Please Move Quickly and Quietly to the Nearest Freeway

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Introduction

We construct a model that expresses total evacuation time for a given route as a function of car density, route length, and number of cars on the route. When supplied values for the last two variables, this function can be minimized to give an optimal car density at which to evacuate.

We use this model to compare strategies and find that any evacuation plan must first and foremost develop a method of staggering traffic flow to create a constant and moderate car density. This greatly decreases the evacuation time as well as congestion problems.

If an even speedier evacuation is necessary, we found that making I-26 one-way would be effective. Making other routes one-way or placing limits on the type or number of cars prove to be unnecessary or ineffective.

We also conclude that other traffic on I-95 would have a negligible impact on the evacuation time, and that shelters built in Columbia would improve evacuation time only if backups were forming on the highways leading away from the city.

Prologue

As Locke asserted [1690], power is bestowed upon the government by the will of the people, namely to protect their property. A government that cannot provide this, such as South Carolina during an act of God as threatening as Hurricane Floyd and his super-friends, is in serious danger of revolutionary

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overthrow by the stranded masses marching from the highways to the capitol. Therefore, South Carolina must find the most effective evacuation program—one that not only provides for the safety of its citizens but also allows for households to rescue as many of their vehicles (read: property) as possible. Pitted against the wrath of God and Nature, one can only hope the power of mathematical modeling can protect the stability of South Carolinian bureaucracy.

Since the goal is to create a useful model that even an elected official can use, our model operates most effectively on the idea that government agencies are poor at higher-level math but good at number-crunching. Our model provides a clear, concise formula for weighing relative total evacuation times and likely individual trip times. This is crucial in deciding how to order a wide-area evacuation while maintaining public approval of the operation and preventing a coup d'état.

Our model shows that of the four strategies for evacuation suggested by the problem statement, staggering evacuation orders is always the most effective choice rather than simply reversing I-26. After that, applying any one of the other options, like lane reversal on I-26 and / or on secondary evacuation routes, can improve the evacuation plan. However, using more than one of these techniques results in a predicted average driver speed in excess of the state speed limit of 70 mph.

Of the three additional methods, we find that the most effective is to make I-26 one-way during peak evacuation times. Implementing the same plan on secondary highways would require excessive manpower from law enforcement officials, and regulating the passenger capacity is too difficult a venture in a critical situation.

We also find that, given the simplifications of the model, I-95 should have a negligible effect.

Furthermore, the construction of shelters in Columbia would facilitate the evacuation only if the highways leading away from Columbia were causing backups in the city.

Analysis of the Problem

To explain the massive slowdowns on I-26 during the 1999 evacuation, our team theorized that substantially high vehicle density causes the average speed of traffic to decrease drastically. Our principal goal is to minimize evacuation times for the entire area by maximizing highway throughput, that is, the highest speed at which the highest density of traffic can travel. Given this fact, we seek to find the relationships between speed, car density, and total evacuation time.

Assumptions

To restrict our model, we assume that all evacuation travel uses designated evacuation highways.

We assume that traffic patterns are smoothly and evenly distributed and that drivers drive as safely as possible. There are no accidents or erratically driving "weavers" in our scenario. This is perhaps our weakest assumption, since this is clearly not the case in reality, but it is one that we felt was necessary to keep our model simple.

Our model also requires that when unhindered by obstacles, drivers travel at the maximum legal speed. Many drivers exceed the speed limit; however, we do not have the information to model accurately the effects of unsafe driving speeds, and a plan designed for the government should avoid encouraging speeding.

As suggested by the problem, we simplify the actual distribution of population across the region placing 500,000 in Charleston, 200,000 in Myrtle Beach, and an even distribution of the remaining approximately 250,000 people.

Multiple-lane highways and highway interchanges are likely to be more complicated than our approximation, but we simplify these aspects so that our model will be clear and simple enough to be implemented by the government.

Distribution of traffic among the interstate and secondary highways in our model behaves according to the results of a survey, which indicates that 20% of evacuees chose to use I-26 for some part of their trip.

The Model

We begin by modeling the traffic of I-26 from Charleston to Columbia, as we believe that understanding I-26 is the key to solving the traffic problems.

We derived two key formulas, the first $s(\rho)$ describing speed as a function of car density and the second $e(\rho)$ describing the total evacuation time as a function of car density:

$$s(\rho) = \sqrt{\frac{1}{k}\left(\frac{5280}{\rho} - l - b\right)}, \qquad e(\rho) = \frac{L\rho + N}{\rho s(\rho)}.$$

The constants are:

k = braking constant,

l = average length of cars in feet, and

b = buffer zone in feet.

The variables are:

ρ = car density in cars/mi,

L = length of highway in feet, and

N = number of cars to be evacuated.

The method is to maximize $e(\rho)$ for a given N and L, which gives us an optimal car density.

Derivation of the Model

The massive number of variables associated with modeling traffic on a micro basis leads to a very complex and difficult problem. Ideally, one could consider such factors as a driver's experience, his or her psychological profile and current mood, the condition of the mode of transportation, whether his or her favorite Beach Boys song was currently playing on the radio, etc. Then one could use a supercomputer to model the behavior of several hundred thousand individuals interacting on one of our nation's vast interstate highways. Instead, our model analyzes traffic on a macro basis.

The greater the concentration of cars, the slower the speed at which the individuals can safely drive. What dictates the concentration of cars? Well, the concentration is clearly related to the distance between cars, since the greater the distance, the smaller the concentration, and vice versa. On any interstate highway, drivers allot a certain safe traveling distance between their car and the car directly in front of them, to allow time to react. Higher speeds require the same reaction time but consequently a greater safe traveling distance. How do we determine what the correct distance between cars at a given speed? The braking distance d of a car is proportional to the square of that car's speed v. That is, $d = kv^2$ for some constant k. The value for k is 0.0136049 ft-hr^{2}mi^2; we derive this value by fitting $\ln d = \ln k + b \ln v$ with data from Dean et al. [2001]; the fit has $r^2 = .99999996$.

However, the distance between cars is an awkward measurement to use. Our goal is to model traffic flow. With our model, we manipulate the traffic flow until we find its optimal value. The distance between cars is hard to control, but other values, such as the concentration or density of the cars in a given space, are much easier to control.

How do we find the value of the car density? To start, any distance can be subdivided into the space occupied by cars and the space between cars. The space occupied by cars can be assumed to be a multiple of the average car length l. The space between cars is clearly related to the braking distance, but the two are not necessarily the same. The braking distance at low speeds (< 10 mph) is less than a foot. However, ordinary experience reveals that even at standstill traffic, the distance between cars is still much greater than a foot; drivers still leave a buffer zone in addition to the safe breaking distance. Then each car has a space associated with it, given by $d + l + b$, where b is the average buffer zone in feet. Since this expression is of the form of 1 car per unit distance, this is in itself a density. We can also convert this to more useful units, such as cars per mile:

$$\rho = \frac{\text{cars}}{\text{mi}} = \frac{\text{cars}}{\text{ft}} \times \frac{5280 \text{ ft}}{mi} = \frac{5280}{d+b+l} \times \frac{\text{cars}}{\text{mi}} = \frac{5280}{kv^2+b+l}\,\frac{\text{cars}}{\text{mi}}.$$

Solving for v gives

$$s(\rho) = v = \sqrt{\frac{1}{k}\left(\frac{5280}{\rho} - l - b\right)}.$$

At this point we can substitute $k = 0.0136049$ ft $\cdot$ h^2/mi^2, $l = 17$ ft (from researching sizes of cars), and $b = 10$ ft (from our personal experience) and graph speed as a function of density (**Figure 1**).

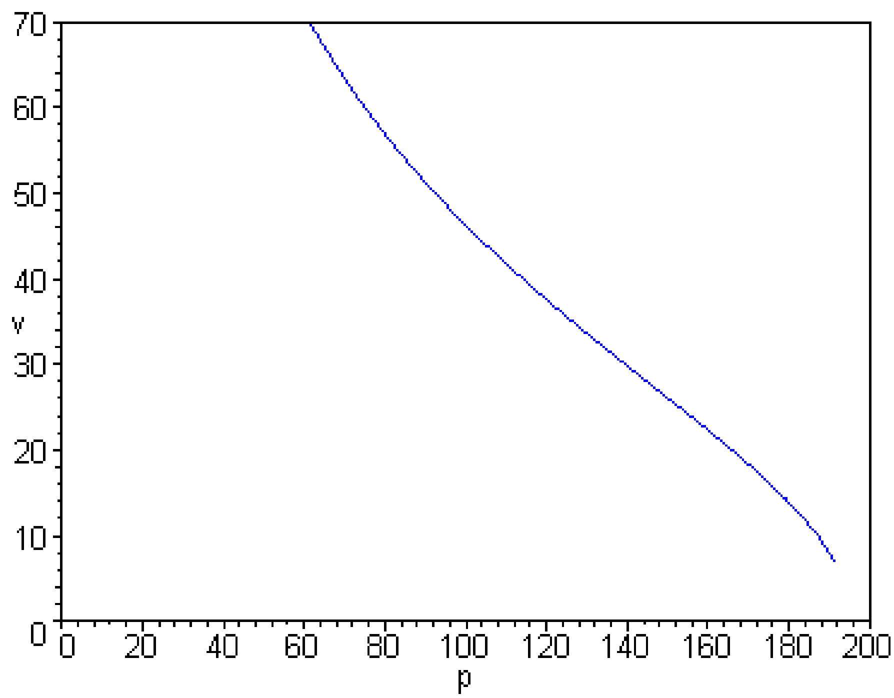

Figure 1. Speed as a function of density.

Note the maximum density at 195.6 cars/mi. To understand why a maximum density exists, consider the case $b = 0$; as $v \to 0$, the distance between the cars approaches zero.

We now determine how long it takes this group of cars to reach their destination. For now, we say that the group reaches its goal whenever the first car arrives. This is a simple calculation: We divide the length of the road L by the average speed of the group

$$t(\rho) = \frac{L}{s(\rho)} = \frac{L}{\sqrt{\frac{1}{k}\left(\frac{5280}{\rho} - l - b\right)}}.$$

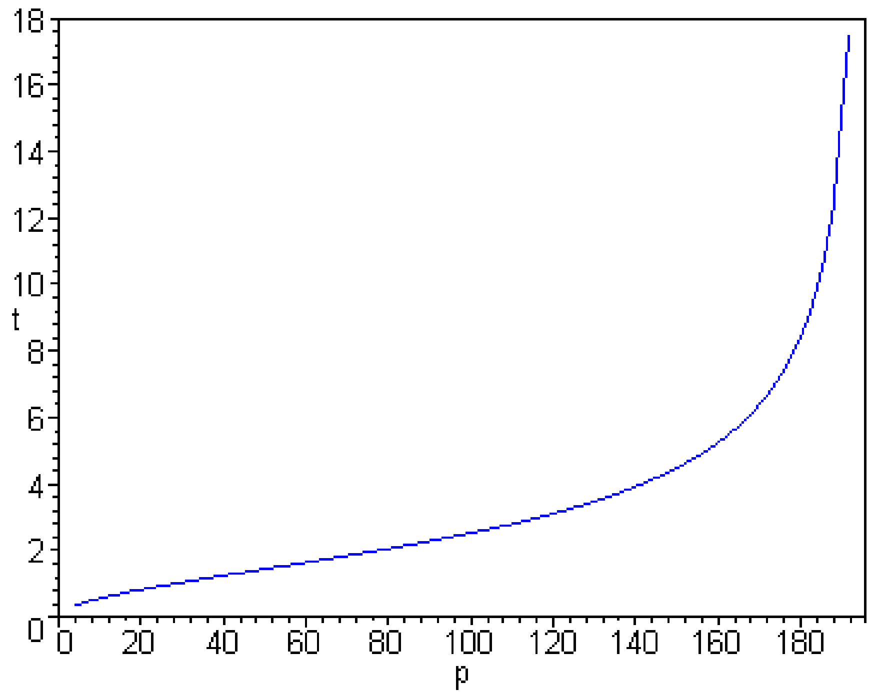

Figure 2. Evacuation time as a function of density.

We refer to this function as t because it gives the time of the trip. **Figure 2** gives t for I-26 between Charleston and Columbia, which has a length of 117 mi.

However, the real problem is not simply evacuating one group of cars, it is quickly evacuating a large number of cars. Our goal poses an interesting dilemma: If we evacuate in a stream of low density, they will travel very fast but evacuation takes an extremely long time. If we evacuate in a stream of very high car density (which is what happened during Hurricane Floyd), many people will move at once but they will move very slowly. We must seek the middle ground.

We express the total evacuation time as a function of car density; then taking the minimum gives the optimum car density.

Unfortunately, the concept of average car density doesn't work as well over large distances. The problem with traffic flow is that it tends to clump, creating high car densities and thus low speeds. However, the instantaneous car density won't vary much for a small distance, such as a mile, compared to the average car density over the entire highway. To look ahead to the problem: Making I-26 one-way will certainly help facilitate evacuation, but it won't help nearly as much as staggering the evacuation flow. Staggering is the only way to realistically create a constant traffic flow and thus an average car density that is more or less constant over the entire length of the highway.

Suppose that we take the N cars that need to be evacuated and subdivide them into groups, each consisting of the number of cars that there are in 1 mi. Call these groups *packets*. Each packet is 1 mi long.

We look at two cases, one where we send only one group and another where we send more than one group. For the first case, we assume that all N cars fit in one packet, so $0 < N < 196$, where 196 is the maximum car density for our values of buffer zone and average car length. Everyone is not technically evacuated until the end of the packet arrives safely, so we need to add to $t(\rho)$ the additional time for the end of the packet to arrive:

$$e(\rho) = t(\rho) + \frac{1}{s(\rho)}.$$

We call this expression e because it express total evacuation time as a function of car density.

For the second case, we can say that the packets travel like a train, as you can't release another packet until the first packet is a mile away. Evacuation time is the sum of the time for the first packet to arrive plus the time until the last packet arrives. Since the packets arrive in order, and they are all one mile long, the time it takes the last packet to arrive is equal to the number of packets times the time it takes the packets to move 1 mi. The equation is

$$e(\rho) = t(\rho) + \frac{1}{s(\rho)} \frac{N}{\rho}.$$

For one packet, this second equation simplifies to the first.

After some algebra, we arrive at

$$e(\rho) = \frac{L\rho + N}{\rho s(\rho)}.$$

The total evacuation time e is simply a function of three variables: the length of the highway L, the number of cars N, and the car density ρ. The speed $s(\rho)$ is itself a function of ρ and three other constants (the braking constant k, the average car length l, and the buffer zone b). Setting $L = 117$ and $N = 65{,}000$ produces the graph in **Figure 3**, which has a clear minimum.

Application of the Model

The model should first deal with I-26. We assume that evacuation along this road takes the longest. We limit our consideration to Charleston, Columbia, and I-26 between them.

Modeling I-26 Traffic Flow

The problem states that the evacuation consists of 500,000 residents from Charleston, 200,000 from Myrtle Beach, and 250,000 others. However, the model need not evacuate all 950,000. The evacuation rate of 64% for Hurricane Floyd was one of the highest evacuation rates ever seen for a hurricane.

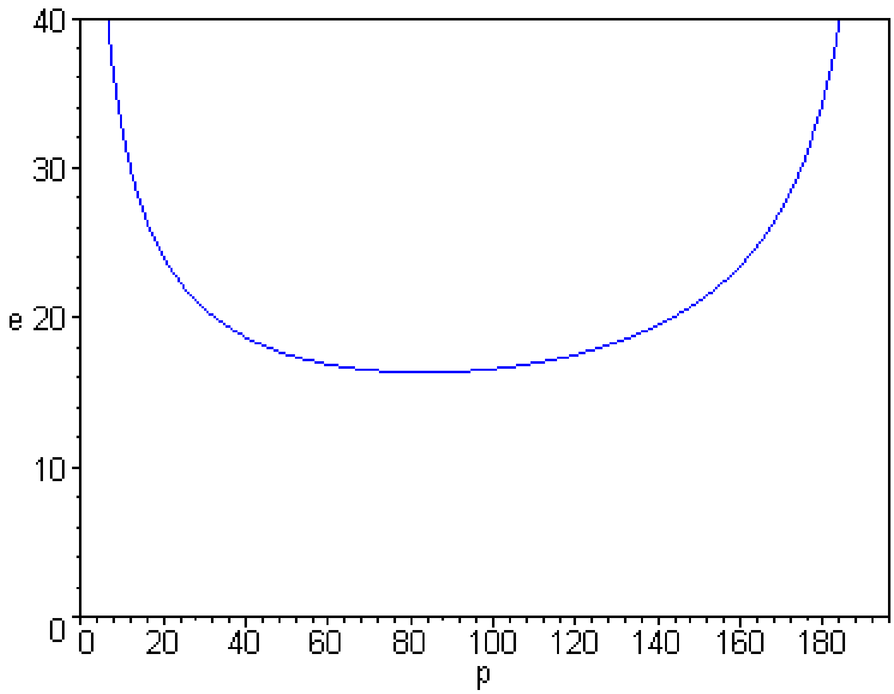

Figure 3. Total evacuation time as a function of density.

Many individuals, whether elderly, financially unable to make the trip, or just darn stubborn remain close to home rather than join the throng of frantic drivers fleeing for their lives. Further, only about 20% of those who left did so along Interstate 26. Taking 20% of 64% of 950,000 gives 122,000 as the number of people that we can expect to evacuate on I-26.

We must convert number of people to cars, the unit in our model. In South Carolina there are 3,716,645 people and 1,604,000 households, or 2.3 people per household. Further research allowed us to find the average number of cars taken for each household. Dow and Cutter [2001] included **Table 2** on the number cars taken by each household.

Table 2.

Cars taken by each household in 1999 Hurricane Floyd evacuation [Dow and Cutter 2001].

Cars	% of households
0	3%
1	72%
2	21%
3+	4%

Since the number of households taking more than three cars is merely a fraction of the 4% of the population evacuating, we assume that a household takes 0, 1, 2, or 3 cars. Thus, we find a weighted average of 1.26 cars/household.

We now calculate the average number of people per car:

$$\frac{\text{people}}{\text{car}} = \frac{\text{people/household}}{\text{cars/household}} = 1.83.$$

We divide the evacuation population by the average number of people per car to find that 66,655 cars need to be evacuated.

Both $s(\rho)$ and $t(\rho)$ are independent of the number of cars to evacuate: $s(\rho)$ is based on highway information that stays constant throughout the model, and $t(\rho)$ is based on the length of the highway. The only function dependent on the numbers of cars is total evacuation time, $e(\rho)$. Substituting values for b, l, N, and L, we arrive at

$$e_l(\rho) = \frac{117\rho + 66655}{\rho \dfrac{1}{0.0136049}\left(\dfrac{5280}{\rho} - 27\right)}.$$

After finding the minimum value, we arrive at

$$\begin{aligned} \min \rho_l &= 83 \text{ cars/mi}, & s &= 52 \text{ mph}, \\ t &= 2 \text{ h } 15 \text{ min}, & e_l &= 18 \text{ h}. \end{aligned}$$

Our model does not evacuate people very quickly, but there is a significant decrease in average trip time $t(\rho)$ for an individual car.

Our model applies to only one lane of traffic. If cars on I-26 were allowed to travel in only one lane but at optimal density, the total evacuation time would be approximately 18 h, with each car making the journey in just over 2 h at an average speed of 52 mph.

We assume that adding another highway lane halves the number of cars per lane and find

$$\begin{aligned} \min \rho_l &= 73 \text{ cars/mi}, & s &= 58 \text{ mph}, \\ t &= 2 \text{ h } 0 \text{ min}, & e_l &= 10 \text{ h}. \end{aligned}$$

Although the average trip time slightly decreased and speed slightly increased, the most striking result of opening another lane is the halving of total evacuation time.

Turning the entire I-26 into one-way traffic turns I-26 into a pair of two-lane highways rather than a four-lane highway; adding another pair of lanes to our already existing pair of lanes again halves the number of cars to be evacuated per lane.

$$\begin{aligned} \min \rho_l &= 58 \text{ cars/mi}, & s &= 68 \text{ mph}, \\ t &= 1 \text{ h } 40 \text{ min}, & e_l &= 6 \text{ h}. \end{aligned}$$

Conclusions from the I-26 Model

The problem explicitly mentions four means by which traffic flow may be improved:

- turning I-26 one-way,
- staggering evacuation,
- turning smaller highways one-way, or
- limiting the number or type of cars.

Staggering

Contrary to the emphasis of the South Carolina Emergency Preparedness Division (SCEPD), the primary proposal should not be the reversal of southbound traffic on I-26 but rather the establishment of a staggering plan. One glance at the total evacuation time vs. car density graph reveals the great benefits gained from maintaining a constant and moderate car density on the highway. The majority of the problems encountered during Hurricane Floyd, such as the 18-hour trips to Columbia or incidents of cars running out of gas on the highway, would be solved if a constant car density existed on the highway.

However, while our model assumes a certain constant car density, it does not provide a method for producing such density. Therefore, the SCEPD should produce a plan that staggers the evacuation to maintain a more constant car density. One proposal was a county-by-county stagger. SCEPD should take the optimal car density and multiply that by the optimal speed to arrive at an optimal value of cars/hour. The SCEPD should then arrange the stagger so that dispersal of cars per hour is as close as possible close to the optimal value.

Making I-26 One-Way

In addition to the staggering plan, making I-26 one-way reduces the total evacuation time from 10 h to 6 h. Thus, while staggering should always be implemented, reversal of traffic on I-26 should supplement the staggering plan when the SCEPD desires a shorter evacuation time.

The Other Options

This leaves two more strategies for managing traffic flow: turning smaller highways one-way and limiting the number or types of car taken per household. Both of these can be implemented easily in our model. Turning smaller highways one-way would encourage evacuees to take back roads instead of I-26, making percentage of evacuees taking I-26 less than 20%, reducing our value for N. Restricting the number of cars per household would also reduce N. Likewise, disallowing large vehicles would reduce l, the average length of vehicles.

Considering that the optimal speed calculated after making I-26 one-way is 68 mph, and that adding other evacuation strategies would raise the optimum average speed above the lawful limit of 70 mph, additional strategies would be unnecessary.

Adding Myrtle Beach

The next route needing consideration is 501 / I-20 leaving Myrtle Beach, with 200,000 people. Unfortunately, we lack statistics on how many people took the 501 / I-20 evacuation route. We can, however, make a guess using a ratio. We assume that an equal proportion of residents of Myrtle Beach evacuate using the main route as in Charleston, so that the number taking the highway leaving a city is directly proportional to the city's size. This suggests that 49,000 people leave Myrtle Beach in 49,000 / 1.84 = 26,600 cars, or 13,300 cars / lane. Combining this with an $L = 150$ mi, the distance between Myrtle Beach and Columbia, we can apply our model to arrive at

$$\begin{aligned} \min \rho_l &= 46 \text{ cars/mi}, & s &= 80 \text{ mph}, \\ t &= 1 \text{ h } 50 \text{ min}, & e_l &= 5 \text{ h } 28 \text{ min}. \end{aligned}$$

We cap the speed at 70 mph and determine a preferred car density of 56 cars / mi. This figure produces a new set of optimal values:

$$\begin{aligned} \min \rho_l &= 56 \text{ cars/mi}, & s &= 80 \text{ mph}, \\ t &= 2 \text{ h}, & e_l &= 5 \text{ h } 30 \text{ min}. \end{aligned}$$

This calculation also confirms one of our basic assumptions, that I-26 between Charleston and Columbia is the limiting factor: Evacuation from Myrtle Beach using two lanes still takes less time (5 h) than evacuation using 4 lanes on I-26 from Charleston (6 h). From this we can conclude that making traffic roads leading from Myrtle Beach one-way would be unnecessary. We assume that applying our model to other smaller highways will lead to similar results.

Adding Intersections and I-95

To simplify the model, we assume that intersecting routes of equal density contribute traffic to the adjoining route so that the difference in densities always seeks a balance. Intersections between roads of similar relative density do not cause unexpected spikes in density on either road. Their effect is therefore negligible.

On the case of two interstates of unequal traffic densities, the volume of traffic on the busier route may cause a substantial change in density on the adjoining route. We assume a normalizing tendency at interchanges, so drivers are not likely to change routes without immediate benefit of higher speed. We also assume that on interstates over long distances with only intermittent junctions, there is a normalizing tendency of traffic to distribute itself. Therefore, though I-95 may cause congestion problems, in our model the traffic on I-95 has a negligible effect on the overall evacuation traffic.

Adding Columbia and the Rest of the World

We view Columbia as a distribution center. Columbia accepts a certain number of cars per unit of time, dispenses another number of cars per unit of time to the rest of the world, and retains a certain amount of the traffic that it receives. In the case of an evacuation, this retention reflects evacuees who stay with families or find hotel rooms.

For example, staggered one-way traffic on I-26 yields optimal car density to be 58 cars/mi and the optimal speed of 68 mph. Multiplying these yields a flow of 3,944 cars/h into Columbia.

If the highways leading to the rest of the world can handle a large traffic flow in cars per unit time, evacuation time will not be affected. If backups outside of Columbia are a problem, building more shelters would help because Columbia would retain more of the incoming traffic. However, if the highways leading elsewhere can handle the flow, shelters are unnecessary.

Strengths and Weaknesses

One strength of the model is its formulaic practicality. With reliable measurements of traffic density and speed, the evacuation volume predictions predictions should be useful.

A second and more important strength of the model is its use for comparison. Its prediction can be considered a reference point for experimentation. After all, we are looking for improvements but not necessarily exact results. The model offers a range of possible values, and this is an advantage.

The simplifications and approximations taken in this model introduce obvious weaknesses, particularly in our concept of car density. In simplifying traffic flow, we assumed a homogeneity over distance, but this isn't likely to happen. Much of our model and its derivation hinge on the assumption that traffic density can be controlled.

With unconvincing generality, we calculate ranges of average speeds and assume that all drivers drive as fast and as safely as possible. However, it is likely that traffic will clump and cluster behind slower drivers.

A potential source of congestion, intersections, was simplified by assuming that relative traffic densities seek a balanced level. In reality, different routes are preferred over others and imbalances in traffic density are likely to occur.

The only way to test the model is to collect data during very heavy traffic. However, traffic as great as during Hurricane Floyd is rare.

In short, the weaknesses of the model are primarily related to its simplicity. However, it is that same simplicity that is its greatest strength.

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Dr. Ann Watkins, President of the Mathematical Association of America, congratulating MAA Winner Adam Dickey after he presented the team's model at MathFest in Madison, WI, in August. [Photo courtesy of Ruth Favro.]

Mathematical Commission Streamlines Hurricane Evacuation Plan

COLUMBIA, SC, FEB. 12— As long as the western coast of Africa stays where it is, global climate patterns will continue to have it in for South Carolina. That proverbial "it" is the hurricane season. Some may point their fingers at God and curse His ways, but cooler heads will eventually surrender—shutter their windows, pack their cars, and drive to Columbia for the next few days, mood unsettled.

However, a report issued from the governor's office earlier this week announced that scientists have designed a new plan for coastal emergency evacuation.

Prompted by public disapproval of evacuation tactics used during the mandatory evacuation ordered during Hurricane Floyd, the new study sought to find the source of the congestion problems that left motorists stranded on I-26 for up to 18 hours wondering why they even attempted evacuation at all.

"The government should have known that we didn't have the roads to get everybody out," one Charleston resident commented, "and we just had the speed limits raised, so I would've expected a quicker escape rout."

Governor Jim Hodges commissioned the evacuation study last Friday, expecting a quick response. So far, the results seem plausible and practical.

The private commission developed a mathematical model to describe the traffic flow that caused the backups responsible for the evacuation problems. Then, by manipulating the model and combining it with statistical survey data collected after the evacuation, the commission developed a report evaluating the various current suggestions to alleviate issues.

The findings of the commission suggest that the best way to avoid evacuation backups is to stagger and sequence the county and metropolitan evacuation orders so that main routes do not exceed the critical traffic density that caused the slowdown.

As explained by one member of the commission, "the model does not get into the gritty details of complicated traffic modeling, but it does present a useful framework for evaluating crisis plans and traffic routing."

The solution garnered some skeptical criticism among mathematics researchers state-wide. "The commission is missing the point here," commented one researcher at the Hazards Research Lab in the Department of Geology at the University of South Carolina. "The problem with Hurricane Floyd was an anomalously high rate of evacuation among coastal residents—this is a sociopsychological problem and not a mathematical one."

Despite the cool reception, the commission is confident in its findings. According to the report, a general staggering of traffic will far exceed the benefits of other methods of congestion control that have been suggested, such as reversing the eastbound lanes of I-26 and some secondary highways.

"We find that advance planning with a mind to reduce the traffic surge associated with quickly ordered mandatory evacuations is the most useful way to improve evacuations," said a spokesman for the commission.

Despite the current concerns, the proving grounds for the findings of the commission will not surface until this fall, when the tropical swells off the coast of Africa begin heading our way again.

—Corey R. Houmand, Andrew D. Pruett, and Adam S. Dickey in Winston–Salem, NC.