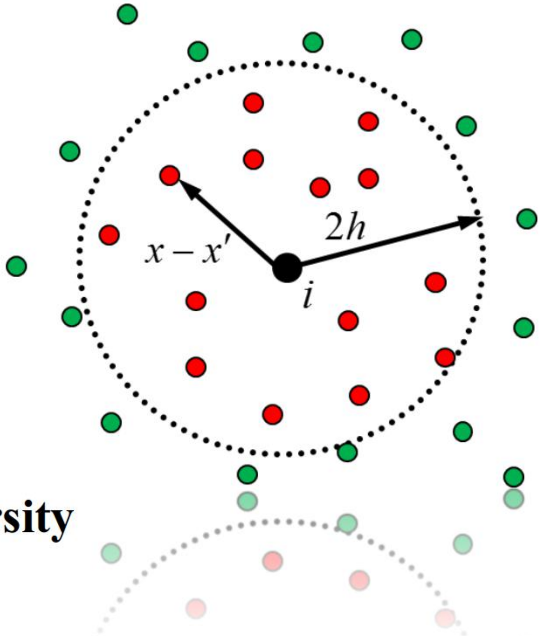
SCHOOL OF AERONAUTICS
NORTHWESTERN POLYTECHNICAL UNIVERSITY

SPHERIC Beijing International Workshop
17-20 October 2017

A dynamic refinement strategy in SPH for simulating the water entry of an elastomer



Lu Wang Fei Xu Yang Yang
School of Aeronautics
Northwestern Polytechnical University
Xi'an, China

Outline

- **Introduction**
- **SPH method for water entry of an elastomer**
- **Dynamic refinement in SPH**
- **Simulation of the entry of an elastic beam into water**
- **Conclusions**

Introduction

Motivation- water entry of elastomers

The seaplane taking off and landing
Spacecraft recycling
Ship slamming

Structural deformation and destroy
Hydrodynamic effect on the motion of the structure

Studying the deformation of the structure when it slams the water is essential to increase the life of the seaplane and ship

A large number of particles need to be involved in the catch of the deformable interface. **Variable resolution SPH** method is applied to water entry of elastomers.

Introduction

variable resolution — a hot issue in SPH

- various smoothing length, $h = h(x, t)$
- Adaptive SPH(ASPH) ,particular ellipsoid kernel, $\kappa = G \cdot r$
- **refining particles:** a general and effective access to obtain varied resolution

dynamic refinement

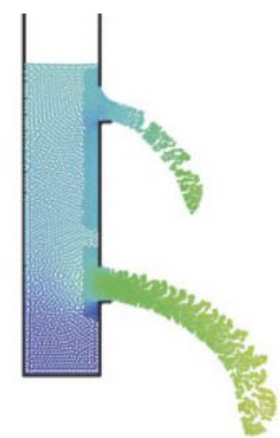
Dynamic refinement is performed locally with the evolution of the physical field. It can adaptively improve the spatial resolution and reduce the computational cost to some extent.

static refinement

Static refinement is accomplished in the initialization. The resolution wouldn't be changed in the subsequent simulation once the refinement is completed

dynamic refinement criteria

velocity gradient
position of the particle → Criterion for two-phase problem
isotropic pressure
strain…



J. Feldman, J. Bonet. Int J for Num Meth in Eng,2007, 72:295–324.

SPH method for water entry of an elastomer

$$
\begin{cases}
\dfrac{d\rho_i}{dt} = \rho_i \sum_j \dfrac{m_j}{\rho_j} (\mathbf{u}_i - \mathbf{u}_j) \nabla_i W_{ij} \\
\dfrac{d\mathbf{u}_i}{dt} = -\sum_j m_i \left(\dfrac{P_i}{\rho_i^2} + \dfrac{P_j}{\rho_j^2} + \prod_{ij} \right) \nabla_i W_{ij} + \boxed{\sum_s m_s \mathbf{x}_{is} f(|\mathbf{x}_{is}|)} + \mathbf{g} \\
P_i = \dfrac{\rho_{0i} {c_{0i}}^2}{\gamma} \left(\left(\dfrac{\rho_{0i}}{\rho_i} \right)^{\gamma} - 1 \right) \\
\dfrac{d\mathbf{x}_i}{dt} = \mathbf{u}_i
\end{cases}
$$

SPH formulas for water

$$
\begin{cases}
\dfrac{d\rho_i}{dt} = \rho_i \sum_j \dfrac{m_j}{\rho_j} (\mathbf{u}_i - \mathbf{u}_j) \nabla_i W_{ij} \\
\dfrac{d\mathbf{u}_i}{dt} = -\sum_j m_i \left(\dfrac{\sigma_i^{\alpha\beta}}{\rho_i^2} + \dfrac{\sigma_j^{\alpha\beta}}{\rho_j^2} + \prod_{ij} \right) \nabla_i W_{ij} + \boxed{\sum_f m_f \mathbf{x}_{if} f(|\mathbf{x}_{if}|)} + \mathbf{g} \\
P_i = c^2 (\rho_i - \rho_{0i}) \\
\dfrac{d\mathbf{x}_i}{dt} = \mathbf{u}_i
\end{cases}
$$

SPH formulas for elastic body

Dynamic refinement in SPH

In order to implement dynamic refinement into the SPH framework, **four main considerations** need to be dealt with:

I. An appropriate criterion is very necessary, which efficiently identifies the candidate particles for refinement.
II. The refinement algorithm should preserve the basic properties of the entire system.
III. The interaction between the coarse particles and fine particles must be consistent.
IV. A better computational accuracy and efficiency are required.

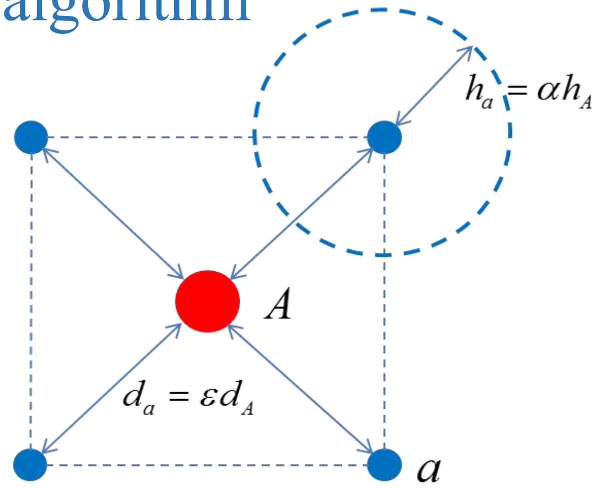
I. Criterion for two-phase problem

$$f(|\mathbf{x}_{if}|) \neq 0 \quad \text{or} \quad f(|\mathbf{x}_{is}|) \neq 0$$

Application of the criterion will describe the fluid-solid interaction minutely

Dynamic refinement in SPH

II. Refinement algorithm



$$\begin{cases} m_a = \dfrac{1}{M} m_A \\ \mathbf{u}_a = \mathbf{u}_A \\ h_a = \alpha h_A \\ d_a = \varepsilon d_A \end{cases}$$

choose optimum values for α and ε by **minimizing the global refinement error** between the refined and unrefined local density field.

density at particle i:

Before refinement:

$$\rho_i = \sum_{j=1}^{N} m_j W(\mathbf{x}_{ij}, h_{ij})$$

After refinement:

$$\rho^*_i = \sum_{j=1}^{N-1} m_j W(\mathbf{x}_{ij}, h_{ij}) + \sum_{k=1}^{Q} m_k W(\mathbf{x}_{ik}, h_{ik})$$

the global refinement error Λ

$$\Lambda = \left(\frac{1}{T}\sum_{i=1}^{T}\rho_i - \frac{1}{R}\sum_{j=1}^{R}\rho_j^*\right)^2$$

Dynamic refinement in SPH

III. Interaction between the coarse particle and fine particle

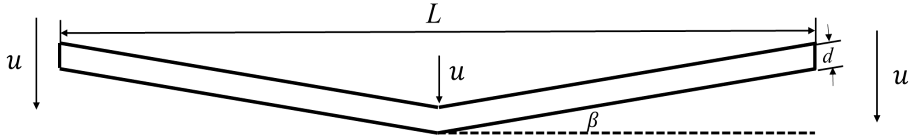
$$h_{Aa} = 0.5 * (h_A + h_a)$$

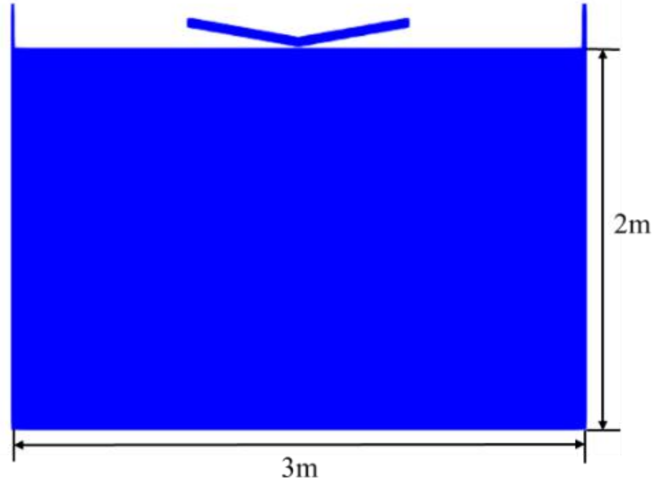
The smoothing length is forced to be taken as the average value to conserve the interactions between the coarse and fine particles.

IV. A better accuracy and efficiency—

Simulation of the entry of an elastic beam into water

- Numerical simulation model





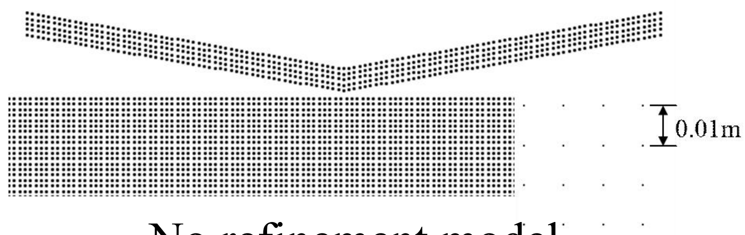
Parameter:

$L = 1.2m \quad d = 0.04m$

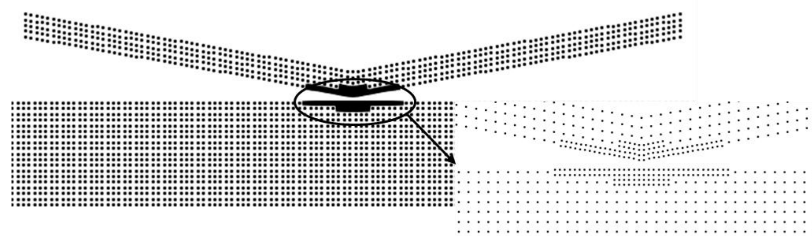
Elastic modulus	$E = 67.5GPa$
Poisson's ratio	$\nu = 0.34$
Density	$\rho = 2700kg / m^3$
Constant velocity	$u = 30m / s$
Particle spacing	0.01m

Simulation of the entry of an elastic beam into water

Difference of Refinement algorithm between elastic and rigid beam

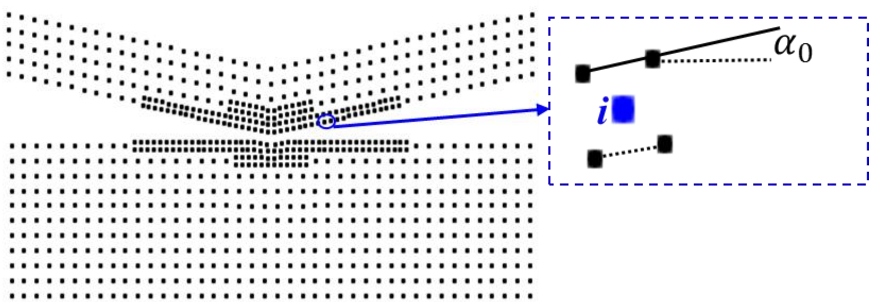


No refinement model

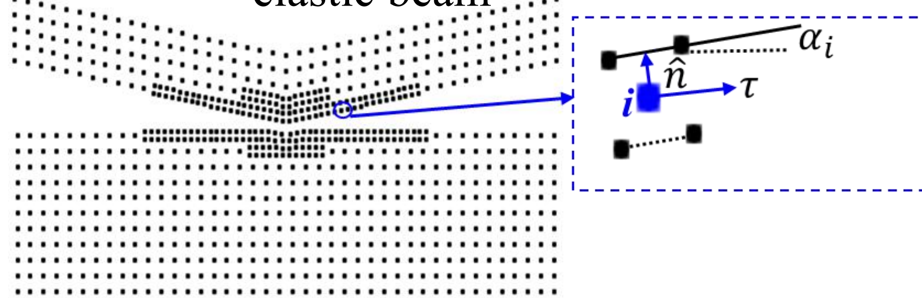


Dynamic refinement model

Rigid beam



elastic beam

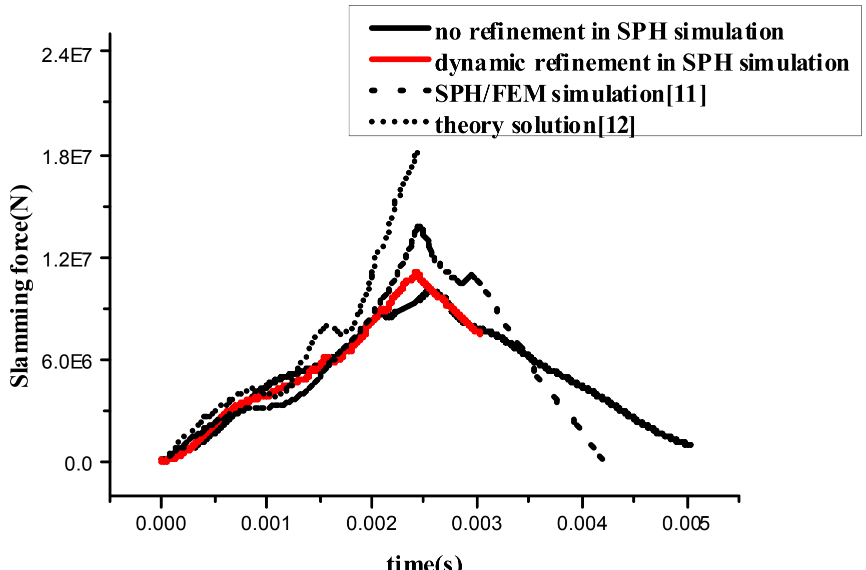


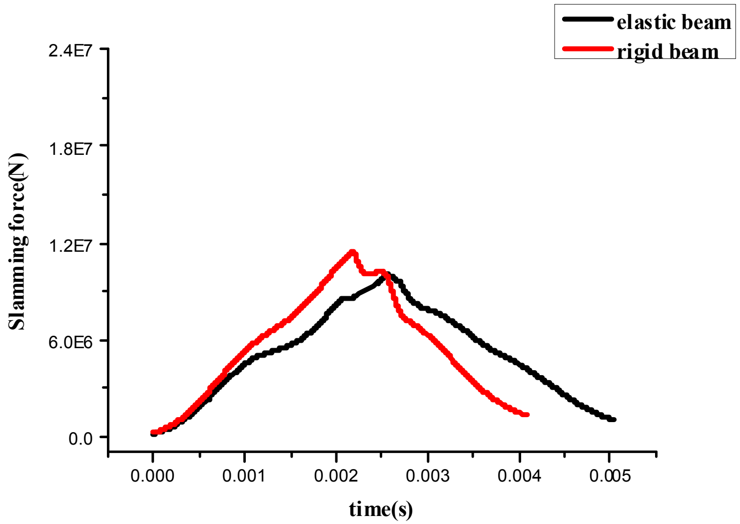
The split degree α_0 for the rigid particle is constant.

The split degree α_i are required to be updated at each time step because of the deformation of the elastomer. The split degree needs to guarantee the connection between two fine "daughter" particles is always parallel to the tangential τ of the coarse particle position.

Simulation of the entry of an elastic beam into water

➢ Results — Slamming force





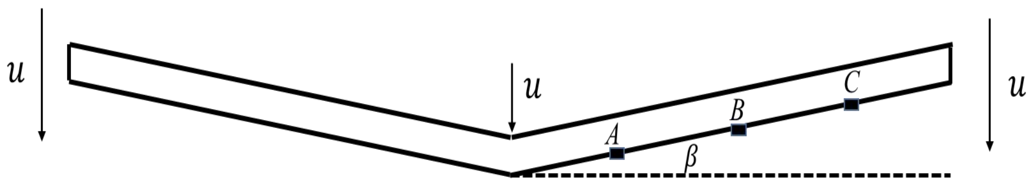
- The results of SPH simulation is smaller than that of the SPH / FEM simulation. This may be due to the different discrete ways for elastic beam in FEM and SPH methods.
- Refinement of water and solid particles at the interface makes the peak value of the slamming force of the beam bigger, which is closer to the SPH / FEM and the theoretical value.

- The peak value of the slamming force for elastic beam is smaller than rigid, and the trend of the force change is slower than rigid beam.
- Elastic effect makes the kinetic energy transformed into the strain energy

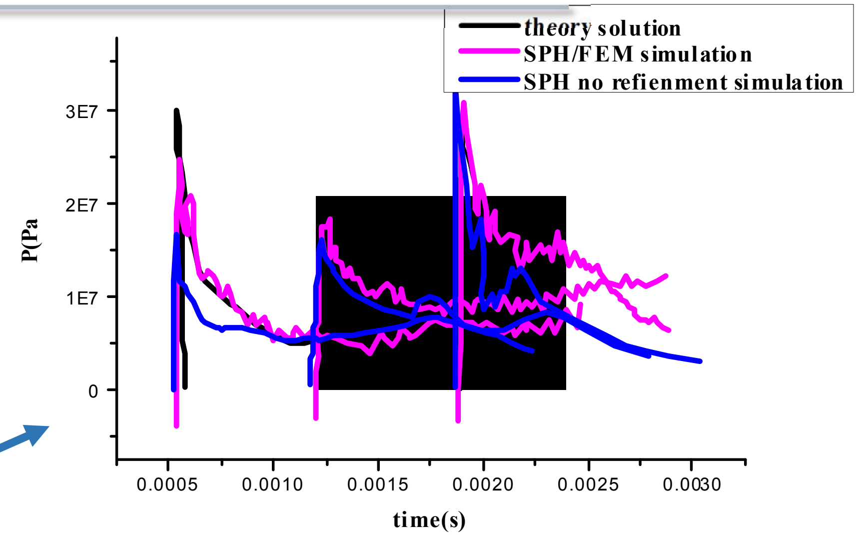
Fourey G, Oger G, Le Touzé D, et al. 2010, 10(1): 012041.
Scolan Y M. 2004, 277(1): 163-203.

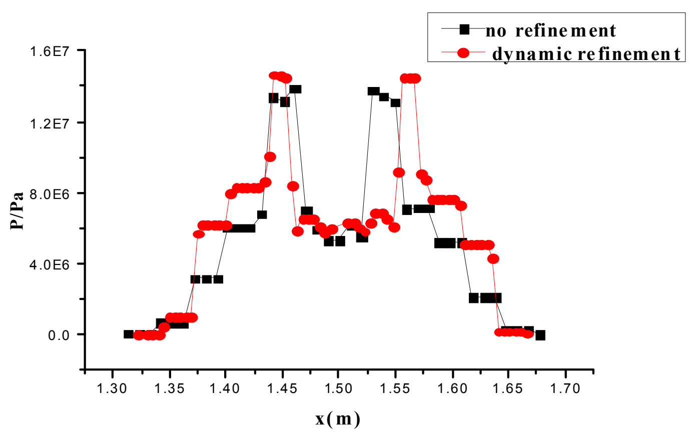
Simulation of the entry of an elastic beam into water

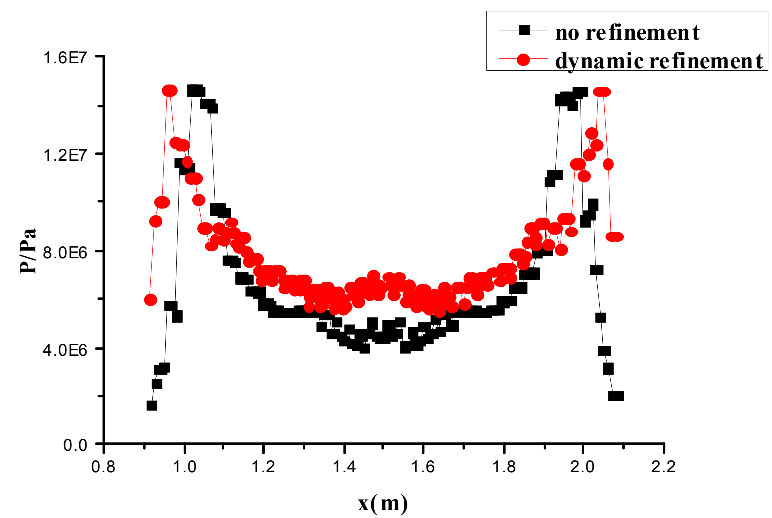
➢ Results — Surface pressure



Pressure change curves at three points of A, B and C





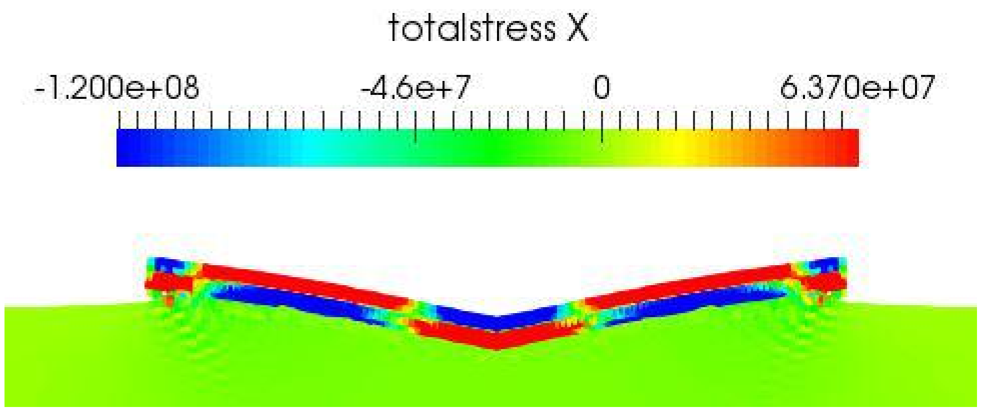


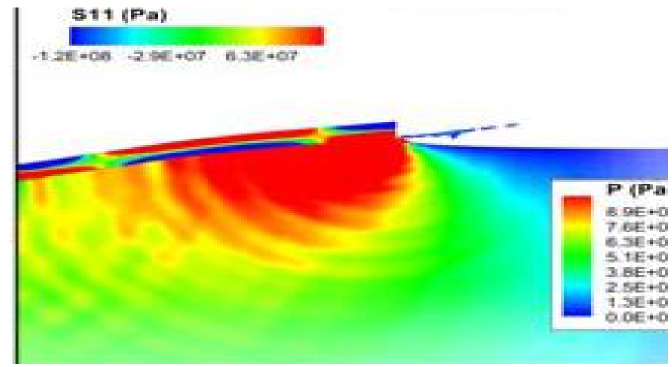
Comparison of the surface pressure distribution at different times (Left:0.05*ms*,Right:2.5*ms*) between the dynamic refinement and no refinement simulations

Simulation of the entry of an elastic beam into water

➤ Results — Stress

Total stress (*X* direction) of the elastic beam between SPH simulation and the literature[13] at t=0.0025s





The tensile force effects on the upper part of the beam while compression force effects on the bottom part of the beam at that moment, which is consistent with the results of the literature.

➤ Results — Computational cost

The **total refinement model** is identical to the no refinement model, while just the initial particle spacing is halved to match the “daughter” particles.

	peak value of slamming force(E7)	computational time（/no refinement）
No refinement	1.004	—
Dynamic refinement	1.111	1.059
Total refinement	1.108	3.891

Conclusions

- SPH could simulate the water entry of an elastomer.
- A refinement strategy aiming at the water entry of elastomers problem is proposed and the effectivity of the strategy is illustrated.
- The slamming force and the surface pressure in the dynamic refinement model are remarkable.
- The dynamic refinement reflects its high efficiency with an equivalent accuracy.

Thanks your attention!