Wave Optics

PHYSICS

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Wave Optics

In previous topic we had discussed about properties of light like reflection, refraction and scattering in this chapter we are going to discuss more above wave nature of light like interference, diffraction and polarization.

As we had studies light posses both wave and particle nature which is extremely important to understand nature of light and develop a theory which could explain all of these properties and there are many attempts were made which are explained below.

Corpuscular theory (Particle nature of light)

- The properties like reflection and refraction was understood by considering rectilinear propagation of light ray. Based on this fact R. Descartes proposed the particle nature of light which was later developed by Sir. Isaac Newton, according to him light is made up of particles which was termed as corpuscles which are massless, hard and elastic.
- According to Newton's theory a light source emits these corpuscles which travels in a straight line in the absence of external force and when these corpuscles hit the reflecting surface, they undergo elastic collision and thus they follow laws of reflection.
- Whereas during refraction, as light propagates from rarer medium to denser medium according to Newton's assumption the speed of light will increase i.e., The speed of light will be greater in denser medium compared to rarer medium as a result light refracts because of this change in speed of corpuscles and this corpuscles are present in different shapes which is used to explain concept of different colours of light present in our nature
- Now in short, the postulates given by Newton can be summarized as
- Newton postulates states that source of light emits massless, elastic and rigid particles which are known as corpuscles

Merits of Newton's corpuscles theory

1. These postulates could successfully explain the rectilinear propagation of light
2. It could explain the properties of light like reflection and refraction.

Drawbacks of Newton's corpuscles theory

1. Newton's corpuscular theory failed to explain phenomenon like simultaneous reflection and refraction of light, interference, diffraction, polarization etc.
2. According to corpuscle theory speed of light increase as light travel from rarer to denser medium but according to experimental result speed of light decrease as it propagates from rarer to denser medium i.e., velocity of light in rarer medium is greater than in denser medium (vr > vd)
3. As particles are emitted from a source because of such continuous emission of corpuscles mass of source must decrease but experimentally we concluded that it is not true.

Huygens wave theory

- Since there are many drawbacks in corpuscular theory, a Dutch physicist C. Huygens in year 1668 purposed that light is a wave. According to Huygens principle light is a wave and it is caused because of vibration of particles in a hypothetical medium and he termed the medium 'ether' which are present everywhere including vacuum.
- Since this theory consider light as a wave because of this Huygens theory not only explained phenomenon like interference and diffraction but it could also explain properties like reflection and refraction. Which was an outstanding result and hence light was considered to be wave and hence this gave rise to new branch of physics known as wave optics.
- But later it was found out that light posses both particle as well as wave nature since photo electric effect could not be explained using wave nature of light.

1) The source emits light in a form of waves
2) Light waves are just like sound waves which are longitudinal waves
3) Speed of light wave remains constant in homogenous medium
4) We preserve different colour of light because of difference in its wavelength
5) Light wave can travel through vacuum due to presence of a homogenous medium which was termed as luminiferous ether

Merits of Huygens wave theory

1. Because of Huygens wave theory we where able to explain phenomenon like interference, diffraction also reflection and refraction which was remarkable in further discovery and study in this field.
2. It also explained phenomenon like partial refraction and reflection which not possible if we assumed light as particles

Drawbacks of Huygens wave theory

1. At to Huygens assumption there exist an hypothetical medium which he termed as ether but from later experiments it was concluded that it does not exist
2. Rectilinear propagation of light was not explained.
3. Huygens theory failed to explain phenomenon like Compton effect and polarization of light (since he assumed light as longitudinal which cannot be polarized and later experiments proved that light is a transverse electromagnetic wave).

Maxwell's electromagnetic theory

Maxwell equations are well known and proved very useful in understanding electrical and magnetic property of matter and using these equations he concluded that light is an electromagnetic wave which have electric and magnetic field vibrating in a direction perpendicular to each other and to that of its propagation as shown below

And speed of light is inversely proportional to square root of permittivity and permeability of the medium

i.e., $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$ $and\ v = \frac{1}{\sqrt{\epsilon \mu}}$

Here,

c = speed of light in vacuum or air

v = speed of light in some medium

ϵ_0 = Permittivity of electric field in free space

μ_0 = Permeability of magnetic field in free space

ϵ = Permittivity of electric field in some medium

μ_0 = Permeability of magnetic field in some medium

Max Planks Quantum theory of light

After so much efforts it was Max plank who proposed that light is an electromagnetic wave and they propagates in a form of small packets of energy which was termed as Quanta

And such quanta of light is called photons and its energy is calculated as

$$E = hf$$

E = energy of Quanta/ Photon

H = planks constant

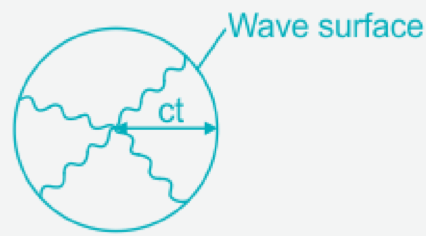
f = frequency of light (we also write frequency as mu (v) or n)

Some important terms used in wave optics

In our previous topic oscillation and sound we had discussed about waves and its nature in detail now this are some terms which are going to used in wave optics in general.

Wave surface

- If a light is emitted from a point source in air or vacuum, waves will travel in all possible direction at speed ‘c’ and all waves will travel certain distance in time ‘t’ is given as ‘ct’. At this instance we can it as spherical surface at which all the light wave will reach simultaneously this surface is termed as wave surface



- The give figure represents wave surface for a spherical wave front.

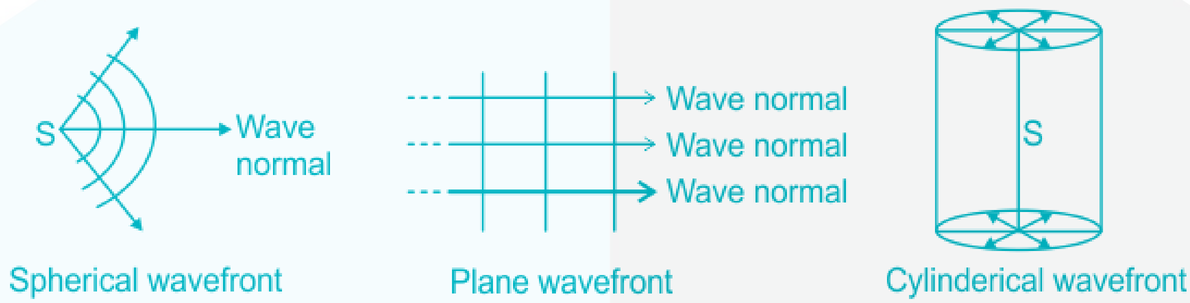
Wave fronts

- The locus of all the point of the medium at which the waves reaches simultaneously, such that all points are in the same phase and it is termed as wave front
- There can be three types of wavefront depending upon its source

1) **Spherical wavefront:** If a waveform is in the form of spherical surface then it is called as spherical wavefront. It can be obtained from a point or a spherical source at some finite distance
2) **Plane wavefront:** A wavefront in the form of plane surface is called plane wavefront. It is obtained by keeping point source at infinite distance or by keeping it at the focus of a convex lens
3) **Cylindrical wavefront:** If a wavefront is in the form of cylindrical surface then it is termed as cylindrical wavefront. It can be obtained by cylindrical source

Wave Normal

- A normal drawn at the surface of wavefront at any point in the direction of propagation are called wave normal.
- The following diagram shows wave normal for spherical, plane and cylindrical surface.

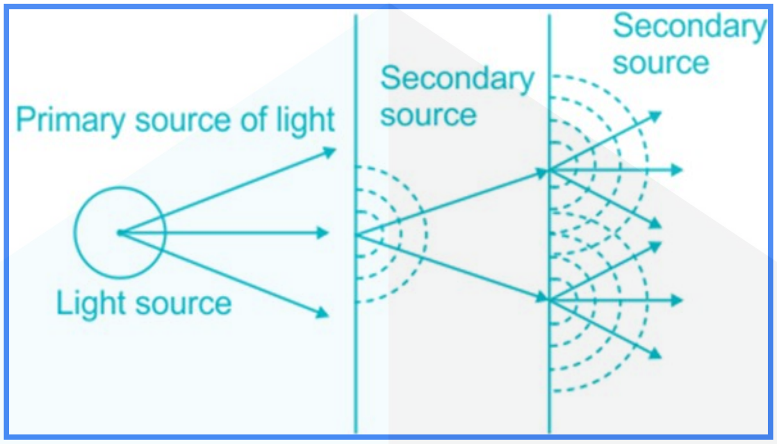


Primary and secondary source

Huygen's principle is simply a geometrical construction which enables us to determine the new position of a wavefront at a later time from its given position at a specific instant.

On the basis of the following assumptions:

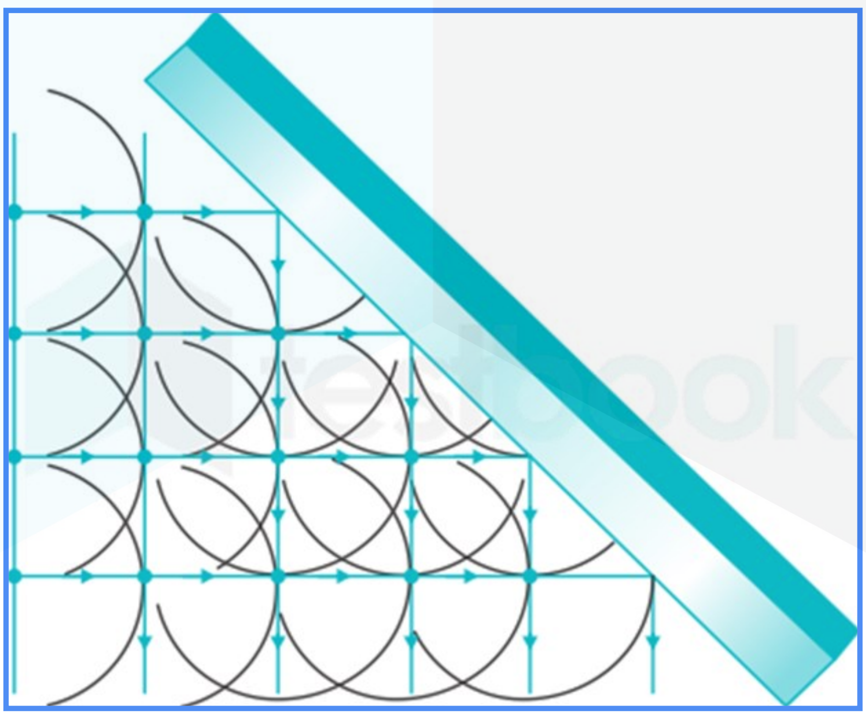
1. Every point on the primary wavefront acts as a source for secondary wavelets which subsequently send out disturbances in all directions similar to the original light source.
2. At a given instant, the new position a wavefront, called the secondary wavefront or secondary source, is formed as a result of all the secondary wavelets at that instant.



From the above figure we can see that, the source from which light was incident is called primary source of light whereas as light passes through slit or narrow gap it behaves like a source and hence, they are termed as secondary source

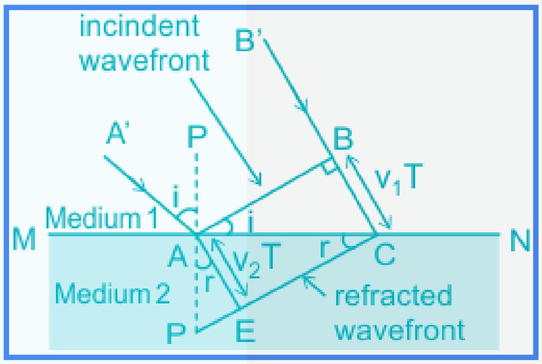
Huygens’ Reflection

The tangent to these wavelets shows that the new wave front has been reflected at an angle equal to the incident angle. The direction of propagation is perpendicular to the wave front, as shown by the downward-pointing arrows.



Huygens Refraction

According to Huygens' principle, a straight wavefront traveling from one medium to another where its speed changes. The ray bends toward or away from the normal since the wavelets have a lower speed in a denser medium because of change in its density (i.e., Frequency remains constant in both medium and velocity of wavelets is maximum in rarer medium vrarer > v-denser).



Here

i = angle of incidence

r = angle of refraction

And from the above figure, we can derive that

$$\frac{\mu_2}{\mu_1} = \frac{\sin\theta_1}{\sin\ \theta_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} \Rightarrow \mu_1 \sin i\ = \mu_2 \sin r$$

Whereas v = n x λ

The above equation is also known as Snell's law of refraction.

Interference

Interference is the phenomenon in which two waves superpose to form a resultant wave.

i.e., The non-uniform distribution of energy due to the superposition of light waves in a media is called the interference of light.

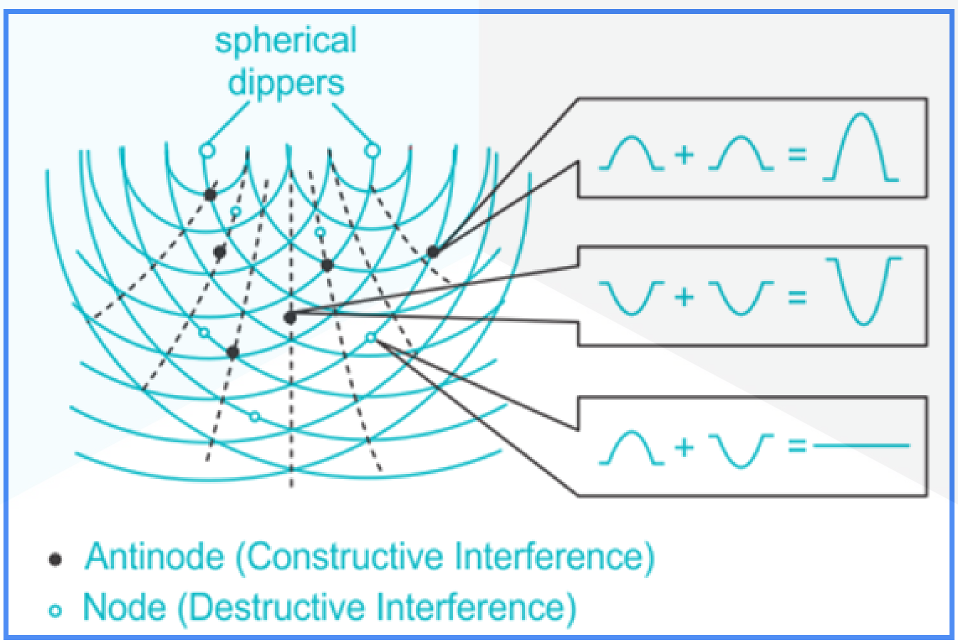
Wave interference is the phenomenon that occurs when two waves meet while traveling along with the same medium. In this process, waves superpose to form a resultant wave of greater, lower, or the same amplitude.

It can be classified into two types

1) destructive interference.
2) constructive interference.

- Constructive interference occurs when the crest of one wave falls on the crest of another wave or the trough of one wave falls on the other. The resultant intensity due to constructive, interference is, therefore, maximum. Bright fringes on the screen correspond to constructive interference which creates a bright band in the interference pattern.

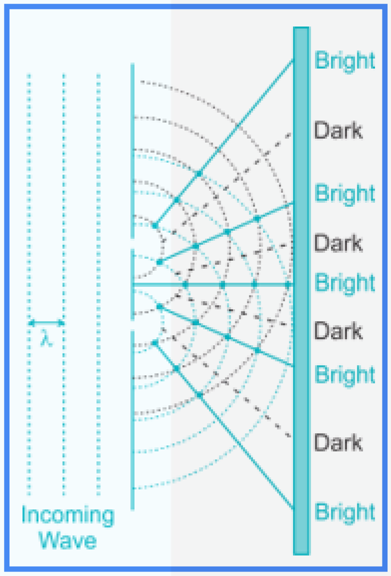
- Destructive interference occurs when the crest of one wave falls on the trough of another wave and vice versa. The resultant intensity due to destructive interference is, therefore, minimum. Dark fringes on the screen correspond to destructive interference which creates a dark band in the interference pattern.



Thomas Young in 1801 through Young's double-slit experiment.

This experiment shows that the observed pattern of interference occurs due to the superposition of light waves which proves the wave nature of light.

In this experiment, two narrow slits that are close to each other, are illuminated by a monochromatic light source. The two slits are responsible for the production of two different wavefronts which then superimpose on a screen forming the definite interference pattern as shown below.

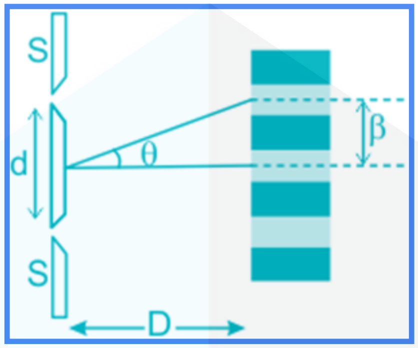


- According to **Young's Double Slit Experiment** consider a Monochromatic light (single wavelength) falls on two narrow slits S_1 and S_2 which are very close together act as two coherent sources, when waves coming from two coherent sources, (S_1, S_2) superimposes on each other, an **interference pattern is obtained on the screen**.
- In **Young's Double Slit Experiment** alternate bright and dark bands were obtained on the screen. These bands are called **Fringes**.

Fringe width (β) –

- The separation between any **two consecutive bright or dark fringes** is called **fringe width**.
- In **Young's Double Slit Experiment** all fringes are of **equal width**.

Hence the Fringe width is expressed as $\beta = \frac{D\lambda}{d}$



d = Distance between slits

D = Distance between slits and screen,

λ = Wavelength of monochromatic light emitted from the source

Example:

Q: The wavelength used in Young's double-slit experiment is 6000 Å. What should be the distance between the slits to get a fringe width of 0.012 cm wide when the screen is 40 cm from the slits?

A:

Given that

$\lambda = 6000 \text{ Å} = 6000 \times 10^{-10}$ m

$\beta = 0.012$ cm $= 0.012 \times 10^{-2}$ m

$D = 40$ cm $= 40 \times 10^{-2}$ m

Now Fringe width is expressed as $\beta = \frac{D\lambda}{d}$

$$\therefore d = \frac{D\lambda}{\beta} = \frac{6000 \times 10^{-10} \times 4 \times 10^{-2}}{0.012 \times 10^{-2}} = 0.0002\ m$$

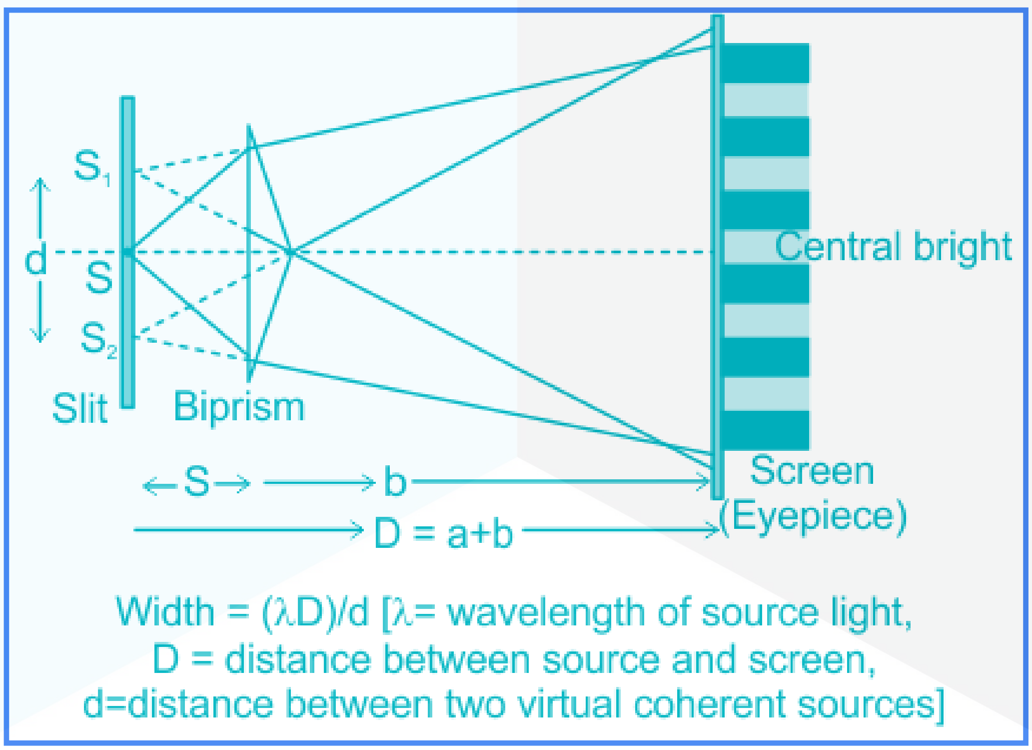
d = 0.02 cm

Fresnel biprism

- A Fresnel biprism is just a variation of Young's double-slit experiment. It is obtained by placing two thin prisms base to base forming an isosceles triangle. It is used to obtain two coherent virtual sources of light waves to produce the interference pattern.
- i.e., It is an instrument that can be used to obtain fringes due to interference and to calculate the wavelength of monochromatic light.
- Bi-prism produces interference patterns from a single source due to the creation of two virtual coherent sources as the light passes through the prism.
- When a monochromatic light source is kept in front of biprism two coherent virtual sources S1 and S2 are produced.

Fringes are of equal width (w).

$$w = \frac{\lambda D}{d}$$



Let the separation between S1 and S2 be 'd' and the distance of slits and the screen from the biprism be a and b respectively i.e., D = (a + b). If the angle of the prism is α and the refractive index is μ then $d = 2a(\mu - 1)\alpha$

$$\lambda = \frac{wd}{D} = \frac{w[2a(\mu - 1)\alpha]}{[a + b]}$$

If a convex lens is mounted between the biprism and eyepiece. There will be two positions of the lens when the sharp images of coherent sources will be observed in the eyepiece. The separation of the images in the two positions are measured. Let these be d1 and d2 then

$$d = \sqrt{d_1 d_2}$$

$$\lambda = \frac{wd}{D} = \frac{w\sqrt{d_1 d_2}}{[a + b]}$$

$$i.e., w = \frac{[a + b]\lambda}{2a(\mu - 1)\alpha}$$

Example:

Q: Interference fringes are observed with a biprism of refracting angle 1° and refractive index 1.5 on a screen 100 cm away from it. The wavelength of light used is 5890 Å. If the distance between the source and the biprism is 20 cm, the fringe width is:

A:

Given that,

α = 1° = $\frac{\pi}{180} \times 1 = \frac{\pi}{180}$

$$\mu = 1.5, a = 20\ cm = 200\ mm;\ \ b = 100\ cm = 1000\ mm$$

$$\lambda = 5890\ A^\circ = 5890 \times 10^{-7} mm$$

Whereas fringe width for Fresnel biprism can be expressed as

$$w = \frac{[a + b]\lambda}{2a(\mu - 1)\alpha} = \frac{5890 \times 10^{-7} \times [200 + 1000]}{2 \times 200 \times (1.5 - 1) \times \frac{\pi}{180}} = 0.202\ mm$$

Interference from thin film

- The bright colors seen in an oil slick floating on water or in a sunlit soap bubble are caused by interference of lightwave reflected from the front and back surface of a thin transparent film.
- The brightest colors are those that interfere constructively. This interference is between the light reflected from different surfaces of a thin film; thus, the effect is known as **thin-film interference.**

i.e., When white light is incident on a thin film, the film appears colored and the color depends upon the thickness of the film and also the angle of incidence of the light.

Consider a thin film made of a transparent material with plane parallel faces separated by a distance d. Suppose a parallel beam of light is incident on the film at an angle i. The wave is divided into two parts at the upper surface, one is reflected and the other is refracted. The refracted part which enters into two parts; one is transmitted out of the film and the other is reflected. Multiple reflection and refraction take place and several reflected waves as well as transmitted waves are sent by the film.

And due to this multiple reflection and refraction between the different surfaces of a thin film, we get maxima and minima as we can see in the above fig.

For **maxima,** the equation can be given as

$$2d + \frac{1}{2}\lambda_n = n\lambda_n \ ...; where, n = 1, 2, 3 \ ...$$

Or

$$2d = \left(\frac{1}{2} + n\right)\lambda_n$$

For **minima,** the equation can be given as

$$2d + \frac{1}{2}\lambda_m = \left(m + \frac{1}{2}\right)\lambda_m ...; where, m = 0, 1, 2, 3 \ ...$$

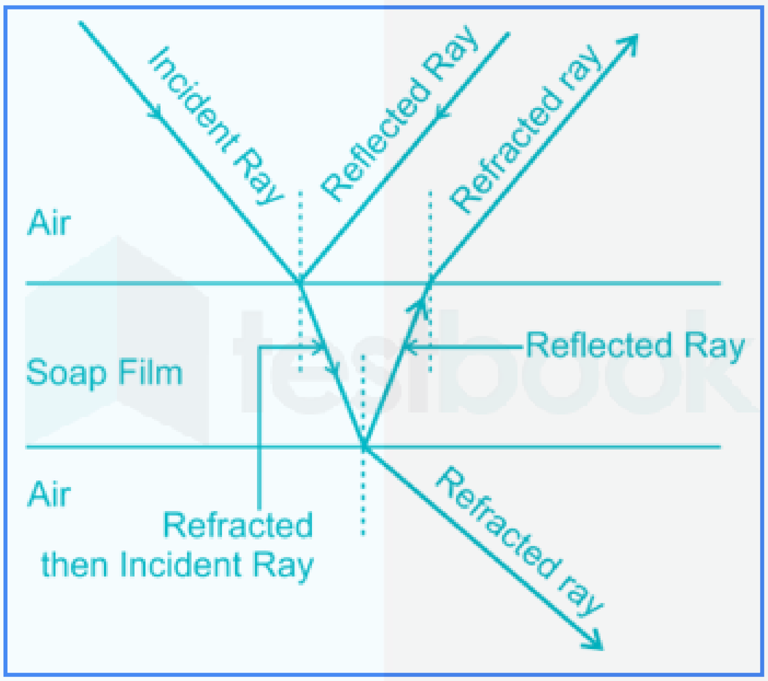
Or

$$2d = m\lambda_m$$

Interference effects are most prominent when light interacts with something having a size similar to its wavelength.

Condition for interference from thin film

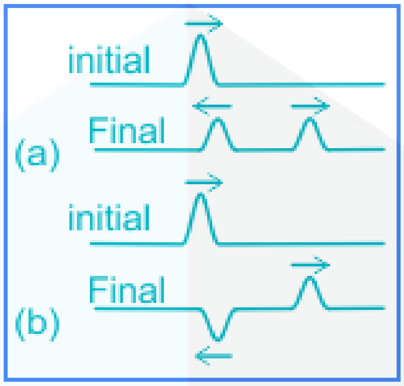
- A thin-film interference pattern can be seen for both monochromatic (single color light) as well as a broadband spectrum (white light or some other combination of monochromatic light).
- In the case of a monochromatic light source, we can see bright and dark bands due to interference of light reflecting from the different surfaces.
- Similarly, for white light, we can see a colorful pattern, which had been discussed earlier.



Now in our case, as we can see above light is undergoing reflection at two different surfaces with two different refractive indices.

Hence these kinds of reflection are termed soft reflection and hard reflection.

- **Soft reflection**: When light is reflected off from the surface within the lower medium of the lower refractive index, as a result, no phase shift can be observed.
- **Hard reflection**: When light is reflected off from the surface within the medium of higher refractive index, as a result, $180° \; or \; \frac{\lambda}{2} \; the$ phase shift is observed.



Case 1:

For soft reflection, there is no phase change due to reflection as we can see in fig (a) and thus the phase difference can be expressed as $2\mu d \cos r$

Thus, the equation for **constructive interference** can be expressed as

$$\frac{(2n+1)\lambda}{2} = 2\mu d \cos r \ldots ; where\ n = 0, 1, 2 \ldots$$

Similarly, for destructive interference, the equation can be expressed as

$$2\mu d \cos r = m\lambda \ldots ; where, n = 1, 2, 3 \ldots$$

Case 2:

For Hard reflection, there is λ/2 phase change due to reflection as we can see in fig (b) and thus the phase difference can be expressed as $2\mu d \cos r \times (\frac{2\pi}{\lambda})$

Thus, the equation for **constructive interference** can be expressed as

$$2n\pi = \left(\frac{2\pi}{\lambda}\right) \times 2\mu d \cos r \ldots ; where\ n = 0, 1, 2 \ldots$$

$$i.e., 2\mu d \cos r = n\lambda$$

Similarly, for destructive interference, the equation can be expressed as

$$2\mu d \cos r = \frac{(2m-1)\lambda}{2} \ldots ; where, n = 1, 2, 3 \ldots$$

Example:

Q: A silicon monoxide (n = 1.5) film of 100 nm thickness is used to coat a glass camera lens (n = 1.56). What wavelength of light in the visible region will be most efficiently transmitted by this system?

A:

Given that,

n =1.5

n' = 1.56

d = 100 nm

In this case, the wavelength of transmitted light can be determined using the equation of hard reflection

i.e., $2\mu d \cos r = n\lambda$

For our simplicity consider r = 0 ⇒ cos 0 =1

$$\therefore\ 2\mu d\ =\ n\lambda\ \Rightarrow \mu d\ = \frac{n}{2}\lambda\ \dots;where, n = 0,1,2,3$$

$$\lambda_1 = 2 \times (1.45 \times 100) = 290\ nm$$

$$\lambda_2 = 4 \times 1.45 \times 100 = 580\ nm$$

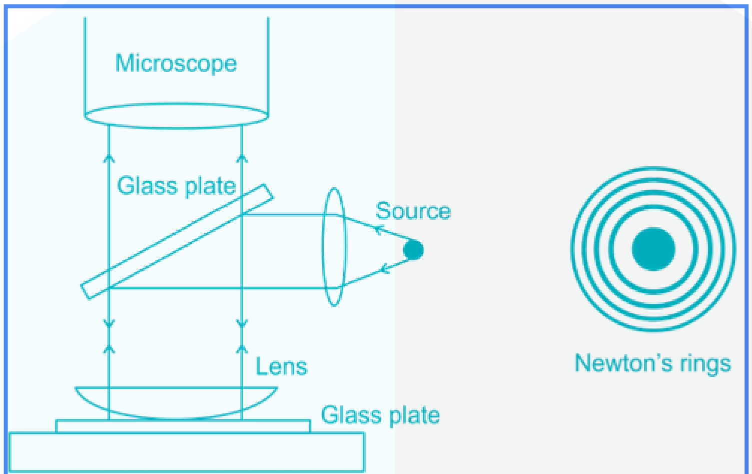
Thus, the visible light region consists of a spectrum of wavelengths that range from approximately 700 nm to approximately 400 nm, hence in this case the wavelength of 540 nm will be most effectively transmitted by this system

Newton's ring

Newton's rings experiment:

- When a parallel beam of monochromatic light is incident normally on a combination of a plano-convex lens L and a glass plate G, as shown in Figure, a part of each incident ray is reflected from the lower surface of the lens, and apart, after refraction through the air film between the lens and the plate, is reflected from the plate surface.

These two reflected rays are coherent; hence they will interfere and produce a system of alternate dark and bright rings with the point of contact between the lens and the plate as the center. These rings are known as **Newton's rings**.



Experimental set-up to observe Newton's ring

And the diameter of such **nth bright band of Newton's ring** can be expressed as

$$D_n = \sqrt{4\left(n - \frac{1}{2}\right)\frac{\lambda R}{\mu}} \;\ldots; where, n = 1,2,3 \ldots$$

Whereas for mth dark band of Newton's ring (Destructive interference in thin-film) can be expressed as

$$D_m = \sqrt{\frac{4m\lambda R}{\mu}} \;\ldots; where\ m = 0, 1, 2, 3$$

Here,

λ = wavelength of the incident light

R = radius of curvature of the lens

μ = refractive index of the medium between lens and glass plate

n = nth band of newton's ring

D_n = Diameter of nth bright band of newton's ring.

Special case:

If there is no medium between the glass plate and lens, the refractive index of a medium can be given as μ = 0.

Thus, the diameter of such n^{th} bright band of Newton's ring in the air can be given as

$$D_n = \sqrt{4\left(n - \frac{1}{2}\right)\lambda R} \ldots; where\ n = 1, 2, 3..$$

$$D_n = \sqrt{2(2n-1)\lambda R} \Rightarrow D_n \propto \sqrt{2n-1}$$

And for m^{th} dark band in the air, the equation can be expressed as

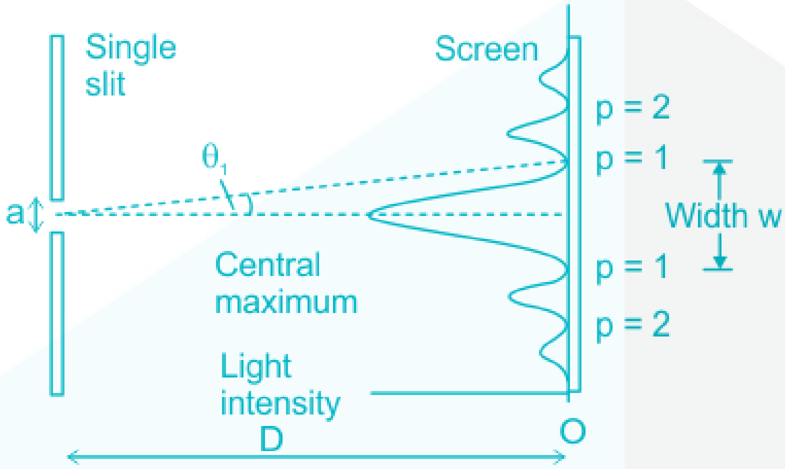
$$D_m = \sqrt{4m\lambda R} \ldots; where\ n = 0, 1, 2 \ldots$$

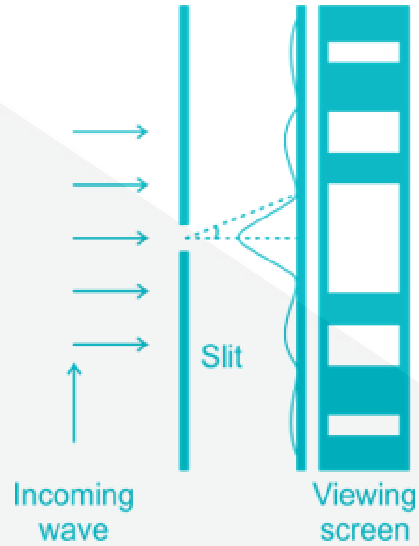
Diffraction of light

- **Diffraction** is the **slight bending of light** as it passes around the edge of an object.
- The amount of bending depends on the relative size of the wavelength of light to the size of the opening.

Single slit diffraction

- When the **monochromatic light ray** falls on a single slit then it gets diffracted from the slit and form a **bright and dark band** on the screen.
- The bright pattern is also called **maxima** and the dark band is called minima.
- At **maxima the intensity is maximum** and at minima the intensity of light is minimum.





The width of the **maxima** is given by**:**

$$a \sin\theta = n\lambda$$

Where λ is the wavelength of the light, n is an integer value, a is slit width and D is the distance of the screen from the slit.

Here if θ is very small (less than 30°) sinθ will be approximately equal to θ.

Thus, for **mth minima,** the equation can be expressed as

$$a\theta = m\lambda, \quad m = \pm1, \pm2 \ldots.$$

i.e., $\theta = \frac{m\lambda}{a}, \quad m = \pm1, \pm2 \ldots.$

The intensity of single-slit diffraction at some mth minima

$$I_\theta = I_m \left(\frac{\sin\frac{m\phi}{2}}{\frac{m\phi}{2}} \right)^2 \ldots.; where, n = \pm1, \pm2, \ldots..$$

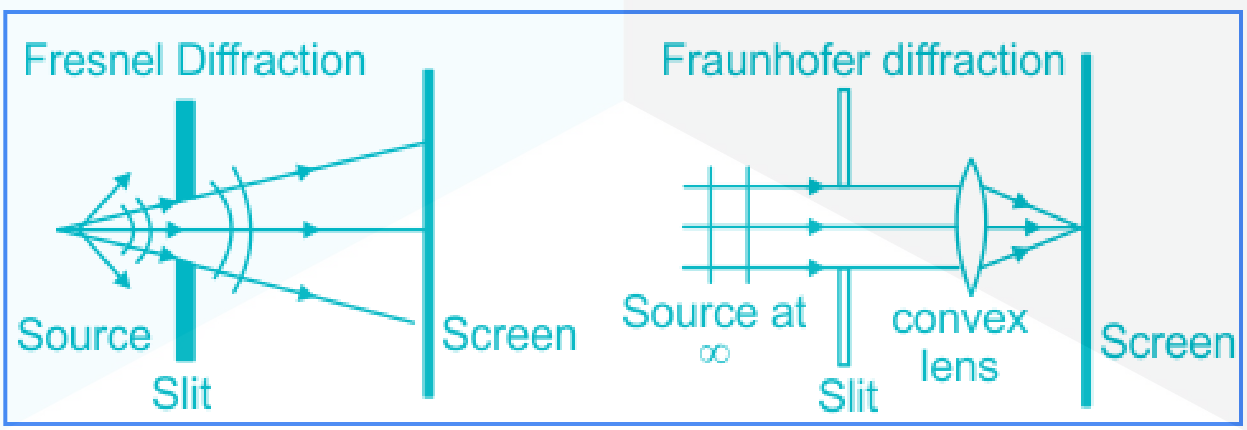
Fresnel & Fraunhofer diffraction

As explained above **Diffraction** of light is the phenomenon of bending light from the sharp corners of a slit or obstacle and spreading into the region of the geometrical shadow.

Diffraction can occur only when the wavelength of light is comparable to the size of the obstacle or the width of the slit.

Diffraction is of two types:

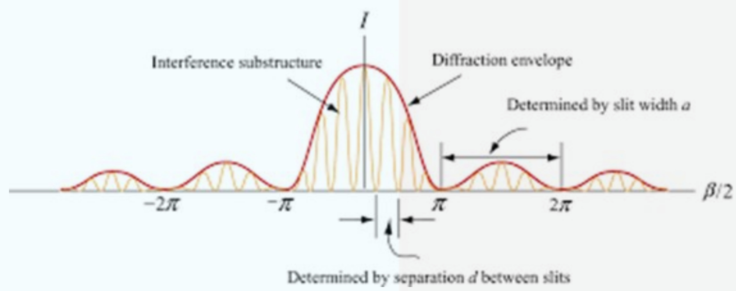
Fresnel Diffraction	Fraunhofer diffraction
• It is the type of diffraction which occurs when the light source lies at a finite distance from the slit. • i.e., The source of light and screen is kept at a finite distance from the diffracting system. • Cylindrical or spherical wavefront is considered. • Fresnel diffraction patterns on flat surfaces.	• it is the type of diffraction which occurs when a plane wavefront is incident on the slit and the wavefront emerging from the slit is also plane. • i.e., The source of light and the screen on which diffraction pattern is obtained is at infinite distance from the diffracting system. • Plane wavefront is considered. • Fraunhofer diffraction patterns on spherical surfaces.



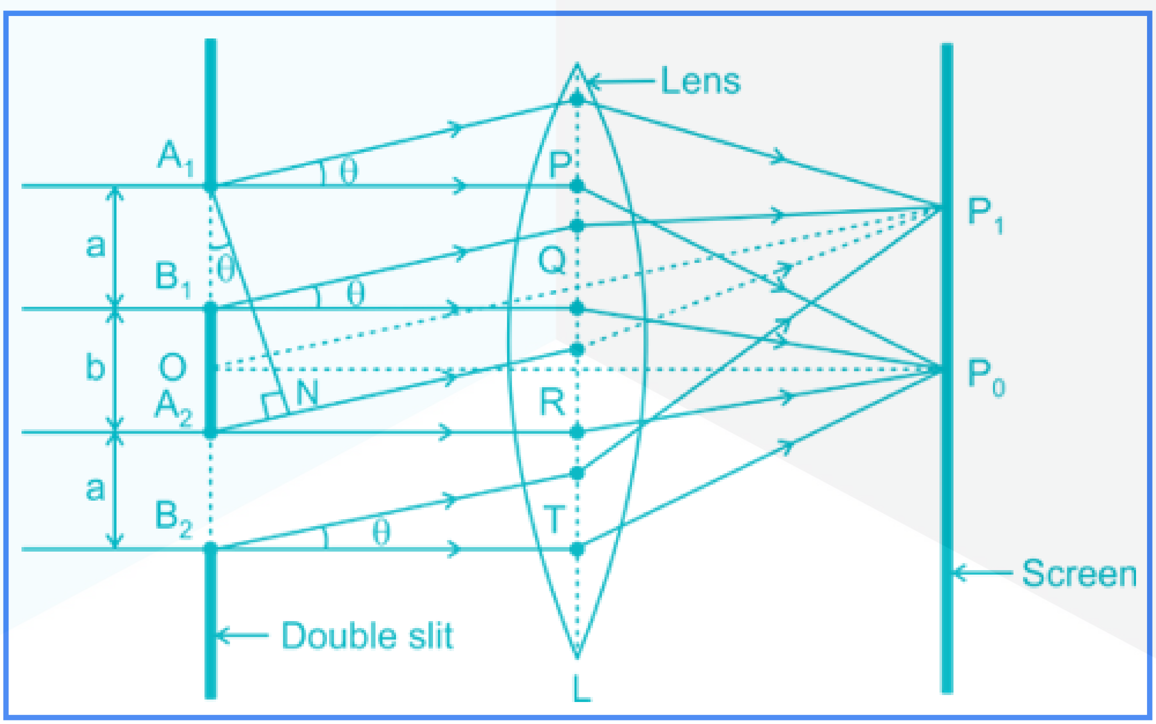
Fraunhofer diffraction at a double slit

In Young's double-slit experiment, an interference pattern comprising alternate dark and bright fringes on-screen was observed. However, in that experiment, we have assumed each slit as behaving like a point source thus we did not consider the width of the slit, as a result, no diffraction pattern was observed due to either slit.

Now if the width of each slit is finite, we shall observe a combined effect of diffraction (through each slit) as well as interference pattern as we can see in the figure below.



Now at any point on the screen, the rays reaching from the first and second slit will have a path difference, where ‘b' is the separation between the center of two slits.



And thus, for double-slit, the intensity pattern of the infinitesimal narrow slit can be expressed as

$$I_{\theta,Int} = I_{m,int}\cos^2\beta \ldots; where, \beta = \frac{\pi d}{\lambda}\sin\theta$$

In which d is the distance between the center of two slits.

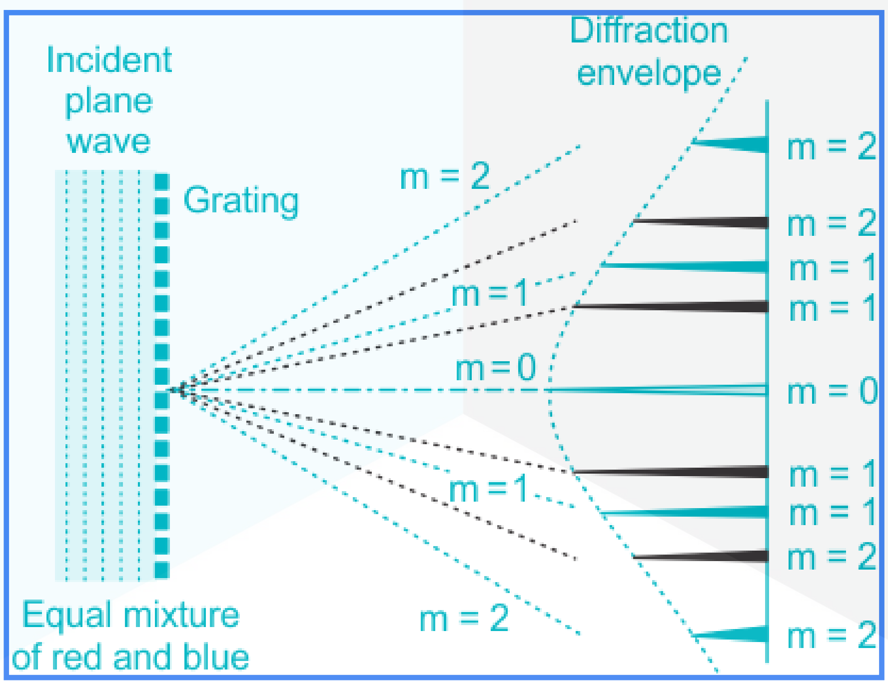
Similarly, intensity for a diffracted wave from either slit can be given as

$$I_{\theta,diff} = I_{m,diff}\left(\left(\frac{\sin\alpha}{\alpha}\right)^2\right) \ldots; where, \alpha = \frac{\pi a}{\lambda}\sin\theta$$

Thus, for the combined pattern, the intensity pattern can be given as

$$I_\theta = I_m\,(\cos\beta)^2\left(\frac{\sin\alpha}{\alpha}\right)^2$$

Diffraction grating



- A diffraction grating is an extremely useful device, and one of it consists of a large number of narrow slits side by side.
- The slits are separated by opaque spaces.
- When a wavefront is an incident on a grating surface, light is transmitted through the slits and obstructed by the opaque portions. Such a grating is called transmission grating.

The formula for the diffraction grating is given by

$$d \sin\theta = m\lambda \quad m = 0, \pm1, \pm2, .\,.,$$

Width of the maxima

Now if d is the separation between two slits, N is the number of diffractions grating and θ is some angle for the first maxima then the angular width can be expressed as

$$\delta\theta = \frac{\lambda}{Nd\cos\theta}$$

Diffraction patterns of air-born water droplets

Rayleigh scattering:

The scattering of light by particles in a medium, without a change in wavelength, is called Rayleigh scattering.

Rayleigh scattering can be considered to be elastic scattering since the photon energies of the scattered photons are not changed.

Based on Rayleigh scattering, the **Rayleigh criterion** for the diffraction limit is used to define to resolution states that two images which are just resolvable when the center of the diffraction pattern of one is directly over the first minimum of the diffraction pattern of the other (or)

The Rayleigh criterion specifies the minimum separation between two light sources that may be resolved into distinct objects.

$$\theta_R = \sin^{-1}\left(\frac{1.22\lambda}{d}\right) \approx 1.22 \times \frac{\lambda}{d}$$

Were

λ is the wavelength of light (or other electromagnetic radiation) and

D is the diameter of the aperture, lens, mirror, etc., with which the two objects are observed

Thus, minimum resolvable details are given by Rayleigh's criterion.

Resolving Power

Resolving power of a telescope

- The reciprocal of the smallest angle subtended at the objective lens of a telescope by two-point objects which can be just distinguished as separate is called the **resolving power of a telescope**.

The **resolving power of a telescope** is given by:

$$R.P = \frac{D}{1.22\lambda}$$

Were

D is the distance between the two-point object

λ is the wavelength of light used

- The resolving power of a microscope is the ability of the microscope to show as separate, images of two-point objects lying close to each other.

$$R.P = \frac{2\mu \sin\theta}{\lambda}$$

Were

λ = Wavelength of light used to illuminate the object,

μ = Refractive index of the medium between object and objective

μ sinθ = Numerical aperture

Resolving Power of a diffraction grating

The capacity of an optical instrument to show separate images of very closely placed two objects is called resolving power.

The resolving power of a diffraction grating is defined as its ability to form separate diffraction maxima of two closely separated wavelengths.

If λ and $\lambda + d\lambda$ are wavelengths of two neighboring spectral lines, the resolving power of the grating is the ratio $\lambda/d\lambda$.

The resolving power of the grating is given by

$$R.P = \frac{\lambda}{d\lambda} = nN$$

Were

n - order of spectrum,

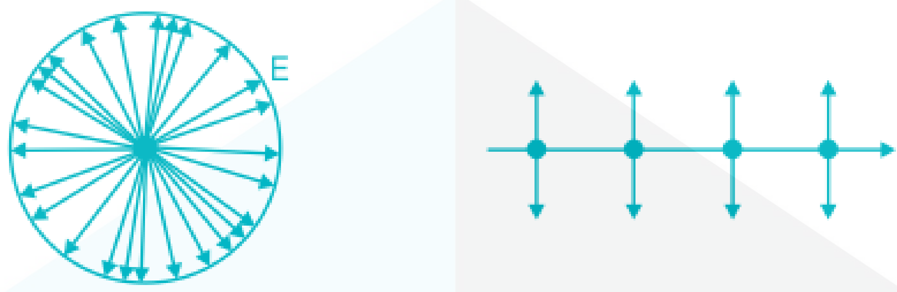
N - total no. of lines in the given grating.

Polarization

As we had seen above light is an electromagnetic wave in which electric and magnetic field vectors are sinusoidally perpendicular to each other as well as they are perpendicular to the direction of propagation of light waves.

Unpolarized light

When light wave propagates it consist of vibration of electric field in all possible direction, which are which are perpendicular to its direction of propagation and such light waves are said to be unpolarized light.

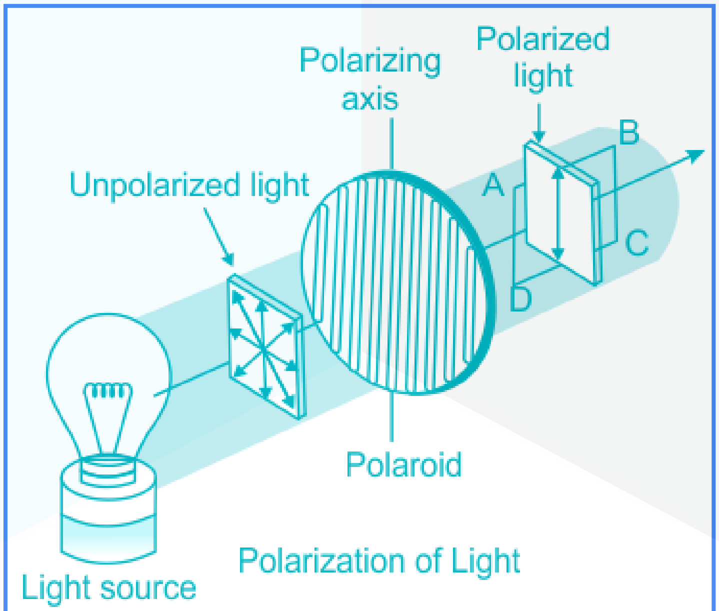


Representation of unpolarized light

The above diagram shown unpolarized light in which arrow represent vertically vibration electric field and dots represents horizontal vibrating electric field and straight line with arrow represents direction of propagation

Polarization of light

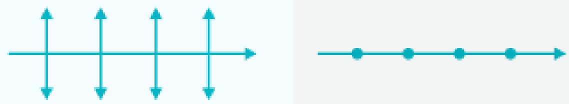
- When we restrict vibration electrical vector in an unpolarized light we get plane polarized light.
- In this the direction of vibration of electric vector and direction of lights propagation are perpendicular to each other as shown below.



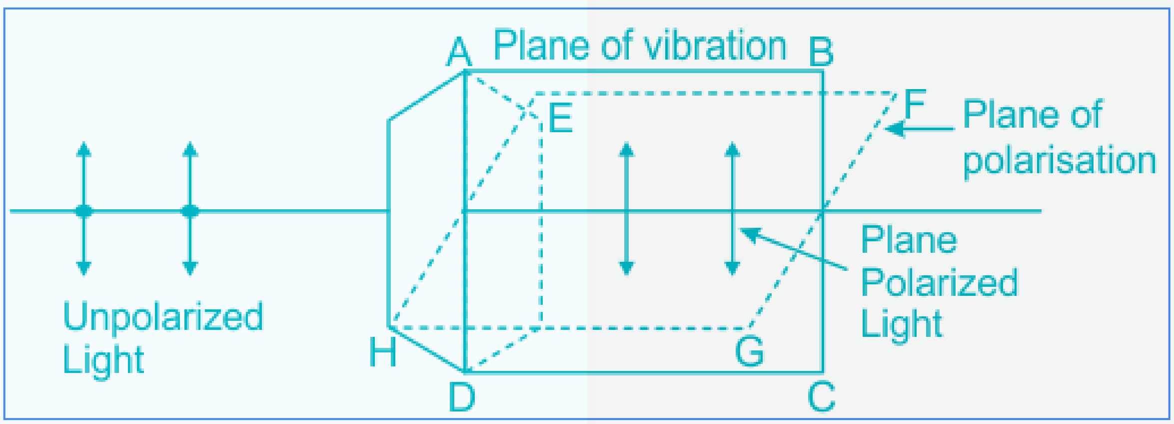
Polarization of Light

Important terms used in polarization

- **Plane of vibration**: The plane in which vibration of electric field vectors are restricted are termed as plane of vibration.



- **Plane of polarization**: The plane which is perpendicular to plane of vibration i.e., the plane in which there is no vibration of electric field vectors are known as plane of polarization.



- **Polarizers**: The device which is used to polarize an unpolarized light are known as polarizers and some of its examples are Nicol prism, Tourmaline crystal, etc.
- **Analyzer**: The device which is used to determine the plane of polarization are termed as analyzer
- **Polaroids**: It is a large sheet made up of microscopic dichroic crystals which can produce a beam of polarized light

Experiment to determine transverse nature of light

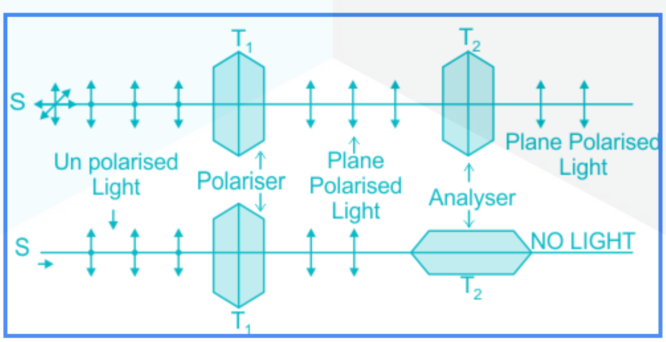
- For this experiment lets consider that we have an unpolarized light passing through two polarizer T1 and T2, here line on polarizer shows plane of vibration i.e., the plane in which electric field vector can vibrate freely, whereas electric field perpendicular to this line will be eliminated completely.
- Now as this unpolarized light passes through polarizer, it will polarize the light in either vertical or horizontal direction.
- This polarized light when again passed through a polarizer there can be two possible outcomes

Case I

- If plane of vibration of both polarizers are parallel to each other in such polarized light from first polarizer can easily pass through second polarizer as well as shown below.

Case II

- Now if we rotate second polarizer in such a way that it becomes perpendicular to first one, in this case as plane polarized light passes through second polarizer it will get eliminated since second polarizer won't allow it to pass and hence it is called analyzer which is used to determine the plane of polarization as shown below



And hence from this experiment we can conclude that electric field are vibrating perpendicular to its direction of propagation, hence light is a transverse wave.

Brewster's law (Polarization by reflection)

- Brewster's law states that, the tangent polarized angle is equal to the refractive index of the refracting medium at which partial reflection of light taken place.
- According to Brewster's law when an unpolarized monochromatic beam of light is incident on a plane refracting surface, part of light will be reflected and rest will be refracted as shown below
- In this we can see that the reflected light will be completely polarized in the plane of incidence at certain angle of incidence, hence this angle is known as polarizing angle.
- Whereas at this angle the reflected and refracted ray are separated by an angle of 90° from each other.
- Now if we consider i_p the polarizing angle, r_p be refracted angle and μ as the refractive index of the medium then from the figure we can see that

$$180° = i_p + 90° + r_p$$

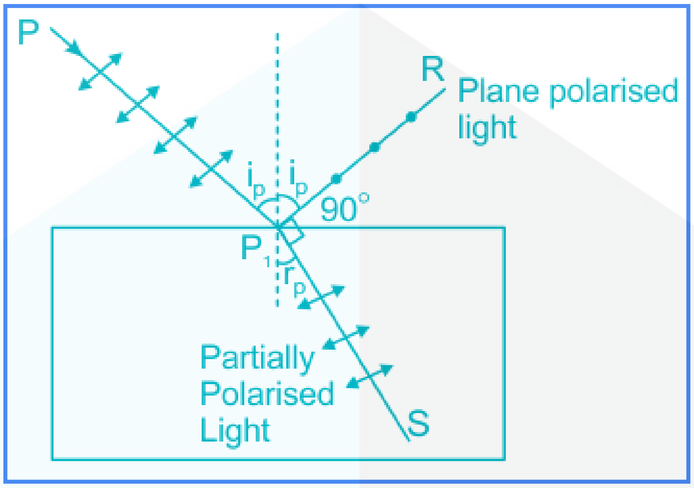
And $r_p = 90° - i_p$

Now by using Snell's law

$$\mu = \frac{\sin i_p}{\sin r_p} = \frac{\sin i_p}{\sin(90 - i_p)} = \frac{\sin i_p}{\cos i_p}$$

$$\therefore \mu = \tan i_p$$

Thus using Brewster's law, we can conclude that polarizing angle depends on frequency of incident ray i.e., colour of incident ray hence polarizing angle will be different for different colour ($\because \tan i_p = \mu = \frac{\lambda_1}{\lambda_2}$).



Example:

Q: What will be the polarizing angle of a fluid with a refractive index of 1.5 with respect to air

A:

Given

Refractive index of fluid with respect to air, n = 1.50

Now according to Brewster's law angle of polarization is given as

$$\tan\theta = \frac{\mu_2}{\mu_1} = n$$

$$\therefore \theta = \tan^{-1} 1.5 = 56.3°$$

Double refraction (Polarization by Refraction)

Double refraction, is also known as birefringence, it is an optical property in which a single ray of unpolarized light entering an anisotropic medium is split into two rays i.e., ordinary & extraordinary rays, each traveling in a different direction.

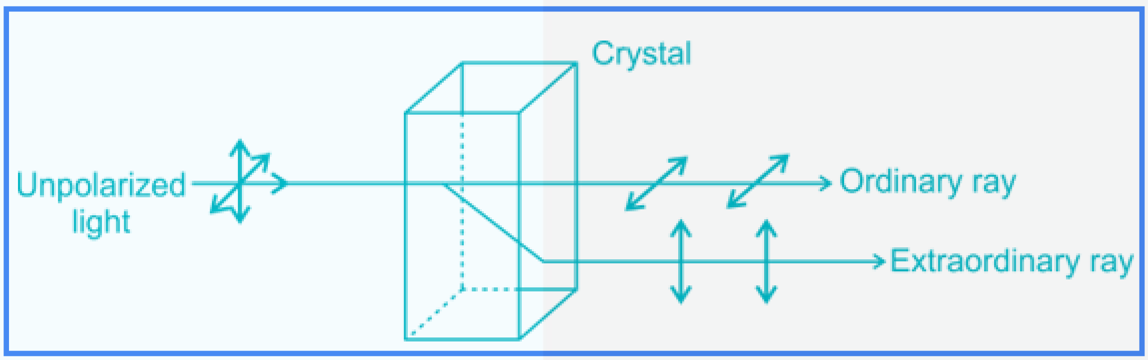
Double refraction can be observed by comparing two materials, glass and calcite.

Suppose if a pencil mark is drawn upon a sheet of paper and then covered with a piece of glass, only one image will be seen but if the same paper is covered with a piece of calcite and the crystal is oriented in a specific direction then two marks will become visible.

- This is because refracted beam acquires some degree of polarization. Most often, the polarization occurs in a plane perpendicular to the surface.

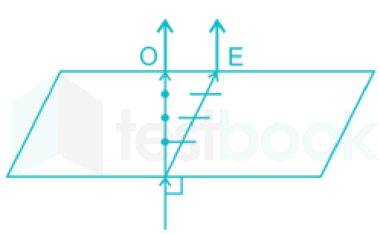
- **Light passing through a calcite crystal is split into two rays as shown below.** This process is called double refraction or polarization by refraction.

The two rays of light are each plane-polarized by the calcite such that the planes of its polarization are mutually perpendicular to each other as shown below.



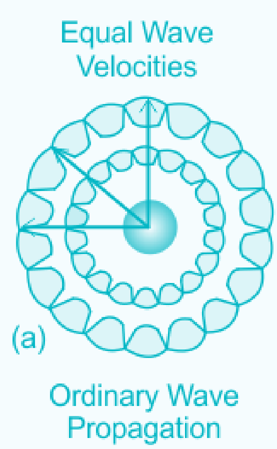
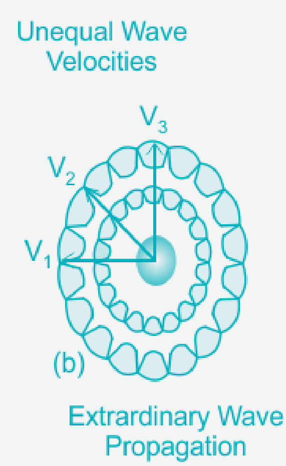
- For calcite, one of the two rays do indeed obey Snell's Law (angle of incidence is equal to the angle of refraction), this ray is called the ordinary ray (or O-ray). The other ray (and any ray that does not obey Snell's Law) is an extraordinary ray (or E-ray).

For normal incidence (a Snell's law angle of 0°), the two planes of polarization are also perpendicular to the plane of incidence.



- The velocity and refractive index of extraordinary waves changes with direction whereas for ordinary ray velocity remains constant trough out the crystal while propagating.

Some other differences between o-ray and e-ray are as shown below

O-ray (Ordinary ray)	E-ray (Extraordinary ray)
It obeys Snell’s law of refraction	It does not obey laws of refraction
It travels at the same speed in all direction inside the crystal	It travels at different speeds in different directions within the crystal. However, the speed of Ordinary and extraordinary ray is the same along its optic axis
The electric vector of O-ray vibrates perpendicular to the principal section of the O -ray	The electric vector of E-ray vibrates parallel to the principal section of the E-ray
	

Extra information:

For monoclinic or tetragonal crystal, we get the above result i.e., One O-ray and one e-ray can be observed.

Whereas for orthorhombic, monoclinic, and triclinic crystals, there will be two E-rays

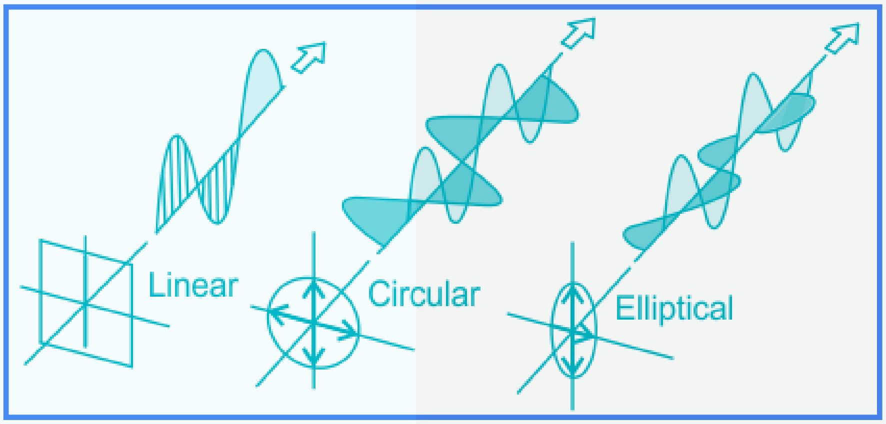
Types of polarization

There are three main types of polarization namely, linear polarization, circular polarization, and elliptical polarization.

1) **Linear polarization** is when the oscillation of a wave is constrained to a single plane. This is the plane of polarization.

2) **Circular polarization** is when two linear components of a wave oscillate perpendicular to each other such that their amplitudes are equal.

3) **Elliptical polarization** is the same as circular polarization except that the amplitudes and phase differences are not the same.



Optical activity and Optical rotation

- Optical activity is the ability of a substance to rotate the plane of polarization of a beam of light that passed through it.
- And polarimeter is an instrument used to determine the angle through which plane -polarized light has been rotated by a given sample.

- The **optical rotation** is the angle through which the plane of polarization is rotated when **polarized light passes** through a **layer of a liquid.**
- Substances are described as dextrorotatory or levorotatory according to whether the plane of polarization is rotated clockwise or counterclockwise, respectively, as determined by viewing towards the light source. Dextrorotation is designated (+) and levorotation is designated (-).
- The specific rotation is a characteristic property of a certain substance and is the standard measurement for the optical rotation of that substance.

The optical rotation and specific rotation are related by $[\alpha] = \frac{\alpha_{observed}}{cl}$

were

[α] = specific rotation

$\alpha_{observed}$ = optical rotation

c = the concentration (g/dl)

l = path length (dm)

Example:

Q: A sugar solution in a tube of length 2.0 dm produces optical rotation of 12°. Then, the sugar solution to one half of its initial concentration. If the dilute solute solution is contained in another tube of length 3.0 dm, the optical rotation produced by it will be:

A:Given $\alpha_{observed}$ = 12° when l = 2 dm and concentration c;

$$[\alpha] = \frac{12}{c \times 2} = \frac{6}{c}$$

Now the concentration is c/2 and l = 3 dm

$$\frac{6}{c} = \frac{\alpha_{observed}}{\frac{c}{2} \times 3}$$

$\alpha_{observed}$ = 9°

Polarization by Scattering

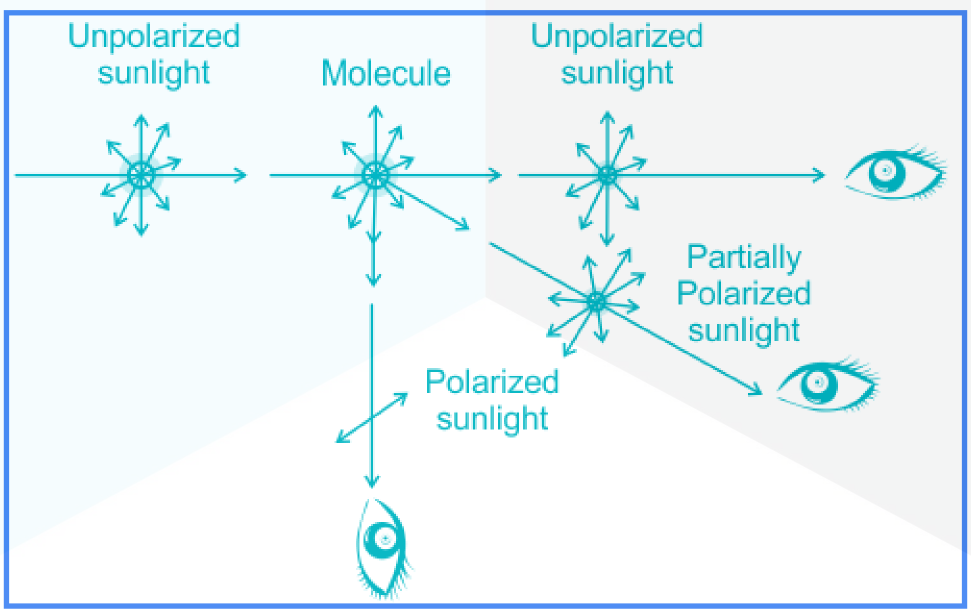
So, as we discussed above light is a transverse wave, and confining the plane of vibration of such transverse wave by the principle of scattering is termed Polarization by scattering.
Also, we know that when an unpolarized light passes through a medium, it undergoes a phenomenon known as scattering.

Such medium contains lots of atoms and when an unpolarized light beam strikes an atom it transfers energy to an atom, as a result, an electron in that atom starts vibrating in response to the electric component of the incident ray.

Now, this vibrating electron emits E.M radiation in all directions, and this forces neighboring electrons to undergo a similar process.

This **phenomenon of absorption and emission of the electromagnetic ration is known as the scattering of light.**

Whereas the scattered light can be unpolarized, partially polarized, or completely polarized based on the direction of the incident ray as shown below.



Polarization by scattering is generally observed in liquid or gas randomness in motion of atom, whereas in perfect crystal we don't see as good results as fluids.

Polarization due to scattering can be seen through the sky as we can see below. When an unpolarized beam of sunlight passes through the atmosphere the light scatters down towards earth has no vertical polarization.

This is the why sky scatters more blue light than red because of the natural frequency of molecules in our atmosphere.