

# Developing Improved Algorithms for Irrigation Systems

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## Summary

Our goal is an algorithm that minimizes the time to irrigate a relatively small field under given conditions.

We focus on minimization of time, uniformity of irrigation, and feasibility. Our effort is divided into five basic parts:

- **We assess the wetted radius** based on experimental results for several typical rotating spray sprinklers.
- **We determine the number of sprinklers** from an empirical formula for sprinkler flow.
- **We simulate the water distribution pattern**, using a  $0.25\text{ m} \times 0.25\text{ m}$  grid.
- **We evaluate the uniformity of water distribution** by Christiansen's uniformity coefficient.
- **We find an optimal irrigation schedule including when and where to move the pipes:** We devise a single-lateral-pipe scheme and a multiple-lateral-pipes scheme; the latter gives better results. To irrigate more uniformly, we adjust the spacing between sprinklers and the spacing from the edge. Using our grid, we move the sprinklers symmetrically on both sides, node by node, to find the optimal positions for an improved multiple-lateral-pipes scheme.

Simulations show that all three schemes perform acceptably in realistic conditions. The multiple-lateral-pipes scheme is superior, with minimum time and

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the highest Christiansen's uniformity coefficient (CU). We conclude that four sprinklers are required, the minimal amount of time is 732 min, and the CU is 90%.

We do a sensitivity analysis of the variation of CU and of minimum time with wetted radius, which shows that our model is robust.

## Introduction

### Structure of a Hand-Move Irrigation System

A hand-move irrigation system has two kinds of pipes: a portable or buried mainline pipe, and a portable aluminum (sometimes plastic) lateral pipe with quick couplers and spray nozzles.

## Definitions and Key terms

**Pipeset:** Pipes that can be connected together in a straight line.

**Working pressure:** Pressure at the water source (kPa).

**Hydraulic pressure:** Equivalent to working pressure but with measurement in meters (m).

**Flow rate:** Volume of water discharged per unit of time at the water source ( $\text{m}^3/\text{h}$ ).

**Sprinkler flow:** Volume of water discharged per unit of time by a sprinkler ( $\text{m}^3/\text{h}$ ).

**Rotating spray nozzle:** Water distribution device equipped with a rotating deflection pad to distribute water.

**Wetted radius:** Farthest distance measured while the spray nozzle is rotating normally, from the spray nozzle centerline to the point where water is deposited (m).

**Precipitation:** How much water reaches the ground, equivalent to natural rainfall (mm/h).

**Distribution pattern:** Pattern showing precipitation by location in the field.

**Uniformity of distribution:** Evenness of water throughout a field.

Symbols are listed in **Table 1**.

**Table 1.**  
Symbols.

Symbol	Description	Units
$Q$	Flow rate	$\text{m}^3/\text{h}$
$Q'$	Sprinkler flow	$\text{m}^3/\text{h}$
$A$	Cross-sectional area of nozzle	$\text{m}^2$
$n$	The number of sprinklers	
$\mu$	Discharge coefficient	
$H_p$	Hydraulic pressure	m
$d$	Diameter of nozzle	mm
$\alpha$	Trajectory angle of nozzle	$^\circ$
$g$	Acceleration due to gravity	$\text{m}/\text{s}^2$
$P$	Precipitation	$\text{mm}/\text{h}$
$R$	Wetted radius	m
$r$	Distance from the sprinkler	m
$\rho$	Average precipitation over the area covered by one sprinkler	$\text{mm}/\text{h}$
CU	Christiansen's uniformity coefficient	
$(x_i, y_j)$	Coordinate of grid node in a network	
$N$	Total number of grid nodes	
$T$	Irrigation time	h
$h_{i,j}$	Precipitation at $(x_i, y_j)$	mm

## General Assumptions

- There is no infiltration, evaporation, or wind.
- Time spent on moving the pipes is negligible.
- Sprinklers used in a pipe set are of the same type.
- The diameter of the sprinkler is small compared to the dimensions of the watered area.
- We ignore the height of sprinklers [Carrión et al. 2001].
- Pressure at each sprinkler equals working pressure.
- Sprinkler flow rate remains stable.

## Model Design

### Wetted Radius

The wetted radius mainly factors in working pressure and nozzle characteristics such as size, trajectory angle, and rotating velocity. Accelerating the rotating speed reduces wetted radius. Wetted radius depends on hydraulic

pressure, diameter, and (not significantly) on trajectory angle, of nozzles via the relationship

$$R = f(\alpha, h_p, d).$$

When the trajectory angle  $\alpha$  is stationary, an empirical formula commonly used by manufacturers is

$$R = \xi h_p^m d^n, \quad (1)$$

where  $\xi$ ,  $m$ , and  $n$  are parameters evaluated by the manufacturer's testing at various water pressures.

Applying least-squares to the experimental wetted radius of three typical rotation spray sprinklers, we get parameter values for four different trajectory angles (**Table 2**). We can substitute parameter values and easily obtain wetted radius.

**Table 2.**  
Values of parameters.

Trajectory angle ( $^{\circ}$ )	$\xi$	$m$	$n$
7	11.46	0.369	0.319
15	5.61	0.225	0.734
22.5	8.63	0.140	0.476
30	4.52	0.128	0.844

## Number of Sprinklers

Sprinkler flow depends mainly on two factors: working pressure and diameter of the nozzle. Flow rate from a nozzle increases with working pressure and can normally be fitted to the equation [Zhao 1999]:

$$Q' = 3600 \mu A \sqrt{2gH_p}, \quad (2)$$

where

$Q'$  is sprinkler flow ( $\text{m}^3/\text{h}$ );

$\mu$  is the discharge coefficient, usually between 0.75 and 0.98; and

$A$  is the cross-sectional area of nozzle ( $\text{m}^2$ ).

The number of sprinklers required is  $n \approx Q/Q'$ .

## Water Distribution

### Distribution Pattern of a Single Sprinkler

Water distribution patterns are usually obtained under controlled no-wind conditions. **Figure 1** is a common pattern for simulating precipitation from a single sprinkler [Mateos 1998].

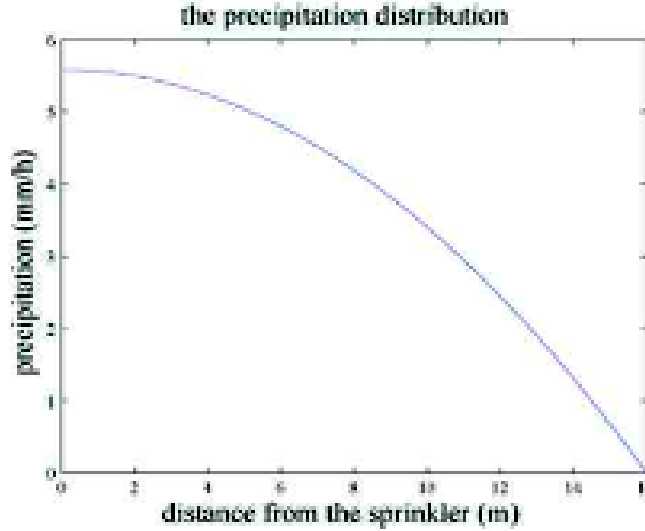


Figure 1. Distribution pattern of an individual sprinkler.

The mathematical function is

$$P = \frac{2Q'T}{R^2\pi} \left( 1 - \frac{r^2}{R^2} \right),$$

where

$P$  is the precipitation (mm/h),

$r$  is distance from the sprinkler (m),

$Q'$  is sprinkler flow ( $\text{m}^3/\text{h}$ ),

$R$  is wetted radius (m), and

$T$  is irrigation time (h).

### Water Distribution over the Whole Field

We divide the field uniformly into sufficiently small grid squares ( $0.25 \text{ m} \times 0.25 \text{ m}$ ) and overlap the precipitation from each sprinkler.

## Average Precipitation

To adjust the amount of water received on each part of the field per hour or per day, we introduce the concept of average precipitation (mm/h):

$$\rho = \frac{Q'}{R^2\pi}, \quad (3)$$

where  $Q'$  is sprinkler flow ( $\text{m}^3/\text{h}$ ) and  $R$  is wetted radius (m).

## Uniformity of Water Distribution

Irrigation uniformity is a major factor in maintaining proper crop growth. We calculate a uniformity coefficient for the field [Wilcox and Swailes 1947], using that of Christiansen [1941]:

$$\text{CU} = \left( 1 - \frac{\sum_i \sum_j (h_{i,j} - \bar{h})}{N \times \bar{h}} \right) \times 100\%, \quad (4)$$

where

$h_{i,j}$  is precipitation at  $(x_i, y_i)$  (mm/h),

$\bar{h}$  is the average value of all  $h_{i,j}$ , and

$N$  is the total number of grid nodes.

## Model Validation on a Small Ranch

### The Specifications

The field is  $80 \text{ m} \times 30 \text{ m}$ . Each pipe has a 10-cm inner diameter with rotating spray nozzles with 6 mm inner diameter, and the pipes connected together are 20 m long. The pressure of the water source is 420 kPa, with a flow rate of 150 L/min. No part of the field should receive more than 0.75 cm/h of water, and each part should receive at least 2 cm of water every 4 days.

### Number of Sprinklers

Given the flow rate and diameter of the nozzle, we calculate the sprinkler flow using (2), then assess that the number of sprinklers should be four.

## The Conditions on the Ranch

Our simulation has  $121 \times 321$  total grid nodes, with grid size  $0.25 \text{ m} \times 0.25 \text{ m}$ . Equation (4) becomes

$$\text{CU} = \left( 1 - \frac{\sum_{i=0}^{120} \sum_{j=0}^{320} (h_{i,j} - \bar{h})}{121 \times 321 \bar{h}} \right) \times 100\%.$$

## Schemes of Positioning and Moving

We examine several typical workable schemes and compare them to find the optimal configuration.

### Single Lateral Pipe

If all pipes are connected into one lateral pipe, then we have the approximate average precipitation rate by (3) that should be satisfied:

$$\frac{4 \times \frac{1}{4}Q}{\pi R^2} < 7.5 \text{ mm.} \quad (5)$$

If we use a rotating spray sprinkler with a trajectory angle of  $30^\circ$ , then by (1) and Table 2, the wetted radius is  $R \approx 20 \text{ m}$ .

### Description

The mainline pipe is located across the field as shown in Figure 2. The lateral pipe is moved across through the field at right angles to the row direction. This lateral pipe has four sprinklers 6.67 m apart.

### Results

We design a schedule for four days. The minimum time is 1228 min, with four cycles, and  $\text{CU} = 78\%$ . The distribution pattern is shown in Figure 3.

### Multiple Lateral Pipes

We try to improve the uniformity of the precipitation by changing the position of the lateral pipe and the spacing between sprinklers, but we can't get a satisfactory result, since CU cannot be improved. In addition, wind normally has a significant impact on sprinklers with a higher trajectory angle. We conclude that more than one lateral pipe should be used.

We conclude that two lateral pipes with two sprinklers on each are appropriate. In light of (3), the approximate average precipitation rate should be satisfied:

$$\frac{2 \times \frac{1}{4}Q}{\pi R^2} < 7.5 \text{ mm.} \quad (6)$$

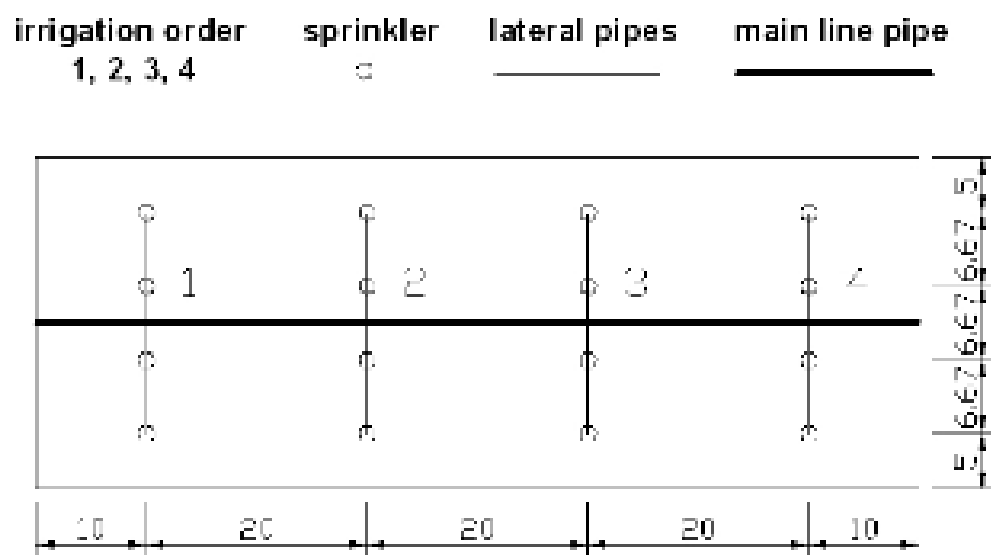


Figure 2. Single-lateral-pipe scheme, with measurements in meters.

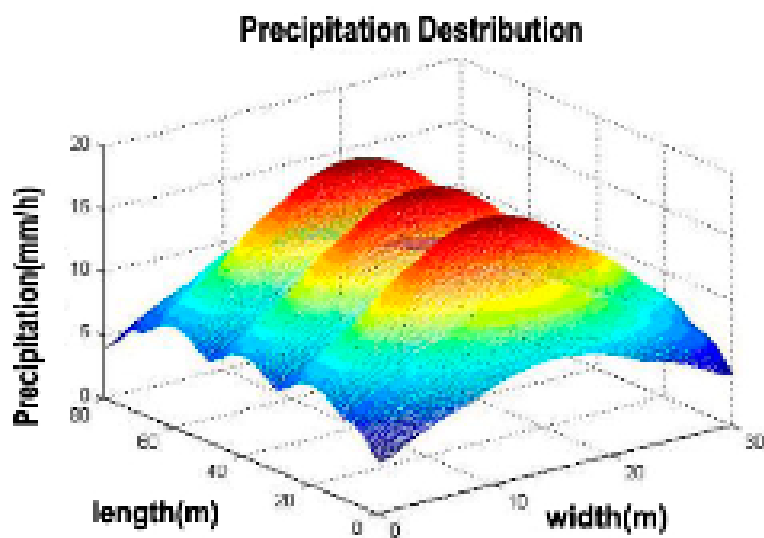


Figure 3. Distribution pattern for the single-lateral-pipe scheme.



If we use a rotating spray sprinkler with a trajectory angle of  $15^\circ$ , then by (1) and Table 2, the wetted radius is  $R \approx 16$  m.

Description

The mainline pipe goes along the edge of field, connected to two lateral pipes. Each lateral has two sprinklers 5 m apart. The two lateral pipes are moved crossways. The irrigation order and positions of sprinklers are presented in Figure 4.

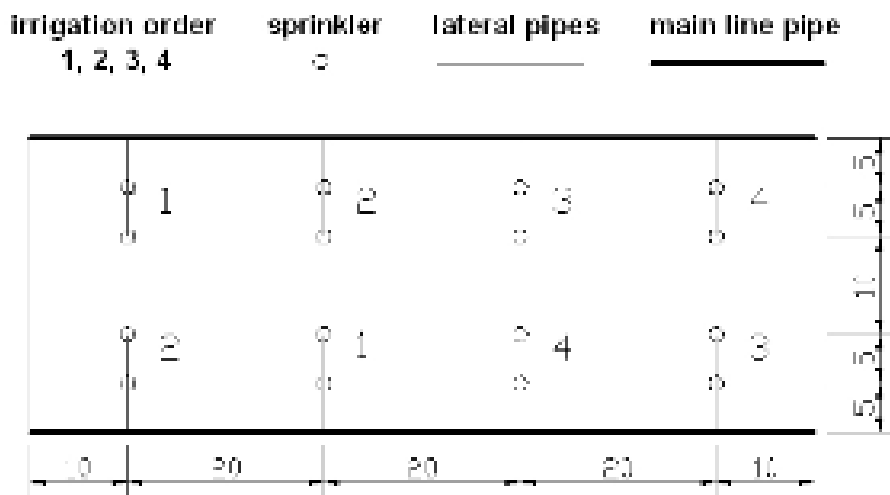


Figure 4. Multiple-lateral-pipes scheme, with measurements in meters.

Results

We design a schedule for four days. The minimum time is 920 min, with four cycles, with  $CU = 83\%$ . The distribution pattern is shown in Figure 5.

Improved Multiple-Lateral-Pipes Scheme

Description

With multiple lateral pipes, precipitation is relatively excessive in the middle of the field, due to overlap. We move the sprinkler closer to the edge to uniformize the precipitation. Using a  $0.25\text{ m} \times 0.25\text{ m}$  network, we move sprinklers on both sides, node by node symmetrically, to determine the optimal position. Two sprinklers 5 m apart on the same lateral, at 3 m or 8 m from the edge of the field, are optimal (Figure 6).

Results

We design a schedule for four days. The minimum time is 732 min, with four cycles, and  $CU = 90\%$ . The distribution pattern is shown in Figure 7.

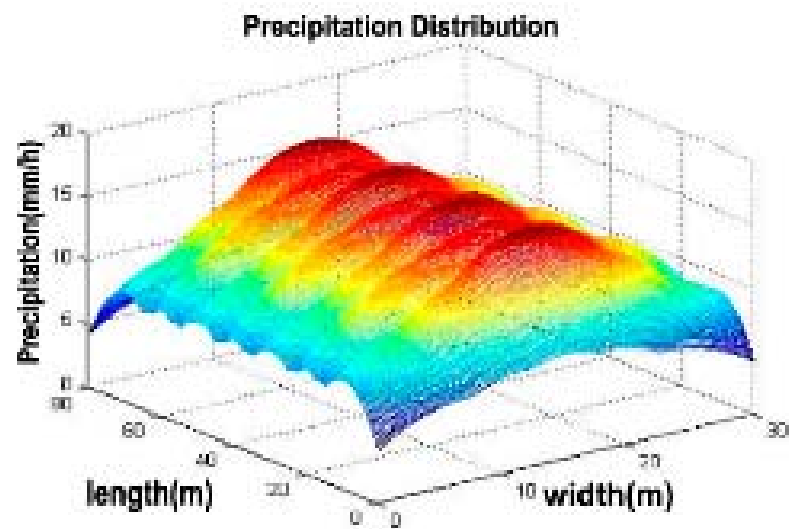


Figure 5. Distribution pattern for the multiple-lateral-pipes scheme.

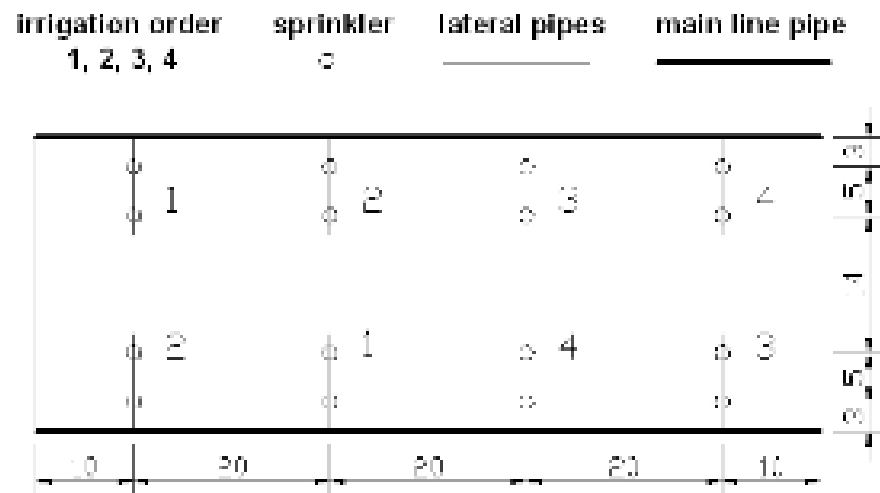


Figure 6. Improved-multiple-lateral pipes scheme, with measurements in meters.

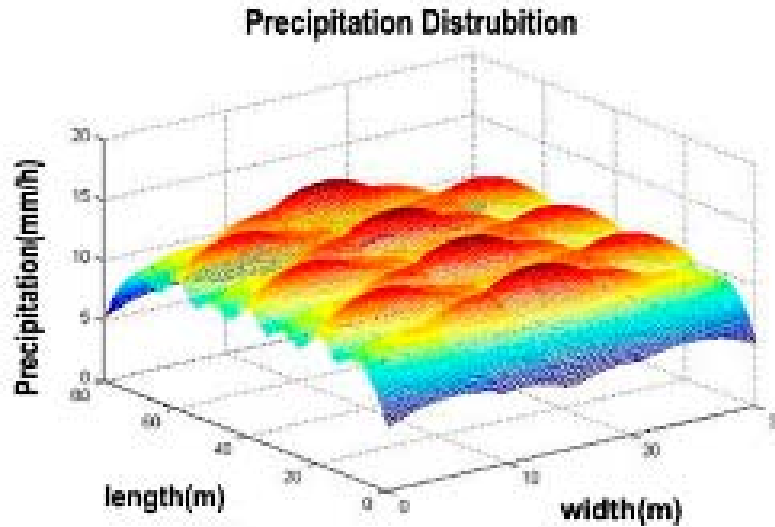


Figure 7. Distribution pattern for the improved multiple-lateral-pipes scheme.

## Conclusions

### Irrigation Schedule

[EDITOR'S NOTE: We omit the schedule.]

### Comparison of Schemes

The multiple-lateral-pipes scheme and the improved multiple-lateral-pipes scheme take less time (920 min and 732 min) than the single-lateral-pipe scheme (1228 min).

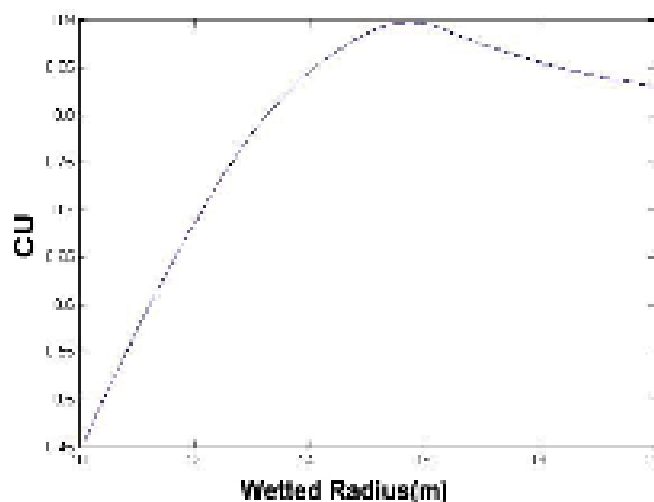
The improved multiple-lateral-pipes scheme is superior, with both minimum time and the highest Christiansen's uniformity coefficient.

### Sensitivity Analysis

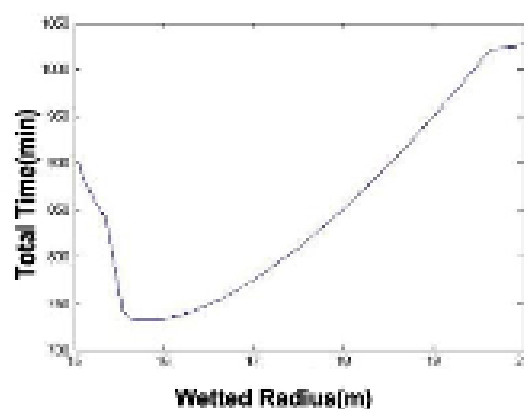
We do a sensitivity analysis of the variation in  $CU$  and in minimum time with value of wetted radius. (Figures 8–9). Both figures show that our model is robust.

To obtain the optimal scheme, we use an algorithm to move sprinklers on both sides node by node symmetrically. Figure 10 shows the sensitivity of  $CU$  to distance of the sprinkler from the main line. In our model, the difference of  $CU$  between two grid nodes is no more than 1.3%.

The sprinklers in the middle of the field can be shut off selectively and don't need to work continuously during one cycle; the minimum time is determined



**Figure 8.** The variation in CU caused by wetted radius (for minimum time). Our improved multiple-lateral-pipes scheme uses a wetted radius of 16 m.



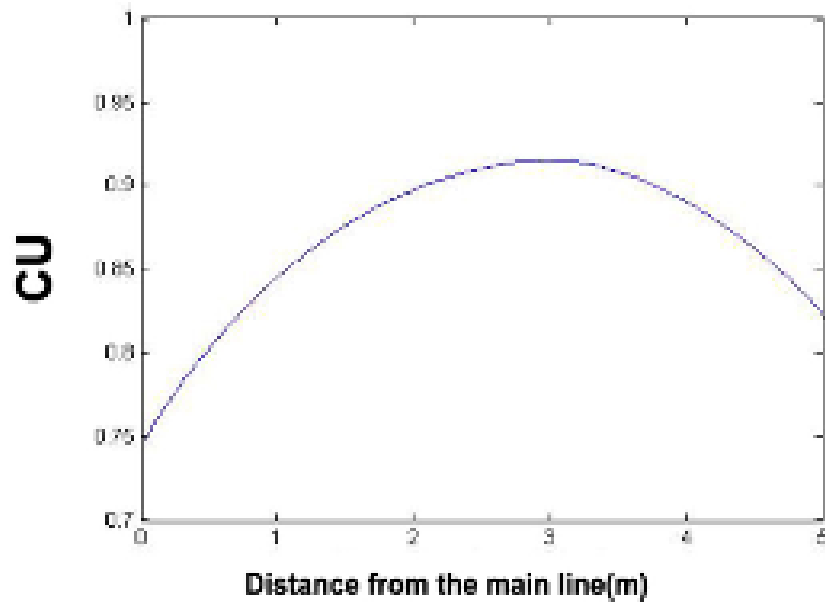
**Figure 9.** The variation of the minimum time caused by wetted radius. Our improved multiple-lateral-pipes scheme uses a wetted radius of 16 m.

by the sprinklers near the edge. That way, we can not only carry out the irrigation more uniformly but also save much water.

## Strengths and Weaknesses

### Strengths

- We find the water distribution pattern from the sprinklers, using simulation.
- We investigate different numbers of lateral pipes and different values of the wetted radius.
- We provide a good result and find an optimal scheme.



**Figure 10.** Sensitivity of uniformity to distance of sprinkler from the main line. In our improved multiple-lateral-pipes scheme, the distance is 3 m.

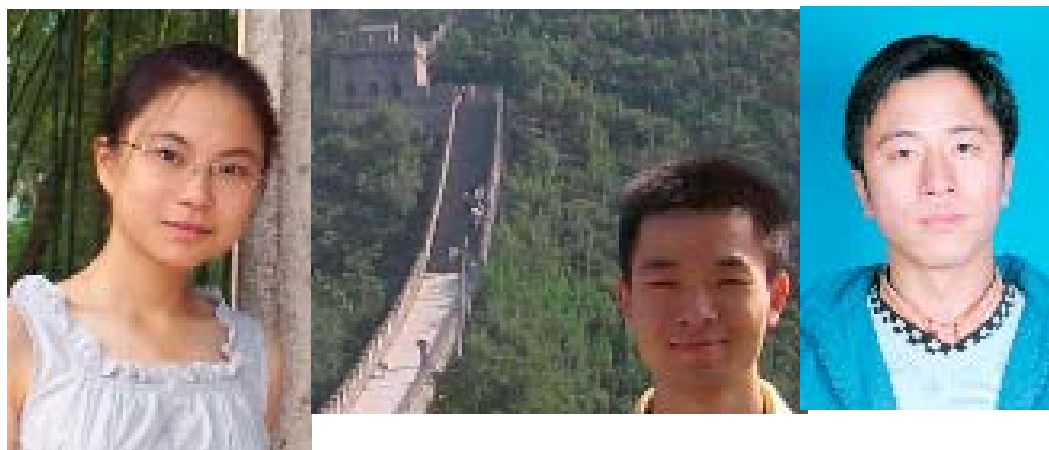
- We examine various approaches and modifications to find the best design for the irrigation system.

## Weaknesses

- We did not incorporate into our model some factors that might have effect in real life, such as infiltration, evaporation, and wind.

## References

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