Boarding at the Speed of Flight

Michael Bauer
Kshipra Bhawalkar
Matthew Edwards
Duke University
Durham, NC

Advisor: Anne Catlla

Summary

We seek an efficient method for boarding a commercial airplane that accommodates unpredictable human behavior, with a framework that allows us to compare and contrast different procedures. Passenger dependencies, bottlenecks, and the rate of interferences are critical factors for airplane boarding time.

Boarding without seating assignments is fastest, since each person is in the correct order for their flexible seat choice; it removes all interferences and makes the boarding time depend solely on the entrance rate of passengers into the plane. Hoping to emulate the performance of this method, which we call "random greedy," we design a new algorithm to model its average seating order: the parabola boarding scheme, which breaks the plane into parabola-shaped zones.

We use a discrete-time simulation engine to model current boarding schemes as well as the parabola and random greedy algorithms. The zone-boarding schemes outside-in, pyramid, and parabola are almost identical in performance; back-to-front and alternating rows are significantly worse.

We examine the effects of scheme-independent parameters on boarding time. Ensuring a fast rate of people entering the plane and fast luggage stowage are both critical; an airline could reduce boarding time by improving either of these regardless of boarding scheme.

By varying both the rate of people entering the plane and time to stow luggage, we find a correlation between average boarding time and the difference between best and worst times. The random greedy algorithm has the smallest difference; outside-in, pyramid, and parabola have equal differences. Faster boarding algorithms are also more reliable and allow for tighter scheduling.

The UMAP Journal 28 (3) (2007) 333–352.

The best boarding algorithms do not have assigned seating. If, however, an airline feels that assigned seating is mandatory for customer satisfaction, then any of outside-in, pyramid, or parabola will result in a consistently fast boarding time with minimum deviation from average times and will be a marked improvement over the traditional back-to-front boarding method.

Introduction

Short of a single minor detail, the airplane boarding problem would be easily solved using a very simple algorithm. Given his performance in the the film *Snakes on a Plane* [2007], we know Samuel L. Jackson is an optimal de-boarder of snakes from planes. Assuming that he maintains equal effectiveness with people, simply invert his role and you have an optimal passenger boarding algorithm. We model people as snakes, play the film in reverse, and determine the boarding time!

Conventions

Terminology

- **Passenger**: A person traveling on the plane who is not part of the crew.
- **Boarding Scheme**: An assignments of zones or groups according to which passengers board the plane.
- **Interference**: An event in which a passenger cannot progress towards their seat because of another passenger blocking the way.

Variables

- C: the number of columns in the plane, which is also the number of seats in a row.
- R: the number of rows in the plane. For the most part, we ignore or treat in a different manner distinctions between classes of seating.
- B: the time for a person to stow luggage, assumed to be constant in our preliminary analysis but allowed to vary in the simulation.
- v: the walking speed of passengers, constant.
- s: the time for an already-seated passenger to get up and get out into the aisle to let another passenger pass, constant for our preliminary analysis but variable in the simulation.

- λ: the rate at which people enter the plane through the main door, constant since any variability among passengers is mitigated by walking down the jet-bridge to the plane.

Assumptions

- Passengers with physical limitations, families with infants, and passengers advanced in years board the plane before other passengers. The time for this "pre-boarding" is a constant overhead that airlines cannot avoid.
- First-class passengers board separately.
- All passengers during general boarding walk at the same speed, limited more by the environment (aisle size, people in the way) than by physical capacity. Passengers board and walk independently, that is, no groups wait for one another. Family members are assigned seats next to one another.
- We confine our analysis to the interior of the plane, ignoring terminal effects beyond requiring that gate agents supply passengers at a certain rate. If the plane cannot "process" passengers quickly enough, they queue in the jet-bridge. The interior of the plane is regular and symmetric, with all rows of equal size.
- All planes fly at maximum capacity and all passengers are present when their zone is called, which they follow obediently. Empty seats only speed up the process. Late or noncompliant passengers can be accounted for by adding a time overhead.
- We confine our recommendations and analysis to methods that do not overly alter the status quo. We analyze ticketless methods for comparison but seek the best boarding method for ticketed contexts. We further consider only zone-based boarding calls, assuming that is logistically impossible to require passengers to line up in any verifiable order.

Motivation and Subproblems

What if all variables involving passenger boarding could be controlled? How would we schedule the boarding optimally? We would use a modified version of the outside-to-inside method. We first order the passengers into groups of equal size R by the following set of criteria in descending order of priority:

- Individual location in row: Window has highest priority, aisle has least.
- Side of plane: left side of plane has priority over right side.

- Row number: Rows in back have priority over those in front.

The following algorithm then boards the plane optimally:

> Each group proceeds down the aisle until each person reaches their row (since people are in order, they all reach their rows simultaneously). They step into the first seat in their row and begin stowing their luggage. During this time, the next group commences down the aisle. The only time when a group might stall in the aisle is if $B > 2R/v$, in which case every other group must wait in the aisle for $B - 2R/v$ seconds. (This accounts for the additional term in the second part of **(1)** below.)

The ideal boarding algorithm places a lower bound on the time to board an airplane:

$$\text{Ideal boarding time} = \begin{cases} C\,\dfrac{R}{v} + B, & B \leq 2\,\dfrac{R}{v}; \\[2ex] C\,\dfrac{R}{v} + B + \left(d\frac{C}{2} - 1\right)(B - 2R), & B \geq 2\,\dfrac{R}{v}. \end{cases} \qquad \textbf{(1)}$$

Key points about the operation of the algorithm are:

- The main aisle is continuously busy unless passengers have to wait for people in their row to finish stowing luggage.
- Passengers are "pipelined" to minimize the blocking effect of stowing luggage.

Imperfect ordering forces us to consider the following:

- **Random orderings**: How out of order are people and how does this impact other dependencies in boarding?
- **Flow rates**: How long does it take people to enter the plane and walk down the aisle without blocking it?
- **Luggage**: How large is the luggage and how long does it take to stow?

All of these introduce dependencies into the system. Randomness prevents us from determining the occurrence or duration of these dependencies and therefore forces us to design boarding schemes capable of tolerating their effects.

One way to remove dependencies is to force people to continue moving as far back in the plane and over in a row as long as they don't get blocked. We return to this random greedy approach later, since it represents the intuitive motivation for our best airplane boarding scheme.

Predicting Bottlenecks with Queuing Theory

One model for airplane boarding is a stochastic process, a collection of random variables that must take on a value at every state, with states indexed by a parameter (in our case, time) [Trivedi 2002]. Queuing theory deals with analyzing how the random variables in stochastic processes interact. Traditionally, queuing theory is used to determine the average throughput of a system. While the plane boarding problem does not possess a quantity directly corresponding to throughput, we gain a better understanding of bottlenecks and their effects by using this approach.

We partition the plane into a series of queues. We place a "processor" at each row. This processor corresponds to a passenger making a decision at this point either to keep walking or to stop and enter their row. Each processor has a queue that stores passengers. A queue has size 1 and if full will stop the processor feeding it; this would represent people backing up if someone stops in the aisle. A diagram for this layout is in **Figure 1**.

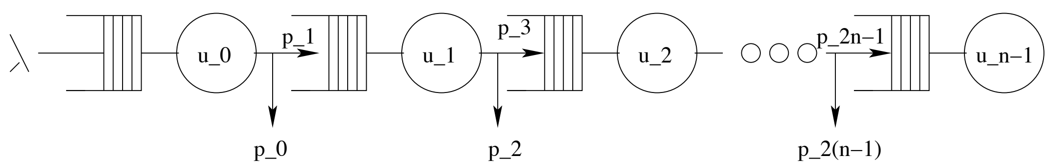

Figure 1. A queuing-theory model of airplane boarding.

In **Figure 1**, u_k is the processing rate, the average walking speed of passengers. Each p_k represents some probability with which passengers divert into the their rows or continue walking in the aisle. In some cases, people will take longer to get into their rows, depending upon how long it takes to stow their luggage. The processor associated with that row then takes longer to process that job, causing the flow of people through the aisle to stall. Downfalls of this model are that all passengers are eventually supposed to leave the system (i.e., get into their seats) and it doesn't accurately reflect that each row should only ever hold C passengers; so we do not use the queuing theory model as our main model. However, it gives us useful knowledge concerning bottlenecks in the aisle.

To convert the open system shown of **Figure 1** to a closed-form system that can be solved by queuing theory, we use Jackson's Theorem (no, not Samuel L. Jackson again!) [1957]:

> *An open system can be represented by a feedback loop if the rate of processing at each processor is augmented proportional to the rate of flow prior to that processor.*

In our application, all rates depend on p_0 because the rate into the next queue depends on the output of the previous processor, so terms cancel under our assumptions for the value of any p_j. Using this theorem, we can redraw our airplane model as in **Figure 2**. The closed form allows us to determine the

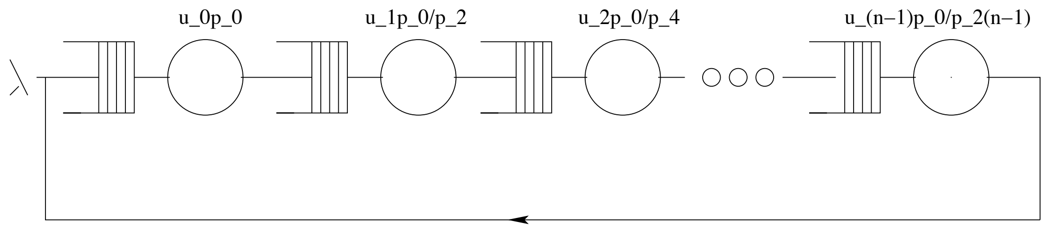

Figure 2. A closed-form queuing model of airplane boarding.

probability of having a given number of passengers at a specific node at a given time. We let $\rho(k_0, k_1, \ldots, k_{n-1})$ be the probability of k_i people in position i in the aisle. Conceptually, this implies that we have an n-dimensional state space, since the number of passengers at each node is potentially different.

We now write down conditions that ρ must satisfy and use these to find an equation for ρ.

The first condition is that ρ must "conserve" passengers by maintaining flow of passengers into and out of each state in the state-space. This ensures that passengers are never "lost" in the system:

$$\left(\lambda + \sum_{j=0}^{n-1} \mu_j\right) \rho(k_0, k_1, \ldots, k_{n-1}) = \lambda\rho(k_0 - 1, k_1, \ldots, k_{n-1}) + \mu_{n-1}\rho(k_0, \ldots, k_{n-2}, k_{n-1} + 1) + \sum_{j=0}^{n-2} \mu_j \rho(\ldots, k_j + 1, k_{j+1} - 1, \ldots).$$

We also need to define the boundary states of the state space, which must ensure that no state can have a negative number of passengers at any processor:

$$\begin{aligned}
(\mu_0 + \lambda)\rho(k_0, 0, 0, \ldots, 0) &= \mu_1\rho(k_0, 1, 0, \ldots, 0) + \\
&\quad \lambda\rho(k_0 - 1, 0, 0, \ldots, 0), \qquad k_0 > 0; \\
(\mu_{n-1} + \lambda)\rho(0, 0, \ldots, k_{n-1}) &= \mu_{n-2}\rho(0, 0, \ldots, 1, k_{n-1} - 1) + \\
&\quad \mu_{n-1}\rho(0, 0, \ldots, k_{n-1} + 1), \qquad k_{n-1} > 0; \\
\lambda\rho(0, 0, \ldots, 0) &= \mu_0\rho(1, 0, \ldots, 0).
\end{aligned}$$

Lastly, all probabilities must sum to 1:

$$\sum_{k_{n-1} \geq 0} \sum_{k_{n-2} \geq 0} \cdots \sum_{k_0 \geq 0} \rho(k_0, k_1, \ldots, k_{n-1}) = 1.$$

We can then extend the solution presented in Trivedi [2002] from a two-processor chain and see that that ρ has the form

$$\rho(k_0, k_1, \ldots, k_{n-1}) = \prod_{j=0}^{n-1} (1 - \rho_j)\rho_j^{k_j},$$

where each ρ_j takes the form

$$\rho_j = \frac{\lambda}{\mu_j}$$

and μ_j is the rate of processing of the jth processor. From Trivedi, we know that the bottleneck of the system occurs at the processor with the largest ρ_j.

We now consider a random ordering of people entering the plane, in which a passenger turns into a given row with probability $1/n$ or continues walking with probability $(n-1)/n$. We assume that in the original system $u_0 = u_1 = \ldots = u_{n-1}$ and therefore all nodes in the closed system must have a rate of $\rho_j = (n-j)\lambda/u_j$ for all j. This implies that ρ_0 is the largest in the system and is therefore the bottleneck. If we recursively apply this for an airplane with $(n-1)$ rows, we see that the bottleneck will always be the first processor. We can then recognize three important properties of airline boarding:

- The critical bottleneck for boarding is always the first row in the plane.
- The criticality of the main bottleneck is linearly proportional to the number of rows in the plane.
- The farther back a row, the less it contributes to bottlenecking.

Effects of Row and Column Interferences

Boarding gets more complicated when people board out of order, which leads to row interference and column interferences that hold up traffic. Here we use probabilistic estimation to assess zone configurations that are affected least by shuffling passengers in a given zone. For the sake of simplicity, we analyze a plane with 6 seats per row, but the analysis generalizes. First, we develop some lemmas, based on assuming that passengers board in random order.

Row interferences occur when a passenger sitting in an aisle or middle seat has to get up to let in the person who has the window seat or the middle seat. We calculate the expected number of times that a passenger has to get up if the passengers sitting in a row of k seats board in random order.

Lemma 1 *The expected number of interferences in a row of k people is $k(k-1)/4$.*

In particular, the expected number of interferences for 3 seats is 3/2.

When a passenger stands in the aisle to stow luggage, the passengers behind must wait. We assume that a passenger can proceed to the right row and stow luggage as long as the passenger is not blocked by another passenger stowing luggage. The lemma below finds the longest sequence of passengers who can be stowing their luggage at once. If the rows are numbered in increasing order from the back of the plane to the front, the problem can be reduced to finding a largest increasing subsequence of row assignments among the passengers, since these passengers then can proceed to their seats and stow their bags.

Lemma 2 (Kiwi 2006) *The expected length of longest increasing subsequence in a permutation of* $\{1, 2, \ldots, k\}$ *is (asymptotically) of size* $2\sqrt{k}$.

The proof is quite involved and we do not discuss it. The lemma tells us that if k passengers sitting in different rows board the plane at once, then $2\sqrt{k}$ of them can proceed to their seats and stow luggage without encountering an interference. If we have m people spread over k rows, then it will take them $\lfloor m/(2\sqrt{k})\rfloor B$ time to stow their luggage.

We use these lemmas to estimate the boarding time for a group of passengers to be seated in different configurations.

Configuration 1: Dense Distribution over Rows

The zone is composed of m passengers spread densely over k rows. For 6 passengers in a row, dense means that all 6 are in the same zone. The expected number of row interferences for this configuration is $\frac{3}{2} \cdot 2k$. The boarding time for this zone is approximately

$$T = \left\lfloor \frac{m}{2\sqrt{k}} \right\rfloor B + 3ks,$$

where B is bag stowage time and s is the time for a passenger to get out of their seat to allow a fellow passenger to pass and then sit down again. The time for people to walk down the aisle can be ignored, since in this case it is overshadowed by bag stowage and reseating.

Configuration 2: Sparse Distribution over Rows

The zone is composed of m passengers sparsely distributed over k rows, meaning at most two passengers in a row, mostly on different sides of the aisle. Having a sparse distribution totally eliminates the effect of reseating time but results in walking time becoming the critical factor. The walking time for this configuration is roughly kv, where v is the time to walk from one row to next. Thus, total time for boarding this group is

$$T = \left\lfloor \frac{m}{2\sqrt{k}} \right\rfloor B + kv.$$

Boarding Schemes

Currently-used boarding systems include:

Back-to-front: (Air Canada, Alaska, American, British Airways, Continental, Frontier, Midwest, Spirit, Virgin Atlantic [Finney 2006]) The most widely-used boarding scheme. Passengers are divided into zones and board at the front door in a back-to-front order.

Outside-in: (Delta and United Airlines) Passengers are boarded windows first, followed by middle seats, with aisle seats boarding last.

Reverse-pyramid: (US Airways on some routes) This scheme boards people in a V-like manner, with rear middle and windows boarding first, followed by rear aisles and front aisle.

No assigned seats: (Southwest Airlines) Ostensibly the fastest boarding scheme. Passengers are not assigned seats and may sit anywhere. This scheme has not been widely copied, since it does not lead to high customer satisfaction and is often likened to a cattle car.

Figure 3 offers a visual comparison. We color seats according to the order in which they fill, with red (dark) earlier and green (light) later. The entry door is at the top and the bottom is the back row. We include an ordering named "Parabolas" that we introduce later.

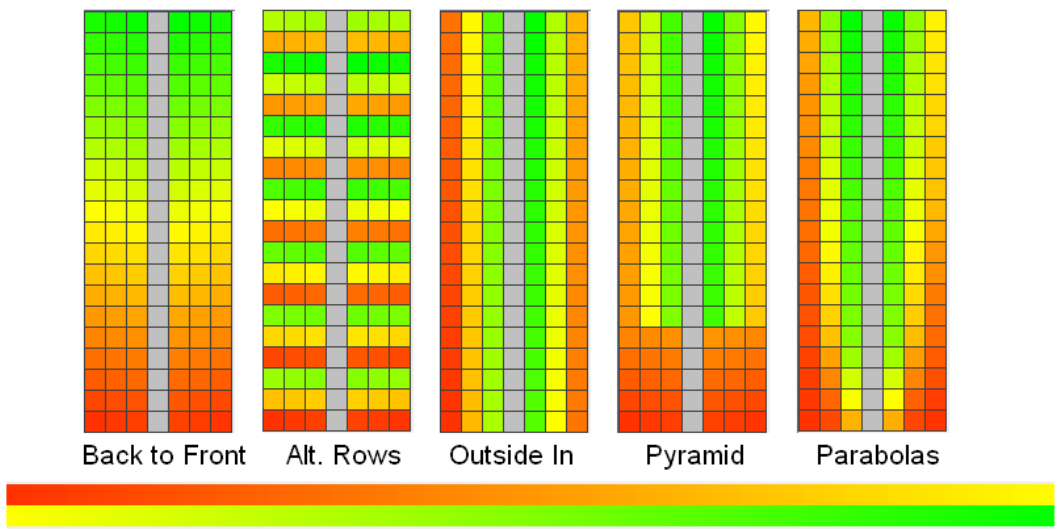

Figure 3. Seat ordering schemes.

Simulation Design and Details

We produced a comprehensive flexible boarding simulator that we use to compare boarding algorithms and the effects of various situations. Our simulation techniques were inspired by stochastic Petri nets, finite time-step simulations, and cellular automata [Marelli et al. 1998].

Process

Our simulation model runs through time in small intervals. At each interval, it moves each participant in the simulation according to rules defined by the input parameters. Certain events take extra time and create blocks for other participants in the model. For example, a passenger stowing luggage blocks the aisle for a certain amount of time.

Plane

The plane is a variably-sized rectangular grid of seats with a single aisle for passenger movement in the center of the columns and a single door for entry at the beginning of the aisle. The space between rows (*pitch* in industry terms) and between columns is adjustable.

Behavior Modeling

Passengers can board with either assigned or unassigned seats.

- **Assigned seats:** Passengers move to their seats as fast as walking speed allows, waiting as necessary for obstacles to clear. They make no mistakes in moving to their assigned seats.
- **Unassigned seats:** Passengers walk as far back as possible before sitting. If the aisle is blocked, they sit in the current row to avoid waiting standing up. When they sit, they are generous and move all the way toward the window to save future passengers' time.

A passenger has an associated seating delay time for moving into their row, which corresponds to the time to stow luggage, wait for already-seated passengers to move out of the way, move in, and get settled. The seating delay rises as more people sit in the row, reflecting decreasing space in the overhead compartment and accompanying longer time to find space for a bag.

When a person stowing luggage blocks the aisle and someone else comes up behind them, there is a certain *pass percentage* representing the chance that the blocked person can pass by and proceed to a seat farther along.

Parameters

The simulation is run with a *passenger input rate*, affected by the gate check-in speed, that is, how fast passengers are processed in the terminal. Passengers have a constant *walking speed* when not blocked.

With assigned seats, passengers are typically called in groups, with each group some approximately contiguous segment of seats. The *group size* is variable, and passengers within each group are randomly ordered. Groups themselves satisfy certain *seating assignment schemes*, for example, ordering groups back to front.

Parameter Estimation

For our simulation trials, we use the following default values and distributions. Estimated values are based on critical thinking; parameters dependent on the plane size will be specified later. Times are in seconds.

- **Walking Speed** = 140 cm / s. This varies based (at least) on the age and gender distribution of the passengers. We used the FAA evacuation simulation requirements that call for a simulated plane's population to be at least 40% female, at least 35% over age 50, and at least 15% both. Our average distribution is balanced male and female with 40% over age 50. The average comfortable walking speeds based on age and gender are from Bohannon [1997].
  Affected by: Passenger demographics, aisle width, ceiling height, number and size of bags per person.

- **Seating Delay** = $U[10, 20] + P_c + P_r$. The seating delay is uniformly distributed and includes the compartment-filling penalty P_c and the row-out-of-order penalty P_r.
  Affected by: Other penalties, plus row spacing, luggage size and number, compartment size and layout, and passenger demographics.

- **Compartment Penalty** = $3p$, where p is the number of people already seated in your row.
  Affected by: Size and layout of overhead compartment, luggage size and number.

- **Pass Rate** = 0.05 (an estimate).
  Affected by: Aisle width, passenger demographics, luggage size and number.

- **Row Out-of-Order Penalty** = $15p$, where p is how many people have to move to let you into your seat.
  Affected by: Row spacing, passenger demographics, aisle width.

- **Entry Delay** = 5.0 (estimate). The time between successive passengers entering the plane.
  Affected by: Check-in procedure, flight attendant behavior, luggage size and number, out-of-plane characteristics.

Summary

The simulation model is configurable. We use it to test different strategies and measure the effects of certain changes on the process. We can model

- different types and sizes of planes with varying interior configurations (aisle width, seat spacing, overhead compartment size)

- passengers with and without assigned seats in many arrangements and zone groupings;

- the effects of luggage count, luggage size, compartment size, and stowing speed; and
- the effects of the gate check-in process speed.

Deriving a New Scheme

Random boarding with unassigned seats tends to be fastest [Finney 2006]. Despite this fact, many airlines do not adopt it because it often leads to low customer satisfaction. We derive a new seating method inspired by the seating patterns of passengers in a random assignment-less environment.

From our earlier analysis, the best strategy would move passengers as far back as possible and also ensure that passengers boarding within a block are spread out over several rows. We use this intuition to develop heuristics.

Seats that are always the first ones to be filled are assigned to the first zone. The next group of seats to get filled are assigned to the second zone, and so on (**Figure 4**). This zone assignment gives a boarding scheme for passengers with assigned seats. Since in the simulation these passengers had minimal interference with one another, we hope that similar results would occur even with shuffling within zones.

We observe that the zones returned by our learning algorithm resemble parabolas, hence we define the zones as seats highlighted by different parabolas centered near the far end of plane and the center of the rows, superimposed on the seating chart. The parabolas get steeper for higher-numbered zones, since we are boarding aisles at that time.

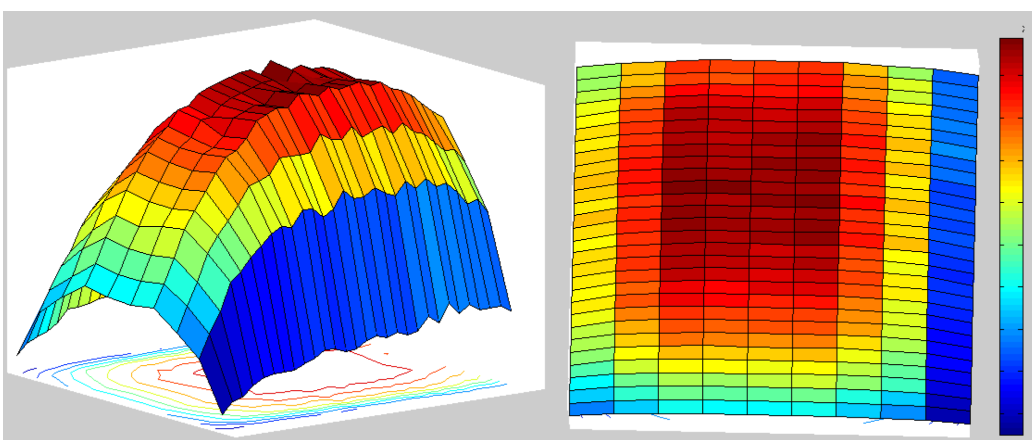

Figure 4. Results of no-assignment seating simulation. Blue-green (light) are passengers seated first and orange-red (dark) are passengers seated last. Traces of equal height take the shape of parabolas.

We wrote a computer program to compute these parabolas for planes of arbitrary size. We refer to this method of assigned seat grouping as the *Parabola boarding method*.

Relative Effect of Parameters

We vary input parameter values to determine their impact. We perform these simulations using the default parameters from above and the plane layout of a Boeing 757-200 (39 rows, 6 columns).

Walking Speed

We analyze the effect of passenger walking speed in **Figure 5**. We vary it from the approximate comfortable walking speed of a 70-year-old female to the approximate maximum walking speed of a 70-year-old male [Bohannon 1997]. Boarding time is not always lowered by increasing walking speed (except in the back-to-front scheme). This fact reflects our key insight from queuing theory analysis that the entry rate is a more critical bottleneck. *Ensuring high walking speed is not critical*.

Luggage Stowage Time

We analyze the effect of changing the luggage stowage time in **Figure 6**. Specifically, we change the average value of the uniform distribution from which we select stowage time. This value has a large effect on the boarding time, following our insight that keeping the aisle full or "pipelined" is important: If we slow the process at this pipeline, performance suffers. *Ensuring quick luggage stowage is critical*.

Plane Entrance Rate

We analyze the effect of changing the plane entrance rate in **Figure 7**. Increasing the delay between successive entries (that is, lowering the rate of incoming passengers) increases the time to board. At a certain large value, all seat assignment methods become equivalent, presumably because no bottlenecks form—since passengers enter so slowly (effectively, each passenger enters independently, one at a time, without conflicts), queueing and overflow effects do not emerge. *Ensuring adequate plane entrance speed is critical*.

Intra-Row Movement Time

In **Figure 8**, we look at the effects of changing the time to shuffle in and out of a row to let in a fellow passenger. Increasing the row movement time raises the boarding time marginally for back-to-front and alternating rows but not for the other algorithms. However, this is to be expected: The other methods are designed to avoid row conflicts, with passengers almost always arriving in outside-in order. *So decreasing row movement time is not critical*, particularly because we can avoid its effects with certain algorithms.

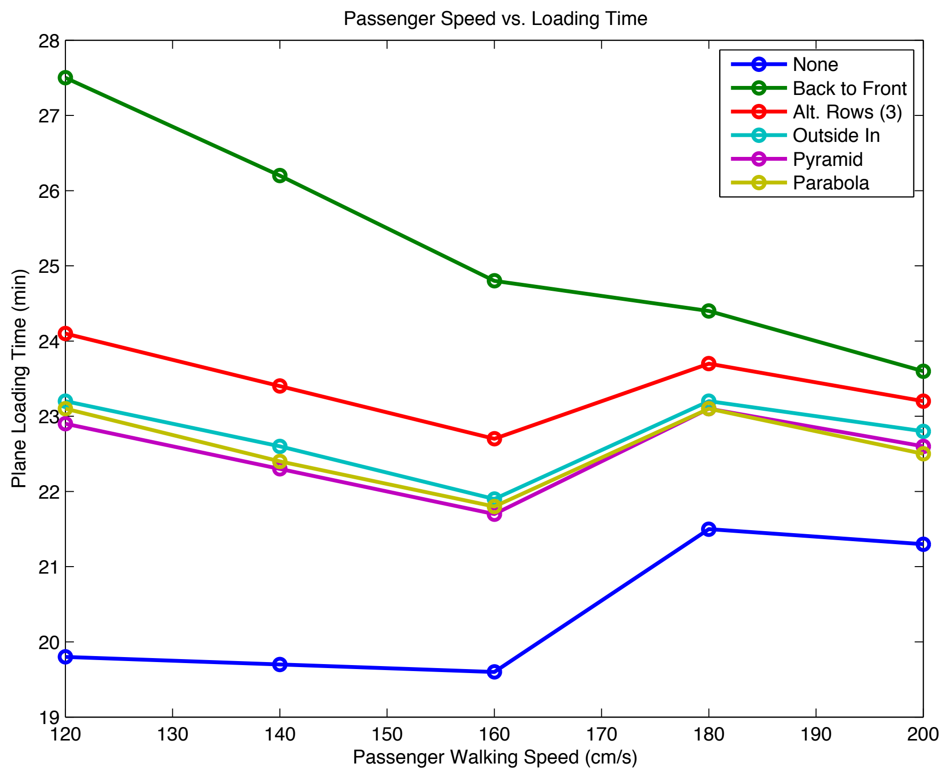

Figure 5. Boarding time as a function of walking speed v.

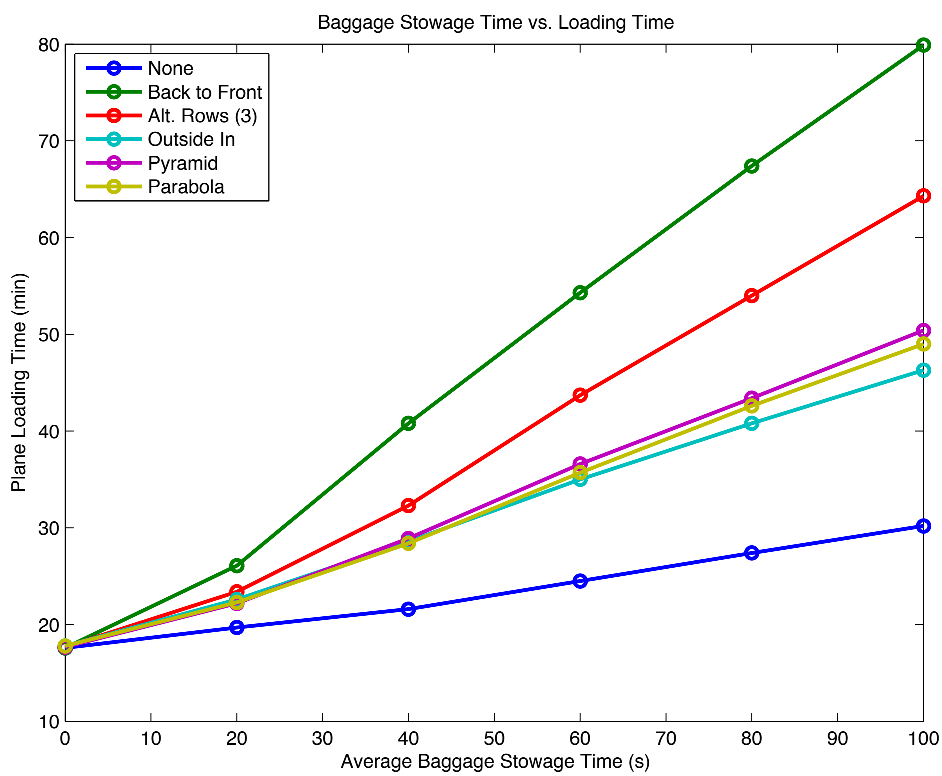

Figure 6. Boarding time as a function of luggage stowage time B.

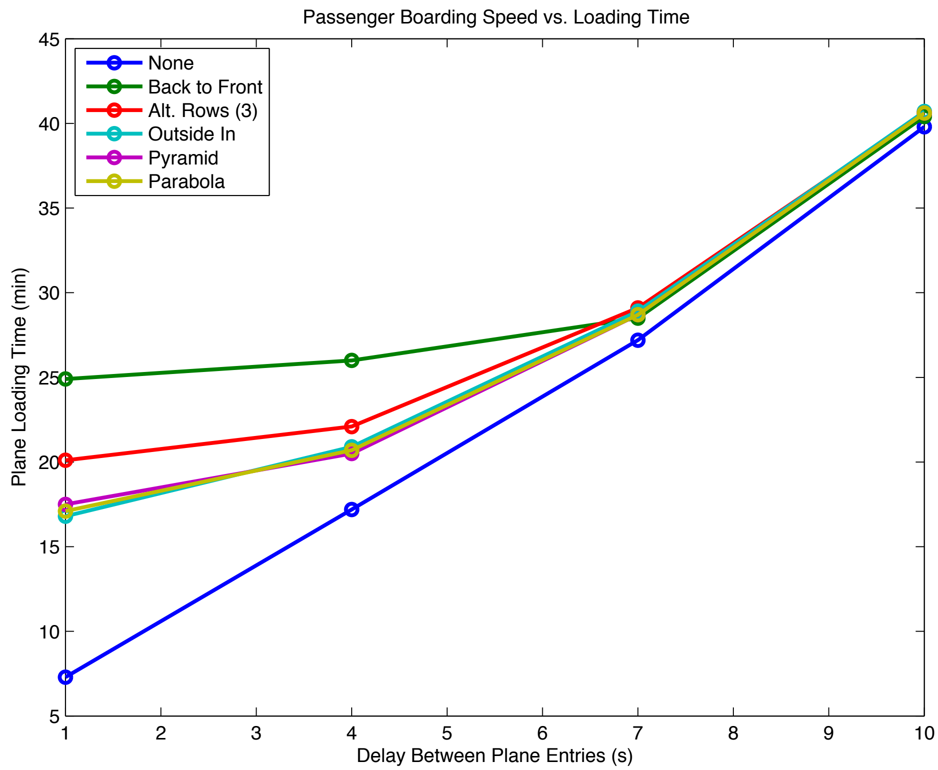

Figure 7. Boarding time as a function of plane entrance rate λ.

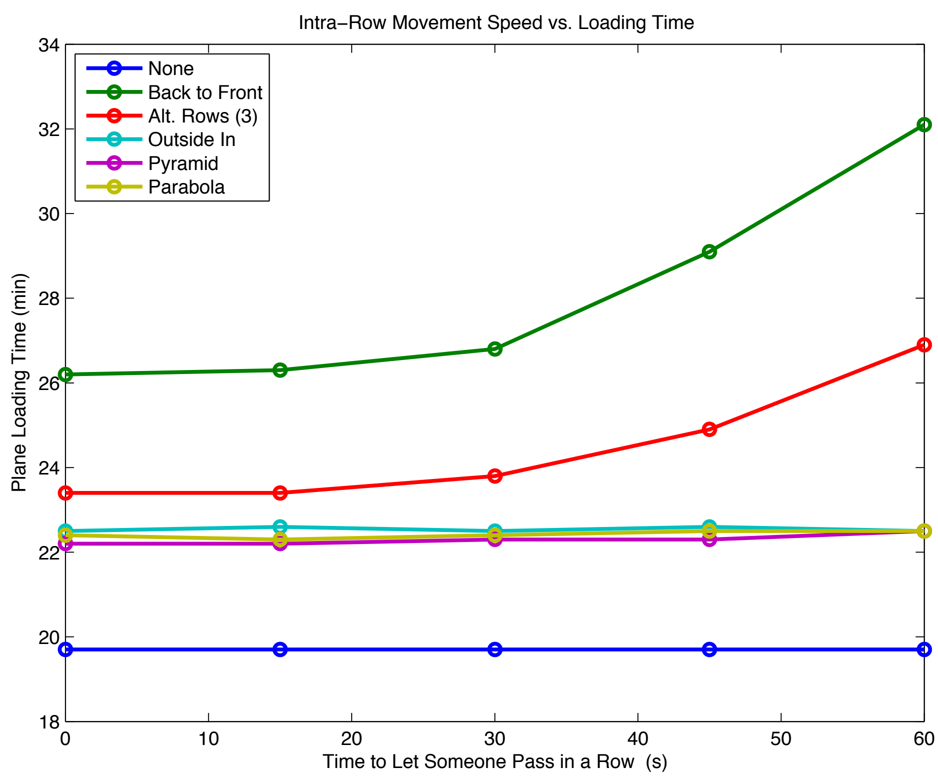

Figure 8. Boarding time as a function of intra-row conflicts.

Summary

We have analyzed the relative impact of the parameters of our model for a representative airplane. Two factors are of key importance: average luggage stowage time and plane entrance rate. Ranking the various strategies in the light of their performance on these factors produces the ordering:

- Outstanding: No assigned seats
- Meritorious: Outside-in, reverse-pyramid, parabola
- Honorable Mention: Alternating rows
- Limited Success: Back-to-front

Some insight into this ordering comes from comparing average seating order after mixing within groups vs. without seat assignments (**Figure 9**). Pyramid and parabola most closely approximate the order achieved by the fast no-assignments method. We conclude that outside-in captures most of the key benefits, since in general it is as fast as the other two while being a less-close approximation of the random greedy model.

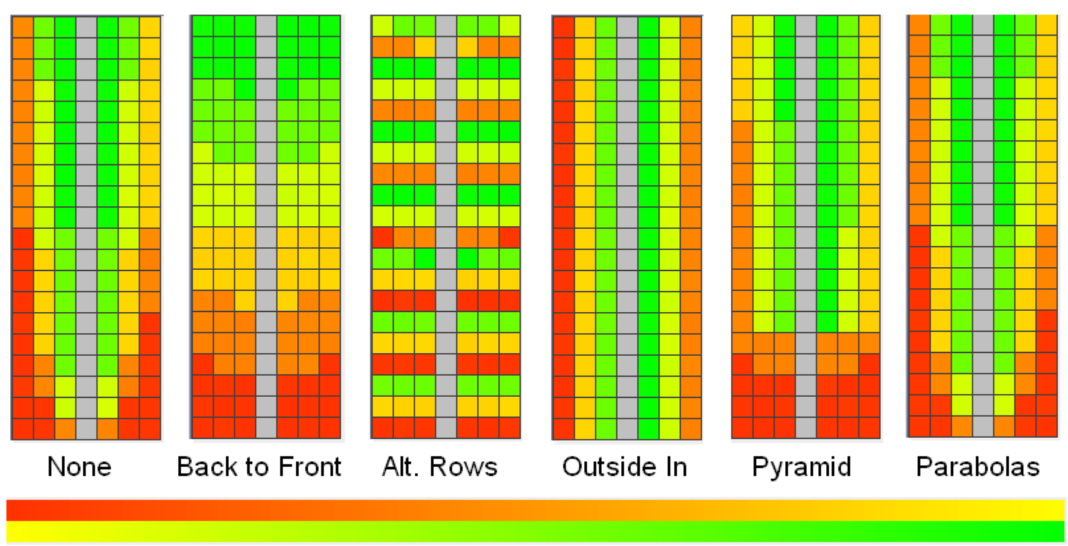

Figure 9. Average seating order with group mixing.

Strategy Robustness and Dependability

Average boarding speed is not the only measure of success. A fast boarding method does no good if once a week it takes twice as long as its average; airlines need to depend on a consistent time to produce achievable and reliable schedules. Therefore, we prefer boarding methods that vary little between

Table 1.
Time range for each boarding method.

Algorithm	Time Range (min)
No assigned seats	0.7
Outside-in	2.6
Parabola	2.8
Reverse-pyramid	3.1
Alternating rows	4.6
Back-to-front	6.2

worst and best cases. We simulated plot boarding times for our various schemes over 500 trials, with results in **Table 1**.

The smallest spread between longest and shortest load times is for no assigned seats. Interestingly, there is a direct correlation between time to board and variability in boarding time. The outside-in, reverse-pyramid, and parabola methods have similar boarding times and distributions. Similarly, Back-to-Front and Alternating Rows take the longest and have the largest spread. To some extent, these results suggest that a faster boarding algorithm is also more dependable; however, this may not be true for all cases.

Model Generalization

Our simulation assumes that the plane is boarded from one end of the seating area with passengers walking down aisles at the center of the rows. But some planes have several aisles or passengers boarding on different levels. Our model can be easily generalized to accommodate for different plane designs and layouts. We divide the problem into sections; each subsection is modeled as its own plane with entry rates changed appropriately. Depending on whether boarding of subsections occurs in serially or in parallel, the times are added or compared (and the maximum taken).

Specific Results

We apply our model to various real-world planes of different sizes to compare the speed of the boarding processes. We let Outside-In serve represent outside-in, reverse-pyramid, and parabola, which are similar in timing. We apply our generalization techniques to model multi-aisle, -class, and -level planes with given configurations [Airbus . . . 2007, Boeing . . . 2007] (**Table 2**).

The results support our previous conclusions. In several cases, back-to-front is quite close to outside-in, perhaps because the plane entry rate was not high enough.

Table 2.
Simulation boarding times for multi-aisle, multi-class, and multi-level planes.

Plane	Passengers	Unassigned	Back-to-Front	Outside-In
DC 9-40	125	12	17	14
Airbus A320	164	14	21	17
Boeing 757-200	234	20	26	23
Boeing 747-400	313	31	31	32
Airbus A380	555	35	35	36

Conclusion

While our approaches and models are effective and produce results, there remain several model weaknesses:

- We assume independent, perfect-knowledge, infallible passengers who always put their luggage directly above themselves, as well as other perfect scenarios (planes of equally-sized rows, jet-bridges of constant flow instead of stairs or buses that bring passengers to the plane).
- There are several areas of the problem that we left untested because they seemed to be of secondary importance, such as varying the number of zones.
- Our comparison of boarding algorithms is simulation-based and therefore by nature not exhaustive. There may be better algorithms that we did not test, such as single-zone random boarding or rotating row-group zones.
- We stay within the current boarding paradigm so as not to produce too much uncomfortable change for passengers. However, greater improvements might obtain if a wider range of choices were available; simple examples might be assigning passengers only to a row and letting them choose a seat there, or hiding money under one seat to encourage speedy boarding.

Overall, we believe the strengths inherent in our approach overcome many of the weaknesses and allow us to make useful recommendations:

- Our multilayered approach produced key insights.
- Our simulation model can be extended to test new algorithms and situations with minimal changes.
- We provide a relative ranking of factors affecting boarding speed, not just a ranked list of algorithms. An airline can still make improvements if they don't want to switch their process, or if they already have a fast process.

Summary

Our key observations are:

- The aisle is the main bottleneck, especially near the entrance, and it is necessary to "pipeline" passengers to maintain a high throughput.
- The rate of passengers entering is also critical, since it determines the maximum rate at which passengers can proceed down the aisle and be seated.
- Sending in passengers with closely-situated seating assignments in short time intervals results in numerous interferences and increases boarding time. Instead, passengers should be enter by zones that distribute seats over several rows.

Our simulations confirm these insights and show that boarding schedules that follow these rules perform better in terms of both speed and reliability.

We offer the following recommendations to airlines to improve their boarding time, turnaround time, and ultimately their bottom line:

- **Passenger entry speed:** The faster passengers enter the plane, the faster it boards. This means that ticket-checking should be as quick as possible, hence with an optimal number of gate agents. Flight attendants should be stationed at critical junctions (such as entrances to aisles in a multi-aisle plane) to direct passengers to the correct rows.
- **Luggage stowage time:** The faster passengers stow their bags and sit down, the faster the plane boards. Stowage time can be reduced by changing or enforcing carry-on luggage limits and by having flight attendants assist passengers with large or heavy bags.
- **Switch from back-to-front to another boarding method.** Outside-in boarding provides a 10%–30% advantage over back-to-front. Foregoing assigned seats results in a further 10%–30% advantage over outside-in. Faster methods are also considerably more reliable: Outside-in has a time range 50% smaller than back-to-front.

References

Airbus aircraft families. 2007. `http://www.airbus.com/en/aircraftfamilies`. Accessed 11 February 2007.

Boeing: Commercial airplanes, products. 2007. `http://www.boeing.com/commercial/products.html`. Accessed 11 February 2007.

Bohannon, R.W. 1997. Comfortable and maximum walking speed of adults aged 20–79 years; reference values and determinants. *Age and Aging* 26: 15–19.

Trivedi, K.S. 2002. *Probability and Statistics with Reliability, Queuing and Computer Science Applications*. 2nd ed. New York: Wiley-Interscience.

Finney, Paul Burnham. 2006. Loading an airliner is rocket science. *New York Times* (14 November 2006). `http://travel2.nytimes.com/2006/11/14/business/14boarding.html`. Accessed 10 February 2007.

Jackson, J.R. 1957. Networks of waiting lines. *Operations Research* 5: 518–527.

Kiwi, M. 2006. A concentration bound for the longest increasing subsequence of a randomly chosen involution. *Discrete Applied Mathematics* 154 (13): 1816–1823.

Marelli, Scott, Gregory Mattocks, and Remick Merry. 1998. The role of computer simulation in reducing airplane turn time. *Aero Magazine* (4th Quarter 1998). `http://www.boeing.com/commercial/aeromagazine/aero_01/textonly/t01txt.html`.

Snakes on a Plane. 2007. `http://www.imdb.com/title/tt0417148/`. Accessed 11 February 2007.

van den Briel, Menkes H.L., J. René Villalobos, Gary L. Hogg, Tim Lindemann, and Anthony V. Mulé. 2005. America West Airlines develops efficient boarding strategies. *Interfaces* 35 (3) (May-June 2005): 191–200.

Kshipra Bhawalkar, Matthew Edwards, Michael Bauer, and advisor Anne Catlla.