The Demand for Securities

- Why would an individual consider owning government equities or any other security?
  - Willingness of an investor to hold a particular security depends on two sets of characteristics:
    - Those of the investor.
    - Those of the security.
- The principal characteristics of securities-particularly the time pattern of returns, the risk of default, and the negotiability-are discussed. How to characterize the behavior of individual investors is the topic here.
- There are two fundamental reasons why individuals demand securities:
  - Pattern of income over time differs from their desired sequence of consumption
  - The desire to increase total rupee wealth- may be "aggressive" attempt and/or "defensive" attempt.
- Demand for securities are examined in terms of these 3 basic attributes:
  - Time characteristics
  - Expected return
  - Risk

Present Value

- Deferent instruments have very different streams of cash flows with very different timing …..how do we compare their values?
- One would require a total rate of return of ‘*r*’ per annum to invest in a particular security.
  - What price would one be willing to pay for a security that returns ₹ CF1 in Year 1, ₹ CF2 in Y2, and so on for 5 years with return of the principal invested ‘*P*’ on maturity?
- The answer is the present value (PV) of the security:
  - *Concept: A return in the future is worth less than a return now. And a commonsense notion.*
- Hence, future returns must be "discounted" by the required rate of return.
- The PV concept can be extended to understand the PV of a single cash flow or the *sum* of a sequence or group of cash flows.

Example-Simple Loan

- *Simple loan???*
- *Example: If you made your friend a simple loan of ₹100 for one year, you would require her/him to repay the principal of Rs100 along with an additional payment for interest; say, ₹10.*
- In this case, the interest rate, *i*, can be measured sensibly.
  - $i = \frac{₹\,10}{₹100} = 0.10 = 10\%$
  - Loan at the end of the year = ₹100 x (1+ 0.10) = ₹ 110
  - If lent it out, at the end of the 2 year :
    - ₹110 x (1+ 0.10) = ₹ 121 or,
    - ₹100 x (1+0.10) x (1+ 0.10) = ₹100 x(1+ 0.10)^2 = ₹121
  - Similarly, at the end of the 3 year:
    - ₹121 x (1+0.10) = ₹100 x(1+ 0.10)^3 = ₹ 133
  - At the end of *n* years, the ₹100 would turn into:-------- ₹100 x (1 + $i)^n$

Simple Loan (Cont.)

- This process can be seen in the following timeline:

Year: 0	1	2	3	n
₹100	₹110	₹121	₹133	$100 \times (1+0.10)^n$

- You are just as happy having ₹100 today as having ₹110 a year from now!!!
- We can also work backward from future amounts to the present.
  - For example, ₹133 = ₹100 x $(1+0.10)^3$ three years from now is worth ₹100 today:
    - $₹100 = \frac{₹\,133}{(₹1+0.10)^3}$ = 0.10 = 10%
  - This process is called *discounting the future*. This can be written as:
    - $PV = \frac{CF}{(1+r)^n}$

Present Value (Cont.)

- The PV can be calculated as follows.
- $PV = \frac{CF_1}{(1+r)} + \frac{CF_2}{(1+r)^2} + \frac{CF_3}{(1+r)^3} + \frac{CF_4}{(1+r)^4} + \frac{CF_5 + P}{(1+r)^5}$
- The value we place on future returns is equal to the amount we would have to invest now at the required rate *r* in order to generate that cash flows in the future.
- More generally, if the required rate of return is allowed to vary over time, the general form of the PV formula becomes:
- $PV = \frac{CF_1}{(1+r_1)} + \frac{CF_2}{(1+r_1)(1+r_2)} + \frac{CF_3}{(1+r_1)(1+r_2)(1+r_3)} + \dots \quad \dots \dots \dots \dots \quad \frac{CF_n + P}{(1+r_1)(1+r_2)(1+r_3) \dots\dots (1+r_n)}$
- In terms of timing of cash flows, the following are different:
  - simple loan; fixed payment loan; coupon bond; discount bond
- How would you decide which of these provides you with more income…they seem very different?
  - The PV is useful to compare the value of two debt instruments with very different timing of their cash flows, at a given interest rate *i*.
  - Need to use the PV concept to measure interest rates of different types of instruments.
  - Alternatively, the concept of yield to maturity (YTM), the most important way of calculating interest rates.

Yield to Maturity (YTM)

- YTM is the interest rate that equates the present value of cash flows received from a debt instrument with its value today.
  - Reflects the total return by holding a bond until it matures.
- It is also called internal rate of return (IRR)/discount rate - equates the future coupon and principal redemption cash flows from a bond to its current market price.
- YTM depends on two assumptions:
  - The bonds are held to maturity
  - The interest payments received are reinvested at the same rate as the YTM
- We now look at how we calculate it for different instruments by using the PV concept.

YTM - Simple Loan

- **Taking earlier example of simple loan: You lent your friend ₹100 and next year you get back ₹ 110 back from him/her.**
- What is the YTM on this loan? It is 10%.
- Need to solve for the YTM *r* that the PV of the future payments must equal today's value of a loan.
  - $\mathrm{PV} = \frac{\mathrm{CF}}{(1+r)^n}$
  - Where, PV = ₹100, CF= ₹110; n = 1 year.
  - Thus,
    - $₹100 = \frac{₹110}{(1+r)}$
    - $(1+r)\ ₹100 = ₹110$
    - $(1+r) = \frac{₹110}{₹100}$
    - $r = 1.10 - 1$
    - $= 0.10 = 10\%$
- For simple loans, the simple interest rate equals the yield to maturity.

YTM- Coupon Bond

- Bond price is the present value of the probable future cash flows, which comprises of the coupon payments and the par value, which is the redemption amount on maturity.

$$\text{Bond Price} = \sum_{i=1}^{n} \frac{C}{(1+r)^n} + \frac{F}{(1+r)^n}$$

where
C = Periodic coupon payment,
F = Face / Par value of bond,
r = Yield to maturity (YTM) and
n = No. of periods till maturity

YTM-Coupon Bond (Cont.)

- Example: Coupon bond
  - FV = ₹1,000; years to maturity = 10; and yearly coupon payments = ₹100 (a 10% coupon rate). What is PV of the bond?
  - At the end of 1year, there is a ₹ 100 coupon with a PV of $\frac{₹100}{(1+r)}$; and so on
  - Setting today's value of the bond (its current price, denoted by PV) equal to the sum of the present values of all the cash flows
    - PV $= \frac{₹100}{(1+r)} + \frac{₹100}{(1+r)^2} + \frac{₹100}{(1+r)^3} + \dots \dots \dots \dots \quad \dots + \frac{₹100}{(1+r)^{10}} + \frac{₹1000}{(1+r)^{10}}$
  - For any coupon bond
    - PV$= \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \dots \dots \dots \dots \quad \dots \frac{C}{(1+r)^n} + \frac{F}{(1+r)^n}$
      - The yield to maturity (r) is the only unknown quantity.
  - YTM calculation for a coupon bond is not easy!!!

YTM by Trail-Error Method

Example: A Five year coupon bond with 8% coupon rate and maturity value of ₹1000 is currently selling at ₹ 925. What is its YTM???

- At its coupon rate, 8%, YTM = $\frac{₹80}{(1+0.08)} + \frac{₹80}{(1+0.08)^2} + \frac{₹80}{(1+0.08)^3} + \frac{₹80}{1+0.08)^4} + \frac{₹80}{(1+0.08)^5} + \frac{₹1000}{(1+0.08)^5}$ = ₹1000
- If YTM is assumed at 9% = $\frac{₹80}{(1+0.09)} + \frac{₹80}{(1+0.09)^2} + \frac{₹80}{(1+0.09)^3} + \frac{₹80}{1+0.09)^4} + \frac{₹80}{(1+0.09)^5} + \frac{₹1000}{(1+0.09)^5}$ = ₹ 961.10
- If YTM is assumed at 11% = $\frac{₹80}{(1+0.11)} + \frac{₹80}{(1+0.11)^2} + \frac{₹80}{(1+0.11)^3} + \frac{₹80}{1+0.11)^4} + \frac{₹80}{(1+0.11)^5} + \frac{₹1000}{(1+0.11)^5}$ = ₹ 889.12
- If YTM is assumed at 10% = $\frac{₹80}{(1+0.10)} + \frac{₹80}{(1+0.10)^2} + \frac{₹80}{(1+0.10)^3} + \frac{₹80}{1+0.10)^4} + \frac{₹80}{(1+0.10)^5} + \frac{₹1000}{(1+0.10)^5}$ = ₹ 924. 18
- The present market value lies between Rs.961.10 and Rs.924.18. By interpolation, through
  - $YTM = lower\ rate + (\frac{Surplus}{Surplus+Deficit})$ x $(\ higher\ rate\ - lower\ rate)$
    - Where
      - Surplus = $Difference\ between\ the\ value\ at\ lower\ rate\ and\ market\ value$
      - Deficit = $Difference\ between\ the\ value\ at\ higher\ rate\ and\ market\ value$
  - $YTM = 9 + \left(\frac{36.1}{36.1 + 0.82}\right)$ x $(\ 10 - 9\ )$
  - YTM = 9.978%
- This method is time consuming.

Exercise

You are considering the purchase of a ₹1,000 face value bond that pays 11 percent coupon interest per year, paid annually. The bond matures in 5 years and has a face value of ₹1,000. The current market price of the bond is ₹931.176. What is the YTM?

$$\text{YTM} = \text{lower rate} + \left(\frac{\text{Surplus}}{\text{Surplus} + \text{Deficit}}\right) \text{x} \left(\text{higher rate} - \text{lower rate}\right)$$

Where

$$\text{Surplus} = \text{Difference between the value at lower rate and market value}$$
$$\text{Deficit} = \text{Difference between the market value and the value at higher rate}$$

YTM - Coupon Bond (Cont.)

- Alternatively,
- $$YTM = \frac{Annual\ Interest\ + \frac{Par\ Value - Market\ Price}{Number\ of\ years\ to\ Maturity}}{\frac{Par\ Value + Market\ Price}{2}}$$

- Example, GOI issued bond with a Par value (face value) with Rs 100. Coupon rate is 10%. Current market price of the bond is Rs 92. Maturity of the bond is 10 years. What is YTM???

- $$= \frac{₹10\ + \frac{₹\ 100 - ₹92}{10}}{\frac{₹100 + ₹92}{2}}$$

- = 11.25 %

Coupon Bond (Cont.)

- *Alternatively (the easiest way), calculator.*
  - Find the price of a 10% coupon bond with a face value of ₹1,000, a 12.25% yield to maturity, and 8 years to maturity.
  - Using a calculator.
    - n = years to maturity = 8
    - FV = face value of the bond = 1,000
    - r = annual interest rate = 12.25%
    - PMT = yearly coupon payments = 100
  - Then push the PV button
    - price of the bond = ₹ 889.20.

Coupon Bond – Perpetuity or consol

- This is one special case of a coupon bond, its YTM is easy to calculate. It has no maturity, no repayment of principal and makes fixed coupon payments of ₹C forever.
- The coupon bond equation (PV= $\frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \ldots + \frac{C}{(1+r)^n} + \frac{F}{(1+r)^n}$) for price of a perpetuity, P_c, is simplified as:
  - $P_c = \frac{C}{i_c}$ or $i_c = \frac{C}{P_c}$
  - where, P_c = price of the perpetuity (consol); C = yearly payment; i_c = yield to maturity of the perpetuity (consol).
    - Thus, the bond formula for perpetuity is:
      - $P_c = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots.$
    - which can be written as
      - $P_c = C\,(x + x^2 + x^3 + \cdots)$ In which $x = (\frac{1}{1+i})$.
      - Taking the formula for an infinite sum:
      - $1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x}\ for\ x < 1$
    - and so
      - $P_c = C\left(\frac{1}{1-x} - 1\right) = C\left[\frac{1}{\frac{1-1}{1+i_c}} - 1\right]$; which by suitable algebraic manipulation becomes
      - $P_c = C\left(\frac{1+i_c}{i_c} - \frac{i_c}{i_c}\right) = \frac{C}{i_c}$
- As i_c goes up, the price of the bond falls.
- Example: A perpetuity pays ₹100 per year forever and the interest rate is 10.
  - Price = ₹ 1000 = ₹ 100/0.10.
  - If the interest rate rises to 20%, its price will fall to ₹ 500 (= ₹ 100/0.20).

Example: What is the yield to maturity on a bond that has a price of ₹2,000 and pays ₹ 100 annually forever?

- The yield to maturity would be 5%.
- $i_c = \frac{C}{P_c}$
- where
  - C = yearly payment = ₹100
  - P_c = price of perpetuity (consol) = ₹ 2,000
- Thus,
  - $i_c = \frac{₹100}{₹\ 2{,}000}$
  - i_c = 0.05 = 5%

Relationship Between Price and Yield to Maturity

- Following table shows Yields to Maturity calculated for several bond prices.
  - YTM on a 10% Coupon Rate Bond Maturing in 10 Years (Face Value = ₹ 1,000)

Price of Bond	Yield to Maturity (%)
1,200	7.13
1,100	8.48
1,000	10.00
900	11.75
800	13.81

- Three interesting facts can be observed:

1. When the bond price is at par/FV, YTM equals the coupon rate (simple reasoning behind the YTM calculation)
2. Price and yield are negatively related- as the YTM rises, the price of the bond falls (all denominators will raise)
3. YTM is greater than the coupon rate when bond price is below par value (follows 1 & 2)

One more Example

- The law of bond pricing is that yields (interest rates) and prices are inversely related.
- The logic is simple:
  - GOI issued a bond: FV= Rs. 100, coupon = 10%; issued in 2015 matures in 2027.
  - 7 years to mature
  - Assume that interest rate has fallen in the economy. As a result GOI can issue 2027 bond with FV Rs. 100 for (say) 7% !
  - Would the old bond still trade for Rs. 100???
  - Unlikely, as it pays 3% extra interest every year for the next 7 years, investor can get Rs. 21 more than on the new bond. So the bond is worth Rs. 121 (you can use bond pricing formula and check this)
  - Similarly, if in 2020, 7-year yields have increased to 12% and a new bond for 2027 is issued at this rate!
    - So, the old bond will depreciate as it pays Rs. 2 less than the new bond.
    - It is worth for Rs. 86 (Rs. 2 lost annually for 7years).
  - Therefore,
    - When interest rate rises, bond prices fall.
    - When interest rate falls, bond prices rise.

YTM- Discount Bond

- The yield-to-maturity calculation for a discount bond is similar to that for the simple loan.
- Example: Govt. T – bill. An issued T-bill will be redeemed at FV of ₹1,000 in one year's time. The current purchase price of this bill is ₹ 900. What is YTM??
  - To equate the current price to the PV of the ₹ 1,000 received in one year
    - $₹900 = \frac{₹1000}{1+r}$
  - Solving for *r*,
    - $(1+r) \times ₹900 = ₹1{,}000$
    - $₹900 + ₹900r = ₹1{,}000$
      - $₹900r = ₹1{,}000 - ₹900$
      - $r = \frac{₹1{,}000 - ₹900}{₹900} = 0.111 = 11.1\%$
  - Generally, for any one-year discount bond, the YTM can be written as
    - $r = \frac{F-P}{P}$; where F = face value of the discount bond; P = current price of the discount bond.
  - In normal circumstances, holding these bonds give positive returns as $P<F$. So, $F - P$ should be positive, and thus the YTM.
  - However, this is not always the case. (extraordinary events in Japan).
  - For a discount bond, the YTM is negatively related to the current bond price.
    - Example: a rise in the bond price from ₹ 900 to ₹ 950 means???
      - $r = \frac{₹1{,}000 - ₹950}{₹950} = 0.053$ or 5.3%, the YTM falls from 11.1%.
  - *So, the YTM for an instrument is the interest rate that equates the PV of the future cash flows on that instrument to its value today.*
  - For longer discount bonds, the YTM can be calculated as: $YTM = \sqrt[N]{\frac{Face\ Value}{Current\ Price}}$ - 1; where N= no. of years to maturity.

Current Yield

- Current yield (CY) is just an approximation for YTM.
- It measures the return, without taking into account the capital gain or loss on redemption.
- Example: GOI Bond with face value ₹ 100 offers 8% coupon.
  - If bank buys the bond for ₹ 105??
  - What is the yield??
  - ₹ 8/105 = 0.0762 or 7.62% $\Rightarrow$ return over the purchase price.

Distinction Between Interest Rates and Returns

- Do interest rate on bonds tell all about the owning them???
- Will you be better off when interest rat rise which is higher than the bond that yields to you?
- Return or the rate of return is a accurate measure of how well of they are.
- Rate of Return = periodic payments + Δ value, expressed as a fraction of its purchase price.
  - *Example: Coupon bond FV = ₹1,000 FV; coupon rate = 10%; Bought it for ₹ 1,000, held for 1 year, and then sold for ₹ 1,200. What is the return look like here???*
    - The payments = annual coupon, ₹ 100, and the change in its value, ₹200 (i.e. ₹ 1,200 – ₹1,000).
    - $Rate\ of\ return = \frac{₹100 + ₹200}{₹1000} = \frac{₹300}{₹1000} = 0.30 = 30\% \Rightarrow$ one-year holding-period return
  - Something surprising! The return equals 30%....., yet our YTM and bond price table discussion, initially the YTM was only 10%.
  - It indicates that the *return on a bond will not necessarily equal the interest rate on that bond.*
- We will see the distinction can be important, although the two may be closely related for many securities.

Discussed in the previous class. Relationship Between Price and Yield to Maturity

- Yields to Maturity on a 10% Coupon Rate Bond Maturing in 10 Years (Face Value = ₹ 1,000)

Price of Bond	Yield to Maturity (%)
1,200	7.13
1,100	8.48
1,000	10.00
900	11.75
800	13.81

- Three interesting facts in the Table above
  1. When bond is at par, yield equals coupon rate
  2. Price and yield are negatively related
  3. Yield greater than coupon rate when bond price is below par value

Distinction Between Interest Rates and Returns (Cont.)

- The return on a bond held from time *t* to time *t*+1 is:
  - $R = \frac{C + P_{t+1} - P_t}{P_t}$
  - where
    - R = return from holding the bond from time t to time $t + 1$
    - C = coupon payment
    - P_{t+1} = price of the bond at time t+1
    - P_t = price of the bond at time t
  - It can be rewritten, into two separate terms:
    - $R = \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t}$
    - $\frac{C}{P_t} = i_c$ $\Rightarrow$ current yield
    - $\frac{P_{t+1} - P_t}{P_t} = g$ $\Rightarrow$ rate of capital gain
    - So, $R = i_c + g$
- Therefore, even the current yield is an near measure of the YTM, the return can differ substantially from the interest rate, if Δ price is substantial (+/-).

Distinction Between Interest Rates and Returns (Cont.)

- *Example: Bond is issued at FV ₹ 1,000 with a coupon rate of 8%. You bought it for ₹1,000 and sold it one year later for ₹ 800. What is the rate of return?*
  - $Rate\ of\ return =$
    - $R = \dfrac{\mathrm{C} + P_{t+1} - P_t}{P_t}$
    - Thus,
      - $R = \dfrac{₹\,80 + (₹800 - ₹1{,}000)}{₹1{,}000} = \dfrac{-₹120}{₹1000} = -0.12 = -12\%$
      - The rate of return is negative….!!!
      - Thus, if Δ price is substantial, the return can differ substantially from the interest rate,.
- Let us explore this point even further.
  - What happens to the returns on bonds of different maturities when $i \uparrow$???

Distinction Between Interest Rates and Returns (Cont.)

- Using $R = i_c + g$ formula, the following table calculates the one-year return on several 10% coupon rate bonds all purchased at par when interest rates on all these bonds rise from 10% to 20%.

(1) Years to Maturity When Bond Is Purchased	(2) Initial Current Yield (%)	(3) Initial Price	(4) Price Next Year*	(5) Rate of Capital Gain (%)	(6) Rate of Return (2 + 5) (%)
30	10	1,000	503	–49.7	–39.7
20	10	1,000	516	–48.4	–38.4
10	10	1,000	597	–40.3	–30.3
5	10	1,000	741	–25.9	–15.9
2	10	1,000	917	–8.3	+ 1.7
1	10	1,000	1,000	0.0	+10.0

If the bond is not sold the capital loss referred as a 'paper loss'.

* *Calcualted using* **Bond Price** $= \sum_{i=1}^{n} \frac{C}{(1+r)^n} + \frac{F}{(1+r)^n}$

- Key findings:
  1. The only bond whose return equals the initial yield to maturity is one whose years to maturity is same as holding period
  2. For bonds with years to maturity > holding period, $i\uparrow$ $P\downarrow$ implying capital loss
  3. Longer a bond's maturity, the greater the change in price, when $i\Delta$
  4. Longer a bond's maturity, the lower the rate of return occurs with $i\uparrow$
  5. Bond with high initial interest rate even can still have negative return if $i\uparrow$

Maturity and the Volatility of Bond Returns: Interest-Rate Risk

- So, prices and returns are more volatile for long-term bonds than those of short-term (due to interest rate risk).
  - No interest-rate risk for any bond whose maturity equals holding period.
- *Reinvestment Risk*
  - If an investor's holding period is longer than the term to maturity of the bond, the investor is exposed to reinvestment risk.
  - Since, the proceeds from the short-term bond need to be reinvested at future *i which is uncertain.*
  - Gain from *i* ↑, lose when *i* ↓
  - *Example: Suppose, you have a holding period of 2 years and decides to purchase a ₹1,000 1 year bond at FV with interest rate 10% and then purchase another one at the end of the 1st year.*
    - At the end of the 1 year, the investment value = ₹ 1,100 ((= ₹1,000 (1+ 0.10)).
    - If *i* on a 1 year bond rises to 20% at the end of the year and ₹ 1,100 will be in another bond!
    - At the end of the 2 year,
      - = ₹ 1320, i.e. ₹ 1100 (1+ 0.20)
    - Return = (₹1320 - ₹ 1000)/ ₹ 1000) = ₹ 320/ ₹1000 = 0.32 = 32%
    - Equals 14.89% at an annual rate! (this can be obtained as: $Return_{annual} = \left[\frac{Amoun\ at\ the\ end}{Amount\ at\ the\ begining}\right]^{\frac{1}{n}} - 1; where\ n = \ number\ of\ years)$
      - $= \left[\frac{₹1320}{₹\ 1000}\right]^{\frac{1}{2}} - 1 = [1.32]^{\frac{1}{2}} - 1$ = 0.1489 = 14.89% an annual rate.
- Earning are more by buying the 1-year bonds than 2-year bond with an interest rate of 10%.

Maturity and the Volatility of Bond Returns: Interest-Rate Risk

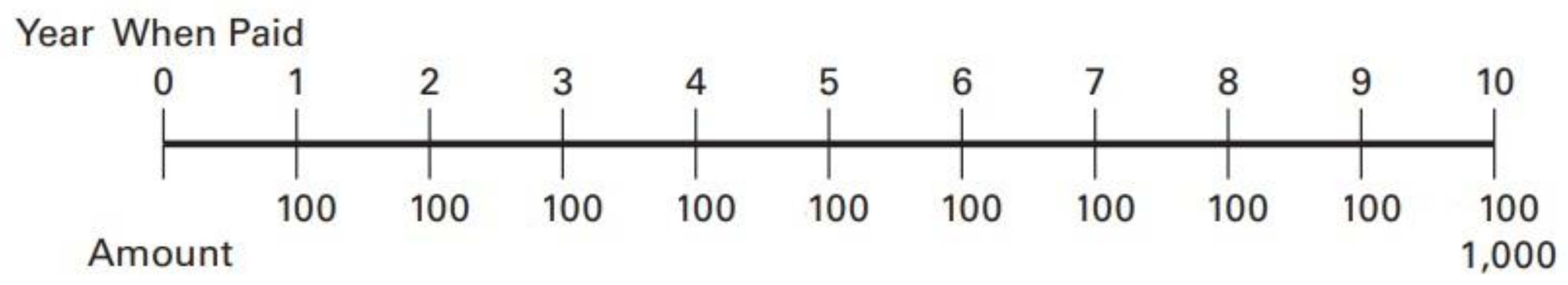
- *Therefore, when your holding period is longer than the term to maturity of the bonds you purchase, you benefit from from i ↑.*
- Conversely, if *i* ↓ on one-year bonds to 5% at the end of the year, the investment value at the 2year end = ₹ 1,155.
  - i.e. ₹ 1100 (1.05) = 1155.
  - Return =(₹1155 - ₹ 1000)/ ₹ 1000) =0.155 = 15.5%
  - The two-year return = 7.2% per annual.
  - i.e. $= \left[\frac{₹1155}{₹\ 1000}\right]^{\frac{1}{2}} - 1 = [1.155]^{\frac{1}{2}} - 1$ = 0.072 = 7.2%
- *Therefore, with a holding period greater than the term to maturity of the bond, you now lose from a i↓*

Measuring Interest rate sensitivity of bonds- Duration

- *r* appears more times and with higher power in the denominator of the PV formula the longer is the bond maturity. So, longer bonds have a higher risk than short bonds.
- However, the YTM is not always the relevant measure of maturity.
- *What is the actual capital gain or loss that occurs when the interest rate changes by a certain amount? Will the price of a bond be more strongly affected by changes in interest rates than the other type of bond?*
  - "yes“ -coupon comes in the early periods (short term) and so is less influenced by interest rate changes.
- To account for these differences we need Duration statistic, the average lifetime of a debt security’s stream of payments.
- Calculated by taking the maturity of each payment the bond returns to its holder and multiplying it by the percentage of the bond's present value paid out at that maturity. So, it is a weighted average of the different maturities of cash payments.

Duration

- A coupon bond is equivalent to a set of zero-coupon discount bonds.
  - For instance, a 10-year 10% coupon bond with ₹1,000 FV has payments similar to a set of zero-coupon bonds:
  - This set of coupon bonds is shown in the timeline:



  - a ₹100 one year zero-coupon bond (which pays the equivalent of the ₹ 100 coupon payment made by the ₹ 1,000 10-year 10% coupon bond at the end of one year),… so on and
  - a ₹1000 10-year zero-coupon bond (pays back the equivalent of the coupon bond's ₹ 1,000 FV).

Calculating Duration $i = 10\%$, 10-Year 10% Coupon Bond

- This same set is listed in column (2) of Table below

Calculating Duration on a ₹1,000 Ten-Year 10% Coupon Bond When Its Interest Rate Is 10%

(1) Year	(2) Cash Payments (Zero-Coupon Bonds)	(3) Present Value (*PV*) of Cash Payments (i = 10%)	(4) Weights (% of total *PV* = *PV*/1,000) (%)	(5) Weighted Maturity (1 × 4)/100 (years)
1	100	90.91	9.091	0.09091
2	100	82.64	8.264	0.16528
3	100	75.13	7.513	0.22539
4	100	68.30	6.830	0.27320
5	100	62.09	6.209	0.31045
6	100	56.44	5.644	0.33864
7	100	51.32	5.132	0.35924
8	100	46.65	4.665	0.37320
9	100	42.41	4.241	0.38169
10	100	38.55	3.855	0.38550
10	1,000	385.54	38.554	3.85500
Total		1,000.00	100.000	6.75850

- So, the duration is the weighted average of the effective maturities of the individual zero-coupon bonds, with the weights equalling the proportion of the total value represented by each zero-coupon bond.
- The figure of 6.76 years is the duration of the 10% 10-year coupon bond because the bond is equivalent to this set of zero-coupon bonds.

Duration (Cont.)

- Simply, duration is a weighted average of the maturities of the cash payments.The procedure done can be written as:

$$DUR = \sum_{t=1}^{n} t \frac{CP_t}{(1+i)^t} \Bigg/ \sum_{t=1}^{n} \frac{CP_t}{(1+i)^t}$$

- where
  - DUR = duration
  - *t* = years until cash payment is made
  - CP_t = cash payment (interest plus principal) at time t
  - *i* = interest rate
  - *n* = years to maturity of the security

- What If the maturity increased to 11 years from 10years???
  - DUR for 11-year 10% coupon bond if *i* is 10%, = 7.14 years, > 6.76 years for the 10-year bond.
  - *Thus,* all else equal, when the maturity of a bond lengthens, the duration rises as well
- is the maturity of a coupon bond enough to tell you what its duration is??

- Let's again calculate the duration for the 10-year 10% coupon bond, but when the current interest rate is 20% not 10%.

(1) Year	(2) Cash Payments (Zero-Coupon Bonds)	(3) Present Value (*PV*) of Cash Payments (i = 20%)	(4) Weights (% of total *PV* = *PV*/580.76) (%)	(5) Weighted Maturity (1 × 4)/100 (years)
1	100	83.33	14.348	0.14348
2	100	69.44	11.957	0.23914
3	100	57.87	9.965	0.29895
4	100	48.23	8.305	0.33220
5	100	40.19	6.920	0.34600
6	100	33.49	5.767	0.34602
7	100	27.91	4.806	0.33642
8	100	23.26	4.005	0.32040
9	100	19.38	3.337	0.30033
10	100	16.15	2.781	0.27810
10	1,000	161.51	27.808	2.78100
Total		580.76	100.000	5.72204

- The DUR has fallen from 6.76 years to 5.72 years!!
- Why ??? ⇒ the i is higher
  - *All else equal, when interest rates rise, the duration of a coupon bond fall.*
- *The DUR of a coupon bond is also affected by its coupon rate.*
  - For example, what is the duration for 10-year 20% coupon bond when the i is 10%?
    - DUR for 10-year 20% coupon bond if i is 10% ⇒ 5.98 years, which is < 6.76 years when the coupon rate is 10%.
    - Higher coupon rate ⇒ a greater earlier payments ⇒ the bond's effective maturity ↓.
  - *All else equal, the higher the coupon rate on the bond, the shorter the bond's duration.*

Duration for Portfolio of Securities

- Duration is additive
- It is the weighted-average of the durations of the individual securities.
- *Example*: A manager of a company is holding 25% of a portfolio in a bond with a 6-year duration and 75% in a bond with a 10-year duration.
- The duration of the portfolio is:
  - DUR = (0.25 x 6) + (0.75 x 10)
  - = 1.5 + 7.5 = 9 years.
  - The duration of the portfolio is 9 years.

Duration and Interest-Rate Risk

- Usage
- provides a good approximation, particularly when Δi are small.
- specified as:
  - $\%\Delta P = -DUR\ (\frac{\Delta i}{1+i})$
- where
  - $\%\Delta P = (P_{t+1} - P_t)\ /\ P_t$ = rate of capital gain/loss
  - DUR = duration
  - i = interest rate

Duration and Interest-Rate Risk (cont.)

- *Exercise: Holding a 10y 10% coupon bond in the portfolio, and the i is currently 10%. What would be the loss if the i ↑from 10% to 11% tomorrow?*
- *Solution*:
  - The duration of a 10-year 10% coupon bond is 6.76 years (seeTable –10% coupon bond).
    - $\%\Delta P = -DUR\ (\frac{\Delta i}{1+i})$
  - Thus,
    - $\%\Delta P = -6.76\ (\frac{0.01}{1+0.10})$
    - $\%\Delta P = -0.0615 = -6.15\%$
  - The approximate $\%\Delta P$ of the bond is -6.15%.

Duration and Interest-Rate Risk (cont.)

- *Exercise 2: suppose, you have the option to hold a 10-y coupon bond with a coupon rate of 20% instead of 10%............Duration for this 20% coupon bond is 5.98 years when i is 10% (as mentioned earlier, table- 20% coupon). What is the* $\%\Delta P$ *when the i* $\uparrow$ *from10% to 11% tomorrow?*
- Solution:
  - $\%\Delta P = -DUR\ (\frac{\Delta i}{1+i})$
  - Thus,
    - $\%\Delta P = -5.98\ (\frac{0.01}{1 + 0.10})$
    - $\%\Delta P = -0.054 = -5.4\%$
    - The change in bond price is much smaller than for the higher-duration coupon bond.
  - Strategy?????.......switch the portfolio.
- Therefore, the greater the DUR, the greater $\%\Delta P$ for a given change in i, …. ⇒ ….the greater its interest-rate risk.

Interest Rates

- Interest Rates (IR): Term Structure and Risk Structure
- Interest Rate levels-Real and nominal interest rates
- Determinants of IR
  - Quoted IR = Nominal IR= $\underbrace{\overbrace{r^*}^{\text{REAL}} + IP}_{\text{Nominal}} + \underbrace{DRP + LP + MRP}_{\text{Risk Premia}}$
  - *r** = REAL risk free IR
    - Rate of return to be compensated for time… for postponing consumption
    - REAL is in terms of goods and services
    - There is no risk to return called as Risk Free IR (RFIR)
    - REAL Risk Free IR = RRIR
    - *r** + Inflation = Nominal RFIR = quoted RFIR
  - IP = Inflation Premium=Expected Inflation (over the maturity) = Inflation Risk
  - DRP = Default Risk Premium: Risk of default
    - Bond Ratings
    - Rates on corporate Bonds
  - LP= Liquidity Premium: Inability to sell at a market price
    - Selling at close to Market Price – Used cars?
    - Search Time
    - Liquidity applies to any asset - Fungible - universal → Liquidity ↑
      - Marketable -
      - Personal - → Liquidity↓
  - MRP = Maturity Risk Premium
    - Maturity ↑ → Higher Volatility → Higher Risk (→ price risk, volatility risk, maturity risk, interest rate risk)

Real Rate vs Current Real Rate: are they same?

- Quoted T bill (current interest rate) = 2.0%
  - Inflation during the latest past/current inflation = 1.0 %
  - Expected Inflation = 1.5%
- *Current Real rate = 2% - 1% = 1.0%*
- *Real Rate = current interest rate – expected inflation = 2% - 1.5% = 0.5%*

Exercise

The r* is 2%, it is expected to remain constant for the next 3 years and inflation is expected to be 3%, 3.5% and 4% for respectively for next three years. The MRP is estimated to be $0.1 \times (t - 1)\%$, where t number of years to maturity. The LP on relevant 3-year securities is 0.25% and the DRP on relevant 3-year securities is 0.6%.

a. What is the yield on a 1-year T-bill?
b. What is the yield on a 3-year government bond?
c. What is the yield on a 3-year corporate bond?

Answer:

a. A Treasury security has no default risk premium or liquidity risk premium. Therefore,

$$r_{T1} = r^* + IP + MRP = 2\% + 3\% + 0.1\ (1-1)\% = \mathbf{5\%}$$

b. A Government bond has no DRP or LP. Therefore,

$$r_{T3} = r^* + IP + MRP = 2\% + [(3\% + 3.5\% + 4\%)/3] + 0.1\ (3-1)\% = 2\% + 3.5\% + 0.2 = \mathbf{5.7\%}$$

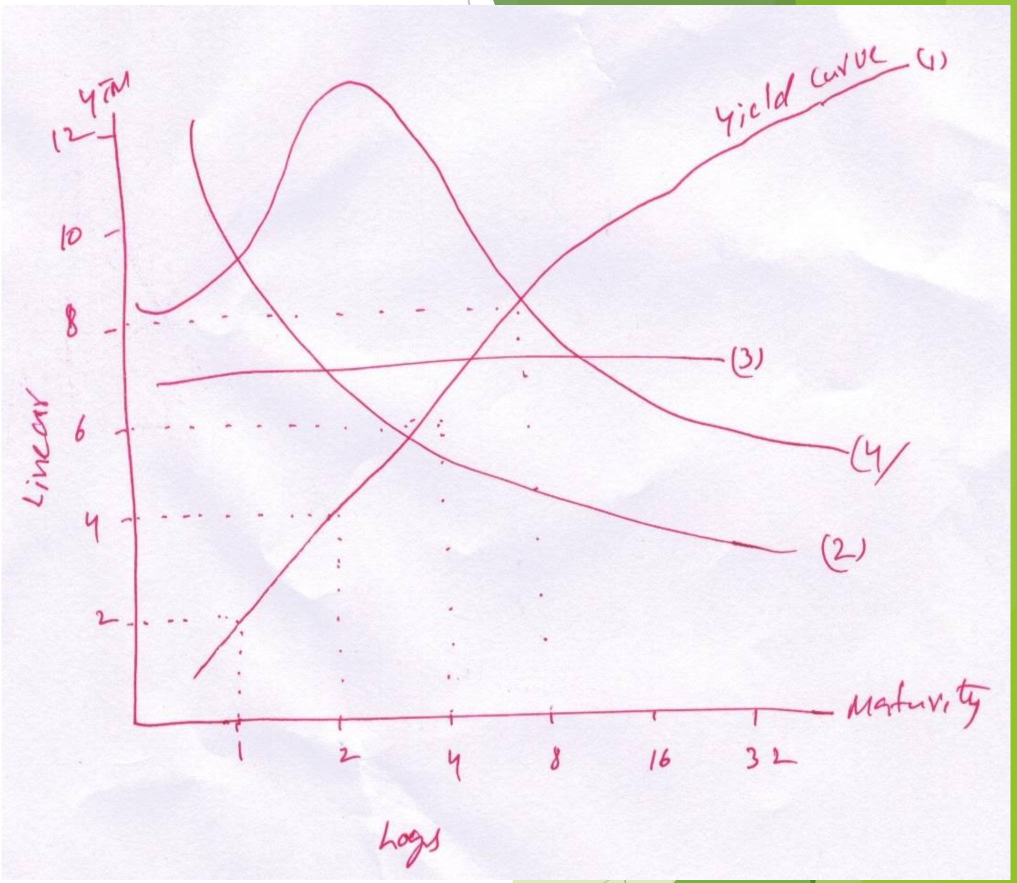
c. Unlike Treasury securities, corporate bonds have both a DRP and a LP. Therefore,

$$r_{C3} = r^* + IP + MRP + DRP + LP$$

$$= 5.7\% + 0.6\% + 0.25\% = \mathbf{6.55\%}$$

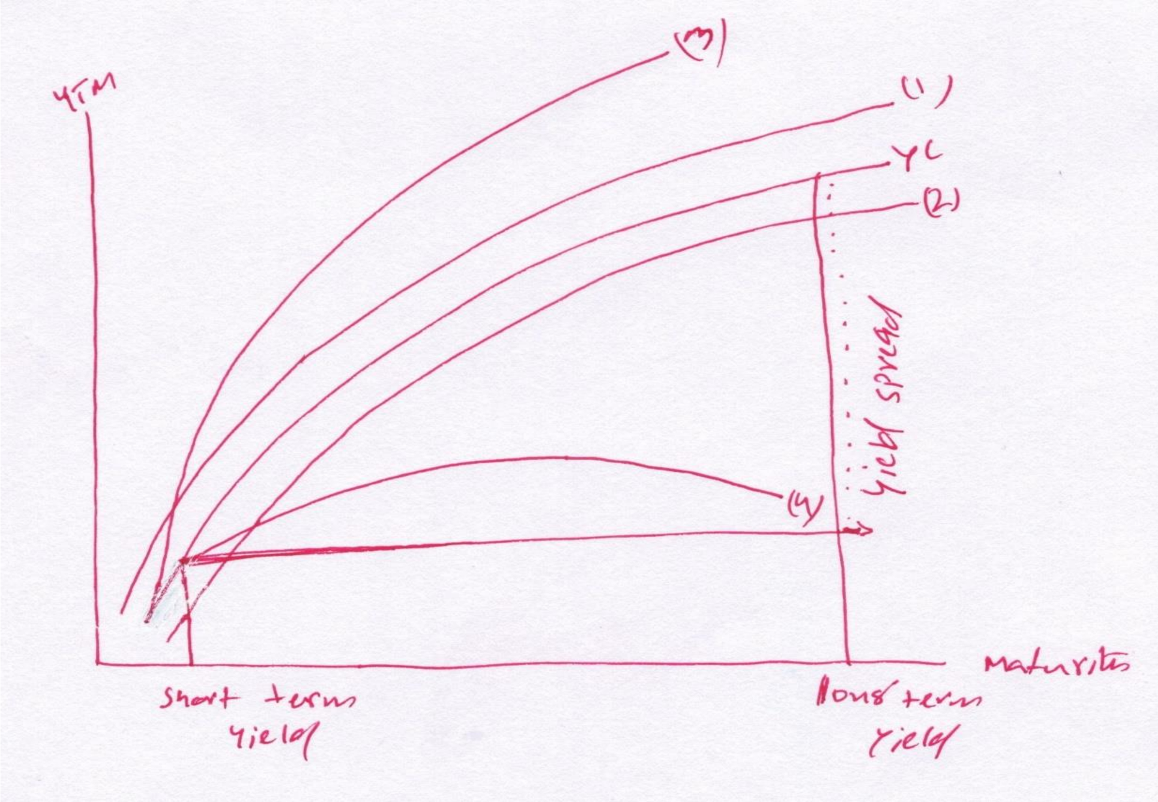
Term Structure of Interest Rates

- Term= Term to Maturity
  - Relationship between bonds yields and their maturity.
  - It is simply how yields change with maturity
- Yield curve: Graph/chart of maturity structure.
- Yield curve shapes:
  - Upward Sloping – yields↑ as the maturity ↑ (1).
  - Horizontal (3)
  - Humped (4)-medium term are higher
  - Downward Sloping –yields ↓ as the maturity raises ↑(2)
- Upward Sloping YC → Normal
- Downward YC→ Abnormal or inverted
- Horizontal YC → Flat



Term Structure of Interest Rates (Cont.)

- YIELD SPREAD – The difference between a short term yield and a long term yield.
- Yield dynamics (shapes): How yield curve changes over time?
  - Raising YC – Yield raises for all maturities (1)
  - Falling YC - Yield falls for all securities (2)
  - Steepening YC – Yield curve gets steeper (3)
    - Yield spread raises but with a steep
    - Spreads increase in the long term than the short term
  - Flattening YC –Falling Yield spread (4)



- Operational twist- Buying long term bonds and selling short term bonds, simultaneously.
- Financial Repression-Artificially keeping short and long term yields low to lower the cost of borrowings.

Term Structure of Interest Rates (Cont.)

- Determinants of shape of YC:
  - Risk free interest rate- monetary policy
  - Risk premium
- Different theories