Percentages

QUANTITATIVE APTITUDE

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Percentages

The Percentage is a fraction whose denominator is always 100. The sign of percentage is **%**.

Example: 9% can be converted to a fraction as 9/100 = 0.09

If, we want to calculate y% of x, then

Percentage Formula: $\boxed{y\% \text{ of } x = x \times \frac{y}{100}}$

Q. If 40% of P = 80, then find the value of P.

A. P × 40/100 = 80

⇒ P = 80 × 100/40

⇒ P = 200.

Fractions and Percentages

To express x% as a fraction

We have, $\boxed{x\% = \frac{x}{100}}$

Thus, $20\% = \frac{20}{100} = \frac{1}{5}; 48\% = \frac{48}{100} = \frac{12}{25}$, etc.

To express $\frac{a}{b}$ as a percent

We have, $\boxed{\frac{a}{b} = \left(\frac{a}{b} \times 100\right)\%}$

Thus, $\frac{1}{4} = \left(\frac{1}{4} \times 100\right)\% = 25\%; 0.6 = \frac{6}{10} = \frac{3}{5} = \left(\frac{3}{5} \times 100\right)\% = 60\%$

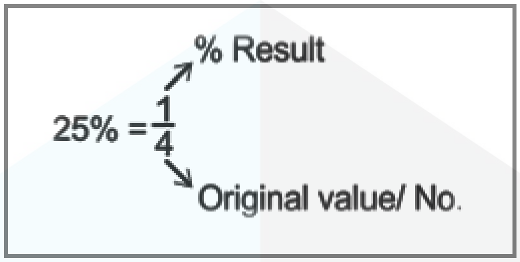

Fig : Representation and interpretation of % in the form of fraction

Fractions	Percentage
$\frac{1}{2}$	50%
$\frac{1}{3}$	$33\frac{1}{3}\%$
$\frac{1}{4}$	25%
$\frac{1}{5}$	20%
$\frac{1}{6}$	$16\frac{2}{3}\%$
$\frac{1}{7}$	$14\frac{2}{7}\%$
$\frac{1}{8}$	$12\frac{1}{2}\%$
$\frac{1}{9}$	$11\frac{1}{9}\%$
$\frac{1}{10}$	10%

Fractions	Percentage
$\frac{1}{11}$	$9\frac{1}{11}\%$
$\frac{1}{12}$	$8\frac{1}{3}\%$
$\frac{1}{20}$	5%
$\frac{1}{25}$	4%
$\frac{2}{3}$	$66\frac{2}{3}\%$
$\frac{3}{4}$	75%
$\frac{3}{8}$	$37\frac{1}{2}\%$
1	100%
2	200%

Tables : Important Fraction to percentage conversions

Testbook Trick

Using the most frequent fraction values in the table above, we can quickly evaluate maximum of the fractions quickly:

$$\rightarrow \frac{31}{6} = \underbrace{5}_{500\%} + \underbrace{\frac{1}{6}}_{16\frac{2}{3}\%} \Rightarrow 516\frac{2}{3}\%$$

Express one quantity as percentage with respect to other quantity

If we want to express quantity 1 as a percentage of quantity 2, then formula will be

$$\left[\frac{\text{Quantity1 to be expressed in percentage}}{\text{Quantity2 in respect of which percentage is to be calculated}}\times 100\right]$$

Q. 20 gram is what percentage of 1 kg?

A. Here, quantity 1 = 20 grams and quantity 2 = 1kg = 1000 grams

Hence, required percentage = 20/1000 × 100 = 2%

Price Change: Consumption and Expenditure

Testbook Trick

If the price of a commodity increases by R%, then the reduction in consumption so that the expenditure remains constant can be find by formula:

$$\left[\frac{R}{(100\pm R)}\times 100\right]\%$$

Note: +ve sign for increase in price and –ve sign for decrease in price.

Q. If the price of sugar is increased by 10%, then by how much percent consumption should be reduced so that the expenditure remains the same?

A. Solution:

Let the price be Rs. x /kg

Consumption be y kg

Hence, expenditure = price × consumption

⇒ Expenditure = xy

Price of sugar is increased by 10%

Hence, new price of sugar = 1.1x per kg

Let new consumption be z kg

Hence, new expenditure = (1.1x) × z

Now, new expenditure = old expenditure

⇒ (1.1x) × z = x × y

⇒ z = y/1.1

Reduction in consumption = (y - z) = y – (y/1.1) = y/11

∴ Percentage reduction in consumption = [(y/11)/y] × 100 = 100/11 = 9.09%

Alternate solution:

By using formula:

$$\left[\frac{R}{(100+R)}\times 100\right]\%$$

Given, $R = 10\%$

$$\left[\frac{10}{(100+10)}\times 100\right]\% = 9.09\%$$

Q. The price of sugar has decreased by 15%. By what percentage can a person increase the consumption so the there is no change in expenditure?

A. $$\left[\frac{R}{(100-R)}\times 100\right]\%$$

Given, $R = 15\%$

$$\left[\frac{15}{(100-15)}\times 100\right]\% = 17.65\%$$

Results on Population

Let the population of a town be P now, then

Population after n years

Population after n years if rate of increment R is same $= P\left(1+\frac{R}{100}\right)^n$

Population after n years if rate of increment R is different $= P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\ldots$

Q. The population of a town 2 years ago was 245000. It increased by 15% in the first year and then increased by 20% in the second year. What is the current population of the town?

A. The population of a town 2 years ago was 245000

It increased by 15% in the first year

∴ The population after first year will be $= \frac{115}{100} \times 245000 = 281750$

The population then increased by 20% in the second year.

∴ The population after second year will be $= \frac{120}{100} \times 281750 = 338100$

Population after n years

Population n years ago $= \frac{P}{\left(1+\frac{R}{100}\right)^n}$

Q. The current population of a town is 25710. If the current population of the town increases by 8% every year, what is the approximate population of the town 7 years ago?

A. Current population = 25710

∴ Population of town 7 years ago = $\frac{25710}{\left(1+\frac{8}{100}\right)^7}$ = 25710/1.7138 = 15001.75 ≅ 15000

Results on Depreciation

Let the present value of a machine be P. Suppose it depreciates at the rate of R% per annum. Then:

Value of the machine after n years

Value of the machine after n years $= P\left(1-\frac{R}{100}\right)^n$

Q. An electric bully was bought at Rs. 4100. Its value depreciates at the rate of 7% per annum. Its value after one year will be:

A. Solution:

Actual price of the electric bully = Rs. 4100

⇒ Depreciation rate = 7%

∴ Value after 1 year = 4100 – 7% of 4100 = 4100 – 4100 × (7/100) = Rs. 3813

Alternate method:

$$P\left(1-\frac{R}{100}\right)^n \Rightarrow 4100\left(1-\frac{7}{100}\right)^n \Rightarrow 4100\left(\frac{93}{100}\right) = \text{Rs. } 3813$$

Value of the machine after n years

Value of the machine n years ago $= \frac{P}{\left(1-\frac{R}{100}\right)^n}$

Q. The value of a machine depreciates 10% annually which was purchased 3 years ago. If its present value is Rs. 6,000, approximately what was its purchase price?

A. We know that, if x_0 is the value at a certain time and r% per annum is the rate of depreciation per year, then the value x_1 at the end of t years is

$$x_1 = x_0\left[1-\frac{r}{100}\right]^t$$

Given, the value of machine depreciates 10% annually. It was purchased 3 years ago and its present value is Rs. 6,000

⇒ x_1 = Rs. 6000, r = 10% and t = 3 years, then value of the machine after 3 years of depreciation =

$$6000 = x_0\left[1-\frac{10}{100}\right]^3$$

$$\Rightarrow x_0 = \frac{6000}{\left[\frac{9}{10}\right]^3} = \text{Rs.}8,230.45$$

Comparison of Percentages

A is R% more than B

Testbook Trick

If A is R% more than B, then B is less than A by $\left[\frac{R}{(100+R)}\times 100\right]\%$

Q. If A's income is 40% more than the income of B, then what percentage of B's income is less than income of A?

A. **Solution:**

Let the income of B be 100

∴ Income of A = 140

B's income is less than income of A by (140 - 100) = 40

Required percentage $= \frac{40}{140}\times 100 = \frac{200}{7} = 28\frac{4}{7}\%$

Alternate method:

By using the formula $\left[\frac{R}{(100+R)}\times 100\right]\%$

$$\Rightarrow \left[\frac{40}{(100+40)}\times 100\right]\% = 28\frac{4}{7}\%$$

A is R% less than B

Testbook Trick

If A is R% less than B, then B is more than A by $\left[\frac{R}{(100-R)}\times 100\right]\%$

Q. If A is 40% less than B, then B is how much percentage more than A?

A. Solution:

Given, A is 40% less than B

Let B be 100

A = B – 40% of B = 100 – 40% of 100 = 100 – 40 = 60

∴ Required % = {(100 – 60)/60} × 100 = (40/60) × 100 = 66.66%

Alternate method:

By using the formula $\left[\frac{R}{(100-R)}\times 100\right]\%$

⇒ [40/(100 – 40)] × 100 = 66.66%

Successive Increase / Decrease

Successive Increase

If some value is increasing at X% and then Y%, then combined increment can be given by

$$\boxed{\left[X+Y+\frac{X\times Y}{100}\right]\%}$$

Q. The population of a town is 18000. It increases by 10% during first year and by 20% during the second year. The population after 2 years will be

A. Here we can use the concept of successive percentage change.

If in the 1st year increase in population is A% and in the 2nd year it is B%, then net increase in population can be given by

$$\left[A + B + \frac{A \times B}{100}\right]\%$$

The question now can be summarized as:

Initial population $(18000) \xrightarrow{+10\%}$ population after 1st year $\xrightarrow{+20\%}$ population after 2nd year

[+ 10% means increased by 10%]

Using above formula:

A = +10 and B = +20

∴ Net increase in population $= \left[10 + 20 + \frac{10 \times 20}{100}\right]\% = 32\%$

∴ Initial population $(18000) \xrightarrow{+32\%}$ Population after 2nd year

∴ Population after 2nd year

$= 18000 + \left[18000 \times \frac{32}{100}\right]$

= 18000 + 5760

= 23760

The population after 2 years will be 23760

Successive Decrease

If some value is decreasing at X% and then Y%, then combined decrement can be given by

$$\boxed{\left[X + Y - \frac{X \times Y}{100}\right]\%}$$

Q. A merchant gives 2 successive discounts of 10% and 14%. What is the overall discount given?

A. Let us assume that the price of an article is `x'

The price of the article after a discount of 10% = x – (10x/100) = 0.9x

Now again a discount of 14% is given.

∴ The price of the article after a discount of 14%

= {0.9x – [(0.9x) × (14/100)]} = 0.774x

We know that percentage increase or decrease $= \frac{\text{Increase or decrease}}{\text{Base value}} \times 100$

∴The net discount offered $= \frac{x - 0.774x}{x} \times 100$

= 22.6%

Alternate solution:

If two successive discounts of a% and b% are offered then the net discount offered =

$$\left[a + b - \frac{(a \times b)}{100}\right]$$

∴ Overall net discount offered for two successive discounts of 10% and 14%

= [10 + 14 – (10 × 14) /100]

= 24 – 1.4

= 22.6%