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# Water Hammer Analysis Using SPH in Density Summation Form

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# Outline

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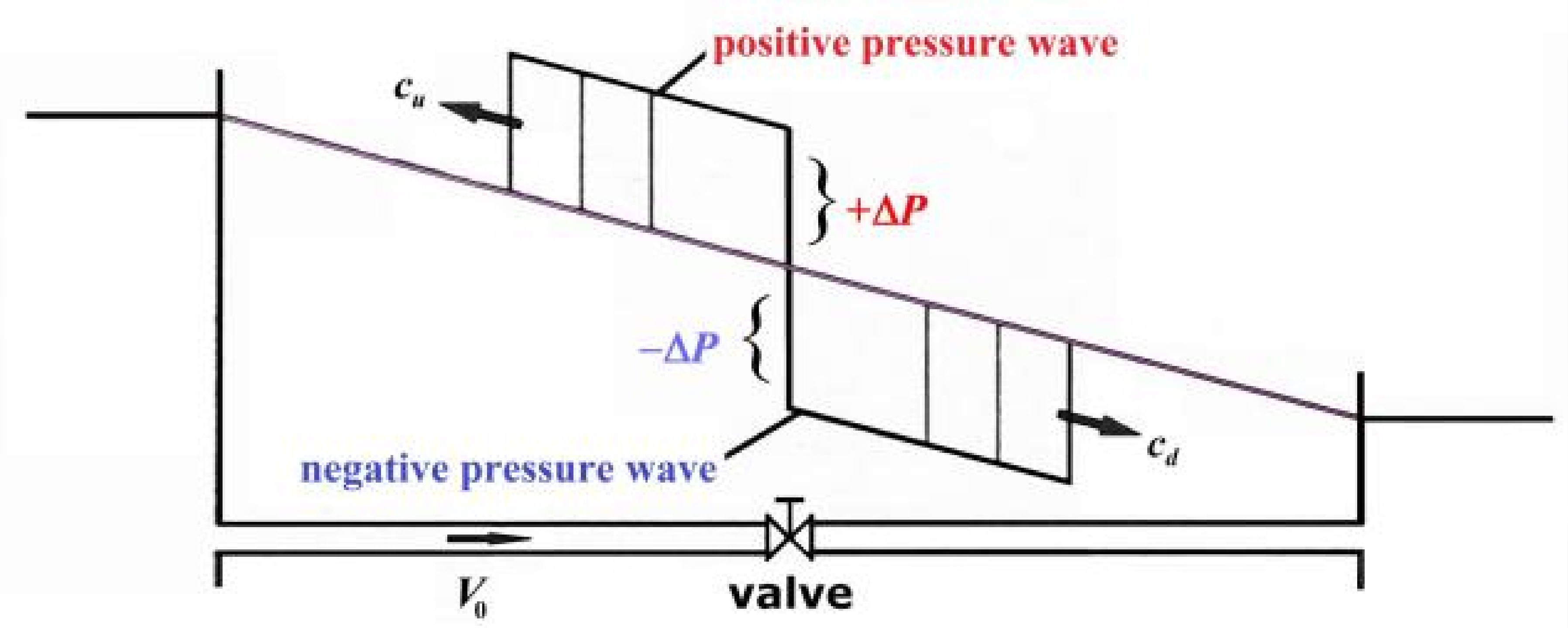
- Governing Equations
- Numerical Method
- Numerical Result
- Summary

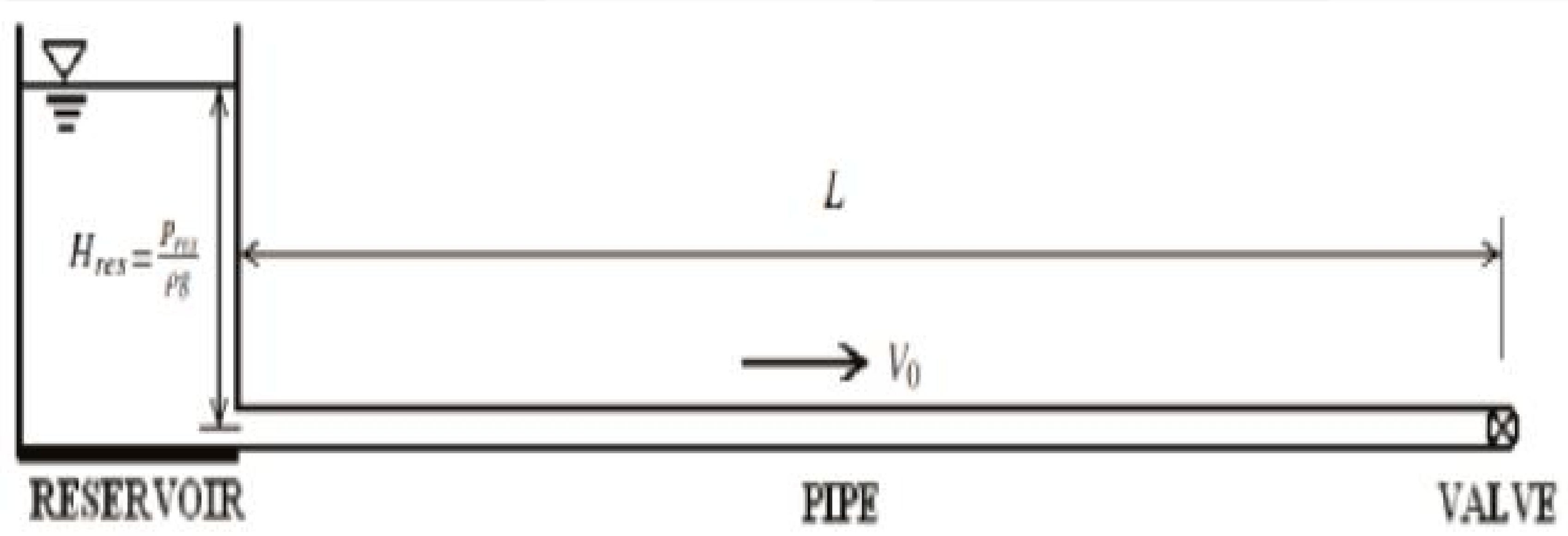
# Background

# Background

## Water hammer



# Physical Model




$L$ = 20 m, $D$ = 797 mm, $\rho$ = 1000 kg/m$^3$, $Q_0 = AV_0$ = 0.5 m$^3$/s, $P_R$ = 1 MPa, $f$ = 0.02, $c$ = 1025.7 m/s.

# Governing Equations

**Continuity equation** $$\frac{\mathrm{D}P}{\mathrm{D}t}+\rho c^2\frac{\partial V}{\partial x}=0$$

**Momentum equation** $$\frac{\mathrm{D}V}{\mathrm{D}t}+\frac{1}{\rho}\frac{\partial P}{\partial x}+g(S-S_0)=0$$

**EoS** $$\mathrm{D}P=c^2\,\mathrm{D}\rho$$

- Downstream boundary
- Upstream boundary

**V = 0 (closed valve) at x = L.** **P = Pr at x = 0.**

# Semi-discrete equation

$$\frac{\mathrm{d}P_i}{\mathrm{d}t} = -\rho_i c_i^2 \sum_j (V_j - V_i) \frac{\mathrm{d}W_{ij}}{\mathrm{d}x_i} \frac{m_j}{\rho_j}$$

$$\frac{\mathrm{d}V_i}{\mathrm{d}t} = -\frac{1}{\rho_i} \sum_j (P_j - P_i) \frac{\mathrm{d}W_{ij}}{\mathrm{d}x_i} \frac{m_j}{\rho_j} - \frac{\lambda V_i |V_i|}{2D}$$

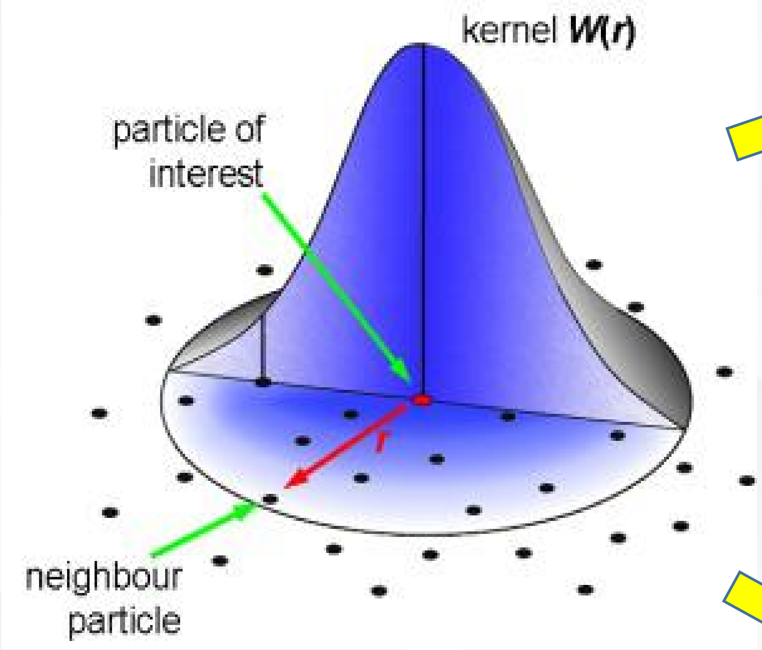
**Density summation form – shock tube**

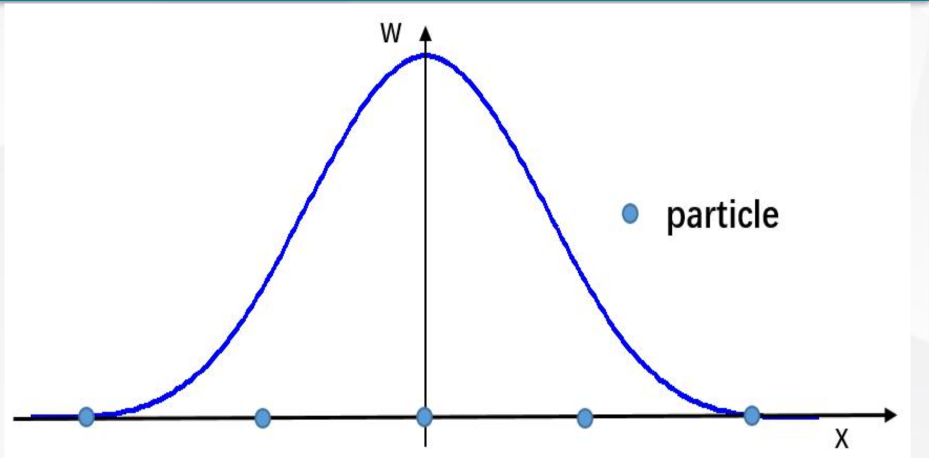
$$\rho_i = m_i \sum_{j=1}^{N} W_{ij}$$

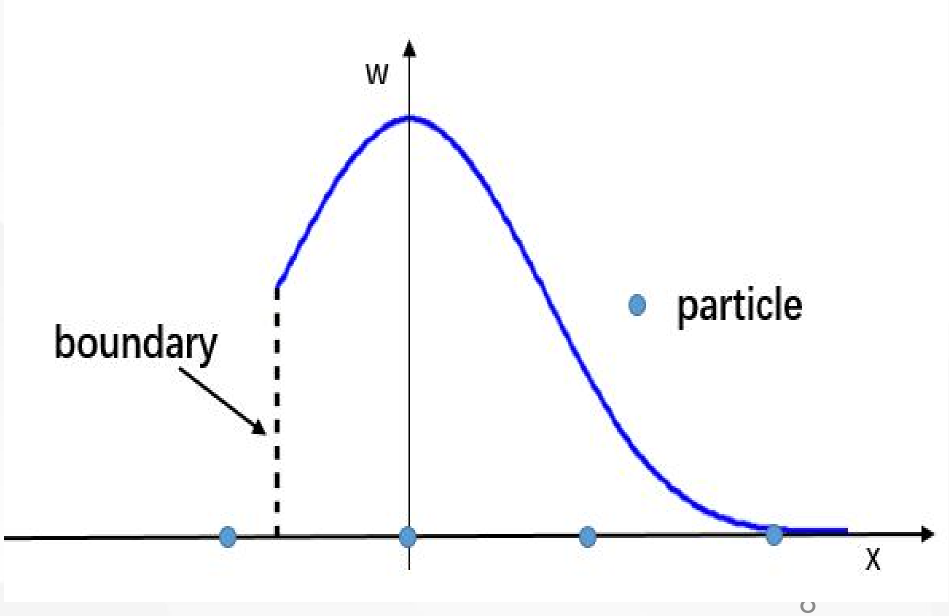
$$\Delta P = \Delta \rho c^2$$

# Numerical Method

- Boundary conditions



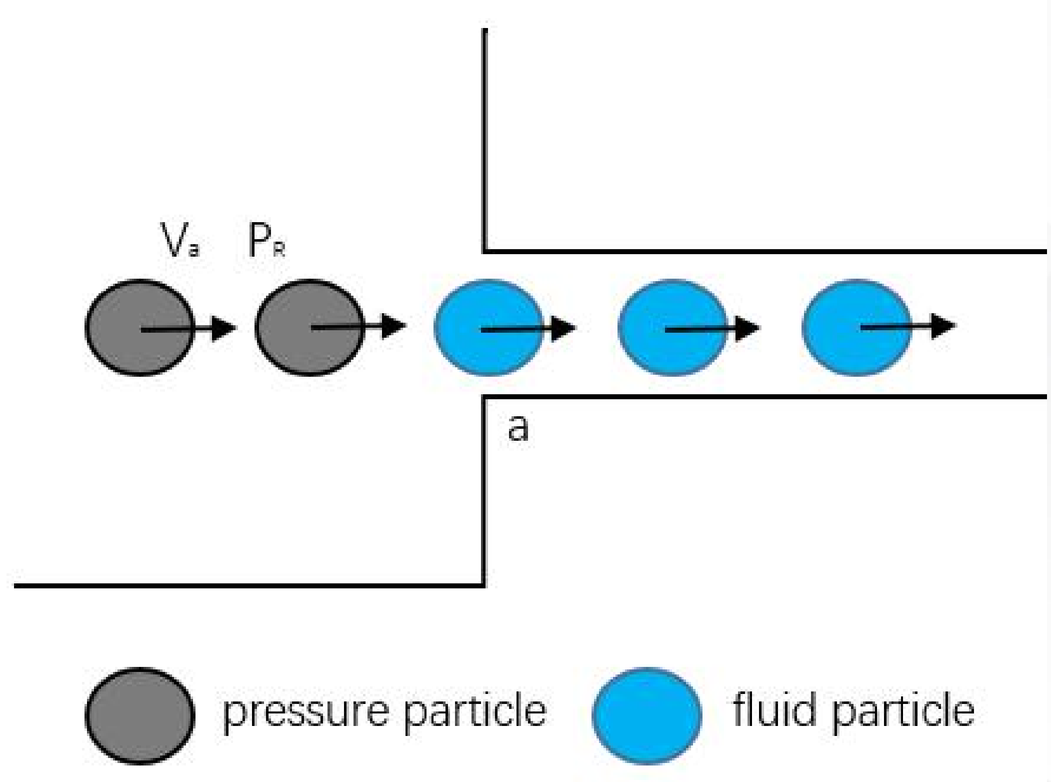



# Numerical Method

- Upstream boundary

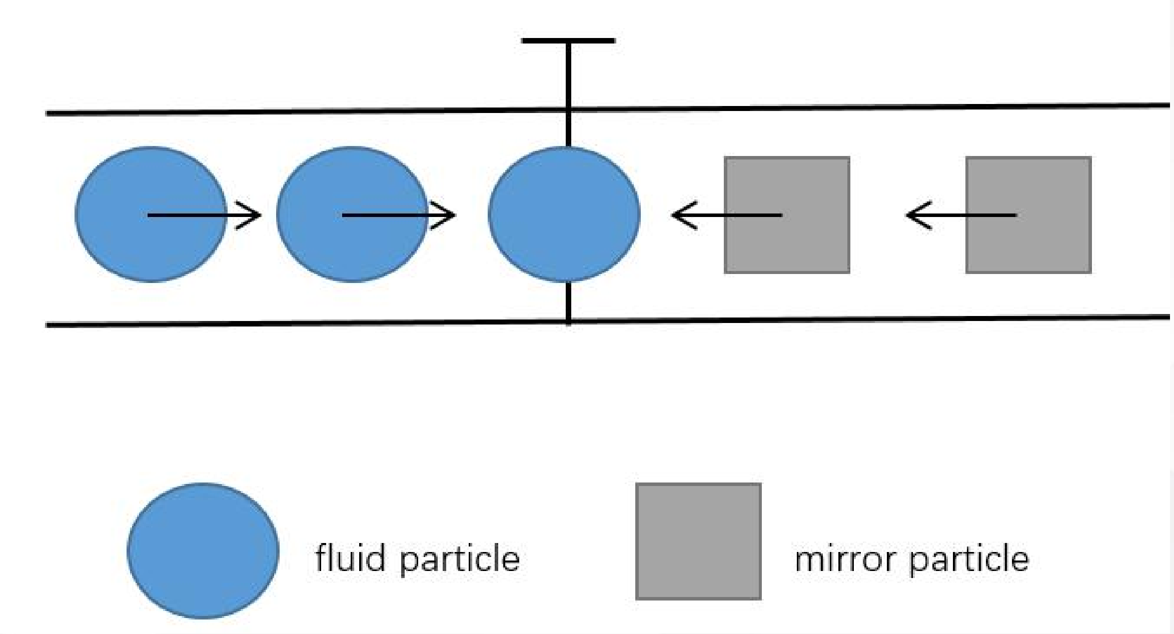
◆ V=Va    $P=P_R$

◆ Key
  Add particle
  Delete particle

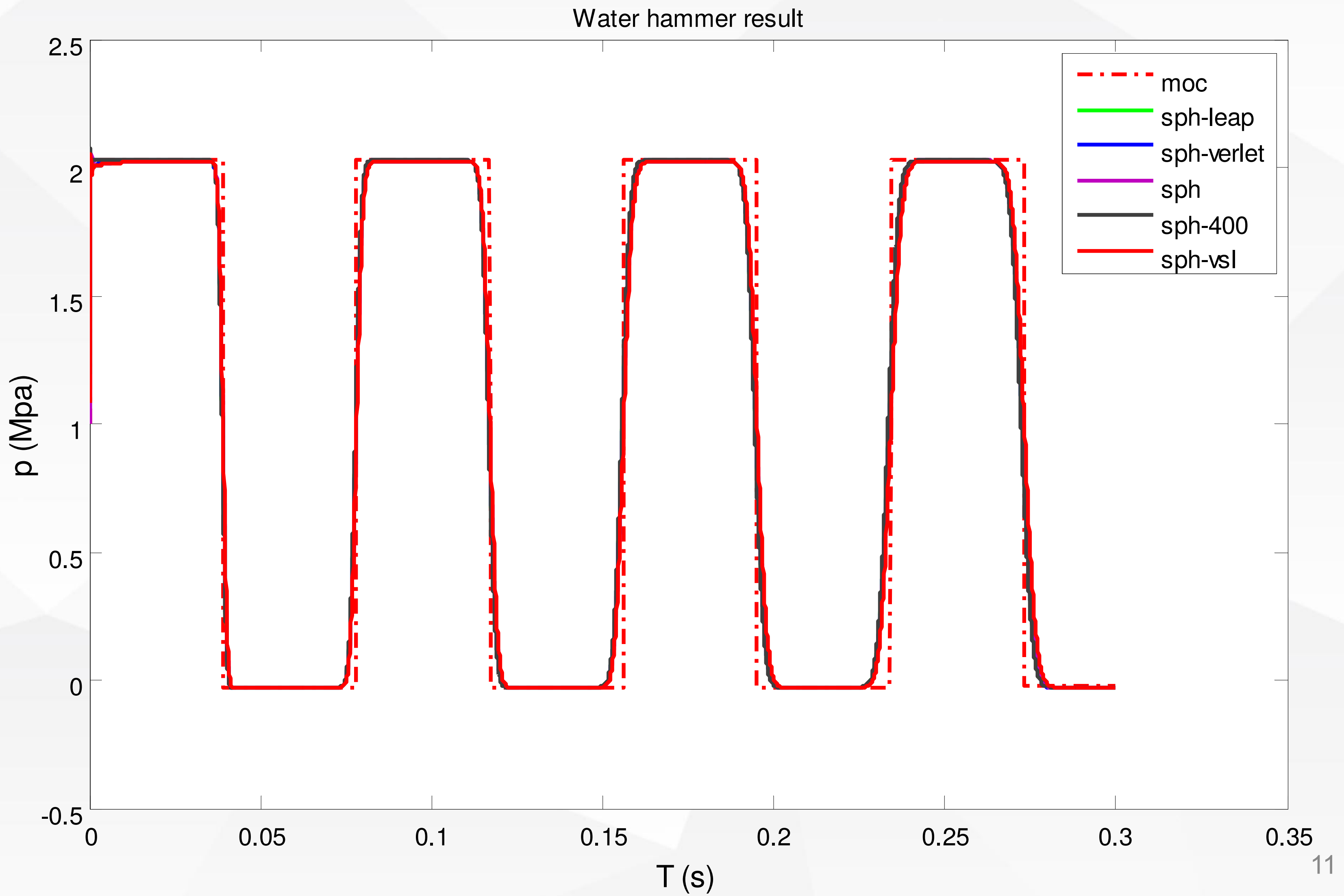


# Numerical Method

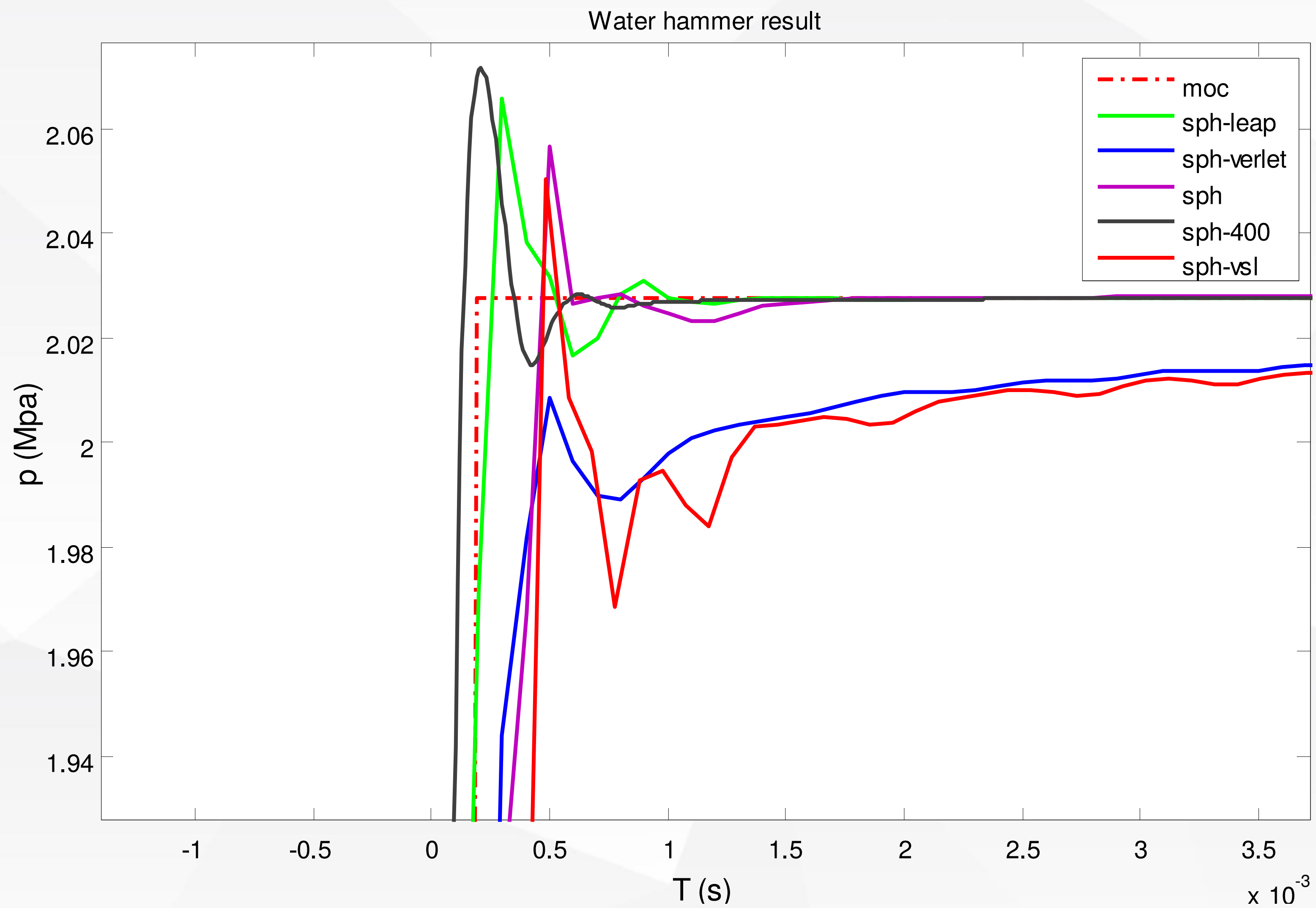
- Downstream boundary



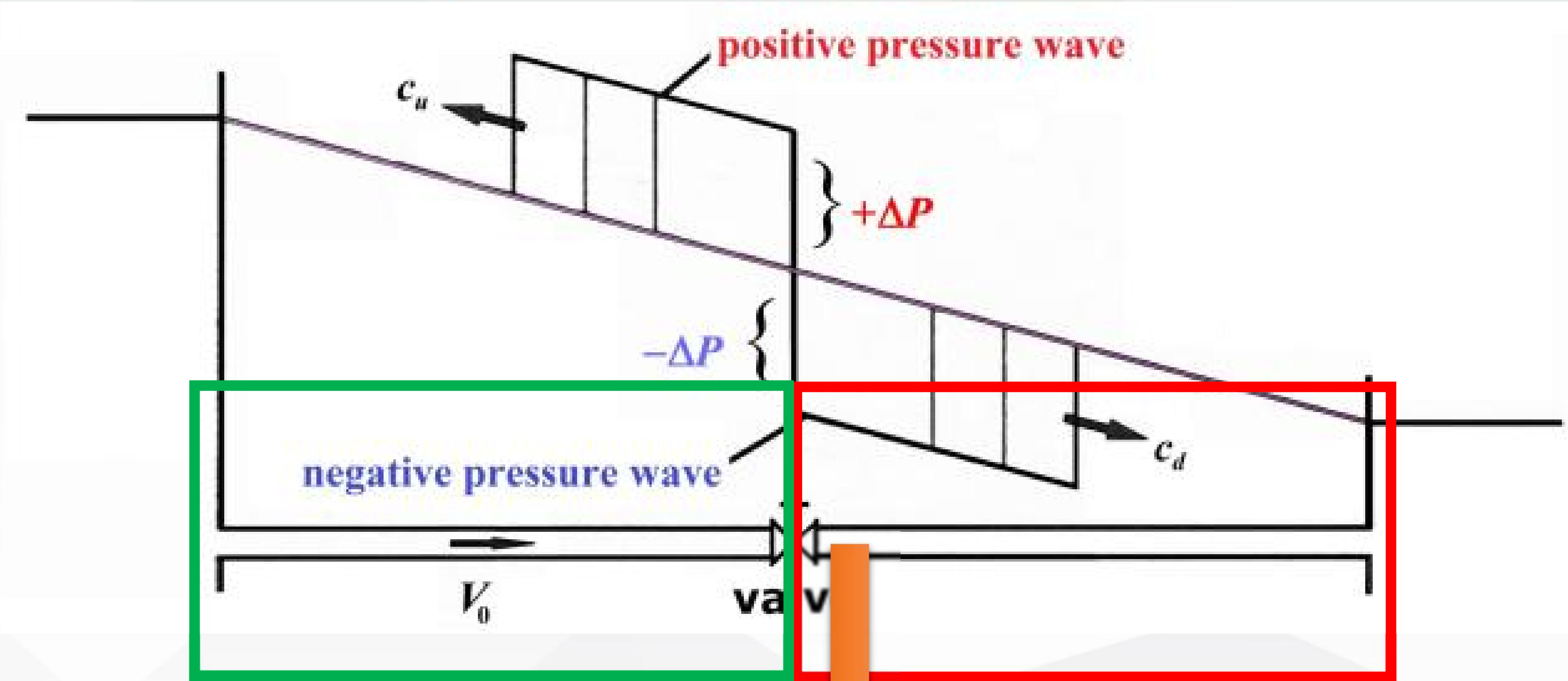
# Numerical Result



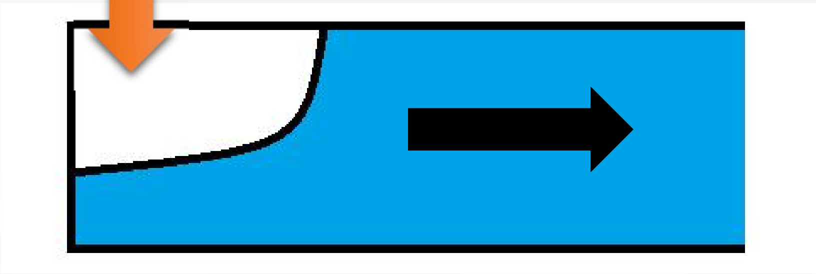
# Experimental Result



# Pipe collapse and cavitation




Cavitation (column separation) due to vaporization



# Summary

- **Density evolution in summation form for water hammer**
- **Effect of time integration method**
- **Variable smoothed length**

THANKS