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Numerical Simulation of Rayleigh-Taylor Instability by MPS Multiphase Method

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spheric
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- **Background**
- **Numerical Methods**
  - Modified MPS method
  - MPS Multiphase solver
- **Numerical Experiments**
  - Numerical model
  - Numerical results
- **Conclusions and Ongoing Work**

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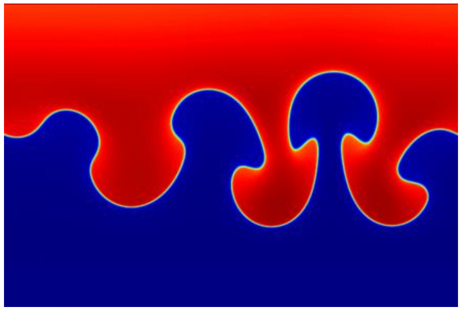
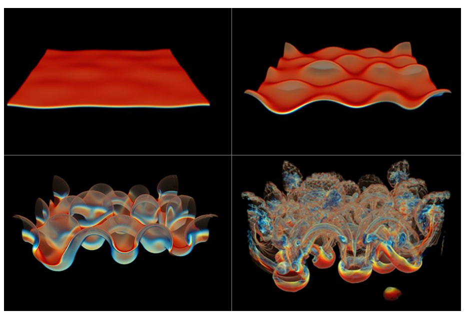
Background

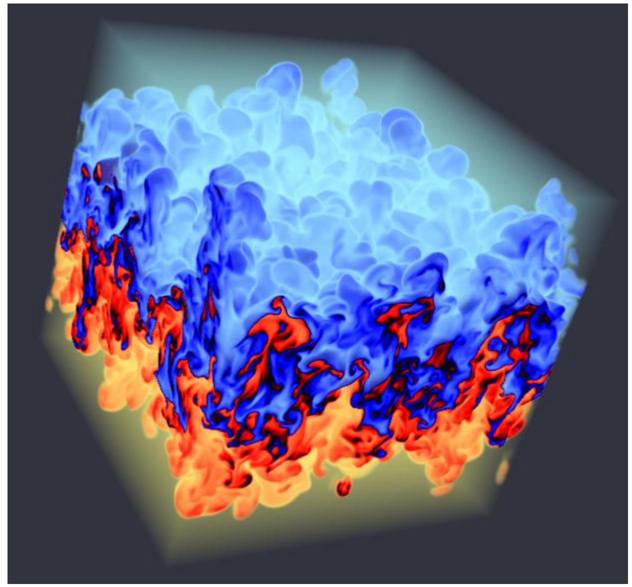
Rayleigh-Taylor instability (RTI)



◇ 2D RTI



◇ 3D RTI



◇ Inertial confinement fusion

◇ Nuclear explosions

◇ Volcanic eruptions

✓ **An instability of an interface between two fluids of different densities which occurs when the lighter fluid is pushing the heavier fluid.**

Picture from internet

Background

Key issues in RTI researches:

- Complicated and large deformation of two –phase interface
- Sensitivity to even tiny experimental or numerical disturbance
- Multiphase flow

Present method

- Modified MPS method for single phase (MLParticle-SJTU solver)
- Special treatments for interface
  - Density and viscosity discontinuity cross the interface
  - Different surface tension
  - Buoyancy inaccuracy

Background

Key issues in RTI researches:

- **Complicated and large deformation of two –phase interface**
- **Sensitivity to even tiny experimental or numerical disturbance**
- **Multiphase flow**

Present method

- **Modified MPS method for single phase (MLParticle-SJTU solver)**
- **Special treatments for interface**
  - **Density and viscosity discontinuity cross the interface**
  - **Different surface tensions**
  - **Buoyancy inaccuracy**

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Modified MPS Method

Semi-implicit algorithm of MPS

Governing Equations

$$\nabla \cdot \boldsymbol{V} = 0 \quad \text{or} \quad \frac{d\rho}{dt} = 0$$

$$\frac{D\boldsymbol{V}}{Dt} = -\frac{1}{\rho}\nabla P + \boxed{\nu\nabla^2\boldsymbol{V} + \boldsymbol{g}}$$

- Prediction step (explicit step):

$$\frac{D\boldsymbol{V}^*}{Dt} = \nu\nabla^2\boldsymbol{V} + \boldsymbol{g}$$

$$\boldsymbol{V}^* = \boldsymbol{V}^n + (\nu\nabla^2\boldsymbol{V} + \boldsymbol{g})\Delta \mathrm{t}$$

$$\boldsymbol{r}^* = \boldsymbol{r}^n + \boldsymbol{V}^*\Delta \mathrm{t}$$

$$\longrightarrow \quad \nabla \cdot \boldsymbol{V} \neq 0, \qquad \frac{d\rho}{dt} \neq 0$$

Modified MPS Method

Semi-implicit algorithm of MPS

Governing Equations

$$\nabla \cdot \boldsymbol{V} = 0 \quad \text{or} \quad \frac{d\rho}{dt} = 0$$

$$\frac{D\boldsymbol{V}}{Dt} = \boxed{-\frac{1}{\rho}\nabla P} + \nu\nabla^2 \boldsymbol{V} + \boldsymbol{g}$$

- Correction step (implicit step):

Poisson Pressure Equation

$$< \nabla^2 P^{n+1} > = \frac{\rho}{\Delta t^2}\frac{< n^* >_i - n^0}{n^0}$$

$$\boldsymbol{V}^{n+1} = \boldsymbol{V}^* + (-\frac{1}{\rho}\nabla P)\Delta t$$

$$\boldsymbol{r}^{n+1} = \boldsymbol{r}^n + \boldsymbol{V}^{n+1}\Delta t$$

$\Longrightarrow$

$$\nabla \cdot \boldsymbol{V} = 0$$

$$\frac{d\rho}{dt} = 0$$

Modified MPS Method

Modifications

Kernel function

$$W(r)=\begin{cases}\dfrac{r_e}{0.85r+0.15r_e}-1 & 0\le r<r_e\\ 0 & r_e\le r\end{cases}$$

- no singularity

Pressure Gradient

$$<\nabla P>_i=\frac{D}{n^0}\sum_{j\neq i}\frac{P_j+P_i}{|\boldsymbol{r}_j-\boldsymbol{r}_i|^2}(\boldsymbol{r}_j-\boldsymbol{r}_i)\cdot W(|\boldsymbol{r}_j-\boldsymbol{r}_i|)$$

- conservation of linear moment

Poisson Pressure Equation

$$<\nabla^2P^{n+1}>_i=(1-\gamma)\frac{\rho}{\Delta t}\nabla\cdot\boldsymbol{V}^*-\gamma\frac{\rho}{\Delta t^2}\frac{<n^*>_i-n^0}{n^0}$$

- instantaneous error
- accumulated error

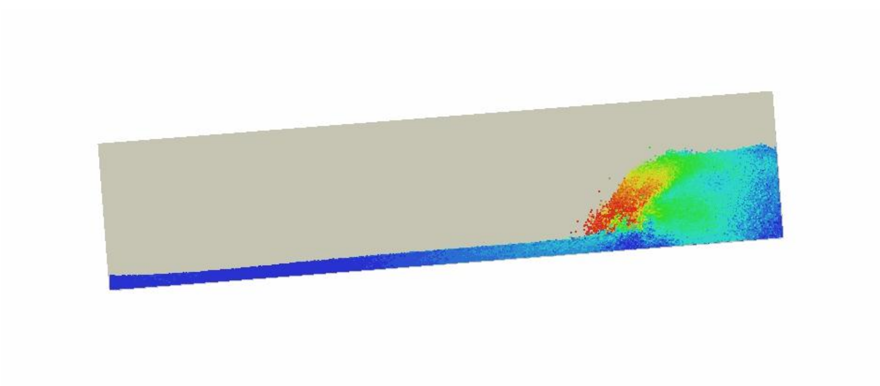
Free surface detection

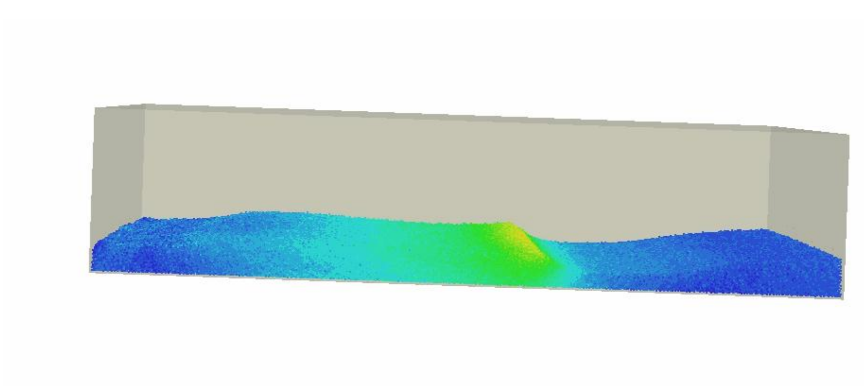
$$<\boldsymbol{F}>_i=\frac{D}{n^0}\sum_{j\neq i}\frac{1}{|\boldsymbol{r}_i-\boldsymbol{r}_j|}(\boldsymbol{r}_i-\boldsymbol{r}_j)W(\boldsymbol{r}_{ij})$$

- accurate capturing of free surface (interface) particles

Modified MPS Method

- **MLParticle-SJTU**

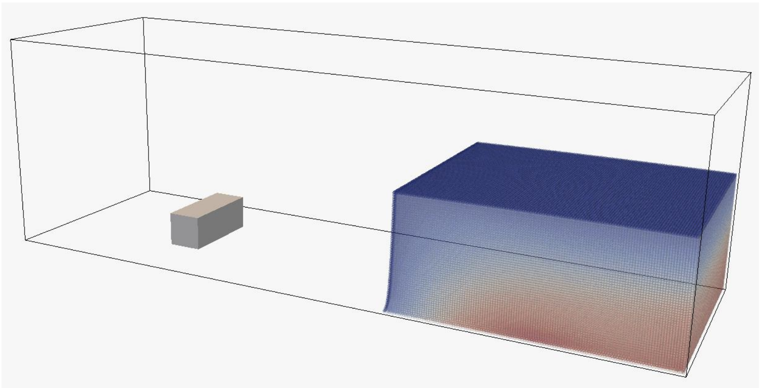




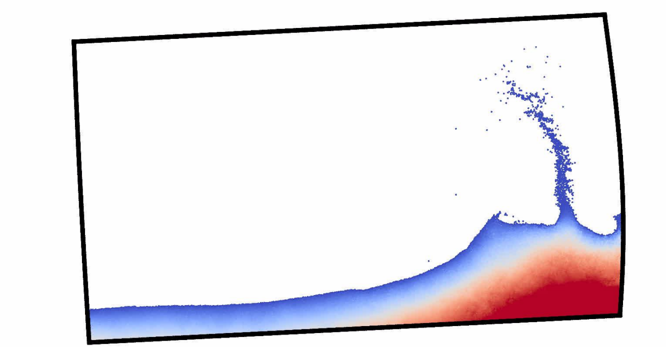
◇ **Liquid sloshing**

Modified MPS Method

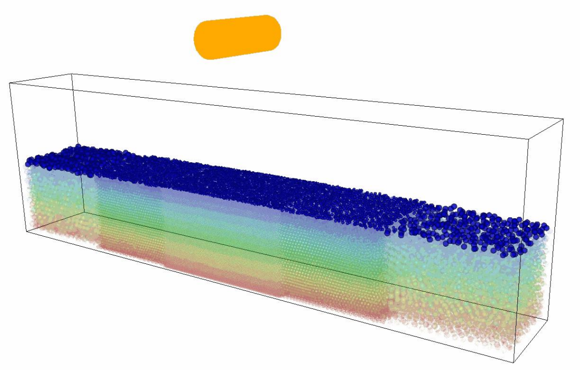
MLParticle-SJTU



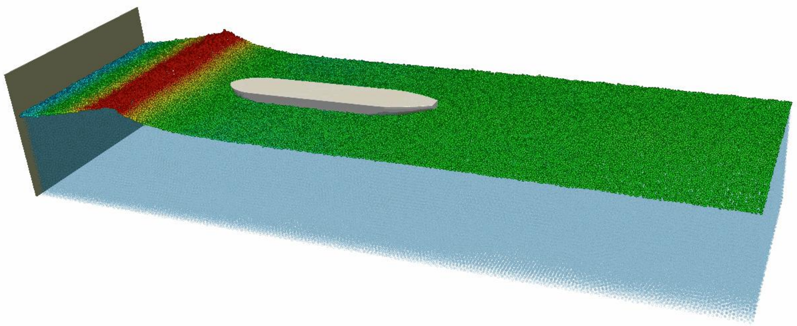
◇ 3D dam break



◇ Fluid-structure interaction



◇ Water entry problem



◇ Wave-Floating body interaction

Contents

Multiphase MPS solver

Multi-density multi-viscosity model

- The form of governing equations for different fluids is completely identical.
- Density and viscosity discontinuity are solved by the setting of transitional region near the interface.
- Continuum Surface Force model (CSF) is applied to the interface particles.
- Buoyancy-Correction model is included to compensate the underestimate of buoyancy.

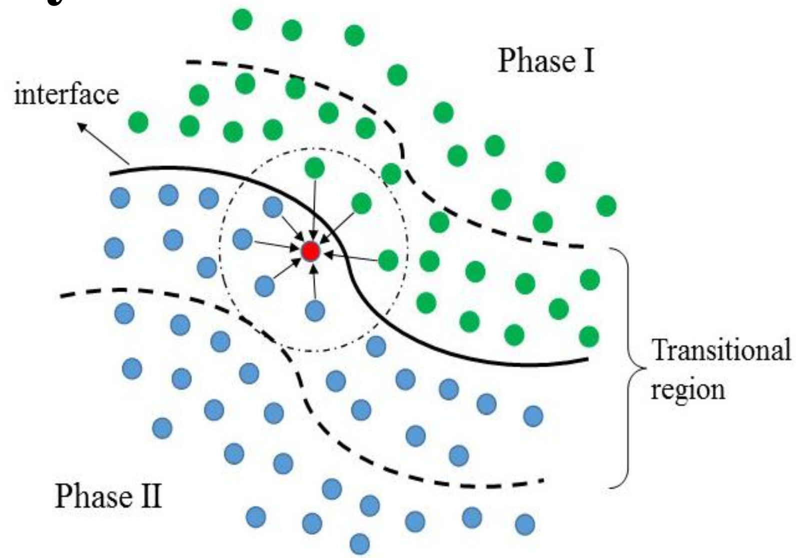
Multiphase MPS solver

Density and viscosity discontinuity

Particles in transitional region :

$$\frac{DV}{Dt} = -\frac{1}{\rho}\nabla P + \nu\nabla^2 V + g$$

$$\frac{DV}{Dt} = -\frac{1}{\langle\rho\rangle}\nabla P + \langle\nu\nabla^2 V\rangle + g$$



◇ **Transitional region**

Smooth Density

$$\langle\rho\rangle_i = \frac{\sum_{j\neq i}\rho_j \cdot W(r_{ij})}{\sum_{j\neq i} W(r_{ij})}$$

Interactive Viscosity

$$\langle\nu\nabla^2 u\rangle_i = \frac{2d}{\lambda n^0}\sum_{j\neq i}\left(\nu_{ij}(\nu_j - \nu_i)\cdot W(r_{ij})\right)$$

$$\nu_{ij} = \left(\left(\nu_i^{-1} + \nu_j^{-1}\right)/2\right)^{-1}$$

Multiphase MPS solver

Density and viscosity discontinuity

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$$\frac{DV}{Dt} = -\frac{1}{\rho}\nabla P + \nu\nabla^2 V + g$$

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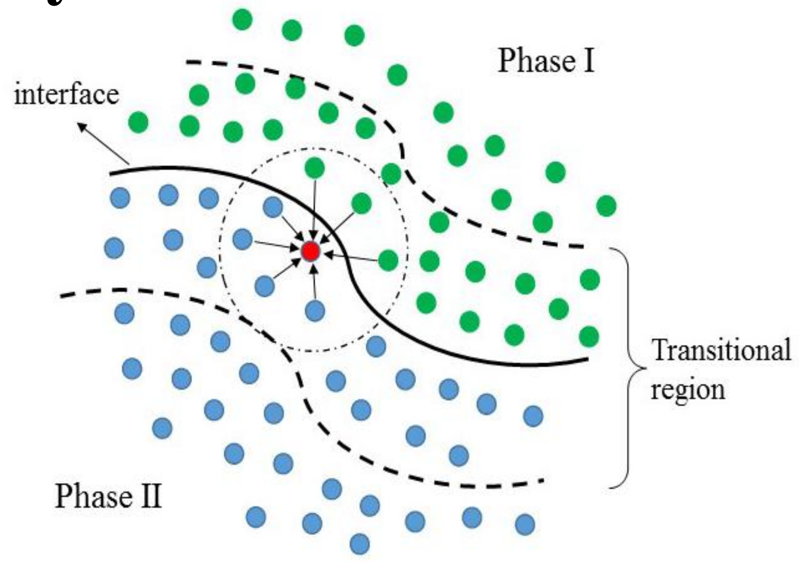

◇ **Transitional region**

➤Smooth Density

$$\langle\rho\rangle_i = \frac{\sum_{j\neq i}\rho_j \cdot W(r_{ij})}{\sum_{j\neq i} W(r_{ij})}$$

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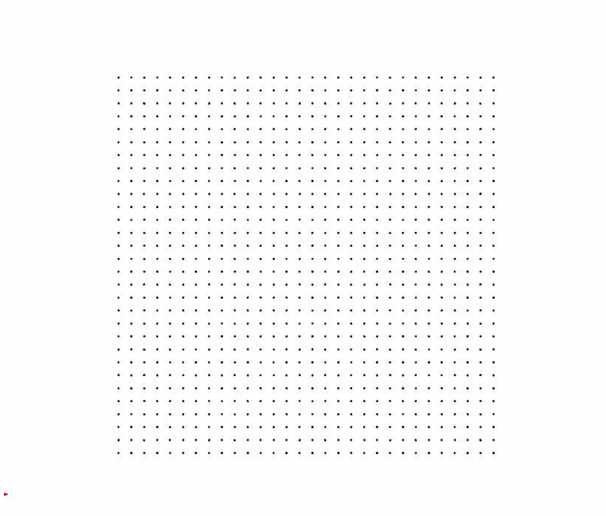
Multiphase MPS solver

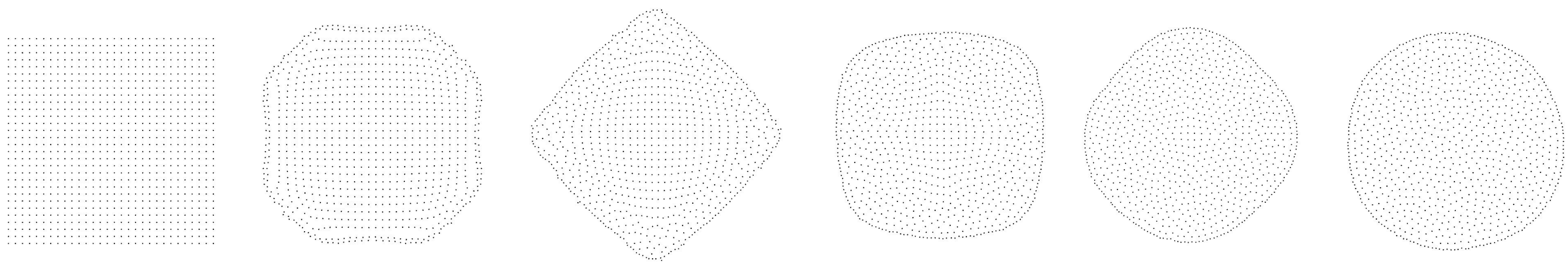
Continuum Surface Force Model (CSF)

$$\frac{DV}{Dt} = -\frac{1}{\rho}\nabla P + \nu\nabla^2 V + \boldsymbol{g}$$

$$\frac{DV}{Dt} = -\frac{1}{\rho}\nabla P + \nu\nabla^2 V + \boldsymbol{g} + \sigma\kappa\vec{n}$$

(Surface tension term)





t=0 s | t=0.25 s | t=0.55 s | t=1.25 s | t=2.5 s | t=5 s

◇ **Transformation of a square droplet (900 particles)**

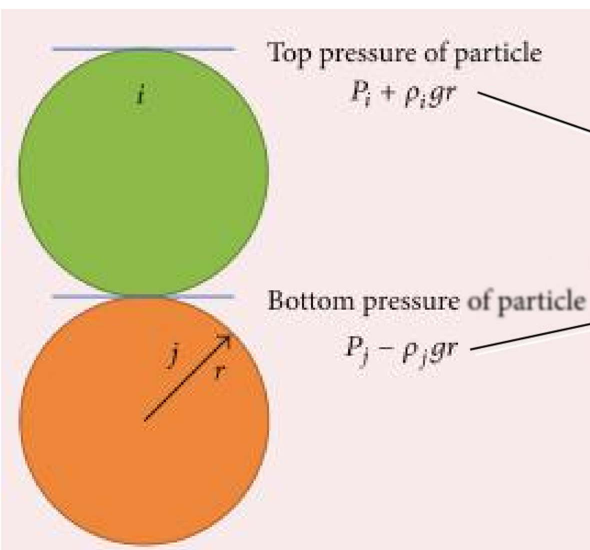
Multiphase MPS solver

Buoyancy-Correction Model

Pressure gradient model with Buoyancy-Correction:

$$<\nabla P>_i = \frac{D}{n^0}\left[\sum_{j\neq i}\frac{P_j+P_i}{|\boldsymbol{r}_j-\boldsymbol{r}_i|^2}(\boldsymbol{r}_j-\boldsymbol{r}_i)\cdot W(|\boldsymbol{r}_j-\boldsymbol{r}_i|)-\sum_{j\neq i}\frac{g(\rho_j-\rho_i)(z_j-z_i)}{|\boldsymbol{r}_j-\boldsymbol{r}_i|^2}(\boldsymbol{r}_j-\boldsymbol{r}_i)\cdot W(|\boldsymbol{r}_j-\boldsymbol{r}_i|)\right]$$

(Buoyancy term)

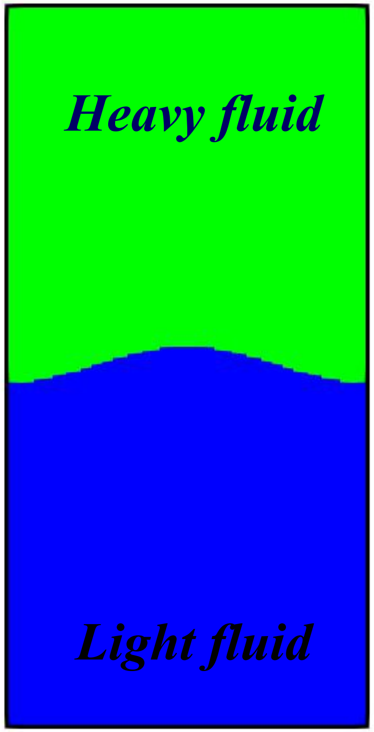


$$(P_j-\rho_j gr)-(P_i+\rho_i gr)$$
$$=(P_j-P_i)-\boxed{(\rho_j-\rho_i)gr}\ \cdots\ \begin{cases} =0 & \text{Single phase} \\ \neq 0 & \text{Multiphase} \end{cases}$$

◇ **Buoyancy Force of particles**

Contents

Numerical model



◇ Scheme of RTI

Initial disturbance

$$y_{\text{interface}} = 0.025\cos(4\pi x)$$

Parameters	Values
Density of light fluid	1000(kg/m^3)
Tank width	0.5(m)
Tank height	1.0(m)
Kinematic viscosity	0 (m^2/s)
Gravitational acceleration	9.81(m/s^2)
Particle spacing	0.005(m)
Fluid number	19701
Total number	25389

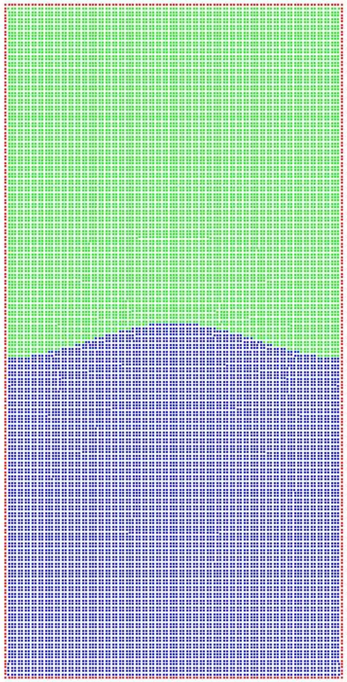
◇ Computational parameters

Case No.	Density of heavy fluid	Density ratio
1	2000(kg/m^3)	2:1
2	3000(kg/m^3)	3:1
3	4000(kg/m^3)	4:1
4	5000(kg/m^3)	5:1

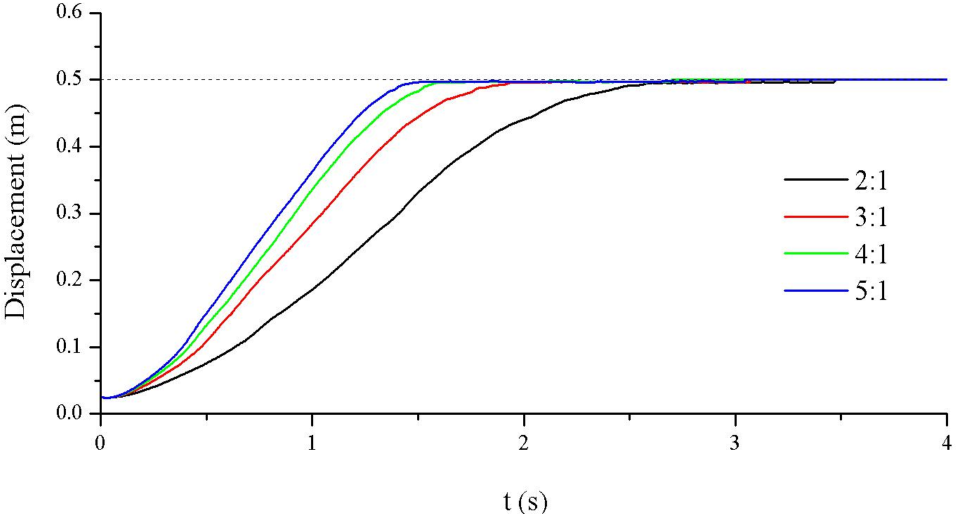
◇ Density ratios in different cases

Numerical results

Dynamic development of RTI



◇ Evolution of RTI

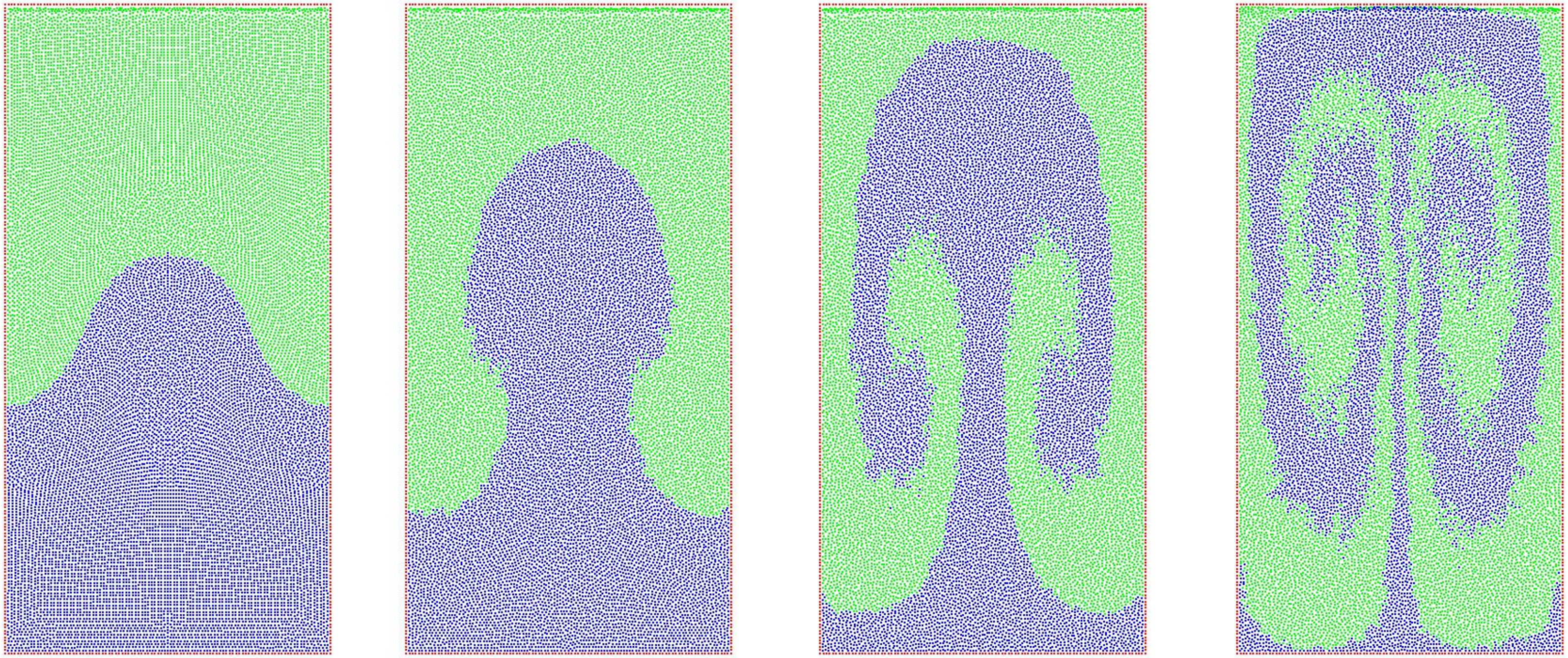


◇ Displacement of the peak of the lighter fluid with different density ratios

- The initial perturbation soon develops into a mushroom-shaped interface.
- With density raio increasing, the development rate of RTI grows fast.

Numerical results

Dynamic development of RTI



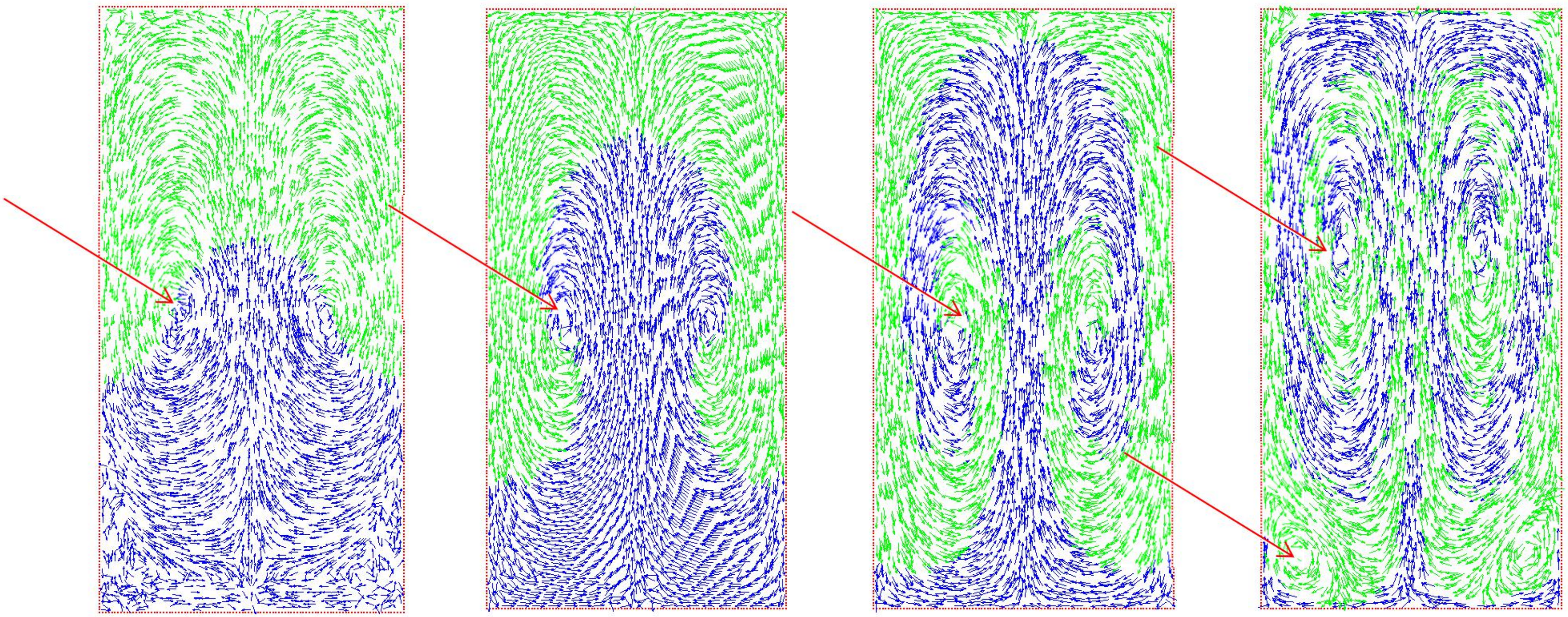
t=0.5 s　　t=1 s　　t=1.5 s　　t=2 s

◇ Evolution of RTI with a density ratio of 3:1

➢ The two-phase interface is clear and natural, even when the interface is quite distorted

Numerical results

Vorticity explanation of RTI



t=0.5 s t=1 s t=1.5 s t=2 s

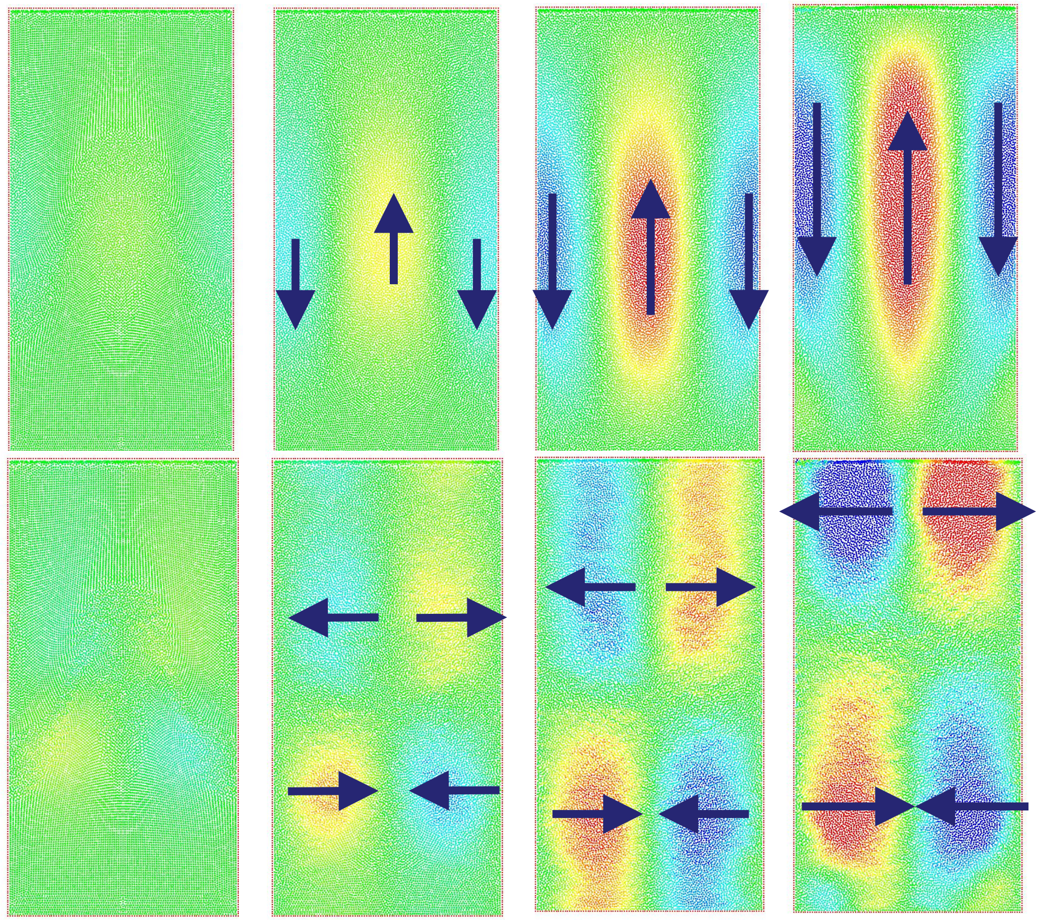
◇ Velocity vector of RTI with a density ratio of 3:1

✓Two vortexes appear and develop fast near the interface

✓Another two vortexes appear at the later stage near the corner

Numerical results

Velocity distribution of RTI



t=0.5 s t=1 s t=1.5 s t=2 s

◇ Y-velocity(top) and X-velocity(bottom) of RTI with a density ratio of 3:1

➢The middle and two sides of the fluid move in opposite directions.

➢Convective flow appears in the lower part, with the upper part inverse.

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Conclusion

- ✓ Through applying special interface treatment to our in-house single phase particle method solver MLParticle-SJTU, a new multiphase MPS solver is developed to simulate the Rayleigh-Taylor Instability.

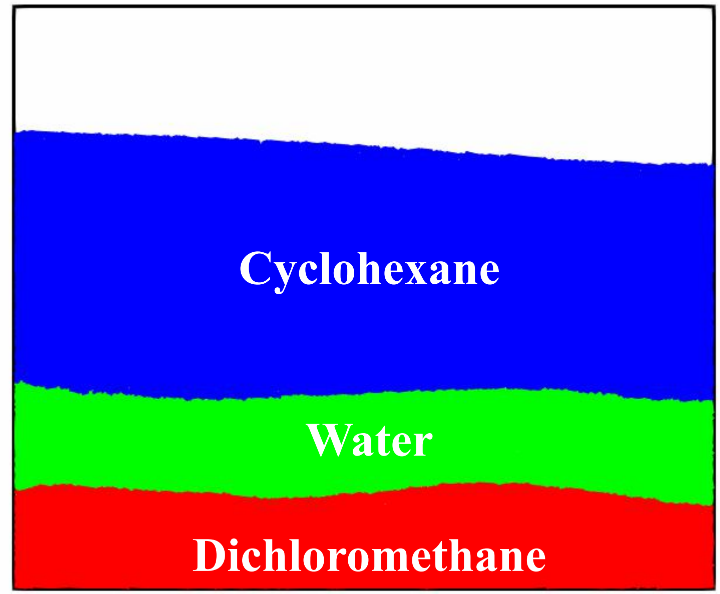
- ✓In RTI simulation, stable and accurate results can be obtained by the new multiphase MPS solver. The unphysical penetrations are well controlled, and the two-phase interface is clear and natural. The vortex motion in RTI problems is well captured and the typical mushroom-shaped interface is observed.

- ✓The simulations of RTI with different density ratios demonstrate the important role of the high density ratio in improving the growth rate of RTI, and also validate the applicability of the MPS multiphase method in different conditions.

Ongoing Work

- Multi-liquid-layer sloshing in separator of different fluids

◇ **Multi-liquid-layer sloshing experiment (Molin et al, 2015)**



◇ **Multiphase MPS solver**

Thank You ！

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