{Put } 3x = \cos \theta \Rightarrow 3dx = -\sin \theta d\theta$$

$$\begin{aligned}
&\therefore -\frac{1}{3} \int \frac{\frac{\cos \theta}{\sin \theta} + \theta^2}{\sin \theta} \cdot \sin \theta d\theta = -\frac{1}{3} \int \left( \frac{1}{3} \cos \theta + \theta^2 \right) d\theta \\
&- \frac{1}{9} \int \cos \theta d\theta - \frac{1}{3} \int \theta^2 d\theta = -\frac{1}{9} \sin \theta - \frac{1}{9} \theta^3 + c \\
&= -\frac{1}{9} \sqrt{1-9x^2} - \frac{1}{9} (\cos^{-1} 3x)^3 + c
\end{aligned}$$

$$\text{Clearly, } A = -\frac{1}{9} \text{ and } B = \frac{1}{9}$$

Hence,  $A + B = 0$ .

$$\begin{aligned}
82. (c) : x &= a \cos^3 t, y = a \sin^3 t, 0 \leq t \leq 2\pi \\
(xg'(t) - yf'(t)) &= a^2 (\cos^3 t \cdot 3 \sin^2 t \cos t + \sin^3 t \cdot 3 \cos^2 t \sin t) \\
&= 3a^2 \cos^2 t \sin^2 t
\end{aligned}$$

$$\begin{aligned}
\Rightarrow \text{Area of the curve} &= \frac{1}{2} \int_0^{2\pi} (xg'(t) - yf'(t)) dt \\
&= \frac{1}{2} \int_0^{2\pi} 3a^2 \cos^2 t \sin^2 t dt = \frac{3}{8} a^2 \pi
\end{aligned}$$

83. (c) : The curve is described when  $t$  varies from  $-1$  to  $1$ . Indeed when  $t = -1$ , both  $x$  and  $y$  are zero. When  $t = 0$ ,  $x = a$ ,  $y = 0$ . Thus the adjoining figure upper portion is described when  $0 \leq t \leq 1$  and the lower portion is described when  $-1 < t < 0$

Required area

$$= - \int_{-1}^1 g(t) f'(t) dt = - \int_{-1}^1 at(1-t^2)(-2at) dt = \frac{8a^2}{15}$$

**84. (a)** : This is a circle of radius 2.

**85. (c)**

**86. (d)**

**87. (a)**

$$I_n = \int x^n \sqrt{a^2 - x^2} dx = \int x^{n-1} \left( x \sqrt{a^2 - x^2} \right) dx$$

Apply integration by parts

$$\begin{aligned} I_n &= -\frac{1}{3}(a^2 - x^2)^{3/2} x^{n-1} + \frac{n-1}{3} \int x^{n-2} (a^2 - x^2)^{3/2} dx \\ &= -\frac{1}{3}(a^2 - x^2)^{3/2} x^{n-1} + \frac{n-1}{3} \\ &\quad \int \left( a^2 x^{n-2} \sqrt{a^2 - x^2} - x^n \sqrt{a^2 - x^2} \right) dx \\ I_n &= -\frac{1}{3}(a^2 - x^2)^{3/2} x^{n-1} + \frac{n-1}{3} a^2 I_{n-2} - \frac{n-1}{3} I_n \\ \Rightarrow I_n &= \frac{-(a^2 - x^2)^{3/2} x^{n-1}}{n+2} + \left( \frac{n-1}{n+2} \right) a^2 I_{n-2} \end{aligned}$$

$$\begin{aligned} \Rightarrow A = n+2, B = \frac{n-1}{n+2} \text{ If } I_n &= \int_0^a x^n \sqrt{a^2 - x^2} dx \\ \Rightarrow I_n &= \left( \frac{n-1}{n+2} \right) a^2 I_{n-2} \Rightarrow I_4 = \frac{3}{6} a^2 I_2 \end{aligned}$$

$$\text{and } I_2 = \frac{1}{4} a^2 I_0. \text{ Also, } I_0 = \frac{\lambda a^2}{4}$$

$$\therefore I_4 = \frac{3}{6} \times \frac{1}{4} \times \frac{\pi}{4} a^6 = \frac{\pi a^6}{32}$$

$$\text{88. (c)} : \frac{dy}{dx} = \frac{1}{x+y} \quad \dots(\text{i}) \text{ [Given]}$$

$$\text{Put } y+x=v \quad \dots(\text{ii})$$

$$\Rightarrow \frac{dy}{dx} + 1 = \frac{dv}{dx} \Rightarrow \frac{dv}{dx} - 1 = \frac{1}{v} \quad [\text{Using (i) and (ii)}]$$

$$\Rightarrow \frac{dv}{dx} = \frac{v+1}{v} \Rightarrow dv \left( \frac{v+1-1}{v+1} \right) = dx$$

Integrating both sides, we get

$$\Rightarrow v - \ln(v+1) = x + c$$

$$\Rightarrow y + x - \ln(x+y+1) = x + c$$

Also  $x=0, y=0 \Rightarrow c=0$

$$\therefore y = \ln(x+y+1) \Rightarrow x = e^y - y - 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{e^y - 1} \Rightarrow \left. \frac{dy}{dx} \right|_{\ln 3} = \frac{1}{e^{\ln 3} - 1} = \frac{1}{2}$$

$$\text{89. (c)} : \text{Area} = \int_0^1 (e^y - y - 1) dy = \left[ e - \frac{1}{2} - 1 \right] - 1 = e - \frac{5}{2}$$

$$\text{90. (b)} : [|\sin \alpha| + |\cos \alpha|]$$

$$\int_{-1}^1 xe^{-y} d(e^y)$$

$$\text{Let } e^y = z \Rightarrow e^y \frac{dy}{dz} = 1 \Rightarrow dy = e^{-y} dz$$

$$\text{Since } 1 \leq |\sin \alpha| + |\cos \alpha| \leq \sqrt{2} \therefore [|\sin \alpha| + |\cos \alpha|] = 1$$

$$\Rightarrow \int_{-1}^1 (e^y - y - 1) dy = e - e^{-1} - 2$$

$$\text{91. (b)} : \text{Let } I(a) = \int_0^1 \frac{x^a - 1}{\log x} dx$$

Diff w.r.t.  $a$  keeping  $x$  as constant

$$\frac{d}{da} I(a) = \int_0^1 x^a dx \Rightarrow \frac{d}{d(a)} I(a) = \frac{1}{a+1}$$

Integrating both sides w.r.t  $a$ , we get

$$I(a) = \log(a+1) + C$$

$$I(0) = 0 \Rightarrow C = 0 \Rightarrow I(a) = \log(a+1)$$

**92. (c)**

$$\text{93. (c)} : \int_0^\pi \frac{dx}{a - \cos x} = \frac{\pi}{\sqrt{a^2 - 1}}$$

Diff w.r.t.  $a$  on both sides treating  $x$  as constant, we get

$$-\int_0^\pi \frac{dx}{(a - \cos x)^2} = \frac{-\pi a}{(a^2 - 1)^{3/2}}$$

Again diff. w.r.t.  $a$  treating  $x$  as a constant, we get

$$2 \int_0^\pi \frac{dx}{(a - \cos x)^3} = \frac{\pi(1+2a^2)}{(a^2 - 1)^{5/2}}$$

$$\therefore \int_0^\pi \frac{dx}{(\sqrt{10} - \cos x)^3} = \frac{\pi(1+2a^2)}{2(9)^{5/2}} = \frac{7\pi}{162}$$

**94. (A)  $\rightarrow$  (r), (B)  $\rightarrow$  (p), (C)  $\rightarrow$  (q)**

$$(A) \int x^2 \cdot \frac{1}{1+x^2} dx = x - \tan^{-1} x + c$$

$$\Rightarrow f(x) = \tan^{-1} x \Rightarrow f(1) = \tan^{-1} 1 = \pi/4$$

$$(B) \int x^2 e^{2x} dx = \frac{1}{2} x^2 \cdot e^{2x} - \int 2x \cdot \frac{1}{2} e^{2x} dx$$

$$= \frac{1}{2} x^2 e^{2x} - \left[ x \cdot \frac{1}{2} e^{2x} - \frac{1}{4} e^{2x} \right] = \left( \frac{x^2}{2} - \frac{x}{2} + \frac{1}{4} \right) e^{2x} + c$$

$$\begin{aligned}
 \text{(C)} \int \frac{(x^4+1)}{x(x^2+1)^2} dx &= \int \frac{(x^2+1)^2 - 2x^2}{x(x^2+1)^2} dx \\
 &= \int \frac{1}{x} dx - 2 \int \frac{x dx}{(x^2+1)^2} = \log|x| + \frac{1}{x^2+1} + c
 \end{aligned}$$

95. (A)  $\rightarrow$  (s), (B)  $\rightarrow$  (r), (C)  $\rightarrow$  (q), (D)  $\rightarrow$  (q)

$$\begin{aligned}
 \text{(A)} \quad \text{Let } I &= 25 \int_0^{\pi/4} (\tan^6 x + \tan^4 x) dx \\
 &= 25 \int_0^{\pi/4} (\tan^4 x \sec^2 x) dx = 5
 \end{aligned}$$

$$\text{(B)} \quad \text{Let } x+1=t \text{ in } I_2 \text{ then } I_1 + I_2 = \int_{-2}^2 (x^2+7) dx = \frac{100}{3}$$

$$\text{(C)} \quad \text{Let } I = \int_{-1}^0 \frac{-\pi}{2} dx + \int_0^3 \frac{\pi}{2} dx = \pi$$

$$\text{96. (8): } \lim_{n \rightarrow \infty} \frac{\sum_{r=1}^n \sqrt{r} \sum_{r=1}^n \frac{1}{\sqrt{r}}}{\frac{n(n+1)}{2}} = \lim_{n \rightarrow \infty} \frac{\sum_{r=1}^n \sqrt{\frac{r}{n}} \sum_{r=1}^n \frac{1}{\sqrt{\frac{r}{n}}}}{\frac{n^2}{2} \left(1 + \frac{1}{n}\right)}$$

$$\begin{aligned}
 &= 2 \times \int_0^1 \sqrt{x} dx \int_0^1 \frac{1}{\sqrt{x}} dx \\
 &= 2 \times \left[ \frac{x^{3/2}}{\frac{3}{2}} \right]_0^1 \left[ \frac{x^{1/2}}{\frac{1}{2}} \right]_0^1 = 2 \times \frac{2}{3} \times 2 = \frac{8}{3}
 \end{aligned}$$

$$\therefore k = 8$$

$$\text{97. (1): } \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{k^{\frac{1}{a}} \left\{ n^{\frac{a-1}{a}} + k^{\frac{a-1}{a}} \right\}}{n^{a+1}}$$

$$\begin{aligned}
 &= \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{n} \cdot \left\{ \left(\frac{k}{n}\right)^{1/a} + \left(\frac{k}{n}\right)^a \right\} = \int_0^1 (x^{1/a} + x^a) dx \\
 &= \left\{ \frac{x^{(1/a)+1}}{\frac{1}{a}+1} + \frac{x^{a+1}}{a+1} \right\}_0^1 = \frac{a}{a+1} + \frac{1}{a+1} = 1
 \end{aligned}$$

$$\text{98. (2): } f(x) = \frac{x^9 - 3x^5 + 7x^3 - x}{\cos^2 x} + \sec^2 x$$

$$\begin{aligned}
 &= \sec^2 x (x^9 - 3x^5 + 7x^3 - x) + \sec^2 x \\
 \Rightarrow \int_{-\pi/4}^{\pi/4} f(x) dx &= \int_{-\pi/4}^{\pi/4} \sec^2 x dx
 \end{aligned}$$

$$= 2 \int_0^{\pi/4} \sec^2 x dx = 2 \tan x \Big|_0^{\pi/4} = 2$$

$$\text{99. (8): } \int_1^4 f'(x) dx = [xf'(x)]_1^4 - \int_1^4 xf''(x) dx$$

$$\begin{aligned}
 \text{Let } I &= \int_1^4 xf''(x) dx = \int_1^4 (5-x)f''(5-x) dx \\
 \Rightarrow I &= 5 \int_1^4 f''(x) dx - I \quad \therefore I = \frac{5}{2}[f'(4) - f'(1)]
 \end{aligned}$$

$$\text{So, } \int_1^4 f'(x) dx = \frac{3}{2}[f'(4) + f'(1)]$$

$$\text{Now, } f''(x) = f''(5-x) \Rightarrow f'(x) = -f'(5-x) + c$$

$$f'(0) + f'(5) = c \Rightarrow c = 8$$

$$\text{So, } f'(x) + f'(5-x) = 8 \Rightarrow f'(4) + f'(1) = 8$$

**100.(3):** Let  $\alpha, \beta$  be roots of the quadratic equation  $18x^2 - 9\pi x + \pi^2 = 0$

$$\therefore \alpha = \frac{\pi}{6}, \beta = \frac{\pi}{3}$$

And,  $(fog)(x) = f(\cos x) = \cos 2x$

$$\text{Area} = \int_{\pi/6}^{\pi/3} \cos^2 x dx = \frac{\pi}{12}$$

$$\text{101. (2): } \text{Let } I_n = \int_0^{\pi/2} (\sin x + \cos x)^{n-1} \cdot (\sin x + \cos x) dx$$

$$I_n = \int_0^{\pi/2} (\sin x + \cos x)^{n-1} (\sin x - \cos x)' dx$$

$$= [\sin x + \cos x]^{n-1} \cdot (\sin x - \cos x) \Big|_0^{\pi/2}$$

$$- \int_0^{\pi/2} (n-1)(\sin x + \cos x)^{n-2} \cdot (\cos x - \sin x)(\sin x - \cos x) dx$$

$$= 2 + (n-1) \int_0^{\pi/2} (\sin x + \cos x)^{n-2} (\cos x - \sin x)^2 dx$$

$$= 2 + (n-1) \int_0^{\pi/2} (\sin x + \cos x)^{n-2} \left[ 2 - (\sin x + \cos x)^2 \right] dx$$

$$= 2 + 2(n-1)I_{n-2} - (n-1)I_n$$

$$\Rightarrow nI_n - 2(n-1)I_{n-2} = 2$$

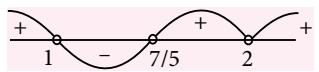
$$\begin{aligned}
 102. (2) : I &= \int_0^{\pi/2} x^2 \operatorname{cosec}^2 x dx \\
 &= \left[ -x^2 \cot x \right]_0^{\pi/2} + \int_0^{\pi/2} 2x \cot x dx \\
 &= 2 \left[ x \ln \sin x \right]_0^{\pi/2} - 2 \int_0^{\pi/2} \ln \sin x dx
 \end{aligned}$$

$$\begin{aligned}
 103. (7) : I &= \int_0^1 (1-x^4)^7 \cdot 1 dx \\
 &= \left[ x(1-x^4)^7 \right]_0^1 + 7 \times 4 \int_0^1 x(1-x^4)^6 x^3 dx \\
 &= 0 - 28 \int_0^1 (1-x^4)^6 [(1-x^4)-1] dx \\
 &= -28 \int_0^1 (1-x^4)^6 (1-x^4) dx + 28 \int_0^1 (1-x^4)^6 dx \\
 &= -28I + 28 \int_0^1 (1-x^4)^6 dx \\
 \Rightarrow 29I &= 28 \int_0^1 (1-x^4)^6 dx
 \end{aligned}$$

$$\Rightarrow \frac{\int_0^1 (1-x^4)^7 dx}{4 \int_0^1 (1-x^4)^6 dx} = \frac{28 \int_0^1 (1-x^4)^6 dx}{4 \int_0^1 (1-x^4)^6 dx} = 7$$

$$\begin{aligned}
 104. (1) : \int_0^{\pi} \frac{dx}{1+2\sin^2 x} &= 2 \int_0^{\pi/2} \frac{dx}{1+2\sin^2 x} \\
 &= 2 \int_0^{\pi/2} \frac{\sec^2 x dx}{1+3\tan^2 x} = 2 \int_0^{\infty} \frac{dt}{1+(\sqrt{3t})^2} \\
 \text{Put } t = \tan x \Rightarrow dt &= \sec^2 x dx \\
 &= \frac{2}{\sqrt{3}} \left[ \tan^{-1} \sqrt{3t} \right]_0^{\infty}
 \end{aligned}$$

$$\begin{aligned}
 105. (1) : f'(x) &= 2(x-1)(x-2)^3 + 3(x-1)^2(x-2)^2 \\
 &= (x-1)(x-2)^2 \{2(x-2) + 3(x-1)\} \\
 &= (x-1)(x-2)^2(5x-7)
 \end{aligned}$$



sign change of  $f'(x)$  from +ve to -ve at  $x = 1$   
 $\therefore$  maximum at  $x = 1$ .



**ATTENTION  
COACHING  
INSTITUTES :**  
a great offer from  
**MTG**

## CLASSROOM STUDY MATERIAL



MTG offers "Classroom Study Material" for JEE (Main & Advanced), NEET and FOUNDATION MATERIAL for Class 6, 7, 8, 9, 10, 11 & 12 **with YOUR BRAND NAME & COVER DESIGN.**

This study material will save you lots of money spent on teachers, typing, proof-reading and printing. Also, you will save enormous time. Normally, a good study material takes 2 years to develop. But you can have the material printed with your logo delivered at your doorstep.

Profit from associating with MTG Brand – the most popular name in educational publishing for JEE (Main & Advanced)/NEET/PMT....

Order sample chapters on Phone/Fax/e-mail.

Phone : 0124-6601200 | 09312680856  
 e-mail : sales@mtg.in | www.mtg.in

\* EXCELLENT  
QUALITY \*  
\* CONTENT  
\* PAPER  
\* PRINTING

# PRACTICE PAPER

# JEE MAIN

2018

Exam Dates

OFFLINE : 8<sup>th</sup> April

ONLINE : 15<sup>th</sup> & 16<sup>th</sup> April

- 1.** Number of 4 digit numbers of the form  $N = abcd$  which satisfy following three conditions:

- (i)  $4000 \leq N < 6000$     (ii)  $N$  is a multiple of 5  
 (iii)  $3 \leq b < c \leq 6$ , is equal to  
 (a) 12    (b) 18    (c) 24    (d) 48

- 2.** The angle between pair of tangents drawn to the curve  $7x^2 - 12y^2 = 84$  from  $M(1, 2)$  is

- (a)  $2 \tan^{-1} \frac{1}{2}$     (b)  $2 \tan^{-1} 2$   
 (c)  $2 \left( \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{2} \right)$   
 (d)  $2 \tan^{-1} 3$

- 3.** Let  $\binom{n}{k}$  represents the combination of 'n' things taken 'k' at a time, then the value of the sum  $\binom{99}{97} + \binom{98}{96} + \binom{97}{95} + \dots + \binom{3}{1} + \binom{2}{0}$  equals  
 (a)  $\binom{99}{97}$     (b)  $\binom{100}{98}$     (c)  $\binom{99}{98}$     (d)  $\binom{100}{97}$

- 4.** For any natural number  $m$ ,  
 $\int (x^{7m} + x^{2m} + x^m)(2x^{6m} + 7x^m + 14)^{1/m} dx$   
 (where  $x > 0$ ), equals

- (a)  $\frac{(7x^{7m} + 2x^{2m} + 14x^m)^{\frac{m+1}{m}}}{14(m+1)} + C$   
 (b)  $\frac{(2x^{7m} + 14x^{2m} + 7x^m)^{\frac{m+1}{m}}}{14(m+1)} + C$   
 (c)  $\frac{(2x^{7m} + 7x^{2m} + 14x^m)^{\frac{m+1}{m}}}{14(m+1)} + C$   
 (d)  $\frac{(7x^{7m} + 2x^{2m} + x^m)^{\frac{m+1}{m}}}{14(m+1)} + C$

where  $C$  is constant of integration.

- 5.** Let  $A = \begin{bmatrix} l & m & n \\ p & q & r \\ 1 & 1 & 1 \end{bmatrix}$  and  $B = A^2$ .

- If  $(l - m)^2 + (p - q)^2 = 9$ ,  $(m - n)^2 + (q - r)^2 = 16$ ,  $(n - l)^2 + (r - p)^2 = 25$ , then the value of  $\det B$  equals  
 (a) 100    (b) 125    (c) 144    (d) 169

- 6.** Let  $\alpha, \beta \in \mathbb{R}$ . If  $\alpha, \beta^2$  be the roots of quadratic equation  $x^2 - px + 1 = 0$  and  $\alpha^2, \beta$  be the roots of quadratic equation  $x^2 - qx + 8 = 0$ , then the value of ' $r'$  if  $\frac{r}{8}$  be arithmetic mean of  $p$  and  $q$ , is

- (a)  $\frac{83}{8}$     (b)  $\frac{83}{4}$     (c)  $\frac{83}{2}$     (d) 83

- 7.** The range of  $k$  for which the inequality  $k \cos^2 x - k \cos x + 1 \geq 0 \forall x$ , is

- (a)  $k < -\frac{1}{2}$     (b)  $k > 4$   
 (c)  $-\frac{1}{2} \leq k \leq 4$     (d)  $-\frac{1}{2} \leq k \leq 2$

- 8.** Let  $g(x) = ax + b$ , where  $a < 0$  and  $g$  is defined from  $[1, 3]$  onto  $[0, 2]$  then the value of

- $\cot(\cos^{-1}(|\sin x| + |\cos x|)) + \sin^{-1}(-|\cos x| - |\sin x|)$   
 is equal to

- (a)  $g(1)$     (b)  $g(2)$   
 (c)  $g(3)$     (d)  $g(1) + g(3)$

- 9.**  $\int_0^\theta \ln(1 + \tan \theta \cdot \tan x) dx$  is equal to

- (a)  $\theta \ln(\sec \theta)$     (b)  $\theta \ln(\cosec \theta)$   
 (c)  $\frac{\theta \cdot \ln 2}{2}$     (d)  $2\theta \ln \sec \theta$

- 10.**  $\frac{\sin x}{\cos 3x} + \frac{\sin 3x}{\cos 9x} + \frac{\sin 9x}{\cos 27x}$  equals

- (a)  $\tan 3x - \tan x$     (b)  $\frac{1}{2} (\tan 9x - \tan 3x)$   
 (c)  $\frac{1}{2} (\tan 27x - \tan x)$     (d)  $\frac{1}{2} (\tan 27x + \tan 9x)$

**11.** Let  $a, b, c, d$  are non-zero real numbers such that  $6a + 4b + 3c + 3d = 0$ , then the equation  $ax^3 + bx^2 + cx + d = 0$  has

- (a) atleast one root in  $[-2, 0]$
- (b) atleast one root in  $[0, 2]$
- (c) atleast two roots in  $[-2, 2]$
- (d) no root in  $[-2, 2]$

**12.** Let  $f: (0, \infty) \rightarrow R$  and  $F(x) = \int_0^x f(t)dt$

If  $F(x^2) = x^4 + x^5$ , then  $\sum_{r=1}^{12} f(r^2)$  is equal to

- (a) 216    (b) 219    (c) 222    (d) 225

**13.** If  $\vec{a}, \vec{b}$  and  $\vec{c}$  are non-zero vectors then the value of the scalar  $((\vec{a} \times \vec{b}) \times \vec{a}) \cdot ((\vec{b} \times \vec{a}) \times \vec{b})$  equals

- (a)  $-(\vec{a} \cdot \vec{b}) |\vec{a} \times \vec{b}|^2$     (b)  $\vec{a}^2 |\vec{a} \times \vec{b}|^2$
- (c)  $|\vec{b}|^2 |\vec{a} \times \vec{b}|^2$     (d)  $(\vec{a} \cdot \vec{b}) |\vec{a} \times \vec{b}|^2$

**14.** If  $Z$  is a non real complex number then the

minimum value of  $\frac{\operatorname{Im} Z^5}{\operatorname{Im}^5 Z}$ , is

- (a) -1    (b) -2    (c) -4    (d) -5

**15.** Suppose that three points on the parabola  $y = x^2$  have the property that their normal lines intersect at a common point  $(a, b)$ . The sum of their  $x$ -coordinates is

- (a) 0    (b)  $\frac{2b-1}{2}$     (c)  $\frac{a}{2}$     (d)  $a+b$

**16.** If the circle  $x^2 + y^2 + (3 + \sin\beta)x + (2\cos\alpha)y = 0$  and  $x^2 + y^2 + (2\cos\alpha)x + 2cy = 0$  touches each other, then the maximum value of  $|c|$  is

- (a)  $1/2$     (b) 1    (c)  $3/2$     (d) 2

**17.** If range of the function  $f(x) = \sin^{-1}x + 2\tan^{-1}x + x^2 + 4x + 1$  is  $[p, q]$  then  $p + q$  equals

- (a)  $-\pi + 4$     (b)  $\frac{3\pi}{4} + 2$     (c) 4    (d) 8

**18.** The complete set of values of the parameter  $\alpha$  so that the point  $P(\alpha, (1 + \alpha^2)^{-1})$  does not lie outside the triangle formed by the lines  $L_1: 15y = x + 1$ ,  $L_2: 78y = 118 - 23x$  and  $L_3: y + 2 = 0$ , is

- (a) (0, 5)    (b) [2, 5]    (c) [1, 5]    (d) [0, 2]

**19.** If the variable line  $y = kx + 2h$  is tangent to an ellipse  $2x^2 + 3y^2 = 6$ , then locus of  $P(h, k)$  is a conic  $C$  whose eccentricity equals

- (a)  $\frac{\sqrt{5}}{2}$     (b)  $\frac{\sqrt{7}}{3}$     (c)  $\frac{\sqrt{7}}{2}$     (d)  $\sqrt{\frac{7}{3}}$

**20.** An ellipse has semi major axis of length 2 and semi minor axis of length 1. The distance between its foci (units) is

- (a)  $2\sqrt{3}$     (b) 3    (c)  $2\sqrt{2}$     (d)  $\sqrt{3}$

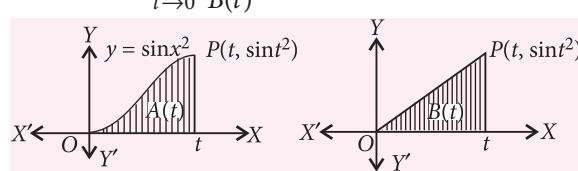
**21.** A coin that comes up head with probability  $p > 0$  and tails with probability  $1 - p > 0$  independently on each flip, is flipped eight times. Suppose the probability of three heads and five tails is equal to  $1/25$  of the probability of five heads and three tails. Let  $p = \frac{m}{n}$ , where  $m$  and  $n$  are relative prime positive integers. The value of  $m + n$  equals

- (a) 9    (b) 11    (c) 13    (d) 15

**22.** Equation of the circle which cuts the circle  $x^2 + y^2 - 2x - 4y + 4 = 0$  and the lines  $xy - 2x - y + 2 = 0$  orthogonally, is

- (a)  $x^2 + y^2 - 2x - 4y + 12 = 0$
- (b)  $x^2 + y^2 - 2x - 4y + 6 = 0$
- (c)  $x^2 + y^2 - 2x - 4y - 6 = 0$
- (d) none of these

**23.** The figure shows two regions in the first quadrant.  $A(t)$  is the area under the curve  $y = \sin x^2$  from 0 to  $t$  and  $B(t)$  is the area of the triangle with vertices  $O, P$  and  $M(t, 0)$ . Then,  $\lim_{t \rightarrow 0} \frac{A(t)}{B(t)}$  equals



- (a)  $3/5$     (b)  $2/3$     (c)  $1/3$     (d)  $1/2$

**24.** Let  $S(t)$  be the area of the  $\Delta OAB$  with  $O(0, 0, 0)$ ,  $A(2, 2, 1)$  and  $B(t, 1, t+1)$ . The value of the definite

integral  $\int_1^e (S(t))^2 \ln t dt$ , is equal to

- (a)  $\frac{2e^3 + 5}{2}$     (b)  $\frac{e^3 + 5}{2}$
- (c)  $\frac{2e^3 + 15}{2}$     (d)  $\frac{e^3 + 15}{2}$

**25.** The number of straight lines equidistant from three non-collinear points in the plane of the points is equal to

- (a) 0    (b) 1    (c) 2    (d) 3



**26.** Two fair dice are rolled simultaneously. It is found that one of them shows odd prime numbers. The probability that remaining dice also shows an odd prime number, is equal to

- (a)  $1/5$  (b)  $2/5$  (c)  $3/5$  (d)  $4/5$

**27.** The straight lines  $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$  and

$$\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}, \text{ will intersect provided}$$

- (a)  $k = \{3, -3\}$  (b)  $k = \{0, -1\}$   
 (c)  $k = \{-1, 1\}$  (d)  $k = \{0, -3\}$

**28.** The length of projection, of the line segment joining the points  $(1, -1, 0)$  and  $(-1, 0, 1)$ , to the plane  $2x + y + 6z = 1$ , is equal to

- (a)  $\sqrt{\frac{255}{61}}$  (b)  $\sqrt{\frac{237}{41}}$  (c)  $\sqrt{\frac{137}{41}}$  (d)  $\sqrt{\frac{155}{61}}$

**29.** The sum  $\frac{1}{2} {}^{10}C_0 - {}^{10}C_1 + 2 \cdot {}^{10}C_2 - 2^2 \cdot {}^{10}C_3 + \dots + 2^9 \cdot {}^{10}C_{10}$

is equal to

- (a)  $\frac{1}{2}$  (b)  $0$  (c)  $\frac{1}{2} \cdot 3^{10}$  (d)  $3^3$

**30.** Let the  $r^{\text{th}}$  term,  $t_r$ , of a series is given by

$$t_r = \frac{r}{1+r^2+r^4}. \text{ Then } \lim_{n \rightarrow \infty} \sum_{r=1}^n t_r \text{ is}$$

- (a)  $1/4$  (b)  $1$   
 (c)  $1/2$  (d) none of these

**31.** Let  $f : [0, 1] \rightarrow [0, 1]$  be a continuous function.

Then

- (a)  $f(x) = x$  for at least one  $0 \leq x \leq 1$   
 (b)  $f(x)$  will be differentiable in  $[0, 1]$   
 (c)  $f(x) + x = 0$  for at least one  $x$  such that  $0 \leq x \leq 1$   
 (d) none of these

**32.** Let  $f(x)$  be a polynomial function of degree 2 and  $f(x) > 0$  for all  $x \in R$ . If  $g(x) = f(x) + f'(x) + f''(x)$  then for any  $x$

- (a)  $g(x) < 0$  (b)  $g(x) > 0$   
 (c)  $g(x) = 0$  (d)  $g(x) \geq 0$

**33.** Let  $f(x) = \tan^{-1}(\phi(x))$ , where  $\phi(x)$  is increasing for  $0 < x < \pi/2$ . Then  $f(x)$  is

- (a) increasing in  $(0, \pi/2)$   
 (b) decreasing in  $(0, \pi/2)$   
 (c) increasing in  $(0, \pi/4)$  and decreasing in  $(\pi/4, \pi/2)$   
 (d) none of these

**34.** If  $f(x) = a \log_e|x| + bx^2 + x$  has extrema at  $x = 1$  and  $x = 3$  then

- (a)  $a = -\frac{3}{4}$ ,  $b = -\frac{1}{8}$  (b)  $a = \frac{3}{4}$ ,  $b = -\frac{1}{8}$   
 (c)  $a = -\frac{3}{4}$ ,  $b = \frac{1}{8}$  (d) none of these

## SOLUTIONS

**1. (c)**: We have,  $N = \boxed{a \ b \ c \ d}$

First place  $a$  can be filled in 2 ways i.e. 4, 5  
 $(\because 4000 \leq N < 6000)$

For  $b$  and  $c$ , total possibilities are '6'  
*i.e.* 34, 35, 36, 45, 46, 56  
 $(\because 3 \leq b < c \leq 6)$   
 Last place  $d$  can be filled in 2 ways i.e. 0, 5  
 $(\because N$  is a multiple of 5)

Hence total numbers =  $2 \times 6 \times 2 = 24$

**2. (c)**: The director circle of given hyperbola  $\frac{x^2}{12} - \frac{y^2}{7} = 1$ , is  $x^2 + y^2 = 5$  and given point  $M(1, 2)$  lies on it.

$\Rightarrow$  The angle between pair of tangents =  $\pi/2$

$$\text{As, } \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{2} = \tan^{-1} 1 = \frac{\pi}{4}$$

$$\Rightarrow 2 \left( \tan^{-1} \left( \frac{1}{3} \right) + \tan^{-1} \left( \frac{1}{2} \right) \right) = \frac{\pi}{2}$$

$$\begin{aligned} \text{3. (d)}: S &= {}^2C_0 + {}^3C_1 + {}^4C_2 + {}^5C_3 + \dots + {}^{99}C_{97} \\ &= {}^4C_1 + {}^4C_2 + {}^5C_3 + \dots + {}^{99}C_{97} (\because {}^2C_0 + {}^3C_1 = {}^4C_1) \\ &= {}^5C_2 + {}^5C_3 + \dots + {}^{99}C_{97} \\ \therefore S &= {}^{100}C_{97} \end{aligned}$$

$$\begin{aligned} \text{4. (c)}: \text{Let } I &= \int (x^{7m-1} + x^{2m-1} + x^{m-1})(2x^{7m} + 7x^{2m} \\ &\quad + 14x^m)^{1/m} dx \end{aligned}$$

$$\text{Now, put } 2x^{7m} + 7x^{2m} + 14x^m = t^m$$

$$14m(x^{7m-1} + x^{2m-1} + x^{m-1})dx = m t^{m-1} dt$$

$$\begin{aligned} \therefore I &= \frac{1}{14} \int t^{m-1} (t^m)^{1/m} dt \\ &= \frac{t^{m+1}}{14(m+1)} + C = \frac{(2x^{7m} + 7x^{2m} + 14x^m)^{\frac{m+1}{m}}}{14(m+1)} + C \end{aligned}$$

**5. (c)**: Det.A is twice the area of the triangle with vertices  $(l, p), (m, q), (n, r)$  with sides 3, 4, 5.

$$\Delta^2 = s(s-a)(s-b)(s-c)$$

$$\Delta^2 = 6(6-3)(6-4)(6-5)$$

$$\Delta^2 = 36 \Rightarrow \Delta = 6$$

Now,  $\det A = 2\Delta = 12$

$$\Rightarrow \det B = (\det A)^2 = 4\Delta^2 = 144$$

- 6. (d):** For the equation  $x^2 - px + 1 = 0$ ,  
the product of roots,  $\alpha\beta^2 = 1$   
and for the equation  $x^2 - qx + 8 = 0$ ,  
the product of roots,  $\alpha^2\beta = 8$   
Hence,  $(\alpha\beta^2)(\alpha^2\beta) = 8 \Rightarrow \alpha^3\beta^3 = 8 \Rightarrow \alpha\beta = 2$   
 $\therefore$  From  $\alpha\beta^2 = 1$ , we have  $\beta = 1/2$  and from  $\alpha^2\beta = 8$ ,  
we have  $\alpha = 4$
- Hence, from sum of roots  $= -\frac{b}{a}$  relation, we have
- $$p = \alpha + \beta^2 = 4 + \frac{1}{4} = \frac{17}{4} \text{ and } q = \alpha^2 + \beta = 16 + \frac{1}{2} = \frac{33}{2}$$
- $$\therefore \frac{r}{8} \text{ is arithmetic mean of } p \text{ and } q$$
- $$\therefore \frac{r}{8} = \frac{p+q}{2} \Rightarrow r = 4(p+q) = 4\left(\frac{17}{4} + \frac{33}{2}\right) = 83$$

- 7. (c):** We have,  $k \cos^2 x - k \cos x + 1 \geq 0 \forall x$   
 $\Rightarrow k(\cos^2 x - \cos x) + 1 \geq 0$
- Now,  $\cos^2 x - \cos x = \left(\cos x - \frac{1}{2}\right)^2 - \frac{1}{4}$   
 $\therefore -\frac{1}{4} \leq \cos^2 x - \cos x \leq 2$   
 $\therefore 2k + 1 \geq 0 \text{ and } -\frac{k}{4} + 1 \geq 0 \Rightarrow -\frac{1}{2} \leq k \leq 4$
- 8. (c):** Consider  $F(x) = \cot(\cos^{-1}(|\sin x| + |\cos x|)) + \sin^{-1}(-|\cos x| - |\sin x|)$

But  $|\sin x| + |\cos x| \in [1, \sqrt{2}]$   
 $\therefore F(x) = \cot(\cos^{-1}(1) + \sin^{-1}(-1))$

$$= \cot\left(0 - \frac{\pi}{2}\right) = 0 = g(3) \text{ (As } F(x) = 0, \forall x \in D_F)$$

**9. (a):** Let  $I = \int_0^\theta \ln(1 + \tan \theta \cdot \tan(\theta - x)) dx$

$$= \int_0^\theta \ln\left(1 + \frac{\tan \theta \cdot (\tan \theta - \tan x)}{1 + \tan \theta \cdot \tan x}\right) dx$$

$$= \int_0^\theta \ln\left(\frac{1 + \tan^2 \theta}{1 + \tan \theta \cdot \tan x}\right) dx$$

$$= \int_0^\theta \ln(1 + \tan^2 \theta) dx - \int_0^\theta \ln(1 + \tan \theta \cdot \tan x) dx$$

$$I = 2\theta \ln \sec \theta - I$$

$$2I = 2\theta \ln \sec \theta \Rightarrow I = \theta \ln (\sec \theta)$$

**10. (c):**  $\frac{\sin x}{\cos 3x} = \frac{\sin 2x}{2 \cos x \cos 3x} = \frac{\sin(3x - x)}{2 \cos x \cos 3x}$

$$= \frac{1}{2}(\tan 3x - \tan x) \text{ etc.}$$

- 11. (b):** We have  $ax^3 + bx^2 + cx + d = 0$   
Let  $f(x) = \frac{ax^4}{4} + \frac{bx^3}{3} + \frac{cx^2}{2} + dx + e$   
 $\therefore f(0) = e$
- $$f(2) = 4a + \frac{8b}{3} + 2c + 2d + e = \frac{(12a + 8b + 6c + 6d)}{3} + e$$
- $$= \frac{2}{3}(6a + 4b + 3c + 3d) + e = 0$$
- $$\Rightarrow f(2) = e$$
- $\therefore$  By Rolle's theorem, there exist atleast one value of  $x \in (0, 2)$  such that  $f'(x) = 0$   
 $\Rightarrow$  The equation  $ax^3 + bx^2 + cx + d = 0$  has atleast one real root in  $[0, 2]$

- 12. (b):** We have  $F(x^2) = \int_0^{x^2} f(t) dt = x^4 + x^5$   
 $\therefore$  On differentiating w.r.t.  $x$ , we get
- $$\Rightarrow f(x)^2 = 2 + \frac{5}{2}x \quad \therefore \sum_{r=1}^{12} f(r^2) = 219$$

**13. (a):**  $((\vec{a} \times \vec{b}) \times \vec{a}) \cdot ((\vec{b} \times \vec{a}) \times \vec{b})$   
 $= ((\vec{a} \cdot \vec{a})\vec{b} - (\vec{b} \cdot \vec{a})\vec{a}) \cdot ((\vec{b} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{b})\vec{b})$   
 $= (\vec{a}^2 \vec{b}^2) \vec{a} \cdot \vec{b} - \vec{a}^2 \vec{b}^2 (\vec{a} \cdot \vec{b}) - (\vec{a} \cdot \vec{b}) \vec{a}^2 \vec{b}^2 + (\vec{a} \cdot \vec{b})^3$   
 $= -(\vec{a} \cdot \vec{b}) |\vec{a} \times \vec{b}|^2$

- 14. (c):** Let  $Z = a + ib$ ,  $b \neq 0$  where  $\operatorname{Im} Z = b$   
 $\therefore Z^5 = (a + ib)^5 = a^5 + {}^5C_1 a^4 bi + {}^5C_2 a^3 b^2 i^2 + {}^5C_3 a^2 b^3 i^3 + {}^5C_4 ab^4 i^4 + i^5 b^5$

$$\Rightarrow \operatorname{Im} Z^5 = 5a^4 b - 10a^2 b^3 + b^5$$

Now,  $y = \frac{\operatorname{Im} Z^5}{\operatorname{Im}^5 Z} = 5\left(\frac{a}{b}\right)^4 - 10\left(\frac{a}{b}\right)^2 + 1$

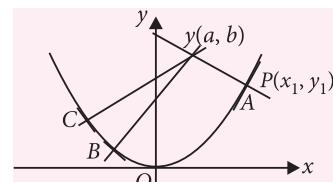
Let  $\left(\frac{a}{b}\right)^2 = x$  (say),  $x \in R^+$

$$y = 5x^2 - 10x + 1 = 5[x^2 - 2x] + 1 = 5[(x - 1)^2] - 4$$

Hence  $y_{\min} = -4$

- 15. (a):**  $y = x^2$

$$\left[ \frac{dy}{dx} \right]_{x_1} = 2x_1$$



$$m \text{ (slope of normal)} = -\frac{1}{2x_1}$$

Equation of normal at  $(x, x_1^2)$  is

$$y - x_1^2 = -\frac{1}{2x_1}(x - x_1) \quad \dots(i)$$

(i) passes through  $(a, b)$

$$b - x_1^2 = -\frac{1}{2x_1}(a - x_1) \Rightarrow 2x_1(b - x_1^2) = x_1 - a$$

$$\Rightarrow 2x_1^3 + x_1(1 - 2b) - a = 0$$

$\therefore$  There is no coefficient of  $x_1^2$

Hence, sum of all the  $x$  coordinates = 0

**16. (b) :** Equation of tangent at  $(0, 0)$  will be same

$$(3 + \sin\beta)x + (2 \cos\alpha)y = 0 \quad \dots(i)$$

$$2 \cos\alpha x + 2cy = 0 \quad \dots(ii)$$

$\therefore$  (i) and (ii) must be identical

$$\Rightarrow c = \frac{2 \cos^2 \alpha}{3 + \sin\beta} \Rightarrow c_{\max} = 1 \text{ when } \sin\beta = -1 \text{ and } \alpha = 0$$

**17. (c) :**  $f(x) = \sin^{-1}x + 2\tan^{-1}x + x^2 + 4x + 1$

Domain of  $f(x)$  is  $[-1, 1]$

$f(x)$  is increasing in the domain

$$\therefore f(x) / \min = -\frac{\pi}{2} + 2\left(\frac{-\pi}{4}\right) + 1 - 4 + 1$$

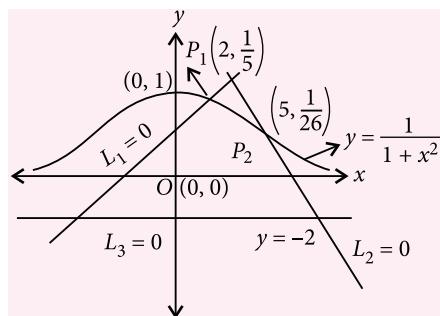
$$= -\pi - 2 \text{ at } x = -1$$

$$f(x) / \max = \frac{\pi}{2} + 2 \cdot \frac{\pi}{4} + 1 + 4 + 1 = \pi + 6 \text{ at } x = 1$$

$\therefore$  Range of  $f(x)$  is  $[-\pi - 2, \pi + 6]$

$$\Rightarrow p + q = 4$$

**18. (b) :** As  $P(\alpha, (1 + \alpha^2)^{-1})$  lie on  $y = \frac{1}{1 + x^2}$



$\therefore$  On solving  $y = \frac{1}{1 + x^2}$  with  $L_1$ ,

$$\text{we get } P_1\left(2, \frac{1}{5}\right) \quad \dots(i)$$

$$\text{and with } L_2, \text{ we get } P_2\left(5, \frac{1}{26}\right) \quad \dots(ii)$$

$\therefore$  From (i) and (ii), we get  $2 \leq \alpha \leq 5$

**19. (d) :** By using condition of tangency, we get

$$4h^2 = 3k^2 + 2$$

$\therefore$  Locus of  $P(h, k)$  is  $4x^2 - 3y^2 = 2$  (which is hyperbola.)

$$\text{Hence, } e^2 = 1 + \frac{4}{3} \Rightarrow e = \sqrt{\frac{7}{3}}$$

**20. (a) :** We have  $d = F_1F_2 = 2ae$

$$d^2 = 4a^2e^2 = 4(a^2 - b^2) = 4(4 - 1) = 12 \Rightarrow d = 2\sqrt{3} \text{ units}$$

**21. (b) :** We have  ${}^8C_3 p^3(1-p)^5 = \frac{1}{25} {}^8C_3 p^5(1-p)^3$

$$\text{or } 1-p = \frac{(1)(p)}{5}$$

$$\text{So, we get } p = \frac{5}{6} \Rightarrow m+n = 11$$

**22. (b) :** We have, Line  $(x - 1)(y - 2) = 0$

$$\Rightarrow x = 1 \text{ and } y = 2$$

Centre  $(1, 2)$  i.e.  $-g = 1$  and  $-f = 2$

Let the equation of the required circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad \dots(i)$$

As (i) is orthogonal to  $x^2 + y^2 - 2x - 4y + 4 = 0$

$$\therefore 2 + 8 = 4 + c \Rightarrow c = 6$$

$$\text{Hence, } x^2 + y^2 - 2x - 4y + 6 = 0$$

**23. (b) :** We have  $A(t) = \int_0^t \sin x^2 dx ; B(t) = \frac{t \sin t^2}{2}$

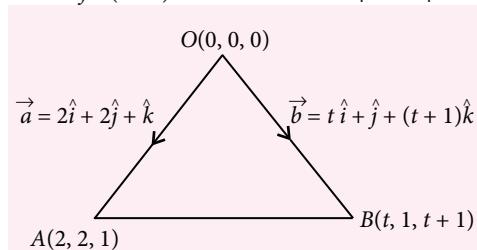
$$\therefore \lim_{t \rightarrow 0} \frac{A(t)}{B(t)} = \lim_{t \rightarrow 0} \frac{\frac{2}{0} \int_0^t \sin x^2 dx}{t \sin t^2}$$

$$= \lim_{t \rightarrow 0} \frac{\frac{2}{0} \int_0^t \sin x^2 dx}{t^3} = \lim_{t \rightarrow 0} \frac{\frac{2}{0} \int_0^t \sin x^2 dx}{t^3}$$

$$\text{Hence } \lim_{t \rightarrow 0} \frac{A(t)}{B(t)} = \lim_{t \rightarrow 0} \frac{2 \sin t^2}{3t^2} = \frac{2}{3}$$

**24. (b) :** We have  $S(t) = \frac{1}{2} |\vec{a} \times \vec{b}|$  where  $\vec{a} = 2\hat{i} + 2\hat{j} + \hat{k}$

$$\text{and } \vec{b} = t\hat{i} + \hat{j} + (t+1)\hat{k} \Rightarrow 4(S(t))^2 = |\vec{a} \times \vec{b}|^2$$



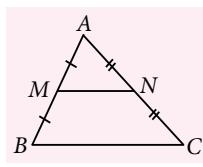
$$\text{But } 4(S(t))^2 = |\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$$

$$= 9(t^2 + 1 + (t+1)^2) - (2t+2+t+1)^2 = 9(t^2 + 1)$$

$$\therefore (S(t))^2 = \frac{9}{4}(t^2 + 1)$$

$$\text{Now } I = \int_1^e \frac{9}{4}(t^2 + 1) \ln t dt, \text{ we get } I = \frac{1}{2}(e^3 + 5)$$

**25. (d) :** Three non collinear points form a triangle and the line joining the mid points of any two sides is equidistant from all the three vertices.



**26. (a) :** 3 and 5 are the only odd prime numbers, among the possible outcomes. The following are the outcomes when one of them show odd prime numbers.

(3, 1), (1, 3), (3, 2), (2, 3), (3, 3), (3, 4), (4, 3), (3, 5), (5, 3), (3, 6), (6, 3), (5, 1), (1, 5), (5, 2), (2, 5), (5, 4), (4, 5), (5, 5), (5, 6), (6, 5).

Out of these 20 equally likely outcomes exactly 4 favour the presence of odd prime numbers on both dice.

Thus, required probability =  $\frac{4}{20} = \frac{1}{5}$

**27. (d) :** Any point on the first line can be taken as  
 $P_1 \equiv (r_1 + 2, r_1 + 3, -kr_1 + 4)$

Similarly any point on second line can be taken as  
 $P_2 \equiv (kr_2 + 1, 2r_2 + 4, r_2 + 5)$ .

These lines will intersect if for some  $r_1$  and  $r_2$  we have  
 $r_1 + 2 = kr_2 + 1, r_1 + 3 = 2r_2 + 4, -kr_1 + 4 = r_2 + 5$ .

$$\therefore r_1 - kr_2 + 1 = 0, r_1 = 2r_2 + 1$$

$$\Rightarrow r_2 = \frac{2}{k-2}, r_1 = \frac{k+2}{k-2}$$

Putting these values in the last condition, we get

$$k^2 + 3k = 0 \Rightarrow k = \{-3, 0\}$$

**28. (b) :** Let  $A \equiv (1, -1, 0), B \equiv (-1, 0, 1)$

Direction ratios of segment  $AB$  are  $2, -1, -1$

If ' $\theta$ ' be the acute angle between segment  $AB$  and normal to plane,

$$\cos \theta = \frac{|2 \cdot 2 - 1 \cdot 1 - 1 \cdot 6|}{\sqrt{4+1+36} \cdot \sqrt{4+1+1}} = \frac{3}{\sqrt{246}}$$

Length of projection =  $|AB| \sin \theta$

$$= \sqrt{6} \cdot \sqrt{1 - \frac{9}{246}} = \sqrt{\frac{237}{41}} \text{ units}$$

**29. (a) :** We have,

$$\frac{1}{2} ({}^{10}C_0 - 2 {}^{10}C_1 + 2^2 {}^{10}C_2 - \dots + 2^{10} {}^{10}C_{10})$$

$$= \frac{1}{2} (2-1)^{10} = \frac{1}{2}.$$

$$\begin{aligned} \text{30. (c) : } t_r &= \frac{1}{2} \cdot \frac{2r}{(r^2+1)^2 - r^2} = \frac{1}{2} \left\{ \frac{1}{r^2 - r + 1} - \frac{1}{r^2 + r + 1} \right\} \\ &= \frac{1}{2} \left\{ \frac{1}{r(r-1)+1} - \frac{1}{(r+1)r+1} \right\} \end{aligned}$$

$$\begin{aligned} \therefore \sum_{r=1}^n t_r &= \sum_{r=1}^n \frac{1}{2} \{f(r) - f(r+1)\}, \text{ where } f(r) = \frac{1}{r(r-1)+1} \\ &= \frac{1}{2} \{f(1) - f(n+1)\} \\ &= \frac{1}{2} \left\{ 1 - \frac{1}{(n+1)n+1} \right\} \rightarrow \frac{1}{2} \text{ as } n \rightarrow \infty \end{aligned}$$

**31. (a) :** Clearly,  $0 \leq f(0) \leq 1$  and  $0 \leq f(1) \leq 1$ . As  $f(x)$  is continuous,  $f(x)$  attains all values between  $f(0)$  to  $f(1)$ , and the graph will have no breaks. So, the graph will cut the line  $y = x$  at one point  $x$  at least where  $0 \leq x \leq 1$ . So,  $f(x) = x$  at that point.

**32. (b) :** Let  $f(x) = ax^2 + bx + c$ .

$$\begin{aligned} \text{Then } g(x) &= ax^2 + bx + c + 2ax + b + 2a \\ &= ax^2 + (2a+b)x + 2a + b + c \end{aligned}$$

As  $f(x) > 0$  for all  $x, a > 0$  and  $D = b^2 - 4ac < 0$ .

$g(x) > 0$  for all  $x$  if  $a > 0$

and  $D = (2a+b)^2 - 4a(2a+b+c) < 0$

Now,  $(2a+b)^2 - 4a(2a+b+c)$

$$= -4a^2 + b^2 - 4ac < 0$$

Hence,  $g(x) > 0$ .

$$33. \text{ (a) : } f'(x) = \frac{\phi'(x)}{1 + \{\phi(x)\}^2} > 0$$

$$\therefore 0 < x < \frac{\pi}{2}, \phi'(x) > 0$$

**34. (a) :** Around  $x = 1, 3$  we have  $|x| = x$ .

$$\therefore f(x) = a \log_e x + bx^2 + x$$

$$\therefore f'(x) = \frac{a}{x} + 2bx + 1$$

According to question,  $f'(1) = 0, f'(3) = 0$

$$\therefore a + 2b + 1 = 0, \frac{a}{3} + 6b + 1 = 0$$

On solving, we get  $a = \frac{-3}{4}, b = \frac{-1}{8}$

### Solution Sender of Maths Musing

#### SET-181

- Khokon Kumar Nandi (West Bengal)
- Gouri Sankar Adhikary (West Bengal)
- N. Jayanthi (Hyderabad)
- Sannibha Pande (West Bengal)

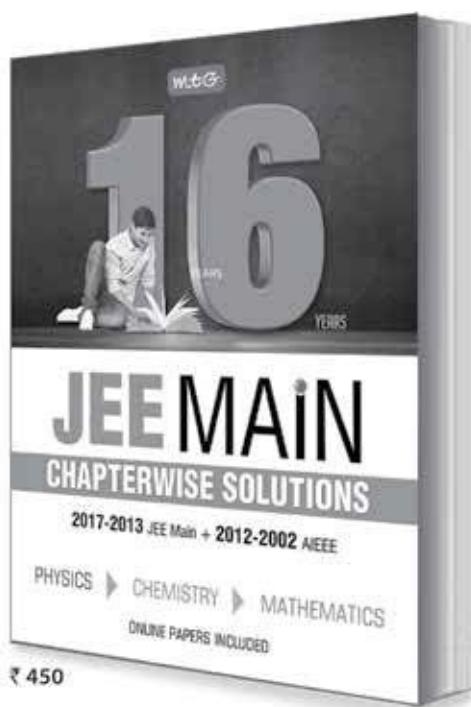
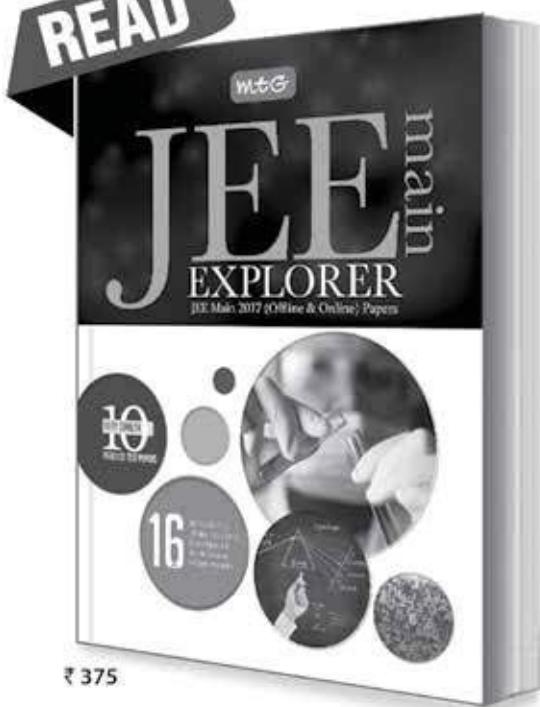
#### SET-180

- Gouri Sankar Adhikary (West Bengal)
- Sannibha Pande (West Bengal)

BEST TOOLS FOR SUCCESS IN

# JEE Main

READ



**10** Very Similar Practice Test Papers

**16** Years JEE MAIN 2017-2015(Offline & Online)-2013  
& AIEEE (2012-2002)



Available at all leading book shops throughout India.  
For more information or for help in placing your order:  
Call 0124-6601200 or email: info@mtg.in

Visit  
[www.mtg.in](http://www.mtg.in)  
for latest offers  
and to buy  
online!



Exam Dates  
OFFLINE : 8<sup>th</sup> April  
ONLINE : 15<sup>th</sup> & 16<sup>th</sup> April

# JEE Main 2018

## MOCK TEST PAPER

Series-8

Time: 1 hr 15 min.

The entire syllabus of Mathematics of JEE MAIN is being divided in to eight units, on each unit there will be a Mock Test Paper (MTP) which will be published in the subsequent issues. The syllabus for module break-up is given below:

Unit No. 8	Topic	Syllabus In Detail
	Integral calculus	Integral as limit of a sum. Fundamental theorem of calculus. Properties of definite integrals. Evaluation of definite integrals, determining areas of the regions bounded by simple curves in standard form.
	Differential equation	Ordinary differential equations, their order and degree. Formation of differential equations. Solution of differential equations by the method of separation of variables, solution of homogeneous and linear differential equation
	Probability	Baye's theorem, Probability distribution of a random variate, Bernoulli trials and Binomial distribution

1.  $\int_{-1}^1 \frac{x^3 + |x| + 1}{x^2 + 2|x| + 1} dx =$ 
  - $\ln 3$
  - $2\ln 3$
  - $\frac{1}{2}\ln 3$
  - none of these
2. Let  $f(x) = \max\{2-x, 2, 1+x\}$ . Then  $\int_{-1}^1 f(x)dx =$ 
  - 0
  - 2
  - $9/2$
  - 3
3.  $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^{2n} \frac{r}{\sqrt{n^2 + r^2}} =$ 
  - $1 + \sqrt{5}$
  - $-1 + \sqrt{5}$
  - $-1 + \sqrt{2}$
  - $1 + \sqrt{2}$
4.  $\int_{\log \frac{\sqrt{\pi}}{2}}^{\log \sqrt{\pi}} e^{2x} \sec^2 \left( \frac{1}{3} e^{2x} \right) dx =$
5. If  $[x]$  stands for the greatest integer function, the value of  $\int_4^{10} \frac{[x^2]dx}{[x^2 - 28x + 196] + [x^2]}$  is
  - 0
  - 1
  - 3
  - none of these
6. The area bounded by the curve  $x^2 + 2x + y - 3 = 0$ , the  $x$ -axis and the tangent at the point where it meets the  $y$ -axis is
  - $\frac{7}{12}$  sq. units
  - $\frac{12}{7}$  sq. units
  - $\frac{7}{6}$  sq. units
  - $\frac{6}{7}$  sq. units.
7. If  $f(x) = \int_0^x \frac{dt}{(f(t))^2}$  and  $\int_0^2 \frac{dt}{(f(t))^2} = \sqrt[3]{6}$ ,  $f(9) =$

By : Sankar Ghosh, S.G.M.C, Mob : 09831244397.

- (a) 2      (b) 0      (c) 3      (d) none of these

8. Let  $\frac{d}{dx}(f(x)) = \frac{e^{\sin x}}{x}$ ,  $x > 0$ . If  $\int_1^4 \frac{2e^{\sin x^2}}{x} dx = f(k) - f(1)$ , then one of the possible value of  $k$  is  
 (a) -4      (b) 0      (c) 2      (d) 16

9. The area between the curves  $y = xe^x$  and  $y = xe^{-x}$  and the line  $x = 1$  is  
 (a)  $2e$       (b)  $e$       (c)  $2/e$       (d)  $1/e$

10. The area bounded by the curve,  $y = f(x) = x^4 - 2x^3 + x^2 + 3$ , the  $x$ -axis and the ordinates corresponding to the minimum of function  $f(x)$  is  
 (a) 1      (b)  $\frac{91}{30}$       (c)  $\frac{30}{9}$       (d) 4

11. The order and degree, respectively of the differential equation whose solution is  $ax^2 + by^2 = 1$ ,  $a$  and  $b$  being arbitrary constants are  
 (a) 1, 1      (b) 2, 1      (c) 2, 2      (d) 1, 2

12. If  $x^3 dy + xydx = x^2 dy + 2ydx$ ,  $y(2) = e$ , then  $y(-1) =$   
 (a)  $\frac{2}{e}$       (b)  $\frac{4}{e}$       (c)  $\frac{2}{e^2}$       (d)  $\frac{4}{e^2}$

13. If  $xdy - ydx + x\cos \ln x \, dx = 0$ ,  $y(1) = 1$ , then  $y(e) =$   
 (a)  $e(1 - \cos 1)$       (b)  $e(1 - \sin 1)$   
 (c)  $e(1 + \cos 1)$       (d)  $e(1 + \sin 1)$

14. If  $(x^2 + y^2)dy = xydx$ ,  $y(1) = 1$ , and  $y(x_0) = e$  then  $x_0 =$   
 (a)  $\sqrt{2(e^2 - 1)}$       (b)  $\sqrt{2(e^2 + 1)}$   
 (c)  $\sqrt{3} \cdot e$       (d)  $\sqrt{\frac{e^2 + 1}{2}}$

15. The solution of  $1 + y^2 + \left( x - e^{\tan^{-1} y} \right) \frac{dy}{dx} = 0$   
 (a)  $x - 2 = ce^{-\tan^{-1} y}$       (b)  $2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + c$   
 (c)  $xe^{\tan^{-1} y} = c + \tan^{-1} y$   
 (d)  $xe^{2\tan^{-1} y} = c + \tan^{-1} y$

16. If  $y - \cos x \frac{dy}{dx} = y^2(1 - \sin x)\cos x$ ,  $y(0) = 1$ , then  
 $y\left(\frac{\pi}{3}\right) =$   
 (a) 1      (b) 2      (c)  $1/2$       (d)  $\sqrt{3}$

17. If  $(2x - 2y + 5)dy = (x - y + 3)dx$ , then  $\ln(x - y + 2) =$   
 (a)  $x - 2y + c$       (b)  $2x - y + c$   
 (c)  $2y - x + c$       (d)  $y - x + c$

18. A curve passes through  $(2, 0)$  and slope of the tangent at the point  $(x, y) = \frac{(x+1)^2 + y - 3}{x+1}$ . The area bounded by the curve and the  $x$ -axis is  
 (a) 1      (b)  $2/3$       (c)  $4/3$       (d) 2

19. A right circular cone with radius  $R$  and height  $H$  contains a liquid which evaporates at a rate proportional to the surface area in contact with the area ( $\lambda > 0$  is proportional constant). Then the time at which liquid evaporates completely is  
 (a)  $\frac{H}{\lambda}$       (b)  $\frac{2H}{\lambda}$       (c)  $\frac{H}{2\lambda}$       (d)  $\frac{R}{2\lambda}$

20. The differential equation which represent the family of curves  $y = ae^{bx}$  where  $a$  and  $b$  are arbitrary constants, is  
 (a)  $y' = y^2$       (b)  $y'' = yy'$   
 (c)  $yy'' = y'$       (d)  $yy'' = (y')^2$

21. A letter is known to have come either from TATANAGAR or CALCUTTA. On the envelope just two consecutive letters are visible. The probability that the letter came from CALCUTTA is  
 (a)  $\frac{4}{11}$       (b)  $\frac{7}{11}$   
 (c)  $\frac{5}{11}$       (d) none of these

22. A coffee connoisseur claims that he can distinguish between a cup of instant coffee and a cup of percolator coffee 75% of time. It is agreed that his claim will be accepted if he correctly identifies at least 5 of the 6 cups. His chances of having the claim accepted is  
 (a) 0.534      (b) 0.466  
 (c) 0.763      (d) none of these

23. If on an average one vessel in every 10 is wrecked, the probability that out of 5 vessels atleast 4 will arrive safely is  
 (a) 0.92      (b) 0.72  
 (c) 0.82      (d) none of these

24. The probability distribution of a random variable is  $P(X) = \begin{cases} \frac{X}{15}; X = 1, 2, 3, 4, 5 \\ 0, \text{ otherwise} \end{cases}$ . Then

$$P\left\{\frac{1}{2} < X < \frac{5}{2} \mid X > 1\right\} =$$

- (a) 1/5 (b) 1/7 (c) 2/15 (d) 1/15

- 25.** A perfect die is thrown twice. The expected value of the product of the number of points obtained in two throws is  
 (a) 7/2 (b) 7  
 (c) 49/4 (d) none of these

- 26.** For a fixed value of  $n$ , the maximum value of variance of binomial distribution  
 (a)  $n/4$  (b)  $n/2$   
 (c)  $n/8$  (d) none of these

- 27.** The variance of a symmetric binomial distribution with mean 5 is  
 (a) 2/5 (b) 5/2  
 (c) 3/2 (d) none of these

- 28.** If a random variable  $X$  follows binomial distribution with mean 2 and  $E(X^2) = 28/5$ , then  $\langle n, p \rangle$  =  
 (a)  $\left\langle 10, \frac{1}{5} \right\rangle$  (b)  $\left\langle 5, \frac{1}{10} \right\rangle$   
 (c)  $\left\langle 10, \frac{4}{5} \right\rangle$  (d) none of these

- 29.** There are 4 white and 3 black balls in a box. In another box there are 3 white and 4 black balls. An unbiased dice is rolled. If it shows a number less than or equal to 3 then a ball is drawn from the first box but if it shows a number more than 3 then a ball is drawn from the second box. If the ball drawn is black then the probability that the ball was drawn from the first box is  
 (a) 1/2 (b) 6/7 (c) 4/7 (d) 3/7

- 30.** A coin is tossed  $n$  times. The probability of getting at least one head is greater than that of getting at least two tails by 5/32. Then  $n$  is  
 (a) 5 (b) 10  
 (c) 15 (d) none of these

### SOLUTIONS

$$\begin{aligned} 1. \quad (d): & \int_{-1}^1 \frac{x^3 + |x| + 1}{x^2 + 2|x| + 1} dx \\ &= \int_{-1}^1 \frac{x^3 dx}{x^2 + 2|x| + 1} + \int_{-1}^1 \frac{|x| + 1}{x^2 + 2|x| + 1} dx \\ &= 0 + 2 \int_0^1 \frac{|x| + 1}{x^2 + 2|x| + 1} dx = 2 \int_0^1 \frac{dx}{1+x} = 2 \ln 2 \end{aligned}$$

**2. (c)**: In  $(-1, 0)$ ,  $2 - x$  is maximum and in  $(0, 1)$ , 2 is maximum.

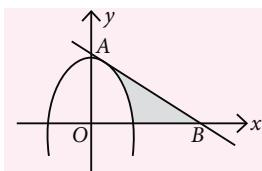
$$\begin{aligned} \therefore \int f(x) dx &= \int_{-1}^0 (2-x) dx + \int_0^1 2 dx \\ &= \left[ 2x - \frac{x^2}{2} \right]_{-1}^0 + 2 = \frac{5}{2} + 2 = \frac{9}{2} \end{aligned}$$

$$\begin{aligned} 3. \quad (b): & \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^{2n} \frac{r}{\sqrt{n^2 + r^2}} = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^{2n} \frac{\frac{r}{n}}{\sqrt{1 + \left(\frac{r}{n}\right)^2}} \\ &= \int_0^2 \frac{x}{\sqrt{1+x^2}} dx = \frac{1}{2} \cdot \left( 2\sqrt{1+x^2} \right)_0^2 = \sqrt{5} - 1 \end{aligned}$$

$$\begin{aligned} 4. \quad (a): & \text{Let } I = \int_{\log \frac{\sqrt{\pi}}{2}}^{\log \sqrt{\pi}} e^{2x} \sec^2 \left( \frac{1}{3} e^{2x} \right) dx \\ &= \frac{3}{2} \tan \left( \frac{1}{3} e^{2x} \right) \Big|_{\ln \frac{\sqrt{\pi}}{2}}^{\ln \sqrt{\pi}} = \frac{3}{2} \left[ \tan \frac{\pi}{3} - \tan \frac{\pi}{6} \right] = \sqrt{3} \end{aligned}$$

$$\begin{aligned} 5. \quad (c): & I = \int_4^{10} \frac{[x^2] dx}{4[x^2 - 28x + 196] + [x^2]} \\ &= \int_4^{10} \frac{[x^2] dx}{4[(14-x)^2] + [x^2]} = \int_4^{10} \frac{[(14-x)^2] dx}{4[x^2] + [(14-x)^2]} \\ &\quad [\because \int_a^b f(x) dx = \int_a^b f(a+b-x) dx] \\ \therefore \quad 2I &= \int_4^{10} dx = 6 \Rightarrow I = 3 \end{aligned}$$

**6. (a)**: The equation of the curve is  $y - 4 = -(x+1)^2$ . The curve meets  $y$ -axis at  $(0, 3)$  and  $x$ -axis at  $(-3, 0)$  and  $(1, 0)$ . The tangent at  $(0, 3)$  is  $y - 3 = -2x$



$$\begin{aligned} \text{The required area} &= \text{Area of } \Delta OAB - \int_0^1 \{4 - (x+1)^2\} dx \\ &= \frac{9}{4} - \left\{ 4x - \frac{(x+1)^3}{3} \right\}_0^1 = \frac{9}{4} - \left\{ \left( 4 - \frac{8}{3} \right) + \frac{1}{3} \right\} = \frac{7}{12} \end{aligned}$$



7. (c) : Here,  $f(x) = \int_0^x \frac{dt}{(f(t))^2} \Rightarrow f'(x) = \frac{1}{(f(x))^2}$

$$\Rightarrow f'(x)(f(x))^2 = 1 \Rightarrow \frac{(f(x))^3}{3} = x + c$$

$$\text{Since } f(2) = \sqrt[3]{6} \Rightarrow \frac{6}{3} = 2 + c \Rightarrow c = 0$$

$$\therefore f(x) = \sqrt[3]{3x} \Rightarrow f(9) = 3$$

8. (d) : Given,  $\frac{d}{dx}(f(x)) = \frac{e^{\sin x}}{x}, x > 0$

$$\text{Now, } I = \int_1^4 \frac{2e^{\sin x^2}}{x} dx$$

$$= \int_1^4 \frac{2xe^{\sin x^2}}{x^2} dx \quad [\text{Put } t = x^2 \Rightarrow dt = 2x dx]$$

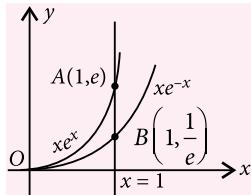
$$= \int_1^{16} \frac{e^{\sin t} dt}{t} = [f(t)]_1^{16} = f(16) - f(1) \quad \therefore k = 16$$

9. (c) : The line  $x = 1$  meets the curves in  $A(1, e)$  and  $B\left(1, \frac{1}{e}\right)$ . Both the curves pass

through origin.

The required area

$$= \int_0^1 (xe^x - xe^{-x}) dx = \frac{2}{e} \text{ sq.units}$$



10. (b) : We have,  $f(x) = x^4 - 2x^3 + x^2 + 3$

$$\therefore f'(x) = 4x^3 - 6x^2 + 2x$$

$$f'(x) = 0 \Rightarrow 4x^3 - 6x^2 + 2x = 0 \Rightarrow x = 0, 1, \frac{1}{2}$$

$$f''(x) = 12x^2 - 12x + 2 = 2(6x^2 - 6x + 1)$$

$f''(x)|_{x=0} > 0 \Rightarrow$  Minimum exists at  $x = 0$  and  $x = 1$

$$f''(x)|_{x=\frac{1}{2}} < 0 \Rightarrow$$
 Maximum exists at  $x = \frac{1}{2}$

$\Rightarrow$  The curve is bounded by the ordinates  $x = 0$  and  $x = 1$

$\therefore$  Required area

$$= \int_0^1 (x^4 - 2x^3 + x^2 + 3) dx = \left( \frac{x^5}{5} - \frac{x^4}{2} + \frac{x^3}{3} + 3x \right)_0^1$$

$$= \frac{91}{30} \text{ sq.units}$$

11. (b) : Differentiating,  $ax^2 + by^2 = 1 \dots(i)$

we get,  $ax + byy_1 = 0 \dots(ii)$

$a + b(y_1^2 + yy_2) = 0 \dots(iii)$

Eliminating  $a$  and  $b$  we get,

$$\begin{vmatrix} x^2 & y^2 & -1 \\ x & yy_1 & 0 \\ 1 & y_1^2 + yy_2 & 0 \end{vmatrix} = 0 \Rightarrow xyy_2 + xy_1^2 - yy_1 = 0$$

$\therefore$  Required order is 2 and degree is 1

12. (d) : We have,  $(x^3 - x^2)dy = (2y - xy)dx$

$$\Rightarrow \frac{dy}{y} = \frac{2-x}{x^2(x-1)} dx = \left( \frac{1}{x-1} - \frac{1}{x} - \frac{2}{x^2} \right) dx$$

$$\Rightarrow \int \frac{dy}{y} = \int \left( \frac{1}{x-1} - \frac{1}{x} - \frac{2}{x^2} \right) dx \Rightarrow \ln y = \ln \frac{x-1}{x} + \frac{2}{x} + c$$

$$\text{Now, } y(2) = e \Rightarrow 1 = -\ln 2 + 1 + c \Rightarrow c = \ln 2$$

$$\therefore \ln y = \ln \frac{x-1}{x} + \frac{2}{x} + \ln 2$$

$$\text{Now, } x = -1 \Rightarrow \ln y = \ln 4 - 2 = \ln \frac{4}{e^2} \Rightarrow y = \frac{4}{e^2}$$

13. (b) :  $\frac{xdy - ydx}{x^2} + \frac{\cos \ln x}{x} dx = 0$

$$\Rightarrow \frac{d}{dx} \left( \frac{y}{x} \right) + \frac{d}{dx} (\sin \ln x) = 0 \Rightarrow \frac{y}{x} + \sin \ln x = c$$

$$\text{Now, } x = 1, y = 1 \Rightarrow c = 1$$

$$\text{and } x = e \Rightarrow y = e(1 - \sin 1)$$

14. (c) : We have,  $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$  [put  $y = vx$ ]

$$v + x \frac{dv}{dx} = \frac{v}{1+v^2} \Rightarrow x \frac{dv}{dx} = \frac{v}{1+v^2} - v = -\frac{v^3}{1+v^2}$$

$$\Rightarrow \frac{1+v^2}{v^3} dv = -\frac{dx}{x} \Rightarrow \int \frac{1+v^2}{v^3} dv = -\int \frac{dx}{x}$$

$$\Rightarrow \ln x + \ln v - \frac{1}{2v^2} = c \Rightarrow \ln y = \frac{x^2}{2y^2} + c$$

$$\text{Now } x = 1, y = 1 \Rightarrow c = -\frac{1}{2}$$

$$\text{Again } y = e \Rightarrow 1 = \frac{x_0^2}{2e^2} - \frac{1}{2} \Rightarrow x_0^2 = 3e^2, x_0 = \sqrt{3} \cdot e.$$

15. (b) : The given equation is

$$1 + y^2 + (x - e^{\tan^{-1} y}) \frac{dx}{dy} = 0 \Rightarrow \frac{dx}{dy} + \frac{1}{1+y^2} \cdot x = \frac{e^{\tan^{-1} y}}{1+y^2}$$

General solution is  $xe^{\tan^{-1}y} = \int \frac{e^{2\tan^{-1}y}}{1+y^2} dy$

$$= \frac{e^{2\tan^{-1}y}}{2} + c_1$$

$$\Rightarrow 2xe^{\tan^{-1}y} = e^{2\tan^{-1}y} + c$$

**16. (b) :** Given,  $y - \cos x \frac{dy}{dx} = y^2(1 - \sin x) \cos x$

Dividing the above differential equation by  $y^2 \cos x$ , we get

$$-\frac{1}{y^2} \frac{dy}{dx} + \sec x \cdot \frac{1}{y} = 1 - \sin x$$

$$\Rightarrow \frac{d}{dx} \left( \frac{1}{y} \right) + \sec x \cdot \left( \frac{1}{y} \right) = 1 - \sin x$$

Here, I.F.  $= e^{\int \sec x dx} = \sec x + \tan x$

General solution is

$$\frac{\sec x + \tan x}{y} = \int \frac{(1 - \sin x)(1 + \sin x) dx}{\cos x}$$

$$= \int \cos x dx = \sin x + c$$

$$\therefore x = 0, y = 1 \Rightarrow c = 1 \Rightarrow y \left( \frac{\pi}{3} \right) = 2$$

**17. (c) :** The given equation is

$$(2x - 2y + 5)dy = (x - y + 3)dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{(x - y + 3)}{(2x - 2y + 5)} \quad \left[ \text{put } x - y = z \Rightarrow 1 - \frac{dy}{dx} = \frac{dz}{dx} \right]$$

$$\Rightarrow 1 - \frac{dz}{dx} = \frac{z+3}{2z+5} \Rightarrow \int dx = \int \left( \frac{2z+5}{z+2} \right) dz$$

$$\Rightarrow x + c = 2z + \ln(z+2) = 2x - 2y + \ln(x-y+2)$$

$$\Rightarrow \ln(x-y+2) = 2y - x + c$$

**18. (c) :** The given differential equation is

$$\frac{dy}{dx} = \frac{(x+1)^2 + y - 3}{x+1} = (x+1) + \frac{y-3}{x+1}$$

[Let  $x+1 = X$  and  $y-3 = Y$ ],  $\frac{dY}{dX} - \frac{Y}{X} = X$ , I.F.  $= \frac{1}{X}$

General solution is  $\frac{Y}{X} = X + c$

$$\Rightarrow \frac{y-3}{x+1} = x+1+c \quad \dots(i)$$

When  $x = 2, y = 0$ , then  $c = -4$

Now (i) becomes  $y - 3 = (x+1)^2 - 4(x+1) = x^2 - 2x - 3$

$$\Rightarrow y = x^2 - 2x \text{ meets } x\text{-axis at } x = 0, 2$$

$$\therefore \text{Area} = \int_0^2 (2x - x^2) dx = 4 - \frac{8}{3} = \frac{4}{3}$$

**19. (a) :** Let  $y$  be the height at time  $t$ .

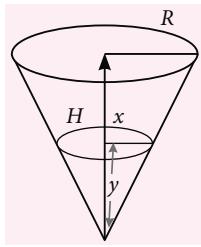
$$\frac{x}{y} = \frac{R}{H}, v = \frac{1}{3}\pi x^2 y = \frac{\pi}{3} \cdot \frac{Hx^3}{R}$$

$$\Rightarrow \frac{dv}{dt} = \frac{\pi Hx^2}{R} \frac{dx}{dt}$$

$$\text{But } \frac{dv}{dt} = -\lambda \pi x^2 \Rightarrow \frac{dx}{dt} = -\frac{\lambda R}{H}$$

$$\Rightarrow x = \frac{-\lambda Rt}{H} + c, t = 0, x = R \Rightarrow c = R$$

$$\Rightarrow x = R \left( 1 - \frac{\lambda t}{H} \right), x = 0 \Rightarrow t = \frac{H}{\lambda}$$



**20. (d) :** We have,  $\ln y = \ln a + bx$

Differentiating twice, we get  $\frac{1}{y} y' = b$ ,

$$\frac{y''}{y} - \frac{1}{y^2} (y')^2 = 0 \Rightarrow yy'' = (y')^2$$

**21. (a) :** Let  $E_1$  and  $E_2$  denote the event that letters came from TATANAGAR and CALCUTTA respectively. Let  $A$  denotes the event that two consecutive visible letters on the envelop are 'TA'. We have

$$P(E_1) = P(E_2) = \frac{1}{2}, P(A|E_1) = \frac{2}{8} \text{ and } P(A|E_2) = \frac{1}{7}$$

Using the Bayes' theorem, we get

$$P(E_2|A) = \frac{P(E_2)P(A|E_2)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)}$$

$$= \frac{\frac{1}{2} \cdot \frac{1}{7}}{\frac{1}{2} \cdot \frac{2}{8} + \frac{1}{2} \cdot \frac{1}{7}} = \frac{4}{11}$$

**22. (a) :** Let  $p$  denotes the probability of a correct distinction between a cup of instant coffee and a cup of percolator coffee.

$$\text{Then, } p = \frac{75}{100} = \frac{3}{4} \Rightarrow q = 1 - p = \frac{1}{4} \text{ and } n = 6$$

Let random variable  $X$  denotes the number of correct distinctions.

$$\text{Then, } P(X=x) = p(x) = {}^6C_x \left( \frac{3}{4} \right)^x \left( \frac{1}{4} \right)^{6-x}, x = 0, 1, \dots, 6$$

The probability of the claim being accepted is

$$P(X \geq 5) = p(5) + p(6) = {}^6C_5 \left( \frac{3}{4} \right)^5 \left( \frac{1}{4} \right)^{6-5} + {}^6C_6 \left( \frac{3}{4} \right)^6$$

$$= 0.534 \text{ (approx.)}$$

**23. (a) :** Probability of a vessel arriving safe = 9/10

$$\text{Required probability} = {}^5C_4 \left(\frac{9}{10}\right)^4 \times \frac{1}{10} + \left(\frac{9}{10}\right)^5 \\ = \frac{9^4 \times 14}{10^5} = 0.92 \text{ (approx.)}$$

**24. (b) :**  $P(X) = \begin{cases} \frac{X}{15}, & X = 1, 2, 3, 4, 5 \\ 0, & \text{otherwise} \end{cases}$

$$\therefore P(X = 1 \text{ or } 2) = P(X = 1) + P(X = 2) = \frac{1}{15} + \frac{2}{15} = \frac{1}{5}$$

$$\text{Now, } P\left(\frac{1}{2} < X < \frac{5}{2} \mid X > 1\right) = \frac{P\left(\left(\frac{1}{2} < X < \frac{5}{2}\right) \cap X > 1\right)}{P(X > 1)} \\ = \frac{P\{(X = 1 \text{ or } 2) \cap (X > 1)\}}{P(X > 1)} = \frac{P(X = 2)}{1 - P(X = 1)} = \frac{\frac{2}{15}}{1 - \frac{1}{15}} = \frac{1}{7}.$$

**25. (c) :** Let  $X$  and  $Y$  denote respectively the number of points obtained in the first and the second throws. Then both of them take the values 1, 2, 3, 4, 5 and 6 with each probability 1/6.

$$\therefore E(X) = (1+2+3+4+5+6) \times \frac{1}{6} = \frac{7}{2}.$$

$$\text{Similarly, } E(Y) = \frac{7}{2}.$$

$$\text{Thus } E(XY) = E(X)E(Y) = \frac{7}{2} \cdot \frac{7}{2} = \frac{49}{4}$$

**26. (a) :** We know, variance of Binomial distribution is

$$\text{Var}(X) = npq = n \left[ \left( \frac{p+q}{2} \right)^2 - \left( \frac{p-q}{2} \right)^2 \right] \\ = n \left[ \frac{1}{4} - \left( \frac{p-q}{2} \right)^2 \right] \leq \frac{n}{4} \quad \left[ \because \left( \frac{p-q}{2} \right)^2 \geq 0 \right]$$

Hence, for a fixed value of  $n$ , the variance of the binomial distribution can never exceed  $n/4$ . So, the maximum value of  $\text{var}(X)$  is  $n/4$ .

**27. (b) :** Let the parameters of the distribution be  $n$  and  $p$ . Since the distribution is symmetric

$$\therefore p = 1/2.$$

Again,  $np = 5$  (Given)

$$\text{So, variance} = np(1-p) = \frac{5}{2}.$$

**28. (a) :** Let a random variable  $X$  follows a binomial distribution with parameter  $n$  and  $p$ .

$$\text{Given that mean} = E(X) = np = 2 \text{ and } E(X^2) = \frac{28}{5}.$$

$$\therefore npq = E(X^2) - [E(X)]^2 = \frac{28}{5} - 2^2 = \frac{8}{5}.$$

$$\text{Now, } \frac{npq}{np} = \frac{8}{5} \div 2 = \frac{4}{5} \Rightarrow q = \frac{4}{5}$$

$$\therefore p = \frac{1}{5} \text{ and hence } n = 10$$

**29. (d) :** Let  $E_1$ : The ball drawn from the first box

$E_2$ : The ball drawn from the second box

$A$ : The drawn ball is black.

$$\therefore P(E_1) = \frac{3}{6} = \frac{1}{2} \text{ and } P(E_2) = \frac{3}{6} = \frac{1}{2}$$

$$\text{Now, } P(A \mid E_1) = \frac{3}{7} \text{ and } P(A \mid E_2) = \frac{4}{7}.$$

$$\therefore P(E_1 \mid A) = \frac{P(E_1) \cdot P(A \mid E_1)}{P(E_1) \cdot P(A \mid E_1) + P(E_2) \cdot P(A \mid E_2)} \\ = \frac{\frac{1}{2} \cdot \frac{3}{7}}{\frac{1}{2} \cdot \frac{3}{7} + \frac{1}{2} \cdot \frac{4}{7}} = \frac{3}{7}$$

**30. (a) :** Let  $X$  be the random variable denoting number of heads obtain. Clearly  $X$  follows Binomial distribution.

$$P(X = x) = {}^nC_x p^x q^{n-x} = {}^nC_x \left(\frac{1}{2}\right)^n \quad \left[ \because p = q = \frac{1}{2} \right]$$

$$\text{Now } P(X \geq 1) = 1 - P(X = 0) = 1 - {}^nC_0 \left(\frac{1}{2}\right)^n = 1 - \frac{1}{2^n}$$

Again, let  $Y$  be the random variable denoting the number of tails obtain.

$$\therefore P(Y = y) = {}^nC_y p^y q^{n-y} \\ = {}^nC_y \left(\frac{1}{2}\right)^n \quad \left[ \because p = q = \frac{1}{2} \right]$$

$$\text{Now, } P(Y \geq 2) = 1 - P(Y = 0) - P(Y = 1)$$

$$= 1 - {}^nC_0 \left(\frac{1}{2}\right)^n - {}^nC_1 \left(\frac{1}{2}\right)^n = 1 - \frac{1}{2^n} - n \frac{1}{2^n}$$

According to the question, we get

$$\left(1 - \frac{1}{2^n}\right) - \left(1 - \frac{1+n}{2^n}\right) = \frac{5}{32} \Rightarrow \frac{n}{2^n} = \frac{5}{32} \Rightarrow n = 5$$



# CONCEPT MAP

# BINOMIAL THEOREM

Class XI

## Expansion

For positive integral index  $n \in N$

- $$(a+b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1} b + \dots + {}^nC_n b^n = \sum_{r=0}^n {}^nC_r a^{n-r} b^r$$

For negative integral index

- $$(a+b)^{-n} = \sum_{r=0}^n (-1)^n {}^nC_r a^{n-r} b^r$$

## Binomial Theorem for Fractional Index

For  $r \geq 0$ ,  $|x| < 1$  and  $n \in Q$

- $$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \dots + \frac{n(n-1)\dots(n-r+1)x^r}{r!} + \dots + \infty$$
- $$T_{r+1} = \frac{n(n-1)(n-2)\dots(n-r+1)}{r!} x^r$$
- $$T_{r+1} \text{ in } (1+x)^{-n} = \frac{(-1)^r n(n+1)(n+2)\dots(n+r-1)x^r}{r!}$$
- $$T_{r+1} \text{ in } (1-x)^{-n} = \frac{n(n+1)(n+2)\dots(n+r-1)x^r}{r!}$$

## General Term

In the binomial expansion for  $(a+b)^n$ , general term

- $T_{r+1} = {}^nC_r a^{n-r} b^r$
- $T_{r+1}$  from the end =  $T_{n-r+1}$  from the beginning

## Binomial Coefficients

Greatest Binomial Coefficient are given by

- $${}^nC_{n/2}$$
, if  $n$  is even,
- $${}^nC_{n-1/2} = \frac{{}^nC_{n+1}}{2}$$
, if  $n$  is odd

Coefficient of

- $(r+1)^{\text{th}}$  term in  $(1+x)^n$  is  ${}^nC_r$ .
- $x^r$  in  $(1+x)^n$  is  ${}^nC_r$ .
- $x^r$  in  $(1-x)^n$  is  $(-1)^r {}^nC_r$ .
- $(r+1)^{\text{th}}$  term in  $(1-x)^n$  is  $(-1)^r {}^nC_r$

## Middle Term

In the expansion  $(a+b)^n$ ,

- If  $n$  is even, then  $\left(\frac{n+1}{2}\right)^{\text{th}}$  term is the middle term.
- If  $n$  is odd, then  $\left(\frac{n+1}{2}\right)^{\text{th}}, \left(\frac{n+3}{2}\right)^{\text{th}}$  are two middle terms.

## Some Important Results

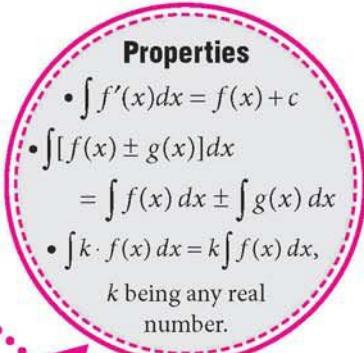
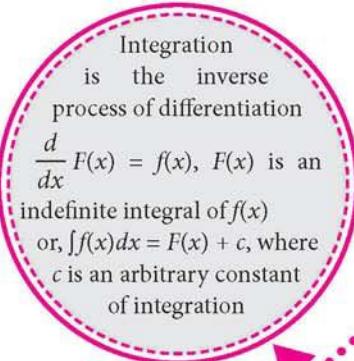
- $${}^nC_r = {}^nC_{n-r}$$
- $${}^nC_r = \frac{n}{r} {}^{n-1}C_{r-1} = \frac{n}{r} \left( \frac{n-1}{r-1} \right) {}^{n-1}C_{r-2}$$
- $${}^nC_x = {}^nC_y \Rightarrow \text{Either } x = y \text{ or } x+y = n$$
- $${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$$
- $$\sum_{r=0}^n {}^nC_r = \sum_{r=0}^n C_r = 2^n$$
- $$C_0 + C_2 + C_4 + \dots = C_1 + C_3 + C_5 + \dots = 2^{n-1}$$
- $$\sum_{r=0}^n r \cdot C_r = n2^{n-1}$$

- $$C_0^2 - C_1^2 + C_2^2 - C_3^2 + \dots + (-1)^n C_n^2 = \begin{cases} 0, & n \text{ is odd} \\ (-1)^{\frac{n}{2}} {}^nC_{\frac{n}{2}}, & n \text{ is even} \end{cases}$$
- $$C_0 - C_2 + C_4 - C_6 + \dots = 2^{\frac{n}{2}} \cos \frac{n\pi}{4}$$
- $$C_1 - C_3 + C_5 - C_7 + \dots = 2^{\frac{n}{2}} \sin \frac{n\pi}{4}$$
- If  $n = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdot p_3^{\alpha_3} \dots p_r^{\alpha_r}$ , where  $p_1, p_2, \dots, p_r$  are prime numbers and  $\alpha_1, \alpha_2, \dots, \alpha_r$  are positive integers. Then sum of positive divisors of  $n = (1 + p_1 + p_1^2 + \dots + p_1^{\alpha_1}) (1 + p_2 + p_2^2 + \dots + p_2^{\alpha_2}) \dots (1 + p_r + p_r^2 + \dots + p_r^{\alpha_r})$ .

# CONCEPT MAP

## Class XII

# INDEFINITE INTEGRALS



## INDEFINITE INTEGRALS

### Methods

#### Using Substitution

The given integral  $\int f(x) dx$  can be transformed into another form by changing the independent variable  $x$  to  $t$  by substituting  $x = g(t)$ .

#### Using by Parts

If  $u$  and  $v$  are two differentiable functions of  $x$ , then

$$\int (uv) dx = \left[ u \cdot \int v dx \right] - \int \left( \frac{du}{dx} \cdot \int v dx \right) dx.$$

In order to choose 1<sup>st</sup> function, we take the letter which comes first in the word ILATE.

I – Inverse Trigonometric Function

L – Logarithmic Function, A – Algebraic Function

T – Trigonometric Function, E – Exponential Function

#### Using Partial Fractions

- If  $f(x)$  and  $g(x)$  are two polynomials such that  $\deg f(x) \geq \deg g(x)$ , then we divide  $f(x)$  by  $g(x)$ .  
 $\therefore \frac{f(x)}{g(x)} = \text{Quotient} + \frac{\text{Remainder}}{g(x)}$
- If  $f(x)$  and  $g(x)$  are two polynomials such that the degree of  $f(x)$  is less than the degree of  $g(x)$ , then we can evaluate  $\int \frac{f(x)}{g(x)} dx$  by decomposing  $\frac{f(x)}{g(x)}$  into partial fraction.

- $\int dx = x + c$
  - $\int x^n dx = \frac{x^{n+1}}{n+1} + c$ , where  $n \neq -1$
  - $\int e^x dx = e^x + c$
  - $\int a^x dx = \frac{a^x}{\log_e a} + c$ , where  $a > 0, a \neq 1$
  - $\int \frac{1}{x} dx = \log_e |x| + c$ , where  $x \neq 0$
  - $\int \sin x dx = -\cos x + c$
  - $\int \cos x dx = \sin x + c$
  - $\int \tan x dx = \log|\sec x| + c$
  - $\int \cot x dx = \log|\sin x| + c$
  - $\int \sec x dx = \log|\sec x + \tan x| + c$
  - $\int \cosec x dx = \log|\cosec x - \cot x| + c$
- where ' $c$ ' is the constant of integration.

### Some Standard Integrals

### Integrals of Some Particular Functions

- $\int \frac{1}{\sqrt{x^2 - a^2}} dx = \log \left| x + \sqrt{x^2 - a^2} \right| + c$
- $\int \frac{1}{\sqrt{x^2 + a^2}} dx = \log \left| x + \sqrt{x^2 + a^2} \right| + c$
- $\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c$
- $\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left| \left( \frac{a+x}{a-x} \right) \right| + c, a > x$
- $\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$
- $\int \frac{1}{x\sqrt{x^2 - 1}} dx = \sec^{-1} x + c = -\cosec^{-1} x + c$ , where  $|x| > 1$
- $\int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log \left| x + \sqrt{x^2 + a^2} \right| + c$
- $\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log \left| x + \sqrt{x^2 - a^2} \right| + c$
- $\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left( \frac{x}{a} \right) + c$
- $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \left( \frac{x}{a} \right) + c$
- $\int e^x (f(x) + f'(x)) dx = e^x f(x) + c$

# YOU ASK WE ANSWER

**Do you have a question that you just can't get answered?**

Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

**1.** Prove that :

$$\begin{vmatrix} a & a+b & a+2b & \dots & a+nb \\ a+nb & a & a+b & \dots & a+(n-1)b \\ a+(n-1)b & a+nb & a & \dots & a+(n-2)b \\ \dots & \dots & \dots & \dots & \dots \\ a+b & a+2b & a+3b & \dots & a \end{vmatrix} = (-b)^n (n+1)^n \left( a + \frac{nb}{2} \right). \quad (\text{Mandeep, Delhi})$$

**Ans.** We have,

$$\begin{aligned} \text{L.H.S.} &= \begin{vmatrix} a & a+b & a+2b & \dots & a+nb \\ a+nb & a & a+b & \dots & a+(n-1)b \\ a+(n-1)b & a+nb & a & \dots & a+(n-2)b \\ \dots & \dots & \dots & \dots & \dots \\ a+b & a+2b & a+3b & \dots & a \end{vmatrix} \\ &= (n+1) \left( a + \frac{nb}{2} \right) \begin{vmatrix} 1 & a+b & a+2b & \dots & a+nb \\ 1 & a & a+b & \dots & a+(n-1)b \\ 1 & a+nb & a & \dots & a+(n-2)b \\ \dots & \dots & \dots & \dots & \dots \\ 1 & a+2b & a+3b & \dots & a \end{vmatrix} \\ &\quad \left[ C_1 \rightarrow C_1 + C_2 + \dots + C_{n+1} \text{ and taking out } (n+1) \left( a + \frac{nb}{2} \right) \right] \\ &= (n+1) \left( a + \frac{nb}{2} \right) \begin{vmatrix} 1 & a+b & a+2b & \dots & a+nb \\ 0 & -b & -b & \dots & a+nb \\ 0 & (n-1)b & -2b & \dots & -2b \\ \dots & \dots & \dots & \dots & \dots \\ 1 & b & b & \dots & -nb \end{vmatrix} \end{aligned}$$

$[R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1, \dots, R_{n+1} \rightarrow R_{n+1} - R_1]$

$$= (-b)^n (n+1) \left( a + \frac{nb}{2} \right) \begin{vmatrix} 1 & a+b & a+2b & \dots & a+nb \\ 0 & 1 & 1 & \dots & 1 \\ 0 & (1-n) & 2 & \dots & 1 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & -1 & -1 & \dots & n \end{vmatrix}$$

[taking  $(-b)$  common from  $R_2, R_3, \dots, R_{n+1}$ ]

$$= (-b)^n (n+1) \left( a + \frac{nb}{2} \right) \begin{vmatrix} 1 & 1 & 1 & \dots & 1 \\ (1-n) & 2 & 2 & \dots & 2 \\ (2-n) & (2-n) & 3 & \dots & 3 \\ \dots & \dots & \dots & \dots & \dots \\ -1 & -1 & -1 & \dots & n \end{vmatrix}$$

[expanding along  $C_1$ ]

$$= (-b)^n (n+1) \left( a + \frac{nb}{2} \right) \begin{vmatrix} 1 & 0 & 0 & \dots & 1 \\ (1-n) & (n+1) & (n+1) & \dots & (n+1) \\ (2-n) & 0 & (n+1) & \dots & (n+1) \\ \dots & \dots & \dots & \dots & \dots \\ -1 & 0 & 0 & \dots & (n+1) \end{vmatrix}$$

$[C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 - C_1, \dots, C_n \rightarrow C_n - C_1]$

$$= (-b)^n (n+1) \left( a + \frac{nb}{2} \right) \begin{vmatrix} 1 & 0 & 0 & \dots & 0 \\ (1-n) & 1 & 1 & \dots & 1 \\ (2-n) & 0 & 1 & \dots & 1 \\ \dots & \dots & \dots & \dots & \dots \\ -1 & 0 & 0 & \dots & 1 \end{vmatrix}$$

[taking common  $(n+1)$  from  $C_2, C_3, \dots, C_n$ ]

$$= (-b)^n (n+1) \left( a + \frac{nb}{2} \right) \begin{vmatrix} 1 & 1 & \dots & 1 \\ 0 & 1 & \dots & 1 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

[expanding along  $R_1$ ]

$$= (-b)^n (n+1) \left( a + \frac{nb}{2} \right) = \text{R.H.S.}$$

**2.**  $\bar{b}z + b\bar{z} = c$  is the equation of a straight line. If  $z_1$  and  $z_2$  be mirror image of each other in this line, then prove that  $\bar{b}z_2 + b\bar{z}_1 = c$ .

(Rituraj, Chandigarh)

**Ans.** If  $z_1$  and  $z_2$  are mirror image of each other in the line

$$\bar{b}z + b\bar{z} = c \quad \dots(i)$$

then the mid-point of  $z_1, z_2$  must lie on (i) ... (ii)

and the line joining  $z_1, z_2$  must be perpendicular to (i) ... (iii)

.....  
Contd. on page no. 52

# THE PERFECT COACH

Now also  
available  
for JEE!

The image shows three MTG JEE Champion books side-by-side. The left book is for Chemistry, the middle for Physics, and the right for Mathematics. Each book has a red cover with the title 'JEE CHAMPION' in large yellow letters. At the top of each cover, it says 'mtG' and 'CLASS 11 & 12'. Below the title, there are four checkmarks: 'Chapterwise', 'Topicwise', 'Error Free', and '100% Solved'. The Chemistry book has a yellow background with a blue circle containing '5000+ CHAPTERWISE-TOPICWISE QUESTIONS FROM PREVIOUS YEARS ENGINEERING ENTRANCE EXAMS (2017-2007)' and a red circle containing 'SOLVE & SECURE TOP RANK IN JEE'. The Physics book has an orange background with a yellow circle containing '5000+ CHAPTERWISE-TOPICWISE QUESTIONS FROM PREVIOUS YEARS ENGINEERING ENTRANCE EXAMS (2017-2007)' and a red circle containing 'SOLVE & SECURE TOP RANK IN JEE'. The Mathematics book has a blue background with a green circle containing '5000+ CHAPTERWISE-TOPICWISE QUESTIONS FROM PREVIOUS YEARS ENGINEERING ENTRANCE EXAMS (2017-2007)' and a blue circle containing 'SOLVE & SECURE TOP RANK IN JEE'. All three books have a similar layout with sections for 'HIGHLIGHTS' and 'CONCISE THEORY FOR EFFECTIVE RECAPITULATION'.

**CHEMISTRY**

**PHYSICS**

**MATHEMATICS**

Skill. Passion. Hard work and determination. As a student sitting for the highly competitive JEE, you need all that. However, only a few will win, very likely with the help of a champion coach.

MTG's Champion Series for JEE is just the coach you need. It will guide you in identifying what's important for success and what's not. And then help you check your readiness with its most comprehensive question bank. So you know your strengths and weaknesses right from the word go and course-correct accordingly. Put simply, MTG's Champion Series will help you manage your preparation effort for JEE for maximum outcome. The best part is you study at a pace you're comfortable with.

Because it's all chapterwise, topicwise.



Visit [www.MTG.in](http://www.MTG.in) to buy online. Or visit a leading bookseller near you.  
For more information, email [info@mtg.in](mailto:info@mtg.in) or call 1800 300 23355 (toll-free) today.



VIT®

Vellore Institute of Technology  
(Deemed to be University under section 3 of UGC Act, 1956)

READY. STEADY.  
**VITEEE**

EXPERIENCE EDUCATION THAT'S AT PAR WITH TOP  
UNIVERSITIES ABROAD

**VITEEE 2018**

**4<sup>th</sup> April to 15<sup>th</sup> April (Computer Based Test)**

Applications are invited for B.Tech Programmes

Biotech., Bioengg., Chemical, Civil, CSE, CSE (Specialisation in Digital Forensics & Cyber security), CSE (Bioinfo.), CSE (Data Analytics), CSE (Gaming Tech.), CSE (Information Security), CSE (Networks & Security), ECE, ECE (Biomedical), ECE (Embedded Systems), ECE (IOT), ECE (Sensors & Wearable Tech.), ECE (VLSI) , EEE, EIE, Electronics & Computer Engg., IT, Mech., Mech (Auto), Mech. (Energy), Production & Industrial Engg., B.Des.Ind.Design

To apply online and to get details like eligibility and fee, please visit [www.vit.ac.in](http://www.vit.ac.in)

**LAST DATE FOR SUBMISSION: 28<sup>th</sup> FEBRUARY 2018**

**ONLINE APPLICATION FEE: ₹1150**

## International Transfer Programme (ITP)

VIT offers students the choice to study the first two years at VIT and the next two in the following universities abroad & get their degree

**United States of America:** • Purdue University • State University of New York (SUNY) • Rochester Institute of Technology (RIT) • University of Massachusetts (UMass) **United Kingdom:** • Queen Mary University of London **Australia:** • Australian National University • Deakin University • Queensland University of Technology • Royal Melbourne Institute of Technology (RMIT) • Curtin University

## Opportunities Unlimited

VIT offers students, opportunity to get placed in companies with regular, dream (5L CTC & above), Superdream (10L CTC & above) offers. In 2016-17, 427 companies came for placement to VIT, which itself is a record, of which 45+ were Super Dream Companies. Recruiting companies include:

- |             |            |              |                    |                  |
|-------------|------------|--------------|--------------------|------------------|
| • Microsoft | • Amazon   | • Maruti     | • ABB              | • Deloitte       |
| • Deshaw    | • Paypal   | • Hyundai    | • L & T            | • Morgan Stanley |
| • Oracle    | • Visa     | • Mahindra   | • ITC              | • KPMG           |
| • SAP       | • Cisco    | • Honda      | • Shapoorji        | • JP Morgan      |
| • Vmware    | • Barclays | • TVS Motors | • Johnson Controls | • PWC            |

## Advantage VIT

-  • Curriculum for Applied Learning (CAL)<sup>TM</sup>
-  • Project Based Learning (PBL)
-  • Opportunity for Semester Abroad Programme
-  • One module of every subject handled by Industry experts



- 35,000 students from 60 countries
- 110 student clubs and chapters
- More than 170 Adjunct Professors from reputed universities abroad

## International Accreditations

- ABET, US, <http://www.abet.org> has accredited 14 B.Tech Programs at VIT Vellore & Chennai Campus
- 3 cycles of NAAC accreditation completed
- Awarded 'A' grade for last two consecutive cycles
- First Engineering Institution in India to get QS 4 Stars Rating



Offline application forms can be obtained: • From the designated branches of Post Offices on cash payment of ₹1200/- (list available on our website) • By handing over a DD for ₹1200/- drawn in favour of Vellore Institute of Technology, payable at Vellore. Applications also available in VIT Chennai, VIT Bhopal & VIT AP.

For enquiries contact us on our helpline number +91-416-330 5555 For more details visit [www.vit.ac.in](http://www.vit.ac.in)



Ranked as 13<sup>th</sup> best engineering institution in India by NIRF, MHRD, GOI, for 2 consecutive years in 2016 & 2017

**FFCS™**  
FULLY FLEXIBLE CREDIT SYSTEM

- Option to Change Branch
- Opportunity to graduate with a double major
- Choice of major and minor programmes
- Degree with honours

## GET THE VIT EDGE

- National Record in Slot 1 companies recruitment for 9 years in a row
- Record 7943 job offers from Slot 1 companies (Accenture, Cognizant, Infosys, TCS, Wipro) at VIT for the year 2016
- Excellent international internship and placement opportunities



**VIT®**

Vellore Institute of Technology  
(Deemed to be University under section 3 of UGC Act, 1956)

[www.vit.ac.in](http://www.vit.ac.in)

VELLORE  
CHENNAI



**VIT®**  
VELLORE INSTITUTE OF TECHNOLOGY

[www.vitbhopal.ac.in](http://www.vitbhopal.ac.in)



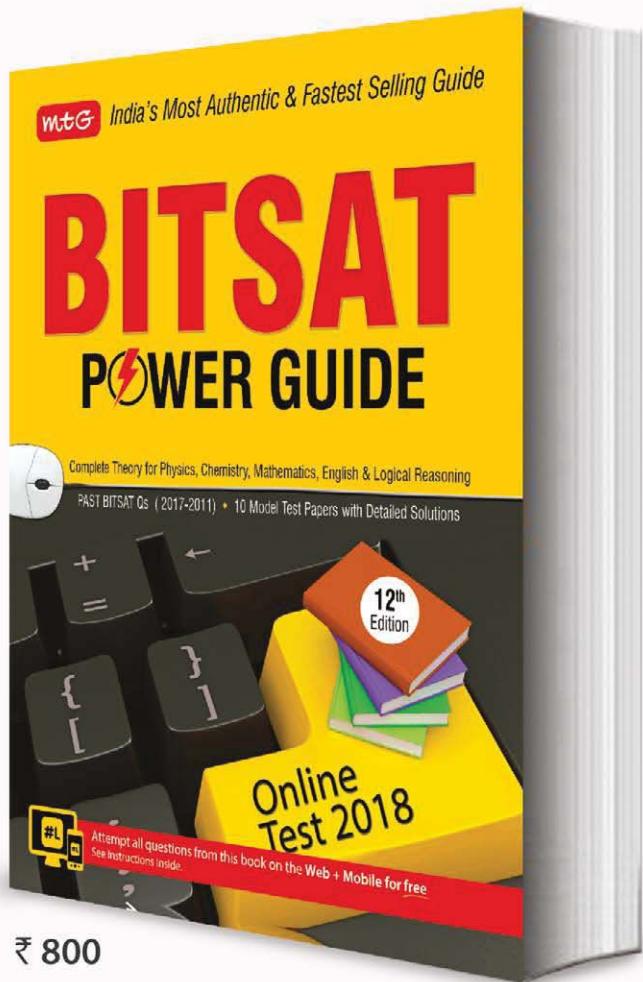
**VIT®**  
AP

[www.vitap.ac.in](http://www.vitap.ac.in)

# FULLY LOADED & COMPLETELY UPDATED

mtG

MTG's BITSAT Power Guide is not only the most exhaustive prep-tool, but also the only book available at present, updated as per the latest BITSAT syllabus for students aspiring for top rank in BITSAT 2018.



₹ 800

Get MTG's BITSAT Power Guide today for a real-world feel of BITSAT. Find out what's different about the BITSAT test, including its pattern of examination and key success factors. Be it with chapterwise MCQs or model test papers, check how good your chances are for glory in BITSAT 2018.

## FEATURES

- Covers all 5 subjects - Physics, Chemistry, Mathematics, English & Logical Reasoning
- Chapterwise MCQs in each section for practice
- Past chapterwise BITSAT Qs (2017-2011)
- 10 Model Test Papers with detailed solutions
- Attempt all questions & tests from this book online, for free with #L

Visit [www.MTG.in](http://www.MTG.in) to buy online. Or visit a leading bookseller near you.  
For more information, email [info@mtg.in](mailto:info@mtg.in) or call 1800 300 23355 (toll-free).

# MATH archives



Math Archives, as the title itself suggests, is a collection of various challenging problems related to the topics of JEE Main & Advanced Syllabus. This section is basically aimed at providing an extra insight and knowledge to the candidates preparing for JEE Main & Advanced. In every issue of MT, challenging problems are offered with detailed solution. The readers' & comments and suggestions regarding the problems and solutions offered are always welcome.

1. Equation of a common tangent to the curves  $y^2 = 8x$  and  $xy = -1$  is
 

(a) $3y = 9x + 2$	(b) $y = 2x + 1$
(c) $2y = x + 8$	(d) $y = x + 2$
2. Integrating factor of the differential equation  $2y \cos y^2 \frac{dy}{dx} - \frac{2}{x+1} \sin y^2 = (x+1)^3$  is
 

(a) $(x+1)^2$	(b) $\log_e(x+1)$
(c) $e^{\log_e(x+1)}$	(d) $\frac{1}{(x+1)^2}$
3. The radius of a right circular cylinder of maximum volume which can be inscribed in a sphere of radius  $R$  is
 

(a) $R$	(b) $\frac{R}{2}$	(c) $\sqrt{\frac{2}{3}}R$	(d) $\sqrt{\frac{3}{2}}R$
---------	-------------------	---------------------------	---------------------------
4. The distance between the line  $\frac{x-2}{1} = \frac{y+2}{-1} = \frac{z-3}{4}$  and the plane  $x + 5y + z = 5$  is
 

(a) $\frac{10}{3\sqrt{3}}$ units	(b) $\frac{10}{3}$ units
(c) $\frac{10}{9}$ units	(d) $\frac{10}{7}$ units
5. The number of ways in which 13 identical gold coins can be distributed among three persons such that each person can get at least two gold coins is
 

(a) 36	(b) 24	(c) 12	(d) 6
--------	--------	--------	-------
6. If  $A, B, C$  are the angles of a triangle, the system of equations  $(\sin A)x + y + z = \cos A$ ,  $x + (\sin B)y + z = \cos B$ ,  $x + y + (\sin C)z = 1 - \cos C$  has
 

(a) no solution
(b) unique solution
(c) infinitely many solutions
(d) finitely many solutions
7. Let  $f(x) = 2x^n + \lambda$ ,  $\lambda \in R$ ,  $f(4) = 133$  and  $f(5) = 255$  then sum of the factors of  $(f(3) - f(2))$  is
 

(a) 20	(b) 60	(c) 21	(d) 59
--------	--------	--------	--------
8. If  $|z^2 - 1| = |z|^2 + 1$ , then  $z$  lies on
 

(a) the real axis	(b) an ellipse
(c) a circle	(d) the imaginary axis
9. If  $x^2 - 4x + 5 - \sin y = 0$ ,  $y \in [0, 2\pi]$ , then
 

(a) $x = 1, y = 0$	(b) $x = 1, y = \pi/2$
(c) $x = 2, y = 0$	(d) $x = 2, y = \pi/2$
10. A solution of the equation  $\tan^{-1}(1+x) - \tan^{-1}(x-1) = \frac{\pi}{2}$  is
 

(a) $x = 1$	(b) $x = -1$	(c) $x = 0$	(d) $x = \pi$
-------------	--------------	-------------	---------------

## SOLUTIONS

1. (d): Any tangent to  $y^2 = 8x$  is  $y = mx + \frac{2}{m}$  ... (i)  
For (i) to be tangent to  $xy = -1$   
 $\Rightarrow x\left(mx + \frac{2}{m}\right) = -1 \Rightarrow m^2x^2 + 2x + m = 0$   
 Now,  $D = 0 \Rightarrow 4 - 4m^3 = 0 \Rightarrow 4m^3 = 1 \therefore m = 1$   
 Hence, tangent is  $y = x + 2$ .
2. (d): I.F. =  $e^{-\int \frac{2}{x+1} dx} = \frac{1}{(x+1)^2}$

By : Prof. Shyam Bhushan, Director, Narayana IIT Academy, Jamshedpur. Mob. : 09334870021

3. (c) : Let  $r$  be radius of cylinder and  $R$  be radius of sphere.

Let  $CM = x$

$$v = \pi(R^2 - x^2)2x$$

$$\text{For Maximum volume } \frac{dv}{dx} = 0$$

$$\Rightarrow x = \frac{R}{\sqrt{3}}; r = \sqrt{R^2 - x^2};$$

$$\text{Also, } \frac{d^2v}{dx^2} < 0$$

$\therefore$  For maximum volume,

$$r = \sqrt{R^2 - \frac{R^2}{3}}; r = \sqrt{\frac{2}{3}R^2} = \sqrt{\frac{2}{3}}R$$

$\therefore$  For maximum volume

4. (a) : Distance of line from the plane

$$= \frac{|(2\hat{i} - 2\hat{j} + 3\hat{k}) \cdot (\hat{i} + 5\hat{j} + \hat{k}) - 5|}{\sqrt{(1)^2 + (5)^2 + (1)^2}} = \frac{10}{3\sqrt{3}}$$

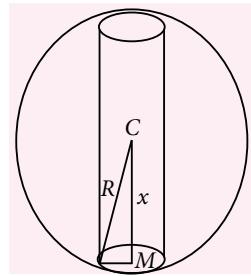
5. (a) :  $x + y + z = 13 - 6 = 7$

$$\therefore \text{No.of non negative solutions} = {}^n + {}^{r-1}C_{r-1}$$

$$= {}^{7+3-1}C_{3-1} = {}^9C_2 = 36.$$

$$6. \text{ (b)} : \text{Let } \Delta = \begin{vmatrix} \sin A & 1 & 1 \\ 1 & \sin B & 1 \\ 1 & 1 & \sin C \end{vmatrix}$$

Apply  $C_2 \rightarrow C_2 - C_1$  and  $C_3 \rightarrow C_3 - C_1$



Then expand along  $C_1$ , we get

$$\Delta = \sin A(1 - \sin B)(1 - \sin C) + (1 - \sin A)(1 - \sin C) + (1 - \sin A)(1 - \sin B)$$

Since  $A, B, C$  are angles of a triangle,

$$0 < \sin A, \sin B, \sin C \leq 1 \Rightarrow \Delta > 0$$

(or) Put  $A = B = C = 60^\circ \Rightarrow \Delta \neq 0$

7. (b) :  $f(x) = 2x^n + \lambda, f(4) = 133, f(5) = 255$

$$\therefore f(4) = 2 \cdot 4^n + \lambda = 133 \quad \dots(i)$$

$$f(5) = 2 \cdot 5^n + \lambda = 255 \quad \dots(ii)$$

$$\therefore f(5) - f(4) = 2(5^n - 4^n) = 122$$

$$\Rightarrow 5^n - 4^n = 61 = 5^3 - 4^3 \therefore n = 3$$

$$\text{Now, } f(4) = 133 = 2 \times 4^3 + \lambda \Rightarrow \lambda = 5$$

$$\therefore f(x) = 2x^3 + 5$$

$$\therefore f(3) = 59 \text{ and } f(2) = 21$$

$$\therefore f(3) - f(2) = 38 = 2 \times 19 = 2^a 19^b, (a=1, b=1)$$

Sum of factors of  $f(3) - f(2)$

$$= \frac{2^{a+1}-1}{2-1} \cdot \frac{19^{b+1}-1}{19-1} = \frac{3 \times 360}{18} = 60$$

8. (d) : Let  $z = x + iy$

$$\text{Given, } |z^2 - 1| = |z|^2 + 1$$

$$\Rightarrow (x^2 - y^2 - 1)^2 + 4x^2y^2 = (x^2 + y^2 + 1)^2 \Rightarrow x = 0$$

9. (d) :  $x^2 - 4x + 5 = \sin y \Rightarrow (x-2)^2 + 1 = \sin y \leq 1$

$$\Rightarrow \sin y = 1, x = 2 \Rightarrow x = 2, y = \pi/2$$

10. (c) :  $\tan^{-1}(1+x) - \tan^{-1}(x-1) = \frac{\pi}{2}$

$$\Rightarrow \tan^{-1} \frac{2}{x^2} = \frac{\pi}{2} \Rightarrow x=0$$



## Y?U ASK WE ANSWER

Contd. from page no. 46

Satisfying condition (ii), we have

$$\bar{b} \left( \frac{z_1 + z_2}{2} \right) + b \left( \overline{\frac{z_1 + z_2}{2}} \right) = c$$

$$\text{i.e., } \bar{b}(z_1 + z_2) + b(\bar{z}_1 + \bar{z}_2) = 2c$$

$$\text{i.e., } (\bar{b}z_1 + b\bar{z}_2) + (\bar{b}z_2 + b\bar{z}_1) = 2c \quad \dots(iv)$$

Putting  $z = x + iy$  in equation (i), it reduces to

$$\bar{b}(x+iy) + b(x-iy) - c = 0$$

$$\text{i.e., } (\bar{b}+b)x + i(\bar{b}-b)y - c = 0$$

$$\text{whose slope} = \frac{(b+\bar{b})}{i(b-\bar{b})}$$

Equation of the line joining  $z_1, z_2$  is

$$\begin{vmatrix} z & \bar{z} & 1 \\ z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \end{vmatrix} = 0$$

$$\text{i.e., } z\bar{a} - \bar{z}a + d = 0$$

$$\text{where } a = z_1 - z_2 \text{ and } d = z_1\bar{z}_2 - \bar{z}_1z_2$$

Putting  $z = x + iy$ , the above equation reduces to

$$\bar{a}(x+iy) - a(x-iy) + d = 0$$

$$\text{i.e., } (\bar{a}-a)x + i(\bar{a}+a)y + d = 0$$

$$\text{whose slope} = \frac{(a-\bar{a})}{i(a+\bar{a})}$$

Satisfying equation (iii), we have

$$\frac{(b+\bar{b})}{i(b-\bar{b})} \times \frac{(a-\bar{a})}{i(a+\bar{a})} = -1$$

$$\Rightarrow (b+\bar{b})(a-\bar{a}) = (b-\bar{b})(a+\bar{a}) \Rightarrow \bar{b}a = b\bar{a}$$

$$\Rightarrow \bar{b}(z_1 - z_2) = b(\bar{z}_1 - \bar{z}_2)$$

$$\Rightarrow \bar{b}z_1 + b\bar{z}_2 = \bar{b}z_2 + b\bar{z}_1 \quad \dots(v)$$

From equations (iv) and (v), we have

$$\bar{b}z_2 + b\bar{z}_1 = c$$

which is the desired result.





# TARGET JEE

## Probability

### DEFINITIONS OF VARIOUS TERMS

- **Random experiment :** An experiment is called random if all possible outcomes of the experiment are known in advance but the exact result of any specific performance cannot be predicted in advance.
- **Sample space :** The set of all possible outcomes of an experiment is known as sample space of the experiment and is usually denoted by  $S$ . The sample space of rolling a die is  $S = \{1, 2, 3, 4, 5, 6\}$  and  $n(S)$  denote the number of numbers in  $S$ . When a die is rolled twice, then  $n(S) = 6^2 = 36$ . Each element of the sample space is called **sample point**. There are two types of sample spaces namely discrete and continuous. The sample space whose elements are countable is discrete and for non-countable it is said to be continuous sample space.
- **Trial and event :** To perform an experiment is known as trial and possible outcomes of the experiment is called event. For example, tossing a coin or rolling a dice (any number of times) is trial and turning up head or tail or occurrence of numbers 1, 2, 3, 4, 5 or 6 are events. In fact, every subset of a sample space is an event.

### TYPES OF EVENT

- **Simple event :** An event is said to be simple if it contains only one sample point of the sample space.
- **Compound event :** An event is said to be a compound event if it contains more than one sample point of the sample space.
- **Equally likely event or equally probable event :** Events are said to be equally likely if there is no

reason for an event to occur in preference to any other event. It means events are said to be equally likely when each event is equally likely to occur as any other.

- **Mutually exclusive events (Disjoint events) :** Two or more than two events are said to be mutually exclusive if the occurrence any one of them exclude/ruled out the occurrence of the others. The events  $A_1, A_2, \dots, A_n$  are mutually exhaustive if  $A_i \cap A_j = \emptyset$  or  $P(A_i \cap A_j) = 0 \forall i \neq j$  and  $1 \leq i, j \leq n$ .  
(Note : Mutually exclusive events are also known as incompatible events).
- **Exhaustive events :** The events  $A_1, A_2, \dots, A_n$  are said to be exhaustive events, if their union form the sample space of the experiment. Thus, events  $A_i$  ( $i = 1, 2, \dots, n$ ) are said to be exhaustive if  $\bigcup_{i=1}^n A_i = S$  (sample space).
- **Mutually exclusive and exhaustive events:** Let  $S$  be the sample space associated with a random experiment.  
Then the events  $A_1, A_2, \dots, A_m$  form a mutually exclusive and exhaustive system of events if
  - (i)  $P(A_i \cap A_j) = 0 (\forall i \neq j)$
  - (ii)  $P(A_i \cup A_j) = P(A_1 \cup A_2 \cup \dots \cup A_m) = P(S) = 1$  ( $i \neq j$ )
- **Independent events :** Events  $A$  and  $B$  are said to be independent if happening or non-happening of  $A$  does not effect the happening or non-happening of  $B$ . In such type of events, we have the following results.
  - $P(A \cap B) = P(A) \cdot P(B)$  and
  - $P(A \cup B) = P(A) + P(B) - P(A)P(B)$

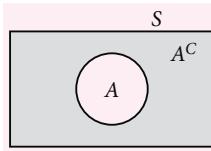
**Note :** If the events  $A, B, C$  are mutually independent,  
 $\Rightarrow P(A \cap B) = P(A)P(B), P(B \cap C) = P(B)P(C)$   
and  $P(C \cap A) = P(A)P(C)$  and  
 $P(A \cap B \cap C) = P(A)P(B)P(C)$

- **Dependent events :** Two or more than two events are said to be dependent if happening of one effects the happening of other. For example, drawing two card from a deck of card without replacement. If  $A$  and  $B$  are dependent events, then  
 $P(A \cap B) = P(A)P(B/A) = P(B)P(A/B)$

- **Complement of an event  $A$  :**

The set of all outcomes which are in  $S$  but not in  $A$  is called complement of  $A$  which is denoted by  $\bar{A}$  or  $A^C$ .

$$\therefore A^C = S - A$$



- **Impossible event :** Event which do not consist any point of the sample space is called impossible event. Throwing a die and hope to obtain tail or head is impossible or null event.

- **Some important points**

On a coin, a die, a chess board and deck of playing cards

- (1) Number of rectangles formed on a normal chess board =  ${}^9C_2 \times {}^9C_2$ .
- (2) Number of non-congruent squares of sides  $1 \times 1, 2 \times 2, \dots, 8 \times 8$  is 1 in each.  
 $\therefore$  Total number of non-congruent squares = 8
- (3) The number of non-congruent rectangles that can be found on a normal chess board are equal to the sum of the non-congruent squares and non-congruent rectangles which are not square =  ${}^8 + {}^8C_2 = 36$
- (4) Number of non-congruent rectangle which are not square =  ${}^8C_2 = 28$

### MATHEMATICAL OR CLASSICAL DEFINITION OF PROBABILITY

- If a random experiment having  $n$  mutually exclusive, equally likely and exhaustive events (outcomes) out of which  $m$  events (outcomes) are favourable to the occurrence of an event, then probability of occurrence of  $A$  is given by

$$P(A) = \frac{\text{No. of favourable outcomes to } A}{\text{No. of total outcomes}} = \frac{m}{n}$$

$$P(\bar{A}) = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(A)$$

**Note :** (i)  $0 \leq P(A) \leq 1$

(ii)  $P(\bar{A}) + P(A) = 1$

- Odd in favour of the event  $E$

$$= \frac{\text{No. of favourable cases (outcomes)}}{\text{No. of non-favourable cases (outcomes)}}$$

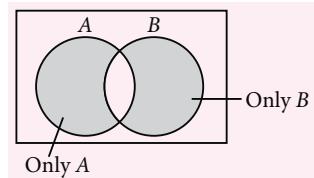
$$\therefore \frac{P(E)}{P(\bar{E})} = \frac{a}{b} \quad (n(S) = a+b)$$

- Similarly, odd against of the event,  $E = \frac{P(\bar{E})}{P(E)}$   
or  $\frac{n(\bar{E})}{n(E)}$

- Let  $A$  and  $B$  are two events then probability that at least one occur is denoted by  $P(A \text{ or } B)$  i.e.  $P(A \cup B)$  and is given by

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= P(A \cap \bar{B}) + P(\bar{A} \cap B) + P(A \cap B) \end{aligned}$$

- Probability that exactly one of the events  $A, B$  occurs  
 $= P(\text{only } A) + P(\text{only } B) = P(A - B) + P(B - A)$   
 $= P(A \cap B^C) + P(B \cap A^C)$   
 $= P(A) - P(A \cap B) + P(B) - P(A \cap B)$



$$= P(A) + P(B) - P(A \cap B) - P(A \cap B)$$

$$= P(A \cup B) - P(A \cap B)$$

$$= P(A^C \cup B^C) - P(A \cup B)^C$$

$$= P(A^C \cup B^C) - P(A^C \cap B^C)$$

- For any three events,  $A, B, C$

$P(\text{at least one of three events occurs})$

$$\begin{aligned} &= P(A) + P(B) + P(C) - [P(A \cap B) + P(B \cap C) \\ &\quad + P(C \cap A) - P(A \cap B \cap C)] \end{aligned}$$

The above result is called addition theorem on probability for three events which can be generalized for  $n$  events.

- $P(\text{at least two of } A, B, C \text{ occurs}) = P(A \cap B) + P(B \cap C) + P(C \cap A) - 2P(A \cap B \cap C)$
- $P(\text{exactly two of } A, B, C \text{ occurs}) = P(A \cap B) + P(B \cap C) + P(C \cap A) - 3P(A \cap B \cap C)$
- $P(\text{exactly one of } A, B, C \text{ occurs}) = P(A) + P(B) + P(C) - 2P(A \cap B) - 2P(B \cap C) - 2P(C \cap A) + 3P(A \cap B \cap C)$
- $P(A \text{ and } B \text{ but not } C \text{ occurs}) = P(A \cap B \cap \bar{C}) = P(A \cap B) - P(A \cap B \cap C)$
- $P(\text{only } A) = P(A \cap B' \cap C') = P(A \cap (B \cup C)')$   
 $= P(A) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C)$

- The results of  $P(\text{only } B)$  and  $P(\text{only } C)$  are similar.
- $P(\text{none of the event } A, B, C) = P(A \cup B \cup C)' = 1 - P(A \cup B \cup C) = 1 - P(\text{At least one of } A, B, C \text{ occurs})$
- If three events  $A, B, C$  are pair wise disjoint i.e. mutually exclusive then  $P(A \cap B) = P(B \cap C) = P(C \cap A) = P(A \cap B \cap C) = 0$  and  $P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C)$
- Addition theorem for  $n$  events : Let  $A_1, A_2, A_3, \dots, A_n$  are  $n$  events associated with a random experiment, then

$$P\left(\bigcup_{i=1}^n A_i\right) = \sum_{i=1}^n P(A_i) - \sum_{i,i \neq j, j=1}^n P(A_i \cap A_j) + \sum_{\substack{i,j,k=1 \\ i \neq j \neq k}}^n P(A_i \cap A_j \cap A_k) + \dots + (-1)^{n-1} P(A_1 \cap A_2 \cap \dots \cap A_n) \quad \dots (*)$$

If the events  $A_1, A_2, \dots, A_n$  form a pairwise disjoint family of events then (\*) reduces the following result.

$$P\left(\bigcup_{i=1}^n A_i\right) = \sum_{i=1}^n P(A_i) = P(A_1 + A_2 + \dots + A_n) = P(A_1) + P(A_2) + P(A_3) + \dots + P(A_n)$$

### CONDITIONAL PROBABILITY (MULTIPLICATION THEOREM ON PROBABILITY)

Let  $A$  and  $B$  are two events associated with a random experiment. Then, the probability of occurrence (happening) of  $A$  when it is known that the event  $B$  has already happened i.e.  $P(B) \neq 0$ , is called the conditional probability, denoted by  $P(A|B)$  and defined as

$$P(A|B) = \frac{n(A \cap B)}{n(B)} = \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}} = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A \cap B) = P(B) P(A|B), P(B) \neq 0$$

Similarly, we have  $P(A \cap B) = P(A) P(B|A), P(A) \neq 0$

### Extension of multiplication theorem

- If  $A, B, C$  are three events of a sample space, then

$$P(A \cap B \cap C) \text{ or } P(ABC) = P(A) P(B|A) P(C|AB)$$

- If  $A_i (i = 1, 2, \dots, n)$  are  $n$  events associated with a sample space then

$$P(A_1 \cap A_2 \cap \dots \cap A_n) = P(A_1) P(A_2|A_1) P(A_3|A_1A_2) \dots P(A_n|A_1A_2 \dots A_{n-1})$$

- Multiplication theorem for independent events**  
If the events  $A$  and  $B$  are independent associated with a random experiment then probability of their simultaneously occurrence is equal to the product of their individual occurrence.

$$\text{i.e. } P(A \cap B) = P(A) P(B)$$

- Extension of multiplication theorem for independent events**

- If  $A, B, C$  are three events associated with a sample space then

$$P(A \cap B \cap C) = P(A) P(B) P(C), \text{ events are pair wise independents.}$$

- If  $A_i (i = 1, 2, \dots, n)$  are  $n$  independent events associated with a sample space of a random experiment, then

$$P(A_1 \cap A_2 \cap A_3 \cap \dots \cap A_n) = P(A_1) P(A_2) \dots P(A_n)$$

- Probability of atleast one event among the two, three or more independent events**

- For two events: probability of happening at least one is given by  $P(A \cup B)$

$$= 1 - P(\bar{A}) P(\bar{B})$$

- For three events  $A, B, C$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B)$$

$$- P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

$$= 1 - P(\bar{A}) P(\bar{B}) P(\bar{C})$$

- For  $n$  events  $A_1, A_2, A_3, \dots, A_n$

$$P(A_1 \cup A_2 \cup \dots \cup A_n) = 1 - P(\bar{A}_1) P(\bar{A}_2) P(\bar{A}_3) \dots P(\bar{A}_n)$$

- Probability of happening of none of them

$$= P(\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3 \cap \dots \cap \bar{A}_n)$$

$$= P(\bar{A}_1) P(\bar{A}_2) P(\bar{A}_3) \dots P(\bar{A}_n)$$

$$= (1 - p_1) (1 - p_2) \dots (1 - p_n) \text{ where } P(A_i) = p_i$$

### LAW OF TOTAL PROBABILITY AND BAYES' THEOREM

- Partition of a sample space

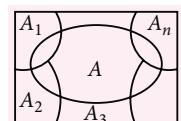
A set of events  $A_i (i = 1, 2, \dots, n)$  is said to represent a partition of the sample space  $S$  is

- $A_i \cap A_j = \emptyset, i \neq j,$

$$j = 1, 2, \dots, n$$

- $A_1 \cup A_2 \cup \dots \cup A_n = S$  and

- $P(A_i) > 0 \forall i = 1, 2, 3, \dots, n$



In otherwords, partitions  $\{A_1, A_2, \dots, A_n\}$  of a sample space  $S$  represent a set of mutually exclusive and exhaustive events having non-zero probabilities.

### Law of total probability

- If  $A_1, A_2, \dots, A_n$  are mutually exclusive and exhaustive and  $A$  be any event which occurs with  $A_1$  or  $A_2$  or ..... or  $A_n$ , then

$$P(A) = P(A_1) P(A|A_1) + P(A_2) P(A|A_2) + \dots + P(A_n) P(A|A_n)$$

### Bayes' theorem

- If  $A_1, A_2, \dots, A_n$  are  $n$  mutually exclusive and exhaustive events, then

$$P(A_i | A) = \frac{P(A_i) P(A | A_i)}{\sum_{i=1}^n P(A_i) P(A | A_i)}$$

### RANDOM VARIABLES

- A random variable is often described as a variable whose values are determined by outcome of a random experiment.
- If to each point of sample space we assign a real number, we have a function defined on the sample space. This function is called random function. Thus, a random variable is a real function whose domain is the sample space of a random experiment and whose range is a real line. For example, the sample space ( $S$ ) of a simultaneous throw of two coins is  $S = \{\text{HH}, \text{TH}, \text{HT}, \text{TT}\}$ .

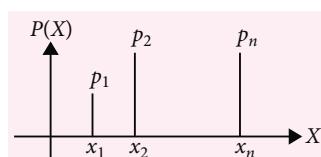
Let  $X$  denote the number of heads. Then  $X$  is a random variable. It can have values  $X(\text{TT}) = 0$ ,  $X(\text{TH}) = 1$ ,  $X(\text{HH}) = 2$  and  $X(\text{HT}) = 1$ . Thus, the domain of  $X$  is  $S$  and range is  $\{0, 1, 2\}$ . On the other hand if we denote  $Y$  as the number of tails then  $Y$  is also a random variable, having domain  $S$  and range is  $\{0, 1, 2\}$ .

### PROBABILITY DISTRIBUTION

- Let  $\{X = x_i\}$  is an event and  $P(X = x_i) = p_i$  ( $1 \leq i \leq n$ ) then the system of numbers

$X_i$	$x_1$	$x_2$	$\dots$	$\dots$	$x_n$
$P(x = x_i)$	$p_1$	$p_2$	$\dots$	$\dots$	$p_n$

is said to be probability distribution of the random variable  $X$  whose graphical representation is



$$\text{Note : (i)} \quad \sum_{i=1}^n P(X = x_i) = \sum_{i=1}^n p_i = 1$$

$$\text{(ii)} \quad P(X = x_i) \text{ lies between 0 and 1} \\ \text{i.e. } 0 \leq P(X = x_i) \leq 1$$

### BERNOULLI TRIALS

The set of  $n$  trials is said to be Bernoulli trial if

- The value of  $n$  is finite i.e. number of trials is finite.
- Each and every trial is independent.
- Trial (experiment) consists of only two outcomes namely success and failure.
- Probability of success and failure for each trial is fixed (same).

### BINOMIAL DISTRIBUTION(B.D.)

- Let  $X$  be a random variable which assume the values  $0, 1, 2, \dots, n$  is said to be in binomial distribution if its distribution function is given by  

$$P(X = r) = {}^n C_r p^r q^{n-r}, r = 0, 1, 2, \dots, n \text{ and}$$

$$p, q > 0 \text{ and } p + q = 1$$
- For the random variable  $X$ , the notation  $X \sim B(n, p)$  is used for binomial distribution with parameters  $n$  and  $p$ , where  $n$  is the number of trials and  $p$  is the probability of success for each trial.

### Mean and variance of binomial distribution

The binomial distribution is

$$\begin{array}{ccccccc} X & 0 & 1 & 2 & \dots & n \\ P(X) & {}^n C_r q^n & {}^n C_1 q^{n-1} p & {}^n C_2 q^{n-2} p^2 & \dots & {}^n C_n q^0 p^n \end{array}$$

- Mean

$$\bar{X} = E(X) = \sum_{i=1}^n x_i p_i = \sum_{r=1}^n r {}^n C_r q^{n-r} p^r = np$$

- Variance : The variance of the binomial distribution is denoted by  $\text{var}(X)$  or  $\sigma^2$  and given by

$$\text{var}(X) = \sum_{r=0}^n r^2 P(r) - \sum_{r=0}^n r P(r) = npq$$

Note : (Mean – variance) =  $np - npq$

$$= np(1 - q) = np^2 > 0, 0 \leq q \leq 1$$

$\therefore$  mean – variance  $> 0 \Rightarrow$  mean  $>$  variance

- Recurrence relation for binomial distribution recurrence relation is given by

$$\frac{P(r+1)}{P(r)} = \frac{{}^n C_{r+1}}{{}^n C_r} \left( \frac{p}{q} \right) \Rightarrow P(r+1) = \frac{n-r}{r+1} \frac{p}{q} P(r) \\ r \in \{0, 1, 2, \dots, n-1\}$$

- Use of multi-nominal expansion : If a die has  $m$  faces marked with numbers  $1, 2, \dots, m$  and if such  $n$  dice are rolled up, then the probability that sum of the numbers appears (exhibited) on the upper faces equal to  $\lambda$  is the coefficient of  $x^\lambda$  in the expansion of

$$\frac{(x^1 + x^2 + x^3 + \dots + x^m)^n}{m^n}$$

## GEOMETRICAL METHOD FOR PROBABILITY

When the number of points in the sample space is infinite it is difficult to use the mathematical definition of probability. For instance, if we need to find the probability that a circle of radius 2 unit lies in a square of side 4 units. We cannot apply the mathematical definition. In such cases we define the probability as follows:

$$P\{y \in A\} = \frac{\text{Measure of region } A}{\text{Measure of sample space}}$$

## SOME IMPORTANT RESULTS

- Letters and envelopes :** Let  $n$  letters corresponding to  $n$  envelopes are placed in the envelopes randomly, then

$$(1) \text{ Probability that all letters are placed in the right envelopes} = \frac{1}{n!}$$

$$(2) \text{ Probability that all letters are not placed in the right envelopes} = 1 - \frac{1}{n!}$$

$$(3) P(\text{no letter goes to right envelopes})$$

$$= \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \dots + (-1)^n \frac{1}{n!}$$

$$(4) P(\text{exactly } m \text{ letters are goes to right envelopes})$$

$$= \frac{1}{m!} \left[ \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \dots + (-1)^{n-m} \frac{1}{(n-m)!} \right]$$

- If  $P(A \cup B) = P(A \cap B) \Rightarrow P(A) = P(B)$

- If  $A \subseteq B \Rightarrow n(A) \leq n(B)$  then

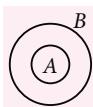
$$(i) P(A) \leq P(B)$$

$$(ii) P(B - A) = P(B) - P(A)$$

- If  $A, B$  are such that  $P(A) > 0, P(B) \neq 1$

$$\text{then } P(\bar{A} | \bar{B}) = \frac{1 - P(A \cup B)}{P(\bar{B})}$$

- If  $A$  and  $B$  are events such that  $P(A|B) = P(B|A)$  then  $P(A) = P(B)$



## PROBLEMS

### Single Correct Answer Type

- A square is inscribed in a circle. If  $p_1$  is the probability that a randomly selected point of the circle lies within the square and  $p_2$  is the probability that point lies outside the square then which of the following is true?

- (a)  $p_1 < p_2$       (b)  $p_1 = p_2$
- (c)  $p_1 > p_2$  and  $p_1^2 - p_2^2 < 1/3$
- (d) none of these

- A natural number  $x$  is chosen randomly from the first 150 natural numbers, the probability that

$$\frac{x^2 - 48x + 540}{x - 25} < 0 \text{ is}$$

- (a)  $\frac{17}{50}$       (b)  $\frac{7}{50}$       (c)  $\frac{4}{50}$       (d)  $\frac{13}{50}$

- A coin is tossed  $n$  times. If the probability of getting at least two heads is less than that of getting at least one tail by  $\frac{3}{32}$ , then the value of  $n$  equals

- (a) 3      (b) 4      (c) 5      (d) 6

- In a  $3 \times 3$  matrix, entries  $a_{ij}$  are selected randomly from the digits 0, 1, 2, ... 9 with replacement. The probability that 3 digit number in each row will be divisible by 15 is

- (a)  $\frac{67}{10^3}$       (b)  $\frac{77}{10^3}$       (c)  $\frac{66}{10^3}$       (d)  $\frac{48}{10^3}$

- An ellipse whose eccentricity is  $\frac{2\sqrt{6}}{5}$  is inscribed in a circle and a point that lies within the circle is chosen randomly, the probability that chosen point lies outside the ellipse is

- (a)  $2/3$       (b)  $3/5$       (c)  $4/5$       (d)  $1/5$

- Two players  $A$  and  $B$  throw a die one after the other for a lottery whose prize worth is ₹ 22. The player who first throws five will win the game. If  $A$  starts the game, the respective expectations of  $A$  and  $B$  are (in ₹) is

- (a) ₹ 12, ₹ 10      (b) ₹ 10, ₹ 12
- (c) ₹ 9, ₹ 13      (d) ₹ 11 each

- Two numbers are selected at random from a set of first 120 natural numbers. Two numbers are selected randomly, the probability that product of selected number is divisible by 3.

- (a)  $\frac{13}{357}$       (b)  $\frac{199}{357}$       (c)  $\frac{158}{357}$       (d)  $\frac{160}{357}$

- A man throws a fair coin a number of times and obtained 2 points for each head he throws and for each tail he gets only a point. The probability that he gets a total of 6 points is

- (a)  $\frac{39}{64}$       (b)  $\frac{43}{64}$       (c)  $\frac{41}{64}$       (d)  $\frac{29}{64}$

- Let  $E$  and  $F$  are independent events such that  $P(E \cap F^c) = 0.12$  and  $P(E^c \cap F) = 0.32$  and  $P(E) > 0.5$  then the value of  $P(E) + P(F)$  equals

- (a) 1.5      (b) 1.6      (c) 1.8      (d) 0.6

- Bag  $A$  consists of 3 red and 4 blue balls and the bag  $B$  consists of 5 red and 8 blue balls. One ball is drawn

randomly from one of the bags and is found to be blue, the probability that ball was drawn from bag  $B$  is

- (a)  $\frac{14}{27}$     (b)  $\frac{13}{27}$     (c)  $\frac{39}{74}$     (d)  $\frac{35}{74}$

#### More than One Correct Answer Type

**11.** Bag  $A$  contains 2 white and 3 red balls and bag  $B$  contains 4 white and 7 red balls, one bag is selected randomly and a ball is drawn from the bag then

- (a) If drawn ball is found to be red, the probability that it was drawn from bag  $B$  is  $\frac{35}{68}$
- (b) If drawn ball is found to be white, the probability that it was drawn from bag  $A$  is  $\frac{11}{21}$
- (c) If drawn ball is found to be red, the probability that it was drawn from bag  $A$  is  $\frac{33}{68}$
- (d) If drawn ball is found to be white, the probability that it was drawn from bag  $B$  is  $\frac{10}{21}$

**12.** The independent probabilities that  $A$ ,  $B$ ,  $C$  can solve the problem are  $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$  respectively then

- (a) Probability that the problem will be solved is  $\frac{3}{4}$
- (b) Probability that problem can not be solved is  $\frac{1}{4}$
- (c) Probability that only two of them solve the problem is  $\frac{1}{4}$
- (d) Probability exactly one of them solve the problem is  $\frac{11}{24}$

**13.** Let  $0 < P(A) < 1$ ,  $0 < P(B) < 1$  and  $P(A \cup B) = P(A) + P(B) - P(A)P(B)$  then which of the following is incorrect?

- (a)  $P(\overline{A \cup B}) = P(\overline{A})P(\overline{B})$
- (b)  $P(A|B) = P(A)$
- (c)  $P(\overline{A} \cup \overline{B}) = P(\overline{A}) + P(\overline{B})$
- (d)  $P(B|A) = P(B) - P(A)$

**14.** For two events  $A$  and  $B$ , the probability that exactly one of them occurs is given by

- (a)  $P(A) + P(B) - 2P(A \cap B)$
- (b)  $P(A \cup B) - P(A \cap B)$
- (c)  $P(A') + P(B') - 2P(A' \cap B')$
- (d)  $P(A' \cap B) + P(A \cap B')$

**15.** A cubical die is thrown 11 times and the numbers obtained are formed as a number whose digits are equal in numbers as many times the die is thrown then probability that number

- (a) begins with 531 is  $\frac{1}{6^3}$
- (b) end with 642 is  $\frac{1}{6^3}$
- (c) begins with 642 and ends with 531 is  $\frac{1}{6^6}$
- (d) begins with 531 and ends with 642 is  $\frac{431}{216 \times 216}$

**16.** An electric component manufactured by 'RAJU' Electronics is tested for its defectiveness by a sophisticated testing device. Let 'A' denote the event "the device is defective" and 'B' denote the event "the testing device reveals the component to be defective". If

$$P(A) = \alpha \text{ and } P\left(\frac{B}{A}\right) = P\left(\frac{B'}{A'}\right) = 1 - \alpha \text{ where } 0 < \alpha < 1$$

then which of the following is true?

- (a)  $P(B) = 2\alpha(1 - \alpha)$     (b)  $P\left(\frac{A'}{B}\right) = \frac{1}{2}$
- (c)  $P(B') = \alpha^2 + (1 - \alpha)^2$
- (d)  $P\left(\frac{A'}{B'}\right) = \left(\frac{\alpha}{1 - \alpha}\right)^2$

**17.** Three numbers are selected at random without replacement from  $\{1, 2, 3, \dots, 11\}$ . Let  $A_1$  be the event that minimum of selected numbers is 4 and  $A_2$  be the event their maximum is 7 then

- (a)  $P(A_1) = \frac{7}{55}$     (b)  $P(A_2) = \frac{1}{11}$
- (c)  $P\left(\frac{A_2}{A_1}\right) = \frac{2}{21}$     (d)  $P\left(\frac{A_1}{A_2}\right) = \frac{2}{15}$

#### Comprehension Type

##### Paragraph for Q.No. 18-20

A bag contains 9 black, 7 white and 4 green balls. If 3 balls are drawn randomly then,

**18.** Probability of getting 3 balls of different colours

- (a)  $\frac{7}{95}$     (b)  $\frac{14}{95}$     (c)  $\frac{21}{95}$     (d)  $\frac{7}{228}$

**19.** Probability of getting balls in order of colour green, white, black respectively.

- (a)  $\frac{7}{190}$     (b)  $\frac{11}{190}$     (c)  $\frac{17}{190}$     (d)  $\frac{29}{190}$

- 20.** Probability of getting 2 black and one green ball  
 (a)  $\frac{7}{95}$  (b)  $\frac{12}{95}$  (c)  $\frac{13}{95}$  (d)  $\frac{20}{95}$

**Paragraph for Q. No. 21-23**

The contents of urns I, II, III are as follows:

Urn I : 1 White, 2 Black, 3 Red balls and

Urn II : 2 White, 1 Black, 1 Red ball and

Urn III : 4 White, 5 Black and 3 Red balls

One urn is chosen randomly and two balls are drawn. If  $E_1, E_2, E_3$  denote the event that urn I, II and III are chosen respectively and  $E$  be the event of selection of two balls from the selecting urn as white and red. Then

**21.**  $P(E)$  equals

- (a)  $\frac{118}{495}$  (b)  $\frac{131}{495}$  (c)  $\frac{137}{495}$  (d)  $\frac{132}{495}$

**22.**  $P\left(\frac{E_1}{E}\right) =$

- (a)  $\frac{118}{495}$  (b)  $\frac{33}{118}$  (c)  $\frac{55}{118}$  (d)  $\frac{15}{59}$

**23.**  $P\left(\frac{E_2}{E}\right) + P\left(\frac{E_3}{E}\right) =$

- (a)  $\frac{55}{118}$  (b)  $\frac{30}{118}$  (c)  $\frac{85}{118}$  (d)  $\frac{33}{118}$

**Paragraph for Q.No. 24-26**

An urn contains 4 white and 3 red balls. Three balls are drawn one by one with replacement. Then,

**24.** Probability of getting 2 white and 1 red ball is

- (a)  $\frac{144}{343}$  (b)  $\frac{199}{343}$  (c)  $\frac{216}{343}$  (d)  $\frac{197}{343}$

**25.** Probability that at most two balls are red is

- (a)  $\frac{199}{343}$  (b)  $\frac{316}{343}$  (c)  $\frac{180}{343}$  (d)  $\frac{319}{343}$

**26.** Probability of getting at least 2 white balls is

- (a)  $\frac{172}{343}$  (b)  $\frac{171}{343}$  (c)  $\frac{208}{343}$  (d)  $\frac{216}{343}$

**Matrix Match Type**

**27.** Match the following.

	<b>Column - I</b>	<b>Column - II</b>
A.	Three number are selected from the set $\{1, 2, 3, \dots, 10\}$ , the probability that they form a G. P.	p. $\frac{5}{18}$

B.	If $x$ and $y$ are selected from the set of numbers $\{1, 2, 3, \dots, 10\}$ the probability that $x^3 + y^3$ is divisible by 3 is	q.	$\frac{1}{4}$
C.	A card is lost from a pack of 52 cards. If a card is drawn from the remaining cards, then the probability that it is a heart is	r.	$\frac{1}{3}$
D.	Two balls are drawn from an urn containing 2 white, 3 red and 4 black balls one by one without replacement. The probability that they are of same colour is	s.	$\frac{1}{15}$

**28.** Match the following.

	<b>Column - I</b>	<b>Column - II</b>
A.	$A$ and $B$ are two independent events such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{5}$	p. $P(A \cup B) \geq \frac{3}{5}$
B.	$A$ and $B$ are two events such that $P(A) = \frac{1}{2}, P(B) = \frac{2}{3}$	q. $P\left(\frac{A}{B}\right) = \frac{1}{2}$
C.	$A$ and $B$ are two independent events such that $P(B) = \frac{1}{5}$ and $P(A \cup B) = \frac{3}{5}$	r. $P(A \cap B) \leq \frac{1}{2}$
D.	$A$ and $B$ are two independent events such that $P(A) = \frac{3}{7}$ and $P(B) = \frac{1}{2}$	s. $P(\text{Exactly one of } A, B) = \frac{1}{2}$

**Integer Type**

**29.** A bag contains 4 balls, two are drawn at random and found to be white and the probability that all balls are white is  $\lambda$  then the value of  $5\lambda$  is

**30.** If  $X$  and  $Y$  are independent binomial variates  $X\left(5, \frac{1}{2}\right)$  and  $Y\left(7, \frac{1}{2}\right)$  and the value of  $P(X + Y = 4)$  is  $m$  then, value of  $\frac{4096}{99}m$  equals

**31.** If the papers of 5 students can be checked by any one of the 5 teachers. If the probability that all the 5 papers are checked by exactly 2 teachers is  $m$  then the

value of  $\frac{125m}{2}$  equals

**32.** Two numbers  $x$  and  $y$  are randomly selected from the set  $A = \{1, 2, 3, 4, 5, \dots, 25\}$  and the probability that  $x^2 - y^2$  is divisible by 3 is  $\lambda$  then the value of  $\frac{75\lambda}{41}$  is

**33.** A pair of dice is rolled once and a total of 8 has appeared. The chance that even number appear on each dice is  $\frac{A}{B}$  and  $B^2 - A^2 = k^2$ ,  $k > 0$  then  $k$  equals

**34.** If  $A$  and  $B$  be any two events such that  $P(A) = \frac{2}{5}$  and  $P(A \cap B) = \frac{3}{20}$  and conditional probability  $P(A | A' \cup B') = \frac{l}{m}$  then  $l$  equals

**35.** If the mean and variance of a binomial variate  $x$  are 2 and 1 respectively and the probability that  $x$  takes the value greater than or equal to 1 is  $\lambda$ , then value of  $\frac{16\lambda}{5}$  is

### SOLUTIONS

**1. (c)** : Let the radius of the circle be  $r$  and side of square be  $x$ .

So, side of square =  $\sqrt{2}r$

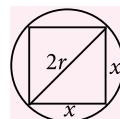
Area of square =  $2r^2$

$$\therefore p_1 = \frac{\text{Area of square}}{\text{Area of circle}} = \frac{2r^2}{\pi r^2} = \frac{2}{\pi}$$

$$\Rightarrow p_2 = 1 - p_1 = \frac{\pi - 2}{\pi} \quad \therefore p_1 > p_2$$

$$\text{Now, } p_1^2 - p_2^2 = (p_1 + p_2)(p_1 - p_2) = (1) \left( \frac{4 - \pi}{\pi} \right)$$

$$p_1^2 - p_2^2 < \frac{1}{3} \quad \text{as } 3 < \pi < 4$$



**2. (b)** : As  $\frac{x^2 - 48x + 540}{x - 25} < 0$

$$\Rightarrow \frac{(x-18)(x-30)}{x-25} < 0$$

$$\Rightarrow x \in \{1, 2, 3, \dots, 17\} \cup \{26, 27, 28, 29\}$$

$$\therefore \text{Required probability} = \frac{21}{150} = \frac{7}{50}$$

**3. (d)** : We have,  $P(X = r) = {}^nC_r p^r q^{n-r}$ ,  $p = q = 1/2$   
 $\therefore$  Probability of getting at least 2 heads

$$= 1 - P(0) - P(1) = 1 - \frac{1}{2^n} - n \cdot \frac{1}{2^n}$$

Probability of getting at least 1 tail

$$= 1 - P(0) = 1 - \frac{1}{2^n}$$

Now, as per question we have

$$\left(1 - \frac{1}{2^n}\right) - \left(1 - \frac{1}{2^n} - \frac{n}{2^n}\right) = \frac{3}{32}$$

$$\Rightarrow \frac{n}{2^n} = \frac{3}{32} = \frac{6}{64} = \frac{6}{2^6} \Rightarrow n = 6$$

**4. (a)** : Here  $n(S) =$  total three digit number with repetition of digit =  $10^3$

Now numbers from 000 to 999 which are divisible by 15 is 67 given by rows

$$000, 015, 030, \dots, 090 = 7$$

$$105, 120, \dots, 195 = 7$$

$$210, 225, \dots, 300 = 7$$

...

...

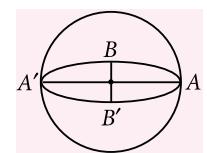
$$915, 930, 945, \dots, 990 = 6$$

$\Rightarrow$  Total favourable numbers =  $7 \times 7 + 6 \times 3 = 67$

$$\therefore \text{Required probability} = \frac{67}{10^3}$$

**5. (c)** : Let the ellipse is  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ,  $b < a$

We know that, if length of major and minor axes are  $a$  and  $b$  units respectively, then the area of ellipse is given by  $\pi ab$  where  $b^2 = a^2(1 - e^2)$



$$\therefore \text{Area of ellipse} = \pi a(a\sqrt{1-e^2})$$

$$= \pi a^2 \left( \sqrt{1 - \frac{24}{25}} \right) = \frac{\pi a^2}{5}$$

Also, area of circle =  $\pi a^2$

$\therefore$  Favourable region = area of circle - area of ellipse

$$= \pi a^2 - \frac{\pi a^2}{5} = \frac{4\pi a^2}{5}$$

$$\therefore \text{Required probability} = \frac{4\pi a^2}{5} \times \frac{1}{\pi a^2} = \frac{4}{5}$$

**6. (a)** : Probability of throwing the number 5 =  $1/6$

Now, it is given that  $A$  starts the game, which means  $A$  gets the chance in first, third, fifth, ... so on throws.

$$\therefore P(A) = \frac{1}{6} \Rightarrow P(\bar{A}) = 1 - \frac{1}{6} = \frac{5}{6}$$

$$\text{Similarly } P(B) = \frac{1}{6} \Rightarrow P(\bar{B}) = \frac{5}{6}$$

Probability of winning of A

$$\begin{aligned}
 &= P(A) + (P(\bar{A}))^2 P(A) + (P(\bar{A}))^4 P(A) + \dots \\
 &= \frac{1}{6} + \left(\frac{5}{6}\right)^2 \left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^4 \frac{1}{6} + \dots \\
 &= \frac{1}{6} \left[ 1 + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^4 + \left(\frac{5}{6}\right)^6 + \dots + \infty \right] \\
 &= \frac{1}{6} \left( \frac{1}{1 - \frac{25}{36}} \right) = \frac{6}{11}
 \end{aligned}$$

Probability of winning of B =  $1 - \frac{6}{11} = \frac{5}{11}$

$\therefore$  Expectation of A and B are

$$E(A) = \frac{6}{11} \times 22 = 12 \text{ (in ₹)}$$

$$E(B) = \frac{5}{11} \times 22 = 10 \text{ (in ₹)}$$

7. (b) : Given set of numbers is {1, 2, 3, 4, ..., 120}

Now, {1, 2, 3, 4, 5, ..., 120}

$$= \underbrace{\{1, 2, 4, 5, \dots, 119\}}_{80 \text{ numbers}} \cup \underbrace{\{3, 6, 9, \dots, 120\}}_{40 \text{ numbers}}$$

The product of two numbers will be divisible by 3 if both the numbers are from the set {3, 6, 9, ..., 120} (i.e. from 40 numbers) or one from the set {1, 2, 4, 5, ..., 119} and other from the set {3, 6, 9, ..., 120}.

Let A = event that product of number is divisible by 3

$$\therefore n(A) = {}^{40}C_2 + {}^{80}C_1 \times {}^{40}C_1 \text{ and } n(S) = {}^{120}C_2$$

$$\therefore P(A) = \frac{{}^{40}C_2 + {}^{80}C_1 \times {}^{40}C_1}{{}^{120}C_2} = \frac{199}{357}$$

8. (b) : Given, the number of points on throwing head = 2 and for tail = 1 point.

$\therefore$  Cases to get a total of 6 points

HHH, HHTT, HTTT and TTTTTT and

$$P(T) = 1/2 = P(H)$$

$\therefore$  Required probability

$$\begin{aligned}
 &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + {}^4C_2 \left( \frac{1}{2} \times \frac{1}{2} \right)^2 + {}^5C_1 \left( \frac{1}{2} \right)^1 \left( \frac{1}{2} \right)^4 + {}^6C_6 \left( \frac{1}{2} \right)^6 \\
 &= \frac{8 + 24 + 10 + 1}{64} = \frac{43}{64}
 \end{aligned}$$

9. (d) : Given,  $P(E \cap F^c) = 0.12$  and  $P(E^c \cap F) = 0.32$ ,  $P(E) > 0.5$

Let  $P(E) = m$  and  $P(F) = n$

Since,  $P(E \cap F^c) = P(E) - P(E \cap F)$

$$\Rightarrow 0.12 = P(E)(1 - P(F)) \quad (\because E \text{ and } F \text{ are independent})$$

$$\Rightarrow 0.12 = m(1 - n) \quad \therefore m = \frac{0.12}{1 - n} \quad \dots(i)$$

$$\text{Also } P(F \cap E^c) = P(F) - P(E \cap F) = P(F)(1 - P(E)) \\ = n(1 - m)$$

$$\Rightarrow 0.32 = n(1 - m) \Rightarrow m = 1 - \frac{0.32}{n} \quad \dots(ii)$$

From (i) and (ii), we get

$$n^2 - 1.2n + 0.32 = 0 \Rightarrow (n - 0.8)(n - 0.4) = 0$$

$$\Rightarrow n = 0.8 \text{ or } n = 0.4$$

When  $n = 0.8 \Rightarrow m = 0.6$  and

when  $n = 0.4 \Rightarrow m = 0.2$

$$\therefore m + n = 1.4 \text{ and } m + n = 0.6$$

$$\Rightarrow P(E) + P(F) = 1.4 \text{ or } P(E) + P(F) = 0.6$$

10. (a) : Let  $E_1$  and  $E_2$  be the event of selecting the bags A and B respectively.

$$\therefore P(E_1) = P(E_2) = \frac{1}{2}$$

$$\text{and } P\left(\frac{\text{Blue}}{E_1}\right) = \frac{4}{7} \text{ and } P\left(\frac{\text{Blue}}{E_2}\right) = \frac{8}{13}$$

Using Baye's theorem

$$\begin{aligned}
 P\left(\frac{E_2}{\text{Blue}}\right) &= \frac{P(E_2)P\left(\frac{\text{Blue}}{E_2}\right)}{P(E_1)P\left(\frac{\text{Blue}}{E_1}\right) + P(E_2)P\left(\frac{\text{Blue}}{E_2}\right)} \\
 &= \frac{\frac{1}{2} \cdot \frac{8}{13}}{\frac{1}{2} \cdot \frac{4}{7} + \frac{1}{2} \cdot \frac{8}{13}} = \frac{\frac{8}{13}}{\frac{4}{7} + \frac{8}{13}} = \frac{14}{27}
 \end{aligned}$$

11. (a, b, c, d) : Let  $E_1$  and  $E_2$  be the events of selecting bags then,

$$P(E_1) = P(E_2) = \frac{1}{2}$$

$$\therefore P\left(\frac{\text{red}}{E_1}\right) = \frac{3}{5}, P\left(\frac{\text{red}}{E_2}\right) = \frac{7}{11}, P\left(\frac{\text{white}}{E_1}\right) = \frac{2}{5}$$

$$\text{and } P\left(\frac{\text{white}}{E_2}\right) = \frac{4}{11}$$

$$\therefore P\left(\frac{E_2}{\text{red}}\right) = \frac{P(E_2)P\left(\frac{\text{red}}{E_2}\right)}{P(E_1)P\left(\frac{\text{red}}{E_1}\right) + P(E_2)P\left(\frac{\text{red}}{E_2}\right)}$$

$$= \frac{\frac{1}{2} \cdot \frac{7}{11}}{\frac{1}{2} \cdot \frac{3}{5} + \frac{1}{2} \cdot \frac{7}{11}} = \frac{35}{68}$$

$$\therefore P\left(\frac{E_1}{\text{red}}\right) = 1 - P\left(\frac{E_2}{\text{red}}\right) = 1 - \frac{35}{68} = \frac{33}{68}$$

Now,  $P\left(\frac{E_1}{\text{white}}\right) = \frac{P(E_1)P\left(\frac{\text{white}}{E_1}\right)}{P(E_1)P\left(\frac{\text{white}}{E_1}\right) + P(E_2)P\left(\frac{\text{white}}{E_2}\right)}$

$$= \frac{\frac{2}{5}}{\frac{2}{5} + \frac{4}{11}} = \frac{11}{21}$$

$$\therefore P\left(\frac{E_2}{\text{white}}\right) = 1 - P\left(\frac{E_1}{\text{white}}\right) = 1 - \frac{11}{21} = \frac{10}{21}$$

**12. (a, b, c, d) :** Given,  $P(A) = \frac{1}{2}$   $\therefore P(\bar{A}) = \frac{1}{2}$

$$P(B) = \frac{1}{3} \therefore P(\bar{B}) = \frac{2}{3}; P(C) = \frac{1}{4} \therefore P(\bar{C}) = \frac{3}{4}$$

Now,  $P(\text{none of } A, B, C \text{ solve the problem}) = P(\bar{A}\bar{B}\bar{C})$

$$= \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} = \frac{1}{4}$$

$\therefore P(\text{Problem will be solved}) = 1 - P(\text{none of them can solve the problem}) = 1 - \frac{1}{4} = \frac{3}{4}$

$P(\text{only two of them solve the problem})$

$$= P(A\bar{B}\bar{C}) + P(A\bar{B}C) + P(A\bar{B}\bar{C}) \\ = \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{3}{4} + \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4} = \frac{6}{24} = \frac{1}{4}$$

$P(\text{exactly one of them solve the problem})$

$$= P(A\bar{B}C) + P(AC\bar{B}) + P(\bar{A}BC) \\ = \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{4} = \frac{11}{24}$$

**13. (c, d) :** We know,  $P(A \cup B)$

$$= P(A) + P(B) - P(A \cap B)$$

But it is given that  $P(A \cup B) = P(A) + P(B) - P(A)P(B)$

$\therefore P(A \cap B) = P(A)P(B)$  i.e. A, B are independent events.

$\therefore A, \bar{B}; \bar{A}, B$  and  $\bar{A}, \bar{B}$  are also pairwise independent events.

$$\therefore P(A \cup B) = P(\bar{A} \cap \bar{B}) = P(\bar{A})P(\bar{B})$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)P(B)}{P(B)} = P(A)$$

**14. (a, b, c, d) :** Probability that exactly one of them occurs

$$= P(\text{only } A \text{ or only } B)$$

$$= P(\text{only } A) + P(\text{only } B)$$

$$= P(A \cap B') + P(A' \cap B) \text{ (option (d))}$$

$$= P(A) - P(A \cap B) + P(B) - P(A \cap B)$$

$$= P(A) + P(B) - 2P(A \cap B) \text{ (option (a))}$$

$$= P(A) + P(B) - P(A \cap B) - P(A \cap B)$$

$$= P(A \cup B) - P(A \cap B) \text{ (option (b))}$$

$$= 1 - P(A \cup B)' - P(A \cap B)$$

$$= 1 - P(A' \cap B') - P(A \cap B)$$

$$= 1 - P(A \cap B) - P(A' \cap B')$$

$$= P(A') + P(B') - P(A' \cap B')$$

$$= P(A') + P(B') - 2P(A' \cap B') \text{ (option (c))}$$

**15. (a, b, c, d) :** Here  $n(s) = 6^{11}$

Now fixing 5, 3, 1 in the first three places  $\frac{531}{\text{fixed}}$  

and remaining 8 places can be filled by  $6^8$  ways as digit can be used as many times

$\therefore$  Required probability the number begins with 531 is  $\frac{6^8}{6^{11}} = \frac{1}{6^3}$

Similarly Probability when number ends with 642 is

$$\frac{1}{216} = \frac{1}{6^3}$$

Again the probability that number begins with 642

$$\frac{531}{632} \text{ and ending with 531 is } \frac{6^5}{6^{11}} = \frac{1}{6^6}$$

$\therefore P(\text{number begins with 531 or ends with 642})$

$$= P(\text{begin with 531}) + P(\text{end with 642}) - P(\text{begin with 531 and end with 642}) = \frac{1}{216} + \frac{1}{216} - \frac{1}{216 \times 216} = \frac{431}{216 \times 216}$$

**16. (a, b, c) :**  $P(B) = P(A)P\left(\frac{B}{A}\right) + P(A')P\left(\frac{B}{A}\right)$

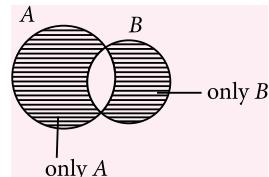
$$= P(A)P\left(\frac{B}{A}\right) + P(A')\left[1 - P\left(\frac{B'}{A'}\right)\right]$$

$$= \alpha(1 - \alpha) + (1 - \alpha)[1 - (1 - \alpha)]$$

$$= \alpha(1 - \alpha) + (1 - \alpha)(\alpha) = 2\alpha(1 - \alpha)$$

$$\text{Again, } P\left(\frac{A'}{B}\right) = \frac{P(B) - P(A \cap B)}{P(B)}$$

$$\therefore P\left(\frac{A'}{B}\right) = \frac{2\alpha(1 - \alpha) - \alpha(1 - \alpha)}{2\alpha(1 - \alpha)} = \frac{1}{2}$$



$$\text{Again } P(B') = 1 - P(B) = 1 - (2\alpha - 2\alpha^2) \\ = \alpha^2 + (\alpha^2 - 2\alpha + 1) = \alpha^2 + (\alpha - 1)^2$$

$$\text{Also, } P\left(\frac{A'}{B'}\right) = \frac{P(A' \cap B')}{P(B')} = \frac{1 - P(A \cup B)}{1 - P(B)} \\ = \frac{1 - (2\alpha - \alpha^2)}{1 - (2\alpha - 2\alpha^2)}$$

$$\Rightarrow P\left(\frac{A'}{B'}\right) = \frac{(\alpha - 1)^2}{2\alpha^2 - 2\alpha + 1} \neq \left(\frac{\alpha}{1 - \alpha}\right)^2$$

**17. (a, b, c, d)** :  $\therefore P(A_1) = P(\text{Selecting 4 and two from the numbers 5 to 11}) = \frac{{}^1C_1 \times {}^7C_2}{{}^{11}C_3} = \frac{7}{55}$

$$P(A_2) = P(\text{Selecting number 7 and two from the numbers 1 to 6}) = \frac{1 \times {}^6C_2}{{}^{11}C_3} = \frac{1}{11}$$

and  $P(A_1 \cap A_2) = P(\text{Selecting 4 and 7 and one between the number 4 and 7})$

$$= \frac{{}^1C_1 \times {}^1C_1 \times {}^2C_1}{{}^{11}C_3} = \frac{2 \times 6}{11 \times 10 \times 9} = \frac{2}{165}$$

$$\therefore P\left(\frac{A_1}{A_2}\right) = \frac{P(A_1 \cap A_2)}{P(A_2)} = \frac{2}{165} \times \frac{11}{1} = \frac{2}{15}$$

$$\text{and } P\left(\frac{A_2}{A_1}\right) = \frac{P(A_1 \cap A_2)}{P(A_1)} = \frac{2}{165} \times \frac{55}{7} = \frac{2}{21}$$

**18. (c)** : Total balls in the bag =  $9 + 7 + 4 = 20$

$P(\text{balls are of different colour})$

$$= \frac{{}^9C_1 \times {}^7C_1 \times {}^4C_1}{{}^{20}C_3} = \frac{9 \cdot 7 \cdot 4 \cdot 6}{20 \cdot 19 \cdot 18} = \frac{21}{95}$$

**19. (a)** :  $P(\text{orderly green, white, black ball})$

$$= P(\text{green, white, black}) = \frac{4}{20} \cdot \frac{7}{19} \cdot \frac{9}{18} = \frac{7}{190}$$

**20. (b)** :  $P(\text{two black and one green ball})$

$$= \frac{{}^9C_2 \times {}^4C_1}{{}^{20}C_3} = \frac{12}{95}$$

**(21-23)** :

From given we have  $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$  and

$$P\left(\frac{E}{E_1}\right) = \frac{1 \times 3}{{}^6C_2} = \frac{1}{5}, P\left(\frac{E}{E_2}\right) = \frac{2 \times 1}{{}^4C_2} = \frac{1}{3},$$

$$P\left(\frac{E}{E_3}\right) = \frac{4 \times 3}{{}^{12}C_2} = \frac{2}{11}$$

**21. (a)** :  $P(E) = \sum_{i=1}^3 P(E_i)P\left(\frac{E}{E_i}\right) = \frac{1}{3} \left[ \frac{1}{5} + \frac{1}{3} + \frac{2}{11} \right] = \frac{118}{495}$

**22. (b)** :  $P\left(\frac{E_1}{E}\right) = \frac{P(E_1)P\left(\frac{E}{E_1}\right)}{P(E)} = \frac{1}{3} \cdot \frac{1}{5} \times \frac{495}{118} = \frac{33}{118}$

**23. (c)** : We have,  $P\left(\frac{E_2}{E}\right) = \frac{P(E_2 \cap E)}{P(E)} = \frac{55}{118}$

$$\text{and } P\left(\frac{E_3}{E}\right) = \frac{P(E_3 \cap E)}{P(E)} = \frac{P(E_3) \cdot P\left(\frac{E}{E_3}\right)}{P(E)} \\ = \frac{1}{3} \times \frac{2}{11} \times \frac{495}{118} = \frac{30}{118}$$

$$\therefore P\left(\frac{E_2}{E}\right) + P\left(\frac{E_3}{E}\right) = \frac{55}{118} + \frac{30}{118} = \frac{85}{118}$$

**24. (a)** : Total balls =  $4 + 3 = 7$

$$\therefore P(\text{red ball}) = \frac{3}{7} \text{ and } P(\text{white ball}) = \frac{4}{7}$$

$$\therefore P(\text{2 white and one red ball}) = 3 \left( \frac{4}{7} \times \frac{4}{7} \times \frac{3}{7} \right) = \frac{144}{343}$$

**25. (b)** :  $P(\text{getting at most 2 red balls}) = P(\text{getting 0 red or 1 red or 2 red balls}) = 1 - P(\text{getting 3 red balls})$

$$1 - \left( \frac{3}{7} \right)^3 = \frac{316}{343}$$

**26. (c)** :  $P(\text{getting at least 2 white balls})$

$$={}^3C_2 \left( \frac{4}{7} \right) \times \frac{4}{7} \times \frac{3}{7} + {}^3C_3 \left( \frac{4}{7} \right)^3$$

$$= 3 \times \left( \frac{4}{7} \right)^2 \left( \frac{3}{7} \right) + \left( \frac{4}{7} \right)^3 = \left( \frac{4}{7} \right)^2 \left[ \frac{3 \times 3}{7} + \frac{4}{7} \right]$$

$$= \frac{16}{49} \times \frac{13}{7} = \frac{208}{343}$$

**27.**  $A \rightarrow s, B \rightarrow r, C \rightarrow q, D \rightarrow p$

**A.** Let  $A = \{1, 2, 3, 4, \dots, 10\}$ .

So, possible G.P's are  $(1, 2, 4), (4, 6, 9), (1, 3, 9), (2, 4, 8)$

$n(S) = {}^{10}C_3$  and  $n(A) = {}^4C_1 \times 2$

$$\therefore \text{Required probability} = \frac{{}^4C_1 \times 2}{{}^{10}C_3} = \frac{4 \times 6 \times 2}{10 \cdot 9 \cdot 8} = \frac{1}{15}$$

**B.** Let set  $A = \{1, 2, 3, \dots, 10\}$

$$\begin{cases} 1 & 4 & 7 & 10 \\ 2 & 5 & 8 & \\ 3 & 6 & 9 & \end{cases} \rightarrow R_1$$

$$\rightarrow R_2$$

$$\rightarrow R_3$$

Now  $x^3 + y^3 = 3 M \Rightarrow x + y = 3 M$

Required case are

1. Selecting  $x$  from  $R_1$  and  $y$  from  $R_2$
2. Selecting  $x$  from  $R_2$  and  $y$  from  $R_1$
3. Selecting  $x$  and  $y$  both from  $R_3$ .

$$\therefore \text{Required probability} = \frac{{}^4C_1 \times {}^3C_1 + {}^3C_2}{{}^{10}C_2} = \frac{1}{3}$$

C. Let HH stand for Heart lost and heart drawn, NH denote non heart lost and Heart draw.

$$\therefore \text{Required probability} = P(\text{HH}) + P(\text{NH})$$

$$= \frac{13}{52} \cdot \frac{12}{51} + \frac{39}{52} \cdot \frac{13}{51} = \frac{1}{4} \left( \frac{12}{51} + \frac{39}{51} \right) = \frac{1}{4}$$

D. Number of white balls = 2, Red balls = 3, Black balls = 4

$P(\text{both are of same colour}) = P(\text{2 white or 2 red or 2 black balls})$

$$= \frac{{}^2C_2}{{}^9C_2} + \frac{{}^3C_2}{{}^9C_2} + \frac{{}^4C_2}{{}^9C_2} = \frac{5}{18}$$

28. A → p, q, r, s, B → p, r, C → p, q, r, s,  
D → r, s

$$\text{A. } P(A \cup B) = 1 - P(A' \cap B') = 1 - \left( \frac{1}{2} \times \frac{4}{5} \right) = \frac{3}{5}$$

$$\text{and } P\left(\frac{A}{B}\right) = P(A) = \frac{1}{2}$$

(∴ A, B are independent events)

Now,  $P(A \cap B) \leq \min\{P(A), P(B)\}$

$$= \min\left\{\frac{1}{2}, \frac{1}{5}\right\} = \frac{1}{5} \leq \frac{1}{2}$$

$P(\text{exactly } A, B) = P(\text{only } A) + P(\text{only } B)$

$$= P(A \cap B') + P(A' \cap B) = P(A)P(B') + P(A')P(B)$$

$$= \frac{1}{2} \times \frac{4}{5} + \frac{1}{2} \cdot \frac{1}{5} = \frac{1}{2}$$

B.  $P(A \cup B) \geq \max\{P(A), P(B)\}$

$$= \max\left\{\frac{2}{3}, \frac{1}{2}\right\} = \frac{2}{3} \geq \frac{3}{5}$$

$$\text{and } P(A \cap B) \leq \min\{P(A), P(B)\} = \frac{1}{2}$$

C.  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow \frac{3}{5} = P(A) + \frac{1}{5} - P(A)\left(\frac{1}{5}\right)$$

(∴ A, B are independent events)

$$\Rightarrow \frac{2}{5} = \frac{4}{5}P(A) \Rightarrow P(A) = \frac{1}{2}$$

$$\therefore P\left(\frac{A}{B}\right) = P(A)$$

$$\text{Also, } P(A \cap B) = P(A)P(B) = \frac{1}{2} \cdot \frac{1}{5} = \frac{1}{10} < \frac{1}{2}$$

So,  $P(\text{exactly one of } A, B) = P(\text{only } A) + P(\text{only } B)$

$$= P(A \cap B') + P(A' \cap B)$$

$$= P(A)P(B') + P(A')P(B)$$

$$= \frac{1}{2} \cdot \frac{4}{5} + \frac{1}{2} \cdot \frac{1}{5} = \frac{1}{2}$$

$$\text{D. } P(A \cup B) \geq \frac{1}{2}, \quad P(A \cap B) \leq \frac{3}{7} \leq \frac{1}{2}$$

$$\therefore P(A|B) = P(A) = 3/7 \text{ (given)}$$

and  $P(B) = 1/2$ . ∴  $P(B') = 1/2$

∴  $P(\text{exactly one of } A, B) = P(\text{only } A) + P(\text{only } B)$

$$= P(A \cap B') + P(A' \cap B)$$

$$= P(A)P(B') + P(A')P(B) = \frac{3}{7} \cdot \frac{1}{2} + \frac{4}{7} \cdot \frac{1}{2} = \frac{1}{2}$$

29. (3) : Let  $E_1$  = the event that bag contains 2 white balls and 2 balls of other colour.

$E_2$  = the event that bag contains 3 white balls and 1 ball of other colour

$E_3$  = the event that bag contains all white balls and A = event of drawing 2 white balls from the bag.

$$\therefore P(E_1) = P(E_2) = P(E_3) = \frac{1}{3} \text{ and}$$

$$P(A|E_1) = \frac{{}^2C_2}{{}^4C_2} = \frac{1}{6}, \quad P(A|E_2) = \frac{{}^3C_2}{{}^4C_2} = \frac{1}{2} \text{ and}$$

$$P(A|E_3) = 1 \text{ so } P(A) = \frac{1}{3} \left( \frac{1}{6} + \frac{1}{2} + 1 \right) = \frac{10}{18}$$

$$\therefore \text{Required probability } P(E_3|A) = \lambda = \frac{P(E_3 \cap A)}{P(A)}$$

$$\Rightarrow \frac{P(E_3) \cdot P(A|E_3)}{P(A)} = \lambda \Rightarrow \frac{1}{3} \times \frac{18}{10} = \lambda$$

$$\Rightarrow \lambda = \frac{3}{5} \Rightarrow 5\lambda = 3$$

30. (5) :  $P(X + Y = 4) = P(X = 0, Y = 4)$

$$+ P(X = 1, Y = 3) + P(X = 2, Y = 2)$$

$$+ P(X = 3, Y = 1) + P(X = 4, Y = 0)$$

$$= P(X = 0)P(Y = 4) + P(X = 1)P(Y = 3) + P(X = 2)P(Y = 2) + P(X = 3)P(Y = 1) + P(X = 4)P(Y = 0)$$

where  $P(X = r) = {}^nC_r \left(\frac{1}{2}\right)^n$ ;  $n = 5, r = 0, 1, 2, 3, 4$  and

# Mad about rehearsing?



Tune. Fine tune. Reach the peak of your readiness for JEE with MTG's 40+16 Years Chapterwise Solutions. It is undoubtedly the most comprehensive 'real' question bank, complete with detailed solutions by experts.

Studies have shown that successful JEE aspirants begin by familiarising themselves with the problems that have appeared in past JEEs as early as 2 years in advance. Making it one of the key ingredients for their success. How about you then? Get 40+16 Years Chapterwise Solutions to start your rehearsals early. Visit [www.mtg.in](http://www.mtg.in) to order online.



Available at all leading book shops throughout the country.  
For more information or for help in placing your order:  
Call 0124-6601200 or email:[info@mtg.in](mailto:info@mtg.in)

Visit  
[www.mtg.in](http://www.mtg.in)  
for latest offers  
and to buy  
online!

$$\begin{aligned}
P(Y=r) &= {}^n C_r \left(\frac{1}{2}\right)^n, n=7, r=0,1,2,3,4 \\
\therefore P(X+Y=4) &= {}^5 C_0 \left(\frac{1}{2}\right)^5 \cdot {}^7 C_4 \left(\frac{1}{2}\right)^7 \\
&\quad + {}^5 C_1 \left(\frac{1}{2}\right)^5 \cdot {}^7 C_3 \left(\frac{1}{2}\right)^7 + {}^5 C_2 \left(\frac{1}{2}\right)^5 \cdot {}^7 C_2 \left(\frac{1}{2}\right)^7 \\
&\quad + {}^5 C_3 \left(\frac{1}{2}\right)^5 \cdot {}^7 C_1 \left(\frac{1}{2}\right)^7 + {}^5 C_4 \left(\frac{1}{2}\right)^5 \cdot {}^7 C_0 \left(\frac{1}{2}\right)^7 \\
&= \left(\frac{1}{2}\right)^{12} \left[ {}^5 C_0 \cdot {}^7 C_4 + {}^5 C_1 \cdot {}^7 C_3 + {}^5 C_2 \cdot {}^7 C_2 + \right. \\
&\quad \left. {}^5 C_3 \cdot {}^7 C_1 + {}^5 C_4 \cdot {}^7 C_0 \right] \\
&= \left(\frac{1}{2}\right)^{12} [1(35) + 5(35) + 10(21) + 10(7) + 5(1)] \\
\therefore m &= \frac{1}{4096} [35 + 175 + 210 + 70 + 5] = \frac{495}{4096} \\
\Rightarrow \frac{4096}{99} m &= 5
\end{aligned}$$

**31. (6)**:  $n(S)$  = The number of ways in which papers of 5 students can be checked by the 5 teachers =  $5^5$   
 $n(A)$  = choosing two teachers out of 5  $\times$  the number of ways in which 5 papers can be checked by exactly 2 teachers

$$= {}^5 C_2 \times (2^5 - 2) = 300$$

$$\therefore \text{Required probability} = \frac{n(A)}{n(S)} = \frac{300}{5^5} = \frac{12}{125} = m \quad (\text{given})$$

$$\text{Hence, } \frac{125m}{2} = 6$$

**32.(1)** :  $A = \{1, 2, 3, 4, \dots, 25\}$

Let  $x$  = subset of  $A$  in which each number is multiple of 3  
 $y$  = subset of  $A$  in which numbers are not multiple of 3  
 $\therefore x = \{3, 6, 9, 12, 18, 21, 24\}$ .  $\therefore n(x) = 8$  and

$$y = A - x = \{1, 2, 4, 5, 7, \dots, 25\}. \therefore n(y) = 17$$

Now, the number  $x^2 - y^2$  will be divisible by 3 if both numbers are selected either from  $x$  or  $y$ .

$$\therefore \text{Number of favourable cases} = {}^8 C_2 + {}^{17} C_2 = 164 \text{ and } n(S) = {}^{25} C_2 = 300$$

$$\therefore \text{Required probability} = \frac{\text{favourable cases}}{\text{total cases}} = \frac{164}{300}$$

$$\therefore \lambda = \frac{164}{300} = \frac{41}{75} \Rightarrow \frac{75\lambda}{41} = 1$$

**33. (4)** : Let  $E_1$  = 'total of 8'

i.e.,  $E_1 = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$

$$\therefore P(E_1) = \frac{5}{36}$$

$E_2$  = even number on each dice

$$\text{i.e., } E_2 = \{(2, 6), (4, 4), (6, 2)\}. \therefore P(E_2) = \frac{3}{36}$$

$$\therefore E_1 \cap E_2 = E_2$$

$$\text{Now, } P(E_2 | E_1) = \frac{P(E_2 \cap E_1)}{P(E_1)} = \frac{3}{5} = \frac{A}{B}$$

$$\therefore B^2 - A^2 = k^2 = 25 - 9 = 16 \Rightarrow k = \pm 4$$

$$\therefore k = 4 \text{ (as } k > 0)$$

$$\text{34. (5)} : \text{Given, } P(A) = \frac{2}{5}, P(A \cap B) = \frac{3}{20}$$

$$\begin{aligned} \text{Now, } P(A' \cup B') &= P((A \cap B)') = 1 - P(A \cap B) \\ &= 1 - \frac{3}{20} = \frac{17}{20} \end{aligned}$$

$$\begin{aligned} \text{Again, } A \cap (A' \cup B') &= A \cap (A \cap B)' \\ &= A - ((A \cap B)')' = A - (A \cap B) \end{aligned}$$

$$\therefore P(A - (A \cap B)) = \frac{2}{5} - \frac{3}{20} = \frac{1}{4} = P(\text{only } A)$$

$$\therefore \frac{l}{m} = P(A | (A' \cup B')) = \frac{P(A - (A \cap B))}{P(A' \cup B')} = \frac{\frac{1}{4}}{\frac{17}{20}} = \frac{5}{17}$$

$$\therefore l = 5$$

**35. (3)** : Given, mean of B.D. =  $np = 2$  and variance =  $npq = 1$

$$\Rightarrow p = q = \frac{1}{2} \text{ and } n = 4$$

$$\text{Now, } \lambda = P(x \geq 1) = P(1) + P(2) + P(3) + P(4)$$

$$= 1 - P(0) = 1 - {}^4 C_0 \left(\frac{1}{2}\right)^4 = 1 - \frac{1}{16}$$

$$\lambda = \frac{15}{16} \therefore \frac{16\lambda}{5} = 3$$

### Your favourite MTG Books/Magazines available in UTTARAKHAND at

- National Book House - Dehradun Ph: 0135-2659430; Mob: 9897830283
- Army Book Depot - Dehradun Ph: 0135-2756683; Mob: 9897927902
- Om Vidya Educational Book - Dehradun Mob: 9897833882
- Career Zone - Haldwani Ph: 05946-262051; Mob: 9412128075, 94123833167
- World Vision Publication - Haldwani Mob: 9927932200, 9027240169
- Diamond Stationers - Haridwar Ph: 0133-4252043; Mob: 9358398035, 9359763348
- Consul Book Depot - Nainital Ph: 0592-235164; Mob: 9412084105
- Indra Book Emporium - Roorkee Ph: 0133-276105; Mob: 8126293314
- Cambridge Book Depot - Roorkee Ph: 272341, 272345; Mob: 9719190955

Visit "MTG IN YOUR CITY" on [www.mtg.in](http://www.mtg.in) to locate nearest book seller OR write to [info@mtg.in](mailto:info@mtg.in) OR call **0124-6601200** for further assistance.

# Challenging PROBLEMS

ON JEE Main



1. Given that  $x_1 + x_2 + x_3 = 0$ ,  $y_1 + y_2 + y_3 = 0$  and  $x_1y_1 + x_2y_2 + x_3y_3 = 0$ . Then
 
$$\frac{x_1^2}{x_1^2 + x_2^2 + x_3^2} + \frac{y_1^2}{y_1^2 + y_2^2 + y_3^2} =$$

(a)  $\frac{1}{3}$    (b)  $\frac{2}{3}$    (c) 1   (d)  $\frac{4}{3}$
  2.  $\sum_{r=1}^n r(r+1)(r+2)\dots(r+p)$  = (where  $n$  and  $p$  are positive integers)
 

(a)  $\frac{n(n+1)(n+2)\dots(n+p+1)}{(p+2)} - (p+1)!$   
 (b)  $\frac{n(n+1)(n+2)\dots(n+p+1)}{(n+p+2)}$   
 (c)  $\frac{n(n+1)(n+2)\dots(n+p)}{(p+2)}$   
 (d)  $\frac{n(n+1)(n+2)\dots(n+p)}{(n+p+2)}$
  3. Let  $z = (18 + 26i)$  and  $z_0 = a + ib$  is the cube root of  $z$  having least positive argument then  $a + b =$ 

(a) 1   (b) 2   (c) 3   (d) 4
  4. Consider the two lines  $L_1, L_2$  and a circle  $C$   
 $L_1 : 2x + 3y + p - 3 = 0, L_2 : 2x + 3y + p + 3 = 0$   
 $C : x^2 + y^2 + 6x + 10y + 30 = 0, (p \in I)$   
 It is given that at least one of the lines  $L_1, L_2$  is a chord of  $C$  then the probability that both are chords of  $C$  is
 

(a)  $\frac{2}{7}$    (b)  $\frac{3}{7}$    (c)  $\frac{4}{7}$    (d)  $\frac{5}{7}$
  5. Let  $f(x) = \log\left(\frac{x^2 + e}{x^2 + 1}\right)$   
 and  $g(x) = \sqrt{\sin f(x)} + \sqrt{\cos f(x)}$ , then range of  $g(x)$  is
- (a)  $(1, 2^{3/4}]$    (b)  $(2^{1/2}, 2^{3/4}]$   
 (c)  $(\pi^{1/2}, \pi^{3/4})$    (d)  $(e^{1/2}, \pi^{1/2}]$
  6. If the parabola  $y = ax^2 + bx + c$  has vertex at  $(4, 2)$  and  $a \in [1, 3]$  then maximum value of product  $(a b c)$  is
 

(a) 144   (b) 12   (c) -12   (d) -144
  7. For  $p \geq 2$ , the equation  

$$\sqrt{2p+1-x^2} + \sqrt{3x+p+4} = \sqrt{x^2+9x+3p+9}$$
 has
 

(a) exactly 1 real root  
 (b) exactly 2 distinct real roots  
 (c) exactly 3 distinct real roots  
 (d) no real roots
  8. If  $f: R \rightarrow R, g: R \rightarrow R$  and  $f(x) + f''(x) = -xg(x)f'(x)$  and  $g(x) > 0 \forall x \in R$  then  $f^2(x) + (f'(x))^2$  has
 

(a) a minima at  $x = 0$   
 (b) a maxima at  $x = 0$   
 (c) a point of inflexion at  $x = 0$   
 (d) data insufficient
  9. All bases of logarithms in which a real positive number can be equal to its logarithm is/ are
 

(a)  $(0, 1) \cup (1, e^{1/e}]$    (b)  $(1, e)$   
 (c)  $(1, e^{1/2})$    (d)  $[e^{1/e}, e^e]$
  10. Let  $f(x) = \lim_{m \rightarrow 0} \frac{1}{m^4} \cdot \int_0^m \frac{(e^{x+t} - e^x)(\log(1+t))^2}{3+2t^3} dt$   
 then  $f(\log 3) =$ 

(a)  $\frac{1}{2}$    (b)  $\frac{1}{3}$    (c)  $\frac{1}{4}$    (d)  $\frac{1}{12}$
  11. The non-negative real numbers  $x_r (r = 1 \text{ to } 5)$  satisfy the following relations :
 

(1)  $\sum_{r=1}^5 r \cdot x_r = a$    (2)  $\sum_{r=1}^5 r^3 x_r = a^2$  and

By : Tapas Kr. Yogi, Visakhapatnam Mob : 09533632105

- (3)  $\sum_{r=1}^5 r^5 x_r = a^3$ , then number of possible values of  $a$  is/are  
 (a) 0 (b) 1 (c) 6 (d) 12
12. Let  $\sum_{k=1}^{\infty} \cot^{-1} \left( \frac{k^2}{8} \right) = \frac{-\pi}{n}$ ,  $n \in I$  then  $n =$   
 (a) 1 (b) 2 (c) 4 (d) 8
13. The natural number  $n$  for which  $2^8 + 2^{11} + 2^n$  is a perfect square is  
 (a) 11 (b) 12 (c) 14 (d) 15
14. If  $f(x)$  is a differentiable function defined  $\forall x \in R$  such that  $(f(x))^3 = x - f(x)$  then  $\int_0^{\sqrt{2}} f^{-1}(x) dx =$   
 (a) 1 (b) 2 (c)  $\sqrt{2}$  (d)  $2\sqrt{2}$
15. Let  $f(x) = \begin{cases} e^{\{x^2\}} - 1, & x > 0 \\ 0, & x = 0 \\ \frac{\sin x - \tan x + \cos x - 1}{2x^2 + \tan x + \log(x+2)}, & x < 0 \end{cases}$
- Lines  $L_1$  and  $L_2$  represent tangent and normal to curve  $y = f(x)$  at  $x = 0$ . Consider the family of circles touching both lines  $L_1$  and  $L_2$ . Then the ratio of the radii of two orthogonally circles of this family is  
 (a)  $2 + \sqrt{2}$  (b)  $2 + \sqrt{3}$  (c)  $2 - \sqrt{2}$  (d)  $2 - \sqrt{3}$
16. Suppose  $a$  and  $b$  are single digit positive integers chosen independently and at random. The probability that the point  $(a, b)$  lies above the parabola  $y = ax^2 - bx$  is  
 (a)  $\frac{17}{81}$  (b)  $\frac{19}{81}$  (c)  $\frac{21}{81}$  (d)  $\frac{23}{81}$
17. Let  $f(x) = ax^2 + bx + c$ ,  $a, b, c \in I$ . Let  $f(1) = 0$ ,  $f(7) \in (50, 60)$ ,  $f(8) \in (70, 80)$  then  $f(2) \in$   
 (a)  $(-2, 0)$  (b)  $(0, 10)$  (c)  $(1, 12)$  (d)  $(20, 30)$
18. In  $\Delta ABC$ ,  $H$  is the orthocentre and  $AH \cdot BH \cdot CH = 3$  and  $AH^2 + BH^2 + CH^2 = 7$  then sum of the possible circumradius ( $R$ ) of the  $\Delta ABC$  is  
 (a)  $\frac{2}{5}$  (b)  $\frac{5}{2}$  (c)  $\frac{3}{2}$  (d)  $\frac{2}{3}$

### SOLUTIONS

1. (b) : Consider three vectors  $\vec{n}_1 = (x_1, x_2, x_3)$ ,  $\vec{n}_2 = (y_1, y_2, y_3)$  and  $\vec{n}_3 = (1, 1, 1)$ . From the given data,

$\vec{n}_1 \cdot \vec{n}_2 = 0$ ,  $\vec{n}_2 \cdot \vec{n}_3 = 0$  and  $\vec{n}_1 \cdot \vec{n}_3 = 0$   
 i.e.,  $\vec{n}_1, \vec{n}_2$  and  $\vec{n}_3$  are mutually  $\perp^r$  vectors.

Now,  $\frac{x_1^2}{x_1^2 + x_2^2 + x_3^2}$ ,  $\frac{y_1^2}{y_1^2 + y_2^2 + y_3^2}$  and  $\frac{1}{3}$  are the squares of the projections of the vector  $(1, 0, 0)$  onto the direction of  $\vec{n}_1, \vec{n}_2, \vec{n}_3$  respectively and hence their sum = 1

$$\text{i.e., } \frac{x_1^2}{x_1^2 + x_2^2 + x_3^2} + \frac{y_1^2}{y_1^2 + y_2^2 + y_3^2} + \frac{1}{3} = 1$$

2. (a) : Since  $r(r+1) \dots (r+p) = (p+1)!^{r+p} C_{p+1}$   
 $= (p+1)!^{[r+p+1]C_{p+2} - r+pC_{p+2]}$   
 and required sum =  $(p+1)!^{[n+p+1]C_{p+2-1]} = \frac{n(n+1)\dots(n+p+1)}{p+2} - (p+1)!$

$$3. (d) : z = 18 + 26i = 10\sqrt{10} [\cos \theta + i \sin \theta]$$

$$\text{where } \tan \theta = \frac{13}{9} \Rightarrow \tan \left( \frac{\theta}{3} \right) = \frac{1}{3}$$

$$\text{and } z_0 = z^{1/3} = \sqrt{10} [\cos(\theta/3) + i \sin(\theta/3)]$$

$$z_0 = \sqrt{10} \left[ \frac{3}{\sqrt{10}} + \frac{i}{\sqrt{10}} \right] = 3+i$$

4. (b) : For  $L_1$  to be a chord of the circle, possible integral value of  $p$  are  $\{17, 18, \dots, 31\}$ . Similarly, for  $L_2$  to be a chord, possible integral value of  $p$  are  $\{11, 12, \dots, 25\}$ . So, in total possible  $p = 21$  and common values are 9.

$$\text{Hence, probability} = \frac{9}{21} = \frac{3}{7}$$

5. (a) : Notice that  $\frac{x^2 + e}{x^2 + 1} \in [1, e]$

Hence,  $f(x) \in (0, 1]$

$$\text{So, } g(\alpha) = \sqrt{\sin \alpha} + \sqrt{\cos \alpha}, \alpha \in (0, 1]$$

$$g'(\alpha) = 0 \text{ gives } \alpha = \pi/4$$

$$\text{So, } g(x) \in (1, 2^{3/4}]$$

6. (d) : From given data,  $\frac{-b}{2a} = 4$  and  $\frac{-D}{4a} = 2$   
 So,  $c = 2 + 16a$

$$\text{and } E = abc = -16(a^2 + 8a^3)$$

$$\text{So, } \frac{dE}{da} = -16(2a + 24a^2) < 0, \text{ for } a \in [1, 3]$$

$$\text{Hence, } E_{\max.} = -16(1^2 + 8 \cdot 1^3) = -144$$

7. (b) : Let  $h = x^2 + x - p$ , then given equation becomes  
 $\sqrt{(x+1)^2 - 2h} + \sqrt{(x+2)^2 - h} = \sqrt{(2x+3)^2 - 3h}$   
 Simplifying further,  $h[2h - 2(x+2)^2 - (x+1)^2] = 0$

If  $2h - 2(x+2)^2 - (x+1)^2 = 0$  then above square root equation is invalid. Hence, only  $h=0$  possible. So, now above equation becomes

$$|x+1| + |x+2| = |2x+3| \Rightarrow x \notin (-2, -1)$$

Hence, the number of real solutions of  $h = x^2 + x - p = 0$

which are not in  $(-2, -1)$  is zero if  $p < -\frac{1}{4}$ , one if  $p = -\frac{1}{4}$  or  $p \in (0, 2)$  and two otherwise. Hence, exactly 2 real roots for  $p \in \left(-\frac{1}{4}, 0\right] \cup [2, \infty)$ .

**8. (b) :** Let  $h(x) = (f(x))^2 + (f'(x))^2$

$$\text{So, } h'(x) = -2x g(x) \cdot (f'(x))^2$$

**9. (a) :** We require  $a \in R - \{1\}$  such that  $x = \log_a x$

$$\text{i.e., } f(x) = \frac{\log x}{x} = \log a$$

$$f'(x) = 0 \Rightarrow x = e$$

So, max.  $f(x) = f(e) = 1/e$ . So,  $a_{\max.} = e^{1/e}$

Hence,  $a \in (0, 1) \cup (1, e^{1/e}]$

$$\text{10. (c) : } f(x) = e^x \cdot \lim_{m \rightarrow 0} \frac{\int_0^m \frac{(e^t - 1) \cdot (\log(1+t))^2}{2t^3 + 3} dt}{m^4} \quad \left(\begin{matrix} 0 \\ 0 \end{matrix}\right)$$

$$\Rightarrow f(x) = e^x \cdot \lim_{m \rightarrow 0} \frac{(e^m - 1) \cdot (\log(1+m))^2}{(3 + 2m^3) \cdot 4m^3}$$

$$\Rightarrow f(x) = e^x \cdot \lim_{m \rightarrow 0} \frac{e^m - 1}{m} \cdot \left(\frac{\log(1+m)}{m}\right)^2 \cdot \frac{1}{4} \cdot \frac{1}{3 + 2m^3}$$

$$\Rightarrow f(x) = \frac{e^x}{12}, \text{ so, } f(\log 3) = \frac{3}{12} = \frac{1}{4}$$

**11. (c) :** Notice that

$$\sum_{r=1}^5 r(a-r^2)^2 x_r = a^2 \sum_{r=1}^5 r \cdot x_r - 2a \sum_{r=1}^5 r^3 x_r + \sum_{r=1}^5 r^5 x_r \\ = a^2(a) - 2a(a^2) + (a^3) = 0$$

Since, L.H.S. terms are non-negative.

Hence, each term in L.H.S. is zero.

So, possible values of  $a$  are  $\{0, 1, 4, 9, 16, 25\}$

$$\text{12. (c) : } T_k = \cot^{-1} \left( \frac{k^2}{8} \right)$$

$$= \tan^{-1} \left( \frac{8}{k^2} \right) = \tan^{-1} \left[ \frac{2}{1 + \frac{k^2}{4} - 1} \right]$$

$$= \tan^{-1} \left[ \frac{k}{2} + 1 \right] - \tan^{-1} \left( \frac{k}{2} - 1 \right)$$

**13. (b) :**  $2^8 + 2^{11} + 2^n = 2^8(9 + 2^{n-8})$

Hence,  $9 + 2^{n-8}$  should be a perfect square.

So,  $9 + 2^{n-8} = k^2$  (say) i.e.  $2^{n-8} = (k+3)(k-3)$

So,  $(k+3)$  and  $(k-3)$  are both powers of 2.

$\Rightarrow k = 5$  being the only possibility. Hence,  $n = 12$

**14. (b) :**  $(f(x))^3 + f(x) = x$ . Hence,  $f^{-1}(x) = x^3 + x$

$$\text{15. (b) : L.H.D.} = \lim_{h \rightarrow 0} \left[ \frac{-\sinh + \tanh + \cosh - 1}{2h^2 - \tanh + \log(2-h)} \right] \times \frac{1}{h} = 0$$

$$\text{and R.H.D.} = \lim_{h \rightarrow 0} \frac{e^{h^2} - 1 - 0}{h} = 0$$

Hence,  $L_1 = y = 0$  and  $L_2 = x = 0$ .

**16. (b) :** If  $(a, b)$  lies above the curve then  $b > y(a)$

$$\text{i.e., } b > a^3 - ba \Rightarrow b > \frac{a^3}{a+1}$$

The only possibilities are  $a = 1, 2, 3$

For  $a = 1, b = 1, 2, 3, \dots, 9$

For  $a = 2, b = 3, 4, \dots, 9$

For  $a = 3, b = 7, 8, 9$ . So, in total there are 19 points out of  $9 \times 9 = 81$  points.

Hence, required probability =  $\frac{19}{81}$

**17. (a) :**  $f(1) = 0 \Rightarrow a + b + c = 0$

$$f(7) \in (50, 60) \Rightarrow 50 < 49a + 7b + c < 60$$

$$\text{or } 50 < 48a + 6b < 60 \text{ i.e., } 8a + b \in \left(\frac{25}{3}, 10\right) \\ \text{i.e., } 8a + b = 9$$

Similarly,  $9a + b = 11$

Hence,  $a = 2, b = -7, c = 5$

$$\text{i.e., } f(x) = 2x^2 - 7x + 5 \Rightarrow f(2) = 8 - 14 + 5 = -1$$

**18. (b) :** If  $\Delta ABC$  is acute, then Cosine rule gives,

$$AB^2 = AH^2 + BH^2 - 2AH \cdot BH \cos(\pi - C) \quad \dots(\text{i})$$

and  $AB = 2R \sin C, CH = 2R \cos C$

$$\Rightarrow AB^2 + CH^2 = 4R^2 \quad \dots(\text{ii})$$

From (i) and (ii),

$$4R^2 = AH^2 + BH^2 + CH^2 + \frac{AH \cdot BH \cdot CH}{R}$$

$$\text{Now, A.T.Q., } 4R^2 = 7 + \frac{3}{R} \text{ i.e., } 4R^3 - 7R - 3 = 0$$

$$\text{i.e., } (R+1)(2R+1)(2R-3) = 0$$

$$\text{Since, } 3 = AH \cdot BH \cdot CH < (2R)^3 \Rightarrow R = 1$$

Similarly when  $\Delta ABC$  is obtuse, we have  $R = \frac{3}{2}$

$$\text{So, sum of possibilities of } R = \frac{3}{2} + 1 = \frac{5}{2}$$



# MATHS MUSING

## SOLUTION SET-181

1. (c) : Let  $x = (1)^{1/n} \Rightarrow x^n - 1 = 0$   
 or  $x^n - 1 = (x - 1)(x - \omega)(x - \omega^2) \dots (x - \omega^{n-1})$   
 $\Rightarrow \frac{x^n - 1}{x - 1} = (x - \omega)(x - \omega^2) \dots (x - \omega^{n-1})$

Putting  $x = 9$  in both sides, we have

$$(9 - \omega)(9 - \omega^2)(9 - \omega^3) \dots (9 - \omega^{n-1}) = \frac{9^n - 1}{8}$$

2. (d) : Since,  $3^{2 \sin 2x - 1}, 14, 3^{4 - 2 \sin 2x}$  are in A.P.

$$\text{So, } 28 = 3^{2 \sin 2x} \cdot \frac{1}{3} + 3^{4 - 2 \sin 2x}$$

Put  $3^{2 \sin 2x} = t$ , we get

$$28 = \frac{t}{3} + \frac{81}{t} \Rightarrow 84t = t^2 + 243$$

$$\Rightarrow t^2 - 84t + 243 = 0$$

$$\Rightarrow (t - 81)(t - 3) = 0 \Rightarrow t = 3, 81$$

$$\Rightarrow 3^{2 \sin 2x} = 3^1, 3^4$$

$$\Rightarrow 2 \sin 2x = 1 \text{ or } 2 \sin 2x = 4$$

$$\Rightarrow \sin 2x = \frac{1}{2} (\because \sin 2x \neq 2)$$

$\therefore$  First term =  $3^{1-1} = 1$ , second term = 14

and third term = 27

Here, common difference = 13

$\therefore$  Fifth term =  $1 + 4 \times 13 = 53$ .

3. (a) :  $\sum_{i=0}^n \sum_{j=1}^n {}^n C_j {}^j C_i = {}^n C_1 ({}^1 C_0 + {}^1 C_1) +$   
 ${}^n C_2 ({}^2 C_0 + {}^2 C_1 + {}^2 C_2) + {}^n C_3 ({}^3 C_0 + {}^3 C_1 + {}^3 C_2 + {}^3 C_3)$   
 $+ {}^n C_4 ({}^4 C_0 + {}^4 C_1 + {}^4 C_2 + {}^4 C_3 + {}^4 C_4)$   
 $+ \dots + {}^n C_n ({}^n C_0 + {}^n C_1 + {}^n C_2 + \dots + {}^n C_n)$   
 $= {}^n C_1 (2) + {}^n C_2 (2)^2 + {}^n C_3 (2)^3 + {}^n C_4 (2)^4 + \dots + {}^n C_n (2)^n$   
 $= (1+2)^n - 1 = 3^n - 1$

4. (b) : Putting  $t = 0$  on both sides, we get  $E = -39$

5. (a) : Let  $A = \begin{pmatrix} -1 & 2 & 5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{pmatrix}$

Applying  $R_1 \rightarrow (-1)R_1$ , we get

$$A = \begin{pmatrix} 1 & -2 & -5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{pmatrix}$$

Applying  $R_2 \rightarrow R_2 - 2R_1$  and  $R_3 \rightarrow R_3 - R_1$ , we get

$$A = \begin{pmatrix} 1 & -2 & -5 \\ 0 & 0 & a+6 \\ 0 & 0 & a+6 \end{pmatrix}$$

For  $a = -6$

$$A = \begin{pmatrix} 1 & -2 & -5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \therefore \text{Rank}(A) = 1.$$

6. (d) : If  $f$  is injective and  $g$  is injective

$\Rightarrow fog$  is injective  $\therefore fof$  is injective.

7. (d) :  $\lim_{n \rightarrow \infty} \left\{ \frac{1}{n} + \frac{n^2}{(n+1)^3} + \frac{n^2}{(n+2)^3} + \dots + \frac{1}{8n} \right\}$   
 $= \lim_{n \rightarrow \infty} \left\{ \frac{n^2}{(n+0)^3} + \frac{n^2}{(n+1)^3} + \frac{n^2}{(n+2)^3} + \dots + \frac{n^2}{(n+n)^3} \right\}$   
 $= \lim_{n \rightarrow \infty} \sum_{r=0}^n \frac{n^2}{(n+r)^3} = \lim_{n \rightarrow \infty} \sum_{r=0}^n \frac{1}{\left(1 + \frac{r}{n}\right)^3 n}$   
 $= \int_0^1 \frac{dx}{(1+x)^3} = - \left[ \frac{1}{2(1+x)^2} \right]_0^1 = - \left\{ \frac{1}{8} - \frac{1}{2} \right\} = \frac{1}{2} - \frac{1}{8} = \frac{3}{8}$

8. (a) : Let  $P = \lim_{n \rightarrow \infty} \left\{ \prod_{r=1}^n \tan \left( \frac{r\pi}{2n} \right) \right\}^{1/n}$

$$\therefore \ln P = \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n} \ln \tan \left( \frac{r\pi}{2n} \right) = \int_0^1 \ln \tan \left( \frac{\pi x}{2} \right) dx$$

$$\Rightarrow \ln P = \frac{2}{\pi} \int_0^{\pi/2} \ln \tan x dx \quad \dots(i)$$

$$\text{and } \ln P = \frac{2}{\pi} \int_0^{\pi/2} \ln \tan \left( \frac{\pi}{2} - x \right) dx \quad (\text{by property})$$

$$= \frac{2}{\pi} \int_0^{\pi/2} \ln \cot x dx \quad \dots(ii)$$

Adding (i) and (ii), we get

$$2 \ln P = \frac{2}{\pi} \int_0^{\pi/2} (\ln \tan x + \ln \cot x) dx = \frac{2}{\pi} \int_0^{\pi/2} \ln 1 dx = 0$$

$$\Rightarrow 2 \ln P = 0 \Rightarrow \ln P = 0 \therefore P = e^0 = 1$$

### MPP-10 CLASS XI

### ANSWER

### KEY

- |            |            |            |           |         |
|------------|------------|------------|-----------|---------|
| 1. (d)     | 2. (c)     | 3. (a)     | 4. (a)    | 5. (c)  |
| 6. (c)     | 7. (c)     | 8. (a, b)  | 9. (a, b) | 10. (c) |
| 11. (a, c) | 12. (a, c) | 13. (a, d) | 14. (d)   | 15. (b) |
| 16. (b)    | 17. (9)    | 18. (5)    | 19. (2)   | 20. (5) |

**9. (2) :** We have,  $\lim_{x \rightarrow 0} \frac{\sin(\sin x) - \sin x}{ax^3 + bx^5 + c} = -\frac{1}{12}$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{2 \cos\left(\frac{\sin x + x}{2}\right) \sin\left(\frac{\sin x - x}{2}\right)}{(ax^3 + bx^5 + c)} = -\frac{1}{12}$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{2 \cos\left(\frac{x + \sin x}{2}\right) \frac{\sin\left(\frac{x - \sin x}{2}\right)}{\left(\frac{x - \sin x}{2}\right)} \left(\frac{x - \sin x}{2}\right)}{\left(a + bx^2 + \frac{c}{x^3}\right)x^3} = \frac{1}{12}$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{2 \cdot 1 \cdot \frac{1}{2} \cdot \frac{1}{6}}{\left(a + bx + \frac{c}{x^3}\right)} = \frac{1}{12} \left( \because \lim_{x \rightarrow 0} \frac{x - \sin x}{x^3} = \frac{1}{6} \right)$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{1}{\left(a + bx + \frac{c}{x^3}\right)} = \frac{1}{2}$$

$\therefore$  R.H.S. is finite quantity.

$$\therefore c = 0, b \in R \text{ and } \frac{1}{a} = \frac{1}{2} \Rightarrow a = 2$$

**10. (a) :** (P)  $\rightarrow$  4, (Q)  $\rightarrow$  3, (R)  $\rightarrow$  1, (S)  $\rightarrow$  2

**P.**  $\because y = \sin^{-1}(3x - 4x^3)$

$$= \begin{cases} -\pi - 3 \sin^{-1} x & , -1 \leq x < -\frac{1}{2} \\ 3 \sin^{-1} x & , -\frac{1}{2} \leq x \leq \frac{1}{2} \\ \pi - 3 \sin^{-1} x & , \frac{1}{2} < x \leq 1 \end{cases}$$

$$\therefore \frac{dy}{dx} = \begin{cases} -\frac{3}{\sqrt{1-x^2}} & , -1 < x < -\frac{1}{2} \\ \frac{3}{\sqrt{1-x^2}} & , -\frac{1}{2} < x < \frac{1}{2} \\ -\frac{3}{\sqrt{(1-x^2)}} & , \frac{1}{2} < x < 1 \end{cases}$$

**Q.**  $\because y = \cos^{-1}(4x^3 - 3x)$

$$= \begin{cases} 3 \cos^{-1} x - 2\pi & , -1 < x < -\frac{1}{2} \\ 2\pi - 3 \cos^{-1} x & , -\frac{1}{2} \leq x \leq \frac{1}{2} \\ 3 \cos^{-1} x & , \frac{1}{2} < x \leq 1 \end{cases}$$

$$\therefore \frac{dy}{dx} = \begin{cases} \frac{-6}{\sqrt{1-x^2}} & , -1 < x < -\frac{1}{2} \\ \frac{6}{\sqrt{1-x^2}} & , -\frac{1}{2} < x < \frac{1}{2} \\ \frac{-6}{\sqrt{1-x^2}} & , \frac{1}{2} < x < 1 \end{cases}$$

**R.**  $\because y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$

$$= \begin{cases} 3 \tan^{-1} x & , -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}} \\ \pi + 3 \tan^{-1} x & , x < -\frac{1}{\sqrt{3}} \\ -\pi + 3 \tan^{-1} x & , x > \frac{1}{\sqrt{3}} \end{cases}$$

$$\therefore \frac{dy}{dx} = \begin{cases} \frac{3}{1+x^2} & , -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}} \\ \frac{3}{1+x^2} & , x < -\frac{1}{\sqrt{3}} \\ \frac{3}{1+x^2} & , x > \frac{1}{\sqrt{3}} \end{cases}$$

**S.**  $f(x) = \left( \frac{\log \sin x}{\log \cos x} \right)^2 + 2 \tan^{-1} x$

$$f'(x) = 2$$

$$\left( \frac{\log \sin x}{\log \cos x} \right) \frac{(\log \cos x) \cot x + (\log \sin x) \tan x}{(\log \cos x)^2} + \frac{2}{1+x^2}$$

$$\therefore f'\left(\frac{\pi}{4}\right) = 2 \cdot 2 \frac{1}{\log \cos \frac{\pi}{4}} + \frac{2}{1+\frac{\pi^2}{16}} = \frac{4}{\log \frac{1}{\sqrt{2}}} + \frac{32}{16+\pi^2}$$

$$= -\frac{8}{\log 2} + \frac{32}{\pi^2+16}$$



### MPP-10 CLASS XII ANSWER KEY

- |           |              |            |            |           |
|-----------|--------------|------------|------------|-----------|
| 1. (c)    | 2. (a)       | 3. (a)     | 4. (a)     | 5. (c)    |
| 6. (c)    | 7. (a,b,c,d) |            | 8. (a, c)  | 9. (a, c) |
| 10. (a,b) | 11. (c)      | 12. (b, d) | 13. (b, c) | 14. (a)   |
| 15. (b)   | 16. (a)      | 17. (8)    | 18. (4)    | 19. (4)   |
| 20. (2)   |              |            |            |           |

# MPP-10 | MONTHLY Practice Problems

Class XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

## Statistics and Probability

Total Marks : 80

Time Taken : 60 Min.

### Only One Option Correct Type

1. If  $\frac{1+3p}{3}, \frac{1-p}{4}$  and  $\frac{1-2p}{2}$  are the probabilities of the three mutually exclusive events, then  $p \in$   
 (a)  $[0, 1]$  (b)  $\left[0, \frac{1}{2}\right]$  (c)  $\left[\frac{1}{3}, 1\right]$  (d)  $\left[\frac{1}{3}, \frac{1}{2}\right]$
2. Mean of 100 items is 49. It was discovered that 3 items 60, 70, 80 were wrongly read as 38, 22, 50 respectively. The correct mean is  
 (a) 48 (b) 78 (c) 50 (d) 80
3. The standard deviation of 25 number is 40. If each of the number is increased by 5, then the new standard deviation will be  
 (a) 40 (b) 25 (c)  $\sqrt{40}$  (d) 1600
4. If  $P(B) = \frac{3}{4}$ ,  $P(A \cap B \cap \bar{C}) = \frac{1}{3}$  and  $P(\bar{A} \cap B \cap \bar{C}) = \frac{1}{3}$ , then  $P(B \cap C) =$   
 (a)  $\frac{1}{12}$  (b)  $\frac{1}{6}$  (c)  $\frac{1}{15}$  (d)  $\frac{1}{9}$
5. Two dice are thrown simultaneously to get the coordinates of a point on  $xy$ -plane. Then the probability that this point lies inside or on the region bounded by  $|x| + |y| = 3$ , is  
 (a)  $\frac{3}{14}$  (b)  $\frac{2}{3}$  (c)  $\frac{1}{12}$  (d)  $\frac{4}{14}$
6. Out of  $(2n + 1)$  tokens consecutively numbered from 1 to  $(2n + 1)$ , 3 tokens are drawn at random. The probability that the numbers on them are in A.P. is  
 (a)  $\frac{2n}{4n^2 - 1}$  (b)  $\frac{4n}{4n^2 - 1}$  (c)  $\frac{3n}{4n^2 - 1}$  (d)  $\frac{5n}{4n^2 - 1}$

### One or More Than One Option(s) Correct Type

7. If the mean deviation of numbers  $1, 1+d, 1+2d, \dots, 1+100d$  from their mean is 255, then the  $d$  is equal to  
 (a) 10.0 (b) 20.0 (c) 10.1 (d) 20.2
8. The variable  $x$  takes two values  $x_1$  and  $x_2$  with frequencies  $f_1$  and  $f_2$ , respectively. If  $\sigma$  denotes the standard deviation of  $x$ , then  
 (a)  $\sigma^2 = \frac{f_1 x_1^2 + f_2 x_2^2}{f_1 + f_2} - \left( \frac{f_1 x_1 + f_2 x_2}{f_1 + f_2} \right)^2$   
 (b)  $\sigma^2 = \frac{f_1 f_2}{(f_1 + f_2)^2} - (x_1 - x_2)^2$   
 (c)  $\sigma^2 = \frac{(x_1 - x_2)^2}{(f_1 + f_2)^2}$  (d) none of these
9. Which of the following is correct statement?  
 (a) The sum of the deviations from arithmetic mean is zero.  
 (b) Computation of arithmetic mean is based on all observations.  
 (c) It gives no importance to extreme values.  
 (d) none of these.
10. The probability that the 13<sup>th</sup> day of a randomly chosen month is a second Saturday, is  
 (a)  $\frac{1}{7}$  (b)  $\frac{1}{12}$  (c)  $\frac{1}{84}$  (d)  $\frac{19}{84}$
11. The chance of an event happening is the square of the chance of a second event but the odds against the first are the cube of the odds against the second. The chances of the events are  
 (a)  $p_1 = 1/9$  (b)  $p_1 = 1/16$   
 (c)  $p_2 = 1/3$  (d)  $p_2 = 1/4$

12. The variance of the numbers 2, 3, 11 and  $x$  is  $\frac{49}{4}$ . Find the value of  $x$ .

(a) 6      (b)  $\frac{14}{5}$       (c)  $\frac{14}{3}$       (d)  $\frac{6}{15}$

13. If runs of two players A and B in 10 cricket matches are such that players A has mean 50 and variance 36 and player B has mean 60 and variance 81 of runs, then the player

(a) A has less coeff. of variation  
 (b) B has less coeff. of variation  
 (c) Both are equally consistent  
 (d) A is more consistent

#### Comprehension Type

In an objective paper, there are two sections of 10 questions each. For 'section 1', each question has five options and only one option is correct and 'section 2' has four options with multiple answers and marks for a question in this section is awarded only if he ticks all correct answers. Marks for each question in 'section 1' is 1 and in 'section 2' is 3. (There is no negative marking).

14. If a candidate in total attempts four questions all by guessing, then the probability of scoring 10 marks is

(a)  $1/15(1/15)^3$       (b)  $3/5(1/15)^3$   
 (c)  $1/15(14/15)^3$       (d) None of these

15. The probability of getting a score less than 40 by answering all the questions by guessing in this paper is

(a)  $(1/75)^{10}$       (b)  $1 - (1/75)^{10}$   
 (c)  $(74/75)^{10}$       (d) None of these

#### Matrix Match Type

16. Match the following.

	Column I	Column II
P.	There are 5 duplicate and 10 original items in an automobile shop and 3 items are brought at random by a customer. The probability that none of the items is duplicate, is	1. $\frac{1}{6}$

Q.	There are 2 teams with $n$ persons in each. The probability of selecting 2 persons from one team and 1 person from the other team is $\frac{6}{7}$ . Then $n=$	2.	$\frac{24}{91}$
R.	If the letters of the word NALGONDA are arranged in arbitrary order, the probability that the letters G, O, D appear in that order is	3.	$\frac{1}{35}$
S.	There are 8 children, 4 boys and 4 girls. They are randomly divided into 2 groups of 4 children each. The probability that all the girls are in one group is	4.	4

P	Q	R	S
(a) 2	1	3	4
(b) 2	4	1	3
(c) 3	1	4	2
(d) 3	4	1	2

#### Integer Answer Type

17. A card is drawn from a pack of 52 cards and a gambler bets that it is a spade or an ace. If the odds against his winning this bet is  $\frac{k}{k-5}$ , then the value of  $k$  is

18. The A.M. and S.D. of 100 items was recorded as 40 and 5.1 respectively. Later on it was discovered that one observation 40 was wrongly copied down as 50. Then the correct S.D is

19. The mean and variance of 7 observations are 8 and 16 respectively. If the observations are  $2, x, 4, 10, 12, y, 14$ . Then the positive value of  $x - y$  is

20. Let  $A = \{1, 3, 5, 7, 9\}$  and  $B = \{2, 4, 6, 8\}$ . An element  $(a, b)$  of their cartesian product  $A \times B$  is chosen at random. If the probability that  $\{a + b = 9\}$  is  $\frac{p}{q}$  then  $\frac{q}{p}$  is



Keys are published in this issue. Search now! ☺

## SELF CHECK

### Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

No. of questions attempted .....  
 No. of questions correct .....  
 Marks scored in percentage .....



YOUR WAY CBSE XII

## PRACTICE PAPER 2018

Time Allowed : 3 hours

Maximum Marks : 100



### GENERAL INSTRUCTIONS

- (i) All questions are compulsory.
- (ii) This question paper contains **29** questions.
- (iii) Questions **1-4** in Section-A are very short-answer type questions carrying **1** mark each.
- (iv) Questions **5-12** in Section-B are short-answer type questions carrying **2** marks each.
- (v) Questions **13-23** in Section-C are long-answer-I type questions carrying **4** marks each.
- (vi) Questions **24-29** in Section-D are long-answer-II type questions carrying **6** marks each.

### SECTION - A

1. Find the domain of  $\sec^{-1}(2x + 1)$ .
2. Find the distance of the point  $P(2, 1, -1)$  from the plane  $x - 2y + 4z = 9$ .
3. If  $A, B, C$  are three non-zero square matrices of same order, then find the condition on  $A$  such that  $AB = AC \Rightarrow B = C$ .
4. Let  $f : R \rightarrow R$  be defined by  $f(x) = 3x^2 - 5$  and  $g : R \rightarrow R$  be defined by  $g(x) = \frac{x}{x^2 + 1}$ . Find  $gof$ .

$$\frac{x+3}{3} = \frac{y-4}{5} = \frac{8-z}{-6}.$$

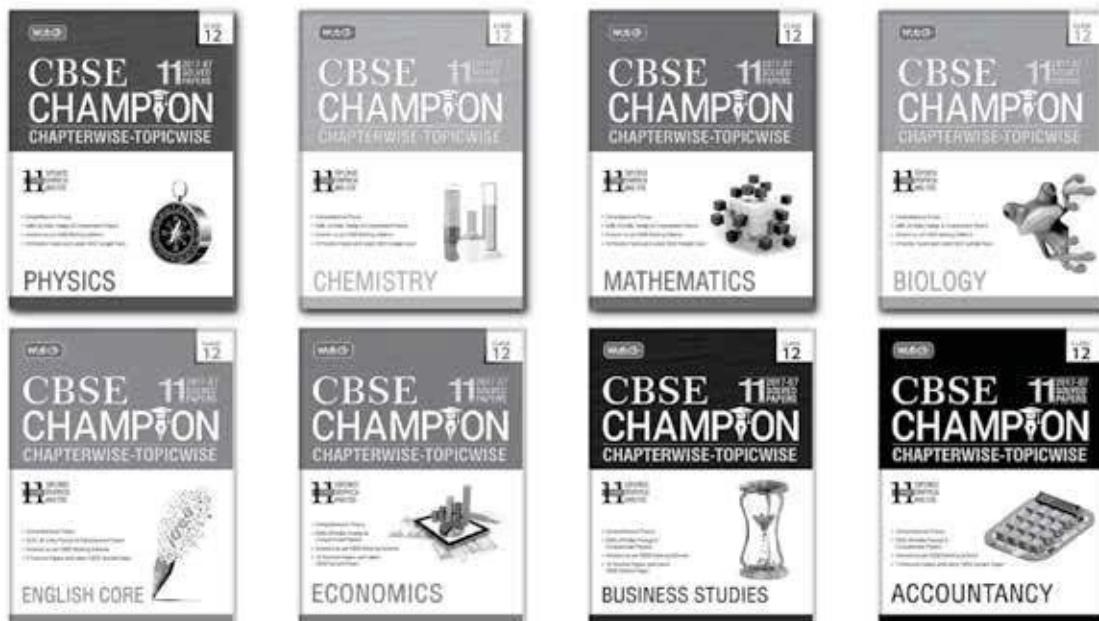
8. Form the differential equation of the family of curves  $y = a \cos(x + b)$ , where  $a$  and  $b$  are arbitrary constants.
9. If  $f(x) = \begin{cases} 1 & , \text{ if } x \leq 3 \\ ax + b & , \text{ if } 3 < x < 5 \\ 7 & , \text{ if } 5 \leq x \end{cases}$ . Determine the values of  $a$  and  $b$  so that  $f(x)$  is continuous.

10. Let  $l_p, m_p, n_p ; i = 1, 2, 3$  be the direction cosines of three mutually perpendicular vectors in space.

Show that  $AA' = I_3$ , where  $A = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}$ .

11. Two balls are drawn at random from a bag containing 5 white and 7 black balls. Then, find the odds in favour of drawing two black balls.
12. Solve the following linear programming problem graphically.  
Maximize  $Z = 3x + 4y$   
subject to  $x + y \leq 4$ ,  $x \geq 0$  and  $y \geq 0$ .

## CBSE CHAMPION Chapterwise -Topicwise Solved Papers



CBSE CHAMPION Chapterwise-Topicwise Solved Papers Series contains topicwise questions and solutions asked over last decade in CBSE-Board examination.

Questions are supported with topicwise graphical analysis of previous years CBSE Board questions as well as comprehensive and lucid theory. The questions in each topic have been arranged in descending order as per their marking scheme. Questions from Delhi, All India, Foreign and Compartment papers are included. This ensures that all types of questions that are necessary for Board exam preparation have been covered.

Important feature of these books is that the solutions to all the questions have been given according to CBSE marking scheme. CBSE sample paper and practice papers are also supplemented.

Examination papers for Class-10 and 12 Boards are based on a certain pattern. To excel, studying right is therefore more important than studying hard, which is why we created this series.



Available at all leading book shops throughout India.  
For more information or for help in placing your order:  
Call 0124-6601200 or email info@mtg.in

Visit  
[www.mtg.in](http://www.mtg.in)  
for latest offers  
and to buy  
online!

### SECTION - C

13. Find the inverse of the matrix, if it exists, using elementary row operations.  $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

**OR**

Using properties of determinants, show that

$$\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix} = (b^2 - ac)(ax^2 + 2bxy + cy^2)$$

14. Using Lagrange's Mean value theorem, show that  $|\cos a - \cos b| \leq |a - b|$ .

15. Find the absolute maximum and minimum values of  $f(x) = x^2 \sqrt{1+x}$  in  $\left[-1, \frac{1}{2}\right]$ .

**OR**

Find the point on the curve  $y = x^3 - 11x + 5$  at which the tangent is  $y = x - 11$ .

16. Evaluate  $\int_1^3 (2x+1)dx$  as limit of sums.

17. Find the area bounded by the curve  $x^2 = 4y$  and the straight line  $x = 4y - 2$ .

18. Solve the following differential equation :

$$\frac{dy}{dx} - y = \cos x, \text{ given that if } x = 0, y = 1.$$

**OR**

Check whether the following differential equation is homogeneous or not.

$$x^2 \frac{dy}{dx} - xy = 1 + \cos\left(\frac{y}{x}\right), x \neq 0$$

Find the general solution of the differential equation using substitution  $y = vx$ .

19. Let  $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$ ,  $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$  and  $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$ . Find a vector  $\vec{d}$  which is perpendicular to both  $\vec{a}$  and  $\vec{b}$  and  $\vec{c} \cdot \vec{d} = 15$ .

20. An urn contains 10 white and 3 black balls. Another urn contains 3 white and 5 black balls. Two balls are drawn from first urn without noticing their colour and put into the second urn and then a ball is drawn from the second urn. Find the probability that it is a white ball.

21. A random variable  $X$  has the following probability distribution :

$X$	0	1	2	3	4	5	6	7
$P(X)$	0	$k$	$2k$	$2k$	$3k$	$k^2$	$2k^2$	$7k^2 + k$

Find each of the following :

- (i)  $k$  (ii)  $P(X < 6)$   
 (iii)  $P(X \geq 6)$  (iv)  $P(0 < X < 5)$

22. Find the equation of the plane passing through the points  $(2, 2, 1)$  and  $(9, 3, 6)$  and perpendicular to the line  $2x + 6y + 6z = 1$ .

23. Solve the following L.P.P. graphically :

$$\text{Minimize } Z = 5x + 10y$$

Subject to constraints  $x + 2y \leq 120$ ,  $x + y \geq 60$ ,  $x - 2y \geq 0$  and  $x, y \geq 0$

### SECTION - D

24. Evaluate :  $\int \frac{1}{\sin x + \sec x} dx$ .

**OR**

$$\text{Evaluate : } \int_0^{\pi/2} (2 \log \sin x - \log \sin 2x) dx$$

25. Find the maximum and minimum values of  $(\sin^{-1} x)^3 + (\cos^{-1} x)^3$ , where  $-1 \leq x \leq 1$ .

**OR**

On the set  $R - \{-1\}$  a binary operation  $*$  is defined by  $a * b = a + b + ab$  for all  $a, b \in R - \{-1\}$ . Prove that  $*$  is commutative as well as associative on  $R - \{-1\}$ . Find the identity element and prove that every element of  $R - \{-1\}$  is invertible.

26. An amount of ₹ 5000 is put into three investments at the rate of interest of 6%, 7% and 8% per annum respectively. The total annual income is ₹ 358. If the combined income from the first two investments is ₹ 70 more than the income from the third, find the amount of each investment by matrix method.

**OR**

Verify :  $A(\text{adj } A) = (\text{adj } A)A = |A| I$  for matrix

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

27. If  $(ax + b)e^{yx} = x$  or  $y = x \log\left(\frac{x}{a+bx}\right)$ , prove that  $x^3 \frac{d^2y}{dx^2} = \left(x \frac{dy}{dx} - y\right)^2$ .

28. Find the shortest distance between the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ and } \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$$

- 29.** Find the cartesian as well as vector equations of the planes through the intersection of the planes  $\vec{r} \cdot (2\hat{i} + 6\hat{j}) + 12 = 0$  and  $\vec{r} \cdot (3\hat{i} - \hat{j} + 4\hat{k}) = 0$  which are at a unit distance from the origin.

### SOLUTIONS

- The domain of  $\sec^{-1} x$  is  $(-\infty, -1] \cup [1, \infty)$ .  
 $\therefore 2x+1 \geq 1 \text{ or } 2x+1 \leq -1$   
 $\Rightarrow 2x \geq 0 \text{ or } 2x \leq -2 \Rightarrow x \geq 0 \text{ or } x \leq -1$   
 $\Rightarrow x \in (-\infty, -1] \cup [0, \infty)$ .  
Hence, the domain of  $\sec^{-1}(2x+1)$  is  $(-\infty, -1] \cup [0, \infty)$ .
- We have point  $P(2, 1, -1)$  and plane  $x - 2y + 4z = 9$   
 $\therefore \text{Required distance} = \frac{|2 - 2 \times 1 + 4 \times (-1) - 9|}{\sqrt{1^2 + (-2)^2 + (4)^2}} = \frac{13}{\sqrt{21}}$ .

3. We have  $AB = AC$

To show:  $B = C$

Since,  $B = C$  is possible if  $\exists A^{-1}$

i.e.,  $A^{-1}AB = A^{-1}AC \Rightarrow B = C$   $[\because A^{-1}A = I]$

Hence,  $A$  should be invertible matrix.

4. Since,  $gof(x) = g\{f(x)\} = g(3x^2 - 5)$

$$= \frac{3x^2 - 5}{(3x^2 - 5)^2 + 1} = \frac{3x^2 - 5}{9x^4 - 30x^2 + 26}.$$

5. Let  $x$  be the length of each edge of the cube,  $S$  be its surface area and  $V$  be its volume at any time  $t$ . Then,

$$S = 6x^2 \text{ and } V = x^3, \text{ Given } \frac{dV}{dt} = k \text{ (constant)}$$

$$\text{Now, } V = x^3 \Rightarrow \frac{dV}{dt} = 3x^2 \frac{dx}{dt}$$

$$\Rightarrow k = 3x^2 \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{k}{3x^2} \quad \dots(i)$$

$$\text{and } S = 6x^2 \Rightarrow \frac{dS}{dt} = 12x \frac{dx}{dt}$$

$$\Rightarrow \frac{dS}{dt} = 12x \left( \frac{k}{3x^2} \right) \quad [\text{Using (i)}]$$

$$\Rightarrow \frac{dS}{dt} = \frac{4k}{x} \Rightarrow \frac{dS}{dt} \propto \frac{1}{x}.$$

6. Equivalence relations could be the following:

$\{(1,1), (2,2), (3,3), (1,2), (2,1)\}$  and  $\{(1,1), (2, 2), (3, 3), (1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3, 2)\}$

So, only two equivalence relations.

7. Given equation of line is  $\frac{x+3}{3} = \frac{y-4}{5} = \frac{8-z}{-6}$

$$\text{or } \frac{x+3}{3} = \frac{y-4}{5} = \frac{z-8}{6}$$

Since a line through  $(-2, 4, -5)$  is parallel to the given line having D.R.'s  $(3, 5, 6)$

$\therefore$  Required equation of line in cartesian form is

$$\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$$

and vector equation of the line is  $\vec{r} = \vec{a} + \lambda \vec{b}$

$$\Rightarrow \vec{r} = (-2\hat{i} + 4\hat{j} - 5\hat{k}) + \lambda(3\hat{i} + 5\hat{j} + 6\hat{k})$$

8. Here,  $y = a \cos(x+b)$   $\dots(i)$

Differentiating (i) w.r.t.  $x$ , we get

$$\frac{dy}{dx} = -a \sin(x+b)$$

Again differentiating w.r.t.  $x$ , we get

$$\frac{d^2y}{dx^2} = -a \cos(x+b) \Rightarrow \frac{d^2y}{dx^2} = -y \Rightarrow \frac{d^2y}{dx^2} + y = 0.$$

9. For  $f(x)$  to be continuous at  $x = 3$ , we must have

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x) = f(3)$$

$$\Rightarrow 1 = 3a + b \quad \dots(i)$$

- For  $f(x)$  to be continuous at  $x = 5$ , we must have

$$\lim_{x \rightarrow 5^-} f(x) = \lim_{x \rightarrow 5^+} f(x) = f(5)$$

$$\Rightarrow 5a + b = 7 \quad \dots(ii)$$

Solving (i) and (ii), we get  $a = 3, b = -8$ .

10. Given,  $l_i, m_i, n_i$  are three mutually  $\perp$  r vector in space.

$$\therefore l_i^2 + m_i^2 + n_i^2 = 1, \text{ for each } i = 1, 2, 3 \text{ and}$$

$$l_i l_j + m_i m_j + n_i n_j = 0 \quad (i \neq j) \text{ for each } i, j = 1, 2, 3$$

$$\begin{aligned} \text{Hence } AA' &= \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \begin{bmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3 \end{aligned}$$

11. The probability of drawing two black balls

$$= \frac{7C_2}{12C_2} = \frac{7 \cdot 6}{1 \cdot 2} \times \frac{1 \cdot 2}{12 \cdot 11} = \frac{7}{22}$$

So, the odds in favour of drawing two black balls are

$$p : (1-p) = \frac{7}{22} : \left(1 - \frac{7}{22}\right) = \frac{7}{22} : \frac{15}{22} = 7 : 15.$$

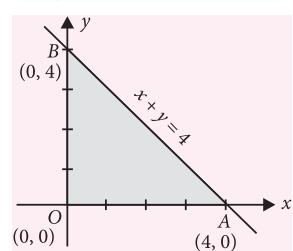
12. We have, maximize  $Z = 3x + 4y$ ,

Subject to the constraints

$$x + y \leq 4, x \geq 0, y \geq 0$$

On plotting the given constraint we get,  $OAB$  bounded region as feasible.

Hence, on evaluating  $Z$  at corner points  $O(0, 0), A(4, 0), B(0, 4)$ , we get



Corner points	$Z = 3x + 4y$
$O(0, 0)$	$3 \times 0 + 4 \times 0 = 0$
$A(4, 0)$	$3 \times 4 + 4 \times 0 = 12$
$B(0, 4)$	$3 \times 0 + 4 \times 4 = 16$ (Maximum)

Hence, the maximum value of  $Z$  is 16 at  $(0, 4)$ .

13. Let  $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ , then  $A = I_3 A$

$$\Rightarrow \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying  $R_1 \leftrightarrow R_2$ , we get

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying  $R_3 \rightarrow R_3 - 3R_1$ , we get

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A$$

Applying  $R_3 \rightarrow R_3 + 5R_2$ , and  $R_1 \rightarrow R_1 - 2R_2$  we get

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix} A$$

Applying  $R_2 \rightarrow R_2 - R_3$ , we get

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ -4 & 3 & -1 \\ 5 & -3 & 1 \end{bmatrix} A$$

Applying  $R_3 \rightarrow \frac{1}{2}R_3$ , we get

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix} A$$

Applying  $R_1 \rightarrow R_1 + R_3$ , we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix} A$$

$$\Rightarrow I_3 = BA$$

$$\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$$

Hence,  $A^{-1} = B = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$ .

OR

$$\text{L.H.S.} = \begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix}$$

Applying  $C_1 \rightarrow C_1 \times x$  and  $C_2 \rightarrow C_2 \times y$ , we get

$$\frac{1}{xy} \begin{vmatrix} ax & by & ax+by \\ bx & cy & bx+cy \\ ax^2+bxy & bxy+cy^2 & 0 \end{vmatrix}$$

Applying  $C_1 \rightarrow C_1 + C_2$ , we get

$$\frac{1}{xy} \begin{vmatrix} ax+by & by & ax+by \\ bx+cy & cy & bx+cy \\ ax^2+2bxy+cy^2 & bxy+cy^2 & 0 \end{vmatrix}$$

Applying  $C_1 \rightarrow C_1 - C_3$ , we get

$$\frac{1}{xy} \begin{vmatrix} 0 & by & ax+by \\ 0 & cy & bx+cy \\ ax^2+2bxy+cy^2 & bxy+cy^2 & 0 \end{vmatrix}$$

Expanding the determinant along  $C_1$ , we get

$$\begin{aligned} & \frac{1}{xy} [(ax^2 + 2bxy + cy^2) \{by(bx + cy) - cy(ax + by)\}] \\ &= \frac{1}{xy} [(ax^2 + 2bxy + cy^2) \{b^2xy + bcy^2 - caxy - bcy^2\}] \\ &= \frac{1}{xy} [(ax^2 + 2bxy + cy^2)(xy)(b^2 - ac)] \\ &= [ax^2 + 2bxy + cy^2] [b^2 - ac] = \text{R.H.S.} \end{aligned}$$

14. Two cases arise:

**Case I:** When  $a = b$

$$|\cos a - \cos b| = 0 = |a - b|.$$

**Case II:** When  $a \neq b$ , let  $a < b$ .

Consider the function  $f(x) = \cos x$  ... (i)

As  $f(x)$  is continuous and differentiable for all  $x \in R$ ,

(i)  $f(x)$  is continuous in  $[a, b]$  and

(ii)  $f(x)$  is differentiable in  $(a, b)$ .

Thus, both the conditions of Lagrange's Mean value theorem are satisfied, therefore, there exists atleast one real number  $c$  in  $(a, b)$  such that

$$f'(c) = \frac{f(b) - f(a)}{b - a} \quad \dots (\text{ii})$$

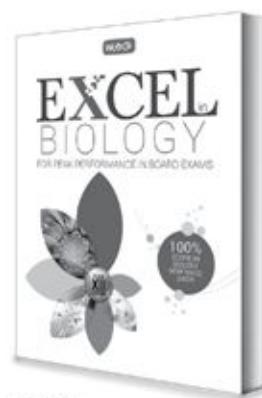
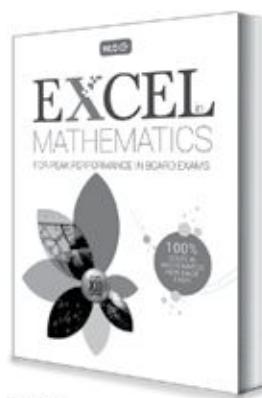
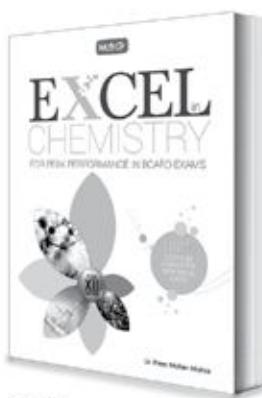
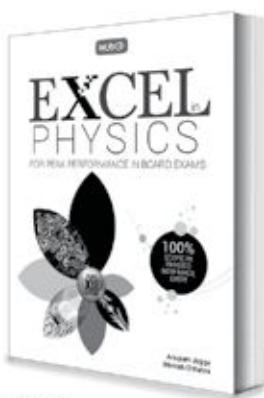
Differentiating (i) w.r.t.  $x$ , we get

$$f'(x) = -\sin x \Rightarrow f'(c) = -\sin c$$

From (ii), we get

$$-\sin c = \frac{\cos b - \cos a}{b - a} \Rightarrow \frac{\cos a - \cos b}{a - b} = -\sin c$$

# Concerned about your performance in **Class XII** Boards?



Well, fear no more, help is at hand.....

To excel, studying in right direction is more important than studying hard. Which is why we created the Excel Series. These books – for Physics, Chemistry, Biology & Mathematics – have been put together totally keeping in mind the prescribed syllabus and the pattern of CBSE's Board examinations, so that students prepare and practice with just the right study material to excel in board exams.

Did you know nearly all questions in CBSE's 2017 Board Examination were a part of our Excel books?  
That too fully solved !

## HIGHLIGHTS:

- Comprehensive theory strictly based on NCERT, complemented with illustrations, activities and solutions of NCERT questions
- Practice questions & Model Test Papers for Board Exams
- Value based questions
- Previous years' CBSE Board Examination Papers (Solved)
- CBSE Board Papers 2017 Included



Scan now with your smartphone or tablet\*

Visit  
[www.mtg.in](http://www.mtg.in)  
for latest offers  
and to buy  
online!



Available at all leading book shops throughout the country.  
For more information or for help in placing your order:  
Call 0124-6601200 or email: [info@mtg.in](mailto:info@mtg.in)

\*Application to read QR codes required

$$\Rightarrow \left| \frac{\cos a - \cos b}{a - b} \right| = |-\sin c| = |\sin c| \leq 1$$

[Since  $|\sin x| \leq 1 \forall x \in R$ ]

$$\Rightarrow \frac{|\cos a - \cos b|}{|a - b|} \leq 1 \Rightarrow |\cos a - \cos b| \leq |a - b|.$$

From cases I and II, we get  $|\cos a - \cos b| \leq |a - b|$ .

**15.** Given  $f(x) = x^2 \sqrt{1+x}$  ... (i)

The given function is differentiable for all  $x$  in  $\left[-1, \frac{1}{2}\right]$ . Differentiating (i) w.r.t.  $x$ , we get

$$f'(x) = x^2 \cdot \frac{1}{2}(1+x)^{-1/2} + \sqrt{1+x} \cdot 2x$$

$$= \frac{x^2}{2\sqrt{1+x}} + 2x\sqrt{1+x} = \frac{5x^2 + 4x}{2\sqrt{1+x}}.$$

$$\text{Now } f'(x) = 0 \Rightarrow \frac{5x^2 + 4x}{2\sqrt{1+x}} = 0 \Rightarrow 5x^2 + 4x = 0$$

$$\Rightarrow x(5x+4) = 0 \Rightarrow x = 0, -\frac{4}{5}.$$

Also  $0, -\frac{4}{5}$  both lie in  $\left[-1, \frac{1}{2}\right]$ , therefore, 0 and  $-\frac{4}{5}$  both are stationary points.

$$\text{Further, } f(0) = 0, f\left(-\frac{4}{5}\right) = \frac{16}{25} \cdot \frac{1}{\sqrt{5}} = \frac{16}{25\sqrt{5}}$$

$$f(-1) = 0, f\left(\frac{1}{2}\right) = \frac{1}{4} \cdot \sqrt{\frac{3}{2}} = \frac{\sqrt{6}}{8}.$$

Therefore, the absolute maximum value is  $\frac{\sqrt{6}}{8}$  and the absolute minimum value is 0.

The point of maxima is  $\frac{1}{2}$  and points of minima are  $\{-1, 0\}$ .

### OR

We have, equation of curve,  $y = x^3 - 11x + 5$

Differentiating w.r.t.  $x$ , we get  $\frac{dy}{dx} = 3x^2 - 11$

Slope of the given tangent line is 1.

Thus,  $3x^2 - 11 = 1$  that gives  $x = \pm 2$

when  $x = 2, y = 2 - 11 = -9$

when  $x = -2, y = 2 - 11 = -13$

Out of the two points  $(2, -9)$  and  $(-2, -13)$ , only the point  $(2, -9)$  lies on the curve  $y = x^3 - 11x + 5$ . Thus, the required point is  $(2, -9)$ .

**16.** We have

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h[f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)], \text{ where } h = \frac{b-a}{n}$$

Here,  $a = 1, b = 3, f(x) = 2x + 1$  and  $h = \frac{3-1}{n} = \frac{2}{n}$

$$\therefore I = \int_1^3 (2x+1) dx$$

$$\Rightarrow I = \lim_{h \rightarrow 0} h[f(1) + f(1+h) + f(1+2h) + \dots + f(1+(n-1)h)]$$

$$= \lim_{h \rightarrow 0} h[(2 \times 1 + 1) + \{2(1+h) + 1\} + \{2(1+2h) + 1\} + \dots + \{2(1+(n-1)h) + 1\}]$$

$$= \lim_{h \rightarrow 0} h[3 + (3+2h) + (3+2 \times 2h) + \dots + (3+2(n-1)h)]$$

$$= \lim_{h \rightarrow 0} h\{3n + 2h(1+2+3+\dots+(n-1))\}$$

$$= \lim_{h \rightarrow 0} h\left\{3n + 2h \frac{n(n-1)}{2}\right\}$$

$$= \lim_{n \rightarrow \infty} \frac{2}{n} \left\{3n + \frac{2}{n} \times n(n-1)\right\}$$

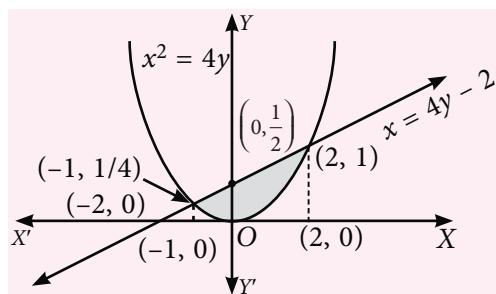
$\left[\because h = \frac{2}{n} \text{ and } h \rightarrow 0 \Rightarrow n \rightarrow \infty\right]$

$$\Rightarrow I = \lim_{n \rightarrow \infty} \left\{6 + 4\left(1 - \frac{1}{n}\right)\right\} = 6 + 4(1 - 0) = 10$$

**17.** The equations of the given curves are

$$x^2 = 4y \quad \dots (\text{i}) \quad \text{and, } x = 4y - 2 \quad \dots (\text{ii})$$

Equation (i) represents a parabola having vertex at the origin and axis along positive direction of  $y$ -axis. Equation (ii) represent a line which meets coordinate axes at  $(-2, 0)$  and  $(0, 1/2)$  respectively. Now, solving the two equations simultaneously we get  $(2, 1)$  and  $(-1, 1/4)$  as their intersection points.



$$\text{So, required area } (A) = \int_{-1}^2 |y_2 - y_1| dx = \int_{-1}^2 (y_2 - y_1) dx$$

$\left[\because y_2 > y_1 \therefore |y_2 - y_1| = y_2 - y_1\right]$

$$\Rightarrow A = \int_{-1}^2 \left( \frac{x+2}{4} - \frac{x^2}{4} \right) dx = \left[ \frac{x^2}{8} + \frac{1}{2}x - \frac{x^3}{12} \right]_{-1}^2$$

$$= \left( \frac{4}{8} + \frac{2}{2} - \frac{8}{12} \right) - \left( \frac{1}{8} - \frac{1}{2} + \frac{1}{12} \right) = \frac{9}{8} \text{ sq. units}$$

**18.** We have,  $\frac{dy}{dx} - y = \cos x$  ... (i)

This is a linear differential equation of the form  $\frac{dy}{dx} + Py = Q$ , where  $P = -1$ ,  $Q = \cos x$

$$\therefore \text{I.F.} = e^{\int P dx} = e^{-x}$$

Hence the solution of (i) is

$$y \cdot e^{-x} = \int e^{-x} \cdot \cos x dx + C$$

$$\text{Now, } \int e^{-x} \cos x dx = -e^{-x} \cos x + e^{-x} \sin x - \int e^{-x} \cos x dx$$

$$\Rightarrow \int e^{-x} \cos x dx = \frac{1}{2} e^{-x} (\sin x - \cos x)$$

Thus the required solution of (i) is

$$y \cdot e^{-x} = \frac{1}{2} e^{-x} (\sin x - \cos x) + C \quad \dots(\text{ii})$$

Putting,  $x = 0, y = 1$  in (ii), we get

$$\Rightarrow 1 = \frac{1}{2}(-1) + C \Rightarrow C = \frac{3}{2}$$

$$\text{From (ii), } y = \frac{1}{2} (\sin x - \cos x) + \frac{3}{2} e^x$$

Which is the required solution.

### OR

Given differential equation can be written as

$$\frac{dy}{dx} = \frac{1+xy+\cos\left(\frac{y}{x}\right)}{x^2} = \frac{y}{x} + \left[ \frac{1+\cos\left(\frac{y}{x}\right)}{x^2} \right] \dots(\text{i})$$

$$\text{Let } F(x, y) = \frac{y}{x} + \left[ \frac{1+\cos\left(\frac{y}{x}\right)}{x^2} \right]$$

Replacing  $x$  to  $\lambda x$  and  $y$  to  $\lambda y$ , we get

$$\begin{aligned} F(\lambda x, \lambda y) &= \frac{\lambda y}{\lambda x} + \left[ \frac{1+\cos\left(\frac{\lambda y}{\lambda x}\right)}{(\lambda x)^2} \right] \\ &= \frac{y}{x} + \left[ \frac{1+\cos\left(\frac{y}{x}\right)}{\lambda^2 x^2} \right] \neq F(x, y) \end{aligned}$$

Hence, the given differential equation is not a homogeneous equation.

Putting  $y = vx$  and  $\frac{dy}{dx} = v + x \frac{dv}{dx}$  in (i), we get

$$v + x \frac{dv}{dx} = v + \frac{1+\cos v}{x^2}$$

$$\Rightarrow \frac{dv}{1+\cos v} = \frac{dx}{x^3} \Rightarrow \sec^2\left(\frac{v}{2}\right) dv = \frac{2}{x^3} dx$$

Integrating both sides, we get

$$2 \tan \frac{v}{2} = -\frac{1}{x^2} + C \quad \text{or} \quad 2 \tan \frac{y}{2x} = -\frac{1}{x^2} + C$$

**19.** We have,  $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$ ,  $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$  and  $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$

$$\vec{a} \cdot \vec{b} = (\hat{i} + 4\hat{j} + 2\hat{k}) \cdot (3\hat{i} - 2\hat{j} + 7\hat{k}) = 3 - 8 + 14 = 9$$

$$\text{Let } \vec{d} = (d_1\hat{i} + d_2\hat{j} + d_3\hat{k})$$

Now,  $\vec{a} \perp \vec{d}$  if  $\vec{a} \cdot \vec{d} = 0$

$$\Rightarrow d_1 + 4d_2 + 2d_3 = 0 \quad \dots(\text{i})$$

Also,  $\vec{b} \perp \vec{d}$  if  $\vec{b} \cdot \vec{d} = 0$

$$\Rightarrow 3d_1 - 2d_2 + 7d_3 = 0 \quad \dots(\text{ii})$$

and  $\vec{c} \cdot \vec{d} = 15$

$$\Rightarrow 2d_1 - d_2 + 4d_3 = 15 \quad \dots(\text{iii})$$

Solving (i), (ii) and (iii) for  $d_1, d_2, d_3$ , we get

$$d_1 = \frac{160}{3}, d_2 = \frac{-5}{3}, d_3 = -\frac{70}{3}$$

$$\therefore \vec{d} = \frac{160}{3}\hat{i} - \frac{5}{3}\hat{j} - \frac{70}{3}\hat{k} = \frac{1}{3}[160\hat{i} - 5\hat{j} - 70\hat{k}]$$

**20.** Urn - I (10 W and 3 B), Urn - II (3 W and 5 B)

Let  $E_1, E_2$  and  $E_3$  and  $A$  be the events as defined below:

$E_1$  = Two black balls are drawn from the first bag.

$E_2$  = Two white balls are drawn from the first bag.

$E_3$  = One white and one black ball is drawn from first bag.

$A$  = One ball drawn from the second bag is white.

$$P(E_1) = \frac{{}^3C_2}{{}^{13}C_2} = \frac{1}{26}, \quad P(E_2) = \frac{{}^{10}C_2}{{}^{13}C_2} = \frac{15}{26},$$

$$P(E_3) = \frac{{}^{10}C_1 \times {}^3C_1}{{}^{13}C_2} = \frac{10}{26}$$

$$\text{Also, } P(A/E_1) = \frac{{}^3C_1}{{}^{10}C_1} = \frac{3}{10}; \quad P(A/E_2) = \frac{{}^5C_1}{{}^{10}C_1} = \frac{5}{10}$$

$$P(A/E_3) = \frac{{}^4C_1}{{}^{10}C_1} = \frac{4}{10} = \frac{2}{5}$$

$$P(A) = P(E_1) P(A/E_1) + P(E_2) P(A/E_2) + P(E_3) P(A/E_3)$$

$$= \frac{1}{26} \times \frac{3}{10} + \frac{15}{26} \times \frac{5}{10} + \frac{10}{26} \times \frac{2}{5} = \frac{59}{130}$$

**21.** (i) Since the sum of all the probabilities in a probability distribution is always unity.

$$\therefore P(X=0) + P(X=1) + \dots + P(X=7) = 1$$

$$\Rightarrow 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$$

$$\Rightarrow 10k^2 + 9k - 1 = 0 \Rightarrow (10k-1)(k+1) = 0$$

$$\Rightarrow 10k-1=0 \quad [\because k \geq 0 \quad \therefore k+1 \neq 0]$$

$$\Rightarrow k = \frac{1}{10}$$

$$\begin{aligned}
 \text{(ii)} \quad P(X < 6) &= P(X = 0) + P(X = 1) + P(X = 2) \\
 &\quad + P(X = 3) + P(X = 4) + P(X = 5) \\
 \Rightarrow P(X < 6) &= 0 + k + 2k + 2k + 3k + k^2 \\
 &= k^2 + 8k = \left(\frac{1}{10}\right)^2 + \frac{8}{10} \quad [\because k = 1/10] \\
 \Rightarrow P(X < 6) &= \frac{81}{100} \\
 \text{(iii)} \quad P(X \geq 6) &= 1 - P(X < 6) = 1 - \frac{81}{100} = \frac{19}{100} \\
 \text{(iv)} \quad P(0 < X < 5) &= P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) \\
 \Rightarrow P(0 < X < 5) &= k + 2k + 2k + 3k = 8k \\
 \Rightarrow P(0 < X < 5) &= \frac{8}{10} = \frac{4}{5} \quad [\because k = 1/10]
 \end{aligned}$$

**22.** Equation of a plane passing through the point  $(2, 2, 1)$  is  $a(x - 2) + b(y - 2) + c(z - 1) = 0$  ... (i)  
It passes through  $(9, 3, 6)$  then

$$7a + b + 5c = 0 \quad \dots \text{(ii)}$$

Since (i) is perpendicular to plane  $2x + 6y + 6z = 1$ , then  
 $2a + 6b + 6c = 0$  ... (iii)

Solving (ii) and (iii) by the method of cross-multiplication, we obtain

$$\begin{aligned}
 \frac{a}{6-30} &= \frac{b}{10-42} = \frac{c}{42-2} \Rightarrow \frac{a}{-24} = \frac{b}{-32} = \frac{c}{40} \\
 \Rightarrow \frac{a}{-3} &= \frac{b}{-4} = \frac{c}{5} = \lambda \text{ (say)} \Rightarrow a = -3\lambda, b = -4\lambda, c = 5\lambda \\
 \text{Putting } a &= -3\lambda, b = -4\lambda \text{ and } c = 5\lambda \text{ in (i), we get} \\
 -3\lambda(x-2) - 4\lambda(y-2) + 5\lambda(z-1) &= 0 \\
 \Rightarrow -3x + 6 - 4y + 8 + 5z - 5 &= 0 \\
 \Rightarrow -3x - 4y + 5z + 9 &= 0
 \end{aligned}$$

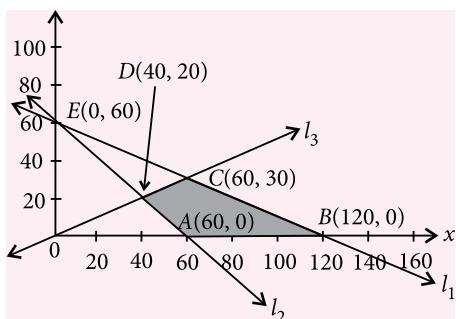
**23.** We have, Minimize  $Z = 5x + 10y$

subject :  $x + 2y \leq 120$ ,  $x + y \geq 60$ ,  $x - 2y \geq 0$  and  $x, y \geq 0$   
To solve L.P.P graphically, we convert inequations into equations.

$l_1 : x + 2y = 120$ ,  $l_2 : x + y = 60$ ,  $l_3 : x - 2y = 0$   
and  $x = 0, y = 0$

$l_1$  and  $l_2$  intersect at  $E(0, 60)$ ,  $l_1$  and  $l_3$  intersect at  $C(60, 30)$ ,  $l_2$  and  $l_3$  intersect at  $D(40, 20)$ .

The shaded region  $ABCD$  is the feasible region and is bounded.



Corner points	Value of $Z = 5x + 10y$
$A(60, 0)$	300 ← (Minimum)
$B(120, 0)$	600
$C(60, 30)$	600
$D(40, 20)$	400

Hence,  $Z$  is minimum at  $A(60, 0)$  i.e., 300.

$$\begin{aligned}
 \text{24. Let } I &= \int \frac{1}{\sin x + \sec x} dx. \text{ Then, } I = \int \frac{\cos x dx}{1 + \sin x \cos x} \\
 &= \int \frac{2 \cos x}{2 + 2 \sin x \cos x} dx \\
 &= \int \frac{\cos x + \sin x}{2 + 2 \sin x \cos x} dx + \int \frac{\cos x - \sin x}{2 + 2 \sin x \cos x} dx \\
 &= \int \frac{\cos x + \sin x}{3 - (1 - 2 \sin x \cos x)} dx + \int \frac{\cos x - \sin x}{1 + (1 + 2 \sin x \cos x)} dx \\
 &= \int \frac{(\cos x + \sin x)}{3 - (\sin x - \cos x)^2} dx + \int \frac{\cos x - \sin x}{1 + (\sin x + \cos x)^2} dx \\
 &= \int \frac{1}{(\sqrt{3})^2 - u^2} du + \int \frac{1}{1 + v^2} dv,
 \end{aligned}$$

where  $u = \sin x - \cos x \Rightarrow du = (\cos x + \sin x)dx$   
and  $v = \sin x + \cos x \Rightarrow dv = (\cos x - \sin x)dx$

$$\begin{aligned}
 \Rightarrow I &= \frac{1}{2\sqrt{3}} \log \left| \frac{\sqrt{3} + u}{\sqrt{3} - u} \right| + \tan^{-1} v + C \\
 \Rightarrow I &= \frac{1}{2\sqrt{3}} \log \left| \frac{\sqrt{3} + (\sin x - \cos x)}{\sqrt{3} - (\sin x - \cos x)} \right| \\
 &\quad + \tan^{-1}(\sin x + \cos x) + C
 \end{aligned}$$

**OR**

$$\begin{aligned}
 \text{Let } I &= \int_0^{\pi/2} (2 \log \sin x - \log \sin 2x) dx \\
 &= \int_0^{\pi/2} \{2 \log \sin x - \log(2 \sin x \cos x)\} dx \\
 &= \int_0^{\pi/2} \{2 \log \sin x - \log 2 - \log \sin x - \log \cos x\} dx \\
 &= \int_0^{\pi/2} \log \sin x dx - \int_0^{\pi/2} \log 2 dx - \int_0^{\pi/2} \log \cos x dx \\
 &= \int_0^{\pi/2} \log \sin x dx - (\log 2) \int_0^{\pi/2} 1 \cdot dx \\
 &\quad - \int_0^{\pi/2} \log \cos \left( \frac{\pi}{2} - x \right) dx
 \end{aligned}$$

$$= \int_0^{\pi/2} \log \sin x \, dx - (\log 2)[x]_0^{\pi/2} - \int_0^{\pi/2} \log \sin x \, dx$$

$$= -(\log 2) \left( \frac{\pi}{2} - 0 \right) = -\frac{\pi}{2} \log 2$$

25. Let  $y = (\sin^{-1} x)^3 + (\cos^{-1} x)^3$ . Then,

$$y = (\sin^{-1} x + \cos^{-1} x)^3 - 3\sin^{-1} x \cos^{-1} x (\sin^{-1} x + \cos^{-1} x)$$

$$\Rightarrow y = \left( \frac{\pi}{2} \right)^3 - \frac{3\pi}{2} \sin^{-1} x \left( \frac{\pi}{2} - \sin^{-1} x \right)$$

$$\qquad \qquad \qquad \left[ \because \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \right]$$

$$\Rightarrow y = \frac{\pi^3}{8} - \frac{3\pi^2}{4} \sin^{-1} x + \frac{3\pi}{2} (\sin^{-1} x)^2$$

$$\Rightarrow \frac{3\pi}{2} (\sin^{-1} x)^2 - \frac{3\pi^2}{4} (\sin^{-1} x) + \left( \frac{\pi^3}{8} - y \right) = 0$$

$$\Rightarrow (\sin^{-1} x)^2 - \frac{\pi}{2} (\sin^{-1} x) + \frac{2}{3\pi} \left( \frac{\pi^3}{8} - y \right) = 0$$

$$\Rightarrow \left( \sin^{-1} x - \frac{\pi}{4} \right)^2 + \frac{\pi^2}{48} - \frac{2y}{3\pi} = 0$$

$$\Rightarrow \left( \sin^{-1} x - \frac{\pi}{4} \right)^2 = \frac{2y}{3\pi} - \frac{\pi^2}{48} \quad \dots(i)$$

We know that

$$-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2} \text{ for all } x \in [-1, 1]$$

$$\Rightarrow -\frac{3\pi}{4} < \sin^{-1} x - \frac{\pi}{4} \leq \frac{\pi}{4} \Rightarrow 0 \leq \left( \sin^{-1} x - \frac{\pi}{4} \right)^2 \leq \frac{9\pi^2}{16} \quad \dots(ii)$$

From (i) and (ii), we find that

$$\Rightarrow 0 \leq \frac{2y}{3\pi} - \frac{\pi^2}{48} \leq \frac{9\pi^2}{16}$$

$$\Rightarrow \frac{\pi^2}{48} \leq \frac{2y}{3\pi} \leq \frac{9\pi^2}{16} + \frac{\pi^2}{48} \Rightarrow \frac{\pi^3}{32} \leq y \leq \frac{7\pi^3}{8}$$

Hence, the maximum and minimum values of

$$(\sin^{-1} x)^3 + (\cos^{-1} x)^3 \text{ are } \frac{7\pi^3}{8} \text{ and } \frac{\pi^3}{32}.$$

**OR**

We observe the following properties of  $*$  on  $R - \{-1\}$ .

**Commutativity:** For any  $a, b \in R - \{-1\}$ , we have

$$a * b = a + b + ab \text{ and } b * a = b + a + ba$$

$$\therefore a + b + ab = b + a + ba$$

$$\Rightarrow a * b = b * a$$

So,  $*$  is commutative on  $R - \{-1\}$ .

**Associativity:** For any  $a, b, c \in R - \{-1\}$ , we have

$$(a * b) * c = (a + b + ab) * c$$

$$= (a + b + ab) + c + (a + b + ab)c$$

$$= a + b + c + ab + bc + ac + abc \quad \dots(i)$$

$$\text{and, } a * (b * c) = a * (b + c + bc)$$

$$= a + (b + c + bc) + a(b + c + bc)$$

$$= a + b + c + ab + bc + ac + abc \quad \dots(ii)$$

From (i) and (ii), we have

$$(a * b) * c = a * (b * c) \text{ for all } a, b, c \in R - \{-1\}.$$

So,  $*$  is associative on  $R - \{-1\}$

**Existence of Identity:** Let  $e$  be the identity element. Then,  $a * e = a = e * a$  for all  $a \in R - \{-1\}$

$$\Rightarrow a + e + ae = a \text{ and } e + a + ea = a \text{ for all } a \in R - \{-1\}$$

$$\Rightarrow e(1 + a) = 0 \text{ for all } a \in R - \{-1\}$$

$$\Rightarrow e = 0.$$

Thus, 0 is the identity element for  $*$  defined on  $R - \{-1\}$ .

**Existence of Inverse:** Let  $a \in R - \{-1\}$  and let  $b$  be the inverse of  $a$ . Then,

$$a * b = e = b * a$$

$$\Rightarrow a * b = e \Rightarrow a + b + ab = 0 \quad [\because e = 0]$$

$$\Rightarrow b = \frac{-a}{a+1}$$

$$\text{Now, } a \in R - \{-1\} \Rightarrow a \neq -1 \Rightarrow a + 1 \neq 0$$

$$\Rightarrow b = \frac{-a}{a+1} \in R$$

$$\text{Also, } \frac{-a}{a+1} = -1 \Rightarrow -a = -a - 1$$

$$\Rightarrow -1 = 0, \text{ which is absurd.}$$

$$\text{Thus, } \frac{-a}{a+1} \in R - \{-1\}.$$

Hence, every element of  $R - \{-1\}$  is invertible and the inverse of an element  $a$  is  $\frac{-a}{a+1}$ .

26. Let ₹  $x$ , ₹  $y$  and ₹  $z$  be the investments at the rate of interest of 6%, 7% and 8% per annum respectively.

$$\text{Total investment} = ₹ 5000$$

$$\Rightarrow x + y + z = 5000 \quad \dots(i)$$

$$\text{Now, income from first investment of ₹ } x = ₹ \frac{6x}{100}$$

$$\text{Income from second investment of ₹ } y = ₹ \frac{7y}{100}$$

$$\text{Income from third investment of ₹ } z = ₹ \frac{8z}{100}.$$

$$\therefore \text{Total annual income} = ₹ \left( \frac{6x}{100} + \frac{7y}{100} + \frac{8z}{100} \right)$$

$$\Rightarrow \frac{6x}{100} + \frac{7y}{100} + \frac{8z}{100} = 358 \quad \dots(ii)$$

$$[\because \text{Total annual income} = ₹ 358]$$

$$\Rightarrow 6x + 7y + 8z = 35800.$$

It is given that the combined income from the first two investments is ₹ 70 more than the income from the third.

$$\therefore \frac{6x}{100} + \frac{7y}{100} = 70 + \frac{8z}{100} \Rightarrow 6x + 7y - 8z = 7000 \quad \dots(\text{iii})$$

From (i), (ii) and (iii), we have

$$\begin{bmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5000 \\ 35800 \\ 7000 \end{bmatrix}$$

or,  $AX = B$ , where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 5000 \\ 35800 \\ 7000 \end{bmatrix}$$

$$\text{Now, } |A| = \begin{vmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{vmatrix}$$

$$= 1(-56 - 56) - (-48 - 48) + 42 - 42 = -16 \neq 0.$$

So,  $A^{-1}$  exists and the solution of the given system of equations is given by  $X = A^{-1}B$ .

$$\text{adj } A = \begin{bmatrix} -112 & 96 & 0 \\ 15 & -14 & -1 \\ 1 & -2 & 1 \end{bmatrix}^T = \begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\text{So, } A^{-1} = \frac{1}{|A|} (\text{adj } A) = -\frac{1}{16} \begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}$$

Hence, the solution is given by

$$X = A^{-1}B = -\frac{1}{16} \begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 5000 \\ 35800 \\ 7000 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1000 \\ 2200 \\ 1800 \end{bmatrix}$$

$$\Rightarrow x = 1000, y = 2200 \text{ and } z = 1800$$

Hence, three investments are of ₹ 1000, ₹ 2200 and ₹ 1800 respectively.

**OR**

$$\text{Here, } |A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{vmatrix} = 1(0+0) + 1(9+2) + 2(0-0) = 11$$

$$\Rightarrow |A|I = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$\text{adj } A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

$$A(\text{adj } A) = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$(\text{adj } A)A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

Thus, it is verified that  $A(\text{adj } A) = (\text{adj } A)A = |A|I$

**27.** We have,  $(ax + b)e^{yx} = x$

$$\Rightarrow e^{yx} = \frac{x}{ax + b} \Rightarrow \frac{y}{x} = \log\left(\frac{x}{ax + b}\right)$$

[On taking log both sides]

$$\Rightarrow \frac{y}{x} = \log x - \log(a + bx)$$

On differentiating with respect to  $x$ , we get

$$\begin{aligned} \frac{x \frac{dy}{dx} - y}{x^2} &= \frac{1}{x} - \frac{b}{a + bx} \\ \Rightarrow x \frac{dy}{dx} - y &= x^2 \left\{ \frac{1}{x} - \frac{b}{a + bx} \right\} \\ \Rightarrow x \frac{dy}{dx} - y &= \frac{ax}{a + bx} \end{aligned} \quad \dots(\text{i})$$

Differentiating both sides of (i) with respect to  $x$ , we get

$$\begin{aligned} x \frac{d^2y}{dx^2} + \frac{dy}{dx} - \frac{dy}{dx} &= \frac{(a + bx)a - ax(0+b)}{(a + bx)^2} \\ \Rightarrow x \frac{d^2y}{dx^2} &= \frac{a^2}{(a + bx)^2} \Rightarrow x^3 \frac{d^2y}{dx^2} = \frac{a^2 x^2}{(a + bx)^2} \\ \Rightarrow x^3 \frac{d^2y}{dx^2} &= \left( \frac{ax}{a + bx} \right)^2 = \left( x \frac{dy}{dx} - y \right)^2 \quad [\text{Using (i)}] \end{aligned}$$

**28.** The equations of two given lines are

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \quad \dots(\text{i})$$

$$\text{and } \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5} \quad \dots(\text{ii})$$

Vector equation of (i) is

$$\vec{r} = \vec{a}_1 + \lambda \vec{b}_1 \quad \dots(\text{iii})$$

where,  $\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$  and  $\vec{b}_1 = 2\hat{i} + 3\hat{j} + 4\hat{k}$ .

Vector equation of (ii) is

$$\vec{r} = \vec{a}_2 + \mu \vec{b}_2 \quad \dots(\text{iv})$$

where,  $\vec{a}_2 = 2\hat{i} + 4\hat{j} + 5\hat{k}$  and  $\vec{b}_2 = 3\hat{i} + 4\hat{j} + 5\hat{k}$ .

Since, S.D. =  $\frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$  ... (v)

Now,  $\vec{a}_2 - \vec{a}_1 = (2\hat{i} + 4\hat{j} + 5\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k}) = \hat{i} + 2\hat{j} + 2\hat{k}$

and,  $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = -\hat{i} + 2\hat{j} - \hat{k}$

$\therefore |\vec{b}_1 \times \vec{b}_2| = \sqrt{1+4+1} = \sqrt{6}$

and,  $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (\hat{i} + 2\hat{j} + 2\hat{k}) \cdot (-\hat{i} + 2\hat{j} - \hat{k}) = -1 + 4 - 2 = 1$

On substituting the values, we get

$$\text{S.D.} = \frac{1}{\sqrt{6}}$$

**29.** The equation of the planes through the intersection of the planes  $\vec{r} \cdot (2\hat{i} + 6\hat{j}) + 12 = 0$  and  $\vec{r} \cdot (3\hat{i} - \hat{j} + 4\hat{k}) = 0$  is

$$[\vec{r} \cdot (2\hat{i} + 6\hat{j}) + 12] + \lambda[\vec{r} \cdot (3\hat{i} - \hat{j} + 4\hat{k})] = 0$$

$$\Rightarrow \vec{r} \cdot \{(2+3\lambda)\hat{i} + (6-\lambda)\hat{j} + 4\lambda\hat{k}\} + 12 = 0 \quad \dots(i)$$

$$\Rightarrow \frac{\vec{r} \cdot \{(-2-3\lambda)\hat{i} + (\lambda-6)\hat{j} + (-4\lambda)\hat{k}\}}{\sqrt{(2+3\lambda)^2 + (\lambda-6)^2 + (4\lambda)^2}} = \frac{12}{\sqrt{(2+3\lambda)^2 + (\lambda-6)^2 + (4\lambda)^2}}$$

It is given that the plane (i) is at a unit distance from the origin.

$$\therefore \frac{12}{\sqrt{(2+3\lambda)^2 + (\lambda-6)^2 + (4\lambda)^2}} = 1$$

$$\Rightarrow 144 = (2+3\lambda)^2 + (\lambda-6)^2 + (4\lambda)^2$$

$$\Rightarrow 26\lambda^2 = 104 \Rightarrow \lambda^2 = 4 \Rightarrow \lambda = \pm 2$$

Putting the values of  $\lambda$  in (i), we obtain

$$\vec{r} \cdot (8\hat{i} + 4\hat{j} + 8\hat{k}) + 12 = 0 \text{ and}$$

$\vec{r} \cdot (-4\hat{i} + 8\hat{j} - 8\hat{k}) + 12 = 0$ , as the equations of the required planes.

These equations can also be written as

$$\vec{r} \cdot (2\hat{i} + \hat{j} + 2\hat{k}) + 3 = 0 \text{ and } \vec{r} \cdot (-\hat{i} + 2\hat{j} - 2\hat{k}) + 3 = 0$$

$$\text{or } 2x + y + 2z + 3 = 0 \text{ and } -x + 2y - 2z + 3 = 0$$



# mcq's

## MEMORY CONTEST

### 1 Who can participate

If you have taken any of the exams given below and possess plenty of grey cells, photographic memory then you are the right candidate for this contest. All you have to do is write down as many questions (with all choices) you can remember, neatly on a paper with name of the exam, your name, address, age, your photograph and mail them to us.

### 2 The Exams

**PMT:** AIIMS, JIPMER, SRMJEEE .... etc.

**Engineering:** VITEEE, UPSEE, AMU, SRMJEEE, BITSAT, COMED-K.... etc.

### 3 The Benefits

Plenty! Each complete question with answer will make you richer by Rs. 100\*. More the questions, the merrier it will be. We will make you famous by publishing your name (photo if possible). Also you can derive psychological satisfaction from the fact that your questions will benefit thousands of readers.

### 4 And Lastly The Pitfalls

Don't send incomplete question. Our panel of experts will cross-check your questions. You have to send it within a month of giving the particular exam.

Mail to : The Editor ,  
**MTG Learning Media Pvt. Ltd.**

Plot 99, Sector 44  
Institutional Area, Gurgaon – 122003(HR)  
Tel. :0124-6601200

#### \*Conditions apply

- Payment will be made after the MCQs are published.
- Kindly note that each question should be complete.
- Payment will be made only for complete questions.
- Preference will be given to the reader sending the maximum complete and correct questions. Other conditions apply. The decision of the Editor, MTG shall be final and binding.

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

## Probability

Total Marks : 80

Time Taken : 60 Min.

### Only One Option Correct Type

1. A determinant is chosen at random from the set of all determinants of order 2 with elements 1 or 0 only. The probability that the value of the determinant is positive is

(a)  $\frac{1}{4}$       (b)  $\frac{2}{9}$       (c)  $\frac{3}{16}$       (d)  $\frac{1}{10}$

2. A bag  $A$  contains 2 white and 3 red balls and bag  $B$  contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. The probability that it was drawn from bag  $B$  is

(a)  $25/52$       (b)  $13/27$       (c)  $8/17$       (d)  $25/51$

3. If  $X$  and  $Y$  are independent binomial variates  $X \left( 5, \frac{1}{2} \right)$  and  $Y \left( 7, \frac{1}{2} \right)$ , then  $P(X + Y = 3)$  is

(a)  $\frac{55}{1024}$       (b)  $\frac{55}{4098}$   
 (c)  $\frac{55}{2048}$       (d) None of these

4. If the integers  $\lambda$  and  $\mu$  are chosen at random between 1 to 100, then the probability that a number of the form  $7^\lambda + 7^\mu$  is divisible by 5, is

(a)  $\frac{1}{4}$       (b)  $\frac{1}{7}$       (c)  $\frac{1}{8}$       (d)  $\frac{1}{49}$

5. A natural number  $x$  is chosen at random from the first hundred natural numbers. The probability that
- $$\frac{(x-20)(x-40)}{(x-30)} < 0$$

(a)  $\frac{1}{50}$       (b)  $\frac{3}{50}$       (c)  $\frac{7}{25}$       (d)  $\frac{9}{50}$

6. Three natural numbers are taken at random from the set  $A = \{x \mid 1 \leq x \leq 100, x \in N\}$ . The probability that the A.M. of the numbers taken is 75, is

(a)  $\frac{77C_2}{100C_3}$       (b)  $\frac{25C_2}{100C_3}$       (c)  $\frac{74C_{72}}{100C_{97}}$       (d)  $\frac{75C_2}{100C_3}$

### One or More Than One Option(s) Correct Type

7. A random variable  $X$  takes values 0, 1, 2, 3, ... with probability proportions to  $(x+1)\left(\frac{1}{5}\right)^x$ , then

(a)  $P(X = 0) = \frac{16}{25}$       (b)  $P(X \leq 1) = \frac{112}{125}$   
 (c)  $P(X \geq 1) = \frac{9}{25}$       (d)  $E(X) = \frac{25}{32}$

8. A fair coin is tossed 99 times. If  $r$  is the number of times tail occurs, then  $P(X = r)$  is maximum when  $r$  is equal to

(a) 49      (b) 51  
 (c) 50      (d) None of these

9. Let  $X$  be a set containing  $n$  elements. If two subsets  $A$  and  $B$  of  $X$  are picked at random, the probability that  $A$  and  $B$  have the same number of elements is

(a)  $\frac{2^n C_n}{2^{2n}}$       (b)  $\frac{1}{2^n C_n}$   
 (c)  $\frac{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)}{2^n \cdot n!}$       (d)  $\frac{3^n}{4^n}$

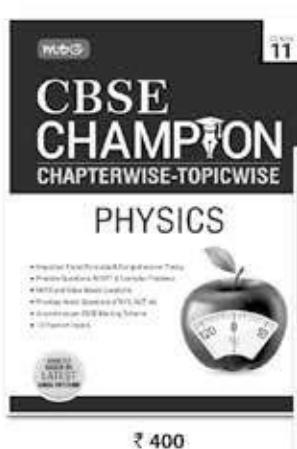
10. Let  $X$  and  $Y$  be two events such that

$P(X / Y) = \frac{1}{2}$ ,  $P(Y / X) = \frac{1}{3}$  and  $P(X \cap Y) = \frac{1}{6}$ .

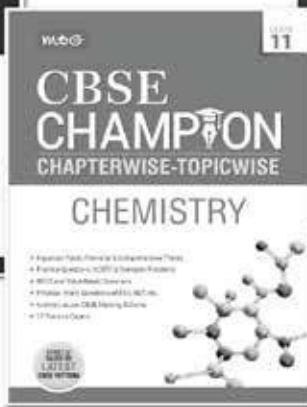
Which of the following is(are) correct?

(a)  $P(X \cup Y) = \frac{2}{3}$   
 (b)  $X$  and  $Y$  are independent

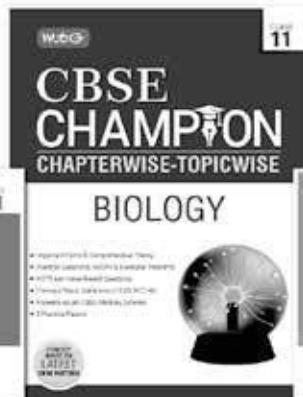
The only thing you NEED for excellence in Class -11



₹ 400



₹ 300



₹ 400



₹ 300

## CBSE CHAMPION Chapterwise -Topicwise Series

CBSE CHAMPION Chapterwise -Topicwise Series contains topicwise practice questions and solutions. Questions are freshly framed as well as question of NCERT, Exemplar and exams like KVS and NCT have also been incorporated.

Important feature of these books is that the solutions to all the questions have been given according to CBSE marking scheme and practice papers are also supplemented.

To excel, studying right is therefore more important than studying hard, which is why we created this series.



Available at all leading book shops throughout India.  
For more information or for help in placing your order:  
Call 0124-6601200 or email info@mtg.in

Visit  
[www.mtg.in](http://www.mtg.in)  
for latest offers  
and to buy  
online!

- (c)  $X$  and  $Y$  are not independent  
 (d)  $P(X^c \cap Y) = \frac{1}{3}$
- 11.** A coin is tossed  $(2n + 1)$  times, the probability that head appear odd number of times is  
 (a)  $\frac{n}{2n+1}$  (b)  $\frac{n+1}{2n+1}$  (c)  $\frac{1}{2}$  (d)  $\frac{n!}{2n+1}$
- 12.** A card is selected at random from cards numbered as 00, 01, 02, ..., 99. An event is said to have occurred. If product of digits of the card number is 16. If cards is selected 5 times with replacement each time, then the probability that the event occurs exactly three times is  
 (a)  ${}^5C_3 \left(\frac{3}{100}\right)^2 \left(\frac{97}{100}\right)^3$  (b)  ${}^5C_3 \left(\frac{3}{100}\right)^3 \left(\frac{97}{100}\right)^2$   
 (c)  ${}^5C_3 \left(\frac{0.3}{100}\right)^3 \left(\frac{9.7}{100}\right)^3$  (d)  $10(0.03)^3(0.97)^2$
- 13.** A coin is tossed repeatedly.  $A$  and  $B$  call alternately for winning a prize of ₹ 30. One who calls correctly first wins the prize.  $A$  starts the call. Then, the expectation of  
 (a)  $A$  is ₹ 10 (b)  $B$  is ₹ 10  
 (c)  $A$  is ₹ 20 (d)  $B$  is ₹ 20

#### Comprehension Type

The probability of happening of an event in one trial being known, then the probability of its happening exactly  $x$  times in  $n$  trials is given by  ${}^nC_x q^{n-x} \cdot p^x$  where  $p$  = probability of happening the event

$q$  = probability of not happening the event =  $1 - p$   
 Now,  ${}^nC_x q^{n-x} p^x$  is  $(x + 1)^{\text{th}}$  term in the expansion of  $(q + p)^n$  whose expansion gives the happening of the event 0, 1, 2, ...,  $n$  times, respectively.

- 14.** In four throws with a pair of dice, the chance of throwing doublets atleast twice is  
 (a)  $\frac{19}{144}$  (b)  $\frac{125}{144}$  (c)  $\frac{17}{144}$  (d)  $\frac{18}{144}$
- 15.** Unbiased coin is tossed 6 times. The probability of getting atmost 4 heads is  
 (a)  $\frac{7}{64}$  (b)  $\frac{57}{64}$  (c)  $\frac{21}{32}$  (d)  $\frac{11}{32}$

#### Matrix Match Type

- 16.** Sixteen players  $S_1, S_2, \dots, S_{16}$  play in a tournament. They are divided into eight pairs at random. From each players a winner is decided on the basis of a game played between the two players of the pair. Assume that all the players are of equal strength.

	Column I	Column II
P.	The probability that player $S_1$ is among eight winners is	1. $\frac{1}{3}$
Q.	If the probability that exactly one of $S_1$ and $S_2$ among eight winners is $\frac{3\lambda}{2}$ , then $\lambda$ is	2. $\frac{7}{30}$
R.	The probability that both $S_1$ and $S_2$ are among the eight winners is	3. $\frac{1}{2}$

- | P     | Q | R |
|-------|---|---|
| (a) 3 | 1 | 2 |
| (b) 3 | 2 | 1 |
| (c) 1 | 2 | 3 |
| (d) 2 | 1 | 3 |

#### Integer Answer Type

- 17.** Let  $A$  and  $B$  be two events such that  $P(A') = 0.3$ ,  $P(B) = 0.4$  and  $P(A \cap B') = 0.5$ , then  
 $P(B/A \cup B') = \frac{2}{n}$ , where  $n$  is
- 18.** If four squares are chosen at random on a chess board. If the probability that they lie on a diagonal line is  $\frac{xyz}{64C_4}$ , then the unit place of  $\lambda$  is
- 19.** The probability that the roots of the equation  $x^2 + nx + \frac{n}{2} = 0$  are real,  $n \in N$  and  $n \leq 5$ , is  $\frac{m}{n}$ , then  $m =$
- 20.** A, B, C and D cut a pack of 52 cards successively in the order given. If the person who cuts a spade first receives ₹ 350, then the value of  $\frac{E(x)}{64}$  is



Keys are published in this issue. Search now! ☺

## SELF CHECK

No. of questions attempted .....  
 No. of questions correct .....  
 Marks scored in percentage .....

#### Check your score! If your score is

> 90%	EXCELLENT WORK ! You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK ! You can score good in the final exam.
74-60%	SATISFACTORY ! You need to score more next time.
< 60%	NOT SATISFACTORY! Revise thoroughly and strengthen your concepts.

# DR. G. VISWANATHAN UNANIMOUSLY ELECTED INDIAN ECONOMIC ASSOCIATION CONFERENCE PRESIDENT



Founder Chancellor of Vellore Institute of Technology (VIT), Dr. G.Viswanathan has been unanimously elected as President of the Centenary Second Year Conference of the Indian Economic Association.

The First Year of the Centenary Celebrations was held at Acharya Nagarjuna University in Guntur, Andhra Pradesh between December 27 and 30, 2017. The selection was made during the General Body to be President of Centenary Second Year Conference of IEA.

Mr. Ram Nath Kovind, President of India, inaugurated the first year centenary celebrations at Acharya Nagarjuna University. E.S.L.Narasimhan, Governor of Andhra Pradesh, Chandrababu Naidu, Chief Minister, Muhammad Yunus, Nobel Laureate for Peace and Founder of Grameen Bank, former RBI Governor C.Rangarajan were among others present on the occasion in the inaugural function.

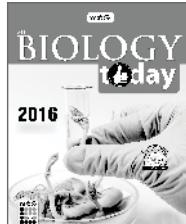
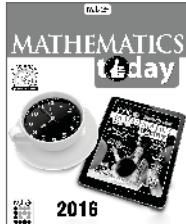
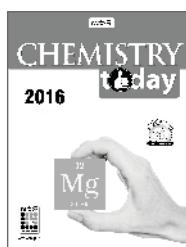
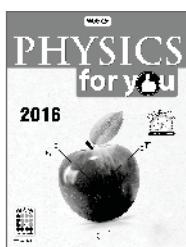
In his address, Mr. Kovind called upon emerging economies, including India, to speak up for an interconnected world with a fair and growing trade, in the wake of some countries turning protectionist. He also said imaginative policy-making was required to overcome the social and economic inequalities between different sections and regions.

During the Conference, Dr. G. Viswanathan presided over a special lecture by Prof Jomo Kwame Sundaram, a leading World Bank and United Nations economist from Malaysia. Mr. Sundaram spoke on the importance of trade for economic development. Dr G.Viswanathan stated that good politics should ensure cooperation among numerous political parties of India to have a common development agenda and clean politics free from corruption. He invited the IEA to guide the nation with good ideas for optimum use of public expenditure and allied fiscal policies.

The IEA was founded by Dr. Gilbert Slater, First Professor of Indian Economics, University of Madras along with Prof. Percy Anstey and C.J. Hamilton of Bombay and Calcutta Presidency Universities in 1917, along with Madras Economic Association. Former Presidents of IEA include former Prime Minister Dr. Manmohan Singh, Nobel Laureate Dr. Amartya Sen, Nobel Laureate, Dr. G.Patel, Director, London School of Economics and Political Science, Vice Chancellors Dr. V.K.R.V. Rao, Dr. Malcolm S Adiseshiah and Dr Yasodha Shanmugasundaram among others.



## AVAILABLE BOUND VOLUMES



buy online at [www.mtg.in](http://www.mtg.in)

Physics For You 2016 (January - December)	₹ 325 12 issues
Mathematics Today 2016 (January - December)	₹ 325 12 issues
Biology Today 2016 (January - June)	₹ 175 6 issues
Chemistry Today 2016 (January - June)	₹ 175 6 issues
Mathematics Today 2013 (January - December)	₹ 300 12 issues

### of your favourite magazines

**How to order :** Send money by demand draft/money order. Demand Draft should be drawn in favour of **MTG Learning Media (P) Ltd.** Mention the volume you require along with your name and address.

### Add ₹ 60 as postal charges

Older issues can be accessed on [digital.mtg.in](http://digital.mtg.in) in digital form.

### Mail your order to :

Circulation Manager, MTG Learning Media (P) Ltd.  
Plot No. 99, Sector 44 Institutional Area, Gurgaon, (HR)  
Tel.: (0124) 6601200  
E-mail : [info@mtg.in](mailto:info@mtg.in) Web : [www.mtg.in](http://www.mtg.in)

# Now, save up to Rs 2,020\*



## Subscribe to MTG magazines today.

Our 2018 offers are here. Pick the combo best suited for your needs. Fill-in the Subscription Form at the bottom and mail it to us today. If in a rush, log on to [www.mtg.in](http://www.mtg.in) now to subscribe online.

\*On cover price of ₹ 30/- each.

For JEE  
(Main & Advanced),  
NEET, AIIMS AND  
JIPMER

## About MTG's Magazines

Perfect for students who like to prepare at a steady pace, MTG's magazines-Physics For You, Chemistry Today, Mathematics Today & Biology Today-ensure you practice bit by bit, month by month, to build all-round command over key subjects. Did you know these magazines are the only source for solved test papers of all national and state level engineering and medical college entrance exams?

Trust of over 1 Crore readers since 1982.

- Practice steadily, paced month by month, with very-similar & model test papers
- Self-assessment tests for you to evaluate your readiness and confidence for the big exams
- Content put together by a team comprising experts and members from MTG's well-experienced Editorial Board
- Stay up-to-date with important information such as examination dates, trends & changes in syllabi
- All-round skill enhancement – confidence-building exercises, new studying techniques, time management, even advice from past JEE/NEET toppers
- **Bonus:** Exposure to competition at a global level, with questions from Intl. Olympiads & Contests

### SUBSCRIPTION FORM

#### Please accept my subscription to:

Note: Magazines are despatched by Book-Post on 4<sup>th</sup> of every month (each magazine separately).

Tick the appropriate box.

Best Offer

#### PCM combo

<input type="checkbox"/> 1 yr: ₹ 1,000 (save ₹ 440)	<input type="checkbox"/> 2 yr: ₹ 1,800 (save ₹ 1,080)	<input type="checkbox"/> 3 yr: ₹ 2,300 (save ₹ 2,020)
--	--	--

#### PCM combo:

<input type="checkbox"/> 1 yr: ₹ 900 (save ₹ 180)	<input type="checkbox"/> 2 yr: ₹ 1,500 (save ₹ 660)	<input type="checkbox"/> 3 yr: ₹ 1,900 (save ₹ 1,340)
--	--	--

#### PCB combo

<input type="checkbox"/> 1 yr: ₹ 900 (save ₹ 180)	<input type="checkbox"/> 2 yr: ₹ 1,500 (save ₹ 660)	<input type="checkbox"/> 3 yr: ₹ 1,900 (save ₹ 1,340)
--	--	--

#### Individual magazines

<input type="checkbox"/> Physics	<input type="checkbox"/> Chemistry	<input type="checkbox"/> Mathematics	<input type="checkbox"/> Biology
<input type="checkbox"/> 1 yr: ₹ 330 (save ₹ 30)	<input type="checkbox"/> 2 yr: ₹ 600 (save ₹ 120)	<input type="checkbox"/> 3 yr: ₹ 775 (save ₹ 305)	

Want the magazines by courier; add the courier charges given below:

1 yr: ₹ 240       2 yr: ₹ 450       3 yr: ₹ 600

Tick the appropriate box.

Student  Class XI  XII  Teacher  Library  Coaching

Name: \_\_\_\_\_

Complete Postal Address: \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

Pin Code  Mobile #

Other Phone # 0



Email \_\_\_\_\_

Enclose Demand Draft favouring MTG Learning Media (P) Ltd, payable at New Delhi.  
You can also pay via Money Orders. Mail this Subscription Form to Subscription Deptt.  
MTG Learning Media (P) Ltd, Plot 99, Sector 44, Gurgaon – 122 009 (H.R.).

E-mail [subscription@mtg.in](mailto:subscription@mtg.in). Visit [www.mtg.in](http://www.mtg.in) to subscribe online. Call (0)8800255334/5 for more info.

Get digital editions of MTG Magazines on <http://digital.mtg.in/>

# EXCEL ACADAMICS®

Bengaluru

A Premier Institute for KARNATAKA  
Integrated PUC | PUC+CET | PUC+IIT-JEE | PUC+NEET

1  
ALL INDIA RANK



SHIVANANDA S  
H.T. No.: 61718963  
All India Quota Cat.

18

18

NEET

44

44

97

97

PRAJWAL. K  
H.T. No.: 61717312  
All India Quota Cat.

CHETHAN. M  
H.T. No.: 61718306  
All India Quota Cat.

SHAMANTH M  
H.T. No.: 903604299

CET Ranks

18



V. CHETAN

36



NAVYA N  
H.T. No.: SH213

66



SACHIN V A  
H.T. No.: WB101

JEE(Main) Score

222



SACHIN VA

192



MADHU J N

187



SANJAY GOWDA M

227



DHEERAJ R

181



V CHETAN

172



NAVEEN RAJ

Helpline : 7676-91-7777, 7676-41-6666

CET, IIT-JEE (Main & Advanced), NEET, AIIMS, JIPMER, COMED-K, BITS, VIT, MAHE

REGISTRATION OPEN

EXCLUSIVE  
NEET-2018

New Batch  
PCB  
Classes with Daily test

K-CET / NEET / JEE  
CRASH COURSE  
2018

2<sup>nd</sup> PUC/12th/+2 Studying Students

46 Days Residential  
SUMMER  
COURSE

1 PUC/11th, +1, Studying Students

1 Month Residential  
BRIDGE  
COURSE

9<sup>th</sup> to 10<sup>th</sup> Studying Students (Maths & Science)

Fully Solved  
OFF-LINE TEST  
SERIES

CET, IIT-JEE (Main & Advanced),  
NEET - 2018 & 2019  
₹ 3,540/- ONLY

Fully Solved  
ON-LINE TEST  
SERIES

CET, IIT-JEE (Main & Advanced),  
NEET - 2018 & 2019  
₹ 7,080/- ONLY

EXCEL ACADEMICS : #326, Opp. People Tree Hospital, Sheshadripuram College Road,  
Yelahanka New Town - 560064, Bangalore, KARNATAKA

Contact: 9535656277 / 9880286824 / 9900836461 / 9036357499

To get free Mock CET/ NEET/JEE SMS your complete postal address to 7676917777



Separate Deluxe Hostel for Boys and Girls





# GAT-2018

## GITAM ADMISSION TEST

ONLINE TEST FOR ADMISSION INTO  
Engineering, Architecture and Pharmacy  
Programs at Bengaluru, Hyderabad &  
Visakhapatnam campuses

### B.Tech. Programs

Aerospace | Biotech. | Civil | CSE | IT | EEE | ECE | EIE | Industrial | Mechanical

### Dual Degree (B.Tech. + M.Tech.) Programs - 6 years

ECE | Mechanical

### M.Tech. Programs

CST | SE | Cyber Forensics | IT | Data Sciences | RF & ME | VLSI | Embedded Systems  
DSSP | EI | PSA | MD | CAD / CAM | Industrial | SE & NDM | Biotech. | FPT

### B.Arch. & M.Arch. Programs

### B.Pharm. & M.Pharm. Programs

#### Record Placements

- Over 200 corporate majors visited GITAM campuses and offered 2250 jobs with lucrative packages ranging from 3.5 to 12.5 lakhs per annum in 2017-18.
- Our recruiters: TCS, Accenture, Deloitte, Microsoft, Amazon, ServiceNow, Futures First, FactSet, Maruti Suzuki, Fiat to name a few.

#### Entrepreneurship Development

Technology incubation and Entrepreneurship development centres provide opportunities in nurturing the young minds of GITAM for innovation and startups.

#### Extra Edge

- Internships in multi national corporations.
- Science activity centre provides an exclusive platform to nourish creativity and unleash the inherent talent.
- Live Classes for GATE / GRE / CAT / Civil Services & Foreign Languages.

**How to Apply :** (i) Apply online at [www.gitam.edu](http://www.gitam.edu)

- (ii) Applications also available at selected branches of Union Bank of India, Indian Bank and Karur Vysya Bank.
- (iii) Filled-in applications along with a DD for ₹ 1,000/- (₹ 600/- for female candidates), drawn in favour of GITAM (Deemed to be University) payable at Visakhapatnam on any scheduled bank, be sent to the Director-Admissions, GITAM, Gandhi Nagar Campus, Rushikonda, Visakhapatnam-530045, Andhra Pradesh.

**Online Admission Test at 48 Centres:** Visit [www.gitam.edu](http://www.gitam.edu) for the list of test centres

**Scholarships :** ♦ 100% Fee waiver to top 10 rankers of GAT-2018 ♦ 50% Fee waiver to top 11-100 rankers of GAT-2018 ♦ Merit - cum - Means Scholarships ♦ Scholarships to every M.Tech. & M.Arch. students ♦ Teaching Assistantships to M.Tech. students ♦ Scholarships to every Aerospace Engineering students ♦ Scholarships to every B.Pharm. & M.Pharm. students

### Important Dates for GAT - 2018

Last date for receipt of Applications	: <b>26<sup>th</sup> March 2018</b>
Online Slot Booking	: <b>5<sup>th</sup> to 8<sup>th</sup> April 2018</b>
Download of E-Hall Tickets	: <b>5<sup>th</sup> April 2018 onwards</b>
Online (Computer based) Tests	: <b>11<sup>th</sup> to 26<sup>th</sup> April 2018</b>



[www.gitam.edu](http://www.gitam.edu)

IVRS: 0891-2866444

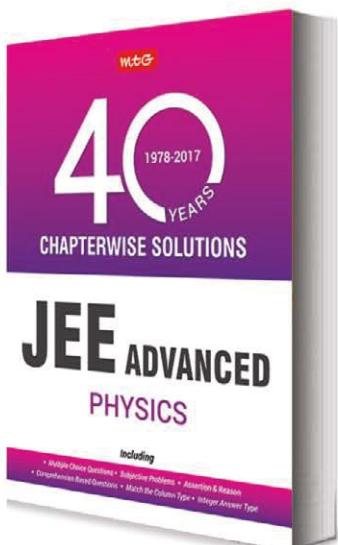
/ GITAMAdmissionTest

**BENGALURU CAMPUS**  
080-28098008, 28098000

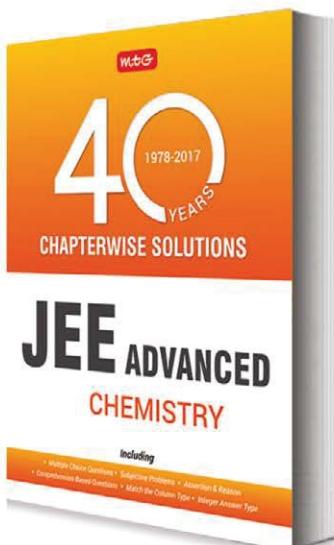
**HYDERABAD CAMPUS**  
08455-221266 / 200 / 204

**VISAKHAPATNAM CAMPUS**  
0891-2866555 / 2730177

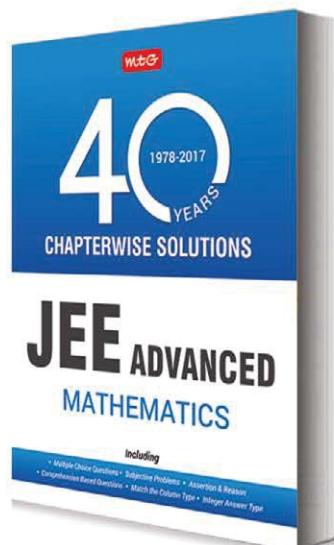
# How can history help to succeed in JEE!



₹ 400



₹ 400



₹ 400

Wouldn't you agree that previous years' test papers provide great insights into the pattern and structure of future tests. Studies corroborate this, and have shown that successful JEE aspirants begin by familiarising themselves with problems that have appeared in past JEEs, as early as 2 years in advance.

Which is why the MTG team created 40 Years Chapterwise Solutions. The most comprehensive 'real' question bank out there, complete with detailed solutions by experts. An invaluable aid in your quest for success in JEE. Visit [www.mtg.in](http://www.mtg.in) to order online. Or simply scan the QR code to check for current offers.

Note: 40 Years Chapterwise Solutions are also available for each subject separately.

Available at all leading book shops throughout India. To buy online visit [www.mtg.in](http://www.mtg.in).

For more information or for help in placing your order, call 0124-6601200 or e-mail [info@mtg.in](mailto:info@mtg.in)

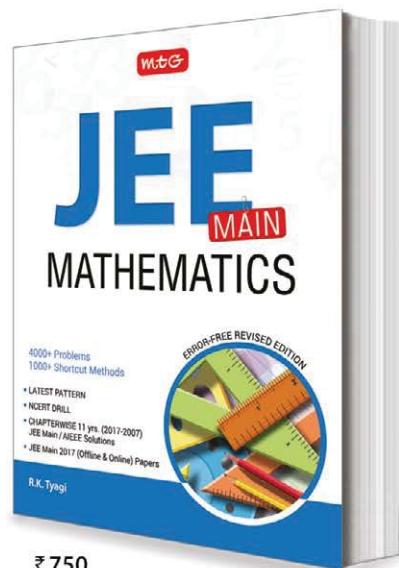
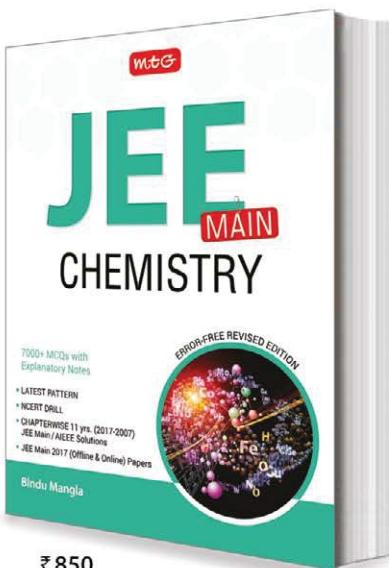
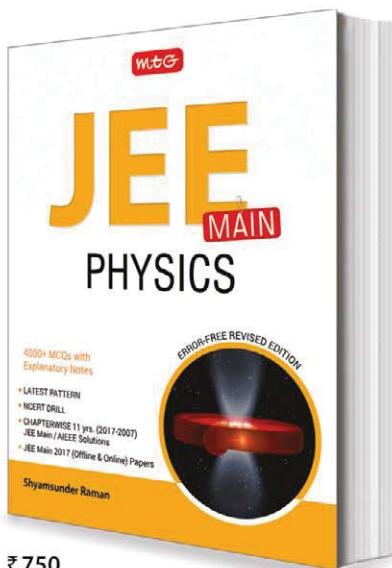


Scan now with your smartphone or tablet

Application to read QR codes required

# Study right. Dig deep.

Build a solid foundation for success  
in JEE Main



**Are you a do-it-yourself type of a student?** Then for success in JEE Main, choose MTG's JEE Main combo, comprising coursebooks for Physics, Chemistry & Mathematics. This combo is all class 11 and 12 students need for a solid and deep understanding of concepts in these three key subjects.

## FEATURES:

- Based on latest pattern of JEE Main
- Full of graphic illustrations & MCQs for deep understanding of concepts
- Covers the entire syllabus
- NCERT Drill MCQs framed from NCERT Books
- 11 Years (2017-2007) Previous Years MCQs of JEE Main / AIEEE
- 2017 JEE Main (Offline & Online) Solved Paper included

Note: Coursebooks are also available separately.

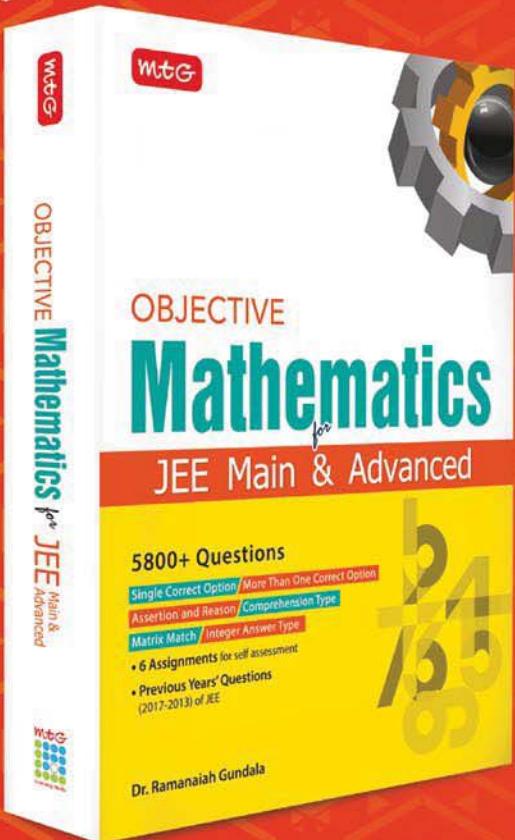
Available at all leading book shops throughout India. To buy online visit [www.mtg.in](http://www.mtg.in).

For more information or for help in placing your order, call 0124-6601200 or e-mail: [info@mtg.in](mailto:info@mtg.in)



Scan now with your  
smartphone or tablet  
Application to read  
QR codes required

# ABRACADABRA



More than magic words, you need help from a magician to pass JEE with flying colours. Which is why MTG has collaborated with Dr Ramanaiah Gundala, the popular Maths professor from Chennai, to bring you the all-new Objective Mathematics for JEE Main & Advanced.

#### Includes

- 6 assignments for self-assessment
- Previous years' JEE questions (2017-13)

After securing his DIIT and Ph.D from IIT Kharagpur, Dr Gundala was elected Fellow of National Academy of Sciences (FNASc). His 50+ years of teaching experience includes distinguished tenures at IIT Kharagpur and Anna University, Chennai. He has authored 7 books and published an astonishing 85 research papers. He's now retired and prepares students for success in IIT-JEE at two leading coaching institutes in Chennai and Warangal.



Dr. Ramanaiah Gundala

#### Highlights of MTG's Objective Mathematics for JEE Main & Advanced

- Well-structured theory covering summary of important concepts and easy-to-understand illustrations
- 5800+ questions
- Unique and brain-stimulating exercises including questions of the following types:
  - Single Correct Option · More Than One Correct Option
  - Assertion and Reason. Comprehension Type
  - Matrix Match · Integer Answer Type

Visit [www.MTG.in](http://www.MTG.in) to buy online or visit a leading bookseller near you.  
For more information, e-mail [info@mtg.in](mailto:info@mtg.in) or call 1800 300 23355 (toll-free) today.



TRUST OF MILLIONS, SINCE 1982



# OUTSTANDING PERFORMANCE BY AAKASHIANS IN KVPY, NSEs & NSO



Achal Kundu  
KVPY  
Abhinav Barnawal  
NSEJS  
Arjun R  
NSO (Level-1)

## Kishore Vaigyanik Protsahan Yojana (KVPY)

It is a scholarship program initiated by Department of Science & Technology, aimed at encouraging students to take up Research Careers in areas of Basic Sciences.

**422**  
379 Classroom | 43 Distance

**Aakashians Qualified in KVPY Aptitude Test 2017**

Our Top Performers from Classroom Programs



Achal Kundu (Stream-SX)  
Aarushi Asrani (Stream-SA)  
Chirag Singh (Stream-SX)  
Dev Chauhan (Stream-SX)  
Dipanjan Das (Stream-SA)  
  
Elamathi V. (Stream-SA)  
Gaurav Vaidya (Stream-SX)  
Govind Sharma (Stream-SA)  
Gurbaaz Singh (Stream-SA)  
Gayathri A. S. (Stream-SA)  
and many more...

## National Standard Examinations (NSEs)

This examination in Junior Science, Physics, Chemistry, Biology & Astronomy is the first step for the worldwide Olympiad. After NSEs, next step for the eligible students is the Indian National Olympiad (INO).

**400**  
Classroom Programs

**Selections in NSEs 2017**

Our Top Performers from Classroom Programs



Uttar Pradesh  
Abhinav Barnawal  
NSEJS  
Delhi  
Samanyu Mahajan  
NSEP  
Uttar Pradesh  
Raman Kumar  
NSEC  
Gujarat  
Sahil Shah  
NSEB  
Uttar Pradesh  
Darpan Kumar Yadav  
NSEA  
and many more...

**Our No. of Selections in NSE (JS/P/C/B/A)**

83	56	111	96	54
NSEJS' 2017	NSEP' 2017	NSEC' 2017	NSEB' 2017	NSEA' 2017

\*NSEJS: National Standard Examination in Junior Science | \*NSEP: National Standard Examination in Physics | \*NSEC: National Standard Examination in Chemistry | \*NSEB: National Standard Examination in Biology | \*NSEA: National Standard Examination in Astronomy

## National Science Olympiad (NSO):

An Annual Scholarship Exam that aims at cultivating scientific reasoning and logical ability among school students.

**667**  
662 Classroom | 5 Distance

**Aakashians Qualified in NSO (Level-1) 2017**

Our Top Zonal Rankers in Various States from Classroom Programs



Tamil Nadu  
Sooraj S.  
Zonal Rank-1  
Class X  
  
Punjab  
Pratham Uppal  
Zonal Rank-1  
Class IX  
  
Punjab  
Sushant Sondhi  
Zonal Rank-1  
Class XI  
  
Kerala  
Arjun R  
Zonal Rank-1  
Class X  
  
Punjab  
Anushka Garg  
Zonal Rank-2  
Class X  
  
Punjab  
Kavya Goyal  
Zonal Rank-3  
Class VIII  
  
Punjab  
Charbhi Gupta  
Zonal Rank-3  
Class XII  
  
Karnataka  
Shrey Jha  
Zonal Rank-3  
Class X  
  
Delhi  
Vaibhav Gupta  
Zonal Rank-3  
Class IX  
  
Tamil Nadu  
Bhagya Sri  
Zonal Rank-3  
Class X  
and many more...

Though every care has been taken to publish the result correctly, yet the institute shall not be responsible for error, if any.

### GET YOUR CHILD ENROLLED TODAY FOR A STRONG FOUNDATION OF HIS / HER CAREER

### ONE YEAR / TWO YEAR INTEGRATED CLASSROOM COURSES FOR 2019, 2020

**MEDICAL**  
NEET, AIIMS  
& Other Medical Entrance Exams

For Class XI students moving to Class XII  
and for Class X students moving to Class XI respectively

**ENGINEERING**  
JEE (Main & Advanced)  
& Other Engg. Entrance Exams

For Class XI students moving to Class XII  
and for Class X students moving to Class XI respectively

**FOUNDATIONS**  
NTSE, Olympiads  
& School / Board Exams

For Class VIII students moving to Class IX  
and for Class IX students moving to Class X

**ACST** **11<sup>th</sup> Feb. 2018**  
ADMISSION CUM SCHOLARSHIP TEST

**DIRECT ADMISSION**

**Crash Courses**

for NEET & AIIMS / JEE (Main & Advanced)  
& Other Medical / Engineering Ent. Exams 2018

**Eligibility:**  
For Class XII studying / passed students

**Test Series Courses**

for NEET & AIIMS / JEE (Main & Advanced) /  
Other Medical / Engineering Ent. Exams 2018

**Eligibility:**  
For Class XII studying / passed students



# Aakash

Medical | IIT-JEE | Foundations

(Divisions of Aakash Educational Services Pvt. Ltd.)

GIVE A MISSED CALL: **9599280605**

TOLL FREE: **1800-212-1238**

SMS Aakash to **53030**

Registered Office : Aakash Tower, 8, Pusa Road, New Delhi-110005. Ph.: (011) 47623456  
E-mail: medical@aesi.in | iitjee@aesi.in | info.afs@aesi.in

[www.aakash.ac.in](http://www.aakash.ac.in)



instagram.com/aakasheducation  
facebook.com/aakasheducation  
youtube.com/AakashEducation  
twitter.com/aakash\_twitted  
aakashinstitute.blogspot.com