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An optimal model of "Sweet Spot" effect

summary

Various definitions on Sweet Spot have been given by former researchers. We take into consideration both the exit velocity and the comfort degree. Hence, the "Sweet Spot" in our model is defined as the optimal hitting location –resulting in high exit velocity, while reducing the impact force on hands to the lowest degree. A concept of Sweet Zone is also defined in our model for deeper study of the performance of a bat.

Based on the Acceleration theorem and the Moment of Momentum Theorem, our optimal model of Sweet Spot is established. The Sweet Spot is found to be 18.21cm apart from the end of a bat, resulting with the exit velocity to be 3915.8cm/s. In contrast, the exit velocity is 3129.6cm/s, which obviously confirms that the Sweet Spot is not located at the end of a bat. The analysis on λ (preference coefficient) shows the insensitivity of the location of Sweet Spot to the value of λ , proving the stability of our model. Sensitivity on the mass of the ball and the swing speed suggests that greater mass of bat and higher swing speed leads to higher exit velocity of a batted ball.

Then, our model is augmented to evaluate the performance of a corked bat, getting the conclusion that the exit velocity of a corked bat is lower than that of a normal bat. Furthermore, the influence that the length of the cork lays on the Sweet Spot effect is discussed. Based on model analysis, we explain why corked bats are banned in most games.

To compare the properties of an aluminum bat with a wooden one, our optimal model of Sweet Spot is developed, showing that the Sweet Spot is 20.15cm apart from the end of the bat, where the exit velocity is 4258.2cm/s. Compared with a wooden bat, an aluminum bat shows obvious superiority—including higher exit velocity, wider Sweet Zone and closer distance to pivot. In sensitivity analysis, the wall thickness is discussed in details. Based on our model, we analyze the reason why an aluminum bat is banned.

Furthermore, the strength and weakness are given and further discussion is expected.

Lastly, we summarize all the conclusions in order to give batters reasonable suggestions from different aspects.

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1. Introduction

1.1. Definition of a sweet spot

There are many definitions for the "Sweet Spot " on a baseball bat. The reason why there isn't an uniform definition is that the research on "Sweet Spot" is not only conducted by different methods and theories, but also considered form different aspects. In most of earlier researches, a "Sweet Spot" is referred to as the center of percussion, which is studied by using Momentum Theorem. The nodes of the first or second modes of vibration has become a popular definition recently, since the collision and vibration theory are introduced. Some define a "Sweet Spot" in order to get the greatest exit velocity of a batted ball, while the others are for the sake of the comfort for batters.

In our model, we take into consideration both the exit velocity and the comfort degree. So, the "Sweet Spot" in our model is defined as the optimal hitting location –resulting in high exit velocity, while reducing the impact force on hands to the lowest degree.

2. Assumption and definition

2.1. General assumptions:

- Unit of length is expressed in cm, time in s and mass in g.
- The mass of a bat is evenly distributed.
- During the collision procedure, the bat doesn't break.
- The rotation of the ball is not taken into consideration.
- Only the factor of material is taken into consideration.

2.2. Definitions:

e is the coefficient of restitution, which is defined as the ratio of the relative velocity after collision to that before the collision

exit velocity is the speed at which a ball moves away after impacting the bat.

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3. Where is the Sweet Spot?

3.1. Assumptions:

- In this section only wooden bat is considered.

3.2. Model establishment

3.2.1. Objective formula

As mentioned in the introduction, a "Sweet Spot" is defined as the optimal hitting location –resulting in high exit velocity, while reducing the impact force on hands to the lowest degree.

Based on the consideration of both factors, we get the objective formula: $f = \lambda E_{batter} + (1-\lambda)E_{ball}$

Where

λ is the Preference coefficient: donates how evaluator lay emphasis on each of the factors taken into consideration (e.g. donates that only the highest exit velocity is to be pursued, ignoring the comfort degree for batters)

E_{batter} and E_{ball} is respectively the comfort index function and exit velocity index function, which is to be established later.

3.2.2. Comfort index function E_{batter} [2]

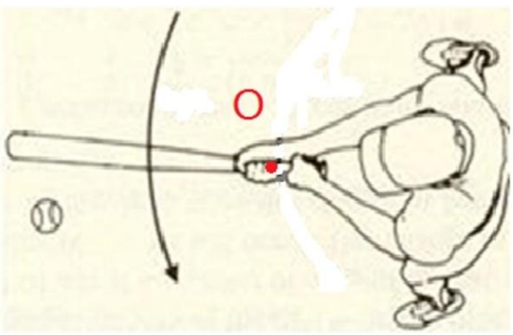

Figure 1 the pivot point[1]

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point O is the point(actually a zone) where a batter holds the bat.

When colliding with the fast-flying ball, a bat is impacted by force F, which will indirectly exert a force on the batter's hands. The force analysis is shown as follows: (see **Figure 2**)

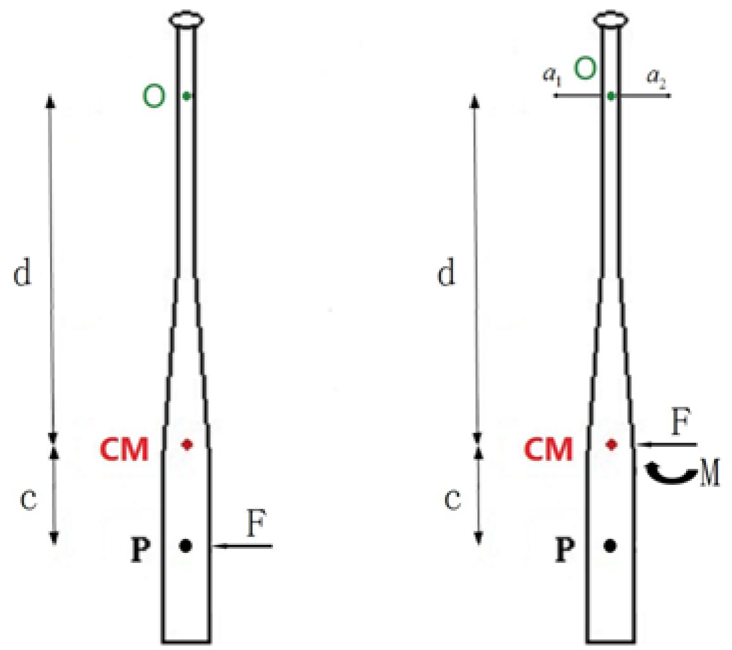

Figure 2

According to the theory of mechanics, force F is equivalent to a moment (equals Fc) and a force (equals F) applied at the mass center of the bat. Shown in （see **Figure 2**）

The acceleration at point O caused by the impulsive force:

$$a_1 = \frac{F}{M_{bat}}$$

Angular acceleration of the bat caused by the equivalent moment equals to $\frac{Fc}{J_{cm}}$，

Then, the acceleration at point O caused by the equivalent moment equals:

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$$a_2 = \frac{Fcd}{J_{cm}}$$

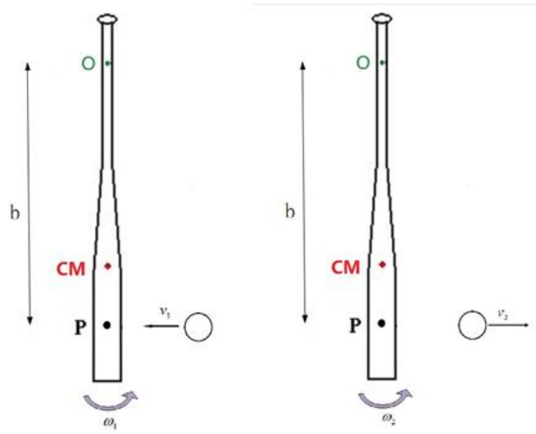

Figure 3 speed before and after collision

Since the acceleration at point O is a composition of accelerations caused by both of the impulsive force and the equivalent moment, a can be expressed as:

$$a = a_1 - a_2 = \frac{F}{M_{bat}} - \frac{Fcd}{J_{cm}}$$

When the impulsive force strikes on a certain point of the bat, the force applied on hands will be zero, since the acceleration at point O will be zero.

So, the acceleration of point O ranges within $[0, \frac{F}{M_{bat}}]$, getting the comfort index:

$$E_{batter} = 1 - 0.3123(\frac{M_{bat}d}{J_{cm}}c - 1)^2$$

- Clearly, the value of E_{batter} ranges from 0 to1.

3.2.3. Exit velocity index function E_{ball}:

It is a index representing the velocity at which the ball moving away after collision

According to the theorem of momentum moment, we can get the following expression:

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$$J_o w_1 - M_{ball} v_1 b = J_o w_2 + M_{ball} v_2 b \tag{1.1}$$

e is the coefficient of restitution, which is defined as the ratio of the relative velocity after collision to that before the collision.

$$e = \frac{v_2 - w_2 b}{v_1 + w_1 b} \tag{1.2}$$

Combining (1.1) and (1.2), v_2 can be expressed as:

$$v_2 = \frac{e(v_1 + w_1 b) J_o - v_1 M_{ball} b^2 + w_1 b J_o}{J_o + M_{ball} b^2}$$

According to Alan M. Nathan's research, the value of e is related to ratio of the force constant of the bat (k_{bat}) and that of the ball (k_{ball}). This ratio varies with the different impact location on a bat.

Hence, e is a function of the length from the hands-holding point to the impact point. The expression above can be overwritten as[3]:

$$v_2 = \frac{e(b)(v_1 + w_1 b) J_o - v_1 M_{ball} b^2 + w_1 b J_o}{J_o + M_{ball} b^2} \tag{1.3}$$

According to (1.3) , we can calculate the maximum velocity and the minimum velocity of the ball moving away and get the velocity index expression as follows:

$$E_{ball} = \frac{v_2 - v_{\min}}{v_{\max} - v_{\min}}$$

- ranging from 0 to 1

3.3. Parameter determination

3.3.1. Parameters in the objective function

λ represents how we lay emphasis on those two factors (the comfort degree for a batter and the batted ball velocity). Here, we attach the same importance to both of them. So, $\lambda = 0.5$

3.3.2. Parameters in the comfort index function

We simplify the shape of a bat as two coaxial cylinders with different diameters.(shown in [4][5])

- length of the bat: $L = 85cm$,
- Radius of the thin part: $r = 1.25cm$,
- Radius of the fat part: $R = 3.5cm$,
- Length of the thin part: $L_1 = 53.55cm$,
- Length of the fat part: $L_2 = 31.45cm$,
- Density of wood: $\rho = 0.6g/cm^3$,

Length from the pivot to the end of the thin part of a bat: 16.8cm.

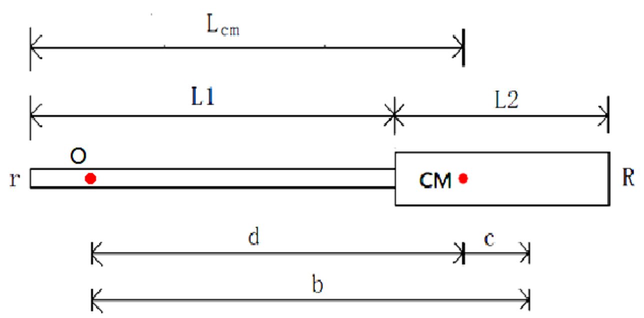

Figure 4 simplified model of a bat

- The mass of the bat can be calculated as:

$$M_{bat} = \pi r^2 L_1 \rho + \pi R^2 L_2 \rho = 883.92g$$

- Length from the center of mass to the end of the thin part:

$$L_{cm}=\frac{\pi r^2 L_1\rho\times\frac{L_1}{2}+\pi R^2 L_2\rho\times(\frac{L_2}{2}+L_1)}{\pi r^2 L_1\rho+\pi R^2 L_2\rho}-16.8=44.89cm$$

- The moment of inertia to the center of mass(CM)：

$$J_{cm}=\pi r^2\rho L_1\times\frac{L_1^2}{12}+\pi r^2\rho L_1\times(L_{cm}-\frac{L_1}{2})^2+\pi R^2\rho L_2\times\frac{L_2^2}{12}+\pi R^2\rho L_2\times(L-L_{cm}-\frac{L_2}{2})^2$$
$$=331590g\cdot cm^2$$

3.3.3. Parameters in batted ball velocity index function

- According to published research, initial velocity of the ball (before collision) equals: $v_1=70mph=112.7km/h=3130.56cm/s$ [6]
- The mass of a ball: $M_{ball}=145g$ [7]
- According to the empirical formula，

$$v=-0.6625M_{bat}+3354=2768.39\text{cm/s}\ ,\qquad (1.4)[8]$$

- So, the angular speed is $w_1=\frac{u_1}{L-b}=\frac{2768.39}{68.2}=40.59rad/s$。
- The moment of inertia to the pivot point：

$$J_o=J_{cm}+M_{bat}b^2=2112800\text{g}\cdot\text{cm}^2$$

- The value of e(coefficient of restitution)

According to the research done by Alan M. Nathan, the value of e varies with the changing impact location along the length of a bat. a set of data is shown as follows:

（Nathan_Trampoline-ISEA2004）

Table 1

68.2-b	12.7	15.24	17.78	20.32	22.84	25.4
e	0.44	0.551	0.589	0.593	0.568	0.525

We use SPSS software to fit this set of data, getting the expression of e:

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$$e = \begin{cases} -0.5294 + 0.113(68.2 - b) - 0.0028(68.2 - b)^2 & when\ e > 0.4 \\ 0.4 & when\ e \le 0.4 \end{cases}$$

Where

$R^2 = 0.962$, $S = 0.007$

$F = 38.01$, showing a high fitting degree.

(Considering all given value of e is greater than 0.4, we assume $e = 0.4$ when $e \le 0.4$.)

3.4. Solution and Analysis

Using the Matlab software we calculate the comfort index, velocity index and the value of our objective function, shown in **Figure 5** and

Table 2

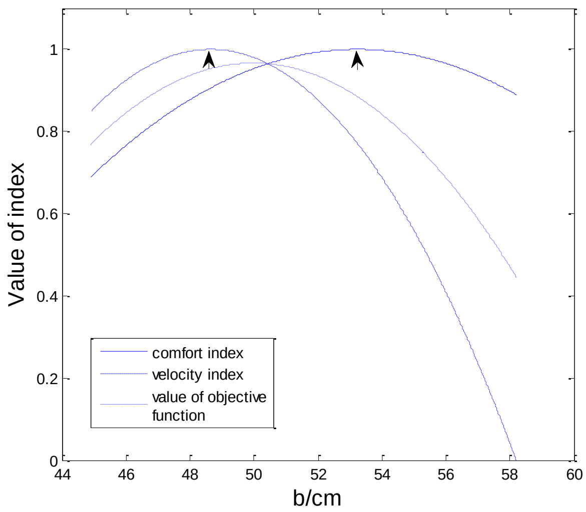

Figure 5

Table 2

b (cm)	E_{batter}	E_{ball}	v_2 (cm/s)	f

53.25	1	0.7693	3665.7	0.8874
48.66	0.9059	1	3938.7	0.9530
49.99	0.9526	0.9809	3915.8	0.9666
68.2	0	0.3163	3129.6	0.1582

Conclusion:

- The most comfortable hitting point is not the maximum-exit-velocity point, with a difference of 6.93% from the maximum speed.
- Vice versa, at the maximum-exit-velocity point, the comfort index equals 0.91, leading to a less comfortable feeling in hands. So, batters have to sacrifice some of the comfort feeling to pursue the highest exit velocity.
- From the curve of the objective function, we can get the optimal solution of the objective formula: The sweet spot is located at the point 18.21cm apart from the end of a bat. Hitting at this point, batters can obtain a very high speed (3915.8cm/s compared with the maximum value 3938.7cm/s), and enjoy a high degree of comfort (0.95) at the same time.
- In contrast, hitting at the end of the bat is clearly not a wise decision, since it brings a relatively low exit velocity (3129.6cm/s compared with the maximum value of 3938.7cm/s) and low comfort degree (0), which does harm to batters' muscles and ligaments and sometimes even breaks the bat its self. The contrast between hitting at the sweet point and the end of the bat is clearly shown in **Table 3**.
- Thus, it has been proved that the sweet spot, hitting which will result in high exit velocity and high degree of comfort, is located 18.21cm apart from the end of a bat.

Table 3

	Comfort index	Exit velocity(cm/s)	Value of objective formula
Sweet point	0.95	3915.8	0.9666
End of a bat	0	3129.6	0.3163

3.5. Sensitivity analysis

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3.5.1. Analysis of λ (Preference coefficient)

In former analysis, we assume the value of λ as 0.5 (i.e. we attach same importance to both comfort and velocity). Here, we change the value of λ from 0.1 to 0.9, getting nine curves to show relation between value of objective formula and impact location. (shown in **Figure 6**)

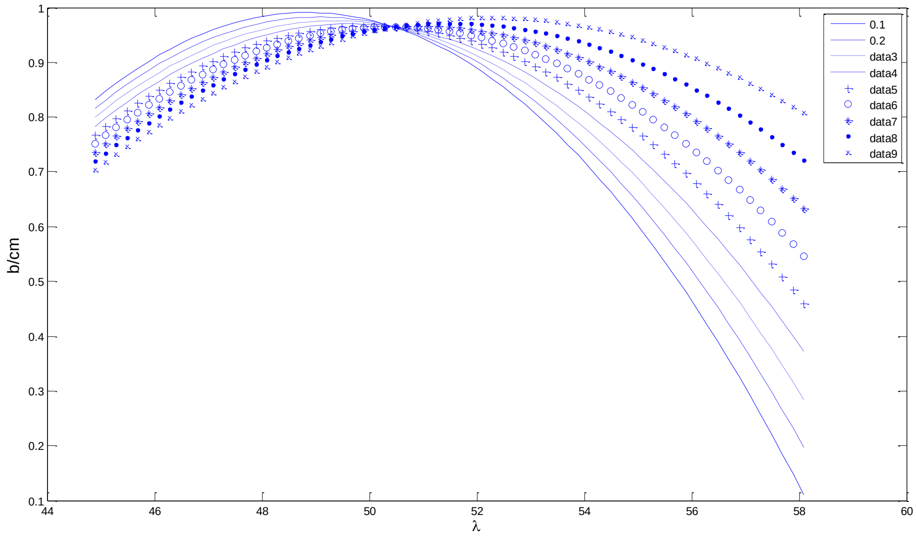

Figure 6

From **Figure 6**, we find that the sweet spots for each value of λ are relatively concentrated.

Figure 7 is used to show the changes of sweet spots' location when the value of λ varies.

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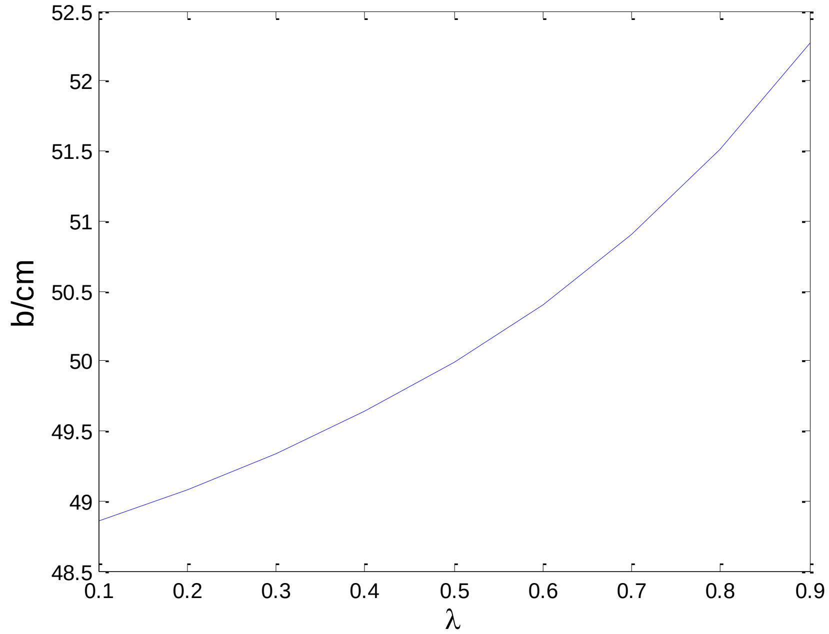

Figure 7

- Locations of Sweet Spots vary within [48.5 , 52.5], which is a zone with a length of 4cm, showing that our model is stable.
- With the increase of the value of λ , the location of the Sweet Spot moves closer to the end of a bat. Thus, in order to obtain higher exit speed of the batted ball, a batter is advised to hit the ball closer to hands within the zone [48.5,52.5], though he will consequently feel harder impact on his hands.

3.5.2. Analysis of the mass of a bat

3.5.2.1. Influence on the location of Sweet Spot

In former analysis, we assume the mass of bat as 883.92g. Here, we change the value of the mass of a bat to see how the location of Sweet Spot changes. (shown in **Figure 8**)

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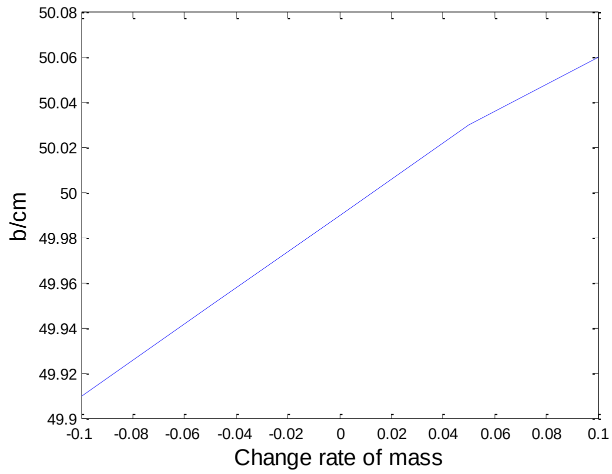

Figure 8

From **Figure 8**, basic conclusions can be given:

- The greater the mass of a bat, the closer the Sweet Spot is located to the end of the bat.
- This influence of it is so slight that can be ignored. Therefore, we can safely say that the location of Sweet Spot stays stable no matter how your bat weighs.

3.5.2.2. Influence on the exit velocity

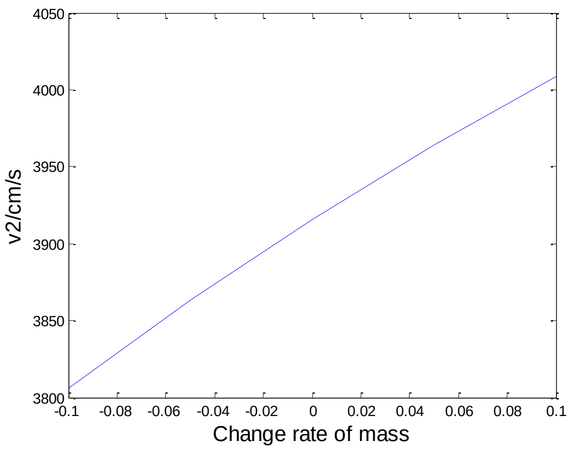

Figure 9

Table 4

Change of the mass of a bat(in ratio)	-0.1	-0.05	0	0.05	0.1

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Location of Sweet Spot	49.91	49.95	49.99	50.03	50.06
Exit velocity	3805.9	3863.3	3915.8	3963.9	4008.5
Changing ratio of exit velocity	-2.81%	-1.34%	0	1.23%	2.37%

From **Figure 9** and **Table 4**, We can get the basic conclusions as follows:

- The heavier a bat is, the higher exit velocity it results in. This conclusion matches our expectation perfectly: with the increase of a bat, the moment of inertia increases, which means that more available energy can be transferred to the ball and consequently a higher speed at which it moves away.
- Hence, in order to obtain higher exit velocity, a batter should choose heavier bat within his own capability, since a heavier bat obviously has higher demand on muscle strength and endurance.

3.5.3. Analysis of the swing speed

In former analysis we identify the swing speed by the empirical formula (1.4). Here, we change the value of swing speed by $\pm 10\%$ in order to research the influence of swing speed.

The influence on the exit velocity: Using our model we calculate the exit velocity curves corresponding to different value of swing speed.(shown in **Figure 10**).

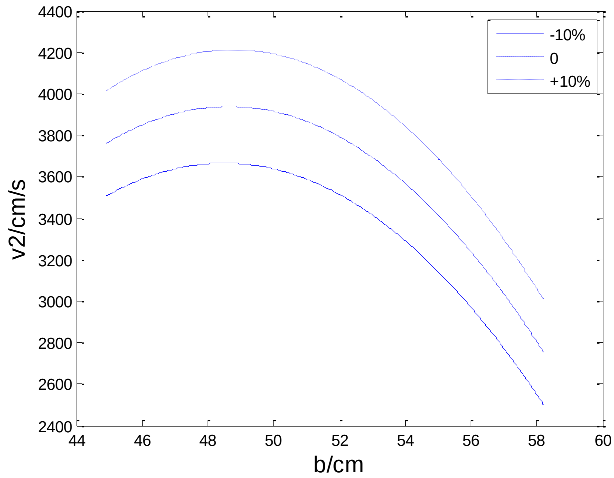

Figure 10

We can see that the overall exit velocity changes with the change of swing speed while the shape of the curves stays stable. The relation between swing speed and exit speed is more clearly shown in **Table 5**

Table 5

The change of swing speed(in ratio)	Exit speed (cm/s)	variance of exit speed(in ratio)
-0.1	3665.5	-6.94%
0	3938.7	0
0.1	4212.4	+6.95%

- From **Table 5**, we can safely say that higher swing speed results in higher exit speed.
- The location of the Sweet Spot stays stable when the swing speed varies.
- Hence, batters are advised to improve their muscle strength in order to get higher swing speed and consequently higher exit speed of the ball.

4. The behavior of a "corked" bat

In history, players using a corked bat perform impressively and even hit home runs with it. People get curious about the property of the corked bat and it has become a controversial issue. Which indeed has better performance, the corked bat or uncorked one?

In this section, we augment our model to solve this problem and further discussion is given based on model analysis. We analyze the influence the cork length lays on a bat's performance and we conclude why the corked bat is banned.

Figure 11

4.1. Assumption

- “replacing a wood cap” is not taken into consideration

4.2. Simplified model of the shape of a corked bat

We continue to use the two-coaxial-cylinder model and simulate the cork as a cylinder with lower density.(see **Figure 12**)

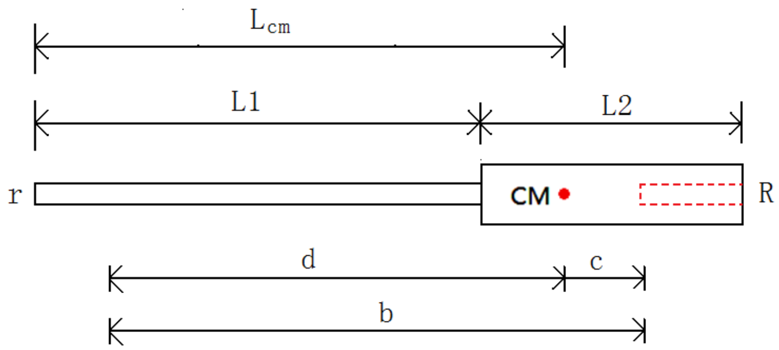

Figure 12 simplified model of a corked bat

Basic parameters can be expressed as follows:

4.2.1. Mass of the bat

Mostly, producers drill a cylinder about 2.54cm in diameter, and 25.4 deep and fulfill the hole with rubber or cork, which reduces the mass of the bat by 42.5g.[9]

Since a 25.4cm long cork can reduce the mass by 42.5g, the mass will be reduced by $1.67x$ when the length of the bat equals x and the diameter is kept as 2.54cm.

Then, the mass of the corked bat decreases to:

$$M_{corked} = 883.92 - 1.67x$$

4.2.2. The center of mass:

The re-distribution of mass will lead to the displacement of the center of mass of a bat.

The length from the pivot point to the center of mass of a corked bat equals:

$$d_{corked} = \frac{M_{normal} d_{normal} - 1.67x \times (68.2 - \frac{x}{2})}{M_{corked}}$$

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Where

x is the length of the bat.

M_{normal} is the mass of a normal (uncorked) bat.

d_{normal} is the length from the pivot point to the center of mass of a normal bat.

M_{corked} is the mass of a corked bat.

4.2.3. Moment of inertia

The displacement of the center of mass and the re-distribution of mass inevitably lead to the decrease of the moment of inertia, which can be expressed as follows:

The moment of inertia to the center of mass:

$$J_{cm-corked} = J_{cm-normal} - 883.92(d_{normal} - d_{corked})^2 - \frac{1}{12} \times 1.67x \cdot x^2 - 1.67x \cdot (68.2 - d_{normal} - 0.5x)^2$$

The moment of inertia to the pivot point:

$$J_{o-corked} = J_{o-normal} - \frac{1}{12} \times 1.67x \cdot x^2 - 1.67x \cdot (68.2 - 0.5x)^2$$

4.2.4. Swing speed:

Being lighter and the moment of inertia of it getting smaller, a bat is more easy to swing. So, a swing speed gets higher, which will necessarily lead to the increase of the batted ball velocity.

The swing speed can be expressed according to the empirical formula[10] :

$$\frac{v}{44.72} = -\frac{M_{corked}}{67.5} + 75$$

Where

v is the swing speed (the speed at the end of a swung bat)

M_{corked} is the mass of a corked bat.

The angular speed of the bat equals:

$$w_1 = \frac{v}{68.2}$$

4.3. Parameter determination

In order to research the influence the length of cork lays on the bat properties, we change the value of the x to see the fluctuation of the Sweet Spot effect.

When x=0,5,10,15,20cm, we calculate the following parameters:

4.3.1. Mass of the bat:

Table 6

Length of cork	0	5	10	15	20
Mass of bat	883.92	875.57	867.22	858.87	850.52

4.3.2. Length from the center of mass to the pivot

Table 7

Length of cork	0	5	10	15	20
Length from center of mass to the pivot	44.89	44.69	44.54	44.43	44.37

4.3.3. Moment of inertia to the center of mass (J_o)

Table 8

Length of cork	0	5	10	15	20
J_o	331590	327920	325740	324670	324320

4.3.4. Swing speed

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Table 9

Length of cork	0	5	10	15	20
Swing speed	40.59	40.67	40.75	40.84	40.92

4.4. Solution and analysis

We substitute the parameters in our basic model with all the above calculated values, getting solutions as follows:

4.4.1. Influence on the location of the Sweet Spot

Table 10

Length of cork	0	5	10	15	20
b	49.99	49.89	49.81	49.76	49.73

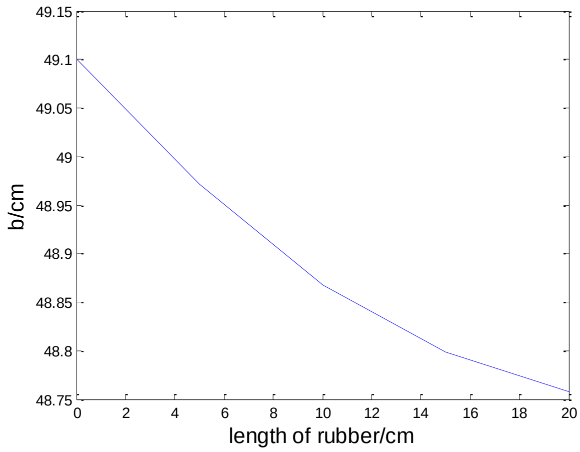

Figure 13

From **Table 10** and **Figure 13**, we find that with the increase of the length of the cork, the location of the Sweet Spot will move towards the pivot though slightly—the length form the Sweet Spot to the pivot is 49.99cm in an uncorked bat, while that in a corked bat is 49.73cm, only 2.6mm shorter.

4.4.2. Influence on the exit speed

Table 11

Length of cork	0	5	10	15	20
v_2	3915.8	3906.7	3889.2	3892.7	3887.8

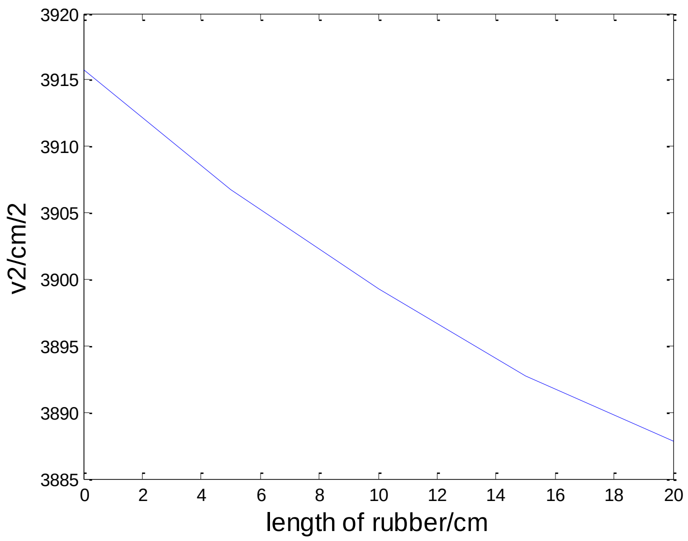

Figure 14

- The longer the cork is, the lower the exit speed will be.
- The exit speed of an uncorked bat is 3915.8cm/s, while that of a 20cm-long-corked bat is 3887.8cm/s, which decreases by 0.72%.
- The result matches our expectation—being corked, a bat gets lighter and the moment of inertia gets smaller (4.47%); that will consequently leads to the decrease of the available energy transferred to the ball. That's why the batted ball moving away at a lower speed when the bat is corked.

4.4.3. Influence on the comfort index

Table 12

Length of cork	0	5	10	15	20
The most comfortable	53.25	53.07	52.97	52.94	52.96

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hitting point

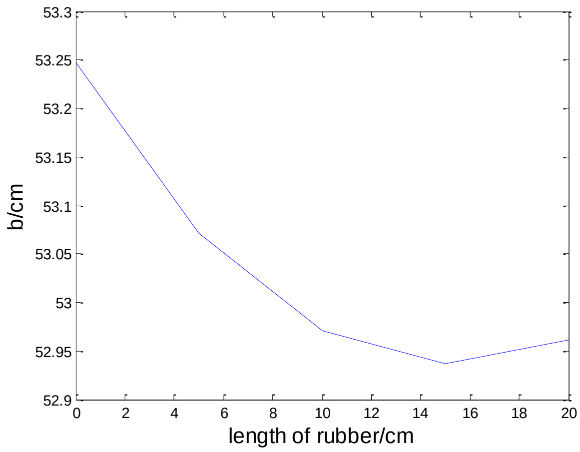

Figure 15

- The length from the most comfortable point to the pivot is 53.25 in an uncorked bat, while that in a corked bat is 52.96, only 0.29 shorter.
- It suggests that a batter, for the sake of comfort, should hit a point that's closer to his hands when the cork in bat gets longer.

4.4.4. Influence on the Sweet Zone

- Sweet Zone is a zone within which the value of objective function is no less than 0.85.

Here, we discuss the influence the length of rubber lays on the span of Sweet Zone.

When x=0，5，10，15，20cm, the Sweet Zone varies as shown in **Figure 16** and **Table 13**:

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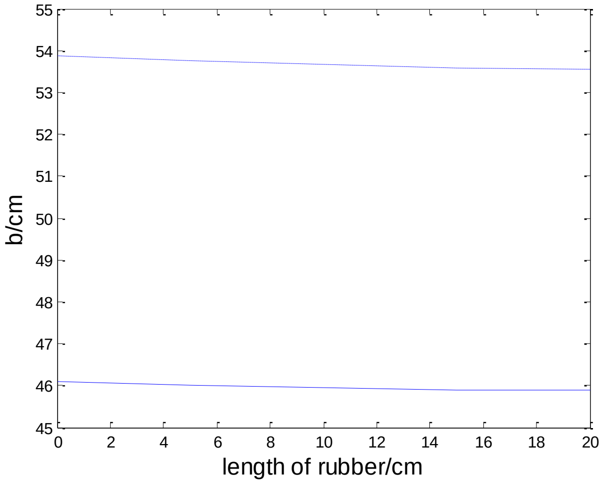

Figure 16 location of Sweet Zone vs. length of rubber

Table 13

X	Lower limit	Upper limit	Span of the zone
0	46.09	53.87	7.78
5	46	53.75	7.75
10	45.95	53.66	7.71
15	45.9	53.59	7.69
20	45.88	53.56	7.68

The location and the span of Sweet Zone both stay stable with the change of the length of cork. Thus a corked bat is not superior to a normal one in the aspect of Sweet Zone.

4.5. Why is corked bat forbidden?

- According to the solution and analysis of our model, a corked bat seems not to show a better performance—with lower (1.5%) exit speed of a batted ball. Nevertheless, there exist other factors that should also be taken into consideration.
- The corked bat is easier to swing since its lighter and the moment of inertia is smaller. Thus, the rise in the swing speed allows a batter longer time to prepare

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and determine the flying path and the speed of the ball, which results in better performance.

- The psychological factor shouldn't be neglected. According to historical data, players using a corked bat perform impressively and even hit home runs with it. Corked bat is more or less something on which a batter lays his psychological dependence.
- Athletes should lay more emphasis on the improvement of Physical Quality and skill practice instead of unduly relying on sports equipments. That's one of the reason why corked bat is forbidden[11].

5. Aluminum vs. wood

5.1. Parameter determination

We simplify the shape of an aluminum bat as two coaxial hollow cylinders with different diameters, using the same basic length parameters with a wooden one[12].(shown in **Figure 17**)

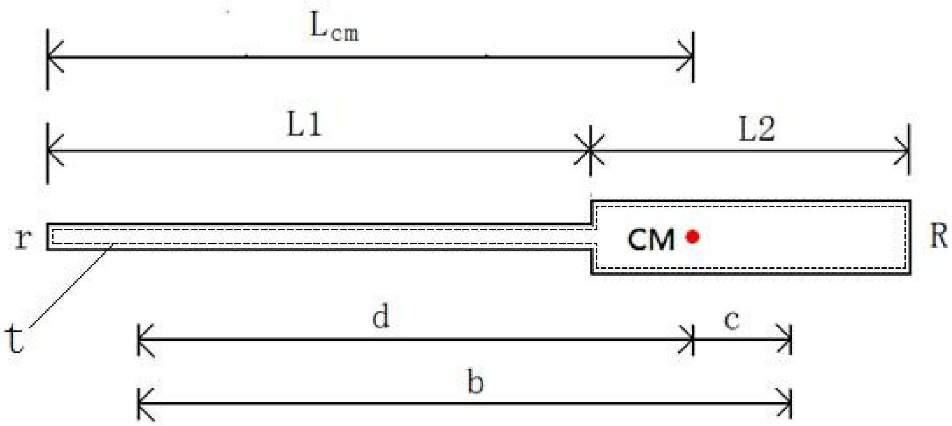

Figure 17 simplified model for an aluminum bat

- According to, we get the mass of an aluminum bat: $M_{al-bat} = 861.63g$.
- The density of aluminum: $\rho_{al} = 2.7g/cm^3$

- From the expression of the mass of an aluminum bat:

$$M_{al-bat} = \pi r^2 L_1 \rho_{al} - \pi (r-t)^2 L_1 \rho_{al} + \pi R^2 L_2 \rho_{al} - \pi (R-t)^2 L_2 \rho_{al}$$

- We calculate the wall thickness of an aluminum bat : $t = 0.31cm$ 。
- Length from the pivot to the center of mass:

$$d_{al} = \frac{\pi[r^2 - (r-t)^2]L_1\rho_{al} \times \frac{L_1}{2} + \pi[R^2 - (R-t)^2]L_2\rho_{al} \times (\frac{L_2}{2} + L_1)}{\pi[r^2 - (r-t)^2]L_1\rho_{al} + \pi[R^2 - (R-t)^2]L_2\rho_{al}} - 16.8 = 37.26cm$$

- Moment of inertia to the center of mass:

$$\begin{aligned} J_{al-cm} &= \pi[r^2 - (r-t)^2]\rho_{al}L_1 \times \frac{L_1^2}{12} + \pi[r^2 - (r-t)^2]\rho_{al}L_1 \times (d_{al} + 16.8 - \frac{L_1}{2})^2 \\ &\quad + \pi[R^2 - (R-t)^2]\rho_{al}L_2 \times \frac{L_2^2}{12} + \pi[R^2 - (R-t)^2]\rho_{al}L_2 \times (L - 16.8 - d_{al} - \frac{L_2}{2})^2 \\ &= 476940 \end{aligned}$$

- Moment of inertia to the pivot ： $J_{al-o} = J_{al-cm} + M_{al-bat} d_{al}^{\ 2} = 1673400$

From literature[9], the average value of e(coefficient of restitution) for an aluminum bat equals to 0.4, which is 1.6 times of that for a wooden bat(0.25).

Therefore, the value of e can be expressed as:

$$e = -0.847 + 0.181(68.2 - b) - 0.0045(68.2 - b)^2$$

All the other parameters keep the same with the wooden bat.

5.2. Solution and analysis:

We substitute the parameters in our basic model with all the above calculated values, getting solutions shown in **Figure 18** and **Table 14**.

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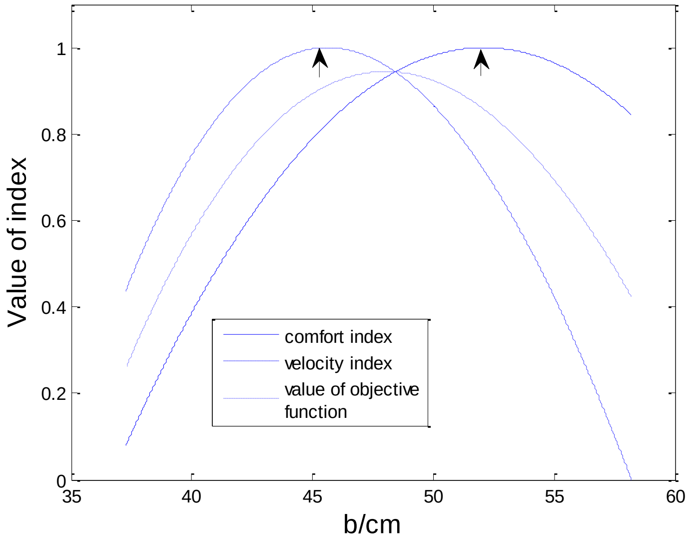

Figure 18

Table 14

b (cm)	E_{batter}	E_{ball}	v_2 (cm/s)	f
52.11	1	0.7145	3632.6	0.8572
45.64	0.825	1	4365.0	0.9125
48.05	0.931	0.9584	4258.2	0.9447

Basic conclusions based on the solution:

- The most comfortable point is 52.11cm apart from the pivot, 1cm closer to the pivot compared with a wooden bat. The exit speed at this point equals to 3632.6cm/s, which is similar to that of the wooden bat.
- The highest exit velocity point is 48.66cm apart from the pivot, 3cm closer to the pivot compared with a wooden bat. The exit speed at this point equals to 4365.0cm/s, which is 11% higher than that of a wooden bat.
- The Sweet Spot is 48.05cm apart from the pivot, 2cm closer to the pivot compared with a wooden bat. The exit speed at this point equals to 4258.2cm/s, which is 9% higher than that of a wooden bat.
- To sum up, all of these optimal points are closer to the pivot compared with those of a wooden bat. Furthermore, The exit velocity increases significantly.

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5.3. Performance comparison between wooden bat and aluminum bat

5.3.1. Exit velocity

The highest exit speed of a wooden bat is 3938.7cm/s, which can be easily obtained by using an aluminum bat, since the exit speed for an aluminum bat is no less than 3938.7cm/s as long as hitting on the bat within a zone $b \subset [41.00, 50.61]$ (with span of 9.5cm).Clearly, the exit velocity for an aluminum bat is much higher than that for a wooden bat.

5.3.2. Sweet Zone

Consider the length of Sweet Zone of aluminum bat and wooden bat, shown in **Figure 19** and **Figure 20**:

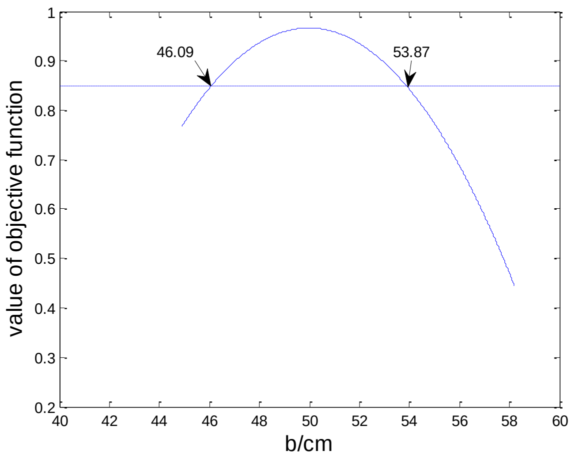

Figure 19 property of wooden bat

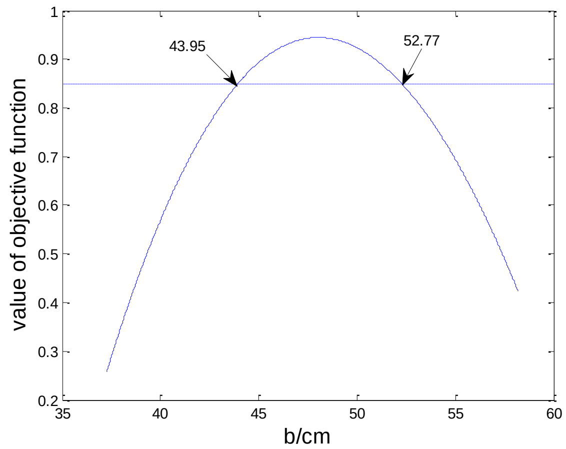

Figure 20 property of aluminum bat

- The Sweet Zone of an aluminum bat is [43.95 , 52.77], with a span of 8.82cm.
- The Sweet Zone of a wooden bat is [46.09 , 53.87], with a span of 7.78cm.
- Clearly, the span of Sweet Zone for an aluminum bat is 1.1cm longer than that for a wooden bat, which means that an aluminum bat is easier to control since it provides higher probability for a batter to obtain an expected exit velocity.

5.3.3. The sensitivity analysis of the wall thickness (t)

Changing the value of t by $\pm 0.01cm, \pm 0.02cm$, we get the following results: (see

Figure 21 and

Table 15)

Table 15

Change of t(in ratio)	Location of Sweet Spot	Exit velocity of ball	Change of the exit velocity(in ratio)
-0.02	48.02	4307.4	0.012
-0.01	48.03	4283.2	0.006
0	48.05	4258.2	0

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0.01	48.06	4234.3	-0.006
0.02	48.07	4210.4	-0.011

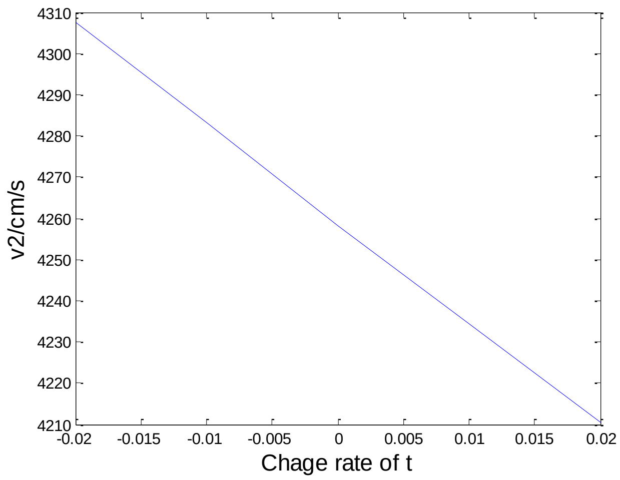

Figure 21

Basic conclusions we get:

- The location of Sweet Spot stays stable with the change of t.
- The exit velocity decreases with the increase of t.
- Hence, producer of aluminum bats should try to make the wall thinner in order to raise the exit speed for the bat. However, there is a lower limit of the wall thickness since too thin the wall is will make the material easily bent and the coefficient of resistant will turn to be very low.

5.4. Why aluminum bat is banned?

Though the aluminum bat has much superiority over a wooden one, it's usually banned in a game. The reason is analyzed as follows:

- The exit speed gets to be as high as 4365cm/s when an aluminum bat is used. We assume the ball weighs 145g and the contact time is 0.1s. It suggests that a impact force of 63.3N is applied on the hands of a batter, threatening the safety of the batter.
- The over high-speed batted ball is also a threat to the safety for the other athletes in a baseball game, which runs counter to the spirit of sports.

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- Since the span of Sweet Zone for an aluminum bat is wider than that for a wooden one, athletes will more or less lay psychological dependence on it, which might lead to a trend of the undue dependence on sports equipments.

6. Superiority and weakness

6.1. Superiorities of our model

- We take into consideration both the exit velocity of the batted ball and the comfort for batters. So, the solution of our model pursues high exit speed, while reducing the harm to the body of athletes to a relatively low degree.
- The results of model match perfectly with the experience, which proves the rationality and correctness of our model.
- Our modal has universal adaptability, since different locations of Sweet Spot are given to different batters who have different preference on the comfort degree and the exit velocity.

7. Further discussion

- In our model, the coefficient of resistant is only determined by the hitting position. Solutions of our model could be more accurate if more factors are taken into account when determining the value of the coefficient of resistant.
- We could take accurate vibration model into consideration: The bat can be simplified as a free-free vibrating elastic beam. Then, the location of the Sweet Spot could be determined by solving the vibration distribution function according to the theory of vibration dynamics.
- We could use finite-element software Ansys to conduct simulation, since the numerical solution would be too complex to obtain.

7.1. Weaknesses of our model

- W e don’t directly consider the effect of vibration, but it has been taken into account in e(the coefficient of resistant), though being rough. More accurate solution could be obtained by experimental methods or dynamical simulation.
- We don’t take into account the influence of the rotation of the ball.

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8. Suggestions given to batters:

Based on the analysis of our model, we provide reasonable suggestions to batters from various aspects:

- The heavier a bat is, the higher exit velocity it results in .In order to obtain higher exit velocity and to hit the ball to a longer distance, a batter should choose heavier bat within his own capability, since a heavier bat obviously has higher demand on muscle strength and endurance
- Practice to improve the swing speed in order to obtain higher exit velocity of ball
- A corked bat allows a batter longer time to prepare and determine the flying path and the speed of the ball, which results in better performance. It's a good choice for new learners
- The span of Sweet Zone for an aluminum bat is wider, making it easier to control. It's also a good choice for new learners
- The over high exit speed of the ball is a threat to the safety of athletes in a baseball game, which should be noticed by athletes.

9. References

[1] http://paws.kettering.edu/~drussell/bats-new/bat-moi.html

[2] http://memo.cgu.edu.tw/yun-ju/CGUWeb/PhyChiu/H201Oscillation/SweetSpotProblem.htm

[3] http://paws.kettering.edu/~drussell/bats-new/nonlinear-ball.html

[4] http://paws.kettering.edu/~drussell/bats-new/alumwood.html

[5] http://zhidao.baidu.com/question/61026308.html

[6] http://paws.kettering.edu/~drussell/bats-new/ban-safety.html

[7] http://zhidao.baidu.com/question/32333384.html

[8] http://paws.kettering.edu/~drussell/bats-new/batw8.html

[9] http://paws.kettering.edu/~drussell/bats-new/corkedbat.html

[10] http://paws.kettering.edu/~drussell/bats-new/batw8.html

[11] http://en.wikipedia.org/wiki/Corked_bat

[12] http://paws.kettering.edu/~drussell/bats-new/alumwood.html

[13] Wu Junchang,Li Minghui,Xiang Ziyuan,Comparison Analysis Research between Striking Results by an Aluminum Bat and a Wooden Bat , learned journal of Taibei 2002(29), 252-271