Through the Breach: Modeling Flooding from a Dam Failure in South Carolina

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Summary

The Saluda Dam, separating Lake Murray from the Saluda River in South Carolina, could breach in the event of an earthquake.

We develop a model to analyze the flow from four possible types of dam breaches and the propagation of the floodwaters:

- instant total failure, where a large portion of the dam erodes instantly;
- delayed total failure, where a large portion of the dam slowly erodes;
- piping, where a small hole forms and eventually opens into a full breach; and
- overtopping, where the dam erodes to form a trapezoidal breach.

We develop two models for the spread of the downstream floodwaters. Both use a discrete-grid approach, modelling the region as a set of cells, each with an elevation and a volume of water. The Force Model uses cell velocities, gravity, and the pressure of neighbouring cells to model water flow. The Downhill Model assumes that flow rates are proportional to the height differences between the water in adjacent cells.

The Downhill Model is efficient, intuitive, flexible, and could be applied to any region with known elevation data. Its two parameters smooth and regulate water flow, but the model's predictions depend little on their values.

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For a Saluda Dam breach, the total extent of the flooding is 106.5 km^2; it does not reach the State Capitol. The flooding in Rawls Creek extends 4.4 km upstream and covers an area of 1.6–2.4 km^2.

Variables and Assumptions

Table 1 shows the variables used in the design and simulation of the flooding model, and **Table 2** lists the parameters in the simulation program.

Table 1.
Variables used in the model.

Variable	Definition
Voume flow rates from the dam	
Q_{TF1}	For instant total failure
Q_{TF2}	For delayed total failure
Q_{PIPE}	For piping failure
Q_{OT}	For overtopping failure m
Q_{peak}	Maximum flow rate
Times when water ceases to flow through the dam	
t_{TF1}	For instant total failure
t_{TF2}	For delayed total failure
t_{PIPE}	For piping failure
t_{OT}	For overtopping failure
ΔV	Total volume of water displaced from Lake Murray by flooding
Vol_{LM}	Normal volume of Lake Murray
Area_{LM}	Normal area of Lake Murray
d_{breach}	Depth of the breach from the top of the dam
t_{breach}	Time from when the breach begins to form until its final formation
m	Slope of the sides of the cone approximating Lake Murray

General Assumptions

- Normal water level is present in the lake prior to a dam breach.
- No seasonal variation of flows occurs in waterways.
- Volume of water in Lake Murray can be accurately approximated by a right circular cone (**Figure 1**).

Dam Assumptions

- Saluda Dam fails in one of four ways:
  - instant total failure,
  - delayed total failure,

Table 2.
Parameters used in the simulation program.

Parameter	Typical value	Meaning
`BREACH_TYPE`	varies	one of `INSTANT_TOTAL_FAILURE`, `DELAYED_TOTAL_FAILURE`, `PIPING`, or `OVERTOPPING`
ΔT	10.0	Length of one time step (s)
`MIN_DEPTH`	0.0001	Depth below which a cell is considered empty (m)
T_{FINAL}	100000	Time for the breach to empty completely the affected portion of the reservoir (s)
T_b	3600	Time until breach reaches maximum size (s)
Q_{peak}	25000	Maximum flow rate of the breach (m^3/s)
d_{breach}	30	Maximum depth of breach below initital reservoir level (m)
$\text{Volume}_{\text{LM}}$	2.714×10^9	Initial volume of Lake Murray (m^3)
Area_{LM}	202×10^6	Initial area of Lake Murray (m^2)
k	0.504	Spreading factor (regulates amount of water exchanged between two cells)
`MAX_LOSS_FRAC`	0.25	Maximum fraction of a cell's water that it can donate in a single time step

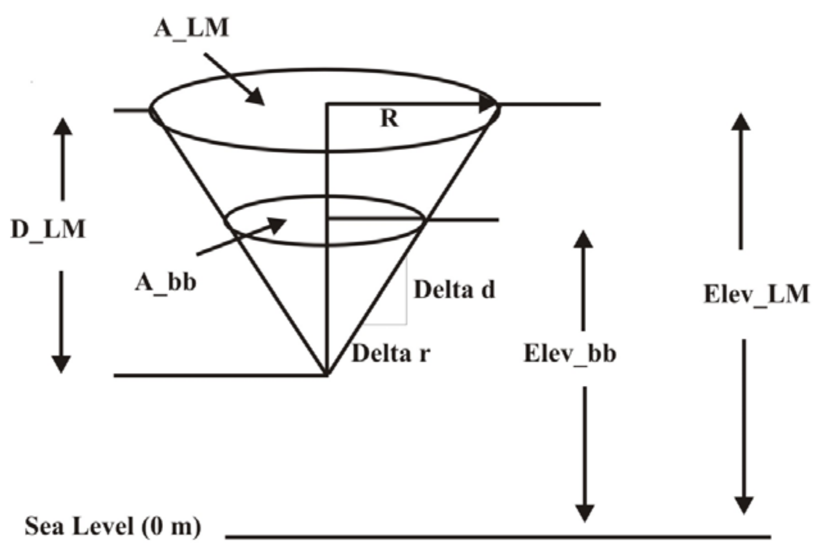

Figure 1. Reservoir approximation using a right circular cone.

  - piping, or
  - overtopping.
- Composition of the earthen dam is uniform throughout.
- Width of the base of the breach is between the height of the dam and three times the height of the dam [U.S. Army Corps of Engineers 1997].
- No human attempt is made to prevent dam breaching.

Downstream Assumptions

- Resistance to water flow due to structures such as bridges and buildings is negligible.
- Water does not alter the terrain significantly as it flows over the floodplain.
- Water does not make alluvial deposits as it flows over the floodplain.
- A negligible amount of water is present in the valley before flooding.
- Negligible water inflow occurs from sources other than the dam breach.
- No human attempt will be made to prevent flooding.

Accepted Facts

- Area of Lake Murray: 200 km^2
- Volume of Lake Murray: 2.710×10^6 m^3
- Height of dam: 63.4 m (crest at 370 ft above sea level)
- Length of dam: 2.4 km
- Elevation of surface of Lake Murray: 106.5—110 m above sea level

Model Design

Dam Breach

Each type of dam breach is described by flow rate as a function of time, with corresponding parameters.

Instant Total Failure

A model of flow rate for instant total failure is right triangular [U.S. Army Corps of Engineers 1997] (**Figure 2**). The parameters are breach depth and peak volume outflow, with values

$$d_{\text{breach}} = 20 \text{ m}, \qquad Q_{\text{peak}} = 30{,}000 \text{ m}^3/\text{s}.$$

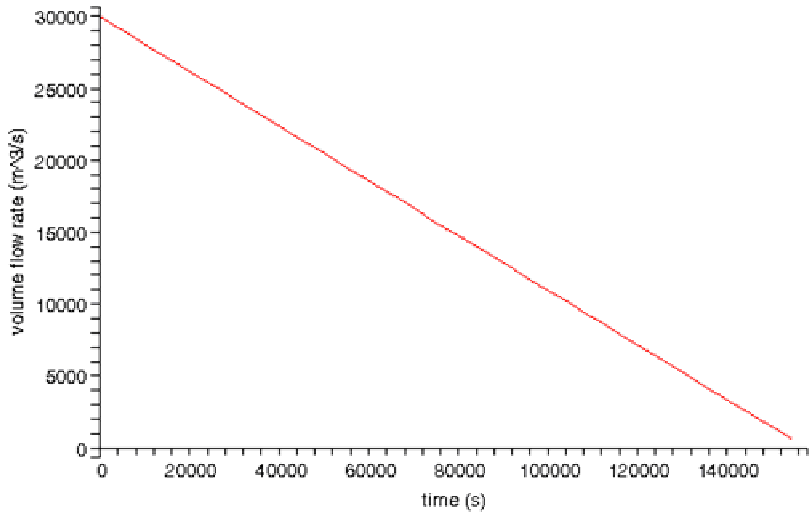

Figure 2. Flow rate for an instant total failure.

Delayed Total Failure

An isosceles triangle model makes sense for delayed failure because it takes half of the total volume of water removed from the lake to erode the dam and the flow rate does not peak until the erosion is complete [U.S. Army Corps of Engineers 1997] (**Figure 3**). Also, for an earthen dam, the erosion time may be longer than for other types of dams, such as concrete.

This model has the same parameters and same values:

$$d_{\text{breach}} = 20 \text{ m}, \qquad Q_{\text{peak}} = 30{,}000 \text{ m}^3/\text{s}.$$

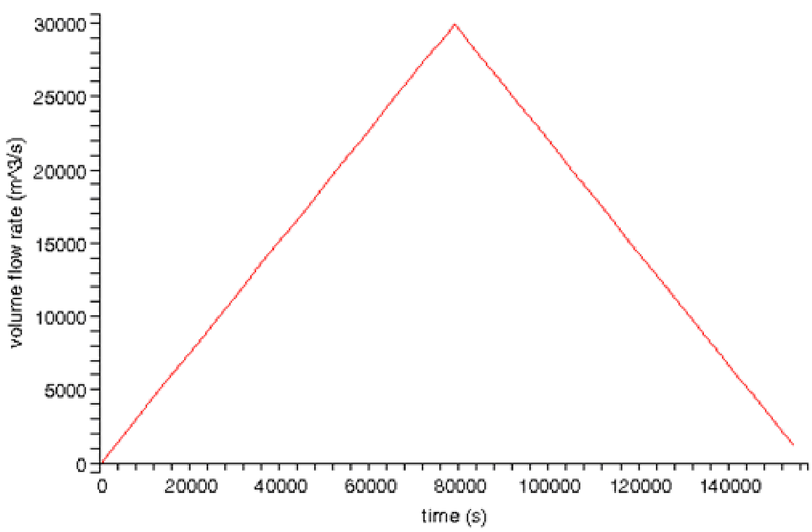

Figure 3. Flow rate for a delayed total failure.

Piping Failure

For a piping failure, the breach begins in the middle of the dam face and grows until the material above the pipe collapses [Sedimentation and River Hydraulics Group 2004]. As the breach grows, the flow rate increases exponentially; the peak flow rate occurs when the material above the pipe collapses. From that point, the flow through the breach is similar to a total failure. We select an exponential decay so as to observe a different effect from the linear decay of the total failure models (**Figure 4**).

We choose the growth rate so that the peak flow rate occurs at the breach time, and the decay rate so that the flow rate is less than 1% of the peak flow rate at the final time. The parameters are the breach depth, the peak volume outflow of the dam, and the breach time, with values

$$d_{\text{breach}} = 20 \text{ m}, \qquad Q_{\text{peak}} = 30{,}000 \text{ m}^3/\text{s}, \qquad t_{\text{breach}} = 50{,}000 \text{ s}.$$

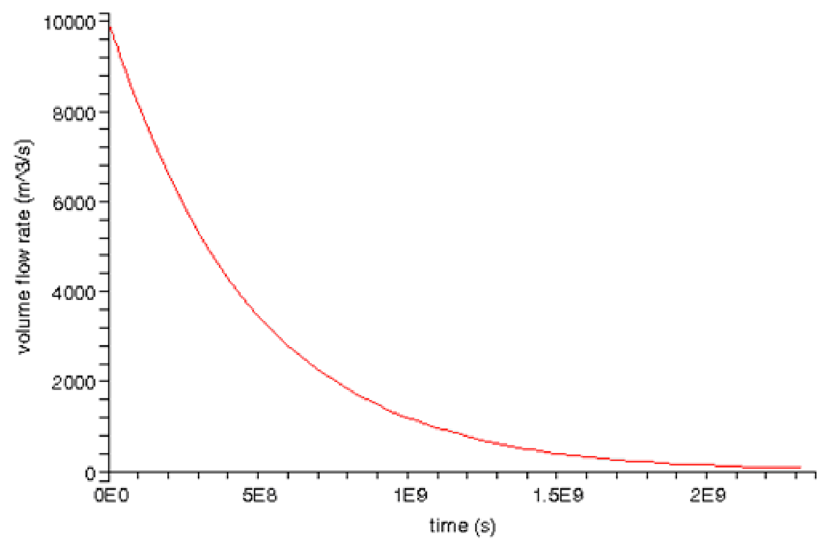

Figure 4. Flow rate for a piping failure.

To demonstrate better the change in flow rate with time when the breach begins to form, we plot over a shorter range of time in **Figure 5**.

Overtopping Failure

For an overtopping failure, the water begins flowing over the top of the breach, eroding the dam from above. We found little information about overtopping failures. From the piping failure, we estimate that the flow rate increases according to a parabolic shape until dam erosion is complete (**Figure 6**). After this point, which corresponds to the breach time, the flow rate behaves as in a total failure.

The parameters are again breach depth, peak volume outflow of the dam, and breach time, with values

$$d_{\text{breach}} = 20 \text{ m}, \qquad Q_{\text{peak}} = 30{,}000 \text{ m}^3/\text{s}, \qquad t_{\text{breach}} = 30{,}000 \text{ s}.$$

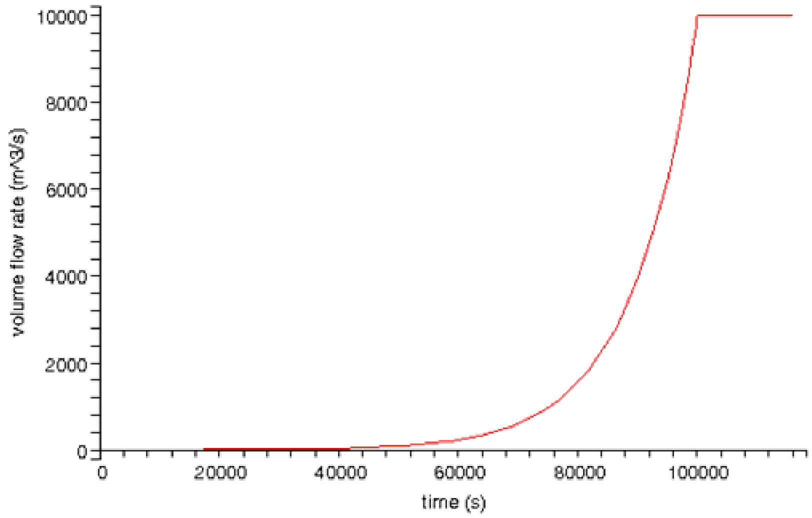

Figure 5. Flow rate for beginning of a piping failure.

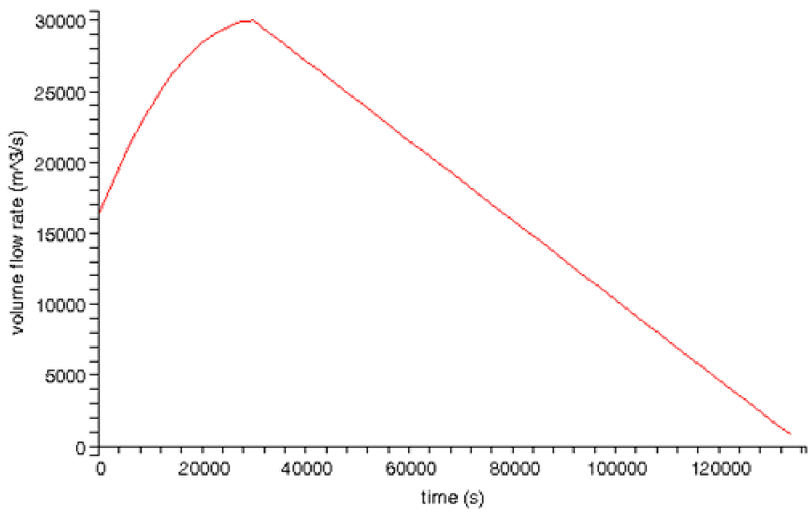

Figure 6. Flow rate for an overtopping failure.

Downstream Flow

We model the behaviour of the water in the region downstream of the breach using a discrete approach. The Force Model uses a physical analogy based on the Bernoulli equation for fluid flow; the Downhill Model uses a simpler, more intuitive mechanism for water flow. The Force Model produces unphysical results; therefore, we use Downhill Model in analysis of the flooding.

For both models, the region surrounding the Saluda Dam is divided into a grid of square cells. Each cell covers a surface area of 210 m by 210 m and has an associated elevation above sea level and a volume of water (based on the mean depth of water in the cell). The elevation data are adapted from the U.S. Geological Survey's National Elevation Data [2004] by (to reduce processing time) averaging together groups of 7×7 cells. Each model simulates the propagation of water among cells; the models differ in how neighbouring cells determine how much water to exchange per unit time.

Force Model

Design

This model performs a force analysis on the water contained in the model cells. Each cell has an associated elevation above sea level, mean depth of water contained in the cell, and mean velocity (x- and y-components) of water within the cell. The force acting on a particular cell is assumed to be due to two effects only: the pressure force exerted by the four cells in direct contact with it, and the gravitational force that accelerates the water to places of lower elevation (that is, downhill).

The main principles of the model are:

- Volume flow between cells is proportional to the difference in pressure between the four adjacent cells with a common face.
- Pressure difference in cells is proportional to the difference in mean depth of each cube.

As demonstrated in the **Figures 7** and **8**, the mean pressure exerted by a cell is assumed to be the pressure at half the depth of the cell, or $P = \frac{1}{2}\rho g d$.

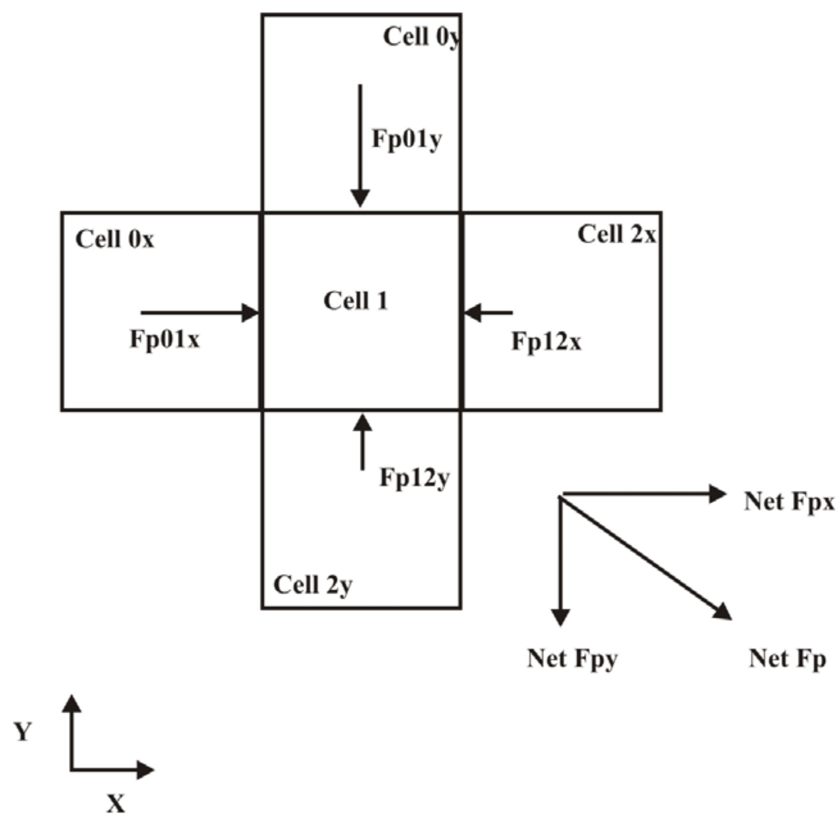

Figure 7. Pressure forces acting on cell matrix.

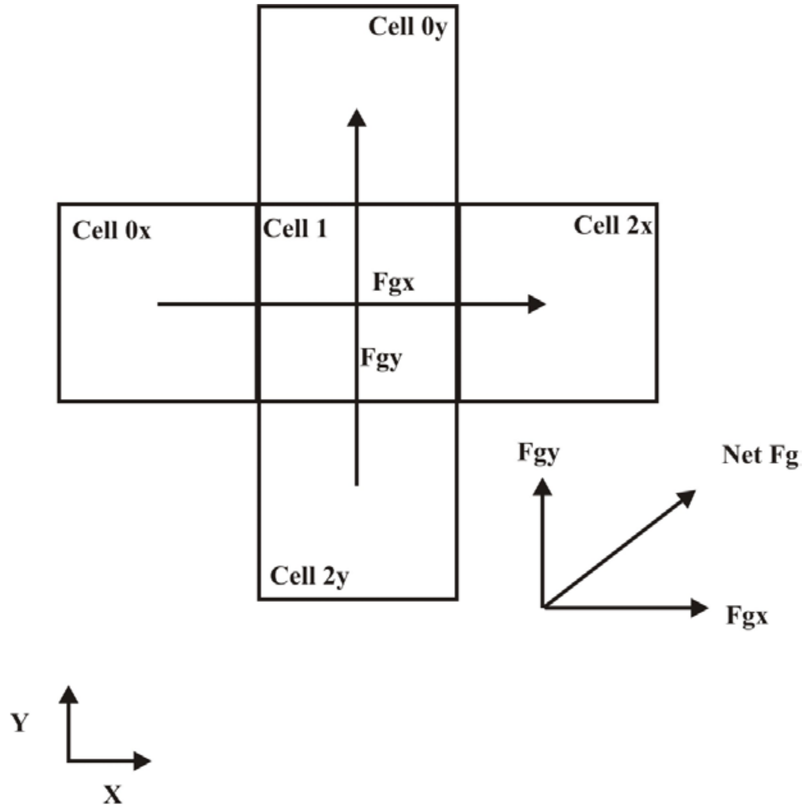

Figure 8. Gravitational forces acting on cell matrix.

We assume that the area over which the pressure acts is the mean depth of the two adjacent cells, so that the depth of water varies linearly between them, with the depth at the boundary the average depth of the two. The force exerted by a neighbouring cell is the mean pressure times the area between the two cells. To find acceleration, we divide the force by the mass of water in the cell, taken to be the volume of the cell times the water density:

$$a_x = \frac{g(d_{0\mathrm{x}}^2 - d_{2\mathrm{x}}^2)}{4wd_1}.$$

We calculate the acceleration due to gravity by estimating the gradient of the ground of the current cell and its four immediate neighbours. We determine the horizontal component of the acceleration geometrically (**Figure 9**) to be

$$a_g = \frac{2\Delta hwg}{4w^2 + \Delta h^2}.$$

The model iterates through a large number of time steps, typically each of 1 s duration. At the beginning of each time step, water is injected into the cells containing the dam breach; the amount is determined by the breach models described above. For each time step, the acceleration (x- and y-components) is calculated for each cell in the region, and the velocity of water in the region is updated according to

$$v_{\text{new}} = v_{\text{old}} + a\Delta t.$$

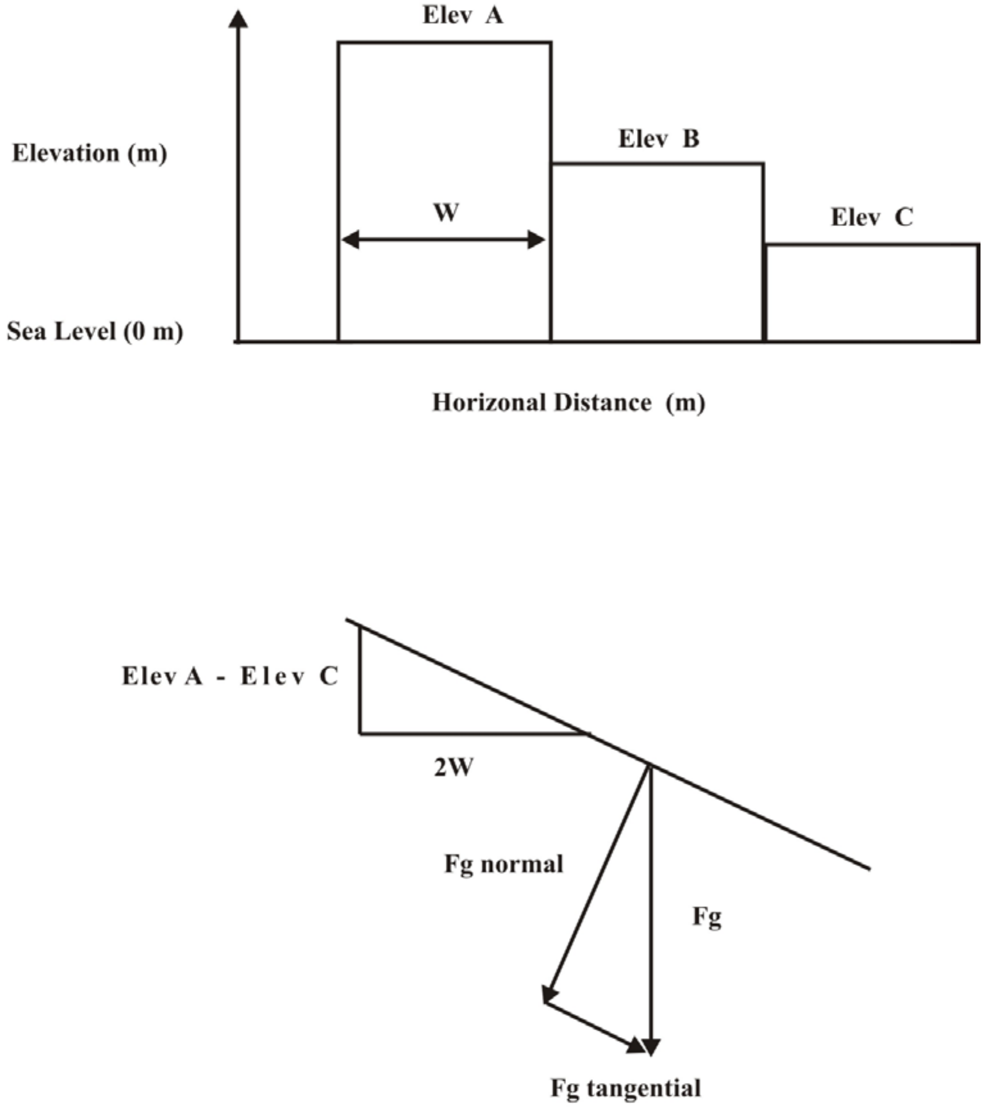

Figure 9. Determination of tangential component of gravity.

The direction of the velocity determines in which direction each cell donates: For $v_x > 0$, the cell donates to the right; for $v_x < 0$, the cell donates to the left. The amount of water donated is proportional to the speed in that direction, so that the change in water depth of the current cell is

$$d_{\text{donated}} = \frac{d_{\text{avg}} \Delta t}{2w}.$$

The water depth of the neighbouring cell receiving the donation is also updated, so that the total amount of water in the model is conserved (except for donations off the edges of the map and the water injected at the breach cell).

For very large velocities, a cell can donate more water than it has. Specifically, if the speed times the time step size is larger than the cell width, the donation would be greater than the cell's current volume. If this occurs, the cell is assumed to donate all its water, and the donations in the x- and y-directions are scaled to account for this.

Justification

This model is intuitively appealing: It models the behaviour of the water using a simple but meaningful physical analogy. The force analysis used is equivalent to taking the gradient of the Bernoulli equation and modeling the fluid discretely.

The model computes and saves velocity information, allowing modeling of the manner in which regions are flooded. For example, the model could predict the speed of the water as it struck a particular building in Columbia, such as the State Capitol.

Reasons for Rejecting the Model

The results from the model are unrealistic. Since cells with large volumes of water have small accelerations, these cells tend to empty very slowly, even if adjacent to completely empty cells; for the same reason, small cells tend to empty too quickly. The result is a checkerboard pattern: Large cells grow larger and their small neighbours grow smaller. This error relates to our assumption that all water within a cell has the same velocity: A single cell cannot spread out in all directions. For a simpler terrain (such as a simple downhill channel), this would not be a problem; however, this terrain is highly complicated and requires the water to propagate in several directions.

Another problem with this model is its complexity. The model juggles a large number of parameters for each cell, making tuning and troubleshooting difficult.

Downhill Model

Design

The Downhill Model assumes that the flow rate between two cells is proportional to the height difference between the centers of mass of those cells multiplied by the effective area between them. The model allows water to be donated in multiple directions by a single cell, if it is higher than several of its neighbours. As in the Force Model, the program iterates through time steps, adding water each step to the cells containing the dam breach.

For each time step, each cell (except those on the bottom and right boundaries of the map, which are handled later) exchanges water with the two cells immediately below and to the right. This ensures that each cell exchanges water with its four neighbours exactly once per time step. To exchange with a neighbour, a cell changes its height according to the formula

$$d_{\text{donated}} = k d_{\text{avg}}(h_0 - h_2).$$

The value of k is based on the assumption that the water speed at the breach during the peak flow rate is 30 m/s. We later describe the model's response to a change in k.

The neighbouring cell then changes its height by the negative of this value. To ensure consistency, the changes in height are not applied until the end of the time step, after all cell height changes have been calculated. If a cell had donated more than `MAX_LOSS_FRAC` of the water that it originally contained, then its donations are scaled down so that it donates exactly this amount. The factor `MAX_LOSS_FRAC` is used to prevent sloshing: Large cells tend to empty completely into empty neighbours, which then donate back on the next turn, so that half of the cells are empty at any one time.

For cells along the boundary, donations on their side(s) against the edges of the map are assumed to be equal to their donations on the opposite sides. Since these cells are far away from the breach or areas of interest, their precise behaviour is less important. Our approach ensures that water reaching the edges of the map leaves smoothly, without piling up unphysically.

Justification

This model affords rapid computation and uses a simple principle that is easy to troubleshoot. Although the equation governing the water exchange between cells lacks a direct physical analogy, it produces results consistent with physical expectations. Water travels most quickly downhill or across the nearly flat floodplain, and creeps uphill only as water levels rise.

Testing and Results

Testing

To test our models, we use National Elevation Data from the U.S. Geological Survey [USGS 2004]. The data are a set of elevation values (in meters above sea level) arranged into rows and columns. Each element represents a square with sides 30 m in length. To reduce computation, we averaged groups of 7×7 cells together, so that the cells that we used were squares 210 m on a side.

We tested both the Force Model and the Downhill Model by placing the breach cell just in front of the dam face and modeling the spread of water for several choices of breach type (instant total failure, delayed total failure, piping, overtopping) and time period (360, 1800, 6240 s). We tested the k dependence and `MAX_LOSS_FRAC` dependence by using the instant total failure breach model.

To prevent errors from very small volumes in cells, we treated a cell as empty if its mean depth was less than 0.0001 m. This cutoff was especially important in the Force Model, where such small cells acquire enormous velocities ($> 10^6$ m/s) when placed next to a cell with a significant amount of water.

We tested the Downhill Model for robustness by running the instant total failure model for 50,000 time steps. The model behaves poorly beyond 40,000 time steps, when the flow rate out of the map becomes much greater than the flow rate of the dam breach (which has slowed by this point). We also tested the model for very large flow rates. For rates that increased the height of the breach cell by more than 10 m per time interval (more than 30 times as large as any flow rate in the simulation), the simulation lasts only 1000 time steps before becoming unstable.

Results

Flooding Extent

The extent of the flooding is largely independent of the type of breach **(Figure 10)***; the difference between breach types is in how quickly flooding spreads.* For instant total failure breach, the flooding has a maximum extent of 106.5 km^2. The flooding is greatest in the Saluda and Congaree valleys, which are quite flat and broad. The flooding in the city of Columbia itself, which is elevated from these valleys, is very minimal. We did not model the effects of the flooding farther down the Congaree, but we expect those to be comparable to the flooding within the region simulated.

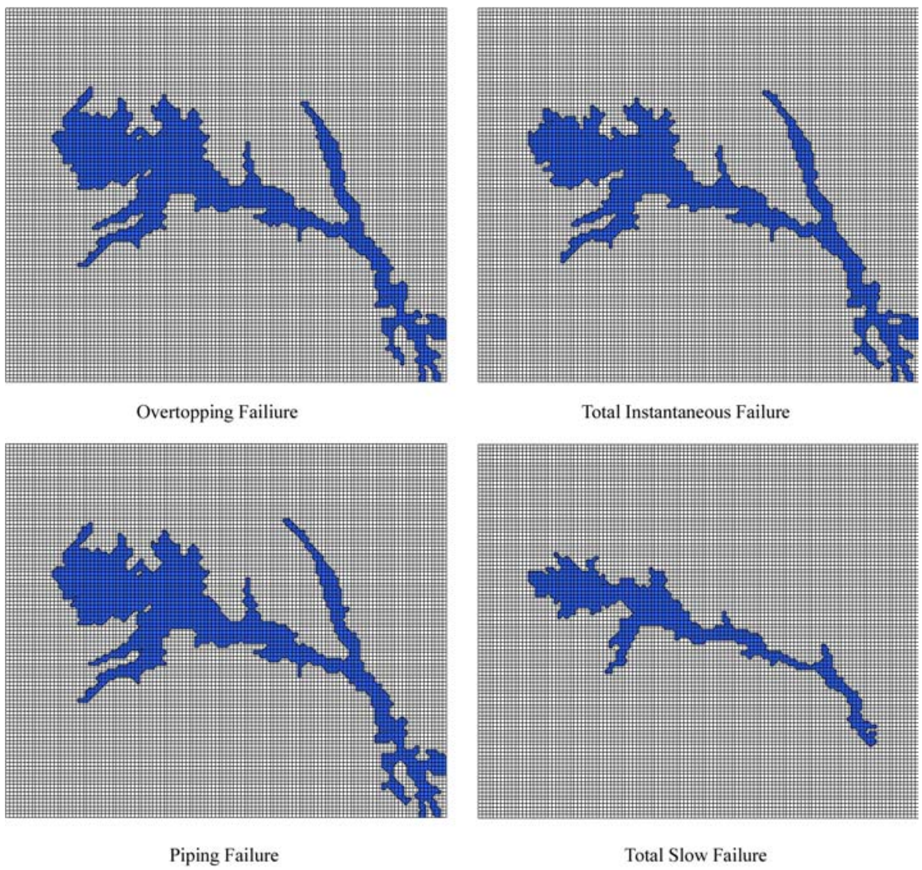

Figure 10. Comparison of dam breach scenarios: flooding area after 24 h.

Rawls Creek

The flooding in Rawls creek is extensive in area but not in upstream extent. Although it is difficult to establish where the flooding of the Saluda river ends and the flooding of Rawls Creek begins, the area around Rawls Creek that becomes flooded we estimate as between 1.6 and 2.4 km^2. The farthest flooded point is 4.4 km upstream.

State Capitol

Flooding does not reach the State Capitol, even for the most extreme case, instant total failure

Error Analysis, Sensitivity, and Robustness

The model depends on the factor k to scale the amount of water donated. Its value is based on the assumption that the water speed at the breach during the peak flow rate is 30 m/s. However, the value of k does not affect the simulation greatly; the total flooded area after 1800 time steps varies by just 17% when k is varied by a factor of 100.

The `MAX_LOSS_FRAC` is used to prevent each cell from donating too much water. However, the extent of the flooding is not strongly dependent on its value, which we took as 0.25 to produce reasonably smooth water distributions.

Strengths and Weaknesses

Strengths

The model is independent of the site simulated: Given elevation data for a region and an equation governing the flow rate of water from a dam breach, it calculates the behaviour of flooding.

The Downhill Model is intuitive. It relies on a simple exchange rule between cells, making it easy to tune and troubleshoot. Tuning may be needed to account for problems associated with more extreme flooding cases, a need to extract additional results from the model, or other unforeseen demands.

The algorithm is efficient; the computation of a single time step is linear in the number of cells in the region. This efficiency makes it possible to model many variations on breach types and flow rates in a short period of time.

The model produces three data sets of grids: 0/1 values describing which cells are flooded; the water depth in each cell to determine the severity of the flooding in a region; and the water depth plus elevation for each cell. From plots of these data sets, the extent and severity of the flooding are easy to see.

Weaknesses

The primary weakness of this model is the tendency of water in the deeper regions of the flooded area to slosh. It should be possible to eliminate this, perhaps by introducing a depth dependence into `MAX_LOSS_FRAC`.

Another weakness that could be corrected with more analysis is the time scale. Since the k-dependence—the only place where the duration of the time step is used explicitly—is weak, the model's time scale is not easily changeable. The time scale could be calibrated by running simulations of an analytical system, such as the propagation of water down a channel, and determining the speed of the water and hence the time scale. Since the time scale was not needed to analyze the extent of the flooding, we did not perform this calibration.

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Appendix: Dam Breach Model Equations

For an instant total failure:

$$Q_{\text{TF1}}(t) = \begin{cases} -\dfrac{Q_{\text{peak}}(t - t_{\text{TF1}})}{t_{\text{TF1}}}, & t < t_{\text{TF1}}; \\ 0 & t_{\text{TF1}} < t, \end{cases}$$

where $t_{\text{TF1}} = \dfrac{2\Delta V}{Q_{\text{peak}}}$.

For a delayed total failure:

$$Q_{\text{TF2}}(t) = \begin{cases} \dfrac{2Q_{\text{peak}}t}{t_{\text{TF2}}}, & t \le \frac{1}{2}t_{\text{TF2}}; \\ \dfrac{2(t_{\text{TF2}} - t)Q_{\text{peak}}}{t_{\text{TF2}}}, & \frac{1}{2}t_{\text{TF2}} - t < 0 \text{ and } t - t_{\text{TF2}} < 0; \\ 0, & t_{\text{TF2}} \le t, \end{cases}$$

where $t_{\text{TF2}} = \dfrac{2\Delta V}{Q_{\text{peak}}}$.

For a piping breach that turns into a total failure:

$$Q_{\text{PIPE}}(t) = \begin{cases} (Q_{\text{peak}} + 1)^{t/t_b} - 1, & t \le t_b; \\ Q_{\text{peak}} \exp\left[\dfrac{5(t - t_b)}{t - t_b}\right], & t_b - t < 0 \text{ and } t + t < 0; \\ 0, & t \le t, \end{cases}$$

where

$$t_{\text{PIPE}} = \Delta V - t_{\text{breach}} \left[\frac{5\left(\dfrac{2 + Q_{\text{peak}}}{\ln(Q_{\text{peak}} + 1)} - 1\right)}{Q_{\text{peak}}(1 - e^{-5})} + 1 \right].$$

For an overtopping breach:

$$Q_{\text{OT}} = \begin{cases} Q_{\text{peak}} + \\ \qquad 15\left(t^2 + 2tt_{\text{breach}} - t_{\text{breach}}^2\right) \times 10^{-6}, & t \le t_{\text{breach}}; \\ \dfrac{Q_{\text{peak}}(t - t_{\text{OT}})}{t_{\text{breach}} - t_{\text{OT}}}, & t_{\text{breach}} t < 0 \text{ and } t - t_{\text{OT}} < 0; \\ 0, & \text{otherwise}, \end{cases}$$

where

$$t_{\text{OT}} = \frac{2(\Delta V + 0.000005t_{\text{breach}}^3 - t_{\text{breach}} Q_{\text{peak}})}{Q_{\text{peak}}} + t_{\text{breach}}.$$

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