

You Too Can Be James Bond

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Abstract

We divide the jump into three phases: flying through the air, punching through the stack, and landing on the ground. We construct four models to minimize the number and the cost of boxes.

In the Ideal Mechanical model, we consider the boxes' force on the motorcycle and stunt person as constant. In the Realistic Mechanical model, we focus on how the boxes support the motorcycle and stunt person, which includes three phases: elastic deformation, plastic deformation, and crush-down deformation. However, in the Ideal Air Box model, the internal air pressure of each box can't be ignored. As a matter of fact, the boxes are unsealed, so we amend the Ideal Air Box model to develop a Realistic Air Box model. We discuss the strengths and weaknesses of each model.

We define a metric U , which is a function of the cost and the number of boxes. By mathematical programming, we calculate the size and the number of the boxes. In normal conditions, we assume the safe speed is 5.42 m/s. For a total weight of stunt person and motorcycle of 187 kg, we need 196 boxes of size 0.7 m \times 0.7 m \times 0.5 m. We analyze the accuracy and sensitivity of the result to such factors as the total weight, the contact area, and the velocity. We also offer some important suggestions on how to pile up the boxes and how to change the shape of the boxes.

Assumptions and Analysis

About Boxes and the Pile of Boxes

- All the boxes are the same size. The ratio of length to width has little effect on the compression strength of a cardboard box; so to simplify the problem, we assume a square cross section.

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Table 1.
Variables, parameters, and physical constants.

Notation	Description	Units
Box		
<i>h</i>	height of box	m
<i>r</i>	length of side of box	m
<i>Z</i>	perimeter around top of box	m
<i>A</i> ₀	total surface area of box	m ²
<i>P</i>	pressure in box (cylinder)	Pa
<i>P</i> _t	pressure in box at time <i>t</i> when it collapses	Pa
<i>k</i>	pressure in box at time <i>t</i> , in atmospheres	atm
σ	rate of air leaking from box	m ³ /s
σ_i	rate of air leaking from box in time interval <i>i</i>	m ³ /s
<i>V</i>	volume of box (cylinder)	m ³
<i>V</i> _{<i>i</i>}	volume of box in time interval <i>i</i>	m ³
Pile		
<i>L</i> _{pile}	length of the pile	m
<i>W</i> _{pile}	width of the pile	m
<i>H</i> _{pile}	height of the pile	m
<i>L</i>	number of layers of boxes in the pile	
Num	total number of boxes	
Cost	cost of boxes	
<i>S</i>	upper surface area of pile	m ²
Jump		
<i>H</i> _{elephant}	average height of the elephant	m
<i>H</i> _{max}	maximum height of the jump	4 m
ν	fraction of <i>H</i> _{max} that a person can reach	
<i>H</i> ₀	height of the ramp	m
θ	angle of the ramp	
<i>v</i> ₀	launch speed	m/s
<i>v</i> _{safe}	safe speed at which to hit the ground	m/s
<i>M</i>	mass of motor plus stunt person	kg
Kellicut formula		
<i>P</i>	compressive strength of the box	
<i>P</i> _x	comprehensive annular compressive strength of the paper	
<i>dx</i> ₂	corrugation constant	
<i>Z</i>	circumference of the top surface of the box	m
<i>J</i>	box shape coefficient	
<i>F</i> ₀	maximum supporting force from the box	N
<i>F</i>	buffering force of the box	N
<i>b</i>	constant concerning the properties of paper	
Other		
<i>x, z</i>	distance that the cylinder is compressed	m
<i>x</i> _t	cylinder displacement when box collapses	m
<i>z</i> _m	compression distance at which box collapses	m
<i>dt</i>	interval of time	s
<i>W</i>	work done	J
<i>D</i> _s , <i>k</i> ₁ , <i>k</i> ₂	quantities related to cost	
λ_h, λ_c	weight factors	
<i>T, U</i>	functions to be optimized	
Constants		
<i>g</i>	acceleration due to gravity, at Earth's surface	m/s ²
<i>P</i> ₀	standard atmospheric pressure at Earth's surface	Pa

- After the box has been crushed down to some extent, we ignore the supporting force that it can still supply.
- Considering the practical production and transport limitations, the size of the box should not be too large.
- The boxes are piled together in the shape of a rectangular solid.
- When the motorcycle impacts one layer, it has little effect on the next layer. The layer below is considered to be rigid flat (its displacement is ignored).
- We ignore the weight of the boxes; they are much lighter than the person plus motorcycle.

About the Stunt Person and the Motorcycle

- We ignore the resistance of the air to the horizontal velocity of the person and motorcycle. The friction is so little that it is negligible.
- The stunt person has received professional training, is skilled, and is equipped with anything allowable for protection.
- The average weight of a stunt person is 70 kg.
- We choose a certain type of motorcycle (e.g., Yamaha 2003 TT-R225), which weighs 259 lb [Yamaha Motor Corp. 2003].

About the Elephant

- The elephant keeps calm during the jump.
- We adopt the classic value of 3.5 m for the height of the elephant [PBS Online n.d.].

About the Weather

The weather is fine for filming and jumping, including appropriate temperature and humidity. On a gusty day, the wind might make the person lose balance in the air.

About the Camera

The most attractive moment is when the person is over the elephant and at maximum height H_{\max} . We have to make sure that the boxes do not appear on camera, namely, we need $H_{\text{pile}} \leq \nu H_{\max}$, where the coefficient ν is best determined empirically. In our model, we set $\nu = 0.625$.

About the Ramp for Jumping

The ramp for jumping is a slope at angle θ of length L_{slope} , as determined by the jump height or horizontal distance for landing.

The Development of Models

When the stunt person begins to contact the cardboard boxes or the ground, he or she may suffer great shock. To absorb the momentum, the contact time must be extended.

We divide the whole process into three independent phases, that is:

1. flying through the air,
2. punching through the stack, and
3. landing on the ground.

We find out the maximum height in phase 1, the greatest speed of hitting the ground in phase 3, and how the person plus motorcycle interact with the boxes is based on the results of phases 1 and 3. Phases 1 and 3 are simple and we solve them first.

Flying through the Air

The stunt person leaves the ramp with initial speed v_0 at angle θ to the horizontal at height H_0 (**Figure 1**).

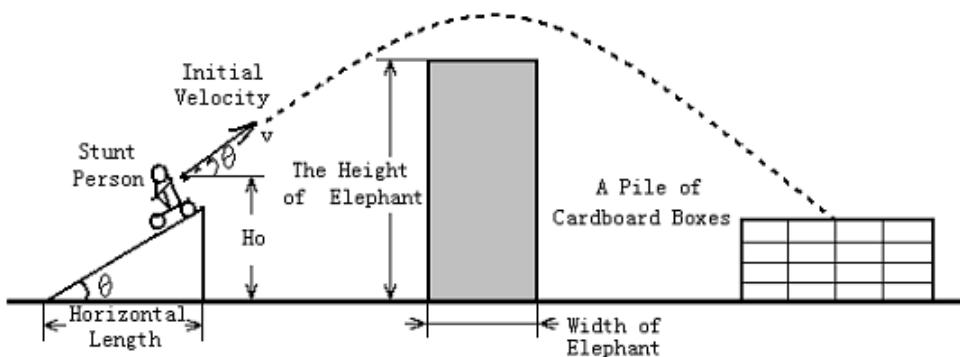


Figure 1. The jump.

In the air, the stunt person on the motorcycle is affected only by the constant acceleration of gravity. Based on Newton's Second Law, we have

$$\begin{aligned}x(t) &= (v_0 \cos \theta)t, \\y(t) &= (v_0 \sin \theta)t - \frac{1}{2}gt^2,\end{aligned}$$

where $x(t)$ and $y(t)$ are the horizontal and vertical displacements from the launch point after t seconds. The launch speed v_0 and the maximum height H_{\max} are related by

$$v_0 \sin \theta = \sqrt{2g(H_{\max} - H_0)}.$$

For an elephant of height 3.5 m, we take $H_{\max} = 4$ m. For $H_0 = 0.5$ m and $\theta = 30^\circ$, we get $v_0 = \sqrt{2 \cdot 9.8 \cdot 3.5} / 0.5 = 16.6$ m/s. With a 2 m-high box-pile, the stunt person hits the pile with vertical speed 6.3 m/s and horizontal speed 14.3 m/s; the distance between the landing point and elephant is $D = 9.2$ m.

Would the Landing Be Safe?

To simplify the problem, we ignore the complex process when the person begins to touch the ground. We consider that there is a critical *safe speed* v_{safe} . If the speed hitting the ground is less than or equal to that speed, the person would not be injured. The safe speed is related to the ground surface (hard, grassplot, mud, etc.) and materials used (paper, rubber etc.). Our simulation uses a typical value, $v_{\text{safe}} = 5.42$ m/s.

Is the Pile Area Large Enough?

The height H_{pile} of the pile of boxes is related to the maximum height H_{\max} that the stunt person reaches and also to the vertical speed of hitting the boxes. The greater H_{\max} , the greater H_{pile} is required, with $H_{\text{pile}} = Lh$, where L is the number of the layers of boxes and h is the height of a single box.

Would L_{pile} equal to the length of the person be enough? The answer is no. When accelerating on the ramp, the stunt person can't make the initial jump speed exactly what we calculate. We think that 3–5 times the length of the person is needed.

The stunt person does not leave the ramp aligned exactly along the central axis and does not keep the motorcycle exactly along that axis after hitting the boxes. That there may be some horizontal movement means that W_{pile} should be 2–4 times the length of the person.

In our simulation, we let $L_{\text{pile}} = 6$ m, $W_{\text{pile}} = 4$ m.

Boxes: How to Cushion the Person

Ideal Mechanical Model

Based on our general assumptions, we suppose that while the stunt person is destroying the boxes of the current layer, boxes in lower layers are seldom affected and keep still.

To illustrate the process of collision with just one layer separately, we suppose that the mutual effect of the stunt person and the box-pile is motion with a constant acceleration. During that, the stunt person plus motorcycle is supported by a constant vertical force F —that is, we treat F as the average force during the whole process.

It can be proved that although the stunt person strikes the boxes of different layers at different initial velocities, the work consumed to fall through each box is the same (**Appendix A**). The number of layers L of the box-pile is determined by the formula

$$\text{Work} \times L - mgh \geq \frac{1}{2}mv_0^2 - \frac{1}{2}mv_{\text{safe}}^2,$$

where L is the smallest integer that satisfies this inequality.

Strength and Weaknesses

This model is simple and efficient. However, we have ignored the detailed process and substituted constant work, though in fact the force changes with time.

Realistic Mechanical Model

First we study the empirical deformed load curve of the cushion system, showing the boxes' deformation under a static load (**Figure 2**) [Yan and Yuan 2000].

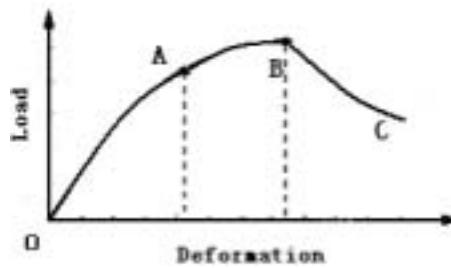


Figure 2. Deformation curve.

The compression process is divided into three phases, as shown in the figure:

- OA phase: Elastic deformation, according to Hooke Law.
- AB phase: Plastic deformation. The compression grows more slowly and reaches the maximum.
- BC phase: Crush-down deformation: After compression reaches the maximum, the rate of deformation starts to fall. The unrecoverable deformation goes on increasing.

According to the *Kellicut formula* [Yan and Yuan 2000; Zhao et al.], the compressive strength of a box is

$$P = P_x \left(\frac{dx_2}{Z/4} \right)^{1/3} Z J,$$

where

P is the compressive strength of the box,

P_x is the comprehensive annular compressive strength of the paper,

dx_2 is the corrugation constant,

Z is the circumference of the top surface, and

J is the box shape coefficient.

For the stunt person plus motorcycle, the maximum supporting force from the box is nearly

$$F_0 = P \frac{s}{(Z/4)^2} = P_x \left(\frac{dx_2}{Z/4} \right)^{2/3} Z J \frac{s}{(Z/4)^2} = b Z^{-5/3} s,$$

where b is a constant concerning the properties of paper.

We assimilate the static loading process to the dynamical process of getting impacted and obtain the following buffering force and its deformation graph (**Figure 3**).

$$F = \begin{cases} \frac{F_0}{a} x, & x \geq a; \\ F_0, & a \leq x \leq b; \\ F_0 \exp\left(\frac{-a(x-b)}{h}\right), & x \leq a. \end{cases}$$

The model describes the mechanical capability of the box and offers an appropriate curve of the relationship between buffering power and deformation. We can measure the energy consumed by the crushing of boxes. One limitation is the Kellicut formula, which applies only to certain kinds of cardboard boxes. Error may also occur in replacing the dynamical process with a static process.

Ideal Air Box Model

We consider the depleting energy consumed by the resistance of air in the process of compression. We divide the process into two phases:

Phase 1: Assume that the cardboard is closed (gas can't escape). The pressure in the box rises from standard atmospheric pressure P_0 to $P_t = kP_0$ (k atmospheres) at time t when the box ruptures. We consider that the impact is so

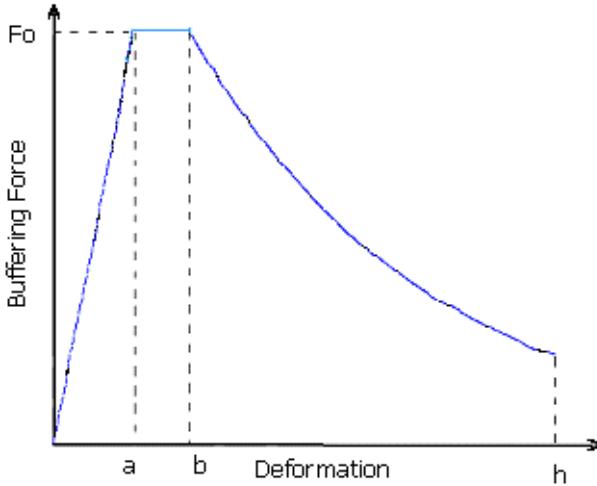


Figure 3. Force-deformation figure.

quick that the gas in the cardboard doesn't exchange energy with environment. So we can just deem it an adiabatic compression process. Using the First Law of Thermodynamics, we get

$$PV^{1.4} = \text{constant.} \quad (1)$$

The proof is in **Appendix B**.

Phase 2: Under the effect of the impact and internal air pressure, cracks appear in the wall of the cardboard box. The internal air mixes with the air outside, quickly falling to standard atmospheric pressure P_0 . We assume that the cardboard box is a rigid cylinder and the compressive face (top surface) of the box is a piston (**Figure 4**).

We calculate the internal pressure when the piston shifts downward a distance x from height h . Let s be the area of the top of the cylinder. According to (1), we have

$$\begin{aligned} P_0 \cdot [hs]^{1.4} &= P \cdot [(h-x)s]^{1.4}, \\ P &= P_0 \left(\frac{h}{h-x} \right)^{1.4}. \end{aligned} \quad (2)$$

The graph of P is shown in **Figure 5**.

Consider Phase 1. According to (2), we have

$$P_t = kP_0 = P_0 \left(\frac{h}{h-x} \right)^{1.4}.$$

We solve for the displacement at time t :

$$x_t = h \left(1 - k^{-5/7} \right).$$

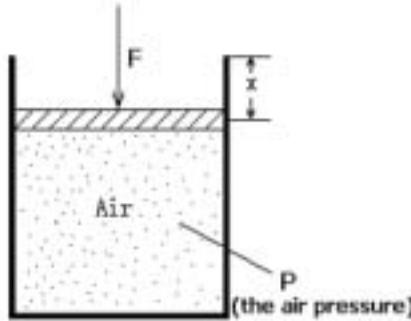


Figure 4. The cylinder model.

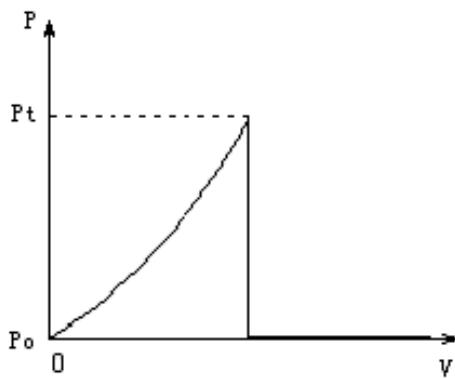


Figure 5. Internal pressure as a function of the volume of compressed air.

The compression work done in Phase 1 is

$$\begin{aligned}
 W_1 &= \int_0^{x_t} P dV = s \int_0^{h(1-k^{-5/7})} P_0 \left[\left(\frac{h}{h-x} \right)^{1.4} - 1 \right] dx \\
 &= P_0 s h \left(\frac{5}{2} k^{2/7} - \frac{7}{2} + k^{-5/7} \right). \tag{3}
 \end{aligned}$$

Consider Phase 2. The compression work of Phase 2 is

$$W_2 = \int (P_0 - P) dV = 0,$$

so the total work done is given by (3).

In this model, first we get the curve of internal pressure and deformation. Based on the graph, we calculate the energy consumed by the resistance of gas. However, considering the box as a rigid cylinder may be inaccurate. Another weakness is the airtightness of the box; actually, the internal gas leaks during the whole process of compression.

Realistic Air Box Model

The height of the box is less than 1 m, so the speed of the motorcycle and stunt person changes little when passing through one box height's distance; hence, we think of it as constant during compression of a single box. The effect of the air in the crushing process is just like the air in a cylinder with a hole to leak air. In **Figure 6**, we have

s is the area of the top of the box (top of the cylinder),

h is the height of the box (cylinder),

σ is the air-leaking rate,

z is the crushing distance from the top of the box (cylinder), and

v is the speed of the motorcycle and stunt person.

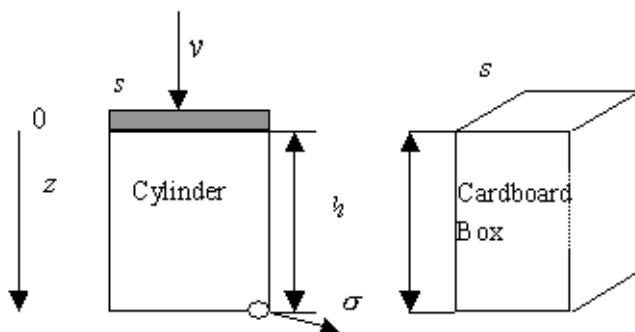


Figure 6. The unsealed cylinder model.

In the crushing process, the cardboard box's deformation becomes larger and larger. As a result, the air leaks more quickly. Generally, when a cardboard box is pressed to about half of its original height, the effect of air-leaking is so prominent that we cannot ignore it.

We assume that the air-leaking rate changes with the crushing distance z via

$$\sigma(z) = 2e^{4z/h}s,$$

for $z \in (0, h)$. Let $x = z/h$ for $x \in (0, 1)$.

The goal is to calculate the total work that the air does to the motorcycle and stunt person. It is necessary to calculate the pressure of the air in a box while the crushing is occurring. We divide the crushing process into N time periods; in each time period, the pressure of the air can be calculated according to the universal gas law $PV = nRT$. We assume that temperature is constant, so PV is constant.

In the first time period, we have

$$P_0 V_0 = P_0 \sigma_0 dt + (V_0 - sv dt) P_1,$$

$$P_1 = \frac{P_0(V_0 - \sigma_0 dt)}{V_0 - sv dt}.$$

In general, we get

$$P_i = \frac{P_{i-1}(V_{i-1} - \sigma_{i-1} dt)}{V_{i-1} - sv dt},$$

where

$$dt = \frac{h}{vN},$$

σ_i is the air-leaking rate in time period i , and

V_i is the volume of the box in time period i .

So in every time period, the pressure of air in the cylinder can be calculated recursively:

$$P_i = \frac{(V_{i-1} - \sigma_{i-1} dt)}{V_{i-1} - sv dt} \cdot \frac{(V_{i-2} - \sigma_{i-2} dt)}{V_{i-2} - sv dt} \cdots \frac{(V_0 - \sigma_0 dt)}{V_0 - sv dt}.$$

Let $h = 0.5$ m and $N = 1000$. The air pressure in the box increases from P_0 (standard atmospheric pressure) to a maximum of $1.66P_0$ at about one-third of the box's height and drops steeply to P_0 at about half the box's height (**Figure 7**).

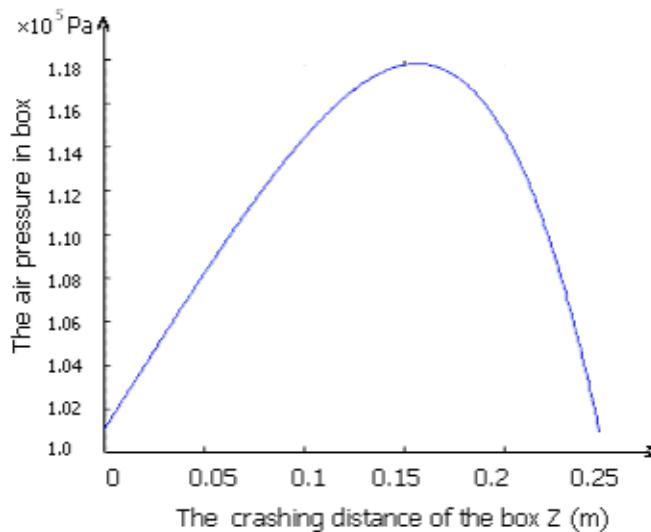


Figure 7. Air pressure in the box as a function of distance crushed.

The energy that the air in box does to the motorcycle and stunt person is

$$W = - \int f dz = - \int_0^{z_m} P(z) s dz,$$

where z_m is the distance at which the air pressure equals P_0 and s is the area of the top of the box. Let $s = 0.5 \text{ m}^2$; then the energy that the air in box does to motorcycle and stunt person is $W = -1322 \text{ J}$.

The consideration of the box's air-leaking effect makes this model more realistic, particularly since an exponential function is used to describe the air-leaking effect. With numerical methods, this model is to solve. However,

there are some parameters to identify, and the assumption of constant speed for motorcycle plus stunt person is not correct.

Use the Model

We need to keep the height of the box-pile lower than the elephant so that the action can be filmed without the box-pile being seen. The height of the pile is the number of layers L times the height h of a single box:

$$H_{\text{pile}} = Lh.$$

The total number of boxes used is

$$\text{Num} = \frac{LS}{r^2},$$

where r is the width of a box (**Figure 8**) and S is the upper surface area of the box-pile.

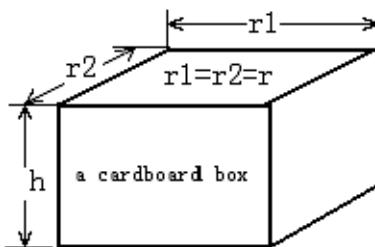


Figure 8. Dimensions of a box.

The cost is

$$\text{Cost} = \text{Num}D_s,$$

where D_s is the unit price, which depends on the materials, transportation cost and other factors. We set $D_s = k_1A + k_2$, where

k_1 is the fabrication cost per square meter.

k_2 includes the average cost for transportation and some other factors, and

A_0 is the total surface area of a single box: $A_0 = 4rh + 2r^2$.

Then the cost is

$$\text{Cost} = \text{Num}D_s = k_2\text{Num} + k_1A_0\text{Num} = k_2\text{Num} + k_1A,$$

where A is the total surface area of all of the Num boxes.

A metric T that takes into account both minimizing cost and the total box-pile height is

$$T = \lambda_h H_{\text{pile}} + \lambda_c \text{Cost},$$

where the λ s are weight factors. So we must solve the following optimization problem:

$$\begin{aligned} \text{minimize} \quad & T(r, h, L) = \lambda h H_{\text{pile}} + \lambda_c \text{Cost} \\ \text{subject to} \quad & \sum_{i=1}^L W_i - Lmgh \geq \frac{1}{2}mv_0^2 - \frac{1}{2}v_{\text{safe}}^2, \end{aligned} \quad (4)$$

$$v_0^2 = 2g(H_{\text{max}} - Lh), \quad (5)$$

where W_i is the energy that the motorcycle and stunt person lose passing through layer i . The inequality (4) makes the piles hard and numerous enough to slow the falling stunt person to the safe speed, while equation (5) defines initial speed in terms of the maximum height of the jump and the height of the pile.

Results and Analysis

Although the Ideal Mechanical model is practical and easy to calculate, it requires that many parameters be determined by experiments. The Realistic Mechanical model gives mechanical support to the Air Box models. The Realistic Air Box model develops from the Ideal Air Box model by considering air-leaking; its results may be more credible, so we give results for the Realistic Air Box Model instead of for the others.

Results of the Unsealed Air Box Model

Case A

To give some typical results and analyze stability, we assume that the maximum height of the box-pile is $H_{\text{pile}} = 2.5$ m and the safe speed is $v_{\text{safe}} = 5.42$ m/s. Considering that the height must be lower than the given maximum height, we ignore its effect in T . Let $k = k_2/k_1$; then the utility function can be written as

$$U = \frac{\text{Cost}}{k_1} = k \text{ Num} + A.$$

We use enumeration to find the best solution that minimizes U (that is, minimizes total cost). The step size for r and h in our enumeration is 0.1 m. Different weights of stunt person plus motorcycle give different solutions; details are in **Table 2**. The solution of r , h , and L changes little with k .

Case B

To find out how contact area influences the solution, we fix $M = 187$ kg and $v_{\text{safe}} = 5.42$ m/s and vary the area s of the top of a box. The solution does not change with k , so we let k equal 0 (**Table 3**).

Table 2.
Results for Case A.

M (kg)	k	Solution			A (m ²)	U
		r (m)	h (m)	L		
150	0	0.4	0.8	1	240	240
	5	"	"	"	"	990
	∞	"	"	3	437	∞
187	0	0.7	0.5	4	466	466
	5	"	"	"	"	1447
	∞	"	"	"	"	∞
200	0	0.5	0.6	3	490	490
	5	0.6	0.5	4	515	1855
	∞	"	"	"	"	∞
230	0	0.4	0.8	3	720	720
	5	"	"	"	"	29700
	∞	"	"	""	720	∞
250	any	No solution				

Table 3.
Results for Case B, with $M = 187$ kg and $v_{\text{safe}} = 5.42$ m/s.

s (m)	r (m)	h (m)	L	A (m ²)	U
1.0	0.8	0.6	1	122	122
0.8	"	0.8	"	146	146
0.6	0.7	0.7	2	288	288
0.5	"	0.5	4	466	466
0.4	0.4	0.7	"	864	864
0.2	No solution				

Case C

To find out how the safe speed influences the solution, we fix $s = 0.5$ m² and $M = 187$ kg. Details can be seen in **Table 4**.

Analysis of the Results

The parameter k influences the solution only slightly; that is, our model is stable, since we can easily get an optimum solution without paying attention to having an accurate value of k , whose value in fact is hard to obtain.

Lightening the weight remarkably decreases the value of U (**Table 2**). So the stunt coordinator should try to reduce the total weight.

Increasing the contact area to some extent reduces cost (**Table 3**). So the stunt

Table 4.Results for Case C, with $M = 187 \text{ kg}$ and $s = 0.5 \text{ m}^2$.

v_{safe} (m/s)	r (m)	h (m)	L (m)	A (m^2)	U
4.64		No solution			
4.85	0.4	0.6	3	576	576
5.05	"	"	"	"	"
5.24	"	0.7	2	432	432
5.42	0.7	0.5	4	466	466

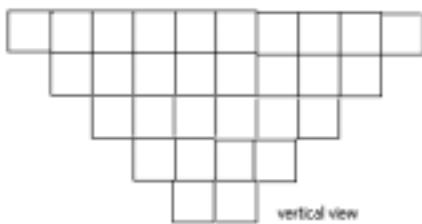
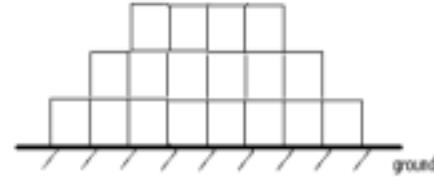
person should try to touch the boxes with both wheels, to enlarger the contact area. The size of wheels also influences the contact area.

The safe speed has a significant effect on the cost (Table 4). To decrease cost, increase the safe speed by selecting a soft landing surface (e.g., grass or sand) instead of cement.

Further Discussion

How to Pile the Cardboard Boxes

The farther the flight in the air, the longer the box-pile needs to be. So the best shape of the underside of the stack may be like a sector, whose further side would be a little wider than the nearer one (**Figure 9**). Another idea is to pile the boxes like a hill whose central part is higher than the other parts, since the contact area between the stunt person plus motorcycle and the box is much smaller than the area of the pile. The cushioning effect is from the contacting area, so we are more interested in the height of the boxes of the contact part of the pile. If we can determine accurately where the landing will be, we can save lots of boxes (**Figure 10**).

**Figure 9.** Vertical view of the box-pile.**Figure 10.** Lateral view of the box-pile.

Tying together boxes in the same layer may be helpful, because doing so supplies a larger horizontal effect and make the layer effect obvious. Another advantage is to prevent the stunt person from falling into a gap between boxes.

Change the Shape of the Boxes

To simplify the problem, we assumed that the cross section is square. Yan and Yuan [2000] give the relationship between the compression strength of the box and the ratio of length to width (**Figure 11**). We could change that ratio for best effect.

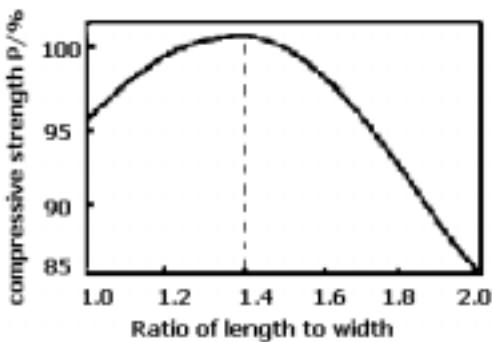


Figure 11. Compression strength as a function of ratio of length to width of the box.

Appendix A: Same Work for Each Box

We analyze the process between boxes of two adjacent layers (**Figure A1**) and show that two boxes do the same amount of work: $W_1 = W_2$, where

$$W_1 = \frac{1}{2}mV_1^2 - \frac{1}{2}mV_0^2 + mgh,$$

$$W_2 = \frac{1}{2}mV_2^2 - \frac{1}{2}mV_1^2 + mgh.$$

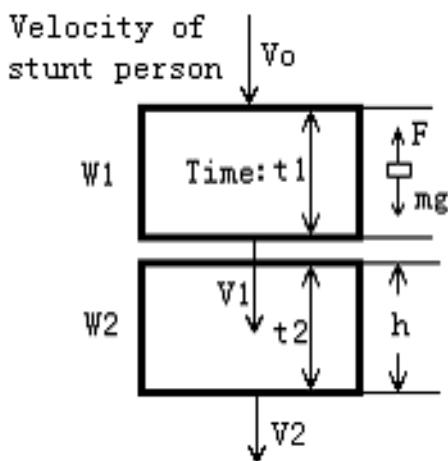


Figure A1. Diagram of two boxes.

Using the average velocity, and since both boxes have the same height h , we have

$$h = \frac{1}{2}(V_0 + V_1)t_1 = \frac{1}{2}(V_1 + V_2)t_2,$$

so

$$\frac{t_1}{t_2} = \frac{V_1 + V_2}{V_0 + V_1}.$$

Use the Newton's Second Law, we have that $F/m = a_1 = a_2$ and

$$V_1 = V_0 + a_1 t_1, \quad V_2 = V_1 + a_2 t_2,$$

so that

$$\frac{t_1}{t_2} = \frac{V_1 - V_2}{V_0 - V_1}.$$

Setting the two expressions for t_1/t_2 equal and cross-multiplying gives

$$V_1^2 - V_0^2 = V_2^2 - V_1^2.$$

Appendix B: $PV^{1.4}$ Is Constant

We assume that the impact is so quick that the gas in the cardboard doesn't exchange energy with environment, so we have an adiabatic compression process. According to First Law of Thermodynamics, we get

$$\Delta E + W_Q = 0,$$

where ΔE is the increment of the gas's internal energy and W_Q is the work done by the adiabatic gas. We also then have

$$dE + P dV = 0,$$

where P is the pressure of the gas and V is the volume. We put them into the Ideal Gas Internal Energy Formula:

$$E = \frac{M}{\mu} \cdot \frac{i}{2} RT \quad \text{and} \quad C_V = \frac{i}{2} R,$$

where

M is the mass of the gas,

i is the number of degrees of freedom of the gas molecule,

μ is the molar mass of the gas.

R is the molar gas constant,

T is the temperature of the gas, and

C_V is the constant volume molar heat capacity.

Then we have:

$$\frac{M}{\mu} C_\nu dT + P dV = 0.$$

We differentiate the ideal-gas state equation

$$PV = \frac{MR}{\mu} T$$

getting

$$P dV + V dP = \frac{MR dT}{\mu}.$$

We eliminate dT from the last two equations to get

$$\frac{dP}{P} + \frac{C_\nu + R}{C_\nu} \cdot \frac{dV}{V} = 0.$$

Integrating, we get $PV^r = \text{constant}$, where

$$r = \frac{C_\nu + R}{C_\nu} = \frac{2+i}{i}.$$

The main composition of the air is nitrogen and oxygen, so $i = 5$ and $r = 1.4$, so $PV^{1.4} = \text{constant}$.

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