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Theory of Measure and Gambling SPA-DRP

Winter 2025

During this quarter, I participated in the UW Statistics Directed Reading Program where we studied the first half of *Probability with Martingales* by David Williams. Our primary focus was on measure theory, Lebesgue integration, and the measure-theoretic foundations of probability. These topics provided a rigorous framework for understanding modern probability theory and built the necessary tools for deeper explorations into stochastic processes.

One of the key concepts we explored was measure theory, where we studied σ -algebras, measurable functions, and integration with respect to a measure. This naturally led to the development of the Lebesgue integral, which generalizes the Riemann integral and allows for stronger convergence theorems. We covered important results such as the Monotone Convergence Theorem and the Dominated Convergence Theorem, which are fundamental in probability theory. Additionally, we examined probability spaces and random variables in a measure-theoretic context, developing a more formal understanding of expectation, independence, and product measures.

A particularly fascinating result that we encountered was the convergence in distribution of the multivariate Gaussian to the uniform distribution on the sphere. This result, which follows from the Central Limit Theorem and properties of high-dimensional Gaussians, highlights how probability behaves counter-intuitively in high-dimensional spaces. As dimension increases, the Gaussian distribution tends to concentrate on a thin shell around the n -sphere, leading to a uniform distribution on the unit sphere after appropriate normalization. Understanding this phenomenon required combining our measure-theoretic probability tools with geometric intuition.

Overall, this quarter provided me with a strong foundation in measure-theoretic probability and deepened my appreciation for rigorous mathematical reasoning in probability theory. The program was an invaluable experience that has prepared me for further studies in stochastic processes and statistical inference. I'm excited to continue this Directed Reading Program with my mentors Nila and Leon, and next quarter we will be studying martingales along with some basic convex geometry.