

# Optimal Overbooking

David Arthur

Sam Malone

Oaz Nir

Duke University

Durham, NC

Advisor: David P. Kraines

## Introduction

We construct several models to examine the effect of overbooking policies on airline revenue and costs in light of the current state of the industry, including fewer flights, increased security, passengers' fear, and billions in losses.

Using a plausible average ticket price, we model the waiting-time distribution for flights and estimate the average cost per involuntarily bumped passenger.

For ticketholders bumped voluntarily, the interaction between the airline and ticketholders takes the form of a least-bid auction in which winners receive compensation for foregoing their flights. We discuss the precedent for this type of auction and introduce a highly similar continuous auction model that allows us to calculate a novel formula for the expected compensation required.

Our One-Plane Model models expected revenue as a function of overbooking policy for a single plane. Using this framework, we examined the relationship between the optimal (revenue-maximizing) overbooking strategy and the arrival probability for ticketholders. We extend the model to consider multiple fare classes; doing so does not significantly alter optimal overbooking policy.

Our Interactive Simulation Model takes into account estimates for average compensation costs. It simulates the interaction between 10 major U.S. airlines with a market base of 10,000 people, factoring in passenger arrival probability, flight frequency, compensation for bumping, and the behavior of rival airlines. Thus, we estimate optimal booking policy in a competitive environment. Simulations of this model with likely parameter values before and after September 11 gives robust results that corroborate the conclusions of the One-Plane Model and the compensation-cost formula.

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Overall, we conclude that airlines should maintain or decrease their current levels of overbooking.

## Terms

- **Ticketholders:** People who purchased a ticket.
- **Contenders:** Ticketholders who arrive in time to board their flight.
- **Boarded passengers:** Contenders who board successfully.
- **Bumped passengers:** Contenders who are not given seating on their flight.
- **Voluntarily bumped passengers:** Bumped passengers who opt out of their seating in exchange for compensation.
- **Involuntarily bumped passengers:** Bumped passengers who are denied boarding against their will.
- **Compensation costs:** The total value of money and other incentives given to bumped passengers.
- **Flight Capacity:** The number of seats on a flight.
- **Overbooking:** The practice of selling more tickets than flight capacity.
- **Waiting time:** The time that a bumped passenger would have to wait for the next flight to the destination.
- **Load factor:** The ratio of the number of seats filled to the capacity.

## Assumptions and Hypotheses

- Flights are domestic, direct, and one-way.
- The waiting time between flights is the amount of time until the scheduled departure time of the next available flight to a given destination.
- The ticket price is \$140 [Airline Transport Association 2002], independent of when the ticket is bought, except when we consider multiple fares.
- Pre-September 11, the average probability of a ticketholder checking in for the flight (and thus becoming a contender) was 85% [Smith et al. 1992, 9].
- The pre-September 11 average load factor was 72% [Bureau of Transportation Statistics 2000].

## Complicating Factors

Each of our models attempts to take into account the current situation facing airlines:

- **The Traffic Factor**

On average, there are fewer flights by airlines between any given locations.

- **The Security Factor**

Security in and around airports has been heightened.

- **The Fear Factor**

Passengers are more wary of the dangers of air travel, such as possible terrorist attacks, plane crashes, and security breaches at airports.

- **The Financial Loss Factor**

Airlines have lost billions of dollars in revenue due to decreased demand for air travel, increased security costs, and increased industry risks.

### The Traffic Factor

Because there are fewer flights, it is likely that the demand for any given flight will increase. Flights are likely to be fuller; the average waiting time between flights to a destination is likely to increase, so bumped passengers will demand higher compensation.

### The Security Factor

The increase in security will likely lead to an increase in the number of ticketholders who arrive at the airport but—due to security delays—do not arrive at their departure gates in time.

Successful implementation of security measures may lead to an improvement in the public perception of the airline industry and an increase in demand for air travel.

### The Fear Factor

Increased fear of flying decreases demand for air travel, so security delays may not be as serious.

On the other hand, if a higher percentage of ticketholders are flying out of necessity, then the probability that a ticketholder becomes a contender may increase because of decreased cancellations and no-shows. However, fewer ticketholders are likely to agree to be bumped voluntarily at any price, so the percentage of involuntarily bumped passengers may increase.

## The Financial Loss Factor

Because companies may seek to increase short-term profits in the face of recent losses, some airlines may implement more aggressive overbooking, which could induce an overbooking war between airlines [Suzuki 2002, 148]. The likely increase in the number of bumped passengers would lead to a rise in compensation costs that would partially offset increased revenue.

Decreasing the number of bumped passengers would improve the airlines' image and might spur demand, which would bolster future revenue.

## One-Plane Model

### Introduction and Motivation

We first consider the optimal overbooking strategy for a single flight, independent of all other flights. We will see later that its results are a good approximation to the results of the full-fledged Interaction Simulation Model.

### Development

Let the plane have a capacity of  $C$  identical seats and let a ticket cost  $T = \$140$  independent of when it is bought. Let the airline's overbooking strategy be to sell up to  $B$  tickets, if possible ( $B > C$ ). We analyze this strategy in the case when all  $B$  tickets are sold.

We model the number of contenders for the flight with a binomial distribution, where a ticketholder becomes a contender with probability  $p$ . The average  $p$  for flights from the ten leading U.S. carriers is  $p = 0.85$  [Smith et al. 1992]. The value of  $p$  for a particular flight depends on a host of factors—flight time, length, destination, whether it is a holiday season—so we carry out our analysis for a range of possible  $p$  values.

With our binomial model, the probability of exactly  $i$  contenders among the  $B$  ticket-holders is  $\binom{B}{i} p^i (1-p)^{B-i}$ .

We assume that each bumped passenger is paid compensation  $(1+k)T = 140(1+k)$ , for some constant  $k$ . Translated into everyday terms, this means that a bumped passenger receives compensation equal to the ticket price  $T$  plus some additional compensation  $kT > 0$ . Later, we relax the assumption that compensation cost is the same for each passenger, when we consider involuntary vs. voluntary bumping.

We define the compensation cost function  $F(i, C)$  to be the total compensation the airline must pay if there are exactly  $i$  contenders for a flight with seating capacity  $C$ :

$$F(i, C) = \begin{cases} 0, & i \leq C; \\ (k+1)T(i-C), & i > C. \end{cases}$$

We calculate expected revenue  $R$  as a function of  $B$ :

$$\begin{aligned} R(B) &= \sum_{i=1}^B \binom{B}{i} p^i (1-p)^{B-i} (BT - F(i, C)) \\ &= 140B - 140(k+1) \sum_{i=C+1}^B \binom{B}{i} p^i (1-p)^{B-i} (i - C) \end{aligned}$$

We use a computer program to determine, for given  $C$ ,  $p$ , and  $k$ , the overbooking strategy  $B_{\text{opt}}$  that maximizes  $R(B)$ . However, it is also possible to produce a close analytic approximation, which we now derive.

The revenue for a bumped passenger,  $T - (k+1)T = -kT$ , has magnitude  $k$  times that for a boarded passenger,  $T$ . Thus, the optimal overbooking strategy is such that the distribution of contenders is in some sense “balanced,” with  $1/(k+1)$  of its area corresponding to bumped passengers and the remaining  $k/(k+1)$  corresponding to boarded passengers.

We approximate the binomial distribution of contenders with a normal distribution:

$$\frac{C - Bp}{\sqrt{Bp(1-p)}} \approx \Phi^{-1} \left( \frac{k}{k+1} \right),$$

where  $\Phi$  is the cumulative distribution function of the standard normal distribution. Clearing denominators and solving the resulting quadratic in  $\sqrt{B}$  gives

$$B'_{\text{opt}} = \left( \frac{-\Phi^{-1} \left( \frac{k}{k+1} \right) \sqrt{p(1-p)} + \sqrt{\Phi^{-1} \left( \frac{k}{k+1} \right)^2 p(1-p) + 4pC}}{2p} \right)^2 \quad (1)$$

as an analytic approximation to  $B_{\text{opt}}$ . For  $k = 1$ , we get  $B'_{\text{opt}} = C/p$ .

This analytic approximation is always within 1 of the optimal overbooking strategy for  $.80 \leq p \leq .90$  and  $1 \leq k \leq 3$ .

## Results and Interpretation

The airline should be able to obtain good approximations to  $p$  and  $k$  empirically. Thus, it can take our computer program, insert its data for  $C$ ,  $T$ ,  $p$ , and  $k$ , and obtain the optimal overbooking strategy  $B_{\text{opt}}$ . **Figure 1** plots expected revenue  $R(B)$  vs.  $B$ .  $C = 150$ ,  $k = 1$ ,  $p = 0.85$ , and  $T = 140$ .

At  $B = 177$ , the airline can expect revenue  $R(177) = \$24,000$ , which is more than 15% in excess of the expected revenue  $R(150) = \$21,000$  from a policy of no overbooking.

Operating at a less-than-optimal overbooking strategy can have serious consequences. For example, American Airlines has an annual revenue of \$20 billion [AMR Corporation 2000]. An overbooking policy  $B$  outside the range of  $[173, 183]$  implies an expected loss of more than \$1 billion over a 5-year period compared with the expected revenue at  $B_{\text{opt}} = 177$ .

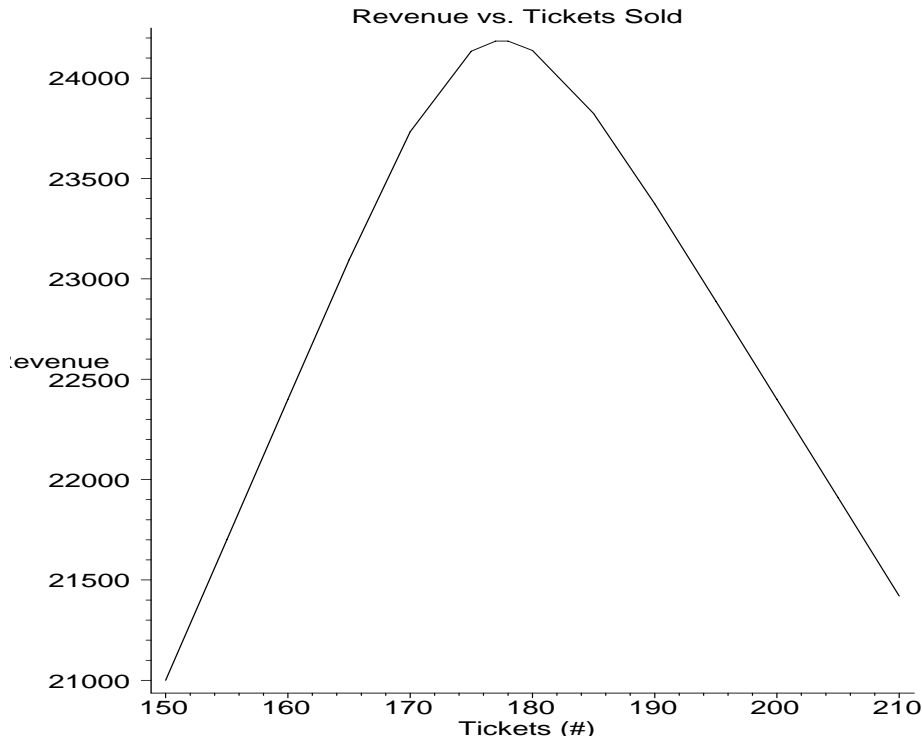


Figure 1. Revenue  $R$  vs. overbooking strategy  $B$  for  $C = 150$ ,  $k = 1$ ,  $p = 0.85$ , and  $T = \$140$ .

## Limitations

The single-plane model

- fails to account for bumped passengers' general dissatisfaction and propensity to switch airlines;
- assumes a simple constant-cost compensation function for bumped passengers;
- ignores the distinction between voluntary and involuntary bumping;
- assumes that all tickets are identical—that is, everyone flies coach;
- assumes that all  $B$  tickets that the airline is willing to sell are actually sold.

Even so, the model successfully analyzes revenue as a function of overbooking strategy, plane capacity, the probability that ticket-holders become contenders, and compensation cost. Later, we develop a more complete model.

## The Complicating Factors

First, though, we use the basic model to make preliminary predictions for the optimal overbooking strategy in light of market changes due to the complicating factors post-September 11.

Of the four complicating factors, only two are directly relevant to this model: the security factor and the fear factor. The primary effect of the security factor is to decrease the probability  $p$  of a ticketholder reaching the gate on time and becoming a contender. On the other hand, the primary effect of the fear factor is that a greater proportion of those who fly do so out of necessity; since such passengers are more likely to arrive for their flights than more casual flyers, the fear factor tends to increase  $p$ .

**Figure 2** plots the optimal overbooking strategy  $B_{\text{opt}}$  vs.  $p$  for fixed  $k = 1$  and  $C = 150$ .

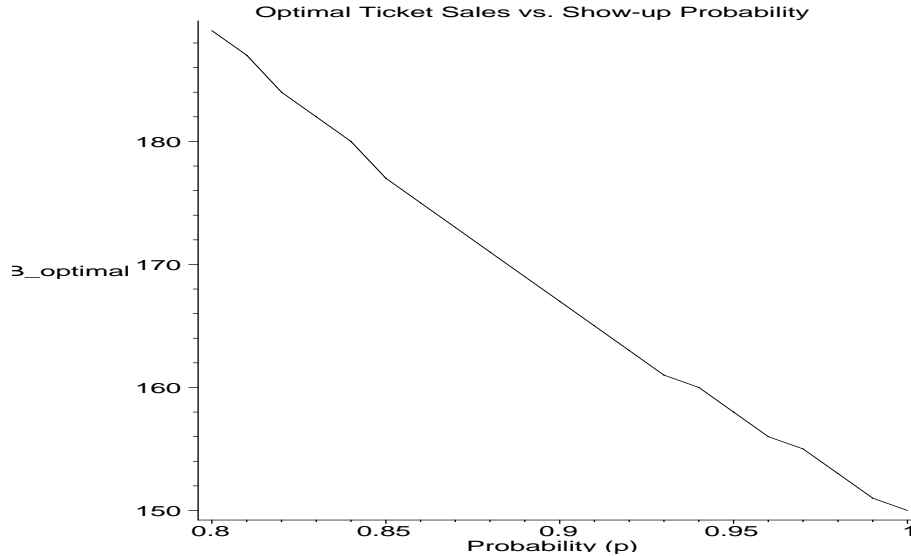


Figure 2. Optimal overbooking strategy vs. arrival probability  $p$ .

It is difficult to assess the precise change in  $p$  resulting from the security and fear factors. However, airlines can determine this empirically by gathering statistics on their flights, then use our graph or computer program to determine a new optimal overbooking strategy.

## One-Plane Model: Multifare Extension

### Introduction and Motivation

Most airlines sell tickets in different fare classes (most commonly first class and coach). We extend the basic One-Plane Model to account for multiple fare classes.

### Development

For simplicity, we consider a two-fare system, with  $C_1$  first-class seats and  $C_2$  coach seats. We assume that a first-class ticket costs  $T_1 = \$280$  and that a

coach ticket costs  $T_2 = \$140$ . We consider an overbooking strategy of selling up to  $B_1$  first class tickets and up to  $B_2$  coach tickets, where the two types of sales are made independently of one another.

We assume that a first-class ticketholder becomes a first-class contender with probability  $p_1$  and that a coach ticketholder becomes a coach contender with probability  $p_2$ . We use two independent binomial distributions as our model. First-class ticketholders are more likely to become contenders than coach passengers, since they have made a larger monetary investment in their tickets; that is,  $p_1 > p_2$ . Thus, the probabilities of exactly  $i$  first-class contenders and exactly  $j$  coach contenders are

$$\binom{B_1}{i} p_1^i (1 - p_1)^{B_1 - i}, \quad \binom{B_2}{j} p_2^j (1 - p_2)^{B_2 - j}.$$

We model compensation costs as constant per bumped passenger but dependent on fare class, with  $(k_1 + 1)T_1$  as compensation for a bumped first-class passenger and  $(k_2 + 1)T_2$  for a bumped coach passenger. We define the compensation cost function:

$$F(i, j, C_1, C_2) = \begin{cases} 0, & i \leq C_1, j \leq C_2; \\ T_1(k_1 + 1)(i - C_1), & i > C_1, j \leq C_2; \\ \max\{T_2(k_2 + 1)((j - C_2) - (i - C_1)), 0\}, & i \leq C_1, j > C_2; \\ T_1(k_1 + 1)(i - C_1) + T_2(k_2 + 1)(j - C_2), & i > C_1, j > C_2. \end{cases}$$

The justification for the third case is that an excess of coach contenders is allowed to spill over into any available first-class seats. On the other hand, excess first-class contenders cannot be seated in any available coach seats; this fact is reflected in the second case.

We model expected revenue  $R$  as a function of the overbooking strategy  $(B_1, B_2)$ :

$$R(B_1, B_2) = \sum_{i=1}^{B_1} \sum_{j=1}^{B_2} \binom{B_1}{i} \binom{B_2}{j} p_1^i (1 - p_1)^{B_1 - i} p_2^j (1 - p_2)^{B_2 - j} \cdot (B_1 T_1 + B_2 T_2 - F(i, j, C_1, C_2))$$

## Results and Interpretation

For fixed  $C_i$ ,  $T_i$ ,  $p_i$ , and  $k_i$  ( $i = 1, 2$ ), we can find  $(B_{1,\text{opt}}, B_{2,\text{opt}})$  for which  $R(B_1, B_2)$  is maximal by adapting the computer program used to solve the one-fare case.

For example, for a plane with  $C_1 = 20$  first class seats,  $C_2 = 130$  coach seats, ticket costs of  $T_1 = \$280$  and  $T_2 = \$140$ , and compensation constants  $k_1 = k_2 = 1$ , we obtain the optimal overbooking strategies listed in **Table 2**.

The optimal strategy involves relatively little overbooking of first-class passengers, since there is a much higher compensation cost. However, the total

**Table 2.**

Two-fare optimal overbooking strategies for selected arrival probabilities.

$p_1$	$p_2$	$B_{1,opt}$	$B_{2,opt}$
0.85	0.80	23	165
0.90	0.80	22	165
0.95	0.80	20	166
0.85	0.85	23	155
0.90	0.85	22	155
0.95	0.85	20	155
0.90	0.90	22	146
0.95	0.90	21	145

number of passengers (coach plus first-class) overbooked in an optimal two-fare situation is virtually the same as the total number overbooked in the one-fare situation. The upshot is that the effect of multiple fare classes on the optimal overbooking strategy is not very significant; so, when we construct our more general model, we do not take into account multiple fares.

## Compensation Costs

The key element that separates different schemes for compensating bumped ticketholders is the degree of choice for the passenger. Airlines often hold auctions for contenders in which the lowest bids are first to be bought off of a flight.

We construct a model for involuntary bumping costs that is based on DOT regulations and takes into account the waiting time distribution for flights. Then we discuss auction methods for voluntary bumping and derive novel results for expected compensation cost for a continuous auction that matches actual ticket auctions fairly well.

## Involuntary Bumping: DOT Regulations

The Department of Transportation (DOT) requires each airline to give all passengers who are bumped involuntarily a written statement describing their rights and explaining how the airline decides who gets on an overbooked flight and who does not [Department of Transportation 2002]. Travelers who do not get to fly are usually entitled to an “on-the-spot” payment of denied boarding compensation. The amount depends on the price of their ticket and the length of the delay:

- Passengers bumped involuntarily for whom the airline arranges substitute transportation scheduled to get to their final destination within one hour of their original scheduled arrival time receive no compensation.

- If the airline arranges substitute transportation scheduled to arrive at the destination between one and two hours after the original arrival time, the airline must pay bumped passengers an amount equal to their one-way fare, with a \$200 maximum.
- If the substitute transportation is scheduled to get to the destination more than two hours later, or if the airline does not make any substitute travel arrangements for the bumped passenger, the airline must pay an amount equal to the lesser of 200% of the fare price and \$400.
- Bumped passengers always get to keep their tickets and use them on another flight. If they choose to make their own arrangements, they are entitled to an “involuntary refund” for their original ticket.

These conditions apply only to domestic flights and not to planes that hold 60 or fewer passengers.

The function for the compensation cost for an involuntarily bumped passenger is

$$C(T, F) = \begin{cases} 0, & \text{if } 0 < T \leq 1; \\ \min(2F, F + 200), & \text{if } 1 < T \leq 2; \\ \min(3F, F + 400), & \text{if } 2 < T, \end{cases}$$

where  $T$  is waiting time and  $F$  is the fare price. We assume that all flights to a given location are direct and have the same flight duration. Thus, the waiting time between flights equals the difference in departure times, and the waiting time  $T$  is the time until the next flight to the destination departs. We assume that involuntarily bumped passengers always ask for a refund of their fare.

## Involuntary Bumping: The Waiting Time Model

To use the compensation cost function to determine the average compensation (per involuntarily bumped passenger), we would need to know the joint distribution of fare prices and waiting times. Because this information would be extremely difficult to obtain, we opt instead for practical compromises:

- We restrict our attention to determining the expected compensation cost for the average ticket price, \$140 [Airline Transport Association 2000].
- We specify a workable model for the distribution of waiting times that allows us to calculate this cost directly.

Our model for the distribution of waiting times is the exponential distribution, a common distribution for waiting times. Let  $T$  be a random variable representing waiting time between flights; then

$$Pr(T \leq t) = 1 - e^{-\lambda t}$$

and  $E(T) = \tau = 1/\lambda$ , where  $\tau$  is the mean waiting time for the next available flight.

The expected cost of compensating an involuntarily bumped passenger who purchased a ticket of price  $P$  can be evaluated directly and is

$$\min(2P, P + 200) [e^{-\lambda} - e^{-2\lambda}] + \min(3P, P + 400) [e^{-2\lambda}].$$

From examining airline booking sites, we estimate the average daytime waiting time  $\tau$  to be 2.6 h, not including the time between the last flight of the day and the first flight of the next day. If we include these night-next-day waiting times in our calculations, we obtain  $\tau \approx 4.8$  h; this value corresponds to five flights per 24-hour period, which is fairly typical. Using the smaller, strictly daytime value  $\tau = 2.6$  h, we obtain an expected compensation cost of \$255.

## Voluntary Bumping: Auction Methods

In 1968, J.L. Simon proposed an auction among ticketed passengers. Each ticketed passenger contending for a seat on a flight would submit a sealed envelope bid of the smallest amount of money for which the contender would give up the seat and wait until the next available one. The airline would compensate the passengers who required the least money and require that they give up their seats. Passengers would never get bumped without suitable compensation, and airlines could raise their overbooking level much higher than they could otherwise. After Ralph Nader successfully sued Allegheny Airlines for bumping him, variants on this scheme have gradually become standard throughout the industry.

There are two reasonable ways to attempt an auction.

- Per Simon, force every contender to choose a priori a price for which they would give up their ticket. The airline could arrange all bumpings immediately.
- The actual practice by most airlines is to announce possible compensation prices in discrete time intervals. Customers can then accept any offer they wish to.

The first is attractive to the airlines because it is instant and minimizes compensation. The second, however, can be started well before a flight departs; and if intervals are increased gradually enough, the difference in cost is negligible. The methods should generate similar results, so for simplicity we concentrate on the second, though with continuous compensation offerings.

## Voluntary Bumping: Continuous-Time Auction

In the literature, it is common to assume that if  $m$  passengers are compensated through an auction, the total cost for the airline should be linear in  $m$ ,

although some authors (such as Smith et al. [1992]) recognize that the function should be nonlinear and convex but do not analyze it further. In fact, we can say a great deal more with only a few basic assumptions. Indeed, suppose that

- $n$  ticketholders check in for a flight with capacity  $C$ , with  $n > C$ .
- Each contender has a *limit price*, the smallest compensation to be willing to give up the seat.
- An airline can always rebook a ticketholder on one of its own later flights at no cost (i.e., it does not have to pay for a ticket on a rival airline).

In an ideal auction, the airline offers successively higher compensation prices; whenever the offer exceeds a contender's limit price, the contender gives up the ticket voluntarily. Suppose that ticketholders  $(\Gamma_1, \Gamma_2, \dots, \Gamma_n)$  are ordered so that  $\Gamma_i$ 's limit price is less than  $\Gamma_j$ 's limit price for  $i < j$ . Define:

- $D(x)$  = the probability that a randomly selected ticketholder gives up the seat for a price  $x$ .
- $Y_m$  = the compensation that the airline must pay  $\Gamma_m$  to give up the ticket.
- $X_m$  = the total compensation that the airline must pay for  $m$  contenders give up their seats.

We have  $X_m = \sum_{i=1}^m Y_i$ . To determine  $E[X_m]$ , we determine  $E[Y_i]$  for  $i \leq m$ . To do this, we need the following result:

$$E[Y_m] = \sum_{i=0}^{m-1} \binom{n}{i} \int_0^\infty (D(x))^m (1 - D(x))^{n-m} dx.$$

[EDITOR'S NOTE: We omit the authors' proof.]

Very little can be done beyond this point without further knowledge about the nature of  $D(x)$ . There is not much recent data on this; but when airlines were first considering moving to an auction-based system, K.V. Nagarajan [1978] polled airline passengers on their limit price. Although he performed little analysis, we find that the cumulative distribution function of this limit price fits very closely exponential curves of the form  $1 - e^{-Ax}$  for a fixed  $A$  (**Figure 3**).

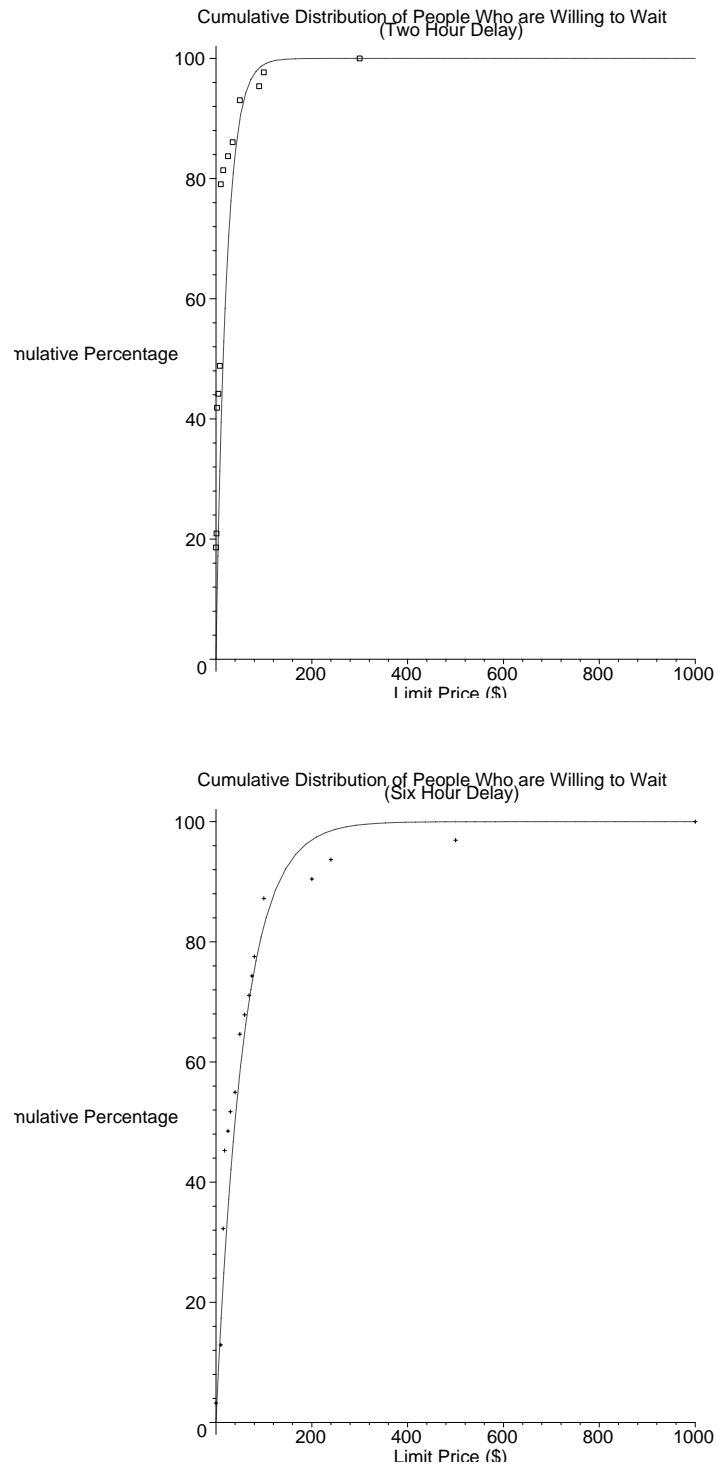
With  $D(x) = 1 - e^{-Ax}$  for some constant  $A$ , then

$$E[X_m] = \frac{1}{A} \left[ m - (n - m) \left( \frac{1}{n} + \frac{1}{n-1} + \frac{1}{n-2} + \dots + \frac{1}{n-m+1} \right) \right].$$

[EDITOR'S NOTE: We omit the authors' proof.]

Using the approximation

$$\frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{n} \approx \ln n,$$



**Figure 3.** Polled distribution of ticketholder limit price, with best fit graphs  $1 - e^{0.046x}$  for 2-hour wait and  $1 - e^{0.0175x}$  for 6-hour wait (data from [Nagarajan 1978, 113]).

this becomes

$$E[X_m] \approx \frac{1}{A} \left[ m - (n - m) \ln \left( \frac{n}{n - m} \right) \right].$$

There is no reason to believe that the value of  $A$  is constant across all scenarios. For example, contenders will certainly accept a smaller compensation if the next flight is departing soon. For our purposes, however, we assume that  $A$  is constant over all situations; and we estimate that on a flight with capacity  $C = 150$  and only a small number of overbooked passengers,  $\Gamma_1$  has a limit price of \$100. Then we have  $\frac{1}{A} \cdot \frac{1}{150} \approx \$100$ , so  $A \approx \$1/15,000$ .

Hence, the expected compensation required to bump  $m$  out of  $n$  ticketholders via auction is approximately

$$\frac{\$1}{15,000} \left[ m - (n - m) \ln \left( \frac{n}{n - m} \right) \right],$$

compared to a cost of  $\$255m$  (plus ill will) for involuntary bumping the same number of ticketholders.

## Effects of Overbooking on Market Share

### Constructing the Model

We focus on the 10 largest U.S. airlines (Alaska, America West, American, Continental, Delta, Northwest, Southwest, Trans World, United, US Air), which comprise 90% of the market. We use 1997–1998 statistics on their flight frequency and market share. [EDITOR'S NOTE: We omit the data table.]

Flights are modeled as identical in all respects except for market interest. The market is simulated as a group of initially 10,000 people, each loyal to one airline, who independently buy tickets on their airline with a fixed probability and meet reservations with a fixed probability. Each member of the market independently chooses to stay with an airline or change airline based on treatment regarding each flight.

Each company choose a number  $r$ , which specifies its overbooking strategy: On a flight of capacity  $C$ , the company will sell up to  $B = Cr$  tickets.

In each time period, precisely one flight is offered. The chance that a given airline will offer that flight is proportional to the number of flights that it offers per year. We also determine a constant  $k$  that indicates the level of interest in this flight. Each flight has capacity  $C = 150$  seats each sold at \$140.

The exact size of the market should have little effect on the result. We assume that the total market is initially made up of 10,000 independent people, each loyal to one carrier. The relative sizes of the company market shares are initialized according to 1997–98 industry data. We assume each person in the market flies on average the same number of times in a year.

We assume that each person in a company's market has probability  $k$  of wanting to buy a ticket for a flight by the company. We have  $k$  follow a normal

distribution with mean fixed so that the average load factor on all flights is the industry average of 0.72 [Bureau of Transportation Statistics 2002].

Industry data prior to September 11 indicate a probability of .85 that a ticketholder will check in for the flight.

If necessary, each airline bumps some passengers voluntarily and some involuntarily, according to its strategy. The immediate cost of bumpings is set to the values that we derived in the previous section. We surmise that voluntarily bumped passengers are relatively happy and thus leave the airline with probability only .05, whereas involuntarily bumped passengers are furious and leave with probability .8.

A person who leaves an airline stays within the market with probability .9 (0.95 if bumped voluntarily) and simply switches to another airline; otherwise, the person leaves the market altogether. People trickle into the market fast enough to compensate for the loss of people due to dissatisfaction, thus allowing the market to grow slowly.

## Simulation Results, Pre-September 11

We investigate the effect of different overbooking rates on profit. For each overbooking rate, we calculate net profit over 500 time periods (ensuring that the same random events occur regardless of the strategy tested). The strategy that maximizes profit for that time period is then determined and tabulated. We repeat this 40 times for each airline.

This leaves open the question of what strategies the companies not being tested should use. To determine this, we initially assume that each company would overbook by 1.17 (as computed in the single-plane model), run the program to get a first estimate of a good strategy, and use the optimal results from that preliminary run to set the default overbooking rates of each company in a final run. Finally, we use the industry figure that 5% of all bumped passengers are bumped involuntarily to set the company compensation strategies.

The optimal overbooking rate for all companies other than Alaska is between 1.165 and 1.176, close to but a little less than the results from the One-Plane Model. This is reasonable, since the most significant improvement that this simulation makes over the One-Plane Model is the consideration of lost customers, whose effect should slightly reduce the optimal overbooking rate.

The program generates very consistent answers on each run for every airline except Alaska. Alaska has far fewer passengers per flight than its competitors and rarely fills any plane entirely, so its overbooking policy has a negligible effect on its overall profit. Thus, the simulation is almost certainly too coarse to generate useful data on Alaska.

## Adjusting the Model Due to September 11

We estimate the effects of the complicating factors after September 11 have on the simulation parameters:

- *Arrival probability  $p$  increases from 0.85 to 0.90.*
- *Flight frequency decreases by 20% [Parker 2002].*
- *Total market size decreases by 15%. Fourth quarter data from 2001 are not yet available, so we make an estimate. Our own experience is that flights are more crowded now, which suggests that the percentage of market size decrease is smaller than the percentage of flight frequency decrease. Thus, we estimate that market size has decreased by 15%.*
- *Market return rate doubles.* The market size has decreased due to the fear factor, but Parker [2002] anticipates that demand will return to pre-September 11 levels by mid-2002. Moreover, public perception of airline safety is improving due to the security factor. Thus, the market return rate should be substantially higher than its pre-September 11 level.
- *Market exit rate decreases by 50%.* The market composition is now more heavily weighted towards those who fly only out of necessity; such fliers are much less likely than casual fliers to leave the market.
- *Percentage of bumps that are voluntary decreases from 95% to 90%.* There are fewer flights, hence the waiting time between flights is greater. Since passengers are more likely to be flying of necessity, they are much less interested in giving up a seat for compensation.
- *Compensation cost of voluntary bumping increases by 20%.*
- *Compensation cost of involuntary bumping increases by 20%.* Bumped passengers face longer waiting times; because of DOT regulations, average involuntary compensation costs must rise.
- *Competitors increase their overbooking levels from  $r$  to  $r + 0.02$ .* Due to financial losses, an airline can expect its competitors to focus more heavily on short-term profits than previously.

## Simulation Results, Post-September 11

Using the parameter changes outlined, we ran the simulation again to estimate the effect of the events of September 11 on optimal overbooking strategies. The results are shown in **Table 3**.

There is again a strong correlation between the simulation results for these parameters and the corresponding results from the One-Plane Model.

From **Table 3**, it is clear that the events of September 11 have indeed had a significant effect on optimal overbooking rates. Indeed, for a company the

**Table 3.**  
Optimal overbooking rates, from simulation results.

Airline	Pre-September 11	Post-September 11
Alaska	1.319	1.260
America West	1.169	1.094
American	1.171	1.094
Continental	1.170	1.096
Delta	1.173	1.095
Northwest	1.174	1.095
Southwest	1.173	1.095
Trans World	1.176	1.096
United	1.168	1.094
US Air	1.165	1.092

size of American Airlines, the 7% change in these rates could easily lead to a difference in profits on the order of \$1 billion.

Thus, if our estimates of parameter changes due to September 11 are reasonable, *all major airlines should significantly decrease their overbooking rates.*

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# Memorandum

Attn: Don Carty, CEO American Airlines  
From: MCM Team 180  
Subject: Overbooking Policy Assessment Results

We completed the preliminary assessment of overbooking policies that you requested. There is a great deal of money at stake here, both from ticket sales and also from compensation that must be given to bumped passengers. Moreover, if too many passengers are bumped, there will be a loss of good will and many regular customers could be lost to rival airlines. In fact, we found that the profit difference for American Airlines between a good policy and a bad policy could easily be on the order of \$1 billion a year.

Using a combination of mathematical models and computer simulations, we considered a wide variety of possible strategies that could be tried to confront this problem. We naturally considered different levels of overbooking, but we also looked at different ways in which airlines could compensate bumped passengers. In terms of the second question, we find that the current scheme of auctioning off compensations for tickets, combined with certain calculated forced bumpings, is still ideal, regardless of changes to the market state.

Although we were forced to work without much recent data, we were also able to achieve reliable and consistent results for the optimal overbooking rate. In particular, we found that prior to September 11, American Airlines stood to maximize profits by selling approximately 1.171 times as many tickets as seats available.

We next considered how this number would likely be affected by the current state of the market. In particular, we focused on four consequences of the events on September 11: all airlines are offering fewer flights, there is heightened security in and around airports, passengers are afraid to fly, and the industry has already lost billions of dollars. Analyzing each of these in turn, we found that they did indeed have a significant effect on the market. In particular, American Airlines should lower its overbooking rate to 1.094 tickets per available seat.

In conclusion, we found that there is indeed a tremendous need to re-evaluate the current overbooking policy. According to our current data, we believe that the rate should be dropped significantly. It would be valuable, however, to supplement our calculations with some of the confidential data that American Airlines has access to, but that we do not.