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A SPH investigation of soil plastic behavior with Mohr-Coulomb constitutive model

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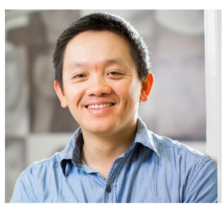
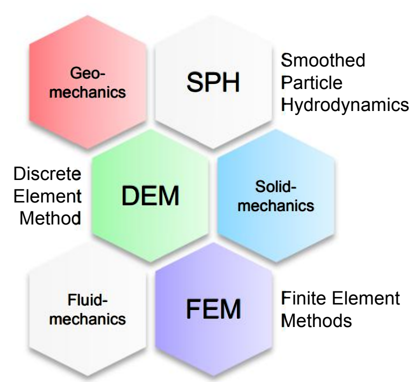


MCG-LAB



GROUP OF EIGHT AUSTRALIA

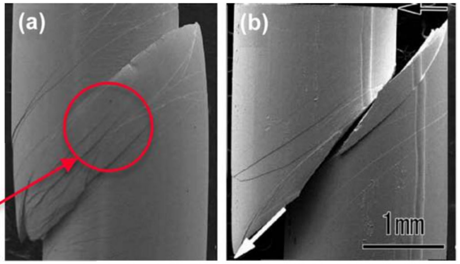
Presentation Content

- Research Background
- Constitutive models in SPH
- A new confining boundary condition
- SPH simulations of soil plastic behaviours

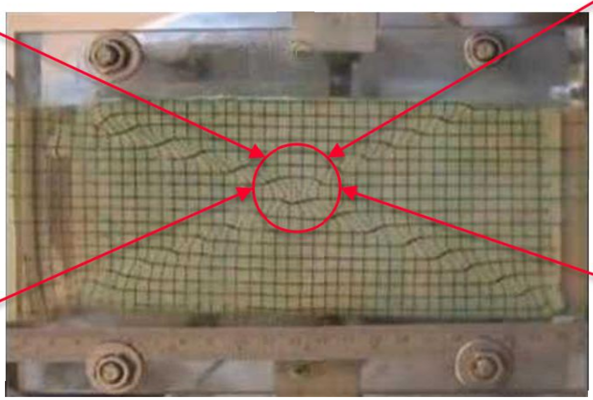
Research Background

Plastic behavior such as strain localisation commonly manifests itself as an area of concentrated deformation in soils. It plays a vital role in civil engineering projects and research since it normally acts as precursor and indicator of ultimate failure of structure materials and relates to significant lost of lives and money.

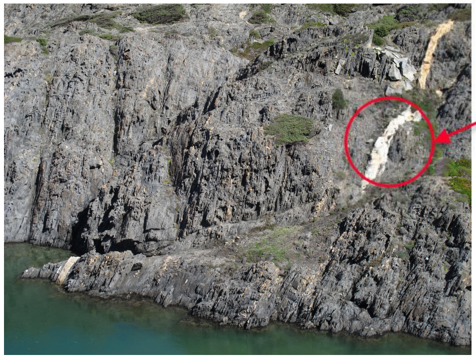
Shear band in dyke rock



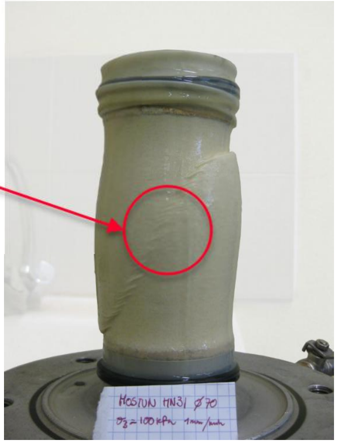
CT image of metallic glass localisation



Strain localisation test with sand



Shear band in geology formation



Clay strain localisation test

Knowledge gap

Mesh-based numerical methods are known for the mesh related pathology including mesh dependency and distortion, which hinder the description of very large deformation of materials with this type of method.

Existing methods

- Mesh-based methods: finite element method
- Meshless methods: discrete element method, Smoothed particle hydrodynamics
- Mesh-based and meshless hybrid methods: material point method

Problems with the existing methods

- Mesh pathology: mesh distortion, mesh dependency
- DEM: computational cost is too high; SPH: boundary condition to apply confinement on sample is needed
- MPM: error in boundary condition to deal with very large deformation and three dimensional space analysis (Steffen, et al. 2008))

Solution

SPH with a new generic confining boundary condition to model soil plastic behaviour

Reference: Steffen, M., Wallstedt, P., Guilkey, J., Kirby, R., & Berzins, M. (2008). Examination and analysis of implementation choices within the material point method (MPM). Computer Modeling in Engineering and Sciences, 31(2), 107-127.

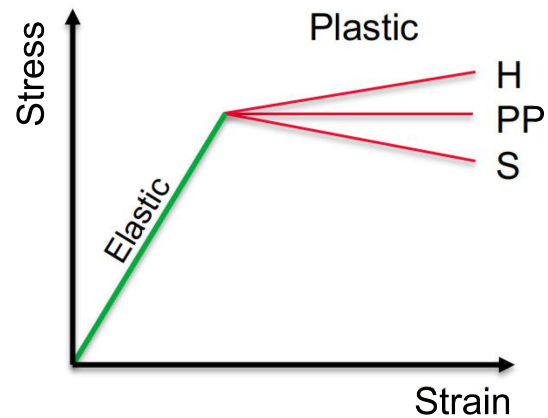
- Like FEM, any constitutive model can be implemented in SPH. In this work, two types of constitutive models (i.e. visco-plastic and elasto-plastic) are tested with SPH:

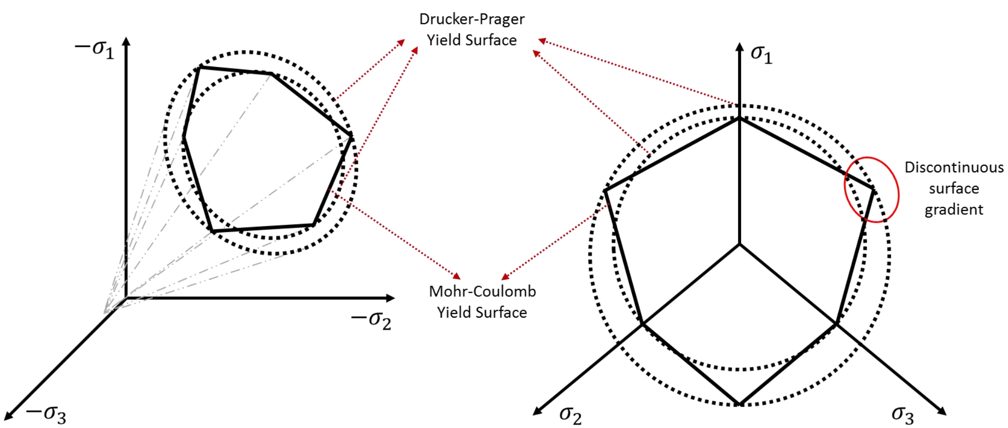
$$\dot{\varepsilon}^{\alpha\beta} = \dot{\varepsilon}_e^{\alpha\beta} + \dot{\varepsilon}_p^{\alpha\beta}$$ Total strain rate tensor

$$\dot{\varepsilon}_e^{\alpha\beta} = \frac{\dot{s}^{\alpha\beta}}{2G} + \frac{1-2\nu}{3E}\dot{\sigma}^{\gamma\gamma}\delta^{\alpha\beta}$$ Hooke's law for elastic part

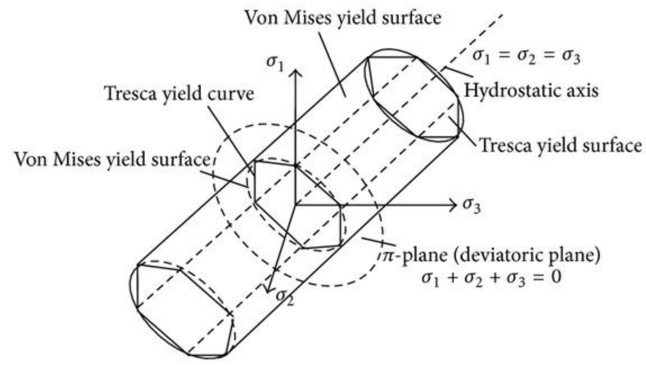
$$\dot{\varepsilon}_p^{\alpha\beta} = \dot{\lambda}\frac{\partial g}{\partial \sigma^{\alpha\beta}}$$ Flow rule for plastic part

$$\dot{\sigma}^{\alpha\beta} = 2G\dot{e}^{\alpha\beta} + K\dot{\varepsilon}^{\gamma\gamma}\delta^{\alpha\beta} - \dot{\lambda}\left[\left(K - \frac{2G}{3}\right)\frac{\partial g}{\partial \sigma^{mn}}\delta^{mn}\delta^{\alpha\beta} + 2G\frac{\partial g}{\partial \sigma^{\alpha\beta}}\right]$$





Mohr-Coulomb and Drucker-Prager yield surface



Von Mises and Tresca yield surface

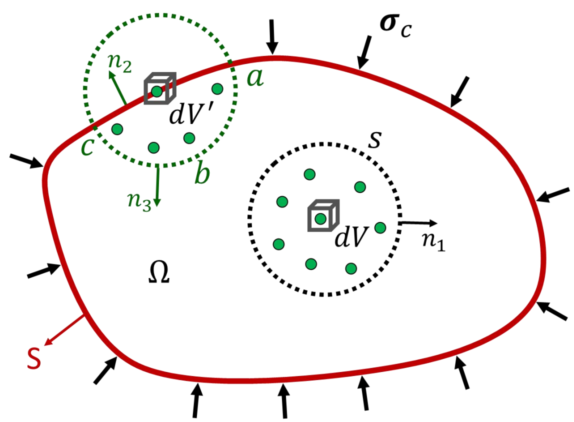
A generic confining boundary condition in SPH

➢ Background for the propose of the new confining boundary condition in SPH.

- The way to impose confining stress currently used in literature is time consuming and unstable during plastic deformation of the domain, to improve this issue, a new confining boundary condition in SPH is proposed.

1. Motion equation:

$$\frac{Dv^{\alpha}}{Dt} = \frac{1}{\rho}\frac{\partial \sigma^{\alpha\beta}}{x^{\beta}} + f^{\alpha}$$



2. Inclusion of confining stress

$$\frac{Dv^{\alpha}}{Dt} = \frac{1}{\rho}\frac{\partial \sigma^{\alpha\beta}}{x^{\beta}} \boxed{+ \frac{1}{\rho}\frac{\partial \sigma^{c}}{x^{\alpha}}\bigg|_{S}} + f^{\alpha}$$

3. SPH approximation of governing equation:

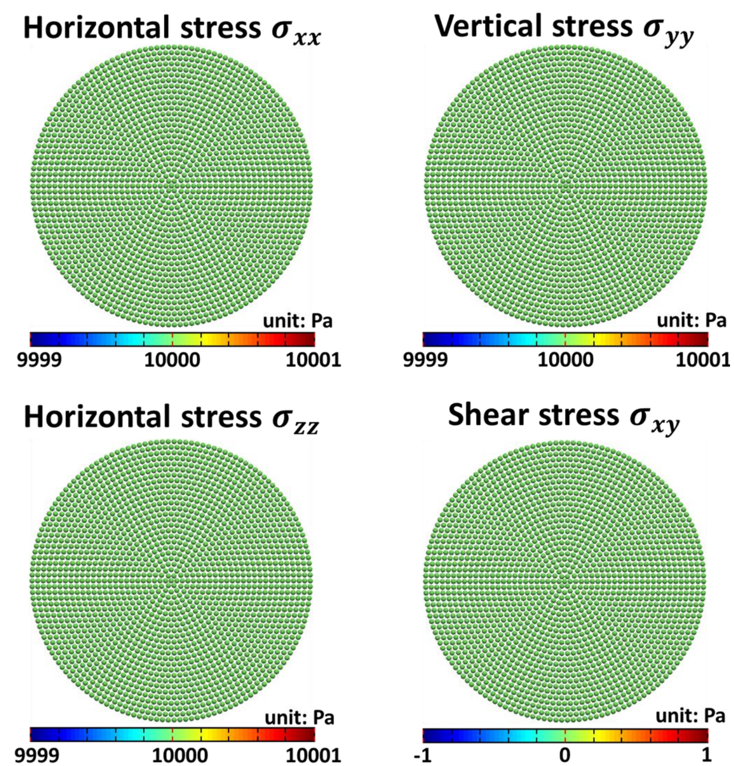
$$\frac{Dv_i^{\alpha}}{Dt} = \sum_{j=1}^{N} m_j \left(\frac{\sigma_i^{\alpha\beta}}{\rho_i^2} + \frac{\sigma_j^{\alpha\beta}}{\rho_j^2}\right)\frac{\partial W_{ij}}{\partial x^{\beta}} + \sum_{j=1}^{N} m_j \left(\frac{\sigma_i^{c}}{\rho_i^2} + \frac{\sigma_j^{c}}{\rho_j^2}\right)\frac{\partial W_{ij}}{\partial x^{\alpha}} + g^{\alpha}$$

Proof:

$$\boxed{\sum_{j=1}^{N} m_j \left(\frac{\sigma_i^{c}}{\rho_i^2} + \frac{\sigma_j^{c}}{\rho_j^2}\right)\frac{\partial W_{ij}}{\partial x^{\alpha}} = \frac{1}{\rho_i}\int_{\Omega} (2\sigma_i^{c})\, d\boldsymbol{W} d\boldsymbol{V}}$$

$$\boxed{-\frac{2\sigma_i^c}{\rho_i}\int_{abc} \boldsymbol{W}\overrightarrow{n_3} ds}$$

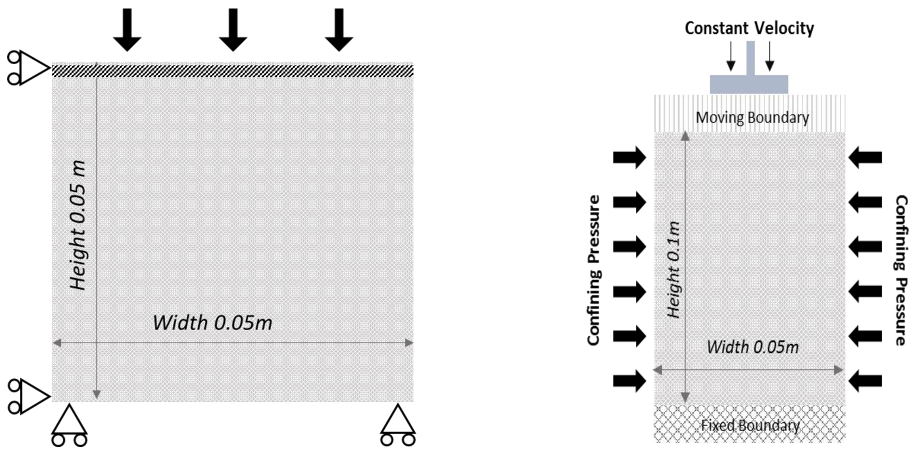
$$\boxed{-\frac{2\sigma_i^c}{\rho_i}\int_{ac} \boldsymbol{W}\overrightarrow{n_2} ds}$$



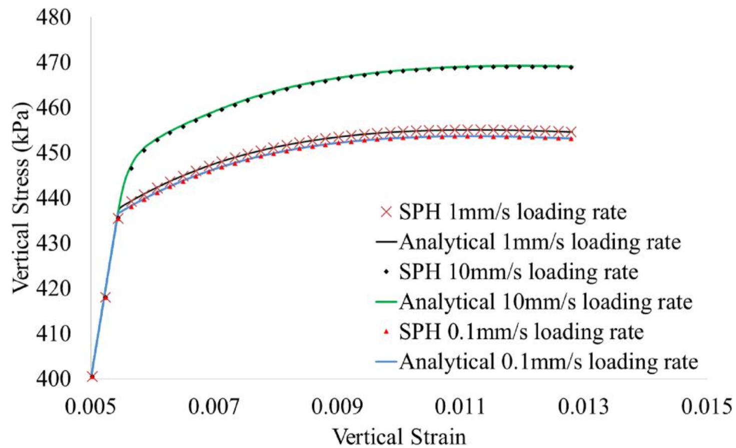
Reference: Bui, H. H., Fukagawa, R., Sako, K. and Wells, J. C. (2011) Slope stability analysis and discontinuous slope failure simulation by elasto-plastic smoothed particle hydrodynamics. Geotechnique, 61, No. 7, 565-574

SPH simulation of strain localisation with viscoplastic model

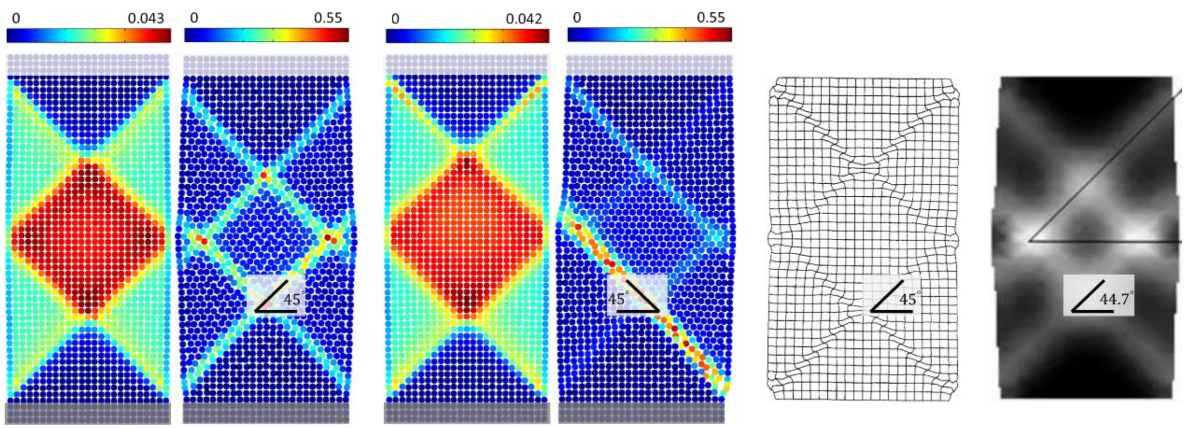
- Biaxial test for Von Mises viscoplastic model



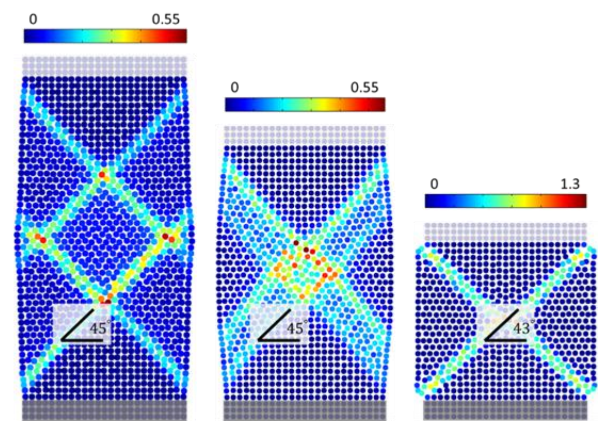
Model set up for viscoplastic SPH tests



Benchmark test for viscoplastic SPH



Biaxial test comparison between SPH and FEM (Fusao et al. 1993, Sayuri et al. 2003)

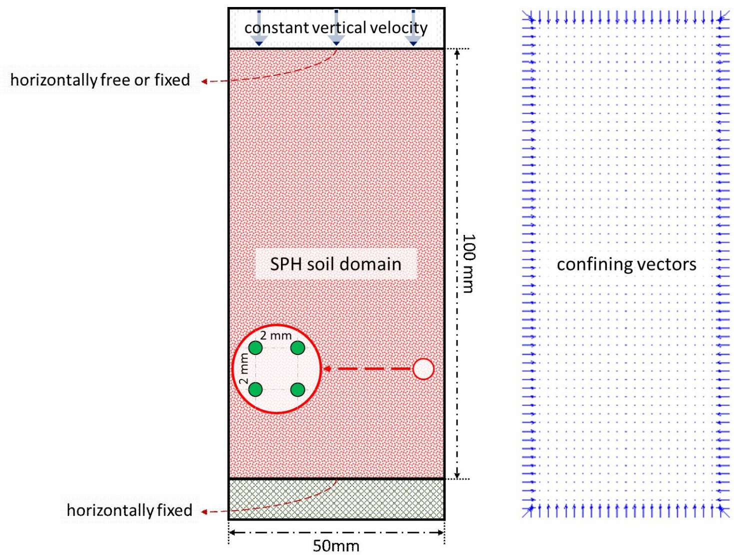


Biaxial test with different aspect ratio

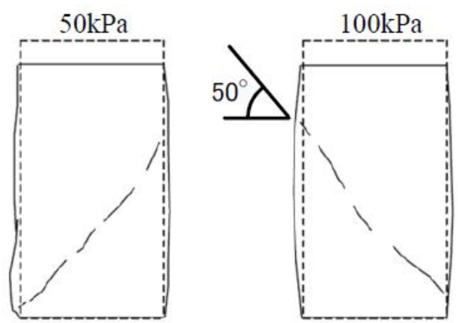
Reference:Fusao, O., Toshihisa, A. and Atsushi, Y. (1993). "Instability of an elasto-viscoplastic constitutive model for clay and strain localization" Mechanics of Materials 18 (1994) 119-129.
Sayuri, K., Fusao, O. and Yosuke, H. (2003). "Strain localization analysis of elastoviscoplastic soil considering structural degradation." Computer Methods in Applied Mechanics and Engineering 193 (2004) 2845-2866.

SPH simulations of biaxial and triaxial test with Mohr-Coulomb model

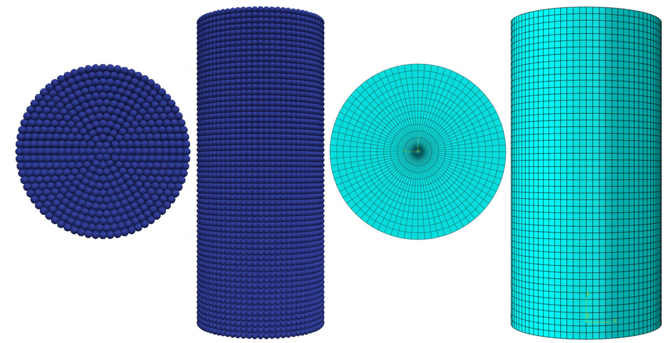
- Biaxial and triaxial tests with Mohr-Coulomb model and stress boundary condition



Simulation setup and confining vector



Experimental results



3D particle and mesh discretisation

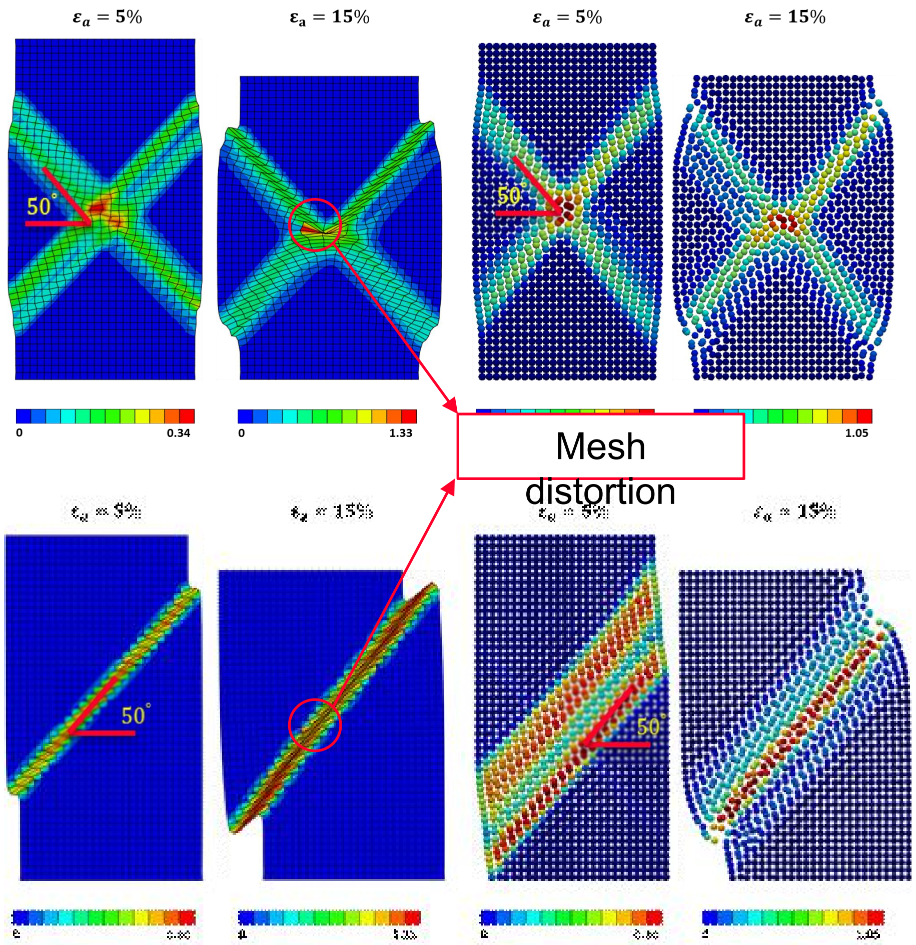
Test No.	Confine-ment (kPa)	Shear modulus (kPa)	Sample density (g/cm³)	Cohesion (kPa)	Internal friction angle (°)
1	50	6401	1.53	8.5	30.5
2	100	8514	1.53		

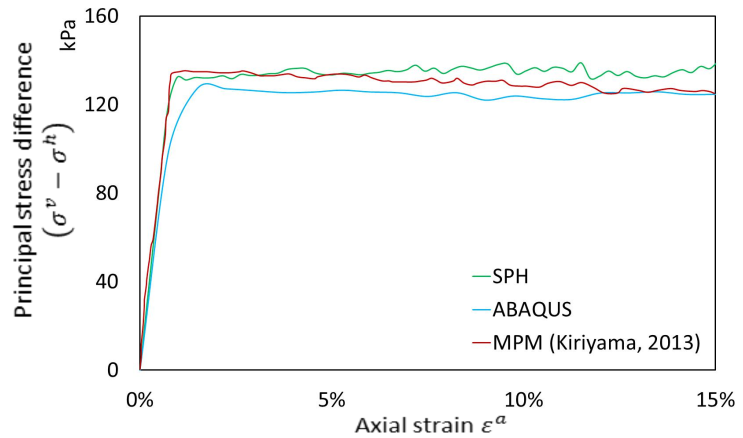
1. Soil sample has an aspect ratio of 2, and the initial particle arrangement in SPH is orthogonal.
2. The second figure in experimental setup shows the plot of confining vectors
3. Two tests with both horizontally fixed and free top boundary are conducted for three different confining stresses.
4. The experimental results shows the inclination angle of shear band is around 50°.

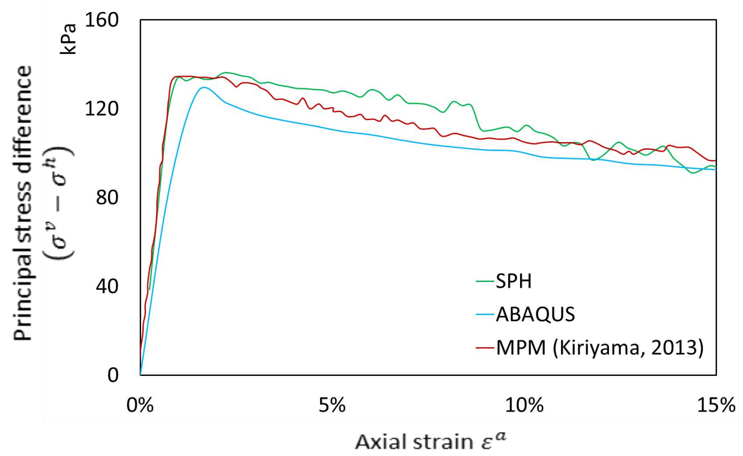
Biaxial test results

- Biaxial test with Mohr-Coulomb model and stress boundary condition

$50\ kPa$



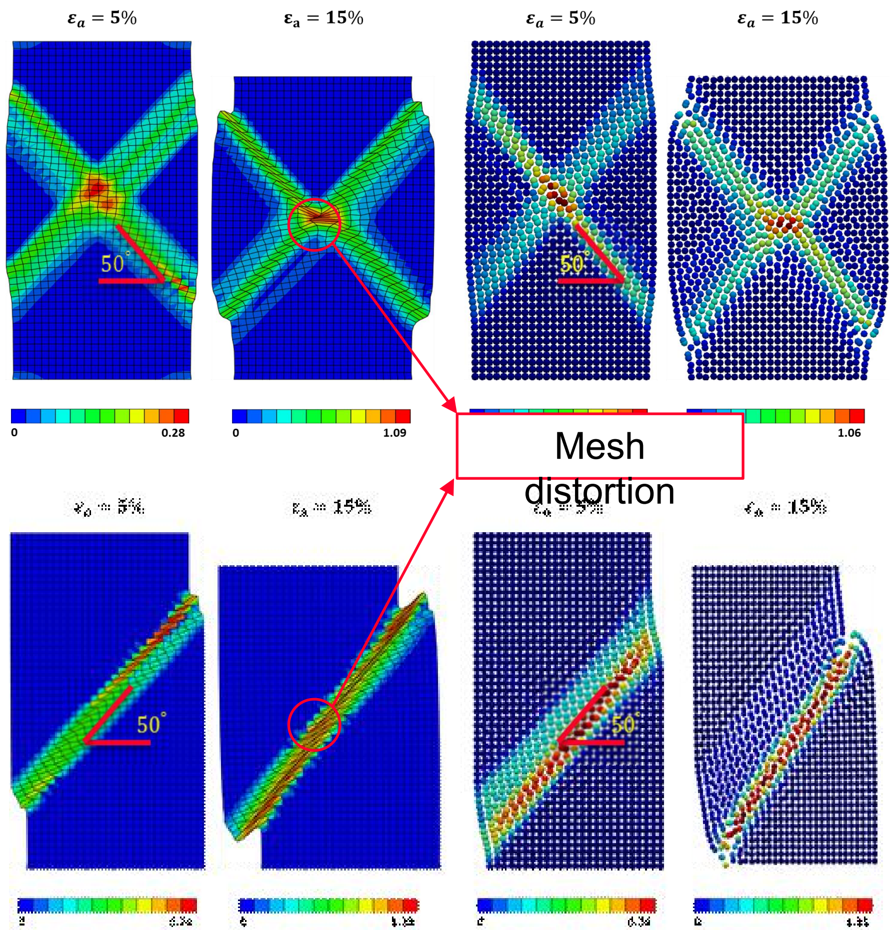


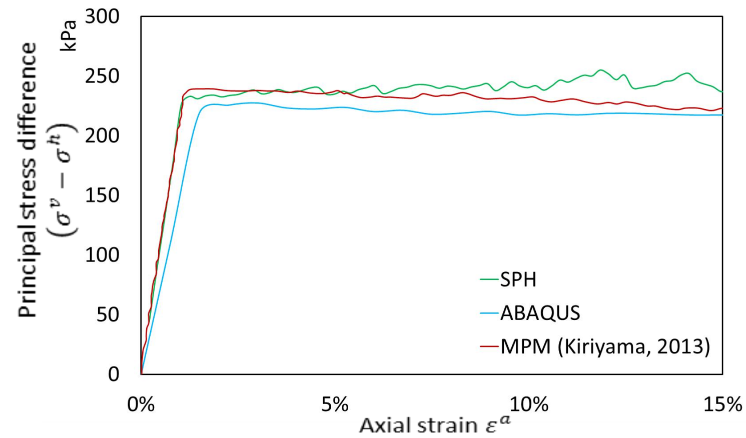


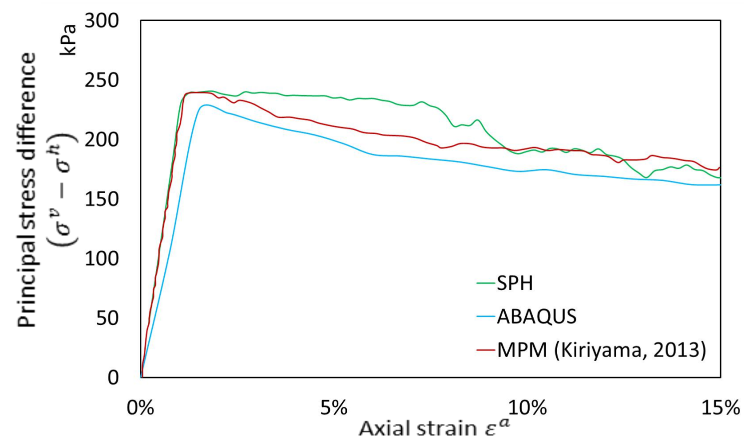
Reference: Kiriyama, T. (2013). "Numerical Simulations of Progressive Failure of Triaxial Compression Tests Using Generalized Interpolation Material Point Method (Japanese Title: GIMPM を用いた三軸圧縮試験の破壊シミュレーション)." Journal of Japan Society of Civil Engineers, Ser. A2 (Applied Mechanics (AM)) **69**.

Biaxial test results

- Biaxial test with Mohr-Coulomb model and stress boundary condition $100\ kPa$



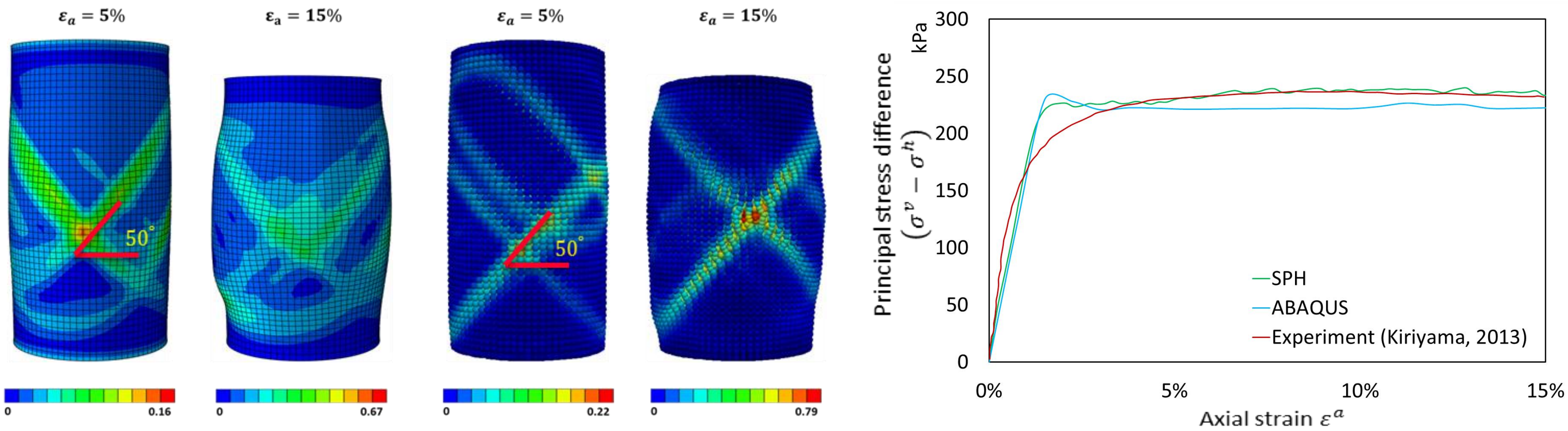


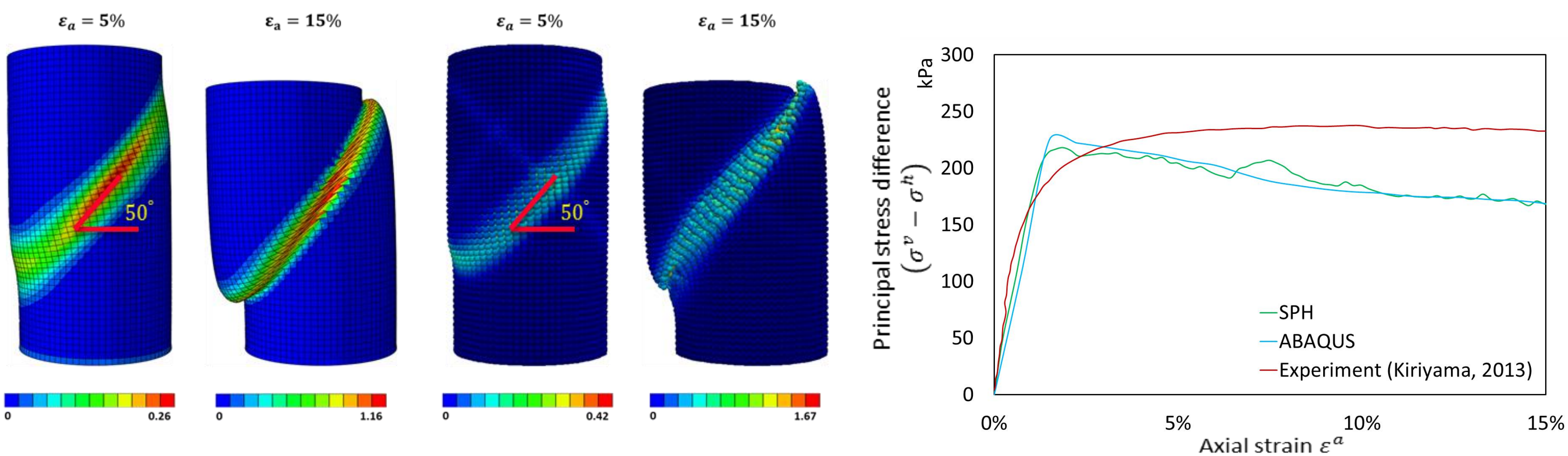


Reference: Kiriyama, T. (2013). "Numerical Simulations of Progressive Failure of Triaxial Compression Tests Using Generalized Interpolation Material Point Method (Japanese Title: GIMPM を用いた三軸圧縮試験の破壊シミュレーション)." Journal of Japan Society of Civil Engineers, Ser. A2 (Applied Mechanics (AM)) **69**.

Triaxial test results

- Triaxial test with Mohr-Coulomb model and stress boundary condition $100\ kPa$





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Conclusion

1. A generic boundary condition to apply confining stress on SPH domain is proposed and validated to be effective and accurate.

2. Von Mises viscoplastic model performs well in SPH, and is able to capture strain localisation. However, Von Mises yield surface oversimplifies soil behaviour and cannot predict shear band inclination angle correctly.

3. SPH with Mohr-Coulomb elastoplastic model is able to capture soil plastic behaviour and yield good results compared with other numerical methods such as FEM and MPM in available literature. The constitutive relation predicted by SPH also agrees well with experimental data

Acknowledgement

- Special thanks to:

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CSIRO

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- Any questions?