A Computational Solution for Elephant Overpopulation

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Introduction

We extrapolate longevity data and explore the long-term behavior of the population age distribution. We determine the number of dartings to fix the long-run stable population at 11,000; about 1,300 dartings are needed for an every-other-year strategy. We employ two simulations, one based on averages and the other tracking each elephant individually, whose results agree closely.

Our modeled population recovers from sudden declines and is not overly sensitive to small changes in survivorship data. The model also allows estimating the number of dartings if up to 250 elephants are relocated each year.

Assumptions

- We are told that emigration and immigration are rare, so in our model no elephants enter the park except those that are born. None leave except those that die or are relocated.
- Fifty percent of the elephants are female, as the problem suggests.
- It is beneficial to the population as a whole, as well as more economically feasible, to use as few contraceptive darts as possible.
- Cows first conceive when they are 11 years old, rather than some time between ages 10 and 12.
- Gestation always takes 22 months exactly, instead of approximately.

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- The darts work, so a cow hit by a dart will not conceive for two years.
- Otherwise, cows give birth every 3.5 years until they reach the age of 60.
- There is a 1.35% chance that a given birth will result in twins.
- The survival rate for the first year is .75.
- The initial population is 11,000 individuals.
- The rangers can readily determine which females are not pregnant, so that no pregnant females are darted, as at Kruger National Park in South Africa, which uses a similar contraceptive program [Purdy 1998].
- Cows normally mate once every 3.5 years. The cycle of a cow darted is not disrupted. If the effect of the dart wears off before she would normally mate and become pregnant, she conceives and gives birth on schedule.
- Previous methods of population control eliminated individuals randomly, so no age group was disproportionately depleted and the relocated elephants have an age distribution that is typical of the population as a whole.
- Since the methods of population control that have been used have no effect on the fertility of the cows, we assume that the initial birth rate is constant.

Analysis of the Problem

We predict the long-term behavior of the elephant population as a function of the number of females. If we track each elephant individually, we must track 11,000 individuals; if instead we look at the population as a whole and take an average-case scenario, we must find formulas for birth and death, mating, aging, and the added effects of the contraceptive darts.

We use both methods. First, we use a computer simulation to track each elephant through its lifespan: We use known probabilities to determine when each elephant is born, reaches maturity, gives birth, and dies. We can use this simulation to test any darting strategy. The results are far less smooth than for an average-case scenario.

We also use another program based on recursive equations to predict the average-case behavior of the population, which we divide into groups of the same age. This method requires far less computer time. The replacement of random events with a deterministic average allows for ready investigation of long-term behavior without interference from individual unlikely events.

Using these two models, we find a mathematical expression for the dynamics of the population and then use these programs to forecast the results of our darting strategy and to demonstrate its stability and flexibility.

Task 1: Predicting Survivorship

There are three distinct phases in the life of an elephant.

- From birth until five years old, the young elephant is very susceptible to predators and accidents and cannot fend for itself while it still nurses from its mother [African Wildlife Foundation 1998].

- After it is weaned, at five years of age, it lives most of the rest of its life in relative safety. Several things can kill an adult elephant, but none has a major effect on the population. There is a low rate of disease, accidents are very rare, and no natural predators can kill something as large as an adult elephant [Hanks 1979, 109]. Therefore, over this period the death rate of the elephant is fairly low, about 2% per year.

- Over the course of its lifetime, the elephant grows six sets of molars; around age 50 the final set of teeth wears out, making it impossible for the elephant to properly chew its food, so that the animal eventually starves to death [Holloway 1994].

We construct a survivorship curve as a piecewise function, with each segment corresponding to one of these phases. Using our assumptions that the given data are a random sample of the elephant population, that the birth rate in the park has been essentially constant, and that the previous killing has been evenly distributed over the population of elephants, we conclude that the demographic shape of this population is typical for an elephant population.

Survivorship l_x is the fraction of the population alive after x years. To compute the survivorship from the data, we sum the data from each year to get a larger sample size and divide the entire data set by the population at age zero. The final survivorship data looks like **Figure 1**.

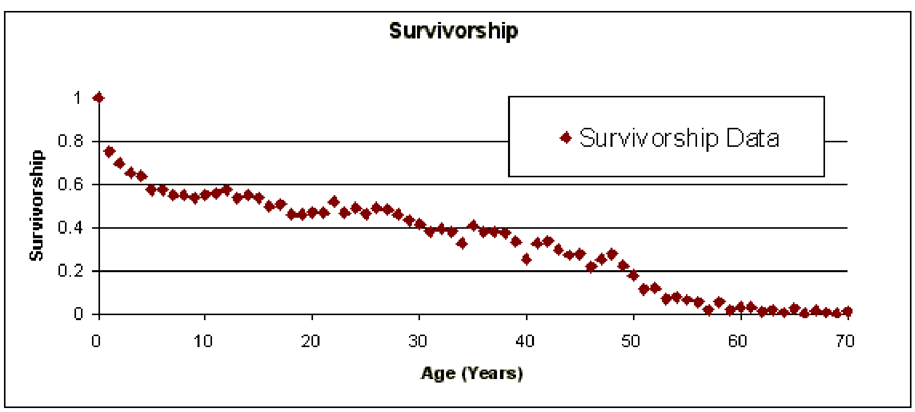


Figure 1. Survivorship function.

The data divide up roughly into three linear sections corresponding to the three stages of the elephant life cycle. These three sections appear to be well approximated by lines, so we generate a piecewise function composed of three linear segments for the ages from 2 to 60, based on a least-squares fit. The two points of discontinuity between the pieces of the function cause little error.

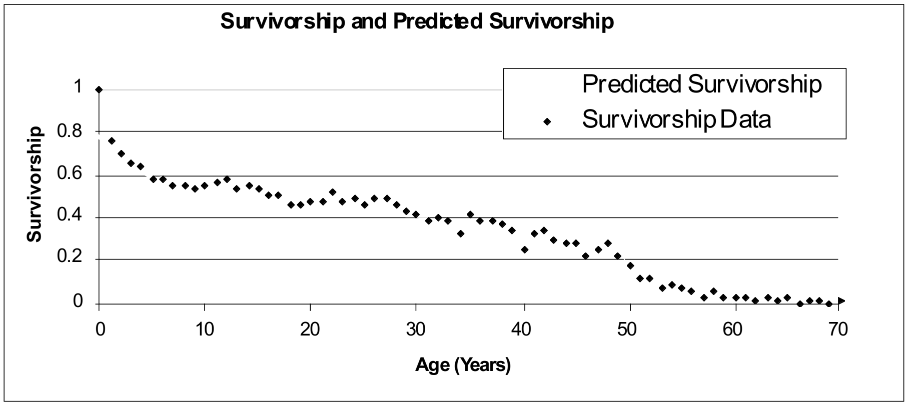


Figure 2. Survivorship data and fitted function.

The survivorship function is

$$l_x = \begin{cases} -0.038806x + 0.77512, & 2 \leq x \leq 5; \\ -0.007818x + 0.640015, & 5 \leq x \leq 50; \\ -0.013116x + 0.799663, & 50 \leq x \leq 60. \end{cases}$$

We calculate the probability of death p_d as the fractional change in l_x:

$$p + d = 1 - \frac{l_{x+1}}{l_x}.$$

The assumption of a constant birth rate is incorrect, as the data are clearly not monotonically decreasing. But given the assumption that previous population control methods (i.e., shooting) did not affect the age distribution, our model is presumably close to the actual profile.

Task 2: Achieving Stability

Birthing cows are females older than 11 and younger than 60 who can give birth; we choose some number D of nonpregnant cows to dart. Because of the additional stress on the darted population and the expense of darting, we should dart as few elephants as necessary.

How often should we dart cows? Darts remain effective for two years. Because the darted elephants are not tagged when they are darted, annual darting would lead to some elephants being darted two years in a row. Darting every two years uses fewer darts and simplifies our solution.

In a population with a stable birth rate, the same distribution occurs among the age groups—each segment of the population has a characteristic percentage.

The only segment that we can directly affect is the fraction f_b that are newborns; the goal is to stabilize the number N of newborns. So, in an ideal setting, after the population stabilizes, we have $N = Ef_b$.

The actual number of newborns is proportional to the number of cows that can give birth in the next year multiplied by the average number of elephants produced at the end of a successful pregnancy and the average chance of a pregnant female surviving long enough to give birth. The average number of elephants born after a pregnancy is one plus the chance p_t of having twins. The average chance of survival $\bar{p_s}$ for up to one year is

$$\bar{p_s} = \frac{\int_0^1 (1-p_d)^t \, dt}{1-0} = \frac{(1-p+d)^1 - (1-p_d)^0}{\ln(1-p_d)} = \frac{-p_d}{\ln(1-p_d)}.$$

The number of pregnant cows that could give birth next year is the number of cows that were not darted two years ago, survived for two years, and are now at least 10 months pregnant. Because cows are distributed randomly throughout the mating cycle, the chance that a pregnant cow is within 12 months of giving birth is 12/42. The chance of a cow having survived for two years is simply $(1-p_d)^2$. The chance that a cow was not darted two years ago is the probability that a nonpregnant cow was not darted two years ago, or one minus the number that were darted two years ago over the number of cows that were not pregnant then. Let P denote the number of pregnant cows. Substituting for the number of cows within a year of giving birth, we find

$$N = (1+p_t)\bar{p_s} \cdot \frac{12}{42} \cdot C(1-p_d)^2 \left(1 - \frac{D}{C-P}\right).$$

We set the real number of newborns equal to the ideal number of newborns and solve for the number of dartings. This tells us the number of elephants that we should have darted two years ago. We base the number of darts to use this year on the effect that the darts had two years ago. Because other terms are constant every year, we can apply the darting equation and find the number of cows to dart this year using this year's C and P. In the case of an excess of newborns, darting increases; if too few births occur then D becomes negative, suggesting that more pregnancies are needed than the population can produce even if no cows are darted.

$$D = (C-P)\left(1 - \frac{11{,}000 f_b}{\frac{12}{42} \cdot C(1+p_t)\bar{p_s}(1-p_d)^2}\right)$$

For values of the parameters, we have $1 + p_t = 1.0135$ and $\bar{p_s}(1-p_d)^2 = 0.94$. The equation should give the number of dartings for tending toward a steady number of newborn elephants How does it behave? To find out, we wrote a program to trace the progress of the population over time. Each year, the number of elephants in one age group times their chance of survival becomes the number in the next age group. We replace the newborn age group with a new generation calculated as the number of pregnant elephants that gave birth in that year times the number of newborns at each birth, $1 + p$. **Figure 3** shows the convergent, oscillating pattern that results.

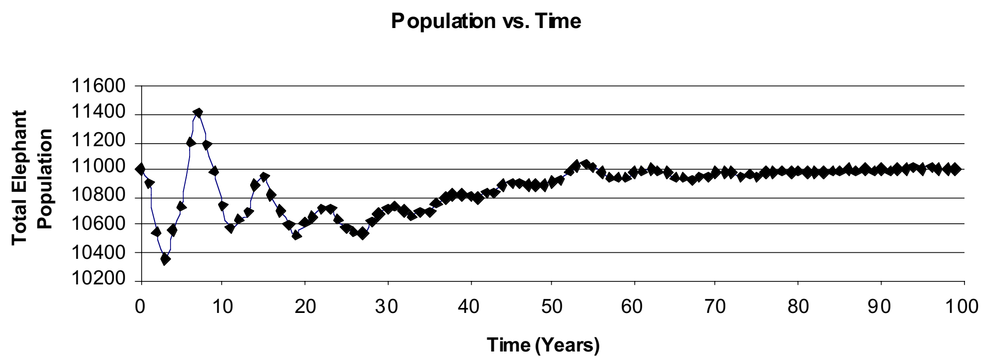


Figure 3. Population over time.

For the first several years after we introduce the contraceptive, the population fluctuates as the model adjusts to compensate by stabilizing the birth rate. In the past, up to 800 elephants were killed every year; here the population never diverges from 11,000 by that much.

How many elephants are darted? While the number initially fluctuates between 0 and 2,000, it levels out to around 1,300 darts per biennial darting, or about 25% of the female population.

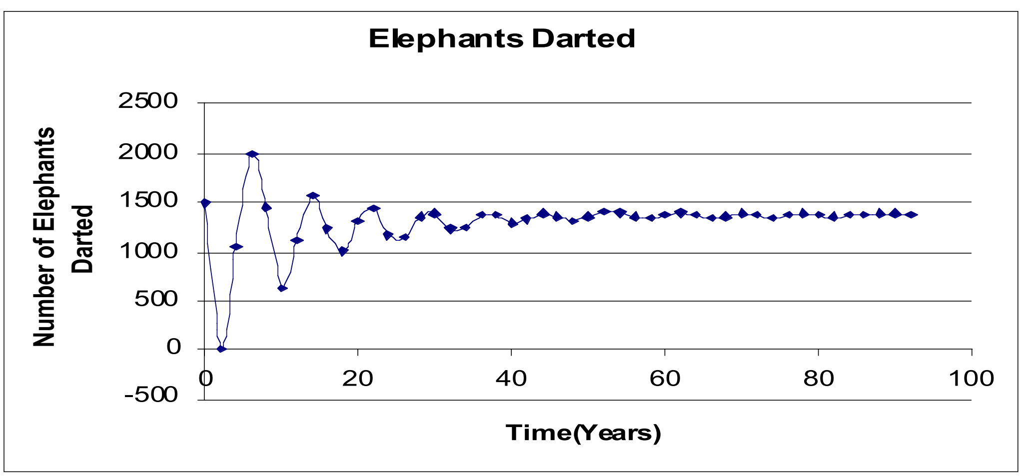


Figure 4. Numbers of elephants darted.

We can simulate the population more accurately by keeping track of each elephant as it ages, gives birth, and dies. Instead of using average probabilities, we use random events to simulate the chaos of the real world. We also keep track of the population on a monthly instead of a yearly basis. **Figure 5** shows a graph produced by our random case simulator. The darting strategy still causes the population converge to 11,000 after some time.

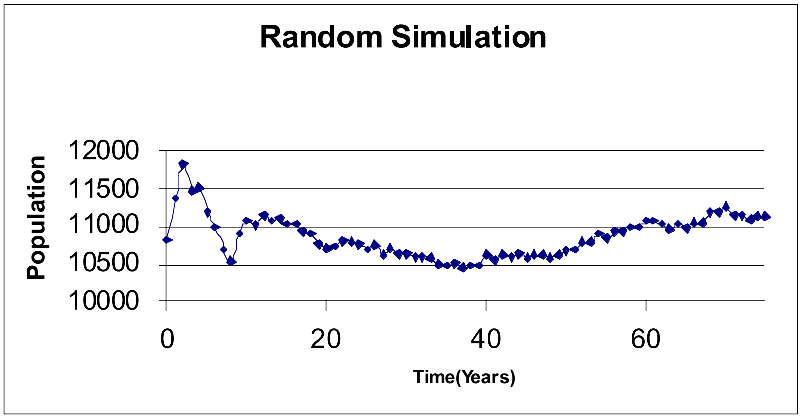


Figure 5. Numbers of elephants darted.

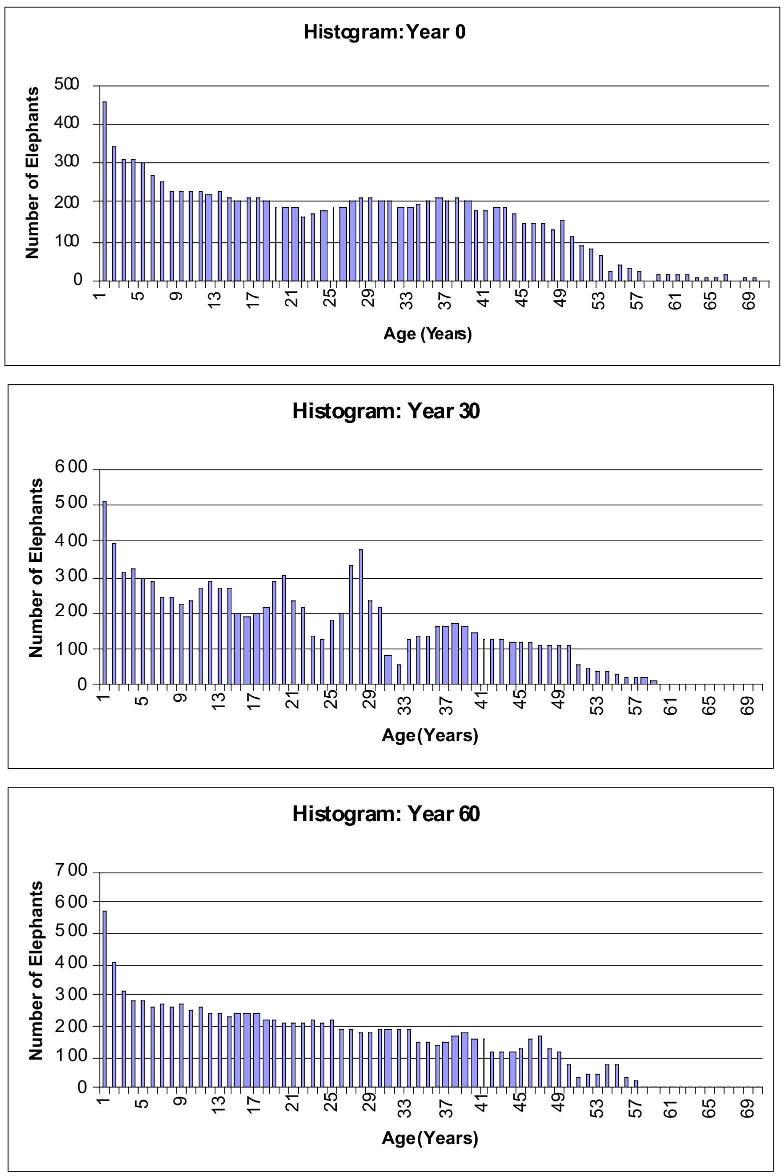


Figure 6. Age distribution initially, after 30 years, and after 60 years of the darting strategy.

Over time, the age distribution tends to shift towards more very young elephants and newborns and fewer old elephants. **Figure 6** shows the initial population distribution and the distributions after 30 years and after 60 years:

- Initially, there is a large number of animals between 25 and 45 and the number of newborn animals is not much larger than all the others.
- After 30 years, there are noticeable spikes in the population due to the large fluctuations that occur during the first several years of the model. There are large numbers of the slightly younger animals, which is good for tourism—tourists usually are attracted to cute animals; additionally, there are still large numbers of the large majestic elephants that everyone wants to see.
- After 60 years, the curve has become much more regular. The only large peak is at the baby elephants. This is the best possible situation for tourists—you can see a good representation of the whole spectrum of young and old, plus a large number of cute babies.

Task 3: Relocation

Relocating elephants each year could make our method more successful, by reducing the number to dart and reducing the stress on females of monthly oestrus. Since we are darting every two years and relocation would remove pregnant and fertile elephants, the combination of darting and relocating has the potential for creating a population disaster; however, we can avoid such a problem by picking the right number of elephants to relocate.

A simulation of relocating 100 elephants per year gives a graph of population much like **Figure 7**.

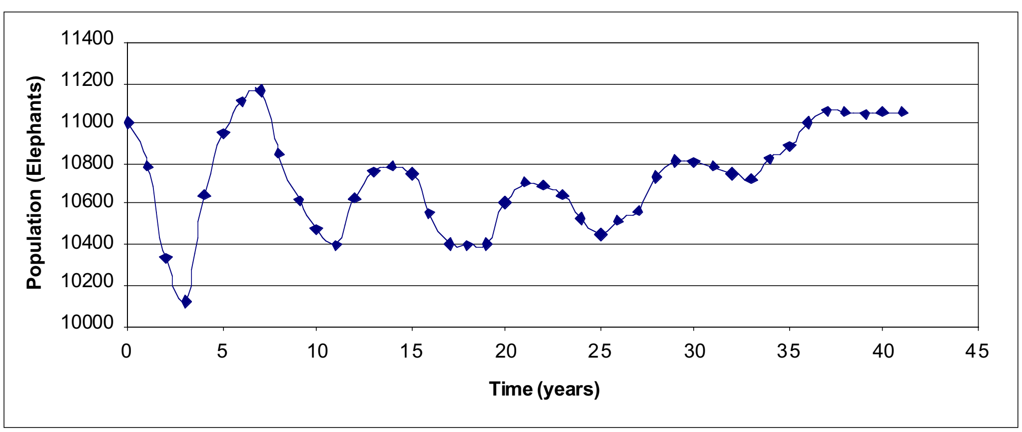


Figure 7. Simulation of removing 100 elephants per year, in addition to darting.

The population drops severely in the first few years but recovers. If this population drop of up to 8% is acceptable, relocation seems to be a viable option. As well as looking at the effects of relocation on population over time, we can also track how many darts we save by relocating various numbers of elephants. The average case results are summarized in **Table 1**.

Table 1.
Darts saved by relocating.

Relocations per year	Darts used in 50 years	Average number of darts per darting	% of darts saved
0	29,900	1,196	0%
50	24,700	988	17%
100	20,200	808	32%
150	16,250	650	46%
200	12,750	510	58%
250	9,700	388	68%

Relocating more than 250 elephants a year could cause an uncontrollable population crash after only a few years.

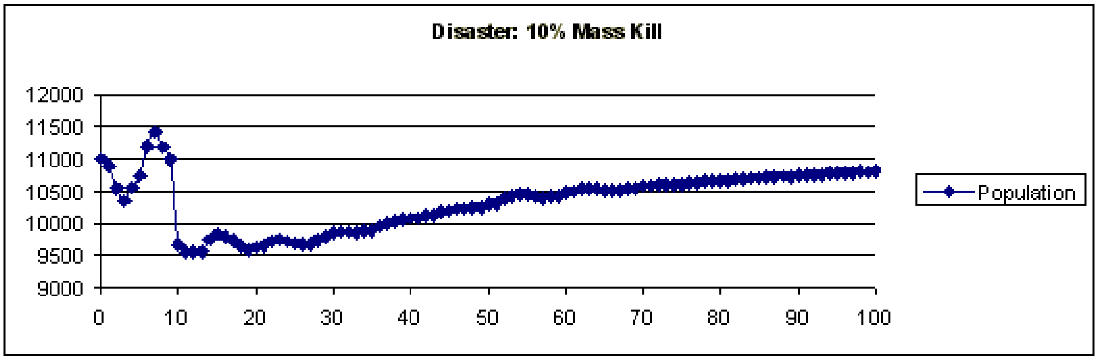
Task 4: Disaster Recovery

Darting may not allow the population to recover from a disaster even if we immediately stop darting. We examine a number of disaster scenarios and see how our model responds to them.

- The first case is a major disaster, such as a rapidly spreading and very deadly disease that indiscriminately kills all segments of the elephant population.
- Next we consider a natural disaster, such as a drought or a famine. In such a disaster, the weakest elephants are most likely to die; these tend to be the youngest and oldest elephants in the population. To model this, we kill portions of the population that are under the age of 10, because they have not yet reached maturity, and portions that are over 50, because they are suffering from the effects of old age.
- Finally, we consider the effect of excessive hunting. Hunters hunt elephants with large tusks, found on very mature elephants. Therefore, we remove parts of the population over the (arbitrary) age of 40.

In each case, we compared removing 10% with removing 50% of the selected population, to simulate moderate and severe disasters. In each scenario, the disaster occurs during year 10.

In the case where 10% of every segment of the population dies, the population hits a minimum of 9,500 and increases fairly steadily thereafter; even for a 50% kill-off, the population still recovers (**Figure 8**). While it might be possible to recover faster, doing so causes dangerously large oscillations once the population has returned to its normal levels. This way, the population makes a steady recovery and reaches normal levels while still remaining under control.



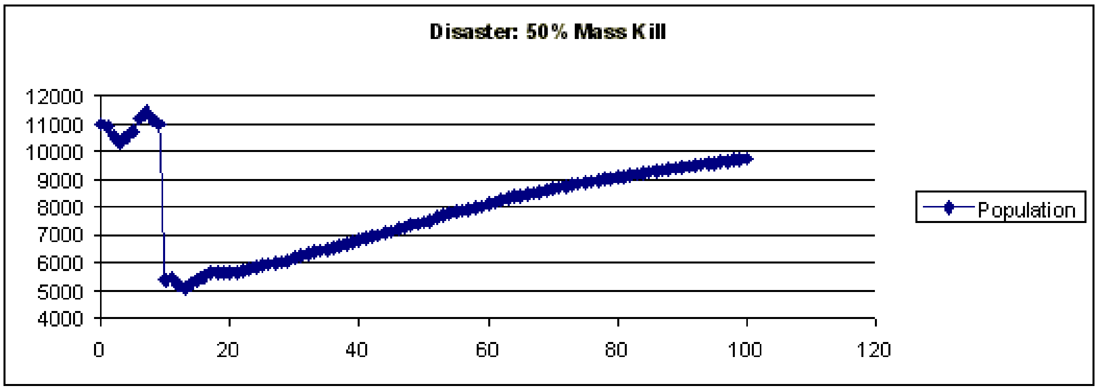


Figure 8. Effects of moderate and severe major disasters (age groups affected equally).

After a natural disaster that kills 10% or even 50% of the very young and very old elephants, the recovery is faster because the young and old are not heavily involved in reproduction (**Figure 9**).

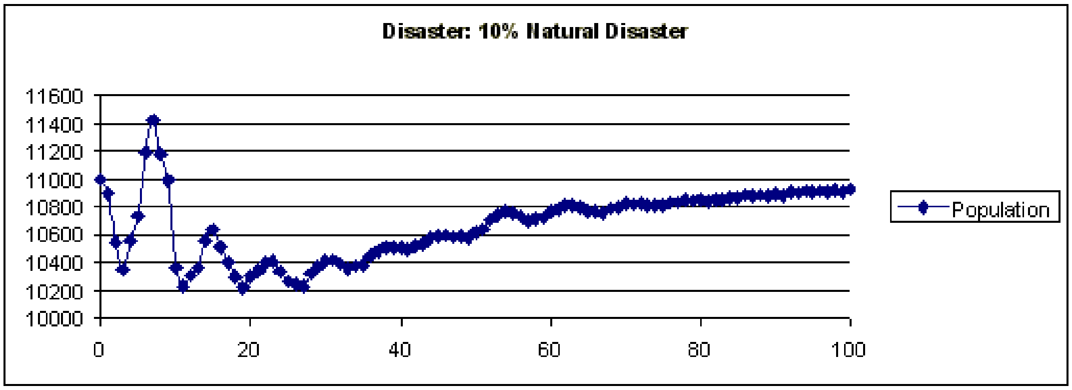
If hunters kill 10% or even 50% of the population over the age of 40, a significant number of reproducing animals are killed, so the recovery is somewhat slower (**Figure 10**).

Our schedule of darting would allow the population to recover from major disasters. Assuming that such disasters occur only rarely, a park using our management policy should have no trouble with population crashes.

Task 5: Justification to the Park Managers

You may well wonder why mathematics is useful in the task of regulating the elephant population in your park. It seems easier to follow a simple set of rules like the following:

- If there are more than 11,000 elephants, dart more than last time.
- If there are fewer than 11,000 elephants, dart fewer than last time.



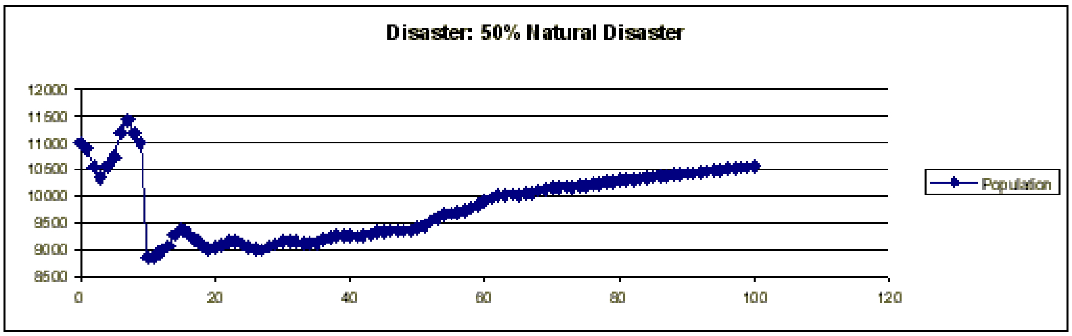


Figure 9. Effects of moderate and severe natural disasters (weakest elephants succumb).

Such a system is simple to understand but difficult to put into practice. For one thing, it is hard to decide on an exact number to increase or decrease the number of darts you are using. The other problem is that changes in the number of dartings does not affect the population for another 22 months. These factors make such a system very problematic in the real world.

Suppose we tried a system of darting a certain percentage of the elephants every two years. If we picked precisely the right percentage, the population would appear to hold steady at 11,000 for a little while, but the fraction of the population that was pregnant would gradually change over time and the population would go out of control faster than the function could compensate. This result can be shown using a simple computer simulation of the population over time.

A better goal than keeping the population constant is keeping the number of elephants born each year constant. Since the rate at which elephants die does not change much, keeping the number of births constant should eventually give a constant number of elephants. Based on elephant birth and death statistics for a healthy herd, we can adjust a healthy population of around 11,000 elephants to a state of equilibrium. By calculating the number of elephants that are less than one year old, we get a good idea of how many elephants were born last year. Dividing by the total number of elephants gives the fraction of the total

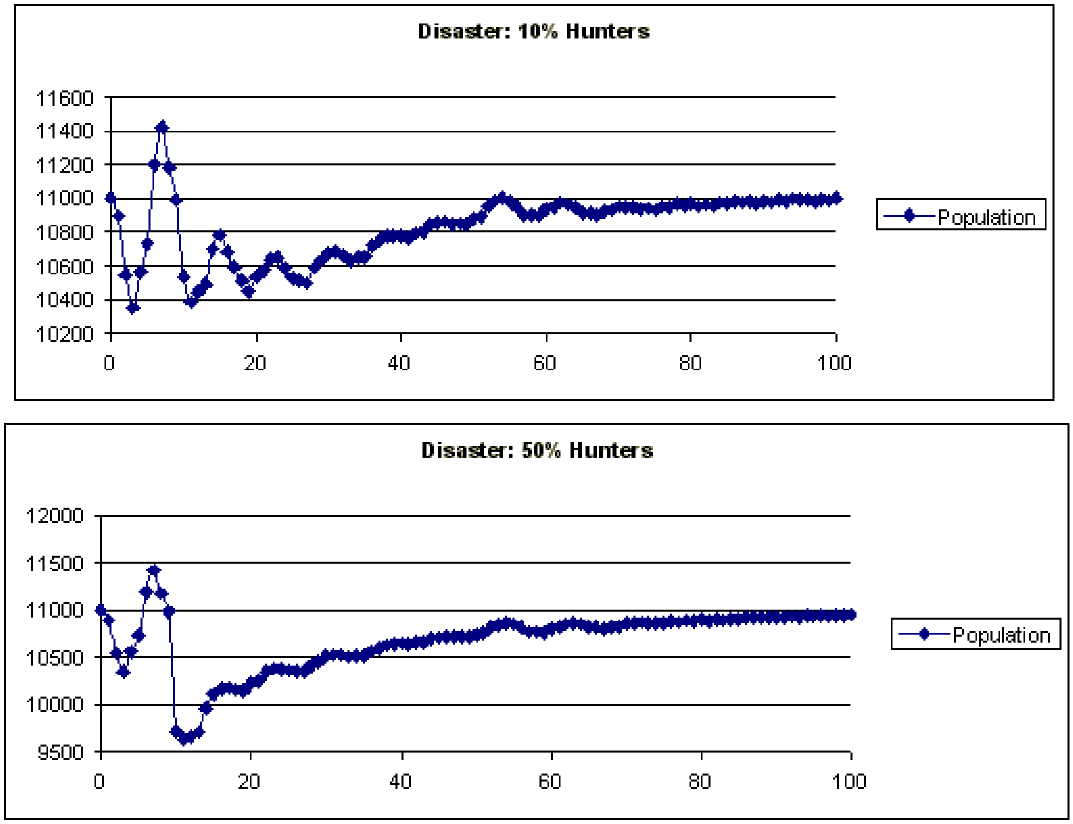


Figure 10. Effects of moderate and severe hunting.

population that must be born each year to keep the population stable.

We constantly have to readjust the number of dartings based on the effect on the future population, for which we provide a formula. We have tried this formula in several simulations and found it extremely adaptable and effective. Its strength lies in the fact that it was derived using sound reasoning; any darting method that does not use mathematics is little better than a wild guess and will not produce satisfactory results. If you use a mathematical model to control your elephant population, you will be satisfied by the long-term behavior of the population. As always, there will be some random fluctuation, but this model provides and effective solution.

Task 6: Generalization

We show that in many cases we can use our model for other parks with different needs.

A key aspect of making our model work is finding an acceptable f_b (the fraction of the population that are newborns) for each target population and set of conditions; this fraction is derived from survivorship data for the individual park's population. Our method forces convergence to the target population.

Suppose that a park has similar conditions but that the death rate among newborn elephants is .35 and the park aims for 25,000 elephants; we find $f_b = 0.046$. This makes sense—the value of f_b must be higher to compensate for the higher death rate, which means that a greater proportion of the population must be newborns in order to maintain the stability of the population.

As a second example, consider a park with a target population of 300 and an infant death rate of 15%. In this case, $f_b = 0.013$—smaller, to compensate for a smaller infant death rate.

Any park with reasonable values for death rates and ideal number of animals should be able to work under this system.

Sensitivity Analysis

For a model incorporating as many parameters as this one does, it is vital to determine which introduce the greatest error. Given a $\pm 10\%$ deviation in the value of the parameter, we calculate the percentage change in the value that the final system converges to. **Table 2** summarizes the parameters that have significant effects; the model is fairly insensitive to the values of other parameters.

Table 2.
Sensitivity of the model to changes in parameters.

Variable	From data	+10%	Equilib. Herd Size	% Diff.	−10%	Equilib. Herd Size	% Diff.
Newborn survival rate	.75	.825	16,200	47%	.675	7,200	−35%
f_b	.0255	.02805	12,100	10%	.02295	9,935	−10%

It is vital to know accurately the newborn survival rate, since the final population is so dependent on this value.

Strengths

- Our methods keep the elephant population under control, which is the main point. The population converges to the ideal number of elephants in a reasonable time.
- Our methods can incorporate various scenarios: contraceptive darting, relocation, compensation for disasters, and application to other similar parks.
- This model is simple enough for the park rangers to understand.
- Our method can produce accurate predictions with very little computer time.

- Our method is robust, so that other variables or situations can be easily introduced.
- After the first five years, under normal conditions the population does not deviate more than 200 elephants from the target value.

Weaknesses

- Our model is somewhat involved, and predictions cannot be generated without a computer.
- The population does not stabilize at exactly 11,000.
- The model responds slowly (though surely) to dramatic changes in the population.
- The method does not allow the relocation of more than 250 elephants per year, which might be possible with a more radical model.

Conclusion

Keeping a dynamic system like an elephant population under control is a very old and difficult problem. It is made more difficult by the long life spans and steady reproductive rates of elephants. We have developed a system that is more humane and more adaptable than simply killing off excess elephants.

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