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A Robust Approach to Model Rock Fracture with SPH

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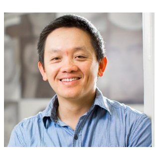


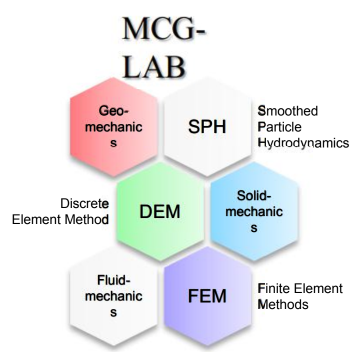

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Outline

- Motivation
- SPH framework of rock
- Constitutive model
- Validation
- Application
- Conclusion and future works

Research Motivation

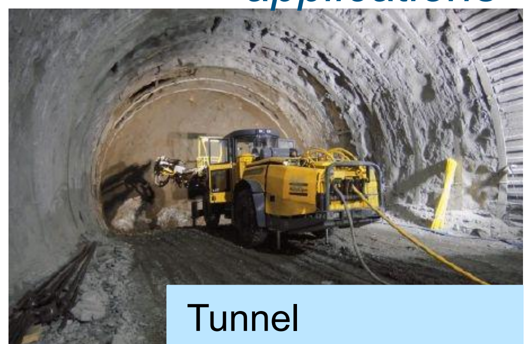
Rock fractures play a vital role in many geophysical processes and engineering applications involving rocks in civil and mining engineering.

Laboratory-scale tests

Compression test

Brazilian disc test

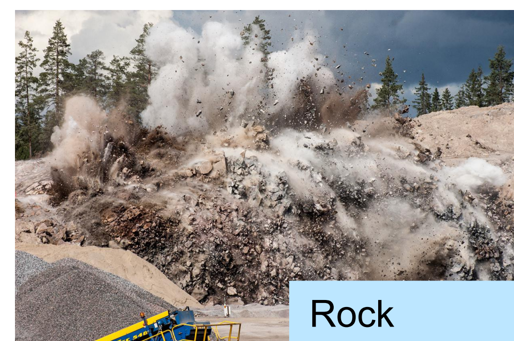
Large-scale practical applications



Tunnel Construction

Rock Excavation

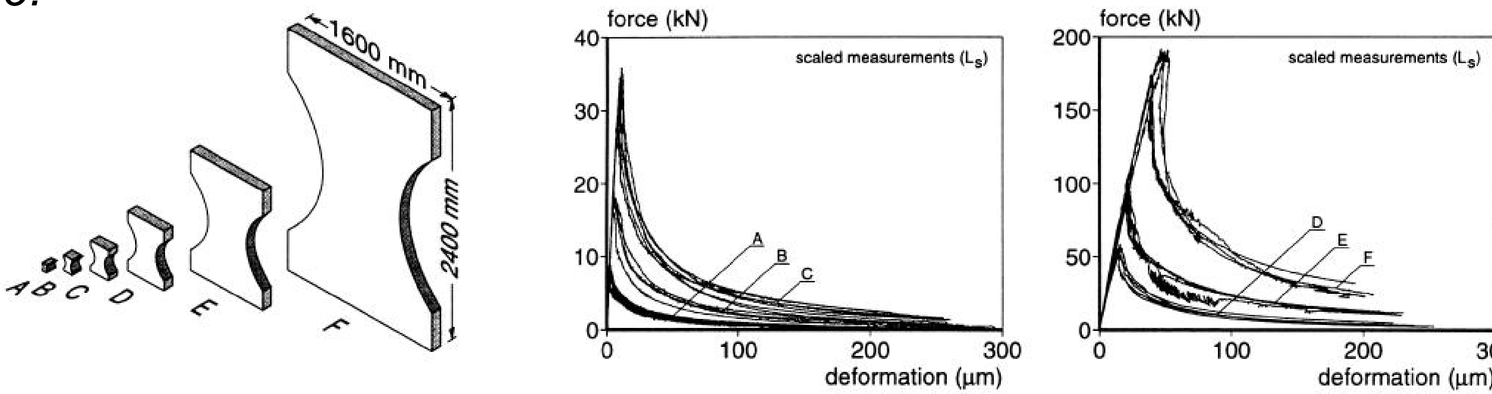
Rock Drilling



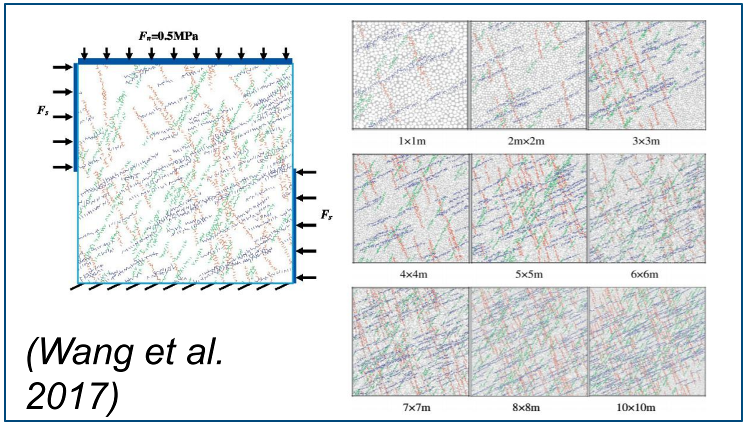
Rock Blasting

Research Motivation

Different approaches have been applied to investigate the mechanism of rock fracture:

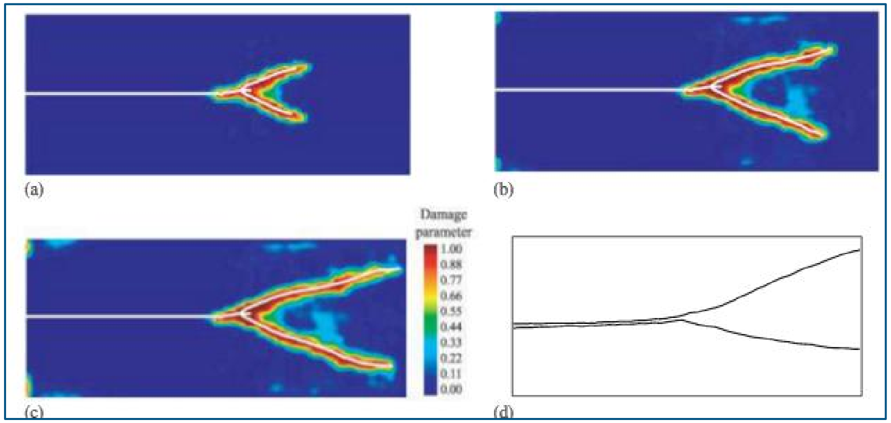


Experiment_Uniaxial tension test (Van Vliet et al. 2000)



(Wang et al. 2017)

DEM_Discrete fracture network



Extended FEM_Crack branching (Song et al. 2006)

SPH framework for rock fracture modelling

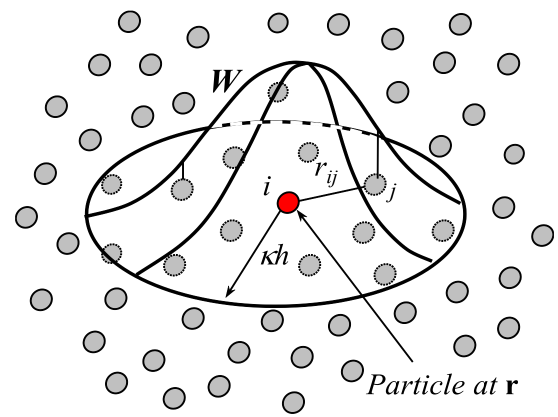
❑ Smoothed Particle Hydrodynamics (SPH) method

- SPH approximation of a function:

$$< f(x) > = \sum_{j=1}^{N} \frac{m_j}{\rho_j} f(x') W(x - x_j, h)$$

- SPH approximation of a spatial derivative:

$$< \nabla \cdot f(x) > = \sum_{j=1}^{N} \frac{m_j}{\rho_j} f(x') \cdot \nabla_i W(x_i - x_j, h)$$

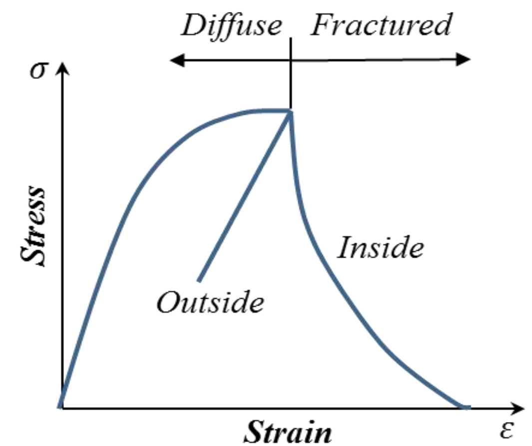


Basic idea of SPH method

❑ Governing equations

- Momentum conservation: $$\frac{Dv^{\alpha}}{Dt} = -\frac{1}{\rho}\frac{\partial \sigma^{\alpha\beta}}{\partial x^{\beta}} + f^{\alpha}$$
- Mass conservation: $$\frac{D\rho}{Dt} = -\rho \frac{\partial v^{\alpha}}{\partial x^{\alpha}}$$

❑ Constitutive relations:



❑ SPH approximation of governing equations:

- Continuity: $$\frac{d\rho_i}{dt} = \sum_{j=1}^{N} m_j ({v_i}^{\alpha} - {v_j}^{\alpha}) \frac{\partial W_{ij}}{\partial {x_i}^{\alpha}}$$
- Momentum: $$\frac{d{v_i}^{\alpha}}{dt} = \sum_{j=1}^{N} m_j \left(\frac{{\sigma_i}^{\alpha\beta}}{{\rho_i}^2} + \frac{{\sigma_j}^{\alpha\beta}}{{\rho_j}^2} \right) \frac{\partial W_{ij}}{\partial x^{\beta}}$$

SPH framework for rock fracture modelling

❑ Time integration scheme

Taylor SPH method is used in this study, which combines Taylor integration scheme, stress particle technique, corrected SPH strategy and traditional SPH approximation.

Taylor integration scheme

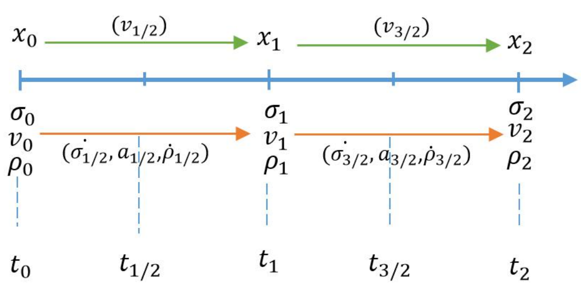
In this method, the time discretisation process is based on a two-step Taylor series expansion of time derivative of the vector of unknowns:

First step:

$$\phi^{n+1/2} = \phi^{n} + \frac{\Delta t}{2} \times \frac{\partial \phi}{\partial t}\Big|_{n}$$

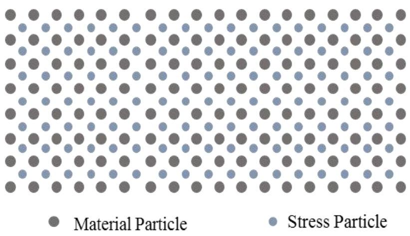
Second step:

$$\phi^{n+1} = \phi^{n} + \Delta t \times \frac{\partial \phi}{\partial t}\Big|_{n+1/2}$$



Particle arrangement

Stress point approach is introduced to overcome tensile instability.



Tensile instability is the attraction when material is stretched, which can result in the SPH particles forming clumps.

Taylor-SPH discretisation

First step:

$$\mathbf{v}_s^{n+1/2} = \mathbf{v}_s^{n} + \frac{\Delta t}{2}\left(-\sum_{m=1}^{N}\frac{m_m}{\rho_m\rho_s}(\boldsymbol{\sigma}_m^{n} - \boldsymbol{\sigma}_s^{n})\cdot\tilde{\nabla}_s W_{sm}) + \sum_{m=1}^{N}\left(\frac{m_m}{\rho_m}\boldsymbol{b}_m^{n}\widetilde{W}_{sm}\right)\right)$$

Second step:

$$\boldsymbol{v}_m^{n+1} = \boldsymbol{v}_m^{n} + \Delta t\left(-\sum_{s=1}^{N}\frac{m_s}{\rho_s\rho_m}(\boldsymbol{\sigma}_s^{n+1/2} - \boldsymbol{\sigma}_m^{n+1/2})\cdot\tilde{\nabla}_m W_{ms}) + \sum_{s=1}^{N}\left(\frac{m_s}{\rho_s}\boldsymbol{b}_s^{n+1/2}\widetilde{W}_{ms}\right)\right)$$

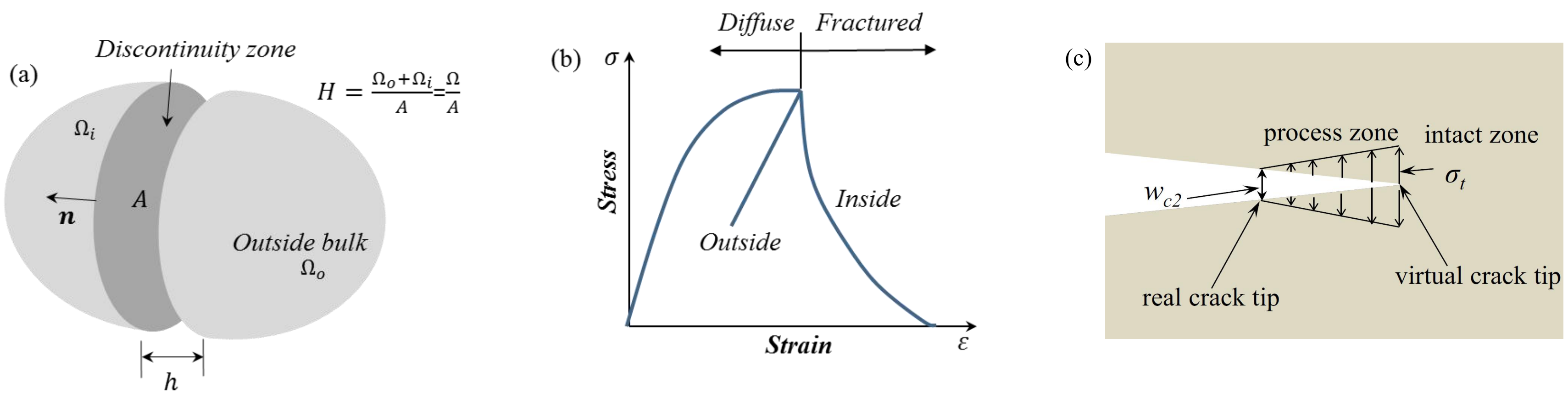
Reference: (Herreros and Mabssout 2011, Dyka and Ingel 1995, Chen et al. 1999)

SPH framework for rock fracture modelling

❑ Size-dependent continuum constitutive model:

A size-dependent continuum constitutive model with embedded cohesive fracture is developed. The model is originated from the double scale continuum constitutive framework for modelling strain localisations in geomaterials:

Constitutive framework | Constitutive response | Cohesive fracture zone

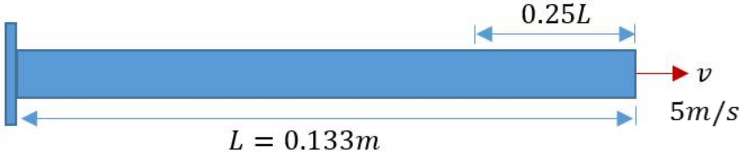


- *Total strain rate is defined as:* $\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}_o + \frac{1}{H}(\boldsymbol{n}\otimes[\![\dot{\boldsymbol{u}}]\!])^{sym}$
- *Cohesive fracture law:* $\dot{\boldsymbol{t}}_i = \boldsymbol{K}_i[\![\dot{\boldsymbol{u}}]\!]$
- *Stress-strain relation for elastic bulk:* $\dot{\boldsymbol{\sigma}} = \dot{\boldsymbol{\sigma}}_o = \boldsymbol{a}_o : \dot{\boldsymbol{\varepsilon}}_o = \boldsymbol{a}_o : [\dot{\boldsymbol{\varepsilon}} - \frac{1}{H}(\boldsymbol{n}\otimes[\![\dot{\boldsymbol{u}}]\!])^{sym}]$
- *Traction continuity across failure plane:* $\boldsymbol{t}_i = \boldsymbol{t} = \boldsymbol{\sigma}\cdot\boldsymbol{n}$
- *Average stress:* $\dot{\boldsymbol{\sigma}} = \boldsymbol{a}_o : \left(\dot{\boldsymbol{\varepsilon}} - \frac{1}{H}\left(\boldsymbol{n}\otimes(\boldsymbol{C}^{-1}\cdot(\boldsymbol{a}_o:\dot{\boldsymbol{\varepsilon}})\cdot\boldsymbol{n}\ )\right)^{sym}\right)$

Validations

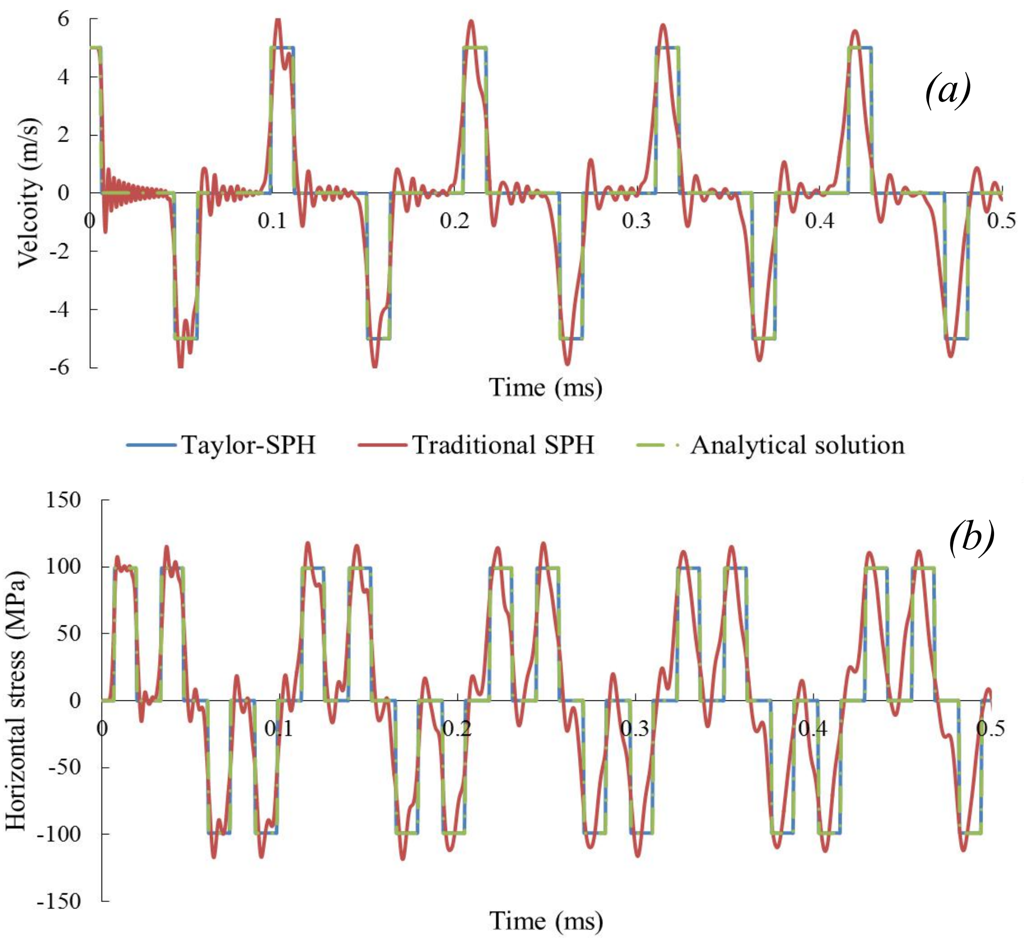
❑ 1D Elastic Tension Test

This test compares traditional SPH and Taylor SPH under tensile loading for on dimensional cases.



Material Properties	
Young's Modulus (E)	$200Gpa$
Density (ρ)	$7800kg/m^3$

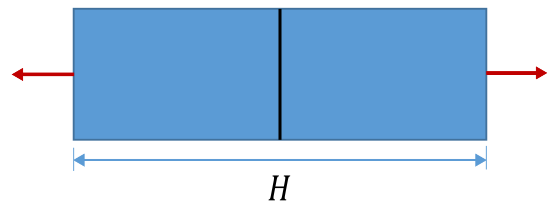
Conclusion: *Taylor SPH is* <u>*more stable*</u> *than traditional SPH for tensile applicatio in one-dimensional cases*

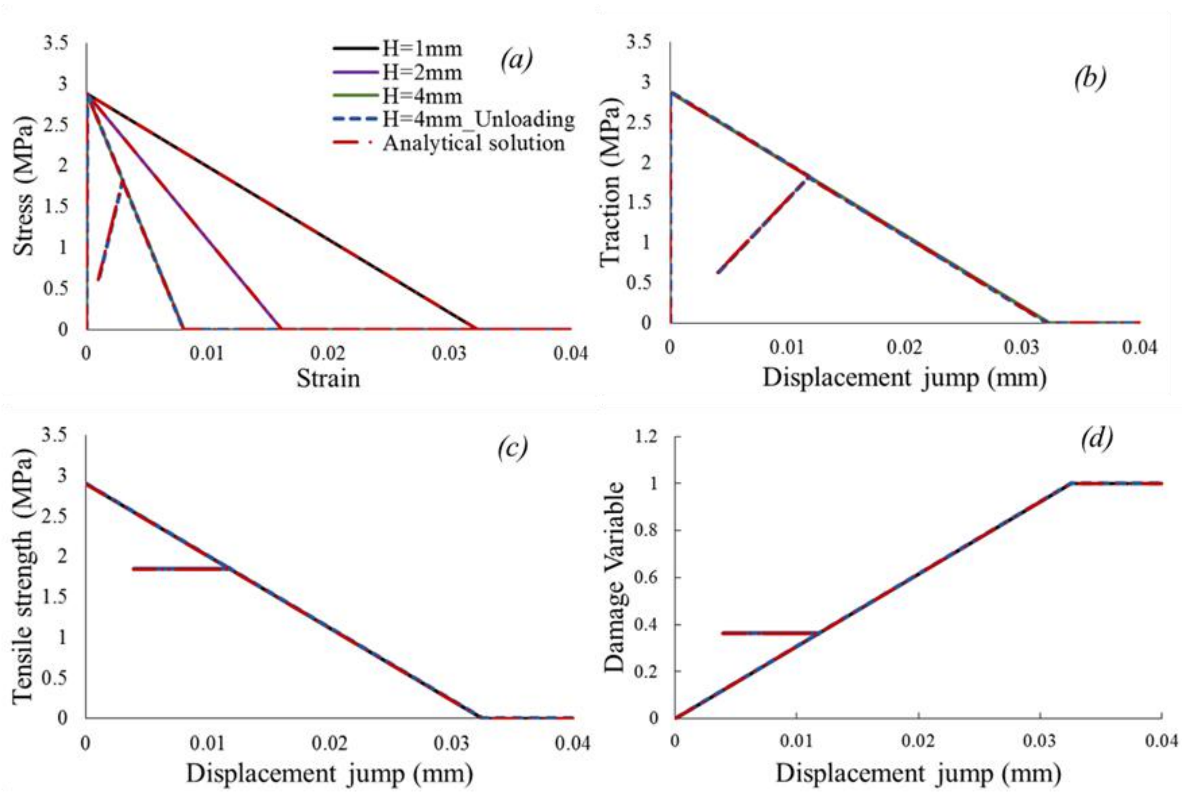


Validations

❑ Constitutive Behaviour

Mode-I behaviour of the size-dependent constitutive model with embedded linear cohesive fracture:





Material parameters	
Young's Modulus (E)	$45Gpa$
Tensile strength (f_t)	$2.9Mpa$
Fracture energy (G_f)	$47.17Nm/m^2$

Key Equations

Displacement jump: $[\![\dot{u}]\!] = C^{-1} \cdot (a_o : \dot{\varepsilon}) \cdot n$

Cohesive fracture law: $\dot{t}_i = K[\![\dot{u}]\!]$

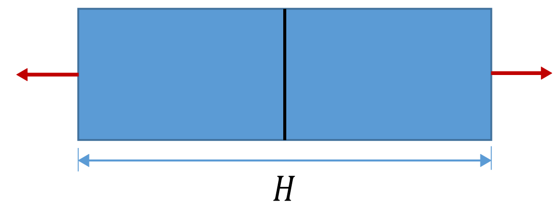
where: $K = -f_t^{\,2}/2G_f$ for linear cohesive law;

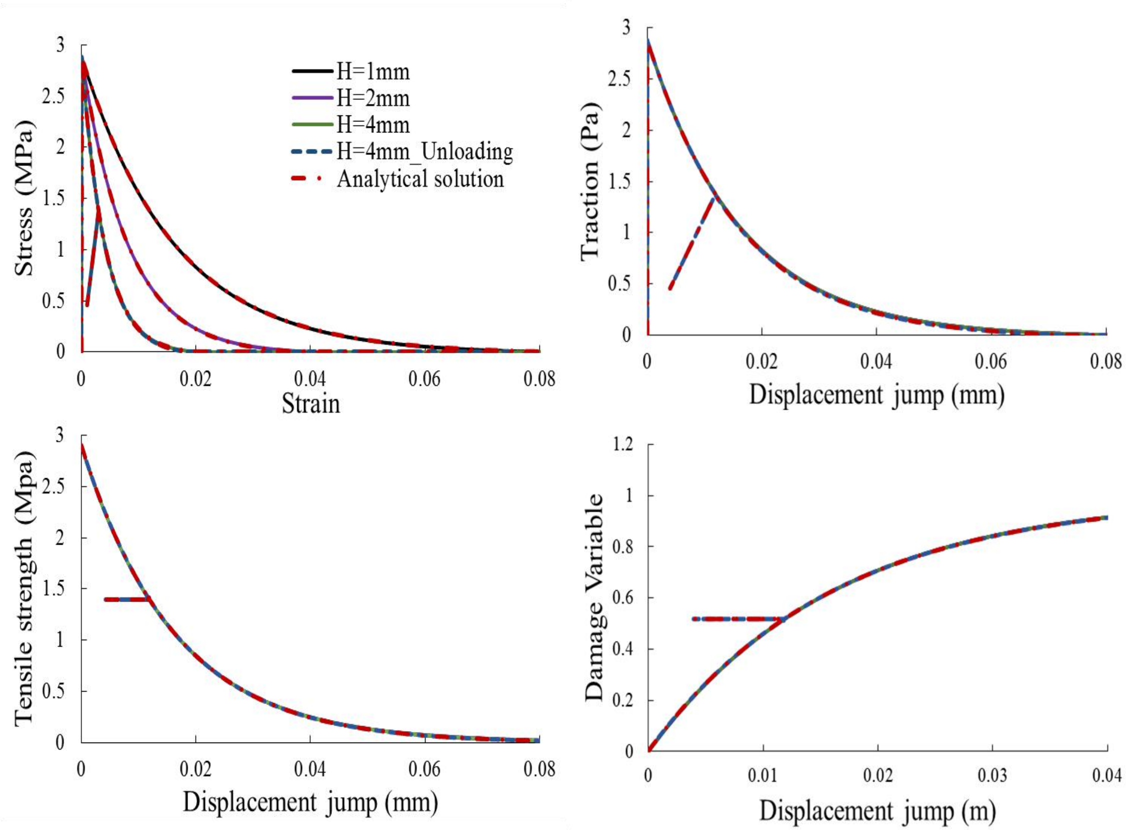
Stress-strain relationship:

$$\dot{\sigma} = a_o : \left(\dot{\varepsilon} - \frac{1}{H} \left(n \otimes \left(C^{-1} \cdot (a_o : \dot{\varepsilon}) \cdot n \right) \right)^{sym} \right)$$

❑ Constitutive Behaviour

Mode-I behaviour of the size-dependent constitutive model with embedded exponential cohesive fracture:





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Key Equations

<u>*Displacement jump*</u>: $[\![\dot{u}]\!] = C^{-1} \cdot (a_o : \dot{\varepsilon}) \cdot n$

<u>*Cohesive fracture law*</u>: $\dot{t}_i = K [\![\dot{u}]\!]$

where: $K = -\frac{f_t{}^2}{G_f} \exp(-\frac{f_t}{G_f} u_{cr1})$ for exponential cohesive law;

<u>*Stress-strain relationship*</u>:

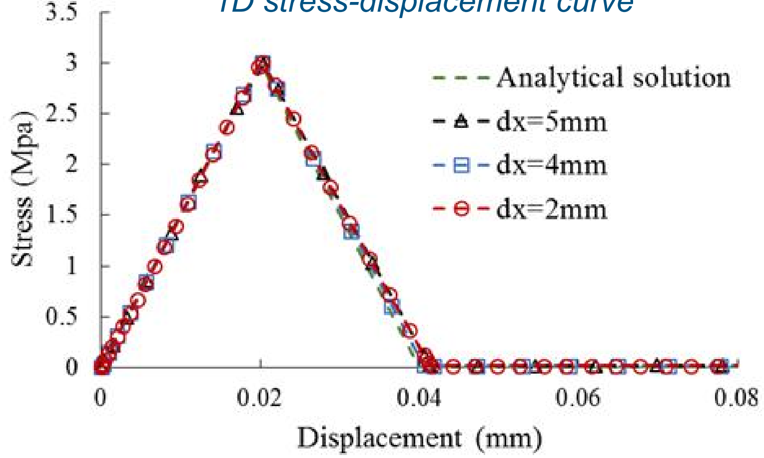
$$\dot{\sigma} = a_o : \left(\dot{\varepsilon} - \frac{1}{H} (n \otimes (C^{-1} \cdot (a_o : \dot{\varepsilon}) \cdot n))^{sym} \right)$$

Validations

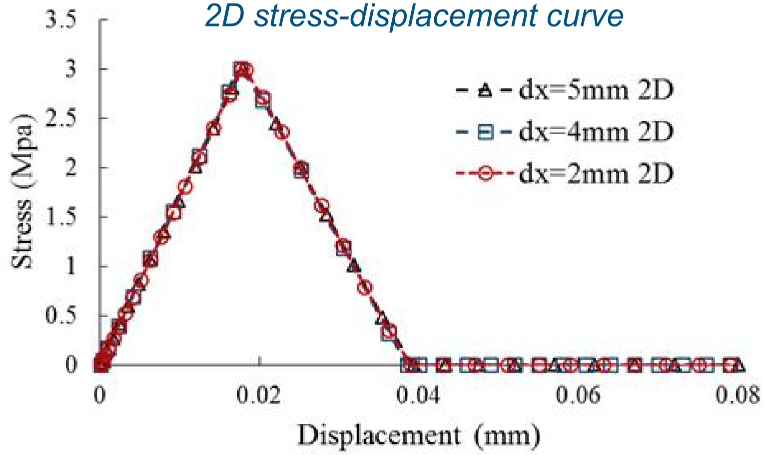
❑ Performance of constitutive model in SPH

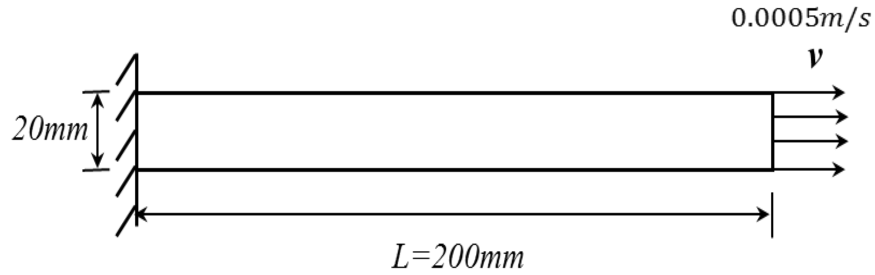
- *Using linear cohesive fracture law*

1D stress-displacement curve



2D stress-displacement curve



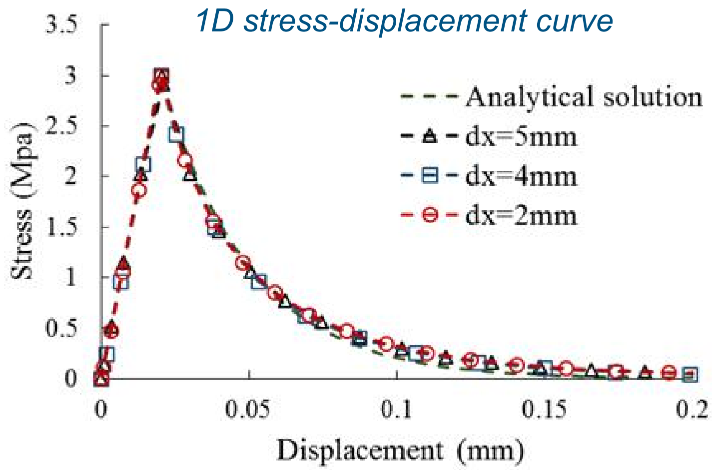


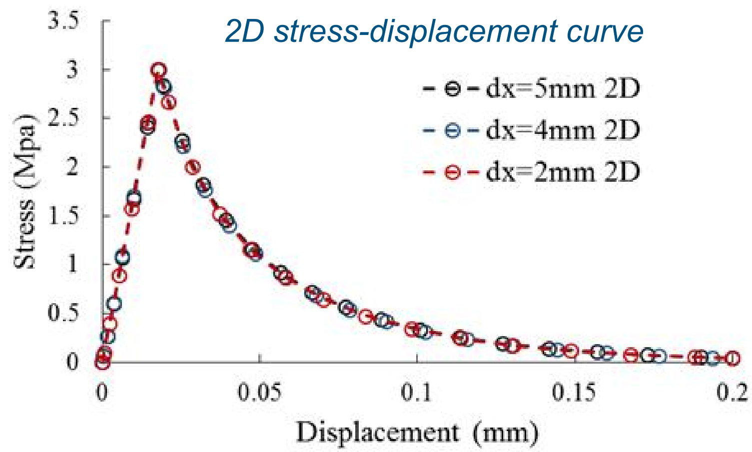
Material parameters	
Young's Modulus (E)	$30Gpa$
Poisson's ratio (v)	0.2
Density (ρ)	$2000kg/m^3$
Tensile strength (f_t)	$3Mpa$
Fracture energy (G_f)	$120Nm/m^2$

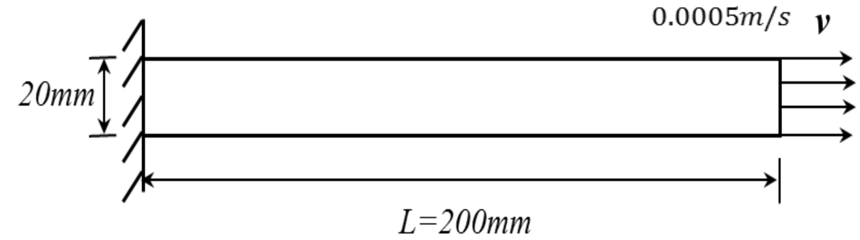
***Conclusion**: This test could prove that the current-developed model is independent of numerical spatial discretisation for 1D and 2D cases.*

❑ Performance of constitutive model in SPH

- *Using exponential cohesive fracture law*







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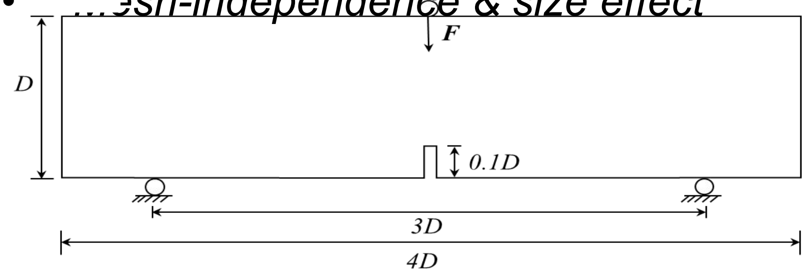
***Conclusion**: This test could prove that the current-developed model is independent of numerical spatial discretisation for 1D and 2D cases.*

Now, it's time for simulating rock fracture!

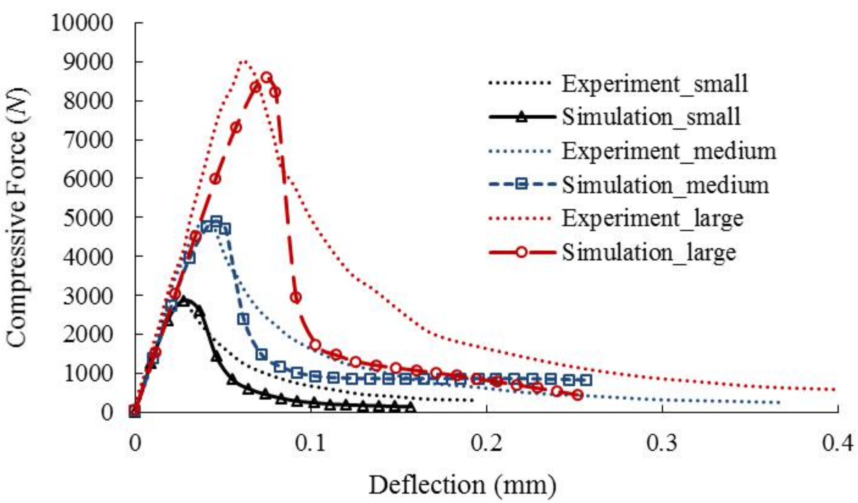
Applications

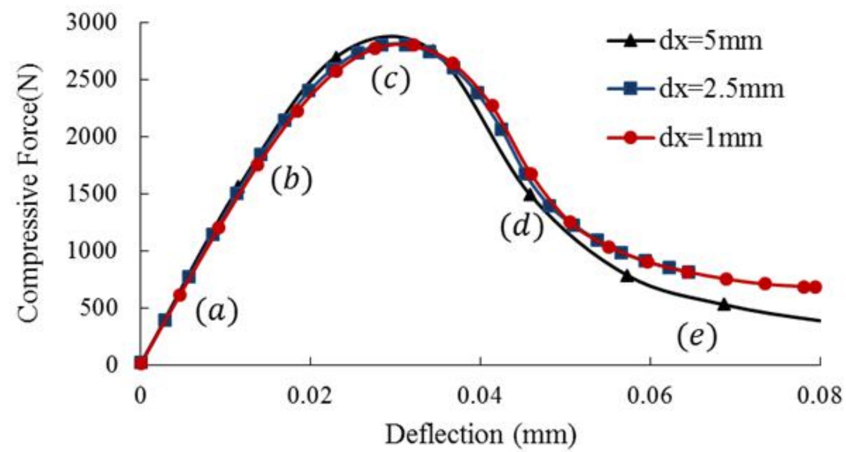
❑ SPH modelling of the three-point bending test:

- *Taylor SPH & continuum constitutive model with embedded linear cohesive fracture law*
- *Mesh-independence & size effect*



Material parameters	
Young's Modulus (E)	$45Gpa$
Poisson ratio (v)	0.24
Tensile trength (f_t)	$2.9Mpa$
Fracture energy (G_f)	$47.17Nm/m^2$
Depth of beam	$80mm, 160mm, 320mm$





Applications

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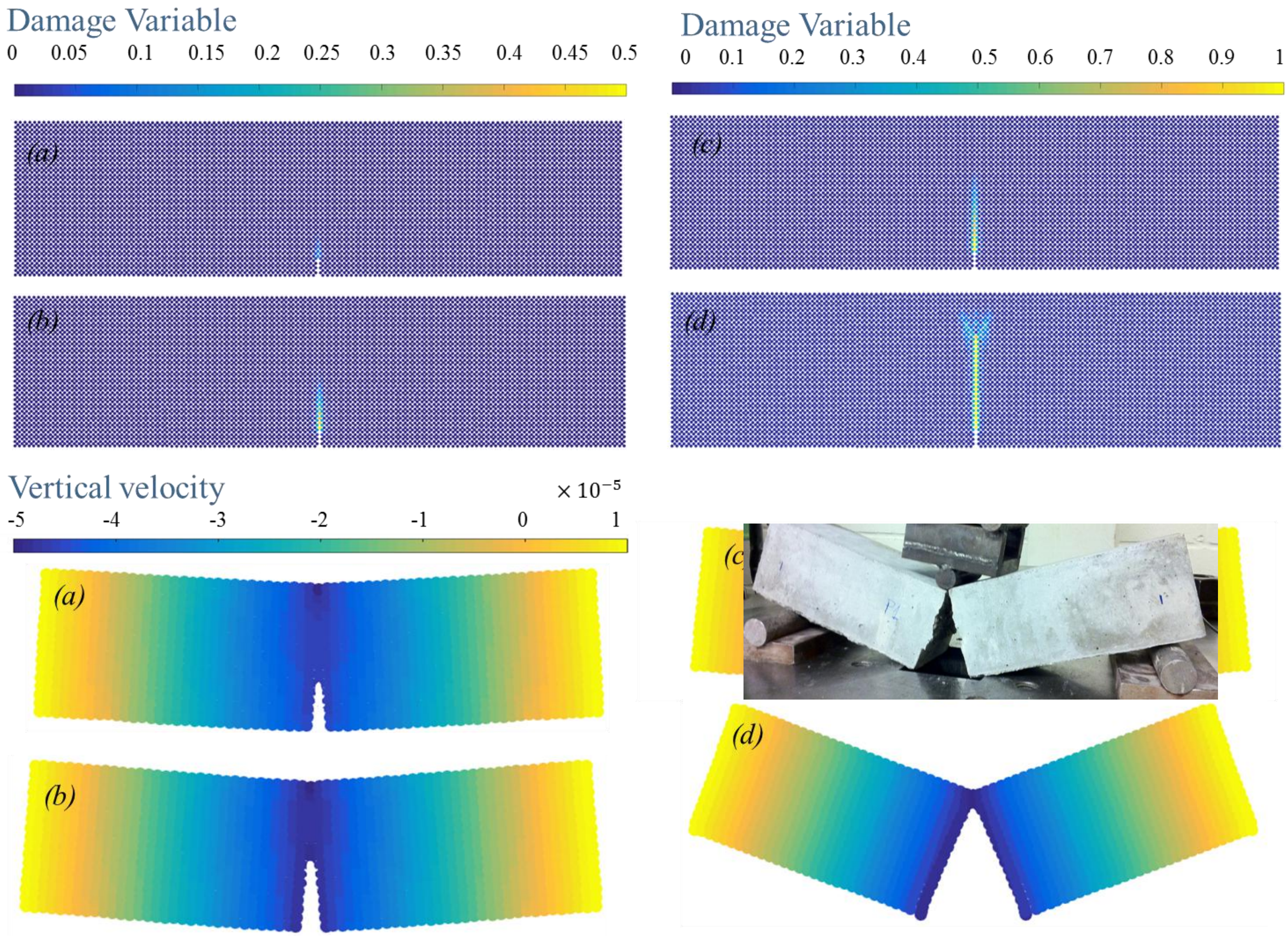
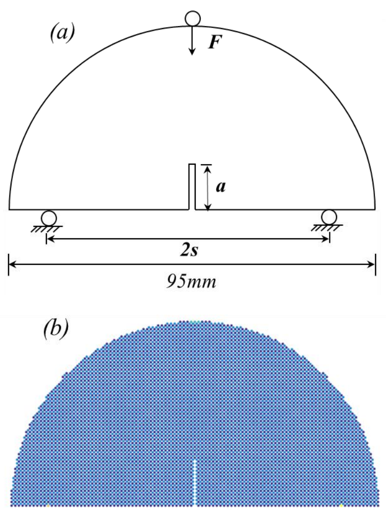


Applications

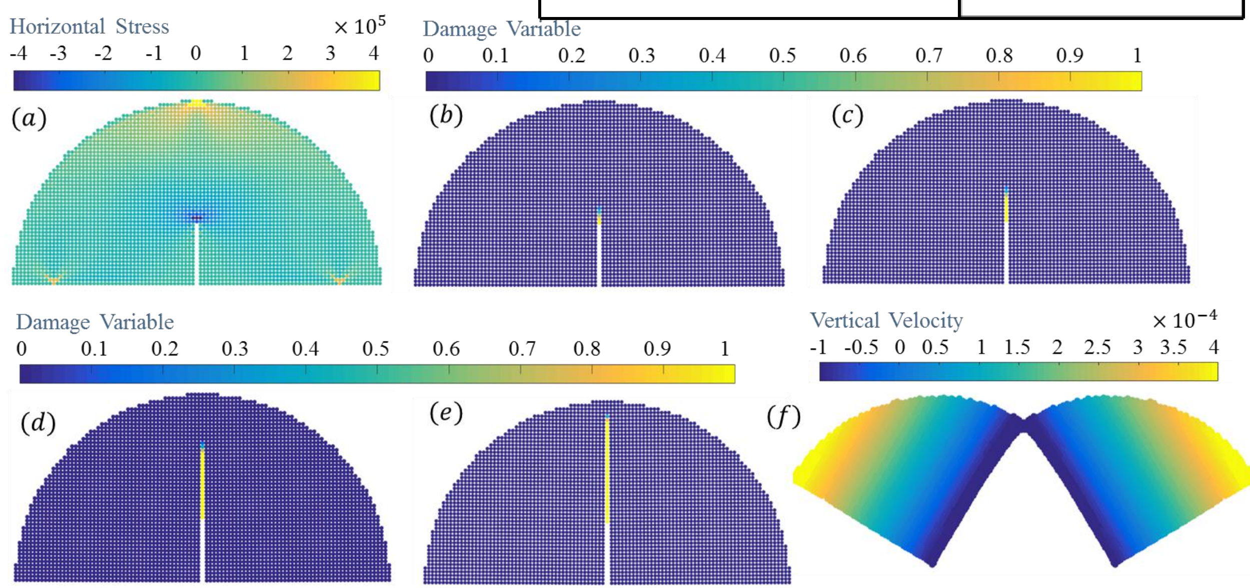
SPH modelling of semi-circular bending test

- *Taylor SPH & continuum constitutive model with embedded linear cohesive fracture law*
- *Notch length & span changed for different cases to further validate model performance in modelling mode I fracture behaviour*

Material parameters	
Young's Modulus (E)	$200Mpa$
Poisson ratio (v)	0.3
Tensile strength (f_t)	$0.42Mpa$
Fracture Toughness (K_{Ic})	$1.1\ \mathrm{MPa}\sqrt{mm}$
Courant Number (C)	1
Particle size (dx)	$0.95mm$



Geometry and boundary conditions



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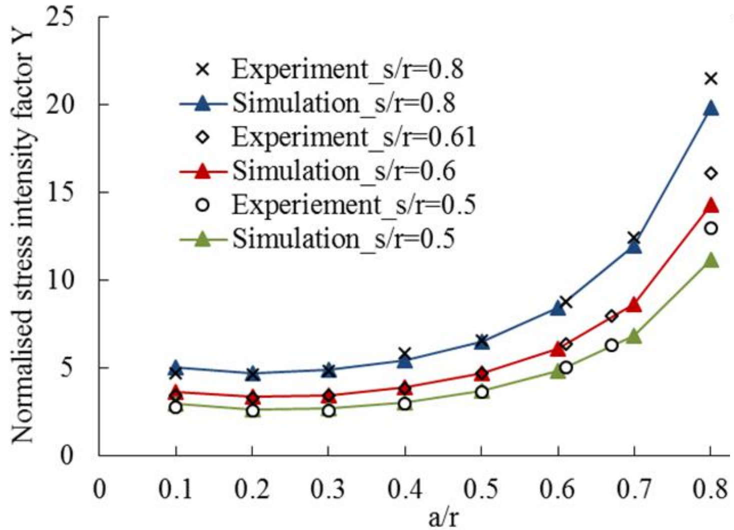

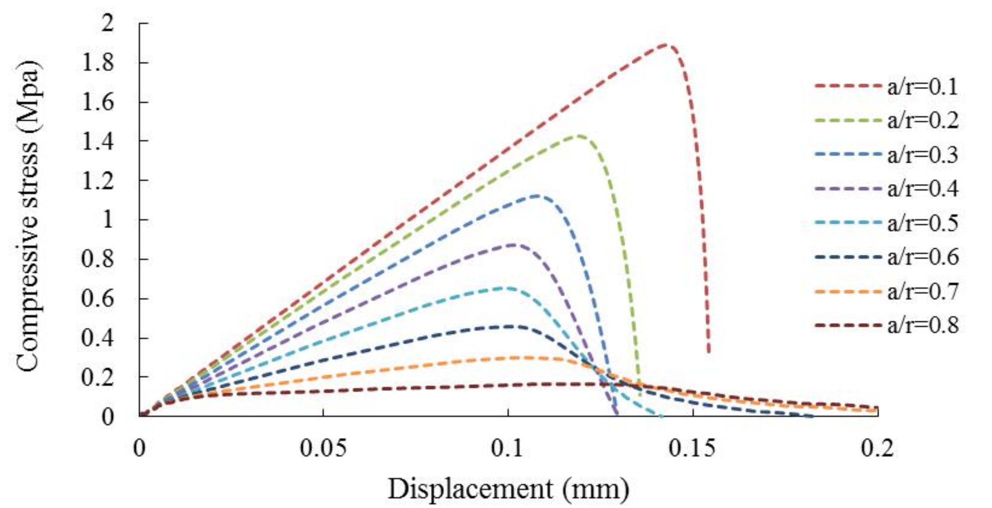

Conclusion

In this study, a new numerical framework is developed to enhance the capability of existing computational tools for modelling mode-I rock fracture behaviour. A set of numerical examples such as three-point bending test, Brazilian disc test and semi-circular bending have been conducted for validating its performance.

Achievement of this model:

1. *Capable of accurately predicting the mode-I rock fracturing processes;*
   - *Does not require an explicit crack representation;*
   - *Allows the fracture to be oriented in any direction;*
2. *Could capture size-dependent behaviour of rock;*
3. *Insensitive with resolution of numerical discretisation.*

In next stage, we will extend this model to mix-mode applications of rock fracturing.

Acknowledgement

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Australian Government

Australian Research Council

MONASH University

THE UNIVERSITY of ADELAIDE

THANK YOU !
ANY QUESTIONS?

Research Motivation

NUMERICAL METHODS

Discontinuum methods
(such as DEM, DDA)
Pros: large deformation
Cons: high computational time

Continuum gird-based methods
(such as FEM, XFEM)
Pros: large scale applications
Cons: limit in multiple fracture network

Mesh-free numerical model
Smoothed Particle Hydrodynamics (SPH)

CONSTITUTIVE MODEL

Classical plasticity and damage theories
Cons: not consider size effect

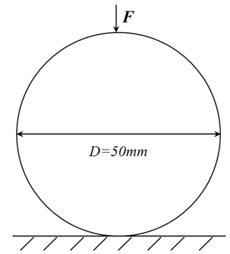
More advanced models such as
Smeared crack approach
Nonlocal/gradient theories
Cons: limit in mesh size

A size dependent constitutive model with embedded cohesive fracture

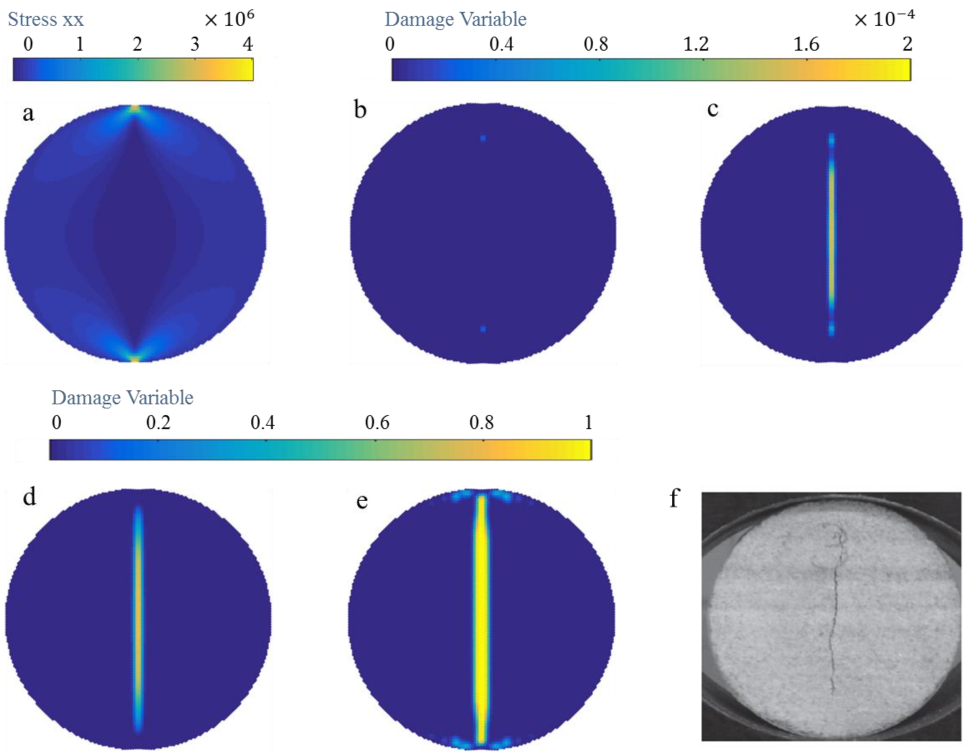
Application

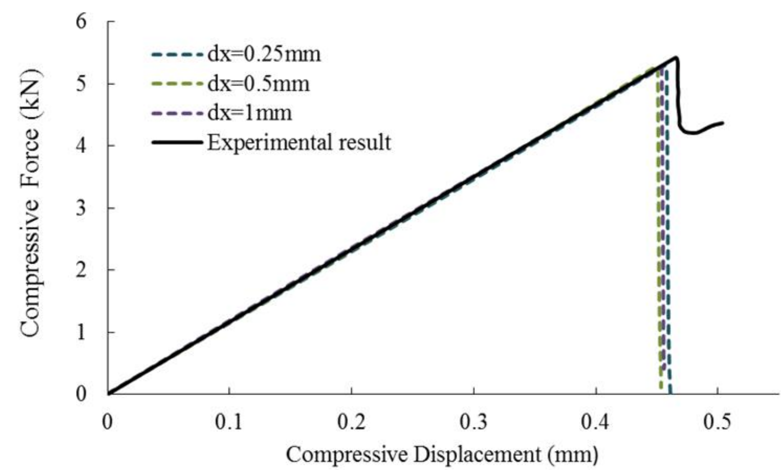
SPH modelling of Brazilian disc (BD) test

- *Taylor SPH & continuum constitutive model with embedded linear cohesive fracture law*
- *Mesh-independence of this model*



Material parameters	
Young's Modulus (E)	$3.43Gpa$
Poisson ratio (ν)	0.25
Tensile strength (f_t)	$4Mpa$
Fracture energy (G_f)	$20Nm/m^2$
Courant Number (C)	1





Macro stress-displacement curve