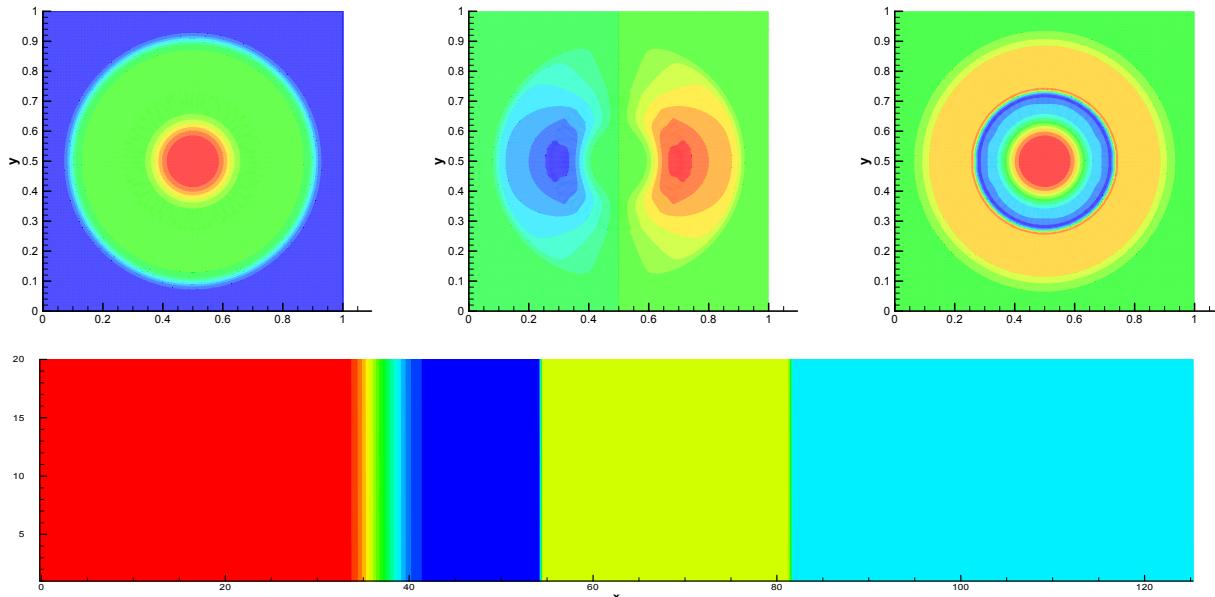


## Multiphase Godunov-typed SPH Method with Approximate Riemann Solvers



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China Ship Scientific Research Center (CSSRC)

Oct. 17, 2017

- 1. Multiphase Riemann problems**
- 2. Multiphase Godunov-typed SPH  
(MGSPH) method**
- 3. Numerical results**
- 4. Conclusions**

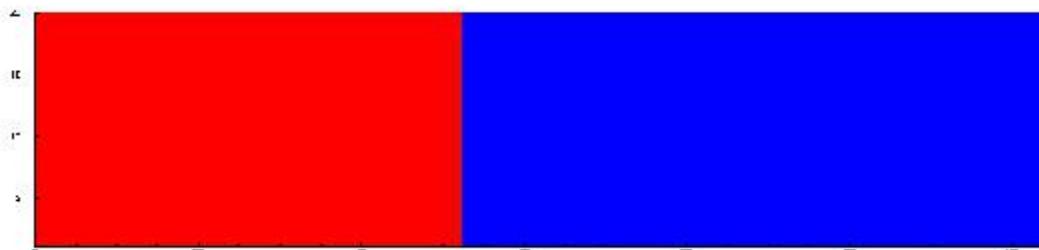
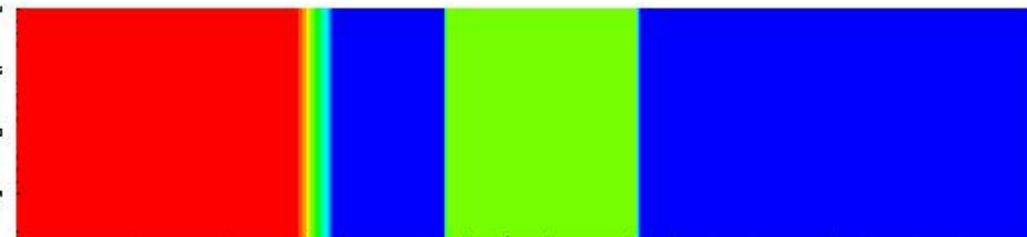
# 1. Multiphase Riemann problems

## ◆ Multiphase Riemann problems

- multiphase flow in an Eulerian framework with discontinuous initial conditions
- shock tube problems, underwater explosion problems and so on

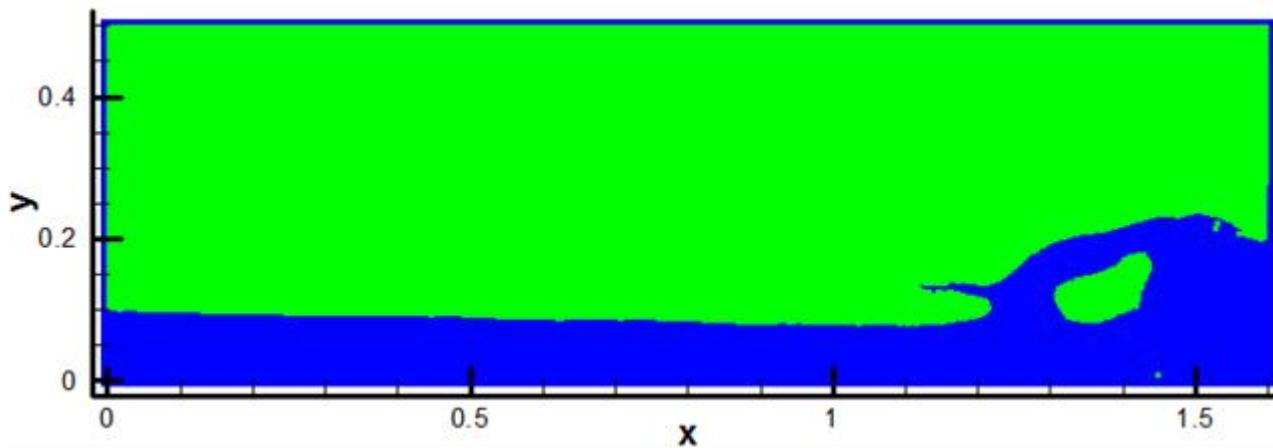
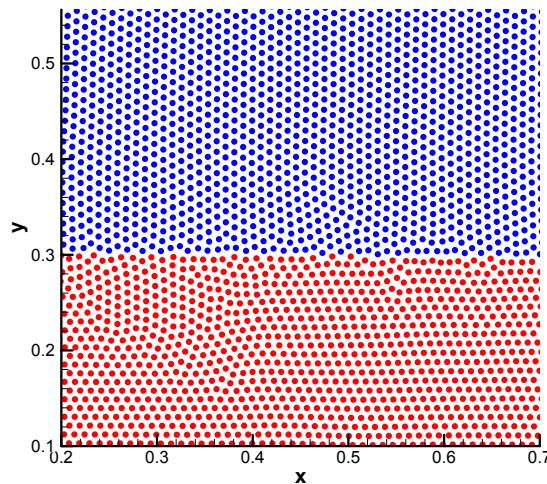
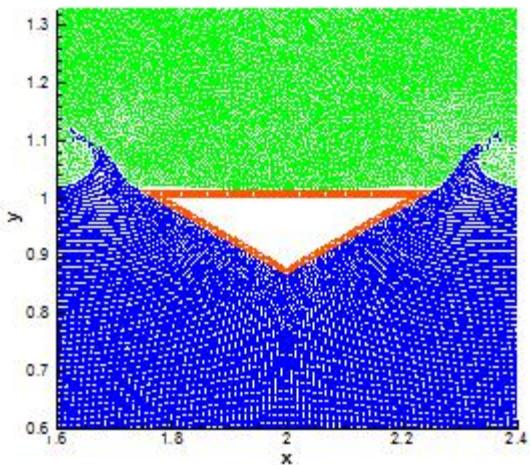
## ◆ Difficulties

- Interface tracking
- Discontinuous initial conditions
- Different Equations of State (EOSs)



## 2. Multiphase Godunov-typed SPH

◆ SPH: interface tracking



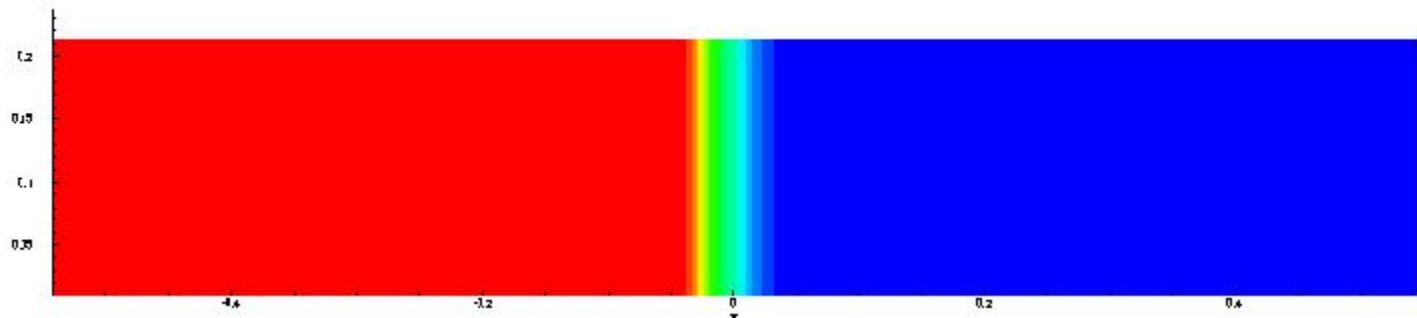
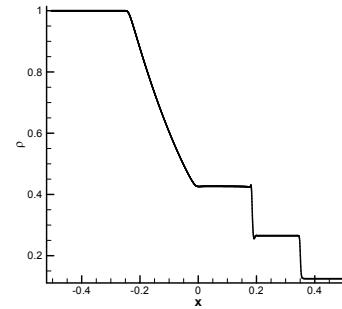
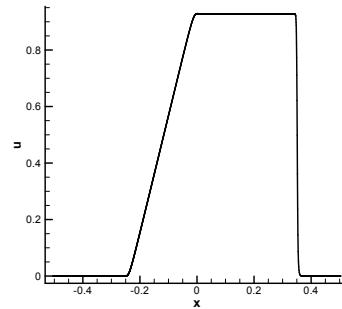
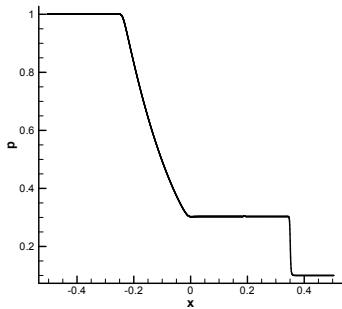
## 2. Multiphase Godunov-typed SPH

◆ GSPH: discontinuous initial conditions

$$\rho_a = \sum_b m_b W_{ab}$$

$$\frac{dv_a}{dt} = -2 \sum_b m_b \frac{P^*}{\rho_a \rho_b} \nabla_a W_{ab}$$

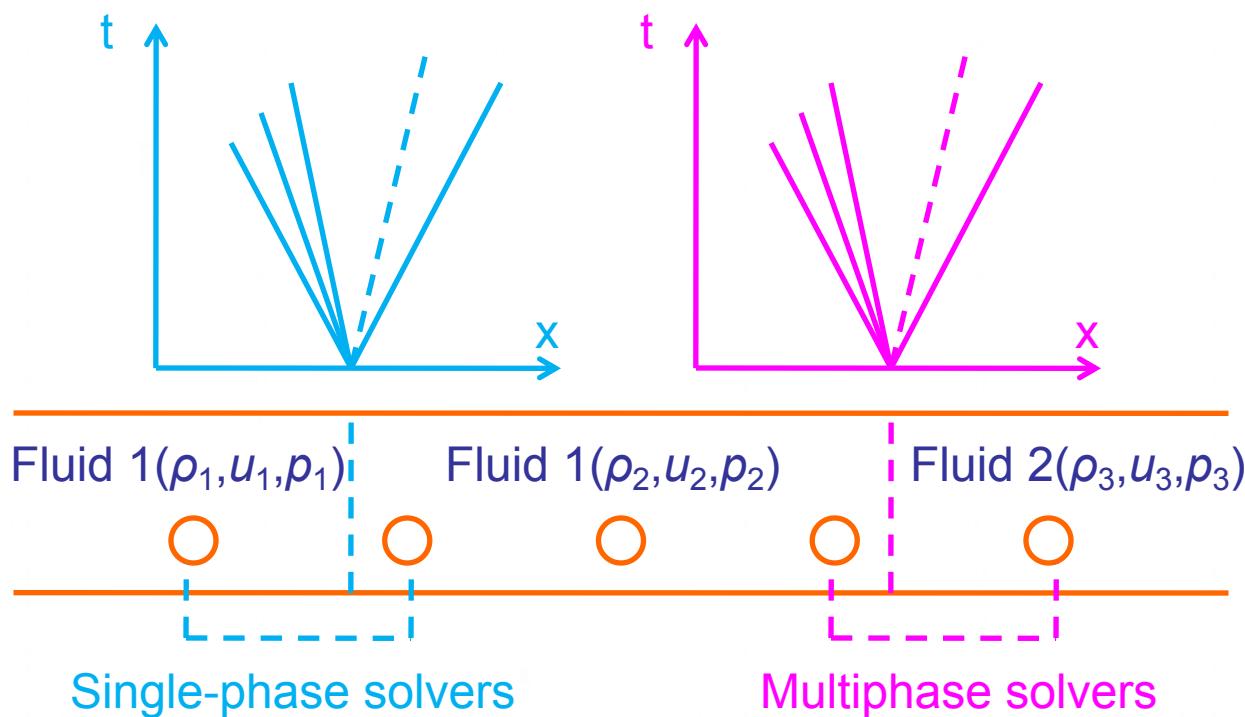
$$\frac{dE_a}{dt} = 2 \sum_b m_b \frac{P^* u^*}{\rho_a \rho_b} \cdot \mathbf{e}_{ab} \cdot \nabla_a W_{ab}$$



## 2. Multiphase Godunov-typed SPH

### ◆ MGSPH: different Equations of State

- single-phase approximate Riemann solver is implemented in the inner area
- multiphase approximate Riemann solver is adopted to the interface



### 3. Numerical results

#### ◆ Single-phase approximate Riemann solvers

- The local Lax–Friedrichs Riemann solver(LLXF)
- Roe's approximate Riemann solver(ROE)
- Harten, Lax, van Leer and Einfeldt approximate Riemann solver (HLLE)
- Harten, Lax and van Leer scheme with contact (HLLC)
- Ducowicz approximate Riemann solver (DUCO)

#### ◆ Multiphase approximate Riemann solvers

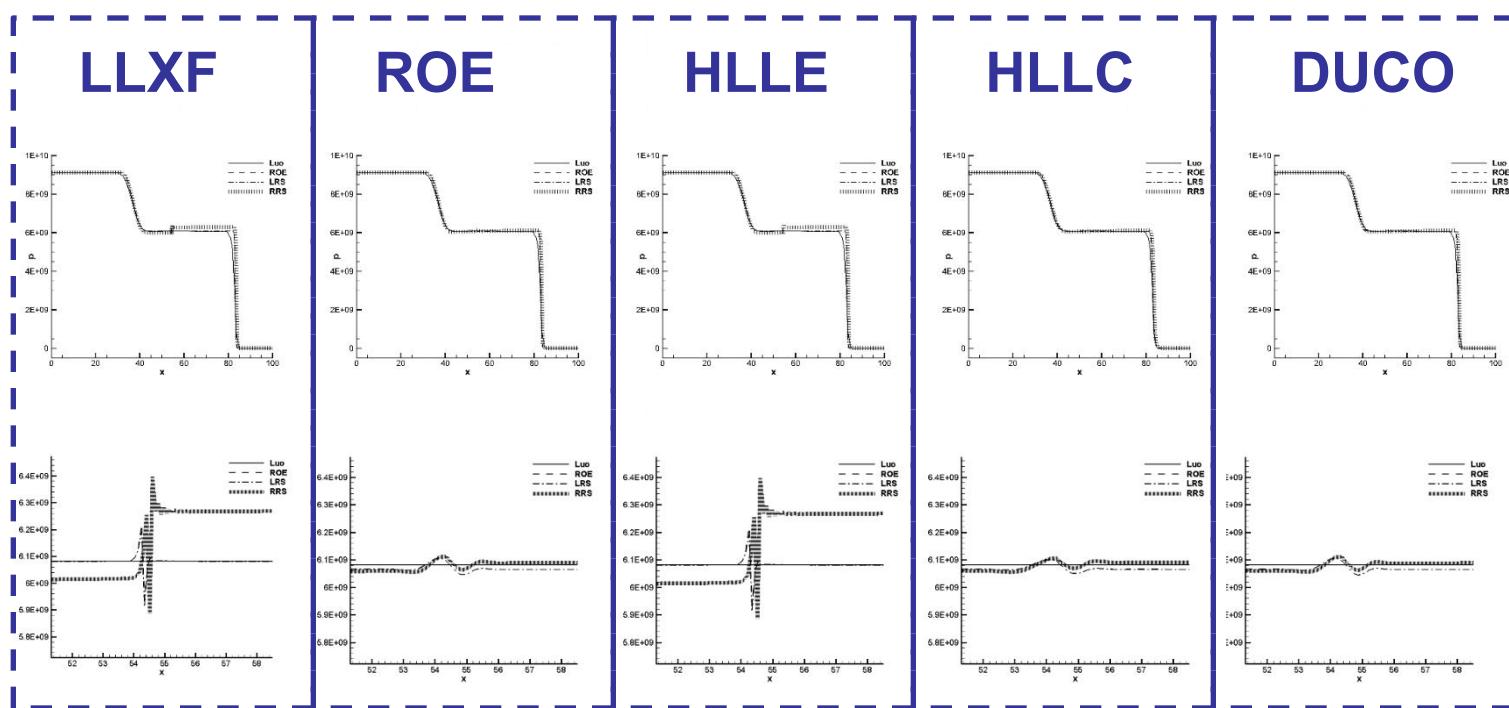
- Linearized wave curves (LRS) by Fechter and Schleper
- Relaxation approach (RRS) by Fechter and Schleper
- Roe's approximate Riemann solver (ROE)

#### ◆ 15 combinations

### 3. Numerical results

#### ◆ Gas-water shock tube

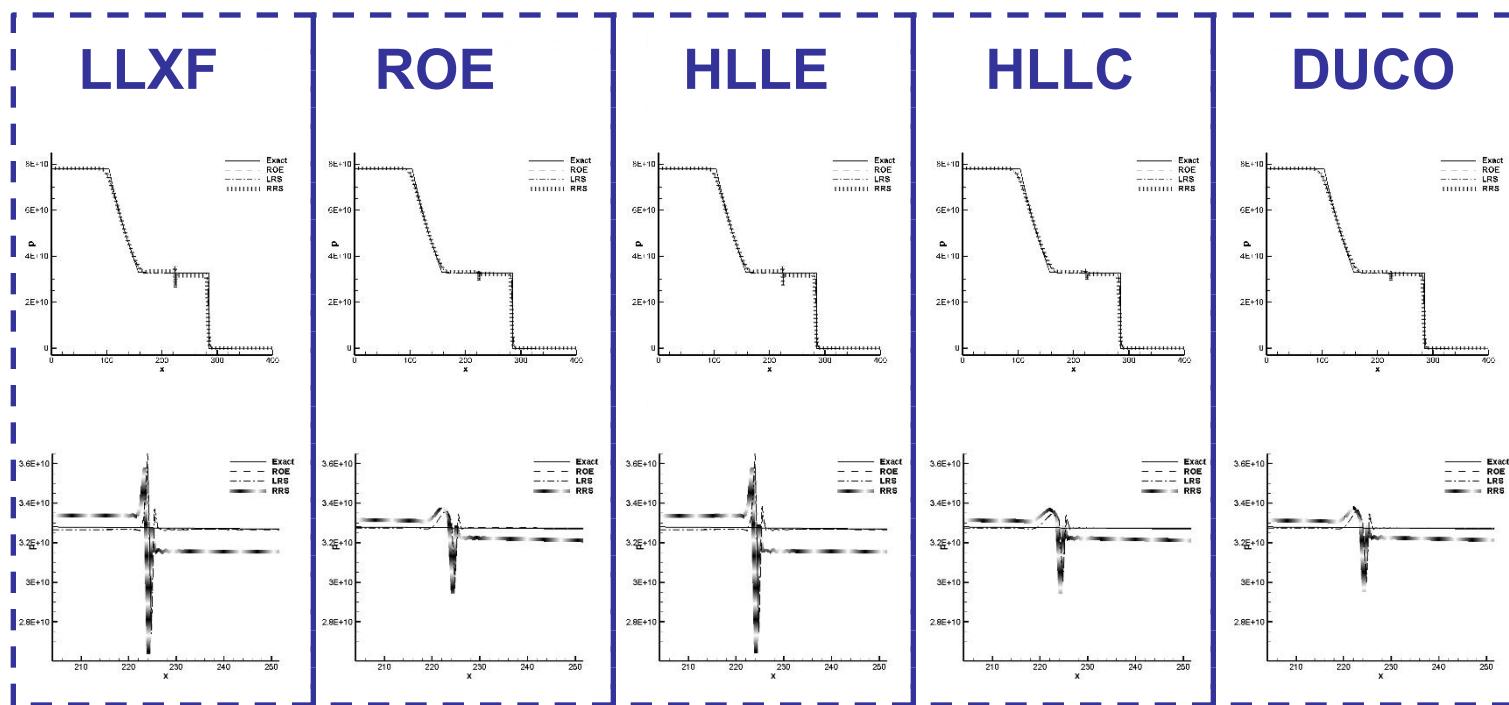
Fluid	$(\rho, u, P)$	EOS	Parameters in EOS
Left	(1.271, 0, 9.119252E9)	$P=(\gamma-1)\rho e - \gamma P_c$	$\gamma=1.4; P_c=0$
Right	(0.999983, 0, 1.01325E6)	$P=(\gamma-1)\rho e - \gamma P_c$	$\gamma=7; P_c=3.03975E9$



### 3. Numerical results

#### ◆ Underwater explosion

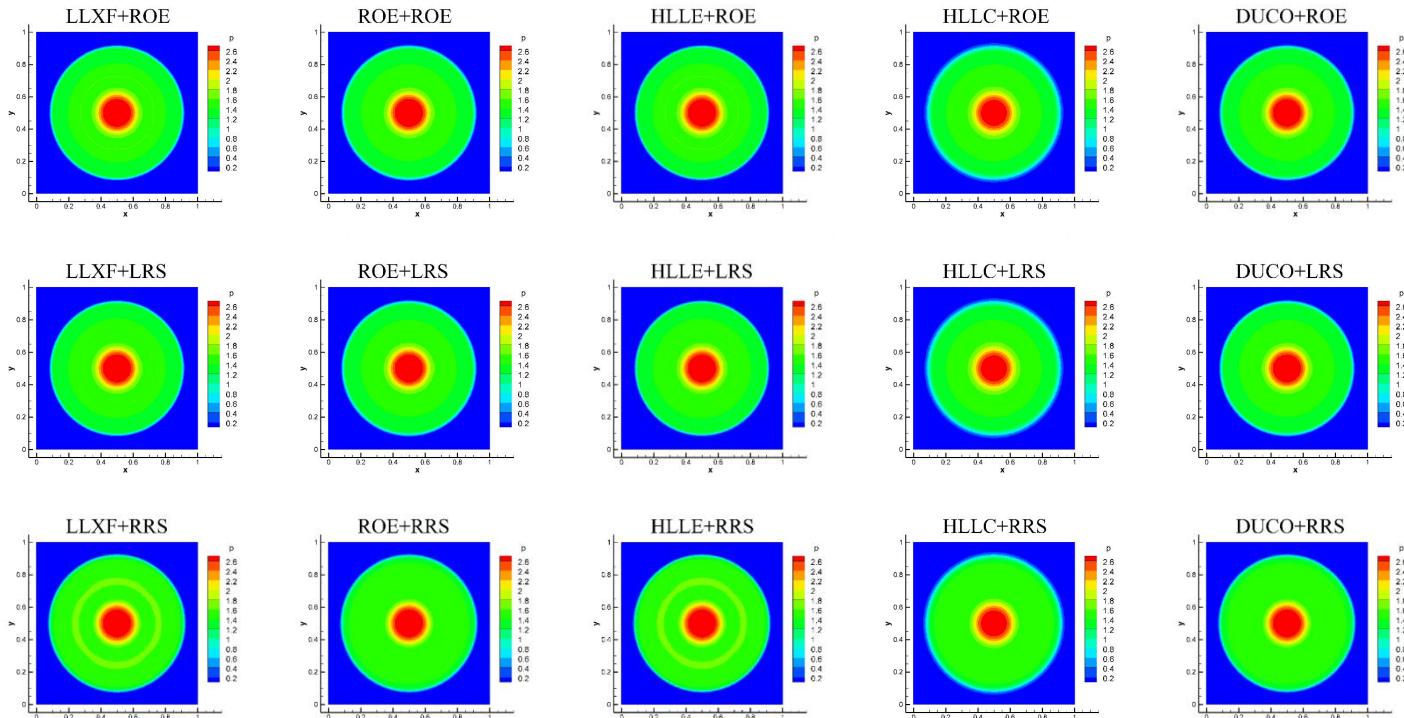
Fluid	$(\rho, u, P)$	EOS
Left	(1.63, 0, 7.81E10)	Tait's EOS
Right	(1, 0, 1E6)	JWL equation



### 3. Numerical results

#### ◆ 2D radially symmetric problem

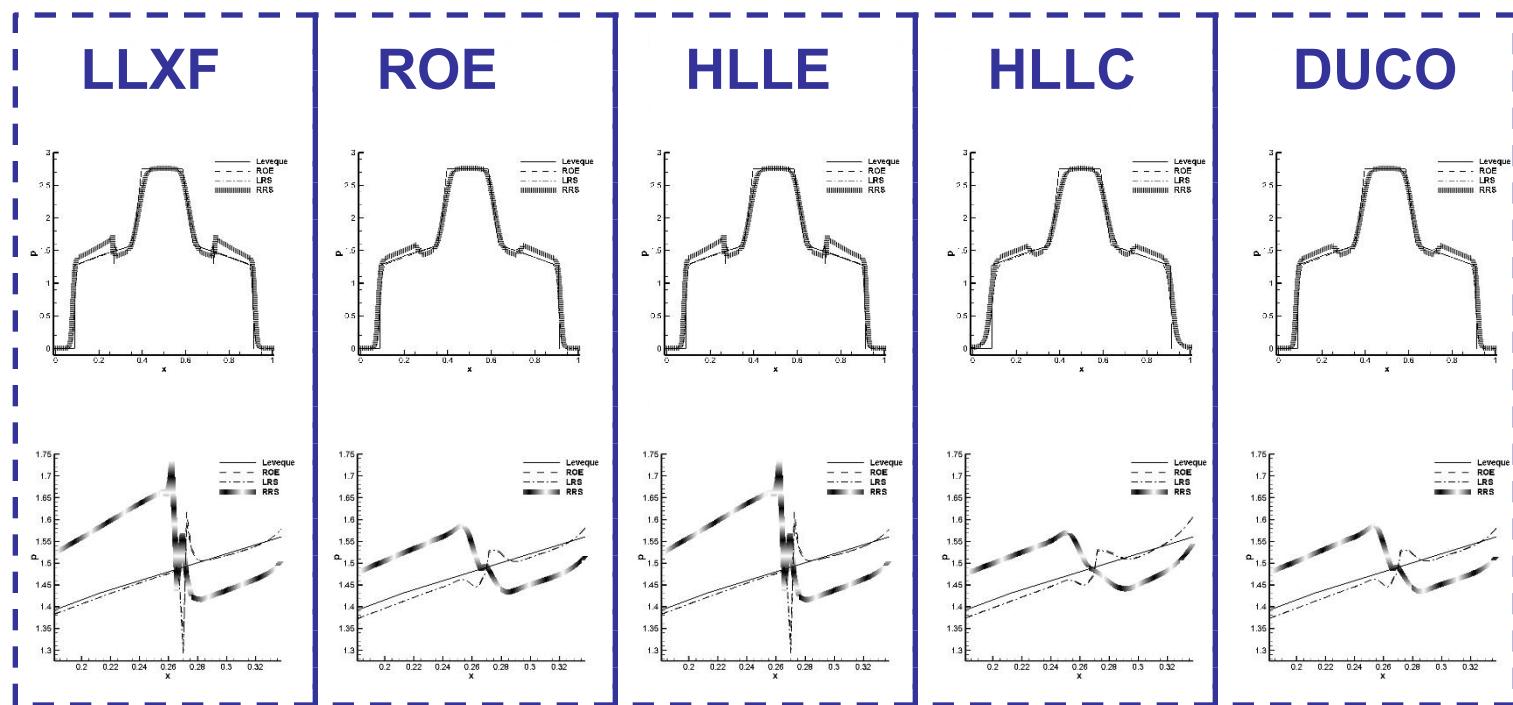
Fluid	$(\rho, u, v, P)$	EOS	Parameters in EOS
Bubble	(1.241, 0, 0, 2.753)	$P=(\gamma-1)\rho e - \gamma P_c$	$\gamma=1.4; P_c=0$
Water	(0.991, 0, 0, 3.059E-4)	$P=(\gamma-1)\rho e - \gamma P_c$	$\gamma=5.5; P_c=1.505$



### 3. Numerical results

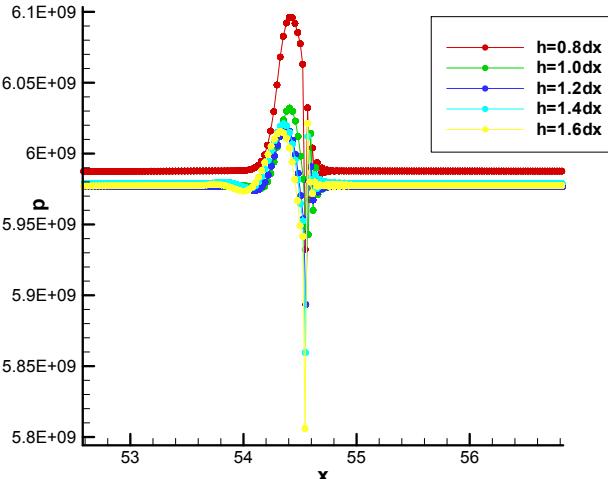
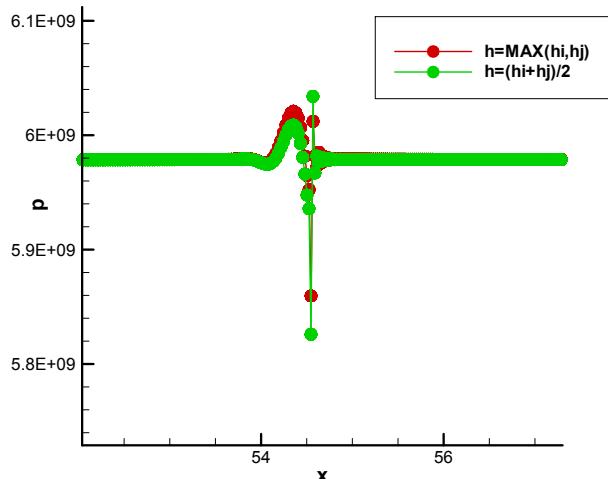
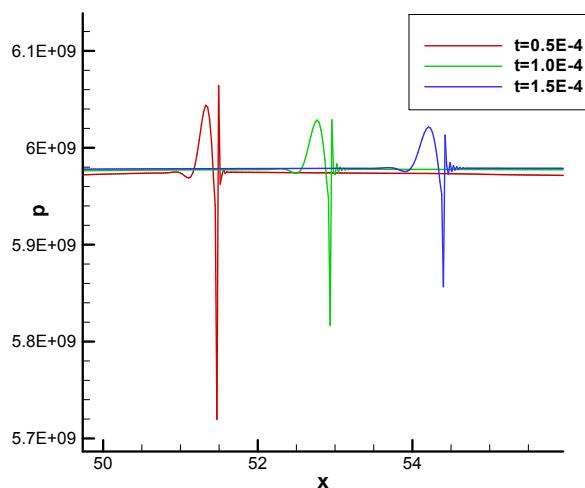
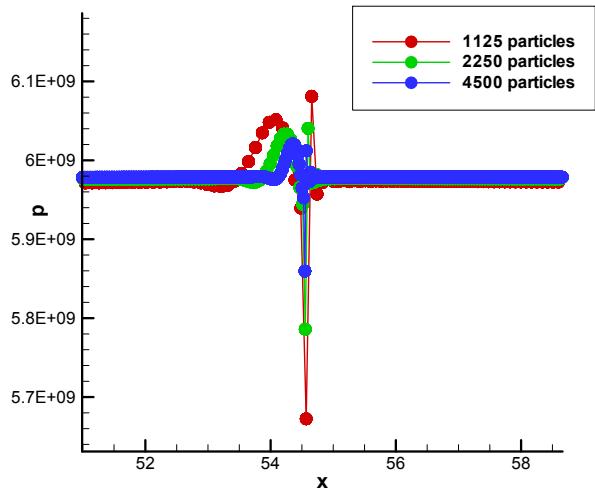
#### ◆ 2D radially symmetric problem

Fluid	$(\rho, u, v, P)$	EOS	Parameters in EOS
Bubble	(1.241, 0, 0, 2.753)	$P=(\gamma-1)\rho e - \gamma P_c$	$\gamma=1.4; P_c=0$
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### 3. Numerical results

#### ◆ Broken features



## 4. Conclusion

### ◆ MGSPH

- MGSPH combined with Godunov-typed SPH model(GSPH), single-phase approximate Riemann solvers and interfacial approximate Riemann solvers is developed for multiphase Riemann problems with multiple Equations of States (EOSs).
- Discontinuous presures is found in the solutions obtained by RRS, which is due to the error introduced by using the relaxed pressure.
- Broken features occur in pressure distribution curves due to the contact discontinuity.

## 4. Conclusions

### ◆ Broken features

- Broken features are sensitive to the approximate Riemann solver, initial particle distance, simulation time and smooth length.
- The broken features of cases with LLXF and HLLE are more severe than other single-phase solvers.
- Smaller initial particle distances lead to less severe broken features.
- Broken features are erased as simulation time goes by.
- The method for computation of smooth length has affection on broken features.
- Smallest broken features occur when  $h=dx$ (fifth-order quintic spline kernel function).

*Thank you  
for your attention !*

