Author's Commentary: The Keep Right Papers

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Introduction

My intent in setting this problem was to begin to understand the global properties of the keep-right-except-to-pass (KRETP) rule. One has the intuitive sense that if everyone kept to this rule without exception, then traffic would eventually become chaotic (not necessarily in the technical sense) and throughput would decrease. If we think of the flow of traffic on a multi-lane highway as a stochastic process, the problem was intended to elicit an understanding of how the application of this particular control scheme alters the properties of the process. For example, does the process with KRETP become unstable or chaotic (in the technical sense) as the traffic density increases, and what implications would that have for throughput and safety? If so, might there be an alternative that works better? Or is any control scheme needed at all?

Disappointment

A solution along these lines in full generality is certainly too much to ask of undergraduates, never mind undergraduates under severe time pressure; but I thought it possible that a simple Poisson process model for each lane, with switching between lanes as prescribed by the KRETP rule, might have been tried. I was disappointed to see that such an approach was not considered by any of the teams. The teams construed the problem very

The UMAP Journal 35 (2–3) (2014) 149–152.

locally and narrowly, and were satisfied to apply a readily-available cellular automaton model, with minor modifications, to study throughput and safety along a linear stretch of freeway having no on- or off-ramps. The best papers did a creditable job with this pedestrian approach, but I couldn't help feeling that there was still something missing. It was as if the teams were solving a consulting problem and gave the client a solution that met the letter of the requirement but nothing more.

I noticed the same phenomenon in the Snowboard Course (half-pipe) Problem from a few years ago [Giordano 2011; Black 2011]. When I set that problem, my intent was to see if the current half-pipe shape was optimal (in whatever sense would be defined by the teams). In the event, however, teams went online and found a standard definition of half-pipe, including shape, in Wikipedia; and thereafter they confined their solutions to fiddling with dimensions—which was far from interesting and far from what was intended.

Leaving Room for Creativity

In the Keep-Right Problem, teams did well in understanding that it was necessary to create specific measures for throughput and safety so that these properties of the rule could be exposed. This activity was consistent with the MCM's intent of leaving room for team creativity in determining the specific mathematical constructs and expressions that teams will use to solve the problem.

Well-written problems play to this strength, allowing teams a lot of freedom in constructing their approaches and solutions. I would prefer to see teams exercise much more creativity in general. I think this goal should motivate us to write problems in such a way that it becomes clear that finding a model that someone else has already constructed, and modifying it in some minor way, may produce a solution to the problem—but that such an approach is not consistent with the spirit of the MCM. I believe that spirit is to draw out the teams' own ideas, even if they are imperfect or not fully formed, so that they get some practice in going beyond the letter of the requirements to test their own skills in a more challenging way. So I believe that those of us who write problems can learn from this experience and pay more attention to whether a problem statement does enough to support this need without unnecessarily constraining the possible approaches that teams may consider. Indeed, properly written, the problem should encourage the team to range widely over possible solution approaches before settling on something to write up for the competition. (Coaches: Train your teams to spend time at the beginning of the contest period in brainstorming mode, looking for other connections—maybe even train using mind-mapping software to explicitly bring unusual ideas out into the open!)

Criteria for the Keep-Right Problem

For the KRETP problem, certain aspects were essential. Papers that did not consider

- three (or more) lanes of traffic,
- behavior in heavy traffic (a problem requirement),
- throughput and safety (another problem requirement), and
- entrances and exits on a limited-access highway

were downgraded. Similarly, teams that found a model online and did not add any value of their own were not ranked well.

Factors causing papers to be looked at more favorably included

- explicit consideration of a tradeoff between throughput and safety,
- explicit consideration of speed variation, and/or
- the influence of new vehicle technologies (smart roads, inter-car communication, etc.).

Formulating Problems to Evoke Creativity

Overall, as an author, I would have to say that I learned from this experience that the way that a problem is stated is important to getting good results. Problem requirements need to be explicitly called out. Perhaps even more important is the idea that problems should be written so as to encourage creativity over routine solution, and this demand places more responsibility on authors to anticipate the possible approaches that teams might take and subtly encourage approaches that promote more "interesting" solutions.

References

Black, Kelly. Judges' Commentary: The Outstanding Snowboard Course papers. *The UMAP Journal* 32 (2) (2011): 123–129.

Giordano, Frank R. 2011. Results of the 2011 Mathematical Contest in Modeling. *The UMAP Journal* 32 (2) (2011): 99–107.

About the Author

Mike Tortorella is Visiting Professor at RUTCOR, the Rutgers Center for Operations Research at Rutgers, the State University of New Jersey, and Managing Director of Assured Networks, LLC. He retired from Bell Laboratories as a Distinguished Member of the Technical Staff after 26 years of service. He holds the Ph.D. degree in mathematics from Purdue University. His current interests include stochastic flow networks, network resiliency and critical infrastructure protection, and stochastic processes in reliability and performance modeling. Mike has been a judge at the MCM since 1993 and particularly enjoys the MCM problems that have a practical flavor of mathematical analysis of social policy. Mike enjoys amateur radio, playing the piano, and cycling.