ELASTO-VISCOPLASTICITY
1D numerical solver

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Consider the following elastic-viscoplastic constitutive model, which is defined – quite generally, in 3D – by the equations:

$$\begin{cases} \dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^{el} + \dot{\boldsymbol{\varepsilon}}^{vp} \\ \dot{\boldsymbol{\varepsilon}}^{el} = \dfrac{1+\nu}{E}\dot{\boldsymbol{\sigma}} - \dfrac{\nu}{E}\mathrm{tr}(\dot{\boldsymbol{\sigma}})\mathbf{I} \\ \dot{\boldsymbol{\varepsilon}}^{vp} = \dfrac{\sqrt{3}}{2\bar{\sigma}}\left[\dfrac{\bar{\sigma}-\sigma_Y}{K\bar{\varepsilon}^n\sqrt{3}}\right]_+^{1/m}\mathbf{s} \end{cases} \tag{1}$$

The strain rate tensor is additively decomposed in an elastic and a viscoplastic part (1a). The Hooke law (1b) links the elastic strain rate to the time derivative of the stress tensor. The viscoplastic flow rule (1c) includes a stress threshold σ_Y: the bracket $[x]_+$ is the positive part of the scalar x ($[x]_+ = x$ if $x > 0$ and $[x]_+ = 0$ otherwise).

The constitutive parameters are K the viscoplastic consistency, n the strain hardening coefficient, and m the strain rate sensitivity.

To begin with, show that the relationship between the von Mises stress, the generalized strain and the generalized strain rate takes the form:

$$\bar{\sigma} = \sigma_Y + K(\sqrt{3})^{m+1}\bar{\varepsilon}^n\dot{\bar{\varepsilon}}^m \tag{2}$$

The problem consists then in solving those equations in the context of a one-dimensional mechanical test, which can be either tensile or compressive, according to Figure 1.

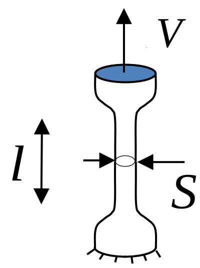

Figure 1: One-dimensional traction/compression test.

The work zone of the specimen is assumed to be a plain cylinder at any instant of the test (no heterogeneity, no necking, no buckling). The boundary condition consists in a controlled upward or downward velocity $V(t)$, which is applied to the upper end of the specimen; the lower end remains fixed. The length of the work zone is denoted l, and S is its transverse section.

It is asked to develop an incremental numerical solver which should be able, starting from a known state at time t (cumulated plastic strain, stress, length of work zone), and considering the applied boundary conditions, to determine the new state at time $t+\Delta t$.

-A first step will be to establish a non linear scalar equation to be solved at each time step.

-After that a numerical solver will be developed and coded (Python, Fortran, Matlab, or C).

-This code will be used for different conditions, such as constant velocity, constant strain rate, stepwise velocity, loading and unloading…

- Newton method, bi-section method for the non-linear equation, check the convergence rate.