ចំលាត់ស្រាវជ្រាវ

១. ដោះស្រាយសមីការ $\frac{dy}{dx}=\frac{y^2}{x^2+1}$ នោះ $\frac{1}{y^2}dy=\frac{1}{x^2+1}dx$

តាង $N(y)=\frac{1}{y^2}$; $M(x)=\frac{1}{x^2+1}$

គេបាន $\int N(y)dy=\int M(x)dx$ នោះ $\int\frac{1}{y^2}dy=\int\frac{1}{x^2+1}dx$ តែ $\int\frac{1}{x^2+1}dx=\arctan(x)+C_2$ និង

$\int\frac{1}{y^2}dy=-\frac{1}{y}+C_1$

នោះ $-\frac{1}{y}+C_1=\arctan(x)+C_2 \Leftrightarrow -\frac{1}{y}=\arctan(x)+(C_2-C_1)$ យក $C_2-C_1=C$

ដូចនេះ $y=-\frac{1}{\arctan(x)+C}$

២. បង្ហាញថា $\tanh^{-1}(x)=\frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$

$$y=\frac{e^x+\frac{1}{e^x}}{e^x-\frac{1}{e^x}}=\frac{e^{2x}+1}{e^{2x}-1}\quad(1)$$

$$y=\frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)\Leftrightarrow 2y=\ln\left(\frac{x+1}{x-1}\right)$$

ប្រើរូបមន្ត $\ln b=y\Rightarrow b=e^y$

$$2y=\ln\left(\frac{x+1}{x-1}\right)\Leftrightarrow\frac{x+1}{x-1}=e^{2y}$$

$$x+1=e^{2y}x-e^{2y}\Leftrightarrow x-e^{2y}x=-e^{2y}-1\ \ x\left(-e^{2y}+1\right)=-e^{2y}-1\Leftrightarrow x=\frac{-\left(e^{2y}+1\right)}{-\left(e^{2y}-1\right)}=\frac{e^{2y}+1}{e^{2y}-1}$$

ប្តូរ x ទៅ y គេបាន $y=\frac{e^{2x}+1}{e^{2x}-1}\quad(2)$

តាម (1) និង (2) គេបាន

$$\frac{e^{2x}+1}{e^{2x}-1}=\frac{e^{2x}+1}{e^{2x}-1}$$

ដូចនេះ $\tanh^{-1}(x)=\frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$ ពិត