Sprinkler Systems for Dummies: Optimizing a Hand-Moved Sprinkler System

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Summary

"Hand moved" irrigation, cheap but labor-intensive systems used on small farms, consists of a movable pipe with sprinklers on top which can be attached to a stationary main. Our goal is a schedule that meets specific watering requirements and minimizes labor, given flow parameters and pipe specifications.

We apply Bernoulli's energy-conservation equation to the flow characteristics to determine sprinkler discharge speeds, ranges, and flow rates. Using symmetry and a model of sprinkler coverage, we find that three sprinklers, operating 57 min at 9 consecutive cycling stations during four 11-hour workdays, with the sprinklers 9 m apart on the 20 m mobile pipe and six mainline stations spaced 15 m apart, will water more than 99% of the field. Our computer model uses a genetic algorithm to improve the efficacy to 100% by changing sprinkler spacing to 10 m and adjusting the mainline station spacing accordingly.

Introduction

Our challenge is to design a movable sprinkler system to meet a set of watering criteria on an 80 m × 30 m field with specified flow characteristics of the main pipe. We must

- decide how a stationary main water pipe should be positioned on the field;

The UMAP Journal 27 (3) (2006) 237–254.

- determine the number of sprinklers, the sprinkler spacing on the movable pipe, and the spacing of the attachments for the movable pipe; and
- schedule moving the pipe.

We make simplifications and formulate models. We determine the stationary pipe position and, ignoring friction, formulate first a simple one-sprinkler model, using the principles of conservation of flow and the Bernoulli energy equation. We use the results in the jet equation to find trajectory and range of a single sprinkler. We apply the same principles to multisprinkler systems.

The area watered can be modeled as either a uniform disk or a ring. We minimize the overlapping areas in the disk model to eliminate underwatered areas and maximize uniform water distribution, for different numbers of sprinklers. This optimization determines sprinkler spacing, movable pipe spacing, and the number of moves to water the entire field.

Using calculated values for flow rates and depth accumulation over time, we determine a schedule that minimizes time and effort.

Assumptions

- The field is flat, so there is no change in energy (or head) of the system due to changes in elevation.
- One 80-m fixed pipe runs the length of the field, to which a lateral arm can be attached perpendicularly.
- There is only one lateral pipe 20 m long.
- The angle at which the water leaves the sprinkler is 30°.
- Sprinklers can be modeled as 360° rotary jets.
- The entire flow does not need to leave the sprinklers; a return system is implied.
- There is no wind; the sprinklers always water evenly over a circular area.
- Rain is not modeled; the system is turned off during rain.
- The sprinkler system is inactive for at least 8 h every night.

Models

System Requirements and Statistics

The aluminum pipe system consists of a stationary main pipe with a diameter of 10 cm. Perpendicular to this pipe, a single 20-m-long (10-cm-diameter) lateral or movable pipe can be attached. On this pipe are a number of sprinklers (0.6 cm diameter) that can either sit directly on the movable pipe or connect to the pipe with 0.6 cm diameter vertical pipes. Water flow at the source is 150 L/min at a pressure of 420 kPa. Every area should receive no more than 0.75 cm depth of water in an hour and no less than 2 cm over four days.

Field Layout

The field is 80 m by 30 m. The lateral or movable pipe is 20 m long when assembled with a number of rotating sprinklers attached to it. We interpret the sprinkler set to mean we can use only one 20-m-long pipe as the lateral movable pipe (multiple 20-m sections cannot be attached to one another). The lateral pipe can be connected perpendicularly to a fixed main pipe with the same diameter (10 cm). We find it efficient and symmetric when the main pipe runs the length of the 80 m field and is inset 5 m onto the field. This ensures that if sprinklers are set on both ends of the lateral pipe, the field can still be watered symmetrically. We can optimize the number of connection points and spacing on the main pipe based on the number of sprinklers on the lateral pipe. We assume that valves can shut off flow to excess length of the main pipe in order to direct the entire flow into the lateral pipe. Our model also assumes that the entire flow does not exit the sprinklers, but instead only the pressure of the system forces the water from the sprinklers; the rest of the water is returned to the reservoir from either a flexible return pipe or irrigation ditch.

One-Sprinkler System

To understand the outflow of the sprinklers, we simplify the model to include only one sprinkler and study the exit speed of the water leaving the rotating sprinkler head. This analysis can be done in terms of conservation of flow, or in terms of conservation of energy.

Conservation of Flow

We assume that there is no speed loss due to friction, the bend in the pipe, or the condition and angle of the sprinkler head.

Flow ($Q = vA = 150 \text{ L/min} = 0.0025 \text{ m}^3/\text{s}$) through the pipe must be conserved, so the flow through the pipes satisfies

$$v_2 A_2 = v_1 A_1,$$

where A_i is the area of pipe i and v_i is the speed in it [Walski et al. 2004, 3]. Taking the main pipe as pipe 1 and measuring in meters, we have

$$v_2 = \frac{0.318 \text{ m/s} \times \pi(0.05)^2}{\pi(0.003)^2} = 88.3 \text{ m/s},$$

which—at almost 200 mph—is faster than a sprinkler could probably handle.

Conservation of Energy

To take energy into consideration, we must make a bold assumption: Not all of the water running through the main pipe ends up on the field. To conserve energy, we must use the Bernoulli equation and neglect friction head loss at both positions 1 (the main pipe) and 2 (the sprinkler head) [Walski et al. 2004, 6]:

$$\frac{P_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{v_2^2}{2g},$$

where

P = pressure (N/m^2),

γ = specific weight of fluid (N/m^3),

z = elevation above a reference point (m),

v = fluid speed (m/s), and

g = gravitational acceleration (m/s^2).

For our parameter values, we have

$$\frac{420\ \text{kN/m}^2}{9.81\ \text{kN/m}^3} + 0 + \frac{(0.318\ \text{m/s}^2)}{2(9.81\ \text{m/s}^2)} = 0 + z_2\ \text{m} + \frac{v_2^2\ (\text{m/s})^2}{2(9.81)\ \text{m/s}^2}.$$

The pressure at point 2 (the sprinkler) is zero because at this point the water is being expelled through the nozzle and is under no pressure from the pipes [Finnemore 2002, 511]. So the exit speed of water out of the sprinkler, as a function of the height z_2 of the sprinkler off the ground is, after some algebra:

$$v_2(z_2) \approx 4.43\sqrt{42.83 - z_2}.$$

The height of sprinklers is usually between 6 in and 4 ft depending on the crop (assuming no braces to support the sprinklers) [National Resources Conservation Service 1997]; therefore, we test the sensitivity of the speed based on height (for our purposes using a range of 0 m to 1 m):

$$v_2(0) = 29.0\ \text{m/s}, \qquad v_2(1) = 28.7\ \text{m/s}.$$

So the speed out of the sprinkler is not sensitive to height. For the remainder of the study, we use $z_2 = 0.5$ m.

Trajectory

First, we make some assumptions:

- The speed found using energy conservation is the same initial speed that would occur through the nozzle at any discharge angle.

- The maximum speed is not affected by the means of dispersing the water, even if the spray is more similar to a fan than a jet.
- The nozzle is frictionless.

We use the single-sprinkler speed to find the range of the water discharged through the nozzle, via the jet equation [Finnemore 2002, 169]:

$$z = \frac{v_{z0}}{v_{x0}}\, x - \frac{g}{2v_{x0}}\, x^2.$$

For each additional sprinkler, the speed is cut down proportionally, based on the conservation of flow equation $v_2 = v_1 A_1/A_2$; the effective outflow area (A_2) is increased proportionally, so the speed decreases to $v = v_0/n = 28.8/n$ m/s. When rearranged, the jet equation gives the range for the number of sprinklers:

$$x = \frac{v\cos\theta\sqrt{v^2\sin^2\theta - 2gn^2 z} + v\sin\theta}{gn^2}, \tag{1}$$

where

x is the outer radius of the sprinkler coverage (range);

θ is the angle to the horizontal at which the sprinkler discharges;

v is the speed of the water at the sprinkler head, found using the conservation of energy equation;

z is the change in height from the jet to the ground;

g is the acceleration due to gravity; and

n is the number of sprinklers on the lateral pipe.

Since most rotary crop sprinklers discharge between 18° to 28° above the horizontal axis, but up to 35°, we assume that $\theta = 30°$ [Fipps and Dainello 2001].

Multiple-Sprinkler System

Given the design, a minimum 5 m radius is needed to reach the edges of the field. Based on our calculations from **(1)**, we find with four sprinkler heads that the radius is 5.32 m. Therefore, no more than four sprinklers can be put on the lateral pipe and still have a sufficient range to reach the edge of the field.

We need to know the rate at which the water flows out of the sprinklers to determine the rate of watering. Since $Q = vA$, the flow Q_n from each sprinkler when there are n sprinklers is

$$Q_n = v_n \pi (0.003)^2.$$

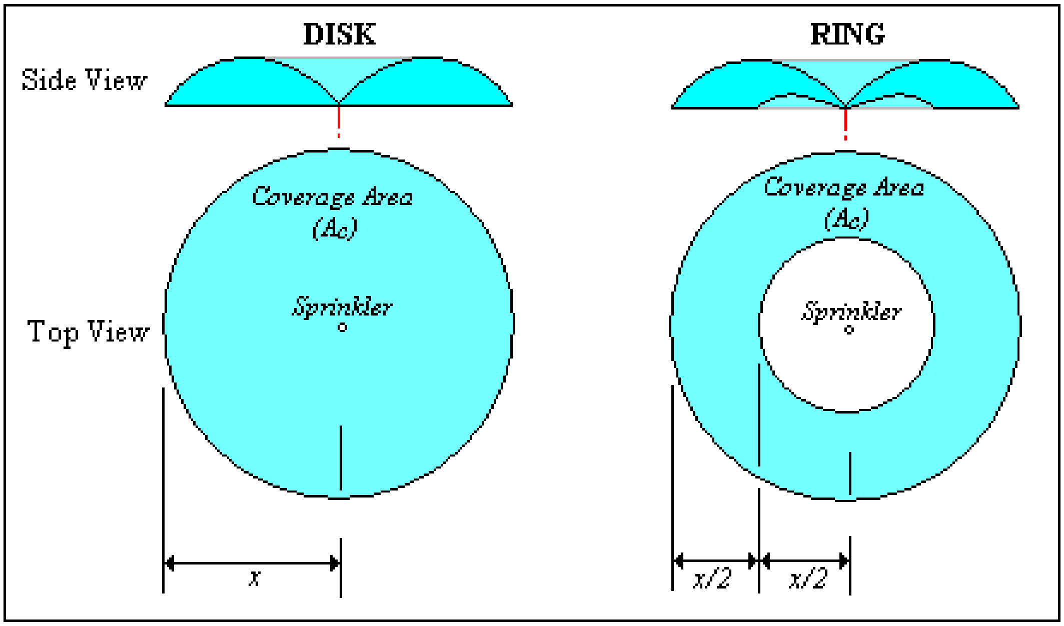

Figure 1. The two sprinkler coverage model extremes.

Two options represent the extremes of sprinkler coverage (**Figure 1**); we assume for both cases that the sprinkler is a well-distributed fan of water, meaning that the entire designated area for each sprinkler is watered uniformly.

- The sprinkler head discharges uniformly over a disk, with outer radius x m and inner radius 0 m.
- The sprinkler discharges uniformly over a ring, with outer radius x m and inner radius $x/2$ m. We can justify this ratio because, realistically, a sprinkler discharging onto an area narrower than this would require too many additional sprinklers to hydrate the unwatered area around the center.

We then find the rate (depth over time) D (cm/h) at which the area covered by the sprinkler receives water:

$$D = \frac{Q_n}{A_c},$$

where A_c is the area covered by the sprinkler, with

$$A_c = \pi(x_o^2 - x_i^2),$$

where $x_o,\ x_i$ are the outer and inner radii of the ring ($x_i = 0$ m for disk).

We calculate the depth over time D distributed over the discharge area A_c (**Table 1**) for both disk and ring models.

As sprinklers are added to the lateral arm, the area covered by each sprinkler decreases; consequently, D increases greatly. Recall our constraints:

- No part of the field should receive more than 0.75 cm/h.

Table 1.
Disk Model: Effective radius, flow/sprinkler, and depth/time for n sprinklers.

Sprinklers (on lateral pipe)	Effective radius (m)	Q_n (m^3/h/sprinkler)	Depth over area (cm/h) Disk	Ring
1	74	2.9	0.02	0.02
2	19	1.5	0.13	0.17
3	9	1.0	0.39	0.52
4	5	0.7	0.82	1.10

- Each part of the field should receive at least 2 cm every 4 days.

One sprinkler is not time-efficient, since the lateral pipe would need to sit about four days to fulfill the minimum. Two sprinklers is also not efficient, since they cover an area far exceeding the field boundaries. Either one or two sprinklers would likely result in far too much pressure for a sprinkler to handle, for both the disk model and the ring model. We study only three- and four-sprinkler systems beyond this point.

Four sprinklers causes a depth rate in excess of 0.75 cm/h. However, we can interpret this to mean the rate can exceed 0.75 cm/h as long as the pipe does not sit long enough for the cumulative depth to exceed 0.75 cm in one hour. In other words, the water can be shut off before the full hour is up.

Optimal Overlap Model (for Disks)

We define a *station* to be one of the lateral positions, a *cycle* to be each station being watered once, and a *watering* to be the process of watering a single station.

We try to optimize the system by arranging overlap of sprinklers so that no area gets hit more than twice in any cycle; doing this maximizes the time that the system can stay in one spot.

We arrange the sprinklers to cover the edges of the field as nearly as possible—though possibly missing some small triangles along the edges of the field as we try to optimize the speed of watering. We assume enough soil permeability so that water seeps from surrounding watered areas.

With four sprinklers, the outer two need to be at the ends of the pipe so that they can cover the edge as much as possible and unwatered area is minimized. With three sprinklers, the two outside sprinklers are 1.1 m from the end point of the lateral line and the third is in the center. It is ideal if the radius is large enough to cover the edge of the field and also hit the next sprinkler over on the mobile line; then the sprinklers can sprinkle a little less water over the edge, minimizing waste.

To determine the move distance from station to station, we use the Pythagorean theorem:

$$R^2 = \left(\tfrac{1}{2}L_s\right)^2 + \left(\tfrac{1}{2}S_s\right)^2, \qquad L_s = 2\sqrt{R^2 - \left(\tfrac{1}{2}S_s\right)^2},$$

where L_s is the lateral spacing of the movable pipe, S_s is the spacing between the sprinklers, and R is the radius of the spray for each sprinkler (**Figure 2**).

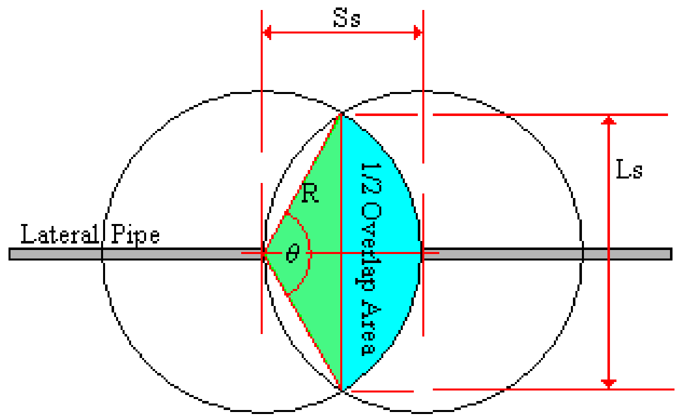

Figure 2. Sprinkler overlap.

Lateral spacing is kept constant for as long as possible through the middle of the field. At the edge of the field, the two connections to the stationary line should be equidistant from the edge while still maintaining enough coverage. The lateral pipe spacing, sprinkler spacing, and number of moves required to cover the field can be seen in **Table 2**.

Table 2.
Pipe spacing, sprinkler spacing, and number of moves for 3 and 4 sprinklers.

Sprinklers	Spray radius (m)	Sprinkler spacing (m)	Pipe spacing (m)	Moves to cover field
3	8.9	8.9	15.4	6
4	5.3	6.7	8.3	10

To calculate the area inside the field that is double-watered, we use L_s to find the length of the opposite edge of the isosceles triangle. The two equal sides are the radius of the disk. From the law of cosines $L_s^2 = 2R^2 - 2R^2\cos\theta$, we can easily find θ; for laterally overlapping wedges, we substitute S_s for L_s. The area of the wedge is found using $A = \theta\pi R^2/360°$ and double the area of the triangle is subtracted from it.

The resulting areas of double, single, missed, and outside property watered are in **Table 3**. The area double-watered by three sprinklers during a cycle is greater than that for four sprinklers, but the area missed is much smaller.

A strength of these models is that there is no specific order in which the field must be watered, since no area is hit more than twice in a cycle. Moving the sprinkler progressively from station to station across the field both minimizes move distance and reduces move time.

The time in each position is calculated such that the lateral pipe is relocated before the overlapping sections receive more than 0.75 cm in an hour. The combined flow rates of the sprinklers for both the three- and the four-sprinkler

Table 3.
Sprinkling imperfections (areas in m^2).

Sprinklers	Double-watered	Single-watered	Missed	Watered outside field
3	1131	1250	19	717
4	1057	1281	59	140

arrangements is faster than the allowed rate, especially in overlapping areas. The water must be shut off in time and left to seep in for the rest of the hour.

Sprinkling Time and Schedule

We calculate the time to accumulate 0.75 cm in the areas hit by two sprinklers (the overlapping sprinkler areas). The D-values for the overlap areas are twice those in **Table 1**. Thus, the time t_n that the lateral pipe with n sprinklers must water an area is

$$t_3 = \frac{0.75}{2D_3} = 0.960 \text{ h} = 58 \text{ min}, \qquad t_4 = \frac{0.75}{2D_4} = 0.455 \text{ h} = 27 \text{ min}.$$

As seen in the **Appendix**, the number of lateral pipe stations to cover the entire field is 6 for three sprinklers and 10 for four sprinklers. [EDITOR'S NOTE: We omit the **Appendix**.] This result is important in realizing how many total moves are required to meet the requirement of the entire area getting at least 2.0 cm in four days.

To find the number of cycles needed to be repeated over four days, we consider the areas that are not overlapped. These areas receive (0.75 cm / cycle) / 2 = 0.375 cm / cycle; so to receive 2 cm over four days requires 2 cm / (0.375 cm / cycle) = 5.3 cycles. Therefore, six complete cycles must occur over four days to meet the minimum water requirement, for either three or four sprinklers.

For simplicity, we assume that the time to move the lateral pipe from one station to the next is uniformly 15 min. We display several statistics about our models in **Table 4**.

Table 4.
Sprinkler statistics for scheduling.

Sprinklers	Watering time (min)	Stations	Cycles in 4 d	Total waterings per 4 d	Waterings per d
3	58	6	6	36	9
4	27	10	6	60	15

Finally, we can make watering schedules for three and for four sprinklers. [EDITOR'S NOTE: We omit the schedules.]

Each model requires six cycles over four days. Therefore, a cycle and a half should be completed each day to keep the workload balanced. For either number of sprinklers, the daily watering time is roughly the same (11 h), as is the total work time over four days (43 h). However, a considerably larger amount of time is spent moving the pipes using four sprinklers (15 stations/d vs. 9). Since one goal is to minimize time moving equipment, we recommend the three-sprinkler system, since watering takes roughly the same amount of time with much less effort and also leaves only 19 m^2 unwatered instead of 59 m^2.

Ring Method of Area Estimation

Some sprinklers water a ring-like area; we assume uniform water distribution over the area and that the outer spray radius is twice the inner radius. Considering the ring areas complicates the model significantly:

- Preventing the same area being hit by the sprinklers more than twice in a cycle is impossible.
- To cover the entire area requires a significant amount of area to be watered three or four times as often as the areas watered once in a cycle (**Figure 3**).

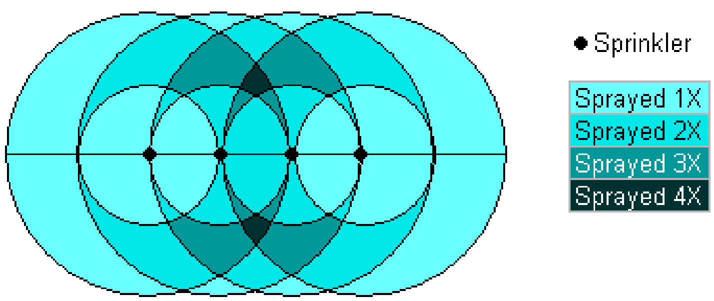

Figure 3. The ring model shows a great imbalance in the area watered.

- The lateral pipe must be moved more frequently and must water for a much smaller length of time to prevent the accumulation in heavily watered areas from exceeding 0.75 cm/h.
- Many more cycles must occur to ensure that lightly watered areas receive a cumulative depth of at least 2.0 cm every four days.
- To prevent areas from getting overwatered while minimizing the underwatered area requires a staggered station progression, thereby increasing the distance that the lateral line must be moved.
- The watering of the field can no longer be considered uniform.

The ring model assumes equally spaced sprinklers arranged such that no area in the center of any radius is left unwatered. We do not explore sprinkler spacing but merely note that the spacing depends on the outer spray radius.

Once the sprinkler spacing is determined, the lateral pipe spacing can be determined as in the disk model (optimal overlap model), as well as the number of stations required for a given number of sprinklers.

For an algorithm on station progression for a cycle, there are several options:

- Minimize the total distance that the lateral pipe must be moved in one cycle. This option
  - progresses similarly to the disk model (simply progress to the next station, with down-time at the endpoints of the main watering line);
  - likely requires some wait time between waterings to meet the maximum watering requirement (no more than 0.75 cm/h on area), since there is significant overlap of watered areas;
  - is not time-efficient.
- Minimize wait time between waterings. This option
  - requires the pipe to be moved to another station immediately after a watering is complete;
  - requires, to avoid overwatering certain areas, that the lateral pipe must be moved beyond an adjacent station upon completing another watering;
  - requires a complex station-progression algorithm to ensure that the field is watered quickly and efficiently.
  - has the weakness that the total distance that the lateral pipe is moved between waterings (as well as total distance moved during a complete cycle) is much greater than in the first option. In theory, the time required to move this extra distance could exceed the time that the farmer would need to wait using the first option.
- Compromise by allowing some wait time between stations and allowing the lateral pipe to be moved beyond the adjacent station during the progression.

Since one objective is to minimize the time and effort moving the pipes, we suggest the first option.

Head Loss

A major weakness of our models is that they do not account for energy losses due to friction between the pipe and the water moving through it, also known as *head loss*. We apply the Hazen-Williams equation solved for meters of head loss per meter of pipe or friction slope [Walski et al. 2004, 17]:

$$S_f = \frac{10.7}{D^{4.87}} \left(\frac{Q}{C}\right)^{1.852}, \qquad \textbf{(2)}$$

where

S_f is friction slope (m/m),

D is the diameter of the pipe (m),

Q is the flow rate (m^3/s), and

C is the Hazen-Williams friction coefficient (for aluminum, $C = 130$).

With our given values of $D = 0.1$ m and $Q = 0.0025$ m^3/s, we have $S_f = 0.00146$ m/m, or a total head loss over the 100 m of pipe of $100S_f = 0.15$ m, which is insignificant.

For the farthest sprinkler position, there will be multiple tees with valves where the lateral line ties into the main line, and each has an associated loss of 0.3–0.4 meters of head [Walski et al. 2004]. These losses can add up and reduce the speed of the water exiting the sprinkler when the lateral arm is far from the water source. The problem is remedied by decreasing the distance between stations as the attachment points progress farther down the mainline.

When we use **(2)** to check the head loss on the small sprinkler pipes, assuming that there is only one outlet for all of the flow, we obtain an astronomical friction slope, $S_f = 1305$ m/m. This calculation is another justification for our assumption that not all of the flow exits the sprinklers and a return line of some sort is necessary. When adjusted for the proper flow based on the number of sprinklers, we obtain a friction slope of 12.5 m/m for four sprinklers and 21.4 m/m for three sprinklers. When we use the Bernoulli equation with losses due to friction accounted for, the appropriate losses are added to each side of the equation and once again solved for v_2. The new speeds are:

Three sprinklers: $v = 8.9$ m/s instead of 9.6 m/s (for no friction)

Four sprinklers: $v = 6.7$ m/s instead of 7.2 m/s (for no friction)

Three sprinklers can cover the field, but with four sprinklers the speed is too low to cover the edges reliably.

Computer Modeling and Solution Approach

We created a computer program to model the effectiveness of solutions and to find its own near-ideal solution via a genetic algorithm.

The Computer Model

We divide the field into a 200×75 cell grid of cells 0.4 m on a side and time into 5-min discrete intervals; each cell records how much water it receives during each time interval over the course of four days. Inputs are:

- The direction that the mainline runs (cross-wise or length-wise). (We later found that length-wise leads to insurmountable pressure-loss.)
- The inset of the mainline from the edge of the field.

- The inset of the first sprinkler from the beginning of the lateral line.
- The inset of the last sprinkler from the end of the lateral line.
- The total number of sprinklers on the lateral line (spaced evenly between first and last).
- A list of steps in the watering schedule. Each step consists of:
  - The distance in cells down the mainline to where the lateral is attached.
  - The number of time intervals that the lateral operates at that location.

In addition, the model accounts for several other variables that are hard-coded into the program but could be changed:

- The time to move the lateral line (15 min).
- The "up-time" per day (16 h), that is, ensuring that the system stops at night.

The program assigns a radius range to the sprinklers and a rate of water accumulation for every cell in that range, based on earlier calculations on sprinkler pressure and speed. After the entire schedule has been simulated, each cell is queried and assigned one of three conditions based on water accumulation:

- Overwatered: If during any one-hour period the cell received more than 0.75 cm of water, it is considered overwatered, even if the total water over four days was less than 2 cm.
- Underwatered: If the cell is not overwatered, and it received less than 2 cm over the course of four days, it is considered underwatered.
- Ideally-watered: If the cell is neither overwatered nor underwatered, it is considered within the ideal watering range.

The Genetic Algorithm

The genetic algorithm attempts to find optimal solutions through evolution. It creates 100 random sets of input for the testing model and tests each set, or "genome"; the 10 best-ranked genomes (see the **Appendix** for the ranking system) are selected as "parents" for another collection of 100 input sets (another "generation") [EDITOR'S NOTE: We omit the **Appendix**.]. Ninety of the new genomes are created from two parents (10 parents allow for 45 unique combinations—two of each are used); the input values are the averages of their parents' values, with a small percentage of random variation added in. The remaining 10 genomes are created directly from one parent, with a slight amount of variation added. The computer proceeds through many generations, constantly improving the solution sets—in theory. In practice, the large number of input variables and the relatively small number of effective solutions means

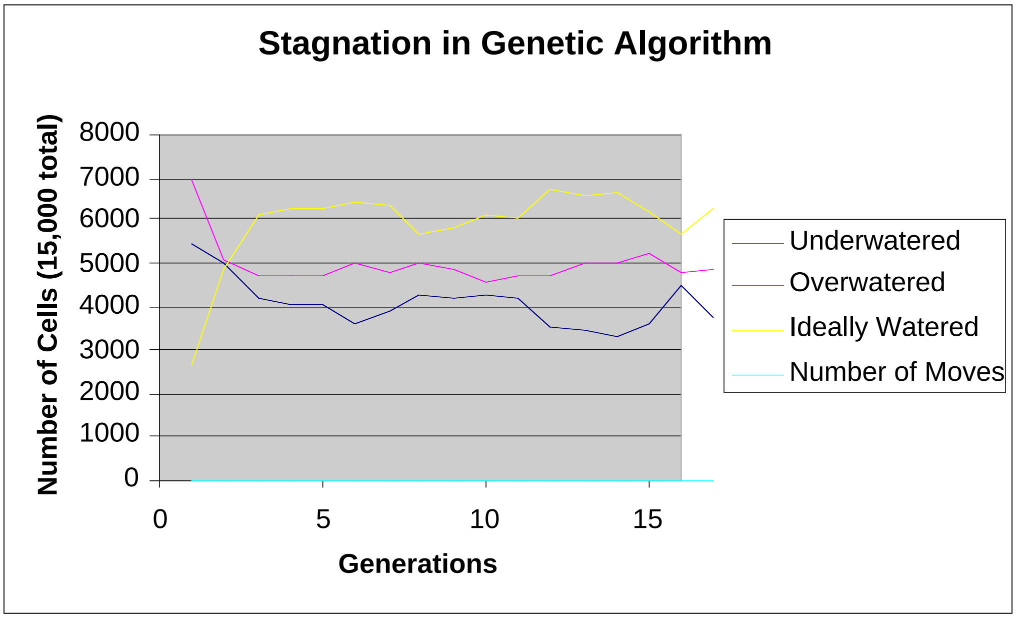

Figure 4. After three to four generations, progress ceases. (Number of Moves hugs the horizontal axis.)

that it is extremely easy to worsen a solution but very difficult to improve it. After a few generations of progress, the model stagnates (**Figure 4**).

With a much larger population size (tens of thousands), the chances increase of finding one or two better solutions in every generation; with our computing power, we are limited to a smaller population and thus cannot use the full range of input variables.

The evolution simulation always favors certain values for particular variables: For three or four sprinkler heads, the simulation always picks three as optimal in the very first generation. In addition, the inset value of the mainline always tends towards centering the length of the lateral in the field, and the inset value of the first and last sprinklers always approaches zero within five to ten generations, suggesting that these sprinklers are best placed at the very beginning and end of the lateral line so as to leave less area unwatered. The simulation does not keep track of water that lands off of the field. By decreasing the areas that are watered twice and dumping a little extra water over the edge, unwatered area decreases to zero if lateral spacing is optimized.

The Station-Based Model

Using noncomputer methods, we had determined that to water the entire field adequately with three sprinklers, six stations had to be visited six times each, with a watering time of less than 57 min for each visit (see **Table 5**). We tested the boundaries of this model by running it with a 60-min and with a 55-min operating time at each schedule step, a situation that should cause overwatering where sprinkler coverage overlaps. **Figure 5** shows the results.

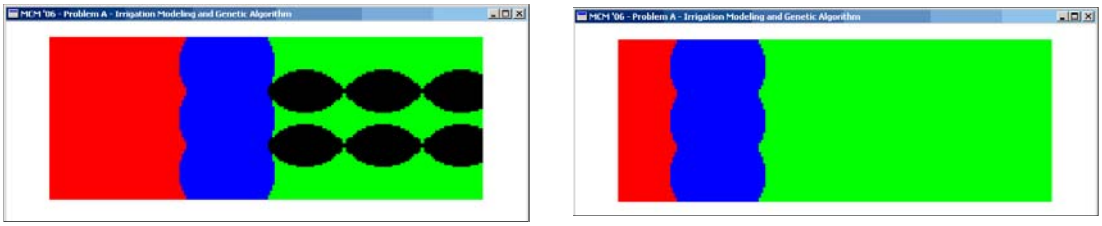

Figure 5. Station-based schedule with 60 min per station (left) and with 55 min per station (right). KEY: Red (leftmost region): cells underwatered or not yet watered. Blue (middle): cells being watered. Black (ovoids; none for 55 min): cells overwatered. Green (right): cells in the ideal range.

For 60 min, our prediction of overwatering in overlaps is proven true; for 55 min, the lack of overwatering shows that our model is accurate.

The next question is, Is our sequence of stations the most optimal? We start at one end of the field and move down by one station every 55 min until we reach the other end, where we stop for one cycle of down-time and then move back up the field. This cycle continues until we hit every station six times, requiring moving the line a total of 432 m. Cycling back and forth seems an obvious and efficient schedule, but is it possible to find something faster?

The Station-Based Genetic Algorithm

We restructured the genetic algorithm to find the optimal station schedule.

The simulation is set up to allow genetic change only in the station number to be visited at each schedule step; each visit must be 55 min long, and each station has to be visited 6 times, but the stations can be visited in any order. All other variables are fixed at the values that we had determined to be the most efficient. The scoring system is adjusted so that the only criterion for selection is moving the lateral line a lesser distance than the other genomes.

The genetic algorithm stabilizes at a single solution within five elapsed generations, as shown in **Figure 6**. The mean distance required to move the lateral line over a four-day cycle drops from more than 900 m to a surprising 72 m, a major improvement over our "optimal" 432 m.

The computer took an approach so simple as to be overlooked: Rather than moving on after turning off the water, the irrigation system is simply turned off for 15 min before resuming at the same location. The lateral line starts at one end of the field and operate for six on-off cycles before moving to the next station.

Of course, this may not be an option for fields that cannot quickly absorb water, but it satisfies the conditions of the problem statement. Over any given hour, the total depth of water applied within the overlapping areas is less than 0.75 cm because of watering for only 55 min plus the 15-min move time.

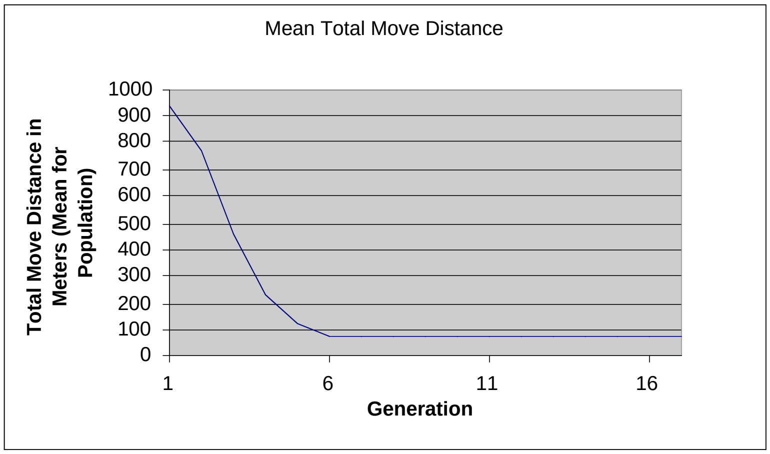

Figure 6. Mean total move distance.

Results and Conclusions

We found spray ranges based on the number of sprinklers in the system. This model assumes no friction loss, and we found speed not to be sensitive to height. We investigated two models of coverage: disk and ring areas. Using the areas and flow rates from sprinklers to approximate depth accumulated in the areas, we eliminated one and two sprinklers based on the overshooting range, poor time efficiency, excessive waste, and large speeds. Calculating the range for five sprinklers showed that the spray would not reach the edge of the field. Therefore, we limited our models to three or four sprinklers on the lateral pipe.

Using geometric methods, we found an optimized spacing of three and four sprinklers along the lateral pipe (using disks to model areas):

- Three sprinklers: placed 1.1 m inset from the end of the 20-m pipe, spaced evenly at 8.9 m between sprinklers.
- Four sprinklers: a sprinkler at each end point of the 20-m pipe, spaced evenly at 6.7 m between sprinklers.

Similarly, the lateral pipe spacing and number of stations/moves required to cover the entire field was calculated for the two cases:

- Three sprinklers: 15.4 m between stations; six stations.
- Four sprinklers: 8.3 m between stations; ten stations.

Advantages to using three sprinklers are fewer moves per cycle and minimization of the area underwatered.

We found that to meet the minimum water requirement of at least 2 cm depth over four days, the areas watered once per cycle are the driving factor, resulting in six cycles required every four days for both three and four sprinklers. Therefore, to balance the workload evenly over four days, one-and-a-half cycles must be completed each day. In accordance with the maximum rate that water can accumulate (0.75 cm/h), the time that a sprinkler could water an overlapping area was calculated to be:

- Three sprinklers: 58 min per station; 9 stations per day.
- Four sprinklers: 27 min per station; 15 stations per day.

We also found lateral pipe can be progressively moved from station to station during a cycle. However, at end stations the best option (while maintaining uniform coverage) involves leaving the pipe at a station for two consecutive turns while scheduling a wait time between the two waterings.

The total work day for these models is approximately the same. Using three sprinklers requires less move time, as well as fewer moves each day.

The ring model requires more frequent relocation of the lateral pipe, many more cycles to ensure the proper amount of moisture in all areas, and a staggered progression of the lateral pipe.

The biggest weakness of our initial models is assuming no friction head loss in the system. After some calculation of head loss, and reapplication of the Bernoulli energy equation, we found the new sprinkler discharge speeds and compared them to speeds calculated with no head loss:

- Three sprinklers: $v = 8.9$ m/s instead of 9.6 m/s (no friction)
- Four sprinklers: $v = 6.7$ m/s instead of 7.2 m/s (no friction)

Though the discharge speeds decrease, the speed of the optimized three-sprinkler system is still large enough to reach the edge of the field when the sprinklers are positioned on the end of the movable pipe.

Our computer simulation found an ideal watering over a four-day period using the parameters of our previous three-sprinkler model, eliminating unwatered area by placing sprinklers on the ends of the lateral pipe and changing the spacing of the lateral attachments accordingly.

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