Judges' Commentary: Lost Plane Problem

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The Problem

Recall the lost Malaysian flight MH370. Build a generic mathematical model that could assist "searchers" in planning a useful search for a lost plane feared to have crashed in open water such as the Atlantic, Pacific, Indian, Southern, or Arctic Ocean while flying from Point A to Point B. Assume that there are no signals from the downed plane. Your model should recognize that there are many different types of planes for which we might be searching and that there are many different types of search planes, often using different electronics or sensors. Additionally, prepare a 1–2 page nontechnical paper for the airlines to use in their press conferences concerning their plan for future searches.

Introduction and Overview

This year's Lost Plane Problem focused on identifying the factors and metrics for developing a coordinated search plan for a lost plane over open water. The problem required teams to develop a modeling approach based on these factors to achieve an undefined objective related to searching for a lost plane. In addition, there was the traditional required nontechnical paper, with this year's focus on providing the airlines information concerning the search plan for use in press conferences.

I start this commentary with a short review of the mechanics of this year's judging process. I follow the mechanics with a discussion and observations from the judging on various elements of the problem. I then discuss the importance of sensitivity analysis, assumptions and identifying

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the strengths and weaknesses of a developed model. I finish by addressing some points concerning communication and conclude with a summary.

The Process

I believe that it is beneficial to once again review several elements of the judging process for this year's contest.

The criteria used to distinguish outstanding papers from good papers gradually changes as the judging progresses through the triage to the final rounds, with the final papers standing out as the best papers submitted under a wide variety of criteria.

Triage

The primary objective of the triage is to identify papers that should be given more detailed consideration from the judges. Every paper is read by at least two judges seeking to determine if the paper contains all of the required and necessary elements that make it a candidate for more detailed readings. If a paper addresses all of the issues and appears to have a reasonable model, then judges are likely to identify it as a paper that deserves more attention.

A paper must be clear and concise to do well in the triage, and the paper's summary is critical at this point in the judging. A good summary provides a brief overview of the problem, the paper's structure, and specific results stated in a clear and concise manner. Small things that make a paper stand out include having a table of contents and ensuring that all required questions are clearly addressed in the paper. Many papers do not do well in the triage because they fail to address all of the questions and the judge decides that a team's efforts will not compare well with the better papers. For example, one critical element overlooked by many papers this year was directly addressing the prompt that the model should recognize that there are many different types of search planes, often using different electronics or sensors.

Fully developing all of the required elements is a critical area often overlooked in papers. For example, sensitivity analysis remains one of the weakest elements and is often entirely missing in many papers—and these papers do not do well during the triage.

In addition, it is vital that the team express their general approach and results as clearly and concisely as possible in the nontechnical position paper. This means providing a broad overview of the problem, the approach, and specific results in clear, concise, nontechnical terms. In other words, can the nontechnical paper be read and understood by someone without an education in mathematics? In this year's problem, the aim of the nontechnical paper was to provide the airlines with a structure to organize a coordinated search process that the airlines could use in future press conferences. Many papers actually provided a press release versus a nontechnical paper to the airlines. This did not hurt papers during the triage process but became more of a factor as papers advanced through the judging process.

These small things make it much easier for a judge to identify the team's effort and for the paper to do well in the triage round. However, the best models and the best effort is not effective if the results are not adequately communicated. It is important to remember that this is a modeling competition and that effective communication is a critical part of the modeling process.

Final

The final consists of multiple rounds of judging over several days. As the rounds progress, the judging criteria shifts from identifying papers that warrant further consideration to a process of identify the very best papers.

The first round of the final begins with each judge reading a set of papers and then all judges meeting to discuss the key aspects of the question and what should be included in a "good" paper. This year these aspects included, in addition to all of the required elements,

- a clear discussion of the search process,
- the development of a truly integrated search plan that incorporated a variety of search assets, and
- emphasis on the sensitivity analysis portion of the paper.

As the final progresses, each paper is read multiple times with the final set of papers being read by all judges. In these last rounds, the modeling process and the mathematical integrity of a paper begin to identify the outstanding papers in the competition.

Components of the Question

This year's Lost Plane Problem consisted of one question, but successful approaches required modelers to address three major components:

- The first component required teams to discuss what happens from the moment of lost contact with the plane until the plane crashes in open water.
- The second component required teams to address potential spatial and temporal conditions that might impact the size and shape of the search area.
- The last component focused on developing an integrated search process to cover the identified search area.

As a general comment, many papers failed to develop a clear overall objective for their model: Was the objective to

- minimize the expected amount of time to find the plane?
- maximize the probability of finding the plane?
- minimize some function of the time and cost of the search?

As is the case with all models, there was a need for data. In many cases, teams forgot, or did not include, specific values for their data or for their model's parameters. For example, saying that "the data can be obtained from the literature" is inadequate.

As another example, many papers developed a model to provide the probability of the plane's location in the ocean, for which it was not unreasonable to use a normal distribution centered around the Intended Flight Path of the plane after lost contact. However, a number of papers including such a model and presenting a lovely picture of the normal distribution did not provide values for the mean and standard deviation of this distribution. These papers did not do as well in the final rounds of judging.

First Component: Loss of Contact

What happens after contact is lost with the plane is one major aspect a team must address in their paper. This question helps to determine the initial size, shape, and location of the search area. Did the plane immediately plummet to the ocean or did it continue to fly for a set amount of time? Did the plane make some aerial maneuvers before its descent or did some combination of scenarios occur? Teams did a good job of addressing this component of the problem, with it typically encompassing the majority of the pages in a paper. In general, modeling this component consisted of two basic approaches:

- The first set assumed that the plane began some form of an immediate descent to the ocean, as if it broke apart in flight or there was an immediate lost of power.
- The second approach assumed some sort of controlled trajectory to the ocean surface.

The judges were not looking for a specific approach but expected to see a modeling process that clearly identified and developed a set of assumptions and first principles that the team could build upon.

Many papers treated this component as a multi-stage problem. Popular first-stage modeling approaches included some type of approach to cover the descent of the plane to the ocean: Bayesian analysis, calculus-based parabolic, or physics-based falling-body. This was then followed by a Monte Carlo, simulation, or particle-distribution element to help shape the initial search area.

The most common first-stage modeling approach was the use of Bayesian analysis with some sort of probability distribution over the map of the initial wreckage area. The next-favored approach by teams was a physics approach that included a wide variety of characteristics including the weight, speed, NS air resistance of the plane, and gravity to determine the impact zone. These approaches produced an initial spherical-to-conical-shaped search area overlaid with a location probability grid. The judges considered fully-developed models with supporting data for their parameters as a critical criterion for "good" papers.

Second Component: Changing Size/Shape of the Search Area

The second component that teams considered consisted of the temporal and environmental impact on the changing shape and size of the initial search area. These considerations included developing various forms of "drifting" models that included multiple factors from prevailing ocean currents, ocean winds, and the Coriolis effect.

Unfortunately, many papers talked in generalities and presented a good deal of theoretical discussion but failed actually to integrate this discussion into a clear model with data for the change in size and shape of their initial search area. The judges considered as better papers those that actually adjusted their findings from the initial search area to account for these factors.

Once a "search region" was found, many teams broke that region into a collection of smaller $N \times N$ rectangular regions. While teams chose a specific value for N, no paper seems to have performed sensitivity analysis on the value of N, which is somewhat important because that value determines the actual size of a search area.

Third Component: Integrated Search Plan

The last component of the problem required the development of an integrated search plan to cover the developed search area. Many papers took some form of optimization approach to address this element. These approaches ranged from nonlinear programming to simulated annealing, with objective functions ranging from minimizing search time or cost to maximizing the probability of successfully finding the plane or maximizing the searched area.

The typical team approached this component using the characteristics of a single search platform. However, these teams failed to recognize that this component contains one of two required elements addressed in the problem prompt. The problem required teams to consider the impact of different types of planes with different search capabilities. The judges considered this prompt to imply that teams should consider an integrated approach to solving this problem that included coordinating the efforts of different search assets in the same plan. Teams that compared the results of different types of search platforms to determine the best platform did not suffer during the process, but the judges felt that comparing several platforms simultaneously in a single integrated search plan was a criterion for better papers.

Sensitivity, Assumptions, and Strengths and Weaknesses

The judges realize the limited time available to the teams to complete their models is a considerable constraint and they do not expect perfect models. However, the judges do expect teams to analyze their models in a structured way and to assess their models critically. A vital part of the mathematical modeling process is this critical analysis of the model.

This analysis ranges from examining the impact of the basic assumptions on the modeled conclusions to examining the shortcomings of the techniques employed in the model. As in previous years, the judging criteria placed a large emphasis on assumptions, sensitivity analysis, and testing. Many papers neglected to fully consider these issues and were scored lower by the judges.

Assumptions

The basic assumptions that a team makes are the starting point for their modeling efforts. The judges did not place restrictions on the basic assumptions other than that they need to make sense and be necessary.

However, simply listing assumptions is not enough; papers should include a discussion of why they are making an assumption and its potential impact/influence on the model.

It is also important to recognize that stating the assumption is not the end of the process; examining the impact on the modeled conclusions if the assumption changes is a vital part of the modeling process.

An example is the assumption of the type of plane that is lost. Many teams, when they developed their model for what happened after contact was lost, assumed that a particular type of plane was lost. However, few teams took advantage to address the second required element, to recognize that there are different types of planes and see how changing the type of plane impacted their analysis.

If changing an assumption results in a change of the results, then the team should indicate that as a potential weakness. The judges considered addressing the impact of different planes being lost as a criterion for "good" papers.

Sensitivity Analysis and Testing

Sensitivity analysis and testing were appropriate and necessary for all modeling approaches. Many papers included a sensitivity analysis section in their paper but only addressed the theoretical aspects of sensitivity analysis versus actually changing the value of an assumption or parameter to understand the impact. This year's problem was a perfect candidate for testing against a historical lost aircraft case study. Many teams provided a historical case study but they only used it to test the development of their initial search area and not against the actual search process. The inclusion of either a robust sensitivity analysis or a model-testing section was viewed as a critical criterion for "better papers."

Communication

Papers were judged on the quality of the writing, with special attention to the summary and to the nontechnical letter. The quality of writing, in general, is continuing to improve. The strongest summaries this year included a clear objective with a description of the search process, a general overview of the modeling process, and an explicit result of the model analysis. The judges continue to be surprised by the number of papers where the summary only describes what the team will attempt or the general theory without describing the results. Similarly, many of the nontechnical articles focused more on providing an actual press release instead of a structure for a search process.

A nontechnical article does not mean that numbers are not included. It means that the article can be read meaningfully by someone without an education in advanced mathematics.

In addition, for this year's problem, the nontechnical letter was supposed to be written as consultants to the airline executives as a guide that they could use in a press conference to describe their efforts in locating the downed aircraft. As such, the letter was supposed to summarize the team's overall approach to solving the problem. Instead, the vast majority of teams wrote this letter in the form of a press release for the airlines, which is not at all what was intended.

Conclusions

The outstanding teams modeled and presented all the aspects of the problem described in the problem statement, including the fully-developed standard elements (assumptions, sensitivity analysis, strengths and weaknesses, etc.), developed an effective model, explained the modeling choices made, and were clearly and concisely written. The judges continue to be impressed with the quality of the submissions, especially considering the time constraints. The growth in the quality and number of submissions is very encouraging to those who work to promote the practice of good mathematical modeling.

About the Author

Robert Burks is a senior lecturer in the Dept. of Defense Analysis at the Naval Postgraduate School. He received his undergraduate degree in Aerospace Engineering from the U.S. Military Academy, his master's in Operations Research from the Florida Institute of Technology, and his Ph.D. in Operations Research from the Air Force Institute of Technology. He has wide-ranging research interests, including diffusion of information, agent-based modeling, and the mathematics of 3D animation and gaming. Dr. Burks served as both a triage and final judge on this year's Lost Plane Problem.