CROSS-RELATION-BASED FREQUENCY-DOMAIN BLIND SYSTEM IDENTIFICATION USING ONLINE ADMM

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ABSTRACT

In this contribution, we propose a cross-relation-based adaptive algorithm for blind identification of single-input multiple-output (SIMO) systems in the frequency domain using the alternating direction method of multipliers (ADMM). The proposed algorithm exploits the separability of the cross-channel relations by splitting the multichannel identification problem into lower-dimensional sub-problems with lowered computational complexity and can be solved in parallel. Each sub-problem yields estimates for a subset of channel spectra, which then are combined into a consensus estimate per channel using general form consensus ADMM in an adaptive updating scheme. With numerical simulations, we show that it is possible to achieve estimation errors and convergence speeds comparable to low cost frequency-domain algorithms.

Index Terms— blind system identification, multichannel signal processing, ADMM, Online-ADMM

1. INTRODUCTION

The problem of blind system identification (BSI), also called blind multichannel identification (BMCI), which aims to estimate channel impulse responses of an unknown system from output signals only has been subject of extensive research over the recent decades. Early methods used higher-order statistics (HOS) [] for channel estimation, however the high complpexity led to research into methods only using second-order statistics (SOS). The first to be proposed was the cross-relation (CR) method [], which this contribution focuses on as well. Other relevant approaches to solve the SIMO BSI problem are subspace methods [], least-squares approach [] and maximum-likelihood [] methods.

Out of various multichannel BSI algorithms which have been proposed, adaptive least mean squares (LMS) algorithms in time [] and frequency domain [] are the ones most widely used. The NMCFLMS algorithm is an efficient algorithm utilizing the fast Fourier transform (FFT) which has been extended to include constraints to improve robustness to noise and performance on acoustic impulse responses (RNMCFLMS [], l_p-RNMCFLMS []).

The alternating direction method of multipliers (ADMM) [] solves convex optimization problems by splitting them into smaller sub-problems, each less complex to solve than the original one. It has found applications in a large number of areas such as machine learning [] or control systems []. Here, we use ADMM to separate the inter-channel cross-relations to form smaller sub-problems involving overlapping subsets of the entire channel set and use constraints to find a consensus on the overlapping estimated parameters of the sub-problems. This reduces the theoretical time effort to find a solution by complexity reduction and additionally enabling parallel processing of sub-problems. The ADMM update steps solving for the sub-problem estimates, the overall consensus estimate and dual variables, are applied in a block processing scheme forming an adaptive algorithm also referred to as Online-ADMM [].

2. PROBLEM STATEMENT

2.1. Signal Model

We define the acoustic SIMO system with the input signal $\mathbf{s}(n) = \begin{bmatrix} s(n) & s(k-1) & \dots & s(k-2L+2) \end{bmatrix}^{\mathrm{T}}$ and $i \in \mathcal{M}$ with $\mathcal{M} \triangleq \{1, \dots, M\}$ outputs $\mathbf{x}_i(n) = \begin{bmatrix} x_i(n) & x_i(k-1) & \dots & x$ Each output $\mathbf{x}_i$ is the convolution of $\mathbf{s}$ with the respective channel impulse response $\mathbf{h}_i$ with additive noise term $\mathbf{v}_i$, assumed to be zero-mean and uncorrelated. The signal model is described by

$$\mathbf{x}_i(n) = \mathbf{H}_i \mathbf{s}(n) + \mathbf{v}_i(n) \tag{1}$$

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where $\mathbf{H}_i$ is the $L \times (2L-1)$ linear convolution matrix of the ith channel using the elements of $\mathbf{h}_i$.

2.2. Cross-relation approach

The cross-relation approach for BSI aims to only use output signals of the system to identify it. This is achieved by exploiting the relative channel information when more than one system output are available and certain identifiability conditions [] (i) the channel transfer functions have no common zeros (i.e. are not co-prime), and (ii) covariance matrix of the input signal $\mathbf{s}(n)$ is of full rank (i.e. the signal fully excites the channels) are satisfied.

The fundamental equality of this approach, here assumed in the noiseless case $\mathbf{v}_i(n) = 0$, is

$$\mathbf{x}_i^{\mathrm{T}}(n)\mathbf{h}_j = \mathbf{x}_j^{\mathrm{T}}(n)\mathbf{h}_i, \quad i,j \in \mathcal{M},\, i \neq j \tag{2}$$

which states that the output signal of one channel convolved with the impulse response of another is equal to the vice-versa. This follows from the commutative nature of the convolution operation. Using the (estimated) covariance matrix $\mathbf{R}_{ij} = \mathrm{E}\left\{\mathbf{x}_i\mathbf{x}_j^{\mathrm{T}}\right\}$ instead of the signal vectors $\mathbf{x}_i$ or data matrices yields a more robust problem formulation. For the subsequent explanations the time index is dropped to make notation more compact.

We can combine all cross-relations (2) into a linear system of equations

$$\mathbf{R}\mathbf{h} = \mathbf{0} \tag{3}$$

where

$$\mathbf{R} = \begin{bmatrix} \sum_{m\neq 1}\mathbf{R}_{mm} & -\mathbf{R}_{21} & \cdots & -\mathbf{R}_{M1} \\ -\mathbf{R}_{12} & \sum_{m\neq 2}\mathbf{R}_{mm} & \cdots & -\mathbf{R}_{M2} \\ \vdots & \vdots & \ddots & \vdots \\ -\mathbf{R}_{1M} & -\mathbf{R}_{2M} & \cdots & \sum_{m\neq M}\mathbf{R}_{mm} \end{bmatrix}, \tag{4}$$

and $\mathbf{h} = \begin{bmatrix}\mathbf{h}_1^{\mathrm{T}} & \cdots & \mathbf{h}_M^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$.

This problem can also be written down in the frequency domain. The derivation is analogous to the time-domain one, using the frame-based overlap-save technique []. This gives the linear system of equations

$$\boldsymbol{\mathcal{R}}(m)\underline{\boldsymbol{h}} = \mathbf{0} \tag{5}$$

where the matrix is recursivly computed by

$$\boldsymbol{\mathcal{R}}(m) = \eta\boldsymbol{\mathcal{R}}(m-1) + (1-\eta)\hat{\boldsymbol{\mathcal{R}}}(m) \tag{6}$$

with $\eta \in [0,1]$ being an exponential smoothing factor and $\hat{\boldsymbol{\mathcal{R}}}(m)$ constructed analogous to (4), however the covariance matrices replaced by the cross-spectrum matrices

$$\boldsymbol{\mathcal{R}}_{ij}(m) = \boldsymbol{\mathcal{S}}_i^{\mathrm{H}}(m)\boldsymbol{\mathcal{S}}_j(m) \tag{7}$$

with

$$\boldsymbol{\mathcal{S}}_i(m) = \boldsymbol{\mathcal{W}}_{L\times 2L}^{01}\boldsymbol{\mathcal{D}}_i(m)\boldsymbol{\mathcal{W}}_{2L\times L}^{10} \tag{8}$$

where $\boldsymbol{\mathcal{D}}_i(m) = \mathrm{diag}\left\{\mathrm{FFT}_{2L}\left\{\mathbf{x}_{i,2L}(m)\right\}\right\}$ is a diagonal matrix of the signal spectrum and the overlap-save matrices

$$\mathbf{W}_{L\times 2L}^{01} = \begin{bmatrix}\mathbf{0}_{L\times L} & \mathbf{I}_{L\times L}\end{bmatrix} \tag{9}$$

$$\mathbf{W}_{2L\times L}^{10} = \begin{bmatrix}\mathbf{I}_{L\times L} & \mathbf{0}_{L\times L}\end{bmatrix}^{\mathrm{T}} \tag{10}$$

$$\boldsymbol{\mathcal{W}}_{2L\times L}^{01} = \mathbf{F}_{L\times L}\mathbf{W}_{L\times 2L}^{01}\mathbf{F}_{2L\times 2L}^{-1} \tag{11}$$

$$\boldsymbol{\mathcal{W}}_{2L\times L}^{10} = \mathbf{F}_{2L\times 2L}\mathbf{W}_{2L\times L}^{10}\mathbf{F}_{L\times L}^{-1} \tag{12}$$

where $\mathbf{F}$ as the DFT matrix for sizes L and 2L. The vector $\underline{\boldsymbol{h}}$ is the stacked vector of the complex-valued spectra of the impulse responses. In the presence of noise, the system of equations (5) is best solved by a minimization problem which seeks to minimize the squared error $\|\underline{\boldsymbol{e}}\|^2 = \|\boldsymbol{\mathcal{R}}\underline{\boldsymbol{h}}\|^2$ with the constraint $\underline{\boldsymbol{h}}^{\mathrm{H}}\underline{\boldsymbol{h}} = a$ to avoid the trivial zero solution. As this effectively seeks the a-scaled eigenvector of the squared hermitian matrix $\boldsymbol{\mathcal{R}}^{\mathrm{H}}\boldsymbol{\mathcal{R}}$ corresponding to the smallest eigenvalue, we can avoid the squared matrix and replace it with its non-squared form $\boldsymbol{\mathcal{R}}$ as the eigenvectors in both cases are equal. To compute the estimate of the spectra, we minimize the cost function

$$J(\underline{\boldsymbol{h}}) = \underline{\boldsymbol{h}}^{\mathrm{H}}\boldsymbol{\mathcal{R}}\underline{\boldsymbol{h}} \tag{13}$$

as

$$\hat{\underline{\boldsymbol{h}}} = \arg\min_{\underline{\boldsymbol{h}}} \quad \underline{\boldsymbol{h}}^{\mathrm{H}}\boldsymbol{\mathcal{R}}\underline{\boldsymbol{h}}, \tag{14}$$

$$\text{s.t.} \quad \underline{\boldsymbol{h}}^{\mathrm{H}}\underline{\boldsymbol{h}} = a. \tag{15}$$

Attention: bit of rewriting necessary. not nice. Also, rayleigh coefficient? $\frac{\underline{\boldsymbol{h}}^{\mathrm{H}}\boldsymbol{\mathcal{R}}\underline{\boldsymbol{h}}}{\underline{\boldsymbol{h}}^{\mathrm{H}}\underline{\boldsymbol{h}}}$

3. PROPOSED METHOD

3.1. Problem Splitting

In state-of-the-art algorithms, the minimization problem (14) is solved in its full form resulting in potentially high computational effort when the number of channels and length of impulse responses or number of DFT bins is large. To counteract this increase, the problem is split into $N \in \mathbb{N}^+$ smaller sub-problems each defined by a subset of the full channel set $\mathcal{C}_i \subseteq \mathcal{M}$. We denote the index set of the sub-problems as $\mathcal{N} \triangleq \{1,\ldots,N\}$. Following from that, we define the sets $\bar{\mathcal{C}}_j = \{i | j \in \mathcal{C}_i\}$ for $i,j \in \mathcal{M}$ which is the set of sub-problems a channel j is part of. This channel-to-sub-problem relation can also be expressed as a $M \times P$ matrix $\mathbf{G}$ (cf. Figure 1). Further, $M_i = |\mathcal{C}_i|$ and $N_j = \left|\bar{\mathcal{C}}_j\right|$ with $i,j \in \mathcal{M}$.

We replace the cost function (13) with the separable cost function

$$\tilde{J}(\underline{\boldsymbol{h}}) = \sum_{i\in\mathcal{N}}\tilde{J}_i(\underline{\boldsymbol{w}}_i) = \sum_{i\in\mathcal{N}}\underline{\boldsymbol{w}}_i^{\mathrm{T}}\boldsymbol{\mathcal{P}}_i\underline{\boldsymbol{w}}_i \tag{16}$$

where $\underline{\boldsymbol{w}}_i$ is defined as the stacked vector of estimated spectra analogous to $\underline{\boldsymbol{h}}$ (cf. subsection 2.2) but using the . Analogous, $\boldsymbol{\mathcal{P}}$ is constructed as defined in (4) with the reduced set

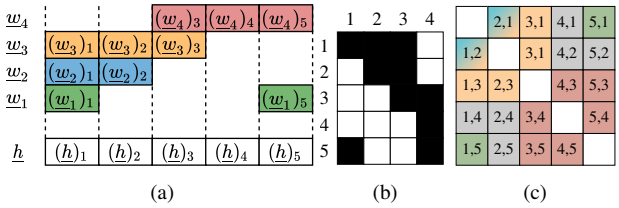


Fig. 1: Problem splitting and parameter mapping for a general case with $M = 5$ channels and $P = 4$ sub-problems. (a) shows the mapping of the local estimate components $(\underline{\boldsymbol{w}}_i)_j$ to the global consensus components $(\underline{\boldsymbol{h}})_j$ via $\mathcal{G}(i,j)$, (b) $\mathbf{G}$ with 1 (■) and 0 (□), (c) shows i,j-cross-relations used by sub-problems compared to full M-channel problem.

of channels. In case the channel subsets are proper $\mathcal{C}_i \subset \mathcal{M}$, this leads to sub-problems each with smaller dimensions than the original centralized problem, reducing complexity. In Figure 1 we try to visualize which information is used to solve the sub-problems in relation to when solving the initial problem. As can be seen, not all cross-relations are used and we conduct a numerical experiment in section 4 to assess this.

3.2. General Form Consensus ADMM

The separability of the problem can be used to take advantage of the well known alternating direction method of multipliers (ADMM). Here more specifically, general form consensus ADMM [], which is used to minimize a problem of form

$$\text{minimize} \quad \sum_{i\in\mathcal{N}} \tilde{J}_i(\underline{\boldsymbol{w}}_i) \tag{17}$$

$$\text{subject to} \quad (\underline{\boldsymbol{w}}_i)_j = \underline{\boldsymbol{h}}_{\mathcal{G}(i,j)}, \quad i \in \mathcal{N},\ j \in \mathcal{C}_i \tag{18}$$

where $\mathcal{G}(i,j) = g$ denotes the mapping of L local variable components $(\underline{\boldsymbol{w}}_i)_j$, i.e. one of the spectra in the stacked vector, to the corresponding global variable components $(\underline{\boldsymbol{h}})_g$ (cf. Figure 1). For brevity, a mapped global variable $\tilde{\underline{\boldsymbol{h}}}_i$ with $(\tilde{\underline{\boldsymbol{h}}}_i)_j = \underline{\boldsymbol{h}}_{\mathcal{G}(i,j)}$ is defined.

The augmented Lagrangian for this particular general-form consensus problem is the real-valued function of a complex variable

$$\mathcal{L}_\rho(\underline{\boldsymbol{w}}, \underline{\boldsymbol{h}}, \underline{\boldsymbol{u}}) = \sum_{i\in\mathcal{M}} \Big(\underline{\boldsymbol{w}}_i^{\mathrm{H}} \boldsymbol{\mathcal{P}}_i \underline{\boldsymbol{w}}_i + \underline{\boldsymbol{u}}_i^{\mathrm{H}} \left(\underline{\boldsymbol{w}}_i - \tilde{\underline{\boldsymbol{h}}}_i\right) + \left(\underline{\boldsymbol{w}}_i - \tilde{\underline{\boldsymbol{h}}}_i\right)^{\mathrm{H}} \underline{\boldsymbol{u}}_i + \left(\underline{\boldsymbol{w}}_i - \tilde{\underline{\boldsymbol{h}}}_i\right)^{\mathrm{H}} \rho\mathbf{I} \left(\underline{\boldsymbol{w}}_i - \tilde{\underline{\boldsymbol{h}}}_i\right) \Big). \tag{19}$$

The ADMM then consists of the steps

$$\underline{\boldsymbol{w}}_i^{k+1} = \underset{\underline{\boldsymbol{w}}_i}{\operatorname{argmin}} \Big\{ \underline{\boldsymbol{w}}_i^{\mathrm{H}} \boldsymbol{\mathcal{P}}_i \underline{\boldsymbol{w}}_i + \underline{\boldsymbol{u}}_i^{k\,\mathrm{H}} \left(\underline{\boldsymbol{w}}_i - \tilde{\underline{\boldsymbol{h}}}_i^{k}\right) + \left(\underline{\boldsymbol{w}}_i - \tilde{\underline{\boldsymbol{h}}}_i^{k}\right)^{\mathrm{H}} \underline{\boldsymbol{u}}_i^{k} + \left(\underline{\boldsymbol{w}}_i - \tilde{\underline{\boldsymbol{h}}}_i^{k}\right)^{\mathrm{H}} \rho\mathbf{I} \left(\underline{\boldsymbol{w}}_i - \tilde{\underline{\boldsymbol{h}}}_i^{k}\right) \Big\} \tag{20}$$

$$\underline{\boldsymbol{h}}^{k+1} = \underset{\underline{\boldsymbol{h}}, \|\underline{\boldsymbol{h}}\|=a}{\operatorname{argmin}} \Big\{ \sum_{i\in\mathcal{M}} \Big(\tilde{\underline{\boldsymbol{h}}}_i^{\mathrm{H}} \underline{\boldsymbol{u}}_i^{k} + \underline{\boldsymbol{u}}_i^{k\,\mathrm{H}} \tilde{\underline{\boldsymbol{h}}}_i + \left(\underline{\boldsymbol{w}}_i^{k+1} - \tilde{\underline{\boldsymbol{h}}}_i\right)^{\mathrm{H}} \rho\mathbf{I} \left(\underline{\boldsymbol{w}}_i^{k+1} - \tilde{\underline{\boldsymbol{h}}}_i\right) \Big) \Big\} \tag{21}$$

$$\underline{\boldsymbol{u}}_i^{k+1} = \underline{\boldsymbol{u}}_i^{k} + \rho \left(\underline{\boldsymbol{w}}_i^{k+1} - \tilde{\underline{\boldsymbol{h}}}_i^{k+1}\right) \tag{22}$$

3.3. Online ADMM

ADMM is originally an iterative method to solve optimization problems, however it has also been applied as an adaptive algorithm, also referred to as *Online ADMM* []. The data term as part of (19) is time-dependent, which from here on will be denoted with the time index m in superscript $\underline{\boldsymbol{w}}_i^{\mathrm{H}} \boldsymbol{\mathcal{P}}_i^m \underline{\boldsymbol{w}}_i$.

The iterative form of the update steps as described in (20)-(22) can be transformed into an adaptive one by computing a (small) finite number of iterations at each time step m with the current data term. In this algorithm, one ADMM iteration is applied per time step, which simply allows us to replace the iteration index k with the time index m.

The minimization problem (20) can be solved by various methods, in this case however we perform the update step

$$\underline{\boldsymbol{w}}_i^{m+1} = \underline{\boldsymbol{w}}_i^{m} - \mu \boldsymbol{\mathcal{V}}_i^m \left(\boldsymbol{\mathcal{P}}_i^m \underline{\boldsymbol{w}}_i^m + \underline{\boldsymbol{u}}_i^m + \rho \left(\underline{\boldsymbol{w}}_i^m - \tilde{\underline{\boldsymbol{h}}}_i^{m} \right) \right) \tag{23}$$

where $0 < \mu \leq 1$ is a step size and $\boldsymbol{\mathcal{V}}_i^m = (\boldsymbol{\mathcal{P}}_i^m + \rho\mathbf{I})^{-1}$ is the inverse Hessian of the problem. As this inverse is costly to compute, it is approximated by a diagonalized matrix

$$\tilde{\boldsymbol{\mathcal{V}}}_i^m = \operatorname{diag}\left\{ \left(\operatorname{diag}\left\{\boldsymbol{\mathcal{P}}_i^m\right\} + \rho\mathbf{1}\right)^{-1} \right\}, \tag{24}$$

similar to NMCFLMS [], which is straightforward to compute.

For the consensus update step, it may be readily verified (cf. Boyd []) that the solution to (21) is given by

$$\underline{\boldsymbol{h}}^{m+1} = a \frac{\bar{\underline{\boldsymbol{w}}}^{m+1} + \frac{1}{\rho}\bar{\underline{\boldsymbol{u}}}^{m}}{\left\| \bar{\underline{\boldsymbol{w}}}^{m+1} + \frac{1}{\rho}\bar{\underline{\boldsymbol{u}}}^{m} \right\|}. \tag{25}$$

where the $ML \times 1$ vectors $\bar{\underline{\boldsymbol{w}}}^{m+1}, \bar{\underline{\boldsymbol{u}}}^{m+1}$ are computed as the mapped averages

$$(\bar{\underline{\boldsymbol{w}}}^{m+1})_g = \frac{1}{N_g} \sum_{\mathcal{G}(i,j)=g} (\underline{\boldsymbol{w}}_i^{m+1})_j, \quad g,i,j \in \mathcal{M}, \tag{26}$$

and

$$(\bar{\underline{\boldsymbol{u}}}^{m})_g = \frac{1}{N_g} \sum_{\mathcal{G}(i,j)=g} (\underline{\boldsymbol{u}}_i^{m})_j, \quad g,i,j \in \mathcal{M}. \tag{27}$$

This results in a computationally inexpensive update step forcing the norm of the consensus to have value a.

4. NUMERICAL EVALUATION

To asses the performance of the proposed algorithm and compare ot to state-of-the-art methods, numerical simulations are done. The error measure to quantify system identification is the normalized projection misalignment (NPM) as used in other BSI-related literature []

$$\mathrm{NPM}(m) = 20\ \log_{10}\left(\frac{\left\|\mathbf{h}(m) - \frac{\mathbf{h}^{\mathrm{T}}(m)\mathbf{h}_{\mathrm{t}}}{\mathbf{h}_{\mathrm{t}}^{\mathrm{T}}\mathbf{h}_{\mathrm{t}}}\mathbf{h}(m)\right\|_2}{\|\mathbf{h}(m)\|_2}\right) \quad (28)$$

where $\mathbf{h}$ is the stacked vector of impulse response estimates (cf. subsection 2.2), which are the inversely Fourier-transformed estimated spectra stacked in $\underline{\boldsymbol{h}}$ and $\mathbf{h}_{\mathrm{t}}$ is the ground truth.

Similar to [], the first experiment aims to evaluate the performance of the algorithm on randomly generated impulse responses under different signal-to-noise ratios (SNR) using white Gaussian noise (WGN) as input signal and uncorrelated WGN independent between channels as additive noise. The SNR is defined as

$$\mathrm{SNR} = 10\log_{10}\left(\frac{\sigma_s^2\|\mathbf{h}_t\|}{M\sigma_v^2}\right) \quad (29)$$

where σ_s^2 and σ_v^2 are the variance of signal and noise respectively.

The impulse responses of length $L = 64$ are drawn from a Normal distribution with unit variance and the signal is 100 seconds of WGN to ensure convergence of all algorithms. Figure 2 shows the median NPM of 30 Monte-Carlo runs where the averaged of the last 100 frames is taken as measurement.

Attention: parameters

It is observable that the proposed algorithm yields a lower steady-state NPM than the compared NMCFLMS, RNMCFLMS, and l_p-RNMCFLMS algorithms.

The second experiment is a small scale assessment of the influence of the overlap of sub-problems. Two base scenarios, $M = 4, P = 4$ and $M = 8, P = 8$, are evaluated using short ($L = 16$) random impulse responses and different values for the sub-problem overlap parameter ζ which is defined as the ratio of ones to zeros of only considering elements of $\mathbf{G}$ that are not part of the main diagonal, superdiagonal and the first element of the last row (marked gray in 3a). Random patterns satisfying this definition for $\zeta \in \{0.0, 0.25, 0.5, 0.75\}$ and random impulse responses with $L = 16$ are generated and the algorithm is applied. Figure 3 shows the median of 30 Monte-Carlo runs for the two setups, which lets us observe the proportional relation of convergence speed, channel number M and sub-problem overlap ζ. Note that the steady state error is not dependent on ζ.

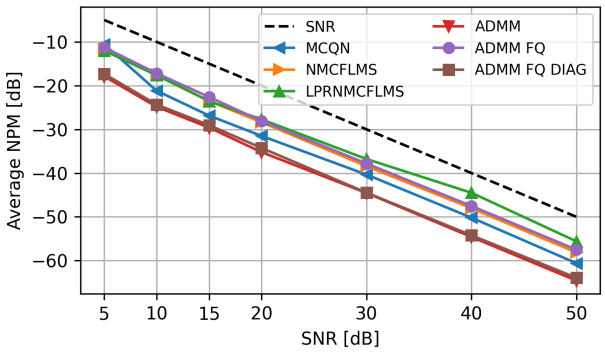
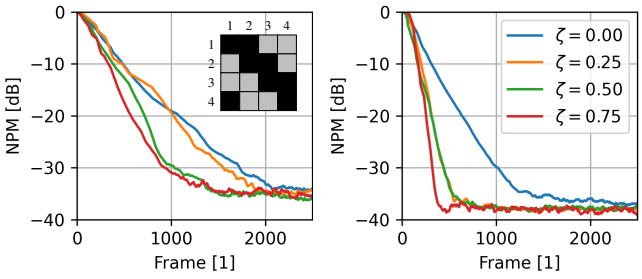


Fig. 2: Steady-state NPM of ADMM-BSI and compared algorithms over different SNRs.



(a) $M = 4, P = 4$ (b) $M = 8, P = 8$

Fig. 3: Comparison of ADMM-BSI convergence behaviour with $L = 16$ for different values of the sub-problem overlap ζ.

Attention: parameters

Attention: Or if figured out, do longer simulated acoustic impulse responses.

5. CONCLUSIONS

In this paper, an adaptive ADMM algorithm for blind system identification was introduced. The algorithm splits the BSI problem into lower-dimensional sub-problems to reduce complexity and allow parallel processing. The the identification performance of the algorithm is compared to state-of-the-art algorithms and shows improved steady-state error measures.