A Schedule for Lazy but Smart Ranchers

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Summary

We determine the number of sprinklers to use by analyzing the energy and motion of water in the pipe and examining the engineering parameters of sprinklers available in the market.

We build a model to determine how to lay out the pipe each time the equipment is moved. This model leads to a computer simulation of catch-can tests of the irrigation system and an estimation of both distribution uniformity (DU) and application efficiency of different schemes of where to move the pipe. In this stage, DU is the most important factor. We find a schedule in which one sprinkler is positioned outside of the field in some moves but higher resulting DU (92%) and saving of water.

We determine two schedules to irrigate the field. In one schedule, the field receives water evenly during a cycle of irrigation (in our schedule, 4 days), while the other schedule costs less labor and time. Our suggested solution, which is easy to implement, includes a detailed timetable and the arrangement of the pipes. It costs 12.5 irrigation hours and 6 equipment resets in every cycle of 4 days to irrigate the field with DU as high as 92%.

Assumptions and Definitions

- The weather is "fine" and the influence of wind can be neglected.
- The whole system is "ideal" in that evaporation, leaking, and other water loss can be neglected.

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- The water source can be put at any position of the field. In practice, a tube can be used to transport water from the pump to the pipe set.
- No mainline exists, so that all pipes join together and can be put at any position of the field.
- The time for a rancher to uncouple, move, and reinstall the pipe set is half an hour.
- The discharge of any sprinkler is the same.
- The design pressure of sprinklers is about 400 kPa and the sprinkler is an impact-driven rotating sprinkler.
- The diameter of the riser is the same as that of the pipe.
- The water pressures in pipes are assumed to be the same. In practice, there is a slight difference.

Table 1.

Variables and constants.

Variable	Definition	Units
p_{in}	Water pressure in the pipe before a junction	kPa
p_{out}	Water pressure in the pipe after a junction	kPa
p_{up}	pup Water pressure at the sprinkler at a junction	kPa
v_{in}	vin Water speed in the pipe before a junction	m/s
v_{out}	Water speed in the pipe after a junction	m/s
v_{up}	Water speed at the sprinkler at a junction	m/s
A_{in}	Section area in the pipe before a junction	cm^2
A_{out}	Section area in the pipe after a junction	cm^2
A_{up}	Section area of the sprinkler at a junction	cm^2
h	Height of the sprinkler above the pipe	m
Δt	Change in time	s
v_{source}	Speed of the water source	m/s
n	Number of sprinklers	–
$\text{distr}(r)$	Distribution function of precipitation profile	–
p	Precipitation rate	–
R	Sprinkling range	m
r	Distance from a sprinkler	m
r_i	Distance from the ith sprinkler	m
α	Obliquity of the precipitation profile	rad
$\text{pr}(r)$	Precipitation function of one sprinkler	cm/min
DU	Distribution uniformity of an irrigation system	–
Constant	**Definition**	**Value**
ρ	Density of water	1.0 kg/L
g	Acceleration of gravity	9.8 m/s^2

Problem Analysis

Our goal is to determine the number of sprinklers and the spacing between them and find a schedule to move the pipes, including where to move them. Our approach can be divided into three stages:

- Determine the number of sprinklers. We figure out the pressure and speed of water from each sprinkler and then determine possible sprinkler numbers from engineering data.
- Determine where to put the pipes. We consider major factors, such as sprinkling time, moving time and distribution uniformity (DU). Since the pipe positions depend on the number of sprinklers and the precipitation profile, we just work out some problem-specific cases. However, our method can be used to solve any practical case.
- Determine the schedule to move the pipes. Referring to the water need of the field, we make a schedule that minimizes the time cost, which, obviously, is closely related to the number of moves of the pipes.

Model Development

Stage 1: Water Pressure and Speed

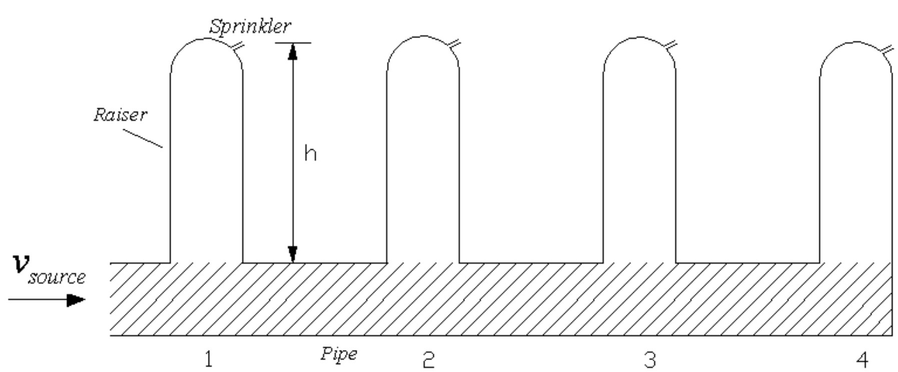

Figure 1. Overall sketch for four sprinklers and four junctions. The pressure throughout the shaded area is the same, due to our assumption.

We apply the law of conservation of energy. The work done by the forces is

$$F_{\text{in}}s_{\text{in}} - F_{\text{up}}s_{\text{up}} - F_{\text{out}}s_{\text{out}} = p_{\text{in}}A_{\text{in}}v_{\text{in}}\Delta t - p_{\text{up}}A_{\text{up}}v_{\text{up}}\Delta t - p_{\text{out}}A_{\text{out}}v_{\text{out}}\Delta t.$$

The decrease in potential energy is

$$-mgh = -\rho g A_{\text{up}}v_{\text{up}}\Delta t h.$$

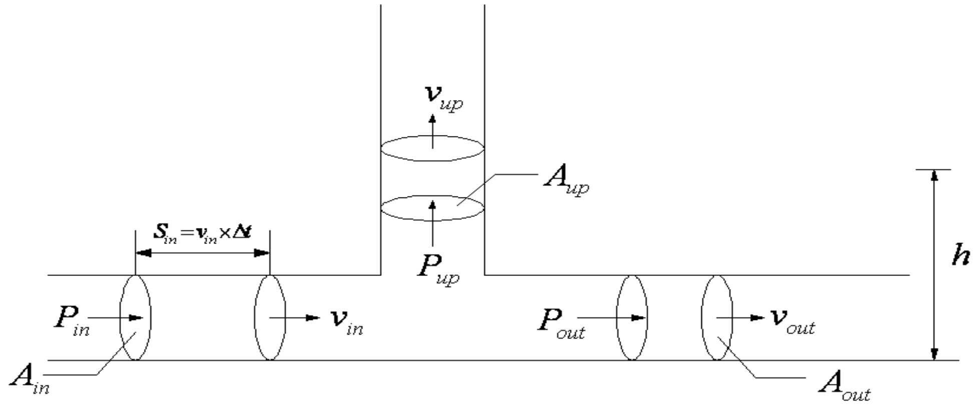

Figure 2. Sketch of one junction.

The increase in kinetic energy is

$$\tfrac{1}{2}mv_{\text{up}}^2 + \tfrac{1}{2}mv_{\text{out}}^2 - \tfrac{1}{2}mv_{\text{in}}^2 = \tfrac{1}{2}\rho A_{\text{up}} v_{\text{up}} \Delta t v_{\text{up}}^2 + \tfrac{1}{2}\rho A_{\text{out}} v_{\text{out}} \Delta t v_{\text{out}}^2 - \tfrac{1}{2}\rho A_{\text{in}} v_{\text{in}} \Delta t v_{\text{in}}^2.$$

Putting these together, because of the law of conservation of energy, yields

$$\begin{aligned} p_{\text{in}} A_{\text{in}} v_{\text{in}} \Delta t - p_{\text{up}} A_{\text{up}} v_{\text{up}} \Delta t - p_{\text{out}} A_{\text{out}} v_{\text{out}} \Delta t - \rho g A_{\text{up}} v_{\text{up}} \Delta t h = \\ \tfrac{1}{2}\rho A_{\text{up}} v_{\text{up}} \Delta t v_{\text{up}}^2 + \tfrac{1}{2}\rho A_{\text{out}} v_{\text{out}} \Delta t v_{\text{out}}^2 - \tfrac{1}{2}\rho A_{\text{in}} v_{\text{in}} \Delta t v_{\text{in}}^2. \end{aligned} \quad \textbf{(1)}$$

Since the fluid is incompressible, we have

$$A_{\text{in}} v_{\text{in}} = A_{\text{up}} v_{\text{up}} + A_{\text{out}} v_{\text{out}}. \quad \textbf{(2)}$$

The diameters are all the same:

$$A_{\text{in}} = A_{\text{up}} = A_{\text{out}} = \pi \left(\frac{10 \text{ cm}}{2} \right)^2. \quad \textbf{(3)}$$

According to the assumptions, at every junction we have

$$p_{\text{in}} = p_{\text{out}} = 420 \text{ kPa}, \quad \textbf{(4)}$$

$$v_{\text{up}} = \frac{v_{\text{source}}}{n}, \quad \textbf{(5)}$$

where

$$v_{\text{source}} = \frac{150 \text{ L/min}}{\pi \left(\frac{10 \text{ cm}}{2} \right)^2} = 0.318 \text{ m/s}.$$

Therefore, from **(2)**, **(3)**, and **(5)**, we have for the ith junction

$$v_{\text{in}} = v_{\text{source}} \left(1 - \frac{i-1}{n} \right), \qquad v_{\text{out}} = v_{\text{source}} \left(1 - \frac{i}{n} \right).$$

Putting **(1)–(5)** together, we can obtain p_{up} at every junction. In fact, at the last (i.e., the nth) junction, we have

$$v_{\text{in}} = v_{\text{up}} = \frac{v_{\text{source}}}{n}, \qquad v_{\text{out}} = 0.$$

Putting these into **(1)**, we get

$$p_{\text{up}} = p_{\text{in}} - \rho g h,$$

which means that the pressure at the last sprinkler is independent of n.

Commonly, h is about 0.5 m to 1.5 m, and even if we assume that $h = 1.5$ m, the v_{up} at the last junction will be 405 kPa, not far from 420 kPa. (If $h = 0.5$ m, the last v_{up} will be 415 kPa.)

From these equations, we know that v_{up} at the last junction differs the most from 420 kPa, that at the first junction is the closest to 420 kPa (and below 420 kPa), and the values are decreasing slowly from junction 1 to junction n. We conclude that the values of v_{up} at every junction are all below 420 kPa but very close to 420 kPa, no matter how many sprinklers. This fact explains our assumption that the design pressure of sprinklers is about 400 kPa.

Information and Analysis of Sprinklers

The impact-driven sprinkler is the most widely used rotating sprinkler and the one that we assume is used. Some rotating sprinklers have a sector mechanism that can wet either a full circle or a circular sector. There are three main structure parameters of sprinklers: intake line diameter, nozzle diameter, and nozzle elevation angle. An empirical formula gives the spraying range of an impact-driven sprinkler:

$$R = 1.70 d^{\,0.487} h_p^{0.45},$$

where d is the nozzle diameter and h_p is the operational pressure head.

Table 2 shows data on impact sprinklers. Since for our problem the design pressure of the sprinklers is 400 kPa, we have medium-pressure sprinklers; in fact, they have the best application uniformity.

Table 2.
Data on sprinklers [Zhu et al. 1989].

Type	Design pressure (kPa)	Range (m)	Discharge (m^3/h)
Low pressure	<200	<15.5	<2.5
Medium pressure	200–500	15.5–42	2.5–32
High pressure	>500	>42	>32

Table 3 shows data on medium-pressure impact-driven sprinklers with 6-mm nozzle diameter. For sprinklers working at 400 kPa (as assumed), the discharge is 2.5–3.5 m^3/h and spraying range is 18.5, 19, or 19.5 m; we use 19 m as the range. The discharge of the source is 150 L/min = 9 m^3/h; thus, to fit every sprinkler's actual discharge to the design discharge, the number of sprinklers should be 3 or 4, because 9/3=3 or 9/4=2.25, which are within the range 2.5–3.5 m^3/h (or close to it).

Table 3.
Data on nozzles with diameter 6 mm [Zhu et al. 1989].

Model	Nozzle diameter (mm)	Design pressure (kPa)	Discharge (m^3/h)	Range (m)
PY-$_1$15	6	200	1.23	15.0
		300	1.51	16.5
PY-$_1$20	6	300	2.17	18.0
		400	2.50	19.5
PY-$_1$S20A (four nozzles)	6 (× 4)	300	2.99	17.5
		400	3.41	19.0
PY$_1$S20	6	300	2.22	18.0
		400	2.53	19.5
15PY$_2$22.5	6	350	2.40	17.0
		400	2.56	17.5
15PY$_2$30	6	350	2.40	18.0
		400	2.56	18.5

Sprinklers with higher design pressure tend to have larger wetted diameters. However, deviations from the manufacturer's recommended pressure may have the opposite effect (increase in pressure, decrease in diameter), and uniformity will probably be compromised. **Figure 3** shows typical precipitation distribution of one sprinkler with low, correct, and high sprinkler pressure.

In practice, people use "catch-can" data to generate a precipitation profile of a "hand-move" irrigation system. That is, they put cans evenly in the field to catch the water; the precipitation profile of the irrigation is given by the amounts of water in the catch-cans.

One measure of how uniformly the water is applied to the field is Distribution Uniformity (DU) [Merkley and Allen 2004]:

$$\mathrm{DU} = \frac{\text{average precipitation of low quarter}}{\text{average precipitation rate}} \times 100\%. \tag{6}$$

Usually, DUs of less than 70% are considered poor, DUs of 70–90% are good, and DUs greater than 90% are excellent. A bad DU means that either too much water is applied, costing unnecessary expense, or too little water is applied, causing stress to crops. There must be good DU before there can be good irrigation efficiency [Rain Bird Agricultural Products n.d.]. To simplify our calculation, we approximate the precipitation profile of a single sprinkler (in **Figure 3b**) to a function distr(r), which means that the relative precipitation rate in the position with a distance r from the sprinkler (**Figure 5**).

Stage 2: Scheduling the Irrigation

A schedule to move the pipes includes both where to move them and how long to leave them. We imagine a fixed irrigation system consisting of several 20 m pipes. If the system can meet the needs of the crops nicely—that is,

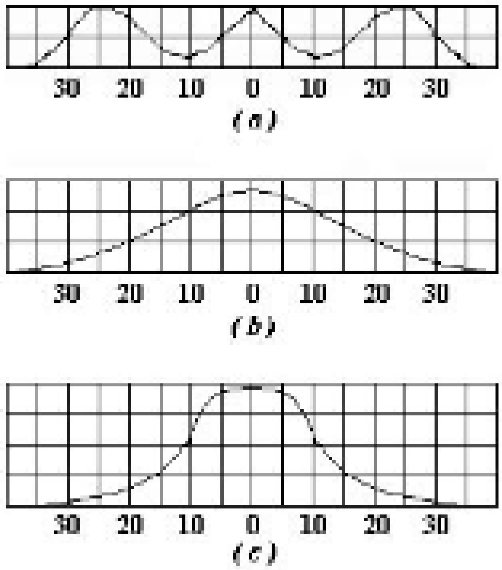

Figure 3. Relation between pressure and precipitation distribution (redrawn from Zhu et al. [1989]). **a)** Pressure is too low. **b)** Pressure is OK. **c)** Pressure is too high.

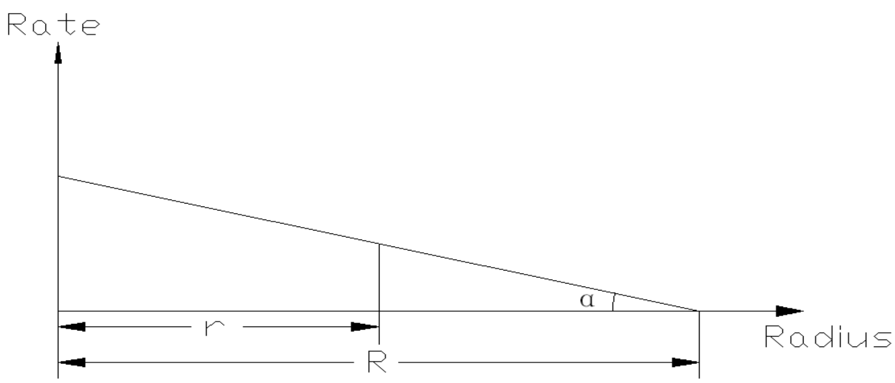

Figure 5. Precipitation rate vs. distance to the sprinkler.

with high Distribution Uniformity (DU)—then we just move the pipe from one position to another. So, we determine where to move the pipe by laying out a system of several 20 m pipes, and then decide for how long we should water the field before making the next move. First, we use a simulation of catch-can analysis to choose a layout with a high DU.

Catch-can Analysis

Since the water sprayed by a sprinkler has a determined distribution distr(r), we use the following method to simulate the catch-can test.

For rectangular spacing (**Figure 6a**), we consider the rectangular region between four adjacent sprinklers. We pick 900 positions evenly distributed in the region. For each position, we calculate its relative precipitation rate p:

$$p = \sum_i \mathrm{distr}(r_i),$$

where $r_i$is the distance from the ith position to the sprinkler. Using **(6)**, we calculate the DU of this irrigation system.

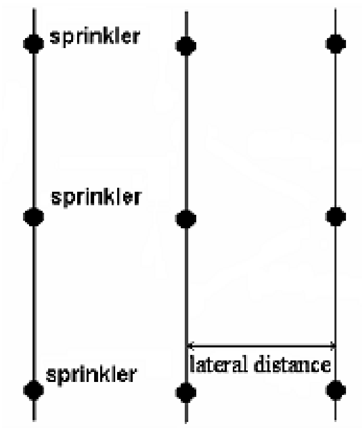

Figure 6a. Rectangular spacing.

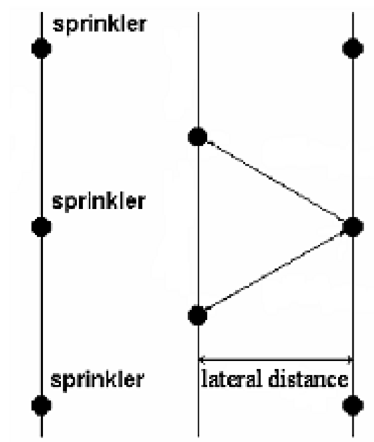

Figure 6b. Triangular spacing.

As we've already deduced, the number of the sprinklers should be 3 or 4, thus the sprinkler distance will be either 10 m $(= 20\ \mathrm{m}/(3-1))$ or 6.67 m $(= 20\ \mathrm{m}/(4-1))$. So the DU is a function of the lateral distance. And this can also be applied to triangular spacing (**Figure 6b**). From the simulation, we get the results in **Figure 7**.

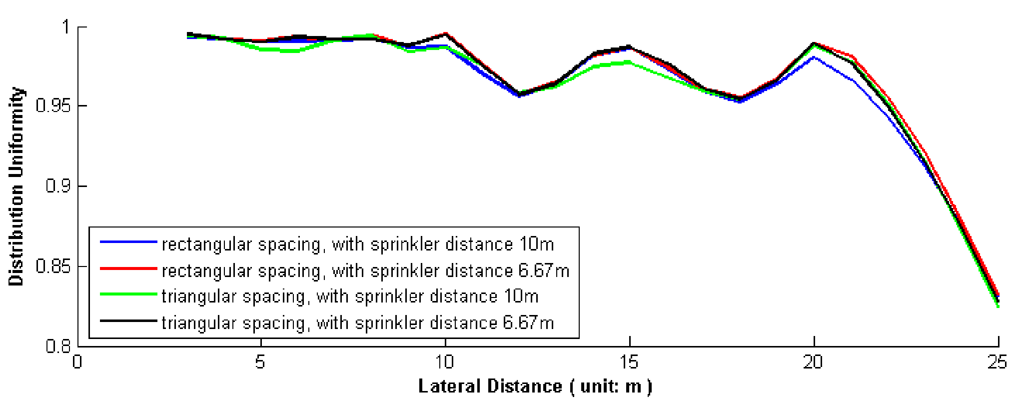

Figure 7. DU vs. lateral distance, in 4 different situations.

The simulation shows that when lateral distance is ≤ 20, DU is acceptable $(\geq 90\%)$, regardless of the spacing and whether the sprinkler distance is 6.67 m or 10 m. But since larger lateral distance results in smaller amount of time required to irrigate the field (the number of moves to make will be fewer, we pick 20 m as the lateral distance. **Figure 8** and **9** show the precipitation profile for the irrigation systems with sprinkler distance 10 m and lateral distance 20 m.

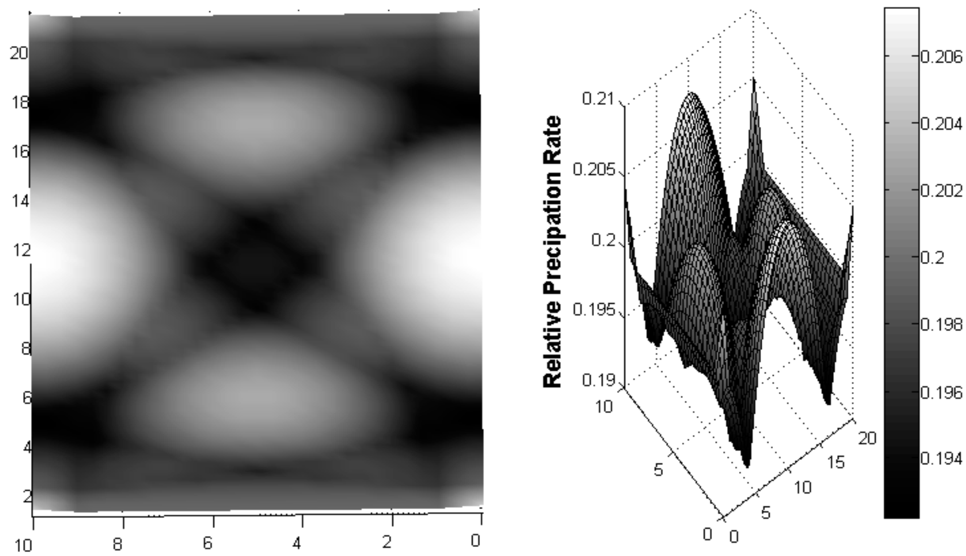

Figure 8. DU=98.1%. Left: Precipitation profile for rectangular spacing with sprinkler distance 10 m and lateral distance 20 m. Right: The 3D form of the precipitation profile.

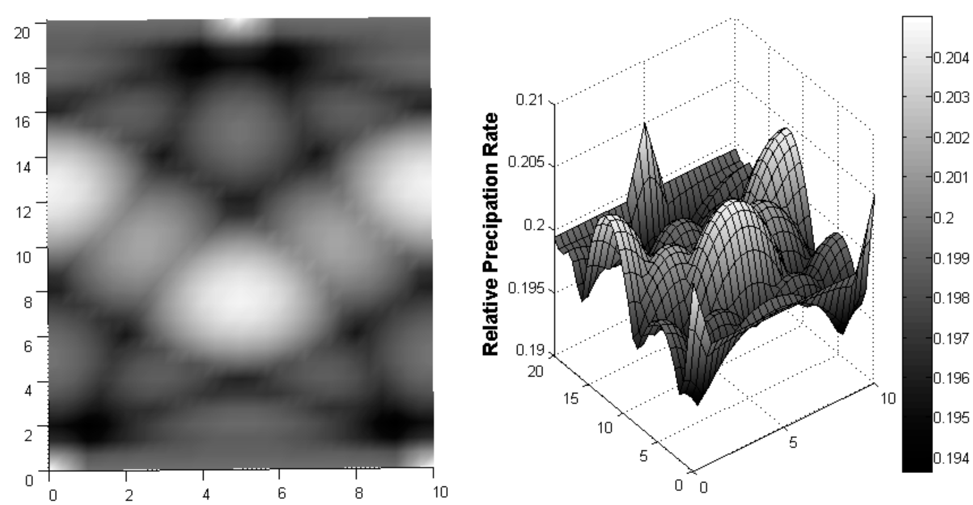

Figure 9. DU=98.7%. Left: Precipitation profile for triangular spacing with sprinkler distance 10 m and lateral distance 20 m. Right: The 3D form of the precipitation profile.

Considering that the field (30 m $\times$ 80 m) is not large enough to implement triangular spacing when the pipe is 20 m long, we use rectangular spacing with only 0.7% negligibly weaker DU. Before we layout the pipe set, we should first determine the maximum distance from the edge of the field to the sprinklers so that the DU is acceptable. We simulate a catch-can test on the rectangular region on the edge of the field (**Figure 10**).

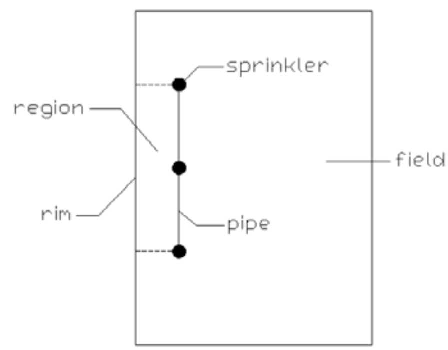

Figure 10. Region to simulate a catch-can test.

The result is in **Figure 11**. The maximum length between the edge of the field and the sprinklers is 5 m, with an acceptable DU of 83%.

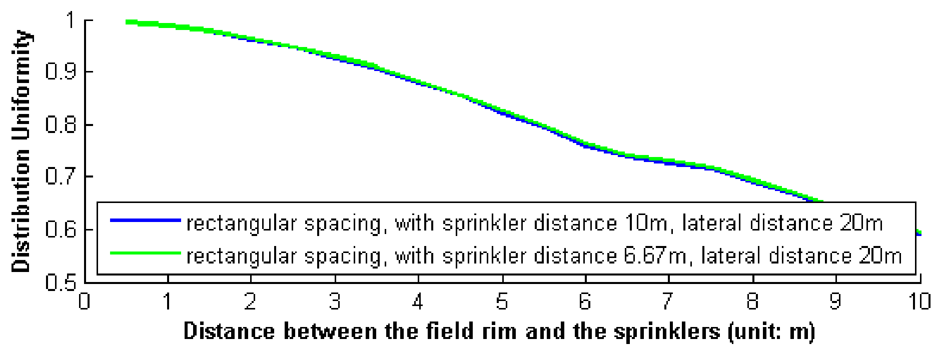

Figure 11. DU vs. distance from the edge to the sprinkler in two different situations.

Layout of the Pipe Set

Having three or four sprinklers along the pipe makes only a slight difference. Considering that the spraying radius (19 m) is too large compared with the sprinkler distance 6.67 m for four sprinklers, we choose to have three. Thus, there are only two feasible layouts (**Figures 12** and **13**). Layout 1 requires five moves and setups of the pipes, while Layout 2 requires six.

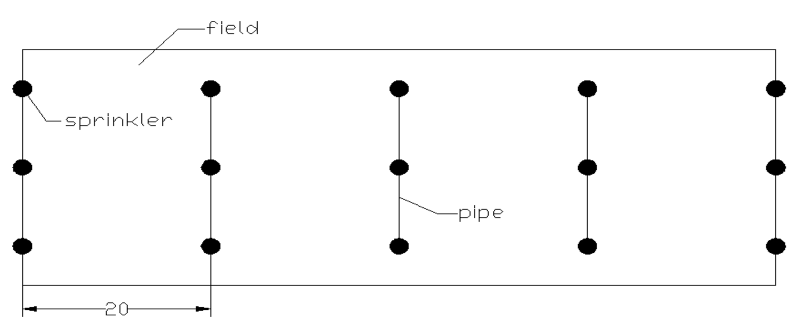

Figure 12. Layout 1.

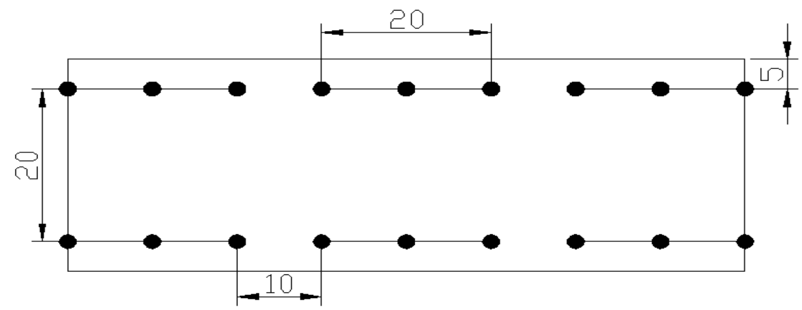

Figure 13. Layout 2.

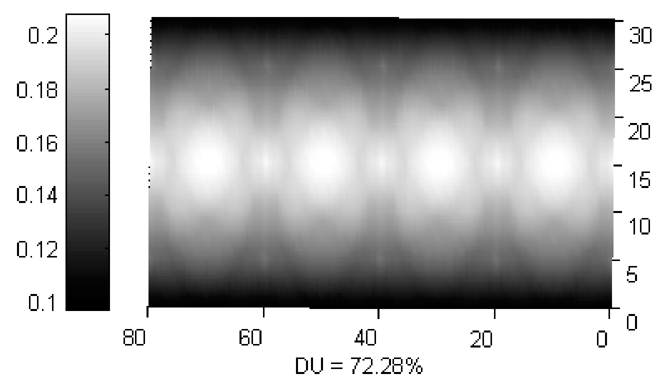

Figure 14. Catch-can test simulation result of Layout 1.

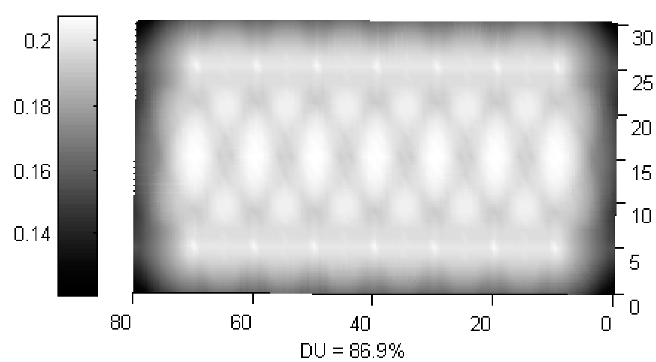

Figure 15. Catch-can test simulation result of Layout 2.

We abandon Layout 1 because it has a very poor DU (**Figure 14**). After slightly changing the lateral distance in Layout 2 (**Figure 15**), we finally achieve a best DU of 89.5% in Layout 3 (**Figure 16**).

Then, if we are brave enough to move some sprinklers outside of of the field, we achieve an even higher DU with Layout 4 (**Figure 17**).

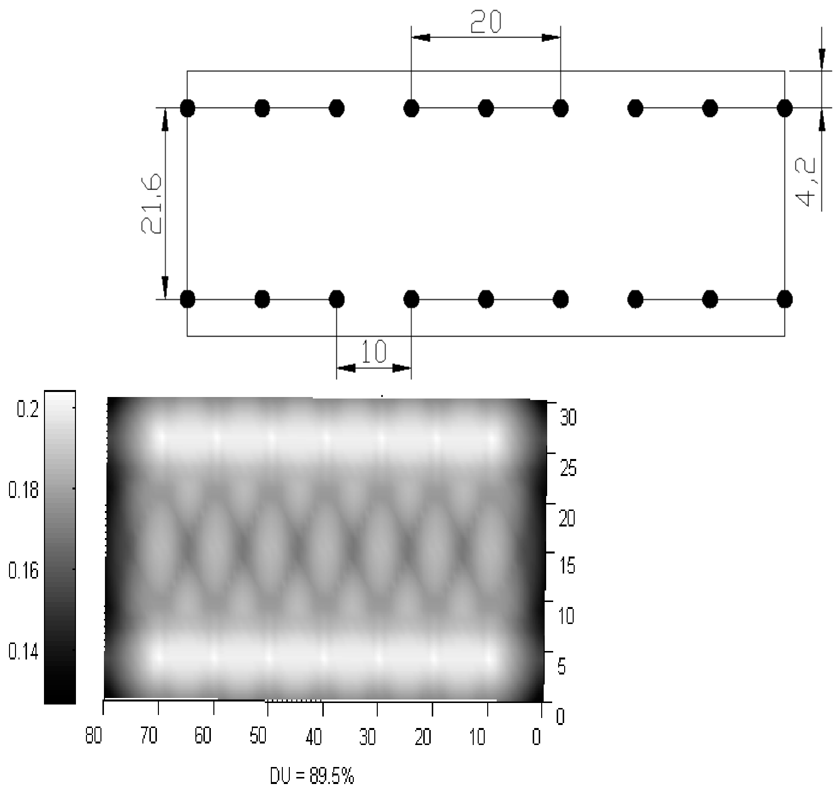

Figure 16. Upper: Layout 3. Lower: Catch-can test simulation result of Layout 3.

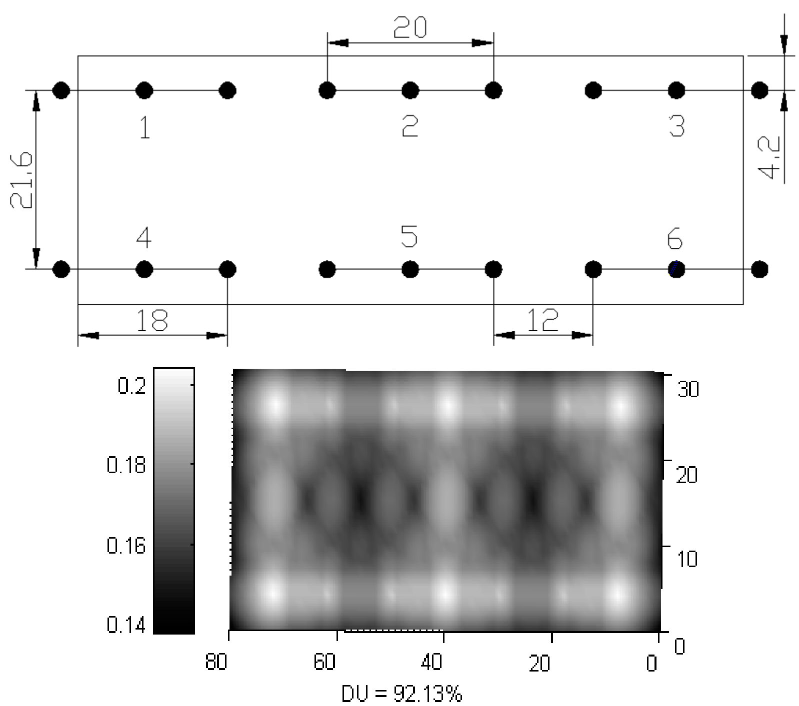

Figure 17. Upper: Layout 4. Lower: Catch-can test simulation result of Layout 4.

Calculation of the Precipitation

To meet the problem's constraints that in any part of the field,

Constraint A: The precipitation rate is ≥ 2 cm/4 d;

Constraint B: The precipitation rate is ≤ 0.75 cm/h,

we should calculate the precipitation rate of the system in Layout 4 before scheduling the interval to irrigating the field and to move the pipe. The precipitation area of a sprinkler should be a circle with a radius R, as **Figure 18** shows.

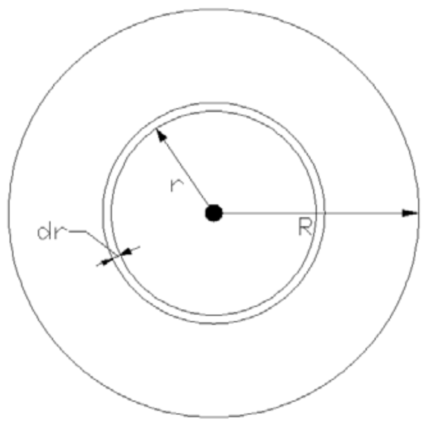

Figure 18. Precipitation area of a single sprinkler.

The profile of the precipitation rate distribution is in **Figure 5**. To figure out the precipitation rate at a point a distance r from the sprinkler, we first calculate the angle α in **Figure 5**. As we normalize the distribution, we get

$$\int_0^R [(R-r)\tan\alpha] 2\pi r\, dr = 1,$$

so

$$\tan\alpha = \frac{3}{\pi R^3}.$$

Then the precipitation rate is

$$\mathrm{pr}(r) = v(R-r)\tan\alpha = \frac{3(R-r)}{\pi R^3}\, v,$$

where $R = 19$ m, $v = 50$ L/min. With Matlab, we calculated the precipitation rate at each point in the 80 m $\times$ 30 m field, with the irrigation system working only once (**Figure 19 Right**) and after a complete cycle of moving the equipment and irrigating (**Figure 19 Left**).

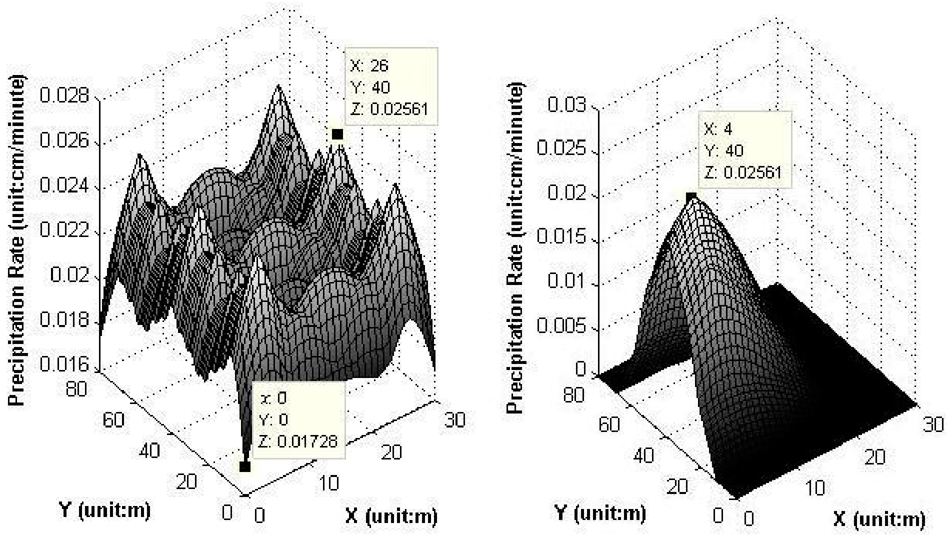

Figure 19. Precipitation rate of water in the field with Layout 4.
Left: The effect of six pipes together. Right: The effect of a 20 m pipe working alone.

Scheduling the Irrigation Time

Figure 19 Right shows that when the sprinklers (only one 20-m pipe at a time) are working, the maximum precipitation rate is 0.02561 cm/min. To satisfy Constraint B, the period of irrigation should be less than

$$\frac{0.75 \text{ cm/h}}{0.02561 \text{ cm/min}} \approx 29 \text{ min/h}.$$

Because the shorter the interval of irrigation, the more frequently the farmer must move the pipe, we choose a large but easy to implement interval: 25 min/h. **Figure 19 Left** shows that after a complete cycle of irrigation of the whole field the minimum precipitation rate is 0.0173 cm/min. To satisfy Constraint A, the period of irrigation should be longer than

$$\frac{2 \text{ cm/4d}}{0.0173 \text{ cm/min}} \approx 116 \text{ min/4d}.$$

To meet this requirement, we irrigate the same place five times, each time for 25 min, or 125 min in all. Using the same method, we calculate the same parameters for Layout 3.

Layout 4 not only has higher DU than Layout 3 but also saves 17% of the water, because Layout 3 has a smaller minimum precipitation rate (0.0160 vs. 0.0173), which leads to more irrigation time (150 min vs. 125 min) in order to satisfy Constraint A. So we choose Layout 4 as our solution.

Stage 3: Schedule Design

Our schedules can achieve a DU as high as 92.1%. We give two concrete timetables; one requires considerably less moving time (3 h vs. 15 h, per 4-days) but waters less evenly on average. Both schedules have a 12.5-h irrigation time in one cycle with a DU of 92%. Using a sprinkler with a sector mechanism, we can control the range of the sprinkler at the edges of the field, to reduce water waste. [EDITOR'S NOTE: We omit the tables.]

Strengths and Weaknesses

Strengths

- We use real data on sprinklers to determine the number of sprinklers.
- We establish a model based on the engineering knowledge of sprinklers, find out the overall precipitation distribution, and then find an optimal schedule.
- Our model for the layout of the irrigation system is sprinkler-independent. If a sprinkler's precipitation profile is known, we can determine the precipitation profile across the whole field.
- The placement and schedule is very clear and easy to implement.

Weaknesses

- Water pressure in the pipe may vary, and so the discharge of the sprinklers may not be exactly the same.

References

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