The study on SPH method with space variable smoothing length and its applications to multi-phase flow

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Catalogue

1.Introduction

2.Methods

3.NNPS performance evaluation

4.Results and Analysis

5.Conclusion and Future work

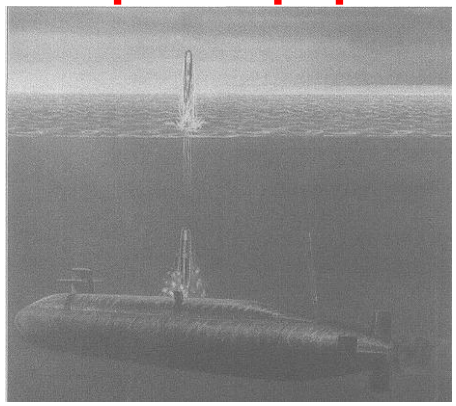
Introduction—multiphase flow

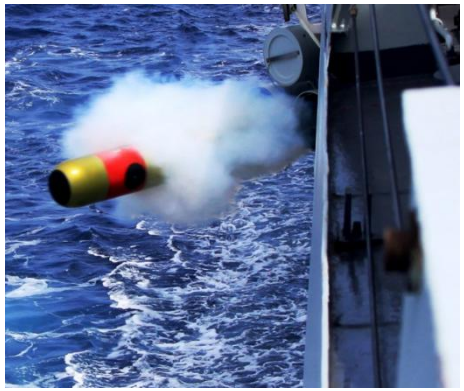
The multiphase flow problem includes a series of unique flow phenomena like free interface, surface tension and component conversion. It's a large class of problems that are widely distributed in military and civilian research fields.

Nature and production life：

National economic field ：

Weapon equipment development ：

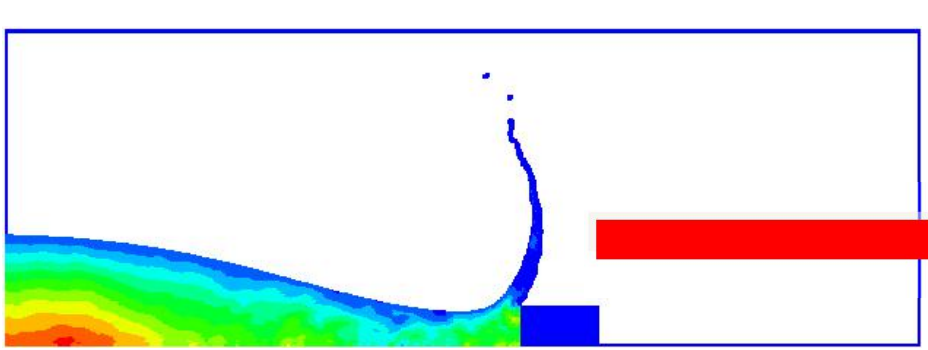


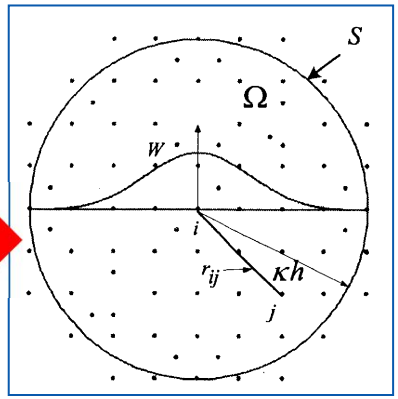


Introduction—SPH

Smoothed particle hydrodynamics (SPH) [1]

- Basic characteristics：meshless and Lagrangian、interpolation kernel function





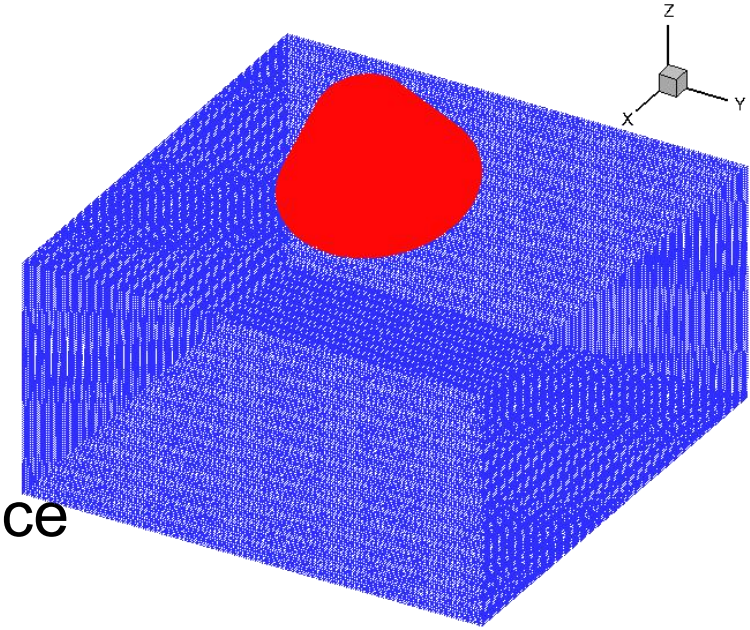
Advantages
- No grids
- Automatic, natural and global flow
- Large free surface deformation
- Fluid-soild coupling procedure

Very suitable for solving multiphase flow problems !

[1] Monaghan J J. Journal of computational physics, 1994, 110(2): 399-406.

Introduction—SPH

- Early SPH simulations：a fixed *h* for all particles was always used

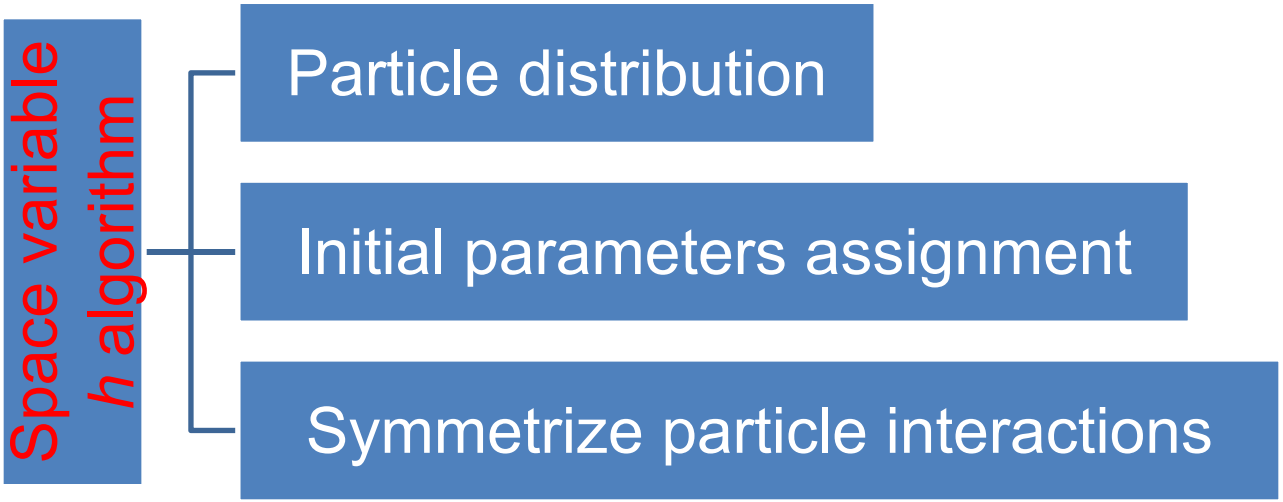


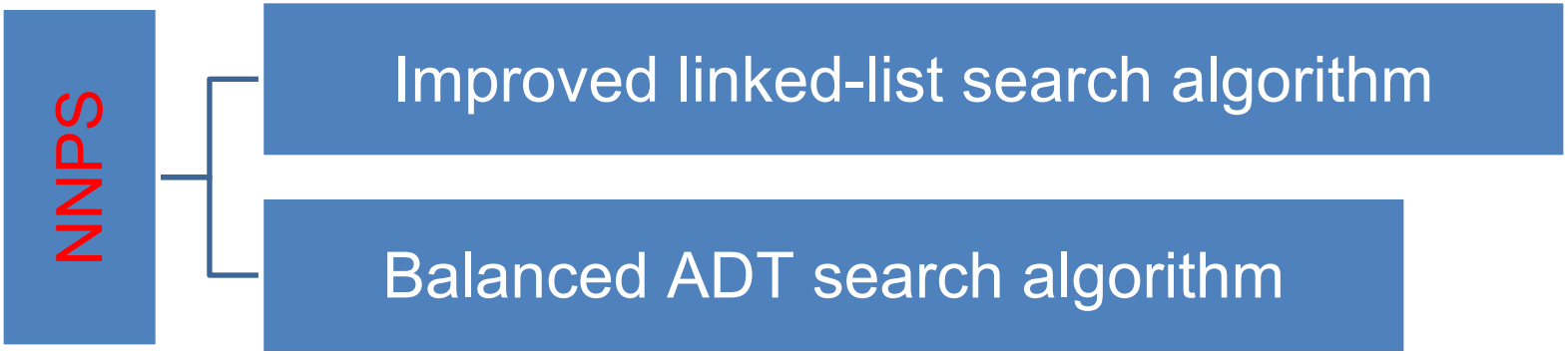
Drawbacks

- Particles are uniformly distributed
- Low computational efficiency
- Low spatial resolution
- Different estimate accuracy in space

- SPH method with space variable smoothing length： to improve the computational efficiency and spatial resolution.
- Balanced ADT search algorithm: to handle variable smoothing length problems

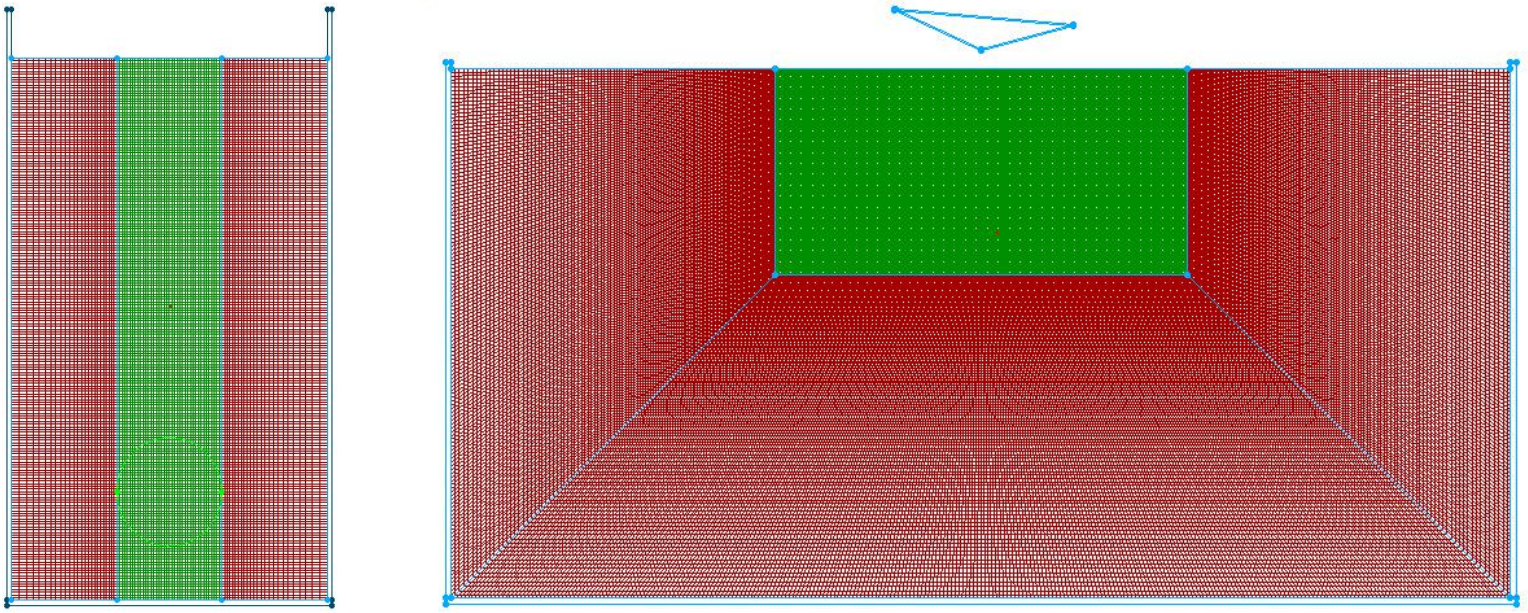
Methods


Space variable *h* algorithm
Particle distribution
Initial parameters assignment
Symmetrize particle interactions


NNPS
Improved linked-list search algorithm
Balanced ADT search algorithm

Methods-- Particle distribution

- Particle diffusion distribution models

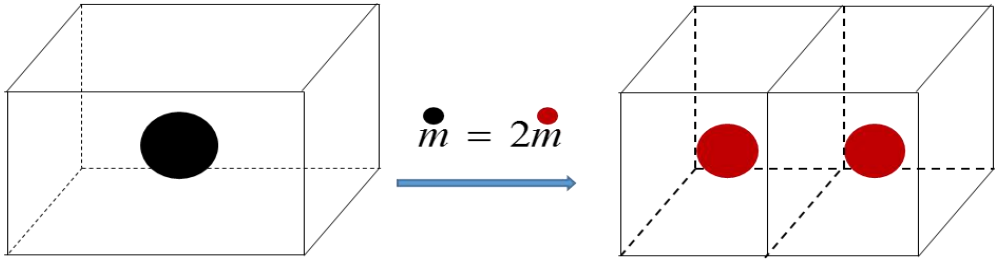


Dense particles: the desired accurate area

Sparse particles: away from the desired accurate area

Methods--Initial particle parameters

- Initial mass assignment: proportional to its own area or volume [2]



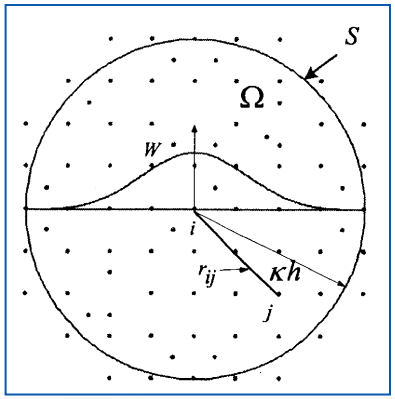
- Initial smoothing length assignment:

$$h = 1.23\Delta x \qquad \Delta x = \sqrt{S} \text{ or } \sqrt[3]{V}$$

For Spline kernel

Interpolation circle radius: 2*h*

Neighbor particle number:21



[2] Omidvar P, Stansby P K, Rogers B D. International Journal for Numerical Methods in Fluids, 2012, 68(6): 686-705.

Methods--Symmetrize particle interactions

- The average value of smoothing length is used to determine the particle interaction pairs in nearest neighbor particle search ,

$$h_{ij} = \frac{h_i + h_j}{2} \tag{1}$$

- Then the kernel and its derivatives can be obtained by substituting (1) into the kernel after the particle pairs are determined,

$$W_{ij} = W(r_{ij}, h_{ij})$$

$$\nabla_i W_{ij} = \frac{\boldsymbol{x_{ij}}}{r_{ij}} \frac{\partial W(r_{ij}, h_{ij})}{r_{ij}} \tag{2}$$

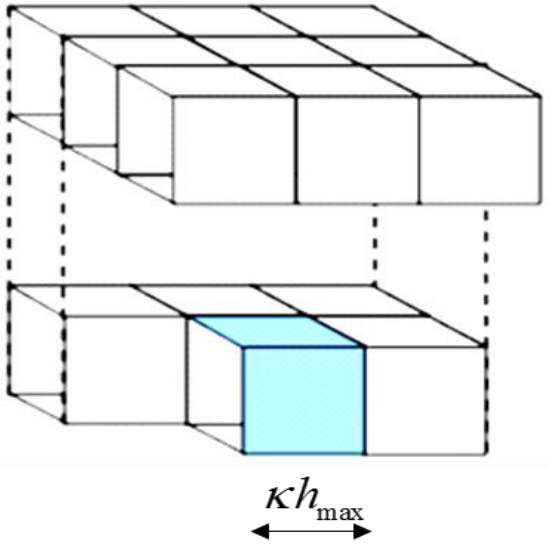
Methods--Symmetrize particle interactions

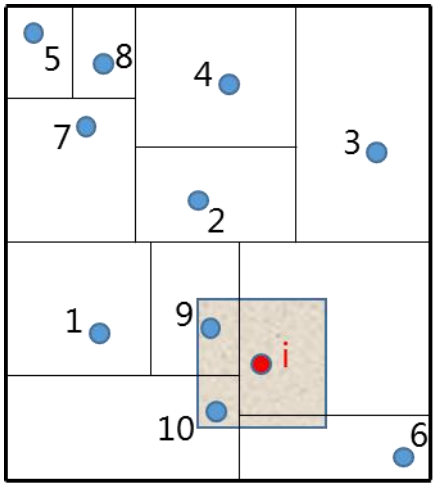
- Finally, the SPH equations with space variable smoothing length can be obtained by substituting (2) into SPH system,

$$
\begin{cases}
\dfrac{d\rho_i}{dt} = \displaystyle\sum_{j=1}^{N} \frac{m_j \rho_i}{\rho_j} v_{ij}^{\ \beta} \cdot \frac{\partial W(r_{ij}, h_{ij})}{\partial x_i^{\beta}} \\
\dfrac{dv_i^{\alpha}}{dt} = \displaystyle\sum_{j=1}^{N} m_j \left(\frac{\sigma_i^{\alpha\beta} + \sigma_j^{\alpha\beta}}{\rho_i \rho_j} + \Pi_{ij} \delta^{\alpha\beta} \right) \frac{\partial W(r_{ij}, h_{ij})}{\partial x_i^{\beta}} \\
\dfrac{de_i}{dt} = \dfrac{1}{2} \displaystyle\sum_{j=1}^{N} m_j \left(\frac{\sigma_i^{\alpha\beta} + \sigma_j^{\alpha\beta}}{\rho_i \rho_j} + \Pi_{ij} \delta^{\alpha\beta} \right) v_{ij}^{\beta} \cdot \frac{\partial W(r_{ij}, h_{ij})}{\partial x_i^{\beta}} + \mathrm{H}_i
\end{cases}
$$

Methods--NNPS

- All-pair search algorithm--efficiency is too low
- Linked-list search algorithm--widely used, but can't solve variable smoothing length problems
- Improved linked-list search algorithm
- Balanced ADT search algorithm



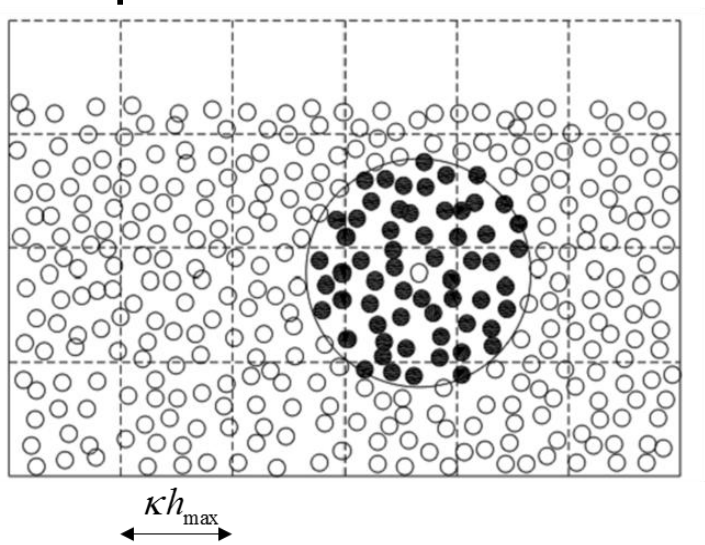


Methods--Improved linked-list search

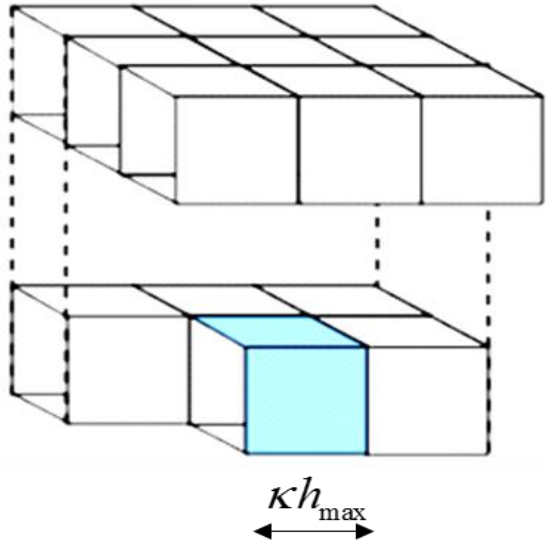
- Improved linked-list search algorithm

  The squared cell size in underlying Cartesian grid equals to the maximum interpolating radius, $\kappa h_{\max}$

  Computing the distance between *i* and *j*, if the distance is less than κh_{mean}(the Symmetric smoothing length), then they are pairs



Two-dimensional



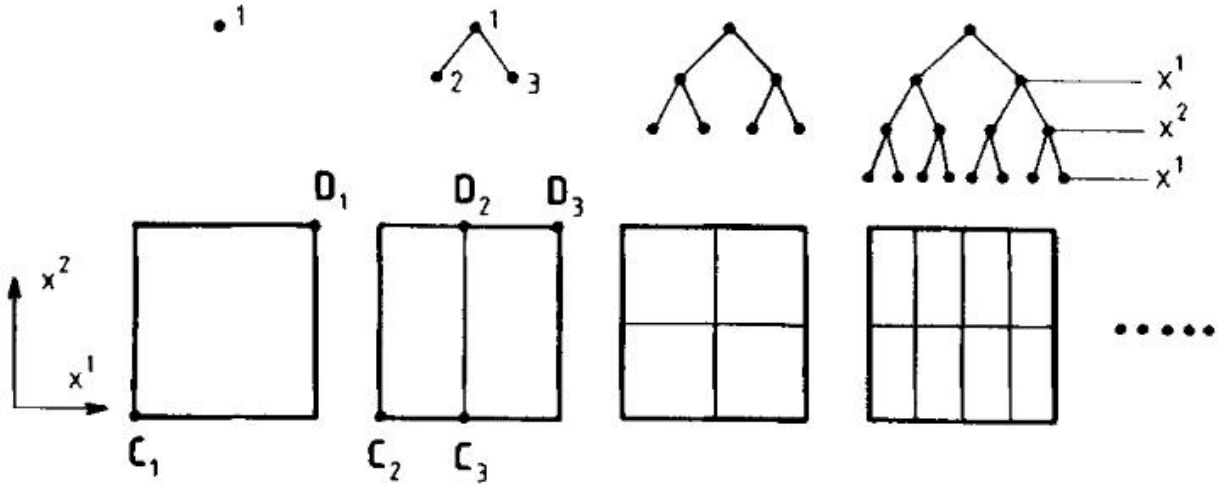
Three-dimensional

Methods--Balanced ADT search

- B-ADT search algorithm—Building tree structure

  For generalized ADT structure, partitions for the ADT were chosen by geometrically bisecting the point set and the resulting sub-regions. [3]

  Besides, the partition direction is alternating chosen from $x^1, x^2, ..., x^N$



Relation between a binary tree and a bisection process

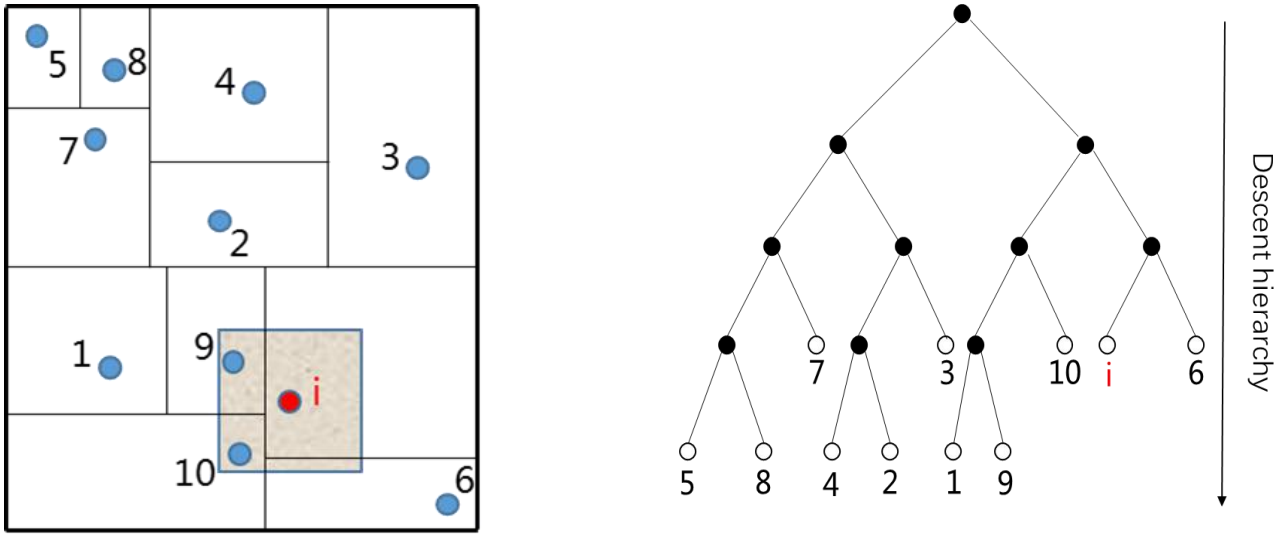
[3] Bonet J, Peraire J. International Journal for Numerical Methods in Engineering, 1991, 31(1): 1-17.

Methods--Balanced ADT search

- B-ADT search algorithm—Building tree structure

  For B-ADT structure in SPH, the partitions are located at positions where assigning half of the list to each child.

  Besides, the partition direction is chosen from the longest Coordinate axis.

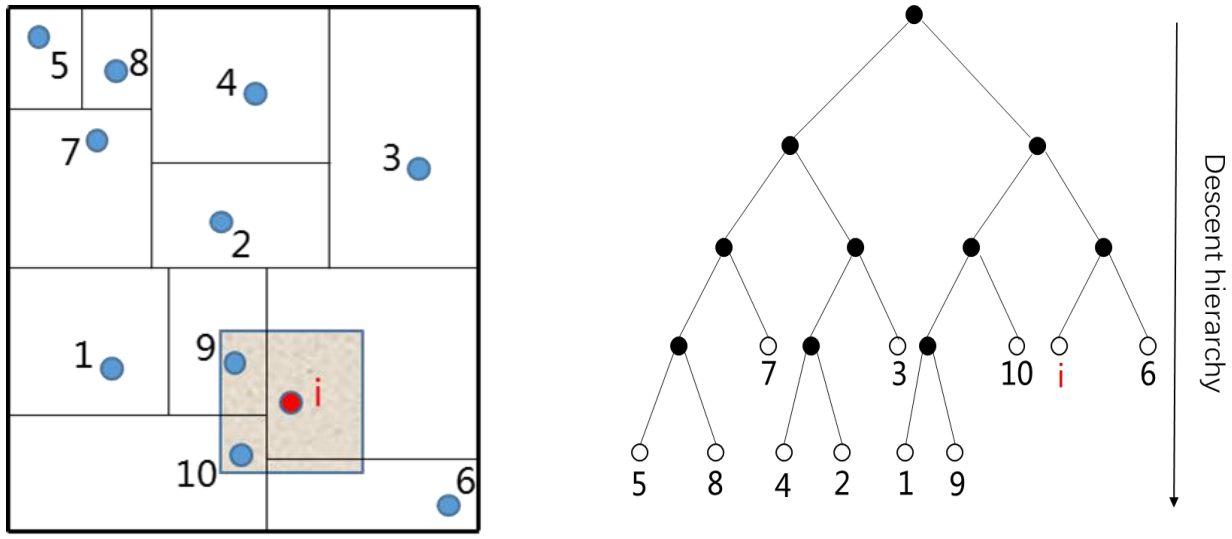


Construction of a balanced ADT and tree search algorithm

Methods--Balanced ADT search

- B-ADT search algorithm—Searching

  For the given particle *i*, take the particle *i* as the center and surround it with a cube with a length of κh_i



Construction of a balanced ADT and tree search algorithm

Methods--Balanced ADT search

- B-ADT search algorithm—Searching

  For the given particle *i*, take the particle *i* as the center and surround it with a cube with a length of κh_i

Accordingly, a tree searching algorithm emerges in a recursive form as:

1)If the left link of the root is not zero and the searching region $(\boldsymbol{c}_{sl}, \boldsymbol{d}_{sl})$ overlaps with (***a***, ***b***), i.e. if $d_{sl}^i \geq a^i$ and $c_{sl}^i \leq b^i$ for *i* = 1,2,…,*N*, search left subtree.

2)If the right link of the root is not zero and the searching region $(\boldsymbol{c}_{sr}, \boldsymbol{d}_{sr})$ overlaps with (***a***, ***b***), i.e. if $d_{sr}^i \geq a^i$ and $c_{sr}^i \leq b^i$ for *i* = 1,2,…,*N*, search left subtree.

CARDC

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2.Methods

3.NNPS performance evaluation

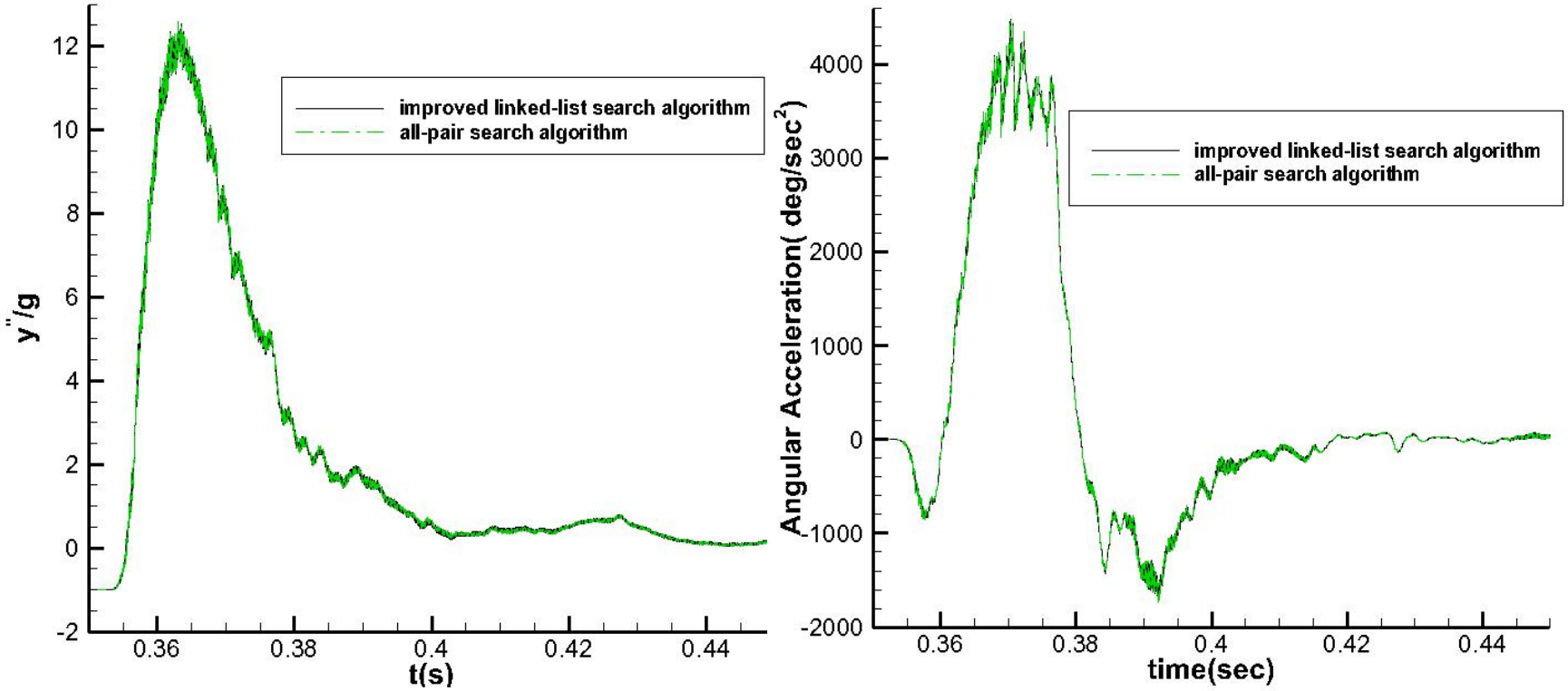
4.Results and Analysis

5.Conclusion and Future work

NNPS performance evaluation

- Validations

Improved linked-list search algorithm

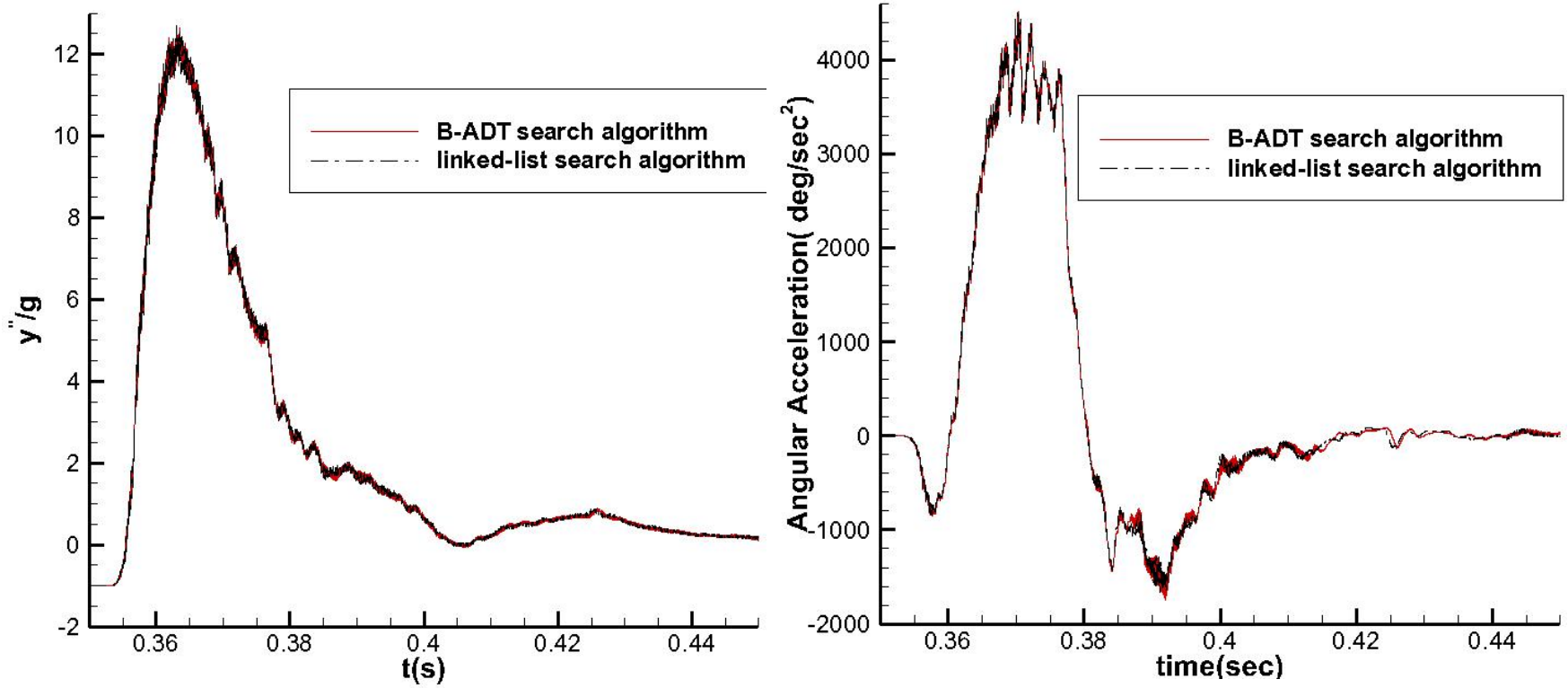


Asymmetric wedge water entry

NNPS performance evaluation

- Validations

Balanced ADT search algorithm

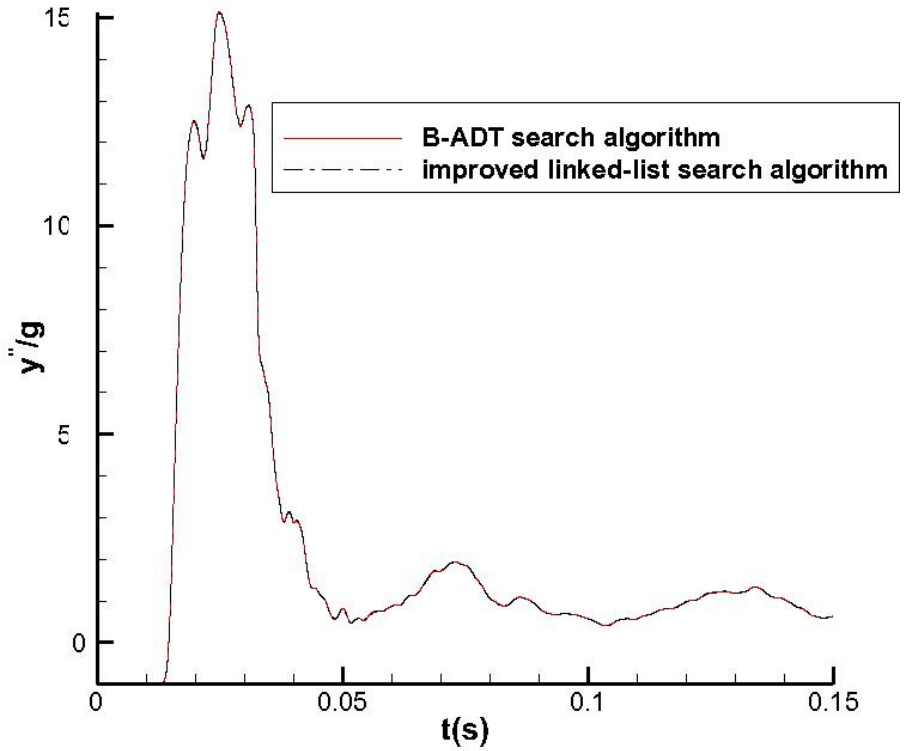


Asymmetric wedge water entry

NNPS performance evaluation

- Validations

Balanced ADT search algorithm



Three-dimensional space capsule water entry

NNPS performance evaluation

- Comparison

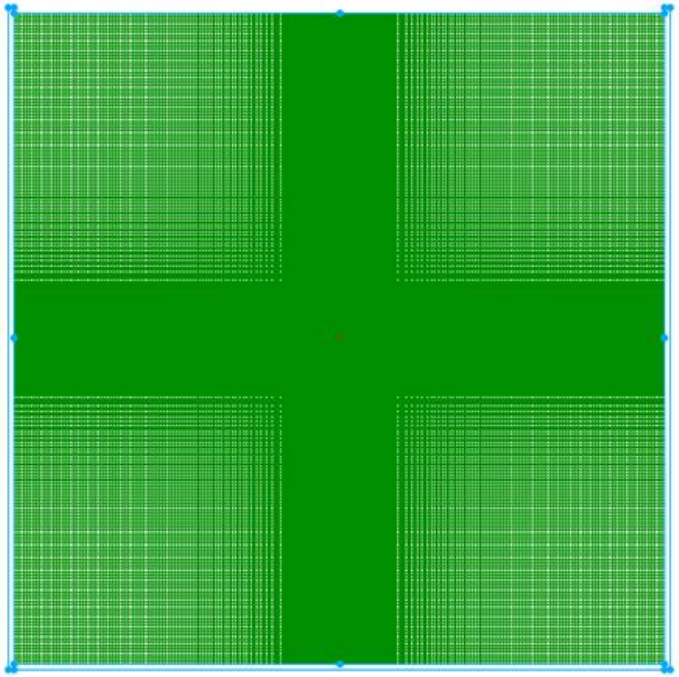
Scope of application

Algorithms / Applications	All -pair	Linked-list	Improved Linked-list	B-ADT
Constant smoothing length problems	√	×	√	√
Variable smoothing length problems	√	√	√	√

NNPS performance evaluation

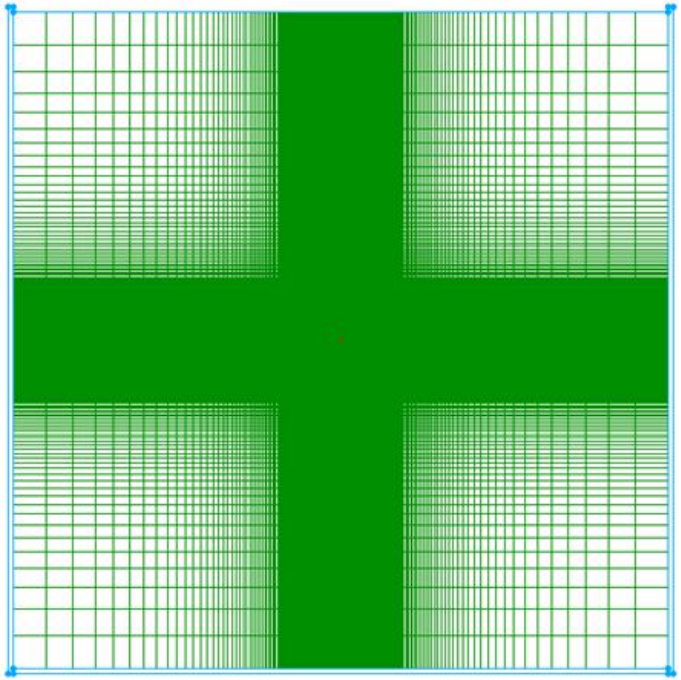
- Efficiency comparison

  Models: only water and boundary particles



particle number: 300×300

△*x:* 0.002-0.01



particle number: 300×300

△*x:* 0.001-0.1

NNPS performance evaluation

- Efficiency comparison

Balanced ADT search algorithm is robust and efficient, it's suitable to handle variable smoothing length problems, especially for those with large stretching ratio. Its order of complexity is $O(N \lg N)$.

Algorithms / Models	All -pair	Improved Linked-list	B-ADT
Model 1	47219.5s	856.2s	1063.1s
Model 2	47203.6s	33871.6s	1079.1s

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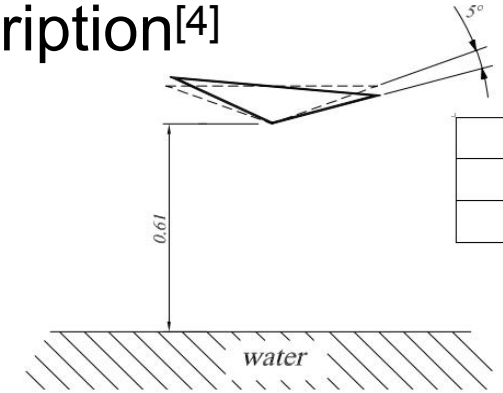
Results and Analysis

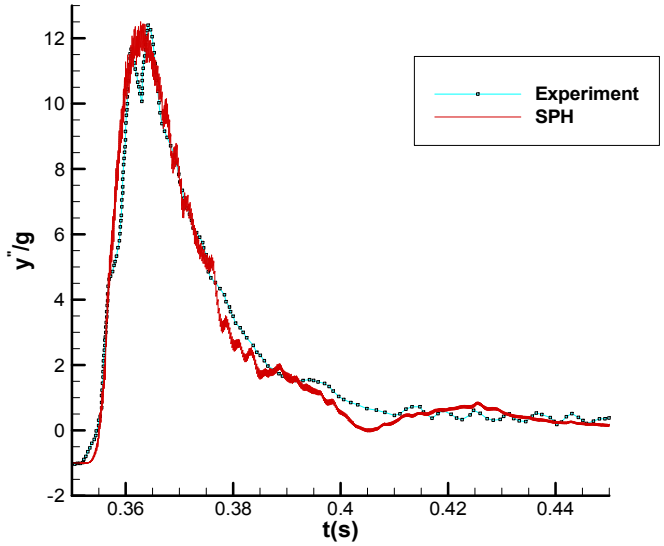
➢ Asymmetric wedge water entry impact

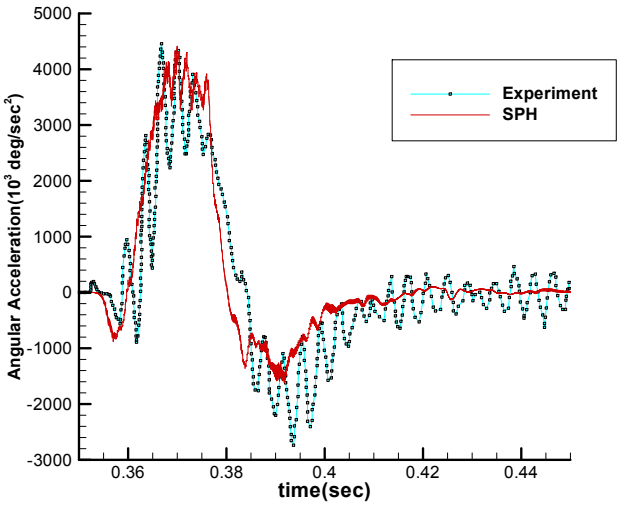
- Test section description[4]

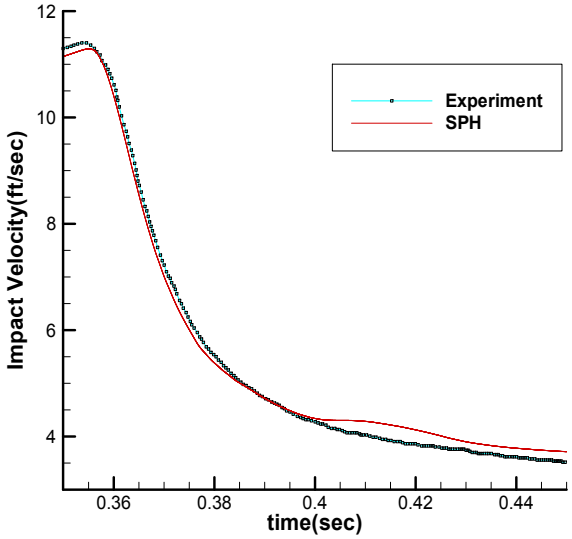


Length	2.44m
Weight	124kg
Inertia	8.85kg/m²

- Validation (uniform particle distribution model)





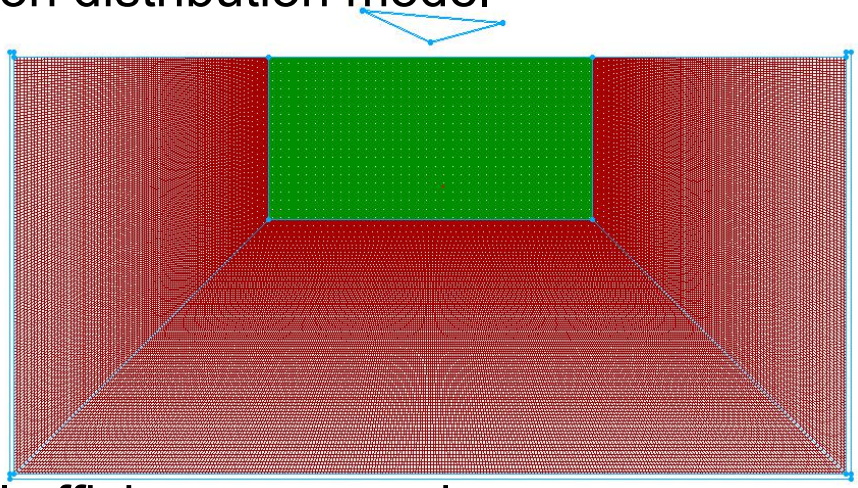


[4] Xu L, Troesch A W, Peterson R. Journal of Offshore Mechanics and Arctic Engineering. Transactions of the ASME, 1999, 121(2): 83-89.

Results and Analysis

➢ Asymmetric wedge water entry impact

- Particle diffusion distribution model



- Computational efficiency comparison

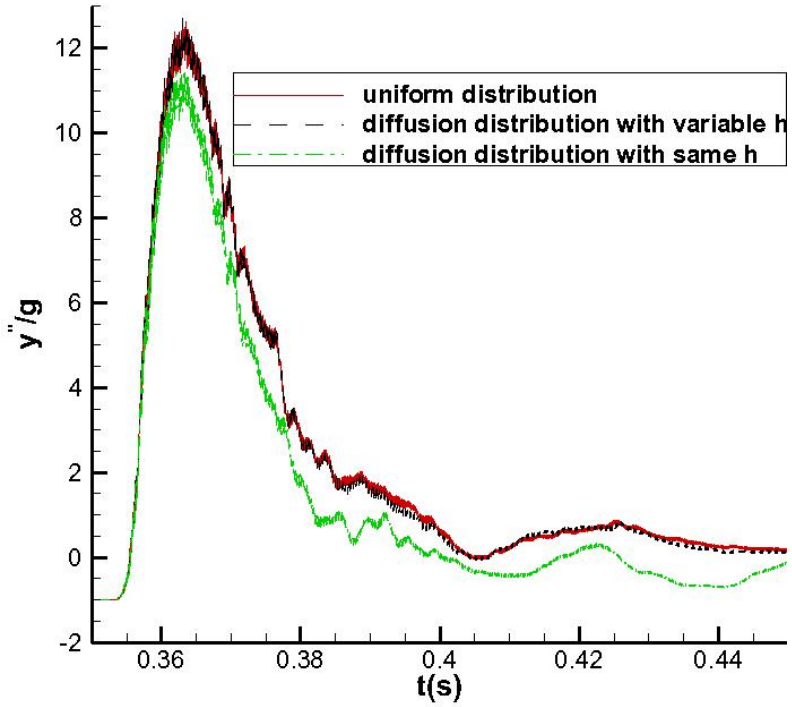
The computational cost decreases by almost 3/5.

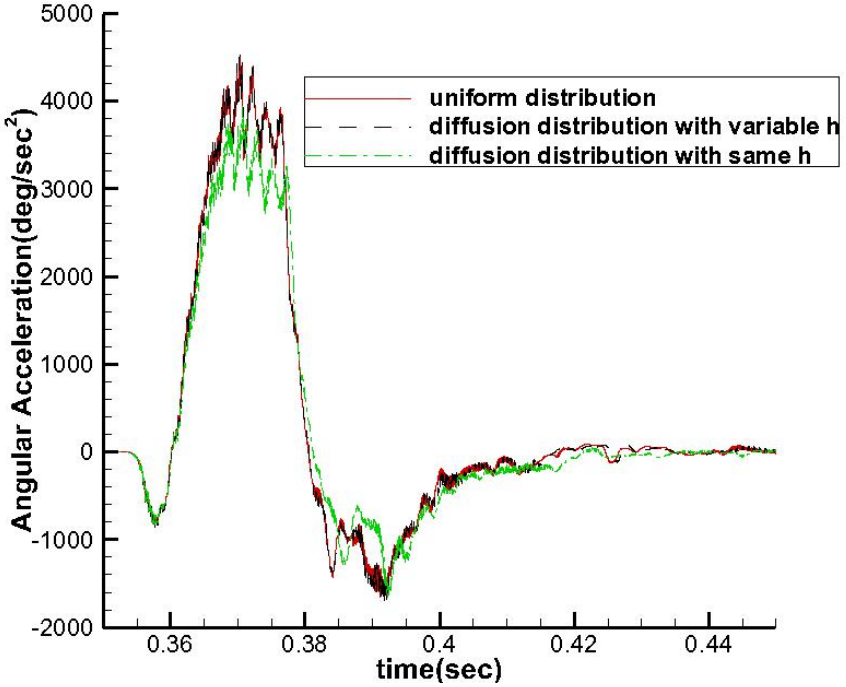
models / 2000 steps	Diffusion model (111357particles)	Uniform model (265277 particles)	Time ratio
Computing time	461.3s	1123.0s	41.0%
Searching time	806.0s	1939.0s	41.6%
Total time	1267.3s	3062.0s	41.4%

Results and Analysis

➢ Asymmetric wedge water entry impact

- Results comparison





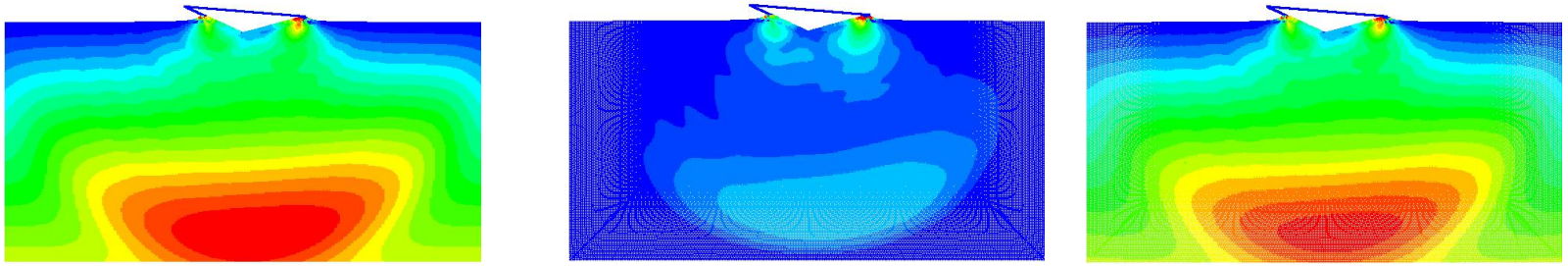
Effects of particle diffused distribution model and space variable smoothing length algorithm

Results and Analysis

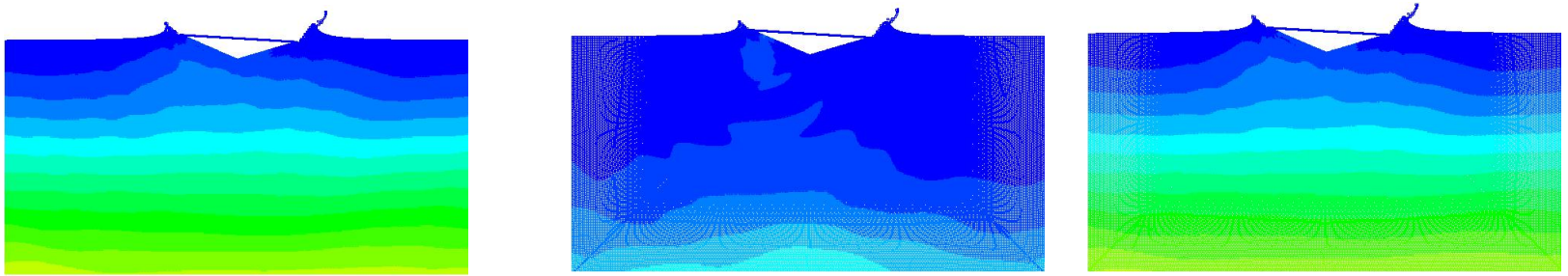
➢ Asymmetric wedge water entry impact

- Results comparison（contour）

0.377s



0.407s



Uniform model

diffusion model with constant *h* algorithm

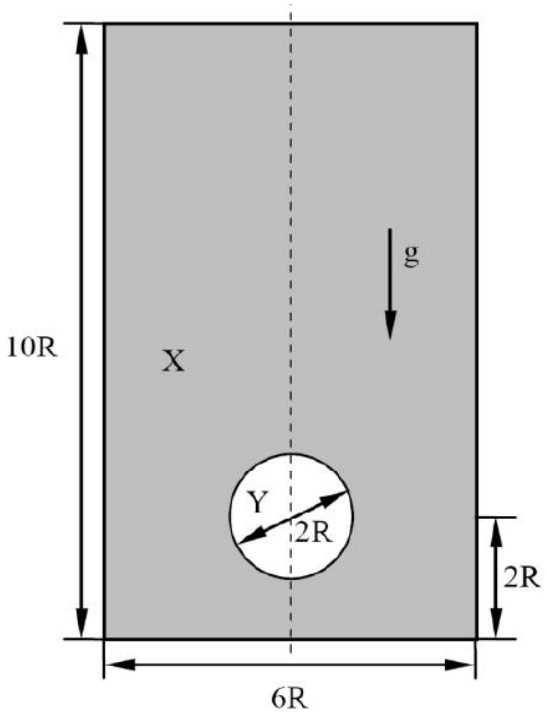
diffusion model with variable *h* algorithm

Results and Analysis

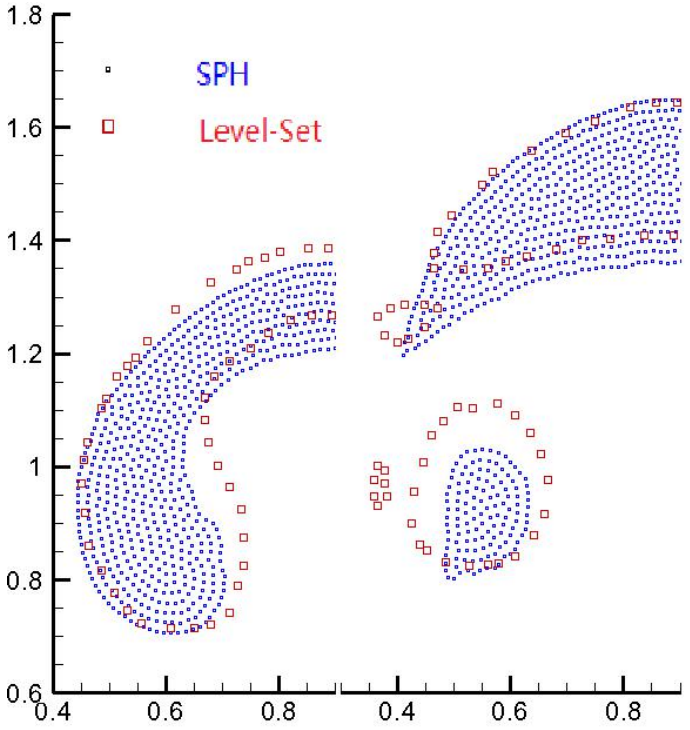
➢ Two-dimensional air bubble rising

- Test section description

$$\rho_X : \rho_Y = 1000 : 1$$



- Validation (uniform particle distribution model) [5]

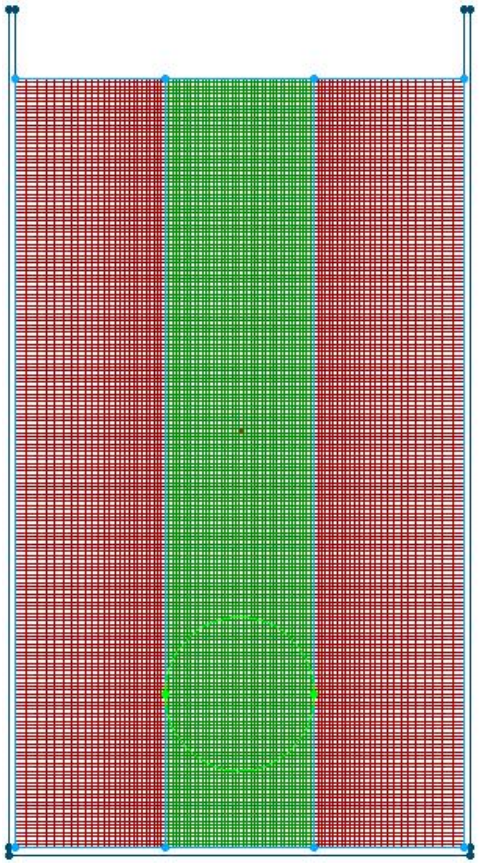


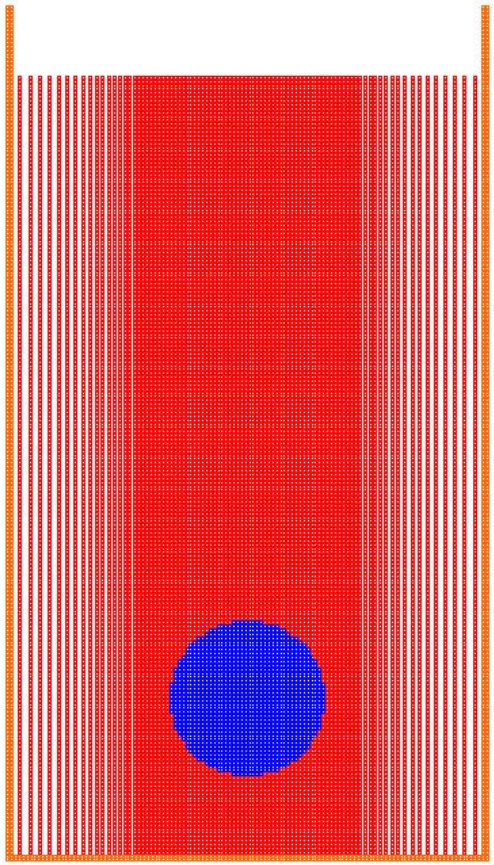
$$t\sqrt{g/R} = 3.2, 4.8$$

[5] Szewc, K., Taniere, A., Pozorski, J., & Minier, J. P. (2012). International Journal of Nonlinear Sciences and Numerical Simulation, 13(6), 383-395.

Results and Analysis

- Two-dimensional air bubble rising
  - Particle diffusion distribution model





Results and Analysis

➢ Two-dimensional air bubble rising

- Computational efficiency comparison

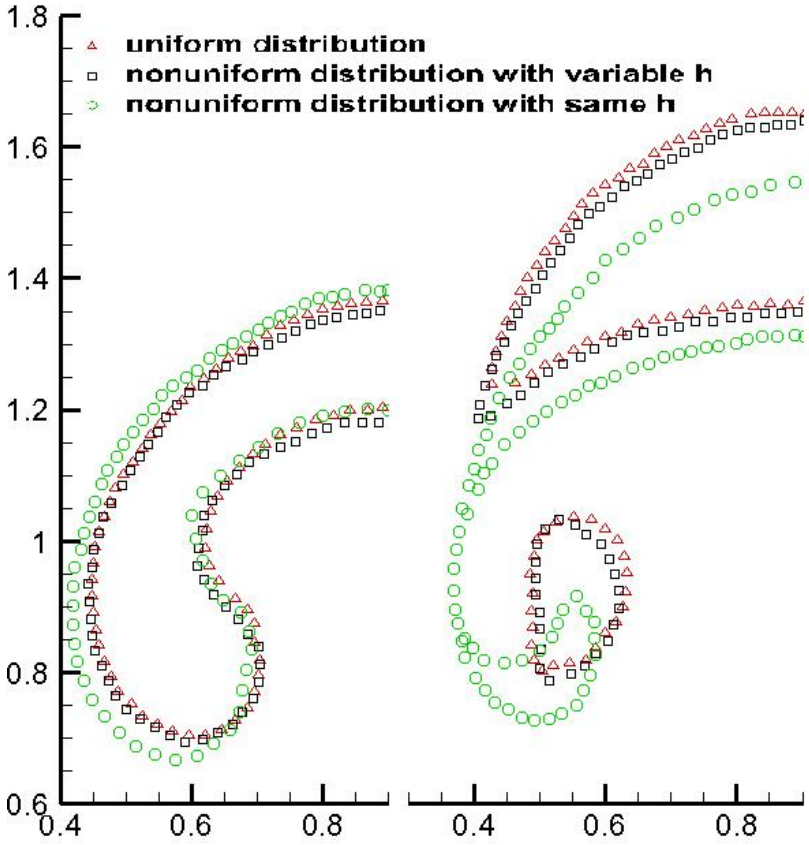
The computational cost decreases by almost 1/4.

Models / 1000 steps	Diffusion model (19120 particles)	Uniform model (24940 particles)	Time ratio
Computing time	36.27s	49.06s	73.9%
Searching time	62.28s	79.32s	78.5%
Total time	98.55s	128.38s	76.8%

Results and Analysis

➢ Two-dimensional air bubble rising

- Results comparison



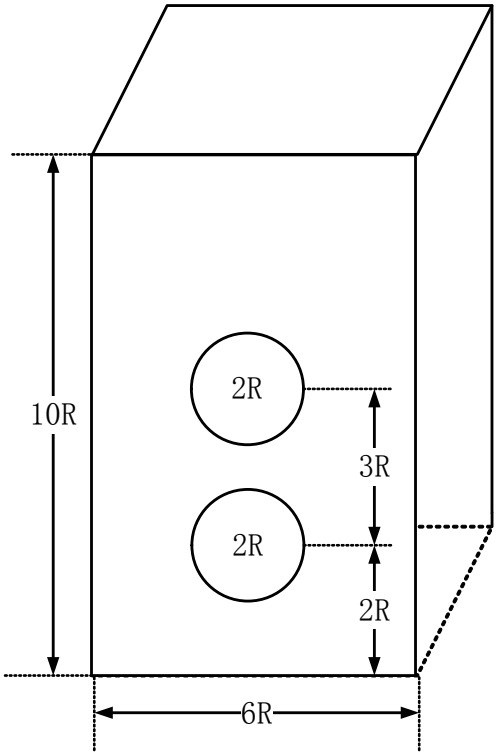
Effects of particle diffused distribution model and space variable smoothing length algorithm

Results and Analysis

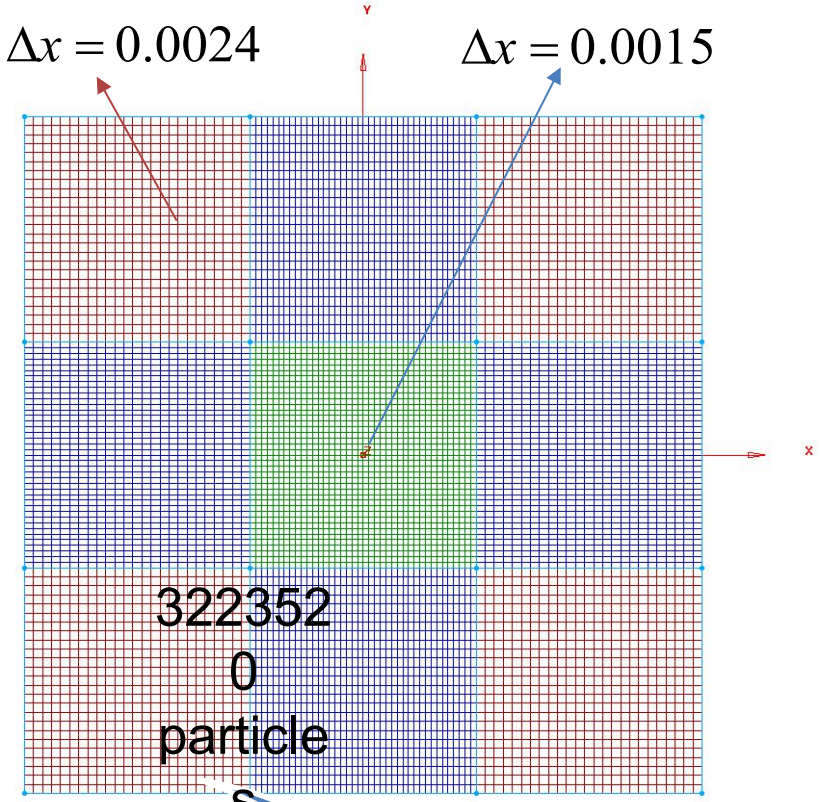
➢ Three-dimensional double air bubble rising

- Test section description

$$\rho_X : \rho_Y = 1000 : 1$$



- Particle diffusion distribution model



Results and Analysis

➢ Three-dimensional double air bubble rising

- Computational efficiency comparison

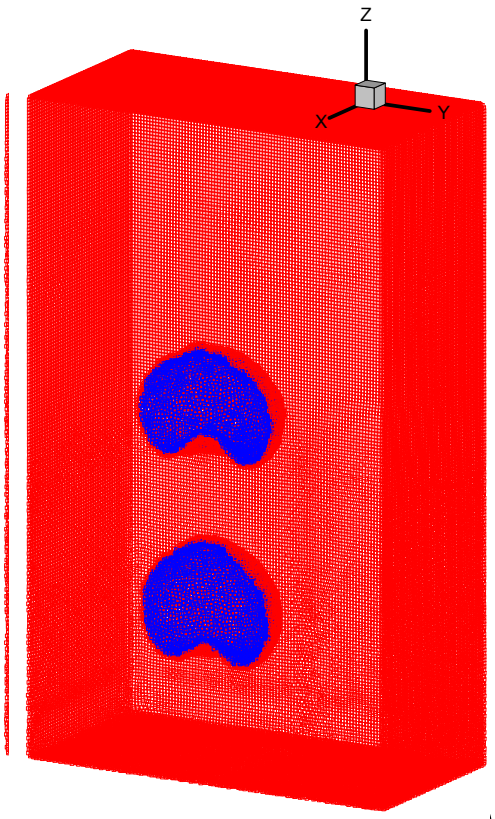
The computational cost decreases by about 1/2.

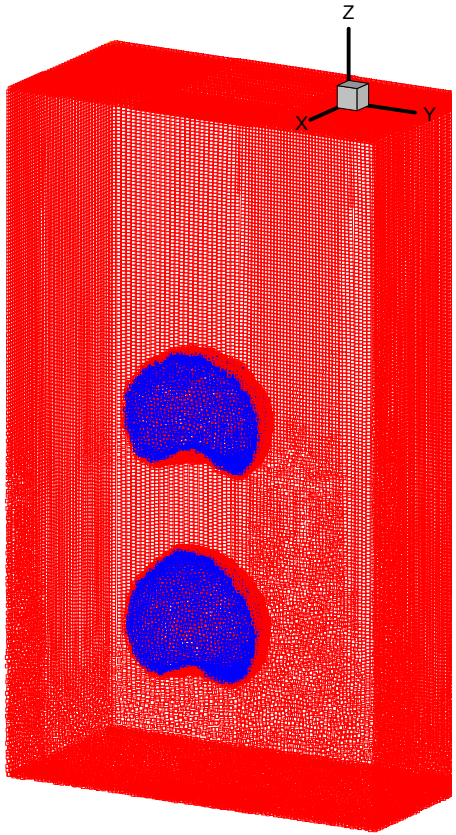
Models / 1000 steps	Diffusion model (1,864,140 particles)	Uniform model (3,223,520 particles)	Time ratio
Computing time	1376. 0s	2368. 0s	58. 1%
Searching time	1312. 0s	3392. 0s	38. 7%
Total time	2688. 0s	5760. 0s	46. 7%

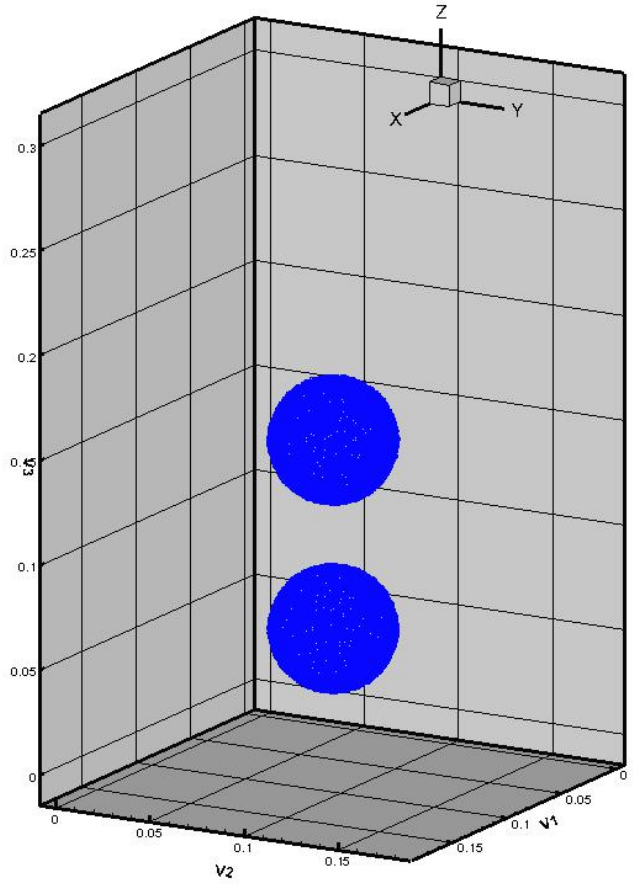
Results and Analysis

➢ Three-dimensional double air bubble rising

- Results comparison







0.15s

0.05s

Left: uniform distribution

Right: diffusion distribution with variable *h* algorithm

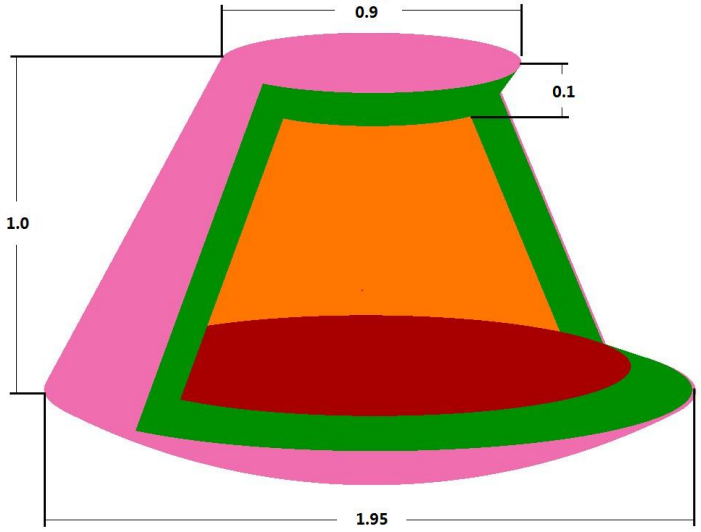
Results and Analysis

➢ Three-dimensional space capsule's water recovery

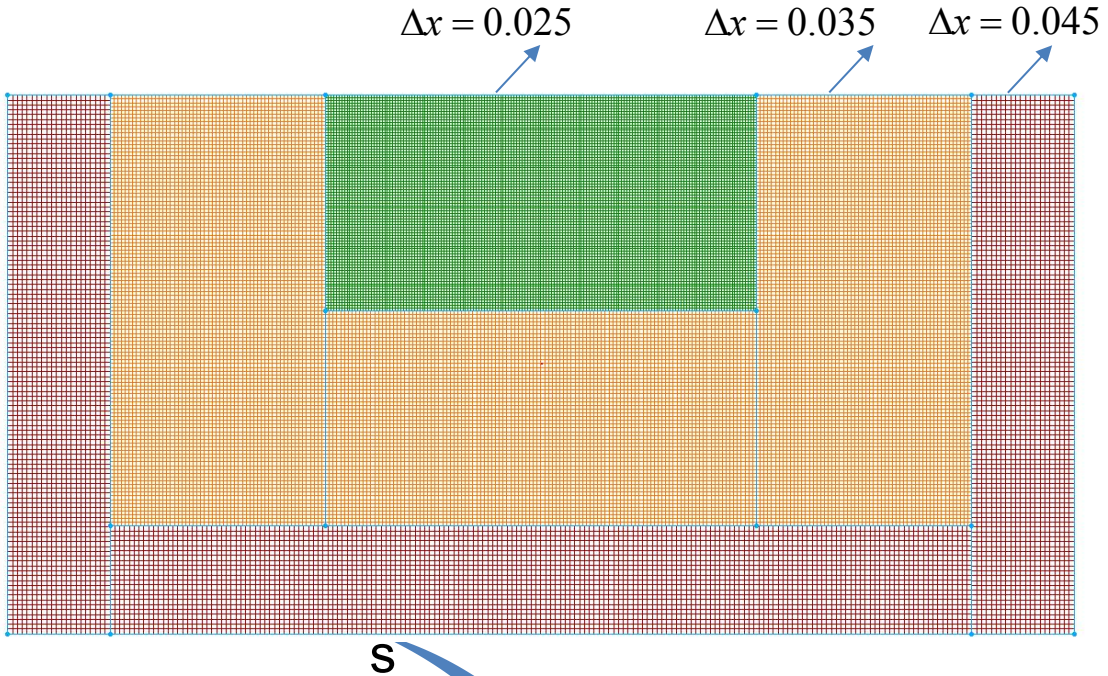
- Test section description[6]

Mass: 3300kg

Impact velocity: 9.5m/s



- Particle diffusion distribution model



13788282 particles

[6] Stubbs S M. National Aeronautics and Space Administration, Washington D. C., 1967.

Results and Analysis

➢ Three-dimensional space capsule water entry

- Computational efficiency comparison

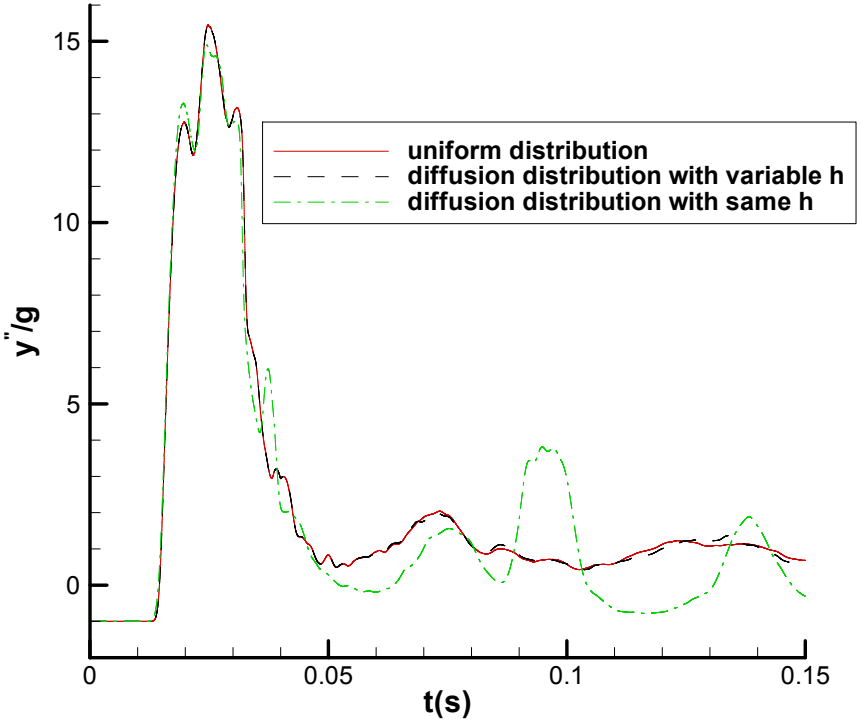
The computational cost decreases by about 3/5.

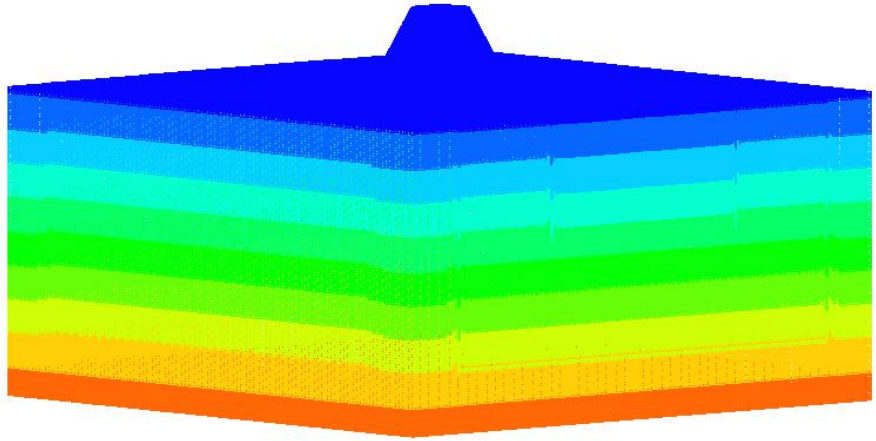
Models / 500 steps	Diffusion model (13,788,282 particles)	Uniform model (32,678,720 particles)	Time ratio
Computing time	4832. 0s	14768. 0s	32. 7%
Searching time	7968. 0s	16720. 0s	47. 7%
Total time	12800. 0s	31488. 0s	40. 7%

Results and Analysis

➢ Three-dimensional space capsule water entry

- Results comparison





CARDC

1.Introduction

2.Methods

3.NNPS performance evaluation

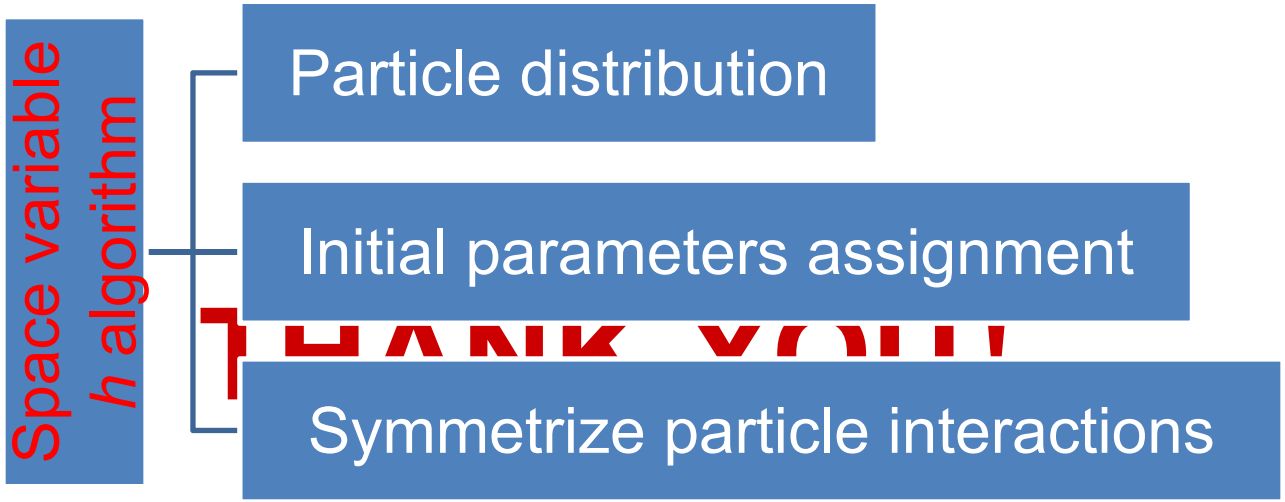
4.Results and Analysis

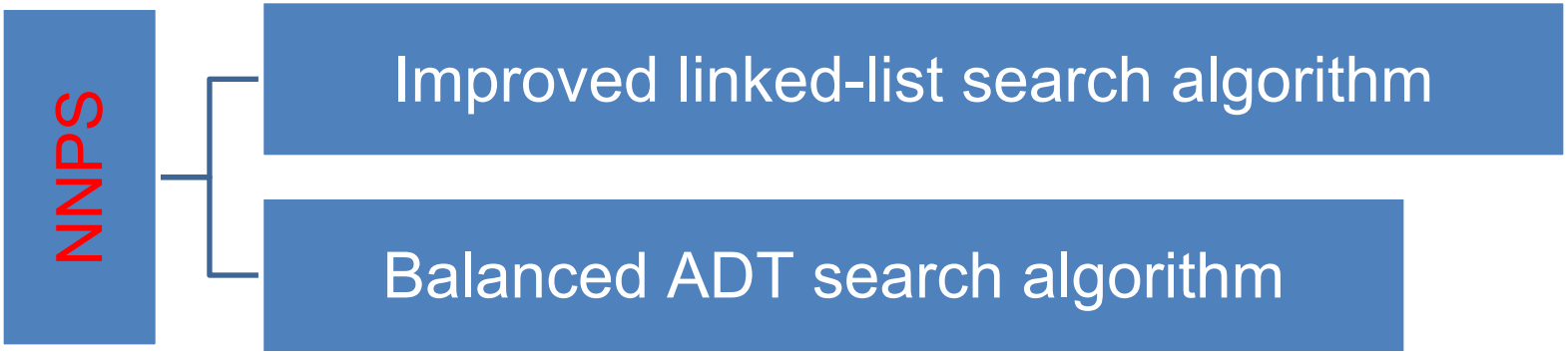
5.Conclusion and Future work

Conclusion and Future work

1. The particle diffusion model can effectively reduce the number of particles, but it also causes strong instability. The space variable smoothing length algorithm can strengthen stability.
2. Computational cost can be greatly reduced by using space variable smoothing length algorithm, while the numerical accuracy is still held.
3. Balanced ADT search algorithm is robust and efficient, especially for variable smoothing length problems.
4. Further research is needed to optimize and standardize the particle distribution model and to suppress the instability caused by the variable smoothing length.

Methods


Space variable h algorithm
Particle distribution
Initial parameters assignment
Symmetrize particle interactions


NNPS
Improved linked-list search algorithm
Balanced ADT search algorithm