

For office use only	Team Control Number	For office use only
T1 _____	26160	F1 _____
T2 _____		F2 _____
T3 _____	Problem Chosen	F3 _____
T4 _____	B	F4 _____

Summary

Aimed to look for the “best all time college coach” for the previous century, we employ methods in grey and fuzzy fields to build an evaluation model and give out coach ranks in various sports fields by massive calculating. Meanwhile, effects of gender and time criterion on evaluation are also considered.

In order to simplify our model, we firstly adopt **AHP** to filter factors and determine seven most influential criterion, including coaching time, gender, etc. With the weight of each criteria worked out, the abstract problem is transferred to a mathematic one.

Based on AHP Model, two advanced models are proposed to rank college coaches.

Grey Correlation Model considers the relevance among evaluation criterion and evaluates all coaches by calculating correlation degree with respect to reference data series.

Fuzzy Comprehensive Model integrates empirical formulas and membership function to get the membership matrix, through which we can figure out scores of each coach. Comparing two results with current ranks, we find that grey model has slight advantages over fuzzy model.

Considering time line horizon’s impact on evaluating results, we augment our model by revising the weights and re-rank, based on membership function. According to the comparison between former rank and the one considering time line horizon, we conclude that it has fewer effects on the top 10 college coaches while has more effects on those ranked behind. The **final rank** table is as follows:

Rank	Basketball	Field hockey	Football
1	Harry Statham	Jan Hutchinson	Joe Paterno
2	Danny Miles	Jan Trapp	Bobby Bowden
3	Herb Magee	Nancy Stevens	Bear Bryant
4	Mike Krzyzewski	Sharon Pfluger	Lou Holtz
5	Bob Knight	Pat Rudy	Pop Warner

Our paper considers multiple factors, employs several models and does plenty of calculations, which makes the ranks more reliable and brings the evaluation model broader applicability. In conclusion, our model successfully achieves our goals that selecting the best college coaches in various sports.

Keywords: AHP Grey Correlation Model Fuzzy Comprehensive Model final rank

Contents

1 Introduction.....	2
2 Assumption	3
3 Symbols Definition	3
4 Model Overview	4
5 AHP Model.....	4
5.1 Establish a Hierarchical Model.....	5
5.2 Structure Comparison Matrix	6
5.2.1 Weights of Evaluation Criterion	6
5.2.2 Select Evaluation Criterion	8
5.3 Explanations about the Seven Criterion.....	8
6 Advanced Model.....	9
6.1 Overview of Evaluation Methods	9
6.2 Grey Correlation Model.....	10
6.2.1 Data Processing.....	10
6.2.2 Grey Correlation Evaluation Model.....	11
6.2.3 Results of Grey Correlation Model.....	12
6.3 Fuzzy Comprehensive Evaluation Model.....	15
6.3.1 Introduction of Fuzzy Comprehensive Algorithm	15
6.3.2 Concrete Calculation.....	17
6.4 Results Review.....	21
7 Is Time Line Horizon Influential?	22
8 Sensitivity Analysis.....	24
8.1 Sensitivity of Grey Correlation Method	24
8.2 Sensitivity of Fuzzy Comprehensive Method.....	25
9 Discussion and Conclusion	26
9.1 Solutions to All Problems	26
9.2 Honor Roll	27
9.3 Strengths and Weaknesses	28
9.4 Conclusion	28
A Letter to Sports Illustrated	29
Reference	30
Appendix.....	31

1 Introduction

The quality of coaches is becoming critical to raise the level of scientific training of competitive sports. A fair and convincing method to evaluate college coaches is demanded. Plenty of researches are made and a number of notable papers are addressed to get a reasonable evaluation method.

MacLean and Chelladurai (1995) proposed a framework for performance evaluation of college coaches. Performance of coaches should be evaluated from two respects, the process of work behavior and the results of work behavior. It is unfair and not convincing to evaluate college coaches only by the result of work behavior. They believe factors of process of work behavior consist of straightforward task behavior, indirect task behavior, daily management behavior and public relations behavior. These factors mainly depend on college coaches' ability to raise the level of sports teams. **George B.Cunningham and Marlene A.Dixion (2003)** considered that good results coming from the joint efforts of athletes, coaches and sports teams. Both results and process of sports team should be considered. Based on the consideration, they proposed evaluation theory in multidimensional performance.

Despite the evaluation method of college coaches has been studied to different extent with different approaches, the researchers' work usually focus on special sport. Meanwhile, their evaluation don't consider the effect of *time*. Therefore, their research may not appropriate to solve the proposed problems. We need to develop a model to look for the "best all time college coach". What's more, the model should be problem-oriented.

The three proposed problems are:

- Develop a model to look for the best all time college coach for the previous century.
- Analyze whether it make a difference with time line horizon or not.
- Present top 5 college coaches in each of 3 different sports.

Meanwhile, the problems request that model should take *gender* into consideration and have a broader applicability in all possible sports.

A reasonable evaluation method should consider many necessary factors. To classify factors appropriately, we apply Analytic Hierarchy Process (AHP) to the problem. Then, we get the weight of several main factors. To evaluate college coaches scientifically, we propose two models based on Grey Correlation Algorithm and Fuzzy Comprehensive Evaluation Algorithm. Then we make a comparison between results of two models. Furthermore, we optimize our model by considering the effect of *time line horizon*. At last, a non-technical explanation letter for sports fans is written to the *Sports Illustrated* magazine.

2 Assumption

1. Assume that *personal quality*, *training quality* and *team achievements* can perfectly reflect college coaches' comprehensive impact.
2. Assume that coaching time and gender can perfectly represent college coaches' *personal quality*. Organizational capability, management capability and race command capability can perfectly represent college coaches' *training quality*. Wins, losses, win-loss percentage and performance in league matches can perfectly represent college coaches' *team achievements*.
3. Assume that continuous wins and the difference of competitors' strength have few effects on rank.
4. The data we used and reference ranks are believable.

3 Symbols Definition

Symbol	Explanation
A	The sequence of college coach
B	Second level criterion
C	Third level criterion
Z	Total score
R	Correlation degree
D	Membership degree
T	Start coaching time

4 Model Overview

The process of developing models is shown in **Figure 1**.

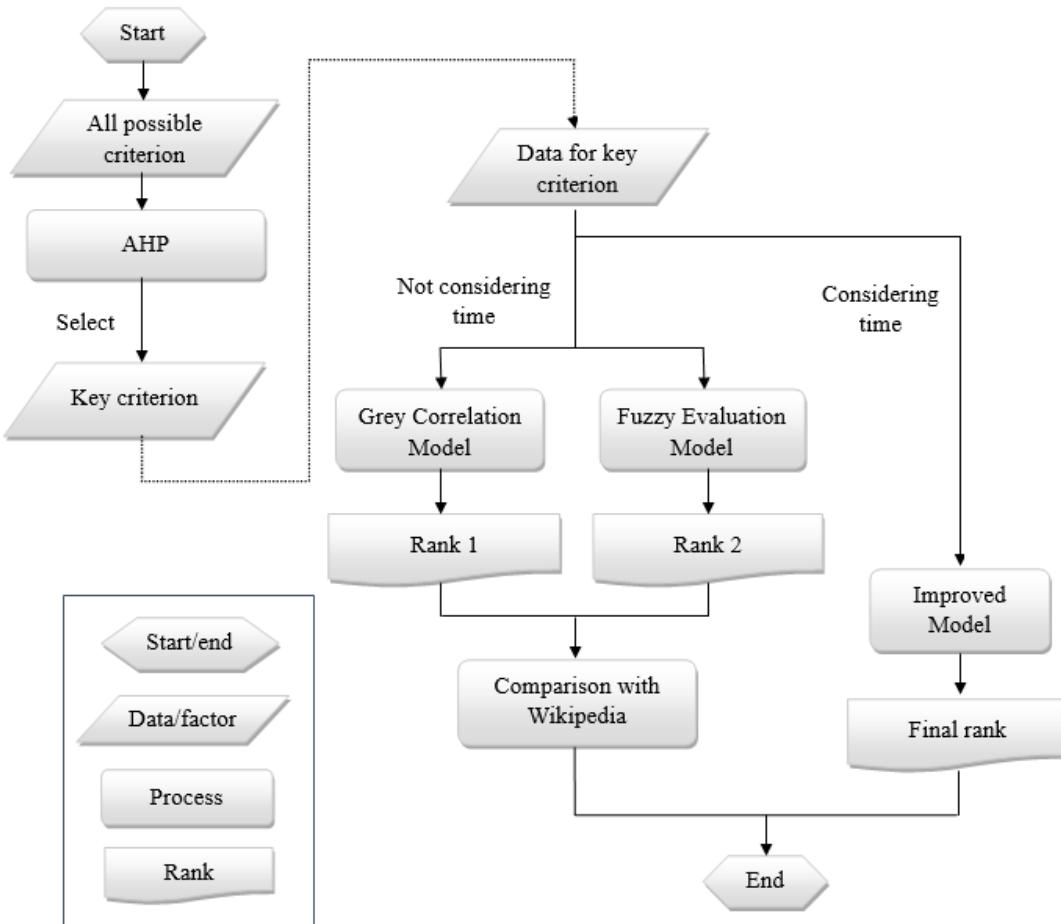


Figure 1. Flow diagram of solution

To determine *the best all time college coach* (past or present) from among either male or female coaches in some sports, we have to know what criterion should be used to evaluate college coaches. As is shown in the problem, we search the predecessors' research and find that many of them are qualitative analysis. What's more, according to some research there exist more than 54 of the evaluation criterion, such as the coach working age, the professional ethics, the ability to communicate with players, management ability statistics and so on. Some evaluation criterion are meaningful, but some of them are of little impact. We must find some methods to process the data in order to distill the crucial criterion.

How we could select the most important ones from 54 initial criterion and process scientifically to make them more useful in evaluating quality is a key problem. However, how to accomplish this goal? The Main Components Analysis method and Analytic Hierarchy Process will be helpful for us to devise our first model.

Combining the key evaluation criterion with the real data from internet, we select coaches in different sports as model evaluation objects and evaluate them with two different algorithms respectively, that is Grey Correlation Algorithm and Fuzzy Evaluation Algorithm. Grey Correlation Model considers the relevance among

evaluation criterion and grades all coaches by calculating correlation degree to reference data. Fuzzy Comprehensive Model integrates empirical formulas and membership function to get the membership matrix, through which we can figure out scores of each coach. Furthermore, we compare the two algorithm and corresponding results and transplant the model to other sports field for evaluating.

We choose college filed hockey, basketball, football sport coaches as examples, and get the rank of the coaches respectively.

5 AHP Model

5.1 Establish a Hierarchical Model

MacLean and Chelladurai (1995) thought that evaluation criterion of college coaches should include the process of work behavior and the results of work behavior. **Patricia (2005)** proposed that different principles should be considered to establish a kind of fair and convincing criterion to evaluate college coaches. With the help of the current literatures, we develop AHP model considering three main factors, these are coaches' personal quality, training quality and team achievements.

Personal quality

Coaches' personal quality consist of coaching time and gender [Yuanming Li, 2009]. College coaches consist of both male and female, we should consider both of them. Coaching time could reflect coaches' time consumption on their jobs.

- Coaching time
- Gender

Training quality

Yu Zhang (2010) proposed the components of training quality in his paper. He thought that training quality should consist of organizational capability, management capability and race command capability. Training quality represent a significant component of the evaluation criterion.

- Organizational capability
- Management capability
- Race command capability

Team achievements

Team achievements are the results of coaches' works, they are obvious to reflect coaches' capability. Meanwhile, people pay more attention to team achievements compared with other factors. However, fair and convincing evaluation criterion should be considered general factors rather than only results.

- Wins
- Losses
- Win-loss percentage
- Performance in league matches

Through above analysis of three main factors, which affect evaluation significantly, hierarchy figure is shown in **Figure 2**.

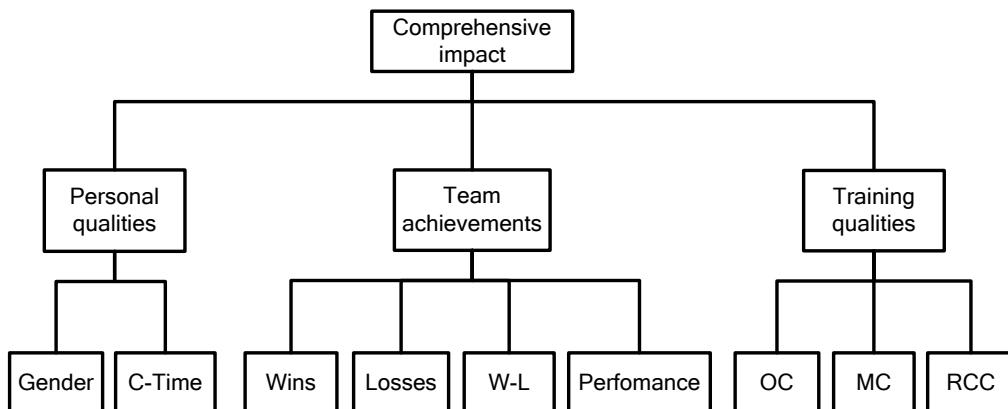


Figure 2. Hierarchy figure

In **Figure 2**, C-Time represents Coaching Time. W-L represents Win-Loss Percentage, OC represent Organization Capability, MC represent Management capability, RCC represent Race Command Capability.

5.2 Structure Comparison Matrix

5.2.1 Weights of Evaluation Criterion

Starting from the second criteria of the above hierarchy figure, we structure comparison matrix by Comparison Method of 1-9. Then, we get the weights of personal quality, training quality and team achievements as is shown in **Table 1**.

Table 1. Pairwise comparison matrix of hierarchy I - II

Comprehensive impact	Personal quality	Training quality	Team achievements	Weight
Personal quality	1	3	1/5	0.2496
Training quality	1/3	1	1/8	0.1565
Team achievements	5	8	1	0.5938

Through calculating the weights of three factors in hierarchy II , we get the maximum eigenvalue is $\lambda = 3.0044$, consistency ratio is 0.0043.

Table 2. Pairwise comparison matrix of hierarchy II -III

Personal quality	Coaching time	Gender	Weight
Coaching time	1	2	0.5498
Gender	1/2	1	0.4502

Through calculating the weights of two factors of personal quality in hierarchy III , we get the maximum eigenvalue is $\lambda = 2.0000$, consistency ratio is 0.2496.

Table 3. Pairwise comparison matrix of hierarchy II -III

Training quality	Organizational capability	Management capability	Race command capability	Weight
Organizational capability	1	2	1/5	0.2473
Management capability	1/2	1	1/6	0.2024
Race command capability	5	6	1	0.5503

Through calculating the weight of two factors of training quality in hierarchy III , we get the maximum eigenvalue is $\lambda=3.0000$, consistency ratio is 0.0001.

Table 4. Pairwise comparison matrix of hierarchy II -III

Team achievements	Wins	Losses	Win-loss percentage	Performance in league matches	Weight
C_6	1	5	1/3	4	0.2853
Losses	1/5	1	1/8	1/3	0.1160
Win-loss percentage	3	8	1	5	0.4256
Performance in league matches	1/4	3	1/5	1	0.1731

Through calculating the weights of two factors of team achievements in hierarchy III , we get the maximum eigenvalue is $\lambda=4.0100$, consistency ratio is 0.0037.

From above tables, we know that all Consistency Test is right. Combination Weight Vector of criterion with respect to evaluation method of college coaches is shown in Table 5. They can used to evaluate college coaches.

Table 5. Criterion's weights to overall objects

Criterion	Weight
Coaching time	0.1373
Gender	0.1124
Organizational capability	0.0387
Management capability	0.0317
Race command capability	0.0861
Wins	0.1694
Losses	0.0689
Win-loss percentage	0.2528
Performance in league matches	0.1028

5.2.2 Select Evaluation Criterion

According to importance of every criteria shown in **Table 5**, we choose several main criterion and decide the final evaluation method. Despite organizational capability and management capability have close relationship with evaluation method in theory. However, from above tables we can find their weights are light. Meanwhile, we lack of quantitative data of them. Therefore, we don't consider them. The final seven criterion are shown in **Table 6**.

Table 6. Final seven criterion to evaluate college coaches

Criterion	Symbol	Weight
Coaching time	C_1	★★★☆☆
Gender	C_2	★★☆☆☆
Race command capability	C_3	★★☆☆☆
Wins	C_4	★★★☆☆
Losses	C_5	★☆☆☆☆
Win-loss percentage	C_6	★★★★★
Performance in league matches	C_7	★★☆☆☆

5.3 Explanations about the Seven Criterion

In the seven criterion we got, only *Losses* is the negative criterion, others are positive.

Data of performance in league matches

Different sports are different in rules and league schedule. Therefore, different sports have different measuring values with respect to the performance in league matches. Based on data, we evaluate performance in league matches of basketball by number of NCAA final four appearances, evaluate performance in league matches of football by the times ranked into top 4, evaluate performance in league matches of college filed hockey by the times ranked into top 4.

Data of gender

Considering the disadvantage of female, we define value of male as 1, while female as 3. There are three reasonable explanation:

- Female have physiological disadvantages in sports.
- Once female are banned to participate sports for a long time. This tradition lasted a long time. Therefore, the tradition may have effects on female now.
- Male have better force and velocity on sports, so they can draw more attention compared with female. What's more, the quantity of male athletes is well larger than female, male athlete prefer male coaches.

6 Advanced Model

6.1 Overview of Evaluation Methods

Through AHP algorithm, we get the weights of final seven criterion to evaluate college coaches. To establish a fair and convincing evaluation method and rank the college coaches for the previous century, we proposed Grey Correlation Model and Fuzzy Comprehensive Evaluation Model.

- **Grey Correlation Model**

Criterion selected by AHP have inner links with each other. And Grey Correlation Model could take the links into consideration and solve the problem of comprehensive evaluation. First normalize the real data of every coach. Then determine the reference data, which is to be regarded as ideal standard. Find the relationship between every evaluated object and reference data series and finally build the comprehensive evaluation model. From this model, we calculate all coaches' scores and rank them. Top 5 coaches in basketball, field hockey and football are represented as:

Rank	Basketball	Field hockey	Football
1	Harry Statham	Jan Hutchinson	Joe Paterno
2	Danny Miles	Jan Trapp	Bobby Bowden
3	Herb Magee	Nancy Stevens	Bear Bryant
4	Mike Krzyzewski	Sharon Pfluger	Lou Holtz
5	Bob Knight	Pat Rudy	Pop Warner

- **Fuzzy Comprehensive Evaluation Model**

There we use multi-level fuzzy comprehensive evaluation model to rank the coaches. Normally, the first step is formulating evaluation criterion. Then determine the evaluation objects and the comment set. Next confirm the membership function and membership degree, through which we get formula of fuzzy evaluation matrix. According to evaluation matrix and weights determined by AHP method, we give a comprehensive evaluation of coaches.

From this model, we calculate all coaches' scores and rank them. Top 5 coaches in basketball, field hockey and football are represented as:

Rank	Basketball	Field hockey	Football
1	Harry Statham	Jan Hutchinson	Bobby Bowden
2	Herb Magee	Jan Trapp	Joe Paterno
3	Danny Miles	Nancy Stevens	Bear Bryant
4	Adolph Rupp	Pat Rudy	Tom Osborne
5	Mike Krzyzewski	Beth Anders	Pop Warner

6.2 Grey Correlation Model

6.2.1 Data Processing

Based on the current evaluation criterion, we collect related data of college coaches. Assume that data sequence of n coaches form the following matrix (**Equation 6.1**). To simplify calculation, we note original data as A_i' , the data after dimensionless process is noted as A_i .

$$(A'_1, A'_2, \dots, A'_n) = \begin{bmatrix} a'_1(1) & a'_2(1) & \cdots & a'_n(1) \\ a'_1(2) & a'_2(2) & \cdots & a'_n(2) \\ \vdots & \vdots & & \vdots \\ a'_1(m) & a'_2(m) & \cdots & a'_n(m) \end{bmatrix} \quad (6.1)$$

$$A'_i = (a'_i(1), a'_i(2), \dots, a'_i(m))^T, i = 1, 2, \dots, n$$

In which, m presents the number of metrics, herein, $m=7$. Based on evaluation and rank of college coaches provided by Wikipedia and other websites, considering several quality of college coaches generally, we choose about 100 coaches of three sports (basketball, football and college field hockey) as evaluated objects.

Then, taking basketball for example, we analyze and dimensionless process data.

We choose 46 college coaches whose achievements are comparatively prominent. Process theirs' data and rank them. Here we just present top 10 of them, who are Harry Statham, Mike Krzyzewski, etc. [<http://www.sports-reference.com/cbb/coaches/>] The data are represented respectively as $A'_1, A'_2, \dots, A'_{10}$.

$$(A'_1, A'_2, \dots, A'_{10}) = \begin{bmatrix} 47 & 1 & 39 & 1079 & 444 & 0.708 & 13 \\ 42 & 1 & 32 & 1000 & 409 & 0.710 & 10 \\ 45 & 1 & 26 & 976 & 391 & 0.714 & 11 \\ 39 & 1 & 29 & 975 & 302 & 0.764 & 11 \\ 38 & 1 & 30 & 942 & 314 & 0.750 & 4 \\ 42 & 1 & 28 & 899 & 374 & 0.706 & 5 \\ 36 & 1 & 27 & 879 & 254 & 0.776 & 11 \\ 40 & 1 & 23 & 877 & 382 & 0.697 & 4 \\ 41 & 1 & 20 & 876 & 190 & 0.822 & 6 \end{bmatrix}$$

Because dimensions of every criteria are different, we normalize every criteria, transfer the absolute value to relative value. For positive (negative) criterion, we normalize data with different algorithms.

- Positive criterion:

$$A_i(j) = [(A'_i(j) - A'_i(j)_{\min}) / (A'_i(j)_{\max} - A'_i(j)_{\min})] \times 100\%$$

- Negative criterion:

$$A_i(j) = [(A'_i(j)_{\max} - A'_i(j)) / (A'_i(j)_{\max} - A'_i(j)_{\min})] \times 100\%$$

In which, i represents the sequence of countries, j represents the sequence of metrics; $A'_i(j)_{\min}$ represents maximum value of the j th metrics, $A_i(j)$ represents normalized value of the j th metrics of the i th coach. Then, we get the normalized value of every criterion of 10 basketball coaches.

Table 7. Standardization of basketball college coaches

Coach	CT	Gender	RCC	Wins	Losses	W/L%	FF
Harry Statham	0.963	0.000	1.000	1.000	1.000	0.619	1.000
Danny Miles	0.778	0.000	0.816	0.912	0.876	0.625	0.769
Herb Magee	0.889	0.000	0.658	0.884	0.812	0.639	0.846
Mike Krzyzewski	0.667	0.000	0.737	0.883	0.496	0.806	0.846
Jim Boeheim	0.630	0.000	0.763	0.845	0.539	0.759	0.308
Bob Knight	0.778	0.000	0.711	0.795	0.752	0.612	0.385
Dean Smith	0.556	0.000	0.684	0.772	0.326	0.846	0.846
Jim Calhoun	0.704	0.000	0.579	0.770	0.780	0.582	0.308
Adolph Rupp	0.741	0.000	0.500	0.769	0.099	1.000	0.462
Eddie Sutton	0.593	0.000	0.658	0.688	0.592	0.625	0.231

In the table:

CT—Coaching time

RCC—Race command capability

FF—number of NCAA final four appearance

Similarly, we can get the normalized value of every criterion of 10 football coaches and college filed hockey coaches.

6.2.2 Grey Correlation Evaluation Model

Because of the relationship between every criterion and total evaluation is sophisticated and interacting, we develop Grey Correlation Evaluation Model to analyze and evaluate general capability of college coaches. Grey Correlation Evaluation Method is branch of Grey Theory, the method are usually used to evaluate interacting factors.

We use the Grey Correlation Analysis to evaluate college coaches, the steps are shown as following:

STEP 1 Note reference data series as ideal comparison criteria. In generally, reference data series consist of the optimal value of every metrics, we can also choose other reference data by different evaluation destination. The relationship is shown in following equation:

$$A_0 = (a_0(1), a_0(2), \dots, a_0(m))$$

In which, we choose every optimal value as reference data series, that is $A_0 = (1, 1, \dots, 1)$.

STEP 2 Calculate the absolute difference between comparison series and reference data series of every evaluated objects. That is $\Delta_i(j) = |a_i(j) - a_0(j)|$. In which, i represents the sequence of college coaches, $i = 1, 2, \dots, n$; j represents the sequence of evaluation criterion, $j = 1, 2, \dots, m$; $a_0(j)$ represents the reference data of the j th evaluation criterion.

STEP 3 determine the value of p and q .

$$p = \min_{1 \leq i \leq n} \min_{1 \leq j \leq m} \{\Delta_i(j)\}$$

$$q = \max_{1 \leq i \leq n} \max_{1 \leq j \leq m} \{\Delta_i(j)\}$$

STEP 4 According to **Equation 6.2.**, We can calculate correlation coefficient with respect to every comparison series and reference data series.

$$y_i(j) = \frac{(p + q\beta)}{(\Delta_i(j) + q\beta)} \quad j = 1, 2, \dots, m \quad (6.2)$$

In which, resolution ratio $\beta \in (0, 1)$, the difference of correlation coefficient increase with β decreasing. Herein, we choose $\beta = 0.5$.

STEP 5 To find the relationship between every evaluated object and reference data series, we calculate the average value of correlation coefficient of every evaluation criterion and reference data series. We define it as Correlation. As is shown in follows.

$$r_j = \frac{1}{n} \sum_{i=1}^n y_i(j)$$

STEP 6 Calculate the weight of every evaluation criterion. That is:

$$r'_j = \frac{r_j}{r_1 + r_2 + \dots + r_m} \quad j = 1, 2, \dots, m$$

STEP 7 Construct general evaluation model

$$Z_i = r'_1 a_i(1) + r'_2 a_i(2) + \dots + r'_m a_i(m) \quad i = 1, 2, \dots, n$$

6.2.3 Results of Grey Correlation Model

Based on above steps, we can get results by MATLAB programming. The evaluation results of basketball, football and college filed hockey coaches can be easily gotten by computer simulation. The results are shown in **Table 8**.

Table 8. Scores of top 15 basketball coaches (Grey Correlation Model)

Rank	Coach	Last School	Scores
1	Harry Statham	McKendree	0.834
2	Danny Miles	Oregon Institute of Technology	0.718
3	Herb Magee†	Philadelphia College	0.710
4	Mike Krzyzewski	Duke	0.662
5	Bob Knight	Texas Tech	0.613
6	Dean Smith	North Carolina	0.599
7	Jim Boeheim	Syracuse	0.585
8	Jim Calhoun	Connecticut	0.570
9	Adolph Rupp	Kentucky	0.540
10	Lou Henson	Illinois	0.519
11	Eddie Sutton	San Francisco	0.517
12	Lute Olson	Arizona	0.507
13	Lefty Driesell	Georgia State	0.474
14	Ray Meyer	DePaul	0.468
15	Hank Iba	Oklahoma State	0.462

To show scores of top 15 basketball coaches intuitively, we draw **Figure 3.** as follows by the data from **Table 8.** According to the figure, we find that the top five basketball college coaches all time are **Harry Statham, Danny Miles, Herb Magee†, Mike Krzyzewski, Bob Knight.**

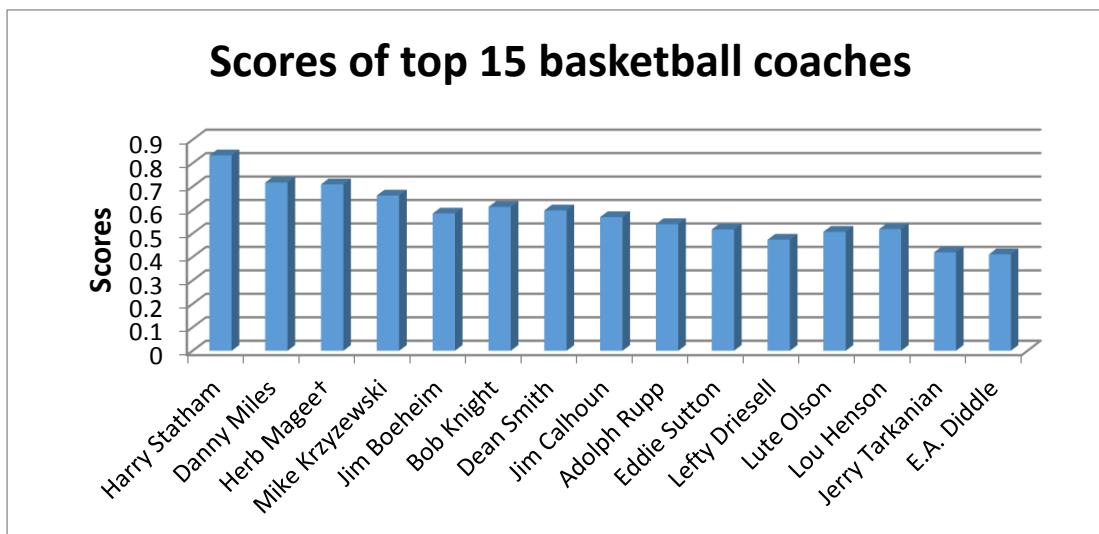


Figure 3. Scores of top 15 basketball coaches

Similarly, by analysis data of the quality of field hockey coaches,[http://en.wikipedia.org/wiki/List_of_college_field_hockey_coaches_with_250_wins] We can get score of top 5 field hockey coaches in Grey Correlation Method, which is shown in **Table 9.**

Table 9. Scores of top 5 field hockey coaches

Coach	Jan Hutchinson	Jan Trapp	Nancy Stevens	Sharon Pfluger	Pat Rudy
Scores	0.847	0.735	0.694	0.674	0.663
Rank	1	2	3	4	5

To show scores of top 5 field hockey coaches intuitively, we draw **Figure 4.** as follows by the data from **Table 9.** According to the figure, we find that the top five field hockey college coaches all time are **Jan Hutchinson, Jan Trapp, Nancy Stevens, Sharon Pfluger, Pat Rudy.**

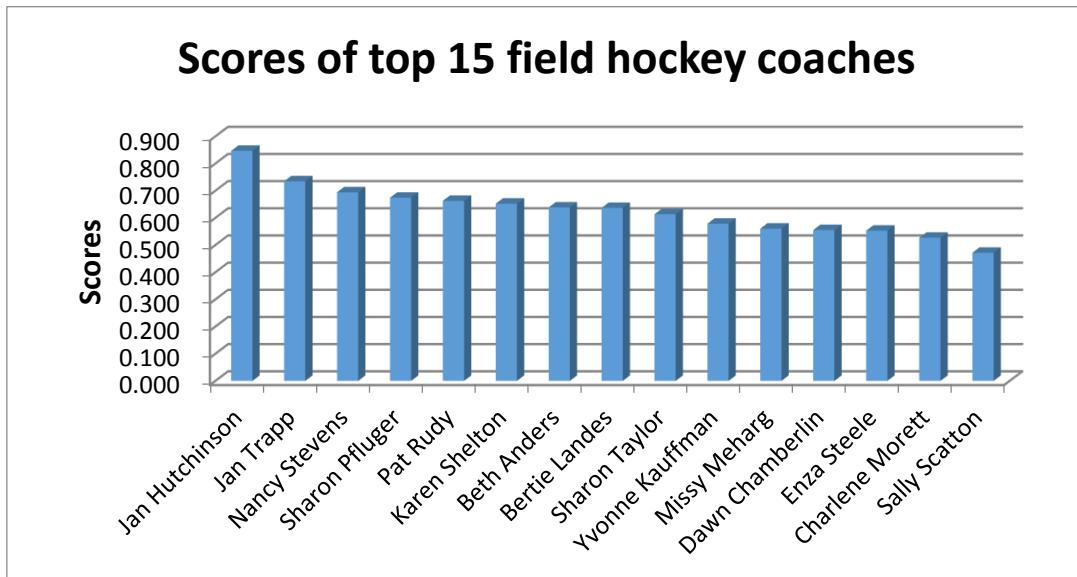


Figure 4. Scores of top 15 field hockey coaches

Similarly, by analysis data of the quality of football coaches, [http://en.wikipedia.org/wiki/List_of_college_football_coaches_with_200_wins] We can get scores of top 5 football college coaches in the same method, which is shown in **Table 10.**

Table 10. Scores of top 5 football coaches

Coach	Joe Paterno	Bobby Bowden	Bear Bryant	Lou Holtz	Pop Warner
Scores	0.834	0.737	0.647	0.525	0.513
Rank	1	2	3	4	5

To show scores of top 5 football coaches intuitively, we draw **Figure 5.** as follows by the data from **Table 10.** According to the figure, we find that the top five football college coaches all time are **Joe Paterno, Bobby Bowden, Bear Bryant, Lou Holtz, Pop Warner.**

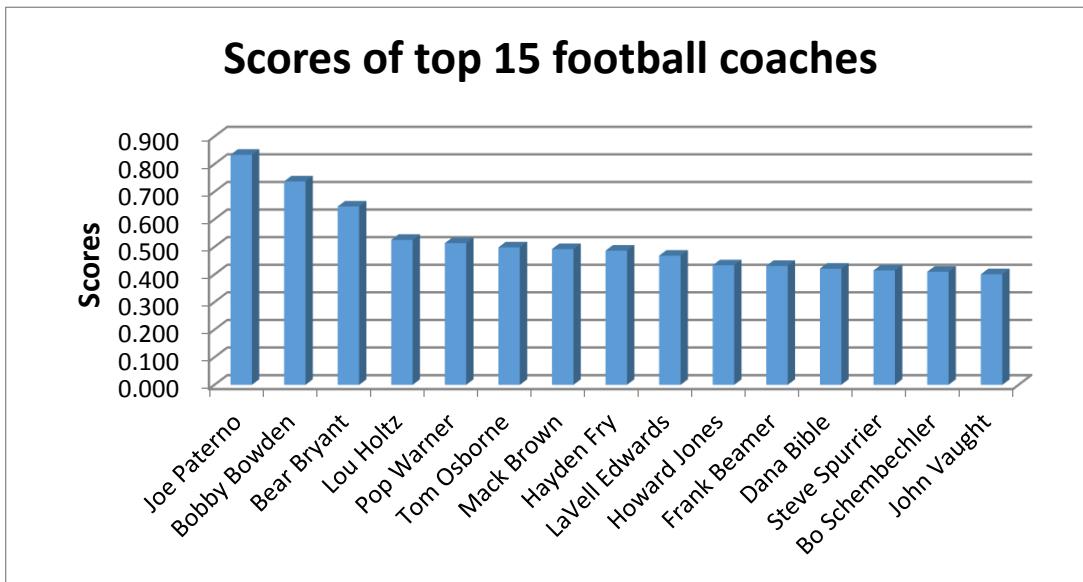


Figure 5. Scores of top 15 football coaches

6.3 Fuzzy Comprehensive Evaluation Model

Giving a rank of things according to their nature is common sense. However, in many cases, the nature of the comparison has no distinct boundary and it is often difficult to compare and sort. As our problem showing, we need to give them a evaluation and rank according to the coaches' accomplishment and ability. But the comprehensive ranking needs to consider personal quality, training quality and team achievements of the coaches, each side is contains a lot of integrated criterion, so direct ranking is difficult. Herein, we use multi-level fuzzy comprehensive evaluation algorithm to evaluate the college coaches of some sports.

6.3.1 Introduction of Fuzzy Comprehensive Algorithm

In this part, we will show the basic steps of multi-level fuzzy comprehensive evaluation model.

1. Giving the set of evaluation objects

According to the problem, we select the top 10 coaches of some field as the evaluation objects $X\{A_1, A_2, \dots, A_n\}, 0 \leq n \leq 10$

2. Determining factor set (also called criteria set)

In many evaluation factors, in most cases, we define $U = \{u_1, u_2, \dots, u_i\}$. What's more, according to the certain attributes, we divide it into some subset B , $U_i = \{u_1^i, u_2^i, \dots, u_i^i\}, i = 1, 2, \dots, B$. Here, B is second criterion.

Meet the conditions:

- $\sum_{i=1}^B n_i = n$. All subsets should be included in total set.
- $\bigcup_{i=1}^B U_i = U$. A subset set should be included in total set.
- $U_i \cap U_j = \emptyset, i \neq j$. No association between subsets.

3. Determining the comment set

The comment set is crucial to evaluate the coaches. In this problem, we define the comment set as follows:

Level	★★★★★	★★★★	★★★	★★	★
Score	5	4	3	2	1

4. Membership functions

For certain properties of object A , there exists differences. We can use a real number which from 0 to 1 in close interval to represent the difference. Particularly, 0 indicates the evaluation properties of an object is minimum, that is to say, the properties of an object don't exist. 1 represents the highest level of evaluation properties of certain object.

Define the difference of object A as membership, represented as: $u(A)$, $0 \leq u(A) \leq 1$.

Furthermore, since the membership is based on the different properties of different objects (as A) obtained, so the study of object properties can determine relative membership function and membership.

Through processing the data, we select top 10 college coaches in a sports field, we can get the relationship between membership and membership function. Then we get the membership. Usually, we integrate empirical formulas and membership functions to figure out the membership.

$$D = \begin{pmatrix} d_{11} & d_{12} & \dots & d_{1j} \\ d_{21} & d_{22} & \dots & d_{2j} \\ \vdots & \vdots & & \vdots \\ d_{i1} & d_{i2} & \dots & d_{ij} \end{pmatrix}$$

In which:

j : the level of evaluation

i : the number of evaluation criterion

5. Getting weights according to AHP

By the basic model studying, we determine the weight vector of this model.

$B = (b_1, b_2, b_3)$ B is the weight of personal quality, training quality and team achievements.

$B_1 = (b_{11}, b_{12})$, $B_2 = b_{21}$, $B_3 = (b_{31}, b_{32}, b_{33}, b_{34})$, $B_i (i=1,2,3)$ is defined as the weights of second criterion.

Table 11. Weight vector of this model

First level criterion (B_i)	The weight (B_i)	Second level criterion (B_{ij})	The weight	★	★	★	★	★
			★	★	★	★	★	★
Personal quality	0.2496	Coaching time	0.5498	d_{11}	d_{12}	d_{13}	d_{13}	d_{15}
		Gender	0.4502	d_{21}	d_{22}	d_{23}	d_{24}	d_{25}
Training quality	0.1565	Race command capability	0.5503	d_{31}	d_{32}	d_{33}	d_{34}	d_{35}
Team achievements	0.5938	Win	0.2853	d_{41}	d_{42}	d_{43}	d_{44}	d_{45}
		Losses	0.1160	d_{51}	d_{52}	d_{53}	d_{54}	d_{55}
		Win-loss percentage	0.4256	d_{61}	d_{62}	d_{63}	d_{64}	d_{65}
		Performance in league matches	0.1731	d_{71}	d_{72}	d_{73}	d_{74}	d_{75}

The training quality include three aspects. However, organizational capability and management capability have little effects. So we ignore them and adjust the proportion when we calculate the concrete rank.

d_{ij} is defined as concrete calculations of different college coaches in different sports field.

6. Comprehensive evaluate coach performance

$$D = BOR = (b_1, b_2, b_3)O \begin{pmatrix} B_1 OR_1 \\ B_2 OR_2 \\ B_3 OR_3 \end{pmatrix} = (b_1, b_2, b_3)O \begin{pmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} \\ d_{31} & d_{32} & d_{33} & d_{34} & d_{35} \end{pmatrix} = (d_1, d_2, d_3, d_4, d_5)$$

O represents $M(\bullet, \otimes)$, \bullet is defined as $a \bullet b = a \times b = ab$, \otimes is defined as $a \otimes b = (a + b) \wedge 1$. The final score is calculated as follows:

$$Z = B \bullet D^T$$

6.3.2 Concrete Calculation

Based on above steps, we calculate membership functions of every quality of every coach. In our model, the membership function is liner function. Then, we get membership by corresponding membership function. Herein, we choose top 15 of college coaches, which are shown as follows:

Table 12. Scores of top 15 basketball coaches' quality

Coach	CT	Gender	RCC	Wins	Lose	W-L%	FF
Harry Statham	5	2	5	5	5	3	5
Herb Magee	5	2	4	5	5	3	4
Danny Miles	4	2	5	5	5	3	4
Adolph Rupp	4	2	3	4	5	5	3
Mike Krzyzewski	3	2	4	5	3	4	4
Dean Smith	3	2	4	4	4	4	4
Bob Knight	4	2	4	4	4	3	3
Jim Boeheim	3	2	4	4	3	4	2
Jerry Tarkanian	2	2	3	3	5	5	2
Roy Williams	1	2	3	2	5	5	3
Jim Calhoun	4	2	3	4	4	2	2
Lute Olson	2	2	4	3	4	3	3
Eddie Sutton	3	2	4	3	3	3	2
Phog Allen	5	2	1	2	4	3	2
Rick Pitino	1	2	3	2	4	4	3
E.A. Diddle	4	2	1	3	3	3	1
Lefty Driesell	4	2	2	3	5	2	1
Lou Henson	4	2	3	3	5	1	2
Ray Meyer	4	2	2	2	4	2	2
Denny Crum	2	2	3	2	3	2	3

In the table:

RCC-- Race command capability

CT—coaching time

FF—number of NCAA final four appearance

The scores of top 15 basketball college coaches are shown in **Figure 6**. From the figure we can find the top 5 basketball college coaches are **Harry Statham, Herb Magee, Danny Miles , Adolph Rupp, Mike Krzyzewski**.

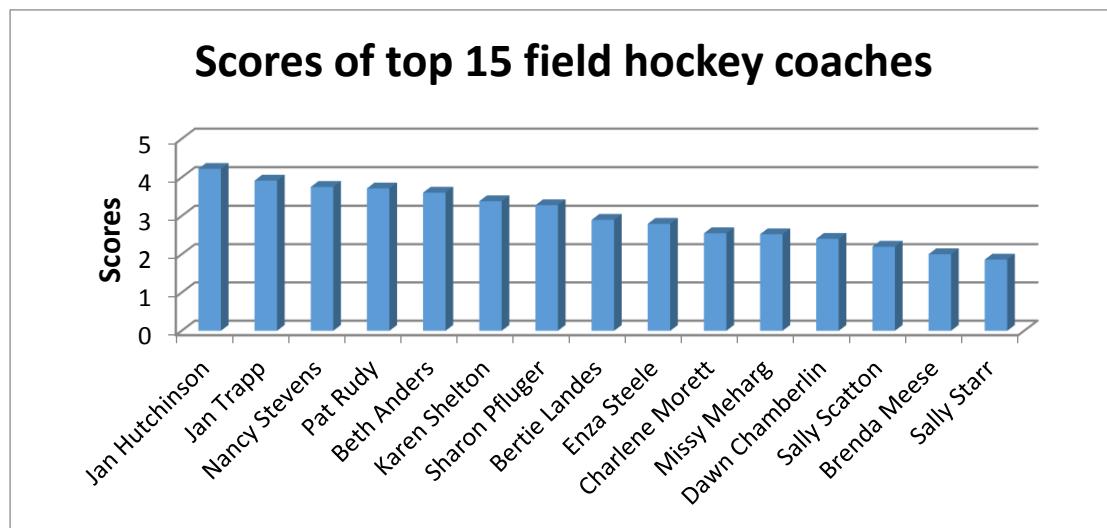


Figure 6. Scores of top 15 basketball coaches

Through normalizing the data, we get Fuzzy Decision Matrix. Finally, we get final grades and ranks by fuzzy calculation of Fuzzy Decision Matrix (R) and Weight Matrix (B), which is shown in **Table 13**.

Table 13. Scores and rank of top 15 basketball coaches
(Fuzzy Comprehensive Evaluation Model)

Ranking	Coach	Last School	Score
1	Harry Statham	McKendree	4.1557
2	Herb Magee	Philadelphia College	3.9668
3	Danny Miles	OTT	3.9156
4	Adolph Rupp	Kentucky	3.9068
5	Mike Krzyzewski	Duke	3.8072
6	Dean Smith	North Carolina	3.6367
7	Bob Knight	Texas Tech	3.4184
8	Jim Boeheim	Syracuse	3.3622
9	Jerry Tarkanian	Fresno State	3.2900
10	Roy Williams	North Carolina	3.0161
11	Jim Calhoun	Connecticut	2.9767
12	Lute Olson	Arizona	2.9044
13	Eddie Sutton	San Francisco	2.8700
14	Phog Allen	Central Missouri	2.7158
15	Rick Pitino	Louisville	2.6944

Similarly, by analysis of data we get from Wikipedia, [http://en.wikipedia.org/wiki/List_of_college_field_hockey_coaches_with_250_wins] We get the scores and rank of top 15 field hockey college coaches by Fuzzy Comprehensive Algorithm. Top 5 of them are listed in **Table 14**.

Table 14. Scores and rank of top 5 field hockey college coaches

Coach	Jan Hutchinson	Jan Trapp	Nancy Stevens	Pat Rudy	Beth Anders
Scores	4.22	3.91	3.75	3.71	3.60
Rank	1	2	3	4	5

To show the top 5 field hockey college coaches Intuitively, we draw **Figure 7.** as follows. From the figure we know that top 5 field hockey coaches are **Jan Hutchinson, Jan Trapp, Nancy Stevens, Pat Rudy, Beth Anders**. By searching the internet for their information, it is found that the number of honor is large enough to support every one them rank into top 5.

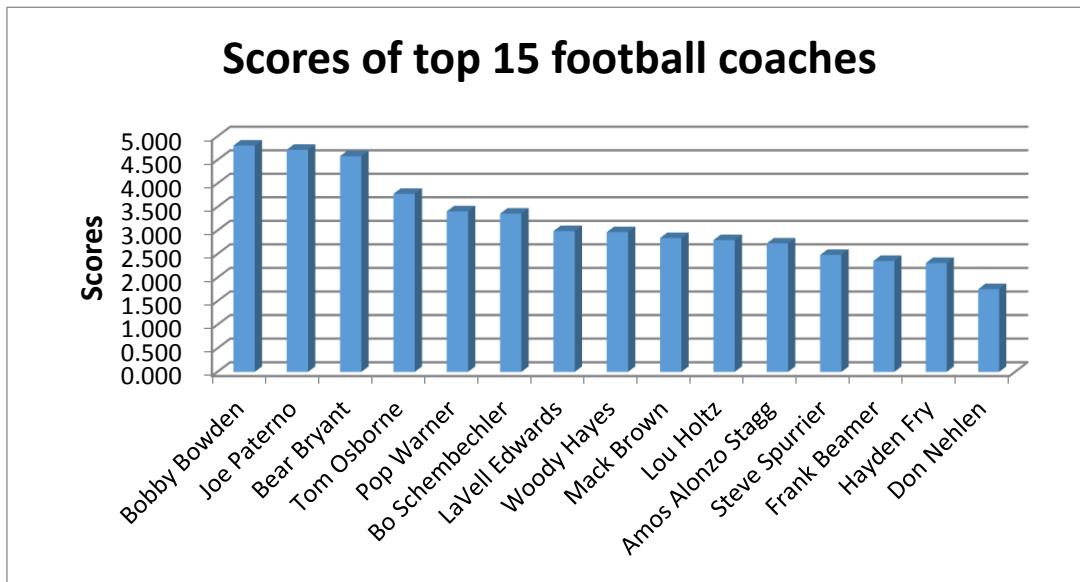


Figure 7. Scores of top 15 field hockey college coaches

Similarly, by analysis of data we get from Wikipedia, [http://en.wikipedia.org/wiki/List_of_college_football_coaches_with_200_wins] We get the scores and rank of top 15 football college coaches by Fuzzy Comprehensive Algorithm. Top 5 of them are listed in **Table 15**.

Table 15. Scores and rank of top 5 football college coaches

Coach	Bobby Bowden	Joe Paterno	Bear Bryant	Tom Osborne	Pop Warner
Scores	4.805	4.715	4.579	3.779	3.407
Rank	1	2	3	4	5

To show the top 5 football college coaches Intuitively, we draw **Figure 8.** as follows. From the figure we know that top 5 football coaches are **Bobby Bowden, Joe Paterno, Bear Bryant, Tom Osborne, Pop Warner**.

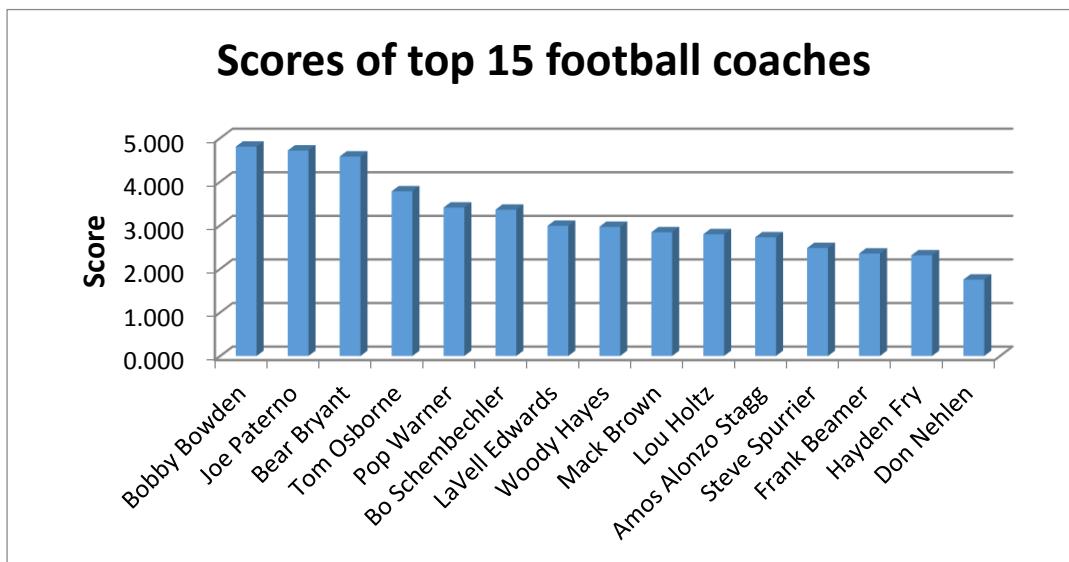


Figure 8. Scores of top 15 football college coaches

6.4 Results Review

We develop the basic model by using AHP algorithm and get the weights of seven main evaluation criterion. Then, we develop two evaluation models to evaluate college coaches of basketball, field hockey and football. Final ranks of three sports are listed respectively. Herein, we discuss the difference of two evaluation models and analyze some reasons. Grey Correlation rank, Fuzzy Comprehensive rank and Current rank are listed in **Table 15**.

Table 15. Ranks comparison

Coaches	Grey Correlation Rank	Fuzzy Comprehensive Rank	Current Rank
Harry Statham	1	1	1
Danny Miles	2	3	2
Herb Magee	3	2	3
Mike Krzyzewski	4	5	4
Jim Boeheim	7	8	5
Bob Knight	5	7	6
Dean Smith	6	6	7
Jim Calhoun	8	11	8
Adolph Rupp	9	4	9
Eddie Sutton	11	13	10

From above rank table, we can find that two evaluation models' ranks reach great agreement with that of Wikipedia. However, there is little difference between two evaluation models. To show the difference apparently, we draw the figure of three ranks, which is shown in **Figure 9**.

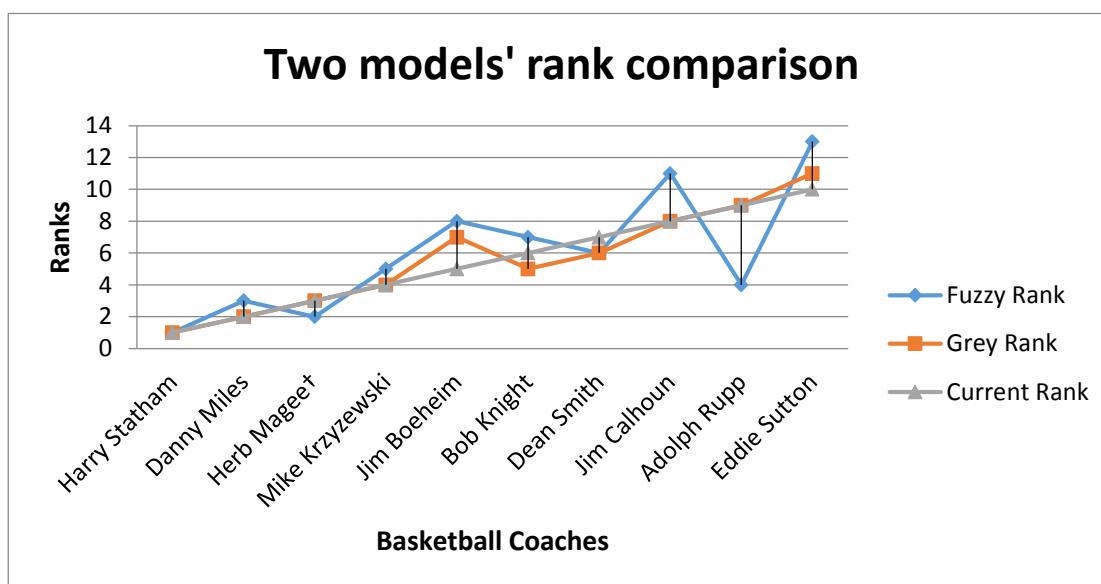


Figure 9. Two models' rank comparison

Through compared with current rank, it's apparent that Grey Correlation rank is better than that of Fuzzy Comprehensive. We analyze reasons for difference as follows:

- Fuzzy Comprehensive Algorithm considers inner link of college coaches' all quality, while the Fuzzy Comprehensive Algorithm doesn't.
- There exists difference in data processing.

Our evaluation ranks are different with that some sports website given. Different websites use different evaluation criterion to evaluate college coaches. For example, criterion in some evaluation systems include the number of consecutive wins, but we didn't take it into consideration. Therefore, it's obvious that ranks are different. Concrete analysis and optimal ranks will be discussed in Conclusion part.

7 Is Time Line Horizon Influential?

In the past 100 years, people's evaluation to coaches may change. One reason is that different society and live environment has different effects on coaches. Meanwhile, evaluation criterion would change with time changing. These reasons all affect fairness of college coaches' ranks.

By considering *time* changing, we discuss whether time line horizon has effects on coaches' ranks or not. Physical environment, spiritual environment and recognition of coaches would change with time changing. Based on considering physical and spiritual environment, we provide new college coaches' ranks.

Through comparing coaches' scores with their start teaching time, we observe whether their changing trends are in agreement.

In AHP model, we ignored approximate 7% weight. In this model, we will assign weight again by considering time changing. We note time as one of the evaluation criterion and assign 7% weight to it. Membership function is used to get normalized membership of time with respect to different coaches. That is :

$$Z_2 = \frac{T - 1900}{100}$$

In which, T represents the start teaching time of coach. Z_2 represents time score after normalized.

On the basis of Grey Correlation Algorithm, we calculate scores again and the weight are $\rho_1=0.93$ and $\rho_2=0.07$. The normalized scores is:

$$Z = \rho_1 \times Z_1 + \rho_2 \times Z_2$$

In which, ρ_1 represents weight of score calculated by Grey Correlation Model. ρ_2 represents weight of time criteria. Z_1 represents the score calculated by Grey Correlation Model. Z represents final total score.

Comparing normalized scores with former scores, the difference between ranks considering coaching start time or not is shown in **Figure 9**.

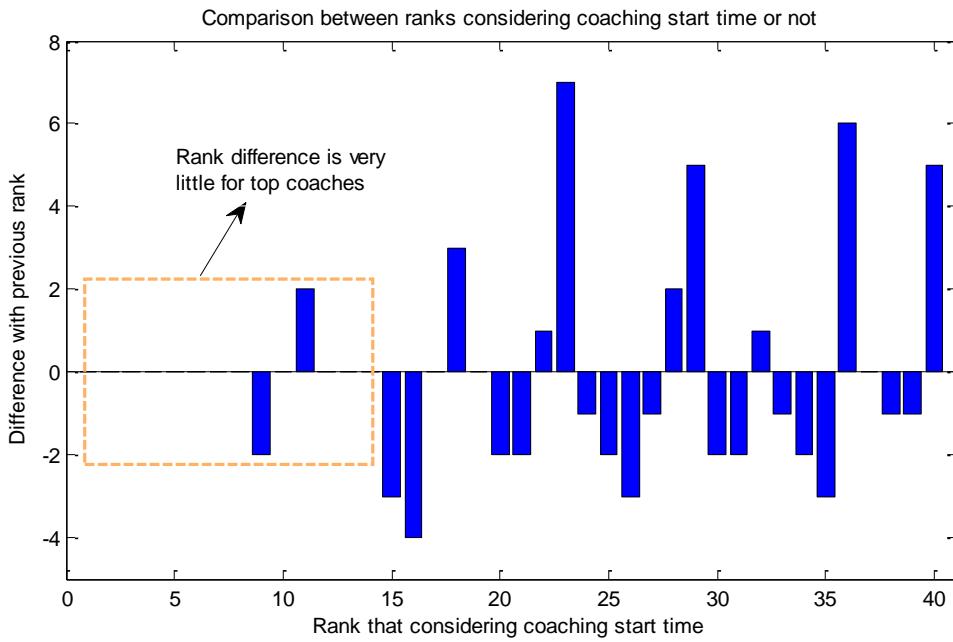


Figure 10. Comparison between ranks considering coaching start time or not of basketball

In **Figure 10.** the horizontal coordinate represents rank that has not considered coaching start time. The vertical coordinate represents difference with previous rank. From the difference value between ranks considering coaching start time or not, we can easily find that time has effects on college coaches' ranks. What's more, time criteria has more effects on coaches whose rank is behind, while it has fewer effects on coaches whose rank is front.

To examine above conclusion, we use football coaches' data to verify the results, which are shown in **Figure 11.** It's obvious that former conclusion is right.

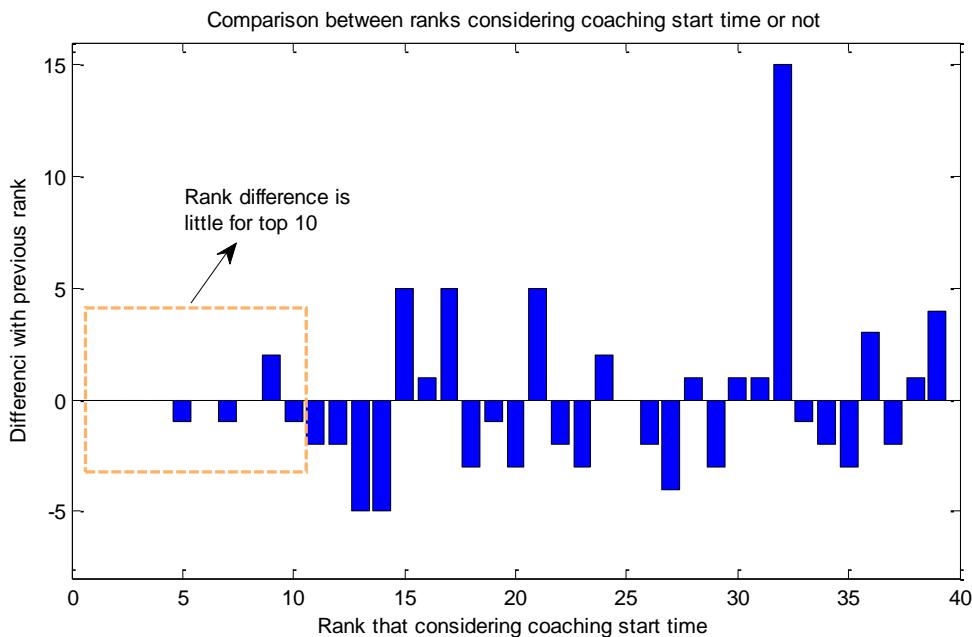


Figure 11. Comparison between ranks considering coaching start time or not of football

In conclusion, time criteria has more effects on coaches whose rank is behind, while it has fewer effects on coaches whose rank is front.

8 Sensitivity Analysis

8.1 Sensitivity of Grey Correlation Method

According to **Equation 6.2**, resolution ratio β is the key parameter to correlation coefficient, which is tightly related to correlation. If β is too small, the difference of correlation coefficient will be too large, which results in failing to give out the correlation of every evaluation criterion and reference data series correctly. On the contrary, if β is too large, the difference of correlation coefficient is too small, leading to the absolute difference having a little impact on evaluation results. Therefore the value of β is crucial to final evaluation results.

In the process of calculating, value of β is uniformly 0.5. In order to analyze its sensitivity, take basketball coaches for example. For resolution ratio β in $[0, 1]$, we calculate the effect of resolution ratio on evaluating by assigning step length as 0.2. After being processed by computer, ranks of top 10 coaches are shown in **Figure 12**. In the figure, the horizontal coordinate represents the top 10 coaches, who are represented by A, B, ..., J, the vertical coordinate Z_i represents correlation of every coach with standard data, β (beta) represents resolution ratio, *beta* represents ranks of top 10 coaches.

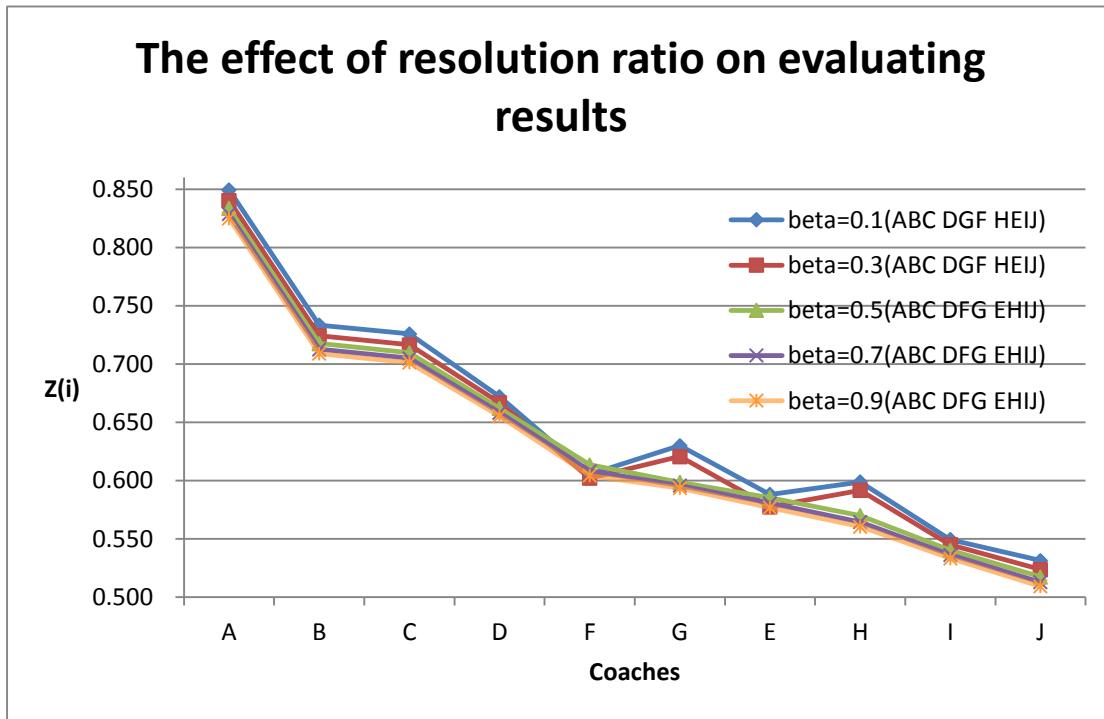


Figure 12. The effect of resolution ratio on evaluating results

From the results of above figure, it is obviously to find that evaluation results are divided into two parts, because of changes of resolution ratio. The erratic stage as $\beta \leq 0.3$ and the stable stage as $\beta > 0.3$.

1. When $\beta \leq 0.3$, result of correlation change greatly and the rank is erratic. If assign resolution ratio small value, stable control capability is weak. Therefore, we couldn't use β in this range.
2. When $\beta > 0.3$, result of correlation change weakly. We find that when $\beta \geq 0.5$, the change of resolution ratio has no effects on ranks, all of the ranks are consistent. Therefore, in our paper, it's reasonable to assign $\beta = 0.5$. meanwhile, the result is stable.

8.2 Sensitivity of Fuzzy Comprehensive Method

The sensitivity analysis is crucial to stability and rank of the reliability evaluation model. Herein, we analyze the sensitivity of evaluation criterion weights. Briefly , to show it's sensitivity, we change the minimum and maximum weight ratio to get the different score of different coach.

We choose basketball coaches' data as an example.

First, we adjust the maximum weight of the evaluation make it increase 0.02, get the score of the coaches.

Then, we adjust the maximum weight of the evaluation make it decrease 0.02, get the score of the coaches.

The results are showing in **Figure 13**.

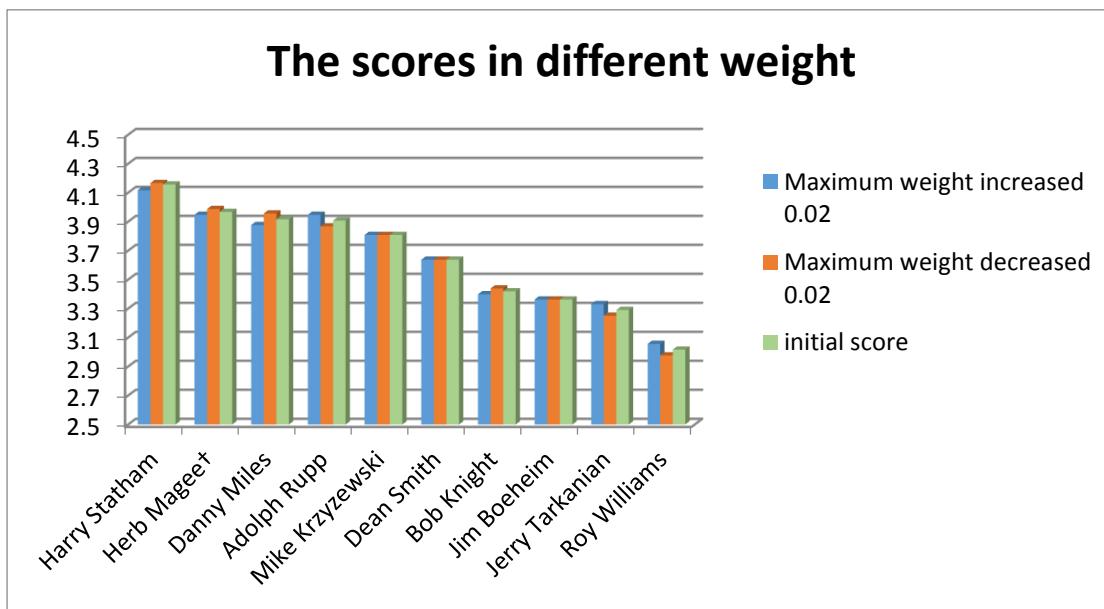


Figure 13. The score in different weight

Perhaps this approach is not scientific, but it is capable to show us a better strength of this kind of model sensitivity. From the figure we see that the weight change will have an impact on score, but the impact is limited and the ranking changes little. Therefore, we conclude that the model is of high stability.

9 Discussion and Conclusion

9.1 Solutions to All Problems

The three proposed problems have been well answered in our models.

- **Develop a model to look for the best college coach for the previous century.**

To answer this problem, AHP is employed to filter seven main criterion, gender and winning courts included, and to get their weights respectively. Then both Grey Relational Model and Fuzzy Comprehensive Evaluation Model are built, each of which is to rank those coaches. The former is about the relevance among the evaluation criterion to get the correlation, while the latter integrates empirical formula and membership functions to figure out the corresponding membership and then to give the evaluation matrix. Furthermore, we compare the results with current ranks and find that grey model has slight advantage over fuzzy model. That is to say that Grey Relational Model is more optimal

- **Analyze whether it make a difference with time line horizon or not.**

We adjust the model obtained earlier and get the conclusion that: time line horizon has more effects on coaches whose rank is behind, while it has fewer effects on coaches who rank top.

- **Present top 5 college coaches in each of 3 different sports.**

Using the model we obtain, we rank the coaches in the basketball. As the evaluation criterion we chose are comprehensive and the coaches who were engaged in different sport fields have the same quality, we apply the above method to rank other coaches in different sports flied. Such as: field hockey and football. Besides, considering time line horizon, we provide the final general ranks, which are shown as follows:

Rank	Basketball	Field hockey	Football
1	Harry Statham	Jan Hutchinson	Joe Paterno
2	Danny Miles	Jan Trapp	Bobby Bowden
3	Herb Magee	Nancy Stevens	Bear Bryant
4	Mike Krzyzewski	Sharon Pfluger	Lou Holtz
5	Bob Knight	Pat Rudy	Pop Warner

9.2 Honor Roll



Honor Roll



After building mathematical models and analysing the results, we find the **top 5 basketball college coaches which are shown as follows:**

Rank	Coach	Teams
1	Harry Statham	McKendree (1967–present)
2	Danny Miles	Oregon Tech (1971–present)
3	Herb Magee	Philadelphia (1967–present)
4	Mike Krzyzewski	Army (1975–1980), Duke (1980–present)
5	Bob Knight	Indiana(1971–2000), Texas Tech (2001–2008)



Top 5 football college coaches are shown in the following:

Rank	Coach	Teams
1	Joe Paterno	Nittany Lions(1966-2011)
2	Bobby Bowden	West Virginia (1970–1975), Florida State (1976–2009)
3	Bear Bryant	Texas A&M (1954–1957), Alabama (1958–1982)
4	Lou Holtz	Notre Dame (1986–1996), South Carolina (1999–2004)
5	Pop Warner	Temple Owls (1933–1938), Stanford Indians (1924–1932)



Top 5 college filed hockey college coaches are shown:

Rank	Coach	Teams
1	Jan Hutchinson	Bloomsburg 1978-2009
2	Jan Trapp	Messiah 1973-2011
3	Nancy Stevens	Northwestern 1981-1989, Connecticut 1990-present
4	Sharon Pfluger	Kean 1983, Montclair St. 1984, TCNJ 1985-present
5	Pat Rudy	SUNY Cortland 1981-95, Lock Haven 1996-present

9.3 Strengths and Weaknesses

Strengths

- Grey Correlation Algorithm and Fuzzy Comprehensive Algorithm are both adopted to evaluate college coaches. Through comparing the ranks, we find that Grey Correlation Algorithm is better. The result is convincing.
- Through AHP algorithm, we select seven main evaluation criterion. What's more, we improve our model by considering gender and time line horizon. Thus, our evaluation method is exhaustive.
- Through plenty of figures and tables, it's intuitively shown that our ranks are in good agreement with that of Wikipedia. That's to say our model is practical to some extent.
- Our data comes from the official website such as National Collegiate Athletic Association (NCAA), it's believable.

Weaknesses

- In the *wins* criteria, we only considered the number of wins and neglected the Simple Rating System (SRS) and Strength of Schedule (SOS). It may lead to little difference with other evaluation method.
- We didn't find much data of female college coaches. So the analysis of *gender* criteria is not exhaustive.
- We only considered objective data, but didn't consider coaches' subjective factors, such as moral, spirit and so on.

9.4 Conclusion

By developing three models, we get an objective rank of college basketball, field hockey and football coaches. Firstly, AHP Model is used to determine seven evaluation criterion. Herein, we consider the effects of gender criteria, which make our evaluation method widespread. Then, we develop Grey Correlation Model and Fuzzy Comprehensive Model to rank college coaches. By comparing with practical rank given by sports fans, we find the above two model both reach well agreement with practical rank. What's more, we find that the rank calculated by Grey Correlation Algorithm is better than that of Fuzzy Comprehensive Algorithm.

Then, we analyze the effects of time criteria on evaluation model. We conclude that time line horizon has more effects on coaches whose rank is behind, while it has fewer effects on coaches whose rank is front. The result is in great agreement with practice. Finally, we provide rank of top 5 college coaches on basketball, field hockey and football sports.

Despite our evaluation model include eight criterion, the modeling domain presented by this problem is vast, and there is a large amount of room for improvement. We believe that the work we have presented here is a significant and successful attempt at solving this problem.

A Letter to *Sports Illustrated*

February 10, 2014

Dear editor,

We all know that *sports illustrated* is one of the most popular sporting magazine in the world. Hearing that you are looking for the “best all time college coach” male or female for the previous century, it’s our pleasure to make some attempt to do it. Different from previous methods of practical survey, we adopt some mathematical method to build an evaluation system. Based on coaches data available, we give out the ranks of college coaches in three sports, basketball, field hockey and football for recent 100 years.

It is thought that an excellent coach must do well in three aspects, that is, personal quality, training quality and team achievements. After filtering from lots of factors, there are 7 key criterion finally determined to be the evaluating standards, which are coaching time, gender, command capability, number of wins, number of losses, win-loss percentage and performance in league matches.

When it comes to evaluating process, we have tried two approaches. For the first model, grey model for short, optimal value for every criteria is firstly determined, and we grade these coaches by calculating the approaching degree to the optimal value. For the second model, called fuzzy model, we grade them by considering that different criterion show different importance to final evaluation. Through the analysis of results derived from two models, it is found that the first one is more objective. Therefore, the ranking from grey model is as follows.

Rank	Basketball	Field hockey	Football
1	Harry Statham	Jan Hutchinson	Joe Paterno
2	Danny Miles	Jan Trapp	Bobby Bowden
3	Herb Magee	Nancy Stevens	Bear Bryant
4	Mike Krzyzewski	Sharon Pfluger	Lou Holtz
5	Bob Knight	Pat Rudy	Pop Warner

With the development of society, the environment which coaches live would change. To analyze whether time has effects on evaluation results or not, we promote above models by considering time criteria. We find that time has fewer effects on the ranking front college coaches.

Finally, we think that best all time college coach should not be one but ones. Since every evaluation method has both advantages and disadvantages. There is only little difference between top 5 college coaches’ scores in every sports field, it is distinguishing in mathematics, but it’s not important for evaluating comprehensive quality of top 5 college coaches. They all have own unique quality, so they are all best. They all have great influence on respective field.

Sincerely yours,
Team 26160

Reference

- [1] Yangping Zhang, Feng Zhai. "Study of a system that evaluates the comprehensive capacities of excellent college track and field coaches." *Journal of Physical Education* 16.11 (2009): 77-80.
- [2] Grisaffe, Christie, Lindsey C. Blom, and Kevin L. Burke. "The effects of head and assistant coaches' uses of humor on collegiate soccer players' evaluation of their coaches." *Journal of Sport Behavior* 26.2 (2003): 103-108.
- [3] Radicchi, Filippo. "Who is the best player ever? A complex network analysis of the history of professional tennis." *PloS one* 6.2 (2011): e17249.
- [4] Knoppers, Annelies, et al. "Opportunity and work behavior in college coaching." *Journal of Sport & Social Issues* 15.1 (1991): 1-20.
- [5] Knoppers, Annelies, et al. "Gender ratio and social interaction among college coaches." *Sociology of Sport Journal* 10.3 (1993): 256-269.
- [6] Pastore, Donna L., Bernie Goldfine, and H. Riemer. "NCAA college coaches and athletic administrative support." *Journal of Sport Management* 10.4 (1996): 373-387.
- [7] Caccese, Thomas M., and Cathleen K. Mayerberg. "Gender differences in perceived burnout of college coaches." *Journal of Sport Psychology* (1984).
- [8] Saaty, Thomas L. *What is the analytic hierarchy process?*. Springer Berlin Heidelberg, 1988.
- [9] Saaty, Thomas L. "How to make a decision: the analytic hierarchy process." *European journal of operational research* 48.1 (1990): 9-26.
- [10] Tao, Yang, and Yang Xinxiao. "Fuzzy comprehensive evaluation, fuzzy clustering analysis and its application for urban traffic environment quality evaluation." *Transportation Research Part D: Transport and Environment* 3.1 (1998): 51-57.
- [11] Shiyun, Liu Luxiang Song Qixin Wang. "A PRELIMINARY STUDY ON MULTIFACTORIAL EVALUTION OF NEW CROP VARIETIES WITH THE APPLICATION OF THE GREY SYSTEM THEORY [J]." *Scientia Agricultura Sinica* 3 (1989): 003.
- [12]<http://www.sports-reference.com/cbb/coaches/>
- [13]http://en.wikipedia.org/wiki/List_of_college_men's_basketball_coaches_with_600_wins
- [14]http://en.wikipedia.org/wiki/List_of_college_football_coaches_with_200_wins
- [15]http://en.wikipedia.org/wiki/List_of_college_field_hockey_coaches_with_250_wins
- [16]<http://www.secsportsfan.com/best-all-time-ncaa-college-football-coach.html>
- [17]<http://www.amstat.org/sections/SIS/Sports%20Data%20Resources/>
- [18]<http://www.basketball-reference.com/coaches/>

Appendix

1. Original data of college basketball coaches

Coach	Last School	C-t	Gen.	RCC	Wins	Loss	W-L	FF
Harry Statham	McKendree	47	1	39	1076	444	0.708	13
Danny Miles	O I T	42	1	32	1000	409	0.71	10
Herb Magee†	Philadelphia College	45	1	26	976	391	0.714	11
Mike Krzyzewski	Duke	39	1	29	975	302	0.764	11
Jim Boeheim	Syracuse	38	1	30	942	314	0.75	4
Bob Knight	Texas Tech	42	1	28	899	374	0.706	5
Dean Smith	North Carolina	36	1	27	879	254	0.776	11
Jim Calhoun	Connecticut	40	1	23	877	382	0.697	4
Adolph Rupp	Kentucky	41	1	20	876	190	0.822	6
Eddie Sutton	San Francisco	37	1	26	806	329	0.71	3
Lefty Diesell	Georgia State	41	1	13	786	394	0.666	0
Lute Olson	Arizona	34	1	28	776	285	0.731	5
Lou Henson	Illinois	41	1	19	775	420	0.649	2
Jerry Tarkanian	Fresno State	30	1	18	761	202	0.79	4
E.A. Diddle	Western Kentucky	42	1	3	759	302	0.715	0
Hank Iba	Oklahoma State	40	1	8	752	333	0.693	4
Ray Meyer	DePaul	42	1	13	724	354	0.672	2
Don Haskins	Texas-El Paso	38	1	14	719	353	0.671	1
Phog Allen	Central Missouri	48	1	4	719	259	0.735	3
Roy Williams	North Carolina	26	1	23	715	187	0.793	7
Rick Pitino	Louisville	28	1	18	681	239	0.74	7
Denny Crum	Louisville	30	1	23	675	295	0.696	6
Ralph Miller	Oregon State	38	1	10	674	370	0.646	0
Mike Montgomery	California	32	1	16	670	312	0.682	1
Gary Williams	Maryland	33	1	17	668	380	0.637	2
Bob Huggins	West Virginia	29	1	20	665	270	0.711	2
John Wooden	UCLA	29	1	16	664	162	0.804	12
Cliff Ellis	Coastal Carolina	36	1	8	662	423	0.61	0
Hugh Durham	Jacksonville	37	1	8	634	430	0.596	2
Norm Stewart	Missouri	32	1	16	634	333	0.656	0
Billy Tubbs	Texas Christian	29	1	12	609	317	0.658	1
Slats Gill	Oregon State	36	1	6	599	393	0.604	2
Tom Davis	Drake	32	1	11	597	356	0.626	0
Stew Morrill	Utah State	28	1	9	597	276	0.684	0
John Thompson	Georgetown	27	1	20	596	239	0.714	3
Thomas Penders	Houston	33	1	11	594	420	0.586	0
Guy Lewis	Houston	30	1	14	592	279	0.68	5

Coach	Last School	C-t	Gen.	Rcc	Wins	Loss	W-L	FF
Bobby Cremins	College of Charleston	31	1	11	586	379	0.607	1
John Calipari	Kentucky	22	1	14	585	171	0.774	4
Rick Barnes	Texas	27	1	20	578	293	0.664	1
Tony Hinkle	Butler	41	1	1	558	394	0.586	0
Norm Sloan	North Carolina State	33	1	6	558	359	0.609	1
Gene Keady	Purdue	27	1	18	550	289	0.656	0
Frank McGuire	South Carolina	30	1	8	549	236	0.699	2
Bill Self	Kansas	21	1	15	524	169	0.756	2
Teresa Lawrence	Tennessee State	28	3	14	212	189	0.523	1

2. Original data of college field hockey coaches

Coach	Teams	C-t	Wins	Losses	W-L%	Ties
Hutchinson	Bloomsburg 1978-2009	32	591	75	0.876	20
Beth Anders	Old Dominion 2004-present	29	561	136	0.802	7
Nancy Stevens	Connecticut 1990-present	33	554	171	0.756	24
Karen Shelton	North Carolina 1981-present	31	550	140	0.793	9
Jan Trapp	Messiah 1973-2011	38	536	179	0.74	28
Pat Rudy	Lock Haven 1996-present	34	532	178	0.742	21
Sharon Pfluger	TCNJ 1985-present	29	498	94	0.836	9
Enza Steele	Lynchburg, 1979-present	33	495	195	0.714	11
Missy Meharg	Maryland 1988-present	24	450	112	0.796	9
Charlene	Penn State 1987-present	28	445	174	0.713	17
Bertie Landes	Shippensburg 1999-present	32	418	119	0.77	17
Chamberlin	Salisbury 1987-present	25	405	99	0.799	8
Sally Starr	Boston U. 1981-present	33	401	259	0.604	21
Brenda Meese	Wooster 2009-present	31	393	217	0.642	10
Sally Scatton	William Smith 1988-present	24	390	121	0.762	3
Murtagh	Bentley 1988-present	26	364	205	0.637	13
Kauffman	Elizabethtown 1967-83	33	346	175	0.654	36
Betty Wesner	Kutztown 1980-present	32	344	257	0.571	15
Sharon Taylor	Lock Haven 1973-1995	28	340	121	0.721	35
Sandy Miller	E Stroudsburg 1984-present	28	338	236	0.587	11
Amy Watson	Keene St. 1994-present	22	319	154	0.672	11
Jennifer	Wake Forest 1992-present	23	319	170	0.651	6
Beth Bozman	Duke 2003-2010	23	318	133	0.702	6
Kathleen	Syracuse 1978-2006	29	316	214	0.594	14
Ann Petracco	FDU-Florham 2003-2009	33	311	306	0.504	40
Madison	Virginia 2006-present	23	312	197	0.612	6
Carol Miller	Lebanon Valley 2011-present	27	309	216	0.584	27
Hallenbeck	Skidmore 2001-present	22	254	132	0.657	2

Coach	Teams	C-t	Wins	Losses	W-L%	Ties
Hawthorne	W & Mary 1987-present	29	306	251	0.549	6
Dana Hall	Washington 1990-2010	23	298	138	0.682	4
Ann Gold	Lafayette 1982-2006	25	294	186	0.607	23
Anna Meyer	Hartwick 1988-present	24	290	184	0.611	3
Wilkinson	Ohio St. 1996-present	25	288	215	0.571	9
Fitzpatrick	Ball State 1980-2000	21	285	130	0.682	11
Linda Wage	Clark (MA) 1985-present	27	283	207	0.574	24
Kostrinsky	Ithaca 1969-1995	27	282	135	0.668	21
Ridinger	Missouri State 1975-1990	18	281	101	0.722	24
Pam Hixon	Massachusetts 1996	17	272	76	0.768	18
DeLorenzo	Middlebury 2001-present	20	271	109	0.712	2
Linda Arena	Wittenberg 1982-1996, 1998	23	262	133	0.65	34

3. Original data of college football coaches

Coach	Last School	C-t	R-c-c	Wins	Losses	W-L%	Per.
Joe Paterno	Penn State	46	37	409	136	0.749	0.662
Bobby Bowden	Florida State	40	33	357	124	0.74	0.682
Bear Bryant	Alabama	38	29	323	85	0.78	0.552
Pop Warner	Temple	42	4	311	103	0.733	0.375
Amos Alonzo	Chicago	42	0	275	121	0.681	0
LaVell Edwards	Brigham Young	29	22	257	101	0.716	0.341
Tom Osborne	Nebraska	25	25	255	49	0.836	0.48
Lou Holtz	South Carolina	33	22	249	132	0.651	0.591
Mack Brown	Texas	29	21	238	117	0.67	0.619
Bo Schembechler	Michigan	27	17	234	65	0.775	0.294
Hayden Fry	Iowa	37	17	230	180	0.56	0.441
Frank Beamer	Virginia Tech	27	21	224	109	0.672	0.429
Steve Spurrier	South Carolina	24	19	219	79	0.733	0.474
Woody Hayes	Ohio State	28	11	205	61	0.761	0.455
Don Nehlen	West Virginia	30	13	202	128	0.609	0.308
Vince Dooley	Georgia	25	20	201	77	0.715	0.45
Dan McGugin	Vanderbilt	30	0	197	55	0.762	0
John Cooper	Ohio State	24	14	192	84	0.691	0.357
Dana Bible	Texas	31	4	190	69	0.715	0.875
John Vaught	Mississippi	25	18	190	61	0.745	0.556
George Welsh	Virginia	28	15	189	132	0.588	0.333
Howard Jones	S,California	28	5	188	63	0.732	1
Jess Neely	Rice	36	7	187	159	0.539	0.571
Johnny Majors	Tennessee	29	16	185	137	0.572	0.563
Darrell Royal	Texas	23	16	184	60	0.749	0.531
Dick Tomey	San Jose State	29	8	183	145	0.557	0.625

Coach	Last School	C-t	R-c-c	Wins	Losses	W-L%	Per.
Jerry Claiborne	Kentucky	28	11	179	122	0.592	0.273
Jackie Sherrill	Mississippi State	26	14	179	121	0.595	0.571
Bill Snyder	Kansas State	22	15	178	90	0.664	0.467
Frank Kush	Arizona State	22	7	176	54	0.764	0.857
Ralph Jordan	Auburn	25	12	175	83	0.674	0.417
Don James	Washington	22	15	175	79	0.687	0.667
Gary Pinkel	Missouri	23	9	175	100	0.635	0.556
Bob Neyland	Tennessee	21	7	173	31	0.829	0.286
Dan Devine	Notre Dame	22	10	172	57	0.742	0.7
George Woodruff	Georgia	17	0	172	41	0.803	0
Wallace Wade	Duke	24	5	171	49	0.765	0.5
Nick Saban	Alabama	18	14	170	57	0.748	0.571
Fisher DeBerry	Air Force	23	12	169	109	0.608	0.5
Jim Sweeney	Fresno State	27	7	169	134	0.557	0.714
Ken Hatfield	Rice	27	10	168	140	0.545	0.4
Bill Mallory	Indiana	27	10	167	130	0.561	0.4
Red Blaik	Army	25	0	166	48	0.759	0
Bobby Dodd	Georgia Tech	22	13	165	64	0.713	0.692

4. Scores and ranks for basketball coaches of two models

C Rank	Coach	Last School	Grey Scores	Grey Rank	Fuzzy Scores	Fuzzy Rank
1	Harry Statham	McKendree	0.834	1	4.1557	1
2	Danny Miles	OIT	0.718	2	3.9156	3
3	Herb Magee†	Philadelphia College	0.710	3	3.9668	2
4	Mike Krzyzewski	Duke	0.662	4	3.8072	5
5	Jim Boeheim	Syracuse	0.585	7	3.3622	8
6	Bob Knight	Texas Tech	0.613	5	3.4184	7
7	Dean Smith	North Carolina	0.599	6	3.6367	6
8	Jim Calhoun	Connecticut	0.570	8	2.9767	11
9	Adolph Rupp	Kentucky	0.540	9	3.9068	4
10	Eddie Sutton	San Francisco	0.517	11	2.87	13
11	Lefty Driesell	Georgia State	0.474	13	2.6173	17
12	Lute Olson	Arizona	0.507	12	2.9044	12
13	Lou Henson	Illinois	0.519	10	2.5534	18
14	Jerry Tarkanian	Fresno State	0.429	21	3.29	9
15	E.A. Diddle	Western Kentucky	0.417	24	2.6462	16
16	Hank Iba	Oklahoma State	0.462	15	2.2568	21
17	Ray Meyer	DePaul	0.468	14	2.4118	19
18	Phog Allen	Central Missouri	0.457	16	2.7158	14
19	Don Haskins	Texas-El Paso	0.444	19	2.1717	22
20	Roy Williams	North Carolina	0.437	20	3.0161	10

5. Scores and ranks for football coaches of two models

C Rank	Coach	Last School	Grey Scores	Grey Rank	Fuzzy Scores	Fuzzy Rank
1	Joe Paterno	Penn State	0.834	1	4.715	2
2	Bobby Bowden	Florida State	0.737	2	4.805	1
3	Bear Bryant	Alabama	0.647	3	4.579	3
4	Pop Warner	Temple	0.513	5	3.407	5
5	Amos Alonzo Stagg	Chicago	0.397	13	2.725	11
6	LaVell Edwards	Brigham Young	0.468	9	2.987	7
7	Tom Osborne	Nebraska	0.498	6	3.779	4
8	Lou Holtz	South Carolina	0.525	4	2.795	10
9	Mack Brown	Texas	0.492	7	2.839	9
10	Bo Schembechler	Michigan	0.410	12	3.358	6
11	Hayden Fry	Iowa	0.487	8	2.309	14
12	Frank Beamer	Virginia Tech	0.431	10	2.352	13
13	Steve Spurrier	South Carolina	0.415	11	2.480	12
14	Woody Hayes	Ohio State	0.388	14	2.964	8
15	Don Nehlen	West Virginia	0.361	15	1.755	15

6. Scores and ranks for field hockey coaches of two models

C Rank	Coach	Grey Scores	Grey Rank	Fuzzy Scores	Fuzzy Rank
1	Jan Hutchinson	0.847	1	4.222	1
2	Beth Anders	0.638	7	3.605	5
3	Nancy Stevens	0.694	3	3.754	3
4	Karen Shelton	0.652	6	3.384	6
5	Jan Trapp	0.735	2	3.918	2
6	Pat Rudy	0.663	5	3.715	4
7	Sharon Pfluger	0.674	4	3.281	7
8	Enza Steele	0.553	11	2.797	9
9	Missy Meharg	0.560	9	2.527	11
10	Charlene Morett	0.527	12	2.552	10
11	Bertie Landes	0.637	8	2.900	8
12	Dawn Chamberlin	0.555	10	2.406	12
13	Sally Starr	0.426	14	1.865	15
14	Brenda Meese	0.412	15	2.004	14
15	Sally Scatton	0.472	13	2.200	13