

# Probabilistically Optimized Airline Overbooking Strategies, or “Anyone Willing to Take a Later Flight?!”

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## Introduction

We develop a series of mathematical models to investigate relationships between overbooking strategies and revenue.

Our first models are static, in the sense that passenger behavior is predominantly time-independent; we use a binomial random variable to model consumer behavior. We construct an auction-style model for passenger compensation.

Our second set of models is more dynamic, employing Poisson processes for continuous time-dependence on ticket purchasing/cancelling information.

Finally, we consider the effects of the post-September 11 market on the industry. We consider a particular company and flight: Frontier Airlines Flight 502. Applying the models to revenue optimization leads to an optimal booking limit of 15% over flight capacity and potentially nets Frontier Airlines an additional \$2.7 million/year on Flight 502, given sufficient ticket demand.

## Frontier Airlines: Company Overview

Frontier Airlines, a discount airline and the second largest airline operating out of Denver International Airport (DIA), serves 25 cities in 18 states. Frontier offers two flights daily from DIA to LaGuardia Airport in New York. We focus on Flight 502.

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## Technical Considerations and Details

We discuss regulations for handling bumped passengers, airplane specifications, and financial interests.

### Overbooking Regulations

When overbooking results in overflow, the Department of Transportation (DOT) requires airlines to ask for volunteers willing to be bumped in exchange for compensation. However, the DOT does not specify how much compensation the airlines must give to volunteers; in other words, negotiations and auctions may be held at the gate until the flight's departure. A passenger who is bumped involuntarily is entitled to the following compensation:

- If the airline arranges substitute transportation such that the passenger will reach his/her destination within one hour of the original flight's arrival time, there is no obligatory compensation.
- If the airline arranges substitute transportation such that the passenger will reach his/her destination between one and two hours after the original flight's arrival time, the airline must pay the passenger an amount equal to the one-way fare for flight to the final destination.
- If the substitute transportation is scheduled to arrive any later than two hours after the original flight's arrival time, or if the airline does not make any substitute travel arrangements, the airline must pay an amount equal to twice the cost of the fare to the final destination.

### Aircraft Information

Frontier offers only one class of service to all passengers. Thus, we base our overbooking models on single-class aircraft.

### Financial Considerations

Airline booking considerations are frequently based on the *break-even load-factor*, a percentage of airplane seat capacity that must be filled to acquire neither loss or profit on a particular flight. The break-even load-factor for Flight 502 in 2001 was 57.8%.

## Assumptions

- **We need concern ourselves only with the sale of restricted tickets.** Frontier's are nonrefundable, save for the ability to transfer to another Frontier flight for \$60 [Frontier 2001]. Restricted tickets represent all but a very small percentage of all tickets, and many ticket brokers, such as Priceline.com, sell only restricted tickets.

- Ticketholders who don't show up at the gate spend \$60 to transfer to another flight.
- Bumped passengers from morning Flight 502 are placed, at the *latest*, 4 h 35 min later on Frontier's afternoon Flight 513 to the same destination. Frontier Airlines first attempts to place bumped passengers on other airlines' flights to the same destination. If it can't do so, Frontier bumps other passengers from the later Frontier flight to make room for the originally bumped passengers.
- The annual effects/costs associated with bumping involuntary passengers is negligible in comparison to the annual effects/costs of bumping voluntary passengers. According to statistics provided by the Department of Transportation, 4% of all airline passengers are bumped voluntarily, while only 1.06 passengers in 10,000 are bumped involuntarily. With a maximum delay for bumped passengers of 4 h 35 min, the average annual cost to Frontier of bumping involuntary passengers is on the order of \$100,000—negligible compared to costs of bumping voluntary passengers.

## The Static Model

Our first model for optimizing revenues is static, in the sense that passenger behavior is predominantly time-independent: All passengers (save no-shows) arrive at the departure gate independently. This model does not account for when passengers purchase their tickets. This system may be modeled by the following steps:

- Introduce a binomial random variable for the number of passengers who show up for the flight.
- Define a total profit function dependent upon this random variable.
- Apply this function to various consumer behavior patterns.
- Compute (for each behavioral pattern) an optimal number of passengers to overbook.

## A Binomial Random Variable Approach

We let the binomial random variable  $X$  be the number of ticketholders who arrive at the gate after  $B$  tickets have been sold; thus,  $X \sim \text{Binomial}(B, p)$ . Numerous airlines consistently report that approximately 12% of all booked passengers do not show up to the gate (due to cancellations and no-shows) [Lufthansa 2000], so we take  $p = .88$ .

$$\Pr\{i \text{ passengers arrive at the gate}\} = \Pr\{X = i\} = \binom{B}{i} p^i (1-p)^{B-i}.$$

## Modeling Revenue

We define the following per-flight total profit function and subsequently present a detailed explanation.

$$T_p(X) = (B - X)R + \begin{cases} \text{Airfare} \times X - \text{Cost}_{\text{Flight}}, & X \leq C_{\$}; \\ \text{Airfare} - \text{Cost}_{\text{Add}} \times (X - C_{\$}), & C_{\$} < X \leq C; \\ \text{Airfare} - \text{Cost}_{\text{Add}} \times (X - C_{\$}) - \text{Bump}(X - C), & X > C, \end{cases}$$

where

$R$  = transfer fee for no-shows and cancellations,

$B$  = total number of passengers booked,

Airfare = a constant

$\text{Cost}_{\text{Flight}}$  = total operating cost of flying the plane

$\text{Cost}_{\text{Add}}$  = cost to place one passenger on the flight

Bump = the Bump function (to be defined)

$C_{\$}$  = number of passengers required to break even on the flight

$C$  = the full capacity of the plane (number of seats)

For Airfare, we use the average cost of restricted-ticket fare over a one-week period in 2002: \$316.  $\text{Cost}_{\text{Flight}}$  is based on the break-even load-factor of 57.8%; for Flight 502, we take  $\text{Cost}_{\text{Flight}} = \$24,648$  [Frontier Airlines 2001]. The average cost associated with placing one passenger on the plane is  $\text{Cost}_{\text{Add}} \approx \$16$ . The break-even occupancy is determined from the break-even load-factor; since Flight 502 uses an Airbus A319 with 134 seats, we take  $C = 134$  and  $C_{\$} = 78$ .

## The Bump Function

We consider various overbooking strategies, the last three of which translate directly into various Bump functions.

- **No Overbooking**

- **Bump Threshold Model** We assign a “Bump Threshold” (BT) to each flight, a probability of having to bump one or more customers from a flight given  $B$  and  $p$ :

$$\Pr\{X > \text{flight capacity}\} < \text{BT}.$$

We take BT = 5% of flight capacity. The probability that more than  $N$  ticket-holders arrive at the gate, given  $B$  tickets sold, is

$$Pr\{X > N\} = 1 - Pr\{X \leq N\} = 1 - \sum_{i=1}^N \binom{B}{i} p^i (1-p)^{B-i}.$$

This simplistic model is independent of revenue and produces (through simple iteration) an optimal number of ticket sales ( $B$ ) for expecting bumping to occur on less than 5% of flights.

- **Linear Compensation Plan** This plan assumes that there is a fixed cost associated with bumping a passenger, the same for each passenger. The related Bump function is

$$\text{Bump}(X - C) = B_{\$} \times (X - C),$$

where  $(X - C)$  is the number of bumped passengers and  $B_{\$}$  is the cost of handling each.

- **Nonlinear Compensation Plan** Steeper penalties must be considered, since there is a chain reaction of expenses incurred when bumping passengers from one flight causes future bumps on later flights. Here we assume that the Bump function is exponential. Assuming that flight vouchers are still adequate compensation at an average cost of  $2 * \text{Airfare} + \$100 = \$732$  when there are 20 bumped passengers, we apply the cost equation

$$\text{Bump}_{\text{NL}}(X - C) = B_{\$} (X - C) e^{r(X-C)},$$

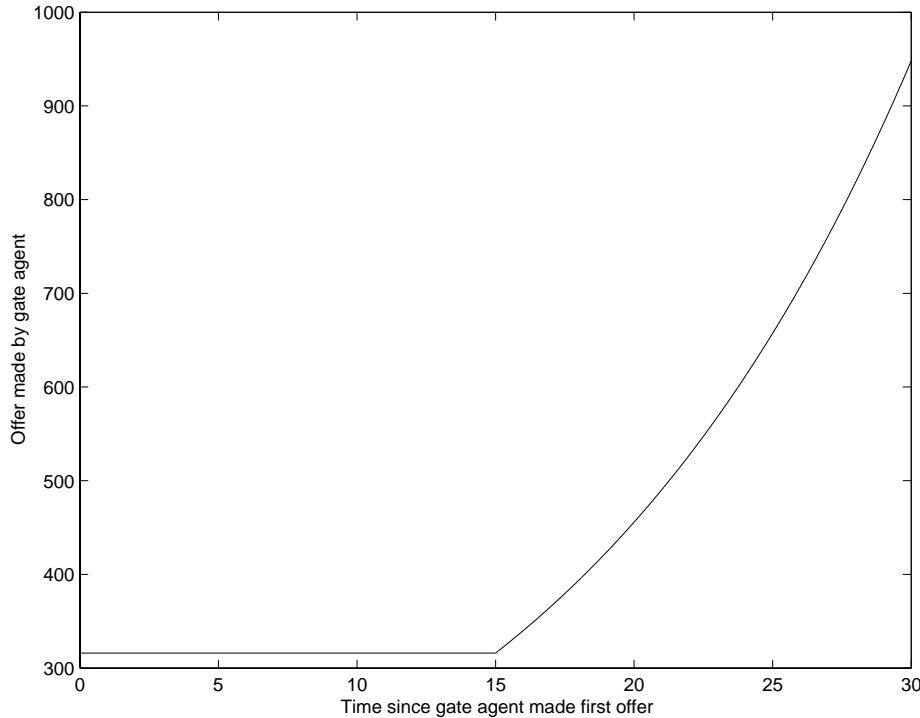
where  $B_{\$}$  is the compensation constant and  $r = r(B_{\$})$  is the exponential rate, chosen to fit the curve to the points  $(0, 316)$  and  $(20, 732)$ .

- **Time-Dependent Compensation Plan (Auction)** The primary shortcoming of the nonlinear compensation plan is that it does not deal with flights with too few voluntarily bumped passengers, where the airline must increase its compensation offering. We now approximate the costs of an auction-type compensation plan.

This plan assumes that the airline knows the number of no-shows and cancellations one-half hour prior to departure. The following auction system is employed. At 30 min before departure, the airline offers flight vouchers to volunteers willing to be bumped, equivalent in cost to the original airfare. This offer stands for 15 min, at which time the offer increases exponentially up to the equivalent of \$948 by departure time. We chose this number as twice the original airfare (which is the maximum obligatory compensation for involuntary passengers if they are forced to wait more than 2 h), plus one more airfare cost in the hope that treating the customers so favorably will result in future business from the same customers. These specifications

are enough to determine the corresponding time-dependent Compensation function, plotted in **Figure 1**.

$$\text{Compensation}(t) = \begin{cases} 316, & 0 \leq t \leq 15 \text{ min}; \\ 105.33e^{0.07324t}, & 15 \text{ min} < t \leq 30 \text{ min}. \end{cases}$$



**Figure 1.** Auction offering (compensation)

Consideration of passenger behavior suggests that we use a Chebyshev weighting distribution for this effort (shown in **Figure 2**). A significant number of passengers will take flight vouchers as soon as they become available. We simulate this random variable, which has probability density function

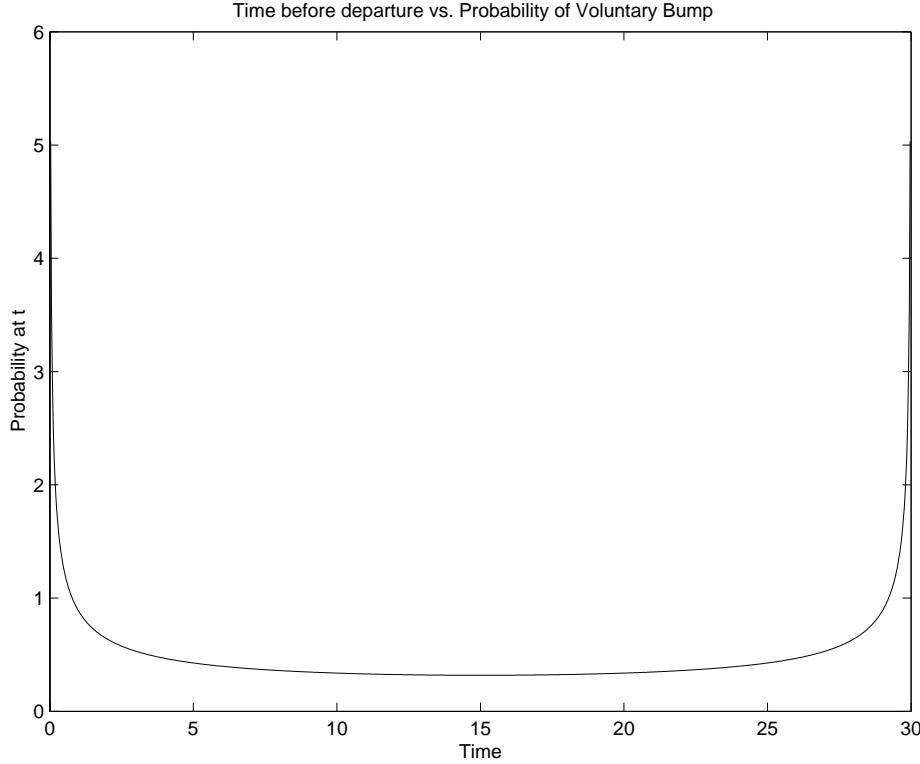
$$f(s) = \frac{1}{\pi\sqrt{1-s^2}}, \quad s \in [-1, 1],$$

and cumulative distribution function

$$F(\tau) = \int_{-1}^{\tau} \frac{1}{\pi\sqrt{1-\eta^2}} d\eta = \frac{1}{2} + \sin^{-1}(\tau),$$

where  $\eta$  is a dummy variable. Inverting the cumulative distribution function results in a method for generating random variables with the Chebyshev distribution [Ross 1990]:

$$F^{-1}(\tau) = \sin \left[ \pi \left( U - \frac{1}{2} \right) \right],$$



**Figure 2.** Chebyshev weighting function for offer acceptance

where  $U$  is a random uniform variable on  $[0, 1]$ .

With a linear transformation from the Chebyshev domain  $[-1, 1]$  to the time interval  $[0, 30]$  via  $t = 15\tau + 15$ , we find a random variable  $t$  that takes on values from 0 to 30 according to the density function  $f(s)$ . **Figure 3** shows the results of using this process to generate 100,000 time values. We use this random variable to assign times for compensation offer acceptance under the auction plan.

The total cost of bumping  $(X - C)$  passengers is  $\sum_{i=1}^{X-C} \text{Compensation}(t_i)$ .

## Optimizing Overbooking Strategies

Our goal is to maximize the expected value of the total profit function,  $E[T_P(X)]$ , given the variability of the bump function and the probabilistic passenger arrival model.

There are competing dynamic effects at work in the total profit function. Ticket sales are desirable, but there is a point at which the cost of bumping becomes too great. Also, the variability of the number of passengers who show up affects the dynamics. The expected value of the total profit function is

$$E[T_P(X)] = \sum_{i=1}^B T_P(i) \binom{B}{i} p^i (1-p)^{B-i}.$$

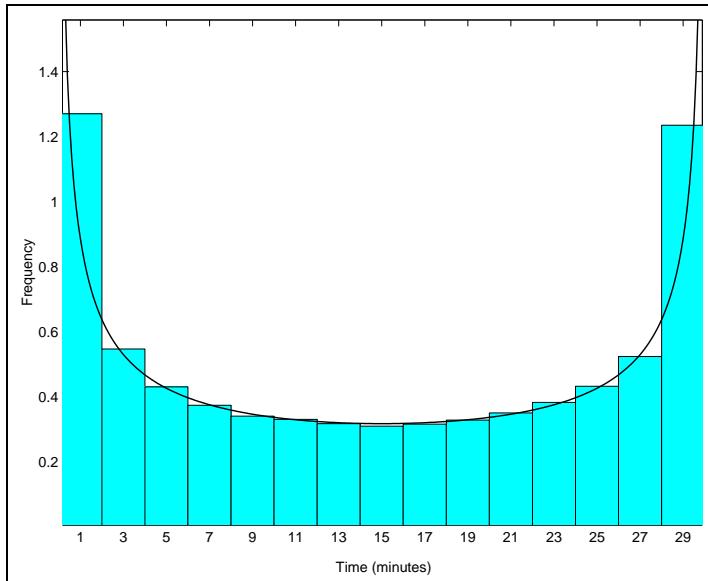


Figure 3. Histogram of 100,000 draws from the Chebyshev distribution.

We optimize the revenue by finding the most appropriate booking limit ( $B$ ) for any bump function. Solving such a problem analytically is unrealistic; any solution would require the inversion of a sum of factorial functions. Therefore, we turn to computation for our results. We wrote and tested MatLAB programs that solve for  $B$  over a range of trivial bump Functions.

## Results of Static Model Analysis

### No Overbooking

If Frontier Airlines does not overbook its flights, it suffers a significant cost in terms of loss of opportunity. If the number of people that booked ( $B$ ) equals plane capacity ( $C$ ), the expected value of  $X$  (number of passengers who arrive at the gate) is  $pB = pC = .88 \times 134 \approx 118$  passengers. Assuming (as in the total profit function) that each passenger beyond the 78th is worth \$300 in profit, the expected profit is nearly

$$(134 - 118) \times \$60 + \$300 \times (118 - 78) = \$12,960$$

per flight. This is only an estimate, since a smaller or larger proportion than 57.8% of ticket-holding passengers may arrive at the gate. The profit is sizeable but there are still (on average) 16 empty seats! The approximate lost opportunity cost is  $\$300 \times 16 = \$4,800$ ! Thus, not overbooking sends Flight 502 on its way with only 63% of its potential profitability.

## Bump Threshold Model

Using a 0.05 bump threshold, we compute an optimal number of passengers to book on Flight 502. Given the Airbus A319 capacity of 134 passengers and a passenger arrival probability of  $p = .88$ , the optimal number of tickets to sell to guarantee that bumping occurs less than 5% of the time is  $B = 145$ , or 107% of flight capacity.

## Linear Compensation Plan

**Table 1** shows the expected profit for various linear bump functions.

**Table 1.**  
Linear bump functions compared.

Bump cost per passenger	Optimal # to book	Expected profit per flight
200	$\infty$	$\infty$
316	162	\$17,817
400	156	\$17,394
500	153	\$17,121
600	152	\$16,940
700	151	\$16,799
800	151	\$16,692
900	150	\$16,601
1000	150	\$16,526

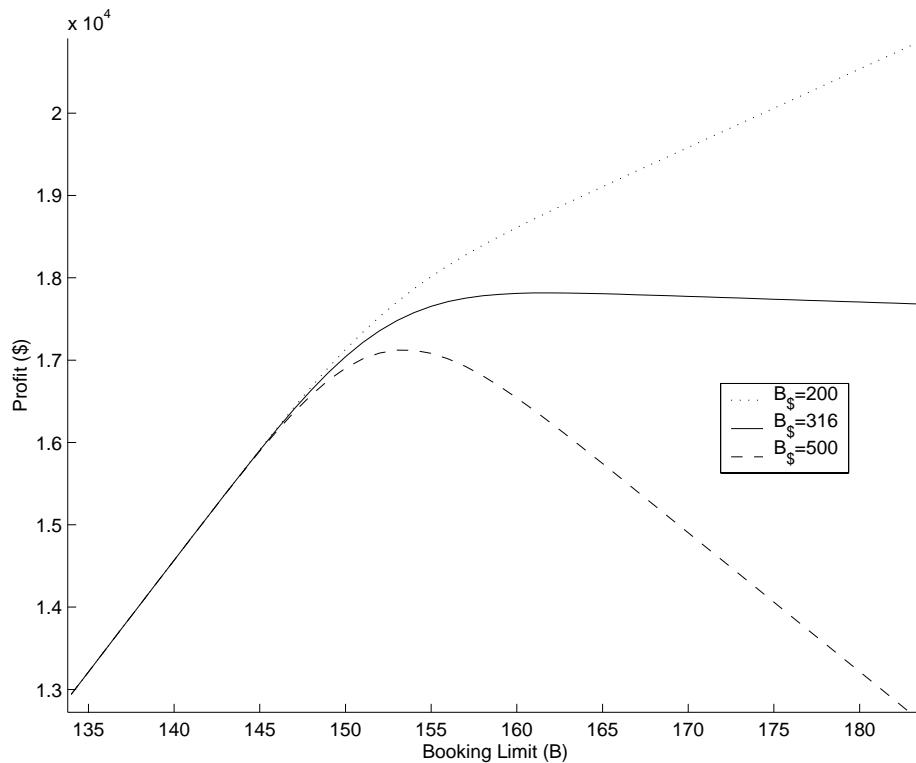
If Frontier were to compensate bumped passengers less than the cost of airfare, bumping passengers would always cost less than revenue gained from ticket sales. Thus, assuming it could sell as many tickets as it wanted, Frontier would realize an unbounded profit on each flight! Obviously, the linear compensation plan is not realistic in this regime, and we must wait for subsequent models to see increased real-world applicability. These results agree with the result of using a simple bump threshold above and indicate an average profit of approximately \$17,000. In comparison with using no overbooking strategy at all, Frontier gains additional profit of \$4,000 per flight!

The actual dynamics of the problem may be seen in **Figure 4**, where competing effects form an optimal number of tickets to sell ( $B$ ) when Frontier assumes a sizeable enough compensation average. We can also see the unbounded profit available in the unrealistic regime.

## Nonlinear Compensation Plan

Numerical results for the more realistic nonlinear model paint a more reasonable picture.

**Table 2** recommends booking limits similar to (though slightly higher than) previous models. The dynamics may be seen in the **Figure 5**.



**Figure 4.** Per-flight profit vs. booking limit ( $B$ ) for different bump costs (Linear Compensation Plan)

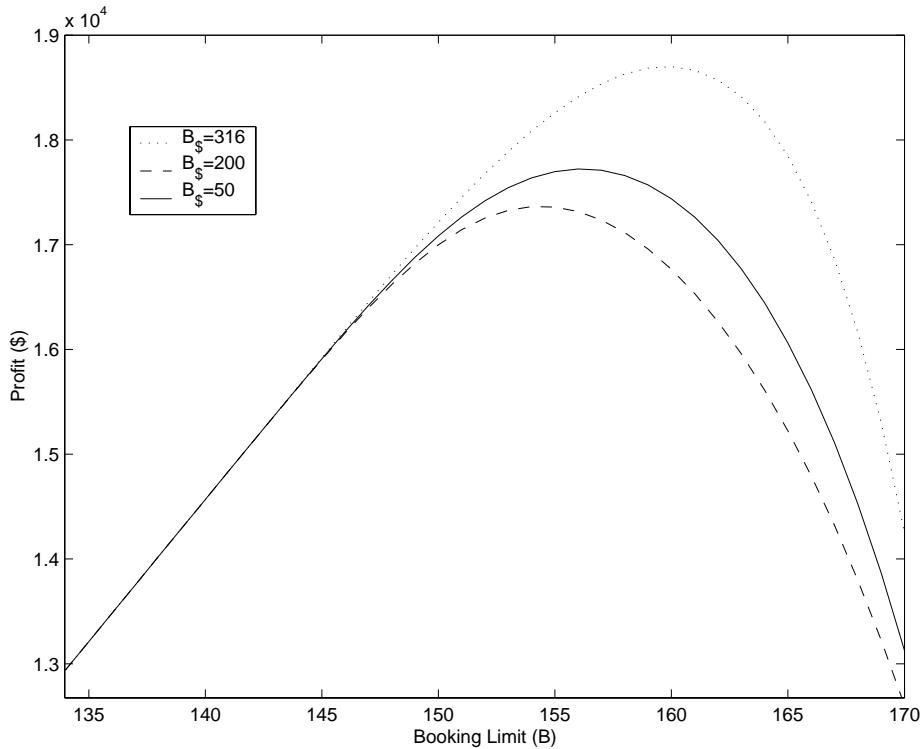
**Table 2.**  
Nonlinear bump functions compared.

Bump function	Optimal number to book	Profit per flight
$50e^{0.134(X-C)}(X - C)$	160	\$18,700
$100e^{0.100(X-C)}(X - C)$	158	\$18,240
$200e^{0.065(X-C)}(X - C)$	156	\$17,722
$316e^{0.042(X-C)}(X - C)$	154	\$17,363

All nonlinear bump functions that we investigated result in a maximum realizable profit, as expected.

## Time-Dependent Compensation Plan

The histogram of 1,000 runs using the time-dependent compensation plan in **Figure 6** shows that the optimal booking limit is most frequently  $B = 154$ . **Figure 7** is a graph of expected total profit versus the optimal booking limit for 15 trials, displaying the randomness due to the Chebyshev draws at higher values of  $B$ . If  $B$  is too low, then all models have the same profit behavior, because the randomness from the overbooking scheme is not introduced until customers are bumped. This graph also shows that regardless of random



**Figure 5.** Per-flight profit vs. booking limit ( $B$ ) for different bump functions (Nonlinear Compensation Plan).

effects, profitability is maximized around  $B = 160$ .

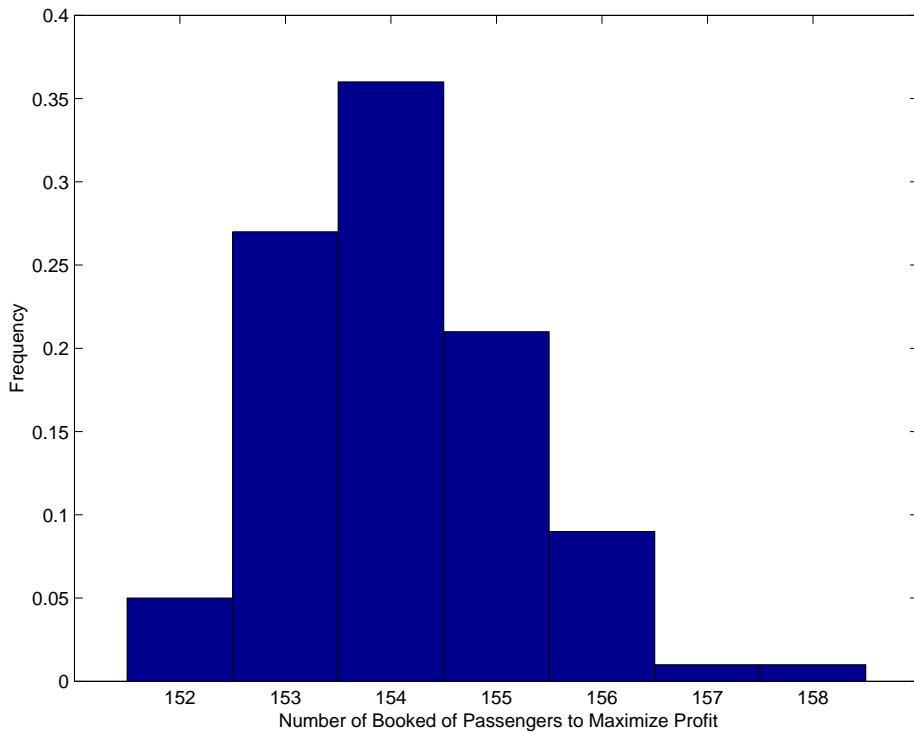
## The Dynamic Model

Many of the assumptions in the binomial-based models are loosened in this dynamic setting. Continuous time allows for more detailed analysis of the order of events in the airline booking problem. Keeping track of the order of reservation requests, ticket bookings, and cancellations results in a model that attempts to recommend what ticketing agents should do at a certain time. In the “Firesale Model,” we attempt to increase revenue by selling the tickets of cancellations to customers who would otherwise be denied tickets due to a fixed booking limit.

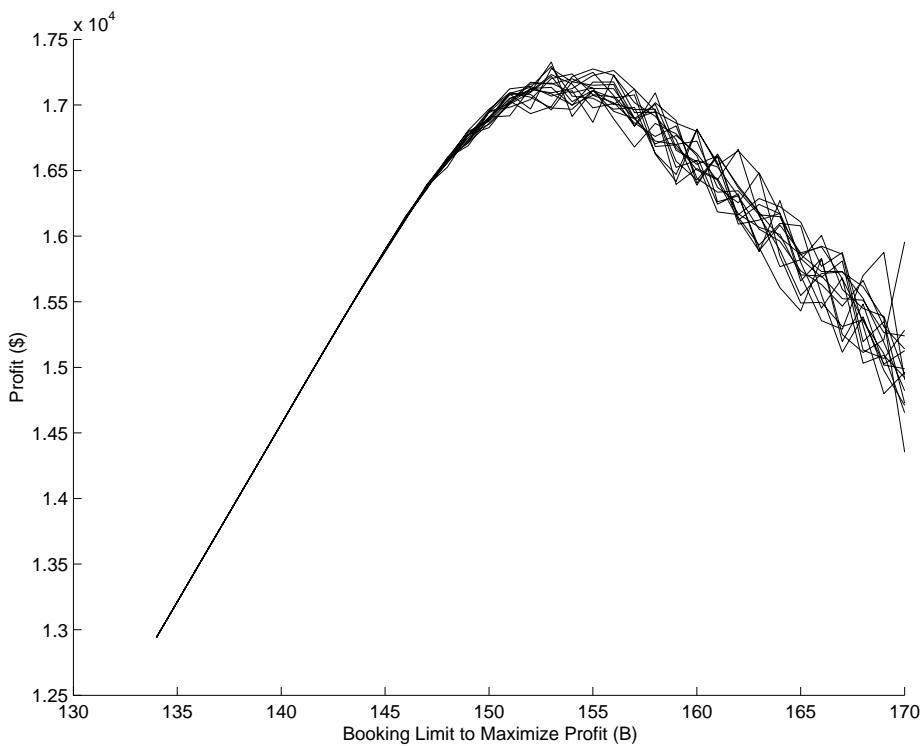
## Reservation Process

We simulate the booking/reservations process, which often begins weeks before departure and continues right up until departure (due, for example, to other airlines booking their bumped customers into Frontiers’ empty seats).

To model the stream of reservation requests, we employ a Poisson process  $\{N(t), t \geq 0\}$ —a counting process that begins at zero ( $N(0) = 0$ ) and has



**Figure 6.** Time-dependent compensation plan simulated 1,000 times



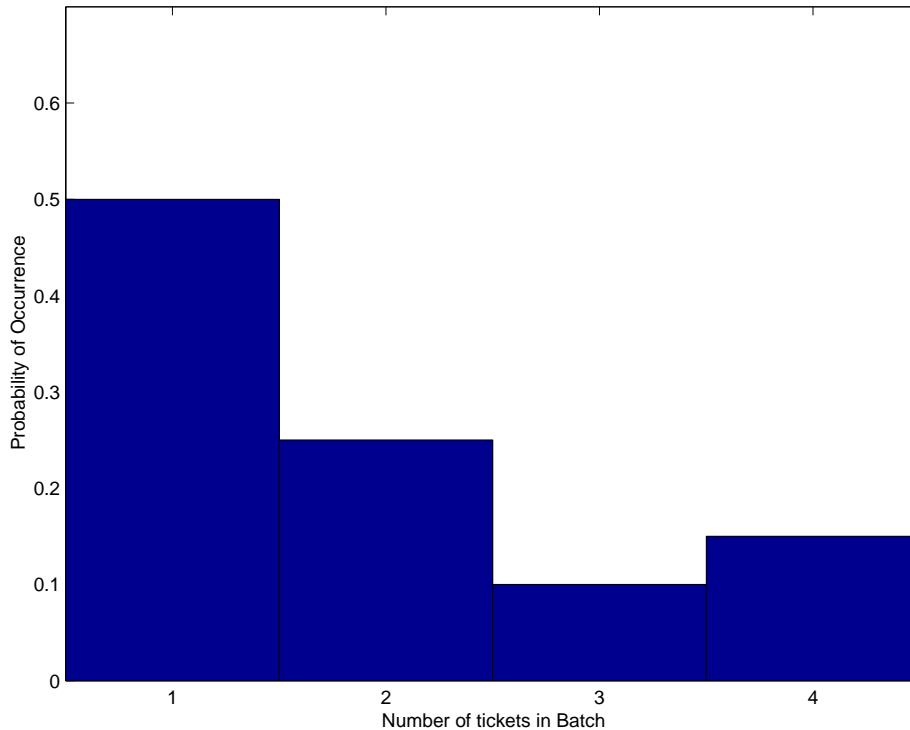
**Figure 7.** 15 time-dependent compensation plan simulations

independent increments, with the number of events in any interval of length  $t$  Poisson-distributed with mean  $\lambda t$  [Ross 2000]. The interarrival times of a Poisson process are distributed according to an exponential distribution with rate parameter  $\lambda$ . Each reservation request comes with a variable number of tickets requested for that reservation. The number of tickets requested is generated from some specified batch distribution, `BatchD`, that we introduce later.

This arrangement results in a compound Poisson process (in this case, a “stuttering” process [McGill and Garrett 1999]), which provides a more reasonable fit to real-world reservation request data than simpler processes.

Simulating the first  $T$  time units of a Poisson process using the method in Ross [1990] results in a vector `at` of arrival times for the  $A = \text{length}(\text{at})$  reservation requests received.

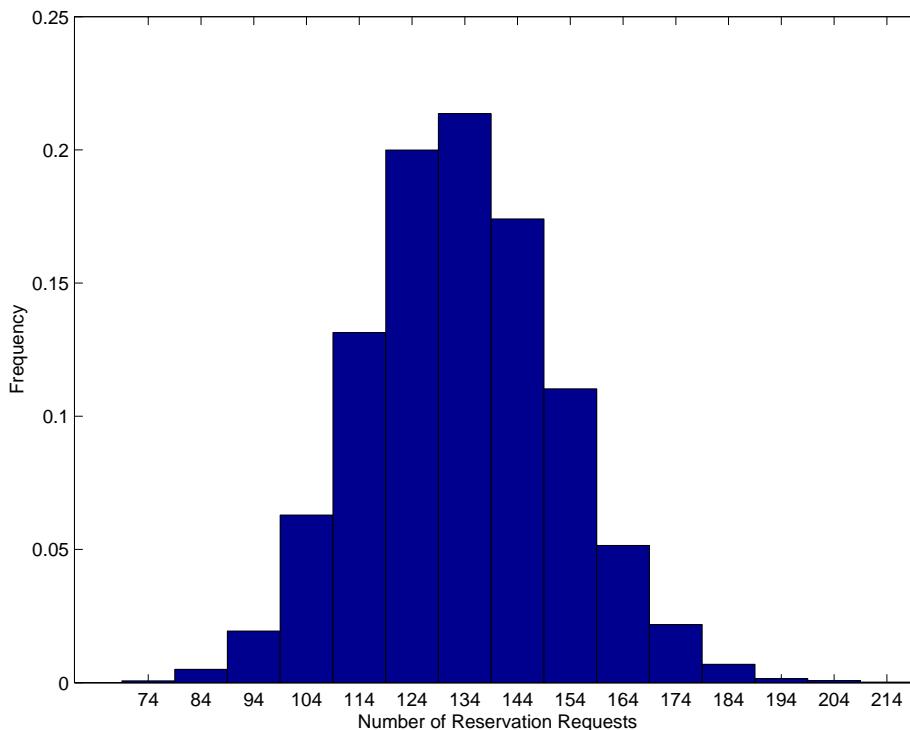
Another vector, `Bnum`, the number of tickets requested in each of the  $A$  reservations, is also generated according to the batch distribution. The density `BatchD` is shown in **Figure 9**; it states that callers reserve anywhere from 1 to 4 tickets at a time, with varying probabilities for each number. The total number of tickets (potential fares) requested is then  $\sum_i (\text{Bnum}(i))$ .



**Figure 9.** Density function for number of tickets in a batch of reservations.

The arrival rates for these reservation requests are derived by setting the expected value of the Poisson process over an interval of length  $T$  equal to the average ticket demand  $A_D$  that we expect. Then a rate of  $\lambda = A_D/E_B T$ , where  $E_B$  is the expected value of `BatchD` (1.9 in this case), will on average generate  $A_D$  tickets. The histogram in **Figure 10** shows the results of a simulation of 10,000

Poisson processes outputting the number of reservations requested when the average demand for tickets was 134 ( $A_D = C$ ).



**Figure 10.** Histogram of number of reservation requests for 10,000 flights with an average demand of 134 tickets.

## Cancellations and No-Shows

The binomial-based static models do not distinguish between cancellations (tickets voided before the flight departs) and no-shows (tickets not used or voided by flight departure); however, the dynamic model is well-suited for monitoring these events. We assume that 75% of unused tickets are cancellations and 25% are no-shows. Additionally, we assume that the time of cancellation for a set of tickets reserved together is uniformly distributed from the time that the tickets are granted to the time that the flight departs. This means that some cancellations occur almost immediately after the ticket(s) are granted (e.g., due to a typo on an online ticket service form), while some occur just before a plane is scheduled to depart (e.g., a last-minute change of plans). Lastly, we assume that multiple tickets in a single reservation behave equivalently (i.e., families act as unbreakable groups!).

To simulate this process, for each requested reservation a biased coin is flipped to determine with probability  $p$  if the group will keep their tickets. If not, another biased coin is flipped to determine whether the unused tickets are cancellations or no-shows. If a cancellation occurs, a cancellation time is drawn uniformly between that batch's arrival time and the flight departure time.

## Dynamic Booking

### Dynamic Test Model

We use the dynamic model to make the binomial-based models more realistic by eliminating some assumptions and introducing randomness. The Dynamic Test allows for “group tickets” (for both reservations and cancellations). The Dynamic Test requires that average ticket demand  $A_D$  be specified, so as to confirm the expected effects of less demand for tickets.

### Firesale Model

The Firesale Model uses cancellation times to sell all possible tickets. If the number of tickets requested (at time  $t$ ) for a particular reservation plus  $Tix$  (the number of tickets approved and still held at time  $t$ ) is less than the predetermined booking limit ( $B$ ), then a reservation request is approved. Conversely, if  $Tix(t)$  is equal to the booking limit or if the sale of the multiple tickets requested in a reservation batch would push  $Tix(t)$  over the booking limit, the request is rejected. Thus, for a process with no cancellations, reservation requests totaling less than the booking limit would be approved while subsequent requests would be rejected. The Fireside Model is highly dependent on the average demand (i.e., if demand is high enough, the airline would end up with an overwhelming majority of no-shows, as opposed to cancellations). The Firesale Model is the most realistic model developed in this paper.

## Results of Dynamic Model Analysis

The Fireside Model attempts to capture a scenario where all tickets of cancellations are sold as long as there are customers willing to buy them. If demand for tickets is high enough, we expect to sell all tickets of cancellations, resulting in a large number of bumped passengers. However, because the airline profits \$60 from each cancellation or no-show and because the numbers of both cancellations and no-shows continue to increase as more tickets are sold, reasonable results are expected for reasonable ticket demand.

**Figure 11** plots expected profit as a function of bumping limit as determined from 1,000 Fireside Model simulations. An average demand twice that of capacity ( $A_D = 268$ ) is used and a maximum profit is realized at a booking limit of 163. Most importantly, this figure displays how a small variation in booking limit could significantly alter profit. A change in either direction of 3 in corresponds to a loss of more than \$1,000 profit.

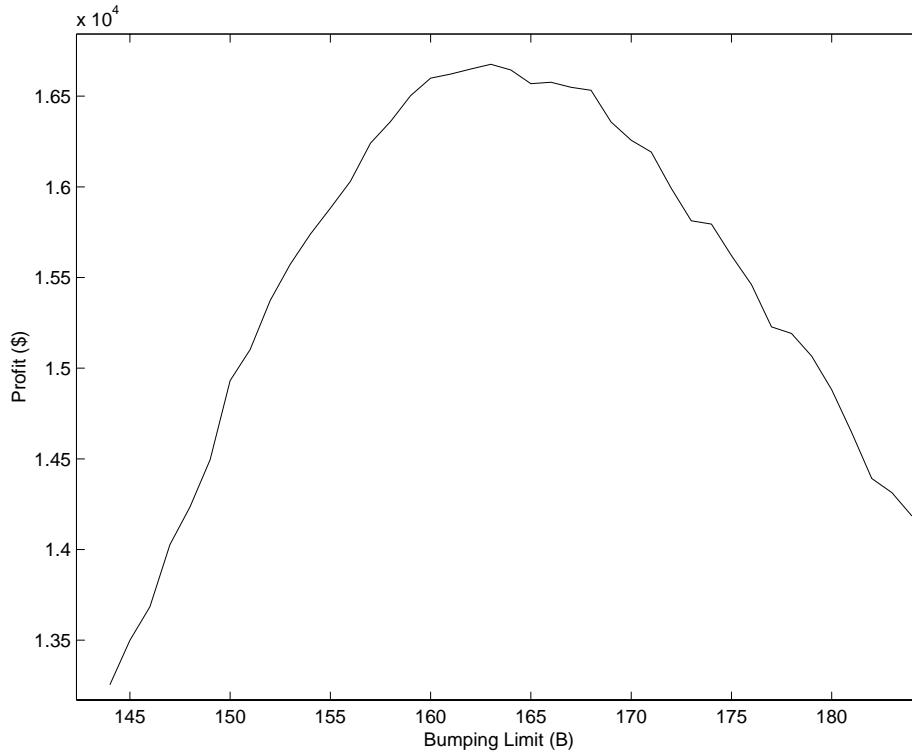


Figure 11. 1,000 simulations of the Fireside Model.

## Dynamic Testing of the Static Model

The dynamic model allows us to test the results from the static (binomial-based) models in a more realistic setting. The Dynamic Test allows tickets to be reserved in batches and introduces the randomness experienced in real-world airline booking.

In all testing, 10,000 simulations are performed for each booking limit ( $B$ ) and then expected profits are computed. Booking limit vs. Profit (\$) is plotted for appropriate booking limit values. The average demand ( $A_D$ ) used in this test is kept constant at twice the capacity of the airplane (so  $A_D = 268$ ), to simulate a very large pool of customers so that the overbooking process could be tested.

## Linear Compensation Plan

We tested two Bump costs ( $B_{\$} = \$316$  and  $B_{\$} = \$600$ ) with different behaviors (as predicted by the static model).

**Figure 12** shows that for this compensation plan, an optimal booking limit is  $B = 155$ , an increase of 3 from the optimal value for the static model. However, profit drops off steeply for booking limits over 155, indicating that a more conservative strategy might be to lower the bumping limit to ensure that this steep decline is rarely reached.

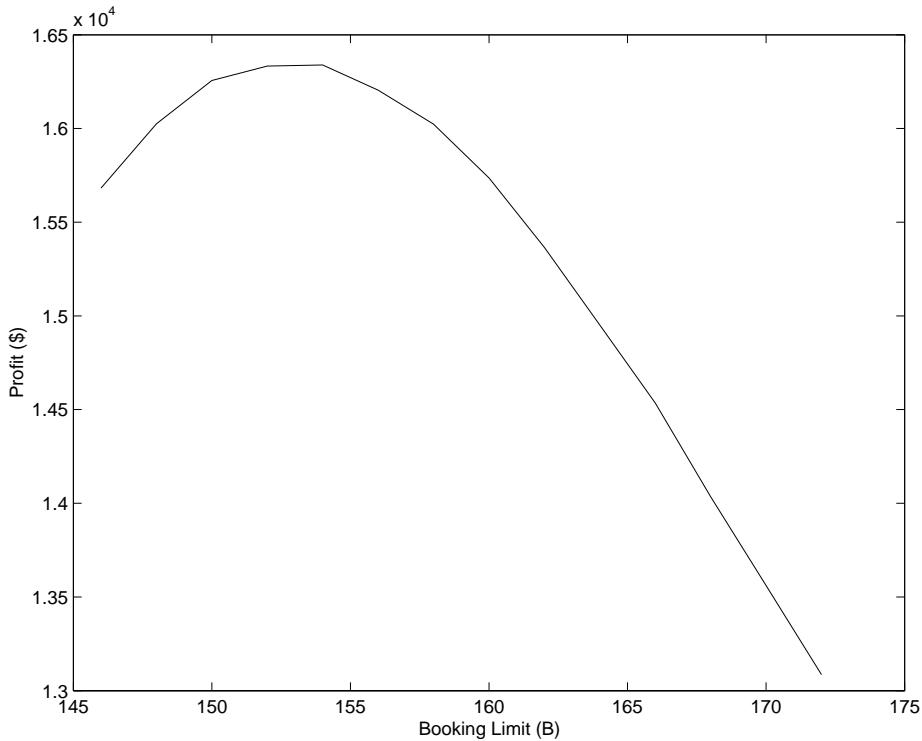


Figure 12. 10,000 simulations of the linear compensation plan with  $B_{\$} = \$600$ .

Figure 13 corresponds to a bump cost of 316; the optimal booking limit is now 166, again an increase (from 162).

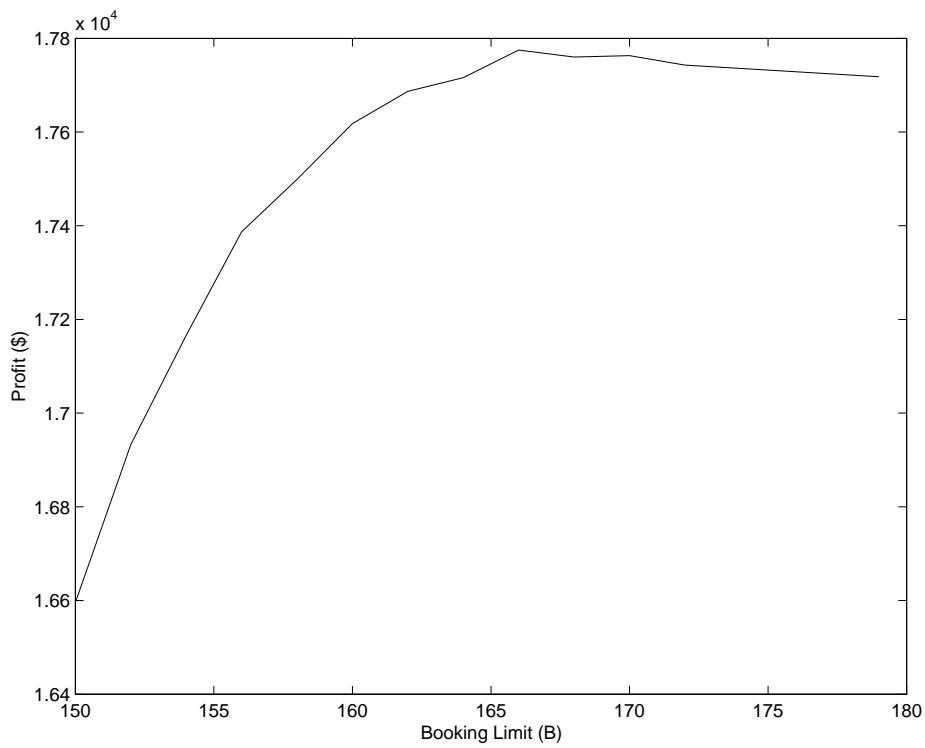
## Nonlinear Compensation Plan

We tested two nonlinear bump coefficients ( $B_{\$} = 316$  and  $B_{\$} = 100$ ) with different behaviors (as predicted by the static model).

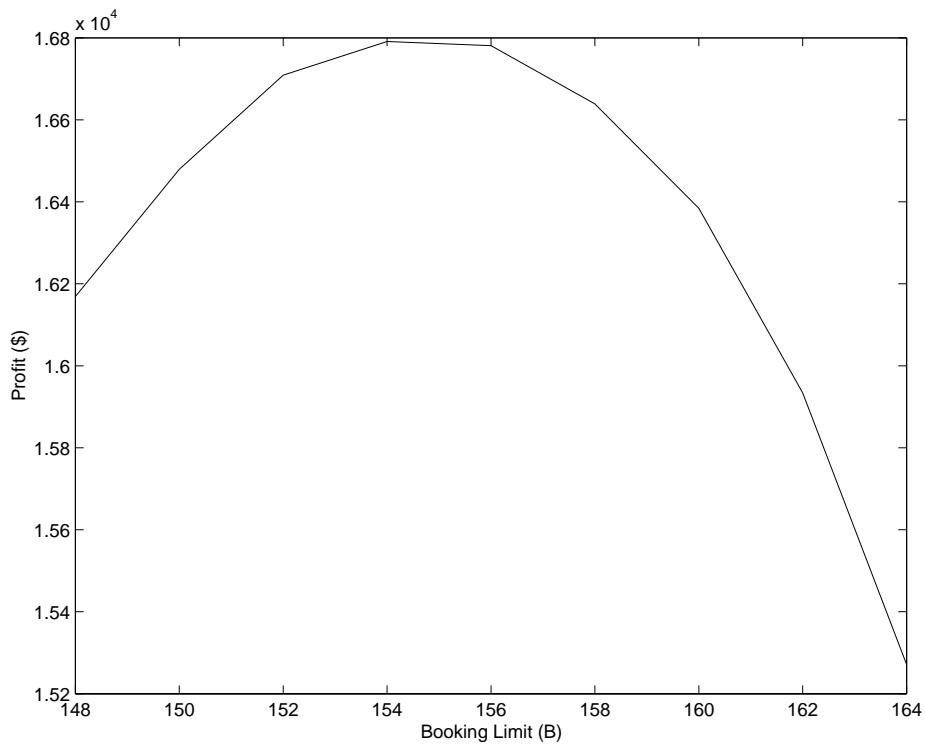
Figures 14 and 15 demonstrate the negative effect of too high a booking limit. For nonlinear bump coefficients  $B_{\$} = 316$  and  $B_{\$} = 100$ , optimal booking limits from the static model are 154 and 160, with Dynamic Test result values of 154 and 158.

## Time-Dependent Compensation Plan

Figure 16 shows that the optimal booking limit for the time-dependent compensation plan is  $B = 155$ , an increase of 1 from the static model. Profit appears to rise relatively steeply until the optimal booking limit is reached and then falls steeply. Thus, in our most realistic static model, a careful overbooking plan matters the most! If the booking limit were altered by 3, the profit would shrink by more than \$1,000, similar to the result detailed in the Fireside Model.



**Figure 13.** 10,000 simulations of the linear compensation plan with  $B_{\$} = \$316$ .



**Figure 14.** 10,000 simulations of the nonlinear compensation plan with  $B_{\$}=316$ .

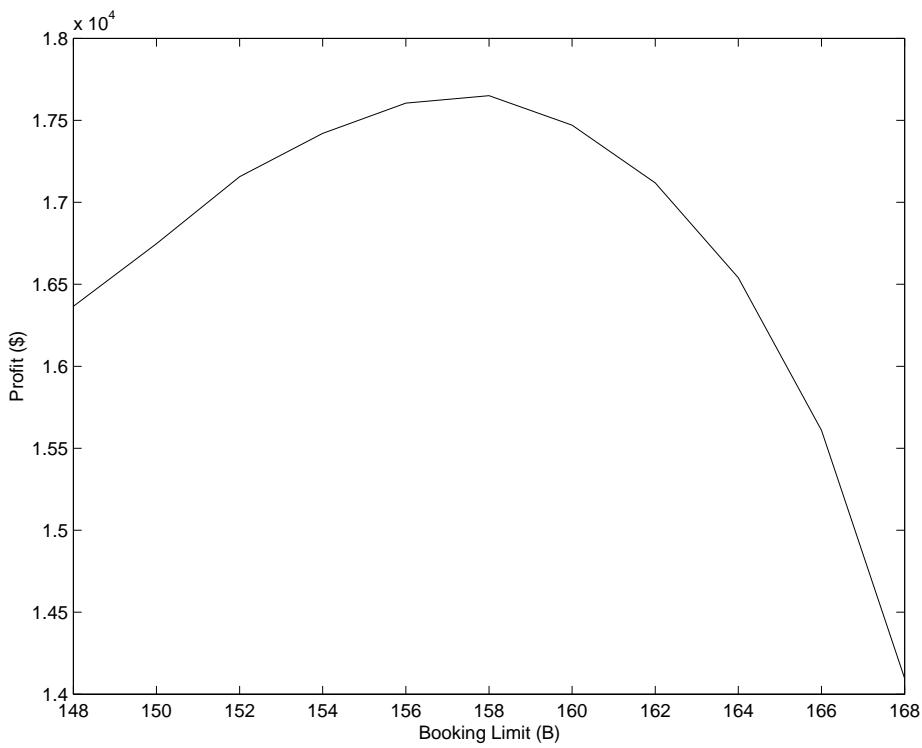


Figure 15. 10,000 simulations of the nonlinear compensation plan with  $B_{\$}=100$ .

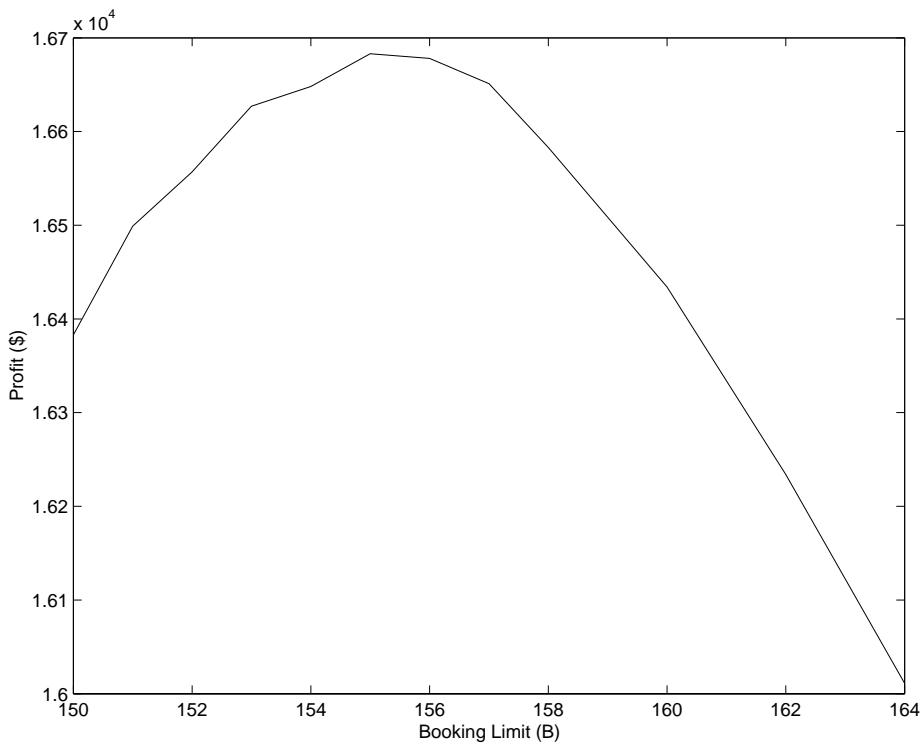


Figure 16. 10,000 simulations of the time-dependent compensation plan.

## Post-September 11 Effects

Security checks (at Denver International Airport) add only 10 min to check-in [“Frontier operating at 80%” 2001], which may be considered negligible.

The most significant post-September 11 effect that the airlines must consider is the consumer fear. The individual probability of passenger arrival  $p$  should not change drastically, since ticket-purchasing customers after September 11 are fully aware of the risks involved. A consequence of September 11 that is difficult to model is the decrease in average demand for flight reservations.

## Model Strengths and Weaknesses

### Strengths

- Time-dependent auction model for pre-flight compensation: When Frontier begins to offer compensation to voluntarily bumped passengers one-half hour before departure, our model allows consumer behavior to influence the financial results.
- Time-dependent decision process in the dynamic model: The dynamic model allows ticketing agents to decide whether or not to accept reservation requests based on the number of tickets sold by then and based on time until departure.
- Multiple considerations of consumer behavior via bump functions: The implementation of multiple bump functions allow for testing alternative strategies for compensation. Profit and customer satisfaction may then be balanced depending upon the company’s short-term or long-term interests.
- Varying degrees of model complexity: Our early models are simple, making sizeable simplifying assumptions to exhibit the most basic dynamics inherent in the problem. We take small steps of increasing complexity towards a more realistic model. The intuitive relationships between the results from each step lead to increased confidence in the stability and applicability of the most involved models.

### Weaknesses

- Absence of a stability analysis: We lack an adequate mathematical understanding of the stability of our models. Varying parameters like  $p$  could potentially alter our results.
- Infinite customer pool in the static model: In our static model, we assume that for any booking limit we set, all tickets will be sold.

- Insufficient data: The only operational data that we could get from Frontier Airlines was its quarterly report, which contains general information on how many people flew, operating costs, revenues, number of flights flown, and occupancy rates. However, our model lacks information regarding cancellation rates, no-show rates, cost per flight, rates of reservation requests, and ratio of restricted tickets sold to unrestricted tickets sold. The lack of this information limits us because our parameters are not based on historical data, and therefore we cannot be confident in the accuracy of our rates.

## Conclusion and Recommendations

Our models are quite consistent in recommending similar booking limits: 154 passengers on 134-seat Flight 502, 115% of capacity. This limit results in an average of \$17,000 per flight; so this one flight alone, by employing one of our overbooking strategies, nets the company an extra \$2.7 million profit per year, under the limiting assumption of an infinite demand pool.

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