Linear Algebra

GATE CE

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Linear Algebra

Properties of determinant of a matrix

- $|A^T| = |A|$
- $|AB| = |A||B|$
- $|A + B| \neq |A| + |B|$
- In a square matrix if each element of any row or column is 0, then its determinant will be 0.
- Determinant of a diagonal matrix or triangular matrix is product of its leading diagonal elements.
- If any two rows or columns of a square matrix are identical then its determinant will be 0.
- $|KA| = K^n |A|$

Where A is square matrix of order n and K is a scalar

Symmetric matrix

$A^T = A$

Skew symmetric matrix

- $A^T = -A$
- Determinant of a skew symmetric matrix of odd order is always zero.
- Determinant of skew symmetric matrix of even order is always a perfect square.

Testbook Trick

- Null matrix is the only matrix which is symmetric matrix as well as skew symmetric matrix.
- Every matrix can be expressed as:

$$A = \underbrace{\left(\frac{A+A^T}{2}\right)}_{\substack{\Downarrow \\ Symmetric \\ matrix}} + \underbrace{\left(\frac{A-A^T}{2}\right)}_{\substack{\Downarrow \\ skew\ symmetric \\ matrix}}$$

Orthogonal matrix

- $AA^T = A^TA = I$
- Determinant of an orthogonal matrix is always 1 or -1.

Inverse matrix

- $A^{-1} = \frac{Adj\ A}{|A|}$

Where Adj A is transpose of cofactor matrix

- $|Adj\ A| = |A|^{n-1}$
- $|Adj\ (Adj\ A)| = |A|^{(n-1)^2}$
- $|A^{-1}| = \frac{1}{|A|}$

Properties of rank (ρ) of a matrix

- If A is a matrix of m × n than $\rho(A) \leq \{m, n\}$.
- If A and B are matrices of same order then
- $\rho(A + B) \leq \rho(A) + \rho(B)$
- $P(AB) \leq \min\{\rho(A), \rho(B)\}$
- $\rho(A) = \rho(A^T)$
- Rank of null matrix is zero
- Rank of matrix having all rows/columns, identical/proportional is always 1.
- If rank of a matrix A is n then ρ(Adj A) = n
- If rank of a matrix A is n – 1 then ρ(adj A) = 1
- If rank of a matrix A is n – 2 or less than ρ(adj A) = 0

Echelon form

$$\begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$$

- The number of zero before non-zero elements should increase as we move downwards.
- The number of non-zero rows in echelon form is equal to rank of the matrix

System of linear equations

Homogenous [AX = 0]	Non homogenous [AX = B]
• Trivial solution $\rho(A) = n$ $\|A\| \neq 0$	• Inconsistent $\rho(A) \neq \rho(A\|B)$
• Non-trivial solution $\rho(A) < n$ $\|A\| = 0$	• Consistent i) unique solution • $\rho(A) = \rho(A\|B) = n$ • $\|A\| \neq 0$ ii) Infinite solution • $\rho(A) = \rho(A\|B) < n$

Testbook Trick

- Homogenous equations are always consistent.
- Free variable or no. of independent variable = Total no. of variable – Rank

Eigen value / Latent roots / Characteristics root

$|A - \lambda I| = 0$ is called characteristic equation. Roots of $|A - \lambda I| = 0$ are called eigen values.

Properties of Eigen values:

i) Sum of eigen values is equal to trace

ii) Product of eigen values is equal to determinant of matrix

iii) Eigen values of A^T and A are same but eigen vectors are different

iv) If λ is eigen value of A then λ^m is the eigen value of A^m

v) If λ is the eigen value of A then Adj(A) has as $\frac{|A|}{\lambda}$ eigen value

vi) If A has eigen value λ then $A \pm KI$ has $\lambda \pm K$ as an eigen value

vii) Eigen values of symmetric or Hermitian matrix are always real

viii) Eigen values of skew symmetric and skew Hermitian matrix are always zero or purely imaginary

ix) Eigen value of orthogonal matrix is ± 1

x) The set of eigen values is called spectrum .

$\{\lambda_1, \lambda_2, \lambda_3 \ldots \lambda_n\} \rightarrow$ Spectrum

xi) maximum absolute value in spectrum is called spectral radius

xii) Number of times a particular eigen value repeats is called Algebraic multiplicity of the eigen value.

xiii) $AX = \lambda X$ always follows, in which X is called eigen vector

xiv) Eigen vectors for distinct λ, of a real symmetric matrix are always orthogonal i.e. $X_1X_2^T = 0$

xv) Eigen vectors A, Adj A, A^m are always same

xvi) Number of linear independent eigen vector = N – Rank of $[A - \lambda I]$; where N is order of A

xvii) Geometrical multiplicity ≤ Algebraic multiplicity

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- Characteristic equation: $\lambda^3 - \lambda^2$ [Trace] + λ [sum of minors of diagonal elements] - $|A| = 0$