Profit Maximizing Allocation of Wheelchairs in a Multi-Concourse Airport

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Summary

To minimize Epsilon Airlines' cost of providing wheelchair assistance to its passengers, we examine the trade-off between explicit costs (chairs and personnel) and implicit costs (losses in market share). Our *Multi-Concourse Airport Model* simulates the interactions between escorts, wheelchairs, and passengers. Our *Airline Competition Model* takes a game-theoretic perspective in representing the profit-seeking behavior of airline companies. To ground these models in reality, we incorporate extensive demographic data and run a case study on 2005 Southwest Airlines flight data from Midland TX, Columbus OH, and St. Louis MO. We conclude that Epsilon Airlines should employ a "hub and spokes" strategy that uses "wheelchair depots" in each concourse to consolidate the movement of chairs. Across different airport sizes and strategies, we find that two escorts per concourse and two wheelchairs per escort are optimal.

Introduction

We study the procedures used by airlines to shuttle passengers from arriving flight to connecting flight. According to the U.S. Department of Transportation, "The delivering carrier shall be responsible for assistance in making flight connections and transportation between gates [for passengers needing assistance]" [2003]. With an aging population, more passengers need help.

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We develop two models. The *Multi-Concourse Airport Model* simulates the interactions of passengers, wheelchairs, and airport staff within an airport, following each passenger and tracking delays. Passengers needing wheelchair assistance are shuttled through the airport using one of three algorithms.

The *Airline Competition Model* simulates a competitive marketplace over a 40-year period. Profit depends on both costs and market share; the latter fluctuates based on customer satisfaction. The *Airline Competition Model* provides a way to judge accurately and objectively the merit of wheelchair allocation strategies.

Key Terminology

- **Gate:** A location where air passengers board flights. A given gate can be represented as an ordered pair (i, j), where i indexes the concourse and j indexes the gates in a concourse.
- **Concourse:** A collection of gates. Concourse i contains k_i gates and is represented as a vector $c_i = \langle (i,1), (i,2), ..., (i,k_i) \rangle$.
- **Airport:** A collection of concourses, which we consider as a graph.
- **Passenger:** A traveler in an airport, associated with an arriving flight and with a connecting flight. We distinguish WPs (wheelchair passengers) from non-wheelchair passengers.
- **Traffic:** The mass of passengers in an airport. The level of traffic affects the number of WPs needing transport between gates.
- **Wheelchair depot:** A location where wheelchairs are stored while not in use. In the *hub and spokes* strategy, there is a depot in each concourse.
- **Escort:** An airline employee responsible for picking up WPs from arrival gates and transporting them to connecting gates.
- **Missed flight:** When a passenger arrives more than 15 min after the connecting flight's departure time, the flight leaves without them.
- **Strategy:** An algorithm for the flow of escorts and wheelchairs throughout the airport.

Basic Assumptions

Airport Layouts

An airport consists of 1 to 10 concourses, each of with consists of 2 to 50 gates. Gates in the same concourse are generally located close to one another, while the travel time between concourses can be quite lengthy. Hence, we assume that inter-concourse travel is much lengthier than intra-concourse travel.

Our model represents concourses and gates as nodes in a graph.

Table 1.
Variables and their meanings.

Variable	Definition
A	Airport, graph of concourses
c_i	Concourse i, a node of A
C	The number of concourses in A
(i, j)	Gate j within concourse i
k_i	Number of gates in c_i
Day	Type of day modeled by the simulation
Year	The year modeled by the simulation
$N_t^{(i)}$	Market share of airline i at time t
$f_t^{(i)}$	Fraction of customers defecting for airline i at time t
P_t	Year t population of the airport market
χ	A strategy
W^*	Ratio of wheelchairs to wheelchair needing customers
E^*	Ratio of escorts to wheelchair needing customers
W_t	Number of wheelchairs in year t
E_t	Number of escorts in year t
U_M	Utility loss of missing a flight (> 0)

Table 2.
Constants and their values.

Constant	Definition	Value
p_E	Escort annual salary (including benefits)	\$40,000 /yr
p_W	Purchase price of a wheelchair	\$ 135
x^*	Maximum time that a plane will wait for a passenger	15 minutes
ω	Proportion of passengers who are WPs	1.6%
p_{inf}	Proportion of WPs informing of arrival	95%
v_F	Velocity of an escort walking alone	250 ft/min
v_S	Velocity of an escort pushing a wheelchair	180 ft/min
U_m	Utility loss for being delayed one minute on the runway	$0.0005U_m$
U_G	Utility loss for a chair idling one minute by a gate	$0.0001U_m$
p_S	Storage cost of a wheelchair	\$50/year
π	Airline profit per customer	\$4.51

Wheelchair Passenger Needs

WPs comprise a proportion ω of the total passenger pool (1.5% in 1996 [Conway 2001] and 1.6% in 2006 (the starting year of our model).

A proportion p_{inf} of WPs inform our airline of their need before arrival; we assume $p_{\text{inf}} = 95\%$. Knowing p_{inf} and $\omega = .015$ and $p_{\text{inf}} = .95$, we use a binomial distribution to find a probability mass function for the arrival of WPs on a flight (**Table 3**).

There are two ways that a WP can miss a connecting flight:

- Late incoming flight: Roughly one-third of flights arrive late and about 5%

Table 3.
Unexpected wheelchair passengers in a flight of 120 people.

Unexpected Passengers	Probability (%)
0	91.39
1	8.23
2	0.37
3	0.01

are at least an hour late [U.S. Department of Transportation n.d.].

- Slow arrival of escorts: We try to minimize this risk.

Intergate Transportation

- Average fast walking speed is 250 ft/min (3 mph), but average speed when arms are immobilized (as when pushing a wheelchair) is only 180 ft/min (2 mph) [Gross and Shi 2001]. We assume that an escort walks at these speeds.
- An escort can operate only one wheelchair at a time. U.S. Dept. of Transportation guidelines discourage leaving WPs unattended. Hence, the escort takes a WP to the connecting flight and remains until the flight leaves.
- Airport customer service employees (escorts) earn on average $11.80/h; the annualized cost with benefits per escort is $p_E = \$40{,}000$ [Bureau of Labor Statistics 2004]. Transport wheelchairs bought in large batches cost $135/chair [Transport Wheelchairs 2005].
- Passengers who arrive more than 15 min late to their connecting flight are left behind; airlines wait just this long for delayed arriving flights.
- Escorts are in contact with one other via radio.

Market Share and Delays

Wheelchair service is not just a legal responsibility but a good idea from a customer-relations standpoint and enhances competition for market share. A passenger who misses their connecting flight (due to poor wheelchair allocation or a delayed arriving flight) must wait (perhaps several hours or overnight) for the next outbound flight. Additionally, that wait could make that passenger and others miss a subsequent connecting flight.

Multi-Concourse Airport Model

Formal Definition

Let A be an airport with C concourses $c_1, ..., c_C$ and gates $\langle(1, 1), ..., (C, k_C)\rangle$, where k_i is the number of gates in concourse c_i. Further, let E^* be the ratio of escorts to total WPs and W^* be the ratio of wheelchairs to total WPs. Escorts and chairs are assigned by some strategy χ.

Several factors in the airport system are beyond our control:

- ω, the proportion of passengers who are WPs;
- Day, i.e., high- or low-traffic days;
- Year (passengers in years past 2006 have different demographics); and
- costs of wheelchairs (p_W) and of wages (p_E) for escorts,

We take these factors as exogenous to our model, so the only control variables are χ, W^*,and E^*.

We seek to minimize cost by reducing both explicit costs (escort wages, wheelchair purchases) and implicit costs (lost business due to frequent delays).

The *Multi-Concourse Airport Model (MCAM)* relates the three control variables to explicit costs and total delays. Delays include both missed flights and late departures as a result of missing passengers.

Let C be daily cost and D be disutility of a delay. We want a function f such that

$$f(\chi, W^*, E^*) \longmapsto (C, D).$$

The MCAM model runs a Monte Carlo simulation for several different days. This process gives the expected daily delays and costs, so the results of MCAM serve as a suitable proxy for f.

Aggregating Delays and Disutility

The airline has a set policy that a plane will wait up to x^* min for a passenger heading toward the gate. Increasing x^* decreases delay due to missing flights but increases delay for passengers waiting aboard planes; similarly, lowering x^* favors boarded passengers at the expense of late passengers.

The airline seeks an optimal value for x^* to balance the discomfort of waiting passengers against the probability that the late passenger will arrive in time.

Let disutility for small unexpected delays be linear in time, so if 120 passengers on a plane wait 15 min, the total utility loss is proportional to the $120 \times 15 = 1{,}800$ min of delay. Also, let the utility loss from missing a flight be $-U_m < 0$ and set time $t = 0$ to be the flight's planned departure time. If a passenger is not at the gate at $t = 0$ but we know that they are on their way, then we wait up to x^* min for them.

Let L be a random variable for the lateness of our passenger; the probability of arrival after $t = 0$ is $P(L \leq x^* | L \geq 0)$. This means the late passenger benefits by $U_m P(L \leq x^* | L \geq 0)$, while the others expect to wait $E[T | 0 \leq T \leq x^*]$, since they will leave in at most 15 min.

With x^* chosen optimally, lost utility from waiting equals benefit to the late passenger. So, when N passengers are waiting, optimality is achieved when

$$NE[T|0 \leq T \leq x^*] = U_m P(L \leq x^* | L \geq 0).$$

Given our past experience, we assume that $x^* = 15$, so that

$$U_m = \frac{N \times E[T|0 \leq T \leq 15]}{P(L \leq 15 | L \geq 0)}.$$

We determine average $P(L \leq x^* | L \geq 0)$ and $E[T|0 \leq T \leq x^*]$ from our simulation results of an airport with a large enough supply of escorts and wheelchairs that every WP is immediately taken to their connecting flight (**Table 4**).

Table 4.

Benefit of waiting 15 min for late passengers.

Airport	*Day*	$P(L \leq 15 \vert L \geq 0)$ (%)	$E[T \vert 0 \leq T \leq 15]$ (min)
Midland	Low-Delay	31.5	5.0
Columbus	Low-Delay	23.2	6.9
St. Louis	Low-Delay	19.8	8.5
Midland	High-Delay	25.9	2.5
Columbus	High-Delay	26.9	6.6
St. Louis	High-Delay	26.3	7.7
Average		**25.6**	**6.2**

An average plane has capacity 120 and load factor .695 (69.5% of seats occupied), so the effective N is $120 \times .695$, we have $U_m = (.695)(120)(6.2)/(.256) \approx 2000$, which means that missing a flight is 2,000 times as bad as one person waiting 1 min—a reasonable result.

A Third Source of Disutility

Idle wheelchairs near gates are an inconvenience to passengers and a liability risk to airlines. Every minute that a wheelchair sits at a gate, it contributes disutility equal to 20% of the disutility of a single individual being delayed 1 min.

Aggregate Disutility

To combine the three disutilities, note that 1 min of flight delay affects the N people waiting at the equivalent of $\frac{N}{2000} U_m = 0.0005 N U_m$, where U_m is the disutility of missing a flight. Also, 1 min of a wheelchair idling by a gate provides disutility 20% as large, or $0.0001 U_m$.

The Strategy

The strategy set χ governs the rules that escorts follow in making their decisions. These include:

- How do escorts choose which WP to pick up?
- How do escorts find a chair to use?
- What do escorts do after they've dropped off their WP?
- Where do escorts leave a wheelchair when they are done using it?

The MCAM tests three strategies, one random and two "intelligent."

Random Strategy

When a WP arrives at the airport, a free escort is randomly chosen to shuttle them to the connecting gate. The escort stays with the wheelchair at the gate until the next assigned WP.

Intelligent Strategies

The two intelligent strategies borrow their names from airline industry terminology: *direct transfer* and *hub and spokes*.

The airport knows in advance about most WPs. Each escort and each gate agent (the airline representative at a gates) has an ordered list of expected WPs. The heuristic for ordering WPs waiting to be taken to connecting gates is:

$$H = \text{time until flight leaves} - \text{time to reach gate (via wheelchair)}.$$

If a WP is expected, an escort anticipates their arrival by waiting at the gate. (In our implementation, expected WPs are inserted into the waiting queue 20 min before their flight lands.) When unexpected WPs arrive, the gate agent there radios over an open channel so that everyone can update their lists.

An escort who becomes free reports to the group. The WP at the top of the list is assigned to the closest free escort, who radios to find available wheelchairs, preferring one close to the WP or on the way to the WP.

Our intelligent strategies differ about what escorts do after shuttling a WP.

- **Direct Transfer:** The gate-based strategy runs all operations out of the gates in an airport. After an escort drops a WP off at gate (i, j), the escort and chair remain at that gate until assigned another WP. For the next assignment, say at gate $(i\prime, j\prime)$, the escort radios to find a chair closer to $(i\prime, j\prime)$. The strategy spreads the wheelchairs out among the gates so that any gate likely has a wheelchair nearby. A disadvantage is unattended chairs near gates.

- **Hub and Spokes**: Each concourse has a wheelchair depot, where wheelchairs are stored. After an escort drops off a WP at gate (i, j), the escort returns the wheelchair to the depot in concourse i. When escorts are idle and there are no WPs to be shuttled, the chairs wait in the depots (instead of at the gates). Wheelchair depots eliminate leaving chairs in high-traffic areas near the gates themselves, and escorts know that all available chairs are at depots.

Long-Term Concerns

The MCAM simulates a single day of airport activities, but to choose a wheelchair/escort strategy we should consider long-term factors. Ideally, our model will take the data from the MCAM and use it to simulate several years of airline operation. The problem is that several factors that are constant in MCAM could change over 10 or 40 years.

Aging Population

In 2000, one-sixth of the US population was over 60 years old [U.S. Census Bureau 2005] and 72% of wheelchair users are in this age group [Conway 2001]; a person over 60 is 13 times as likely to need a wheelchair as a person under 60.

The over-65 age group will grow by 40% in the next 20 years [U.S. Census Bureau 2005]. We assume that in 2006 1.6% of passengers are WPs; using the Census Bureau's demographic data, we calculate the fraction of future air travelers who will be WPs.

It is not clear whether this growing older group will take proportionately more or fewer plane flights in the future (a question of saved income and free time vs. health). We assume a middle ground: The proportion of wheelchair users in the country and on flights are directly proportional. In our models, we use the appropriate ω for each year.

Lower Profit Margins

In 2004 and 2005, airline earnings were reduced by high jet-fuel prices.If high prices persist, airlines will be forced to raise ticket prices (sales will fall) or cut profit margins. We simulate this effect by using a smaller marginal profit per passenger in the long-run model than in the short-run model.

Airline Competition Model

The MCAM model simulates the flows of escorts, chairs, and WPs. But we are searching for a cost-minimizing strategy. To do this, we need to model the changing market share of our airline (based on customer satisfaction) and derive a profit function for its operations. Maximizing profit is equivalent to minimizing costs if we view lost future business as a cost.

The *Airline Competition Model (ACM)* uses the output of the MCAM to determine market share and profitability for a group of competing airlines.

Market Share

A principal factor in an airline's long-run profitability is market share, the proportion of the market the airline holds at an airport. After experiencing flight delays or missed flights due to limited availability of wheelchairs, WPs and non-WPs may defect to other airlines. We model defection as a function of total disutility to an airline's passenger base.

Let there be M airlines and let $N_t^{(i)}$ be airline i's market share at time t, with $f_t^{(i)}$ the fraction of its customers who defect at the end of period t. Assuming that defecting passengers choose one of the $M-1$ other airlines with equal frequency, the stochastic process for market share is

$$\Delta N_t^{(i)} = N_{t+1}^{(i)} - N_t^{(i)} = \left(\sum_{j \neq i} \frac{1}{M-1} N_t^{(j)} f_t^{(j)} \right) - N_t^{(i)} f_t^{(i)}. \qquad \textbf{(1)}$$

Alternatively, we could suppose that defecting passengers distribute themselves among the $M-1$ other airlines in proportion to airline market share:

$$\Delta N_t^{(i)} = \left(\sum_{j \neq i} \frac{N_t^{(i)}}{1 - N_t^{(j)}} N_t^{(j)} f_t^{(j)} \right) - N_t^{(i)} f_t^{(i)}$$

Without data on how consumers choose airlines, we assume that ticket price is the overwhelming factor. Ticket pricing is complex but does not vary with market share—the giant Southwest and tiny Vanguard Airlines offer comparable rates [Airline pricing data 2005]. Because of the importance of prices, we believe that **(1)** is more accurate.

If the total market grows by a proportion r, new passengers choose carriers so that market shares remain unchanged. We can express this assumption nicely using matrix notation. Letting $\mathbf{N_t}$ denote the vector of market shares and define the elements of $M \times M$ matrix $\mathbf{A}$:

$$a_{ij} = \begin{cases} \dfrac{1}{M-1}, & \text{if } i \neq j; \\ -1, & \text{if } i = j. \end{cases}$$

This simplifies **(1)** to:

$$\mathbf{N_{t+1}} = (\mathbf{I_M} + \mathbf{AF_t})\mathbf{N_t},$$

where $\mathbf{F^{(t)}}$ is the matrix whose diagonal entries are $f_t^{(j)}$, and whose off-diagonal entries are zero and $\mathbf{I_M}$ is the $M \times M$ identity matrix. This formula iterates nicely to give the closed form

$$\mathbf{N_T} = \left[\prod_1^T (\mathbf{I_M} + \mathbf{AF_t})\right] \mathbf{N_0}.$$

The distribution of the multivariate random variable $\mathbf{F_t}$ is fixed in the short run while the proportion of handicapped individuals remains constant. In the short run, therefore, $\mathbf{N_T}$ is (up to a constant) the product of T uniformly distributed random variables, all distributed as $(\mathbf{I_M} + \mathbf{AF})$ (we drop the subscript on $\mathbf{F}$ because its distribution is independent of time in the short run).

Passenger Defection

The rationale behind a profit-based model is to quantify the trade-off between the cost of accommodating WPs and the loss in market share associated with customer dissatisfaction. In the short term, airlines would be more prone to provide less accommodations because the resultant effect on market share would not be seen until the next period. However, in the long term, the market share for airlines providing poor service will suffer and profit will fall. Moreover, short-run costs include the fixed cost of purchasing wheelchairs, while long-run costs are the smaller costs of chair maintenance and replacement. Our profit function measures only the profit gained (or lost) from wheelchair allocation strategy.

Let there be J different types of days, each of with its distribution of delays. For example, days before holidays and weekends probably have longer delays. A day of type j experiences a defection of $f_t^{(i,j)}$ from total market share, based solely on total disutility of passengers of airline i traveling on that day. We also assume that traffic is constant over days of each type but varies across types; these traffic differences affect the underlying value for $f_t^{(i,j)}$. Let there be V_j days of type j per year.

The MCAM simulates total disutility of passengers given the parameters (χ, W, E), and we obtain $f_t^{(i,j)}$, the daily defection rate. This gives a distribution for the random variable $g_t^{(i,j)} = \log\left(1 - f_t^{(i,j)}\right)$, for various days t, implying a mean and variance for such a distribution, which we denote by the ordered pair $\left(\mu(i,j), \sigma(i,j)^2\right)$:

$$g_t^{(i,j)} \sim \left(\mu^{(i,j)}, (\sigma^{(i,j)})^2\right).$$

The retention rate for a day is given by $1 - f_t^{(i,j)}$, so total retention for a year is given by

$$1 - F_t^{(i)} = \prod_{j=1}^{J} \prod_{v=1}^{V_j} \left(1 - f_v^{(i,j)}\right).$$

Taking logarithms yields

$$\log\left(1 - F_t^{(i,j)}\right) = \sum_{j=1}^{J} \sum_{v=1}^{V_j} g_v^{(i,j)}.$$

By the Central Limit Theorem, if the values for V_j are sufficiently large (they are 35 and 330 for our program), we have the approximate distribution

$$\sum_{v=1}^{V_j} g_v^{(i,j)} \sim \mathrm{N}\left[V_j \mu^{(i,j)}, V_j \left(\sigma^{(i,j)}\right)^2\right],$$

where $N[\mu, \sigma^2]$ is the normal distribution. In our implementation, we use random draws for the realizations of $g_v^{(i,j)}$.This implies that $F_t^{(i)}$ is approximately distributed as

$$F_t^{(i)} \sim 1 - \exp\left(\sum_{j=1}^{J} N\left[V_j \mu^{(i,j)}, V_j \left(\sigma^{(i,j)}\right)^2\right]\right).$$

Our profit model is constructed to hold all factors constant except wheelchair strategy. Because of this feature, when a WP misses a flight, it is always the result of poor wheelchair allocation and not of another factor. We assume that missing a flight causes a WP to defect from the airline with probability $p_d = 1/4$. This high probability is reasonable, since to the WP it appears as if the airline has neglected them by not shuttling them to their connecting gate.

On day t, airline i has $P_t N_t^{(i)}$ passengers in its market share, of whom $n_t^{(i)}$ total are traveling on day t with airline i.

The probability of not defecting after missing a flight is $1 - p_d$. We assume that the probability of not defecting is multiplicative in the number of missed flights, that is, after missing m flights, there is a $1 - (1 - p_d)^m$ probability of defection.

We also assume that the disutility D_t is uniformly distributed across all passengers, so each passenger has disutility $u_t = D_t / n_t^{(i)}$, measured as a multiple of U_m, the disutility of missing a flight. An individual defects with probability $1 - (1 - p_d)^{u_t}$.

By the Law of Large Numbers, the total number of individuals defecting on a particular day approximately equals the expected value of this random variable. We therefore have:

$$f_t^{(i,j)} = n_t^{(i)} [1 - (1 - p_d)^{u_t}].$$

Implementation of Delay Distributions

In our implementation, we use actual 2005 daily data on average delays for Southwest Airlines. At each of three airports, about 10% of days have particularly high delays [Southwest Airlines 2006]. So we categorize days as *high-delay* (the top 10%, 35 days) or *low-delay* (the bottom 90%, 330 days).

Southwest Airlines reports an average load factor of 69.5% (the percentage of occupied seats on a flight). Since high-delay days are often during peak travel times, we assume that flights operate at full capacity on such days; a weighted average calculation gives 100% and 66% for the load factors on high- and low-delay days, respectively.

Present Value of Profits

We assume that costs and airline profits per passenger grow at a constant annual rate r_C, and we let r_D be the nominal interest rate.

Let $\Pi_t^{(i)}$ denote the real (adjusted for inflation) profit in year t for airline i. If costs and profits per unit of good grow at a constant inflation rate, then $\Pi_t^{(i)}$ can be determined assuming zero inflation. Inflation will be included in the discount factor for the long-term calculation of firm profit.

So, airline i maximizes the expected present value of profits given the discount rate δ:

$$\Pi^{(i)} = \sum_{t=0}^{T} \delta^t \Pi_t^{(i)}.$$

We let $T = 40$ and make projections out to 2046. The current inflation rate is approximately 2%, so we estimate r_C as 2%, and r_D is the forward risk-free rate in the future, which we assume to be constant and equal to 4.5%.

$$\delta = \frac{1}{1 - r_C + r_D} = \frac{1}{1.025} =\approx 0.975.$$

The Profit Function

Profit related to wheelchair policy can be split up into the following contributive factors:

- total profits from passengers, which is proportional to market share;
- wheelchair purchase and replacement costs;
- wheelchair storage costs; and
- escort salary and benefits payments.

We derive from these contributive factors a formula for profit, given by:

$$\Pi_t^{(i)} = \begin{cases} P_t N_t^{(i)} \pi - p_W R_t - p_S W_t - p_E E_t, & \text{for } t > 0; \\ P_t N_t^{(i)} \pi - (p_W + p_S) W_t - p_E E_t, & \text{for } t = 0, \end{cases}$$

where:

P_t = total number of airline passengers in the market,

$N_t^{(i)}$ = market share of airline i,

π = profit per passenger,

R_t = number of wheelchairs replaced in year t,

p_W = price of a wheelchair,

p_S = annual storage cost of a wheelchair ($50), and

p_E = annual cost of an escort.

Earlier, W^* and E^* were the proportions of chairs and escorts to the population, but here we need W_t and E_t, the actual number of chairs and escorts used.

The cost in the first period differs because there is an initial purchase cost of wheelchairs. In later periods, wheelchairs need replacement at 20% per year, so at the end of year t we have $0.8W_t$ usable wheelchairs. Due to changes in market share, we may desire more or fewer wheelchairs in year $t+1$; the target number of chairs is

$$W_{\text{target},t+1} = W^* P_t N_t^{(i)},$$

and similarly, $E_{\text{target},t+1} = E^* P_t N_t^{(i)}$. We won't throw away good chairs, so the number of chairs we use in year $t+1$ is:

$$W_{t+1} = \max\{0.8W_t, W_{\text{target},t+1}\}.$$

This gives the necessary number of chair replacements:

$$R_{t+1} = \max\{0, W_{\text{target},t+1} - 0.8W_t\}.$$

Implementation

The computer simulation of ACM relies heavily on the results produced by MCAM. We take M airlines, each with a different (χ, W^*, E^*), and for a given year we estimate their total costs and total disutilities.

Starting with the first year, ACM simulates the operation of airline i by using its strategy (which remains fixed through the end year, T) as the input for $MCAM$. Realizing that high-delay and low-delay days affect airport operations differently, we simulate 35 high days and 330 low days in each year.

This gives and output of total disutility which determines gain or loss in market share by **(1)**. (In the first year, we start every airline with an equal market share.) We can calculate each company's profit for year t and use the updated market shares in the calculation for year t.

This simulation runs for 40 years and result is a profit vector (across time) for each airline. We discount future periods at the rate δ and compare the present values that the various strategies (χ, W^*, E^*) produce. Recall that our profit function does not determine the airline's actual profit (which involves buying planes, pilots, flight attendants, etc.) but only the profit related to wheelchair use in airports. The relative value of discounted profit is how we gauge which strategy is most attractive.

Case Study

Southwest Airlines reported a 2005 profit of \$313 million from 70.9 million passengers, or \$4.41/passenger. However, this number is already reduced by costs included in our profit function, namely wheelchair and escort costs, which we estimate at \$0.10/passenger.

The data for our case study are quite extensive, including flight times, airport layouts, load factors, and average delays per airport, for airports in Midland TX, Columbus OH, and St. Louis MO.

Results and Observations

Multi-Concourse Airport Model

We applied the random, direct-transfer, and hub-and-spokes strategies to the case-study data, using 5 escorts with 5, 10, or 20 wheelchairs, and 10 escorts with 20 chairs. There are several notable results:

- Low-delay days and high-delay days give about the same disutility across all strategies when only 5 escorts are used. When 10 escorts are used, high-delay days give an average of 40 more disutility equivalents than low-delay days. A possible explanation is that 5 escorts are kept busy on both low- and high-delay days, but on a low-delay day 10 escorts is sufficient to handle all of the wheelchair traffic. On a high-delay day, some every late passengers will miss their flights even with 10 escorts available.

- The random strategy performs nearly identically to direct transfer under all chair/escort configuration combinations and both delay types. This could be due to the fact that the direct transfer strategy distributes wheelchairs so sparsely (at all the gates) that the assignment is essentially random.

- The hub-and-spokes strategy is less effective than the other two strategies when 5 escorts are used but more effective with 10 escorts. The hub-and-spokes algorithm involves streamlining wheelchair movement, so without adequate personnel it may not be efficient. In essence, the hub-and-spokes strategy requires a base number of escorts to be effective.

Airline Competition Model

For each of the three strategies and four combinations of chairs and escorts, we calculate the market share (reported as N_{year}) as well as the final profit.

The *Hub and Spokes* strategy with 10 escorts and 20 chairs earns the top market share in 2046 (11.1%), while the same strategy with 5 escorts and 20 chairs does the worst (7.2%). We believe that this is because the hub-and-spokes system is labor-intensive, since escorts must walk longer in taking chairs back to their depots.

In terms of profit, the strategy (Hubs, 20, 10) wins again—by a lot! (106 vs. 101.9 for second-best). Interestingly the strategy (Random, 20, 10), which had the 3rd-best market share, also had the 2nd-worst profit.

The winner is the hub-and-spokes configuration with 20 chairs and 10 escorts.

Analysis of the Models

Strengths of Model

The model uses actual data, including flight delay distributions for all 365 days in 2005, to rank wheelchair policies in terms of market share and long-term profits. Over the long-term, it also accommodates projections of changes in the proportion of WPs in the airline passenger market.

The model realistically captures uncertainty about prior information about need for wheelchairs.

The model converts total passenger disutility into a defection rate, which captures the effects of quality of service in a competitive market. The dynamics show both the long-term and short-term effects of the trade-offs between budgeting and loss/gain in market share.

Weaknesses of Model

Growth in passengers depends on many factors, including changing demographics, which may change from predicted values.

Conclusion

We recommend the hub-and-spokes configuration with two escorts per concourse and two wheelchairs per escort, to maximize both gain in market share and long-term profit. These suggestions are a fast and effective formula for higher profits. After all, at the end of the day, all we want is to make Epsilon greater than Delta.

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