Airliner Boarding and Deplaning Strategy

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Summary

To reduce airliner boarding and deplaning time, we partition passengers into groups that board in an arranged sequence. We assume that first-class and business-class passengers board first; our model treats only economy class. Since deplaning is the converse process of boarding, a strategy for boarding gives a strategy for deplaning.

We develop a model of interferences among passengers, which determine boarding time. We try to find a strategy with the least interferences. By running Lingo, we tackle the resulting nonlinear integer programming problem and obtain near-optimal strategies for fixed numbers of groups. This model supports the outside-in and reverse-pyramid strategies.

We develop another model to give a global lower bound for interferences. We also prove that individual boarding sequence, which boards passengers one by one in a particular order, attains that lower bound.

We develop code in C++ to simulate boarding strategies and test various strategies for three airliners: Canadair CRJ-200 (small), Airbus A320 (midsize) and Airbus A380 (large). Individual boarding sequence, reverse-pyramid, and outside-in are the best three strategies in terms of both average boarding time and its standard deviation.

We test strategies under various luggage loads and levels of occupancy, with and without late passengers and those with special needs. Outside-in and reverse-pyramid are stable under variation of parameters, whereas individual boarding sequence is extremely sensitive, though not to luggage.

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Our conclusions discredit traditional back-to-front strategies and support individual boarding sequence, reverse-pyramid, and outside-in. The more groups, the worse the situation with back-to-front. Taking cost into consideration, random sequencing should also be recommended.

Finally, we analyze deplaning and see how its time can be minimized.

Introduction

Airliner turnaround time is an important factor in determining airplane productivity [van den Briel et al. 2005], and boarding time constitutes an important part of turnaround time. *Boarding time* is the period between when the first passenger enters the plane and when the last passenger is seated.

Deplaning is also an essential part of turnaround time. We regard deplaning as the converse process of boarding, so an efficient boarding strategy brings out an equally efficient deplaning strategy.

A common approach in boarding is to partition passengers into several groups and board the groups in a sequence:

- **Back-to-front:** This is a traditional strategy.
- **Outside-in:** Window seats first, middle seats second, and aisle seats last.
- **Reverse-pyramid:** Discovered by van den Briel et al. [2005]. It is in use.
- **Individual boarding sequence:** Passengers are called to board one by one according to their seat number. This strategy is criticized as impractical; no known airline uses it.
- **Random:** There is no sequencing at all.
- **Rotating:** The deck is divided into blocks from back to front. The back block is called first, then the front block, the next back block third, continuing until the blocks meet in the middle.
- **Free seat choice:** Some airlines do not preassign seats to passengers; passengers choose their seat after boarding [Ferrari and Nagel 2005].

Previous Work

The back-to-front procedure is used by many major airlines [Finney 2006]. Recent research [Ferrari and Nagel 2005; Van Landeghem n.d.; Van Landeghem and Beuselinck 2002; van den Briel et al. 2005] discredits the effectiveness of back-to-front and recommends a version of outside-in, such as reverse-pyramid. These versions perform with similar efficiency, as substantiated by simulations.

Previous work tends to compare known boarding strategies. Research based purely on simulation is not fully satisfactory because of its lack of rigor. Among previous work, van den Briel et al. [2005] seems to offer the only well-rounded research. They combine analytical model, simulation, and practical implementation, but use a complex nonlinear integer programming problem. We simplify their model and do similar nonlinear integer programming.

Ferrari and Nagel [2005]:

- point out deficiencies in the simulation of Van Landeghem and Beuselinck [2002];
- offer more details on passenger behavior, such as a bin occupancy model and seating model, which they use to describe the effect of seat interferences;
- consider the passengers' seat preferences, while van den Briel et al. do not; and
- pay special attention to robustness.

Van Landeghem and Beuselinck [2002] is an excellent work, with data from the national carriers' database and from interviewing gate agents and method engineers. They give distributions for walking speed, time to store luggage, and time to sit. They also consider the distribution of the number of luggage items. They conclude that the most effective strategy is the outside-in by-seat strategy but do not offer a rigorous argument to defend that conclusion. They find that random sequencing, used frequently today, preforms well compared to most other strategies, which is somewhat surprising; only 9 of the 46 strategies that Van Landeghem and Beuselinck study do better than random. They conclude that in taking a structured approach to boarding, one should beware of making things far worse by choosing a wrong way of sequencing.

The data from Van Landeghem and Beuselinck [2002] and Van Landeghem [n.d.] have a strong impact on our simulation.

Problem Analysis and Basic Approach

It is difficult to arrange passengers precisely in a designated sequence. One basic solution is to partition the passengers into several groups, such as by row number or column letter.

Our basic boarding strategy is based on group partition. Groups board in a particular order, but passengers in the same group enter in a random sequence.

Assumptions

- Within a group, all sequences of passengers are equally likely; and permutations within different groups are independent of one another.

- We simplify the behavior of passengers. In reality, a passenger walks at a speed between zero and an upper bound; in our models, a passenger either keeps still or moves at a constant speed, but different passengers walk at their personal constant walking speeds. A static passenger does not require any time to accelerate to usual walking speed.
- The airliner deck is a rectangle. In reality, the shape resembles a rectangle with its corners truncated.
- We consider deplaning to be the converse of boarding except in the cases of random sequencing and free-seat-choice strategy. Thus, we consider only boarding.
- We follow the traditional strategy to assign a constant time for first-class and business-class passengers to board before economy class.
- Some large airliners, such as the Boeing 747 and Airbus A330-300, have two parallel aisles. We divide a two-aisle airline into two equal halves and treat each half as a single plane with only one aisle. (Sometimes the two halves are not exactly symmetric, as with the Boeing 747.)

Model I: Interferences

We describe passenger behavior with two parameters, the walking speed and the expected time to stow luggage. It seems impossible to obtain an explicit closed formula for expected boarding time, so we use expected boarding interference instead.

We use the definitions of van den Briel et al. [2005]. *Boarding interference* is a passenger blocking another passenger's access to his or her seat. There are two types: *seat interferences* and *aisle interferences*. Seat interferences occur when passengers seated close to the aisle block other passengers to be seated in the same row. Aisle interferences occur when passengers stowing luggage block other passengers' access to seats.

Model Description

Suppose that the plane has a single aisle. Let the groups be numbered and let groups enter the airliner in sequence 1, 2, ...

Assumption To avoid seat interference as often as possible, if passenger A of Group i and passenger B of Group j sit on the same half of a row (the same left row or the same right row), with $i < j$, we assume that A sits closer to the window than B.

With this assumption, we can describe a group partition and a sequence of groups by two matrices:

$$(x^j_{i,\mathrm{r}}),\ (x^j_{i,\mathrm{l}}), \qquad j = 1, \ldots, n;\ i = 1, \ldots, m,$$

where n is the number of groups and m is the number of rows. Group j consists of $x^j_{i,\mathrm{r}}$ passengers from the right half of row ith and $x^j_{i,\mathrm{l}}$ passengers from the left half of the row i. The sum of all the entries in the two matrices is the number of passengers on the plane

Due to the assumption, seat interferences occur only within the same group, so we have:

$$\text{E[seat interferences]} = \sum_{j=1}^{n}\sum_{i=1}^{m}\left[\tfrac{1}{2}\binom{x^j_{i,\mathrm{r}}}{2} + \tfrac{1}{2}\binom{x^j_{i,\mathrm{l}}}{2}\right], \tag{1}$$

$$\text{E[interferences within Group } i\text{]} = \frac{1}{s_j}\left[\binom{s_j}{2} + \sum_{i=1}^{m}\binom{x^j_{i,\mathrm{r}} + x^j_{i,\mathrm{l}}}{2}\right], \tag{2}$$

where

$$s + j = \sum_{i=1}^{m}(x^j_{i,\mathrm{r}} + x^j_{i,\mathrm{r}}),$$

and the expected number of aisle interferences between consecutive groups Group j and Group $j+1$ is

$$\frac{1}{s_j s_{j+1}}\sum_{i=1}^{m}\left[\left(x^j_{i,\mathrm{r}} + x^j_{i,\mathrm{l}}\right)\sum_{t=1}^{m}\left(x^{j+1}_{t,\mathrm{r}} + x^{j+1}_{t,\mathrm{r}}\right)\right]. \tag{3}$$

Then the expected number of aisle interferences is the sum of expected aisle interferences within groups plus the sum of expected aisle interferences between groups.

Equation **(1)** is interpreted as each pair of passengers in the same group and the same row on the same side (both right or both left) have probability $1/2$ of seat interference, since exactly one of their two possible boarding orders causes a seat interference.

For aisle interference, for each ordered pair of passengers there are $s_j - 1$ positions for the two passengers to board one after another, leaving $(s_j - 2)!$ ways for the remaining passengers in this group to board. Thus, the probability of such a situation is $(s_j - 1)(s_j - 2)!/s_j! = 1/s_j$, and there are

$$\binom{s_j}{2} + \sum_{i=1}^{m}\binom{x^j_{i,\mathrm{r}} + x^j_{i,\mathrm{l}}}{2}.$$

pairs that can cause interference; this gives **(2)**. In a similar way, we can calculate the expected aisle interferences of two consecutive groups. The only case of two passengers in different groups to cause interference is that they are the last of the previous group and the first of the next one. This happens with probability $1/s_j s_{j+1}$. Calculating all the interfering pairs gives **(3)**.

We define the evaluation of a strategy to be

$$\text{(expected aisle interferences)} + \lambda\text{(expected seat interferences)}. \qquad \textbf{(4)}$$

where λ is a positive number determined by the time needed to stow luggage and the constant walking speed of passengers.

Optimal Strategy in a Weak Sense

Because the number of aisle interferences between each pair of consecutive groups is no more than 1, the total of aisle interferences between different groups is less than the number of groups. When the number of groups is not very large, which is often the case, we neglect aisle interferences between different groups and concentrate on seat interferences and aisle interferences within the same group.

The first term in **(2)** is in fact constant when summed over j. The other terms in **(1)** and **(2)** are convex and monotonically increasing functions with respect to $x^j_{i,\mathrm{r}}$ and $x^j_{i,\mathrm{l}}$.

With aisle interferences between different groups neglected, and the number of groups and number of passengers in each group all fixed, the total number of expected interferences is a sum of convex functions. Therefore, the strategy with the fewest interferences must have the property that in the same group the difference of passengers in different half-rows is no more than 1. Moreover, the difference of passengers in different *rows* is also no more than one in the best strategy.

For instance, outside-in and reverse-pyramid strategies have the above properties. This indicates that outside-in and reverse-pyramid strategies might be optimal strategies when the number of groups is not large.

Results

As in van den Briel et al. [2005], let $\lambda = 1$. For each strategy, we could compute the total expected interferences using **(4)**. We use Lingo 8.0 to search for the optimal strategy with the least expected interferences with the number of groups fixed. The task is to determine the two matrices representing a strategy with the least interferences. We are faced with a nonlinear integer programming problem. The objective function is

$$\text{(expected aisle interferences)} + \text{(expected seat interferences)}.$$

Such a problem is NP-hard [van den Briel et al. 2005].

Due to the limitation of our computers, we could not determine the global optimal solution for a 60-passenger or larger plane, even in several hours.

Nevertheless, we ran Lingo, stopped the software after some tens of minutes of search, and observed the best solution found to that point. In rare cases, the lower bound of the objective function equaled the least interferences, which means that our computer found a global minimum. In many cases, the interferences of the best strategies found by computer is slightly greater than the lower bound of objective function.

Table 1 gives the results from Lingo for different airliner structures. Triples in the table denote structures of airliners; for instance, “2,3,11” means an airliner with 2 columns of seats on one side of aisle, 3 columns of seats on the other, and 11 rows. The incompleteness of the results of computation set aside, the best strategies found, as anticipated, are consistent with the theoretical results of the previous subsection.

Table 1.

Results from LINGO.

Airliner type	Structure	Number of groups	Best known strategy	Bound by Lingo
Airbus A380	2,3,15	2	47.0	46.4
		3	39.6	37.5
		4	40.4	33.1
		5	40.7	30.8
		6	40.3	29.2
	2,3,11	2	35.0	35.0
		3	29.7	28.5
		4	29.8	25.3
		5	30.1	23.5
	2,3,9	2	29.1	29.1
		3	24.6	24.1
		4	24.8	21.3
		5	24.9	19.4
Airbus A320	3,3,26	2	106.5	101.0
		3	80.5	80.5
		4	81.0	71.4
		5	81.3	65.6
		6	81.8	61.7
		7	81.4	58.9
		8	83.3	56.8
Canadair CRJ-200	2,2,14	2	29.5	29.5
		3	29.8	25.1
		4	29.1	22.7
		5	30.1	21.5
		6	30.6	20.3
Part of Boeing 747	2,2,19	2	39.5	39.5
		3	39.8	33.3
		4	40.3	30.1
		5	40.8	28.2
		6	45.1	27.0

Surprisingly, we got three exactly outside-in strategies, with the rest resembling either reverse-pyramid or outside-in.

Model II: Individual Boarding Sequence

Practical problems set aside, the most efficient strategy comes out of the finest group partition, in which each passenger corresponds to a group and each group consists of exactly one passenger. Passengers are arranged to enter the airliner in an expected order. Using this partition, the best solution is the individual boarding sequence strategy.

In minimizing interferences, there is an obvious lower bound: the back-seat passenger in each column must be blocked when the one just before is stowing luggage. Also, the front-seat passenger while stowing luggage must block the next passenger in sequence. Hence, the interference at least occurs $(n-1)$ times, where n is the number of columns, since every back-seat passenger of a column causes an interference except the first to board. Such a minimum can be attained actually when each back-seat passenger follows a front-seat passenger and there are no other interferences—which is an individual boarding sequence.

Certainly, this strategy is often considered seriously impractical. Van Landeghem and Beuselinck [2002] argue that comparable systems exist today. We devise a system that can be used to make the finest partition: At the airport, there is plenty of time between check-in and when passengers are allowed to enter. Airlines can assign numbers and letters to waiting seats at the airport. Passengers can be seated there according to their seats on the airliner. The airline can call passengers to board in sequence by the numbered seats.

Stochastic Simulations

To establish a simulation that can test various strategies and treat various kinds of airliners, we wrote a program in C++.

To test boarding strategies, our simulation should reproduce the boarding process as closely as impossible, so that the assumptions about human behavior are tenable and data describing passenger boarding behavior accords with reality. Our simulation is based on discrete time and continuous space; each time step is 0.5 s.

Assumptions and Details in the Simulation

- Business/economy assumption: First-class and business-class passengers board before economy-class passengers, in an assigned constant time.

- The aisle of economy class is narrow and cannot contain two passengers abreast. Thus, if a passenger is stowing luggage, the passenger behind in the aisle must wait and cannot pass.

- Between consecutive boarding groups, there is no time interval. That is, Group i boards in the wake of Group $(i-1)$.

- In free-seat-choice boarding strategies, passengers prefer seats more toward the window. Thus, the window seat is passengers' favorite and the aisle seat is the most unpopular. However, there is no preference for a particular row.
- There is only one way to reach each seat. Later, we discuss validity of this assumption in multi-aisle airliners.
- In the aisle, passengers can move only toward the back. If a passenger's seat is more toward the front than where the passenger is, the passenger has to go to the back and wait for the aisle to clear at the end of boarding.
- A passenger walks at an individual constant walking speed, but different passengers have different walking speeds. The walking speed distribution is a triangular distribution with lower limit 0.28 m / s, mode 0.365 m / s, and upper limit 0.45 m / s, based on observations by Van Landeghem and Beuselinck [2002]. This distribution is the sum of two continuous uniform distributions.
- The distance between consecutive passengers must not be smaller than 0.6 m. A passenger who walks fast enough to violate this limitation stops where the minimum is attained.
- We exclude several low-probability events: a passenger falls down, passenger mistakes another's seat for the passenger's own, or a seated passenger leave the seat voluntarily (e.g., for the toilet).
- The distance between rows is 33.25 in = 0.84 m [Wikipedia n.d.].

Bin Occupancy Model

As in Ferrari and Nagel [2005], we suppose that there is an overhead bin for each row on each side of the aisle and every passenger is assigned a random number of pieces of luggage, according to the probabilities in **Table 2**.

Table 2.
Luggage distribution at normal and high load.

	Number of pieces 0	1	2	3
Normal load	5%	55%	30%	10%
High load	5%	20%	55%	20%

The time that a passenger needs to stow luggage depends on how much luggage and the occupancy of the overhead bin, as follows:

$$t_{\mathrm{sl}} = 2.4\left(2 + \frac{n_{\mathrm{bin}} + n_{\mathrm{l}}}{2} \times n_{\mathrm{l}}\right)$$

when n_{l} is positive, with

- t_{sl} is the time to stow all pieces of luggage (seconds),
- n_{bin} is the number of pieces of luggage already in the bin, and
- n_{l} is the number of pieces of luggage carried by the passenger.

We let $t_{\text{sl}} = 0$ when $n_{\text{l}} = 0$.

Fractional results for t_{sl} are rounded to the nearest half-integer. The values of n_{bin} refer to the corresponding half-rows overhead; passengers always put their luggage into the bin corresponding to their half-row. In reality, if the overhead bin gets full, passengers have to move to other rows to find a bin. This fact is not reproduced directly by the simulation; however, note that t_{sl} becomes rather large for full bins.

The Seating Model

Our seating model too is inherited from Ferrari and Nagel [2005]. The time that passengers need to sit down depends on the number of interfering passengers (seat interferences) who are already seated. Those interfering passengers passengers have to get out of their row and then sit down again after the new passenger sits. The mathematical form of this is

$$t_s = t_p + 2t_p n_s = t_p(1 + 2n_s)$$

when n_s is positive, where

- t_s is the total time for seating (seconds);
- t_p is time used to get from the seat into the aisle or back (seconds), $tp = 3.6$; and
- n_s is the number of occupied seats in front of the passenger's seat.

We let $t_s = 0$ when $n_s = 0$.

Additional Assumptions of Free Seat Choice

Modeling free-seat-choice strategies is not easy. We need more assumptions.

- Passengers are supposed to be sagacious. That is to say, they know the best kind of seats that they can obtain under the worst situation. They know instantly how many of such kind of seats are guaranteed to be available to them. They sit at a seat of such kind with possibility of $1/n$, where n is the estimated number of available seats of that kind after then.
- Queued passengers may lose patience and accept a "bad" seat.
- If a passenger arrives at the last row with free seats, the passenger sits there.
- Passengers do not change their walking direction to find seats.
- We do not consider free seat choice in a two-aisle airliner, where two passengers could reach a seat from two aisles at the same time.

Simulation Results

Using the simulation software that we built in C++, we simulated each boarding strategy 50 times. Average boarding time indicates the performance of a strategy, while the standard deviation reflects its robustness.

The airliner in these simulations has full occupancy. Passengers are considered to carry luggage of normal load as indicated in **Table 2**.

We tested 14 strategies for Canadair CRJ-200, 16 for Airbus A320, and 9 for Airbus A380. These strategies come from back-to-front, outside-in, reverse-pyramid, random sequencing, rotating-zone, free-seat-choice, individual sequencing, and two strategies produced by computer from Model I. Except for individual boarding sequence, all strategies are currently practicable, with several in wide use. The notation "back-to-front 3" means a back-to-front strategy with 3 groups.

Canadair CRJ-200

We consider the CRJ-200 as a typical small airliner, with a rectangular 14-row deck and two columns on either side of the aisle. We tested 14 boarding strategies, with the results in **Table 3**.

Table 3.
Statistics of simulation of the boarding time of strategies toward CRJ-200 (min).

Strategy	Average	SD
Individual boarding sequence	3.7	0.3
Reverse-pyramid 3	6.5	0.7
Strategy 1 from Model I	6.8	0.6
Strategy 2 from Model I	6.8	0.5
Free seat choice	6.8	0.3
Outside-in 2	6.8	0.6
Outside-in 4	7.7	0.6
Random	7.8	0.8
Back-to-front 2	8.1	0.6
Back-to-front 3	8.2	0.7
Back-to-front 4	8.4	0.7
Rotating-zone 4	8.5	0.9
Rotating-zone 3	8.6	0.8
Rotating-zone 5	9.1	0.6

- The simulation supports the claim from Model II that individual boarding sequence has the shortest boarding time, only 3.7 min; all other strategies need at least 6 min. Moreover, individual boarding sequence's standard deviation is the smallest.

- The simulation results of Strategy 1 and Strategy 2 from Model I are both satisfactory, which substantiate Model I greatly.

- Among all strategies currently in use, reverse-pyramid, outside-in, and free-seat-choice are the soundest, with boarding time approximately 6.5 min. The standard deviation for free-seat-choice is quite small.
- The traditional back-to-front strategy is most disappointing, with average boarding time over 8 min.
- Considering the ease to perform free-seat-choice strategy and random sequencing (do not require any extra effort), they are acceptable choices.

Airbus A320 (Midsize)

The Airbus A320 is a typical midsize airliner. We consider it to have a rectangular 26-row deck with three columns on either side of aisle. We tested 16 boarding strategies; the two new ones are back-to-front 5 and back-to-front 6.

The results are completely analogous to those for the CRJ-200. Ironically, the more groups, the worse the situation is for the rotating-zone strategy and for back-to-front. Both are even worse than random sequencing.

Airbus A380 (Large-Size)

We take the Airbus A380 as a typical large airliner, with two decks. We divide its upper deck into two halves; we divide the lower deck into three parts horizontally and then divide each of three into two halves. Thus, the two-deck economy class is divided into eight parts and each part is treated as a single airliner. We assume that the A380 has two entrances in the front. Passengers are not allowed to cross between halves, so there is only one way to reach each seat.

Our strategy is a combination of strategies for each of the eight parts of the A380. Thus, there are many possible combinations, of which we selected nine to test. Individual boarding sequence performs best again, with the least standard deviation. We also find that large airliners are less sensitive to boarding strategies than smaller ones.

Sensitivity Analysis

In our simulations to this point, we assume that the airliner is full and passengers carry a normal load of luggage. Also, we exclude the possibility that passengers are late to board and neglect passengers with special needs, who are usually board first. Here we analyze the effect of these possibilities.

More Luggage

We compare normal load with high load of luggage for a full A320 with no late passengers. Individual boarding sequence still performs excellently with great robustness; its increase is below 10%. The remaining strategies all increase by 25% to 30%. These results indicate that sensitivity to luggage load should not be considered an important criterion in choosing boarding strategies.

Occupancy

We ran simulations with an A320 at 62.5% occupancy, as in Van Landeghem and Beuselinck [2002], with no late passengers and normal load of luggage. We randomly selected 37.5% of seats to be unoccupied. All strategies perform well. In comparison with full occupancy, however, individual boarding sequencing show the least sensitivity. Among the remaining strategies, reverse-pyramid, outside-in, and strategies from Model I have greater sensitivity; and back-to-front and rotating strategies are quite sensitive.

Late Passengers

A late passenger is one who does not board when the passenger's group is allowed to board but who reaches the gate before boarding ends. We assume that late passengers are not allowed to board before non-late passengers. We randomly choose passengers to be late, with probability 25%, for a full A320 with normal load of luggage. As expected, individual boarding, which requires the most precise sequencing, is the most sensitive to late passengers, requiring 70% more time than without late passengers—but still requiring less time than other strategies, all of which vary by at most 12% (increase or decrease) from their times without late passengers. One can think of increasing late passengers as making a strategy more like random sequencing; in fact, random sequencing has almost the same average boarding time with or without late passengers.

Passengers with Special Needs

We follow the tradition that passengers with special needs, such as the handicapped and mothers with children, board first. We compare an A320 with 5% special-need passengers with one with no such passengers, with full occupancy and normal load of luggage. At this level of passengers with special needs, average boarding time is not sensitive.

Strengths and Weaknesses

Strengths

- Our simulation can deal with different types of airlines, ranging from a 20-passenger jet to a two-deck Airbus-380.

- We give theoretical proof to support the excellence of individual boarding sequence; this proof is not found elsewhere.

Weaknesses

- Interferences overestimate boarding time.
- In Model I, our computer did not allow us to make a complete computation to obtain the global minimum.
- The data that we use to simulate passenger boarding behavior is specifically intended to model a certain type of airliner. However, passenger behavior, such as walking speed and time to stow luggage, varies in different types of airliner [Marelli et al. 1998].
- In the simulation of free-seat-choice, a passenger is unrealistically expected to foresee blocking ahead.
- We oversimplify the preferences for seats and rows, neglecting the possibility that some passengers love aisle seats and some passengers prefer front rows.
- Considering the need to maintain balance in flight, passengers are not allowed to be seated randomly in a non-full occupancy airliner. However, we neglect this point.

Conclusions

- With Model I, we translate the original problem into a nonlinear integer programming. Running Lingo, we almost get the optimal strategies with the number of groups fixed. The results from Lingo are outside-in strategies and many of them are only slightly different from reverse pyramid.
- With Model II, we give a proof that individual boarding sequence is the best strategy except for its impracticability.
- We test several kinds of boarding strategies. The results are in accordance with results from Model I and II, in terms of average and standard deviation of boarding time and in terms of robustness. Moreover, random sequencing is acceptable.
- Unfortunately, the traditional back-to-front is worst in many major aspects.
- Sensitivity toward luggage load should not be considered important.
- The more late passengers, the more a best strategy moves toward random sequencing. Individual sequencing is extremely vulnerable to late passengers.

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