Author's Commentary: The Outstanding Exhaustible Resource Papers

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Introduction

This modeling problem was inspired by revisions that I was making in the last chapter in COMAP's overwhelmingly successful college-level textbook in applied mathematics for liberal arts students [Garfunkel et al. 2006]. That chapter, "The Economics of Resources," applies concepts and formulas from simple finance to assess how large the Earth's population may become, how long nonrenewable resources can last, and why renewable resources are extinguished in pursuit of economic gain. The revision includes M.K. Hubbert's model for exhaustion of oil and ends with a retelling of the ecological and human tragedy of the despoilment of Easter Island [Diamond 1995; Diamond 2005].

The compound interest formula serves as a basic model for growth of a biological population. The chapter also considers the logistic model, used by many teams in this year's ICM. The other main formula, for savings at interest, provides a way to estimate cumulative usage of a nonrenewable resource whose rate of use is increasing at a fixed rate.

The chapter introduces concepts and terminology of Michael Olinick (Middlebury College) [1991]: The *static reserve* of a resource is how long a fixed supply S will last at a constant annual rate of use U, namely, S/U years. The *exponential reserve* is how long the supply will last at an initial rate of use U that is growing at rate r per year (that is, growing exponentially), namely,

$$\frac{\ln\left(1+\dfrac{S}{U}\right)}{\ln(1+r)}.$$

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In keeping with the spirit of the book, the chapter illustrates these concepts with data on real resources. For example, as several teams in this year's ICM noted, the U.S. has recoverable reserves of coal that would last about 250 years—at the current rate of use. However, the coal will last only 85 years if the rate of use increases at 2.25% per year, as it did from 2002 to 2003. Exercises ask students to calculate the exponential reserves for oil and natural gas under the curious projections of the U.S. Geological Survey that world consumption will increase at respectively 1.9% and 2.2% per year through 2025.

Perhaps the most eye-opening exercise for students is the exercise that asks, "Can our energy problems be solved by increasing the supply?" This exercise asks students to compare the exponential reserve for an amount S of a resource with those for $10S$ and $100S$, under a rate of consumption that is increasing 2.5% per year (as U.S. oil use has been doing since 1973) [Nering 2001].

> **Exercise.** U.S. oil consumption in 2004 was 7.5 billion barrels (bbl), of which almost 60% was imported. As I write this in spring 2005, the U.S. House of Representatives has approved harvesting oil from the Alaska National Wildlife Refuge (ANWR) and the Senate is to take up the matter. Seismic estimates put the amount of recoverable oil in the ANWR at 5 to 8 billion bbl [Korpela 2002], which can become available starting in 5 years or so (so perhaps in 2010). How many months of supply of oil for the U.S. will the entire output of ANWR oil amount to in 2010, if U.S. usage continues to increase at 2.5% per year? (Predicts Korpela [2002], "That it will be drilled one day is a foregone conclusion, for when shortages appear every argument against drilling it will be dismissed by the public's clamour for oil. . . . The most appealing argument is to save it as long as possible, for once all efforts have been made to shift into a thrifty living, any oil from it would go further.")

Searching for data for the book's examples and exercises led me to the literature on models for exhaustion of nonrenewable resources.

The Outstanding Papers

The Outstanding papers go into these matters far more deeply than a book that cannot presume knowledge of calculus or differential equations.

The paper from the Olin School of Engineering examines data on U.S. water withdrawals and correctly concludes that the data are too variable to allow extrapolation. It then uses a common parametric family of models to model water withdrawal in the agricultural, industrial, and municipal sectors. This model incorporates a factor of "economic and political influence"—the fraction of under/over-draw of water relative to the renewable amount—taken to a power that is a parameter for the sector. This feature is a clever idea that avoids directly considering pricing, lets the three sectors "compete" for the same water, and (via different powers of the fraction) lets the model adapt to differential "pressures" in the three sectors. Using linear regressions of change in irrigation area and of population growth on log time and of GDP on time, plus data on U.S. water usage 1990–1995, the paper identifies the power parameters. These values vary considerably in size but the paper does not go into why this is the case. The paper then projects water usage for the U.S. and (with the same power parameter values) for an imaginary country with different initial withdrawal amounts and a different amount of renewable water. The projections are point projections, without any specification of range of variability. The paper concludes with a thorough roundup of literature on alternatives to drawing down nonrenewable water sources.

The other two papers treat oil. The paper from the Maggie Walker Governor's School follows the ideas of Hubbert [Laherrère 2000; Deffeyes 2001; 2005], fitting a logistic curve to oil discovery and a normal-distribution curve to oil production (a Hubbert curve is the derivative of a logistic curve). The paper uses a spreadsheet and Euler's method to project estimates for cumulative discovered, harvested, demanded, and remaining untapped oil. A key assumption that avoids considering price (and its volatility) is simple inverse proportionality of cumulative oil discovered and cumulative demand for oil, in the form $y = c/x$. The rationale offered for this assumption is simply the law of supply and demand; but the paper does not try to argue for the functional form used, nor for why past discoveries and past demand should be so affected. The paper concludes with applications of the model to various alternative scenarios, including disasters and alternative fuels.

The paper from East China University of Science and Technology begins with a simple system of linear first-order differential equations involving oil supply, demand, and price over time. Surprisingly, the (analytic) solution for demand fluctuates; with addition of an exponential forcing term, the demand function fits historical data well. The paper then fits demand as just an exponential plus a constant but does not return to the system to examine the consequences for supply and price (they are exponentially driven, too). The paper uses data for 1995–2003 and finds linear regressions of demand on world GDP, on population, and (rather obviously) on carbon dioxide emission from oil consumption. The linear fits are excellent in part because the time interval is short enough to mask the exponential trend in demand fitted earlier. The paper projects the date of oil exhaustion under various assumptions of growth in GDP and under a logistic model of population growth, as well as the allowable oil consumption under various rates of growth of carbon dioxide emissions. Utterly innovative is the paper's idea to allocate oil between generations (setting aside some oil for the future, by smoothly decreasing the amount of oil used) and between countries in terms of refinery capacity. Balancing conservation with development also suggests optimizing for GDP produced per barrel of oil. Countries vary enormously in the energy used per dollar of GDP; China uses 3 times as much as the U.S., and Ukraine uses 17 times as much. But some differences are unavoidable because natural resources (e.g., aluminum) help determine industries (energy-intensive smelting of ore, as in Jamaica). Countries also differ in how severely changes in the price of oil would affect their GDP (growth or decrease) [Bacon 2005, 48–52].

The two Outstanding papers on oil model the world as a whole; a finer analysis would disaggregate the world into geographical sectors, as the Olin paper did with water and economic sectors.

One concept only implicit in the Outstanding papers is *price elasticity of demand* [Nievergelt 1987]. How elastic is the demand for oil, water, or other exhaustible resources? For example, by how much did the 2004 increase in oil price from \$30 to over \$50 per barrel lessen demand in the U.S.? How much would a gasoline tax of \$4/gallon (as in Europe) affect demand? For price P and demand Q, the *price elasticity of demand* is the relative change in price quantity divided by the relative change in demand:

$$\epsilon = \frac{dQ/Q}{dP/P} = \frac{dQ}{dP}\frac{P}{Q}.$$

The quantity is always nonpositive; values farther from 0 correspond to greater elasticity of demand; values above -1 indicate inelastic demand. For the U.S., elasticity of demand for oil is -0.06, about the same as for coffee or cigarettes; demand for oil is more elastic than for pet food or breakfast cereal, but less elastic than for ice cream, beer, or wine. Elasticity is less when there is no good substitute (not yet for oil in transport), when consumers spend only a little on it at a time (as for gasoline), and when it is seen as a necessity (as gasoline and home heating oil are) [Besanko and Braeutigam 2005, 44–52].

The elasticity of -0.06 is for the short term, during which consumers don't have time to adjust completely to the price change. In the long run, U.S. price elasticity for oil is -0.45, reflecting opportunity to plan for reduced use of oil.

Another consideration that the Outstanding papers do not handle (and it would be very difficult to do so) is that the market for oil is not completely subject to supply and demand principles. The market is partly manipulated by OPEC, whose members account for about 40% of production and (if they cooperate) can adjust their output to meet targets for world supply and price. Despite the run-up in oil prices from 2002 (\$20/bbl) through 2005 (over \$50/bbl), OPEC revenues today are far less than in 1980 (\$66/bbl in 2004 dollars). One result—about whose other consequences one can speculate—is that per capita income in Saudi Arabia declined by 70% from 1980 to 1999 [Wikipedia 2005], the worst ever for any nation in history. Oil prices are denominated in U.S. dollars; the decline of the dollar since 2002 would have required a 30% increase in the price of oil just to maintain purchasing power of the producers in other currencies. A key question for producers is how to optimize present value of future revenues from oil—and over how long a time frame. As I explain in the chapter in *For All Practical Purposes*, if economic returns from other investments are expected to be higher, it is more profitable to pump oil now (all of it, if possible) and invest the proceeds (e.g., buy the U.S. economy). On the other hand, if the cost of oil can be expected to rise faster than the returns on other investments, it pays to keep it in the ground as long as possible.

Action?

The U.S. faces no countrywide shortage of water, but over 30 years ago it received a "wake-up" call about oil. America has adjusted to rising energy costs (gas, coal, and wood all go up with oil) by gradually improving efficiencies of industrial production, home heating, and home appliances—but not cars.

Americans feel they have a right to cheap gasoline and are highly averse to increased taxes on it; the American Automobile Association (AAA) demands that such taxes go for highways. The $0.50/gal energy conservation tax recommended in 1992 by presidential candidates Paul Tsongas and Ross Perot helped sink their candidacies, and Bill Clinton settled in 1993 for a $0.04/gal increase.

Last fall, I estimated the cost and economic benefit of putting photovoltaic cells or solar water heating on the roof of our house in Wisconsin. Despite enough sun and some incentives from Wisconsin, neither is a "good investment," and fewer than 10 of each are installed each year in the state. Is economics the only basis for economic decision-making? Are there peculiar economic, political, and particularly religious [Moyers 2005] considerations in U.S. culture that lead us to focus on short-term profits, economies, and pleasures?

But we are not alone in our indifference and in our ambivalence about providing for the future. I am currently living in a building in a Western European country completely dependent on oil imports, where gasoline and electricity cost almost three times as much as in the U.S. This country produces half as much solid waste per person, and uses half as much energy per dollar of GDP, as the U.S. An enormous solar collector field built by farmers south of town is economically viable because the government encourages expansion of non-fossil-fuel electric capacity by requiring electric companies to buy such power at twice what they can sell it at (there is probably a governmental subsidy to the utilities). Yet in our brand-new "energy-efficient" building (it has amazingly good insulation), the hall lights are not on timers, the light bulbs in our apartment are not compact fluorescents but incandescents and halogens (120 to 180 W—I won't go into their short lifetimes), and the environmental organizations on the floor below do not always completely separate waste office paper from trash.

Meanwhile, are Americans still debating the question, from my mother's generation or earlier, and long since definitively settled [Greenfield 2004; Rea et al. 1987]: Should they shut off the light if they leave a room for a few minutes?

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