Cryptographic Primitives for Zero-Knowledge: Theory and Implementation

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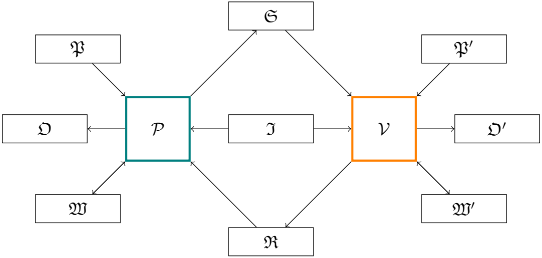
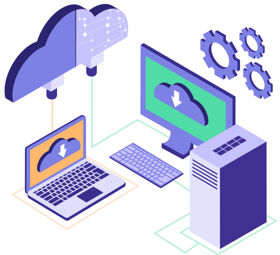
Interactive Proof Systems [GMR89]



- Prover: wants to prove a statement by creating a proof.
- Verifier: wants to check the soundness of the proof.
- Modeled as *interactive I/O probabilistic Turing machines*.
- Verifier is polynomially bounded.
- Verifier might be fooled with *negligible* probability.
- IP = PSPACE [Sha92].

Verifiable Computation



Proof systems can be used for *verifiable computation*:

- Delegating heavy loads to the cloud [ACK$^+$02].
- Calculate household due bills [PGHR13].
- **Verifying transactions on the blockchain.** [BSCG$^+$14]

ZK-SNARK systems

Completeness

Soundness

Zero-Knowledge

ZK-SNARK systems:

- Prover might be **dishonest** $\implies$ proof system.
- Verifier might be **curious** $\implies$ *Zero-Knowledge*.
- Verification must be fast $\implies$ *Succinct*.
- There may be many verifiers $\implies$ *Non-interactive*.
- Prover is polynomially bounded $\implies$ *Argument of Knowledge*.

SNARKs via QAPs [GGPR12, PGHR13, Gro16]

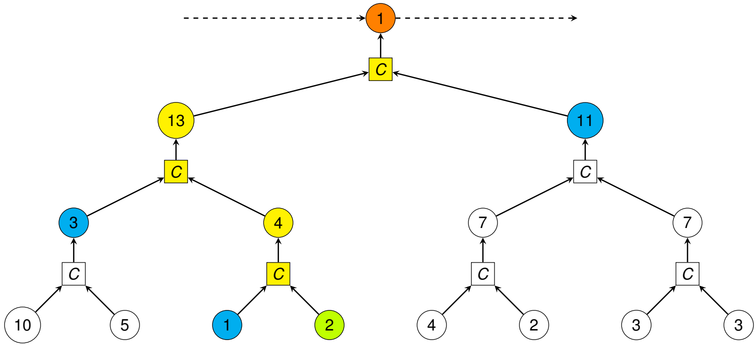
high-level language		low-level logic	satisfiability encoding	verification protocol
C →	IL →	ARITH GATES →	QAP →	ECC
	C exprs	+, *, *split*	*polynomials*	verification

Setting up a SNARK for some computable function:

1. Bounded computations represented through *arithmetic circuits*.
2. *Rank-1 constraint systems (R1CSs)* encode circuit invariants.
3. *Quadratic Arithmetic Programs (QAPs)* “compress” R1CSs.
4. *Private key* to build the proof, *public key* to verify it.
5. Exploit *bilinear maps*, work in the exponent: discrete $\log$ is hard!
6. Inject random noise for statistical zero-knowledge.

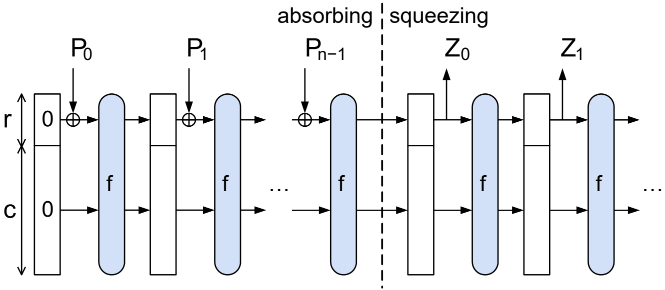
Generating the keys incurs into the *toxic waste* problem. . .

The Blockchain



- Groups of transactions are leaves of a *Merkle tree* [Mer88].
- Bottom-up computation using a **compression function**.
- The root contains the *commitment* (among other data).
- Verify a commitment following the *authentication path*.

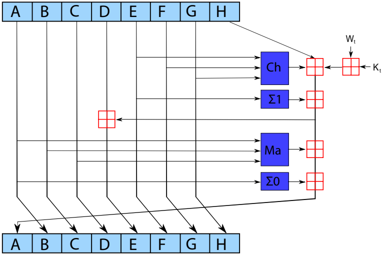
Cryptographic Compression Functions



One-way compression functions (OWCF):

- Many inputs reduced to a few outputs (e.g. 2-to-1).
- Easy to compute, but hard to invert and find collisions.
- Usually derived from one-way permutations.
- Constructed through secure schemes, like Sponge [BDPVA07].

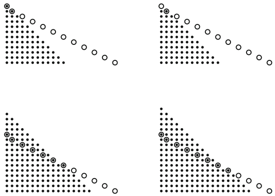
SHA [Dan15]



Standard designs, like SHA, are designed over *boolean fields*:

- Bitwise AND, XOR, rotation, modulo 2^k addition...
- Extremely efficient hardware and software implementations.
- However, ZK-SNARKs work over prime fields $\implies$ emulation.
- SHA-256 $\approx$ 25000 constraints.
- Can we do better?

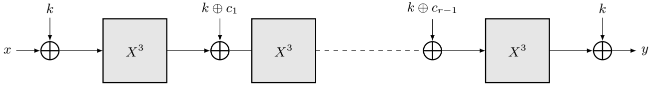
Arithmetization Oriented Primitives



Arithmetization-Oriented (AO) cryptographic primitives:

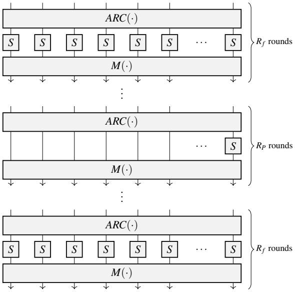
- Build keyed permutations using prime field sum and multiplication.
- Apply a secure scheme to get a compression function.
- AO primitives can be modeled as polynomials.
- Must be protected against *classic* and *algebraic* attacks.

MiMC [AGR+16]



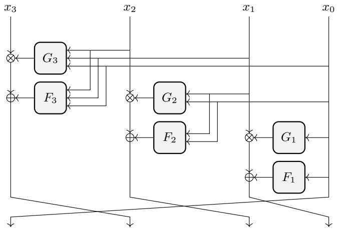
- MiMC: Minimal Multiplicative Complexity.
- Extremely simple: *round function* is $x^3 + c$.
- Exponent is the lowest integer in $\mathbb{F}_p$ coprime with $p - 1$.
- Many rounds to be secure against *algebraic attacks*.
- MiMCHash-256: 640 constraints.

POSEIDON [GKR+21]



- POSEIDON: Partial *substitution-permutation network* (SPN).
- Full rounds defend against classic attacks.
- Partial rounds defend against algebraic attacks.
- POSEIDON-256: 240 constraints.

GRIFFIN [GHR+22]



- GRIFFIN is based on the `Horst` scheme: $(x, y) \mapsto (y, x \otimes G(y))$.
- Circulant MDS matrix in the linear layer.
- Inverse power to achieve faster degree growth [AABS+19].
- GRIFFIN-256: 96 constraints.

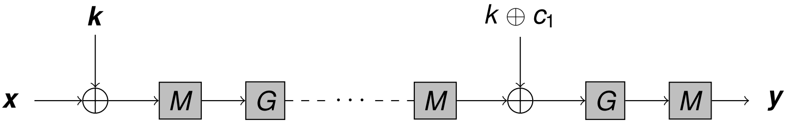
The GTDS [RS22]

$$
\begin{array}{lcl}
x_1 & \longmapsto & f(x_n, x_{n-1}, \ldots, x_2, x_1) \\
x_2 & \longmapsto & f(x_n, x_{n-1}, \ldots, x_2) \\
\cdots & \longmapsto & \cdots \\
x_{n-1} & \longmapsto & f(x_n, x_{n-1}) \\
x_n & \longmapsto & f(x_n)
\end{array}
$$

The new *Generalized Triangular Dynamical System* (GTDS):

- Includes and improves previous design strategies.
- $f(x_n) = y_n = x^{1/d_2}; \quad f(x_n, \ldots, x_i) = y_i = x_i^{d_1} g_i(\sigma_{i+1}) + h_i(\sigma_{i+1}).$
- $\sigma_i = \sum_{j=i}^{n} x_j + y_j; \quad g_i(x) = x^2 + \alpha_i x + \beta_i; \quad h_i(x) = x^2 + \gamma_i x.$
- π-equivalence: constraint systems unaffected by permutations.

Arion and ArionHash [RST23]



Arion, a new keyed permutation from the GTDS:

- Exponent d_2: easy to exponentiate by, inverse is big.
- Affine layer is an MDS circulant matrix easy to multiply by.
- Achieves degree overflow in just one round.
- ArionHash: OWCF based on Arion in sponge mode.
- α-ArionHash: 76 constraints, same guarantees as competitors.

Comparisons

`libsnark`: used by ZCash [BSCG$^+$14] for its blockchain.
We used it to implement:

- Several primitives designed for ZK-SNARK, including ours.
- A self-parametrizing Merkle tree.
- A new mode of hash, the Augmented Binary tRee [ABR21].

Proof generation times for MT commitments over 256-bit prime fields

Tree height	α-ArionHash	GRIFFIN	POSEIDON
4	73 ms	88 ms	186 ms
8	145 ms	181 ms	386 ms
16	278 ms	338 ms	745 ms
32	509 ms	622 ms	1422 ms

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