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Traffic Flow Models and the Evacuation Problem

Samuel W. Malone
Carl A. Miller
Daniel B. Neill
Duke University
Durham, NC

Advisor: David P. Kraines

Introduction

We consider several models for traffic flow. A steady-state model employs a model for car-following distance to derive the traffic-flow rate in terms of empirically estimated driving parameters. We go on to derive a formula for total evacuation time as a function of the number of cars to be evacuated.

The steady-state model does not take into account variance in speeds of vehicles. To address this problem, we develop a cellular automata model for traffic flow in one and two lanes and augment its results through simulation.

After presenting the steady-state model and the cellular automata models, we derive a space-speed curve that synthesizes results from both.

We address restricting vehicle types by analyzing vehicle speed variance. To assess traffic merging, we investigate how congestion occurs.

We bring the collective theory of our assorted models to bear on five evacuation strategies.

Assumptions

- Driver reaction time is approximately 1 sec.
- Drivers tend to maintain a safe distance; tailgating is unusual.
- All cars are approximately 10 ft long and 5 ft wide.
- Almost all cars on the road are headed to the same destination.

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Terms

Density d**:** the number of cars per unit distance.

Occupancy n**:** the proportion of the road covered by cars.

Flow q**:** the number of cars per time unit that pass a given point.

Separation distance s**:** the average distance between midpoints of successive cars.

Speed v**:** the average steady-state speed of cars.

Travel Time: how long a given car spends on the road during evacuation.

Total Travel Time: the time until the last car reaches safety.

The Steady-State Model

Development

Car-following is described successfully by mathematical models; following Rothery [1992, 4-1], we model the average separation distance s as a function of common speed v:

$$s = \alpha + \beta v + \gamma v^2, \tag{1}$$

where α, β, and γ have the physical interpretations:

$$\begin{aligned} \alpha &= \text{the effective vehicle length } L, \\ \beta &= \text{the reaction time, and} \\ \gamma &= \text{the reciprocal of twice the maximum average deceleration} \\ &\quad\ \text{of a following vehicle.} \end{aligned}$$

This relationship allows us to obtain the optimal value of traffic density (and speed) that maximizes flow.

Theorem. *For $q = kV$ (the fundamental equation for traffic flow) and* **(1)**, *traffic flow q is maximized at*

$$q^* = (\beta + 2\gamma^{1/2}L^{1/2})^{-1}, \qquad v^* = (L/\gamma)^{1/2}, \qquad k^* = \frac{\beta(\gamma/L)^{1/2} - 2\gamma}{\beta^2 - 4\gamma L}.$$

Proof: Consider N identical vehicles, each of length L, traveling at a steady-state speed v with separation distance given by **(1)**. If we take a freeze-frame picture of these vehicles spaced over a distance D, the relation $D = NL + Ns'$

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must hold, where s' is the bumper-to-bumper separation. Since $s' = s - L$, we obtain $k = N/D = N/(NL + Ns') = 1/(L + s') = 1/s$. We invoke **(1)** to get

$$k = \frac{1}{\alpha + \beta v + \gamma v^2}.$$

This is a quadratic equation in v; taking the positive root yields

$$v(k) = \frac{1}{2\gamma}\sqrt{4\,\frac{\gamma}{k} + (\beta^2 - 4\gamma L)} - \frac{\beta}{2\gamma}.$$

Applying $q = kv$, we have

$$q(k) = \frac{k}{2\gamma}\sqrt{4\,\frac{\gamma}{k} + (\beta^2 - 4\gamma L)} - \frac{k\beta}{2\gamma}.$$

Differentiating with respect to k, setting the result equal to zero, and wading through algebra yields the optimal values given. □

Interpretation and Uses

We can estimate q^*, k^*, and v^* from assumptions regarding car length (L), reaction time (β), and the deceleration parameter (γ). If we let $L = 10$ ft, $\beta = 1$ s, and $\gamma \approx .023$ s^2/ft (a typical value [Rothery 1992]), we obtain

$$q^* = 0.510 \text{ cars/s}, \qquad v^* = 20.85 \text{ ft/s}, \qquad k^* = 0.024 \text{ cars/ft}.$$

A less conservative estimate for γ is $\gamma = \frac{1}{2}(a_f^{-1} - a_l^{-1})$, where a_f and a_l are the average maximum decelerations of the following and lead vehicles Rothery [1992]. We assume that instead of being able to stop instantaneously (infinite deceleration capacity), the leading car has deceleration capacity twice that of the following car. Thus, instead of $\gamma = 1/2a = .023$ s^2/ft, we use the implied value for a to compute $\gamma' = \frac{1}{2}\left(a^{-1} - 2a^{-1}\right) = \frac{1}{2}\gamma = 0.0115$ s^2/ft and get

$$q^* = 0.596 \text{ cars/s}, \qquad v^* = 29.5 \text{ ft/s} \approx 20 \text{ mph}, \qquad k^* = .020 \text{ cars/ft}.$$

Going 20 mph in high-density traffic with a bumper-to-bumper separation of 40 ft is not bad.

The 1999 evacuation was far from optimal. Taking 18 h for the 120-mi trip from Charleston to Columbia implies an average speed of 7 mph and a bumper-to-bumper separation of 7 ft.

Limitations of the Steady-State Model

The steady-state model does not take into account the variance of cars' speeds. Dense traffic is especially susceptible to overcompensating or undercompensating for the movements of other drivers.

A second weakness is that the value for maximum flow gives only a first-order approximation of the minimum evacuation time. Determining maximum flow is distinct from determining minimum evacuation time.

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Minimizing Evacuation Time with the Steady-State Model

Initial Considerations

The goal is to keep evacuation time to a minimum, but the evacuation route must be as safe as possible under the circumstances. How long on average it takes a driver to get to safety (Columbia) is related to minimizing total evacuation time but is not equivalent.

A General Performance Measure

A metric M that takes into account both maximizing traffic flow and minimizing individual transit time T is

$$M = W\,\frac{N}{lq} + (1-W)\,\frac{D}{v},$$

where $0 \leq W \leq 1$ is a weight factor, D is the distance that to traverse, l is the number of lanes, and N is the number of cars to evacuate. This metric assumes that the interaction between lanes of traffic (passing) is negligible, so that total flow is that of an individual lane times the number of lanes. Given W, minimizing M amounts to solving a one-variable optimization problem in either v or k. Setting $W = 1$ corresponds to maximizing flow, as in the preceding section. Setting $W = 0$ corresponds to maximizing speed, subject to the constraint $v \leq v_{\text{cruise}}$, the preferred cruising speed; this problem has solution $M = D/v_{\text{cruise}}$. The model does not apply when cars can travel at v_{cruise}.

Setting $W = 1/2$ corresponds to minimizing the total evacuation time

$$\frac{N}{lq} + \frac{D}{v}.$$

The evacuation time is the time D/v for the first car to travel distance D plus the time N/lq for the N cars to flow by the endpoint.

To illustrate that maximizing traffic flow and maximizing speed are out of sync, we calculate the highest value of W for which minimizing M would result in an equilibrium speed of v_{cruise}. This requires a formula for the equilibrium value v^* that solves the problem

$$\text{minimize } M(v) = W\,\frac{N(L+\beta v+\gamma v^2)}{lv} + (1-W)\,\frac{D}{v}$$

$$\text{subject to } 0 < v \leq v_{\text{cruise}}.$$

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The formula for $M(v)$ comes from **(1)**, $q = kv$, and $k = 1/s$. Differentiating with respect to v, setting the result equal to zero, and solving for speed yields

$$v^* = \min\left\{ v_{\text{cruise}}, \sqrt{\frac{1}{\gamma}\left[L + \frac{(1-W)}{W}\cdot\frac{Dl}{N}\right]}\right\}.$$

For v_{cruise} to equal the square root, we need

$$W = \left(1 + \frac{N}{Dl}\left(v_{\text{cruise}}^2\gamma - L\right)\right)^{-1}.$$

Using $N = 160{,}000$ cars, $D = 633{,}600$ ft (120 mi), $l = 2$ lanes, $v_{\text{cruise}} = 60$ mph $=$ 88 ft/s, $\gamma = .0115$ s^2/ft, and $L = 10$ ft, we obtain $W \approx 1/11$. Thus, minimizing evacuation time in situations involving heavy traffic flow is incompatible with allowing drivers to travel at cruise speed with a safe stopping distance.

Computing Minimum Evacuation Time

From the fact that $T = 2M$ when $W = 1/2$, we obtain

$$q^* = k^*v^*, \qquad v^* = \sqrt{\frac{1}{\gamma}\left[L + Dl/N\right]},$$

$$k^* = \frac{\beta\gamma^{1/2}[L + Dl/N]^{-1/2} - 2\gamma\frac{[L+\frac{1}{2}Dl/N]}{[L+Dl/N]}}{[\beta^2 - 4\gamma L] - \gamma\frac{[Dl/N]^2}{[L+Dl/N]}}.$$

The minimum evacuation time is

$$T^* = \frac{N}{lq^*} + \frac{D}{v^*}.$$

Predictions of the Steady-State Model

For N large, evacuation time minimization is essentially equivalent to the flow maximization (**Figure 1**), and it can be shown analytically that

$$\lim_{N\to\infty}\frac{T_{\text{flow}}(N)}{T_{\min}(N)} = 1.$$

The predicted evacuation time of 40 h for $N = 160{,}000$ seems reasonable. We can evaluate the impact of converting I-26 to four lanes by setting $l = 4$ in the equation for minimum evacuation time, yielding $T \approx 23$ h. For the steady-state model, this prediction makes sense, since the model does not deal with the effect of the bottleneck that will occur when Columbia is swamped by evacuees. The bottleneck would be compounded by using four lanes instead of two. On balance, however, doubling the number of lanes would lead to a net decrease in evacuation time.

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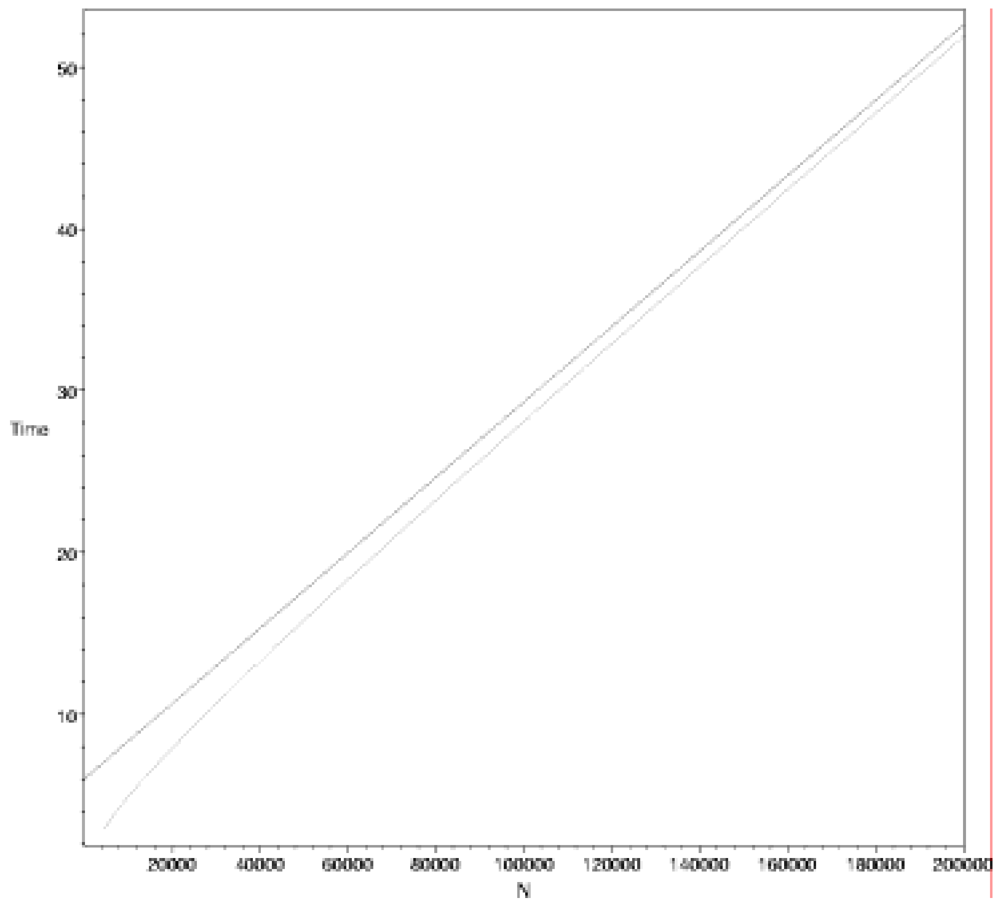

Figure 1. Comparison of minimum evacuation time (lower line) and maximum flow evacuation time (upper line).

One-Dimensional Cellular Automata Model

Development

In heavy traffic, cars make repeated stops and starts, with somewhat arbitrary timing; a good model of heavy traffic should take this randomness into account but also be simple enough to give an explicit formula for speed.

We divide a single-lane road into cells of equal length. A cell contains one car or no car. A car is *blocked* if the cell directly in front of it is occupied. At each time state, cars move according to the following rules:

- A blocked car does not move.
- If a car is not blocked, it advances to the next cell with probability p.

The decisions of drivers to move forward are made independently.

A traffic configuration can be represented by a function $f : \mathbb{Z} \to \{0, 1\}$, where $f(k) = 1$ if cell k contains a car and $f(k) = 0$ if not. Probability distributions on the set of all such functions are called *binary processes*.

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Given a process X, define a process $I_p(X)$ according to the following rule:

$$\text{If} \qquad \big(X(i), X(i+1)\big) = (1,0)$$

$$\text{then} \qquad \big(I_p(X)(i), I_p(X)(i+1)\big) = \begin{cases} (0,1), & \text{with probability } p; \\ (1,0), & \text{with probability } 1-p. \end{cases}$$

This rule is identical to the traffic flow rule given above: If X represents the traffic configuration at time t, $I_p(X)$ gives the traffic configuration at time $t+1$.

We are interested in what the traffic configuration looks like after several iterations of I. Let $I_p^n(X)$ mean I_p applied n times to X. The formula for traffic speed in terms of density comes from the following theorem[1]:

Theorem. *Suppose that X is a binary process of density d. Let*

$$r = \frac{1 - \sqrt{1 - 4d(1-d)p}}{2pd}$$

and let $\mathcal{M}_{p,d}$ denote the Markov chain with transition probabilities

$$0 \longrightarrow \begin{cases} 0, & \text{w/ prob. } 1-r; \\ 1, & \text{w/ prob. } 1-r; \end{cases} \qquad 1 \longrightarrow \begin{cases} 0, & \text{w/ prob. } r; \\ 1, & \text{w/ prob. } r. \end{cases}$$

The sequence of processes $X, I_p(X), I_p^2(X), I_p^3(X), \ldots$ converges to $\mathcal{M}_{p,d}$.

Here "density" means the frequency with which 1s appear, analogous to the average number of cars per cell. This theorem tells what the traffic configuration looks like after a long period of time.

Knowing the transition probabilities allows us to compute easily the average speed of the cars in $\mathcal{M}_{p,d}$: the average speed is the likelihood that a randomly chosen car is not blocked and advances to the next cell at the next time state:

$$\begin{aligned} v &= Pr\left[I(\mathcal{M}_{p,d}(i)) = 0 \,|\mathcal{M}_{p,d}(i) = 1\right] \\ &= Pr\left[\mathcal{M}_{p,d}(i+1) = 0 \,|\mathcal{M}_{p,d}(i) = 1\right] \\ &\quad \cdot Pr\left[I(\mathcal{M}_{p,d}(i)) = 0 \,|\mathcal{M}_{p,d}(i) = 1 \text{ and } \mathcal{M}_{p,d}(i+1) = 0\right] \\ &= rp = \left(\frac{1 - \sqrt{1 - 4d(1-d)p}}{2pd}\right) p = \frac{1 - \sqrt{1 - 4d(1-d)p}}{2d}. \end{aligned} \qquad \textbf{(2)}$$

The model does not accurately simulate high-speed traffic and does not take into account following distance, and the stop-and-start model of car movement is not accurate when traffic is sparse. The model is best for slow traffic (under 15 mph) with frequent stops.

[1] This theorem is the result of previous research [Miller 2001] by an author of this paper. The **Appendix** on binary processes (written during the contest period) gives a relatively complete proof of the result. [EDITOR'S NOTE: We unfortunately must omit the **Appendix**.]

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Low Speeds

We must set three parameters:

Δx = the size of one cell, the space taken up by a car in a tight traffic jam; we set it to 15 ft, slightly longer than most cars.

Δt = the length of one time interval, the shortest time by a driver to move into the space in front; we take this to be 0.5 s.

p = the movement probability, representing the proportion of drivers who move at close to the overall traffic speed; we let $p = 0.85$.

In **(2)** we insert the factor $(\Delta x/\Delta t)$ to convert from cells/time-state to ft/s:

$$v = \left(\frac{\Delta x}{\Delta t}\right) \frac{1 - \sqrt{1 - 4d(1-d)p}}{2d}.$$

Density d is in cars per cell, related to *occupancy* n by $d = (15\text{ ft}/10\text{ ft}) \times n = 3n/2$. **Table 1** gives v for various values of n and k.

Table 1.
v for various values of n and k.

n	K ft^{-1}	d	v (mph)
0.60	0.060	0.90	2
0.55	0.055	0.83	4
0.50	0.050	0.75	5
0.45	0.045	0.68	8
0.40	0.040	0.60	10
0.35	0.035	0.53	12
0.30	0.030	0.45	14
0.25	0.025	0.38	15
0.20	0.020	0.30	16

One Lane

To explore the one-dimensional cellular automata model, we wrote a simple simulation in C++. The simulation consists of a 5,000-element long (circular) array of bits, with a 1 representing a car and a 0 representing a (car-sized) empty space. The array is initialized randomly based on a value for the occupancy n: An element is initialized to 1 with probability n, or to 0 with probability $1 - n$. The array is iterated over 5,000 time cycles: On each cycle, a car moves forward with probability p if the square in front of it is empty. The flow q is calculated as the number of cars N passing the end of the array divided by the number of time cycles (i.e., $q = N/5000$), and thus the average speed of an individual car in cells per time cycle is $v = q/n = N/5000n$.

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Table 2.

Comparison of simulation and equation values for speed.

	Speed			
	$p = 1/2$		$p = 3/4$	
n	Simulation	Equation	Simulation	Equation
0.2	0.433	0.438	0.694	0.697
0.4	0.356	0.349	0.592	0.589
0.6	0.234	0.232	0.392	0.392
0.8	0.108	0.110	0.171	0.174

Table 2 shows results for various values of the occupancy n and probability p, verifying the accuracy of the one-dimensional cellular automata equation **(2)**.

The value for p should be related to the mean and standard deviation of v_{cruise}. The mean and standard deviation of a binary random variable are p and $\sqrt{p(1-p)}$. We have

$$\frac{p}{\sqrt{p(1-p)}} = \frac{\mu}{\sigma} \qquad \longrightarrow \qquad p = \frac{1}{1 + \left(\frac{\sigma}{\mu}\right)^2}.$$

For $\mu(v_{\text{cruise}}) = 60$ mph and $\sigma(v_{\text{cruise}}) = 5$ mph, we get $p = 144/145$. Now, $L = \mu t p$; so assuming $L = 10$ ft, $p = 144/145$, and $\mu = 60$ mph $= 88$ ft/s, we obtain a time step of 0.113 s.

We now use the model to predict how fast (on average) a car moves in a single lane, as a function of the occupancy. We consider the "relative speed" v_{rel}, the average speed divided by the (mean) cruise speed. The average speed is given by the one-dimensional cellular automata equation, and the cruise speed is p cells per time cycle, so this gives us

$$v_{\text{rel}} = \frac{v_{\text{avg}}}{v_{\text{cruise}}} = \frac{1 - \sqrt{1 - 4n(1-n)p}}{2pn}.$$

Using $p = 144/145$, we calculate v_{rel} and v_{avg} as a function of n (**Table 3**).

The model predicts that for low occupancy the average speed will be near the cruise speed, but for occupancies greater than 0.5 the average speed will be significantly lower. The cellular automata model does not take following distance into account; thus, it tends to overestimate v_{avg} for high speeds and is most accurate when occupancy is high and speed is low.

Table 3 also shows flow rate $q = nv_{\text{avg}}/L$, in cars/s, as a function of occupancy. The flow rate is symmetric about $n = 0.5$. Each car movement can be thought of as switching a car with an empty space, so the movement of cars to the right is equivalent to the movement of holes to the left.

The model fails, however, to give a reasonable value for the maximum flow rate: 4.1 cars/s $\approx$ 14,600 cars/h, about seven times a reasonable maximum rate [Rothery 1992]. The reason is that the cell size equals the car length, a correct

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Table 3.

Relative speed, average speed, and flow rate as a function of occupancy.

n	v_{rel} (ft/s)	v_{avg} (ft/s)	flow rate (cars/s)
0.1	.999	88	0.9
0.2	.998	88	1.8
0.3	.995	88	2.6
0.4	.987	87	3.5
0.5	.923	81	4.1
0.6	.658	58	3.5
0.7	.426	38	2.6
0.8	.249	22	1.8
0.9	.111	10	0.9

approximation only as car speeds approach zero and occupancy approaches 1. For $n \geq 0.5$, we should have a larger cell size; so we assume that cell size equals car length plus following distance and that following distance is proportional to speed. Assuming a 1 s following distance, we obtain cell size as

$$C = L + v_{\text{avg}} \times (1 \text{ sec}).$$

But we do not know the value of v_{avg} until we use the cell size to obtain it! For n large, we can assume that $v_{\text{avg}} \approx v_{\text{cruise}}$ and find an upper bound on cell size:

$$C = L + v_{\text{cruise}} \times (1 \text{ sec}) = 98 \text{ ft}.$$

We divide the original flow rate by the increased cell size to obtain a more reasonable flow rate:

$$q = \frac{4.063 \text{ cars/s}}{98 \text{ ft}/10 \text{ ft}} = 0.415 \text{ cars/s} \approx 1{,}500 \text{ cars/h}.$$

This is likely to be an underestimate; for greater accuracy, we must find a method to compute the correct cell size before finding the speed. We address this problem later.

Two Lanes

We expand the one-dimensional model. The simulation consists of a two-dimensional (1000×2) array of bits. The array is initialized randomly and then iterated over 1,000 time cycles: On each cycle, a car moves forward with probability p if the cell in front of it is empty. If not, provided the cells beside it and diagonally forward from it are unoccupied, with probability p the car changes lanes and moves one cell forward.

Like the one-lane simulation, the two-lane one is correct only for high densities and low speeds, since it uses cell size equal to car length. Hence, we do not use the two-lane simulation to compute the maximum flow rate. However,

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since cell size affects flow rate by a constant factor, we can compare flow rates by varying parameters of the simulation. In particular, we use this model to examine how the flow rate changes with the variance of speeds.

"But I Want to Bring My Boat!"

There are two main types of variance in speeds:

- σ_t^2 of traveling speed (random fluctuations in the speed of a single vehicle over time), and
- σ_m^2 of mean speed (variation in the mean speeds of all vehicles).

In the one-lane simulation, we assumed that $\sigma_m = 0$ and $\sigma_t = 5$ mph; this choice was reflected in the calculation of p, since $p = 1/[1 + (\sigma_t/\mu)^2]$ for every car. When we take σ_m into account, each car gets a different value of p:

- Choose the car's mean speed μ randomly from the normal distribution with mean v_{cruise} and standard deviation σ_m.
- The car's traveling speed will be normally distributed with mean μ and standard deviation σ_t.
- The car's transition probability p is

$$p = \frac{\mu}{v_{\text{cruise}} + \lambda\sigma_m}\left(\frac{1}{1 + \left(\frac{\sigma_t}{\mu}\right)^2}\right),$$

  where λ is a constant best determined empirically. We use $\lambda = 0$, a conservative estimate of the change in flow rate as a function of σ_m.

We consider what effect σ_t and σ_m have on the speed at a given occupancy. Considering the cars' movement as a directed random walk, increasing σ_t increases randomness in the system, causing cars to interact (and hence block one other's movement) more often, decreasing average speed.

The effects of σ_m are even more dramatic: Cars with low mean speeds impede faster cars behind them.

We ran simulations in which we fixed $n = 0.5$ and varied both σ_m and σ_t from 0 to 15 mph. For each pair of values, we calculated average flow rate for the one- and two-lane simulations.

Table 4 shows the effects of lane-changing by comparing maximum flows for the two-lane model with twice that for the one-lane model. For small σ_t and σ_m, allowing cars to switch lanes does not increase the flow rate much; for high values, the two-lane model has a much higher flow rate. Each 5 mph increase in σ_m results in an 11–16% decrease in flow rate (two-lane model, $\sigma_t = 0$), while each 5 mph increase in σ_t results in a 5–7% decrease in flow rate (two-lane model, $\sigma_m = 0$). Both variances dramatically affect flow rate, and σ_m is more significant than σ_t.

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Table 4.
Two-lane average flow / Twice the one-lane average flow.

σ_m	σ_t 0	5	10	15
0	979/976	923/908	856/830	805/776
5	822/757	817/746	790/729	753/683
10	699/537	691/518	659/485	642/469
15	588/393	569/366	540/319	518/292

"So, Can I Bring My Boat?"

We consider how variations in vehicle type affect σ_m, σ_t, and flow rate. Most large vehicles (boats, campers, semis, and motor homes) travel more slowly than most cars. A significant proportion of large vehicles results in increased σ_m and hence a lower flow rate.

As a simplified approximation, we assume that there are two types of vehicles: fast cars ($\mu = \mu_1$) and slow trucks ($\mu = \mu_2$), with proportion α of slow trucks α. We calculate

$$\begin{aligned}
\sigma_m^2 &= \alpha(\mu_2 - \bar{\mu})^2 + (1-\alpha)(\mu_1 - \bar{\mu})^2 \\
&= \alpha\big[\mu_2 - \big(\mu_1 - (\mu_1 - \mu_2)\alpha\big)\big]^2 + (1-\alpha)\big(\mu_1 - (\mu_1 - \mu_2)\alpha\big)^2 \\
&= (\mu_1 - \mu_2)^2\big(\alpha^2(1-\alpha) + \alpha(1-\alpha)^2\big) = (\mu_1 - \mu_2)^2\alpha(1-\alpha).
\end{aligned}$$

Thus, $\sigma_m = (\mu_1 - \mu_2)\sqrt{\alpha(1-\alpha)}$. We now assume that fast cars travel at $\mu_1 = 70$ mph and slow trucks at $\mu_2 = 50$ mph and find σ_m as a function of α. Random fluctuations in vehicle speed are likely to depend more on driver psychology than on vehicle type, so we assume $\sigma_t = 5$ mph. We interpolate linearly in **Table 4** to find the flow rate (cars per 1,000 time cycles) as a function of the proportion of slow vehicles α (**Table 5**).

Table 5.
Flow rate as a function of proportion of slow vehicles.

α	σ_t	flow rate	% reduction in flow
0	0	923/908	0/0
.01	1.99	881/844	4.6/7.0
.02	2.80	864/817	6.4/10.0
.05	4.36	831/767	10.0/15.5
.1	6.00	792/700	14.1/22.9
.2	8.00	741/609	19.7/32.9
.5	10.0	691/518	25.1/43.0

The flow rate is decreased significantly by slow vehicles: If 1% of vehicles are slow, the flow rate decreases by 5%; if 10% of vehicles are slow, the flow rate decreases by 15%. The effects of σ_m are magnified if vehicles are unable to pass slower vehicles; so if the highway went down to one lane at any point (due to

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construction or accidents, for example), the flow rate would be reduced even further. Hence, we recommend no large vehicles (vehicles that may potentially block multiple lanes) and no slow vehicles (vehicles with a significantly lower mean cruising speed). Exceptions could be made if a family has no other vehicle and for vehicles with a large number of people (e.g., buses). Slow-moving vehicles should be required to stay in the right lane and families should be encouraged to take as few vehicles as possible.

The Space-Speed Curve

To determine optimal traffic flow rates, we can combine the one-dimensional cellular automata and the steady-state models to get a good estimate of the relationship between speed v and the separation distance s.

$s \leq 15$: There is essentially no traffic flow: $v(s) = 0$.

$15 \leq s \leq 30$: Traffic travels at between 0 and 12 mph and the one-dimensional cellular automata model applies; v is approximately a linear function of s.

$30 \leq s \leq 140$: Traffic travels between 12 and 55 mph and the steady-state model is appropriate; v is again approximately a linear function of s, with less steep slope.

$140 \leq s$: Traffic travels at the speed limit of 60 mph.

Incoming Traffic Rates

The optimal flow is determined by the optimal flow through the smallest bottleneck. However, the time of travel (which is a more important measure for our purposes) is affected by other factors, including the rate of incoming traffic. If incoming traffic is heavy, congestion occurs at the beginning of the route, decreasing speed and increasing travel time for each car.

How does congestion occur and how much does it influence travel time? Consider the one-dimensional cellular automata model with $p = 1/2$. Represent the road by the real line and let $F(x,t)$ denote the density of cars at point x on the road at time t. (For our purposes now, the cells and cars are infinitesimal in length.) Suppose that the initial configuration $F(x,0)$ is given by

$$F(x,0) = \begin{cases} 1, & \text{if } x < 0; \\ 0, & \text{if } x \geq 0. \end{cases}$$

This represents a dense line of cars about to move onto an uncongested road.

We omit units for the time being. By formulas derived earlier, the speed $v(x_0, t_0)$ at position x_0 and time t_0 is given by

$$v(x_0, t_0) = \frac{1 - \sqrt{1 - 2F(x_0,t_0)[1 - F(x_0,t_0)]}}{2F(x_0,t_0)},$$

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while speed must also equal the rate at which the number of cars past point x is increasing; that is,

$$v(x_0, t_0) = \frac{d}{dt}\left(\int_x^\infty F(x,t)dx\right)(t_0).$$

Thus,

$$\frac{dF}{dt} = -\frac{dv}{dx} = -\frac{d}{dx}\left(\frac{1-\sqrt{1-2F(1-F)}}{2F}\right).$$

This is a partial differential equation whose unique solution is

$$F(x,t) = \begin{cases} 1, & \text{if } x/t < -\frac{1}{2}; \\ \frac{1}{2} - \frac{(x/t)}{\sqrt{2-4(x/t)^2}}, & \text{if } -\frac{1}{2} \le x/t \le \frac{1}{2}; \\ 0, & \text{if } \frac{1}{2} < x/t. \end{cases}$$

Thus, after a steady influx of cars for a period of Δt, the resulting congestion is

$$\frac{1}{2} - \frac{\frac{x}{\Delta t}}{\sqrt{2-4\left(\frac{x}{\Delta t}\right)^2}}$$

and the congestion ends at $x = \Delta t/2$.

Thus, the extent of the congested traffic is *linear* in Δt. So if there is a steady influx of cars onto a highway, the extent of the resulting congestion is directly proportional to how long it takes them to enter, thus to the number N of them. Likewise, the time for the congestion to dissipate is proportional to N.

This allows us to evaluate staggering evacuation times for different counties. Suppose that n counties have populations $P_1, \ldots, P_n$. If all evacuate at the same time, the effect of the resulting traffic jam on total travel time is proportional to the product of the extent of the jam and the time before it dissipates:

$$\begin{aligned} \Delta T_{\text{travel time}} &= c_1 \cdot c_2 (P_1 + \cdots + P_n) \cdot c_3 (P_1 + \cdots + P_n) \\ &= c_1 c_2 c_3 (P_1 + \cdots + P_n)^2 \end{aligned}$$

for some constants c_1, c_2, c_3. If the evacuations are staggered, the effect is

$$\Delta T_{\text{travel time}} = c_1 c_2 c_3 P_1^2 + \cdots + c_1 c_2 c_3 P_n^2 = c_1 c_2 c_3 \left(P_1^2 + \cdots + P_n^2\right).$$

Now, $P_1^2 + \cdots + P_n^2 < (P_1 + \cdots + P_n)^2$; so unless one of the counties has a much larger population than the rest, the difference between these two values is relatively large. We therefore recommend staggering counties.

The Effects of Merges and Diverges

While the steady-state model is a reasonably accurate predictor of traffic behavior on long homogeneous stretches of highway, we must also consider how to

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deal with the effects of road inhomogeneities: merges of two lanes into a single lane and "diverges" of one lane into two lanes. To do so, we apply the principle of conservation of traffic [Kuhne and Michalopolous 1992]. Assuming that there are no sources or sinks in a region, we have

$$\frac{\partial q}{\partial x} + \frac{\partial k}{\partial t} = 0,$$

where q is flow rate (cars/s), k is density (cars/ft), x is location (ft), and t is time (s). Let the merge or diverge occurs at a specific point x. In the steady state (i.e., for $\partial D/\partial t = 0$), we have $\partial q/\partial x = 0$, so flow conserved at a junction.

For a flow q_s merging or diverging flows q_1 and q_2, we have $q_s = q_1 + q_2$. If proportion P ($0 < P < 1$) of the flow q_s going to (or coming from) q_1, and we know either density (k_s or k_1), we can solve for the other density:

$$q_1 = Pq_s, \qquad k_1 v_1 = Pk_s v_s, \qquad k_1 v(k_1) = Pk_s v(k_s).$$

From the steady-state model, for the given values of the constants, we know

$$v(k) = \begin{cases} 88 \text{ ft/s}, & 0 < k < 0.0056; \\ 21.7\left(\sqrt{.08 + \frac{.092}{k}} - 1\right), & 0.0056 < k < 0.1. \end{cases}$$

Assuming that both densities are greater than the free-travel density $k = .0056$, we can set

$$k_1\left(\sqrt{.08 + \frac{.092}{k_1}} - 1\right) = Pk_s\left(\sqrt{.08 + \frac{.092}{k_s}} - 1\right).$$

Given either k_s or k_1, we can solve numerically for the other. Then we can find the speeds associated with each density using the above expression for $v(k)$.

Solving the equation gives two values; we assume that the density is greater on the single-lane side of the junction (i.e., density increases at a merge and decreases at a diverge). Also, if solving produces a speed v_1 that is larger than v_{cruise}, we set $v_1 = v_{\text{cruise}}$ and calculate $n_1 = q_1/v_1$.

How is the steady-state flow rate determined on a path with merges and diverges? Following Daganzo [1997], we consider a bottleneck to be a location (such as a merge or diverge) where queues can form and persist with free flow downstream. The *bottleneck capacity* is the maximum flow rate through the bottleneck, which (with Daganzo) we assume to be constant. If a steady-state flow greater than the bottleneck capacity attempts to enter the bottleneck, the queue size will increase until it stretches all the way back to its origin. At that point, the steady-state flow is blocked by the queue of cars and decreases to the bottleneck capacity. Hence, the maximum steady-state flow rate along a path is the minimum capacity of all bottlenecks along the path.

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Parallel Paths

From A to B, let there be multiple parallel paths $p_1, \ldots, p_m$ with bottleneck capacities $c_1, \ldots, c_m$. The maximum steady-state flow rate from A to B is the minimum of:

- the bottleneck capacity of the diverge at point A,
- the bottleneck capacity of the merge at point B, and
- the sum of the bottleneck capacities of all paths p_i.

If a path has no bottlenecks, its capacity is the maximum flow rate predicted by the steady-state model.

We focus on maximizing flow rate rather than minimizing evacuation time, since for a large number of cars, maximizing flow gives near-minimal evacuation time. The maximum flow rate $q_{\max}$ from Charleston to Columbia is

$$q_{\max} = \min\left(\sum_i q_i, c_0, c_f\right),$$

where q_i is the maximum flow rate of path i, c_0 is the bottleneck capacity of Charleston, and c_f is the bottleneck capacity of Columbia. The flow rates q_i are

$$q_i = \min(b_1 \ldots b_n, q_{i,ss}),$$

where $b_1, \ldots, b_n$ are the capacities of bottlenecks along the given route and $q_{i,ss}$ is the maximum flow along that route as predicted by the steady-state model.

We first consider evacuation with no bottlenecks along I-26. Denoting the steady-state value $q_{\text{I-26},ss}$ by q_I, this gives us $q_{\max} = \min(q_I, c_0, c_f)$. Which factor limits $q_{\max}$? We cannot achieve $q_I \approx 2{,}000$ cars/h if if either $c_0 < 2{,}000$ (traffic jam in Charleston) or $c_f < 2{,}000$ (traffic jam in Columbia). With the traffic in Columbia splitting into three different roads, there should be less congestion there than with everyone merging onto I-26 in Charleston. Hence, we assume that $c_0 < c_f$, so the limiting factor is c_0 if $c_0 < 2{,}000$ and q_I if $c_0 > 2{,}000$. The value of c_0 is best determined empirically, perhaps by extrapolation from Charleston rush-hour traffic data or from data from the 1999 evacuation.

Effects of Proposed Strategies

Reversing I-26

Reversing I-26 doubles q_I to 4,000 cars/h. It is likely to increase c_0 as well, since cars can be directed to two different paths onto I-26 and are thus less likely to interfere with the merging of cars going on the other set of lanes. On the other hand, twice as many cars will enter the Columbia area simultaneously, and the

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unchanged capacity c_f may become the limiting factor. It may be possible to increase c_f by rerouting some of the extra traffic to avoid Columbia, or even turning around traffic on some highways leading out of Columbia.

Thus, this strategy is likely to improve evacuation traffic flow.

Reversing Other Highways

A similar argument applies to turning around the traffic on the smaller highways. Each highway adds some capacity to the total $\sum_i q_i$, increasing this term, but each highway's capacity is significantly less than q_I, and increasing the number of usable highways has unclear effects on c_0. It may increase capacity by spreading out Charleston residents to different roads, or crossing evacuation routes may lead to traffic jams. As with the reversal of I-26, reversal of smaller highways does not affect c_f (unless crossing evacuation routes becomes a problem in Columbia). More important, the interactions between highways (merges and diverges) may lead to bottlenecks, reducing capacity. In fact, interactions between these highways and I-26 could cause bottlenecks that slow the flow on I-26, perhaps offsetting the extra capacity of the smaller highways. Thus, it is safer not to turn around traffic on the secondary highways or to encourage using these as evacuation routes.

Temporary Shelters

Establishing temporary shelters in Columbia, to reduce the traffic leaving that city, could be useful if only some of the cars are directed into Columbia; thus the flow of traffic in the Columbia area would be split into four streams rather than three, possibly increasing c_f. Nevertheless, we hesitate to recommend this strategy, since the actual effects are likely to be the opposite. Evacuees entering Columbia are likely to create congestion there, making it difficult for traffic to enter, resulting in a major bottleneck. Without careful regulation, more people will try to stay in Columbia than the available housing, and frantic attempts of individuals driving around looking for housing will exacerbate the bottleneck. Hence, it is most likely that c_f will decrease significantly, probably becoming the limiting factor on maximum flow rate.

Staggering Traffic Flows

Staggering is likely to reduce the time for an average car to travel from Charleston to Columbia while leaving the value of the steady-state flow rate q_I unchanged. Hence, staggering decreases total evacuation time; it may also increase maximum flow rate, since it decreases the number of cars traveling toward I-26 at any one time, reducing the size of the c_o bottleneck. Increasing the capacity c_o, however, increases the flow rate only when c_o is the limiting factor.

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Evacuees from Florida and Georgia

Since evacuation time is proportional to number of cars over flow rate, out-of-state evacuees add to total evacuation time unless they take a route that does not intersect the paths of the South Carolina evacuees. However, it is very hard to constrain the routes of out-of-state evacuees, since they come from a variety of paths and are unlikely to be informed of the state's evacuation procedures. In particular, major bottlenecks are likely at the intersections of I-26 with I-95 and of I-95 with I-20 and U.S. 501. If many cars from I-95 attempt to go northwest on I-26 toward Columbia, q_I will no longer equal 2,000 cars/h but instead the capacity of the I-26/I-95 bottleneck. This is likely to reduce q_I significantly and likely make q_{I-26} the limiting factor.

A similar argument suggests that I-95 traffic will impede the flow of traffic west from Myrtle Beach by causing a bottleneck at the I-95/I-20 junction. Traffic flow from Myrtle Beach is less than from Charleston, and many of the cars from I-95 may have already exited at I-26; so the bottleneck at the I-20 junction is likely to be less severe. Nevertheless, the flow of evacuees from Florida and Georgia has the potential to reduce dramatically the success of the evacuation.

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Modeling Hurricane Evacuation Strategies

DURHAM, NC, FEB. 12— Hurricanes pose a serious threat to citizens on the South Carolina coastline, as well as other beach dwellers in Florida, Georgia, and other neighboring states. In 1999, the evacuation effort preceding the expected landfall of Hurricane Floyd led to a monumental traffic jam that posed other, also serious, problems to the more than 500,000 commuters who fled the coastline and headed for the safe haven of Columbia. Several strategies have been proposed to avoid a future repeat of this traffic disaster.

First, it has been suggested that the two coastal bound lanes of I-26 be turned into two lanes of Columbia bound traffic. A second strategy would involve staggering the evacuation of the coastal counties over some time period consistent with how hurricanes affect the coast, instead of all at once. Third, the state might turn around traffic flow on several of the smaller highways besides I-26 that extend inland from the coast. The fourth strategy under consideration is a plan to establish more temporary shelters in Columbia. Finally, the state is considering placing restrictions on the type and number of vehicles that can be brought to the coast.

In the interest of the public, we have developed and tested several mathematical models of traffic flow to determine the efficacy of each proposal. On balance, they suggest that the first strategy is sound and should be implemented. Although doubling the number of lanes will not necessarily cut the evacuation time in half, or even double the flow rate on I-26 away from the coast, it will significantly improve the evacuation time under almost any weather conditions.

Our models suggest that staggering the evacuation of different counties is also a good idea. Taking such action on the one hand will reduce the severity of the bottleneck that occurs when the masse of evacuees reaches Columbia, and on the other hand could potentially increase average traffic speed without significantly increasing traffic density. The net effect of implementing this strategy will likely be an overall decrease in coastal evacuation time.

The next strategy, which suggests turning traffic around on several smaller highways, is not so easy to recommend. The main reason for this is that the unorganized evacuation attempts of many people on frequently intersecting secondary roads is a recipe for inefficiency. In places where these roads intersect I-26, the merging of a heightened volume of secondary road traffic is sure to cause bottlenecks on the interstate that could significantly impede flow. To make a strategy of turning around traffic on secondary roads workable, the state would have to use only roads that have a high capacity, at least two lanes, and a low potential for traffic conflicts with other highways. This would re-

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quire competent traffic management directed at avoiding bottlenecks and moving Charleston traffic to Columbia with as few evacuation route conflicts as possible.

The fourth proposal, of establishing more temporary shelters in Columbia, is a poor idea. Because it is assumed that travelers are relatively safe once they reach Columbia, the main objective of the evacuation effort should be minimizing the transit time to Columbia and the surrounding area. It is fairly clear that increasing the number of temporary shelters in Columbia would lead to an increased volume of traffic to the city (by raising expectations that there will be free beds there) and exacerbate the traffic problem in the city itself (due to an increased demand for parking). Together, these two factors are sure to worsen the bottleneck caused by I-26 traffic entering Columbia and would probably increase the total evacuation time by decreasing the traffic flow on the interstate.

The final proposal of placing limitations on the number and types of vehicles that can be brought to the beach is reasonable. Families with several cars should be discouraged from bringing all of their vehicles and perhaps required to register with the state if the latter is their intention. Large, cumbersome vehicles such as motor homes should be discouraged unless they are a family's only option. Although buses slow down traffic, they are beneficial because they appreciably decrease the overall number of drivers. In all cases, slow-moving vehicles should be required to travel in the right lane during the evacuation.

In addition to the strategies mentioned above, commuters in the 1999 evacuation were acutely aware of the effect on traffic flow produced by coastal residents of Georgia and Florida traveling up I-95. We have concluded that, when high-volume traffic flows such as these compete for the same traffic pipeline, the nearly inevitable result is a bottleneck. A reasonable solution to this problem would be to bar I-95 traffic from merging onto I-26 and instead encourage and assist drivers on I-95 to use the more prominent, inland bound secondary roads connected to that interstate.

To conclude, we think that combining the more successful strategies suggested could lead to a substantial reduction in evacuation time, the primary measure of evacuation success. Minimizing the number of accidents that occur en route is also important, but our models directed at the former goal do not make compromises with the latter objective. In fact, the problem of minimizing accidents is chiefly taken care of by ensuring that traffic flow is as orderly and efficient as possible.

— Samuel W. Malone, Carl A. Miller, and Daniel B. Neill in Durham, NC