Fastidious Farmer Algorithms (FFA)

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Summary

An effective irrigation plan is crucial to "hand move" irrigation systems. "Hand move" systems consist of easily movable aluminum pipes and sprinklers that are typically used as a low-cost, low-scale watering system. Without an effective irrigation plan, the crops will either be watered improperly, resulting in a damaged harvest, or watered inefficiently, using too much water.

We determine an algorithm for "hand move" irrigation systems that irrigates as uniformly as possible in the least amount of time. We physically characterize the system, determine a method of evaluating various irrigation algorithms, and test these algorithms to determine the most effective strategy.

Using fluid mechanics, we find that we can have at most three nozzles on the 20-m pipe while maintaining appropriate water pressure. We model our sprinkler system after the Rain Bird 70H 1" impact sprinkler, which works at the desired pressure and has approximately a 0.6 cm diameter. Combining data and analysis, we confirm that the radius of the sprinkler will be 19.5 m. Researchers have proposed several models for the water distribution pattern about a sprinkler; we consider a triangular distribution and an exponential distribution.

We do not consider schemes that do not water all areas of the field at least 2 cm every 4 days or water areas more than 0.75 cm/h. The largest cost in time and labor is in moving the pipe. Thus, we look for a small number of moves that still gives the desired time and stability. From these configurations, computer analysis determines which is most uniform.

For various situations, we propose an optimal solution. The bases of the sprinkler placement patterns are triangular and rectangular lattices. We craft three patterns to maximize application to the difficult edges and corners.

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- For calm conditions and a level field, the field can be watered with just two moves (the "Lazy Farmer" configuration). However, this approach is unstable, and even weak wind would leave parts of the field dry. With three moves, little stability is gained; so four positions is best.
- The "Creative Farmer" triangular lattice gives both stability and uniformity. The extra time is warranted because of its ability to adapt.
- We obtain even more stability using the "Conservative Farmer" model but at the price of a decrease in uniformity from the "Creative Farmer" approach.

Description of Problem

The goal is to irrigate a 30 m × 80 m field as uniformly as possible while minimizing the labor/time required. We assume the following equipment and specifications:

- Pipes of 10-cm diameter with rotating spray nozzles of 0.6 cm diameter
- Nozzles are raised about 1 m from the pipe and can spray at angles ranging from 20° to 30°.
- Total length of the pipe is 20 m.
- A water source with a pressure of 420 kPa and a flow rate of 150 L/min.

We consider the following guidelines and assumptions:

- No part of the field should receive more than 0.75 cm/h.
- Every part of the field should receive at least 2 cm every four days.
- Overwatering should be avoided.
- Sprinklers are in working order and rotate 360°, spraying uniformly with respect to rotational symmetry.
- The soil is approximately uniform and the terrain is flat.
- Wind is considered only in terms of stability.
- We can place a water supply pipe through the field along either its width or its length, which has multiple connection spots for the movable pipes.
- For such a small field, any move requires approximately equal time, so we need only minimize the total number of moves M_T.
- In particularly arid areas, evaporation reduces the total water application but with no more than 5 % loss.
- We ignore rainfall, assuming that it is accounted for by delaying waterings.

Definitions and Notation

Let D be a distribution of sprinklers, including placement on the pipe and locations in the field. We consider accumulation over a region R. Let

$M_T(D)$ = total number of moves by the farmer required for a distribution,

$Aver(D, R)$ = average application rate over region R,

$\text{Var}(D, R)$ = variance of the rate of application over region R,

$\max(D, R)$ = maximum rate of application over the region R, and

$\min(D, R)$ = minimum rate of application over the region R.

Pipe Capacity and Resulting Pressure/Radii

Watch out for the Rain Bird

We derive the exit speed, flow, and water-drop drag coefficient for a sprinkler with our conditions and show that it agrees with the Rain Bird 70H 1″ Brass Impact Sprinkler [Rain Bird Agricultural Products n.d.]. We assume laminar flow and use Bernoulli's equation:

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2,$$

where

P_i is absolute pressure,

ρ is the density of water,

v_i is speed,

g is the gravitational constant, and

y_i is height.

Because our field is flat, we have $y_1 = y_2$, so the height of our source relative to our sprinklers does not affect the exit speed v_2:

$$v_2 = \sqrt{\frac{2}{\rho}P + v_1^2},$$

where P is the relative pressure. We must first find the speed v_1 of water at our source:

$$v_1 = \frac{150\ \text{L}}{\text{min}} \times \frac{1\ \text{min}}{60\ \text{s}} \times \frac{1\ \text{m}^3}{1000\ \text{L}} \times \frac{1}{\pi(0.05)^2\ \text{m}^2} = \frac{1}{\pi}\ \text{m/s}.$$

Plugging v_1 into the equation for v_2, we obtain

$$v_2 = \sqrt{\frac{2}{1000} \times 420 \times 1000 + \frac{1}{\pi^2}} \approx \sqrt{840} \approx 28.9 \text{ m/s}.$$

That's fast (about 60 mph)! It may be too fast. This exit speed does not take into account friction in the pipes, for which we propose an attenuation factor. The volume out of the sprinkler is the speed times the cross-sectional area of the sprinkler times the attenuation factor:

$$Q = C_s A_c \sqrt{\frac{2}{\rho} P},$$

where

Q is the discharge in (m^3/s),

C_s is the attenuation factor, and

A_c is the cross-sectional area (m^2).

Using pressure and discharge data from Rain Bird Agricultural Products [n.d.], we find the attenuation factor to be

$$C_s = \frac{Q}{A_c\sqrt{\frac{2}{\rho P}}} = \frac{3.17 \times \frac{1}{3600}}{\pi(0.003175)^2\sqrt{800}} \approx 0.983.$$

This value shows very little loss due to friction. The escape speed with friction is

$$v = 0.983 \times 28.9 \approx 28.5 \text{ m/s}.$$

How many liters flow out of each sprinkler per minute is simply the speed multiplied by the area, converted to liters per minute:

$$\frac{\text{Volume}}{\text{unit time}} = 28.5 \times \pi(0.003)^2 \times \frac{1000 \text{ L}}{\text{m}^3} \times \frac{60\text{s}}{\text{min}} = 48.35 \text{ L/min}.$$

We can therefore use up to three sprinklers without using more than 150 L/min; with more than 3 sprinklers, there will be a pressure drop. To find the new pressure, we use the continuity principle, which states that the volume of water flowing in equals the volume of water flowing out:

$$A_s v_s = n A_N v_N,$$

where

A_s is cross-sectional area of the source,

v_s is speed of water at our source,

n is number of sprinklers,

A_N is cross-sectional area of the sprinkler nozzle, and

v_N is speed out of the sprinkler nozzle.

Solving for v_N, we obtain

$$v_N = \frac{r_s^2}{n\pi r_N^2} = \frac{(5 \times 10^{-2})^2}{n\pi(3 \times 10^{-3})^2} \approx \frac{88}{n} \text{ m/s},$$

where $n > 3$, r_s is the radius of the pipe at the source, and r_N is the radius of the sprinkler nozzle.

For four sprinklers, the exit speed would be 22 m/s and the pressure would be 252 kPa. The pressure needs to be above 280 kPa [Rain Bird Agricultural Products n.d.]. Since too low a pressure would result in a low degree of uniformity, we limit ourselves to at most three sprinklers.

Kinematics Equations

Because water droplets are small and the escape speed is above the terminal speed, drag must be taken into account. We have the following differential equations for speeds in the x- and y-directions:

$$\frac{dv_x}{dt} = -kv_x, \qquad \frac{dv_y}{dt} = -g - kv_y,$$

whose solutions are

$$y(t) = \frac{-g}{k}t + \left(\frac{v_0 k \sin\theta + g}{k^2}\right)\left(1 - e^{-kt}\right) + y_0,$$
$$x(t) = \frac{v_0 \cos\theta}{k}\left(1 - e^{-kt}\right) + x_0.$$

We use the following initial conditions (with some from Rain Bird Agricultural Products [n.d.]) to determine the drag constant:

$$y_0 = 1 \text{ m}, \qquad x_0 = 0, \qquad v_0 = 1 \text{ m}, \qquad \theta = 21^\circ.$$

The published value for the radius for our system is approximately 19.5 m [Rain Bird Agricultural Products n.d.]. Using this distance and the above initial conditions, we determine the drag constant to be $k = 1.203$.

We thus have an equation for how the radius of the water emitted by the sprinkler is determined by the height and angle of the sprinkler. Although we keep our sprinkler at factory settings, the farmer could modify the sprinkler to adjust the radius if needed.

Distribution from Standard Sprinkler

While the sprinklers under consideration cover a disk of radius 19.5 m, the distribution need not be uniform over that area. Large droplets tend to travel farther, but the area near the perimeter is much larger than near the sprinkler head. We discuss various models for this behavior based on empirical data.

Triangular Model

Smajstrla et al. [1997] propose that the water distribution can be modeled as a triangle. That is, the application rate falls linearly as a function of distance from the sprinkler head, disappearing outside the radius. **Figure 1** shows an example with radius 25 ft.

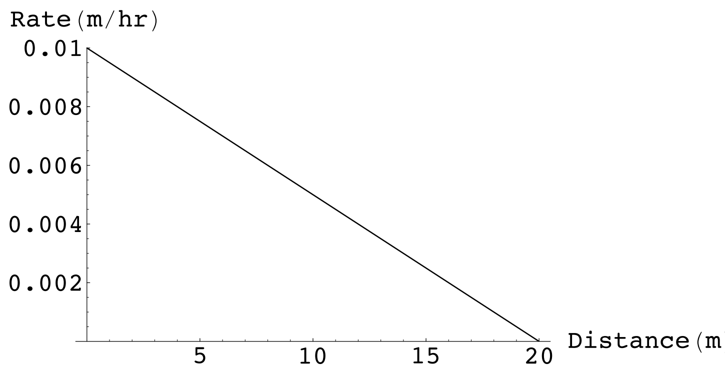

Figure 1. Triangular water distribution (redrawn from Smajstrla et al. [1997]).

In three dimensions, the distribution is cone-shaped, centered about the sprinkler nozzle. To analyze a grid pattern, we sum the water distribution over a cone and analyze the resulting surface.

Experimental/Exponential Decay Model

Louie and Selker [2000] experimentally tested the performance of a Rain Bird 4.37-mm nozzle. The distribution spikes within 2 m of the sprinkler head, then maintains an approximately uniform rate before decaying near the edge of the radius (**Figure 2**). We use exponentials to fit a curve to this graph. We then scale the width and the height of the function to correspond to the radius and water flow of our larger sprinkler:

$$f(r) = (3 \times 0.00267 \times e^{-0.7r} + 0.00267) \times e^{-(r/19.5)^{20}}.$$

To get the three-dimensional distribution, we rotate the function about the z-axis by replacing r with $\sqrt{(x-a)^2 + (y-b)^2}$ for a sprinkler centered at (a, b). In some ways, this distribution is a worst-case scenario because of the large peak about the sprinkler. The curve is based experimentally on where drops landed but does not take into account possible spread on landing.

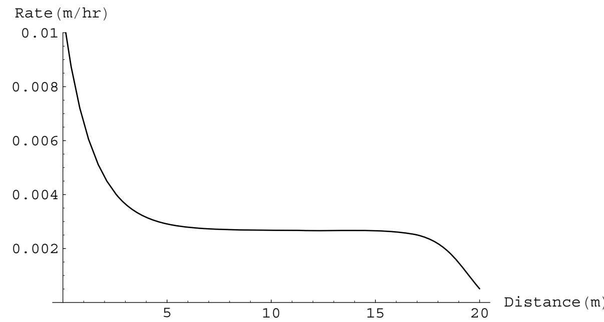

Figure 2. Exponential water distribution (redrawn from Louie and Selker [2000].

Comparison of Models

The exponential decay model is the more realistic of the two models. It forces careful consideration of how long a sprinkler can be left. Any configuration acceptable for this model will most likely work under the triangular model too.

Conclusions

With the exponential model, the application rate near the sprinkler head increases to 0.01 m/h = 10 cm/h. We are constrained to a maximum rate of 7.5 cm/h to avoid damage to the soil and crops. Thus, using the exponential model results in configurations where sprinklers run for less than the full 60 min each hour. We later discuss several methods to minimize the inconvenience that this constraint causes the farmer.

A similar difficulty arises in the triangular model with three sprinklers. The best that we can do for the sprinkler in the middle is to space the sprinkler heads evenly, with one at each end. The distance of separation is then 10 m. Scaling the triangle for the values of the Rain Bird (3.2 m^3/h, 19.5-m radius), we get a peak height of about 8 cm/h; so at 10 m, we get 4 cm/h. The middle sprinkler head would be receiving 16 cm/h, which is over twice the acceptable amount.

For either model, three sprinklers can only be run for a limited time every hour. Thus, our proposed solutions have exactly two sprinklers on the pipe.

Analysis of Standard Grid Patterns

We analyze standard grid patterns. Symmetric designs that cover a rectangular field include squares, rectangles, and triangles. To counteract the effects of varying models of distribution (triangular or exponential), all patterns employ overlapping sprinkler patterns. In most cases, researchers recommend 40–60% overlap of radii to obtain the most uniform distribution, which also tends to be the most stable under windy conditions [Eisenhauer et al. n.d.].

We use the triangular distribution to evaluate grid patterns with the goal of finding the ideal side lengths as a ratio of the radius. Mathematical analysis shows that in terms of uniformity, the ideal rectangle is a square with side $1.1\times$(radius); however, a triangle grid pattern obtains better uniformity, though with smaller spacing, $0.85\times$ (radius).

Evaluation Methods

We have two primary concerns in evaluating a grid pattern. The minimum value on the surface determines the time required to water the field, so we must watch for too low a minimum value. We measure uniformity by calculating the variance of the distribution. In each case, we consider a unit of the grid, that is, one square or one triangle, and plot the distributions of all sprinklers that water that square. The average rate and variance are

$$\begin{aligned} Aver(D,R) &= \frac{1}{\text{Area}(R)}\int_R D(x,y), \\ \text{Var}(D,R) &= \frac{1}{\text{Area}(R)}\int_R (D(x,y)-Aver(D,R))^2, \end{aligned}$$

where $D(x,y)$ is the distribution and R is the unit region. A large $\text{Var}(D,R)$ means water will be applied nonuniformly and could result in poor growth. To aid in assessing the extent of variation, we also calculate $\max(D,R)$. The difference between $\max(D,R)$ and $\min(D,R)$ gives a measure of how large the variation is.

Rectangular Grids

For each of several vertical separations between sprinklers, from 0.8 to 1.2 times the radius, we consider a range of possible horizontal separations. Generally, the variance decreases and then increases as the horizontal separation increases, defining a clear minimum value, which for all configurations is approximately a horizontal separation of 1.1 times the radius. The best rectangular configurations turns out to be a square of side length $1.1\times$(radius). In the tests, the difference in maximum and minimum correlates closely with the variance, so we use that as the basis for comparison.

Triangular Grids

For the triangular lattice, we must also model surrounding triangles because nearby sprinklers have a significant effect.

We do not consider distances less than $0.8\times$ (radius) because significant overwatering would take place. Thus, the most uniform configuration is for $0.85\times$ (radius). This separation distance results in higher uniformity than the rectangular configuration.

The exponential distribution gave similar results on the tests, indicating that triangular set-ups are a good enough approximation for comparing uniformity.

Proposed Irrigation Methods

We proceed to design the pipe network and watering schemes. We make the following assumptions:

- Water is distributed according to the exponential water distribution.
- The sprinklers are unmodified, and we place two of them at the ends of the pipe, separated by 20 m.
- The efficiency of the sprinkler irrigation system is at least 95%, no more than 5% is lost to evaporation and other factors.
- There exists infrastructure that can supply water along the center of the field.

Our goal is a system that provides at least 2 cm of water at every point of the field every 4 days, and no more than 0.75 cm during any single hour. In addition, we would like the pattern of watering to be periodic with period of four days. We compare the different systems using the following criteria:

- required number of moves M_T of the pipes;
- hours of operation of the system;
- stability with respect to factors like wind and equipment malfunctions; and
- uniformity of irrigation.

The water falling right next to the sprinkler is 1 cm/h, which means we cannot have a sprinkler operational for more than 45 min in an hour; so the farmer must come and stop the sprinklers 45 min after they were turned on and to turn them on again (or move them) 15 min later. Since the pipes have valves that can easily be closed and opened, even under pressure, turning them on or off doing consumes only a few minutes. In addition, if one sprinkler is within the radius of another, the time that they can be operational will be severely reduced; we want to avoid such a situation by positioning the sprinklers at the ends on the 20 m pipe.

We divided the field into four 20 m $\times$ 30 m rectangular pieces, each of which is further subdivided into triangles by the two diagonals (**Figure 3**).

It is impossible to water the whole field using our pipe of length 20 m and our sprinklers with their radius of irrigation of 19.5 m, since we cannot water two points separated by more than 19.5 m + 20 m + 19.5 m = 59 m. Therefore, we must move the pipe at least once; and since after 4 days the pipe should be in its initial position, we must move it twice per period. Therefore, our lower

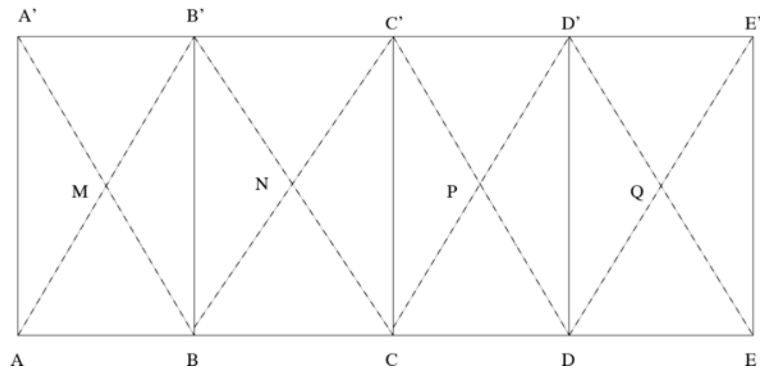

Figure 3. The field, subdivided.

bound on M_T is 2. It would be nice to achieve this minimum. Insight into how to do this can be obtained by drawing circles with radius 19.5 at the points A, C, E, A', C', and E' in **Figure 4**. We must position the sprinklers so that in each circle there is at least sprinkler. This leads to a scheme with two moves.

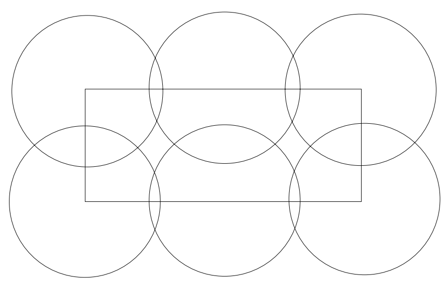

Figure 4. Covering the edges.

The Case $M_T = 2$

Suppose that we position the pipe as shown in **Figure 5**.

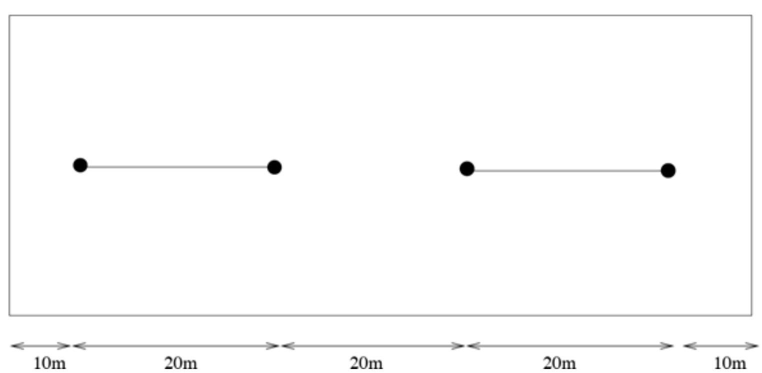

Figure 5. The Lazy Farmer configuration.

The greatest distance from a sprinkler (denoted by a dot) is $\sqrt{10^2 + 15^2} = 18.02$ m, and these points are precisely A, C, E, A', C', and E' in **Figure 5**.

From our exponential water distribution graph (**Figure 2**), the amount of water falling at those points is 2.25 mm/h; but since we operate a sprinkler for at most 45 min, the actual value is 1.68 mm/h. Thus, if we operate the system for 13 hours at each location, we get a minimum of 2.18 cm of water at every point, for more than 2 cm everywhere when we subtract loss due to evaporation. Therefore, the total time the system would be operational is 26 hours, the pipes would have to be moved twice, and the amount of water used would be $(2)(26\text{ h})(45\text{ min/h})(48.35\text{ L/min}) = 113 \times 10^3$ L per period. If the watering were optimal, the required amount of water would be $(30\text{ m})(80\text{ m})(2\text{ cm}) = 48 \times 10^3$ L of water. Therefore, the water efficiency is 42%. As for uniformity, we calculate that the variance is 1.7×10^{-6}, which corresponds to a high degree of uniformity. However, this configuration has a major disadvantage: Even a small change of 2 m in the sprinkler radius (for instance, due to wind or to decrease in pressure in the pipes) can result in distant points such as A receiving no water. Therefore, this configuration, although very uniform and with minimal M_T, is not very stable.

The case $M_T = 3$

We want to have a smaller maximum distance $d_{\max}$ between a point in the field and the nearest sprinkler. Using a similar argument to the the previous case, the resulting configuration should look something like **Figure 6**.

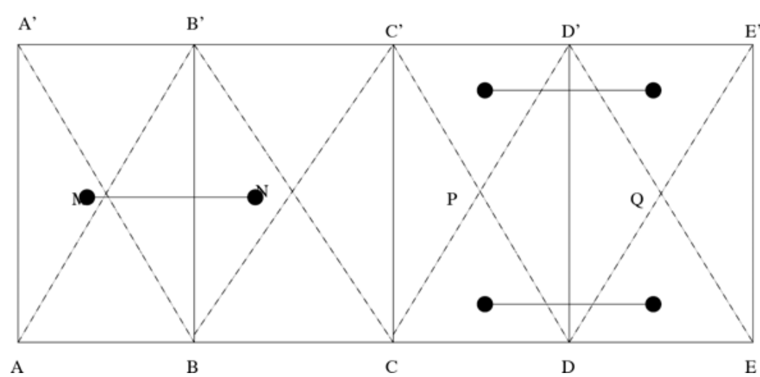

Figure 6. Configuration for $M_T = 3$.

In such a configuration, $d_{\max} \geq 16.5$, so the gain in stability is slight. In addition, there is a huge increase in operational time and water required, to 39 h and 169×10^3 L. Therefore, the case $M_T = 3$ results in bad configurations.

The case $M_T = 4$

With the increase in the number of times that we can move the pipes, the complexity of positioning them increases dramatically, making it nearly impossible to consider all configurations. However, since we want a stable configuration, we should have sprinklers close to the points A, A', E, and E'. In addition, for uniformity, we should preserve some symmetry. The earlier triangular and rectangular patterns can be successfully applied in this case. The best way to reduce peaks in watering is to use a triangular pattern, like the one in **Figure 7**.

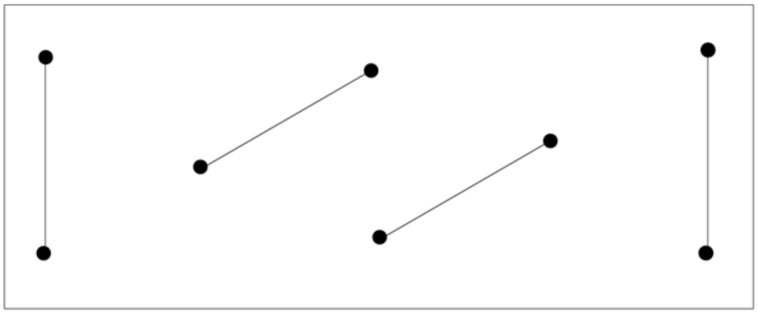

Figure 7. Creative Farmer layout.

The sprinklers are first set at the vertices of equilateral triangles of side 20 m. After that, to minimize instability, the leftmost pipe is translated 5 m to the right and the rightmost 5 m to the left. Then $d_{\max} \leq 14$, which implies that this scheme would work well provided that the wind does not result in more than 25% deviation. This layout has a variance of 3.35×10^{-6}, period of operation of 52 h, and water consumption of 226×10^3 L per period.

Another possibility is sprinklers in a rectangular pattern (**Figure 8**).

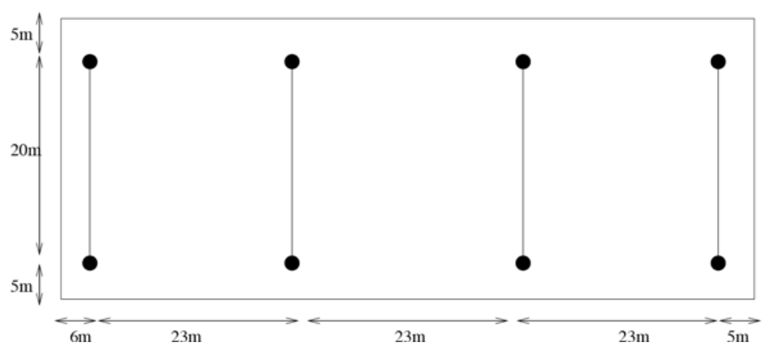

Figure 8. Conservative Farmer layout.

The distance from the sprinklers to the points on the sides is less than 12 m, and the area between two pipe positions is within the radius of four sprinklers and thus would be watered no matter what the direction of the wind is. This irrigation would be good provided that the wind doesn't alter the area covered by more than 7 m. The layout's variance is 4.17×10^{-6}, and the hours of operation and water consumption are the same as in the previous case.

The case $M_T > 4$

Since moving the pipes takes a lot of time, and in addition we have observed how the variance increases, even for the triangular configuration, we can conclude that the case $M_T > 4$ would not lead to a good layout.

Numerical Analysis of Proposed Strategies

With the same criteria used to evaluate the standard grid patterns, we diagnose the algorithms on our 30 m $\times$ 80 m test field. We set up the field on a grid with endpoints (0,0), (80,0), (0,30) and (80,30). This setup allows us to evaluate the entire field and take into account edge effects. The following four strategies are the best performers of several that we analyzed. The corresponding sprinkler placements are shown in **Figure 9**.

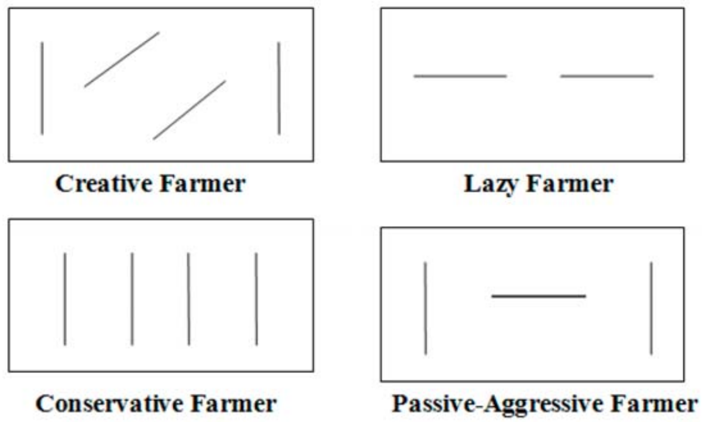

Figure 9. Sprinkle placements for several strategies.

Lazy Farmer System

This strategy uses few moves, has a high degree of uniformity, and can irrigate the entire field—perfect for farmers who would rather be shooting soda cans off of a fence post than lugging around a heavy aluminum tube. It takes 26 h and uses the least amount of water.

The Passive-Aggressive Farmer System

This approach neither improves much upon the stability of the Lazy Farmer system nor saves time by using few pipe moves. Therefore, it would be perfect for an indecisive or passive-aggressive farmer.

The Conservative Farmer System

This strategy is very stable, perfect for a farmer who is very careful and untrusting of the wind. It takes twice as much time as the Lazy Farmer approach and uses twice as much water.

The Creative Farmer System

This is the second most uniform system. The setup is somewhat complicated, but some farmers may be up to the task. It is perfect for a farmer who regularly plays Sudoku and stopped watching the TV show *MacGyver* (1985–1992) because the farmer felt MacGyver lacked ingenuity. It takes just as long as the Conservative Farmer system and uses just as much water.

Conclusion

There are only two worthwhile strategies. The Conservative Farmer system should be used in windy conditions or if the level of the field is somewhat nonuniform. The Lazy Farmer system should be used otherwise, because it is the fastest, easiest, and most uniform.

We base our strategies and conclusions on data from a sprinkler manufacturer We also examined specifications of sprinklers from other manufacturers and found little change in our results.

The methods that we used to evaluate proposed strategies are general. Our method of analysis could be repeated to obtain optimal strategies in other cases with different parameter values (a different pipe, field, or water pressure).

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Team members Matthew Fischer, Brandon Levin, and Nikifor Bliznashki, in Duke apparel.

Team advisor William Mitchener (on right), with friend.