Interaction between Solitary Wave and Horizontal Plate based on MPS-FEM coupled Method

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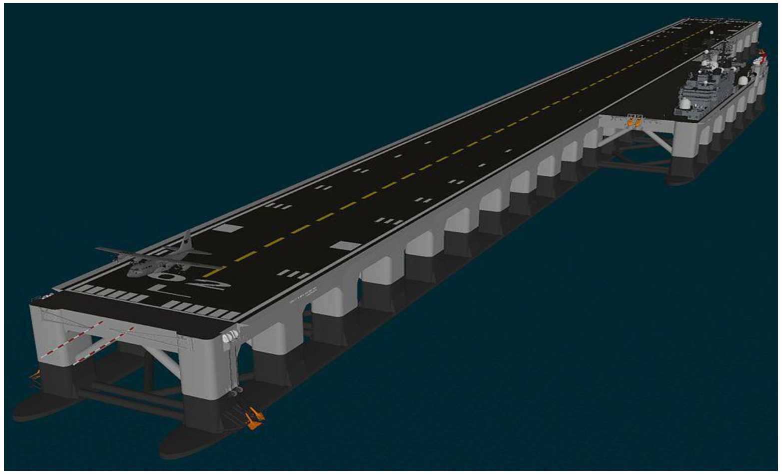
Background

Significance of the wave-plate interaction

- ✓ Structure of plate is commonly seen in naval architecture and ocean engineering, such as VLFS or bay bridge;
- ✓ Solitary wave model can be used to simulate extreme sea condition, such as Tsunami;
- ✓ When encountering severe wave load, these structures are likely to suffer from large deformation or damage.

VLFS



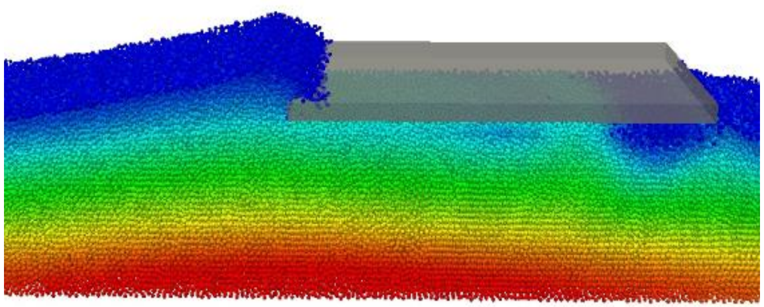
Floating runway

Picture from internet

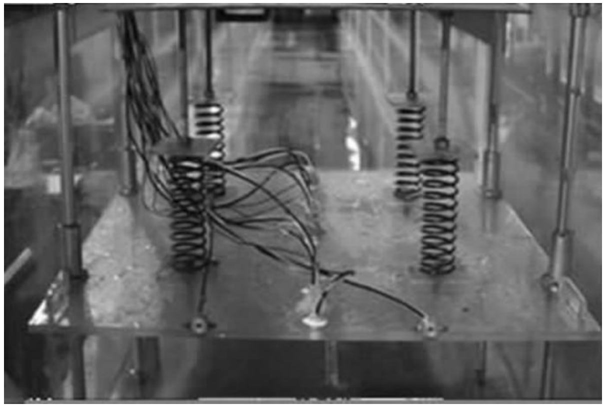
Background

Research methods

- Empirical method: Wave-induced force is decomposed into different components, such as slowly varying load and short-duration impact load;
- Experimental method: Record the wave-induced force history based on physical model;
- CFD method: Calculate the time-dependent loads using computational method, such as FVM, FDM, SPH, MPS, etc.



■ MPS method



■ Experimental method (Song et al. 2014)

Song, Zi Lu, et al. "Experimental study of the wave impact pressure on horizontal deck with elastic braces." Chinese Journal of Hydrodynamics, 2014, 29(4): 435-443

Present method

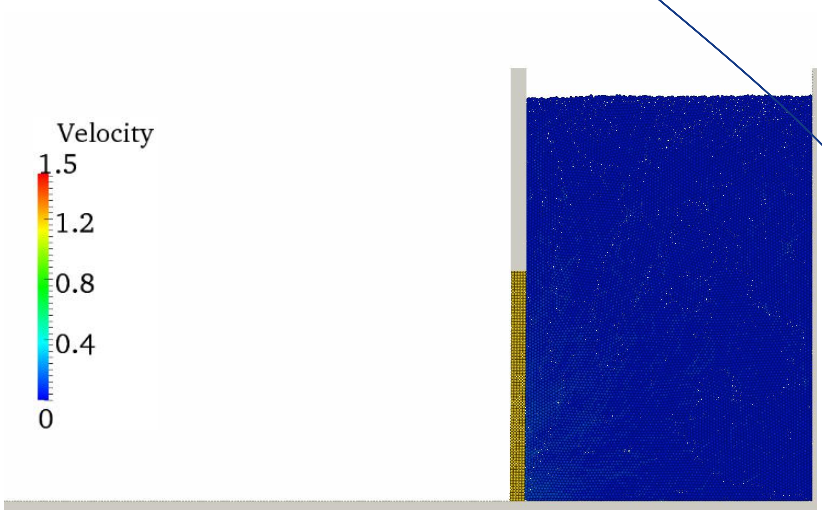
FSI-MPSFEM-SJTU

Fluid Analysis

Structure Analysis

Based on MPS method:
- ✓ Unnecessary for *re-meshing/ mesh-updating*, reduce error produced by grids deformation;
- ✓ Easy to *track* the new *interface* and *free surface*, easy to simulate breaking/splashing phenomenon.

Based on FEM:
- ✓ Reliable and be widely employed in commercial softwares;
- ✓ Capable for *large/nonlinear deformations*.



- ✓ Fully Lagrangian method;
- ✓ Partition coupled method without *interpolation on* fluid and structural *interface*;

Zhang, YL., Chen, X, and Wan, DC (2016a). "An MPS-FEM Coupled Method for the Comparative Study of Liquid Sloshing Flows Interacting with Rigid and Elastic Baffles," Applied Mathematics and Mechanics, 37(12), 1359-1377.

Contents

Numerical Method

➢ MPS method for fluid

Governing equation

$$\nabla \cdot \boldsymbol{V} = 0$$

$$\frac{\mathrm{D}\boldsymbol{V}}{\mathrm{D}t} = -\frac{1}{\rho}\nabla P + \nu \nabla^2 \boldsymbol{V} + \boldsymbol{g}$$

Kernel function

$$W(r) = \begin{cases} \dfrac{r_e}{0.85r + 0.15r_e} - 1 & 0 \le r < r_e \\ 0 & r_e \le r \end{cases}$$

Gradient model

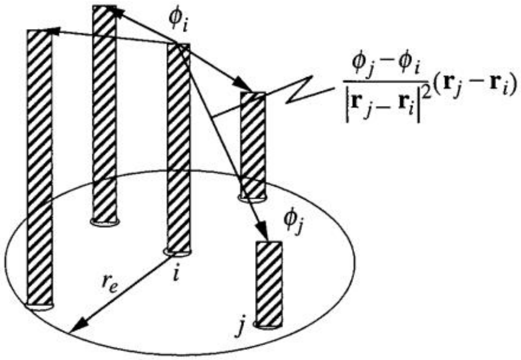
$$\langle \nabla \phi \rangle_i = \frac{D}{n^0} \sum_{j \ne i} \frac{\phi_j + \phi_i}{|\boldsymbol{r}_j - \boldsymbol{r}_i|^2} (\boldsymbol{r}_j - \boldsymbol{r}_i) \cdot W(|\boldsymbol{r}_j - \boldsymbol{r}_i|)$$

Laplacian model

$$\langle \nabla^2 \phi \rangle_i = \frac{2D}{n^0 \lambda} \sum_{j \ne i} (\phi_j - \phi_i) \cdot W(|\boldsymbol{r}_j - \boldsymbol{r}_i|)$$

Free surface detection

$$< \boldsymbol{F} >_i = \frac{D}{n^0} \sum_{j \ne i} \frac{1}{|\boldsymbol{r}_i - \boldsymbol{r}_j|} (\boldsymbol{r}_i - \boldsymbol{r}_j) W(\boldsymbol{r}_{ij})$$



- **Gradient model (*Koshizuka et al. 1998*)**

$$\lambda = \frac{\sum_{j \ne i} W(|\boldsymbol{r}_j - \boldsymbol{r}_i|) |\boldsymbol{r}_j - \boldsymbol{r}_i|^2}{\sum_{j \ne i} W(|\boldsymbol{r}_j - \boldsymbol{r}_i|)}$$

Koshizuka, Seiichi, Atsushi Nobe, and Yoshiaki Oka. "Numerical analysis of breaking waves using the moving particle semi-implicit method." International Journal for numerical methods in fluids 26.7 (1998): 751-769.

Numerical Method

➢ FEM method for structure

Governing equation

$$\mathbf{M}\,\ddot{y} + \mathbf{C}\,\dot{y} + \mathbf{K}\,y = F(t)$$

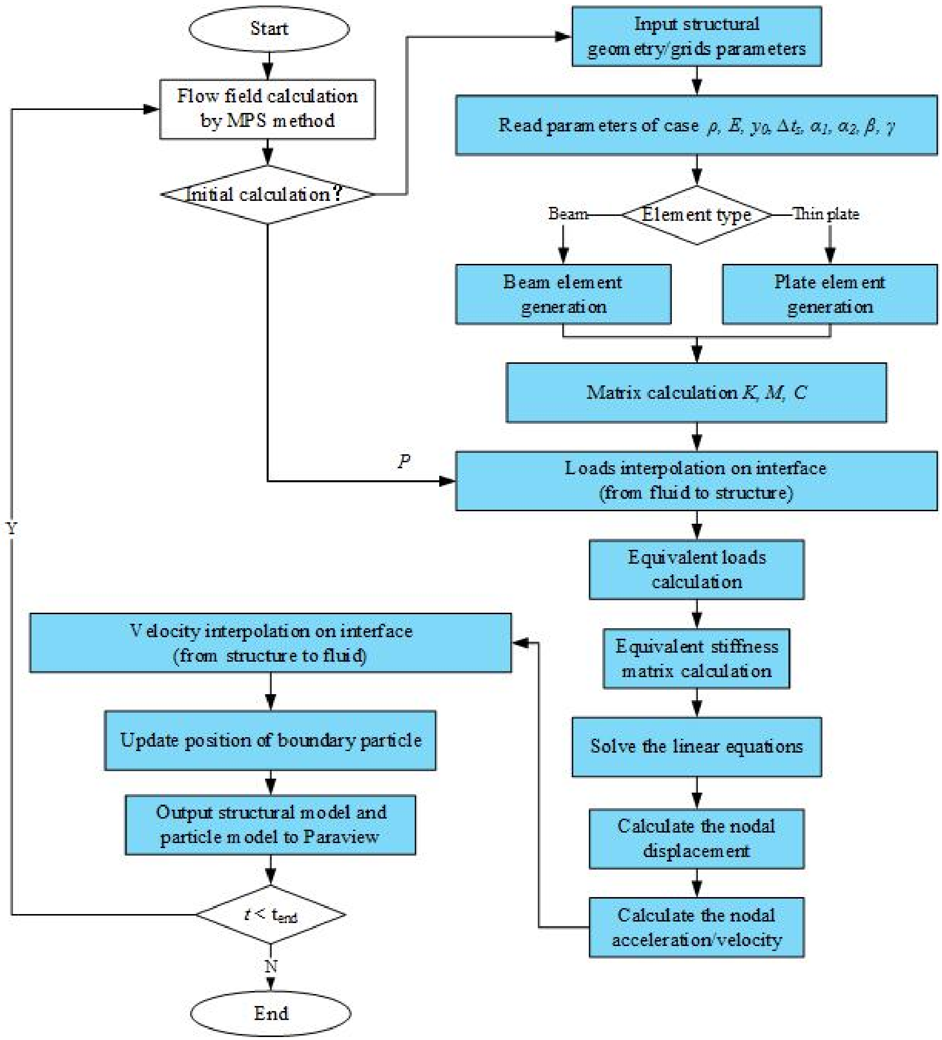
$$\mathbf{C} = \alpha_1\,\mathbf{M} + \alpha_2\,\mathbf{K}$$

Newmark-β scheme for structure

$$\bar{\mathbf{K}}\,y_{t+\Delta t} = \bar{F}_{t+\Delta t}$$

$$\ddot{y}_{t+\Delta t} = a_0\left(y_{t+\Delta t} - y_t\right) - a_2\dot{y}_t - a_3\ddot{y}_t$$

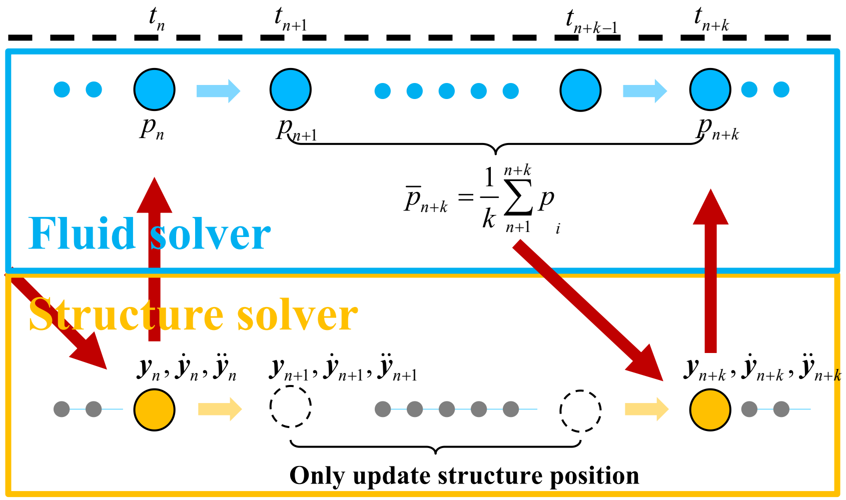
$$\dot{y}_{t+\Delta t} = \dot{y}_t + a_6\ddot{y}_t + a_7\ddot{y}_{t+\Delta t}$$

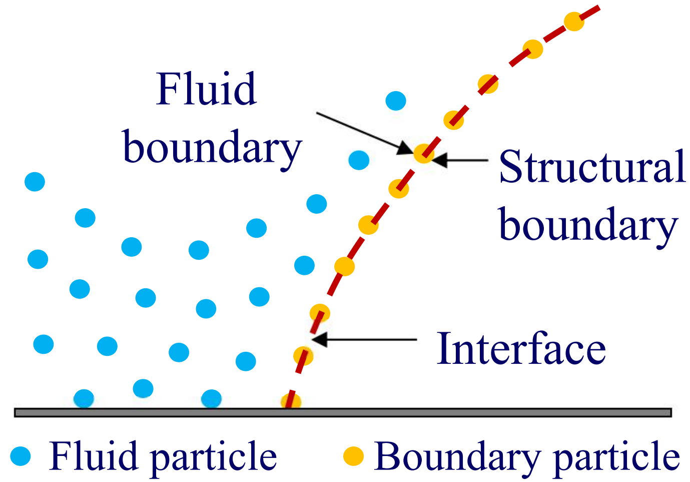


■ **Flowchart of the MPS-FEM Solver**

Numerical Method

➢ Coupling strategy for FEM & MPS





- **Time step sizes:** different sizes for fluid & structure analysis ($t_s = k * t_f$)
- **Interface:** consistent boundary for both fluid and structure
- **Data transfer:** unnecessary of interpolation on interface
  - ✓ *P* (loads acting on structural boundary), be calculated directly by the solving of PPE
  - ✓ ***y*** (displacements of boundary particles), be solved with interval t_s, updated with interval t_f

Numerical Method

➢ Numerical wave generation

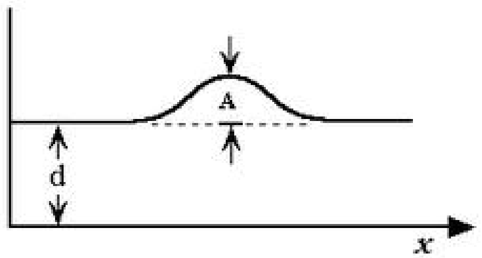
H wave height, ***d*** water depth, ***x*** horizontal coordinate, ***η*** wave elevation, ***c*** wave speed, ***t*** time

Equation for solitary wave

Wave profile: $\eta = H\mathrm{sech}^2(k(x-ct))$

Wave number: $k = \sqrt{3H/4d^3}$

Wave speed: $c = \sqrt{g(H+d)}$



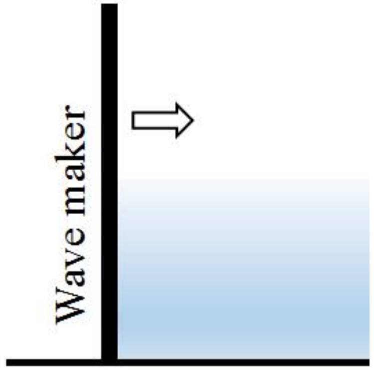
- **Profile of solitary wave**

Method for wave generation *(Goring et al. 1978)*

Speed of the wavemaker: $U(t) = \dfrac{\mathrm{d}X(t)}{\mathrm{d}t} = \dfrac{cH\mathrm{sech}^2(k(X-ct))}{d + H\mathrm{sech}^2(k(X-ct))}$

Position of the wavemaker: $X(t) = \dfrac{H}{kd}\tanh(k(ct-X))$

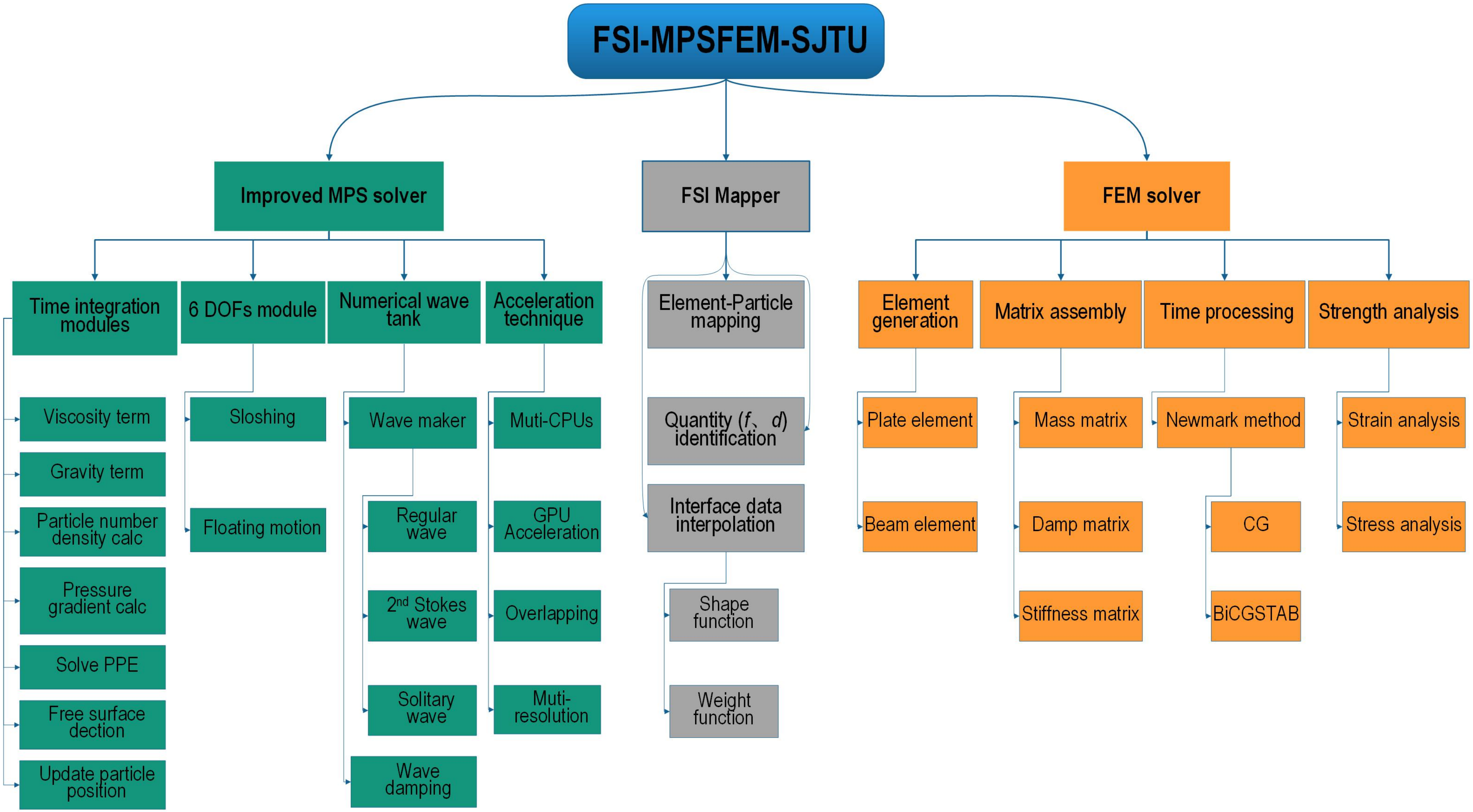
Stroke length: $S = X(t\to\infty) - X(t\to-\infty) = \sqrt{\dfrac{16Hd}{3}}$



- **Piston-type wavemaker**

Goring, DG (1978). "Tsunamis-the Propagation of Long Waves onto a Shelf," Pasadena, California, USA, California Institute of Technology.

Framework of the FSI solver



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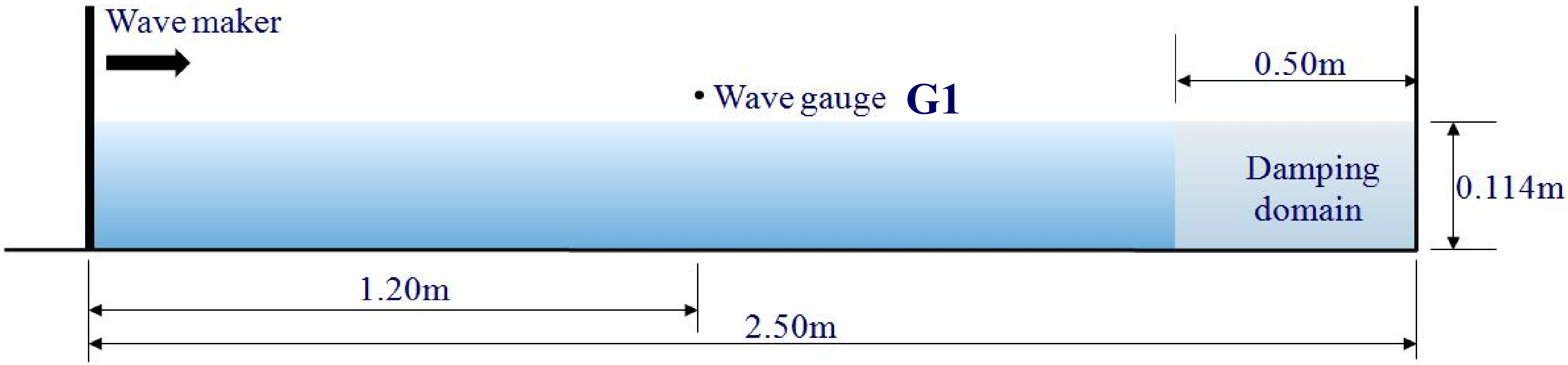
Numerical Examples

➢ Numerical wave generation



■ **The scheme of numerical wave tank**

Parameters	Values
Water density	1000(kg/m^3)
Water depth	0.114(m)
Kinematic viscosity	1×10^{-6}(m^2/s)
Gravitational acceleration	9.81(m/s^2)
Particle spacing	0.002(m)
Fluid number	71193
Total number	75762

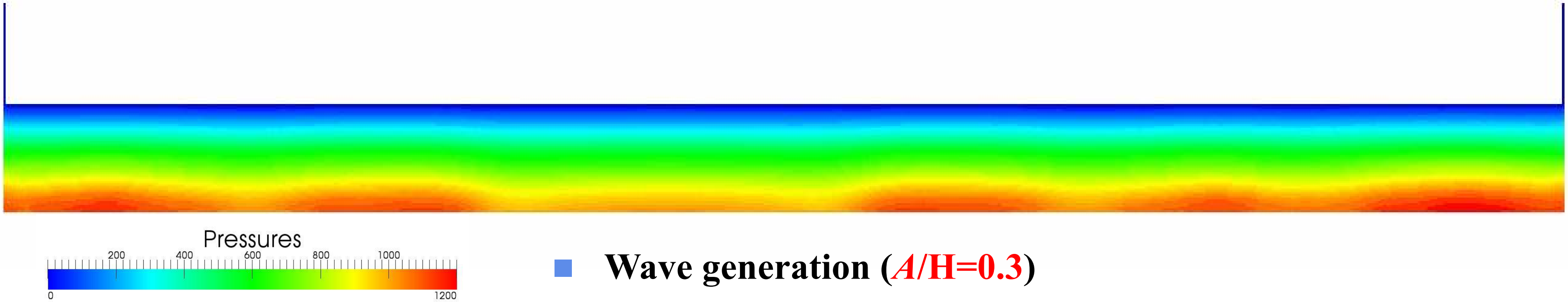
■ **Computational parameters**

Case No.	Amplitudes (A)	A/H ratio
1	0.0228 (m)	0.2
2	0.0342 (m)	0.3
3	0.0456 (m)	0.4
4	0.0570 (m)	0.5

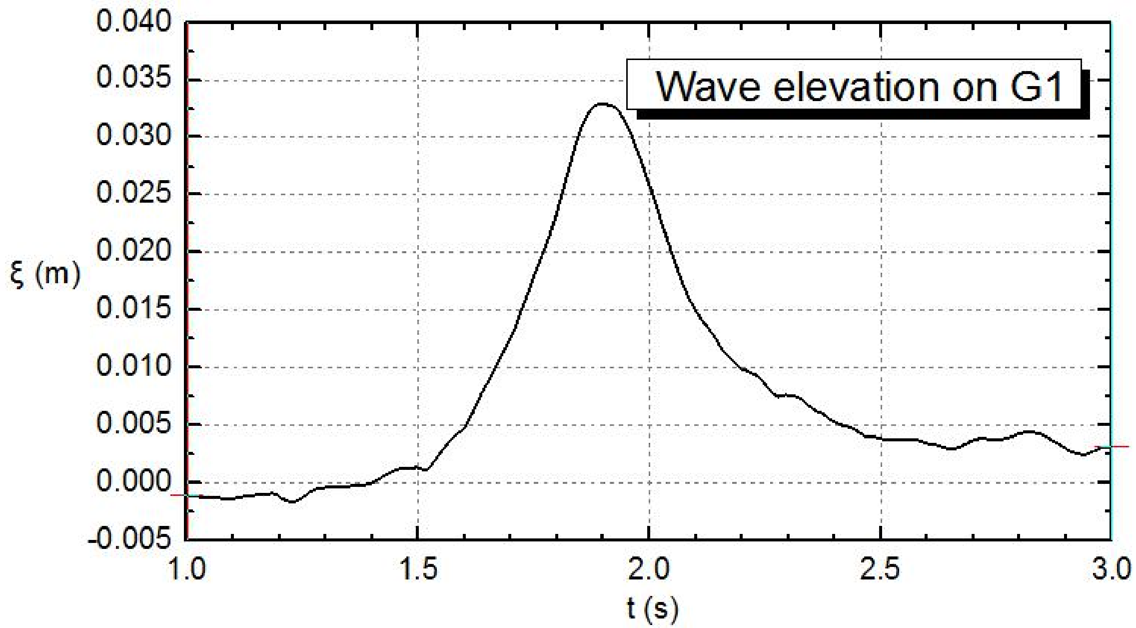
■ **Wave parameters**

Numerical Examples

➢ Numerical wave generation



■ **Wave generation (*A*/H=0.3)**

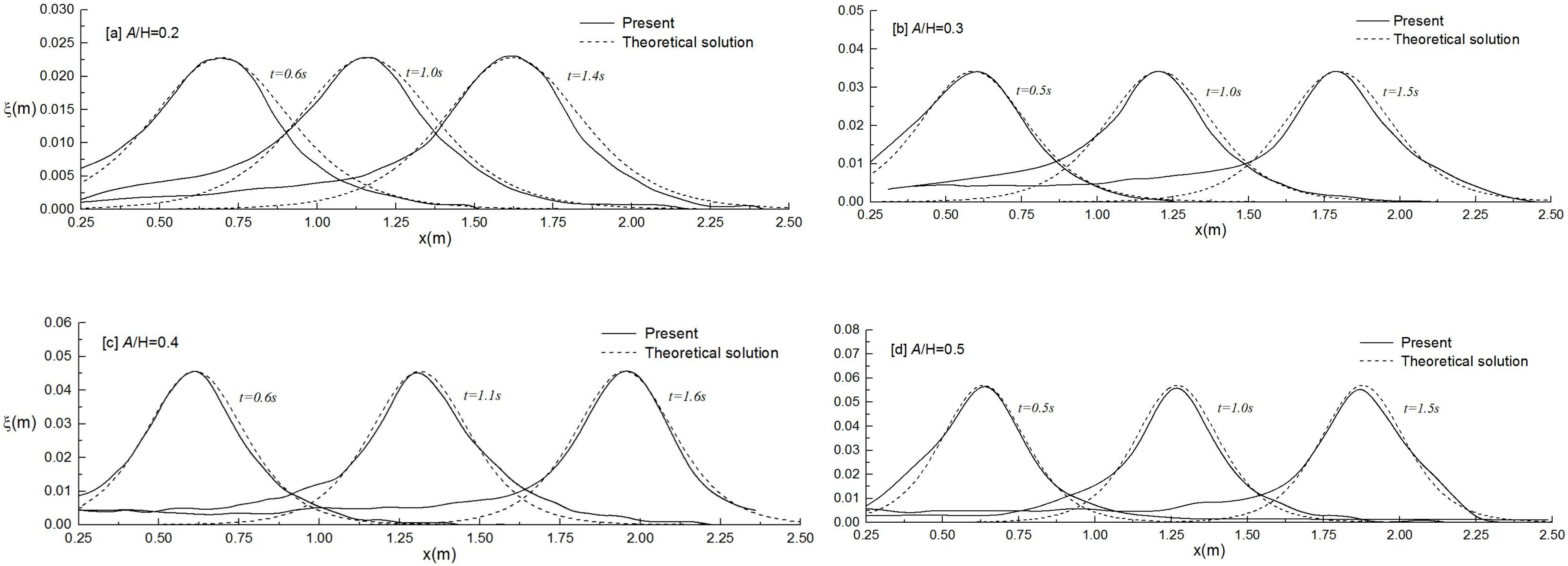


■ **Wave elevation history on wave gauge**

Numerical Examples

➢ Numerical wave generation

Wave elevation (m)



Distance (m)

- **Wave profile of the obtained result and theoretical solution**

<u>Desired solitary wave can be generated using present solver</u>

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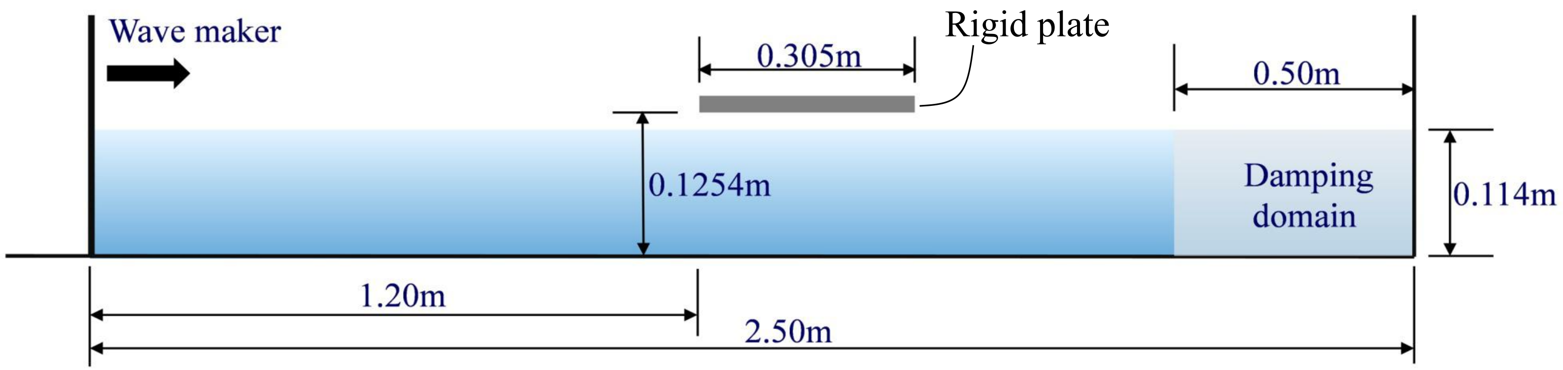
Numerical Examples

➢ Interaction between Solitary Wave and Rigid Plate



■ **The scheme of numerical simulation**

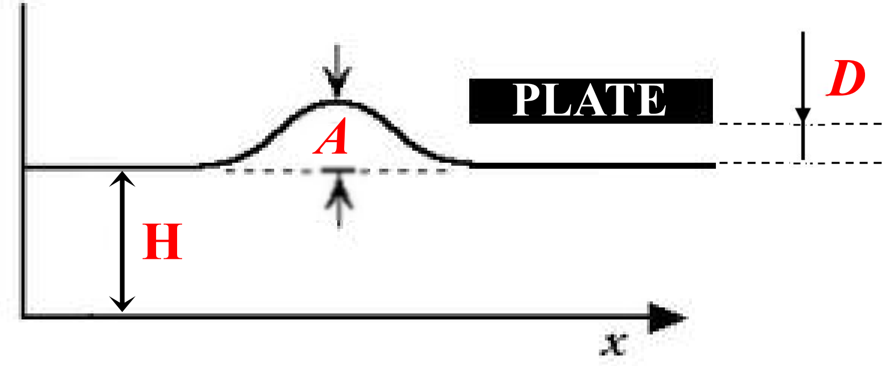
Parameters	Values
Water density	1000(kg/m^3)
Water depth	0.114(m)
Kinematic viscosity	1×10^{-6}(m^2/s)
Gravitational acceleration	9.81(m/s^2)
Particle spacing	0.002(m)
Fluid number	71193
Total number	75762

■ **Computational parameters**

Numerical Examples

➢ Interaction between Solitary Wave and Rigid Plate

Wave amplitude (A/H=0.2, 0.3, 0.4, 0.5)
Plate elevation (D/H=0.03, 0.06, 0.1)

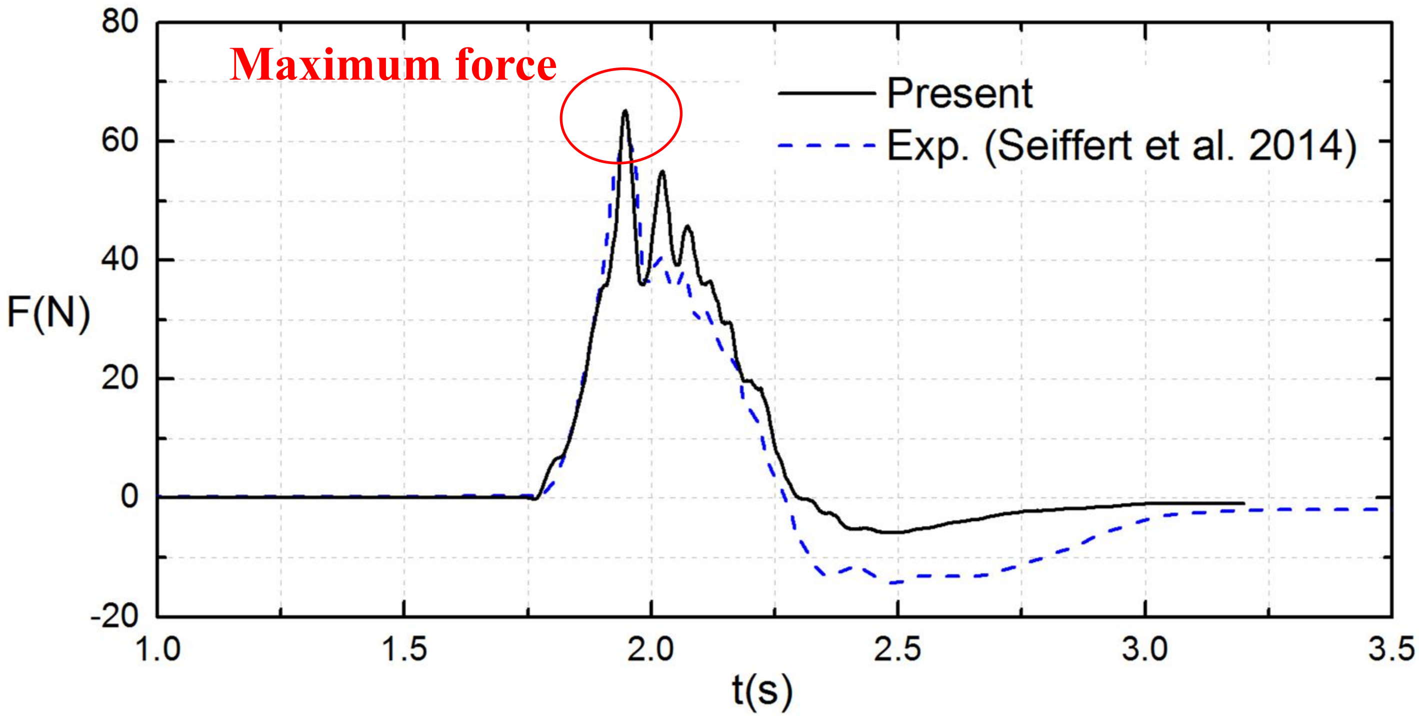


Case No.	Amplitude (A/H)	Elevation (D/H)
1	0.2	0.03
2	0.2	0.06
3	0.2	0.1
4	0.3	0.03
5	0.3	0.06
6	0.3	0.1
7	0.4	0.03
8	0.4	0.06
9	0.4	0.1
10	0.5	0.03
11	0.5	0.06
12	0.5	0.1

- List of cases

Numerical Examples

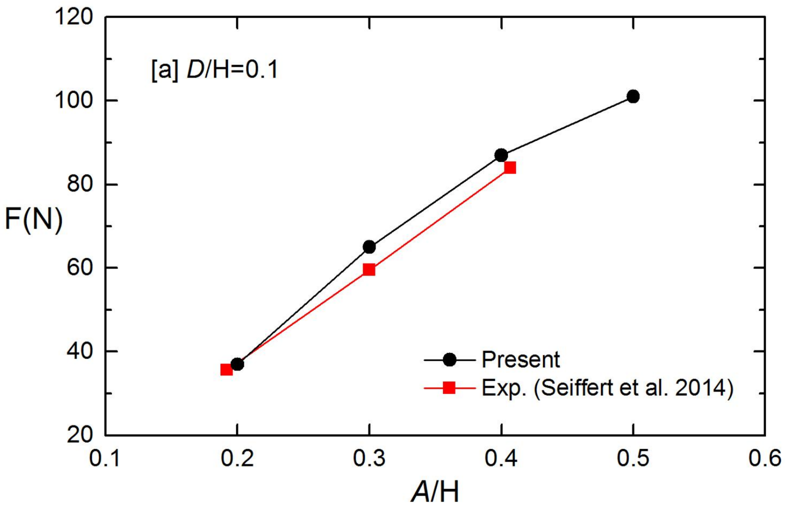
➢ Interaction between Solitary Wave and Rigid Plate

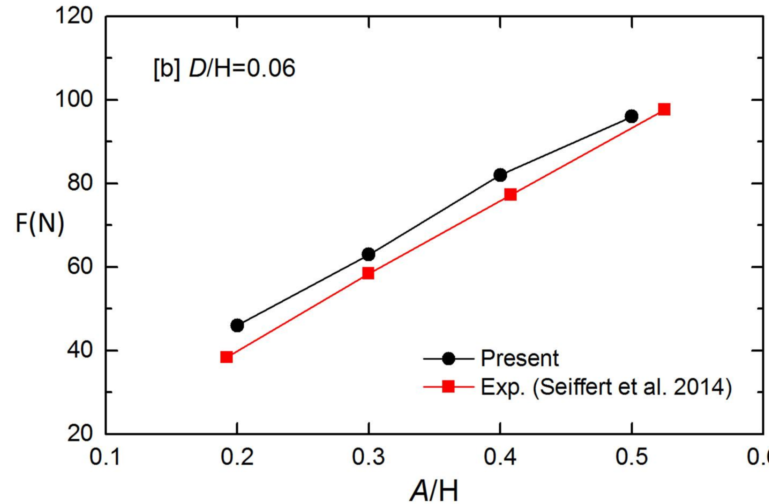


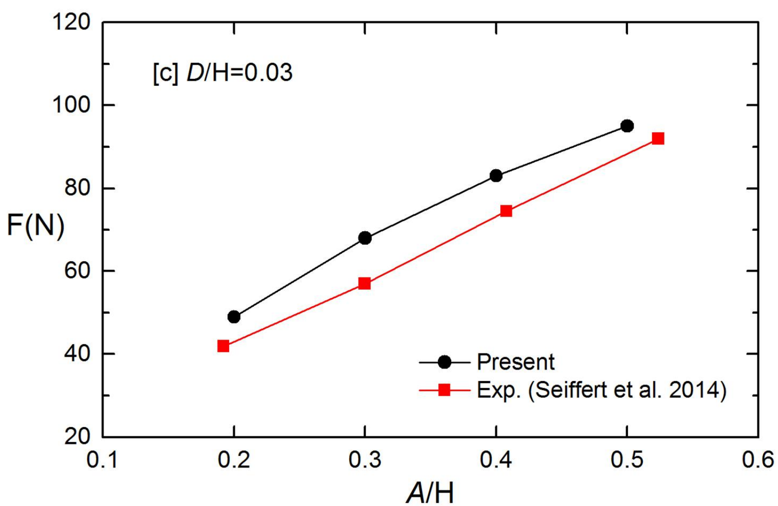
- Comparison of the vertical force on plate (*A*/H=0.3, *D*/H=0.1)

Numerical Examples

➢ Interaction between Solitary Wave and Rigid Plate







Computational wave-induced forces agrees with the experimental results (*Seiffert et al. 2014*)

■ **Comparison of the maximum force on plate**

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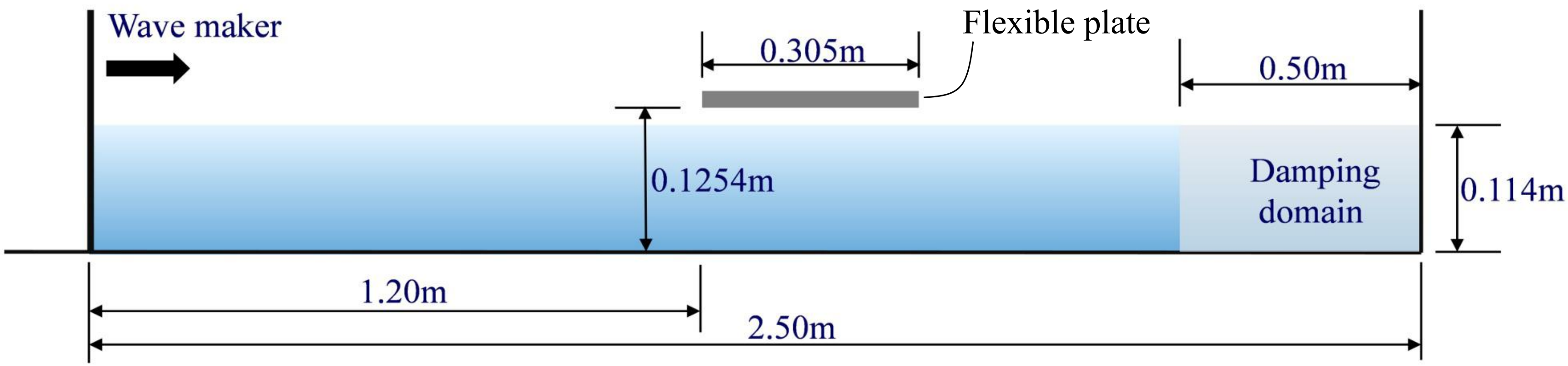
Numerical Examples

➢ Interaction between Solitary Wave and Flexible Plate



■ **The scheme of numerical simulation**

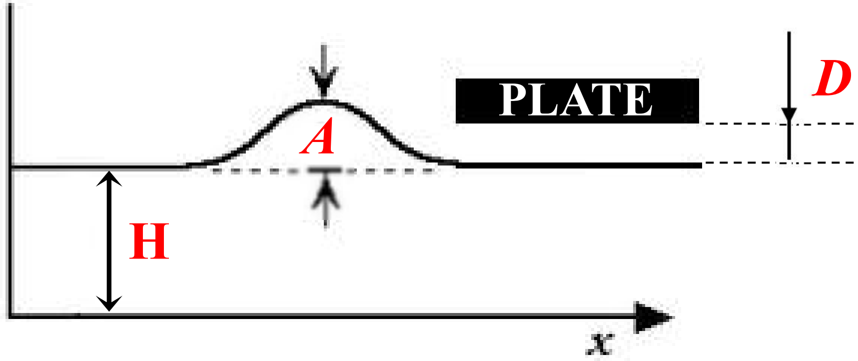
Fluid	Parameters	Values	Structure	Parameters	Values
	Water density	1000(kg/m^3)		Structural density	1040 (kg/m^3)
	Water depth	0.114(m)		Elastic modulus	1(MPa)
	Kinematic viscosity	1×10^{-6}(m^2/s)		Cross area	2.5×10^{-5}(m^2)
	Gravitational acceleration	9.81(m/s^2)		Inertia moment	1×10^{-3}(m^4)
	Particle spacing	0.002(m)		Damping ratios α_1, α_2	1.6646, 0.00096
	Particle number	76686		Element length	0.002(m)
				Element number	152

■ **Computational parameters**

Numerical Examples

➢ Interaction between Solitary Wave and Flexible Plate

Wave amplitude (A/H=0.2, 0.3, 0.4, 0.5)
Plate elevation (D/H=0.03, 0.06, 0.1)



Case No.	Amplitude (A/H)	Elevation (D/H)
1	0.2	0.03
2	0.2	0.06
3	0.2	0.1
4	0.3	0.03
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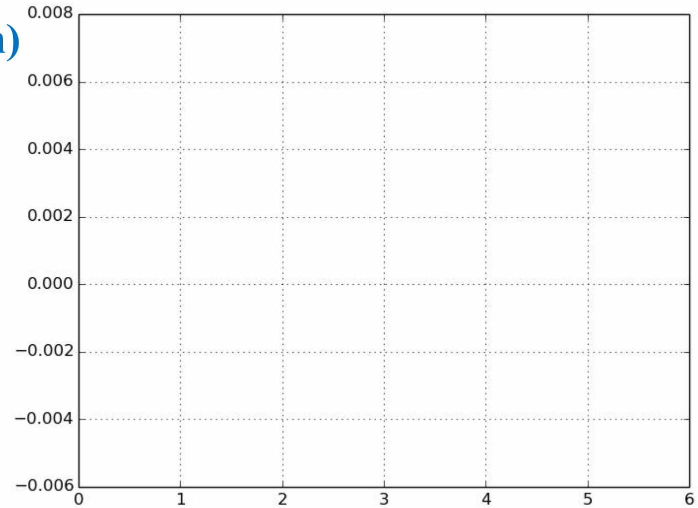
6	0.3	0.1
7	0.4	0.03
8	0.4	0.06
9	0.4	0.1
10	0.5	0.03
11	0.5	0.06
12	0.5	0.1

- List of cases

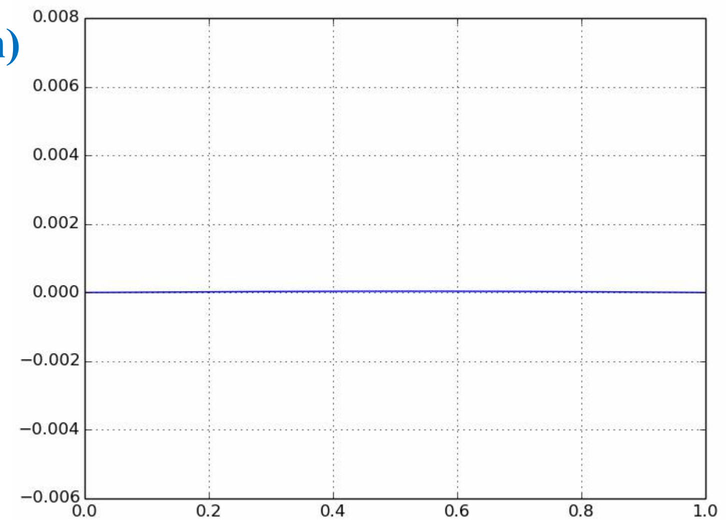
Numerical Examples

➢ Interaction between Solitary Wave and Flexible Plate

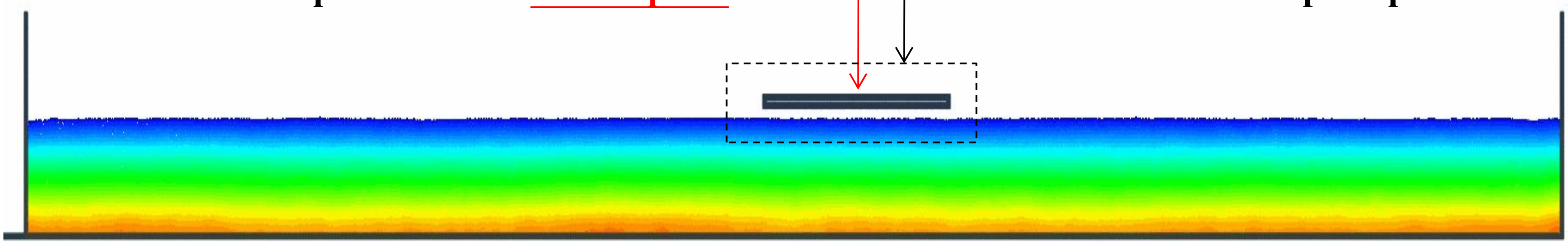
Disp. (m)

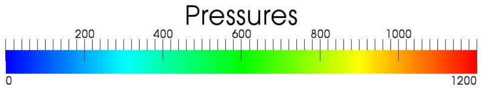


Disp. (m)



- Displacement on middle point
- Deformed shape of plate

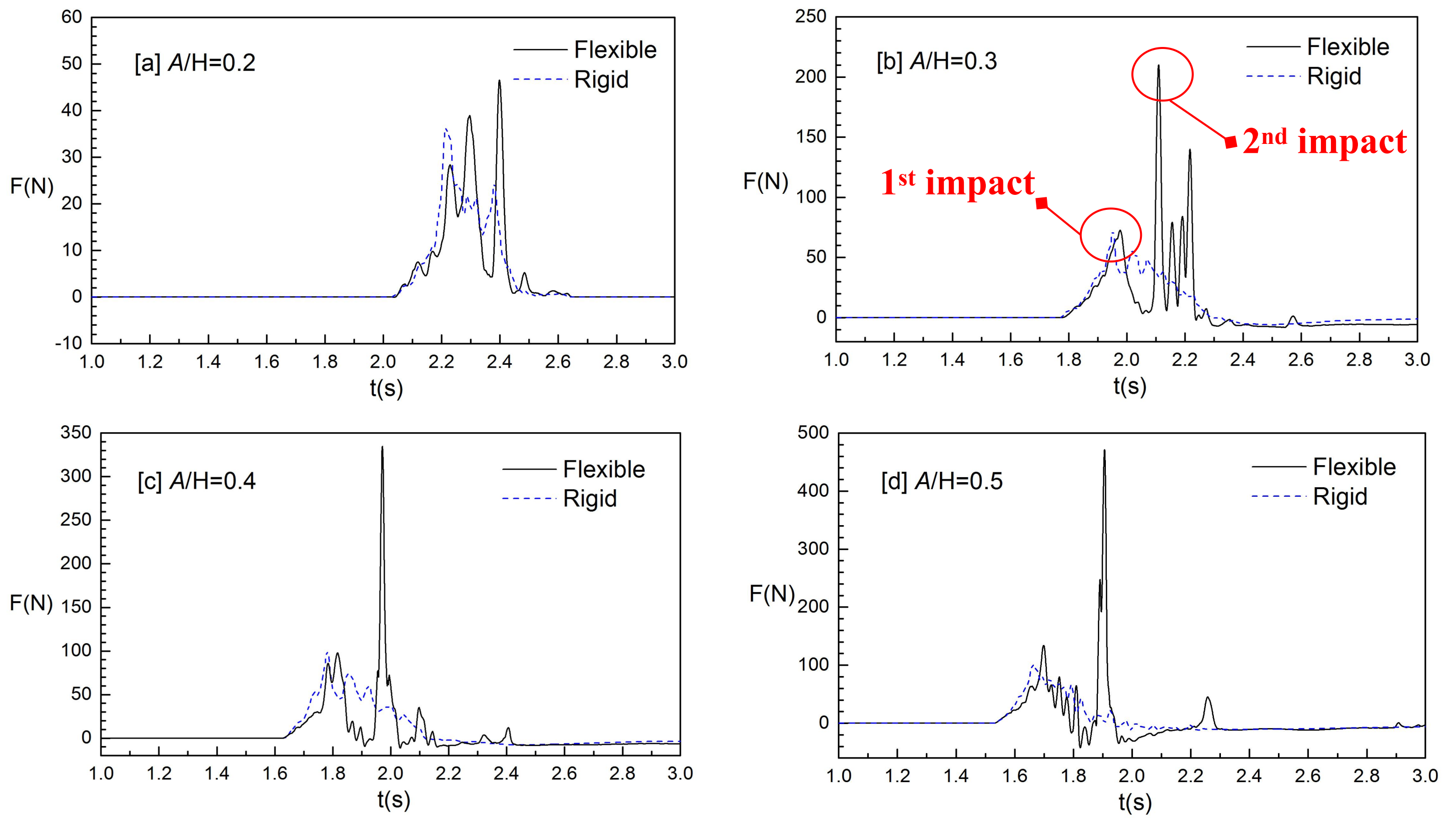




- The scheme of numerical simulation (*A*/H=0.3, *D*/H=0.1)

Numerical Examples

➢ Interaction between Solitary Wave and Flexible Plate

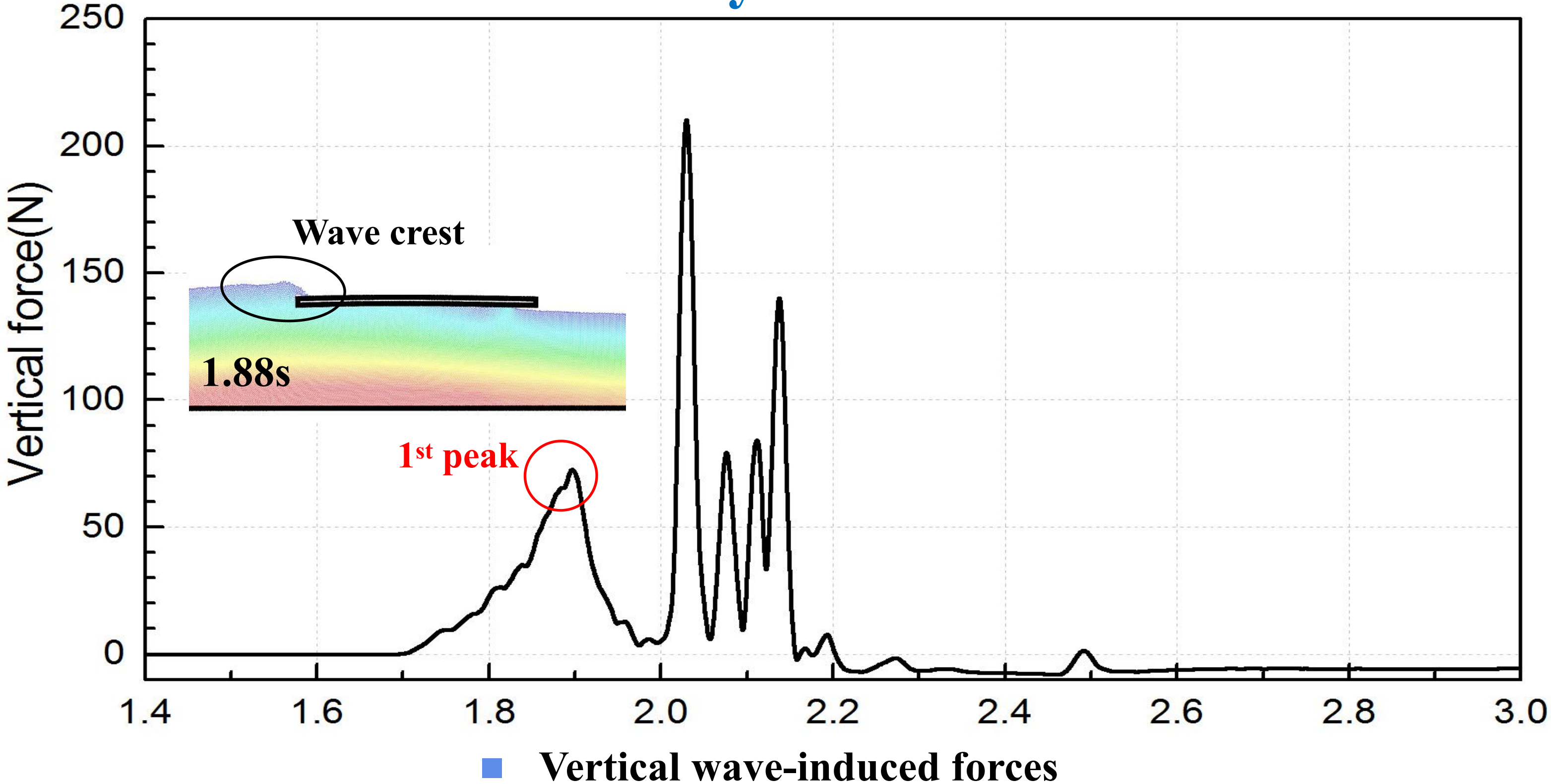


- **Wave-induced force on plate (Dash: rigid; solid: flexible)**

A 2nd impact force can be observed for the flexible plate

Numerical Examples

➢ Interaction between Solitary Wave and Flexible Plate

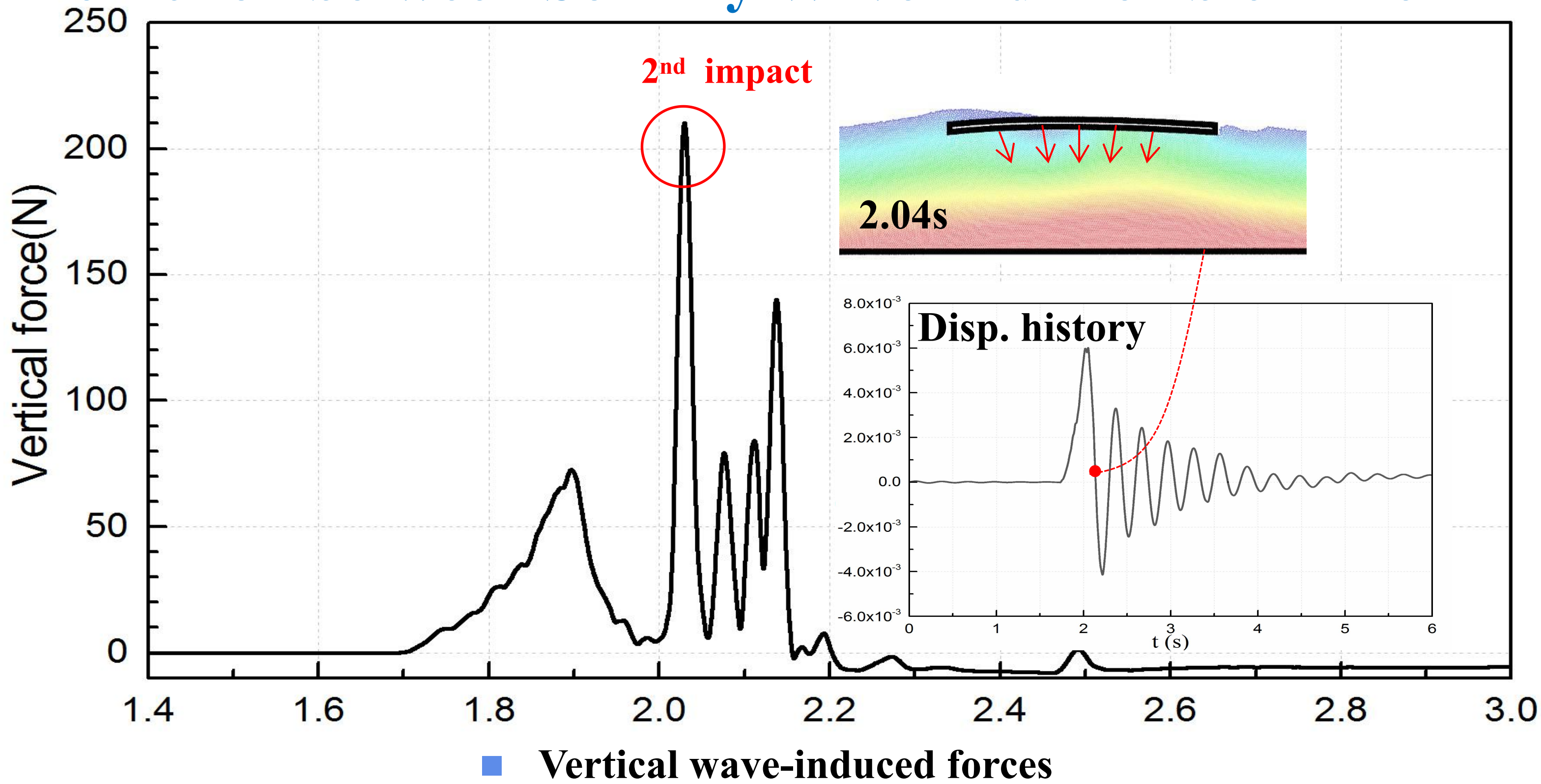


DISCUSSIONS

① The 1st peak happens when the wave crest arrives the leading edge (1.88s);

Numerical Examples

➢ Interaction between Solitary Wave and Flexible Plate

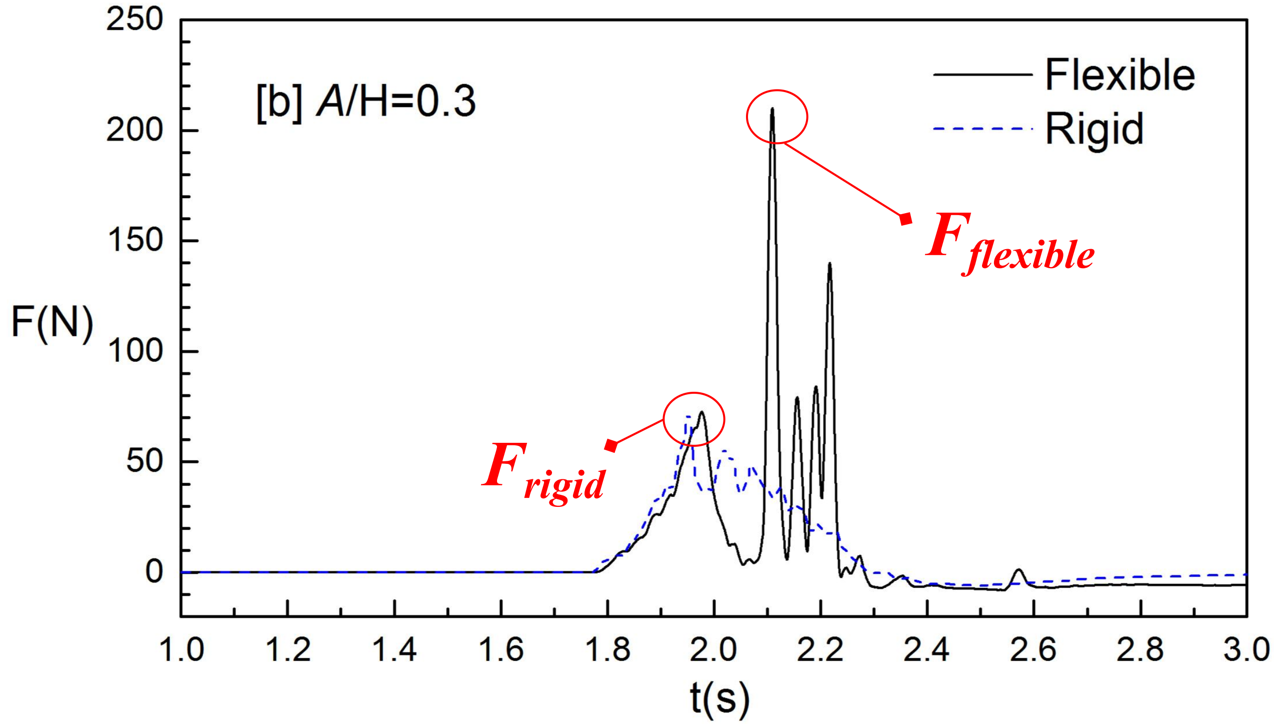


DISCUSSIONS

② The 2nd impact happens when the plate possesses the largest downward velocity;

Numerical Examples

➢ Interaction between Solitary Wave and Flexible Plate

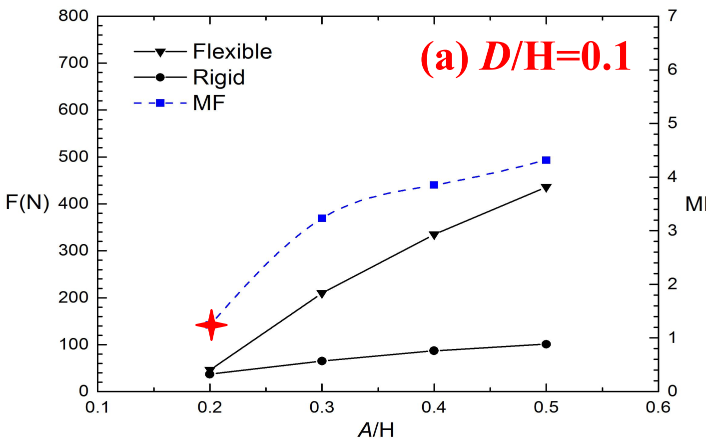


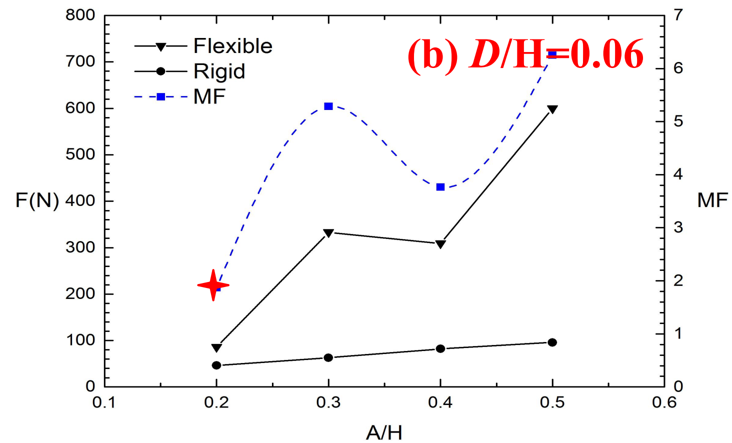
- **Wave-induced force on plate (*A*/H=0.3, *D*/H=0.1)**

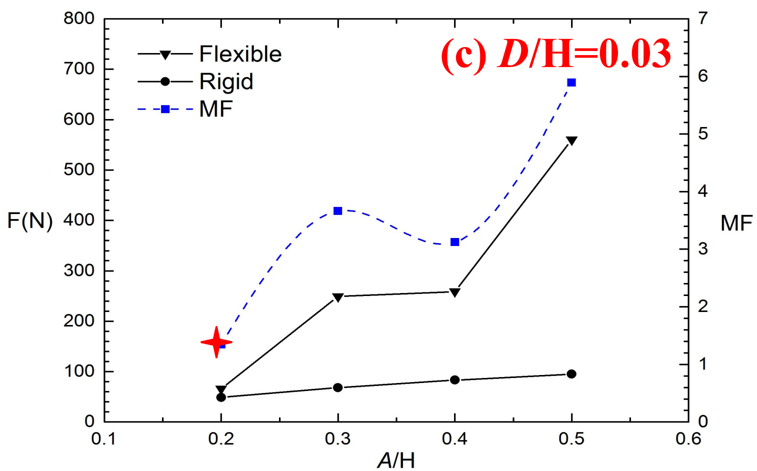
$$MF=F_{flexible}/F_{rigid}$$

MF: Magnification Factor

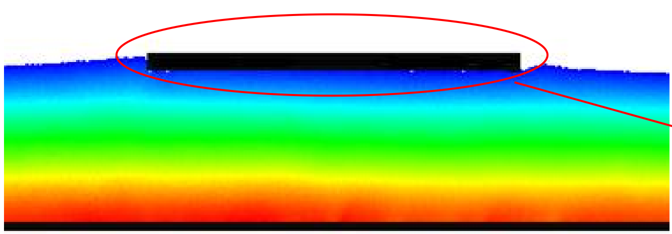
Numerical Examples







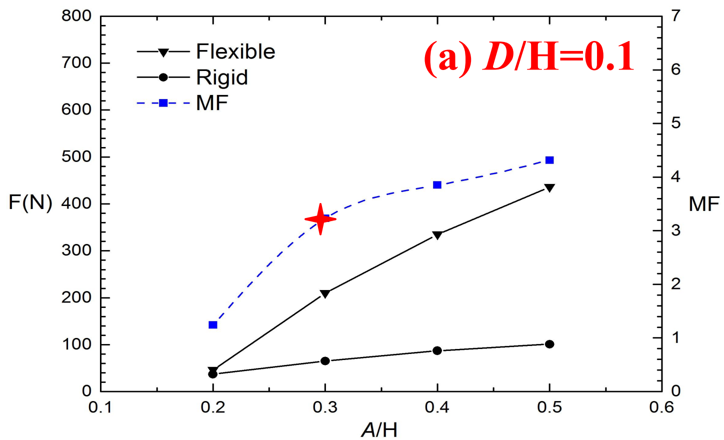
① **For the case of *A*/H=0.2, the magnification factors are relatively low owing to the small wave-induced deformation;**

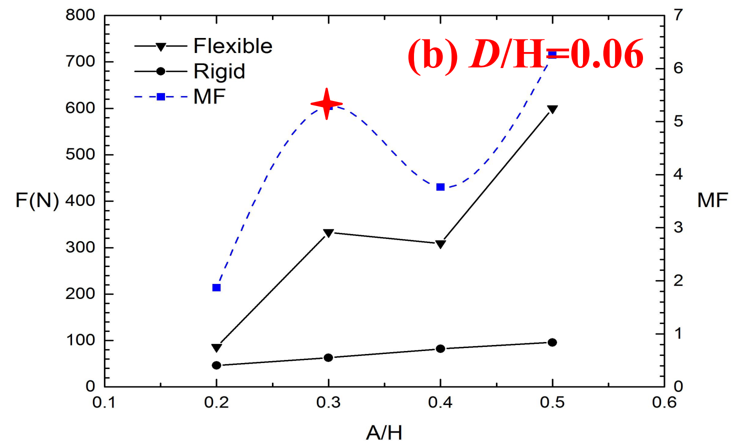


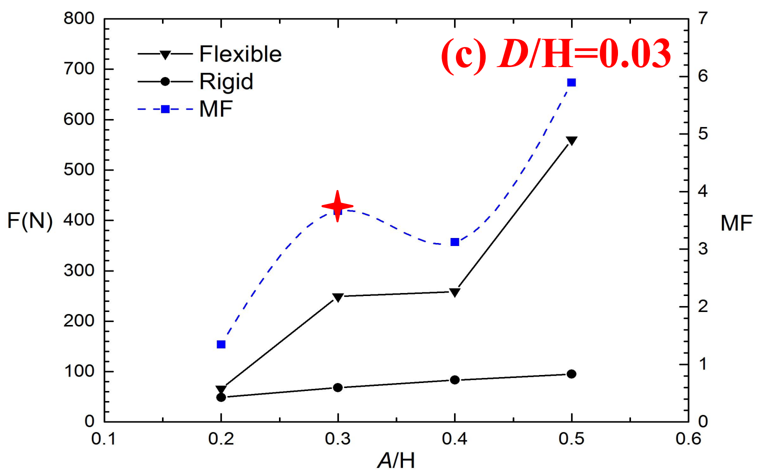
Small deformation

- **The maximum vertical force and the corresponding MF**

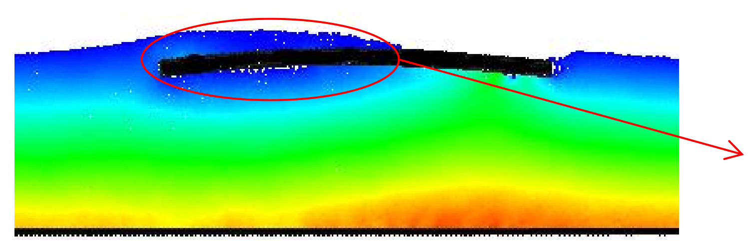
Numerical Examples







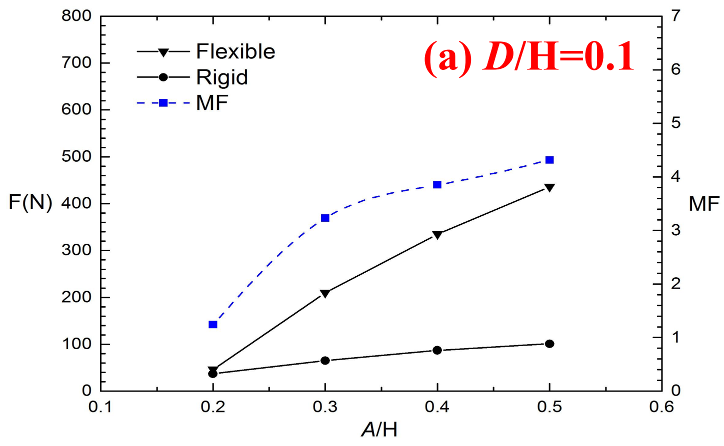
② **For the case of *A*/H=0.3, the magnification factors are all greater than 3 owing to the strong impact onto the surface;**

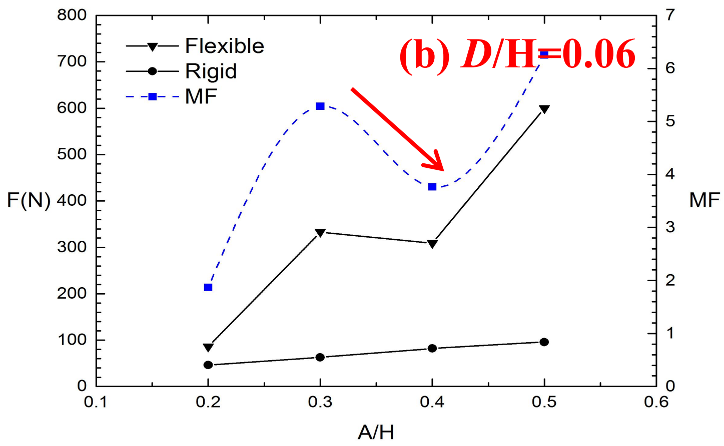


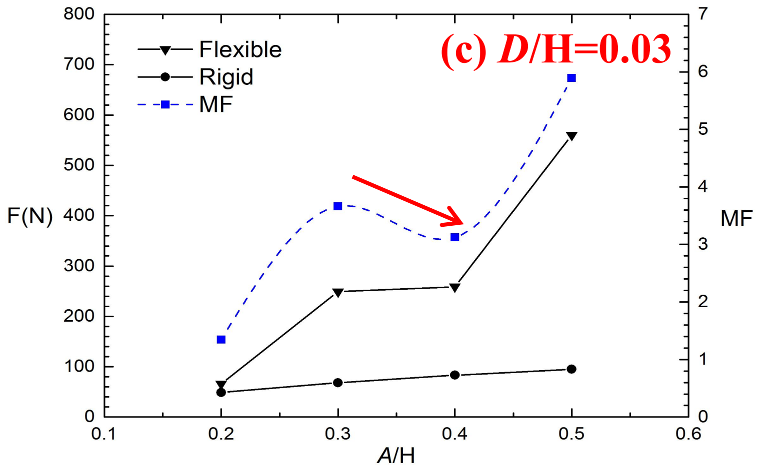
Free surface & slamming

- **The maximum vertical force and the corresponding MF**

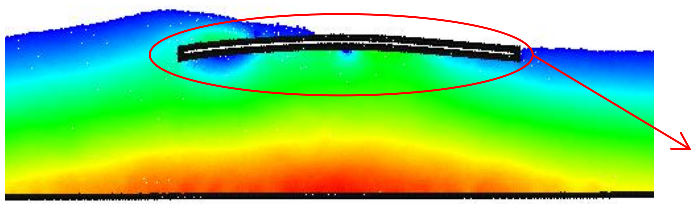
Numerical Examples







③ **For the case of small elevation (*D*/H=0.03 or 0.06), the *MF* drops in spite of an enlarged wave (*A*/H=0.04).**



No slamming

- **The maximum vertical force and the corresponding *MF***

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  - Interaction between Solitary Wave and Rigid Plate
  - Interaction between Solitary Wave and Flexible Plate
- **Conclusions**

Conclusion

- ✓ FSI-MPSFEM-SJTU, an in-house solver based on the fully Lagrangian MPS-FEM coupled method is proposed to solve the wave-plate interaction.
- ✓ The magnification effects of structural flexibility on wave-induced force can be observed; it is mainly owing to that deformed plate impacting onto the surface (slamming). Special attention should be paid on this intensified impact.
- ✓ The *MF* shows strong nonlinearity with regard to wave amplitude and plate elevation. In the case of D/H=0.03 or 0.06, the *MF* decreases in spite of a larger wave (A/H=0.4). It is because the plate is so close to the surface that the slamming doesn't happen.
- ✓The proposed MPS-FEM coupled solver is powerful for problems of structural deformation induced by violent free surface flow.

Thank You !

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