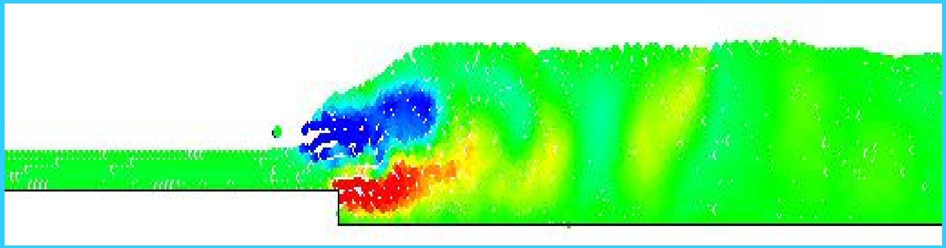
SPH numerical investigation of oscillating characteristics of hydraulic jumps at an abrupt drop



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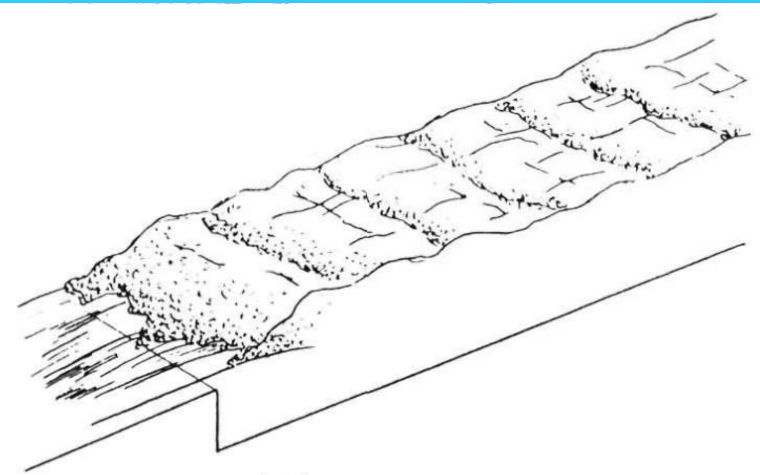
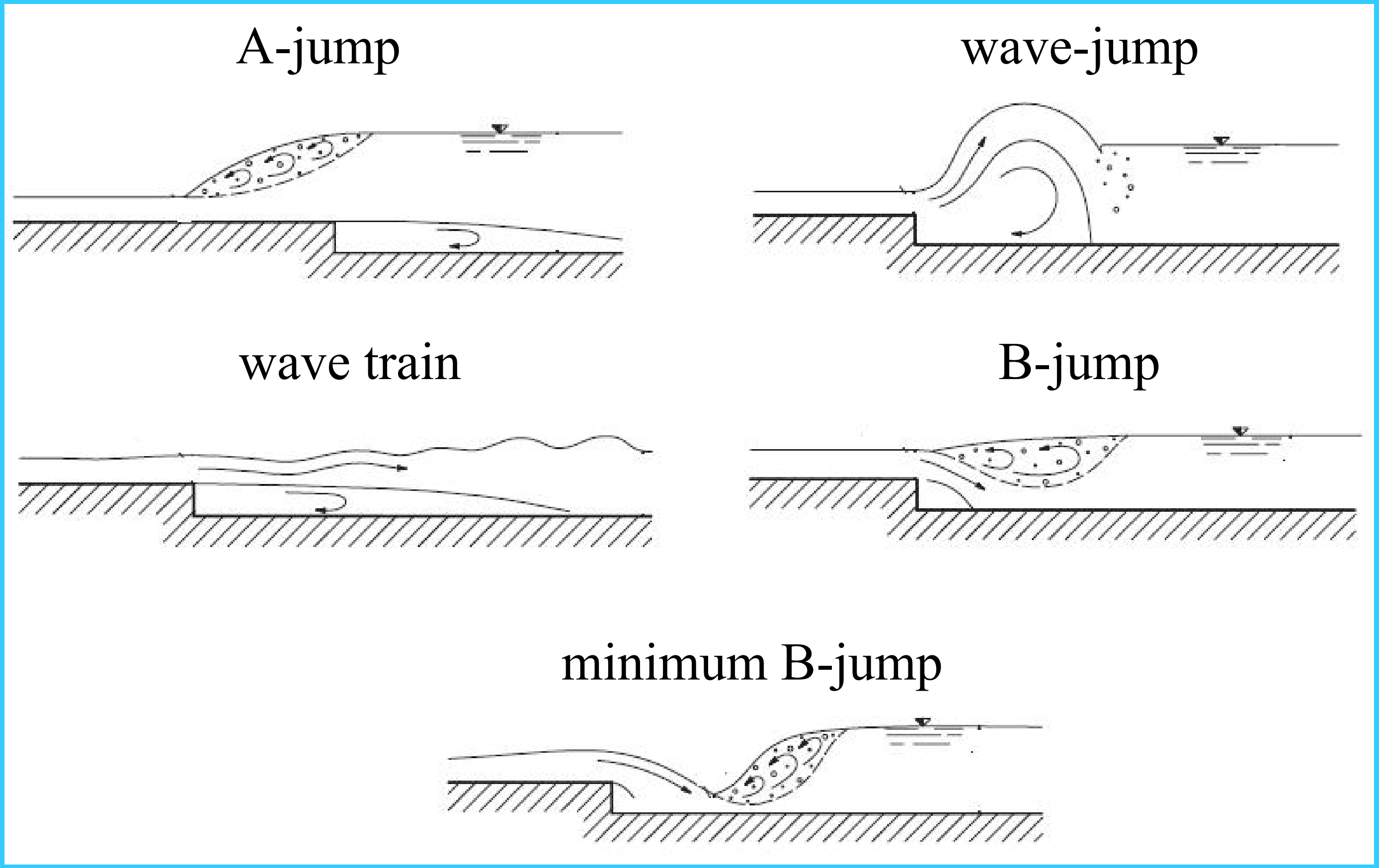
Hydraulic jump on a step

- Stilling basins are designed to dissipate flow energy through an hydraulic jump
- An abrubt drop (step) is often introduced to stabilize jump position
- The onset of different flow patterns depends on inflow and tailwater conditions
- Sometimes, oscillations occur between different flow patterns
- Oscillations propagate waves which can be harmful to downstream structures

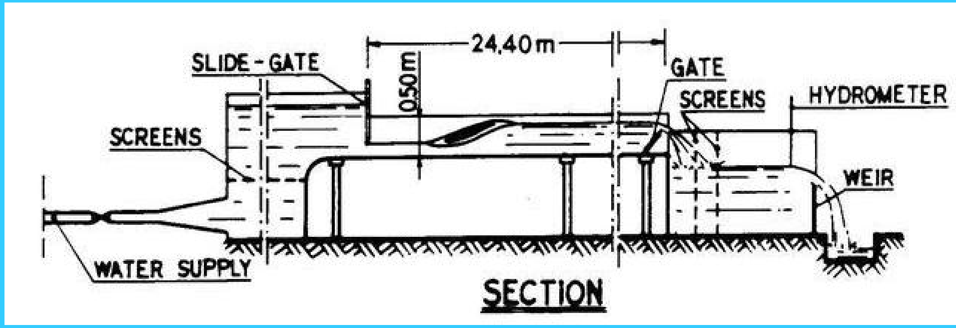


Hydraulic jump on a step: flow conditions



Aim of the work

- Confirm feasibility of the SPH simulation of unstable conditions where oscillations between different regimes occur
- Compare with lab experiments



Flume 0.4 m wide, 24.4 m long

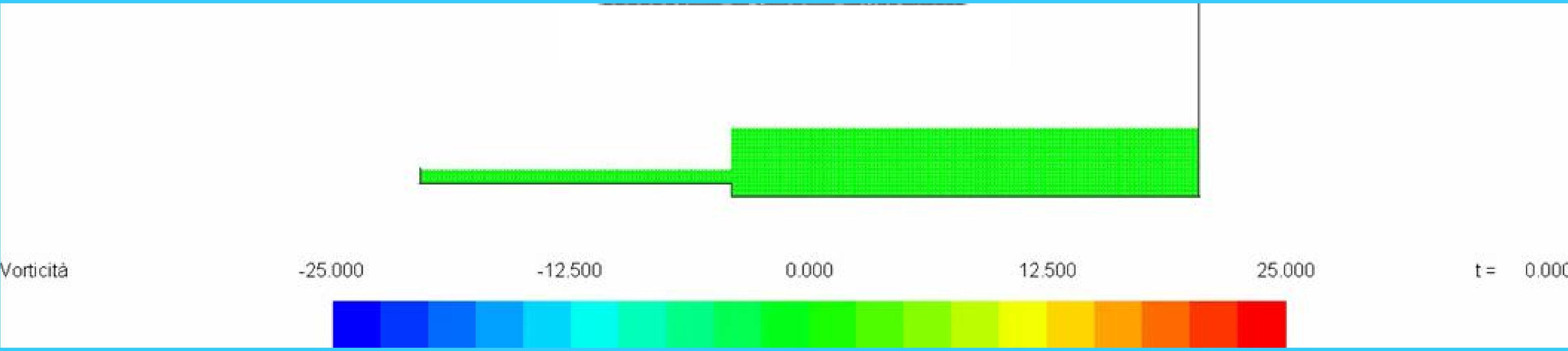
Electric hydrometers (accuracy ±0.1mm)

Pressure transducers (range 7.5 kPa)

- Determine minimal resolution to obtain correct characteristics of the oscillating flow
- Validate SPH as a “numerical experiment” to study the oscillating flow physics

Oscillating regimes

- The onset of oscillations depends on inflow Froude number and tailwater depth ratio
- Example: SPH simulation of oscillating regime (B/wave)



SPH discretization

- WC-SPH Navier-Stokes equations
- Linearized state equation ($c \approx 30$ m/s)
- C2 Wendland kernel function with renormalized gradient $\hat{\nabla} W_{ij}$
- Stress tensor computed according to algebraic/k-ε turbulence models

$$\begin{cases} \langle \dfrac{\mathrm{D}\rho_i}{\mathrm{D}t} \rangle = \sum_j m_j \left(\boldsymbol{v}_i - \boldsymbol{v}_j \right) \cdot \hat{\nabla} W_{ij} \\ \langle \dfrac{\mathrm{D}\boldsymbol{v}_i}{\mathrm{D}t} \rangle = -\sum_j m_j \left(\dfrac{p_i}{\rho_i^2} + \dfrac{p_j}{\rho_j^2} \right) \nabla W_{ij} + \sum_j m_j \left(\mathcal{T}_i - \mathcal{T}_j \right) \cdot \hat{\nabla} W_{ij} + \mathbf{g} \\ p_i - p_0 = c_i^2 \left(\varrho_i - \rho_0 \right) \\ \mathcal{T}_i = \mu_{T_i} \mathcal{S}_i \end{cases}$$

Kernel renormalization

- Evaluation of derivatives according to Liu&Liu (IJNMF, 2006)
- Applied only to restore 1st order consistency (2nd order accuracy)
- Applied only to continuity and stress terms
- Not applied to pressure term to conserve linear momentum
- Can be extended to higher order and derivatives (Sibilla, CAF 2015)

$$A\,F = B \qquad F = [\tilde{f}(x_i) \quad \tilde{f}'(x_i)]^T$$

$$A = \begin{bmatrix} \sum_{j=1}^{N} W(x_j - x_i, h)\Delta\Omega_j & \sum_{j=1}^{N} (x_j - x_i) W(x_j - x_i, h)\Delta\Omega_j \\ \sum_{j=1}^{N} W'(x_j - x_i, h)\Delta\Omega_j & \sum_{j=1}^{N} (x_j - x_i) W'(x_j - x_i, h)\Delta\Omega_j \end{bmatrix} \qquad B = \begin{bmatrix} \sum_{j=1}^{N} f(x_j) W(x_j - x_i, h)\Delta\Omega_j \\ \sum_{j=1}^{N} f(x_j) W'(x_j - x_i, h)\Delta\Omega_j \end{bmatrix}$$

SPH details

- XSPH, no particle shifting
- Pressure smoothing on deviation from hydrostatic pressure (tested on several hydraulic jump/ standing wave cases)

$$p_i = \frac{\sum_j \frac{m_j}{\rho_j} \left[\hat{p}_j + \rho_i g (z_j - z_i) \right] W_{ij}}{\sum_j \frac{m_j}{\rho_j} W_{ij}}$$

- Velocities and water levels imposed both at inlet and outlet
- Ghost particles for wall b.c.

Turbulence models

- Algebraic (mixing length)

$$\mu_T = c_\mu \rho l^2 \|\mathcal{S}\| \qquad\qquad l_i = \min\left[1, \left|\sum_j \frac{m_j}{\rho_j}\nabla W_{ij}\right|^{-3}\right]\min(\kappa y, l_{max})$$

- Two-equation (k-ε)

$$\mu_T = c_\mu \frac{k^2}{\varepsilon}$$

$$\frac{Dk_i}{Dt} = P_{k_i} + \frac{1}{\sigma_k}\sum_j m_j \frac{\nu_{T_i} + \nu_{T_j}}{\rho_i + \rho_j}\frac{k_i - k_j}{r_{ij}^2 + 0.01h^2} r_{ij}\cdot\nabla\hat{\mathrm{W}}_{ij} - \varepsilon_i$$

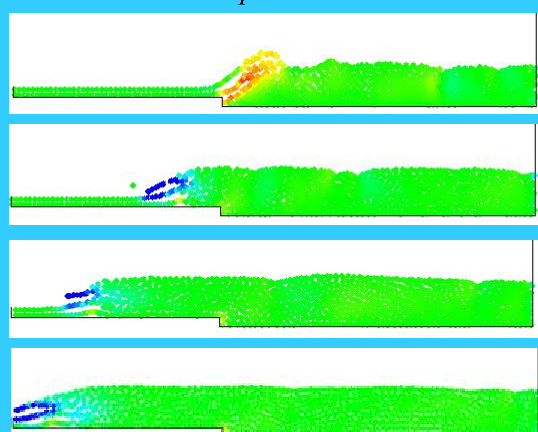
$$\frac{D\varepsilon_i}{Dt} = \frac{1}{\sigma_\varepsilon}\sum_j m_j \frac{\nu_{T_i} + \nu_{T_j}}{\rho_i + \rho_j}\frac{\varepsilon_i - \varepsilon_j}{r_{ij}^2 + 0.01h^2} r_{ij}\cdot\nabla\hat{\mathrm{W}}_{ij} + $$

$$+ C_{\varepsilon_1}\frac{\varepsilon_i}{k_i}P_{k_i} + C_{\varepsilon_2}\frac{\varepsilon_i}{k_i}\sum_j \frac{m_j}{\rho_j}\varepsilon_j \hat{\mathrm{W}}_{ij}$$

Sensitivity to particle resolution

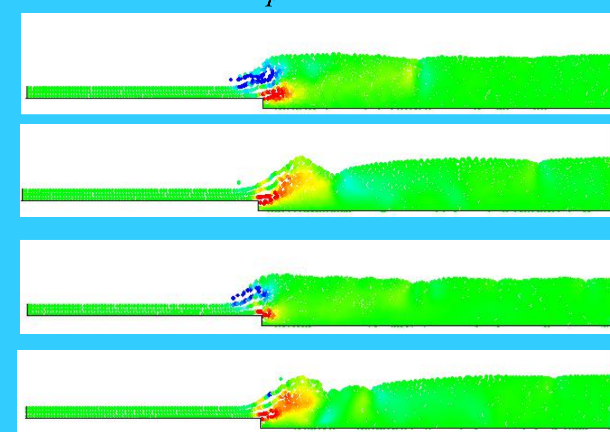
- Aim: determination of the coarsest allowable 2D resolution
- Focus: global flow regimes and downstream wave propagation
- Minimal number of partilces on upstream flow heigth

$N_p = 2$



No onset of flow oscillations

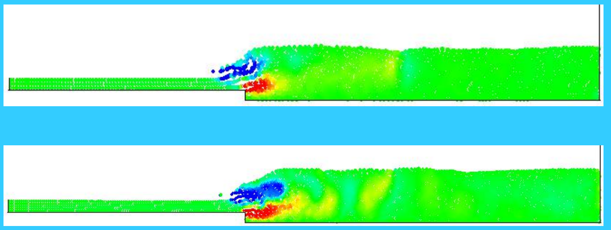
$N_p = 3$



Oscillating regime

Sensitivity to particle resolution

- Aim: determination of the coarsest allowable 2D resolution
- Focus: global flow regimes and downstream wave propagation
- Minimal number of partilces on upstream flow heigth



$N_p = 3$

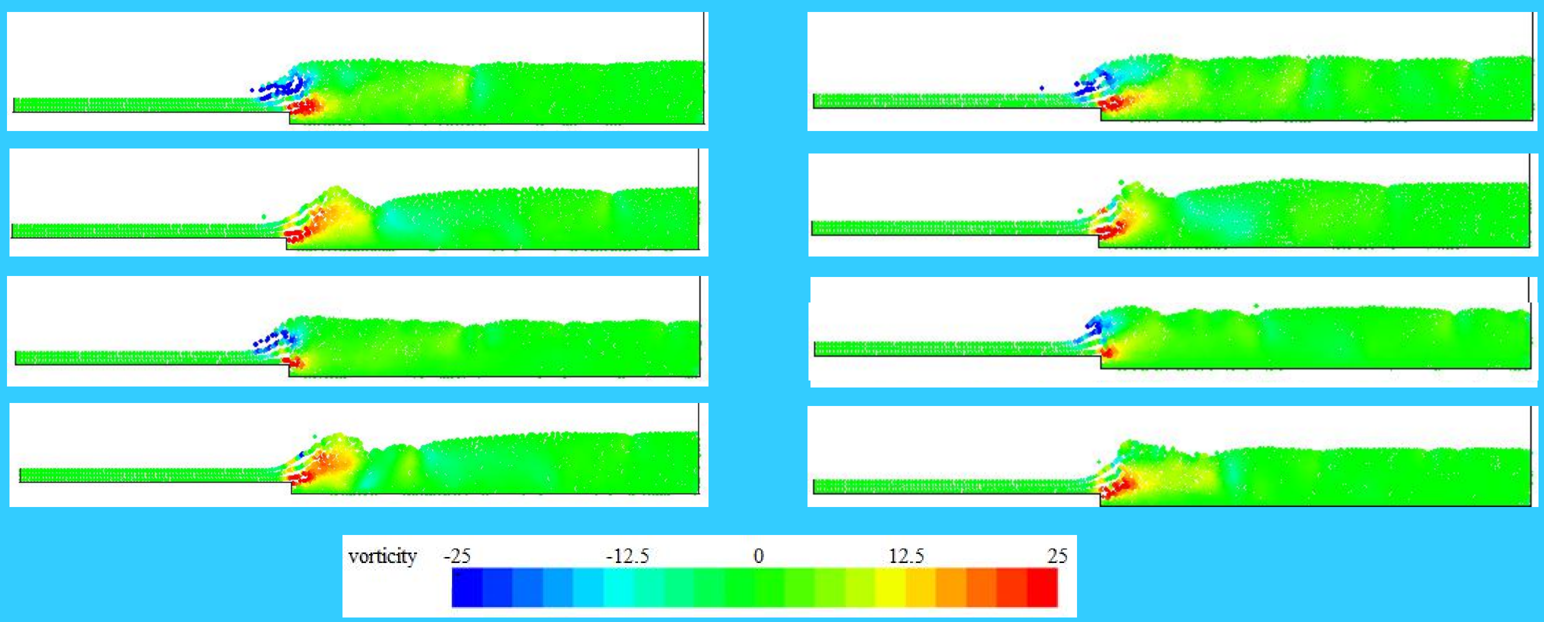
$N_p = 4$

Higher resolution shows (at higher detail) the same regime and frequencies of flow oscillation

Sensitivity to turbulence models

- Tested on the oscillating B-wave case ($Fr = 3.3$, $y_1/y_t = 4.75$)
- Same (correct) oscillation pattern predicted

Mixing-length | *k-ε*



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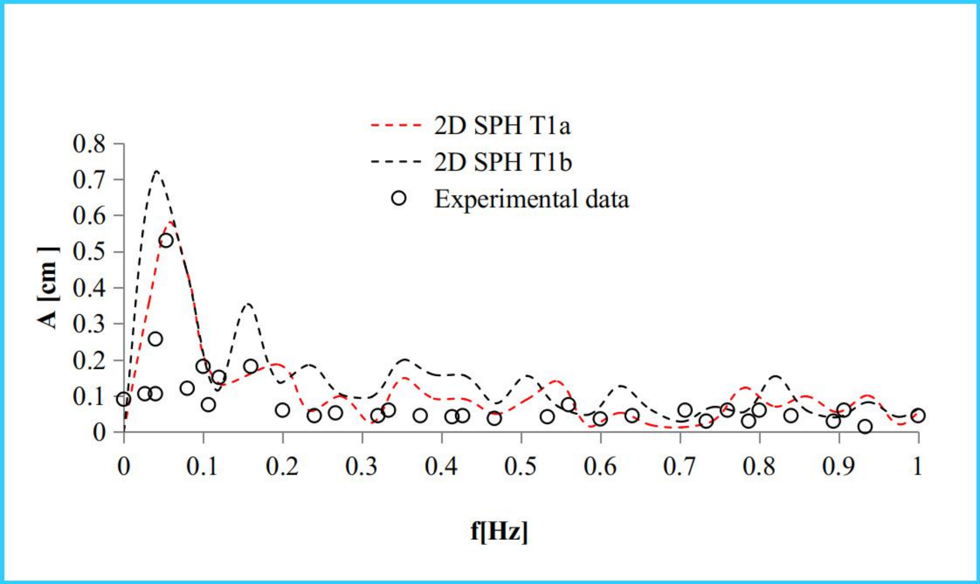
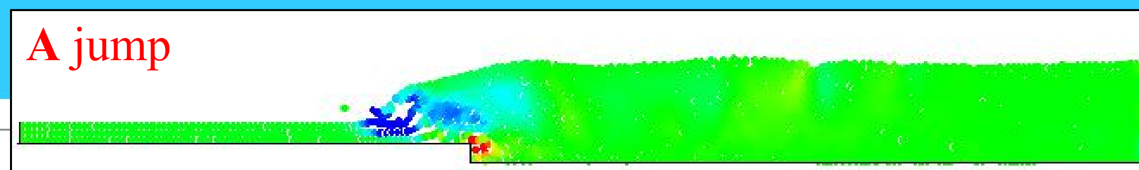
Sensitivity to turbulence models

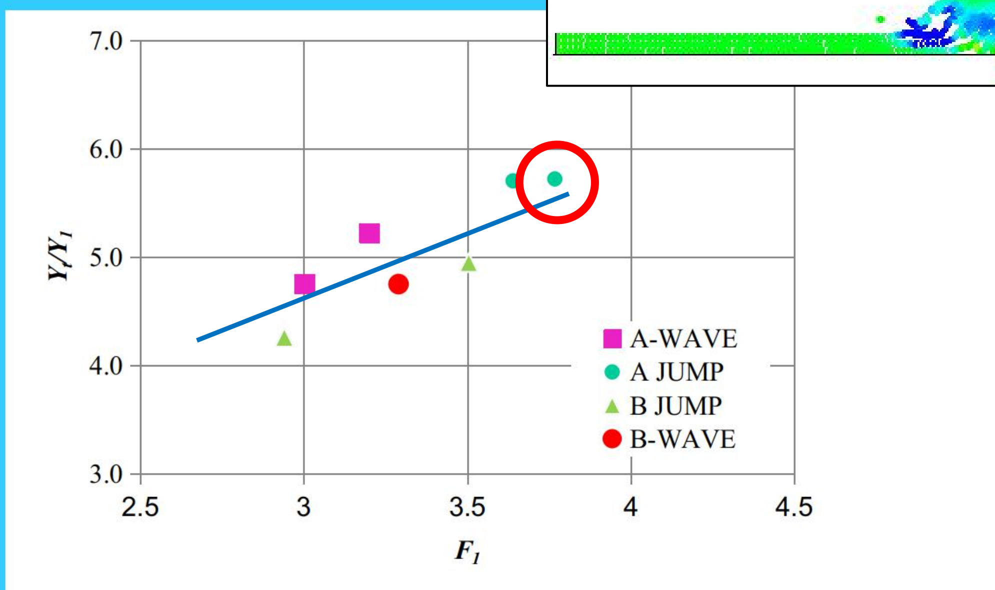
- Tested on the oscillating B-wave case ($Fr = 3.3$, $y_1/y_t = 4.75$)
- Same (correct) oscillation pattern predicted
- However the two-equation model overestimates the peak amplitude of the pressure (level) oscillations



Analysis of the different regimes

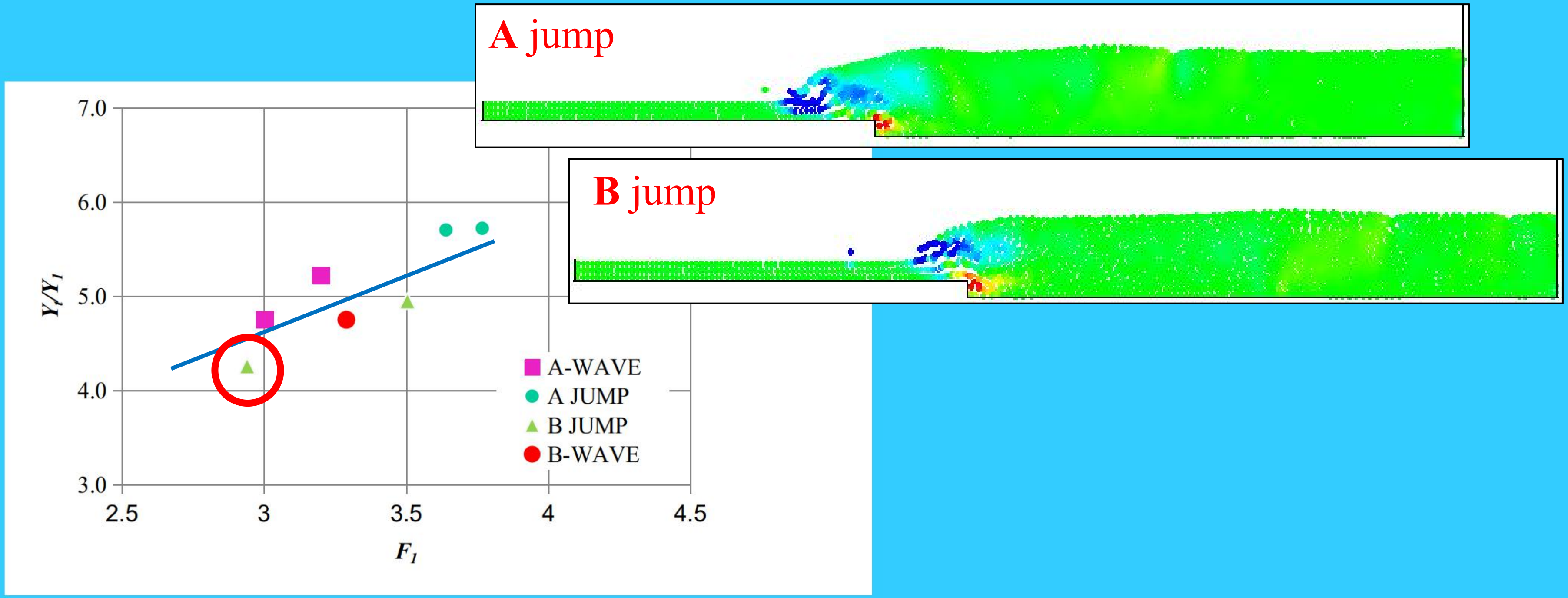
- Quasi-steady A-jump (Fr = 3.8, y_1/y_t = 5.72)





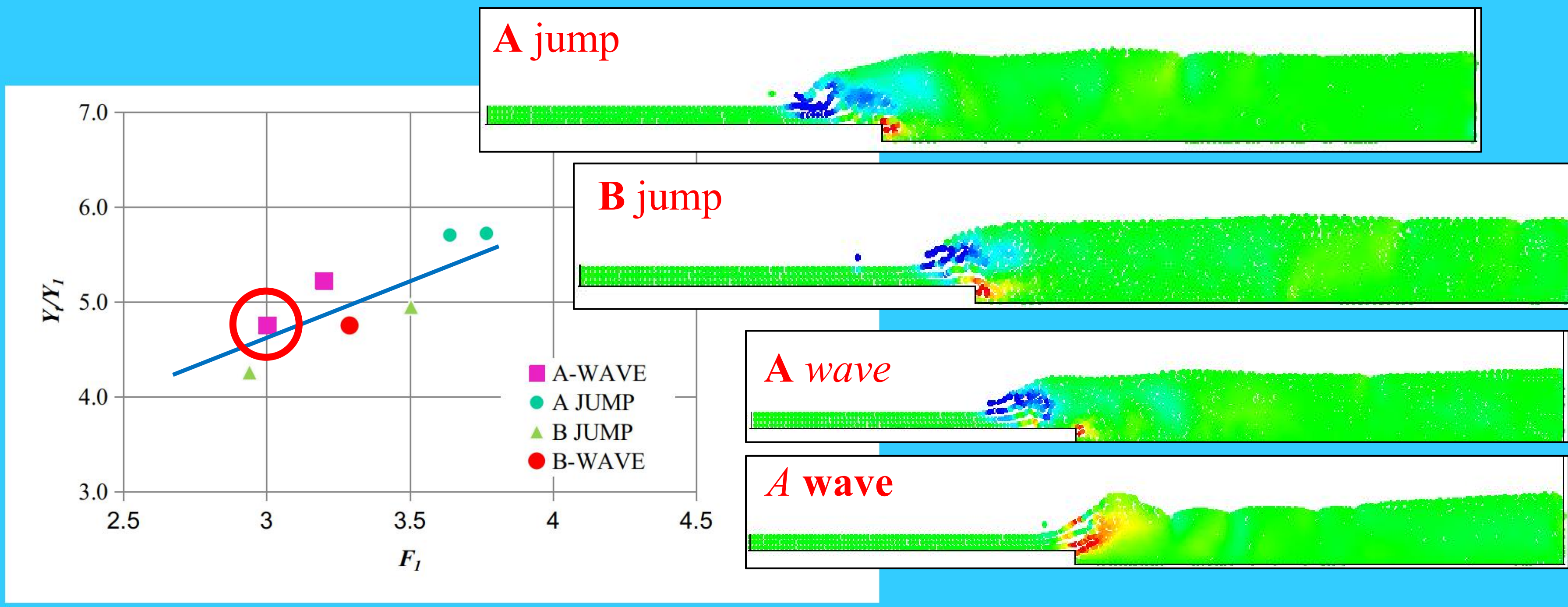
Analysis of the different regimes

- Quasi-steady B-jump ($Fr = 2.8$, $y_1/y_t = 4.26$) [maximum plunging condition]



Analysis of the different regimes

- Oscillating A-wave (Fr = 3.0, y_1/y_t = 4.76)



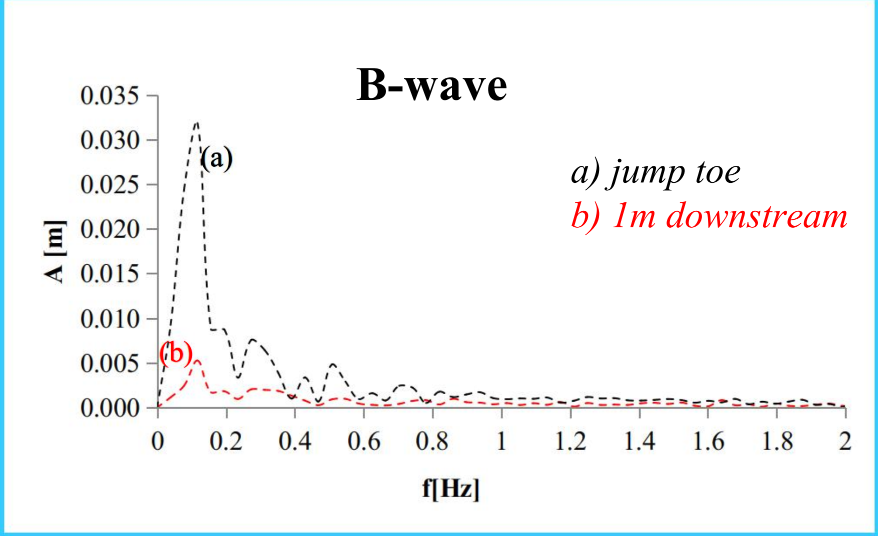
Detailed analysis of flow features

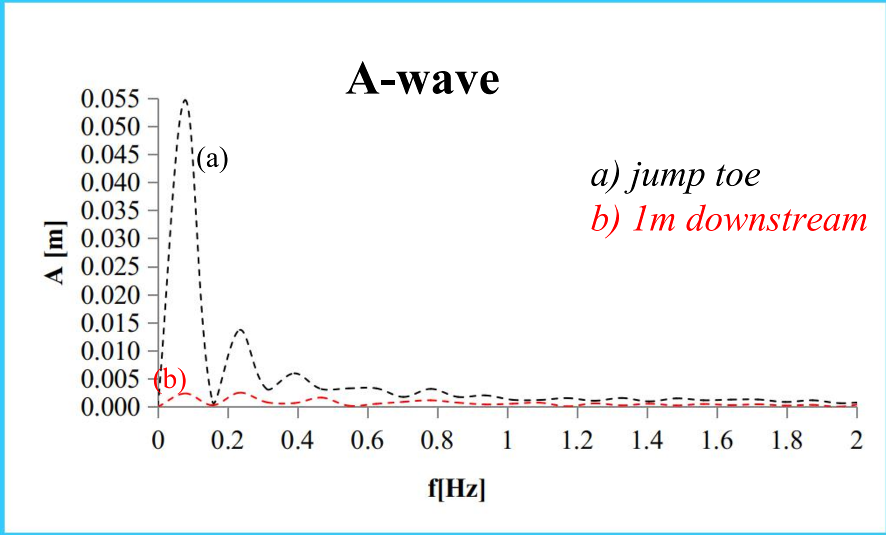
Correct SPH quantitative and qualitative reproduction of experiments

→ "numerical experiments" to complement flow description

- Analysis of elevation spectra to characterize wave propagation:
  - Power amplitude at different locations downstream of the jump toe
  - Detection of wave non-linearities (higher-order harmonics)

Wave propagation





- Same fundamental frequency (0.1 Hz, i.e. $St = f\, y_1 / V_1 = 0.002$)
- Wave propagation downstream at the same frequency
- A-wave case exhibits higher non-linearity (higher A_2/A_1 ; A_3/A_1)

Detailed analysis of flow features

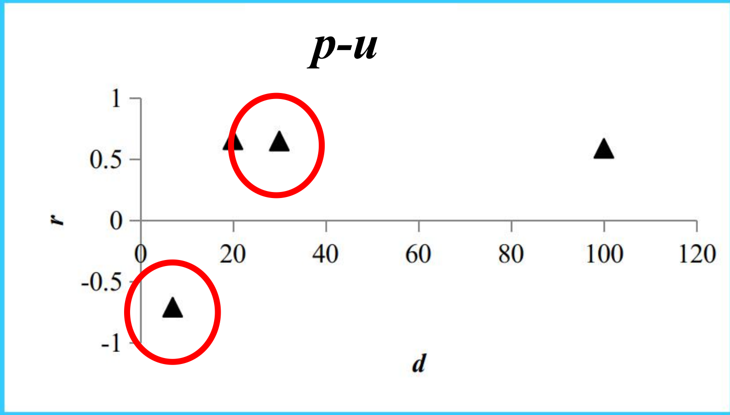
Correct SPH quantitative and qualitative reproduction of experiments

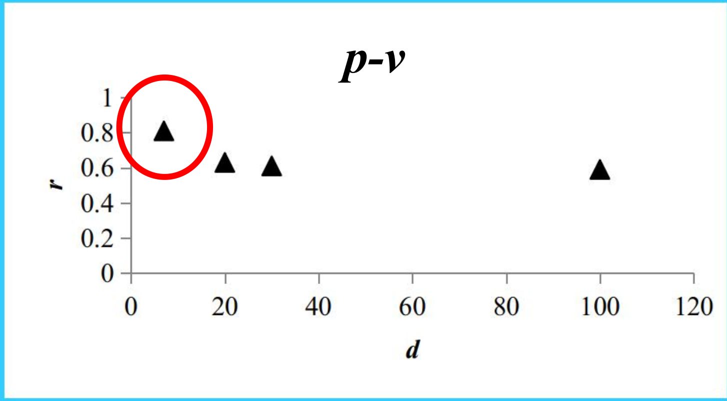
⇨ "numerical experiments" to complement flow description

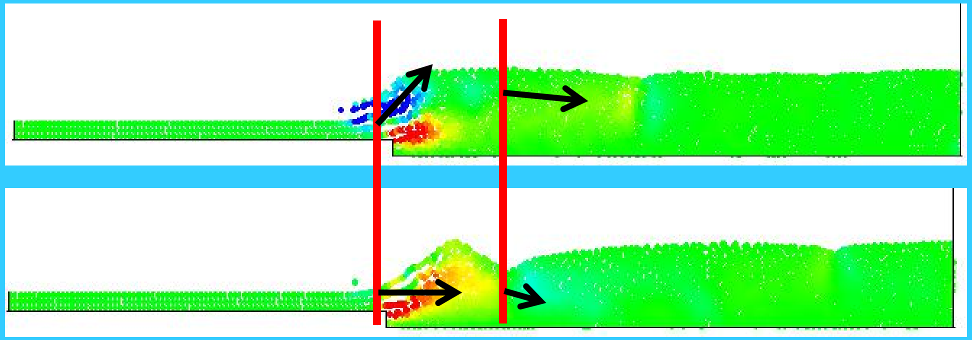
- Analysis of cross-correlations to characterize flow structure:

$$r=\frac{\sum_{i=1}^{n}\left[\left(x_{1_i}-\overline{x_1}\right)\left(x_{2_i}-\overline{x_2}\right)\right]}{\sqrt{\sum_{i=1}^{n}\left(x_{1_i}-\overline{x_1}\right)^2\sum_{i=1}^{n}\left(x_{2_i}-\overline{x_2}\right)^2}}$$

Flow structure (B-wave)







Conclusions

- SPH reproduces correctly the global physical characteristics of the unsteady hydraulic jump flow
- even at a rather low resolution, the flow instability and its dependence on *Fr* and tailwater depth ratio are correctly identified
- SPH can therefore serve as a “numerical experiment” to investigate flow physics

Thank you!