Bounded Model Checking

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Automatic System Verification

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Introduction

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  - it costed $\approx$ US $475 million;
  - big investment in formal verification.

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- distinctive features:
  - fully automatic;
  - exhaustive;
  - it generates a counterexample trace if the specification does not hold.

Linear Temporal Logic

We consider LTL model checking.

- LTL syntax:

$$\begin{aligned} p \mid \neg\phi \mid \phi_1 \vee \phi_2 \mid \phi_1 \wedge \phi_2 \\ \mid \mathsf{X}\,\phi_1 \mid \phi_1\,\mathcal{U}\,\phi_2 \\ \mid \phi_1\,\mathcal{R}\,\phi_2 \mid \mathsf{F}\,\phi_1 \mid \mathsf{G}\,\phi_1 \end{aligned}$$

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\end{aligned}
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- shortcuts:
  - $\phi_1\,\mathcal{R}\,\phi_2 \equiv \neg(\neg\phi_1\,\mathcal{U}\,\neg\phi_2)$,
  - $\mathsf{F}\,\phi_1 \equiv \top\,\mathcal{U}\,\phi_1$
  - $\mathsf{G}\,\phi_1 \equiv \neg\,\mathsf{F}\,\neg\phi_1$

Linear Temporal Logic

- Semantics. LTL formulas are interpreted over infinite *state sequences* $\sigma = \langle \sigma_0, \sigma_1, \ldots \rangle \in (2^\Sigma)^\omega$ of sets of propositions $\sigma_i \in 2^\Sigma$:

$$
\begin{array}{lll}
\sigma \models_i p & \text{iff} & p \in \sigma_i \\
\sigma \models_i \mathsf{X}\,\phi & \text{iff} & \sigma \models_{i+1} \phi \\
\sigma \models_i \phi_1\,\mathcal{U}\,\phi_2 & \text{iff} & \text{there exists } j \geq i \text{ such that} \\
 & & \sigma \models_j \phi_2 \text{ and } \sigma \models_k \phi_1 \text{ for all} \\
 & & i \leq k < j \\
\ldots & & \\
\sigma \models_i \mathsf{F}\,\phi & \text{iff} & \exists j \geq i\ .\ \sigma \models_j \phi \\
\sigma \models_i \mathsf{G}\,\phi & \text{iff} & \forall j \geq i\ .\ \sigma \models_j \phi
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$$

LTL model checking

- LTL model checking:
  - decide if $\mathcal{M}, s \models \phi$, where $\mathcal{M} = (S, I, T, L)$ is a Kripke structure, $s \in I$ is an initial state and ϕ is an LTL formula; in many contexts, you may find the notation: $\mathcal{M}, s \models A\phi$;
  - PSPACE-complete.

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$$\mathcal{M}, s \not\models \phi$$

State-space Explosion Problem

- the previous algorithm belongs to the class of explicit model checking algorithms:
  - the Kripke Structure $\mathcal{M}$ is represented as a set of memory locations, pointers ecc...

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is exponential in n;
- the size of system that could be verified by explicit model checkers was restricted to $\approx 10^6$ states.

Tackling the explosion...

Three main techniques have been proposed:

- BDD-based symbolic model checking
- partial order reduction
- SAT-based symbolic model checking, aka Bounded Model Checking.

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They allowed for the verification of systems with $> 10^{20}$ states.

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- why not in DNF?

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- there are several efficient algorithms for solving SAT (*e.g.*, DPLL, CDCL...) along with many heuristics (*e.g.*, 2 watching literals, glue clauses...)
- some numbers:
  - $>$ 100˙000 variables;
  - $>$ 1˙000˙000 clauses;

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The corresponding symbolic Kripke structure is the tuple $(\bar{s}, f_I, f_T, \{f_{p_1}, \dots, f_{p_k}\})$.

Symbolic Transition Systems

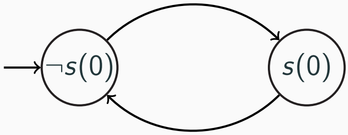
- we will write simply $\mathcal{M} = (S, I, T, L)$, meaning a symbolic transition system
- a path (or trace) $\pi = m_0, m_1, \dots$ is an infinite sequence of assignment to the state variables such that:
  - $m_0 \models I(s)$;
  - $m_i, m'_{i+1} \models T(s, s')$ holds, for all $i \geq 0$.

  where $\bar{s}' := \{s'(0), \dots, s'(n)\}$.

Example 1

simple-example.smv

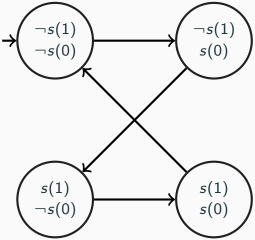
Example 1 - SMV




```
MODULE main
VAR
  s0 : boolean;
INIT
  !s0;
TRANS
  s0 <-> next(!s0);
```

modulo-4-counter.smv

Example 2 - SMV




```
MODULE main
VAR
  s0 : boolean;
  s1 : boolean;
INIT
  !s0 & !s1;
TRANS
  (next(s0) <-> !s0)
    &
  (next(s1) <-> ((s0 & !s1) | (!s0 & s1)));
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Bounded Model Checking

- recall that we can reduce $\mathcal{M}, s \models \psi$ to checking the emptiness of $\mathcal{M} \times \mathcal{A}_{\neg\psi}$;
  - the universal problem $\mathcal{M}, s \models A\psi$ is reduced to the existential problem $\mathcal{M}, s \models E\phi$, where $\phi := \neg\psi$;
- Bounded Model Checking (BMC) solves the problem $\mathcal{M}, s \models E\phi$ by proceeding incrementally:
  - we start with $k = 0$;
  - check if there exists and execution π of $\mathcal{M}$ of length k that satisfies ϕ; encode this problem into a SAT instance and call a SAT-solver;
  - if so, we have found a counterexample to ψ; if not, $k{+}{+}$.

Loop-backs

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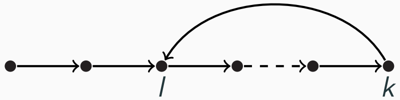
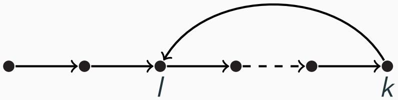
Crucial observation:

- a finite trace can still represent an infinite state sequence, if it contains a loop-back.





Definition (k-loop)

A path π is a (k, l)-loop, with $l \leq k$, if $T(\pi(k), \pi(l))$ holds and $\pi = u \cdot v^{\omega}$, where:

- $u = \pi(1) \dots \pi(l-1)$;
- $v = \pi(l) \dots \pi(k)$.

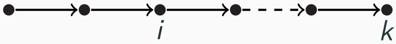
We call π a k-loop if there exists $l \leq k$ for which π is a (k, l)-loop.

BMC

Given a finite trace π of the system $\mathcal{M}$, BMC distinguishes between two cases:

- either π contains a loop-back (π is lasso-shaped):
  - $\Rightarrow$ apply standard LTL semantics to check if $\pi \models \phi$;
- or π is loop-free:
  - $\Rightarrow$ apply bounded semantics
  - $\Rightarrow$ if a path is a model of ϕ under bounded semantics then <u>any extension of the path</u> is a model of ϕ under standard semantics (conservative semantics)

Bounded Semantics for LTL



If π is not a k-loop, we introduce bounded semantics for LTL.

Definition (Bounded semantics for LTL)

Let $k \geq 0$ and π a path that is not a k-loop. An LTL formula ϕ is valid along π with bound k, written $\pi \models_k^0 \phi$, iff:

- $\pi \models_k^i p$ *iff* $p \in L(\pi(i))$
- $\pi \models_k^i \neg p$ *iff* $p \notin L(\pi(i))$

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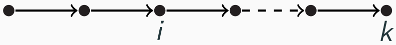

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- $\pi \models_k^i \phi_1 \vee \phi_2$ *iff* $\pi \models_k^i \phi_1$ *or* $\pi \models_k^i \phi_2$
- $\pi \models_k^i \phi_1 \wedge \phi_2$ *iff* $\pi \models_k^i \phi_1$ *and* $\pi \models_k^i \phi_2$

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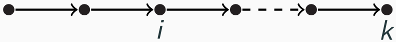

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- $\pi \models_k^i \mathsf{X}\, \phi_1$ *iff* $i < k$ *and* $\pi \models_k^{i+1} \phi_1$
- $\pi \models_k^i \phi_1 \,\mathcal{U}\, \phi_2$ *iff* $\exists i \leq j \leq k$ *such that* $\pi \models_k^j \phi_2$ *and* $\forall i \leq n < j$ *it holds that* $\pi \models_k^n \phi_1$

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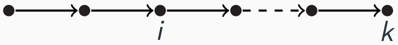

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- $\pi \models_k^i \mathsf{G}\,\phi_1$ *iff* *???*
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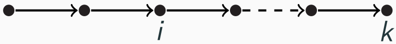

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- $\pi \models_k^i \mathsf{G}\,\phi_1$ *is always false*
- $\pi \models_k^i \mathsf{F}\,\phi_1$ *iff* $\exists i \leq j \leq k$ *such that* $\pi \models_k^j \phi_1$

SAT-based encoding of BMC

Now we see how to reduce BMC to SAT.

- the first thing to do is to define a Boolean formula that encodes all the paths of $\mathcal{M}$ of length k.

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Definition (Unfolding of the Transition Relation)

For a Kripke structure $\mathcal{M}$ and $k \geq 0$, we define:

$$[\![\mathcal{M}]\!]_k := I(s_0) \wedge \bigwedge_{i=0}^{k-1} T(s_i, s_{i+1})$$

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What does a model of $[\![\mathcal{M}]\!]_k$ represent?

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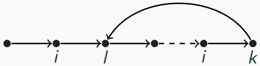
We have seen that BMC distinguishes between lasso-shaped (k-loop) and loop-free paths:

- we start with the encoding in case of k-loops.

Encoding of a loop



Definition (Loop Encoding)

Let $l \leq k$. We define:

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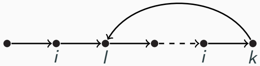

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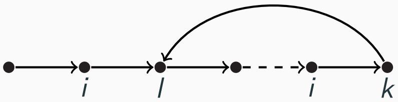
Definition (Successor in a Loop)

Let $l, i \leq k$ and π be a (k, l)-loop. We define the successor $succ(i)$ of i in π as:

- $succ(i) := i + 1$ *if $i < k$;*
- $succ(i) := l$ *if $i = k$.*

Encoding in case of Loop

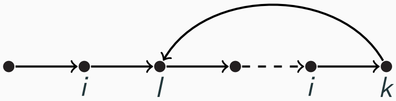


Definition (Encoding of an LTL formula for a (k, l)-loop)

Let ϕ be an LTL formula and $l, i, k \geq 0$ such that $l, i \leq k$. We define ${}_l[\![\phi]\!]_k^i$ recursively as follows:

- ${}_l[\![p]\!]_k^i := p(s_i)$
- ${}_l[\![\neg p]\!]_k^i := \neg p(s_i)$

Encoding in case of Loop



Definition (Encoding of an LTL formula for a (k, l)-loop)

Let ϕ be an LTL formula and $l, i, k \geq 0$ such that $l, i \leq k$. We define ${}_l[\![\phi]\!]_k^i$ recursively as follows:

- ${}_l[\![\phi_1 \vee \phi_2]\!]_k^i := {}_l[\![\phi_1]\!]_k^i \vee {}_l[\![\phi_2]\!]_k^i$
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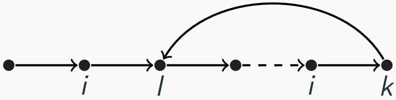

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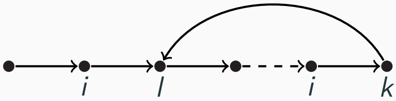
- ${}_l[\![\mathsf{X}\,\phi_1]\!]_k^i := {}_l[\![\phi_1]\!]_k^{succ(i)}$
- ${}_l[\![\phi_1\,\mathcal{U}\,\phi_2]\!]_k^i := {}_l[\![\phi_2]\!]_k^i \vee \left({}_l[\![\phi_1]\!]_k^i \wedge {}_l[\![\phi_1\,\mathcal{U}\,\phi_2]\!]_k^{succ(i)}\right)$

Encoding in case of Loop



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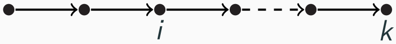
- ${}_l[\![\mathsf{F}\,\phi_1]\!]_k^i := {}_l[\![\phi_1]\!]_k^i \vee {}_l[\![\mathsf{F}\,\phi_1]\!]_k^{succ(i)}$

Encoding in case of NO Loops

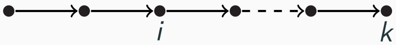


Definition (Encoding of an LTL formula for a loop-free path)

Let ϕ be an LTL formula and $i, k \geq 0$. We define $[\![\phi]\!]_k^i$ recursively as follows:

- $[\![\phi]\!]_k^{k+1} := \bot$

Encoding in case of NO Loops



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- $[\![p]\!]_k^i := p(s_i)$
- $[\![\neg p]\!]_k^i := \neg p(s_i)$

with $i \leq k$

Encoding in case of NO Loops

$$\bullet \longrightarrow \bullet \longrightarrow \underset{i}{\bullet} \longrightarrow \bullet \dashrightarrow \bullet \longrightarrow \underset{k}{\bullet}$$

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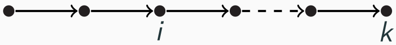

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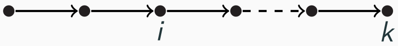

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Overall encoding

Definition (Overall encoding)

Let ϕ be an LTL formula, $\mathcal{M}$ be a Kripke structure and $k \geq 0$:

$$[\![M, \phi]\!]_k := \underbrace{[\![\mathcal{M}]\!]_k}_{\substack{\text{encoding of}\\ \text{the machine}}} \wedge \Bigg(\big(\underbrace{\neg L_k \wedge [\![\phi]\!]_k^0}_{\substack{\text{loop-free}\\ \text{models}}} \big) \vee \underbrace{\bigvee_{l=0}^{k} \big({}_l L_k \wedge {}_l[\![\phi]\!]_k^0 \big)}_{\substack{\text{lasso-shaped}\\ \text{models}}} \Bigg)$$

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Theorem (Soundness)

$[\![\mathcal{M}, \phi]\!]_k$ *is satisfiable iff* $\mathcal{M} \models_k E\phi$.

Completeness

Algorithm:

- start with $k = 0$
- call a SAT-solver on $[\![\mathcal{M}, \phi]\!]_k$
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What happens if $\mathcal{M} \not\models \phi$?

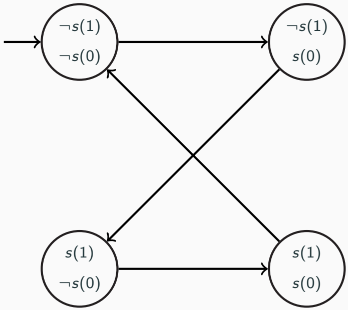
- the procedure does not terminate
- in order to be complete, BMC needs to compute the recurrence diameter: very costly
- BMC is mainly used as a bug finder, rather than as a prover.

Questions?

Appendix

Example

modulo-4-counter.smv



- $\phi_1 \coloneqq \mathsf{G}\,\mathsf{F}(s(0) \wedge s(1))$ ✓
- $\phi_2 \coloneqq \mathsf{F}\,\mathsf{G}(\neg s(0) \wedge \neg s(1))$ ✗

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- model checking:

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- satisfiability checking

$$\top \wedge \left((\neg L_k \wedge [\![\phi]\!]_k^0) \vee \bigvee_{l=0}^{k} ({}_l L_k \wedge {}_l[\![\phi]\!]_k^0) \right)$$

BLACK

- we developed this tool based on the idea of *bounded satisfiability checking*
- BLACK = Bounded Ltl sAtisfiability ChecKer [1]

[1] https://github.com/black-sat/black