Midterm1

Graded

Student

Jesse Zhang

Total Points

28.5 / 74 pts

Question 1

Multiple choice **12** / 27 pts

1.1 MC1 **3** / 3 pts

✔ **− 0 pts** Correct

− 3 pts Incorrect

1.2 MC2 **3** / 3 pts

✔ **− 0 pts** Correct

− 3 pts Incorrect

1.3 MC3 **3** / 3 pts

✔ **− 0 pts** Correct

− 3 pts Incorrect

1.4 MC4 **3** / 3 pts

✔ **− 0 pts** Correct

− 3 pts Incorrect

1.5 MC5 **0** / 3 pts

− 0 pts Correct

✔ **− 3 pts** Incorrect

1.6 MC6 **0** / 3 pts

− 0 pts Correct

✔ **− 3 pts** Incorrect

1.7 MC7 **0** / 3 pts

− 0 pts Correct

✔ **− 3 pts** Incorrect

1.8 MC8 **0** / 3 pts

− 0 pts Correct

✔ **− 3 pts** Incorrect

1.9 MC9 **0** / 3 pts

− 0 pts Correct

✔ **− 3 pts** Incorrect

Question 2

True/False **11** / 20 pts

2.1 TF1 **0** / 2 pts

– 0 pts Correct

✔ **– 2 pts** Incorrect

2.2 TF2 **2** / 2 pts

✔ **– 0 pts** Correct

– 2 pts Incorrect

2.3 TF3 **2** / 2 pts

✔ **– 0 pts** Correct

– 2 pts Incorrect

2.4 TF4 **2** / 2 pts

✔ **– 0 pts** Correct

– 2 pts Incorrect

2.5 TF5 **0** / 2 pts

– 0 pts Correct

✔ **– 2 pts** Incorrect

2.6 TF6 **0** / 2 pts

– 0 pts Correct

✔ **– 2 pts** Incorrect

2.7 TF7 **2** / 2 pts

✔ **– 0 pts** Correct

– 2 pts Incorrect

2.8 TF8 **1** / 2 pts

– 0 pts Correct

✔ **– 1 pt** Incorrect

2.9 TF9 **0** / 2 pts

– 0 pts Correct

✔ **– 2 pts** Incorrect

2.10 **TF10** **2** / 2 pts

- ✔ **− 0 pts** Correct
- **− 2 pts** Incorrect

Question 3

Recurrences **0** / 12 pts

3.1 (a) **0** / 6 pts

– 0 pts **Part 1: Recurrence relation for running time [3 pts/3 pts]**

Correctly wrote down the recurrence relation for running time as $T(n) = 64T(n/4) + \Theta(n^3)$.

– 1 pt **Part 1: Recurrence relation for running time [2 pts/3 pts]**

Could not correctly write down the entire recurrence relation, but the student was on the right track to do so by expressing the running time T as a function of input size n and showing some of the terms in recurrence correctly.

✔ **– 3 pts** **Part 1: Recurrence relation for running time [0 pts/3 pts]**

Did not attempt at all or did not express the recurrence relation for running time T as a function of input size n (no mentioning of $T(n)$ at all).

– 0 pts **Part 2: Solution for recurrence relation in O [3 pts/3 pts]**

Correctly wrote down the solution of the recurrence relation as $O(n^3 logn)$ using Master Theorem.

– 1 pt **Part 2: Solution for recurrence relation in O [2 pts/3 pts]**

Did not apply the Master Theorem and attempted to unfold the recurrence relation but could not reach the correct solution. Or found the right values for a,b, and d but could not reach the correct solution.

✔ **– 3 pts** **Part 2: Solution for recurrence relation in O [0 pts/3 pts]**

Did not attempt at all or incorrectly **applied the Master Theorem** to solve the recurrence relation.

3.2 **(b)** **0** / 6 pts

− 0 pts **Part 1: Recurrence relation for running time [3 pts/3 pts]**

Correctly wrote down the recurrence relation for running time as $T(n) = T(19) + T(29) + T(39) + T(n-1) + 1.9^n$ and *optionally* simplified the expression to $T(n) = T(n-1) + 1.9^n$.

− 1 pt **Part 1: Recurrence relation for running time [2 pts/3 pts]**

Could not correctly write down the entire recurrence relation, but the student was on the right track to do so by expressing the running time T as a function of input size n and showing some of the terms in recurrence correctly.

✔ **− 3 pts** **Part 1: Recurrence relation for running time [0 pts/3 pts]**

Did not attempt at all or did not express the recurrence relation for running time T as a function of input size n (no mentioning of $T(n)$ at all).

− 0 pts **Part 2: Solution for recurrence relation in O [3 pts/3 pts]**

Correctly wrote down the solution of the recurrence relation as $O(1.9^n)$ either by *mentioning HW1 Q5(e)* or unfolding the recurrence.

− 1 pt **Part 2: Solution for recurrence relation in O [2 pts/3 pts]**

Did not apply the Master Theorem and attempted to unfold the recurrence relation but could not reach the correct solution.

✔ **− 3 pts** **Part 2: Solution for recurrence relation in O [0 pts/3 pts]**

Did not attempt at all or incorrectly **applied the Master Theorem** to solve the recurrence relation.

Question 4

Algorithms **5.5** / 15 pts

4.1 **(a)** **2.5** / 5 pts

- **– 0 pts** Correct
- ✔ **– 2.5 pts** Only one of M_L or M_R is incorrect
- **– 5 pts** Both M_L and M_R are incorrect/not written

4.2 **(b)** **3** / 5 pts

- **– 0 pts** Correct
- **– 1 pt** Justification for relation between M_L and M_R to majority of A not provided/incorrect
- ✔ **– 2 pts** Incorrect answer for how M_L and M_R are related to majority of A, or provided the relation specific to the example not in general
- **– 3 pts** Missing answer for how M_L and M_R are related to majority of A
- **– 2 pts** Missing/Incorrect answer for majority of A

4.3 **(c)** **0** / 5 pts

- **– 0 pts** Correct
- ✔ **– 5 pts** wrong algorithm and time complexity
- **– 4.75 pts** only running time correct
- **– 0.5 pts** wrong running time
- **– 3 pts** idea of the algorithm
- **– 1 pt** paritially correct algorithm
- **– 4 pts** attempt on algorithm
- **– 2 pts** mostly correct

Midterm 1 - D

Name: Jesse Zhang

Penn State access ID (xyz1234) in the following box:

zfz5311

Student ID number (9XXXXXXXX):

975557148

TA and/or section time:

MWF 4:35

Instructions:

- Answer all questions. Read them carefully first. Be precise and concise. Handwriting needs to be neat. Box numerical final answers.
- Please clearly write your name and your PSU access ID (i.e., xyz1234) in the box on top of **every page**.
- Write in only the space provided. You may use the back of the pages only as scratch paper. **Do not write your solutions in the back of pages!**
- **Do not write outside of the black bounding box**: the scanner will not be able to read anything outside of this box.
- We are providing two extra page at the end if you need extra space. Make sure you mention in the space provided that your answer continues there.

Good luck!

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Multiple choice questions *(27 points)*

For each of the following questions, select the right answer by filling the corresponding grading bubble.

1. If $T(n) = 3T(n/2) + O(n^2)$, which of the following is the tightest upper bound for $T(n)$?

   Answer.

   ○ $T(n) = O(n^{\log_3 2})$
   ○ $T(n) = O(n^{\log 3})$
   ● $T(n) = O(n^2)$
   ○ $T(n) = O(n^2 \log n)$
   ○ None of the above

2. While performing InsertionSort to sort the array $[4, 1, 2, 3, 7, 8, 5, 6]$ in **descending** order, which of the following is NOT a transition state of the array?

   Answer.

   ○ $[4, 2, 1, 3, 7, 8, 5, 6]$
   ○ $[8, 7, 4, 3, 2, 1, 5, 6]$
   ○ $[4, 3, 2, 1, 7, 8, 5, 6]$
   ○ $[7, 4, 3, 2, 1, 8, 5, 6]$
   ○ $[8, 7, 5, 4, 3, 2, 1, 6]$
   ● None of the above

3. If $T(n) = 9T(n/3) + O(n^2 + n(\log n)^5)$, which of the following is the tightest upper bound for $T(n)$?

   Answer.

   ○ $T(n) = O(n^2)$
   ● $T(n) = O(n^2 \log n)$
   ○ $T(n) = O(n(\log n)^5)$
   ○ $T(n) = O(n^2(\log n)^5)$
   ○ None of the above

4. Suppose that $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$. Which of the following must be true?

   Answer.

   ● $f(n) = O(g(n))$
   ○ $f(n) = \Omega(g(n))$
   ○ $f(n) = \Theta(g(n))$
   ○ None of the above

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5. Consider the Partition function used in QuickSort. If we choose the last element as the pivot, what is the resulting array after calling Partition on $[5, 8, 6, 7, 1, 3, 2, 9, 4]$?

Answer.

○ $[1, 2, 3, 4, 5, 6, 7, 8, 9]$
○ $[1, 3, 2, 4, 5, 8, 6, 7, 9]$
○ $[1, 2, 3, 4, 9, 8, 7, 6, 5]$
○ $[1, 3, 2, 4, 5, 8, 6, 9, 7]$
○ $[1, 3, 2, 7, 5, 8, 6, 9, 4]$
● None of the above

4 5 8 6 1
4 1 8 6 5
1 4 8 6 5 7
1 4 7 6 5 8
4 5 8 6 7 1
4 5 1 6 7 8
1 5 4 6 7 8 3
1 5 3 6 7 8 4 2
1 5 3 2 7 8 4 6
1 2 3 5 7 8 4 6 9

~~6. Run MergeSort to sort the array $[5, 2, 9, 4, 6, 1, 10, 7, 3, 11]$ in ascending order, which two subarrays are the inputs to the final Merge call that produces the fully sorted array?~~

Answer.

○ $[5, 2, 9, 4, 6]$ and $[1, 10, 7, 3, 11]$
○ $[2, 5, 4, 9, 6]$ and $[1, 10, 3, 7, 11]$
● $[1, 2, 4, 5, 6, 9]$ and $[3, 7, 10, 11]$
○ $[2, 4, 5, 6, 9]$ and $[1, 3, 7, 10, 11]$
○ $[3, 5, 6, 9, 10]$ and $[1, 2, 4, 7, 11]$
○ None of the above

1 5
1 2 5 4 9
1 2 3 4 5 6 7 9 10 11

7. How many integer multiplications does Strassen's algorithm need to do to multiply two 4×4 matrices of integers?

Answer.

○ 7
○ 8
○ 9
○ 49
● 64
○ None of the above

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8. Running the Median-of-Medians algorithm with group size 5 to select the 3rd smallest element from the array [8, 2, 4, 13, 5, 1, 7, 10, 14, 15, 9, 6, 11, 3, 12], which number is the first pivot used to partition the array?

Answer.

- ○ 7
- ○ 12
- ○ 9
- ● 5
- ○ 8
- ○ None of the above

9. ~~Consider running InsertionSort (for sorting in ascending order) on an array with the first~~ $\cancel{n - \lceil n^{1/3} \rceil}$ ~~elements already sorted in ascending order. What is the tightest upper bound of its running time?~~

Answer.

- ○ $O(n^{1/3})$
- ○ $O(n - (n)^{1/3})$
- ○ $O(n^{4/3})$
- ○ $O(n \log n)$
- ● $O(n^2)$

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True/False *(20 points)*

True or false? Fill in the correct bubble. No justification is needed.

T	F	
●	○	$n^{0.1} \log n = O(\log n)$.
●	○	$\log n / n = O(n^{1/n})$.
●	○	$(\sqrt{n})^n = \Omega(n^{\sqrt{n}})$.
●	○	$\log(n!) = \Omega(n)$.
●	○	If $f(n)$ and $g(n)$ are non-negative functions, then either $f(n) = O(g(n))$ or $g(n) = O(f(n))$.
●	○	If $f(n) = O(g(n))$, then there must be some value of $n > 0$ where $f(n) \leq g(n)$.
○	●	If $f(n)$ is a nonnegative function, then $f(n+1) = \Theta(f(n))$.
○	●	The best-case time complexity of InsertionSort is $O(n^2)$.
●	○	The information theory lower bound of the comparison-based insertion problem is $\Omega(n)$.
●	○	The median-of-medians selection algorithm runs in $O(n)$ time with group size 7.

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Recurrences *(12 points)*

Each of the following scenarios outlines a divide-and-conquer algorithm. In each case, write down the appropriate recurrence relation for the running time as a function of the input size n and give its solution. Giving your solution in big-O notation. You need not give a full derivation of your solution, but you should indicate how you arrived at it (e.g., by using the Master Theorem). You may assume that n is of some special form (e.g., a power of two or of some other number), and that the recurrence has a convenient base case with cost $\Theta(1)$.

(a) An input of size n is broken down into 64 subproblems, each of size $n/4$. The time taken to construct the subproblems, and to combine their solutions, is $\Theta(n^3)$.

r $\frac{n}{4}$

let $n=1$

$64 \times \frac{1}{4} = 13.5$

so the conclusing is $\Theta(n^3)$

(b) An input of size n is broken down into four subproblems of sizes 19, 29, 39, and $n-1$ respectively. The time taken to construct the subproblems, and to combine their solutions, is 1.9^n.

because it is alway be definition Θ $(n \log n)$

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Algorithms. *(15 points)*

Suppose the instructor is considering several possible grading curves for the exam, and you are asked to collect students' opinions. Let an array $A[1..n]$ represent the choices of n students. A *majority* choice is one that is selected by more than $n/2$ students. For example, $A = [a, a, b, a, c, c, a]$ has a majority choice a because it appears in $4 > 7/2$ positions; on the other hand, $A = [1, 4, 3, 1, 3, 3]$ does not have a majority choice because no choice appears in more than $6/2 = 3$ positions.

You would like to design a divide-and-conquer algorithm to find the majority choice (if it exists):

(a) Consider the following array $A[1..16] = [5, 2, 2, 5, 5, 2, 5, 2, 3, 5, 3, 5, 5, 5, 2, 5]$. Divide it into two equal-sized halves. Let M_L and M_R be the majority choices of the left and right halves of the array, respectively. Do M_L and M_R exist? If so, what are they?

$M_L = 8$ $M_R = 8$

only the part contain more than 4 times are exists 5 and

M_{left} didn't exit because no mchoice more than

$= [3, 5, 3, 5, 5, 5, 2, 5]$

(b) In the example of part (a), does the array A have a majority choice? In general, how is the majority choice of A related to M_L and M_R? Justify your answer.

Yes, A majority choice is $5 > \frac{16}{2}$ position

Only 9 times 5 appear and there is 16 position, so only the side have more than 4 times about 5 have M_R or M_L

(c) Based on your answer for part (b), describe a general divide-and-conquer algorithm that finds a majority choice of an array or return NULL if it doesn't exist. What is the running time of your algorithm?

3

3 3 5 5 5 5 5 5 5 5 2 2 2 2

Should be NULL

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45768 1

45867

45867

45

3

15367842

1532784 6

1235784 69

1234 458671

4 33
4|64
4
14
12
20

4

45 45867

45867

451678

1546783

1536784

8

478

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492

4 2 1 3 7 8 5 6

1

17 123

2 9 4 3

2 3 4 9 1

2 1 4 9 3

2

1 25 8 7 4

4 15

432

7 8 1

1 2 4 9 3 7

9

a=3 b=2 d=2

$\log_2 3 < d = 2$

85

$\log_3 9 = 3$ d=5

1 2 4 9 3 7 6