Over the Spring 2025 quarter I participated in the UW Statistics Directed Reading Program where I studied the latter half of the textbook *Probability with Martingales* by David Williams. Martingales are sequences of random variables that are not necessarily independent, but instead related to each other in a specific way. Studying the theory of martingales leads to insights into various real world situations, like gambling and stock prediction.

Before jumping into martingales, it was important to lay the necessary groundwork. I had learned most of the measure theoretic concepts last quarter, but I still had to review conditional expectation (which felt wieldy to work with). With this tool, we are able to properly describe the defining characteristic of a martingale, namely that each variable is the conditional expectation of the previous one given some set of information. After this is defined, we are able to obtain various convergence theorems (Doob's Convergence theorem) about martingales. An important type of variable called a stopping time also arises. After satisfying a set of conditions that rigorously describe the intuitive notion of checking when an event has occurred, we are also able to obtain a set of powerful theorems related to the stopping time (Optional stopping theorem).

A particularly illuminating example was that of random walks and the precursor to stochastic calculus, Brownian motion. We started by defining a random walk and noting that it is a martingale (revealing that the martingale condition actually captures a lot of potential processes). Then, we conducted various analyses on the random variable and the stopping time associated with crossing some interval. I learned that it can also be difficult to conjure the martingale that leads to the quantity you are interested in (for example to determine the expected value of the stopping time, we end up considering $M_n = S^2_n - n$, which seemingly comes out of nowhere)! We also briefly discussed how turning this into a continuous process is what gives rise to Brownian motion, again highlighting the applicability of martingales.

It was certainly a lot of fun to learn about martingales, and I enjoyed thinking about exercises related to gambling, processes, or thought experiments about monkeys typing Shakespeare. Overall, the DRP helped build my mathematical maturity and gain exposure to some more rigorous topics. It also highlighted some nuances in probability that I didn't know about before. I'm excited to see how I can apply what I've learned in my research and classes. I look forward to what comes next!