Summary

Our goal was to design a model that could account for the dynamics of vehicles in a traffic circle. We mainly focused on the rate of entry into the circle to determine the best way to regulate traffic. We assumed that vehicles circulate in a single lane and that only incoming traffic can be regulated (i.e. the incoming traffic could never have the right-of-way).

For our model, the adjustable parameters were the rate of entry into the queue, the rate of entry into the circle (service rate), the maximum capacity of the traffic circle, and the rate of departure from the circle (departure rate). We used a compartmental model with the queue and the traffic circle as comparments. Vehicles first enter the queue from the outside world, then enter the traffic circle from the queue, and lastly exit the traffic circle to the outside world. We modeled both the service rate and the departure rate as dependent on the number of vehicles inside the traffic circle.

In addition, we ran computer simulations to have a visual representation of what happens in traffic circles during different situations. This allowed us to examine different cases, such as unequal traffic flow coming from the different queues or some intersections having a higher probability of being a vehicle destination than others. It also implements several life-like effects, such as the way vehicles accelerate when on an empty road but decelerate when another vehicle is in front of them.

In many cases, we found that a fast service rate was the optimal way to maintain traffic flow, signifying that a yield sign for incoming traffic would be most effective. However, when the circle became more heavily trafficked, a slower service rate better accommodated the traffic, indicating that a traffic light should be used. Thus, a light should be installed in most circle implementations, with variable timing depending on the expected amount of traffic.

The main advantage of our approach was that the model was very simple and allowed us to clearly see the dynamics of the system. Also, the computer simulations that we ran provided more in-depth information about traffic flow under conditions that the model could not easily show, and enabled visual observation of the traffic. Some disadvantages to our approach were that we could not analyze the affects of multiple lanes nor stop lights that controlled the flow of traffic inside the circle. In addition, we had no way of analyzing singularities in the situation, such as vehicles that drive faster or slower than the rest of the traffic circle and pedestrians.

One Ring to Rule Them All: The Optimization of Traffic Circles

MCM Contest Question A

Team # 5180

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Contents

1 Introduction

Traffic circles, often called rotaries, are used to control vehicle flow through an intersection in both small towns and large cities. Depending on the goal

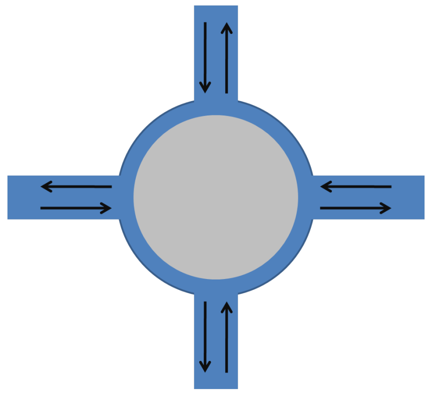

Figure 1: This figure illustrates a simple traffic circle. Traffic circles may have more than one lane and may have a different number of intersections.

of the rotary, it can take different forms. Figure 1 shows a simple model of a traffic circle upon which variations build. A circle can have one or more lanes; vehicles that enter the traffic circle can be met by a stop sign, a traffic light, or a yield sign; the circle can have a large or small radius; a circle can confront roads containing different amounts of traffic. These features of the traffic circle affect the cost of the circle to build, the congestion that a vehicle is confronted with as it circles the rotary, the travel time of a vehicle in the circle, and the size of the queue of vehicles waiting to enter the rotary. Each of these variables could be metrics for evaluating the efficacy a traffic circle.

Our goal is to determine how to best control traffic flow entering, exiting, and traversing a traffic circle. The design that we created modeled vehicles circling a rotary. We modeled the dynamics of a traffic circle by taking as given the traffic circle capacity, the arrival rates at each of the roads, the rate of departure from the rotary at each road, and the initial number of vehicles circulating in the rotary. Our metric is the queue length, or buildup, at each of the entering roads. We try to minimize the queue length by allowing the rate of entry from the queue into the circle to vary. In order for a vehicle to traverse the rotary efficiently, its time spent in the queue should be minimized.

We present a simple model that along with our approach makes the following assumptions:

- We assume a certain time of day so that the parameters are constant.
- There is a single lane of circulating traffic (all moving in the same

direction).

- Nothing impedes the exit of traffic from the rotary.
- There are no singularities such as pedestrians trying to cross.
- The circulating speed is constant (i.e. a vehicle does not accelerate/decelerate to enter/exit the rotary).
- Any traffic light in place regulates only traffic incoming to the circle.

2 The Models

2.1 A Simplified Model

We modeled the system as being continuous. This can be thought of as modeling the vehicle mass dynamics of a traffic circle. The simplest model that we created made the assumptions the rate of arrival to the back of the entering queue and the rate of departure from the queue into the traffic circle were given and independent of time t. Thus, the rate of change in the length of the queue is given by

$$\frac{dQ_i}{dt} = a_i - s_i \tag{1}$$

where Q_i is the length of the queue coming in from the i^{th} road, a_i is the rate of arrival of vehicles into the i^{th} queue, and s_i is the rate of removal, also called the service rate, from the i^{th} queue into the traffic circle.

We also modeled the vehicular flow circulating the traffic circle. To do this, we introduce the parameter d_i, the rate at which vehicles exit the traffic circle. We let be C the number of vehicles traveling in the circle. Then we model the change in traffic in the rotary by the difference between the influx and outflux of vehicles, where the outflux of vehicles is dependent on the amount of traffic in the rotary:

$$\frac{dC}{dt} = \sum s_i - C \sum d_i. \tag{2}$$

2.2 An Intermediate Model

The model proposed above simplifies the dynamics of a traffic circle. The most glaring simplifications that it makes are that there is no way to indicate that the circle has a maximum capacity and that the flow rate into the traffic circle s_i is not dependent on the amount of traffic already circulating. These are both corrected by proposing that the traffic circle has a maximum capacity C_{max}. As the number of vehicles circling approaches this maximum capacity, it should become more difficult for another vehicle to merge into the circle. At the extreme, when the traffic circle is operating at capacity, no more vehicles should be able to be added. Now, the s_i in the previous model can be represented logistically as $s_i = r_i(1 - \frac{C}{C_{max}})$, where r_i is how fast vehicles would join the circle if there were no traffic slowing them down. Thus, the equation governing the rate at which the i^{th} queue length changes becomes

$$\frac{dQ_i}{dt} = a_i - r_i\left(1 - \frac{C}{C_{max}}\right). \tag{3}$$

The equation governing the number of vehicles in the traffic circle becomes

$$\frac{dC}{dt} = \sum r_i\left(1 - \frac{C}{C_{max}}\right) - \sum d_i C. \tag{4}$$

2.3 A Congestion Model

The previous two models that we proposed failed to take into account congestion. We consider that congestion will alter the circulation speed. The circulation speed directly affects the departure rate d_i of the vehicles from the circle. Eq. 3 above still holds, but we need to find some way to indicate that d_i is not fixed. The vehicles will travel faster if there is no congestion, so they will be able to depart at their fastest rate $d_{i,max}$. When circle is operating at maximum capacity, the departure rate will decrease to be $d_{i,min}$. Then, the number of vehicles present in the circle is affected positively in the same manner as in Eq. 4, but the lessening factor changes to the weighted average of the $d_{i,max}$ and $d_{i,min}$:

$$\frac{dC}{dt} = \sum r_i\left(1 - \frac{C}{C_{max}}\right) - C\left(\sum d_{i,max}\left(1 - \frac{C}{C_{max}}\right) + \sum d_{i,min}\left(\frac{C}{C_{max}}\right)\right). \tag{5}$$

2.4 Extending the Model Using Computer Simulation

Concurrent and supplemental to our mathematical model, we created a computer simulation in MATLAB®. The simulation was designed to be able to account for variables that would be too complicated to use in the mathematical model. That model does not deal with the vehicle's speeds while inside the traffic circle, so the computer simulation focused mostly on areas related to vehicle speed. The computer simulation focused on the following:

- Enabling drivers to accelerate to fill gaps in the traffic (with a maximum speed)
- Forcing drivers to decelerate to maintain distance between cars
- Requiring that drivers accelerate and decelerate when entering and exiting the circle
- Giving probabilistic weights to the different directions of travel
- Keeping track of time spent within the traffic circle for each vehicle
- Giving each intersection a different vehicle introduction rate

Figure 2 on page 6 shows an outline of the program flow and design.

2.4.1 Simulation Assumptions

This model makes several key assumptions about the vehicles:

- There is only one lane of traffic
- No traffic control signals exist within the traffic circle
- Vehicles are all the same size
- Vehicles all have the same top speed
- Vehicles all accelerate and decelerate at the same rate
- Drivers all have the same spatial tolerance
- There are only four intersections
- There is only one circle size
- There are no pedestrians trying to cross the circle

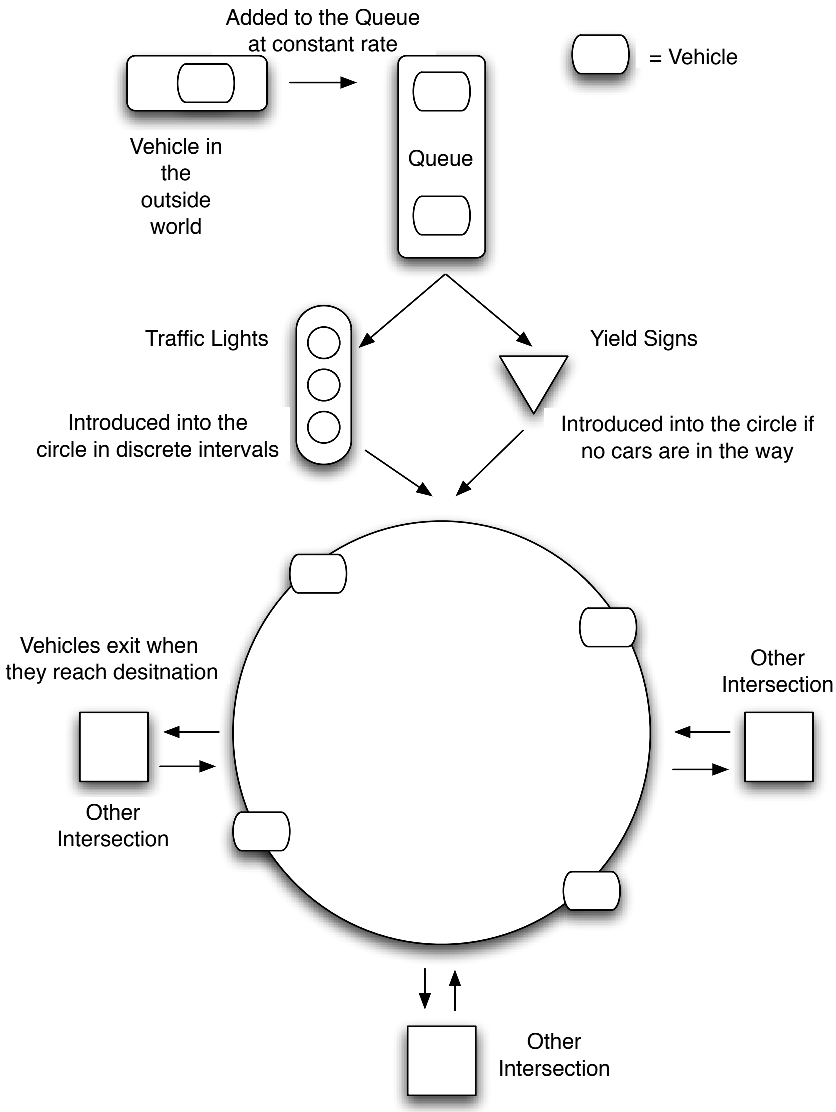

Figure 2: Chart showing program flow. Each intersection is modeled as a queue of vehicles with a traffic control device. Vehicles are added to the queue at a constant rate. For a vehicle to leave the queue and enter the traffic circle, the area in the circle must be clear of other vehicles. Additionally, if the queue has a stop light, the light must be active.

2.4.2 Limitations

The assumption of one lane is not a key factor because of our other assumptions. Since we do not allow for different vehicle speeds, we do not need to put the slow vehicles in one lane and the fast vehicles passing them in another lane. However, we do have slower vehicles if they ever become backed up behind vehicles trying to exit, or begin to exit themselves. This would be an opportunity for another vehicle to use a different lane to maintain their faster speed. Additionally, we cannot let emergency vehicles through the circle if there is only one lane. The vehicles present in the circle have no way to let the emergency vehicle by unless they exit. For a more detailed discussion of emergency vehicles and traffic circles, see [2].

By not allowing traffic control devices inside the traffic circle, we restrict the possible circle configurations we can explore. We also limit the effectiveness of our stoplight model. The model only prevents vehicles from entering the circle, it does not stop vehicles that are currently in the circle. They act much like a stop light on a busy highway that restricts the flow of vehicles entering the highway, but does not affect the vehicles currently on the highway.

Since we do not allow for different vehicle properties (size, acceleration, top speed, etc.), we cannot model the effect of large trucks, motorcycles, or other nonstandard verhicles on the flow of traffic. Since emergency vehicles are often large and slow, this is another factor preventing us from modeling them.

Giving all of the vehicles the same acceleration and top speed, along with forcing all of the drivers to have the same spatial tolerance prevents us from modeling aggressive drivers and their interaction with timid drivers. One kind of aggressive driver might only have a smaller spatial tolerance (they would put their vehicle in smaller gaps in traffic), while another might only accelerate and decelerate faster. It would be interesting to see how these accelerations affected the bunching together of vehicles, discussed on page 12 in section 3.4. Additionally, since cars decelerate before exiting even if they are already moving slowly, we generate small amounts of false traffic backups.

Only having four intersections limits our applicability in the real world. If the circle is created to help slow traffic, it is likely to have only four intersections. [2] Many traffic circles are not created just as a speed control device, and they often contain more than four intersections. Without further testing, we cannot determine how adding another intersection would actually affect the flow of traffic. We think it is likely that adding an intersection would give rise to a similar difference in traffic flow from increasing one intersection's service rate. However, we have to consider the fact that there is another exit for the vehicles as well. Without further development, this simulation is limited to four intersections.

Limiting the size of the circle does not really limit our ability to model real world traffic circles. Since we are mostly looking at driver behavior with the computer simulation, we should see the same behaviors as we scale up the circle and its corresponding traffic.

3 Analyzing the Models

3.1 The Simplest Model

In all of the above models, the rate r_i is indicative of the sort of regulation imposed at the i^{th} intersection. A near zero r_i indicates that a traffic light is in use. A longer r_i indicates that a yield sign regulating only the incoming traffic is in place.

For the simplest model, we were able to find explicit formulae for the queue length and the number of vehicles in the rotary by integrating with respect to time. We found that

$$Q_i = [a_i - s_i]\, t + Q_{i0}, \tag{6}$$

and

$$C = \frac{\sum s_i}{\sum d_i} + \left(C_0 - \frac{\sum s_i}{\sum d_i}\right) e^{-\sum d_i t}. \tag{7}$$

Therefore, given all of the inputs of the system, we would be able to predict the queue length. We note that to minimize the queue length, we solve Eq. 1 for when the queue length is decreasing $\frac{dQ_i}{dt} < 0$. This indicates that in order to minimize the queue length, the s_i term should be maximized.

3.2 Intermediate Model

For the model wherein we added a carrying capacity of the system. Again, the model was simple enough for us to find explicit formulae for the queue length and the number of vehicles in the rotary by integrating with respect to time. We found that

$$Q_i = \left[a_i - r_i\left(1 - \frac{C}{C_{max}}\right)\right] t + Q_{i0}, \tag{8}$$

and

$$C = \frac{\sum r_i}{\frac{\sum r_i}{C_{max}} + \sum d_i} + (C_0 - \frac{\sum r_i}{\frac{\sum r_i}{C_{max}} + \sum d_i} e^{-\left(\frac{\sum r_i}{C_{max}} + \sum d_i\right)t}. \tag{9}$$

Therefore, if given the other inputs of the system, we would be able to predict the length of the queues. We could also solve for where Eq. 3 is less than zero to find for what service rate the queue lengths are decreasing . We found that the queue length decreases when

$$r_i > \frac{a_i}{1 - \frac{C}{C_{max}}}. \tag{10}$$

3.3 Congestion Model

When we began to model congestion, the model was sufficiently complex that we were unable to intuit what conditions would optimize (minimize) the queue length. We noticed that the differential equation Eq. 5 was quadratic:

$$\frac{dC}{dt} = AC^2 + BC + D, \tag{11}$$

where

$$A = \frac{\sum d_{i,max}}{C_{max}} - \frac{\sum d_{i,min}}{C_{max}},$$
$$B = -\left(\frac{\sum r_i}{C_{max}} + \sum d_{i,max}\right),$$
$$D = \sum r_i.$$

Since $\sum d_{i,max} > \sum d_{i,min}$, it will always be the case $A > 0$. In addition, $B < 0$ and $D > 0$. This means that the curve for $\frac{dC}{dt}$ is a concave up quadratic curve with a positive y-intercept and a global minimum at some $C > 0$. Furthermore, notice that for $C = C_{max}$, $\frac{dC}{dt} = -\frac{d_{i,min}}{C_{max}}$ which is always negative for $d_{i,min} > 0$. Thus, the global minimum for the curve must be in the fourth quadrant. Figure 3 shows an example of such a curve using sample parameters.

We notice from Figure 3 that there are two equilibrium points for this differential equation: $C = \frac{-B-\sqrt{B^2-4AD}}{2A}$ is a stable equilibrium point and $C = \frac{-B+\sqrt{B^2-4AD}}{2A}$ is an unstable equilibrium point. Also, since for $C = C_{max}$, $\frac{dC}{dt} < 0$, the number of vehicles will eventually decrease to an equilibrium value less than C_{max}. We will call this equilibrium point for the number of vehicles C_{limit}.

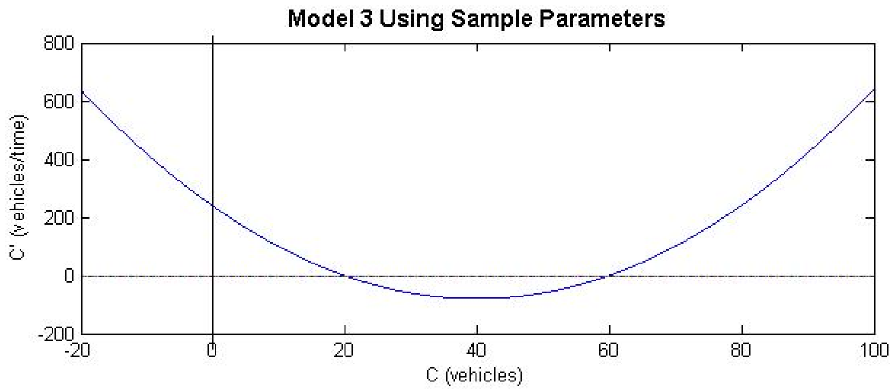

Figure 3: This graph shows the relationship between $\frac{dC}{dt}$ and C for the congestion model using some sample parameters: $r_1 = 60$, $r_2 = 60$, $r_3 = 60$, $r_4 = 60$, $d_{1,max} = 2$, $d_{2,max} = 2$, $d_{3,max} = 2$, $d_{4,max}$, $d_{1,min} = 0.5$, $d_{2,min}$, $d_{3,min}$, $d_{4,min}$, and $C_{max} = 30$.

Since our metric for how well a traffic circle is maintained depends on how many vehicles are in the queues, we would like the queue flow $(a_i - s_i)$ to be as small as possible. In other words, we would like s_i to be as large as possible. In the congestion model, this is given by Eq. 3.

Without loss of generality, we can analyze queue 1 because the equations for each queues only differ by their a_i and r_i. We will keep these constant among the queues in the mathematical simulations. Since the only changing variable in Eq. 3 is C, when $C = C_{limit}$, Q_1 will also be at its equilibrium.

Using this fact, we can evaluate whether we would like to use a traffic light or not and for how long the light would be red. Thus, we compared different values for the service rate constant r_1 and the value of $\frac{dQ_1}{dt}$ at $C = C_{limit}$. The results can be seen in Figure 4. The graph shows that when r_1 increases $\frac{dQ_1}{dt}$ decreases.

A situation that occurs in real life is congestion of the traffic circle causing the vehicles in the circle to move very slowly. Decreasing the value of $d_{1,min}$ would cause the vehicles to depart the traffic circle at a slower rate when there is more congestion in the circle. Using lower departure rates to approximate slower vehicle speeds inside the traffic circle, we can examine what happens for decreasing values of $d_{1,min}$. The results from this are shown in Figure 5. For values of $d_{1,min} < 0.5$ the smallest value for $\frac{dQ_1}{dt}$ is not at $r_1 = 60$, but for smaller values of r_1.

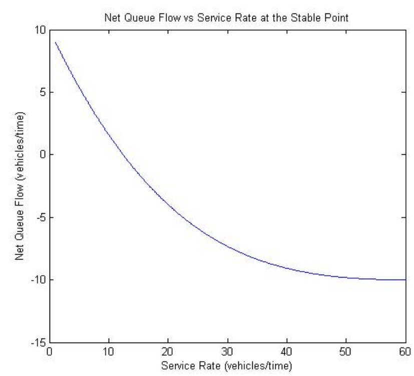

Figure 4: This graph shows the relationship between r_1 and $\frac{dC}{dt}$ for the congestion model with $C = C_{limit}$. The constant parameter values are $d_{1,max} = 2$, $d_{2,max} = 2$, $d_{3,max} = 2$, $d_{4,max}$, $d_{1,min} = 0.5$, $d_{2,min}$, $d_{3,min}$, $d_{4,min}$, and $C_{max} = 30$. r_1, r_2, r_3, r_4 are being changed from 1 to 60

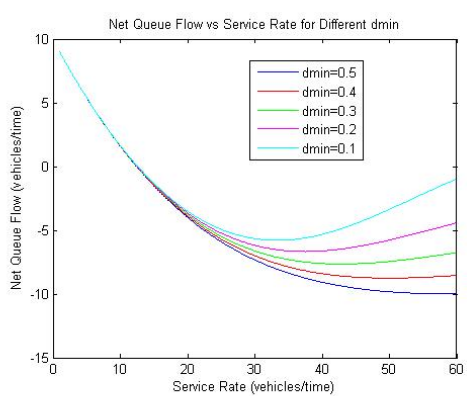

Figure 5: This graph shows the relationship between r_1 and $\frac{dC}{dt}$ for the congestion model with $C = C_{limit}$. The constant parameter values are $d_{1,max} = 2$, $d_{2,max} = 2$, $d_{3,max} = 2$, $d_{4,max}$, and $C_{max} = 30$. The values of r_i are ranging from 1 to 60 for different values of $d_{i,min}$.

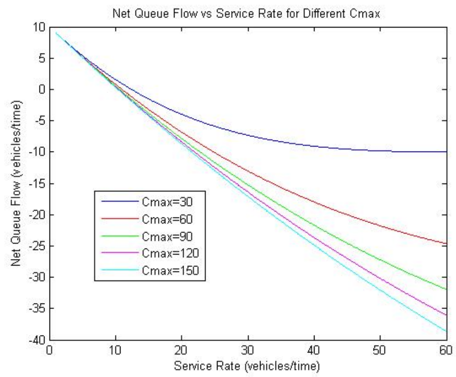

Figure 6: This graph shows the relationship between r_1 and $\frac{dC}{dt}$ and C for the congestion model with $C = C_{limit}$. The constant parameter values are $d_{1,max} = 2$, $d_{2,max} = 2$, $d_{3,max} = 2$, $d_{4,max}$, $d_{1,min} = 0.5$, $d_{2,min}$, $d_{3,min}$, and $d_{4,min}$. The values of r_i are ranging from 1 to 60 for different values of C_{max}

Another situation that the congestion model can approximate is the addition of extra lanes. We can make a crude approximation of this event by saying that the every time a lane is added, the capacity would increase by a factor of C_{max}. If $C_{max} = 30$ for one lane, then for two lanes $C_{max} = 60$. Figure 6 shows the results of plotting r_1 versus the C_{max} for adding different amounts of lanes. Like in the previous plots, the correlation is negative.

3.4 Simulation Results

One of the most interesting things effects we saw in our MATLAB simulation was the buildup of vehicles in front of each of the exits. As the vehicles slow down in preparation of their exit, they force other vehicles behind them to decelerate in order to maintain a safe distance. This buildup creates a higher queue at the intersection before the exit, as the buildup prevents those vehicles from exiting their queue because the space is taken. In Figure 7, we can see the large number of vehicles in the fourth queue, and the buildup in the fourth quadrant.

Another interesting part of real life that the simulation shows are the bunching and expanding effects that vehicles experience. Because they can decelerate more quickly than they accelerate, the vehicles bunch up behind a slow moving vehicle, then expand again as that vehicle accelerates into the free space ahead. Figure 8 shows an example of this compaction.

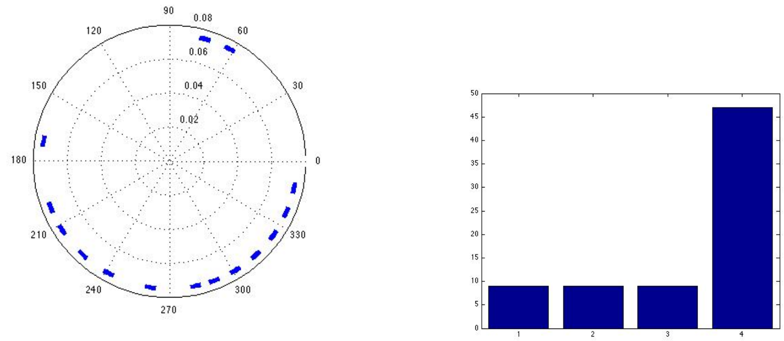

Figure 7: Shows the buildup of vehicles in front of the first intersection as vehicles slow down to exit. Additionally, the queue at the fourth intersection is quite high because vehicles cannot enter into full traffic circle.

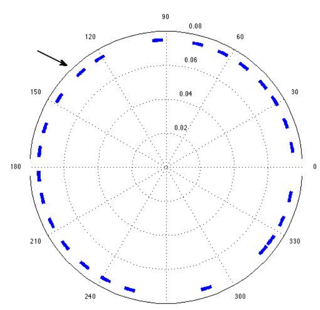

Figure 8: The arrow points out bunching occurring in the second quadrant. Bunching happens because drivers decelerate faster than they accelerate.

We tested several different traffic circle and vehicle setups to explore the problem of optimal circle design. The first setup we tested was having a single intersection with high arrival and service rates. It created a large traffic buildup in the quadrant immediately following it, even though the vehicles all had random destinations. The effects of random destinations could not overcome the vehicles slowing down to exit. Interestingly, the queue for the intersection was not appreciably higher than the other queues. Figure 9 shows the buildup in quadrant 1 when the first intersection (at angle 0) had a high arrival and service rate, but it also shows that queue 1 was not appreciably higher than the others.

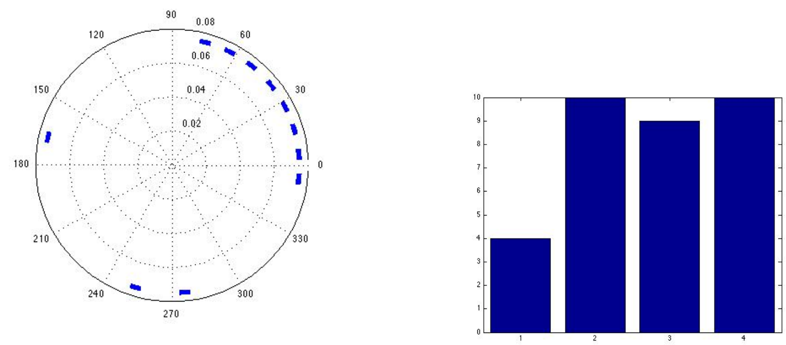

Figure 9: The first intersection has both high arrival and service rates. This creates a traffic buildup before the next intersection. However, the queue for the first intersection does not increase, since there is limited traffic coming from the intersection behind it.

Our second test focused on one intersection having a much higher chance of being a destination. This is very possible in the real world if one of the roads leads to an important commercial center or highway. This creates the expected buildup in front of the likely exit, as seen in Figure 10. However, it also creates a substantial buildup in front of the previous exit, and a severe buildup in that intersection's queue as vehicles are prevented from entering. The buildup in the adjacent road must be taken into account when constructing a traffic circle with a high volume intersection.

We also tested the outcome if one intersection had a high service rate and the standard arrival rate, and another intersection had a high arrival rate and standard service rate. The traffic distribution was mostly random, with a slight tendency towards backups in quadrant following the intersec-

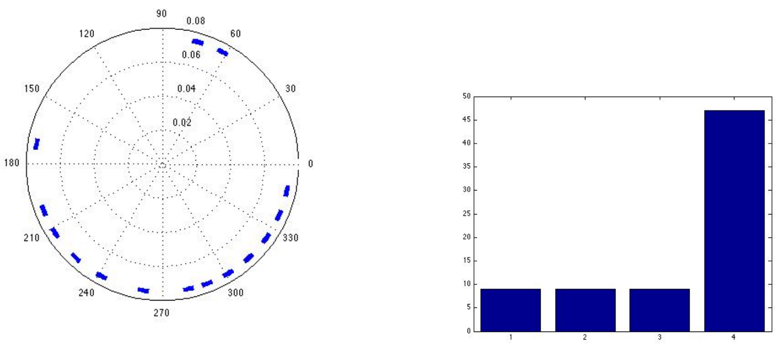

Figure 10: The first intersection has a higher probability of being chosen as a destination. This creates a buildup in front of that intersection and a smaller buildup in front of the previous intersection. It also creates a very large increase in the queue of the previous intersection since those vehicles cannot enter the full circle.

tion with high service rate. This was expected, since the intersection with high service rate could only add as many vehicles as it had in its queue, which was limited by its low arrival rate. The intersection with high arrival rate and low service rate also had a much larger queue than the other intersections, entirely as expected.

4 Conclusion

We modeled the dynamics of a traffic circle in order to determine how to best regulate traffic coming into the circle. As shown in figure6, increased capacity decreases the queue flow which leads to a decrease in queue size. This indicates that a multiple lane traffic circle will better accommodate more people by decreasing the length of the queue in which they must wait. However, as shown in the same figure, the marginal utility of increasing the maximum capacity does decrease. Using a cost function (where cost varied proportionally to the amount of space that the circle takes up), then there would exist an optimum size of the traffic circle.

Although the simpler models that we created indicate that letting vehicles into the rotary as fast as possible would be optimum, analysis of the congestion model showed that if $d_{i,min}$ is sufficiently small given all other parameters, then the highest service rate is no longer optimal. The implications of this result in terms of construction is that traffic lights could make travel through the rotary more efficient in certain cases. During times when many vehicles would use the traffic circle, such as during the morning and evening commutes, there would be enough vehicles so that the C_{limit} is reached. In this case, using stop lights would help to alleviate the flow of traffic. However, the duration of the red light should be adjusted according to the $d_{i,min}$ for the specific traffic circle.

In addition to the mathematical models, we created a computer simulation that tracked individual vehicles progress through the traffic circle, and their effect on other vehicles. Our simulation showed several traffic effects that can be observed in real life, namely a buildup of vehicles in front of the exits, and vehicles bunching together and expanding apart as drivers accelerate and brake. We also tested several traffic circle configurations. If a single intersection had both a high service rate and a high arrival rate, traffic built up heavily in the area of the circle immediately following. However, if the intersection only had a high service rate or high arrival rate, the resulting traffic was mostly random, and only the intersection with high arrival rate built up a large queue. Our final configuration gave one intersection a higher likelihood of being a destination. This created a traffic buildup in front of that intersection, but also created a very large queue at the previous intersection. This large queue buildup should be considered when building any circle with more popular intersections.

While both of our modeling techniques have limitations, they enabled us to considers the problems of designing a traffic circle. Combining the results of both techniques, we can provide a fairly comprehensive look at the dynamics of a traffic circle, and to draw conclusions about how to optimally implement such a circle in a real world situation.

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5 Technical Summary

In order to explore issued related to traffic circle design, we have created a model to simulate vehicle dynamics within a traffic circle. Our model describes the net rate of vehicles in each queue (the vehicles on the incoming roads waiting to be let into the traffic circle) as well as the net rate of vehicles in the traffic circle. The equations for the rates are as follows:

$$\frac{dQ_i}{dt} = a_i - r_i\left(1 - \frac{C}{C_{max}}\right),$$

$$\frac{dC}{dt} = \sum r_i\left(1 - \frac{C}{C_{max}}\right) - C\left(\sum d_{i,max}\left(1 - \frac{C}{C_{max}}\right) + \sum d_{i,min}\left(\frac{C}{C_{max}}\right)\right)$$

where Q_i is the number of vehicles in the i^{th} queue, C is the number of vehicles in the traffic circle, a_i is the rate at which vehicles enter the queue, r_i is the rate at which vehicles enter the traffic circle when the circle is empty, C_{max} is the capacity of traffic circle, and $d_{i,max}$ and $d_{i,min}$ are the maximum and minimum departure rates from the circle based on congestion. The rates can be found empirically except for r_i for each queue.

We made several assumptions in our model. One assumption is that the traffic circle is a single lane with all vehicles moving in the same direction. Our model takes into account the lowering of the speed due to congestion (through $d_{i,max}$ and $d_{i,min}$), but not due to the random acceleration and deceleration of vehicles.

The r_i for each queue can be found using the model. The smaller r_i is, the more time each vehicle takes at the front of the queue before entering the traffic circle, which models a stop light with longer red light periods.

One limitation of this model is that it cannot be used to examine the possibility of a traffic light that gives priority to the vehicles in the queue. Another is that we are modeling the vehicles deterministically which does not allow for the special behavior of individual vehicles (such as one vehicle speeding).

We also created a computer simulation to enhance and extend the capabilities of the mathematical model. The simulation tracked every vehicle's progress as it traversed the traffic circle, and modeled their interaction with other vehicles. We were able to observe real world effects such as traffic bunching together into groups, and traffic build up in front of the exits.

Based on testing of both our mathematical and computer models, we would recommend the following:

- Most of the time, letting vehicles enter the circle as quickly as possible is optimal, which means that yield signs should be the standard traffic control device.

- During periods of high traffic, slowing the rate of entry into the circle helps prevent congestion, which decreases the efficiency of the circle. Therefore, in areas of high traffic, stop lights should be used as traffic control devices.

- If any single road has a high traffic, its vehicles should be given preference in entering the circle. This will help prevent a large queue.

- Traffic often builds up in front of each intersection as cars exit, so a separate exit lane could help keep traffic moving.