A SPH model for Root growth

2017 SPHERIC Beijing International Workshop

Matthias MIMAULT

The James Hutton Institute

19 Oct 2017

Introduction

Describe the root system

Plants develop complex and efficient root architectures

- Access water and nutrients
- Linear expansions and lateral branching
- Soil properties dependant

Introduction

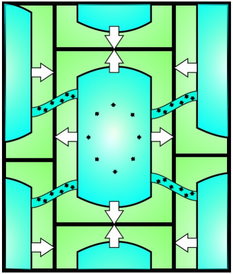
Scope of this work

Dynamics of growth

- Turgor pressure
- Cell wall mechanics
- Cell division

Identification

- Root cells ≡ SPH particles
- Turgor pressure ≡ Pore pressure
- Cell wall rheology ≡ Elasticity
- Incompressible materials ≡ Weakly compressible equations



Modelling

Governing equations

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\rho\,\frac{\partial u}{\partial x} + \check{\rho}$$
$$\frac{\mathrm{d}u}{\mathrm{d}t} = \frac{1}{\rho}\frac{\partial \sigma}{\partial x}$$
$$\frac{\mathrm{d}p}{\mathrm{d}t} = \kappa\left(\frac{\partial^2 p}{\partial x^2} - B\frac{\partial u}{\partial x}\right)$$

- ρ Density
- u Velocity
- σ Total stress
- p Pore pressure
- κ permeability coefficient
- c_0 sound speed
- B Biot coefficient

Constitutive law: $\sigma = -(p + P)$

$P(\rho) = c_0^2\,(\rho - \rho_0)$ hydrostatic pressure

$\check{\rho} = -\lambda\,(\rho - \rho_0)$ source term

Modelling

SPH formulation

At particle a,

$$\left\langle \rho \frac{\partial u}{\partial x} \right\rangle_a = -\sum_b m_b \left(u_a - u_b\right) \frac{\partial W_{ab}}{\partial r}$$

$$\left\langle \frac{1}{\rho} \frac{\partial \sigma}{\partial x} \right\rangle_a = \sum_b m_b \left[\left(\frac{\sigma_a}{\rho_a^2} + \frac{\sigma_b}{\rho_b^2} \right) + \Pi_{ab} \right] \frac{\partial W_{ab}}{\partial r}$$

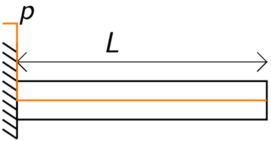
$$\left\langle \frac{\partial^2 p}{\partial x^2} \right\rangle_a = 2 \sum_b \frac{m_b}{\rho_b} \left(p_a - p_b\right) \frac{1}{\left| r_{ab} + \left(0.1 h_a\right)^2 \right|} \frac{\partial W_{ab}}{\partial r}$$

with the artificial viscosity Π_{ab}

FD-SPH formulation of Laplacian [Monaghan 2005]

Numerical Application

Model settings

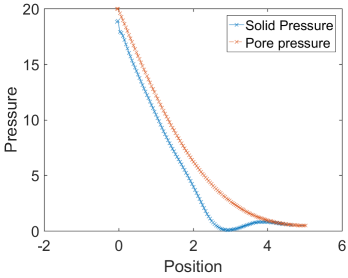


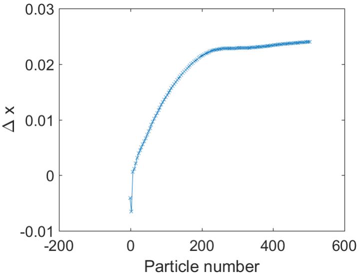
- *Cubic spline* kernel, Euler integration
- $L = 5\,mm$ - $\rho_0 = 1\,mg.mm^{-3}$ - $c_0 = 316\,mm.ms^{-1}$
  $\kappa = 1$ - $B = 1$
- Density boundary: replicate
- Speed boundary: mirror
- Pressure boundary: fixed $p_0 = 20\,MPa$

Numerical test

Parameters and outputs

- $N = 500$ - $T = 2$

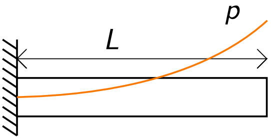




- Extension $\Delta x = 0.025$
- Convergence of hydrostatic pressure to pore pressure

Numerical Application

Model settings



- *Cubic spline* kernel, Euler integration
- $L = 5\,mm$ - $\rho_0 = 1\,mg.mm^{-3}$ - $c_0 = 316\,mm.ms^{-1}$
  $\kappa = 1$ - $B = 1$
- Density boundary: replicate
- Speed boundary: mirror
- Variable smoothing length h
- Pore pressure distribution $p(x) = p_0 \frac{|x|}{L} g(v)$

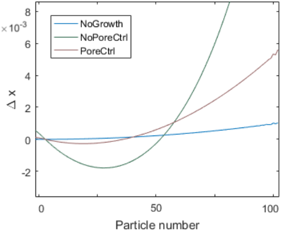
  $g(v)$ cell volume dependant factor, $p_0 = 20\,MPa$

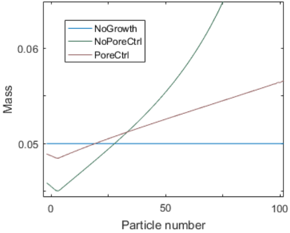
Numerical test

Parameters and outputs

- $N = 100$ - $T = 50$
- Mass evolution
- Equilibrium

	λ	$g(v)$
NoGrowth	0	1
NoPoreCtrl	0.5	1
PoreCtrl	0.5	$1 - v$





Discussion and perspectives

The tests show

- Smooth extension
- Fast damping oscillations
- Good separation of dynamics

In the future

- Biological meaningful turgor pressure distribution (solute concentration)
- Mass growth, visco-elasticity, cell division
- 3D extension, parallelization

Thank you for your attention !

Contact - matthias.mimault@hutton.ac.uk