# The Spreading HIV/AIDS Problem

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## Summary

We propose four main focus areas in which the world can win its battle against AIDS:

- **Identifying** individuals who are HIV positive, through blood testing (in batches, to save money). People who are HIV-positive will recognize that they are infected and take measures to ensure that they do not spread the virus to other people. We plan to test everybody in sub-Saharan Africa by spending $1.5 billion/year for 3 years.
- **Educating** the public on how HIV can be prevented, in order to keep the incidence rate down. Showing people that AIDS is incurable, along with the ABCs of prevention, our hope is that they will inculcate the practices in order to stop the epidemic. These practices include remaining abstinent ("A"); being faithful ("B") to one uninfected partner if one chooses to be sexually active; and the use, availability, and effectiveness of condoms ("C") during intercourse.
- **Antiretroviral treatments** (ART) for women who are or become pregnant before our plan is implemented. These treatments help reduce the risk of an infected mother passing the virus to her child during pregnancy, birth, and nursing. This area will be the most difficult to support financially due to the cost of producing and distributing treatments. However, surplus funds will alleviate this problem after the testing phase is complete.
- A **vaccine**; one might be available by 2011. Though this date could be pushed forward with more funding, we suggest keeping vaccine research resources stable and focusing on the 100% guaranteed cure, the ABCs. The vaccine

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however, might not be completely effective and accidentally allow for drug-resistant HIV strains to evolve. By 2011, the testing phase will be complete, and hopefully governments would have established a basic national education program, thus freeing up additional assets, which can be put into manufacturing and dispensing the vaccine.

# Problem Approach

- Task 1: To get the rate of change in HIV/AIDS for 2006 to 2050, we develop a model of the growth/decay of the population for each country. We use the prevalence rate, number of new cases, and the population each year (as the carrying capacity) to develop a logistic growth model for the virus, assuming that there is no intervention in the spread of HIV.
- Task 2: To account for the introduction of treatments, vaccinations, and a combination of the two, we develop state diagrams to show the possible paths of HIV. Using the diagrams and determining rates of change from one state to another, we formulate new plots of the total population and the spread of AIDS.
- Task 3: There is no way to predict the rate at which the virus would become drug-resistant. We would have to make too many assumptions such as the number of people being treated, the amount of treatment they were receiving, the effectiveness of that treatment, the frequency of each treatment, the adherence to the treatment regimen, and the probability of the virus to mutate. Hence, we offer a qualitative approach rather than a quantitative one. We look at what effect drug-resistant strains would have on our model, specifically the rates of change from one state to another.
- Task 4: In our white paper to the United Nations, we focus on allocating resources to treat the problem and not to the symptoms. The surest way to keep from getting HIV/AIDS is practicing abstinence, being faithful, and using condoms during intercourse. Therefore, HIV testing and education are our primary objectives in combating the epidemic.

# Assumptions

- The population with HIV/AIDS is homogeneously distributed. This assumption is critical. I those infected with HIV and AIDS interact only with others with HIV and AIDS, then the infection will never spread to the uninfected populace; with homogeneity, the infection could spread to everyone.
- The number of people infected with HIV/AIDS includes those who have been diagnosed and those who have yet to be diagnosed. Without this

assumption, the number of people affected each year only from diagnosed patients would be too low.

- The probability of a person who has been vaccinated contracting HIV from someone who is undergoing ARV treatment is negligible.
- Each country to be modeled is a closed system. There is no immigration or emigration of infected individuals.

# Task 1: The Rate of Change from 2006 to 2050

We chose the following countries: Haiti, Guyana, South Africa, Ukraine, India, and Australia, for reasons noted below.

The following is a state diagram that is the base for our models. We reserve the technical details of formulation of a system of differential equations to the **Appendix**.

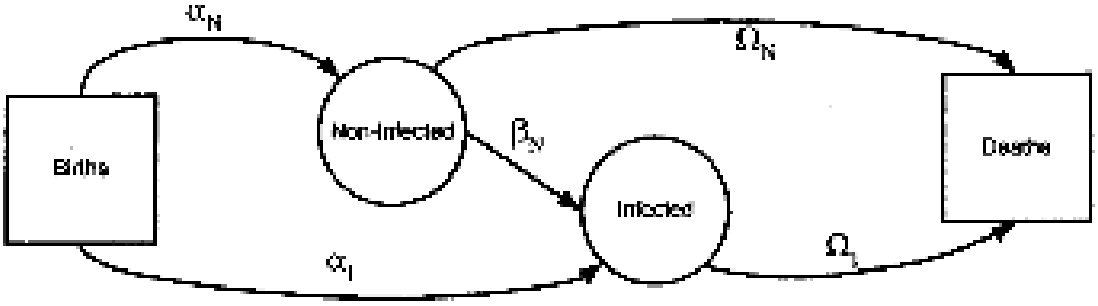


**Figure 1.** Simplistic HIV / AIDS state diagram.

People are born either infected with HIV or non-infected. A person who is not infected can become infected; but once a person becomes infected, there is no known way to rid oneself of the infection. Both states transition to deaths, but at different rates.

We chose to model Haiti from North America because of its relatively high prevalence rate of 2.6%—high compared to the 1.4% prevalence across the border in the Dominican Republic or the 0.15% and 0.29% prevalence in more developed countries such as Canada and the U.S. Haiti is also one of the poorest countries on the continent, with a GNP per capita of only $510 per year. With treatment cost at well over $1,000 per person per year [Adams et al. 2001], the average Haitian cannot afford HIV medication.

Guyana represents South America in our model. Guyana has a prevalence rate of 2.0%. While Brazil has 36 times as many cases of as Guyana, Guyana's small population makes the problem more widespread. In addition, Brazil has already taken steps to institute a solution to their AIDS problem, spending $444 million to provide 100% of the HIV infected with treatment [Andrews 2004]. Guyana, however, has a far lower GNP than Brazil and fewer resources to dedicate to HIV.

Although Africa is by far the hardest hit continent in the world in terms of HIV/AIDS, South Africa stands out even among African countries, leading the world with 4.2 million cases and a 9.33% prevalence. Additionally, 29.5% of pregnant women tested in South Africa test positive, meaning that the disease is being propagated to children [AVERT.org 2006].

In 1999, Ukraine reported 240,000 cases, with a total population that is decreasing rather than increasing.

India has begun to show disturbing signs that it may be the next hotbed of HIV/AIDS. It has the second-largest population, 1.1 billion, as well as the second-largest HIV-positive population, 3.7 million. This leads to a deceptively low prevalence rate——0.47%——but India is expected to surpass South Africa as the world's-leading HIV-positive population by 2010 [AVERT.org 2006].

From the continent of Australia, having not much choice, we selected Australia. All of the countries in the region have low rates of HIV infection; Australia has only 14,000 cases, the most in the region.

## Assumptions

- For a simplified model, we assume that countries will not intervene to stem the spread of the virus through medical treatments.
- AIDS spreads through a population according to a logistic function.

This second assumption is realistic because many similar models, such as the spread of technology and the growth of a population, are based on logistic growth. Each infected person makes contacts with a fraction of the non-HIV population in a way that could transmit the virus, and only a fraction of those contacts actually transmit the virus. So the number of people whom an infected person infects in a year is the number of contacts per non-infected persons, times the infections per contact, times the non-infected population:

$$\left(\frac{\text{contacts}}{\text{non-HIV}}\right) \times \left(\frac{\text{transmissions}}{\text{contacts}}\right) \times (\text{non-HIV}). \qquad \textbf{(1)}$$

We call the product of the first two fractions the transmission rate $\beta_N$. So the total transmissions per year is $\beta_N NI$: the transmission rate times the non-infected population times the number of infected people $I$. Since the total population is growing logistically, HIV also spreads logistically.

Expression **(1)** assumes that the transmission rate is constant. However, since spreading HIV is much more common among certain human behaviors, such as sexual encounters and intravenous drug use, the transmission rate could be different for each infected person. Additionally, certain types of contacts have a higher fraction of transmissions per contact. For example, anal intercourse has a higher fraction of transmission per contact than vaginal intercourse; likewise, transmission of the virus from a male to a female is more likely than from a female to a male [World Bank Group 2006]. To simplify the model, we just consider an average fraction of transmissions per contact.

# Logistic Models for Critical Countries

We now consider a graphical model of the spread of HIV in Haiti (**Figure 2**).

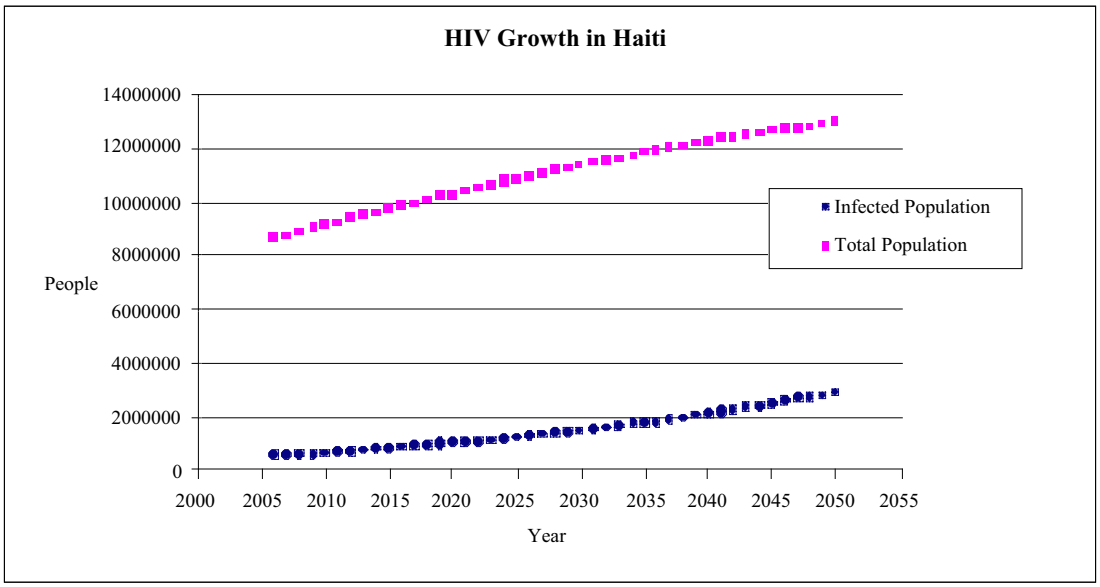


**Figure 2.** HIV and total population projections for Haiti.

To develop this graphical model, we follow our assumption that HIV follows simple logistic growth.

$$\frac{dI}{dt} = \alpha_I I - \frac{\alpha_I I^2}{K_I}, \tag{2}$$

where $\alpha_I$ is the initial growth rate of the infected population and $K_I$ is its carrying capacity. We can find values for both of these by fitting a second-order polynomial to a graph of the infected population rate of change vs. the total population. To do so, we created a table of data for Haiti from 1985 to 2005: population, number of HIV infections, and prevalence rate [Central Intelligence Agency 2001; UNICEF 2005]. We graphed the change in the number of infections each year against the total number of infections that year. Based on the values obtained, we projected the infected population from 2006 to 2050.

Since the carrying capacity is based on the population, and the population increases, the graph doesn't appear to follow an exactly logistic model. However, it does show how quickly AIDS epidemic is spreading. Since HIV / AIDS has been around for only 25 years, predictions 50 years into the future, can only give an idea of the potential severity of the spread.

Following the same procedure, we created graphs for each of the other countries (**Figure 3**).

## Guyana

The overall population is projected to decrease, which one would expect to lead to the AIDS level also decreasing. However, our models project the

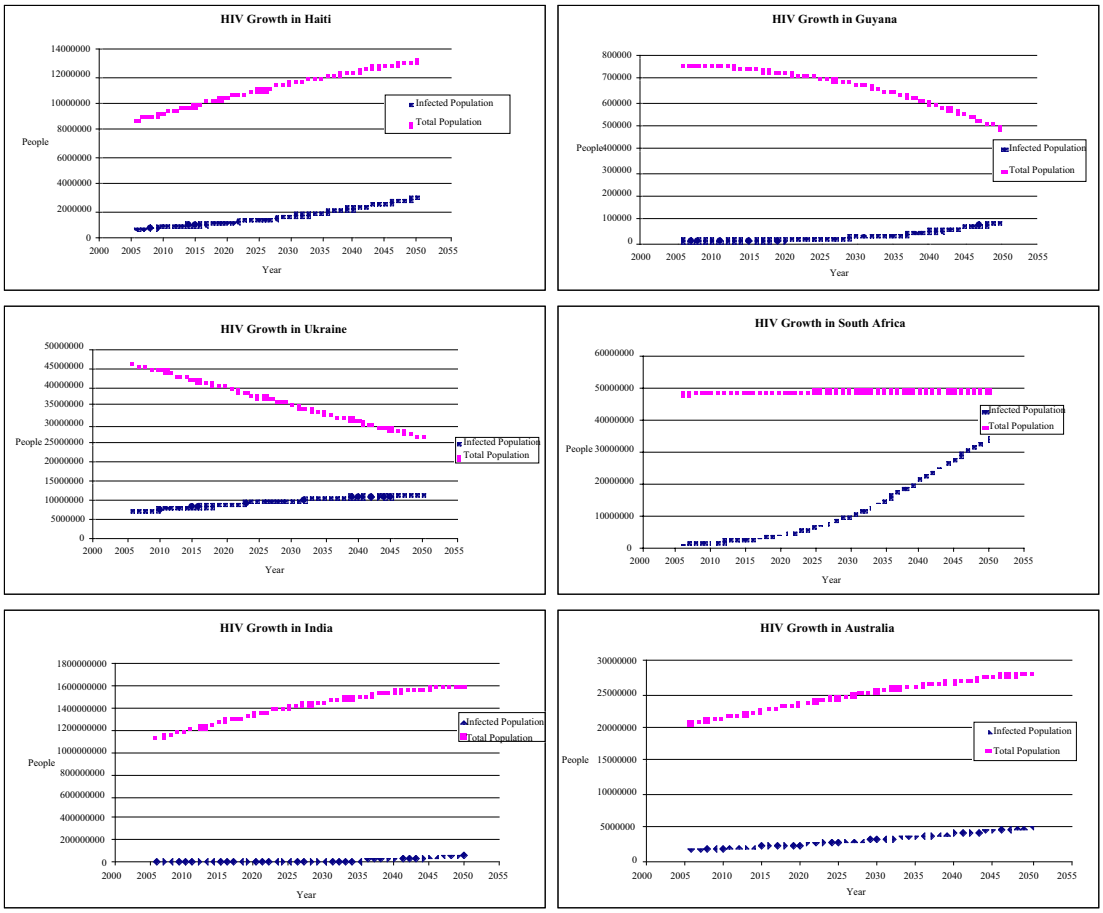


**Figure 3.** Total population and HIV population projections for the critical countries.

infected population growth rate and then calculate the number of cases based on this growth rate and the population. Therefore, as the infection growth rate increases and population decreases, the number of AIDS cases can either increase, decrease, or remain the same, depending entirely on the relationship between the growth rate and the rate of decrease of population.

## Ukraine

presents a scenario similar to Guyana's. Were the population to remain stable, the number of infected people would rise much more quickly. This graphic model still presents an interesting issue: the infected population will eventually reach the total population in time.

## South Africa

Based on the initial quick growth of HIV, the model predicts continued alarming increases in cases, barring intervention, to 69.3% of the population in 2050.

### India

The large population creates a deceptively small prevalence rate. The graph makes it look as if the number of infected people is small and growing slowly, yet the numbers are large, increasing quickly, and perched to surpass South Africa by 2010.

### Australia

HIV is seen to be growing steadily, yet more slowly than the population. This is a stark contrast to almost all of the other critical countries.

## Task 2: Drug Therapy and Vaccination

Therapy refers to antiretroviral treatment(ART), which is currently prolonging the lives of many people living with HIV. ARV treatments lower the concentration of HIV in the bloodstream, allowing the body to fight the effects longer and making the disease more difficult to transmit to others. The decreasing transmission rate among infected individuals means that not as many people each year will contract HIV compared to the same scenario without ART. Introducing ARV treatment changes the state diagram to **Figure 4**.

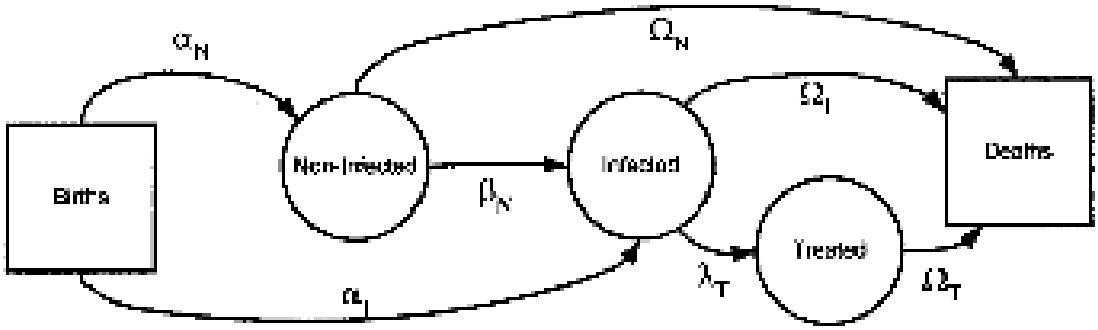


**Figure 4.** Treatment state diagram.

As treatments become available, infected people begin receiving them. Non-Infected people will not need treatment, which is why there is no transition from the Non-Infected state to the Treated state.

Although no vaccine exists yet, the state diagram in **Figure 5** depicts the changes in the model for a vaccine.

A vaccine would be administered only to the Non-Infected population or those being born. Since the vaccine is not an elixir of life, there will still be some death rate.

ARV treatments are available now, and should vaccines become available in the future, the two methods would be used together to attempt to eliminate AIDS. This creates still another model from which we can make predictions (**Figure 6**).

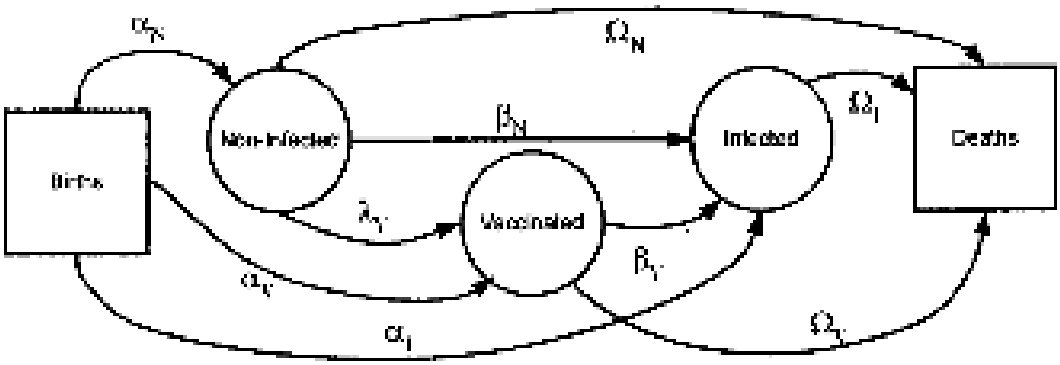


**Figure 5.** Vaccination state diagram.

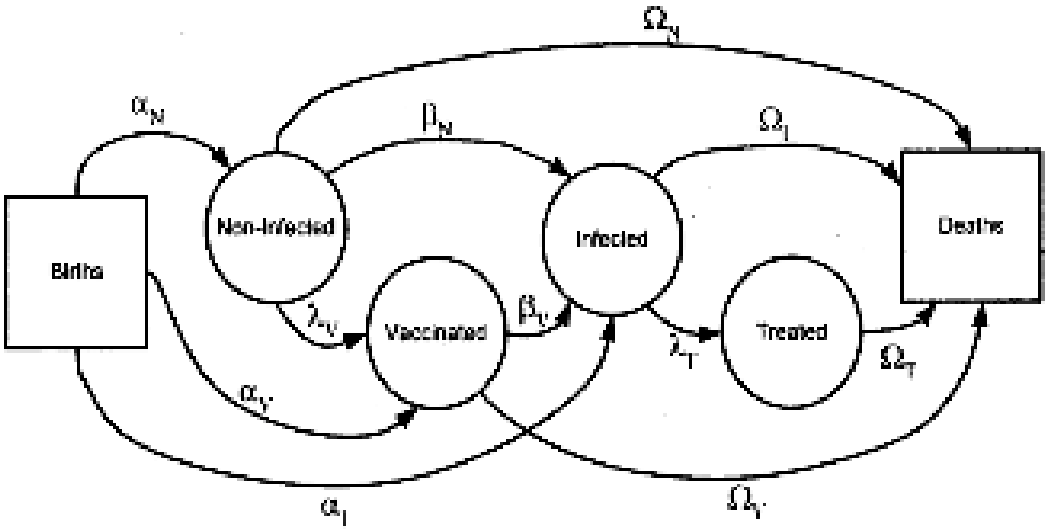


**Figure 6.** Vaccination and treatment state diagram.

Without strong data on ART and an actual vaccine, it is difficult to make a quantitative extrapolation of their effects. Qualitatively, however, the factors can be graphed and trends predicted. **Figure 7** shows a generic graph based on the differential equations developed at the end of the **Appendix**.

The graph suggests that ART and a vaccine will be fairly successful in battling AIDS but with some major drawbacks:

- ART is expensive and not widespread. It currently costs \$800 per person per year to treat AIDS using the name-brand medications; this cost could drop as low as \$300 with generic drugs [Andrews 2004].

- If price were the only obstacle, the problem would be clearer. However, ART also makes it much more likely that a patient will develop resistance to a drug, or that the virus will mutate. Both of these are extremely serious considerations that must be weighed against the effectiveness of the treatments themselves. We consider drug resistance in Task 3.

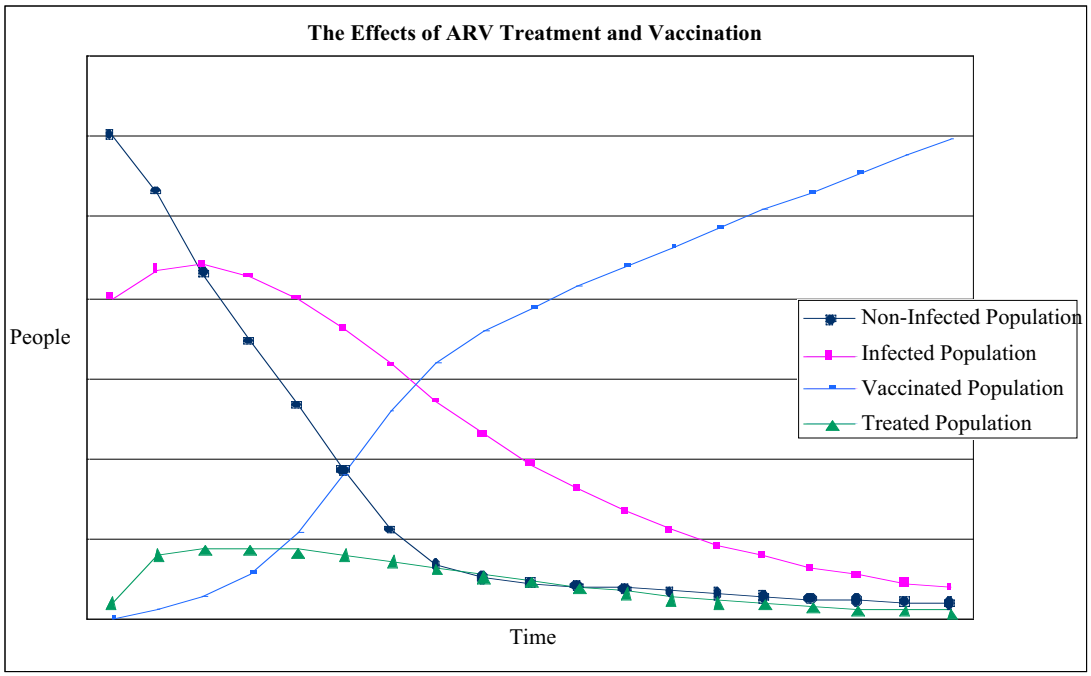


**Figure 7.** Effects of ARV treatment and vaccination.

- Cost and resistance also surface with a vaccine, as well as mutation that would render the vaccine useless. We discuss mutations in Task 3.

- Availability of financial resources has been a problem; a hopeful sign is that with the worldwide concern for AIDS growing quickly, the level of financial resources and relief is also on the rise.

**Figure 8** shows a generic example of what can be expected to happen in South Africa with ART and prevention education. When compared with **Figure 2** (spread of AIDS left unchecked), the difference is immediately evident as well as inspiring.

Why, then, do people in the world still suffer from AIDS if it is so simple to make the line on the graph go down? Aside from the unavailability of a vaccine yet, the main factor is funding. South Africa, for instance, has a GNP per capita of only $3,020. While this is enough to purchase ART, an individual would have to make substantial sacrifices to do so. Additionally, wealth is concentrated in a small percentage of the population.

In deciding where to send aid and which areas represent the largest threat of infection expansion, GNP and a country's economic strength should be taken into consideration as major factors. Not all countries that are struggling against AIDS have even the mediocre GNP per capita of South Africa. Haiti, for example, is expecting a much sharper increase in the number of cases than Australia. Australia has a GNP per capita of $20,240, allowing individuals infected with HIV to afford ART. Haitians, with a per capita GNP of $510, cannot even afford a year worth of ART with an entire year's GNP. This is one major reason why

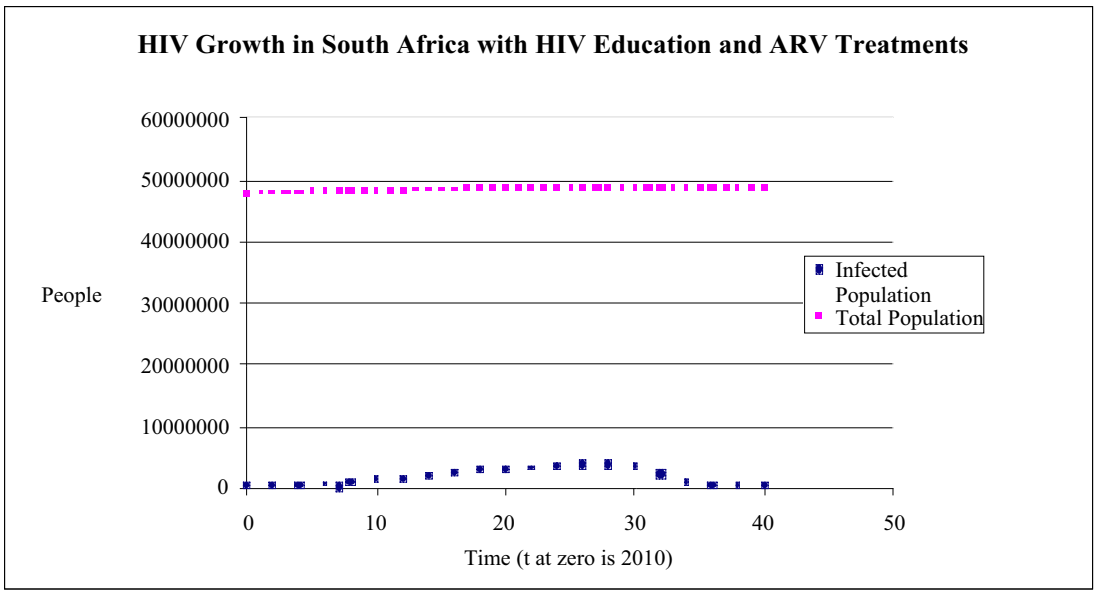


**Figure 8.** HIV growth with education and ARV treatments.

HIV prevalence is increasing so quickly in the Third World——the countries cannot stop it. This is why foreign aid is imperative and needs to be directed to the poorest countries.

# Task 3: Drug-Resistant Strains

When treatments are weak or are not taken correctly, viruses can develop resistance to the treatments. Both $\beta_T$ and $\beta_V$, the coefficients for transmission in the cases of treatment and of a virus would be changed by virus resistance. We can consider such a transmission rate as the reciprocal of the drug or vaccine's effectiveness. As resistance develops, effectiveness decreases; as effectiveness decreases, transmission (and the value of $\beta$) increases. In time, it is possible for this transmission rate to catch up with $\beta_N$, which would mean that vaccinating or treating people would make them more susceptible to the virus.

In addition to drug resistance, HIV could mutate as a result of treatment. Mutations could produce strains of the virus that are more active and destroy the body more quickly.

# Conclusion: Recommendations

Sub-Saharan Africa is the decisive point for stopping the spread of AIDS. Of the expected $7 billion dollars to be spent on AIDS worldwide in 2006, we recommend that $4.5 billion be spent in sub-Saharan Africa on testing the population, educating the population about prevention, distributing preventive

measures, limited dispersal of treatment, and providing baby formula to nursing mothers infected with HIV. This would be a three-year program, spending the same proportion of money each year.

Following the three years of intensive effort in Africa, we would conduct an analysis of gains to determine success. If unsuccessful, the plan would be altered; if successful, focus would shift to India and Southeast Asia, though Africa would continue to receive money for education, prevention, treatment, and new mothers. In India, the distribution of resources would be similar to that in Africa, focusing initially on testing, education, prevention, limited treatment, and new mothers. This phase is expected to take longer in India than in sub-Saharan Africa, five years instead of three.

Southeast Asia and sub-Saharan Africa comprise the largest threat to world health due to AIDS. After they have been treated, our program will hopefully be able to enter a phase of vaccination, pending development. It is important to focus on testing, education, and prevention up to this point, using ARV treatments sparingly so as to avoid resistance and mutated strains. At this point, the plan will once again be assessed and adjusted to meet new threats.

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# Appendix

We model the total population $P = N + V + I + T$ as composed of non-infected non-vaccinateds $N$, vaccinateds $V$, infected but untreateds $I$, and infected and treateds $T$, with corresponding subscripts for parameters. We let $\alpha$

and $\Omega$ be rates of birth into and death out of each subpopulation; $\beta_N$, $\beta_T$, $\beta_V$ be infection rates for non-infecteds, infected but treateds, and vaccinateds; and $\lambda_V$ and $\lambda_T$ be rates of vaccination of non-infecteds and treatment of infecteds. The resulting system of differential equations, corresponding to the state diagram of **Figure 6** and assumption that $\beta_V = 0$, is:

$$\begin{aligned}
\frac{dN}{dt} &= \alpha_P(N+V+I+T)\left(1-\frac{N+V+I+T}{K}\right) \\
&\qquad -(\alpha_I-\Omega_I)-(\alpha_V-\Omega_V)+\Omega_T-\beta_N NI-\beta_T NT-\lambda_V, \\
\frac{dV}{dt} &= \alpha_V+\lambda_V-\beta_V VI, \\
\frac{dI}{dt} &= \alpha_I+\beta_N NI+\beta_T NT-\lambda_T-\Omega_T, \\
\frac{dT}{dt} &= \lambda_T-\Omega_T.
\end{aligned}$$

Systems of equations for the state diagrams of the other figures are obtained by setting other appropriate parameter values to zero.

# About the Authors

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