**SPHERIC 北京**
**2017 Oct 17-20**

# Augmenting SPH simulations using the Material Point Method:

*Modeling mixed-mode failure in rocks*

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Geonumerics

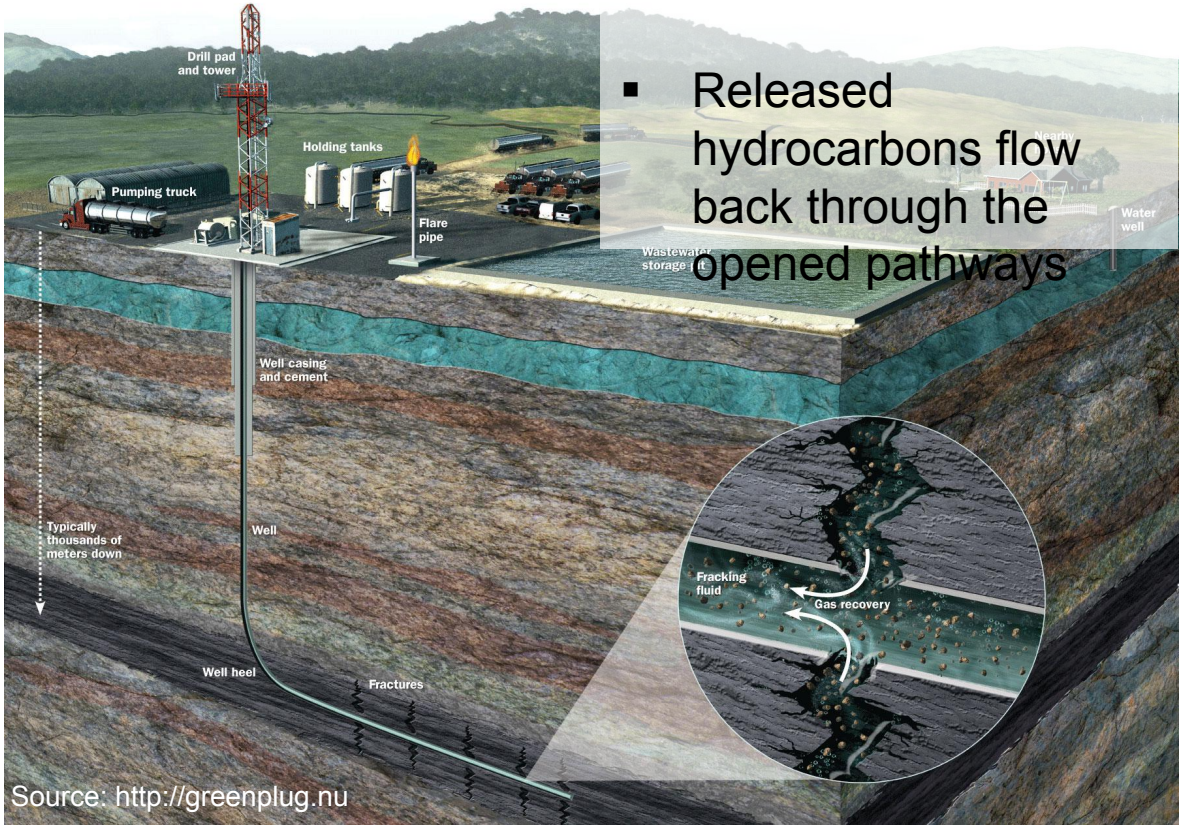
# Hydraulic Fracturing – What is it?

**Why use numerical models?**

- ❑ State of material deep underground is hard to measure and reproduce
- ❑ Fracture behavior is heavily dependent on material properties

**How does it work?**

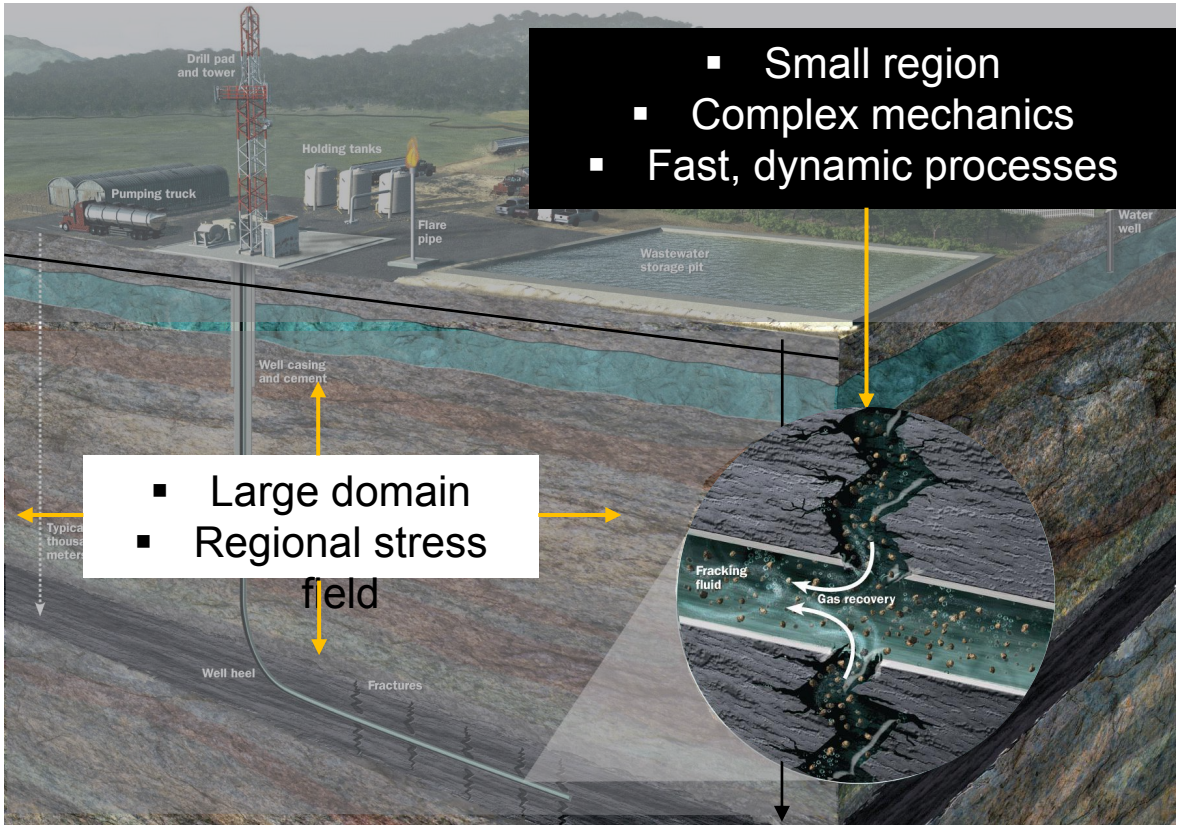
- Fluid is used to open fractures in dense rock
- Released hydrocarbons flow back through the opened pathways



# Hydraulic Fracturing – Modeling

## Modeling Challenges

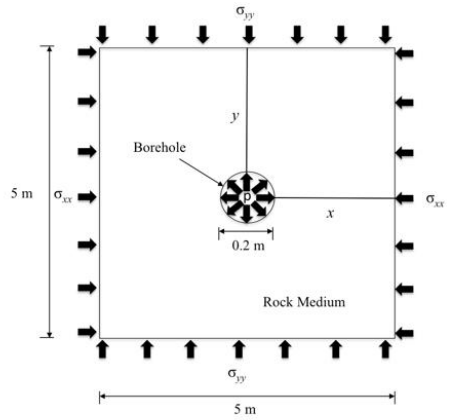
- Two scales of behavior:
  - Surrounding rock (~1000's of meters)
  - Resource-rich rock around borehole (~1 meter)
- Complex behavior:
  - Fracture creation and propagation
  - Fracture interaction
  - Fluid-solid interactions



# Hydraulic Fracturing – SPH Modeling

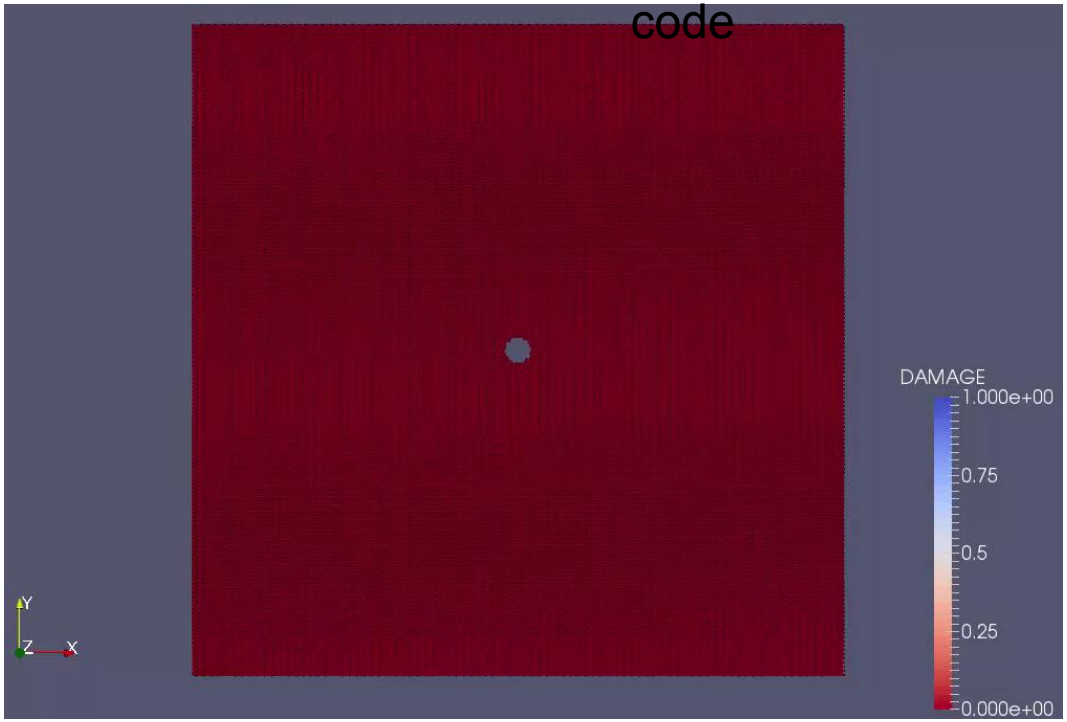
## Borehole Physics

- ❑ Fracture creation and propagation
- ❑ Multiple, interacting fractures
- ❑ Interaction of multiphase flows




Schematic of the circular opening problem

Borehole model of fracture propagation using MIT in-house SPH code



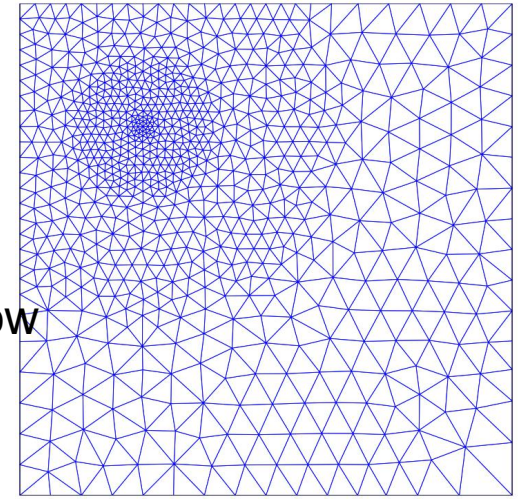

## Model Limitations

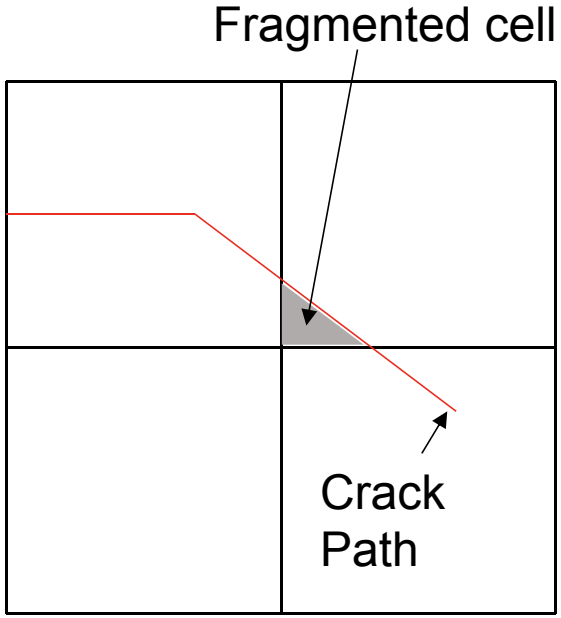
- 5m x 5m simulation ~ 100,000 particles
- SPH scaling up to reservoir scale is inefficient
- Boundary conditions

# Hydraulic Fracturing – FEM Modeling

**Reservoir Model**

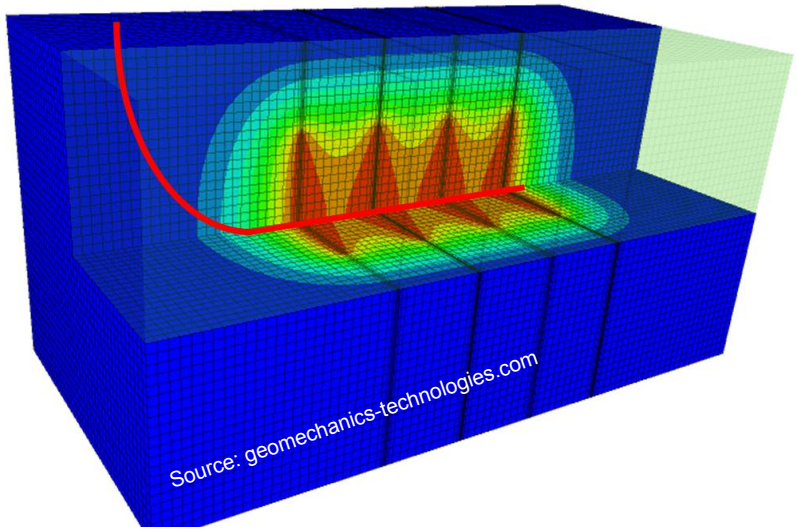
- Geometry is smoothly varying
- Dynamics are mostly elastic, slow






**FEM Simulation**

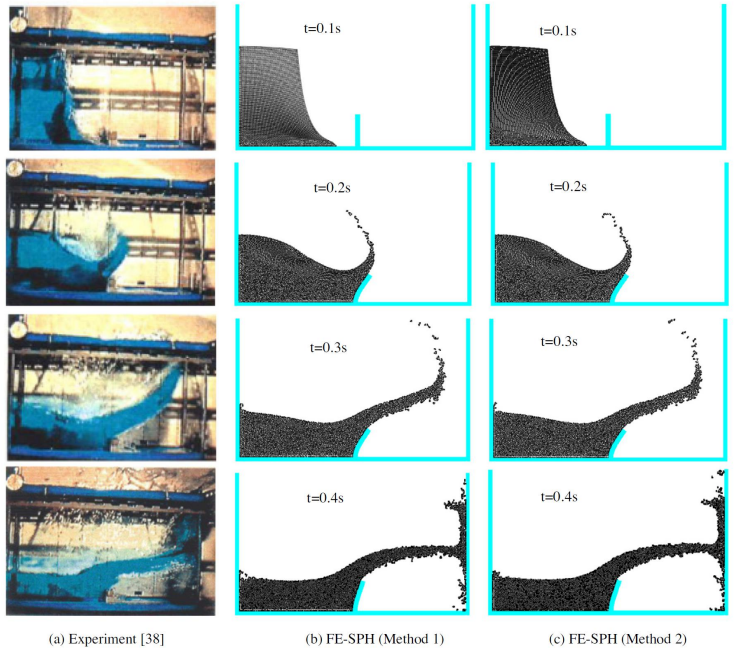
- More efficient at simpler physics and geometries
- Can scale more effectively from borehole to reservoir
- Far field boundary conditions are straightforward



# SPH/FEM Coupling

**Coupling a particles to a mesh:**

- ❑ Allows for deformation to be simulated more efficiently
- ❑ Reduces total computation time




**Some issues:**

- Implementation is complex
- Flexibility of interface interactions

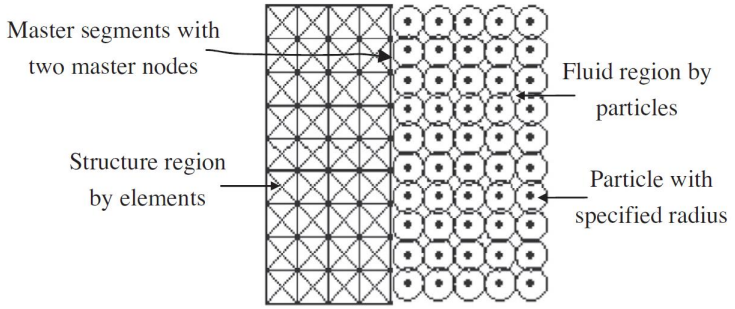
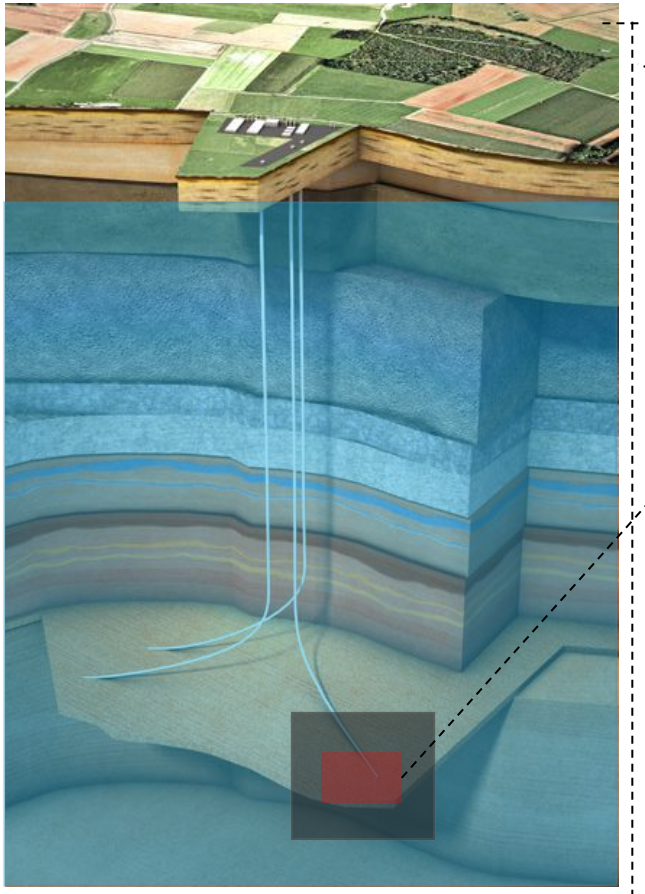



Fig. 1. Schematism of interface between fluid and structure.

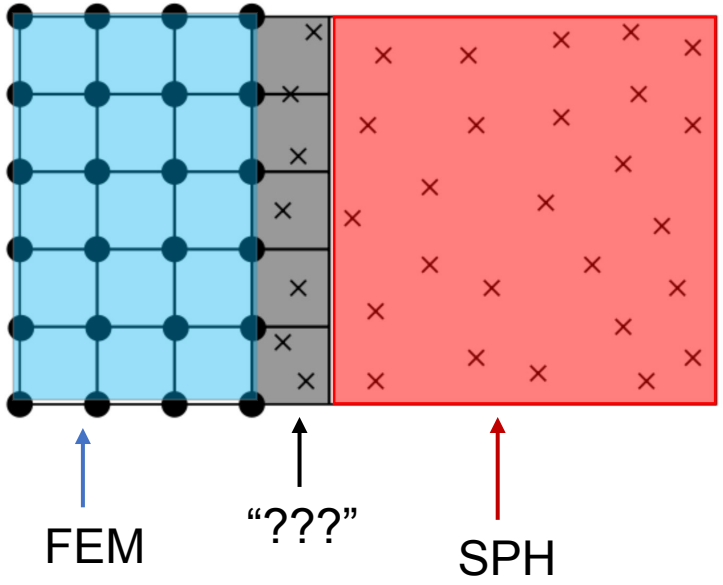
Source: Comput. Methods Appl. Mech. Engrg. 276 (2014) 266–286

# Proposed Solution SPH+"???"+FEM



Full scale 10's of km

Area of interest 10's of meters

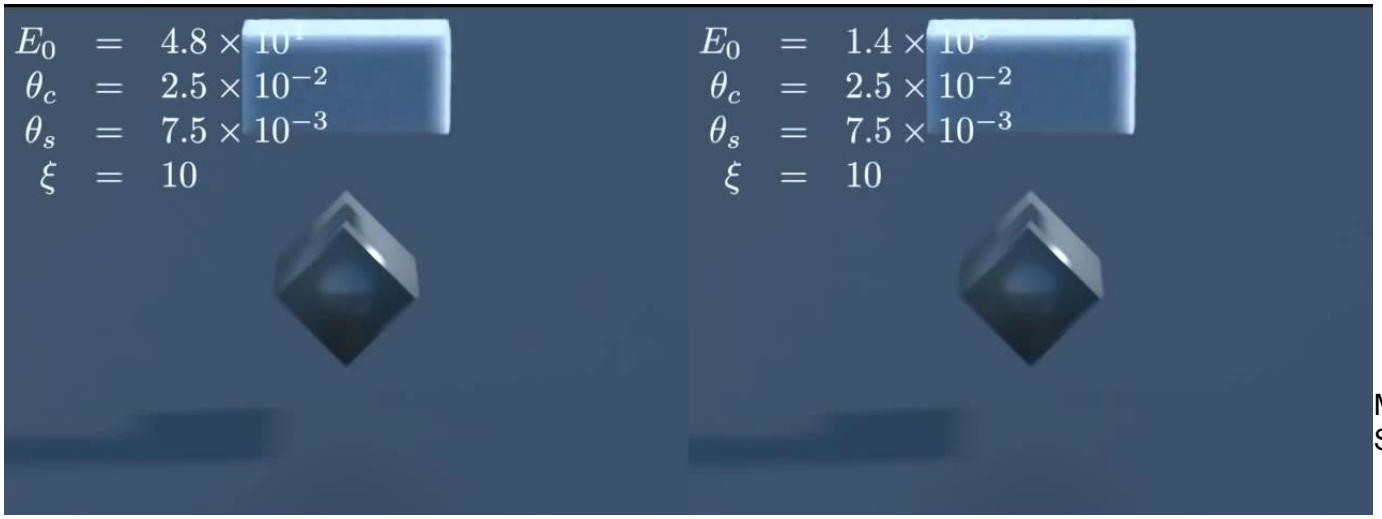



**Transitional Method between SPH & FEM**

- ❑ Hybrid particle/grid technique
- ❑ Couple grid to FEM for fast scaling
- ❑ Couple particles to SPH for complex mechanics

# The Material Point Method (MPM)

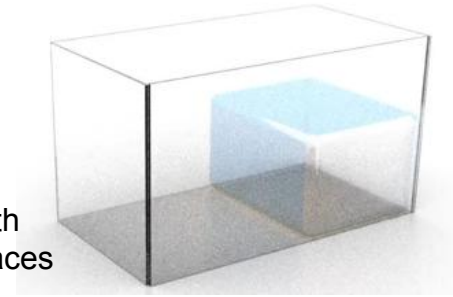
MPM has been used in the animation industry for movies like Disney's Frozen, Big Hero 6, Zootopia, Moana and more.

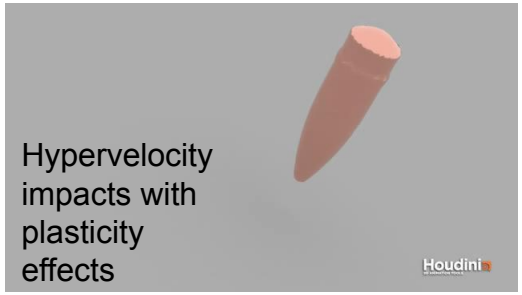



Made by Walt Disney Studios Animation Group

Can be used for a wide range of problems in continuum mechanics

Fluids with free surfaces






Made with our code

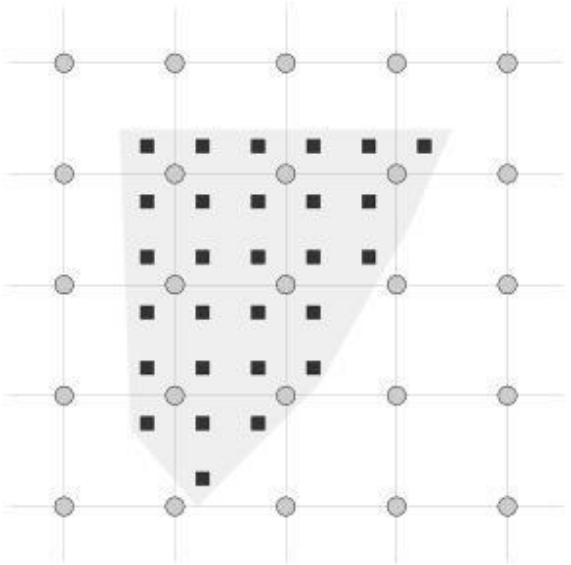
# The Material Point Method (MPM)

**MPM Algorithm**

**Step 1** – Interpolate to nodes

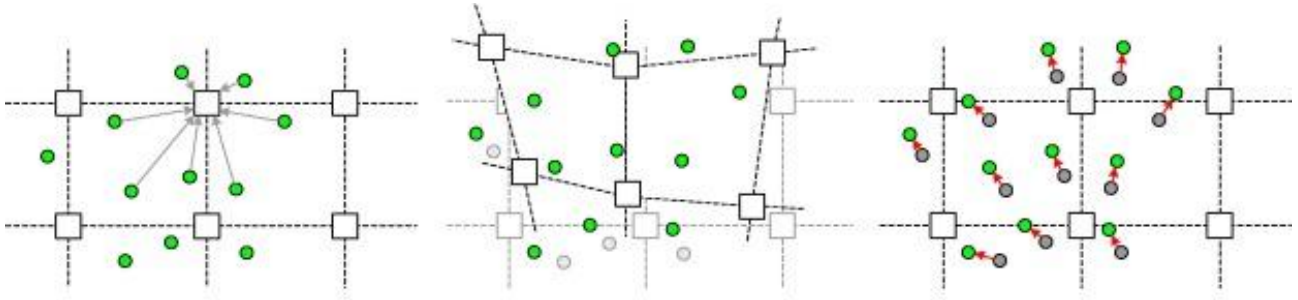
*Mass:* $M_i = \sum^{N_p} m_p N_{ip}$

*Force:* $f_i^{int} = -\sum^{Np} \sigma_p G_{ip} V_p \quad (G_{ip} \equiv \nabla N_{ip})$



**Step 2** – Solve F=ma on nodes

**Step 3** – Interpolate back to particles and update



**Step 1** **Step 2** **Step 3**

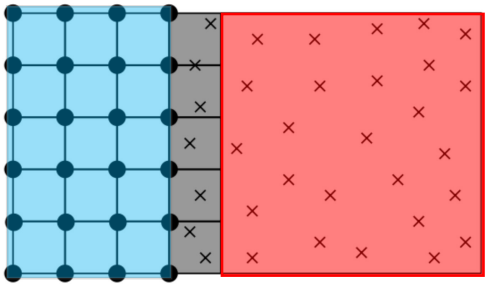
# Checklist

For MPM to be a useful candidate its must be capable of:

? MPM – FEM coupling

? MPM – SPH Coupling

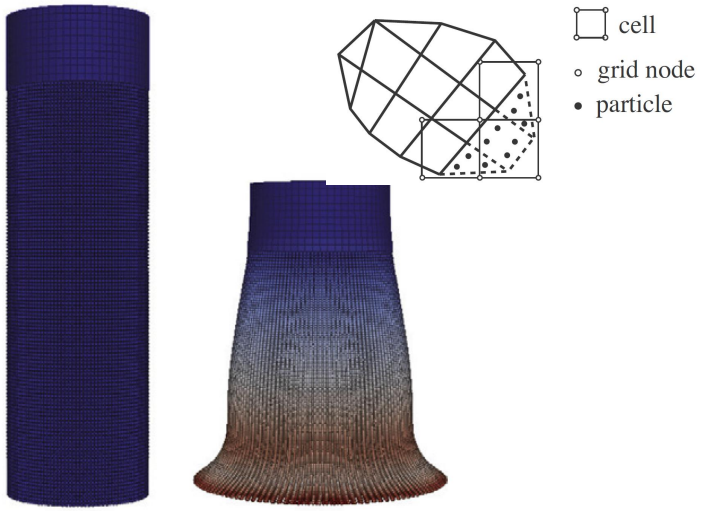
? Modeling same physics as SPH



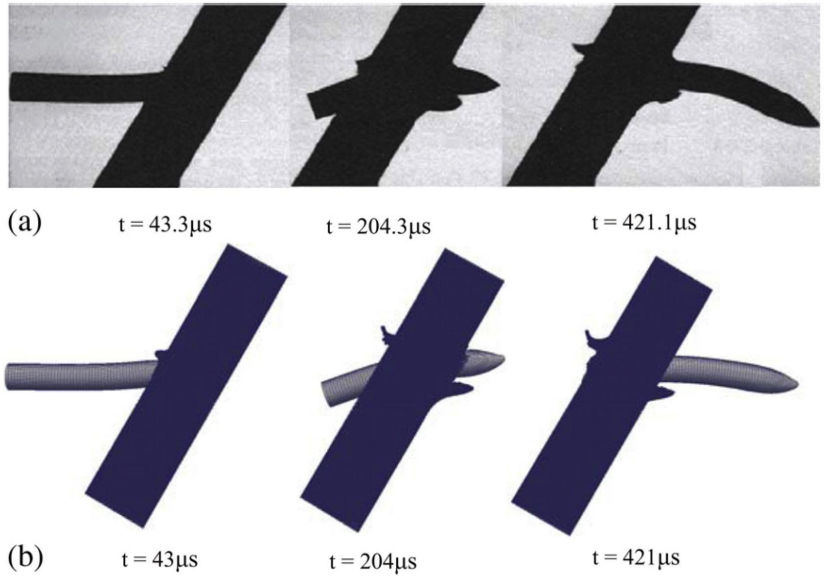
# MPM/FEM Coupling

## MPM-FEM Coupling

- Used to augment FEM simulations (below)
- Used to model different material behavior (right)




Source: *Comput. Methods Appl. Mech. Engrg. 241–244 (2012) 275–285*




Source: Comput. Methods Appl. Mech. Engrg. 200 (2011) 3482–3494

**Nodal values during MPM timestep**

**=**

**FEM nodal values**

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- ? MPM – SPH Coupling
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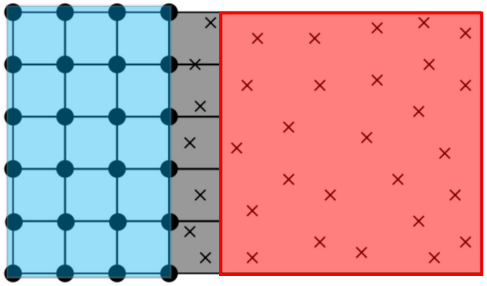


# MPM/SPH Coupling

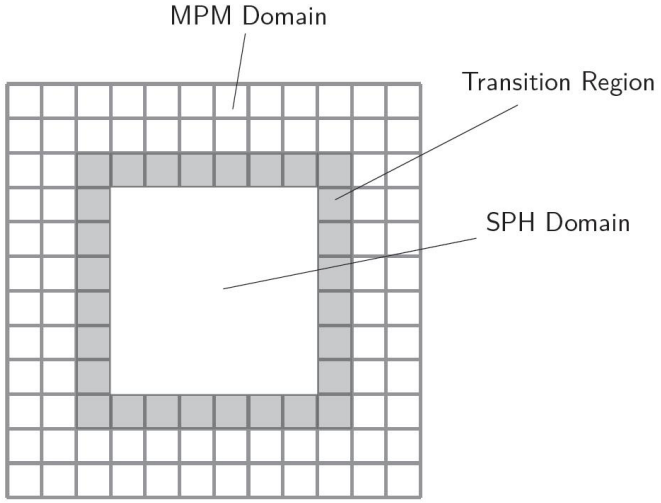
**Coupling Philosophy**

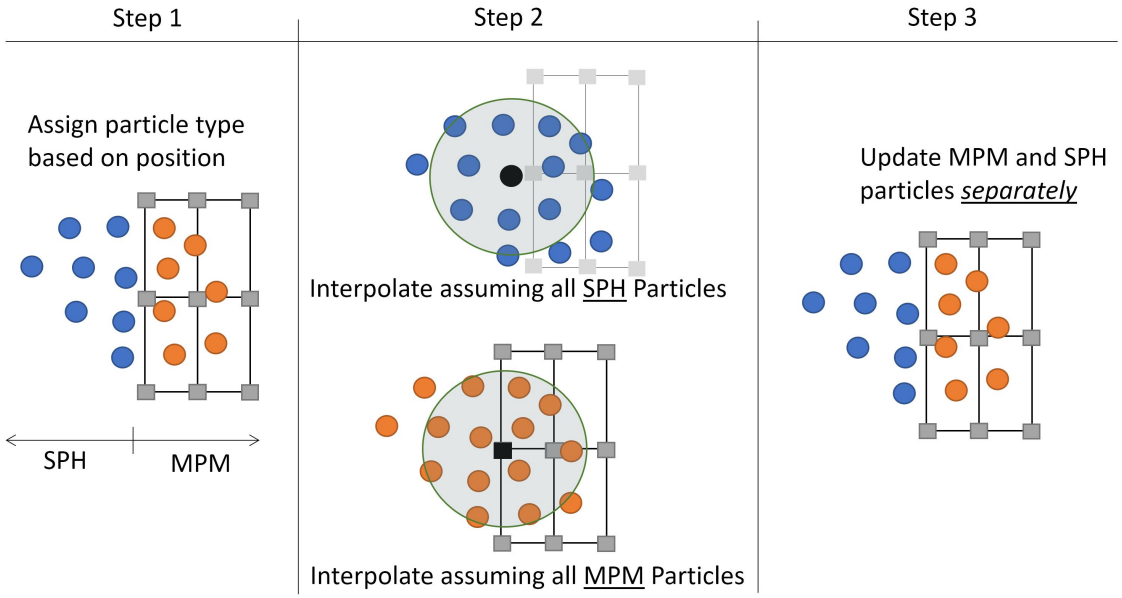
**=**

*Assume each particle is data for interpolation*

**+**

*Update particles based on type*



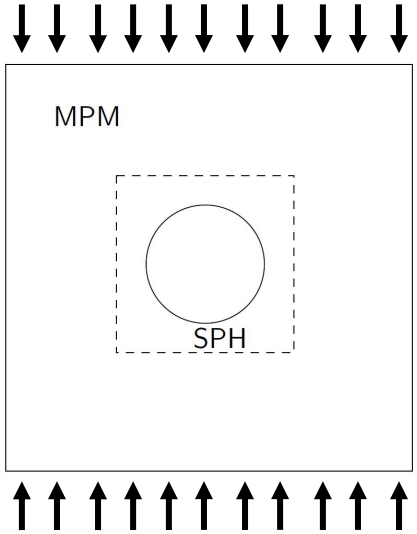

**Coupling Algorithm:**

- ❑ Assign particle type based on position.
- ❑ Interpolate from all surrounding points.
- ❑ Update particles using respective algorithms, SPH/MPM

# MPM/SPH Coupling

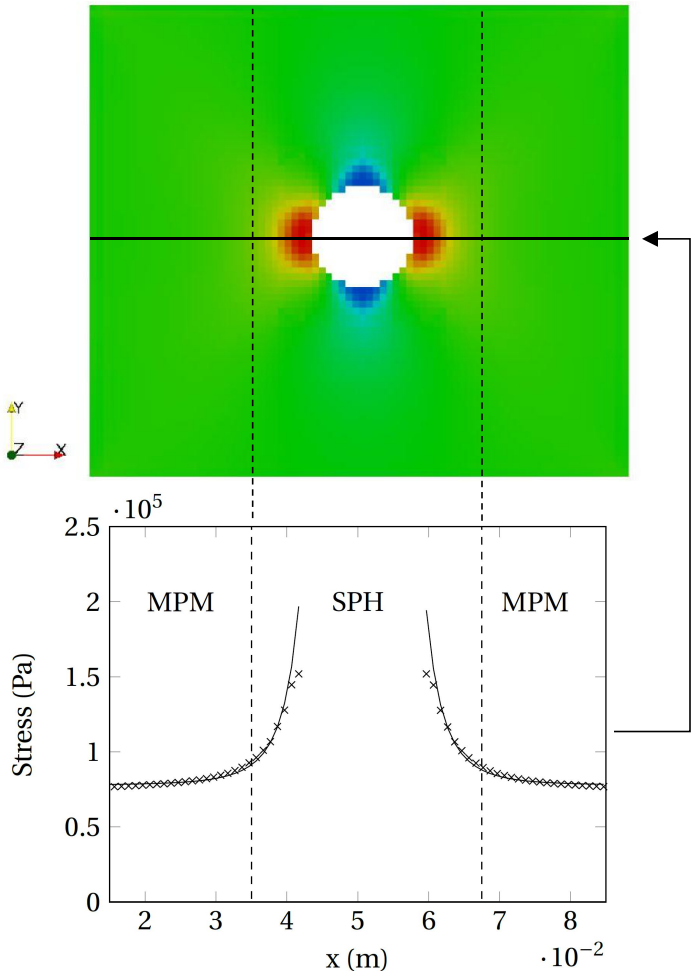
**Coupling Verification:**
*2D Hole in a plate*

- ❑ Stress concentration
- ❑ Stress gradient across transition region
- ❑ Analytical solution




**MPM-SPH Simulation:**

- ✓ Smooth transition across stress gradient
- ✓ Matches analytical solution




Stress along centerline

Full paper in **Computational Particle Mechanics**:
Raymond, S.J., Jones, B. & Williams, J.R. Comp. '*A strategy to couple the material point method (MPM) and smoothed particle hydrodynamics (SPH) computational techniques*' (2016).

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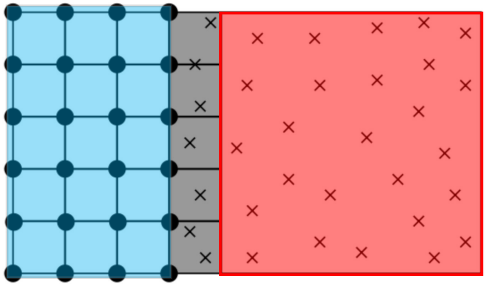
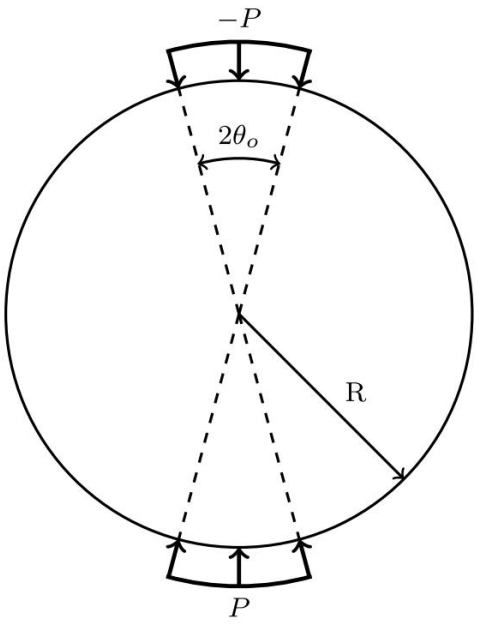


# Validating MPM against SPH

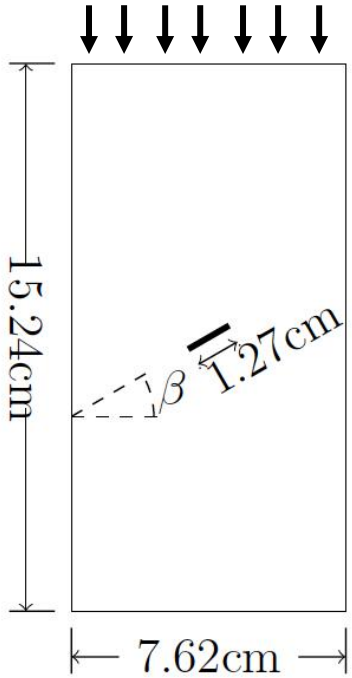
## Brazil Test

- ❑ Analytical solution in elastic region
- ❑ Separate plastic and damage regions



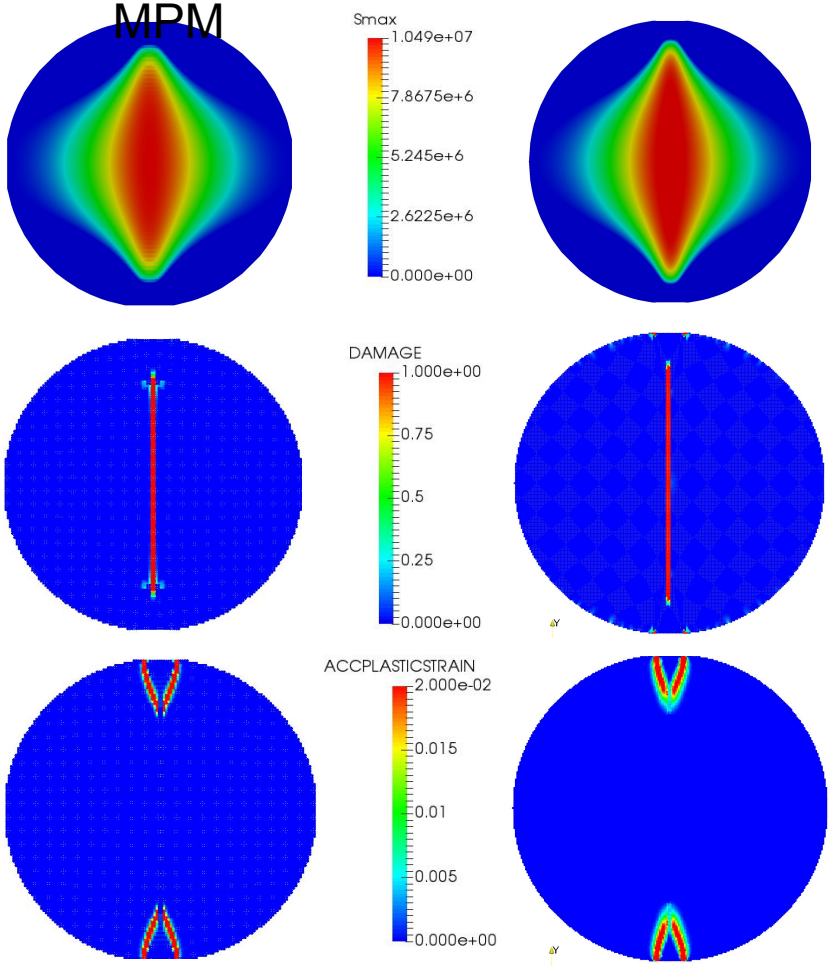

## Single Flaw Uniaxial Compression

- ❑ Comparison with experimental results
- ❑ Complex mixed-mode failure paths



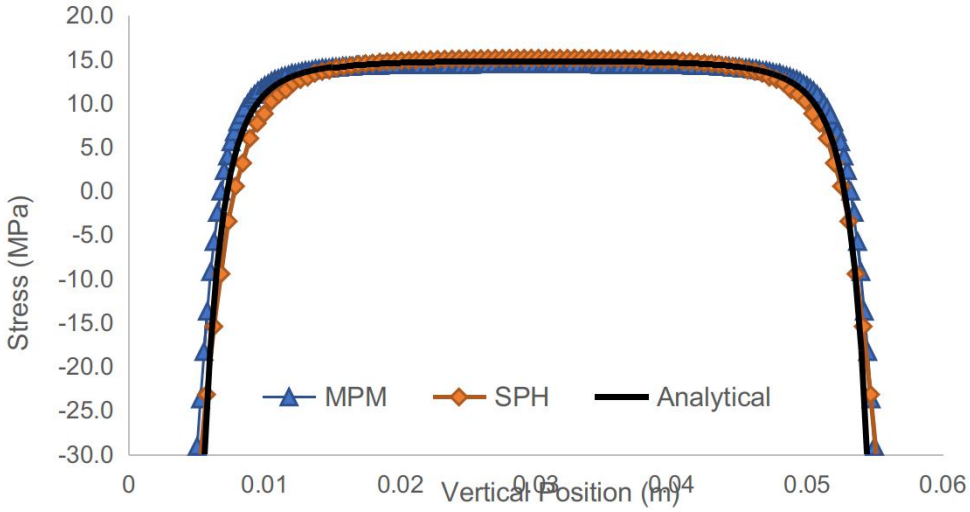
# Brazil Test – SPH vs. MPM

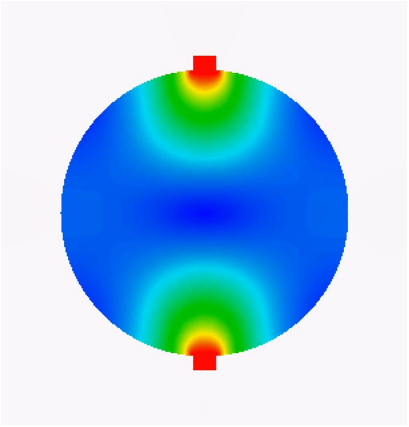
SPH
MPM




## Maximum Tensile Stress

$$\sigma_x(0,y) = \frac{2p}{\pi}\left[\frac{(1-y^2/R^2)\sin 2\theta_o}{1-2y^2/R^2\cos 2\theta_o + y^4/R^4} - \tan^{-1}\left(\frac{1+y^2/R^2}{1-y^2/R^2}\tan\theta_o\right)\right]$$



# Single Flaw Specimen – SPH vs. MPM

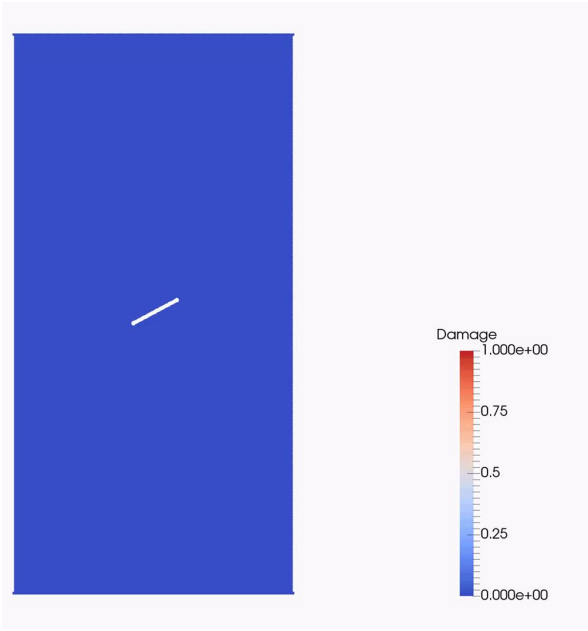
MPM

SPH

(d) $\beta = 30°$ at $t = 1ms$

(e) $\beta = 30°$ at $t = 1.5ms$

(f) $\beta = 30°$ at $t = 2ms$



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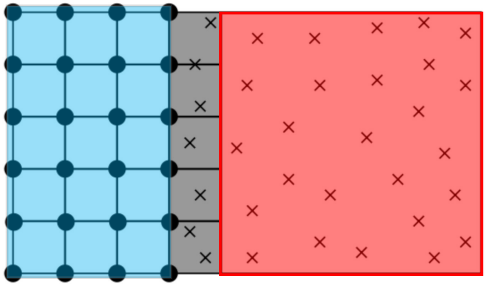

# Conclusions and moving forward

**Summary:**

- ✓ MPM is a potential candidate for smooth particle to mesh method coupling
- ✓ SPH and MPM can accurately model mixed mode failure in rock material

**Next Steps:**

- ➢ Combine all three methods into a single simulation
- ➢ MPM and SPH only with scaled MPM regions