

Final Report

Introduction of Gaussian Processes

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This quarter, under my mentor's guidance, I studied the foundations of Gaussian Processes. We began by reviewing the multivariate normal distribution, because GP models rely on the key fact that for any finite set of input points, the corresponding function values follow a multivariate Gaussian distribution. This perspective helped me understand how a GP is specified by its mean function and covariance kernel, which together describe the behavior and smoothness of the functions we want to model.

From there, I studied how a Gaussian Process is fully characterized by its mean function and covariance (kernel) function, and how different kernels encode smoothness or structural assumptions about the unknown function. In GP regression, these components allow us to update prior beliefs using observed data to obtain a posterior mean and covariance. I found the posterior mean especially intuitive, since it represents a kernel-weighted combination of the training data.

A major theme of this quarter was understanding the connections between GPs, Reproducing Kernel Hilbert Spaces (RKHS), and Kernel Ridge Regression (KRR). One of the most interesting results I learned is that the GP posterior mean is mathematically equivalent to the KRR estimator under a simple mapping between parameters. This made the relationship between Bayesian and frequentist approaches much clearer to me.

I also explored some introductory ideas in causal inference, including potential outcomes, causal effects, and propensity scores. Even though causal inference is a separate area, I enjoyed seeing how both GPs and causal methods rely on modeling conditional structures.

Overall, this DRP gave me a valuable introduction to Gaussian Processes, their mathematical foundations, and their connections to both optimization and causal reasoning. I would also like to express my sincere gratitude to my mentor for the tremendous support throughout this quarter. Beyond guiding me through the mathematical foundations, my mentor also helped me understand how to structure ideas in a more formal, paper-like style. This guidance was invaluable when I was preparing written materials, including how to present definitions, motivations, and mathematical results clearly. This experience has strengthened my interest in statistical modeling and prepared me well for future study and research.