CMPSC 465 Fall 2025

Data Structures & Algorithms
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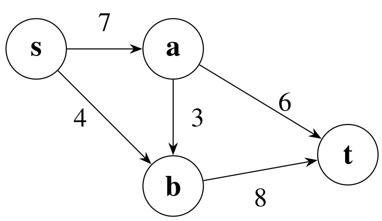
Quiz 7 Solution

Lecture Section: **Monday, Oct 20, 2025**

Student Name: **PSU Email ID:**

1. (2 pts.) Using Ford-Fulkerson, what is the value of a maximum flow in the following flow network?



(a) 9

(b) 10

(c) 11

(d) 12

Solution: (c) 11.
Using Ford-Fulkerson with augmenting paths $s \to a \to t$ (flow 6), $s \to b \to t$ (flow 4), and $s \to a \to b \to t$ (flow 1), the resulting maximum flow has value **11**.

2. (2 pts.) In the network of question 1, what is the minimum cut with respect to the maximum flow you found?

(a) $(\{s,a,b\},\{t\})$

(b) $(\{s,a\},\{b,t\})$

(c) $(\{s,b\},\{a,t\})$

(d) $(\{s\},\{a,b,t\})$

(e) $(\{s,t\},\{a,b\})$

Solution: (d) $(\{s\},\{a,b,t\})$ The min-cut produced by Ford-Fulkerson splits the vertex set into those reachable from s in the residual graph and those not reachable from s. In the above example, the max-flow saturates both edges $s \to a$ and $s \to b$. Therefore the only reachable nodes from s is s itself.

3. (2 pts.) If the augmenting $s-t$ paths are found by BFS, then the Ford–Fulkerson algorithm runs in polynomial time with respect to the number of vertices and edges in the graph.

○ True

○ False

Solution: True.
If the augmenting paths are selected arbitrarily, in the worst case, Ford-Fulkerson's running time is $O(C(|V|+|E|))$, which is not polynomial in the input size as it depends on the total capacity out of the source node. However, we mentioned in lecture (without proof) that simple modifications such as selecting the path with the largest bottleneck, or selecting the path with the fewest edges (i.e., using BFS, this is the Edmonds-Karp algorithm) yields polynomial running time.

4. (2 pts.) The Ford–Fulkerson algorithm can be adapted to work correctly when:

(a) The graph contains cycles.

(b) Edge capacities are rational numbers.

(c) There are multiple source nodeds.

(d) All of the above.

Solution: (d) All of the above.
We learned in lecture how to handle each of (a)–(c).

5. (2 pts.) Every edge (u,v) in a flow network is converted into either a forward edge (u,v) or a backward edge (v,u) in Ford-Fulkerson's residual graph. In other words, (u,v) and (v,u) never exist simultaneously in the residual graph.

○ True

○ False

Solution: False.
If an edge in the flow network has some flow routed through it that is greater than zero but less than the capacity of the edge, it will have both a forward and a backward edge in the residual graph.