Erosion in Florida: A Shore Thing

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Abstract

Rising sea levels and beach erosion are an increasingly important problems for coastal Florida. We model this dynamic behavior in four discrete stages: global temperature, global sea level, equilibrium beach profiles, and applications to Miami and Daytona Beach. We use the Intergovernmental Panel on Climate Change (IPCC) temperature models to establish predictions through 2050. We then adapt models of Arctic melting to identify a model for global sea level. This model predicts a likely increase of 15 cm within 50 years.

We then model the erosion of the Daytona and Miami beaches to identify beach recession over the next 50 years. The model predicts likely recessions of 66 m in Daytona and 72 m in Miami by 2050, roughly equal to a full city block in both cases. Regions of Miami are also deemed to be susceptible to flooding from these changes. Without significant attention to future solutions as outlined, large-scale erosion will occur. These results are strongly dependent on the behavior of the climate over this time period, as we verify by testing several models.

Introduction

The northern ice cap plays an important role in global climate and oceanic conditions, including interactions with the global oceanic currents, regulation of the atmospheric temperature, and protection from solar radiation [Working Group II 2007]. There are significant recent trends in polar melting, global temperature, and global sea level.

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By correlating the effects of an increasing sea level on beach erosion, we can strategically develop our coast for the future so that homes and businesses can remain untouched by disaster.

Approach

- Analyze existing arctic and climate models to determine the most reasonable predictions for future changes.
- Identify the best available models for global change.
- Relate the future trends and physical melting processes to time and predicted temperatures.
- Examine and apply the Bruun model for beach erosion.
- Establish realistic physical models and parameters of Daytona Beach and Miami.
- Model the long-term erosion of the beach shores in those beaches.
- Propose cost-effective solutions to minimize the impact of erosion.

Arctic Melting

Justified Assumptions

- The northern ice cap includes the North Polar ice cap (over seawater) and the Greenland ice sheet (over land).
- The IPCC temperature models are accurate and stable within the time period of interest.
- The melting of the North Polar ice cap does not contribute directly to global water levels.
- Tectonic considerations within the IPCC model are relevant to the coast of Florida.
- Changes in oceanic salinity cause negligible changes in sea levels.
- Changes in ocean temperature will lead directly to increases in sea level within the time period of interest.

Polar Ice Cap

The North Polar ice cap is essentially a source of fresh water. Because of its composition and unsupported status, 90% [Stendel et al. 2007] of it is largely suspended beneath the surface of the Arctic Ocean. Since the density of ice is only 10% lower than that of water (0.92 g/cm^3 vs 1.0 g/cm^3), *any melting of the North Polar ice cap contributes negligibly to global water levels*.

The primary effect of the North Polar ice cap is to regulate global and oceanic temperatures, through solar deflection and melting. As the ice cap melts further, this capability is diminished, and temperatures change. Current models for the ice cap, atmosphere, and global temperatures are complex; we capture the time-dependent effects through existing temperature predictions.

Greenland Ice Sheet

Since the Greenland ice sheet is supported on a land mass, its contribution to global climate and sea level is considerably different from the polar ice caps (which are floating ice). Melting ice from the Greenland ice sheet contributes directly to the total volume of water in the oceans. This contribution to global sea levels is not captured directly by existing temperature models and hence must be related back to historic data.

Temperature Effects

The density of water is temperature dependent. As the temperature of the oceans increase, the corresponding decrease in water density will lead to an overall increase in volume.

Salinity Changes

Since both the Greenland ice sheet and the North Polar ice cap are pure freshwater sources, any melting will result in slight reductions in the salinity of the global oceans. The two effects of this interaction are a slight change in density due to the reduced salt content and a possible decrease in the rate at which the North Polar ice cap melts (due to osmotic forces based on the salt concentrations, an effect commonly observed in chemistry).

However, according to the IPCC [Working Group II 2007], these changes are relatively small compared to the thermal effects of the warming process. Thus, these effects are included in our model through the sea level predictions of the IPCC and only applied as a direct relationship to global temperatures.

Tectonic Effects

In addition to global trends from the rising sea level, shifts within the tectonic plates of the Earth have been argued to cause an upward movement of some of the ocean bottoms, and thus contribute to local deviations in the sea level change [Nerem et al. 2006]. Such effects are outside our scope here.

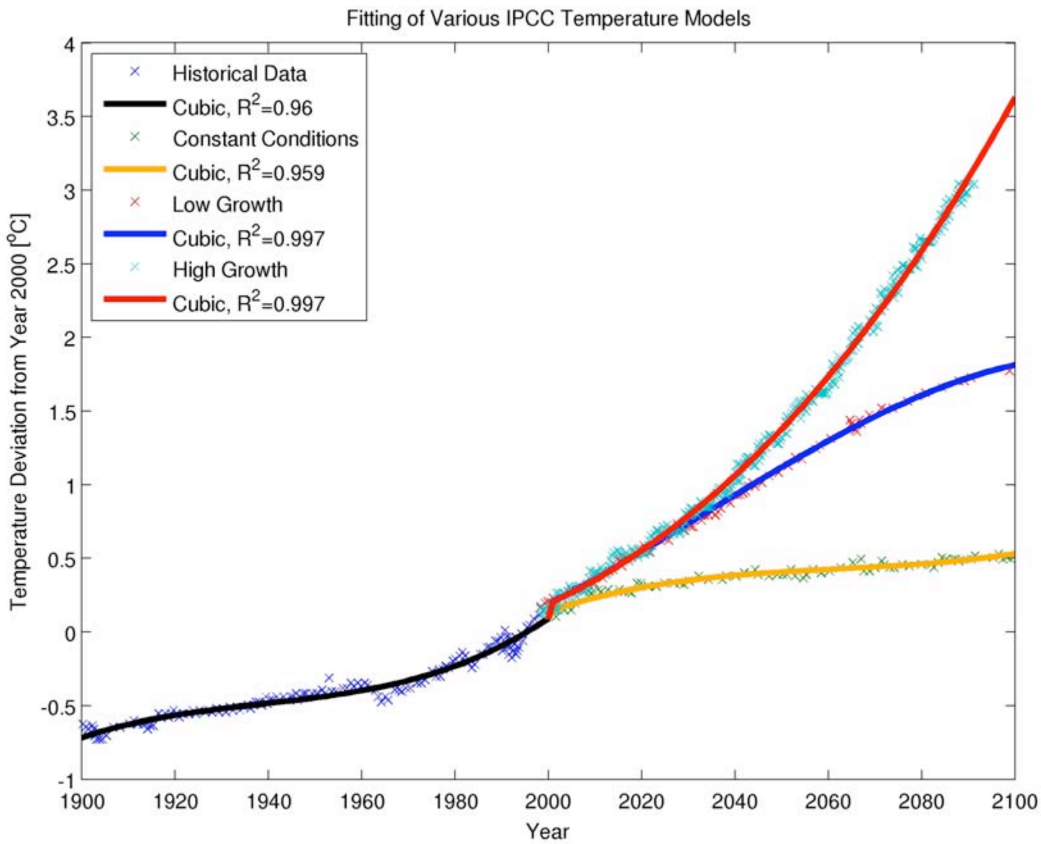

Figure 1. A global temperature model endorsed by the IPCC [Working Group II 2007]

Global Temperature Model

Many large-scale computer simulations and models have been proposed to predict the effects of arctic melting. These results have been compiled and studied by the IPCC fourth assessment report [Working Group II 2007], and its predictions for global temperature are used within this report. Criticism of IPCC modeling is common due to its simplified assumptions, but however we have not seen a better alternative.

We use the temperature models shown in **Figure 1**, which shows historical data and several scenarios for the future. We make graphical fits and show corresponding information. We conclude that simulations that use constant conditions prevailing in 2000 are unrealistic. Therefore, we consider only the low-growth and high-growth model cases. We assume a cubic growth model for temperature change:

$$\Delta T(t) = at^3 + bt^2 + ct + d.$$

Modeling Sea-Level Changes

Justified Assumptions

- The IPCC temperature and sea-level estimates are accurate.
- Sea-level change is global and equal everywhere.
- Sea-level changes can be broken into factors directly related to temperature, and factors whose rate is dependent on temperature.

Sea Level Model

While the IPCC [Working Group II 2007] predicts temperature changes for the next century, the only predictions for sea level changes are possible ranges at the end of the century. To develop time-dependent models for the sea level rise, we correlate these changes to the temperature model.

The IPCC simulations include ranges for the effects of various parameters on the global sea level change [Working Group II 2007]. These effects can be broken roughly into 55% indirect effects leading to temperature change (and thermal expansion), and 45% other volume effects, such as the melting of the Greenland ice sheet (see **Table 1**).

Table 1.
Results from the third IPCC report for 2100 [Working Group II].

Source	Sea Rise (mm)	Mean rise (mm)
Thermal expansion	110–430	270
Glaciers	10–230	130
Greenland ice	20–90	35
Antarctic ice	170–20	−95
Terrestrial storage	83–30	−26.5
Ongoing contributions from ice sheets	0–55	27.5
Thawing of permafrost	0–5	2.5
Total global-average sea level rise	110–770	440

For the 55% of changes related directly to temperature, we consider the corresponding sea level to be proportional to temperature:

$$S_1 = \gamma \Delta T(t),$$
$$\gamma = \frac{\Delta S(2100)}{\Delta T(2100)}.$$

Since the Greenland ice sheet is noticeably devoid of water (whatever melts, runs off the ice sheet), the primary limitation on ice melting is assumed to be limitations of heat transfer from the air above the ice shelf.

To model this, we use a generic heat exchanger rate equation, with an arbitrary thermal coefficient U_a. To determine the rate, we use the average summer temperature of Greenland, 6°C [Vinther et al. 2006]. We integrate the resulting equation and obtain scaling coefficients:

$$\frac{dS_2}{dt} \propto q = U_a(T_1 - T_2),$$

$$\Delta S_2 = \alpha \int_{2000}^{t_f} U_a\big(T_1(t) - T_2\big)\,dt$$

$$= \alpha \int_{2000}^{t_f} U_a\big(T + (ax^3 + bx^2 + cx + d) - 0\big)\,dt$$

$$\beta = \alpha U_a = \frac{\Delta S(2100)}{\int_{2000}^{t_f}\big(T + (ax^3 + bx^2 + cx + d)\big)\,dt}.$$

We determine the scaling coefficient β for each simulation, and calculate the overall sea-level rise as follows:

$$\Delta S(t) = 0.55 S_1(t) + 0.45 S_2(t).$$

The resulting predicted sea-level rises are shown in **Figure 2**. The lower and upper bounds on the predictions are shown by calculating the rises for the lower range of the low-growth model and the upper range of the high-growth model. The predicted sea-level rises for the mean rises of both scenarios through 2050 are quite similar, and using either is sufficient. However, such engineering modeling questions often need to err on the side of caution, so we consider the upper extreme in later models. Historical data are included for comparison and agree reasonably with the predicted trends.

The predicted sea-level increases are shown in **Table 2**.

Table 2.

Model predictions for future sea level rises.

Year	Sea Level Increase (cm)		
2010	4.1	4.4	2.6
2020	6.8	7.7	4.4
2030	9.6	11.5	6.2
2040	12.5	15.6	8.0
2050	15.3	20.2	9.9

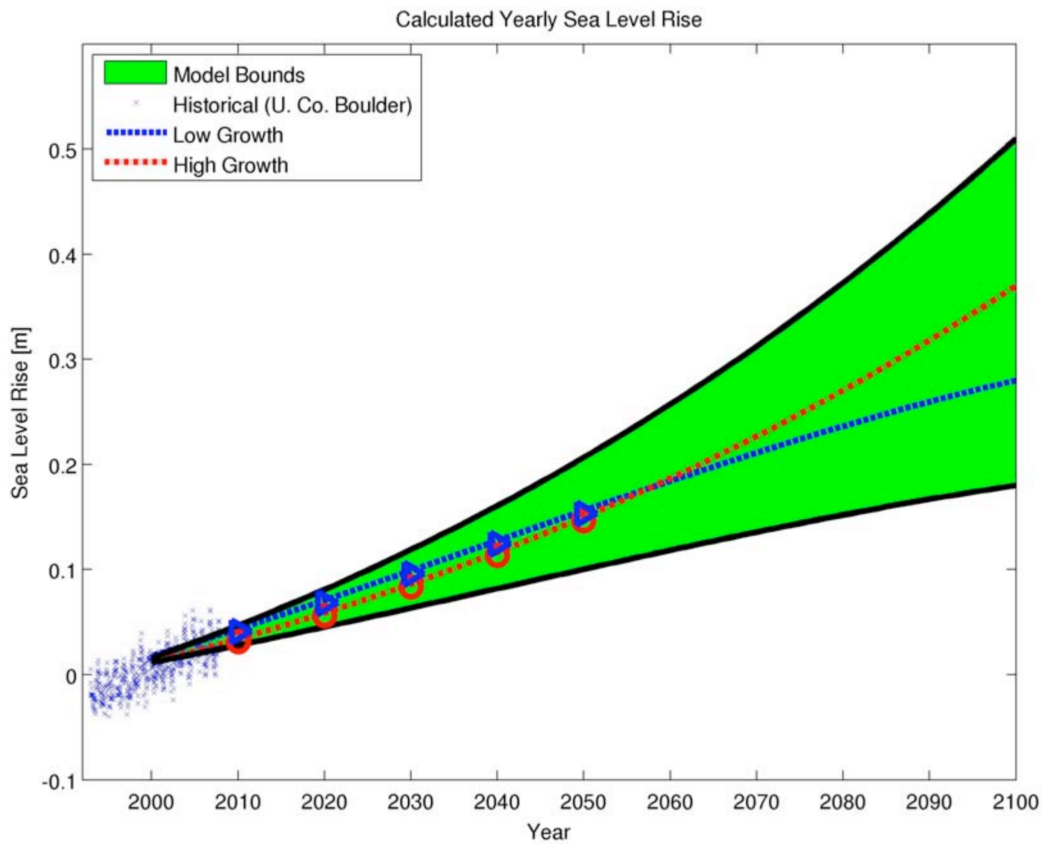

Figure 2. The model for global sea-level changes through 2100.

Beach Erosion Models

Justified Assumptions

- Beach erosion is continuous when observed over long time periods.
- Beach profiles do not change.
- Only direct cause of erosion is sea-level change.

Overview

Beach erosion is complex, since the behavior of the beach depends on a huge number of local beach and weather parameters, as well as being linked to the physical bathymetry of the surrounding sea bed.

Seasonal and Weather Effects

Seasonal temperature changes can cause differing rates of erosion, and winter weather has been observed to cause formation of offshore bars, af-

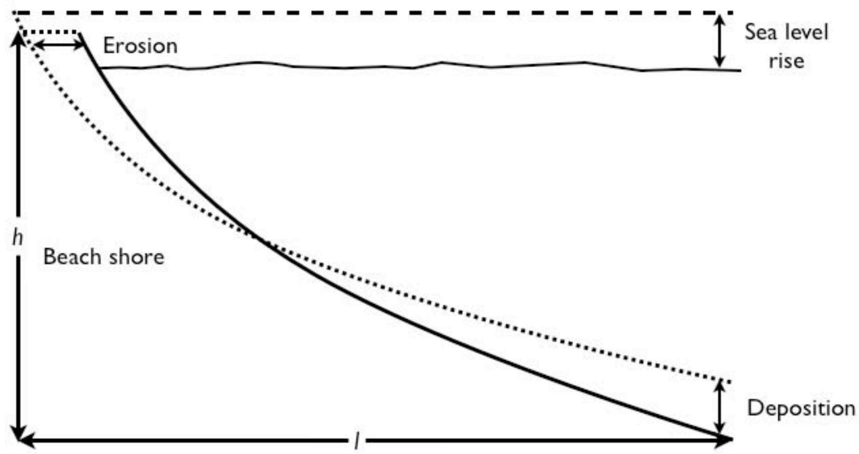

Figure 3. The Bruun model for equilibrium beach profiles [Bruun 1983].

fecting the relative rates of erosion. Storms and hurricanes generally show no lasting long-term effect on the state of a beach [Walton, Todd L. 2007].

Thus, for the purposes of this model, these effects are unimportant. Predicting weather activity is impossible on a short time scale, and attempting to simulate any sort of effects over a long (50-year) period would be unreasonable.

Bruun Model

Instead of modeling transient effects on beach erosion, we use the well-known Bruun model of beach profiles [Herbich and Bretschneider 1992]. At the core of the model is the observation that many beaches fit the general profile:

$$h(x) = Ax^{2/3},$$

where h is the depth of the water, x is the distance from the shoreline, and A is a static parameter related to the average particle size of the beach material. We illustrate the model in **Figure 3**.

Using this model, Bruun found that the ratio between the rise R in sea level and the recession ΔS of a beach front are linearly related through a constant K,

$$R = K\Delta S. \tag{1}$$

The constant K can be calculated using the long-range profile of the coast [Herbich and Bretschneider 1992] via

$$K = \frac{l}{h},$$

where l is the distance from the shoreline and h is the depth at l. We fit the parameter K and use this linear relation to predict future erosion.

Justification of Erosion Model Choice

There has been widespread criticism of the assumptions made by Bruun in his constant-profile model. However, it is the only beach erosion model to have received significant experimental testing. A thorough review of the current state of the Bruun model and additions was performed in Slott [2003], with modifications proposed by Dean in Miller and Dean [2004].

Effects on Florida

Justified Assumptions

- Beach profiles are consistent for all locations on a given beach (city location).
- The profile parameters are time-independent.

Geographical Overview

Florida sits on a shelf projected between the Atlantic Ocean and the Gulf of Mexico. The topography is characterized by extremely low elevation. There are significant urban areas situated along most of the coastline, with significant centers at Tampa Bay on the west coast and at Miami and Daytona on the East coast. In addition, barrier islands are present on much of Florida's east coast, with large implications for modeling.

Primary Effects

We consider two primary effects within our model and examine the flooding implications of a rise in sea level.

We conclude that beach erosion is be the primary effect of a rising sea level. We present these results for several scenarios.

Daytona Beach

Physical Profile

We show a topographical and bathymetric map in **Figure 4** [NOAA 2007]. The elevation is at least several meters for all major inhabited areas, so we neglect the likelihood of direct flooding from the predicted rise.

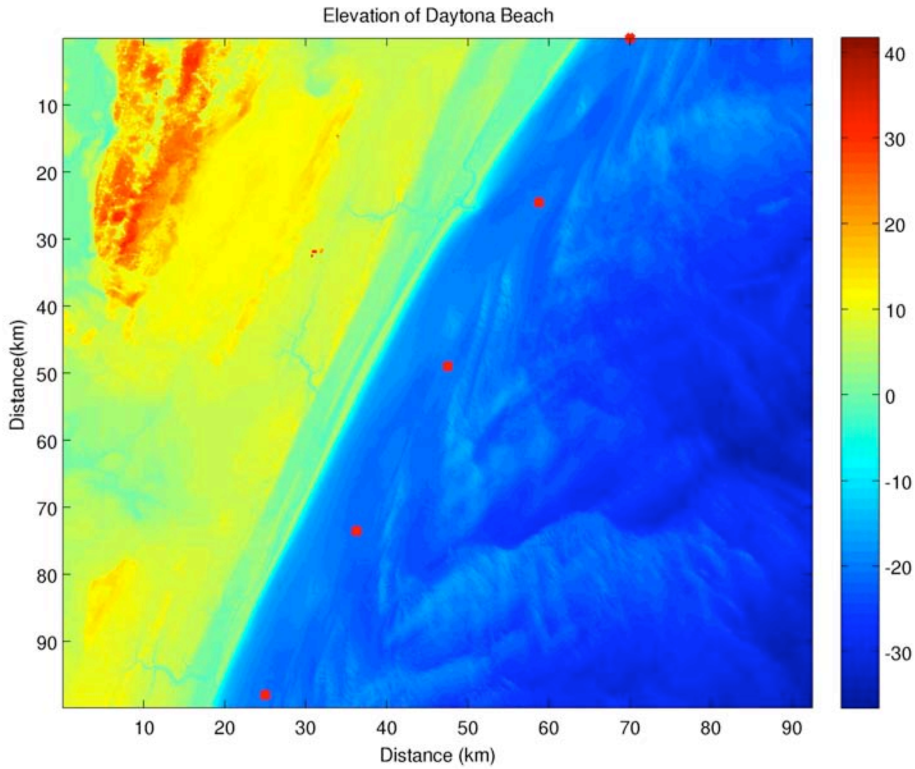

Figure 4. Topography and bathymetry of Daytona Beach, with five sampled points (in red) lying along a line from $(25\ \ 98)$ to $(70\ \ 0)$.

Beach Profile

To determine the constant K in **(1)** for Daytona Beach, we collect sample points (shown in red in **Figure 4**. We use these results with the corresponding elevation and position of the shoreline to determine the ratio as follows:

$$K_i = \frac{\sqrt{(\Delta x)^2 + (\Delta y)^2}}{\Delta h}.$$

We show the results of this calculation for all five points in **Table 3** and arrive at a mean value $K = 452$.

We observe the effectiveness of the Bruun approximation when we fit an averaged profile for Daytona Beach (**Figure 5**).

Future Erosion of Daytona Beach

We use the sea levels in **Table 2** to calculate values for the beach recession at the necessary intervals. Daytona Beach contains a series of barrier islands, and we assume that the small separation between them and the mainland will prevent any significant erosion on the Daytona mainland.

Table 3.
Determination of the scaling coefficient K for Daytona Beach.

Point	Distance (km)	Elevation difference (m)	K
1	9.65	20.29	475.7
2	9.39	20.51	457.8
3	9.66	21.15	456.6
4	9.64	22.18	434.44
5	9.22	21.13	436.31
Mean			452 ± 17

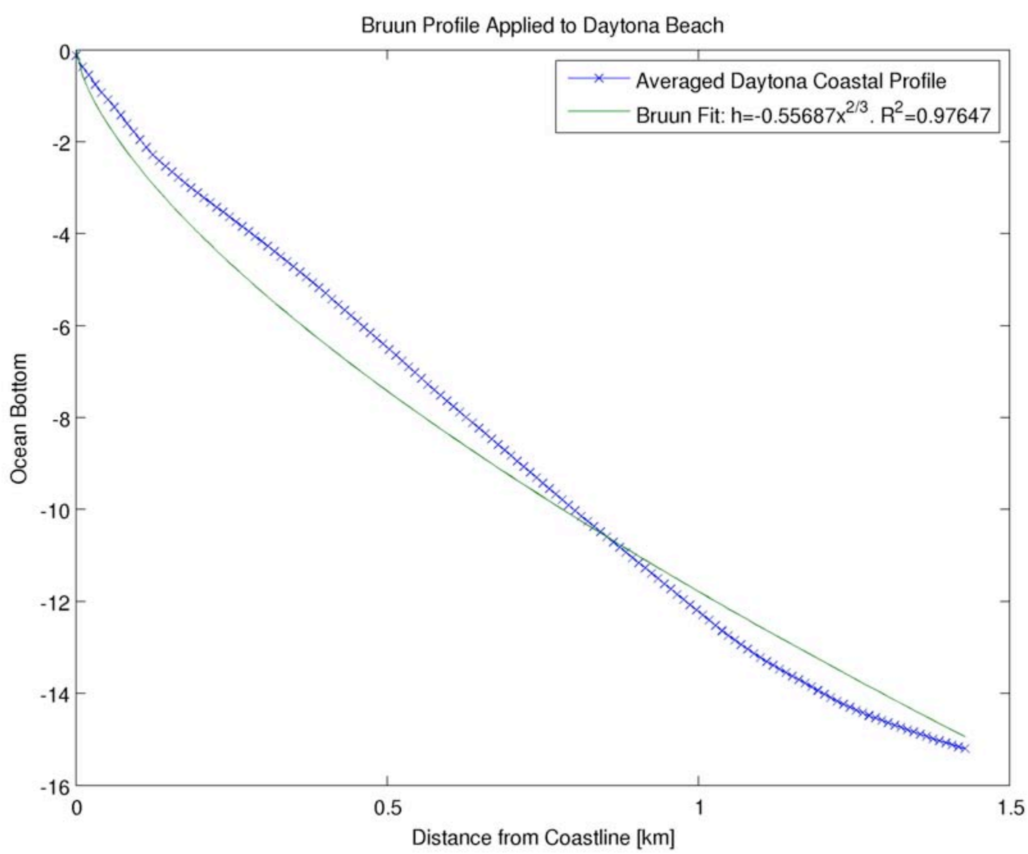

Figure 5. Appropriateness of the Bruun model.

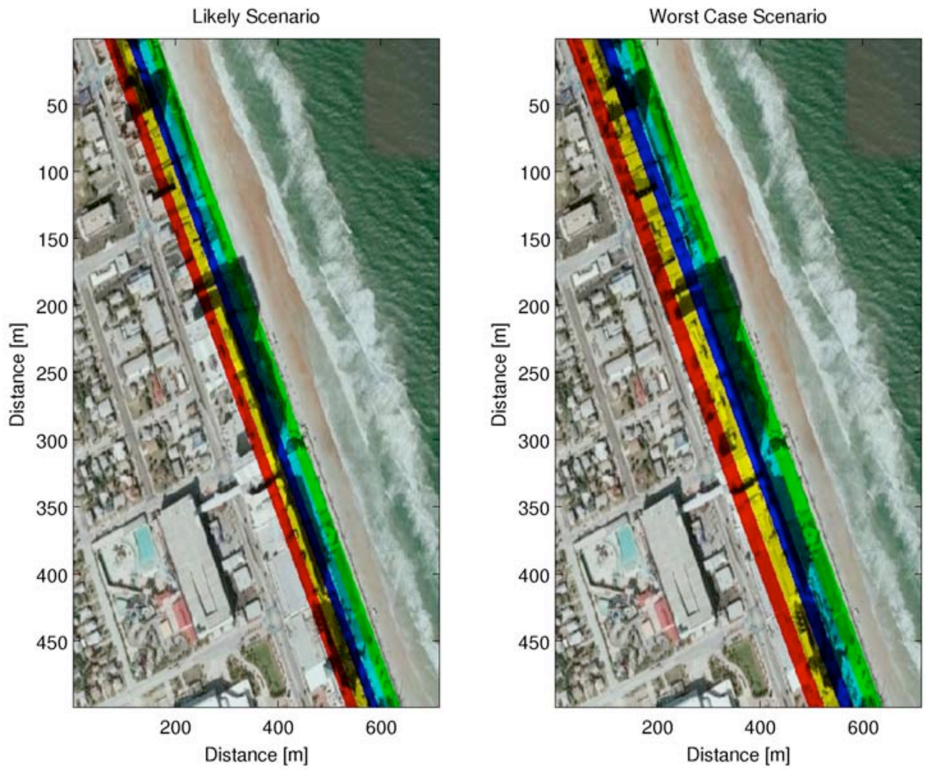

Figure 6. Effect of two climate scenarios on the erosion of Daytona Beach (Overlay: Google Earth [2008]). Shaded regions indicate increments of 10 years from 2000.

To gauge the impact of this erosion, we overlay the results for the likely- and worst-case scenarios for each decade onto a Google Earth [2008] map of Daytona Beach (**Figure 6**). *Nearly a full block width of the city will be destroyed by 2050 if no precautions are put into place.*

Miami Beach

Physical Profile

Again we work with topographical and bathymetric representations [NOAA 2008]. The low elevation of the boundaries of Miami yield problems for the city with the rise in sea level. The effects of the likely 17 cm rise in sea level are visualized in **Figure 7**.

The regions of concern are already surrounded by high walls. They should be reinforced.

Beach Profile

We determine the constant K for Miami in a similar manner to that for Daytona Beach; but rather than using multiple samples, we obtain an aver-

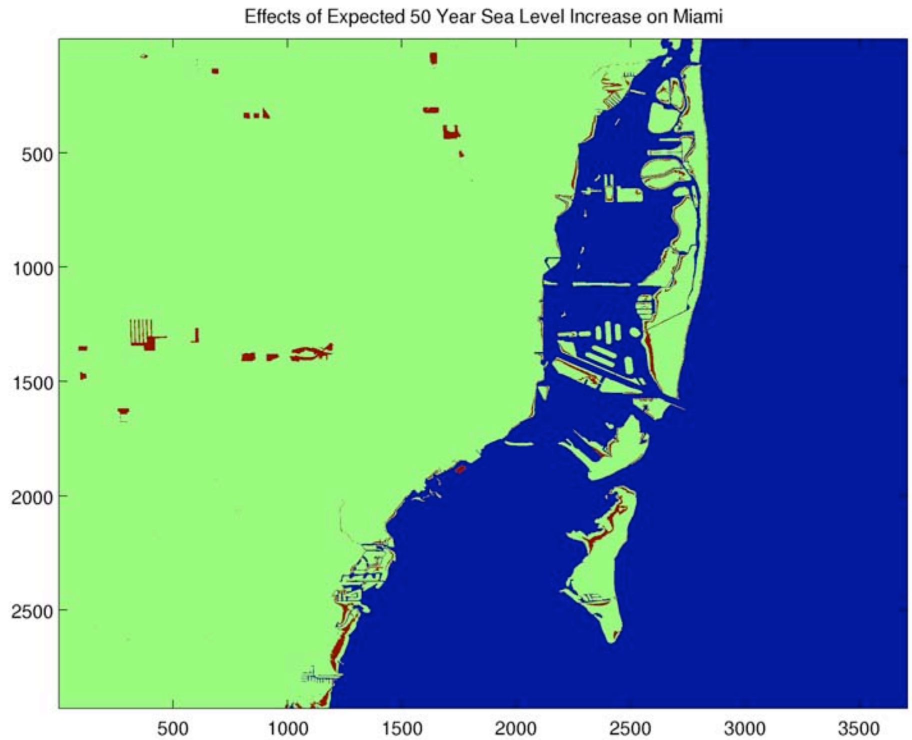

Figure 7. Regions of Miami susceptible to a 17 cm rise in sea level. Dark (blue) is existing water, light (green) is safe land, and dark (red) regions inside the light is susceptible land.

age beach profile through averaging. This results in $K = 520.83$, a higher value than for Daytona Beach, due to the significantly greater gradual slope in the coastal area just off the shore of Miami.

Future Erosion of Miami Beach

We show the results in **Figure 8**. *As with Daytona Beach, without intervention a width of nearly a city block width will be lost to the ocean.*

Common Solution for Daytona and Miami

Our beach erosion model is grounded in the observation that most beaches return to an equilibrium profile based on the average particle sizes reflected in the coefficient A. To take advantage best of the predictions of our model, we propose a solution for Miami and Daytona, based on raising the average height of the curve at the bottom of the slope to allow for a more stable beach front. This is visualized in **Figure 9**.

There are several key benefits to this design. The use of a retainer along the bottom allows the natural tendency of the waves to carry sand and sedimentation to fill in the beach naturally, without the need for costly and

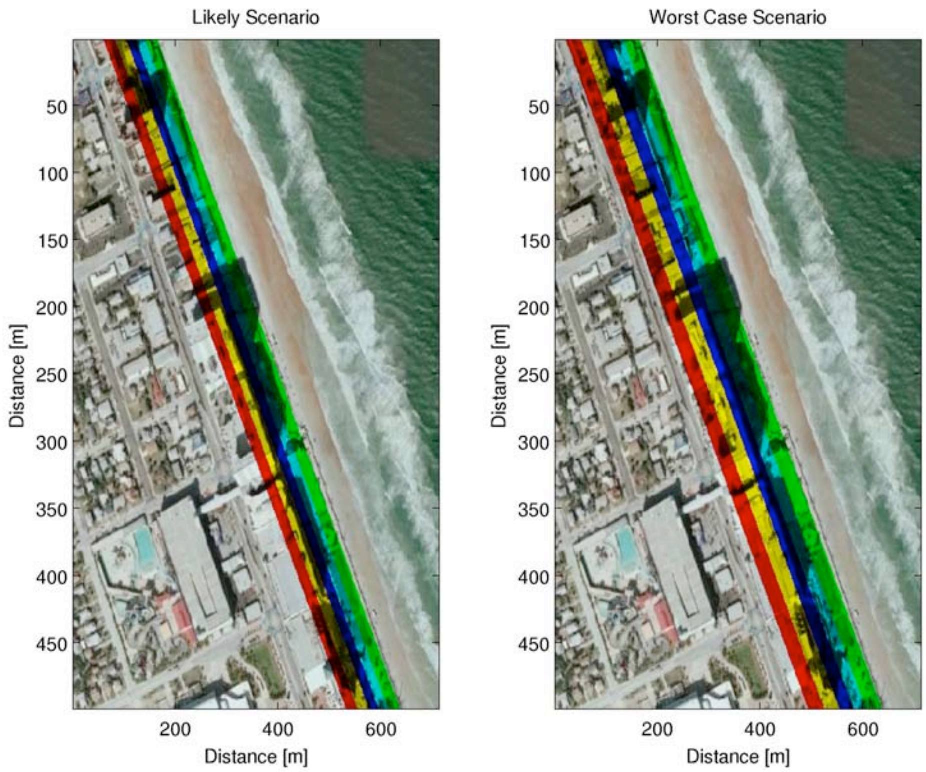

Figure 8. Effect of two climate scenarios on the erosion of Miami Beach. Shaded regions indicate increments of 10 years from 2000.

continuous additions of sand and filler. The ideal design for these retainers would be anchored concrete shapes, built to withstand the continuous force of the waves over long periods.

Conclusion

Several important conclusions can be made about future problems for the coastal cities of Florida. The sea level is definitely rising, and our model linking this activity to changes in the northern ice caps suggest an acceleration of this trend. Our model predicts a likely beach recession of 60 m by 2050, with up to 90 m possible. This recession would severely damage the first block nearest the ocean in each city unless there is intervention. Due to its lower elevation, Miami is significantly more at risk than more northern cities like Daytona, so it should be more concerned.

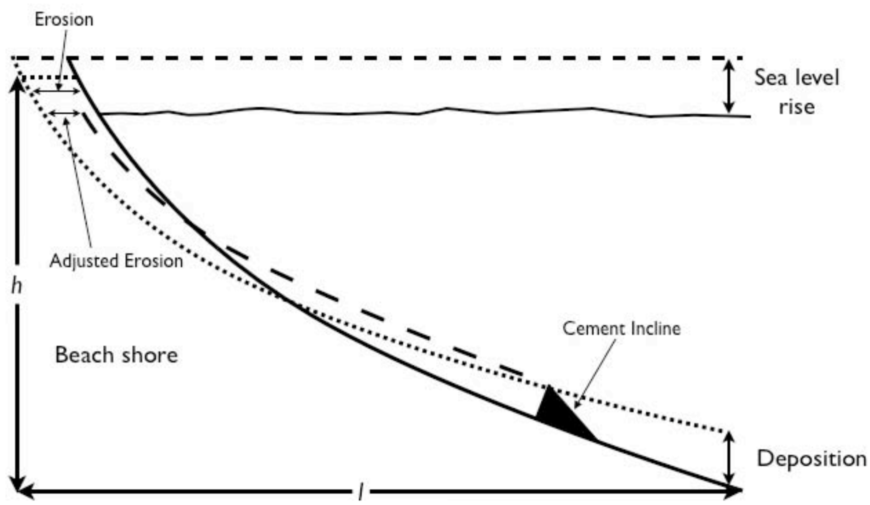

Figure 9. Proposed solution for Daytona and Miami.

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Coach Lou Rossi (seated) with Mathematical Modeling teams' members (from left) senior Matthew Thies, senior Zachary Ulissi, junior Bob Liu, and freshman Kyle Thomas (kneeling).

Beach, FL 1/3 arc-second tsunami inundation DEM.
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