As an overview, my DRP focused on Measure Theory with an intent to dive into Martingales next quarter. I'll overview/highlight the essential parts of the content and follow with my thoughts

We started by realizing that not all sets can be assigned a volume (Vitali Sets, Banach Tarski paradox) so it is important to restrict the kinds of sets one deals with. This gave birth to sigma algebras and measure spaces. We also covered the fundamental Borel sigma algebra and problem solving strategy of proving facts on building blocks (pi systems, lambda system) to get results on the whole.

We then proceeded to look at results on sigma algebras, including monotone convergence of measure, notions of almost surely, the First Borel Cantelli Lemma, and measurable functions. With those tools we could define random variables, the law and distribution functions. We also quickly transitioned to what independence is formally, Borel Cantelli 2, and Tail Sigma Algebras (which contain information about "limiting" events).

Moving on from the building blocks, we turned to integration. We about how one could use simple functions (sum of indicators) to not only define integration on measurable functions but also how they could be used to prove general results via the standard machine. I'll also name drop the main theorems here: (MON), (FATOU), (DOM), (SCHEFFE), (BDD).

Finally, we discussed expectation, the monotonicity and vector spaceness of Lp norms, and other fundamental facts/inequalities of note. Using this, we proved a first Strong Law (If the independent RVs' fourth moments are bounded, they converge to the expectation).We also used Chebyshev's to derive one of the cleanest proofs of WAT that I've seen.

The exercises were also notably interesting. For example, one of our exercises trailed off into a discussion on high dimensional geometry in statistics (did you know that as the number of dimensions increases, the distribution of a random gaussian vector matches that of a uniform distribution on the boundary of a high dimensional ball). Quite surprising but makes more intuitive sense when you notice that most of the volume of the ball is concentrated around the boundary.

Overall, I enjoyed the exploration of measure theory and the cool applications we saw like the one above. The 524 professor told me that (roughly) "without measure theory, any statement about probability is just intuition". After examining edge cases and wrinkles while studying the foundations, I understand him more. It is cool to have the tools to formally treat and understand the weird cases. Aside from that, I very much look forward to next quarter, where we learn about martingales and betting strategy.