Judges' Commentary: The Outstanding Repeater Coordination Papers

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Overview

This year's problem dealt with finding the number of repeaters needed to create a VHS network to cover a circular region of radius 40 miles and simultaneously serve first 1,000, then 10,000, users. Naturally, there is quite a bit of literature available related to this topic.

Approaches

The approaches used could be broken down into two categories. Some papers focused first on covering the area, others on covering the population.

Covering the Area

There is much to be said for the simplicity and directness of the method of covering the area first. The most common approach was to tile the region with hexagons inscribed within circles of radius equal to the distance that a user's signal will reach effectively. Some papers shifted their hexagonal lattice back and forth to capture the minimum number of hexagons needed to cover the 40-mile circle. Good papers then generated simulated populations, generally uniformly distributed, to check if the number of repeaters was adequate for the usage load. Most then added more repeaters

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for the 10,000-user case. The better papers tested their results against non-uniformly distributed populations, either following some other distribution or concentrated in groups or towns. Some of the population generation that we saw was quite creative and demonstrative of good modeling.

Covering the Population

Many of the papers simulated user populations first, then attempted to cover all (or a high percentage of) the users with a minimal number of repeaters. Of course, if a population distribution is simulated and then covered with K repeaters, in general additional argument is needed before one can conclude that K repeaters will work for any such distribution. Most of the papers used repeated simulations as their argument. There were some interesting approaches used to cover the populations minimally, including greedy and genetic algorithms. One of the more creative papers assumed that although the goal was to cover 1,000 or 10,000 simultaneous users, there were in fact, more users than that; and that team's algorithm was designed to capture just the required number of users. Although this was a simplifying assumption that dramatically changed the problem, and judges felt that the uncovered users might not appreciate this approach, we could find nothing in the problem statement to preclude it; and the paper in question stated and justified the assumption.

The most disappointing feature of these papers, which were in general creative and presented interesting modeling, was that although their approaches clearly relied on advance knowledge of the locations of all the users in the region, virtually none of the papers highlighted this fact, either in the assumptions or in the weaknesses of their models. Although (as one might expect) this approach generally (especially in the 1,000-user case) required fewer repeaters than the area-first approach, almost without exception teams that took this approach failed to indicate that such an approach requires collecting and entering great amounts of data that may not even be available in a real-world application. While this fact does not necessarily negate the validity of the model or its results, the papers should have clearly stated the assumption that these locations must be known for the model to be useful and should also have mentioned this requirement as a disadvantage. Almost no teams made the reader aware of this critical fact.

Generally, the judges in the final stages, referring to flaws in papers, call a flaw that keeps a very good paper from being outstanding a "fatal flaw"; and our discussions and deliberations frequently come down to arguing whether or not a discovered flaw should be considered "fatal." Some felt that requiring knowledge of users' locations, while neither including such knowledge in the assumptions nor acknowledging the need as a weakness, should be a "fatal flaw"; but eventually the desire to have some Outstanding papers outweighed those feelings.

Determining the Required Spacing for the Repeaters

There was quite a bit of disagreement in the ranges used for the repeaters and the users. Papers generally correctly assumed that the range of the repeaters would be greater that the range of the users' equipment, making the latter the determining factor. But we saw ranges for repeaters going from about 3 miles to 100 miles. It is possible, using the radius of the Earth (and assuming that the Earth is perfectly spherical) to compute the "line of sight" distance to the horizon as a function of the height of the repeater. Some papers found this relationship, either in the literature or by computing it themselves. Others made reference to online sources giving the ranges for repeaters. Given the time constraints and the fact that this is a modeling contest, not a contest to distinguish engineering prowess, we did not use the range value as a discriminator, even though we suspected that some of the sources referenced may not have actually referred to VHS repeaters.

Use of Sources

In a contest of this nature, it is expected that participants will rely on sources; but it is also expected that the participants will cite and evaluate those sources. Many papers used graphics that—since we saw them in a number of papers—must have come from some online source, but they failed to specifically credit the source for the graphic. Also, many used models that they found in the literature, such as the Hata Model. This is appropriate; however, if you choose a model from the literature, then you should explain why you choose that equation to use, what assumptions led you to that equation, and what value-added you gave it as you adapted it to the given problem. It is also important that if you use equations from the literature, that you adapt the notation to match what you use in the rest of the paper, and that you clearly explain any notation that you use.

Mountainous Terrain

Most papers that considered mountainous terrain spent some time dealing with line-of-sight issues relating to the terrain. A few simulated some mountainous terrain or found some sample elevation maps and indicated what changes would be necessary in repeater placement for these samples. Some papers discussed changes to the population distribution caused by the terrain. The judges acknowledged that it would have been unreasonable to expect models that would independently deal with any terrain, but we looked for papers that indicated how one would approach uneven ground.

General Modeling Principles

One of the things teams needed to do for this problem—and which many neglected—was to decide consciously which portions of their model should be deterministic and which should be stochastic.

Assumptions were also important factors. When you make assumptions, you need to justify them—not simply state them. You should not include assumptions that are unnecessary for your model or have nothing to do with it. But even with the assumptions that you do need, you should indicate how sensitive your results are to those assumptions. It is OK to justify an assumption by indicating that it was necessary for your model, even though in reality it may not hold (for example, in this problem the assumption that population is uniformly distributed might fall into this category); but in that case, it is essential to discuss how the results depend on that assumption.

It is important not to make assumptions that defeat the purpose of the problem. Some papers assumed that repeaters were connected by wires. That was not at all in keeping with the statement of the problem, and it eventually eliminated some otherwise well-written papers.

Sensitivity Analysis, Error Analysis, and Model Testing

An important area that turned out to be one of the major discriminators at the end was testing and sensitivity analysis. How does the number of repeaters change if your population is distributed in a different fashion? If you used normal distributions, for example, how do your results change given an $x\%$ change in the assumed standard deviation? What if the range of a repeater is less than assumed? The better papers also tested their results, some by comparison with the actual distribution of users and repeaters in various locations, and others by simulations of one sort or another.

Finally, always do a commonsense check. If you are running out of time, and your commonsense check fails, you should at least acknowledge that. We had some beautifully written papers that had results where you needed on the order of 2,000 repeaters for 1,000 users. One could argue that such might be possible if the area coverage was what was driving the need. But when the same paper then required 15,000+ repeaters for 10,000 users, the reality check certainly failed. This was a "fatal flaw"! Always ask if the results "make sense" logically.

Executive Summary

Every year, we seem to reiterate the importance of a good executive summary. We continue to threaten not to read beyond a poor summary; and while we have yet to live up to that threat, it is certainly the case that the summary sets the expectation of the reader for the rest of the paper. The summary should

- be the last thing written,
- stand alone,
- make sense, and
- be satisfying, even if the reader has not read the problem and never intends to read the paper.

The results, a description of the model, any key assumptions, and recommendations should be clearly included. Important strengths and weaknesses should be highlighted. It takes some skill to write a good executive summary, but it is a skill that will take you far. Out in that "real world," you frequently need to boil down months of work into a well-crafted executive summary for the decision makers. Your MCM summary should be good practice. Look at the Outstanding paper printed in this issue, which exemplifies what we look for in a good summary. That paper consistently, through all rounds of judging, received the highest marks.

Writing and Organization

Even a brilliant team will not go far if the members cannot convey their work effectively. A few tips for the writing:

- Even when you divide up tasks such as sections to write, have your best writer do a final edit. Do this *after* you have run the grammar and spellchecker, and then run them one more time after the final edit.
- If you try some additional models and abandon them, not using them in the final analysis, put them in an appendix rather that in the body of the paper, where they distract the reader.
- Keep in mind that judges have very limited time to read your paper. The salient points need to be easy to find. If your paper is long, it may be that although many judges have looked at it, no single judge has had time to read the whole thing.
- Avoid unnecessary repetition, use good section headings, and offset/display important parts.

- Label graphics in such a way that a reader flipping through your paper will see what they represent.
- Have conclusions at the end of each section, and make sure that results are easy to find.

Conclusion

This problem led to a variety of solution techniques and approaches. It allowed for a great deal of creativity, and in the end the creativity in the solution was one of the primary factors for bringing papers recognition. Mathematical modeling is an art, and in the long run it will be the kind of creativity we see in these papers that will help solve the problems facing the world. We commend all the participants for developing these crucial skills. We are proud of your accomplishments and the drive that led you to devote your time and energy to this endeavor.

About the Author

Kathleen Shannon is Professor of Mathematics at Salisbury University and former chair of the Department of Mathematics and Computer Science. She earned her bachelor's degree at the College of the Holy Cross with a double major in Mathematics and Physics and her Ph.D. in Applied Mathematics from Brown University in the mid 1980s under the direction of Philip J. Davis. Since then, she has been primarily interested in undergraduate mathematics education and mathematical modeling. She has been involved since 1990 with the MCM as, at different times, a team advisor, a triage judge, and a final judge (sometimes as an MAA or SIAM judge). She has also been a co-Principal Investigator on two National Science Foundation Grants for the PascGalois Project (`http://www.PascGalois.org`) on visualizing abstract mathematics.