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Implement of the MPS-FEM Coupled Method for 3-D FSI Problem

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- Validation of the coupled method
  - ✓Dam-break flow in rigid tank
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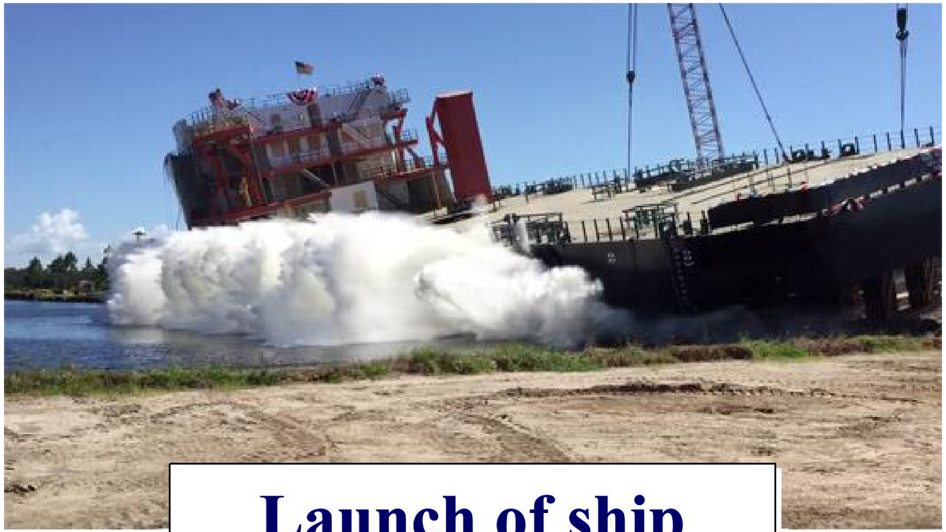
  - ✓Shape function based data transmission technique
- Application of MPS-FEM coupled method
- Conclusions

Background

Ship in wave

Platform in storm

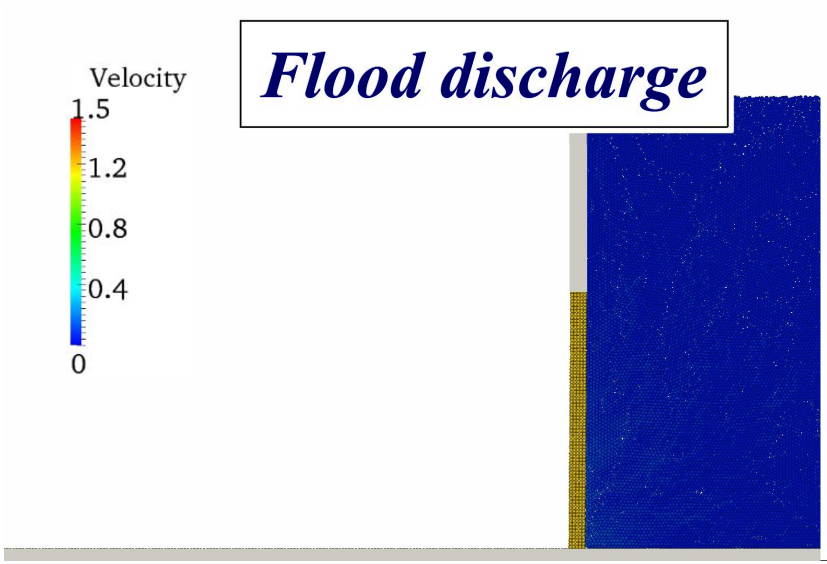


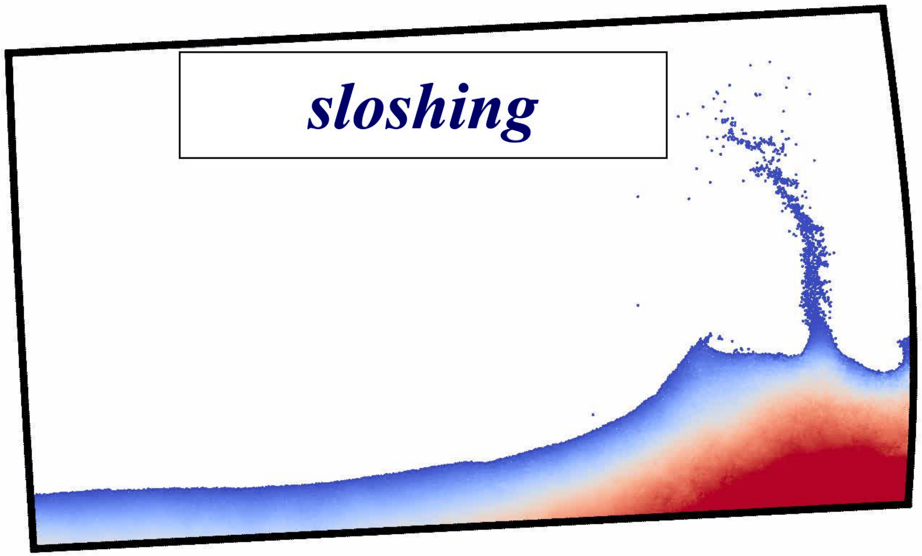
Launch of ship

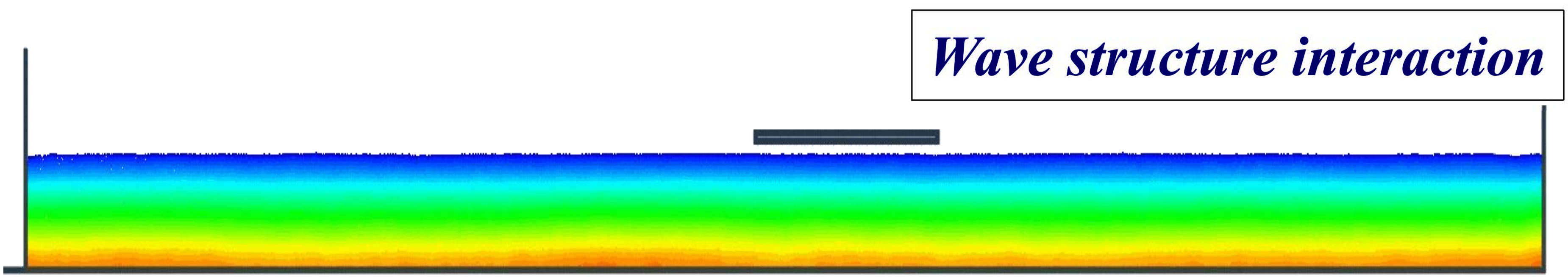
Trestle bridge in tsunami

Background

- Fully Lagrangian particle approach



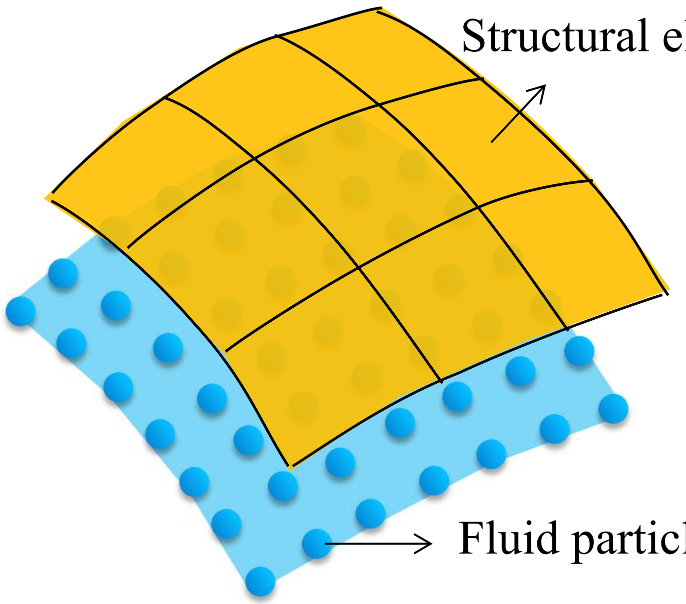




2-D FSI problems simulated by *MPSFEM-SJTU*

Background

Challenge for 3-D FSI simulations



<u>How to implement the data transmission between the two boundaries?</u>

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Formulas of MPS and FEM methods

Fluid domain

Fluid governing equations

$$\nabla \cdot \boldsymbol{V} = 0 \qquad \frac{D\boldsymbol{V}}{Dt} = -\frac{1}{\rho}\nabla P + \nu\nabla^2\boldsymbol{V} + \boldsymbol{g}$$

Kernel function

$$W(r) = \begin{cases} \dfrac{r_e}{0.85r + 0.15r_e} - 1 & 0 \le r < r_e \\ 0 & r_e \le r \end{cases}$$

Pressure Gradient

$$\langle \nabla P \rangle_i = \frac{D}{n^0}\sum_{j \ne i}\frac{P_j + P_i}{|\boldsymbol{r}_j - \boldsymbol{r}_i|^2}(\boldsymbol{r}_j - \boldsymbol{r}_i)\cdot W(|\boldsymbol{r}_j - \boldsymbol{r}_i|)$$

Source term

$$\langle \nabla^2 P^{n+1} \rangle_i = (1-\gamma)\frac{\rho}{\Delta t}\nabla \cdot \boldsymbol{V}^* - \gamma\frac{\rho}{\Delta t^2}\frac{\langle n^* \rangle_i - n^0}{n^0}$$

Free surface detection

$$\langle \boldsymbol{F} \rangle_i = \frac{D}{n^0}\sum_{j \ne i}\frac{1}{|\boldsymbol{r}_i - \boldsymbol{r}_j|}(\boldsymbol{r}_i - \boldsymbol{r}_j)W(\boldsymbol{r}_{ij})$$

Structure domain

Structure governing equations

$$\Pi = \frac{1}{2}\int_V \boldsymbol{\varepsilon}^T\boldsymbol{\sigma}\,dv - \int_V \boldsymbol{u}^T\boldsymbol{f}\,dv - \int_{S_\sigma}\boldsymbol{u}^T\bar{\boldsymbol{f}}\,ds$$

$$\delta\Pi = 0$$

Structural dynamic equation

$$\boldsymbol{M}\ddot{\boldsymbol{y}} + \boldsymbol{C}\dot{\boldsymbol{y}} + \boldsymbol{K}\boldsymbol{y} = \boldsymbol{F}(t)$$

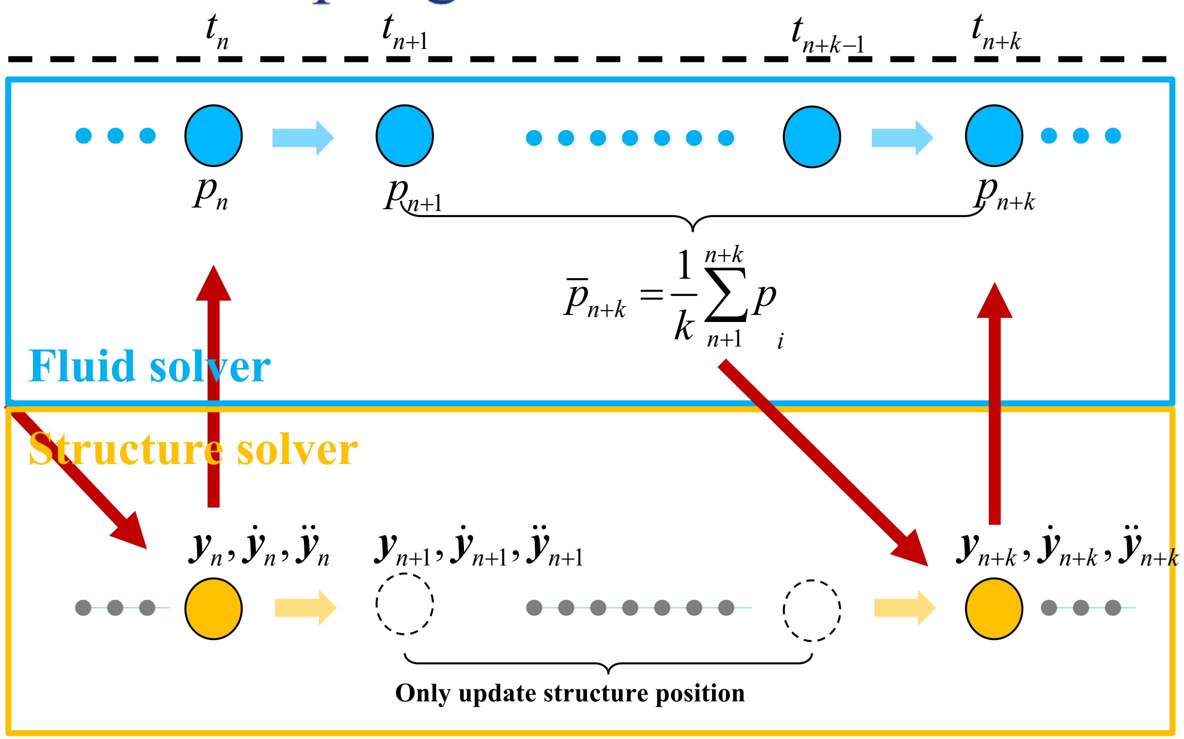
Newmark-beta method

$$\dot{\boldsymbol{y}}_{t+\Delta t} = \dot{\boldsymbol{y}}_t + (1-\gamma)\ddot{\boldsymbol{y}}_t\Delta t + \gamma\ddot{\boldsymbol{y}}_{t+\Delta t}\Delta t$$

$$\boldsymbol{y}_{t+\Delta t} = \boldsymbol{y}_t + \dot{\boldsymbol{y}}_t\Delta t + \frac{1-2\beta}{2}\ddot{\boldsymbol{y}}_t\Delta t^2 + \beta\ddot{\boldsymbol{y}}_{t+\Delta t}\Delta t^2$$

Coupling strategy

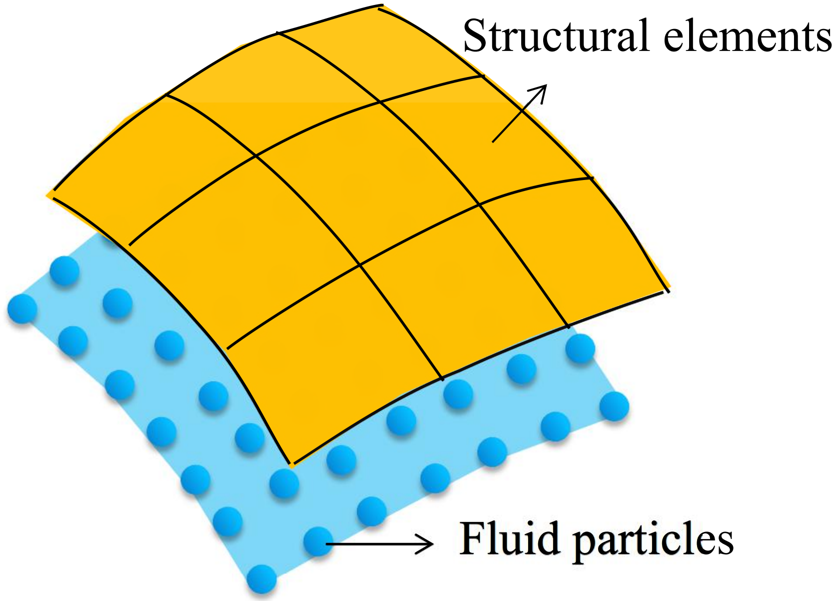
- Implement the coupling between the two domains



- Partitioned coupling strategy:
  - ✓ the fluid and structure fields are self-governed by different equations.
  - ✓ separated fluid and structure codes are used for each computational domain.
  - ✓ different time step sizes can be used for fluid & structure analysis ($t_s = k \times t_f$).

Data transmission on F-S interface

- Data transmission between the F-S boundaries



Different types and sizes of discretization for fluid and structure boundaries.

Interpolation of quantities is necessary.

Boundary conditions on the F-S interface:

- ✓ displacement compatibility, $\mathbf{d}_f = \mathbf{d}_s$
- ✓ traction equilibrium, $\mathbf{f}_f = \mathbf{f}_s$

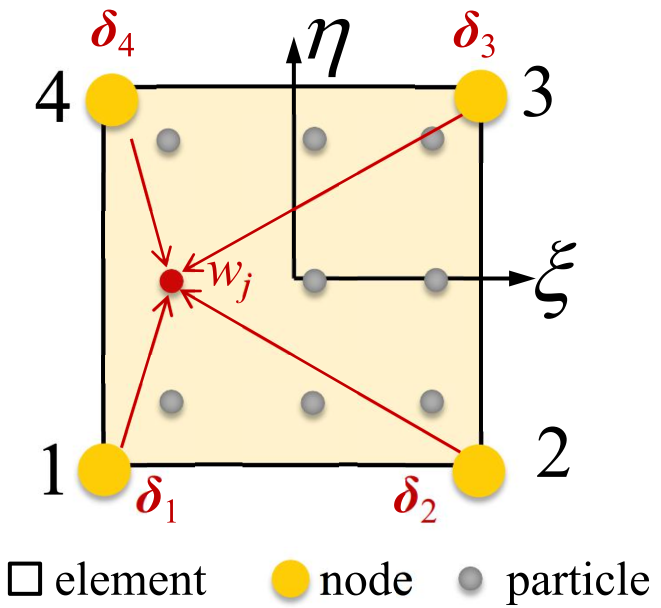
Shape function based interpolation technique

Kernel function based interpolation technique

Data transmission on F-S interface

Shape function based interpolation technique

- Displacement interpolation on the F-S interface:



$$w = N^e \delta^e$$ Calc particle displacement

$$\delta^e = \left[\delta^1\ \delta^2\ \delta^3\ \delta^4\right]$$ Nodal displacement

$$= \left[w_1\ \theta_{x1}\ \theta_{y1}\ w_2\ \theta_{x2}\ \theta_{y2}\ w_3\ \theta_{x3}\ \theta_{y3}\ w_4\ \theta_{x4}\ \theta_{y4}\right]^{\mathrm{T}}$$

$$N^e = \left[N_i\ N_j\ N_m\ N_n\right]$$ Shape function

$$= \left[N_i\ N_{xi}\ N_{yi}\ N_j\ N_{xj}\ N_{yj}\ N_m\ N_{xm}\ N_{ym}\ N_n\ N_{xn}\ N_{yn}\right]$$

$$N_i = \frac{1}{8}(1+\xi_i\xi)(1+\eta_i\eta)(2+\xi_i\xi+\eta_i\eta-\xi^2-\eta^2)$$

$$N_{xi} = -\frac{1}{8}b\eta_i\,(1+\xi_i\xi)(1+\eta_i\eta)(1-\eta^2)$$

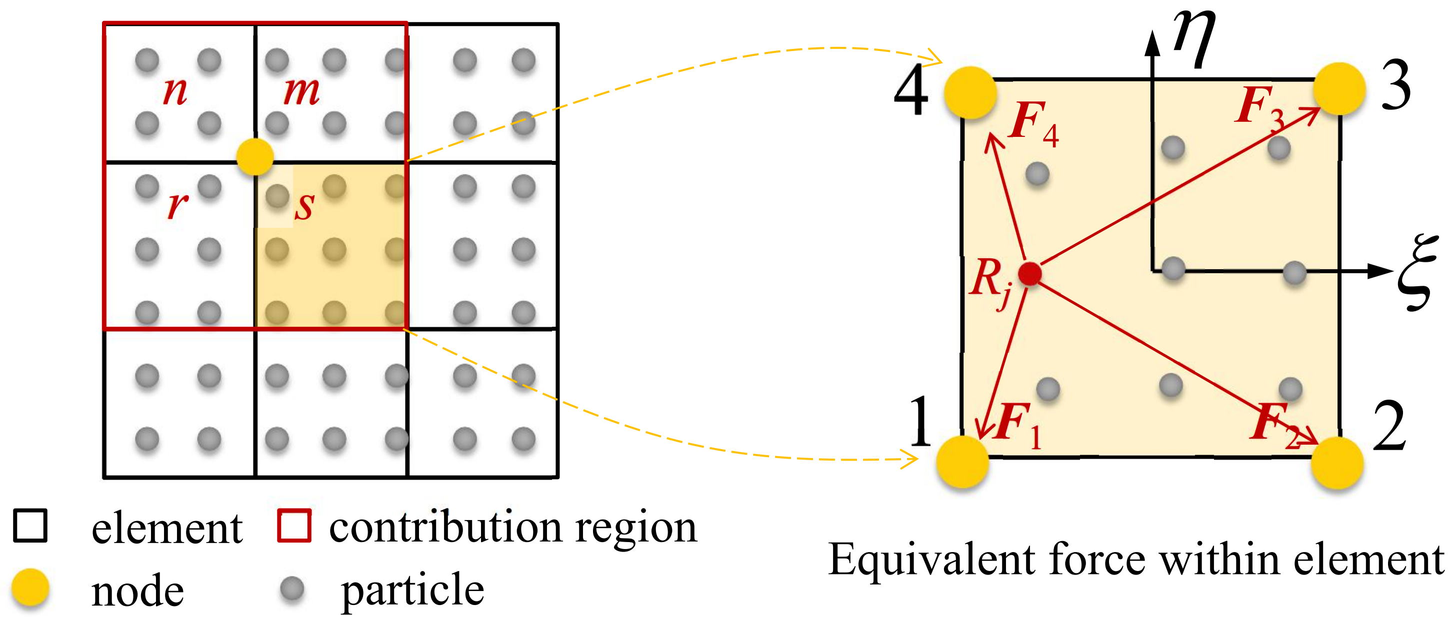
$$N_{yi} = \frac{1}{8}a\xi_i\,(1+\xi_i\xi)(1+\eta_i\eta)(1-\xi^2)$$

$$\xi = \frac{x}{a}, \xi_i = \frac{x_i}{a}, \eta = \frac{y}{b}, \eta_i = \frac{y_i}{b}$$

Data transmission on F-S interface

Shape function based interpolation technique

- Force interpolation on the F-S interface:



Equivalent force within element *s*

Total nodal force

$$\boldsymbol{F} = \boldsymbol{F}_r + \boldsymbol{F}_s + \boldsymbol{F}_m + \boldsymbol{F}_n$$

Force component within element *s*

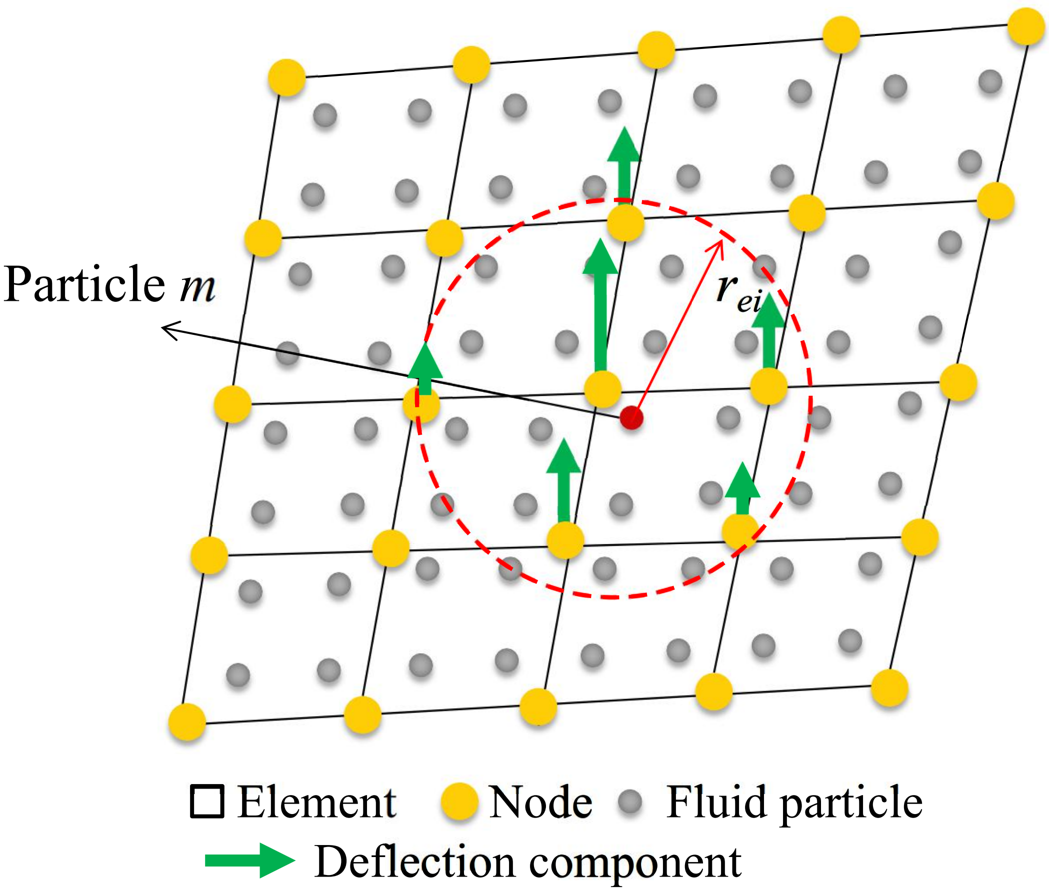
$$\boldsymbol{F}_s = \boldsymbol{N}^{\mathrm{T}} R_j \ (j = 1, 2 \cdots n)$$

Force subcomponent of particle *j* $\quad R_j = P_j \cdot l_0^2$

Data transmission on F-S interface

Kernel function based interpolation technique

- Displacement interpolation on the F-S interface:



Calculate the nodal displacements by FEM

↓

Search neighbor nodes of particle m

↓

Calculate particle's $\mathbf{d}_f$ based on $\mathbf{d}_s$ of neighbor nodes

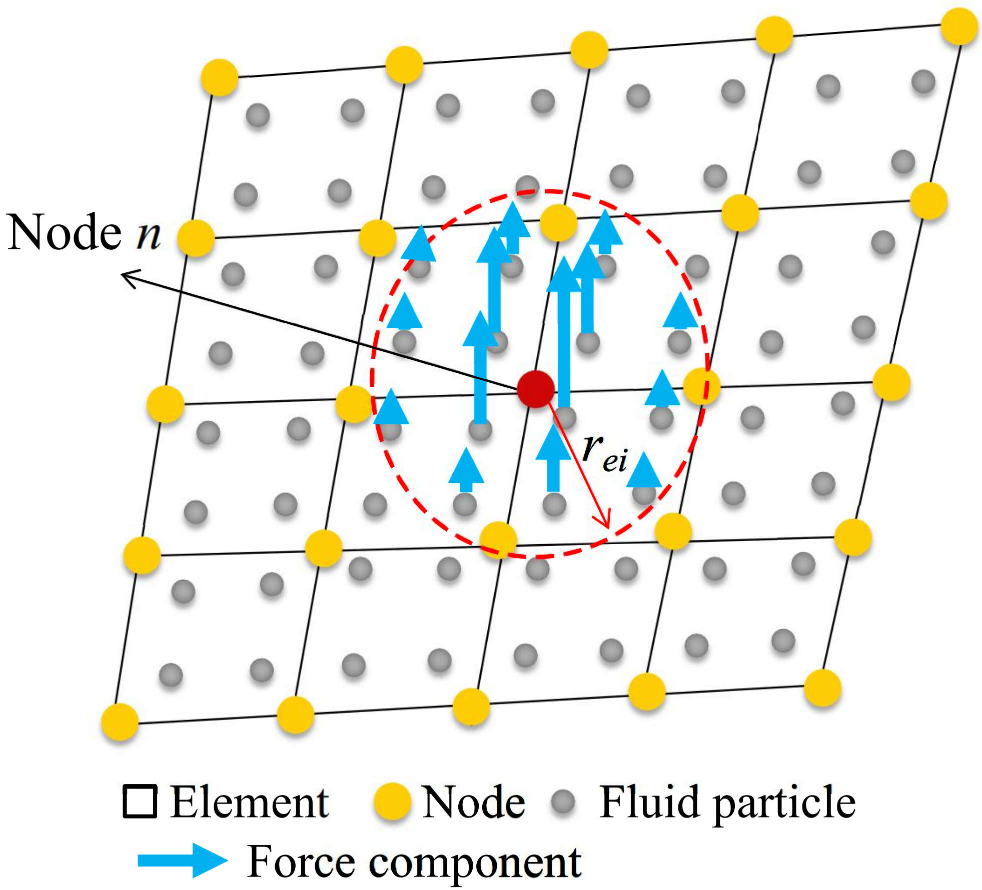
$$w_m = \frac{\sum_i \delta_i \cdot W(|\boldsymbol{r}_i - \boldsymbol{r}_m|)}{\sum_i W(|\boldsymbol{r}_i - \boldsymbol{r}_m|)}$$

$$W(r) = \begin{cases} \dfrac{r_e}{0.85r + 0.15r_e} - 1 & 0 \le r < r_e \\ 0 & r_e \le r \end{cases}$$

Data transmission on F-S interface

Kernel function based interpolation technique

- Force interpolation on the F-S interface:



Calculate the particle's pressure by MPS

↓

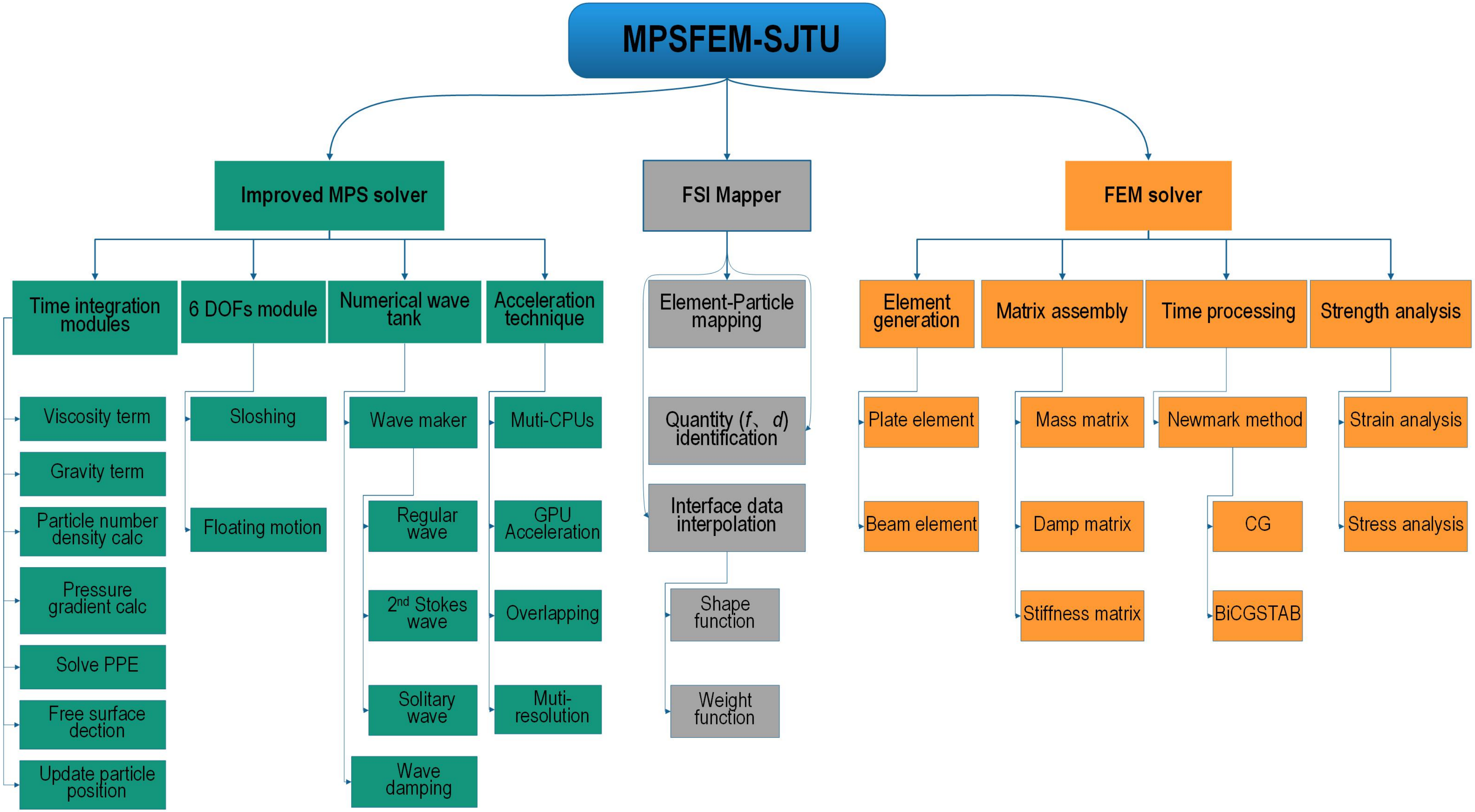
Search neighbor nodes of particle m

↓

Calculate nodal $\mathbf{f}_s$ based on $\mathbf{f}_f$ of neighbor particles

$$F_n = \frac{\sum_i P_i \cdot l_0^2 \cdot W(|\boldsymbol{r}_i - \boldsymbol{r}_n|)}{\sum_i W(|\boldsymbol{r}_i - \boldsymbol{r}_n|)}$$

FSI solver- Framework



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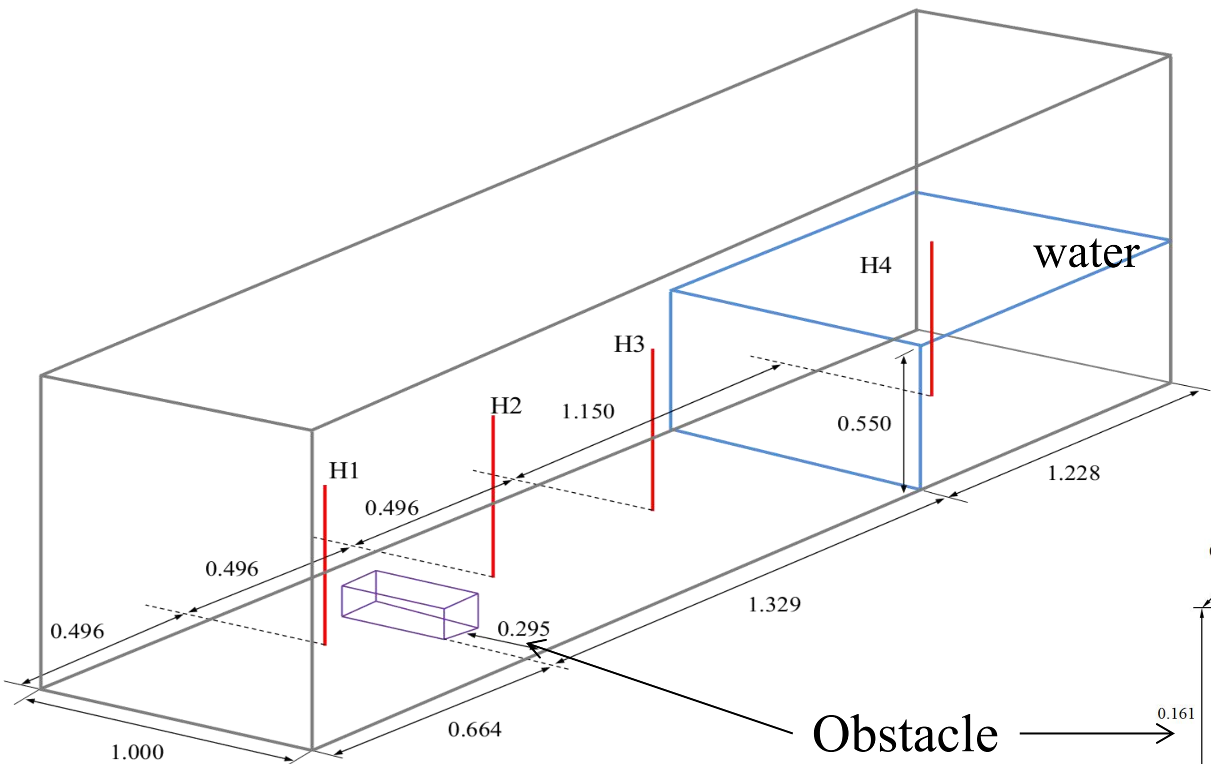
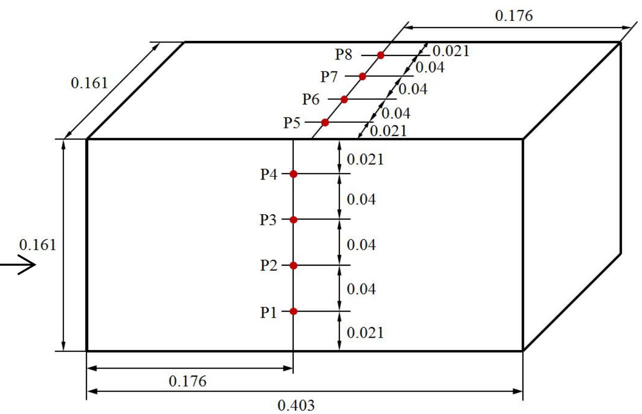
Validation of solver-MPS

- MPS method for dam-break flow in rigid tank



Particle size

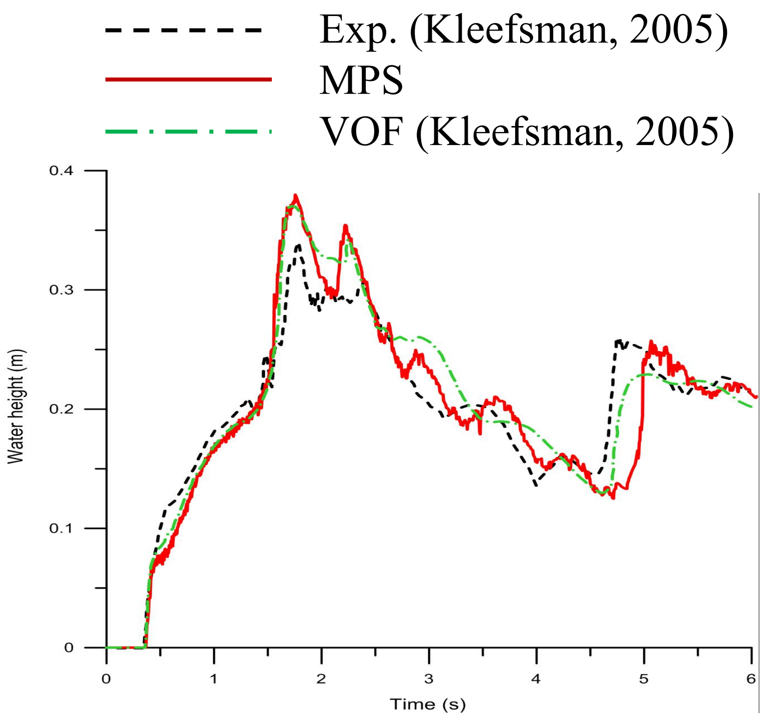
N_{total}	1.01 million
N_{fluid}	0.67 million
N_{wall}	0.33 million
l_0	0.01 m



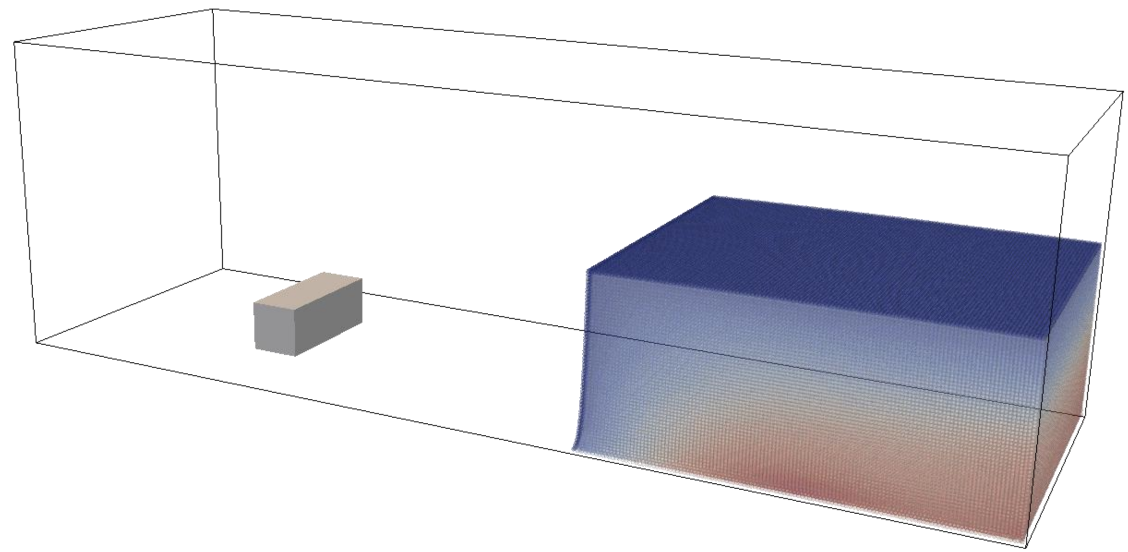
Schematic diagram of 3D tank

Validation of solver-MPS

MPS method for dam-break flow in rigid tank



Comparison of wave height (H2)



Animation of dam break

Zhang Yuxin. Improved MPS Method and Its 3D Parallel Computation. Shanghai: Shanghai Jiao Tong University, 2014.

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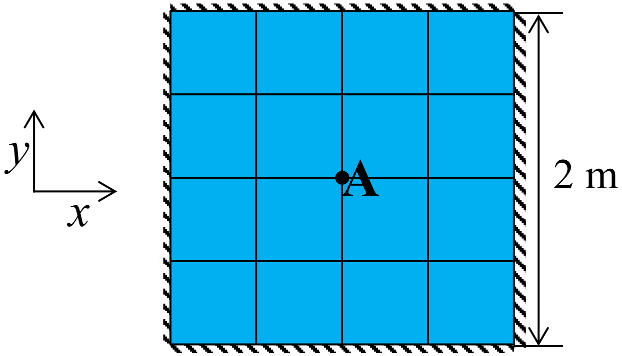
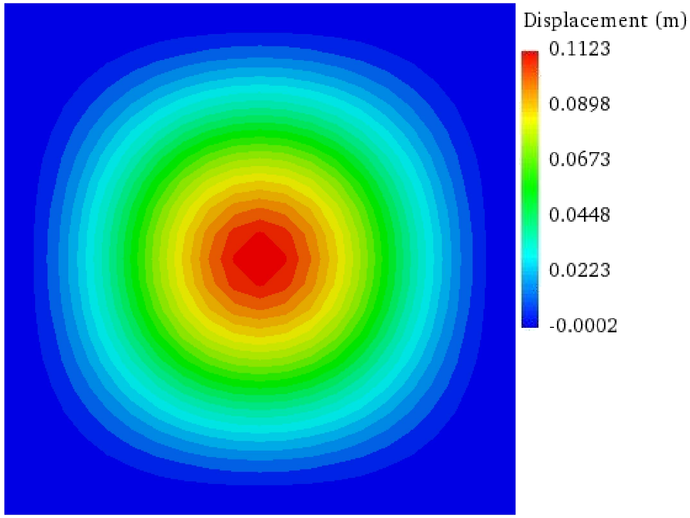
Validation of solver-FEM

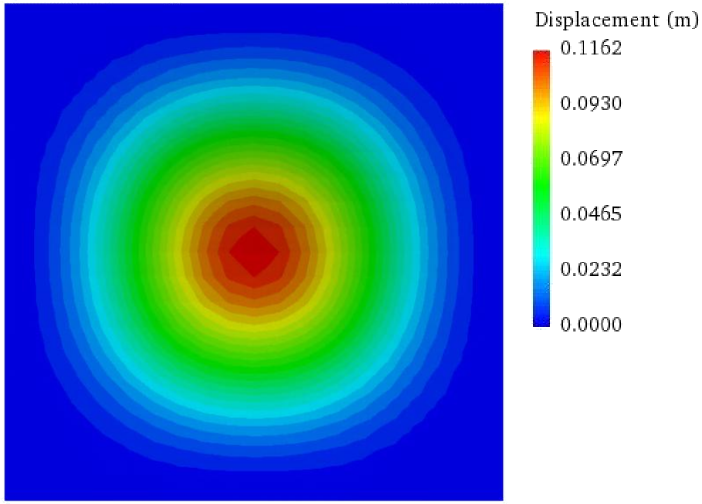
FEM method for structural response



Elements: 20×20 **Thickness**: 1 mm
Density: 1800kg/m^3 **Stiffness**: E = 40 GPa
Poisson's ratio: 0.3
Constraint: boundary nodes fixed with six DoFs
Load: force applied at the point A $f(z, t) = 10$ N



(a) Calculated by ANSYS

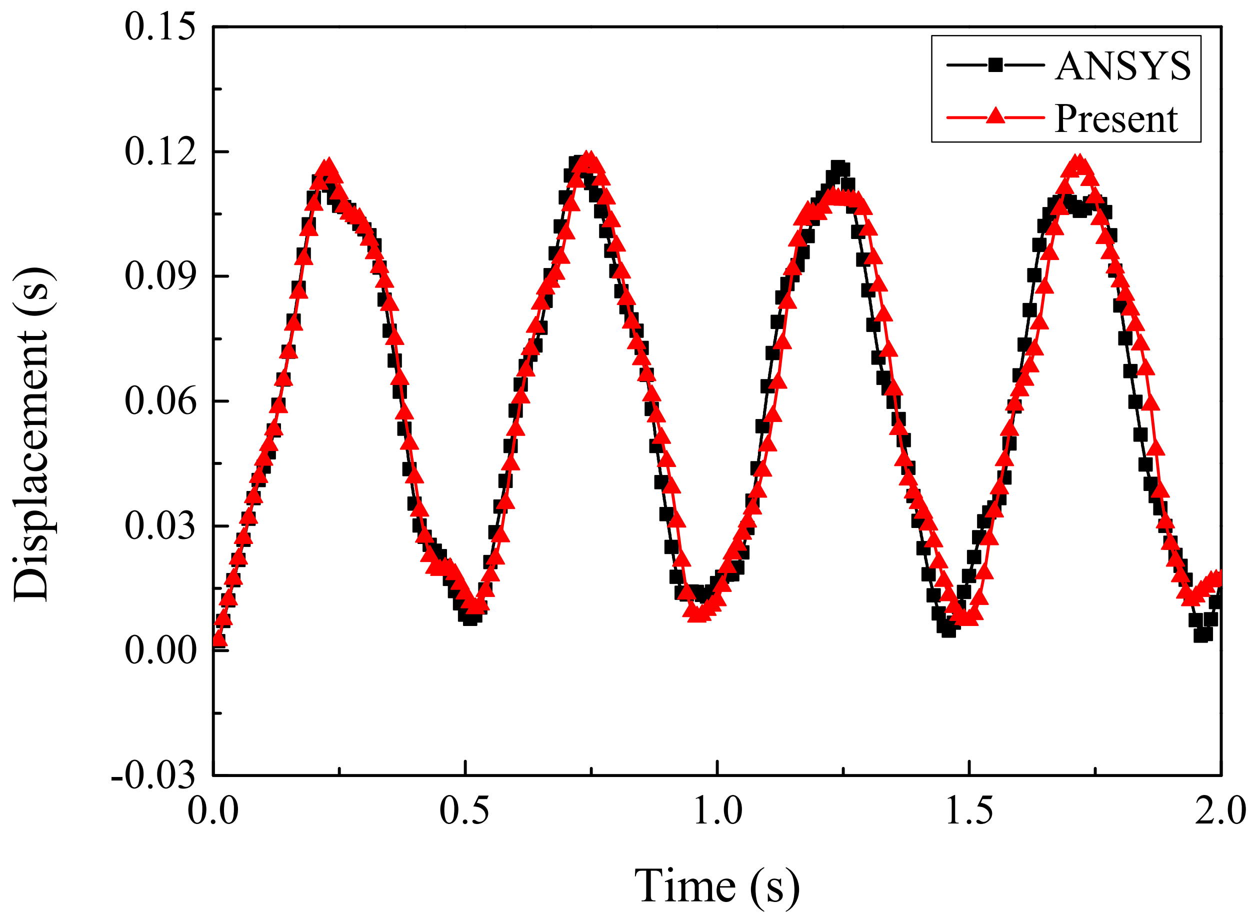


(b) Present solver

Contours of Structural deformation at $t = 0.75$ s

Validation of solver-FEM

- FEM method for structural response



Comparison of time histories of structure vibrations at the measuring point A

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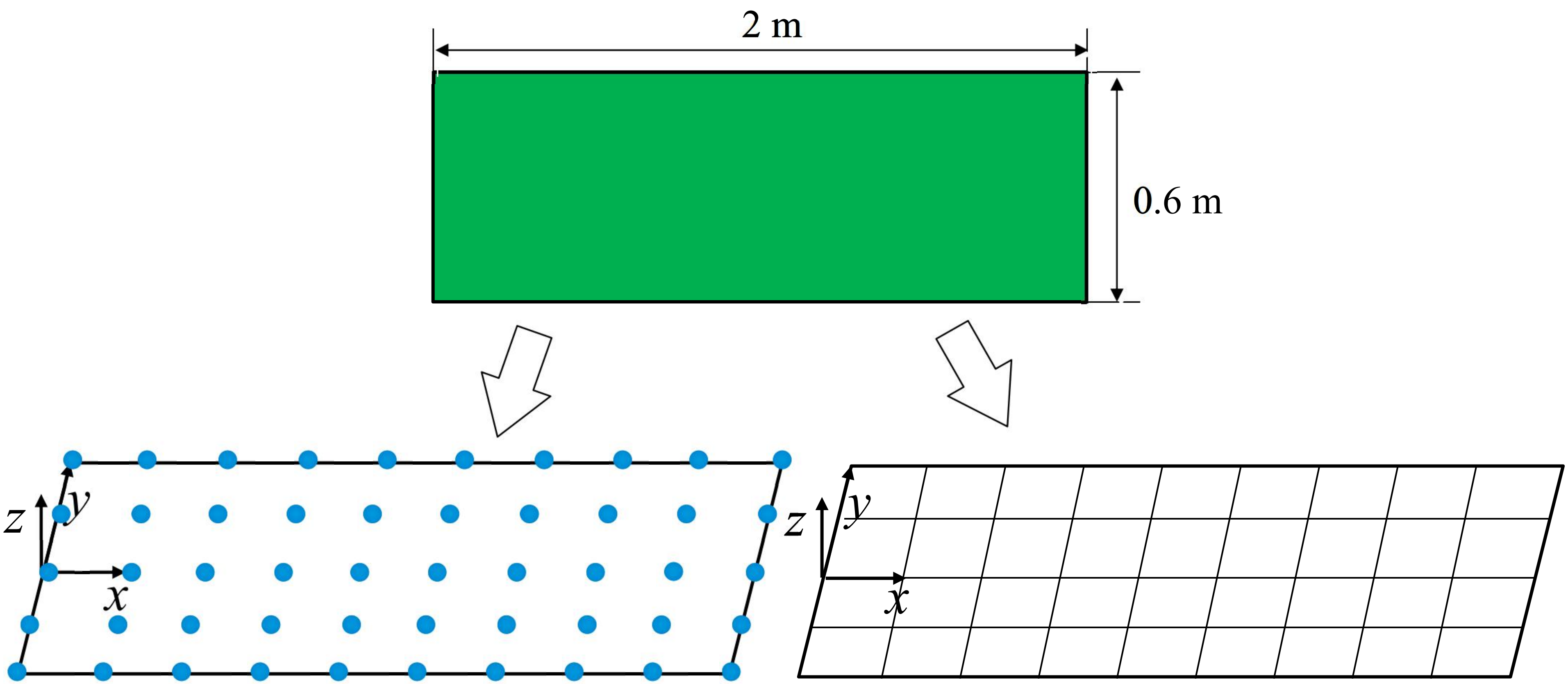
Validation of solver-Data transmission

- Fluid and structure boundary models



particle model (spacing size l_0=0.02 m)

element model (spacing size l_s m)

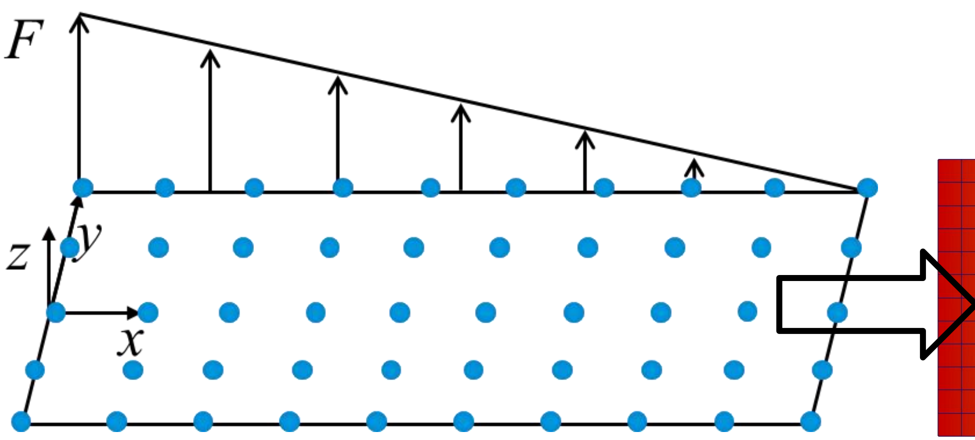
Validation of solver-Data transmission

- Data transmission between F-S boundaries

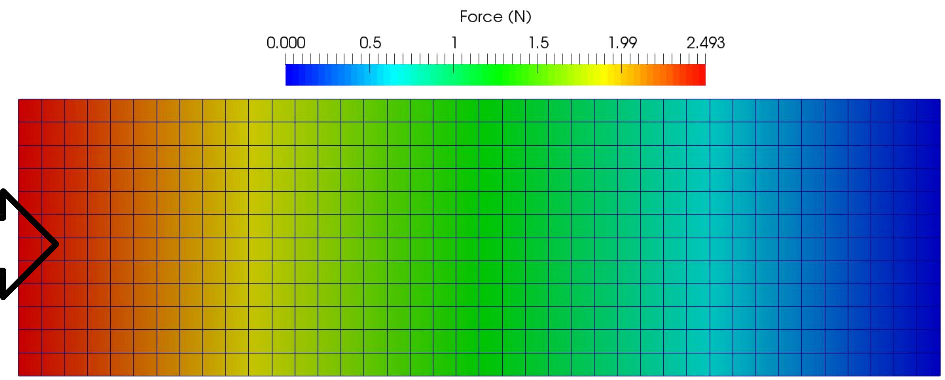
Test 5: Force transmission from fluid particles to structure nodes

Force distribution function $F=1000\text{-}500x$

Theoretical max value: 2.5 N



Initial force distribution on particle model



Force distribution on element model after transmission

Validation of solver-Data transmission

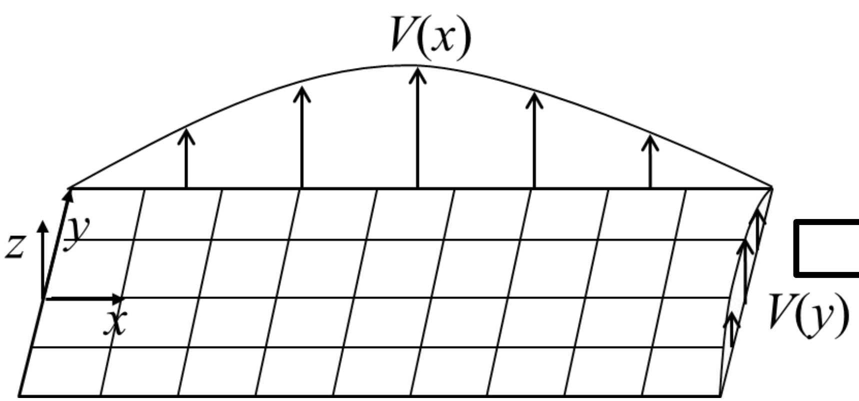
- Data transmission between F-S boundaries

Test 6: Displacement transmission from structure nodes to fluid particles

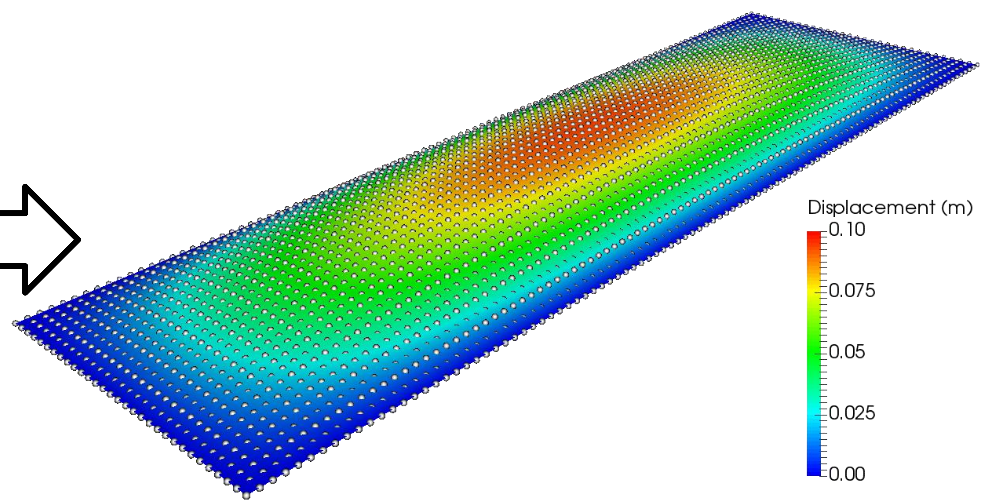
Displacement distribution function

$$V = 2\pi f(x) f(y) \cos(\omega t)$$

$$where\ f(x) = 2x - x^2,\ f(y) = 1 - \frac{100}{9} y^2$$



Initial displacement distribution on element model



Deformation of the particle model and element model after transmission (t=0.12s)

Validation of solver-Data transmission

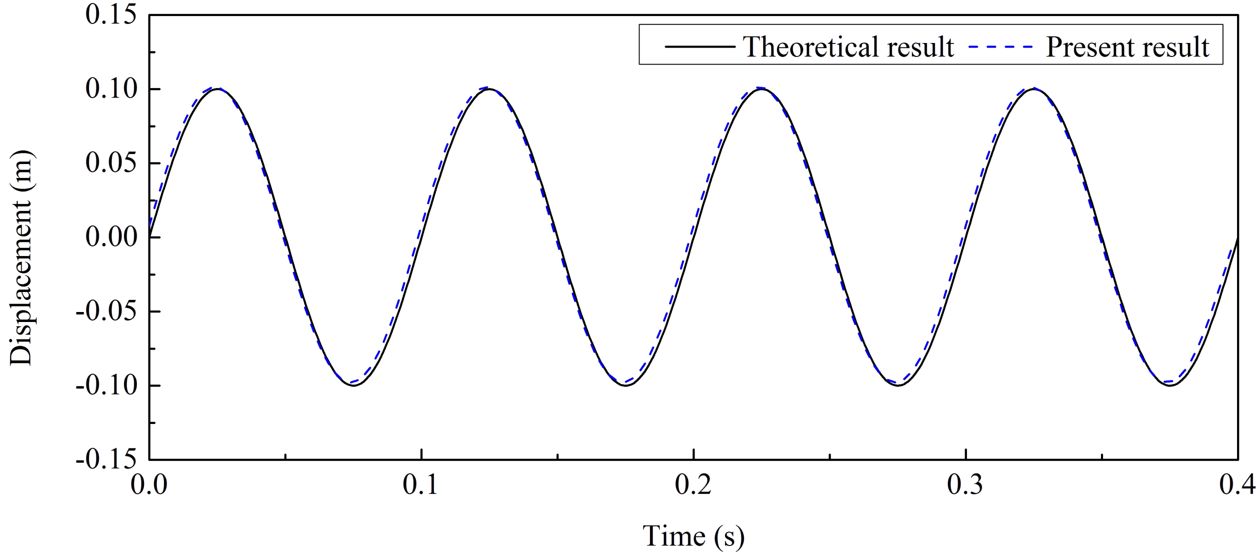
- Data transmission between F-S boundaries

Test 6: Displacement transmission from structure nodes to fluid particles

Displacement distribution function

$$V = 2\pi f(x) f(y) \cos(\omega t)$$

$$where\ f(x) = 2x - x^2,\ f(y) = 1 - \frac{100}{9} y^2$$



Displacement of the geometric center point of the **particle** model

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Validation of solver-Data transmission

Test 3: Displacement transmission from structure nodes to fluid particles (Shape function based data transmission technique)

Test conditions:

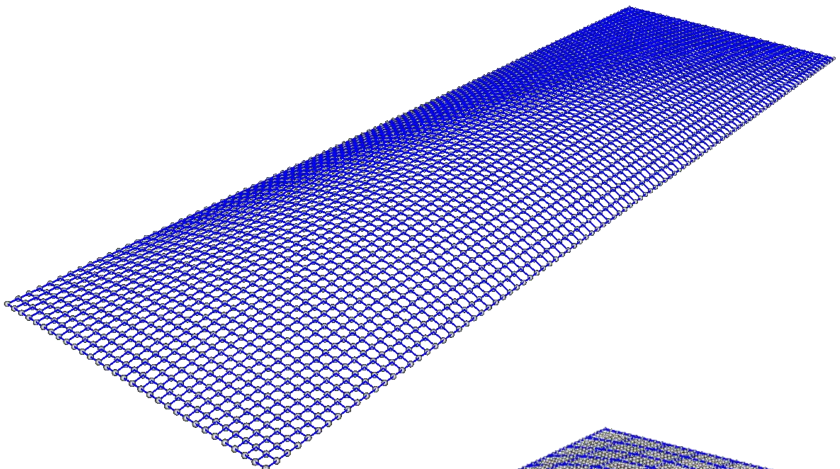
spacing size of particle model l_0=0.02 m

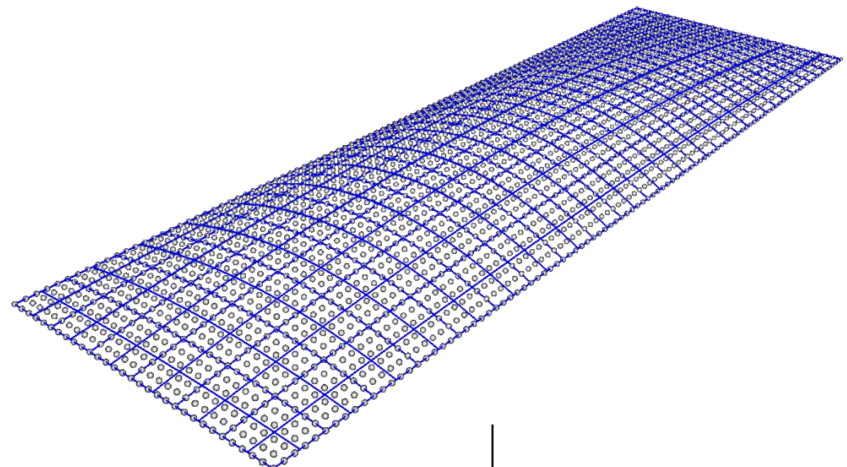
spacing size of element model dx×dy

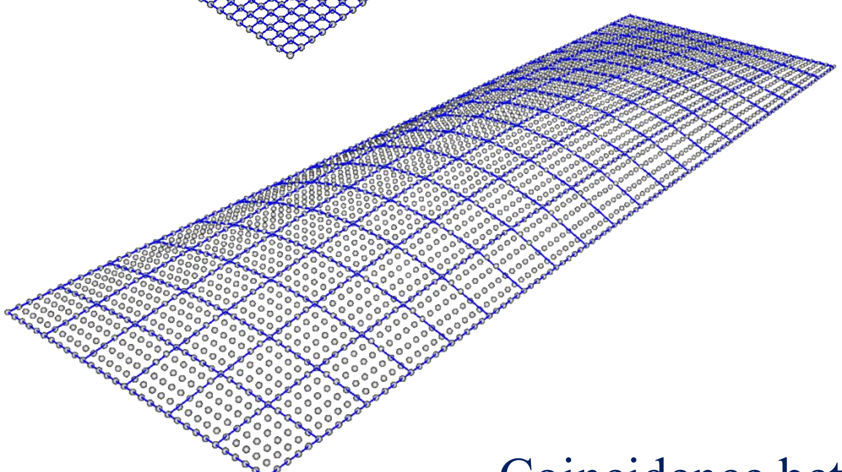
Case	1	2	3
dx×dy	0.02×0.02	0.05×0.05	0.1×0.1
Nx×Ny	1×1	2.5×2.5	5×5

Validation of solver-Data transmission

Test 3: Displacement transmission from structure nodes to fluid particles (Shape function based data transmission technique)





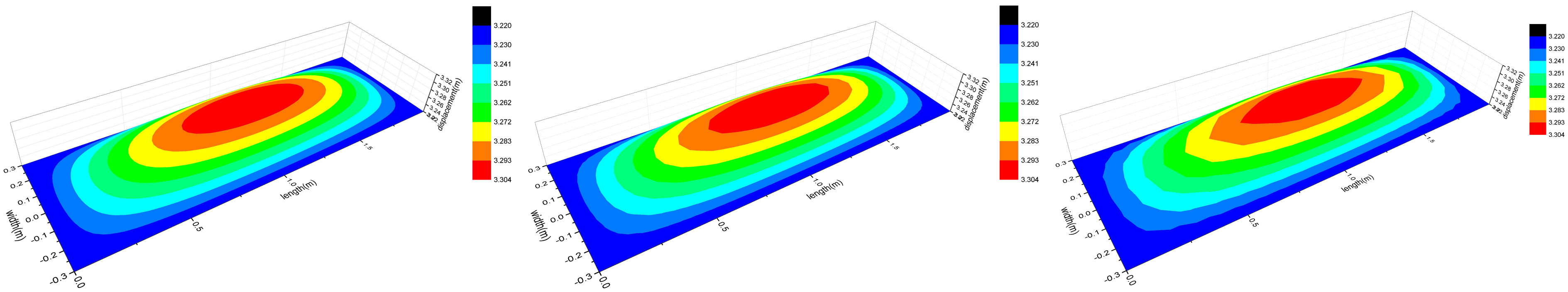


Case 1 0.02×0.02	Case 2 0.05×0.05
Case 3 0.1×0.1	

Coincidence between particle and element boundaries

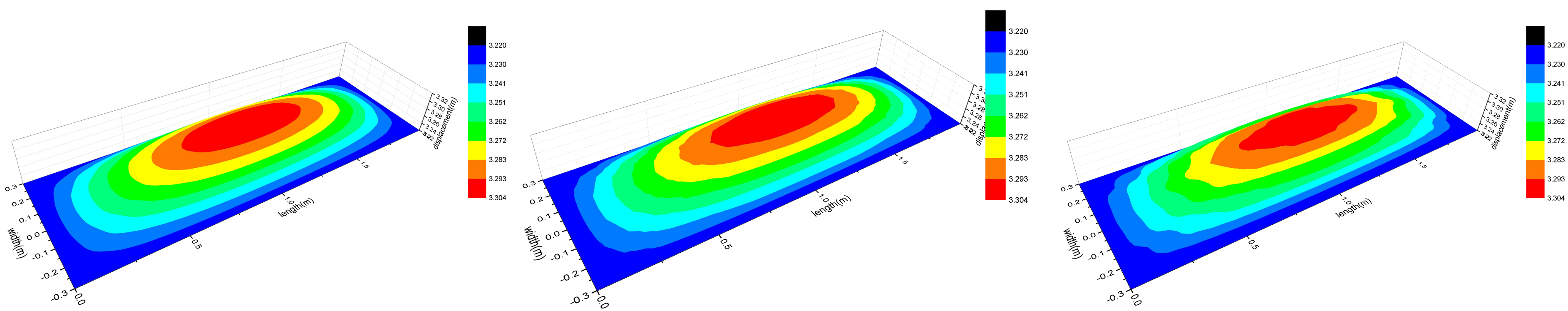
Validation of solver-Data transmission

- Comparison of displacement distribution



Displacement distribution on element boundary

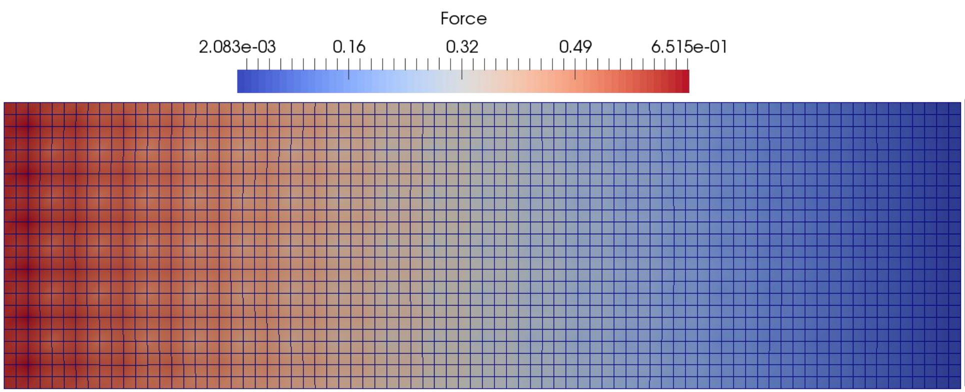
Displacement distribution on particle boundary

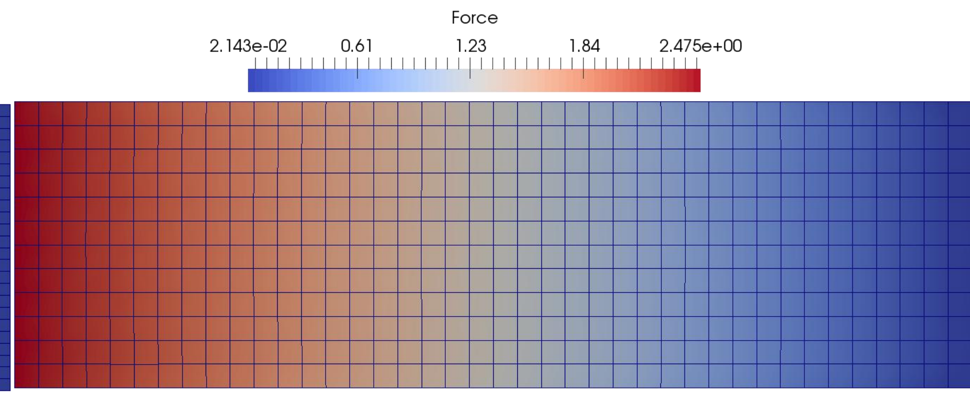


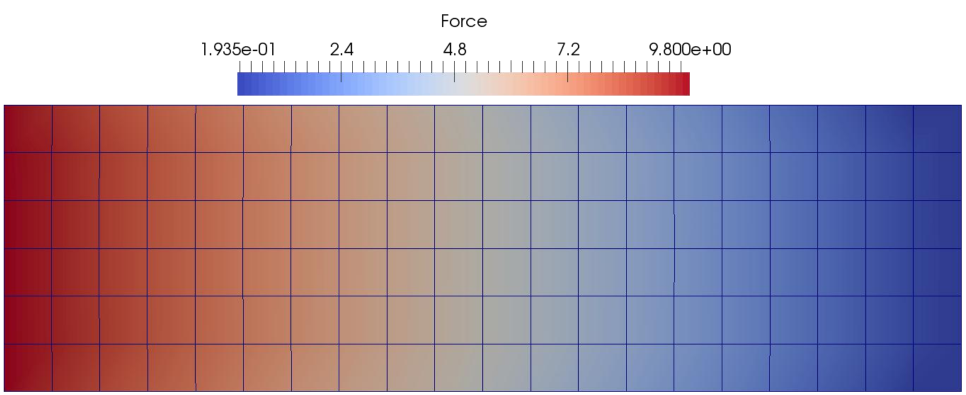
Case 1: 0.02×0.02 | Case 2: 0.05×0.05 | Case 3: 0.1×0.1

Validation of solver-Data transmission

Test 4: force transmission from fluid particles to structure nodes (Shape function based data transmission technique)







Case 1 0.02×0.02	Case 2 0.05×0.05
Case 3 0.1×0.1	

Force distributions on element boundaries

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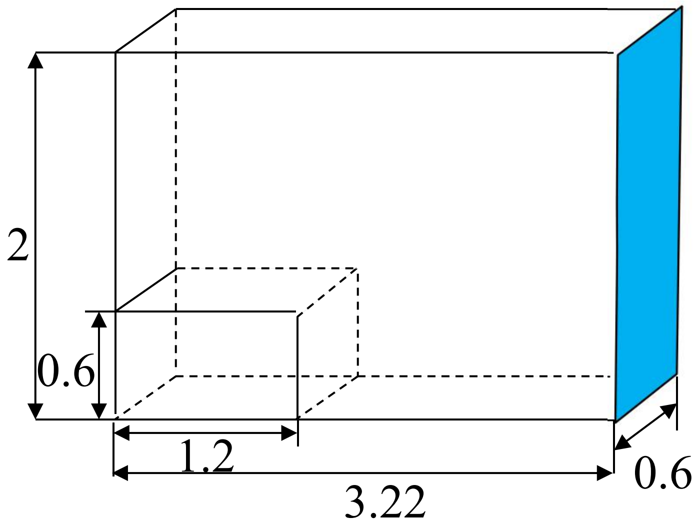
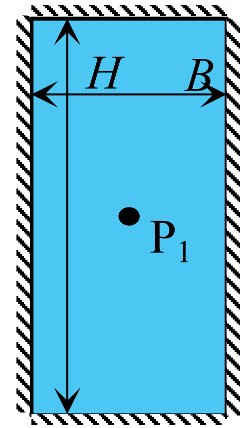
Dam-break flow interacting with elastic wall

Numerical setup



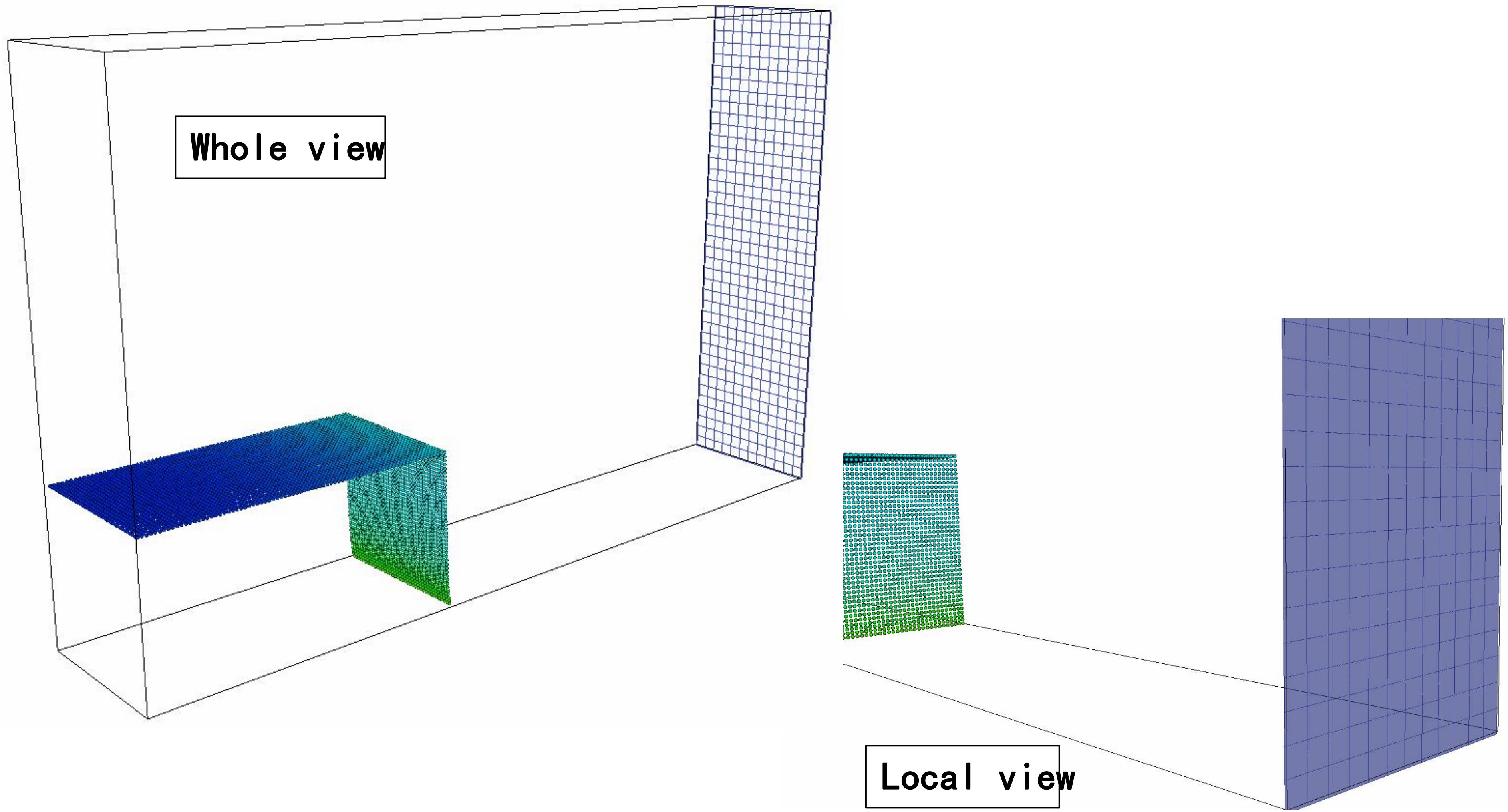
(a) Schematic diagram of tank (Unit: m)



(b) Elastic lateral wall

Fluid parameters	**Value**	**Structural parameters**	**Value**
Fluid density (kg/m³)	1000	Structure density (kg/m³)	1800
Kinematic viscosity (m²/s)	1×10^{-6}	Young's modulus (GPa)	40
Gravitational acceleration (s/m²)	9.81	Poisson's ratio	0.3
Particle spacing (m)	0.03	Element size (m)	0.05
Number of fluid particles	15200	Damping coefficients α_1	0.025
Total number of particles	44371	Damping coefficients α_2	0.0005
Time step size (s)	5×10^{-4}	Time step size (s)	5×10^{-4}

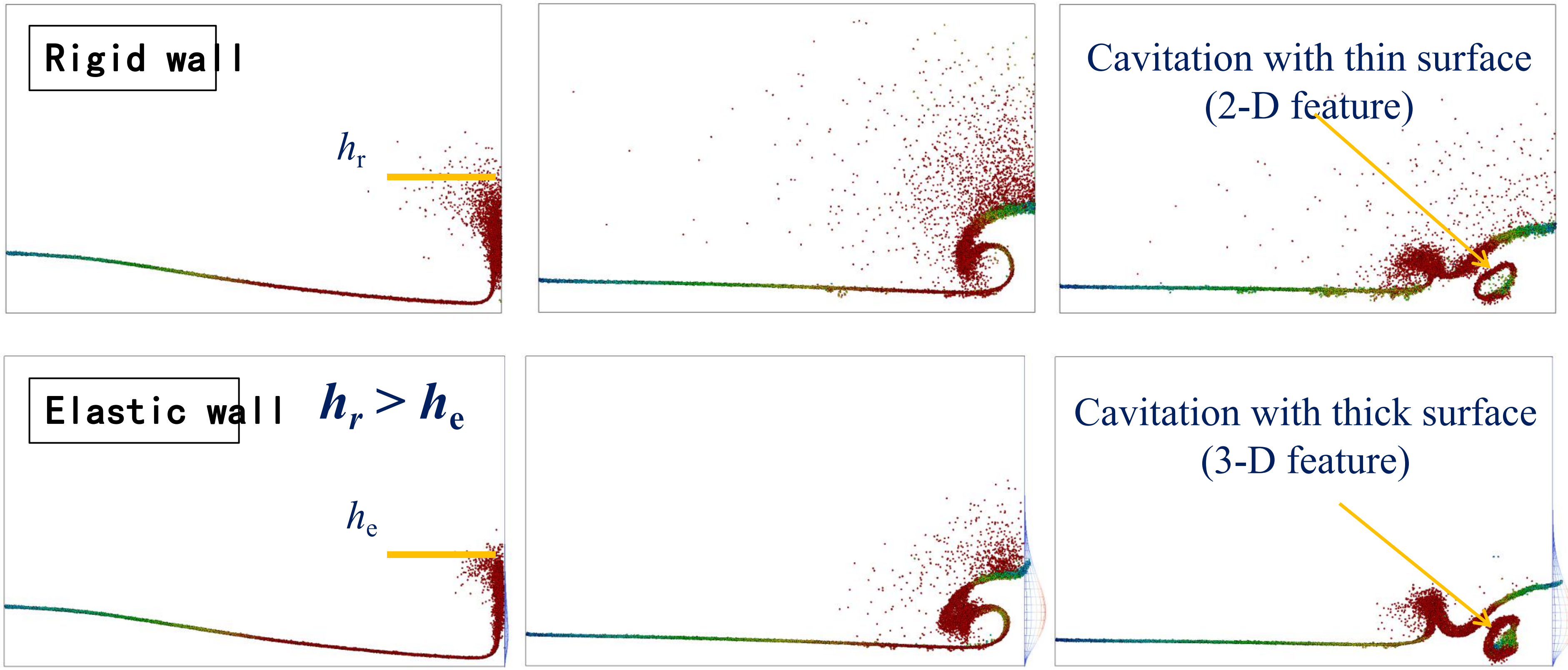
Dam-break flow interacting with elastic wall



Fluid structure interaction process

Dam-break flow interacting with elastic wall

Evolution of free surface

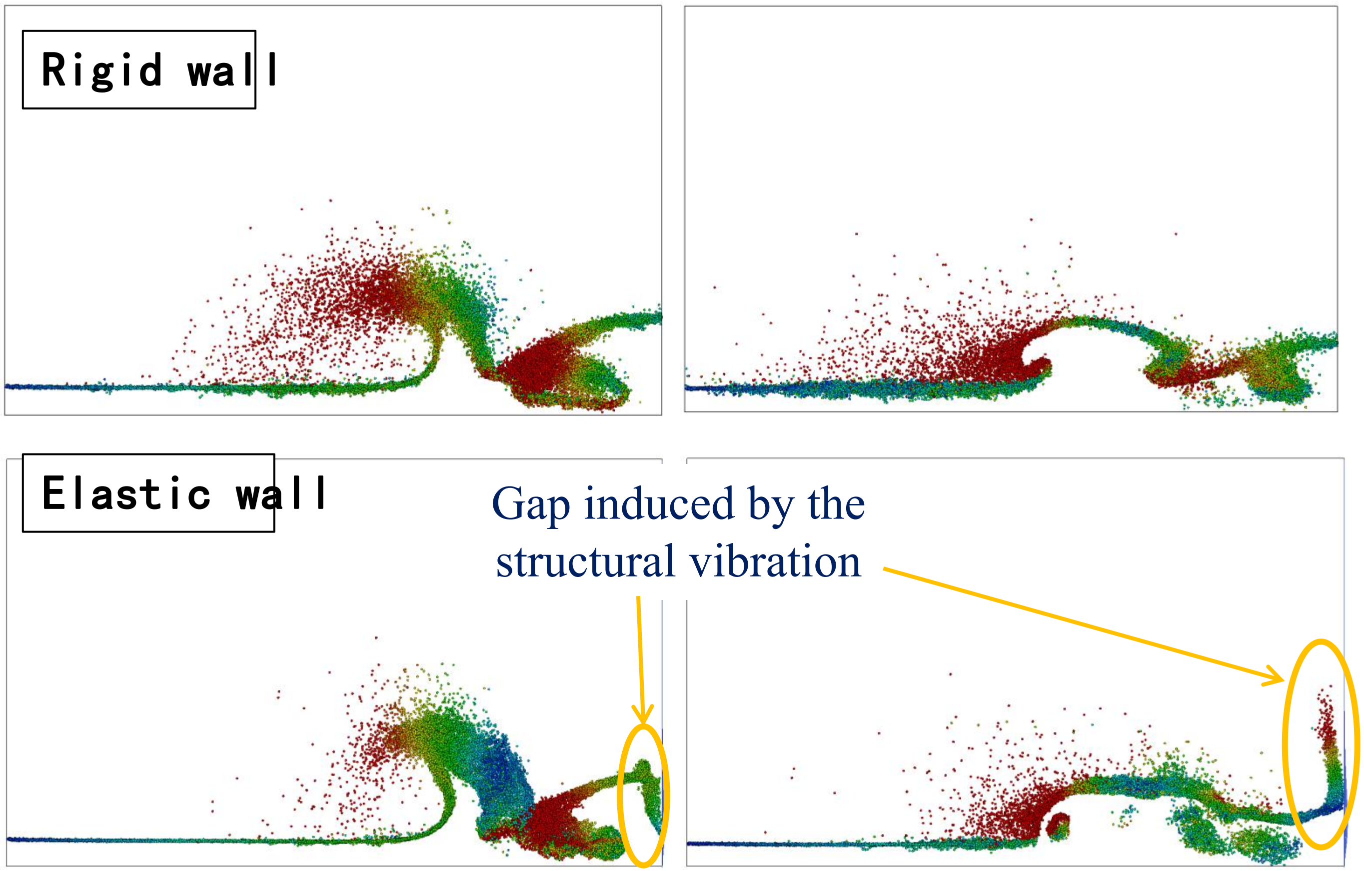


t=0.8s t=1.4s t=1.6s

Free surface in side view（top：rigid wall；bottom：elastic wall）

Dam-break flow interacting with elastic wall

Evolution of free surface

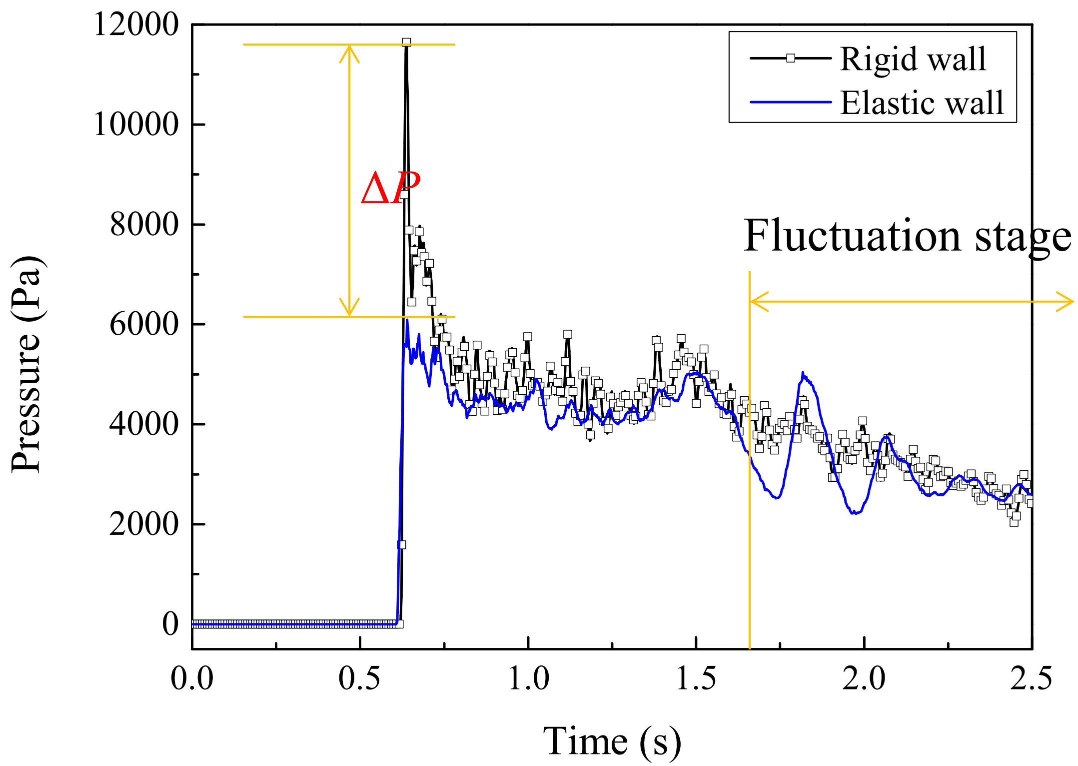


t=1.9s   *t*=2.15s

Free surface in side view（top：rigid wall；bottom：elastic wall）

Dam-break flow interacting with elastic wall

Impact loads

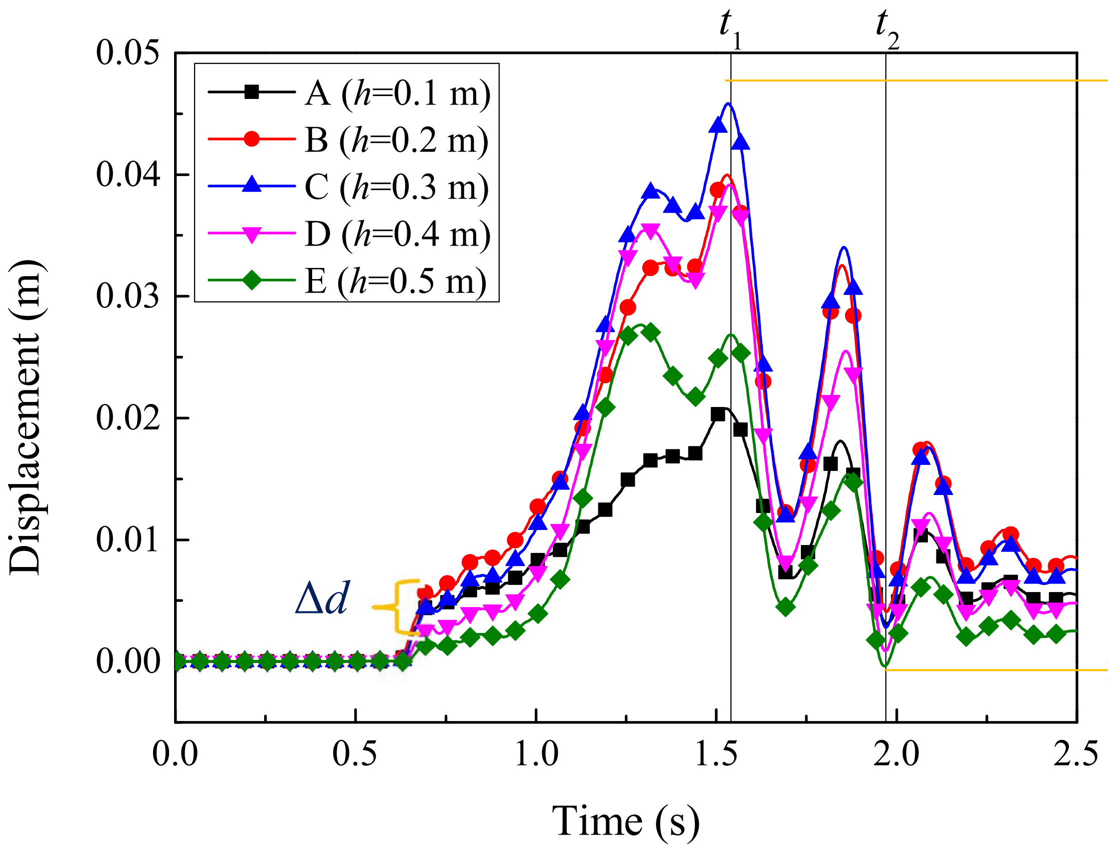


Time histories of impact pressures

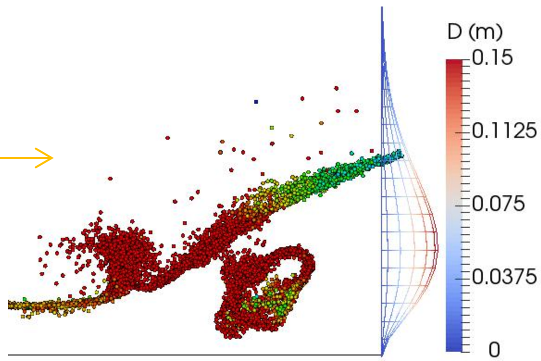
- ✓ The amplitude of slamming pressure (6096.89Pa) concerning the elastic tank is 47.77% smaller than that of rigid tank (11673Pa)
- ✓ The pressure curve fluctuates after the instance 1.5 s.

Dam-break flow interacting with elastic wall

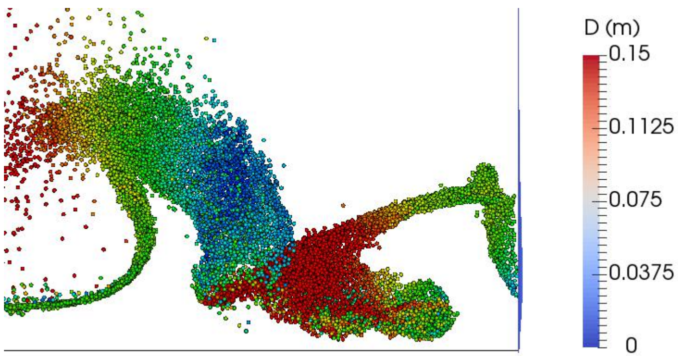
Vibrations of elastic wall



Time histories of structural vibrations



t_1



t_2

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- Backgrounds
- MPS-FEM coupled method
- Validation of the coupled method
  - ✓Dam-break flow in rigid tank
  - ✓Structural dynamic response
  - ✓Kernel function based data transmission technique
  - ✓Shape function based data transmission technique
- Application of MPS-FEM coupled method
- Conclusions

Conclusions

- An in-house solver based on the fully Lagrangian MPS-FEM coupled method for FSI problems with free surface, is developed.
- Two techniques, the kernel function and the shape function based interpolation techniques, for the data transmission between the fluid and structure boundaries are proposed.
- Accurate fluid force and structure displacement interpolation results can be achieved by the two techniques.
- With the help of the two data transmission techniques, the 3-D dam-break FSI problem can be simulated by the coupled MPS-FEM method.

Thank You ！

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