AMATH 482 HW1: Finding Submarine

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Abstract

Our task is to determine the location and moving path of a submarine. We use fast Fourier transform to get the frequency signature generated by the submarine, design a Gaussian filter around the frequency center, and clear the noise of acoustic data.

1 Introduction and Overview

There exists an unknown submarine in the Puget Sound emitting an unknown acoustic frequency. Our task is to detect the location and moving path of this submarine.

We are provided three dimensional dynamic data, that is acoustic pressure measurements in 3D and as a function of time. To calculate the average frequency, we transform the signal from spatial domain to frequency domain, by applying fast Fourier transform (FFT) onto the matrix for each time step, and calculate the average signal over time. After that, we can find the indices that emits the maximum frequency.

After determine the center frequency, we design a Gaussian filter and apply that onto the frequency domain signal. Then we use inverse transform to convert the frequency domain data back to spatial domain. We will get a denoised acoustic data that can help us determine the location of the submarine for each time step. Using the location of each time step, we can draw a path of the submarine in the past 24 hours.

2 Theoretical Background

1. Fourier Transform:

   For a function $f(x)$, we define its Fourier transform $\hat{f}(k)$ as:

   $$\hat{f}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-ikx} dx$$

   The Fourier transform will take a function $f(x)$ in time or space domain, and convert it into a function in frequency domain $\hat{f}(k)$. In the definition, we assume a infinite domain.

   The inverse Fourier Transform is:

   $$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}(k) e^{ikx} dk$$

2. Frequency Domain:

   A function within a time domain shows how a signal change over time, where as a function within a frequency domain shows how much of the signal lies within each given frequency band over a range of frequencies.

3. FFT and Inverse FFT:

   The Fast Fourier Transform(FFT) can convert an acoustic data from time domain to frequency domain. The inverse process can be done by inverse Fast Fourier Transform. Python's numpy package has built-in functions for multi-dimensional FFT.

4. White noise:

   In signal processing, white noise is a random signal around the more dominant frequency band. Removing the white noise gives the frequency band a higher visibility.

5. Gaussian Filter:

   There are multiple ways to filter the noisy frequency data. We will use Gaussian filter to remove the undesired frequency – white noise. When we apply the filter, we need to find the frequency signature emitted by the submarine, and filter out frequency signals other than the center frequency.

3 Algorithm Implementation and Development

1. Find the center frequency:

   The provided acoustic data is a $64^3 \times 49$ array where 49 represents number of time steps within 24 hours. It is hard to analyze 4D data, hence we separate the signal by each time step.

   For each of the 49 time steps, we reshape the signal to a $64 \times 64 \times 64$ 3D matrix, and convert that spatial domain data into frequency domain. Then we calculate an average frequency data over time. By taking the absolute value of the data, we can then find the indices with the maximum frequency value. Then we reflect the indices of matrix onto the frequency domain, and get the center frequency.

2. Design a Gaussian Filter:

   Derived from 1D and 2D filters, the 3D Gaussian Filter center at the origin is:

   $$g(x, y, z) = e^{-\frac{x^2+y^2+z^2}{2\sigma^2}}$$

   We want the filter to be centered around the frequency signature (f_x, f_y, f_z), therefore removing lower frequencies around it. The filter becomes:

   $$g(x, y, z) = e^{-\frac{(x-f_x)^2+(y-f_y)^2+(z-f_z)^2}{2\sigma^2}}$$

   The σ value determines the radius of the filter. Since the frequency domain is relatively small, we choose $\sigma = 2$ to filter out noise outside the circle with radius of 4.

3. Find the submarine:

   Similar to what we have done to find the center frequency, for each time step, we use numpy's fftn() function to transform the data into frequency domain. To help visualization, we use python's built-in fftshift() function.

For each time step, we multiply each frequency data by the filter, getting a denoised frequency. Then we use ifftshift() first, and then ifftn() to convert the frequency back to its spatial domain. Then we calculate the absolute value of the data, finding the prominent acoustic signal, which corresponds to the location of the submarine.

In each time step, we record the spatial coordinate into a 3×49 matrix, representing the submarine's location at each time step.

4 Computational Results

1. Center Frequency:

   The frequency signature is located at [5.3407, 2.1991, -6.9115], corresponding to indices of [39, 49, 10] in the 3D matrix.

2. Gaussian Filter:

   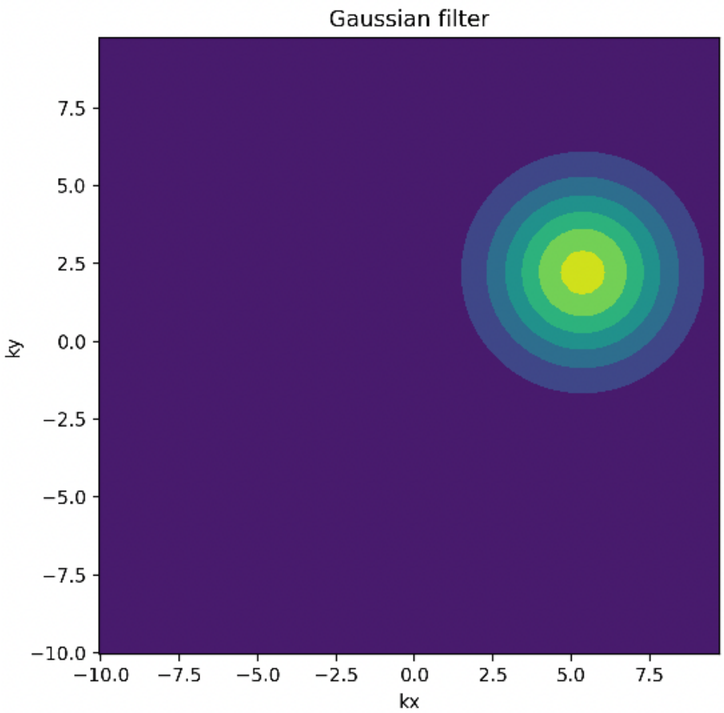

   Figure 1: The x-y plane of the Gaussian filter at z = 32:

3. Moving trajectory:

   The x, y coordinates of the submarine during the 24 hour period is this:

Time	x	y
0	3.125	-8.125
1	3.125	-7.8125
2	3.125	-7.5
3	3.125	-7.1875
4	3.125	-6.875
5	3.125	-6.5625
6	3.125	-6.25
7	3.125	-5.9375
8	3.125	-5.625
9	2.8125	-5.3125
10	2.8125	-5
11	2.5	-4.6875
12	2.1875	-4.375
13	1.875	-4.0625
14	1.875	-3.75
15	1.5625	-3.4375
16	0.9375	-3.125
17	0.625	-2.8125
18	0.3125	-2.5
19	0	-2.1875
20	-0.625	-1.875
21	-0.9375	-1.5625
22	-1.25	-1.25
23	-1.875	-1.25
24	-2.1875	-0.9375
25	-2.8125	-0.625
26	-3.125	-0.3125
27	-3.75	0
28	-4.0625	0.3125
29	-4.375	0.625
30	-4.6875	0.9375
31	-5.3125	1.25
32	-5.625	1.5625
33	-5.9375	1.875
34	-5.9375	2.1875
35	-6.25	2.5
36	-6.5625	2.8125
37	-6.5625	3.125
38	-6.875	3.4375
39	-6.875	3.75
40	-6.875	4.0625
41	-6.875	4.375
42	-6.5625	4.6875
43	-6.5625	5
44	-6.25	5.3125
45	-6.25	5.625
46	-5.9375	5.9375
47	-5.3125	5.9375
48	-5.3125	6.25

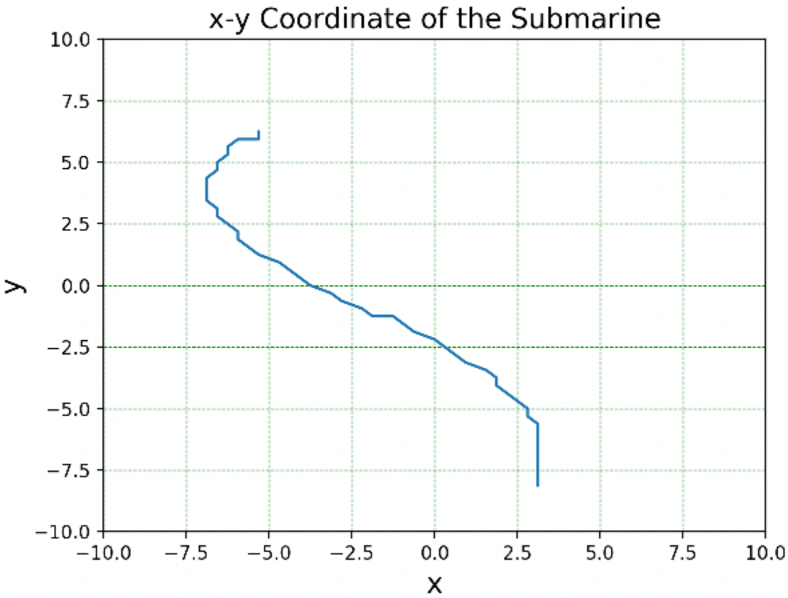

Figure 2: The visual representation of submarine's moving path in x-y dimension:

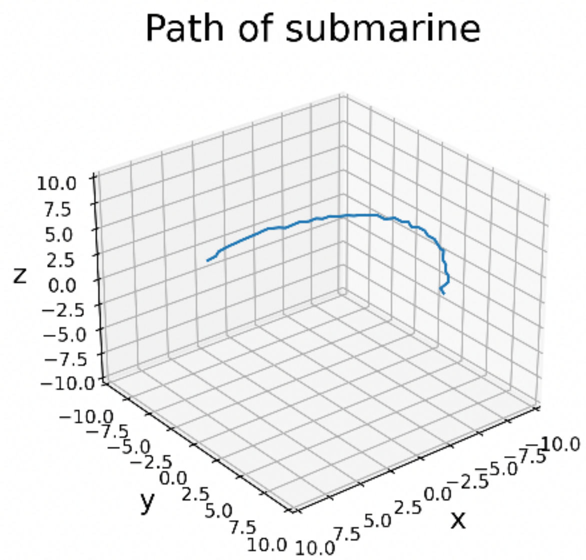

Figure 3: The 3D representation of the submarine's moving path:

5 Summary and Conclusions

After filtering the frequency data, we get a clear acoustic signal highlighting the movement of the submarine. In x-y dimension, the submarine starts at the bottom right of the x, y coordinate, and moves upward and leftward. In z-direction, the submarine first moves upward, then moves downward.

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