An SPH Model for Fluid-Solid Interaction

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OUTLINE

1) Background
2) SPH method
3) Numerical examples
4) Concluding remarks

Background

Impact of violent wave

Liquid sloshing in LNG ship

Underwater movement of torpedo (military)

Dam break

SPH method

Governing equations

- PDE form
- SPH equations

Continuity equation

$$\frac{d\rho}{dt} = -\rho \frac{\partial v^{\beta}}{\partial x^{\beta}}$$

$$\frac{d\rho_i}{dt} = \rho_i \sum_{j=1}^{N} \frac{m_j}{\rho_j} \left(v_i^{\beta} - v_j^{\beta}\right) \frac{\partial W_{ij}}{\partial x_i^{\beta}}$$

Momentum equation

$$\frac{dv^{\alpha}}{dt} = \frac{1}{\rho} \frac{\partial \sigma^{\alpha\beta}}{\partial x^{\beta}}$$

$$\frac{dv_i^{\alpha}}{dt} = -\sum_{j=1}^{N} m_j \left(\frac{\sigma_i^{\alpha\beta}}{\rho_i^2} + \frac{\sigma_j^{\alpha\beta}}{\rho_j^2} \right) \frac{\partial W_{ij}}{\partial x_i^{\beta}}$$

Energy equation

$$\frac{de}{dt} = \frac{\sigma^{\alpha\beta}}{\rho} \frac{\partial v^{\alpha}}{\partial x^{\beta}}$$

$$\frac{de_i}{dt} = \frac{1}{2} \sum_{j=1}^{N} m_j \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} \right) \left(v_i^{\beta} - v_j^{\beta}\right) \frac{\partial W_{ij}}{\partial x_i^{\beta}} + \frac{1}{\rho_i} S_i^{\alpha\beta} \varepsilon_i^{\alpha\beta}$$

SPH method

rigid body

The movement of any point A can be composed of tanslation and rotation

$$\boldsymbol{u}_a = \boldsymbol{u}_c + \boldsymbol{u}_{ca} = \boldsymbol{u}_c + \omega_c \times \boldsymbol{r}_{ca}$$

The translational and rotational speed can be determined by the following equation

$$\frac{d\boldsymbol{u}_c}{dt} = \frac{\sum m_j \boldsymbol{a}_j}{M} + \boldsymbol{g} \qquad \frac{d\omega_c}{dt} = \frac{\sum m_j \left(r_j - \boldsymbol{R}_0\right) \times \boldsymbol{a}_j}{I_c}$$

elastic body

The shear stress can be obtained by the linear elastic relation

$$\frac{d\tau^{\alpha\beta}}{dt} = 2\mu_s (D^{\alpha\beta} - \frac{1}{3}\delta^{\alpha\beta} D^{\alpha\beta}) + \tau^{\alpha\gamma} R^{\beta\gamma} + \tau^{\gamma\beta} R^{\alpha\gamma}$$

where, $$D_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \qquad \omega_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} - \frac{\partial v_j}{\partial x_i}\right)$$

SPH method

Deficiency of conventional SPH

- low accuracy and instability in boundaries and irregular domains
- Stress instability.

Improvement

- Kernel function
- Boundary conditon
- Stress

kernel fuction correction

Recovering the conservation of the particles

$$\rho_i^{new} = \sum_{j=1}^{N} \rho_j W_{ij}^{new} \frac{m_j}{\rho_j} = \sum_{j=1}^{N} m_j W_{ij}^{new}$$

- Shepard filter method (Re-normalizing the kernel function)[1]

$$\int W(\mathbf{r}-\mathbf{r}', h)dx' \neq 1 \longrightarrow W_{ij}^{new} = \frac{W_{ij}}{\sum_{j=1}^{N} W_{ij} \frac{m_j}{\rho_j}} \longrightarrow \int W(\mathbf{r}-\mathbf{r}', h)dx' = 1$$

- Moving least square (MLS)[2]

$$W_{ij}^{MLS} = W_j^{MLS}(\vec{r}_i) = \beta(\vec{r}_i) \bullet (\vec{r}_j - \vec{r}_j) W_{ij} \qquad A = \sum_{j=1}^{N} W_j(\vec{r}_i) A' V_j$$

$$\beta(\vec{r}_i) = \begin{bmatrix} \beta_0 \\ \beta_x \\ \beta_y \end{bmatrix} = A^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad A' = \begin{bmatrix} 1 & x_i - x_j & y_i - y_j \\ x_i - x_j & (x_i - x_j)^2 & (x_i - x_j)(y_i - y_j) \\ y_i - y_j & (x_i - x_j)(y_i - y_j) & (y_i - y_j)^2 \end{bmatrix}$$

For different situations, we should choose different density correction

[1] A. Colagrossi and M. Landrini, “Numerical simulation of interfacial flows by smoothed particle hydrodynamics,” Journal of computational physics, 2003, vol. 191, pp. 448-475.
[2]T. Rabczuk, T. Belytschko and S. Xiao, “Stable particle methods based on Lagrangian kernels,” Computer Methods in Applied Mechanics and Engineering, 2004. vol. 193, pp. 1035-1063.

Boundary treatment

- Repulsive force boundary

1) adapt to complex condition

2) a initial pressure disturbance

3) the coefficient of repulsive force is uncertain, low accuracy

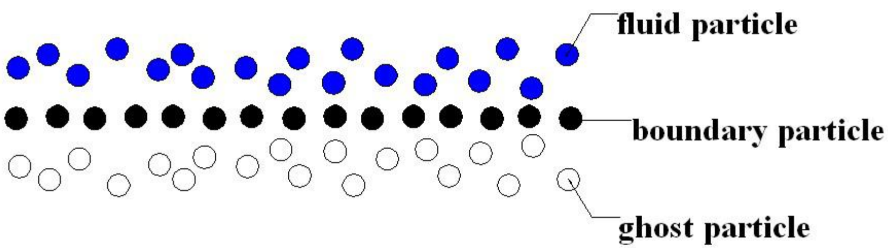
- Dynamic boundary

1) adapt to complex condition

2) pressure oscillation in boundary

3) can't guarantee no penetration

- Ghost particle boundary (free-slip)



tangential and normal velocity

$$\begin{cases} x_g = 2x_w - x_i \quad y_g = 2y_w - y_i \\ v_{g,t} = v_{i,t} \quad v_{g,n} = -v_{i,n} \\ p_g = p_i \quad \rho_g = \rho_i \end{cases}$$

1) higher accuracy

2) guarantee no penetration

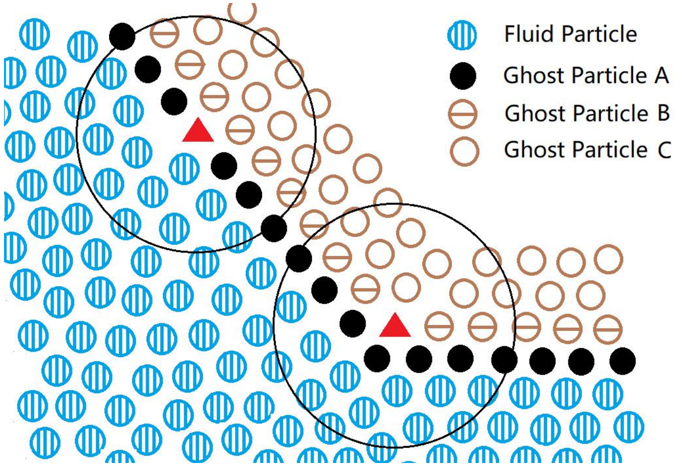
3) not adapt to complex condition

The three conditions can not be realized at the same time.

Boundary treatment

Improved fluid-solid interface treatment algorithm

- Three types of ghost particles, field variable of the particles can be dynamically evolved and obtained from SPH approximation.
- Ghost particles were genetated in a regular or irregular distribution at the first time step, while ghost particle positions do not need to change during following steps.
- The computation domains of different ghost particles are different.



The pressure of the ghost particles can be obtained as follows

$$P_i = \sum_j P_j W_{ij}^I \frac{m_j}{\rho_j}$$

W^I is the improved kernel function, which represents the new function obtained by the Shepard Filter or Moving Lease Square method separately.

Boundary treatment

For approximations with particle from different materials, different density may produce large numerical oscillation in the interface region. The density change rate (continuum equation) can be corrected as follows

$$\frac{d\rho_j}{dt} = \sum_j m_j v_{ij} \nabla_i W_{ij} \frac{\rho_i}{\rho_j}$$

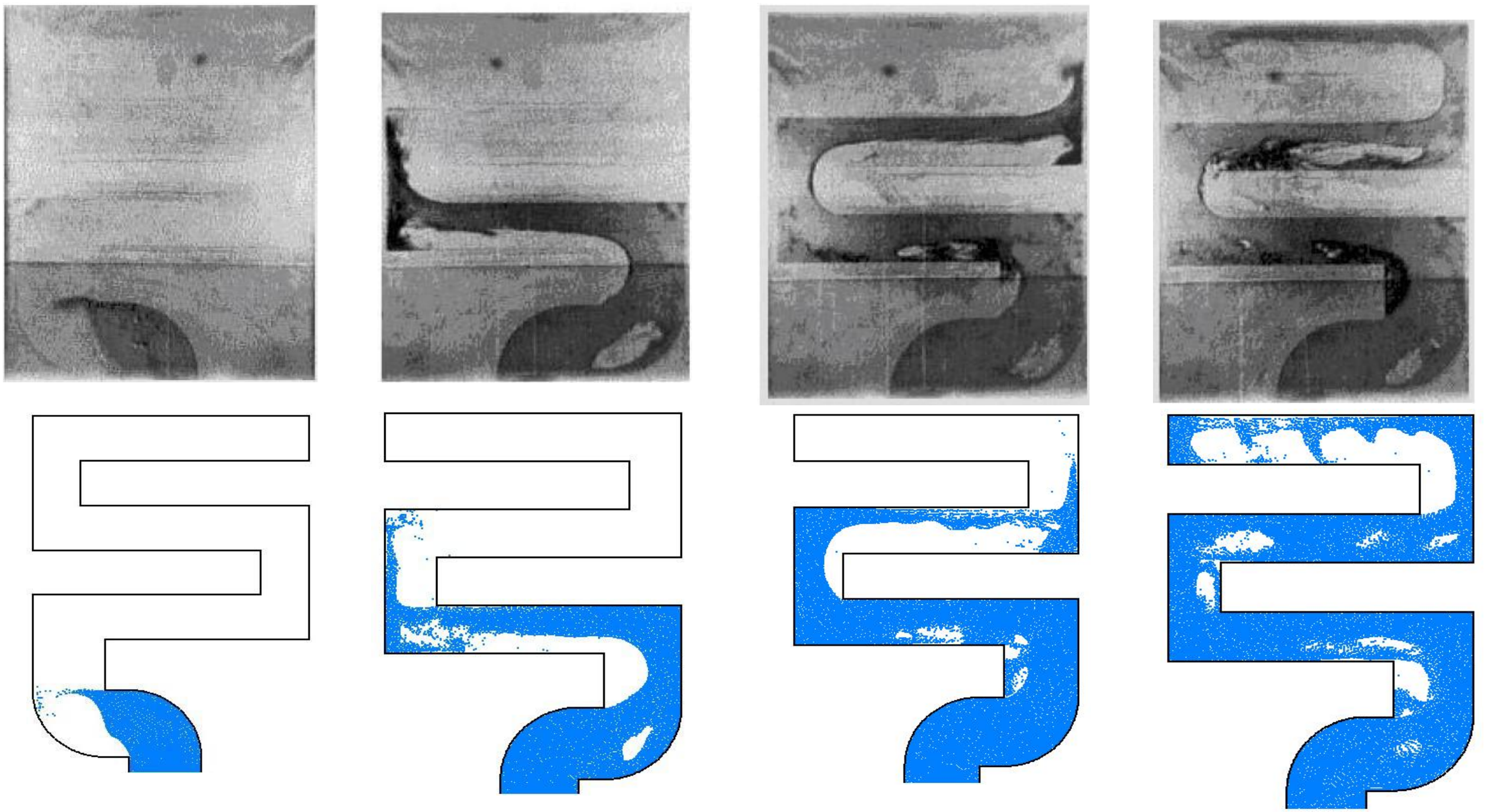
Adding a corrective term can help to balance the underestimation of density change rate

For the particles from fixed solid boundary, the density is usually set to be the same as the fluid particles. Take non-slip boundary condition for example, the variables of the boundary particles can be obtained from the following equations

$$v_i = -\sum_{j=1}^{N} v_j W_{ij}^I \frac{m_j}{\rho_j}$$

Numerical examples

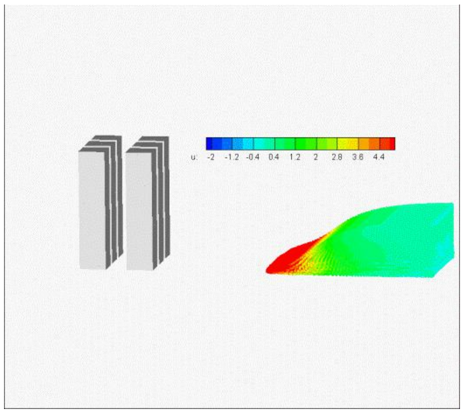
Water injection problem in complex booundary

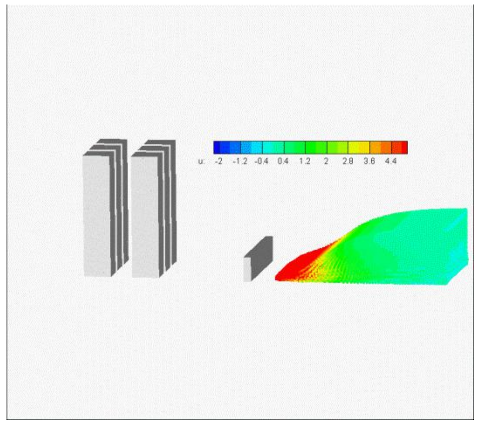


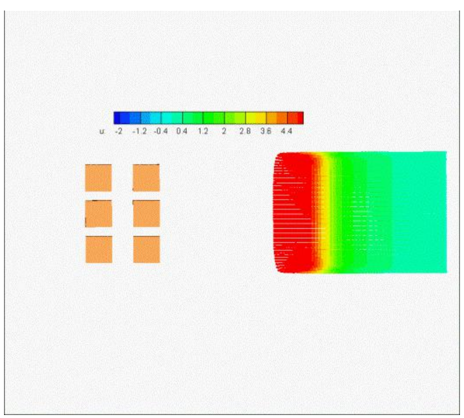
SPH results agree well with the experimental results, they have similiar flow pattern and cavity. It validates that SPH can predict the volient free surface flow.

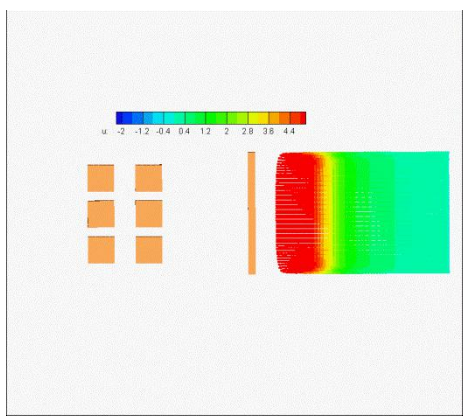
Numerical examples

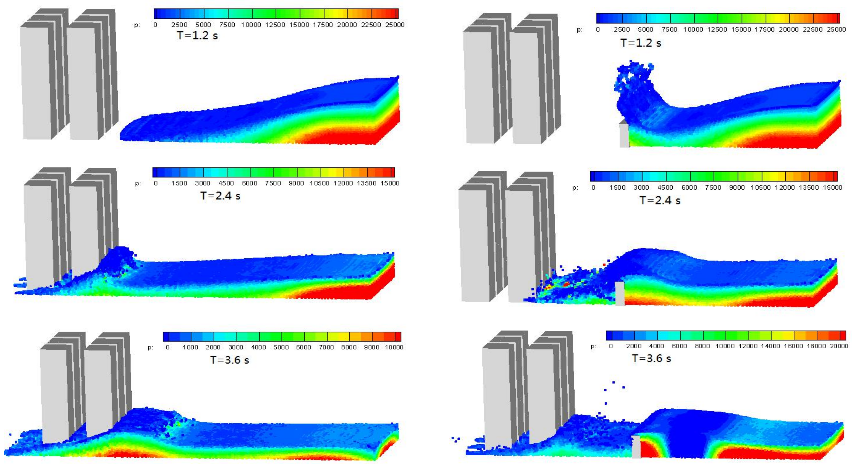
3D Flooding flow model with and without obstacle (Simplified model of a street)







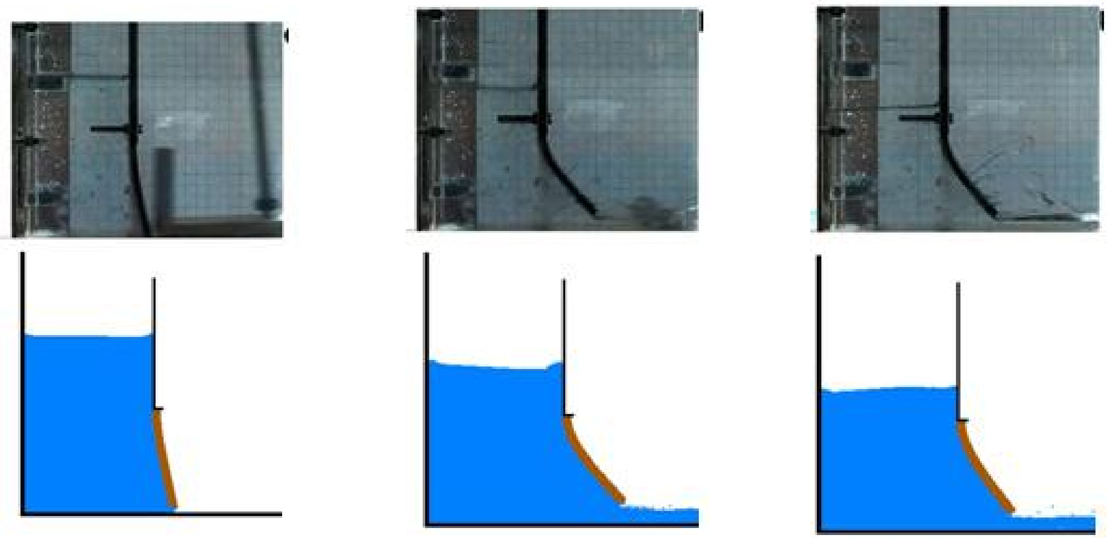


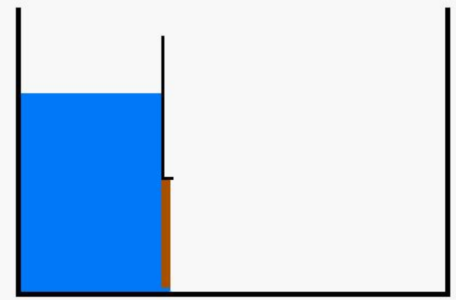


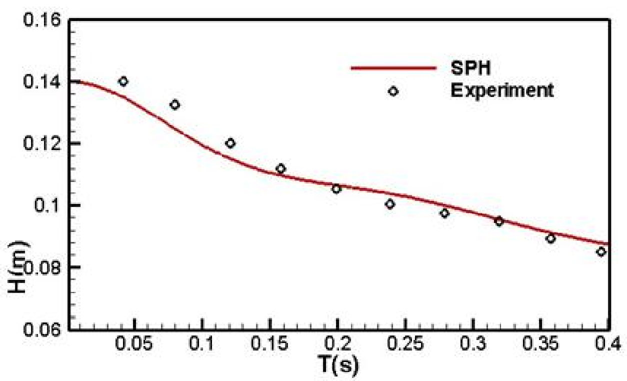
The SPH model can predict complex 3D flow character , and can predict the flood flow in the engineering.

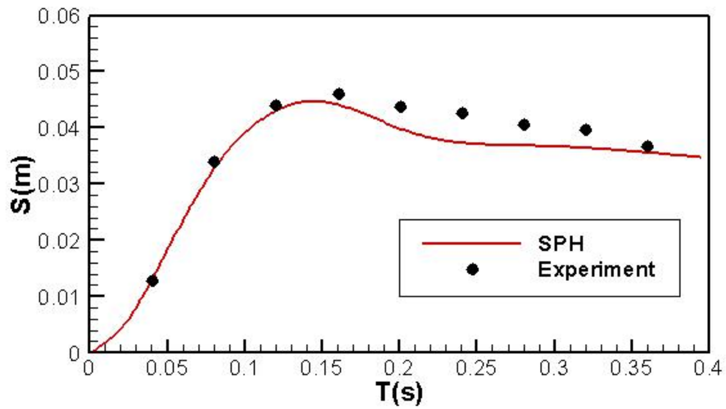
Dam break with an elastic gate

The pressure will cause the deformation of the elastic gate



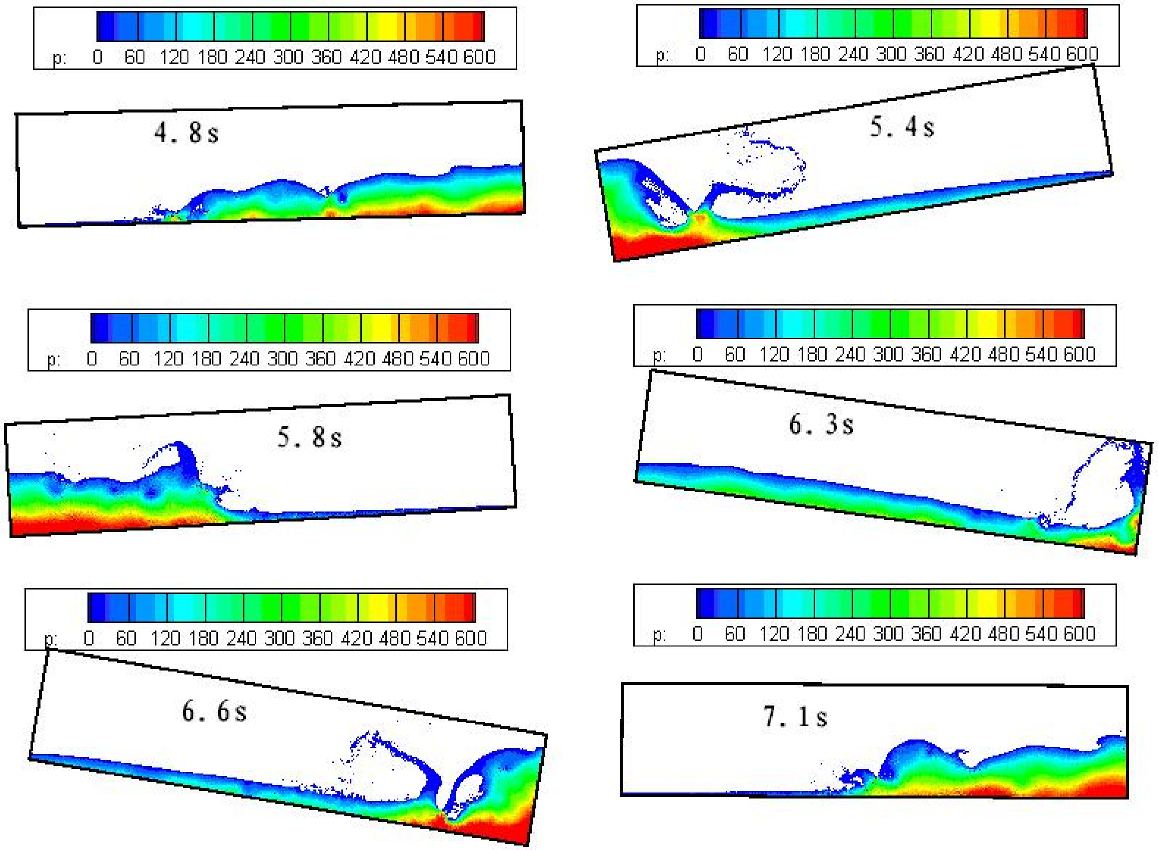


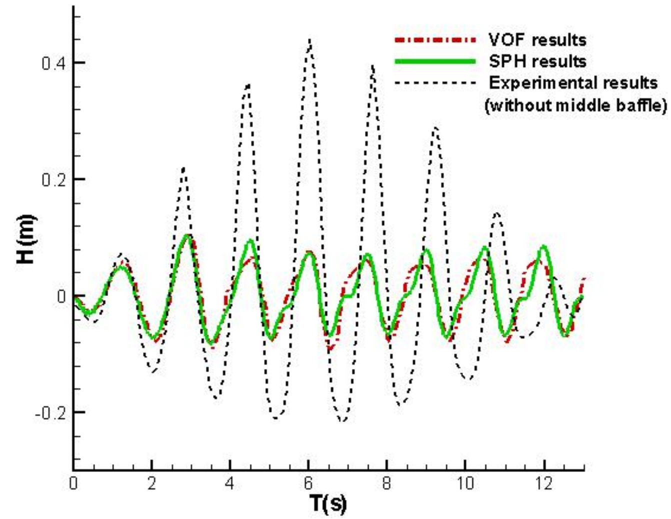




Horizontal and vertical displacements of the free end of the elastic gate

Liquid sloshing

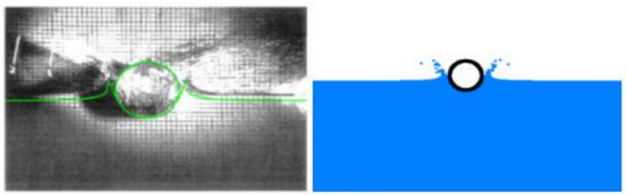




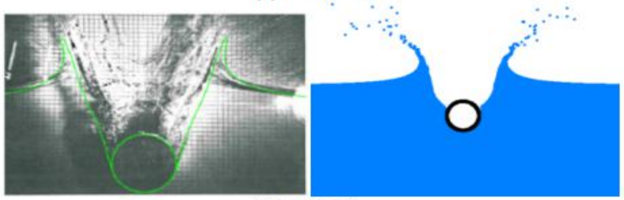
The SPH model can predict the liquid sloshing under arbitrary external excitation, and can obtain the accurate pressure curve.

Water entry

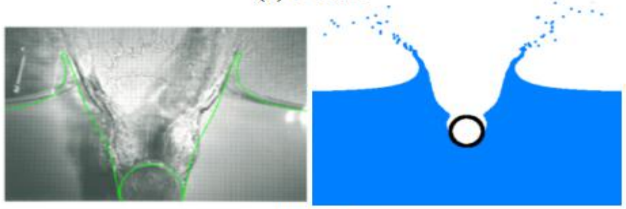
water entry of a cylinder



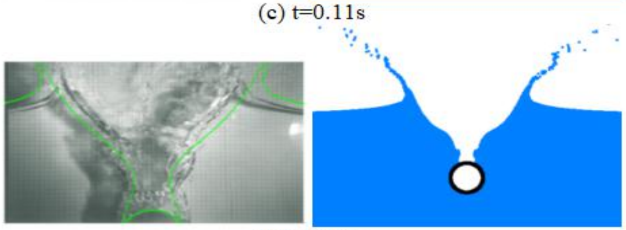
(a) t=0.015s



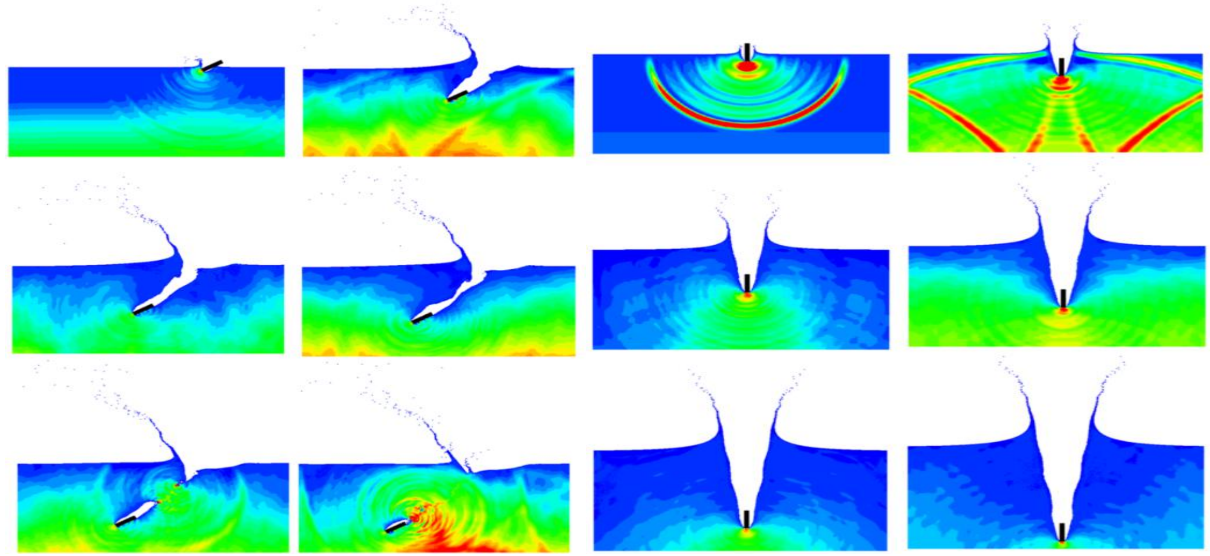
(b) t=0.09s

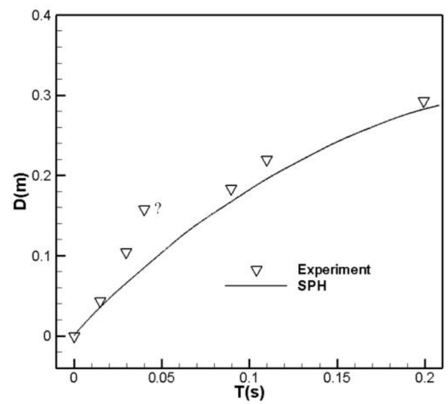


(c) t=0.11s

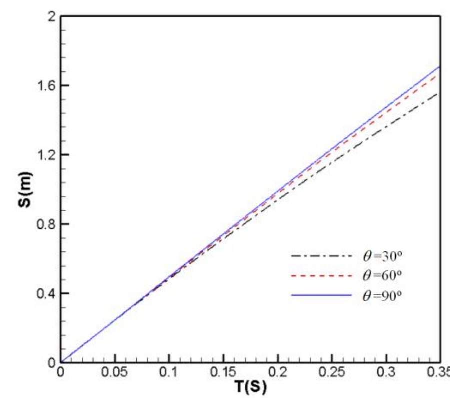


(d) t=0.20s





Penetration depths



Displacement of the centroid with different angle

Concluding remarks

1) SPH method can be applied to simulate fluid solid interaction problem, such as water entry, liquid sloshing, dambreak problems.

2) The improved fluid solid interface treatment algorithm can obtain accurate flow form, smoothed pressure field, and accurate pressure curve.

Thank you！