Optimization of Irrigation

Bryan J.W. Bell
Yaroslav Gelfand
Simpson H. Wong
University of California
Davis, CA

Advisor: Sarah A. Williams

Summary

We determine a schedule for a hand-move irrigation system to minimize the time to irrigate a 30 m × 80 m field, using a single 20-m pipeset with 10-cm-diameter tube and 0.6-cm-diameter rotating spray nozzles. The schedule should involve a minimal number of moves and the resulting application of water should be as uniform as possible. No part of the field should receive water at a rate exceeding 0.75 cm per hour, nor receive less than 2 cm in a four-day irrigation circle. The pump has a pressure of 420 KPa and a flow-rate of 150 L/min.

The sprinklers have a throw radius of 13.4 m. With a riser height of 30 in, the field can be irrigated in 48 h over four days. Moreover, a single sprinkler is optimal. The pipes should be moved every 5 h and be at least 21 m apart. The resulting irrigation has precipitation uniformity coefficient of .89 (where 1 would be maximum uniformity).

We deal with each constraint in turn. Using geometrical analysis, we convert the coverage problem to determining the least number of equal-sized circles that could cover the field. We perturb the solution to optimize uniformity, by applying a Simultaneous Perturbation Stochastic Approximation (SPSA) optimization algorithm. We perturb this solution further to find the minimal number of pipe setups, by experimentally "fitting" the pipesets through the sprinklers. The rationale for perturbation is that some drop in uniformity can be tolerated in favor of minimizing the number of setups while still ensuring that we irrigate the entire field. We feed the optimal layout of pipe setups to another algorithm that generates an irrigation schedule for moving the pipes.

Assumptions

Main Assumptions

- The sprinkler has a throw radius of 13.4 m.

- Zero wind conditions. While wind affects precipitation uniformity, we do not explore this option.

- The field is reasonably flat, which allows us to assume equal water pressure at sprinkler nozzles.

- The rancher operates in 12-h workdays.

- All sprinklers operate on a 30-in riser. This is the most common riser configuration that we found.

- The rancher does not use the sprinklers when it rains.

- The sprinkler application rate profile is semi-uniform for the rotating spray nozzle sprinkler.

Other Assumptions

- There is only one accessible water source.

- The boundary of the field is lined with pipes that connect to the water source.

- Each pipe placement must be perpendicular to and touching one of the field boundaries.

- Set up time for the pipeset does not take more than an hour.

- The flow rate and water pressure from the source remains constant.

- As nozzle size increases, so does the flow rate loss.

- All sprinklers operate identically and do not malfunction.

- The throw radius of the sprinklers may exceed the bounds of the field.

- No sprinkler is placed outside the boundaries of the field.

- At the end of the workday, the sprinklers are shut off; the pipeset need not be disassembled.
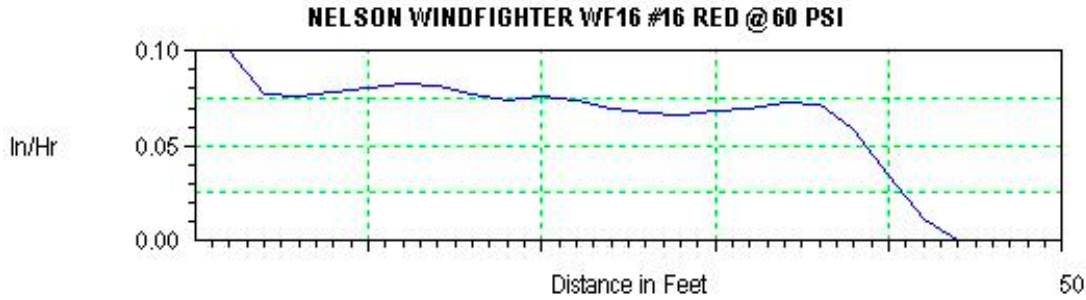
Figure 1. Sprinkler profile [Zoldoske and Solomon 1988].

Possible Sprinkler Profiles

We did not find any sprinkler application rate profiles for a 0.6 cm nozzle but we did find a profile for the Nelson Wind Fighter WF16 with a #16 Red nozzle (1/8″ ≈ 0.3 cm); **Figure 1** shows its application rate profile at 60 psi.

We assume that the profile for a 1/4″ (≈ 0.6 cm) nozzle would be similar but with a higher application rate. The flow rate for the WF16 with a 1/8″ nozzle is 3.42 gal/min; for our nozzle diameter of 0.6 cm = 0.236″, we have a flow rate of 12.57 gal/min, so we estimate that the application rate is 3.3 times as great, taking into account an increased loss due to the sprinkler (see later for the formulas used).

Model

Overall Approach

First, we tackle the requirement of sprinkling the entire field. Using geometrical analysis, we reduced this problem to a covering problem, which translates to finding the least number of equally-sized circles that can cover any given area.

However, this solution results in placing the sprinklers outside the field boundaries, so we perturb the solution to readjust the placement while maintaining complete coverage of the field.

We then use this new solution as a blueprint for finding the minimal number of pipe setups by experimentally "fitting" the pipesets through the sprinklers (if possible). We use an algorithm that iteratively perturbs the sprinkler layout and finds the minimum number of pipe setups. After a specified number of iterations, the algorithm outputs the minimum found. The rationale for perturbation is that we are willing to sacrifice some uniformity in order to find the least number of setups, while simultaneously ensuring that we still irrigate the entire field.

We feed the layout of pipe setups to another algorithm to generate an irrigation schedule.

Simulating Sprinkler Irrigation

Given the sprinkler positions, the sprinkler precipitation profile, and the length of time that they are on, we simulate in Matlab the sprinkler irrigation of the field. **Figure 2** shows the the output for the sprinkler profile of **Figure 1** with the sprinkler running for 1 h.
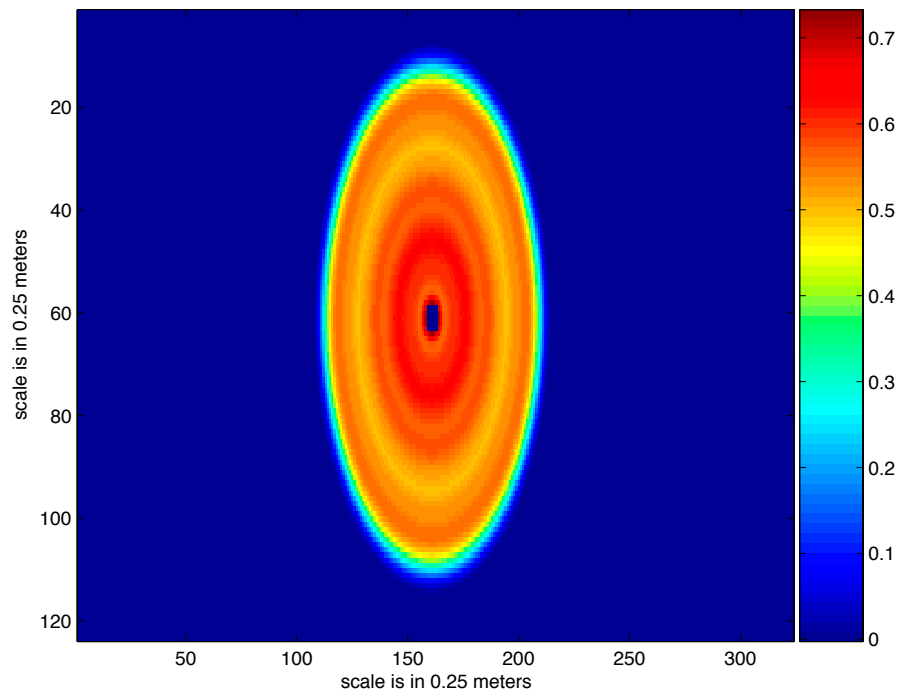

Figure 2. Matlab simulation of sprinkler irrigation over 1 h.

To represent the field, we use a matrix of cells. For the simulation, we have a list of sprinkler positions, and for each sprinkler specify how long it runs.

We iterate through the list of sprinkler positions; for each, we simulate the precipitation due to the sprinkler.

To simulate precipitation from a sprinkler, we use a simple nested `for` loop to iterate through the cells within the wetted radius of the sprinkler. For each cell, we compute the distance to the sprinkler and then use the given sprinkler precipitation rate profile and the length of time the sprinkler runs to calculate the additional precipitation received by that cell.

Complete Sprinkler Coverage of Field

No area of the field may receive less than 2 cm of water every four days. To accomplish this, we think of a sprinkler's wetted area as a circular disk located in a rectangle representing the field. The problem then reduces to covering this rectangle with disks. However, because the distribution profile for the sprinkler is nearly uniform (see **Figure 2**), allowing for radial overlaps

will disturb overall uniformity and increase the number of sprinklers needed to cover the field. Hence, it is best to minimize overlap by minimizing the number of sprinklers while ensuring that every part of the field is completely covered. This can, however, be restated as a covering problem, in which we find the smallest number of equally-sized circular disks that can cover a given rectangle. **Figure 3** displays this solution; no other configuration of circular disks can cover this rectangle more efficiently [Kershner 1939].
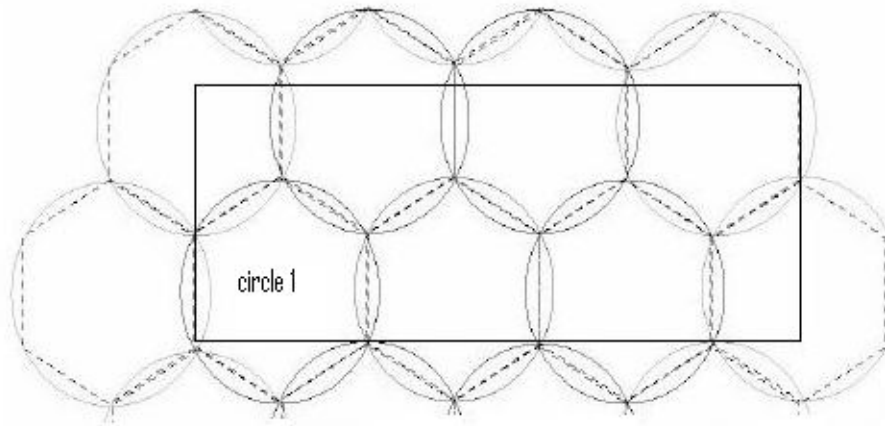

Figure 3. Hexagonal covering.

Adjustment of the Covering Problem Solution

This solution, however, is not completely useful, since it would result in some sprinklers being positioned outside the field. Moreover, adjusting sprinkler positions can also increase uniformity. In **Figure 4**, we show a possible placement of the field with the solution of the covering problem.

We need to adjust this solution, to ensure that the sprinklers are inside the field while covering it as uniformly as possible. From the hexagonal covering pattern (**Figure 3**), we know that the center of circle 1 is supposed to be located at a distance of $X = R\sqrt{3}/2$ to the right of field's left boundary. From spline interpolation of the data from the sprinkler coverage profile, we estimate $R \approx$ 13.4 m. But at 13.4 m, the precipitation rate is zero. We not only want the rate greater than zero but also for the total application to reach 2 cm in 5 h. Placing this constraint on the precipitation rate, we find that it has a radius of only 11.5 m; and anything beyond this will not provide the required coverage. Thus, to achieve the most complete coverage, we need to calculate the x-coordinate of circle 1, using $R = 11.5$ m, and then place all the other circles on the left row with the regular spacing of 13.4 m. This will result in the shift of the left
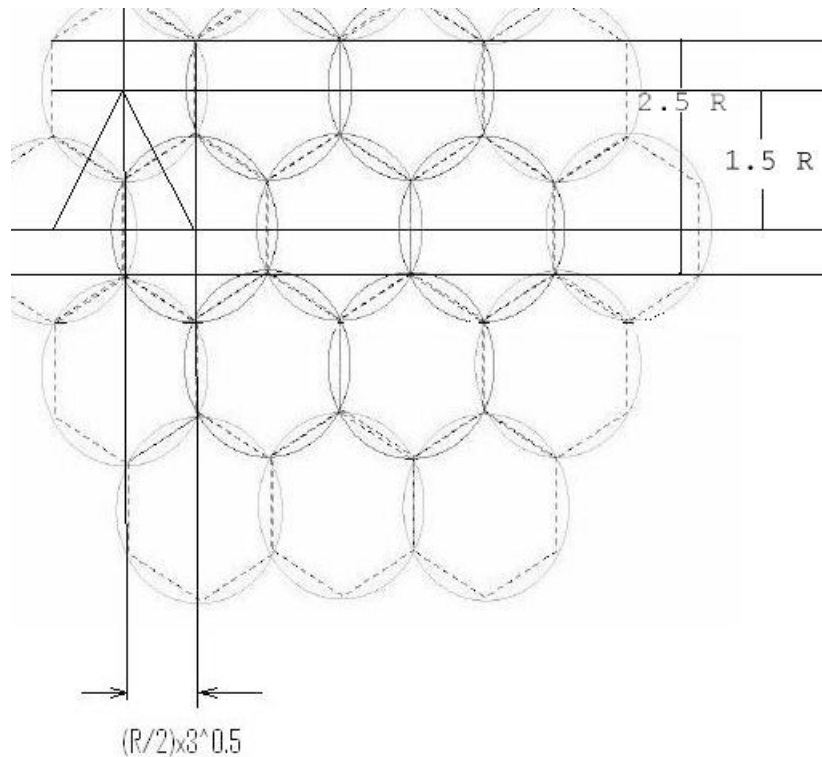
Figure 4. Hexagonal covering of the field.

row 13.4 m − 11.5 m = 1.93 m to the left, thus not only keeping the sprinklers within bounds but also increasing precipitation uniformity.

Maximizing Precipitation Uniformity

Applying water to the field as uniformly as possible is a main concern, since doing so leads to efficient crop yields [Lamm 1998]. We tackle this optimization problem using a Simultaneous Perturbation Stochastic Approximation (SPSA) optimization algorithm [Spall 2996], with which we minimize the standard deviation from the average precipitation in the field. **Figure 5** shows the result.

After 5,000 iterations, the solution seems to mimic the shifted covering problem solution—that is, our shifting method achieves a uniformity coefficient that adequately approximates the one from the SPSA, hence yields a sprinkler output that approximately maximizes uniformity.

Algorithm: Minimization of Pipe Setups

This algorithm takes as input the coordinates of sprinklers positions as determined by our approximation to the SPSA solution. From this layout, it minimizes the number of pipe setups by first selecting a sprinkler closest to the upper corner of the field, calculating the distances of all other sprinklers from it, and selecting the sprinkler with the shortest *lateral* distance (we cannot place a pipe diagonally). If this distance is less than or equal to the pipe length
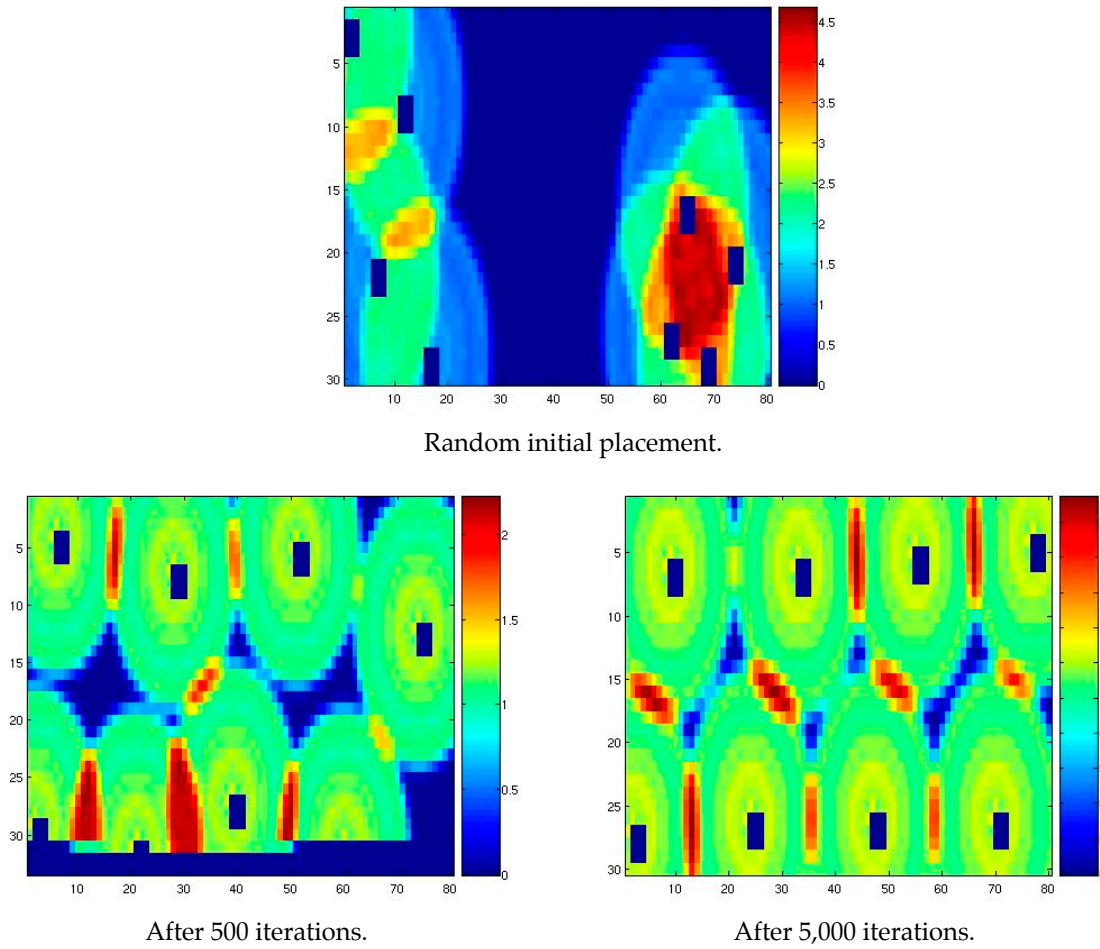
Random initial placement.

After 500 iterations.

After 5,000 iterations.

Figure 5. Sprinkler placement from the SPSA algorithm after a specified number of iterations.

(20 m), the algorithm calculates the precipitation rates at points located within the overlapping radii; if a rate exceeds 0.75 cm/h, the algorithm goes onto the next closest sprinkler.

Irrigation System Calculations

The problem statement specifies that the mainline pipe used in the hand-move system is aluminum with diameter 10 cm, the sprinkler nozzle size is 0.6 cm, and the water source has pressure of 420 kPa and possible flow rate 150 L/min. For our calculations, we use the following formulas from Rain Bird Agri-Products Co. [2001].

Hazen-Williams

$$\text{Pressure Loss (psi)} = 4.55 \, \frac{\left(\frac{Q}{C}\right)^{1.852}}{(ID)^{4.87}} \, L,$$

where

$$Q = \text{pipe flow (gal/min)},$$
$$C = \text{roughness coefficient (aluminum w/ couplers = 120)},$$
$$ID = \text{pipe inside diameter (in), and}$$
$$L = \text{pipe length (ft)}.$$

Nozzle Discharge

$$\text{Discharge (gpm)} = 29.82\sqrt{P}D^2C_d,$$

where

$$P = \text{nozzle pressure (psi)},$$
$$D = \text{nozzle orifice diameter (in), and}$$
$$C_d = \text{nozzle discharge coefficient (tapered} \approx 0.96 \text{ or } 0.98).$$

Since 0.145 kPa $= 1$ psi, the system pressure is at most 60.9 psi. The nozzle size is $(0.6 \text{ cm})/(2.54 \text{ cm/in}) = 0.236$ in; and assuming a nozzle discharge coefficient of 0.97, we obtain a flow rate per sprinkler of 12.6 gal/min$= 47.58$ L/min. The pressure loss due to the mainline pipe assuming four sprinklers is only 0.012 psi, which can be neglected. We assume that each sprinkler is on a 30 in riser and that the riser is a 1 in-diameter steel pipe. The pressure loss assuming a flow of 12.6 gal/min is 0.058 psi; thus, we also can neglect pressure loss due to the riser.

Results

Our model generates the optimal pipeset configuration shown in **Figure 6**. This configuration consists of 8 pipe movements each in intervals of 5 h (with an assumed 1 h time for moving and set up of equipment) and results in a total irrigation time of 48 h every 4 days, or approximately 12 h/d. Each pipeset contains only one sprinkler; if the sprinklers were closer than 21 m apart, the overlap of their wetted areas would yield precipitation greater than 75 cm/h. Less than 1% of the field receive an insufficient amount (2 cm) of water, in areas at the edges, where the crop could easily be damaged by other factors.

Table 1 shows the generated irrigation schedule for the repositioning of the sprinklers, given a 12-h work day for a rancher. Each pipe is set in place for 5 h. [EDITOR'S NOTE: We omit the table giving the irrigation schedule and coordinates for the sprinklers.]

Based on our assumptions and the design of our algorithm, there is no faster way to irrigate this field while maintaining such a high measure of uniformity.

As expected, this result is consistent with the earlier analysis, given our assumed sprinkler distribution profile. Our solution yields a uniformity coefficient of 0.89, unsurprisingly close to the optimal value of .90 generated by the SPSA after 5,000 iterations.
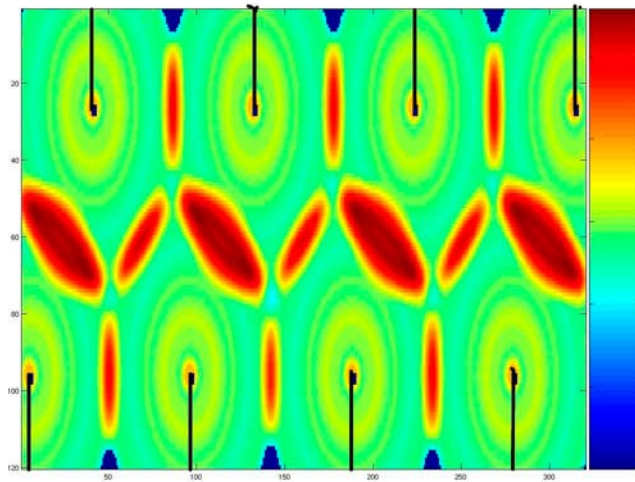
Figure 6. Best sprinkler placement, with pipe sets shown as black lines.

Weaknesses

- The model does not account for change in the sprinklers' profile due to wind, which could lead to a completely different model for sprinkler placement.

- SPSA ideally could have produced the best possible solution, but we did not have time to run it for enough iterations. Using FORTRAN would increase the speed of calculations by a factor of about 10,000.

- The rancher has to work 12 h/d, not 8 h/d.

- The rotating spray nozzle profile in our model is half the size of the one prescribed in the problem statement. Even though we scale the precipitation rate, there is no guarantee that the sprinkler profile would not change with flow rate.

- Coefficient of Uniformity (CU) provides an average deviation from mean coverage. Iff the area is overwatered due to the overlap of different sprinklers placed at different times, or overwatered in one spot and underwatered at another, the CU may come out the same [Zoldoske and Solomon 1988]. Thus, CU is not the most accurate way to measure the uniformity and water application, especially for our purposes, since we care less about minor overwatering than about underwatering.

- We could not validate our model in real-life conditions.

References

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Team members Bryan Bell, Simpson Wong, and Yaroslav Gelfand.

Team advisor Sarah A. Williams.