A Convenient Truth: Forecasting Sea Level Rise

Jason Chen
Brian Choi
Joonhahn Cho

Duke University
Durham, NC

Advisor: Scott McKinley

Abstract

Greenhouse-gas emissions have produced global warming, including melting in the Greenland Ice Sheet (GIS), resulting in sea-level rise, a trend that could devastate coastal regions. A model is needed to quantify effects for policy assessments.

We present a model that predicts sea-level trends over a 50-year period, based on mass balance and thermal expansion acting on a simplified ice-sheet geometry. Mass balance is represented using the heat equation with Neumann conditions and sublimation rate equations. Thermal expansion is estimated by an empirically-derived equation relating volume expansion to temperature increase. Thus, the only exogenous variables are time and temperature.

We apply the model to varying scenarios of greenhouse-gas-concentration forcings. We solve the equations numerically to yield sea-level increase projections. We then project the effects on Florida, as modeled from USGS geospatial elevation data and metropolitan population data.

The results of our model agree well with past measurements, strongly supporting its validity. The strong linear trend shown by our scenarios indicates both insensitivity to errors in inputs and robustness with respect to the temperature function.

Based on our model, we provide a cost-benefit analysis showing that small investments in protective technology could spare coastal regions from flooding. Finally, the predictions indicate that reductions in greenhouse-gas emissions are necessary to prevent long-term sea-level-rise disasters.

Introduction

There is strong evidence of global warming; temperatures have increased by about 0.5°C over the last 15 years, and global temperature is at its highest level in the past millennium [Hansen et al. 2000]. One of the feared consequences of global warming is sea-level rise. Satellite observations indicate that a rise of 0.32 ± 0.02 cm annually 1993–1998 [Cabanaes et al. 2001]. Titus et al. [1991] estimate that a 1-meter rise in sea levels could cause \$270–475 billion in damages in the U.S. alone.

Complex factors underlie sea-level rise. Thermal expansion of water due to temperature changes was long implicated as the major component, but it alone cannot account for observed increases [Wigley and Raper 1987]. Mass balance of large ice sheets, in particular the Greenland Ice Sheet, is now believed to play a major role. The mass balance is controlled by accumulation (influx of ice to the sheet, primarily from snowfall) and ablation (loss of ice from the sheet, a result of sublimation and melting) [Huybrechts 1999].

Contrary to popular belief, floating ice does not play a significant role. By Archimedes' Principle, the volume increase ΔV of a body of water with density ρ_{ocean} due to melting of floating ice of weight W (assumed to be freshwater, with liquid density ρ_{water}) is

$$\Delta V = W \left(\frac{1}{\rho_{\text{water}}} - \frac{1}{\rho_{\text{ocean}}} \right).$$

The density of seawater is approximately $\rho_{\text{ocean}} = 1024.8 \text{ kg/m}^3$ [Fofonoff and Millard 1983]; the mass of the Arctic sea ice is 2×10^{13} kg [Rothrock and Jang 2005]. Thus, the volume change if all Arctic sea ice melted would be

$$\Delta V = 2 \times 10^{13} \text{ kg} \left(\frac{1}{1000 \text{ kg/m}^3} - \frac{1}{1024.8 \text{ kg/m}^3} \right).$$

Approximating that 360 Gt of water causes a rise of 0.1 cm in sea level [Warrick et al. 1996], we find that volume change accounts for a rise of

$$4.84 \times 10^8 \text{ m}^3 \times \frac{1000 \text{ kg}}{\text{m}^3} \times \frac{1 \text{ Gt}}{9.072 \times 10^{11} \text{ kg}} \times \frac{0.1 \text{ cm}}{360 \text{ Gt}} \approx 0.00015 \text{ cm}.$$

This small change is inconsequential.

We also neglect the contribution of the Antarctic Ice Sheet because its overall effect is minimal and difficult to quantify. Between 1978 and 1987, Arctic ice decreased by 3.5% but Antarctic ice showed no statistically significant changes [Gloersen and Campbell 1991]. Cavalieri et al. projected minimal melting in the Antarctic over the next 50 years [1997]. Hence, our model considers only the Greenland Ice Sheet.

Models for mass balance and for thermal expansion are complex and often disagree (see, for example, Wigley and Raper [1987] and Church et al. [1990]). We develop a model for sea-level rise as a function solely of temperature and time. The model can be extended to several different temperature forcings, allowing us to assess the effect of carbon emissions on sea-level rise.

Model Overview

We create a framework that incorporates the contributions of ice-sheet melting and thermal expansion. The model:

- accurately fits past sea-level-rise data,

- provides enough generality to predict sea-level rise over a 50-year span,

- computes sea-level increases for Florida as a function of only global temperature and time.

Ultimately, the model predicts consequences to human populations. In particular, we analyze the impact in Florida, with its generally low elevation and proximity to the Atlantic Ocean. We also assess possible strategies to minimize damage.

Assumptions

- Sea-level rise is primarily due to the balance of accumulation/ablation of the Greenland Ice Sheet and to thermal expansion of the ocean. We ignore the contribution of calving and direct human intervention, which are difficult to model accurately and have minimal effect [Warrick et al. 1996].

- The air is the only heat source for melting the ice. Greenland's land is permafrost, and because of large amounts of ice on its surface, we assume that it is at a constant temperature. This allows us to use conduction as the mode of heat transfer, due to the presence of a key boundary condition.

- The temperature within the ice changes linearly at the steady state. This assumption allows us to solve the heat equation for Neumann conditions. By subtracting the steady-state term from the heat equation, we can solve for the homogeneous boundary conditions.

- Sublimation and melting processes do not interfere with each other. Sublimation primarily occurs at below-freezing temperatures, a condition during which melting does not normally occur. Thus, the two processes are temporally isolated. This assumption drastically simplifies computation, since we can consider sublimation and melting separately.

- The surface of the ice sheet is homogeneous with regard to temperature, pressure, and chemical composition. This assumption is necessary because there are no high-resolution spatial temperature data for Greenland. Additionally, simulating such variation would require finite-element methods and mesh generation for a complex topology.

Defining the Problem

Let M denote the mass balance of the Greenland Ice Sheet. Given a temperature-forcing function, we estimate the sea-level increases (SLR) that result. These increases are a sum of M and thermal expansion effects, corrected for local trends.

Methods

Mathematically Modeling Sea-Level Rise

Sea-level rise results mostly from mass balance of the Greenland Ice Sheet and thermal expansion due to warming. The logic of the simulation process is detailed in **Figure 1**.

Temperature Data

We create our own temperature data, using input forcings that we can control. We use the EdGCM global climate model (GCM) [Shopsin et al. 2007], based on the NASA GISS model for climate change. Its rapid simulation (10 h for a 50-year simulation) allows us to analyze several scenarios.

Three surface air temperature scenarios incorporate the the low, medium, and high projections of carbon emissions in the IS92 series resulting from the IPCC Third Assessment Report (TAR) [Edmonds et al. 2000]. The carbon forcings are shown in **Figure 2**. All other forcings are kept at default according to the NASA GISS model.

One downside to the EdGCM is that it can output only *global* temperature changes; regional changes are calculated but are difficult to access and have low spatial accuracy. However, according to Chylek and Lohmann [2005], the relationship between Greenland temperatures and global temperatures is well approximated by

$$\Delta T_{\text{Greenland}} = 2.2\Delta T_{\text{global}}.$$

The Ice Sheet

We model the ice sheet as a rectangular box. We assume that each point on the upper surface is at constant temperature T_a, because our climate
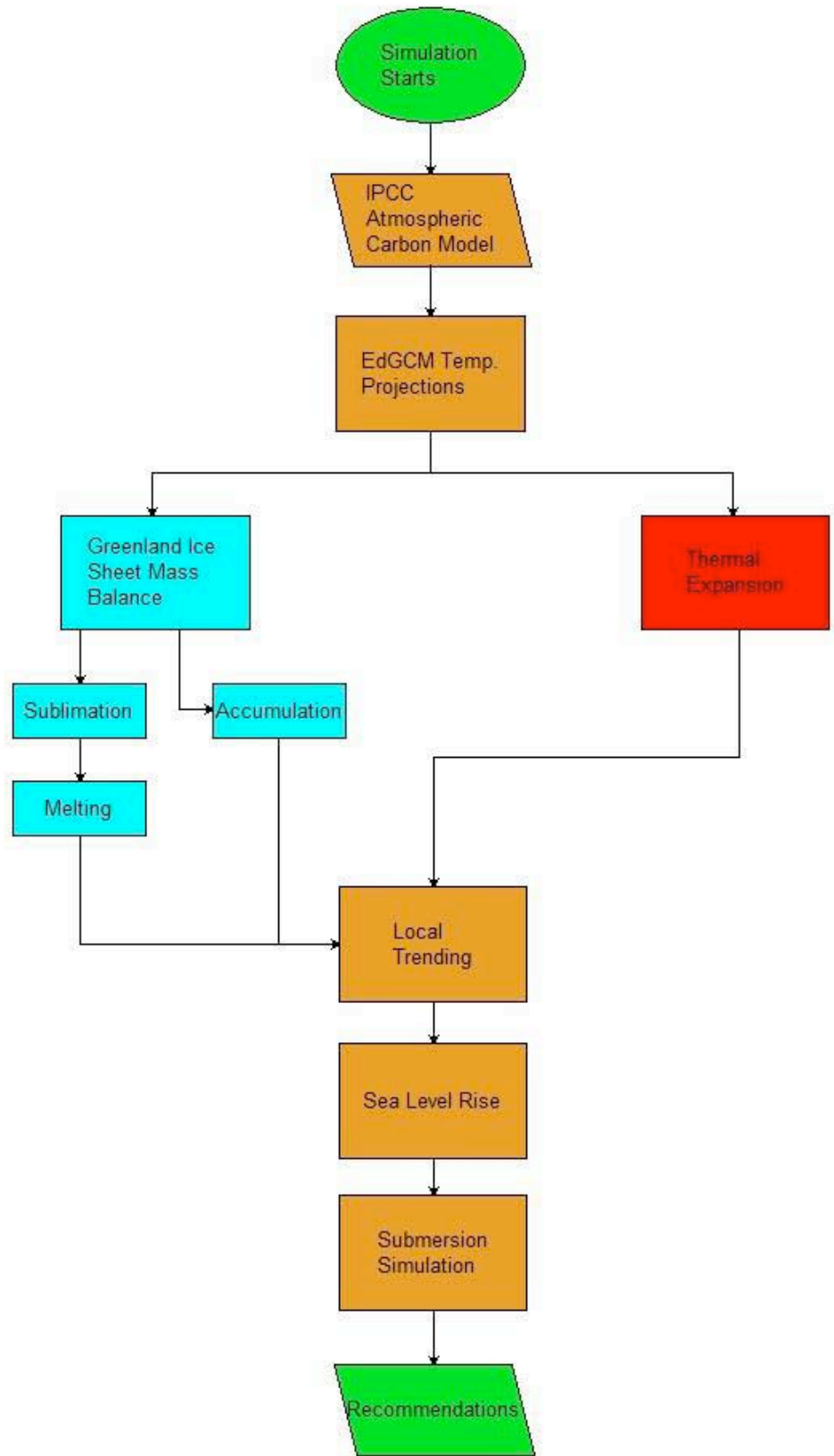
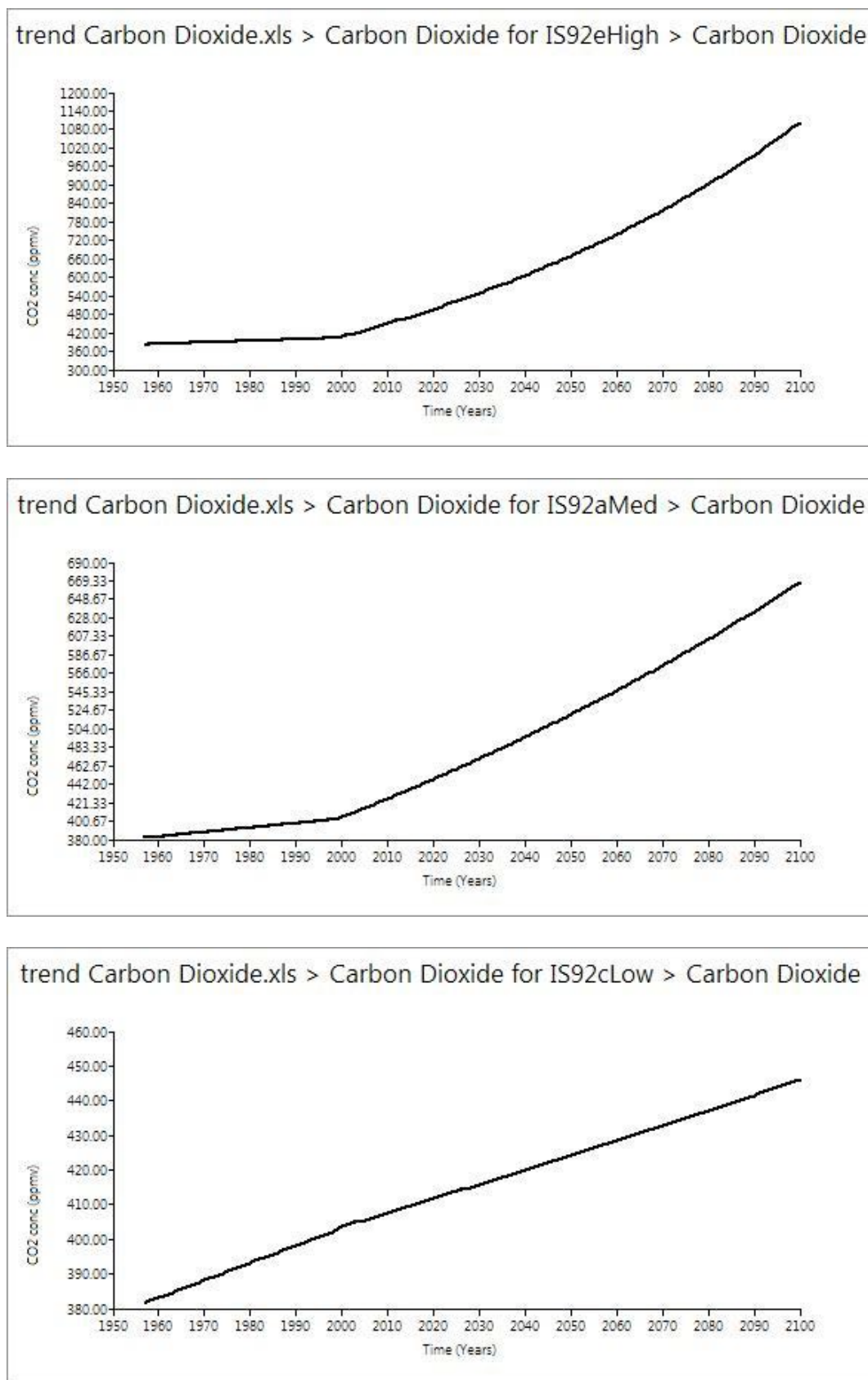**Figure 1.** Simulation flow diagram.

Figure 2. Carbon dioxide forcings for the EdGCM models.

model does not have accurate spatial resolution for Greenland. The lower surface, the permafrost layer, has constant temperature T_l.

To compute heat flux, and thus melting and sublimation through the ice sheet, we model it as an infinite number of differential volumes (**Figure 3**).
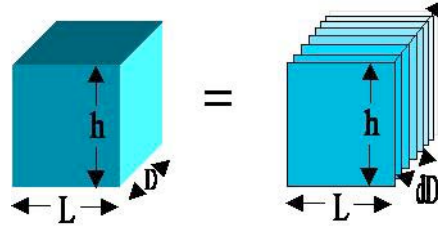

Figure 3. Differential volumes of the ice sheet.

The height h of the box is calculated using data provided by Williams and Ferrigno [1999]:

$$h = \frac{\text{Volume}_{\text{ice}}}{\text{Surface}_{\text{ice}}} = \frac{2.6 \times 10^6 \text{ km}^3}{1.736 \times 10^6 \text{ km}^2} = 1.5 \text{ km}.$$

The primary mode of sea-level rise in our model is through mass balance: accumulation minus ablation.

Mass Balance: Accumulation

Huybrechts et al. [1991] show that the temperature of Greenland is not high enough to melt significant amounts of snow. Furthermore, Knight [2006] shows that the rate of accumulation of ice is well-approximated by a linear relationship of 0.025 m/month of ice. In terms of mass balance, we have

$$M_{\text{ac}} = 0.025LD,$$

where L and D are the length and width of the rectangular ice sheet.

Mass Balance: Ablation

We model the two parts of ablation, sublimation and melting.

Sublimation

The sublimation rate (mass flux) is given by:

$$S_0 = e_{\text{sat}}(T) \left(\frac{M_w}{2\pi RT} \right)^{1/2},$$

where M_w is the molecular weight of water and T is the temperature in kelvins. This expression can be derived from the ideal gas law and

the Maxwell-Boltzmann distribution [Andreas 2007]. Substituting Buck's [1981] expression for e_{sat}, we obtain:

$$S_0 = 6.1121 \exp\left[\frac{\left(18.678 - \frac{T}{234.5}\right)T}{257.14 + T}\right]\left(\frac{M_w}{2\pi R(T + 273.15)}\right)^{1/2},$$

where we now scale T in $°C$. Buck's equation is applicable over a large range of temperatures and pressures, including the environment of Greenland. To convert mass flux into rate of change of thickness the ice, we divide the mass flux expression by the density of ice, getting the rate of height change as

$$S_h = \frac{6.1121d}{\rho_{\text{ice}}} \exp\left[\frac{\left(18.678 - \frac{T}{234.5}\right)T}{257.14 + T}\right]\left(\frac{M_w}{2\pi R(T + 273.15)}\right)^{1/2},$$

where d is the deposition factor, given by $d = (1 - \text{deposition rate}) = 0.01$ [Buck 1981].

The thickness of the ice sheet after one timestep (= one month) of the computational model is

$$S(t) = h - S_h t,$$

where h is the current thickness of the ice sheet and t is one timestep. Substituting for S_h the expression above and the molecular weight of water yields

$$S(t) = h - \frac{6.1121 \times 10^{-2} t}{\rho_{\text{ice}}} \exp\left[\frac{\left(18.678 - \frac{T}{234.5}\right)T}{257.14 + T}\right]\left(\frac{M_w}{2\pi R(T + 273.15)}\right)^{1/2}.$$

Melting

To model melting, we apply the heat equation

$$U_t(x,t) = kU_{xx}(x,t),$$

using $k = 0.0104$ as the thermal diffusivity of the ice [Polking et al. 2006]. For the Neumann conditions, we assume a steady-state U_s with the same boundary conditions as U and that is independent of time. The residual temperature V has homogeneous boundary conditions and initial conditions found from $U - U_s$. Thus, we can rewrite the heat equation as

$$U(x,t) = V(x,t) + U_s(x,t).$$

The steady-state solution is

$$U_s = T_l + \frac{T_a - T_l}{S(t)} x,$$

subject to the constraints $0 < x < S(t)$ and $0 < t < 1$ month. Directly from the heat equation we also have

$$V_t(x,t) = kV_{xx}(x,t) + f, \quad \text{where } f \text{ is a forcing term; and}$$
$$V(0,t) = V\big(S(t),t\big) = 0, \quad \text{for the homogeneous boundary equations.}$$

Since no external heat source is present and temperature distribution depends only on heat conduction, we take as the forcing term $f = 0$. To calculate change in mass balance on a monthly basis, we solve analytically using separation of variables:

$$V(x,t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \exp\left[\frac{-n^2\pi^2 t}{s^2}\right] \cos\left(\frac{n\pi x}{s}\right),$$

where

$$a_0 = \frac{2}{s} \int_0^s \left(T_l + \frac{T_a - T_l}{s} x\right) dx = 2T_1 + T_a - T_l = T_l + T_a$$

and

$$\begin{aligned}
a_0 &= \frac{2}{s} \int_0^s \left(T_l + \frac{T_a - T_l}{s} x\right) \cos\left(\frac{n\pi x}{s}\right) dx \\
&= \left(\frac{s}{n\pi}\right)^2 \big(\cos(n\pi) - 1\big) \\
&= \left(\frac{s}{n\pi}\right)^2 \big((-1)^n - 1\big).
\end{aligned}$$

Therefore,

$$V(x,t) = \frac{T_l + T_a}{2} + \sum_{n=1}^{\infty} \frac{2(T_a - T_l)}{(n\pi)^2}\big((-1)^n - 1\big) \exp\left[\frac{-n^2\pi^2 t}{s^2}\right] \cos\left(\frac{n\pi x}{s}\right).$$

Having found $V(x,t)$ and $U_s(x,t)$, we obtain an expression for $U(x,t)$ from

$$U(x,t) = V(x,t) + U_s(x,t).$$

Since U is an increasing function of x, and for $x > k$, we have $U(x,t) > 0$ for fixed t; the ice will melt for $k < x < h$. To determine ablation, we solve $U(k,t) = 0$ for k using the first 100 terms of the Fourier series expansion and the Matlab function `fzero`. We use the new value of k to renew h as the new thickness of the ice sheet for the next timestep.

With these two components, we can finalize an expression for ablation and apply it to a computational model. The sum of the infinitesimal changes

in ice sheet thickness for each differential volume gives the total change in thickness. To find these changes, we first note that

$$\text{Mass balance loss due to sublimation} = (h - S)LD,$$
$$\text{Mass balance loss due to melting} = (S - k)LD,$$

where the product LD is the surface area of the ice sheet. In these equations, the "mass balance" refers to net volume change. Thus, ablation is given by

$$M_{\text{ab}} = (h - S)LD + (S - k)LD = (h - k)LD.$$

Mass Balance and Sea-Level Rise

Combining accumulation and ablation into an expression for mass balance, we have

$$M = M_{\text{ac}} - M_{\text{ab}} = 0.025LD - (h - k)LD.$$

Relating this to sea-level rise, we use the approximation 360 Gt water = 0.1 cm sea-level rise. Thus,

$$\text{SLR}_{\text{mb}} = M\rho_{\text{ice}} \frac{0.1 \text{ cm}}{360 \text{ Gt}},$$

which quantifies the sea-level rise due to mass balance.

Thermal Expansion

According to Wigley and Raper [1987], for the current century thermal expansion of the oceans due to increase in global temperature will contribute at least as much to rise in sea level as melting of polar ice [Huybrechts et al. 1991; Titus and Narayanan 1995]. So we incorporate thermal expansion into our model.

Temperature plays the primary role in thermal expansion, but the diffusion of radiated heat, mixing of the ocean, and various other complexities of ocean dynamics must be accounted for in a fully accurate description. We adapt the model of Wigley and Raper [1987]. Based on standard greenhouse-gas emission projections and a simple upwelling-diffusion model, the dependency of the model can be narrowed to a single variable, temperature, using an empirical estimation:

$$\Delta z = 6.89\Delta T k^{0.221},$$

where

Δz is the change in sea level due to thermal expansion (cm),

ΔT is the change in global temperature (°C), and

k is the diffusivity.

Localization

A final correction must be added to the simulation. The rise in sea level will vary regionally rather significantly. The local factors often cited include land subsidence, compaction, and delayed response to warming [Titus and Narayanan 1995]. We thus assume that previous patterns of local sea-level variation will continue, yielding the relationship

$$\text{local}(t) = \text{normalized}(t) + \text{trend}(t - 2008),$$

where

- $\text{local}(t)$ is the expected sea level rise at year t (cm),

- $\text{normalized}(t)$ is the estimate of expected rise in global sea level change relative to the historical rate at year t, and

- trend is the current rate of sea-level change at the locale of interest.

The normalization prevents double-counting the contribution from global warming.

In our model, the rates of sea-level change are averaged over data for Florida from Titus and Narayanan [1995] to give the trend. This is reasonable because the differences between the rates in Florida are fairly small. The normalized (t) at each year is obtained from

$$\text{global}(t) - \text{historical rate}(t - 2008),$$

where $\text{global}(t)$ is the expected sea-level rise at year t from our model and historical rate is chosen uniformly over the range taken from Titus and Narayanan [1995].

Simulating Costs of Sea-Level Rise to Florida

To model submersion of regions of Florida due to sea-level rise, we created a raster matrix of elevations for various locations, using USGS data (GTOPO30) [1996]. The 30-arc-second resolution corresponds to about 1 km; however, to yield a more practical matrix, we lowered the resolution to 1 minute of arc (approx. 2 km).

The vertical resolution of the data is much greater than 1 m. To model low coastal regions, the matrix generation code identified potential sensitive areas and submitted these to the National Elevation Dataset (NED) [Seitz 2007] for refinement. (NED's large size and download restrictions restrict its use to sensitive areas.) The vertical resolution of NED is very high [USGS 2006]. We use these adjustments to finalize the data.

We measure the effect of sea-level rise on populations by incorporating city geospatial coordinates and population into the simulation. We

obtained geospatial coordinates from the GEOnames Query Database [National Geospatial Intelligence Agency 2008] and population data from the U.S. Census Bureau [2000].

We used the sea-level rise calculated from our model as input for the submersion simulation, which subtracts the sea-level increase from the elevation. If rising sea level submerges pixels in a metropolitan area, the population is considered "displaced."

A key limitation of the model is that the population is considered to be concentrated in the principal cities of the metropolitan areas, so a highly accurate population count cannot be assessed. This simplification allows quick display of which cities are threatened without the complexity of hard-to-find high-resolution population distribution data.

We checked the model for realism at several different scenarios. As shown in **Figure 4**, our expectations are confirmed:

- 0 m: No cities are submerged and no populations or land areas are affected.

- 10 m: This is slightly higher than if all of the Greenland Ice Sheet melted (approx. 7 m). Many cities are submerged, especially in southern Florida.
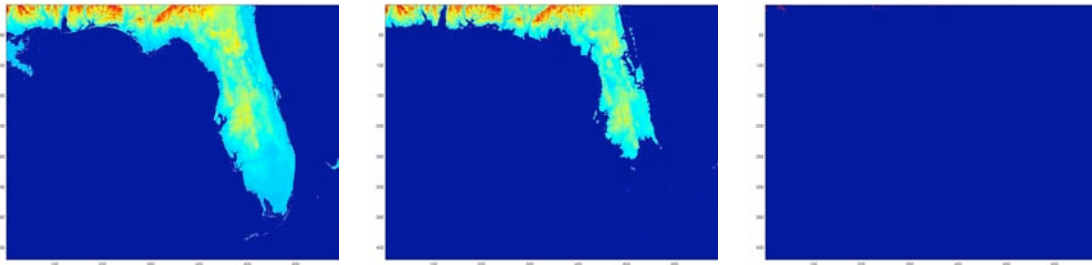
- 100 m: Most of Florida is submerged.

Figure 4. Effects of 0, 10, and 100-meter sea-level rise.

Results

Output Sea-Level-Rise Data

We ran the program with a Matlab script for the IS92e (high), IS92a (intermediate), and IS92c (low) carbon-emissions models. The program produces a smooth trend in sea-level increase for each of the three forcings, as shown in **Figure 5**: Higher temperature corresponds to higher sea-level rise, as expected. The sea-level output data are then used to calculate submersion consequences.
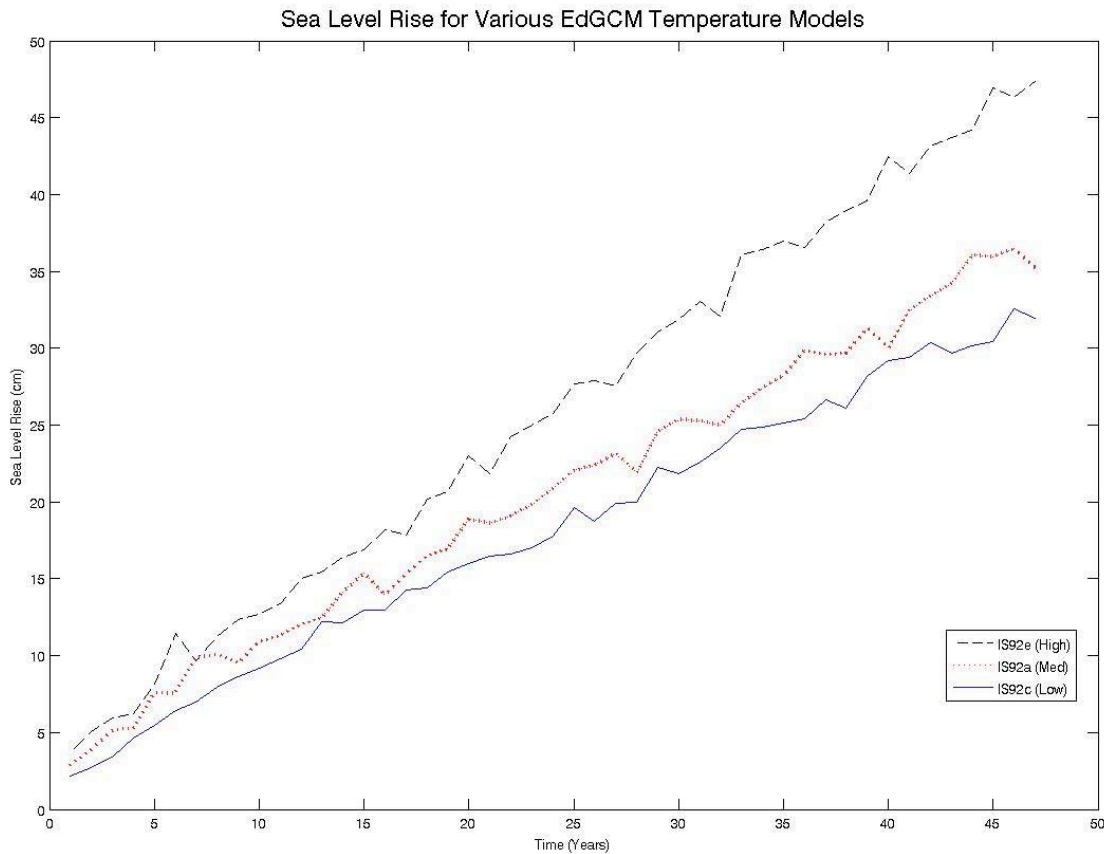
Figure 5. Sea level rise as a function of time for the three temperature models.

Submersion Simulation Results

Output consists of the submerged land area and displaced population statistics. The program quantified the effects noted in **Table 1**. For the low and medium scenarios, no metropolitan areas are submerged until after 30 years. In all scenarios, Miami Beach and Key Largo are submerged after 40 years.

Discussion and Conclusion

The estimated sea-level rises (**Figure 5**) for the three scenarios seem reasonable. The 50-year projection is in general agreement with models proposed by the IPCC , NRC, and EPA (less than 10 cm different from each) [Titus et al. 1991]. Additionally, the somewhat-periodic, somewhat-linear trend is similar to past data of mean sea-level rise collected in various locations [Titus et al. 1991]. Thus, the projections of our model are reasonable.

The high-emission scenario results in a 40–50 cm rise in sea level by 2058, with results from the intermediate scenario 6–10 cm lower and the

Table 1.

Effects under different scenarios (using current population values).

Time	High		Medium		Low	
	Displaced	Submerged	Displaced	Submerged	Displaced	Submerged
(yrs)	($\times 10^3$)	(km^2 $\times 10^3$)	($\times 10^3$)	(km^2 $\times 10^3$)	($\times 10^3$)	(km^2 $\times 10^3$)
10	0	6.5	0	6.4	0	6.2
20	12	7.5	0	6.9	0	6.8
30	100	9.2	12	7.7	0	7.1
40	100	9.7	100	9.0	100	8.0
100	135	10.0	100	9.5	100	9.2

low-emission scenario trailing intermediate by 5–8 cm. The model thus works as expected for a wide range of input data: Higher temperatures lead to increased sea level rise.

Overall, the damage due to sea-level change seems unremarkable. Even in the worst-case scenario, in 50 years only 135,000 people are displaced and 10,000 square kilometers are submerged, mostly in South Florida.

However, these projections are only the beginning of what could be a long-term trend. As shown by the control results, a sea-level increase of 10 m would be devastating. Further, not all possible damages are assessed in our simulation. For example, sea-level increases have been directly implicated also in shoreline retreat, erosion, and saltwater intrusion. Economic damages are not assessed. Bulkheads, levees, seawalls, and other structures are often built to counteract the effect of rising sea levels, but their economic impacts are outside the scope of the model.

Our model has several key limitations. The core assumption of the model is the simplification of physical features and dynamics in Greenland. The model assumes an environment where thickness, temperature, and other physical properties are averaged out and evenly distributed. The "sublimate, melt, and snow" dynamics are simulated with a monthly timestep. Such assumptions are too simplistic to capture fully the ongoing dynamics in the ice sheets. But we do not have the data and computing power to perform a full-scale 3-D grid-based simulation using energy-mass balance models, as in Huybrechts [1999].

With regard to minor details of the model, the assumed properties regarding the thermal expansion, localization, and accumulation also take an averaging approach. We make an empirical estimate adapted from Wigley and Raper [1987]. Consequently, our model may not hold over a long period of time, when its submodels for accumulation, thermal expansion, and localization might break down.

The assumptions of the EdGCM model are fairly minimal, and the projected temperature time series for each scenario are consistent with typical carbon projections [Edmonds et al. 2000]. Although the IS92 emissions scenarios are very rigorous, they are the main weakness of the model. Because

all of the other parameters depend on the temperature model, our results are particularly sensitive to factors that directly affect the EdGCM output.

Despite these deficiencies, our model is a powerful tool for climate modeling. Its relative simplicity—while it can be viewed as a weakness—is actually a key strength of the model. The model boasts rapid runtime, due to its simplifications. Furthermore, the model is a function of time and temperature only; the fundamentals of our model imply that all sea-level increase is due to temperature change. But even with less complexity, our model is comprehensive and accurate enough to provide accurate predictions.

Recommendations

In the short term, preventive action could spare many of the model's predictions from becoming reality. Key Largo and Miami Beach, which act as a buffer zone preventing salinization of interior land and freshwater, are particularly vulnerable. If these regions flood, seawater intrusion may occur, resulting in widespread ecological, agricultural, and ultimately economical damage. Titus and Narayanan [1995] recommend building sand walls.

In the long term, carbon emissions must be reduced to prevent disasters associated with sea-level rise.

References

Andreas, E.L. 2007. New estimates for the sublimation rate of ice on the Moon. *Icarus* 186, 24–30.

Buck, A.L. 1981. New equations for computing vapor pressure and enhancement factor. *Journal of Applied Meteorology* 20: 1527–1532.

Cabanes, C., A. Cazenave, and C.L. Provost. 2001. Sea level rise during past 40 years determined from satellite and in situ observations. *Science* 294: 840–842.

Cavalieri, D.J., P. Gloersen, C.L. Parkinson, J.C. Comiso, and H.J. Zwally. 1997. Observed hemispheric asymmetry in global sea ice changes. *Science* 278: 1104–1106.

Church, J.A., J.S. Godfrey, D.R. Jackett, and T.T. McDougall. 1990. Sea level rise during past 40 years determined from satellite and in situ observations. *Journal of Climate* 4: 438–456.

Chylek, P., and U. Lohmann. 2005. Ratio of the Greenland to global temperature change: Comparison of observations and climate modeling results. *Geophysical Research Letters* 32: 1–4.

Edmonds, J., R. Richels, and M. Wise. 2000. Ratio of the Greenland to global temperature change: Comparison of observations and climate modeling results: A review. In *The Carbon Cycle*, edited by T.M.L. Wigley and D.S. Schimel, 171–189. Cambridge, UK: Cambridge University Press.

Fofonoff, P., and R.C. Millard, Jr. 1983. Algorithms for computation of fundamental properties of seawater. *UNESCO Technical Papers in Marine Sciences* 44: 1–53.

Gloersen, P., and W.J. Campbell. 1991. Recent variations in arctic and antarctic sea-ice covers. *Nature* 352: 33–36.

Hansen, J., M. Sato, R. Ruedy, A. Lacis, and V. Oinas. 2000. Global warming in the twenty-first century: An alternative scenario. *Proceedings of the National Academy of Science* 97: 9875–9880.

Huybrechts, P. 1999. The dynamic response of the Greenland and Antarctic ice sheets to multiple-century climatic warming. *Journal of Climate* 12: 2169–2188.

_____, A. Letreguilly, and N. Reeh. 1991. The Greenland ice sheet and greenhouse warming. *Palaeography, Palaeoclimatology, Palaeoecology* 89: 399–412.

Knight, P. 2006. *Glacier Science and Environmental Change*. Oxford, UK: Wiley-Blackwell.

National Geospatial-Intelligence Agency. 2008. GEOnames Query Database. `http://earth-info.nga.mil/gns/html/index.html` .

Polking, J., A. Boggess, and D. Arnold. 2006. *Differential Equations with Boundary Value Problems*. 2nd ed. Upper Saddle River, NJ: Pearson Education.

Rothrock, D.A., and J. Zhang. 2005. Arctic Ocean sea ice volume: What explains its recent depletion? *Journal of Geophysical Research* 110: 1–10; `http://psc.apl.washington.edu/pscweb2002/pubs/rothrockJRG05.pdf` .

Seitz, M. 2007. Lat/lon to elevation. `http://www.latlontoelevation.com/` .

Shopsin, M., M. Shopsin, and K. Mankoff. 2007. *Education Global Climate Modeling*. New York: Columbia University Press.

Titus J.G., and V.K. Narayanan. 1995. *The Probability of Sea Level Rise*. Washington, DC: U.S. Environmental Protection Agency.

Titus, J.G., R.A. Park, S.P. Leatherman, J.R. Weggel, M.S. Greene, P.W. Mausel, S. Brown, G. Gaunt, M. Trehan, and G. Yohe. 1991. Greenhouse effect and sea level rise: The cost of holding back the sea. *Coastal Management* 19: 171–204; `http://www.owr.ehnr.state.nc.us/ref/16/15086.pdf` .

U.S. Census Bureau. 2000. Census 2000 datasets. `http://www2.census.gov/census_2000/datasets/` .

U.S. Geological Survey (USGS). 1996. GTOPO30 Tile W100N40. Earth Resources Observation and Sciences. `http://edc.usgs.gov/products/elevation/gtopo30/w100n40.html` .

———. 2006. Accuracy of the National Elevation Dataset. `http://ned.usgs.gov/Ned/accuracy.asp` .

Warrick, R., C.L. Provost, M. Meier, J. Oerlemans, and P. Woodworth 1996. *Climate Change 1995: The Science of Climate Change.* Cambridge, UK: Cambridge University Press.

Wigley, T.M.L., and S.C.B. Raper. 1987. Thermal expansion of sea water associated with global warming. *Nature* 330: 127–131.

Williams, Richard S., Jr., and Jane G. Ferrigno. 1999. Estimated present-day area and volume of glaciers and maximum sea level rise potential. U.S. Geological Survey Professional Paper 1386-A. In *Satellite Image Atlas of Glaciers of the World*, Chapter A, edited by Richard S. Williams, Jr., and Jane G. Ferrigno. Washington, DC: U.S. Government Printing Office. `http://www.smith.edu/libraries/research/class/idp108USGS_99.pdf` . An updated version was in press in 2007 and is to be available from `http://pubs.usgs.gov/fs/2005/3056/fs2005-3056.pdf` .

Brian Choi, Joonhahn Cho, and Jason Chen.