Final report: Measure Theory

This semester, I completed a research project on Measure Theory under the guidance of my mentor, Nila Cibu. This research has been incredibly rewarding, as it not only deepened my understanding of foundational concepts in statistics but also taught me to approach problems with a statistical mindset.

As a non-statistics major, I had almost no prior knowledge of the field at the beginning of the research. During the initial meetings with my mentor, I often felt overwhelmed due to my unfamiliarity with statistical terminology and concepts. However, with my mentor's patient guidance, I started from the basics of statistics and gradually built my understanding of its core theoretical framework and applications.

We began our studies with Sets and Events. I learned about the definitions and theories of sets from a statistical perspective and mastered the basic computational methods and logical reasoning involved. This laid a solid foundation for my subsequent learning. Next, we delved into the concept of fields, focusing particularly on σ-fields and their associated theories, such as the properties and proofs of Borel Sets. This phase of study helped me grasp the mathematical structure and rigor of statistics.

Afterward, we explored the basic definitions and properties of Probability Spaces, with a special focus on the proof and application of Dynkin's Theorem. This topic formed the core of our discussions, providing me with an initial understanding of how to construct probability spaces through mathematical methods. From there, we progressed to more advanced topics, such as Measure Constructions. Towards the end of the semester, our focus shifted to the construction of measures, where I studied in detail how to construct the Lebesgue measure on the interval (0, 1). This process allowed me to appreciate the profound connections between statistics and analysis while equipping me with the critical steps and logical reasoning required for measure construction.

Throughout the research process, I not only gained theoretical knowledge of statistics but also came to deeply appreciate the patience and rigor needed to tackle complex problems. With the support of my mentor, I gradually built confidence and was able to independently complete key theoretical derivations and applications. This research experience has not only given me a deeper understanding of Measure Theory but also sparked a strong interest in the field of statistics, paving the way for potential interdisciplinary exploration in the future.