Stable Sharp Interface Method for SPH

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Content

- Motivations: why we develop SIM for SPH
- SIM for SPH: Details of the method
- Benchmark tests: Performance of SIM for SPH
- Concluding remarks: future work

Interface

Worthington jet

Soap bubble

Ocean waves

(From website of Argonne NL)

- Interface: Surface separating two phases of matter, each of which may be solid, liquid, or gas. An interface is not a geometric surface but a thin layer that has properties differing from those of the bulk material on either side of the interface. (ref: Encyclopædia Britannica)

Key Issues in Interface Treatment

- Interface position
  - Track interface explicitly
  - capture interface implicitly
- Interface reconstruction: curvature, normal vector
- Material discontinuity (or Jump condition)

Existing Interface Numerical Models

- **SIM**: C.-H. Chang, X. Deng and T. G. Theofanous, Direct numerical simulation of interfacial instabilities: A consistent, conservative, all-speed, sharp-interface method, Journal of Computational Physics, 242 (2013) 946–990.
- **DIM**: J. U. Brackbill, D. B. Kothe and C. Zemach, A continuum method for modeling surface tension, Journal of Computational Physics, 100 (2) (1992) 335-354.
- **VOF**: C. W. Hirt and B. D. Nichols, Volume of fluid (VOF) method for the dynamics of free boundaries, Journal of Computational Physics, 39 (1) (1981) 201-225.
- **LSM**: J. A. Sethian and P. Smereka, Level set methods for fluid interfaces, annual Review of Fluid Mechanics, 35 (2003) 341-372.
- **Front Tracking** - A Front-Tracking Method for the Computations of Multiphase Flow, Journal of Computational Physics, 169 (2) (2001) 708-759.
- **Phase field**: D. Jacqmin, Calculation of Two-Phase Navier-Stokes Flows Using Phase-Field Modeling, Journal of Computational Physics, 155 (1) (1999) 96-127.

Interface Models in SPH

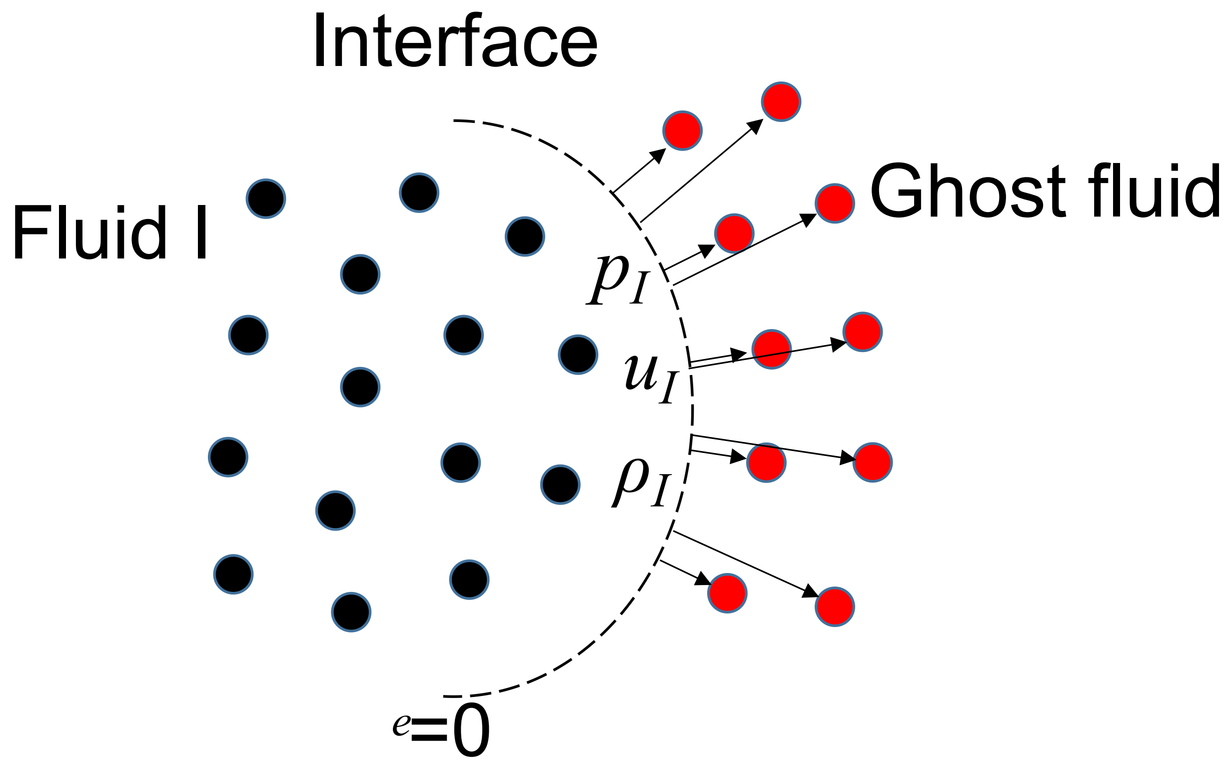
- Color funtion
  - J. P. Morris, Simulating surface tension with smoothed particle hydrodynamics, International Journal for Numerical Methods in Fluids, 33 (3) (2000) 333-353.
  - S. Adami, X. Y. Hu and N. A. Adams, A new surface-tension formulation for multi-phase SPH using a reproducing divergence approximation, Journal of Computational Physics, 229 (2010) 5011-5021.
- Particle-pair interaction
  - Van der Waals: S. Nugent and H. A. Posch, Liquid drops and surface tension with smoothed particle applied mechanics, Physical Review E, 62 (4) (2000) 4968-4975.
  - Cosine force: A. M. Tartakovsky and P. Meakin, Simulation of Unsaturated Flow in Complex Fractures Using Smoothed Particle Hydrodynamics, Vadose Zone Journal, (2005)

Current states in interface treatment for SPH

- Description of interface: Sometimes **No clear definition**
- Geometric information calculation: normal vector, especially curvature. **Low accuracy**
- Including surface tension: **Continuum method** for surface tension
- Jump condition: **No exact consideration**

Main idea (SIM for SPH)

- Description of the interface: level set method
  - Smoothness
  - high accuracy of interface geometric calculation
- Handling the material discontinuity: Ghost fluid method
  - Smooth and stable calculation around interface



Description of the interface : 1st Version

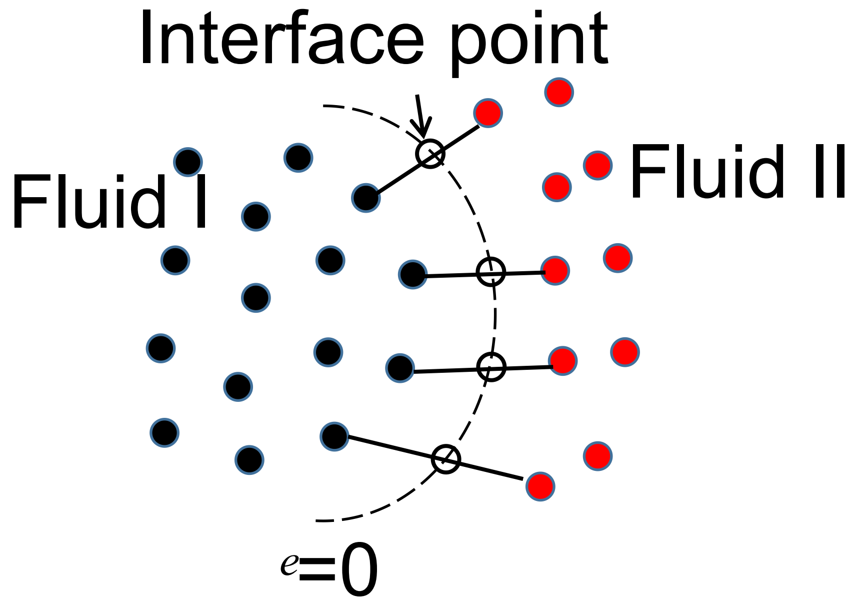
- Initialization: Level set function is set to be the signed distance from the interface. **The interface is given by the user.**
- Evolving the level set function (Level set equation)

$$\frac{\partial \phi}{\partial t} + \overrightarrow{V_\phi} \cdot \nabla \phi = 0 \Rightarrow \frac{d\phi}{dt} = (\vec{V} - \overrightarrow{V_\phi}) \cdot \nabla \phi$$

- Re-initialization: every few time steps to keep the signed distance property, geometric method

Description of the interface : 2nd Version(Mass Conserving)

- Initialization: Level set function is set to be the signed distance from the interface. **The interface is calculated via geometric information of the SPH particles.**

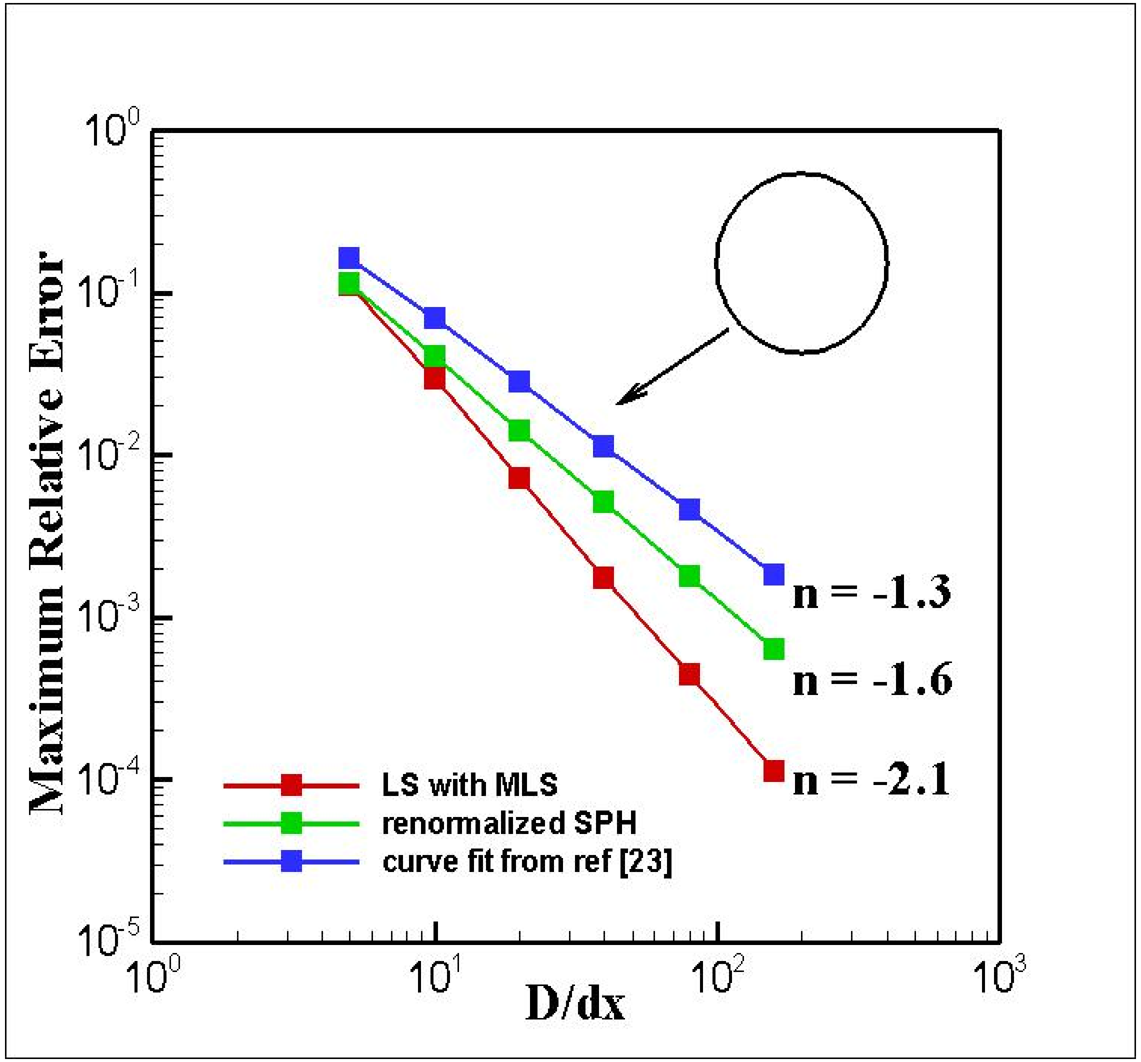


- Evolving the level set function: After the interface position is updated, level set function is set to be the signed distance from the interface.

Handling the material discontinuity: Ghost fluid method

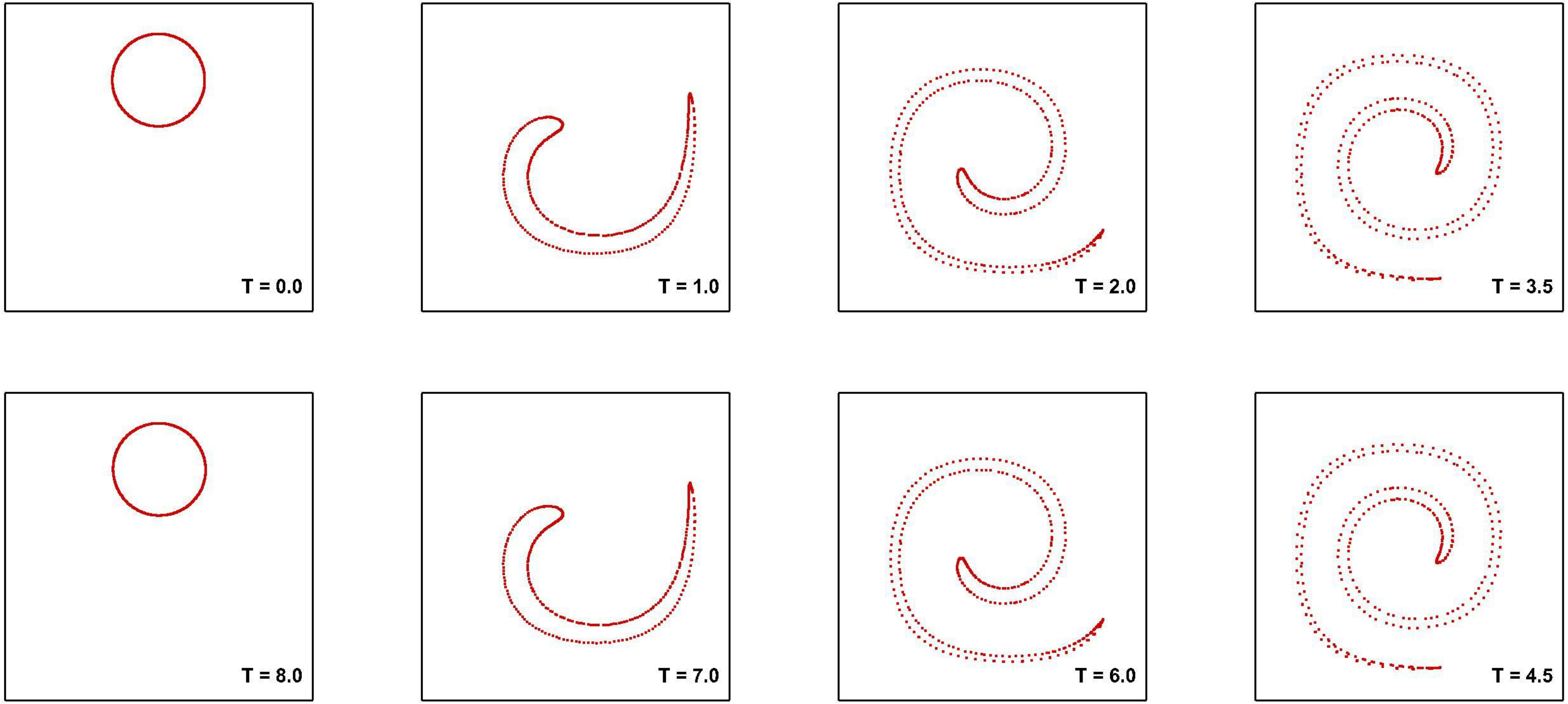
- The interface state:
  - Stresses: $[(\vec{n}\cdot\tau)\vec{n}-p]+\sigma\kappa=0$ $\quad$ $[(\vec{n}\cdot\tau)\vec{s}]=0$
  - Velocity continuity: $[\vec{V}]=0$
  - Property jump: $[\rho]=given$ $\quad$ $[\mu]=given$
- The ghost fluid state: extension of the interface status to the ghost fluid particle

Convergence Order of Curvature Computation: 1st Version

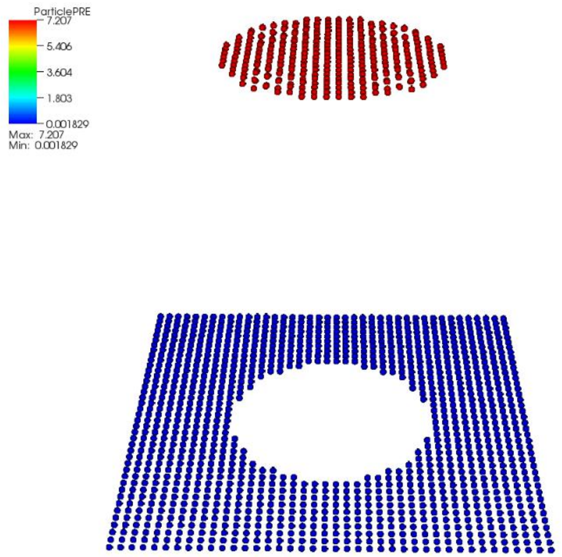


2nd order convergence of curvature computation

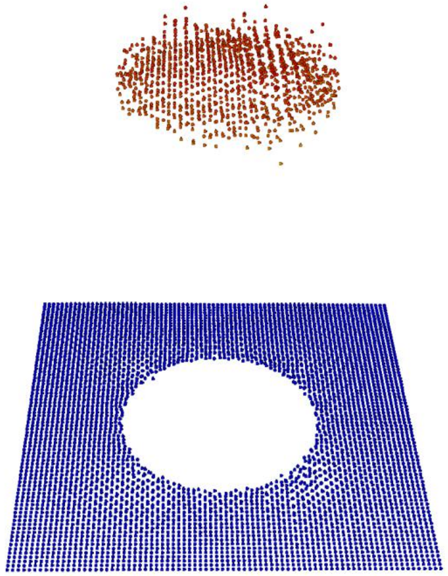
Stretching circle: 1st Version



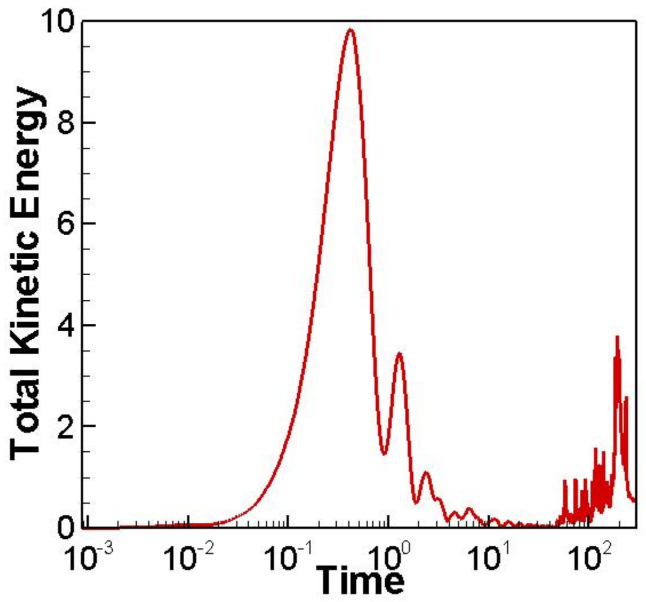
Parasitic current: 1st vs. 2nd Version



1st Version at stable



2nd Version at t=44.04

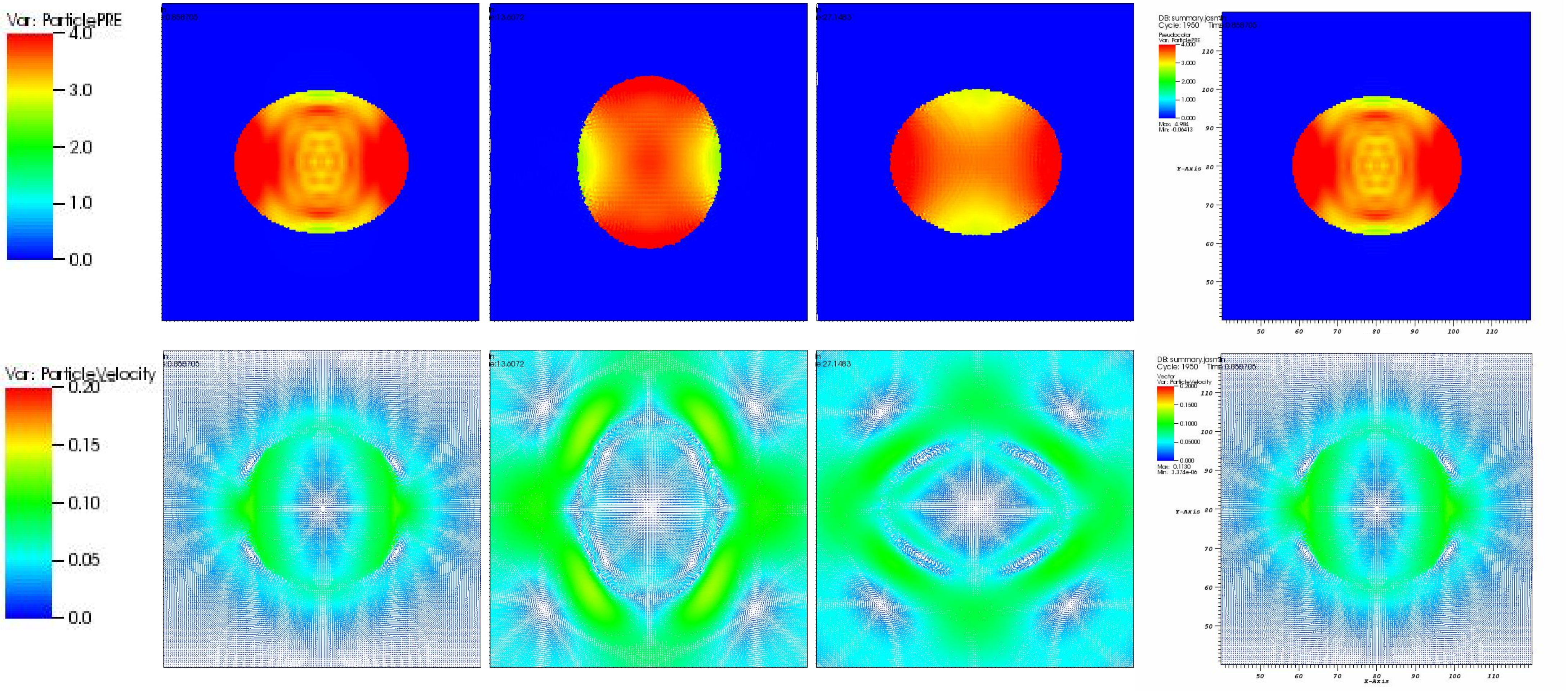


TKE for 2nd Version

Density ratio = 775 viscosity ratio = 55.5 La = 1440

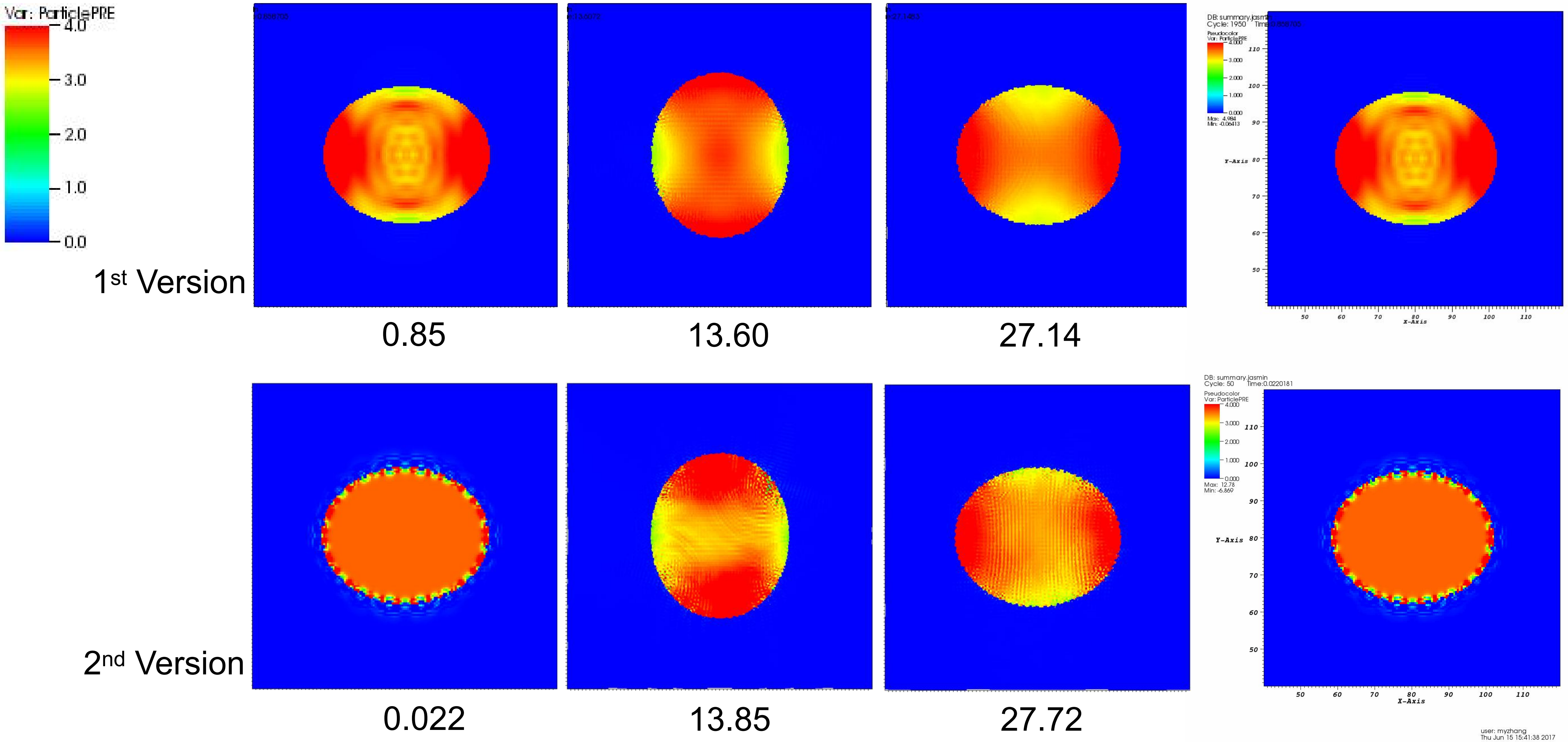
Ca = 3.9×10^{-6} Pressure fluctuation = 0.07%(1st version)

Oscillation of inviscid cylinder: 1st Version

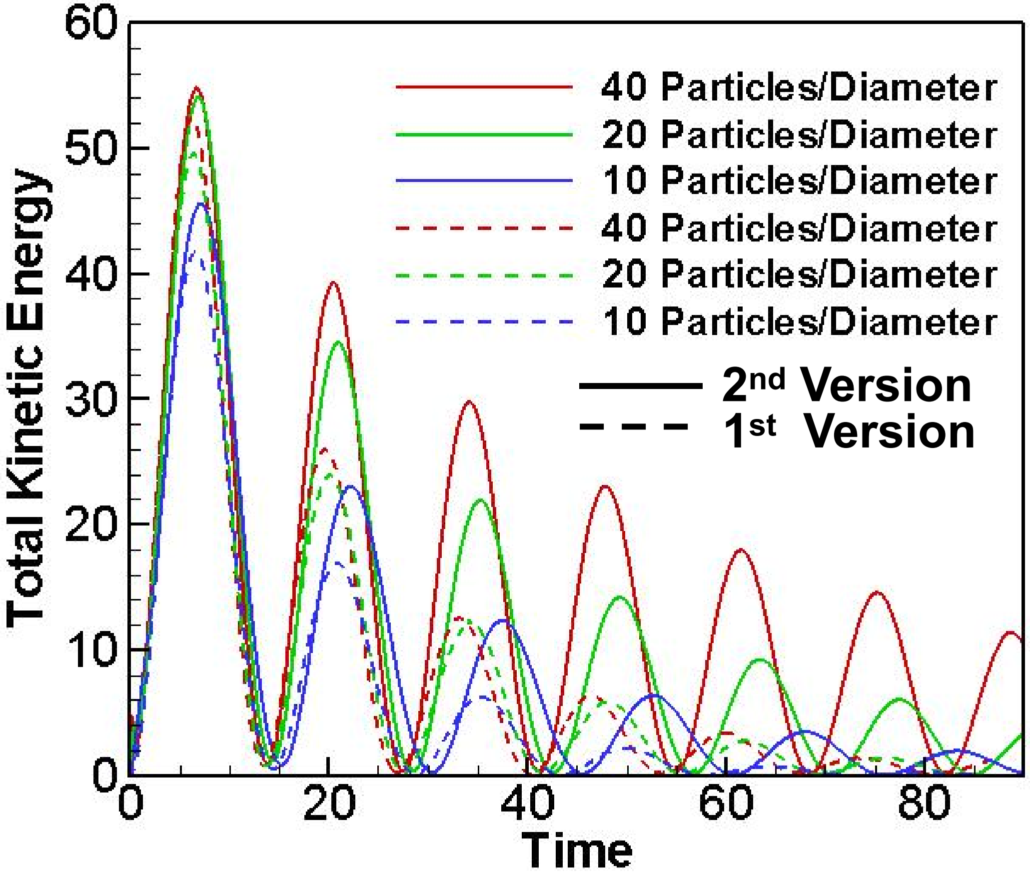


0.85 13.60 27.14

Oscillation of inviscid cylinder: 1st vs. 2nd Version



Oscillation of inviscid cylinder



Perturbation $r = r_0\left(1 + \varepsilon cos(n\theta)\right)$

Period of oscillation

$$T = 2\pi\sqrt{\frac{(\rho_d + \rho_e)r_0^3}{\sigma n(n^2 - 1)}}$$

n = 2 ε = 0.1

D/dx	10	20	40
Error(1st Version)	0.072	0.026	0.0094
Error(2nd Version)	0.12	0.037	0.0047

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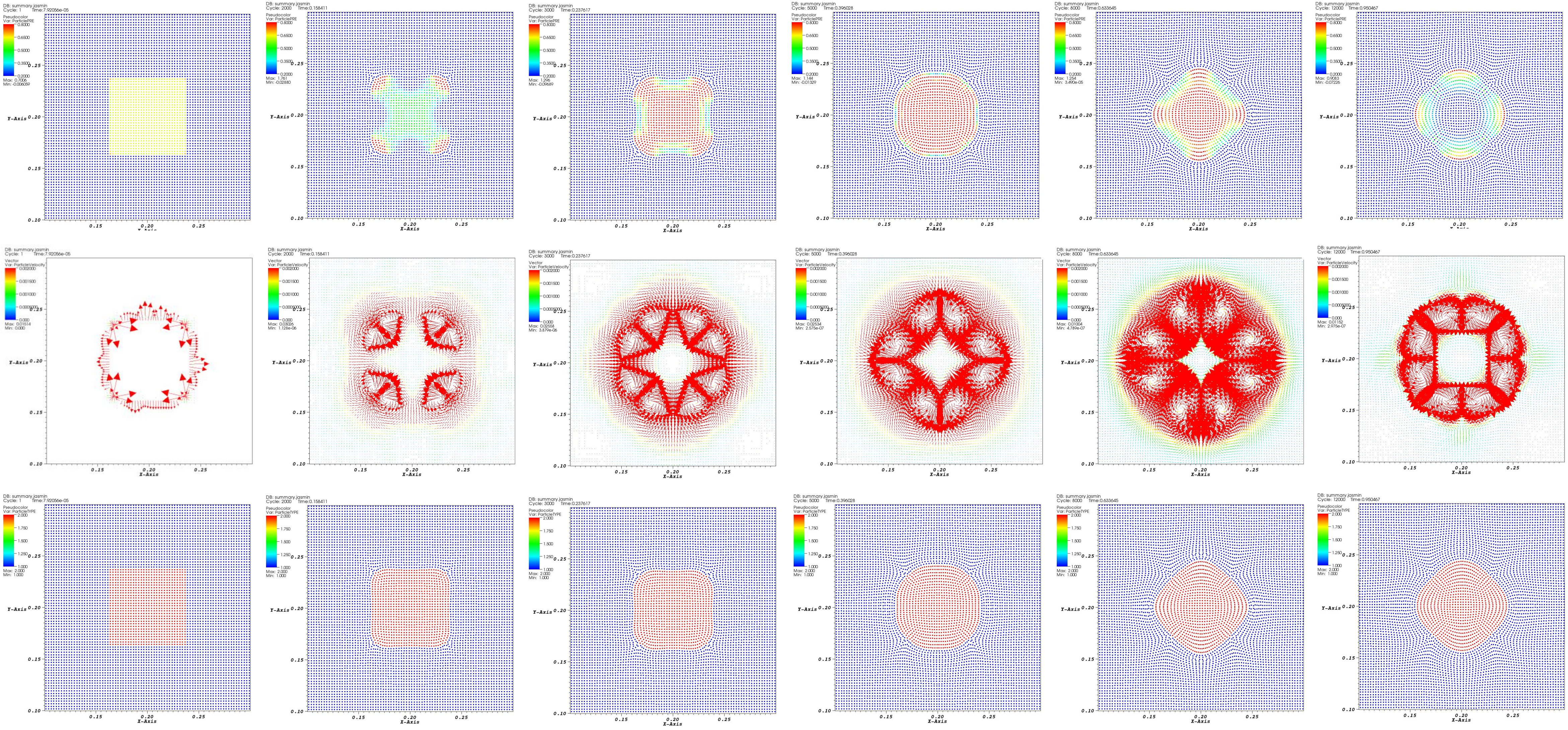
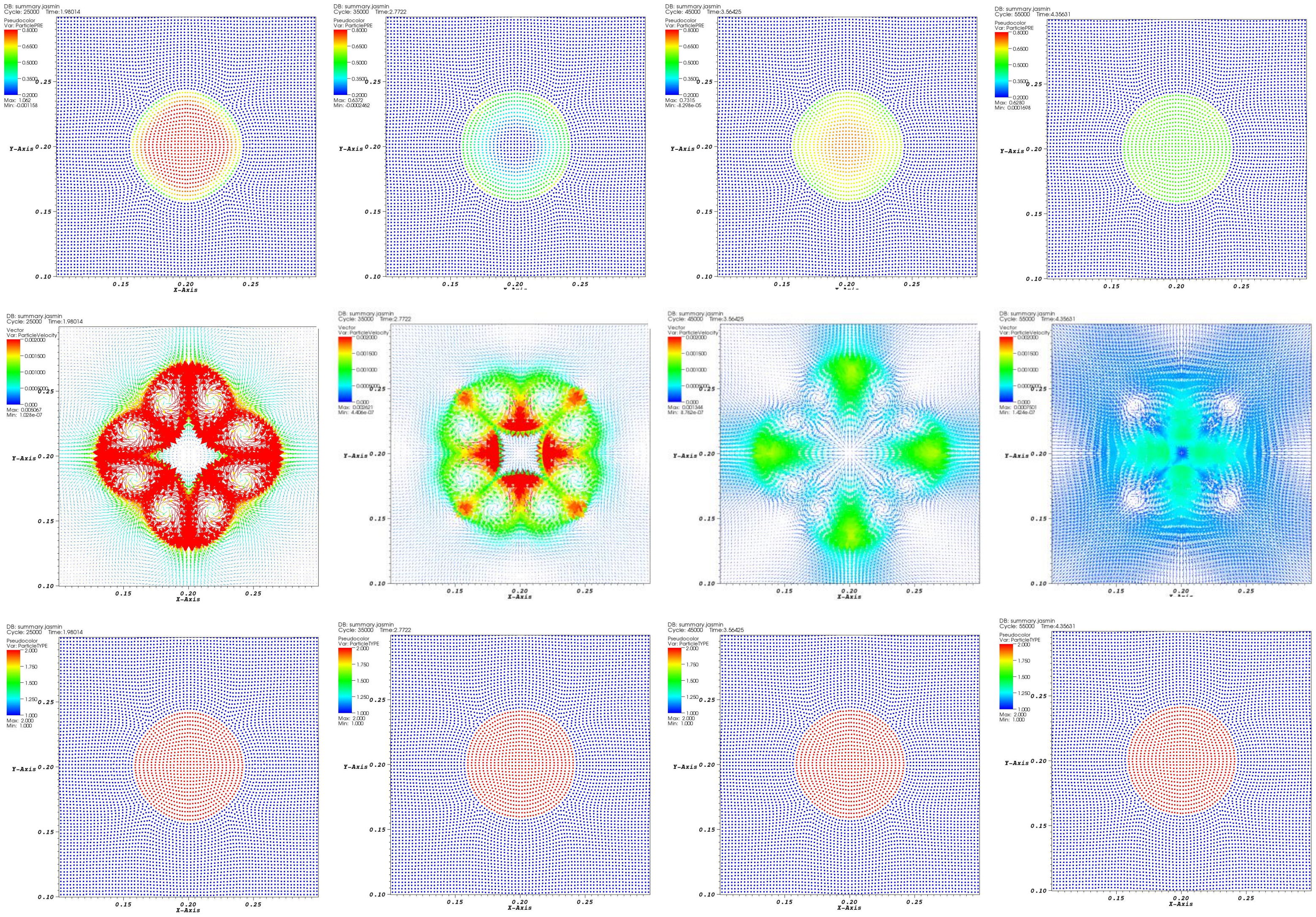
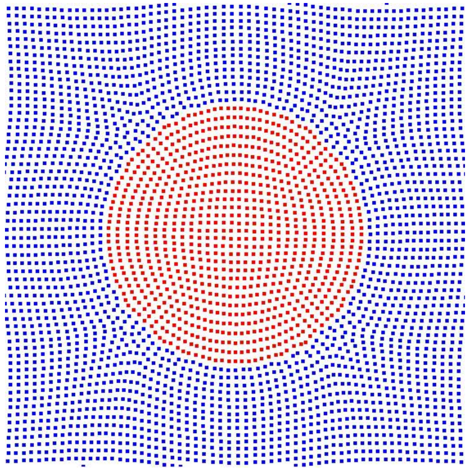
Oscillation of square drop(2nd Version)

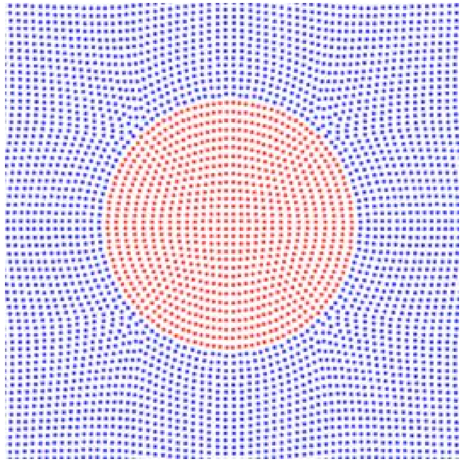


Oscillation of square drop(2nd Version continued)





1st Version



2nd Version

Concluding remarks

- The developed SIM for SPH can be applied to simulate low speed two-phase flows of high density ratios with clear interface accurately and stably.
- The mass conserving version of SIM for SPH is more stable than the original version.
- In the future, the developed method will be applied in the study of complex multiphase flow. The compressible version will be developed.

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Thank you！