Evaluation System for College Coaching Legends

Summary

In order to evaluate the performance of a coach, we describe metrics in five aspects: *historical record, game gold content, playoff performance, honors* and *contribution to the sports*. Moreover, each aspect is subdivided into several secondary metrics. Take playoff performance as example, we collect postseason result (Sweet Sixteen, Final Four, etc.) per year from NCAA official website, Wikimedia and so on.

First, Analytic Hierarchy Process (**AHP**) Model is established to determine the weight of each metric to coaches' evaluation grade. All metrics are adequately filled into the three-hierarchy structure, and then we obtain the metric weight based on which evaluation grade is calculated. Second, Fuzzy Synthetic Evaluation (**FSE**) is built to overcome weakness of excess subjective factors in AHP. This model takes data processing by membership function to generate fuzzy matrix. After that, **entropy method** and linear weighted method are applied to obtain evaluation grade.

To evaluate the accuracy of the two models, ***hit score*** is defined. It is supposed to reflect the difference between our results and standard rankings from several authorities such as ESPN and Sporting News. Take NCAA basketball as a case study, AHP receives 78.77 hit score while FSE gets 81.81, which indicates that FSE performs better than AHP. Afterwards, **Aggregation Model (AM)** can be developed by combining the two models based on hit score. The top 5 college basketball coaches, in turn, are *John Wooden, Mike Krzyzewski, Adolph Rupp, Dean Smith and Bob Knight.*

Time line horizon does make a difference. According to turning points in NCAA history, we divide the previous century into six periods with different **time weights** which lead to the change of ranking. We apply our model into college women's basketball only to find that **genders do not matter**. Model proves to be **efficient in other sports**. The ranking of college football is: *Bear Bryant, Knute Rockne, Tom Osborne, Joe Paterno , Bobby Bowden*, and the top 5 coaches in college hockey are *Bob Johnson, Red Berenson, Jack Parker, Jerry York, Ron Mason.*

We conduct sensitivity analysis on FSE to find best membership function and calculation rule. Sensitivity analysis on **aggregation weight** is also performed. It proves AM performs better than single model. As a creative use, **top 3 presidents** (U.S.) are picked out: *Abraham Lincoln, George Washington, Franklin D. Roosevelt*.

At last, the strength and weakness of our mode are discussed, non-technical explanation is presented and the future work is pointed as well.

I. Introduction

1.1 Problem Background

Sports Illustrated is an American sports media franchise owned by media conglomerate Time Warner [1]. This magazine is looking for the "best all time college coach" male or female for the previous century. The best college coach or coaches can be from among either male or female coaches in different fields, such as college hockey or field hockey, football, baseball or softball, basketball, or soccer.

We face mainly four problems:

- Articulate our own **metrics** and build a mathematical model;
- Set up the evaluation system for the **performance** of the model.
- Discuss how our model can be applied with **time factor** or across both **genders** and all possible **sports**;
- Analyze the influences of the parameters, then discuss whether your model could be applied into wide fields.

1.2 Previous Research

Some magazines or websites that focus mainly on college sports have ranked the top college coaches of different sports. For example, rivels.com has made a basket-ball power rankings [2] which shows the top 25 coaches of college basketball.

Considering the best college coaches is an evaluation problem. There are some models which can solve such problem. One is the Analytic hierarchy process (AHP), which was developed by Thomas L. Saaty [3] in the 1970s. The AHP provides a comprehensive and rational framework for structuring a decision problem, for representing and quantifying its elements, for relating those elements to overall goals, and for evaluating alternative solutions [4]. Another is the Fuzzy Synthetic Evaluation Model. Fuzzy mathematics forms a branch of mathematics related to fuzzy set theory and fuzzy logic [5]. It started in 1965 after the publication of Lotfi Asker Zadeh's seminal work Fuzzy sets [6].

1.3 Our Work

In this paper we determine the best college coaches from among either male or female coaches in different sports. In Section 2, we provide the terminology definitions and assumptions that will be utilized in the rest of the paper. In Section 3, we give the definitions of evaluation standard and specific evaluation norms which we used in our models, and show some of the data we have collected. In Section 4, we build two mathematical models to choose the best college coaches, and Section 5 considers combination of the two models mentioned above. In Section 6 we extend our models and take time, genders and types of sports into consideration. Section 7 provides further discuss of our models. In Section 8, we provide an overview of our approach and give a non-technical explanation of our models that sports fans will understand. Section 9 shows some work we can do in the future.

II. Symbols, Definitions and Assumptions

2.1 Symbols and Definitions

- Symbols for evaluation norms:

Symbol	Definition
a_i	wins for the i^{th} year
b_i	losses for the i^{th} year
$\mathbf{R}$	the average SRS
$\mathbf{O}$	the average SOS
n_k	the times for each class of playoff
k_i	the weight of each award
c_i	point for each aspect of contribution

- Symbols for Analytic Hierarchy Process:

Symbol	Definition
$\mathbf{A}$	the judging matrix
λ_{max}	the greatest eigenvalue of matrix A
$\mathbf{CI}$	the indicator of consistency check
$\mathbf{CR}$	the consistency ratio
$\mathbf{RI}$	the random consistency index
$\mathbf{CW}$	the weight vector for criteria level
$\mathbf{AW}$	the weight vector for alternatives level
Y_1	the evaluation grade for model I

- Symbols for Fuzzy Synthetic Evaluation:

Symbol	Definition
X_i	the grades for each aspect
$\mu_j(X_{ij})$	the membership function

$\boldsymbol{X_f}$	the fuzzy matrix
$\boldsymbol{p_{ij}}$	the characteristic weight
$\boldsymbol{e_j}$	the entropy for the j^{th} evaluation grade
$\boldsymbol{EW}$	the weight vector in entropy method
$\boldsymbol{Y_2}$	the evaluation grade for model II

- Symbols for Aggregation Model:

Symbol	Definition
$\mathbf{D}$	the average offset distance
$\boldsymbol{W_1}$	the weight for model I
$\mathbf{Y}$	the evaluation grade for aggregation model

2.2 General Assumptions

- The elements that we already have taken into consideration play a vital role in the evaluation.
- The ignored elements of coach do not influence the ranking.
- The data that we have collected is enough and accurate and the quantification is correct.
- There exists objective and accurate ranking for coaches, and the rankings from selected media could reflect the accurate ranking to some extent.

III. Articulate our metrics

3.1 Specify evaluation norms

As for the evaluation standard for players, there are mainly five aspects [9] that count: strength, speed, skill, defense and attack. Similarly a coach could be evaluated from following five aspects: historical record, game gold content, play-off performance, honors and contribution to the sports. What follows in the chapter will hammer at accounting for the five aspects.

- **Historical record**: The team's record undoubtedly accounts for the largest proportion in the coach evaluation. According to the mainstream statistic indexes for the team record, *wins* and *losses* are most notable. The team's historical record could directly reflects the coaching ability.

  The total wins a could be calculated as follows:

$$\mathrm{a} = \sum_i a_i \qquad (3.1)$$

-Where a_i denotes wins for the i^{th} year.

The total loses b could be calculated as follows:

$$\mathrm{b} = \sum_i b_i \qquad (3.2)$$

-Where b_i denotes losses for the i^{th} year.

- **Game gold content**: If all wins are produced during the fights with weak teams, apparently the wins could not illustrate the real coaching ability. At the same time, the average point difference also makes a difference. It reflects the coaching style that whether a coach is conservative or radical. To illustrate the upon two points, we choose the following two norms:
  - ✓ *Simple Rating System* (SRS) [8]: The simple rating system works by first finding how many points, on average, a team wins/loses by. For each game, the point differential is then weighted based on how much better or worse than average their opponent's point differential is.

    Let R denote the total SRS, then it could be calculated as follows:

$$\mathrm{R} = \frac{\sum_i SRS_i}{t} \qquad (3.3)$$

-Where SRS_i denotes the SRS value for the i^{th} year, t denotes the number of the years.

  - ✓ *Strength of Schedule* (SOS) [8]: In sports, strength of schedule (SOS) refers to the difficulty or ease of a team's/person's opponent as compared to other teams/persons. This is especially important if teams in a league do not play each other the same number of times.

    Let O denote the total SOS, then it could be calculated as follows:

$$\mathrm{O} = \frac{\sum_i SOS_i}{t} \qquad (3.4)$$

-Where SOS_i denotes the SOS value for the i^{th} year, t denotes the number of the years.

- **Playoff performance**: Generally, during the regular season, teams play more games in their division than outside it, but the league's best teams might not play against each other in the regular season [11]. Therefore, in the postseason any group-winning team is eligible to participate thus making the playoff performance extremely important in the coach evaluating [12]. For college basketball in U.S., the playoff performance could be divided as follows:
  - ✓ *First round*: The team is eliminated in the first round.
  - ✓ *Second round*: The team is eliminated in the second round.

- ✓ *Sweet sixteen*: The last sixteen teams remaining in the playoff tournament
- ✓ *Final four*: The last four teams remaining in the playoff tournament.
- ✓ *Runner-up*: The team loses in the finals.
- ✓ *Champion*: The team wins in the finals.

To quantify the aspect, we count the number of times for each class using the symbol n_k. Let a binary variable m_{ki} denote whether the team get the k^{th} (for first round $\mathrm{k} = 1$, champion $\mathrm{k} = 7$) class in the i^{th} year. Thus n_k could be calculated as follows:

$$n_k = \sum_i m_{ki} \qquad (3.5)$$

- **Honors**: There are various awards in this field which make up the honors of the coach, at the same time, the basketball hall of fame and college basketball hall of fame [7] are also honors. To quantify the honor, we count the *times of main award* such as the *Naismith College Coach of the Year, Basketball Times National Coach of the Year* [9] and so on. Different awards have different gold content. To determine the weight of each award ($\mathcal{k}_i$), we collect its time period, based on which weights are designated. Let $\mathcal{H}$ denote the total weights of all the awards a coach has got:

$$\mathcal{H} = \sum_i \mathcal{k}_i \qquad (3.6)$$

  Namely $\mathcal{H}$ reflects the how much honor a coach have ever obtained.

- **Contribution to sports**: This concept covers a wide range. In order to quantify the contribution, we divide the contribution into five parts:
  - ✓ *Star players*: Evaluate the number of the star players the coach have trained.
  - ✓ *Coaching age*: When the coaching career start and how long does it last.
  - ✓ *Tactical Innovation*: Have the coach invented tactical innovation?
  - ✓ *Performance in international competitions*: Have the coach ever fight in the international competitions? Then How many gold or silver medals?
  - ✓ *Popularity*: The number of the results when search its name in *Google*.

  We give c_i points for each aspect above: 0 for mediocre, 1 for good, 2 for excellent. Then add the points up to form the final grade $\mathcal{C}$ in this aspect (the full mark is 10):

$$\mathcal{C} = \sum_i c_i \qquad (3.7)$$

A figure is prepared to conclude the evaluation norms above. (See figure)

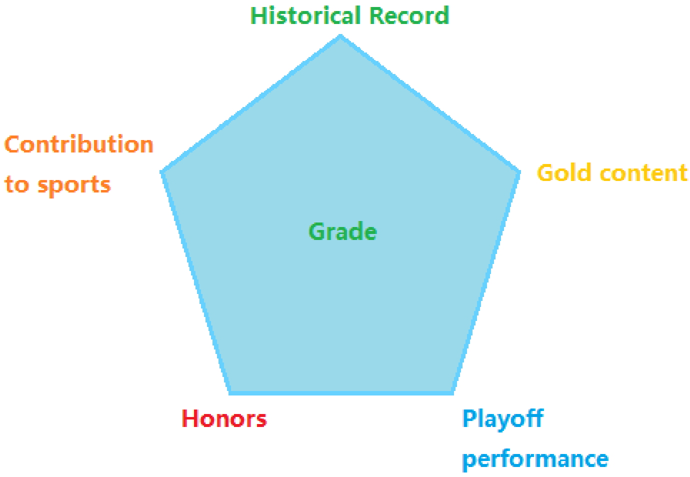


Figure 3.1 First level evaluation norms

Figure 3.2 Second level evaluation norms

3.2 Collect data

We use men's college basketball that will be utilized in the following models discussion as an example, and collect relative data from the Internet. We choose those 70 coaches who were in the list of the National Collegiate Basketball Hall of Fame [7], because those coaches had gained tremendous glory and are more competitive to be chosen as the best coaches. What's more, we select other 5 college coaches who are not in the Hall of Fame but still made significant contributions.

Searching from the sports-reference.com [8], a website that can provide specific data about coaches, we can find relative data of our specific evaluation norms. Combining those data with the statistics we search from the Wikipedia, we finally conclude the relative statistics of those 75 college coaches and list them in a form. Here we give statistics of 10 coaches as an example.

In the following table, "*FR*", "*SR*", "*SS*", "*EE*", "*FF*", "*RU*", and "*CH*" refer to "*First*

Round", "*Second Round*", "*Sweet Sixteen*", "*Elite Eight*", "*Final Four*", "*Runner-Up*", and "*Campion*", respectively.

Name	from	to	year	win	lose	SRS	SOS	FR	SR	SS	EE	FF	RU	CH
Jim Boeheim	**1905**	**1995**	**48**	**719**	**259**	**15.81**	**7.27**	**5**	**8**	**11**	**2**	**1**	**2**	**1**
Jim Calhoun	**1972**	**2001**	**40**	**877**	**382**	**12.64**	**4.74**	**5**	**5**	**4**	**5**	**1**	**0**	**3**
Larry Brown	**1979**	**2013**	**9**	**210**	**83**	**13.08**	**5.95**	**0**	**3**	**1**	**0**	**1**	**1**	**1**
…														
Mike Krzyzewski	**1975**	**2013**	**39**	**975**	**302**	**20.16**	**8.78**	**2**	**6**	**6**	**2**	**3**	**4**	**4**

Table 3.1 the relevant data of the "best college coaches" candidates

We also collect college basketball coaching record about each season of every candidate. Here we take *Larry Brown* as an example.

Season	win	lose	SRS	SOS	AP Pre	AP High	AP Final	Result
1979-80	22	10	15.67	6.1	8	7	—	NCAA Runner-up
1980-81	20	7	14.89	5.26	6	3	10	NCAA Second Round
1983-84	22	10	9.76	5.86	17	17	—	NCAA Second Round
1984-85	26	8	11.84	6.27	19	9	13	NCAA Second Round
1985-86	35	4	23.18	10.42	5	2	2	NCAA Final Four
1986-87	25	11	13.36	7.73	8	6	20	NCAA Sweet Sixteen
1987-88	27	11	15.71	10.77	7	7		NCAA Champions
2012-13	15	17	-0.59	-1.33	—	—	—	—
2013-14	18	5	13.88	2.45	—	—	—	—

Table 3.2 the college basketball coaching record about each season of *Larry Brown*

3.3 Preprocess data

When we collect data from Internet, we notice that some data is missing due to age. Given the fact, we have to preprocess the data from Internet. As for the data for college basketball, SRS&SOS are sometimes missing. The solution adopt by us is filling the data mainly based on interpolation according to the ranking generated by the other metrics.

IV. Two models for coach ranking

4.1 Model I: Analytic Hierarchy Process (AHP)

When we try to obtain the weight of mainly five aspects as the first class index and the weight of several second class index, subjective judgment is ill-considered. So we choose the Analytic Hierarchy Process [4] (AHP) as the way to conform the weighting coefficient of all the indicators in the evaluation system.

4.1.1 The three-hierarchy structure

The three hierarchy structure which contains criteria level and alternatives level is shown in following table.

Goal	Criteria	Alternatives
The influence of coach	Historical Record	Wins
		Losses
	Game Gold Content	SRS
		SOS
	Playoff Performance	First Round
		…
		Champion
	Honors	Different Awards
		Hall of Fame
	Contribution to sports	Star Player
		Coaching Age
		Tactical Innovation
		International Games
		Popularity

Table 4.1 the three hierarchy structure of our model

4.1.2 Obtain the index weight

- Determine the judging matrix

We use the pairwise comparison method and one-nine method to construct judging matrix $A = (a_{ij})$.

$$a_{ik} * a_{kj} = a_{ij} \qquad (4.1).$$

Where a_{ij} is set according to the one-nine method

- Calculate the eigenvalues and eigenvectors

The greatest eigenvalue of matrix A is λ_{max}, and the corresponding eigenvector is $u = (u_{1,}u_{2,}u_{3,} \dots u_n)^T$. Then we normalize the u by the expression:

$$x_i = \frac{u_i}{\sum_{i=0}^{n} u_j} \qquad (4.2)$$

- Do the consistency check

The indicator of consistency check formula:

$$\mathrm{CI} = \frac{\lambda_{max} - n}{n - 1} \qquad (4.3)$$

Where n denotes the exponent number of matrix.

The expression of consistency ratio:

$$\mathrm{CR} = \frac{CI}{RI} \qquad (4.4)$$

As we have confirmed the weighting coefficient of all the indicators in the evaluation system, now we quantify the importance of coaches.

CW_i denotes the weight of i^{th} criteria level factor, where AW_j is the weight of j^{th} secondary critical level factor, and F_j denotes the j^{th} secondary critical level factor.

The evaluation grade Y_1 should be:

$$Y_1 = \sum_{i=1}^{5} CW_i * \sum_{j=1}^{mi} AW_j * F_j \qquad (4.5)$$

4.1.3 Results & analysis

Based on the data we have already collected in section 3.2, we solve the model and obtain the following results:

- Judging matrix:

$$\mathrm{A} = \begin{bmatrix} 1 & 5 & 5/9 & 1 & 1 \\ 1/5 & 1 & 1/9 & 1/5 & 1/5 \\ 7/5 & 7 & 1 & 7/5 & 9/5 \\ 1 & 5 & 5/7 & 1 & 9/5 \\ 1 & 5 & 5/7 & 1/5 & 1 \end{bmatrix}$$

- Weight vector of criteria level:

$$\mathrm{CW} = [0.1996 \quad 0.0399 \quad 0.3093 \quad 0.2419 \quad 0.2092]$$

For this level, CI=0.301, CR=0.0269 satisfying $\frac{CI}{RI} < 0.1$.

- Weight vector of alternatives level:

✓ Historical Record:$AW_1 = [1.5 \quad -0.5]$

✓ Game Gold Content:$AW_2 = [0.75 \quad 0.25]$

✓ Playoff Performance: $AW_3 = [0.0079 \quad 0.0157 \quad 0.0315 \quad 0.126 \quad 0.252 \quad 0.5039]$

All of these eight vectors satisfy $\frac{CI}{RI} < 0.1$.

Finally, we can obtain the final rankings of the top ten college basketball coaches using AHP models.

Rank	Name	Grade(Y_1)	Rank	Name	Grade(Y_1)
1	Mike Krzyzewski	0.8426	**6**	Roy Williams	0.5637
2	John Wooden	0.7334	**7**	Bob Knight	0.5479
3	Adolph Rupp	0.6048	**8**	Phog Allen	0.4788
4	Jim Boeheim	0.5985	**9**	Rick Pitino	0.4683
5	Dean Smith	0.5844	**10**	Lute Olson	0.4132

Table 4.2 the top ten college basketball coaches' rankings

Conclusion:

- Analyzing the weight vector of criteria level, we can know that the highest value is the weight of Playoff Performance, so coaches with better game results have more chance in top ranking.
- SOS plays a less important role than SRS when determining the Game Gold Content, and the weight of the Game Gold Content is the lowest value in criteria value.

4.2 Model II: Fuzzy Synthetic Evaluation (FSE)

4.2.1 Quantify grades in the five aspects

Fuzzy set theory [6] has been developed and extensively applied since 1965 (Zadeh, 1965). It was designed to supplement the interpretation of linguistic or measured uncertainties for real-world random phenomena.

In section III, we have already articulate our metrics for ranking. Totally, there are five aspects: *historical record, game gold content, playoff performance, honors, contribution to sports*. Before using the fuzzy set theory, we calculate the grades $\{X_1, X_2, X_3, X_4, X_5\}$in each of the 5 aspect using the collected data.

- Calculation rule for *historical record*:

a denotes the number of wins, b denotes the number of loses, λ_{win} denotes the weight for single win, and λ_{lose} denotes the weight for single lose.

$$X_1 = \lambda_{win} a - \lambda_{lose} b \qquad (4.6)$$

This formula provides a comprehensive assessment for wins and losses, obviously $\lambda_{win} > \lambda_{lose}$.

- Calculation rule for *game gold content*:

R denotes the value of SRS, O denotes the value of SOS， O_{max} denotes the maximum value of SOS in the *Strength of Schedule* system.

$$X_2 = R\left(1 + \frac{O}{O_{max}}\right) \qquad (4.7)$$

If a coach has higher SRS, it will have higher grade in this aspect because his team is always far ahead its opponents. At the same time, the higher SOS is, the harder games are. So we let the SOS be an addition to SRS.

- Calculation rule for *playoff performance*:

n_k denotes the number of times for each class of the playoff results.

$$X_3 = \sum_{k=1}^{7} 2^k n_k \qquad (4.8)$$

The number of teams decreases exponentially with power of 2, thus making the weight increase exponentially with power of 2. Sum up the times by designated weight then we could finally draw X_3.

- Calculation rule for *honors*:

We have already count up the awards by weight ($\mathcal{H}$) in section III, so the formula:

$$X_4 = \mathcal{H} \qquad (4.9)$$

- Calculation rule for *contribution to sports*:

We have already give a final grade $\mathcal{C}$ for this aspect in section III, so the formula is:

$$X_5 = \mathcal{C} \qquad (4.10)$$

In conclusion, we form the following quantification rules:

$$\begin{cases} X_1 = \lambda_{win}\mathcal{a} - \lambda_{lose}\mathcal{b} \\ X_2 = R\left(1 + \frac{O}{O_{max}}\right) \\ X_3 = \sum_{k=1}^{7} 2^k n_k \\ X_4 = \mathcal{H} \\ X_5 = \mathcal{C} \end{cases} \qquad (4.11)$$

4.2.2 Determine membership functions

A fuzzy set is defined in terms of a membership function [6] which maps the domain of interest, e.g. concentrations, onto the interval [0, 1]. The shape of the curves shows the membership function for each set. The membership functions represent the degree, or weighting, that the specified value belongs to the set.

Let X_{ij} denote the X_j value for the i^{th} coach and $X_{j(max)}$ denote the maximum X_j value for all the coaches.

Here we use the normalization function as membership function:

$$\mu_j(X_{ij}) = \frac{X_{ij}}{X_{j(max)}} \qquad (4.12)$$

After calculating $\mu_j(X_{ij})$ for each of the X_{ij}, we could concluded the fuzzy matrix X_f (N denotes the total number of the coaches).

$$X_f = \begin{bmatrix} \mu_1(X_{11}) & \dots & \mu_1(X_{15}) \\ \dots & \mu_j(X_{ij}) & \dots \\ \mu_N(X_{N1}) & \dots & \mu_N(X_{N5}) \end{bmatrix}$$

4.2.3 Determine the weights using entropy method

The principle of entropy method [13] states that, subject to precisely stated prior data (such as a proposition that expresses testable information), the probability distribution which best represents the current state of knowledge is the one with largest entropy. To use entropy method, there are mainly 5 steps:

- Calculate the characteristic weight $p_{ij}$for the i^{th} coach's j^{th} evaluation grade (X_{ij}) based on the normalized fuzzy matrix X_f:

$$p_{ij} = \frac{X_{f(i,j)}}{\sum_{i=1}^{N} X_{f(i,j)}} \qquad (4.13)$$

- Calculate the entropy for the j^{th} evaluation grade:

$$e_j = -\frac{1}{\ln(N)} \sum_{i=1}^{N} p_{ij} ln(p_{ij}) \qquad (4.14)$$

- Calculate the diversity factor for the j^{th} evaluation grade:

$$g_j = 1 - e_j \qquad (4.15)$$

- Determine the weight for each evaluation grade:

$$w_j = \frac{g_j}{\sum_{j=1}^{5} g_j} \qquad (4.16)$$

- Determine the final score Y for each coach;

$$Y_2 = \mathrm{W} * X_f \qquad (4.17)$$

4.2.4 Results & analysis

Characteristic weight, entropy, diversity factor and weight are shown as follows:

p_{ij}	X_1	X_2	X_3	X_3	X_5	Y_2
John Wooden	0.04	0.06	0.15	0.13	0.08	0.8708
Mike Krzy	0.07	0.06	0.10	0.13	0.08	0.8629
Adolph Rupp	0.06	0.06	0.08	0.11	0.02	0.675
Dean Smith	0.06	0.06	0.08	0.04	0.05	0.609
Bob Knight	0.05	0.06	0.06	0.07	0.05	0.6052
Roy Williams	0.06	0.05	0.06	0.04	0.08	0.5872
Jim Boeheim	0.06	0.03	0.05	0.03	0.01	0.5864
Phog Allen	0.04	0.04	0.05	0.05	0.05	0.4874
Henry Iba	0.04	0.04	0.04	0.04	0.02	0.4664
Lute Olson	0.06	0.04	0.04	0.06	0.07	0.4538
g_j	-0.99	-1.00	-1.04	-0.96	-0.93	0.4336
w_j	0.18	0.16	0.23	0.20	0.22	0.8708

Table 4.3 the results for FSE

- The weights for each aspect is near to each other.
- The playoff performance (X_3) plays the most important role (with 0.23 weight) in FSE evaluating.
- The coaches whose playoff performance is better will enjoy priority to some extent. At the same time, the coaches who have amazing game gold content (with only 0.16 weight) might not outstand.

V. Models Combination

5.1 Evaluation of individual model

In order to evaluate the accuracy of our two individual models, ***average offset distance*** D is defined.

We collect ranking lists of top 10 NCAA basketball coaches from several authoritative media such as ESPN, Bleacher Report, Yahoo Sports, and Sporting News [15]. Then compare our results to those lists and average offset distance D reflects the difference.

Here we use the first-order Minkowski distance to denote the average offset distance of the top 10.

$$D = \frac{1}{10n}\sum_{i=1}^{n}\sum_{j=1}^{10}\left|j - r_j\right| \qquad (5.1)$$

Where n is the number of the top 10 ranking lists, j is the ranking in the i^{th} list, and r_j is the ranking of j^{th} coach in our results. So $\left|j - r_j\right|$ denotes the difference between result of media and ours, and D means the average difference. If our results are the same as all media selection results, then D is equal to zero.

D_α is the average offset distance of top 5

$$D_\alpha = \frac{1}{5n}\sum_{i=1}^{n}\sum_{j=1}^{5}\left|j - r_j\right| \qquad (5.2)$$

D_β is the average offset distance of 6^{th} to 10^{th}.

$$D_\beta = \frac{1}{5n}\sum_{i=1}^{n}\sum_{j=6}^{10}\left|j - r_j\right| \qquad (5.3)$$

Obviously model with smaller average offset distance should get higher score. So We can define hit score

$$g = \frac{900}{9+D}\ (0 < g < 100) \qquad (5.4)$$

When D = 0, g= 100, means if there is no average offset distance, this model can get full marks 100. Here are our results:

	AHP	FSE
D_α	1.75	1.15
D_β	3.1	2.85
D	2.425	2
g_α	83.72	88.67
g_β	73.38	75.94
g	78.77	81.81

Table 5.1 the results for evaluation

Conclusions:

- Vertical comparison: Either AHP or Fuzzy Synthetic Evaluation D_α is obviously smaller than D_β. It means that the results are more reasonable in top 5 than in top 10.

➢ Horizontal comparison: Fuzzy Synthetic Evaluation performs better than AHP in both top 5 and top 10. It proves that Fuzzy Synthetic Evaluation is more accurate than AHP. Because AHP depends on artificial scoring which is too subjective.

5.2 Aggregation Model

AHP is a subjective method, it largely depends on artificial scoring; Relatively, Fuzzy Synthetic Evaluation is an objective method, it all depends on the data. To comprehensively consider the effect of subjective and objective factors, we adopt **linear weighted method**:

$$\begin{cases} W_1 + W_2 = 1 \\ Y = W_1 Y_1 + W_2 Y_2 \end{cases} \qquad (5.5)$$

Y_1 is the evaluation grade of AHP model , Y_2 is the evaluation grade of Fuzzy Synthetic Evaluation model. All of them range from 0 to 1.

To determine the weight W_1 and W_2, we take D(the average offset distance) into consideration. Since smaller average offset distance means the more accuracy results, we can assign higher weight to the mode with smaller D. Then we get

$$\begin{cases} W_1 = \dfrac{D_2}{D_1 + D_2} \\ W_2 = \dfrac{D_1}{D_1 + D_2} \end{cases} \qquad (5.6)$$

In conclusion, our final model can be defined as:

$$Y = W_1 Y_1 + W_2 Y_2 \qquad (5.7)$$

5.3 Results & analysis

	AHP	FSE	AM
Rank 1	Mike Krzyzewski	John Wooden	John Wooden
Rank 2	John Wooden	Mike Krzyzewski	Mike Krzyzewski
Rank 3	Adolph Rupp	Adolph Rupp	Adolph Rupp
Rank 4	Jim Boeheim	Dean Smith	Dean Smith
Rank 5	Dean Smith	Bob Knight	Bob Knight
Rank 6	Roy Williams	Roy Williams	Jim Boeheim
Rank 7	Bob Knight	Jim Boeheim	Roy Williams
Rank 8	Phog Allen	Phog Allen	Phog Allen
Rank 9	Rick Pitino	Rick Pitino	Rick Pitino
Rank 10	Lute Olson	Henry Iba	Henry Iba

Top5 Hit score	83.72	88.67	88.67
Top10 Hit score	78.77	81.81	82.57

Table 5.2 the ranking comparison among the models

Conclusion:

- All our models perform better in top 5 than in top 10. It proves that the top 5 coaches in college basketball history are less controversial than top 10.
- The result of AM is very similar to FSE. They have the same hit score 88.67 in top 5; but in top 10, AM have highest hit score 82.57 in these three models. It proves the combination can improve our model.
- According to our final result, our model's top 5 coaches in college basketball are *John Wooden, Mike Krzyzewski, Adolph Rupp, Dean Smith and Bob Knight.*

VI. Extend Our Models

6.1 Genders do not matter

Now we take genders into consideration. We still use basketball as an example, and rank the top ten college women's basketball coaches [20] for the previous century. Searching from the internet, we collect the relative data about 50 college women's basketball coaches [18] with 600 and other 5 coaches who have established outstanding traditions, earned many awards and garnered recognition for their colleges. Then we rank them with our models mentioned above.

Coaches' ranking with the Aggregation Model:

Rank	Name	Grade	Rank	Name	Grade
1	Pat Summitt	0.8532	**6**	Sylvia Hatchell	0.5875
2	Geno Auriemma	0.8434	**7**	Jody Conradt	0.5673
3	Tara VanDerveer	0.7465	**8**	Kay Yow	0.5486
4	Leon Barmore	0.7236	**9**	Sue Gunter	0.4783
5	C. Vivian Stringer	0.6074	**10**	Gail Goestenkors	0.4379

Table 6.1 the ranking for coaches of women's basketball

From the sports.yahoo.com [19], we get a list of the all-time top ten NCAA women's basketball coaches, and the list is shown in following table.

Rank	Name	Rank	Name
1	Pat Summitt	**6**	Jody Conradt
2	Geno Auriemma	**7**	Kay Yow

3	Leon Barmore	8	Sylvia Hatchell
4	C. Vivian Stringer	9	Gail Goestenkors
5	Tara VanDerveer	10	Sue Gunter

Table 6.2 the ranking from Yahoo

Using the average offset distance mentioned in section 5; we can measure the hit score for our models. All results of our models are in agreement within reasonable error range (hit score = 87.57), so that we can safely address the conclusion that our models can be applied in general across both genders.

6.2 Time factor does make a difference

6.2.1 Why time factor matters?

National Collegiate Athletic Association Basketball Tournament [14] started at 1939, during the 74 years' development, while the number of teams participating in the tournament increasing a lot, the competition becomes fiercer. Also in different historical periods, the NCAA Basketball Tournament gained different popularity, and this also influences the quality of the evaluation grades.

To quantify the time factor, we attach weight w_i (1-10) to different time periods mainly based on the turning points that occurred in the period.

The following table shows the critical years in the NCAA history [14]:

Year	Turning points	w_i
1913-1939	There are no national college basketball competition.	5
1939-1951	NCAA Basketball Tournament started, and 8 teams anticipated. There are two college tournament: NIT and NCAA.	6
1951-1975	16 teams anticipated, NIT became second class competition.	7
1975-1980	32 teams anticipated, especially in 1979, Magic Johnson fight with Larry Bird in the finals, achieving 24.1% audience rating, then a golden age came.	8
1980-1985	48 teams anticipated,	9
1985-2013	64 teams anticipated.	10

Table 6.3 the time weights for each time period

6.2.2 How time factor matters?

The whole metric system will change after introducing the time weight. What follows in the chapter will be devoted to explaining the changes in detail.

- The evaluation norms will change after introducing the time weight (w_i).

$$\begin{cases} a = \sum_i w_i a_i \\ b = \sum_i w_i b_i \\ R = \dfrac{\sum_i w_i SRS_i}{t} \\ O = \dfrac{\sum_i w_i SOS_i}{t} \\ n_k = \sum_i w_i m_{ki} \\ \mathcal{H} = \sum_i \mathcal{k}_i \\ \mathcal{C} = \sum_i c_i \end{cases} \qquad (6.1)$$

Where

- ✓ a denotes the wins, a_i denotes the wins per year.
- ✓ b denotes the loses, b_i denotes the loses per year.
- ✓ R denotes the average SRS, SRS_i denotes the losses per year.
- ✓ O denotes the average SOS, SOS_i denotes the losses per year, t denotes the number of years.
- ✓ The binary variable m_{ki} denotes whether the team get the k^{th} class in the i^{th} year. n_k denotes the number of times for each class.
- ✓ $\mathcal{k}_i$ denotes the weight for each award, $\mathcal{H}$ denotes the total weights of all the awards a coach has ever got.
- ✓ c_i denotes the points for each aspect, $\mathcal{C}$ denotes the total points.

- Accordingly, the results for AHP (model I) & FSE (model II) will change.

The following table shows how AHP (model I) will change (The names in bold are the people whose rank has changed):

AHP(without w_i)	Grades (Top 10)	AHP(with w_i)	Grades (Top 10)
Mike Krzyzewski	0.8426	Mike Krzyzewski	0.8894
John Wooden	0.7334	John Wooden	0.7601
Adolph Rupp	0.6048	**Jim Boeheim**	0.6465
Jim Boeheim	0.5985	**Adolph Rupp**	0.6322
Dean Smith	0.5844	Dean Smith	0.6251
Roy Williams	0.5637	Roy Williams	0.6137
Bob Knight	0.5479	Bob Knight	0.5922

Phog Allen	0.4788	**Rick Pitino**	0.5171
Rick Pitino	0.4683	**Phog Allen**	0.5062
Lute Olson	0.4132	Lute Olson	0.4606
Top10 Hit score	78.77	**Top10 Hit score**	76.60
Top5 Hit score	83.72	**Top5 Hit score**	83.56

Table 6.4 what is different in AHP introducing time weight?

Conclusion:

- In model I (AHP), Top5 hit score changes from 83.72 to 83.56, namely, Top5 hit score nearly remains unchanged.
- Top10 hit score changes from 78.77 to 76.60, namely, Top10 hit score falls to some extent.
- The changes in rankings occur only locally, not globally.

The following table shows how FSE (model II) will change (The names in bold are the people whose rank has changed):

FSE (without w_i)	Grades (Top 10)	FSE (with w_i)	Grades (Top 10)
John Wooden	0.8708	**Mike Krzyzewski**	0.9337
Mike Krzyzewski	0.8629	**John Wooden**	0.7850
Adolph Rupp	0.6750	**Roy Williams**	0.6556
Dean Smith	0.6090	**Jim Boeheim**	0.6260
Bob Knight	0.6052	Bob Knight	0.6207
Roy Williams	0.5872	**Dean Smith**	0.5984
Jim Boeheim	0.5864	**Adolph Rupp**	0.5445
Phog Allen	0.4874	**Rick Pitino**	0.5164
Rick Pitino	0.4664	**Lute Olson**	0.4603
Henry Iba	0.4538	**Tom Izzo**	0.4292
Top10 Hit score	81.81	**Top10 Hit score**	75.31
Top5 Hit score	88.67	**Top5 Hit score**	84.51

Table 6.5 what is different in FSE introducing time weight?

Conclusion:

- In model II (FSE), Top5 hit score changes from 88.67 to 84.51, namely, Top5 hit score falls to some extent.
- Top10 hit score changes from 81.81 to 75.31, namely, Top10 hit score falls a lot. The model appears to be more inaccurate.
- The changes in rankings occur globally. The model appears to be easily influenced by the time weights.

- There is no doubt that the results for Aggregation Model (AM) will also change.

The following table shows how aggregation model (final model) will change (The names in bold are the people whose rank has changed):

AM (without w_i)	Grades (Top 10)	AM (with w_i)	Grades (Top 10)
John Wooden	0.8568	**Mike Krzyzewski**	0.9204
Mike Krzyzewski	0.8296	**John Wooden**	0.7775
Adolph Rupp	0.6539	**Roy Williams**	0.6430
Dean Smith	0.6016	**Jim Boeheim**	0.6322
Bob Knight	0.5900	Bob Knight	0.6122
Jim Boeheim	0.5880	**Dean Smith**	0.6064
Roy Williams	0.5802	**Adolph Rupp**	0.5708
Phog Allen	0.4848	**Rick Pitino**	0.5166
Rick Pitino	0.4669	**Lute Olson**	0.4604
Henry Iba	0.4308	**Phog Allen**	0.4354
Top10 Hit score	82.57	**Top10 Hit score**	76.6
Top5 Hit score	88.67	**Top5 Hit score**	85.47

Table 6.6 what is different in AM introducing time weight?

Conclusion:

- The changes in rankings occur globally. The model appears to be easily influenced by the time weights.
- The performance for AM appears to be easily influenced by FSE because of the weight distribution for the two models.

Take Bob Knight for example, the following figure shows how evaluation grade changes in the two different situations:

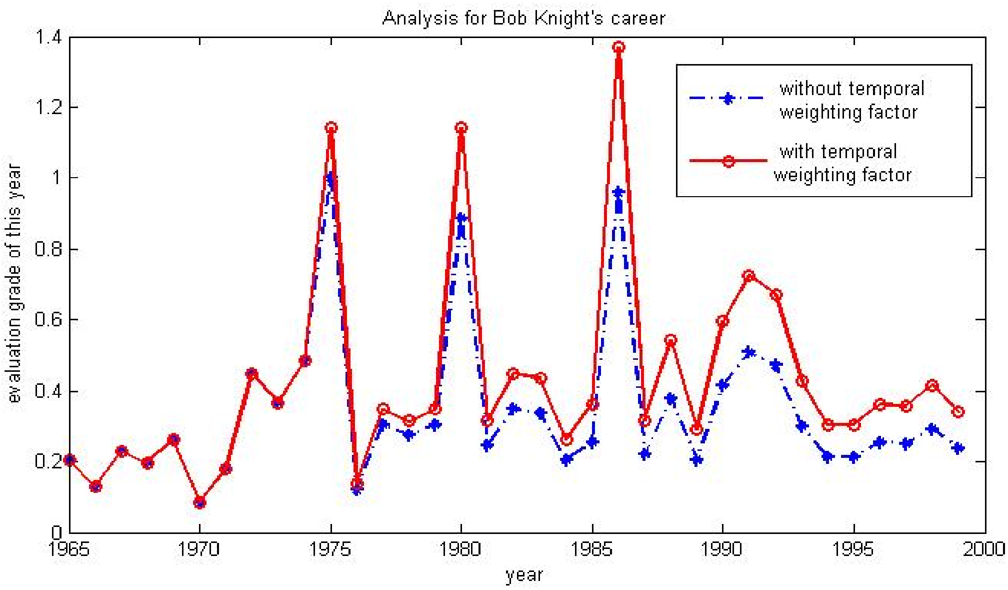


Figure 6.1 every year's evaluation of Bob Knight

6.2.3 What is the variation tendency?

From the results in section 6.2.2, variation tendency could be concluded as follows:

- For coaches in **earlier ages**, their rankings will **fall** to some extent.

Take *Phog Allen* [21]for example, he is known as the "Father of Basketball Coaching", but most of his games occurred in 1920-1959, which means that NCAA had not started or though started the teams were few. The time weight for his age is relatively low thus making the ranking fall.

- For coaches in **recent years**, they will enjoy some **superiority**.

Take *Roy Williams* [22]and *Adolph Rupp* [20]for example, the two coaches' performance are quiet close to each other, *Adolph Rupp* was even better in in historical record, but due to the time weight, the historical record for *Adolph Rupp* does not count that much, and Roy Williams is ahead of him.

- Introducing time weights does **not** necessarily mean **higher** hit score.

In section V, We have stated that we choose some existed rankings as criterion, but these rankings generally do not take coaching ages into consideration, moreover, some authorities hold the viewpoints that the coaches in early ages are of more authority, leading to the hit score falling.

6.3 Model also works in other sports

It obviously could not live up to our expectations if the model could only be used in basketball. This chapter we will explain in detail how our models can be applied in general across all possible sports.

There are mainly ***4 steps*** to apply the model in any sport as you want.

✧ **Step1**: Adjust the **metrics** according to the feature of the sports.

Main differences are in the *Playoff Performance*, different sports may have different playoff rules, so the metrics in this aspect should be adapted according to the rule. Take football as example:

5-aspect norms	Metrics for basketball	Metrics for football
Historical record	Wins	Wins
	Losses	Losses
Gold content	SRS	SRS
	SOS	SOS

Playoff Performance	First round	Fiesta Bowl
	Second round	Orange Bowl
	Sweet sixteen	Sugar Bowl
	Elite eight	Rose Bowl
	Final four	National Championship Game
	Runner-up	
	Champion	Non-BSC bowls
Honors	Awards	Awards
Contribution to sports	Star players	Star players
	Coaching ages	Coaching ages
	Tactical innovation	Tactical innovation
	International	International
	Popularity	Popularity

Table 6.7 the metrics for college football

✧ **Step2**: Adapt the **calculation rules** according to the feature of the sports.

For example, the metrics for football in *playoff performance* have already changed, each class of performance should be assigned another weight according to the gold content of different bowls. At the same time, the awards and their weights under consideration should also be adjusted for the “*Honors*” aspect.

✧ **Step3**: Adjust the **time weight** according to the history of the sport.

For example, as for football, before 2006, there are no BSC bowls. After 2006, 5 BSC bowls came into being, enjoying extremely high gold content.

✧ **Step4**: **Solve** the Aggregation model again and analyze the results.

The following figure shows the steps of applying the model into all possible sports:

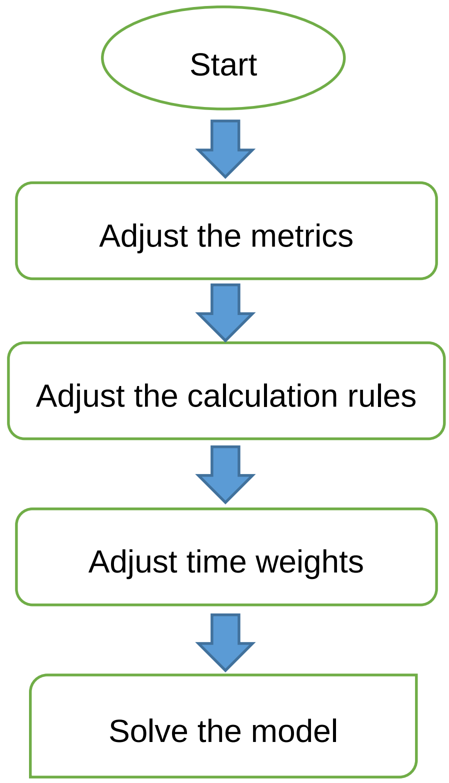


Figure 6.2 the flow chart of the four steps

Following the 4 steps presented above, we apply the model in other 2 different sports: football and hockey.

Top5 for football	Grades	Top5 for hockey	Grades
Bear Bryant	0.8874	Bob Johnson	0.8963
Knute Rockne	0.8664	Red Berenson	0.8732
Tom Osborne	0.8538	Jack Parker	0.8525
Joe Paterno	0.7872	Jerry York	0.7756
Bobby Bowden	0.7864	Ron Mason	0.7632

Table 6.8 the rankings for football & hockey

VII. Further discussion

7.1 Sensitivity Analysis on FSE

7.1.1 Vary Membership function

In FSE discussed in section 4.2, we choose (4.12) as the membership function. But there are also other available membership functions. The following table shows the membership functions that are taken into consideration in this part.

Type	Membership function
1	$\mu_j(X_{ij}) = (\frac{X_{ij}}{X_{j(max)}})^k$
2	$\mu_j(X_{ij}) = (\frac{X_{ij} - X_{j(min)}}{X_{j(max)} - X_{j(min)}})^k$
3	$\mu_j(X_{ij}) = 1 - e^{-k(X_{ij} - X_{j(min)})}$

Table 7.1 membership functions

- For type 1, we will analyze how the weight distribution (only for FSE) for different aspects will change while varying k.

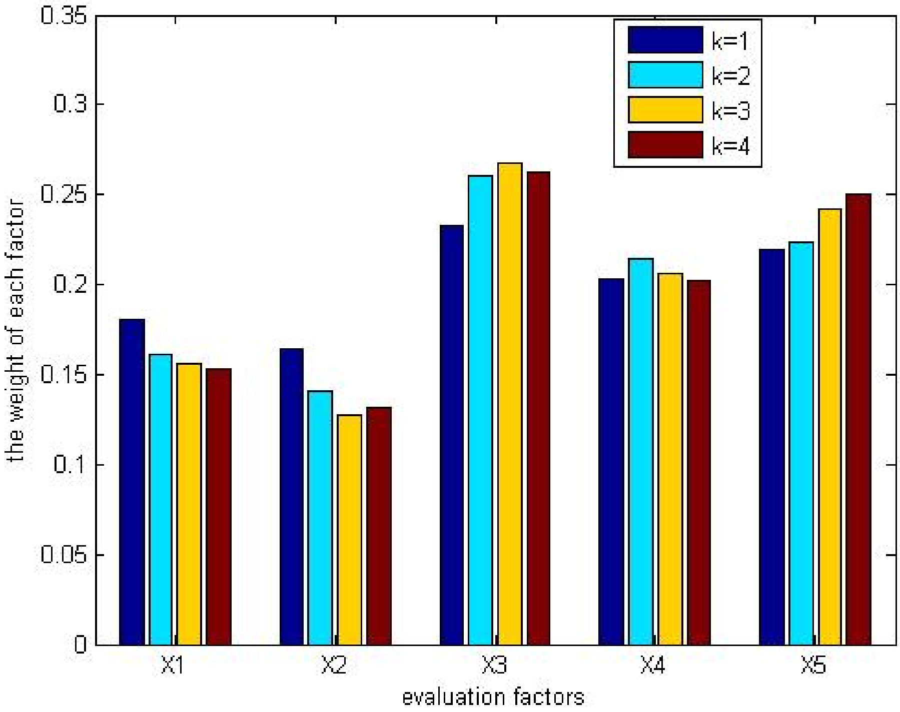


Figure 7.1 the weight of each factor for each aspect

Conclusions:

- Weight distribution is sensitive to the value of k and changes of weight distribution can be seen clearly in **Figure 7.1**。
- As the value of k increases, X_1 and X_2 tend to be less important but X_3 and X_5 tend to be more important.
- When $1 \le \mathrm{k} \le 4$, X_3 and X_5 have the highest weight, X_1 and X_2 have the lowest weight.

- For all three types, we will analyze how hit score (only for FSE) will change varying k.

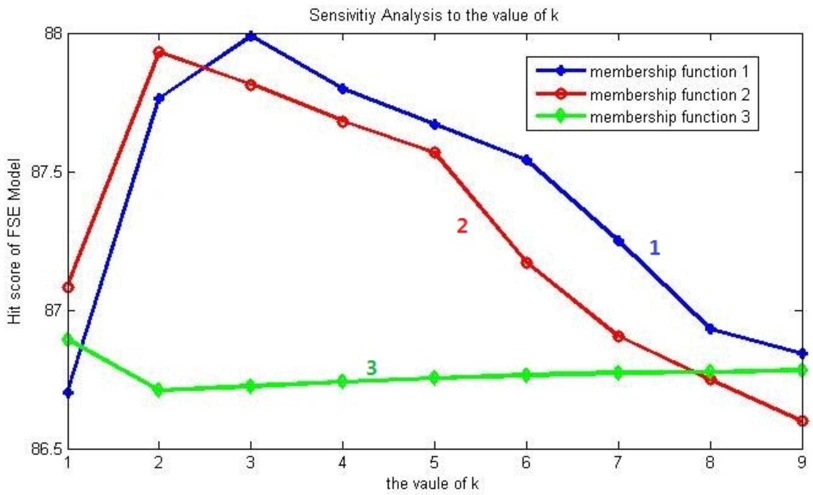


Figure 7.2 sensitivity analysis on k for FSE

Conclusions:

- Membership function 1 and member function 2 are sensitive to the value of k, while membership function 3 is not sensitive to the value of k.
- Membership function 1 reaches its maximum value at k = 3; membership function 2 reaches its maximum value at k = 2.
- Obviously, membership function1 and membership function 2 perform much better than membership function 3. And membership function 3 is not suitable for our model.
- For best results, membership function1 with k = 3 is most appropriate in this model.

7.1.2 Vary calculation rule

Here we focus on figuring out how hit score will change to the ratio $\frac{\lambda_{win}}{\lambda_{lose}}$.

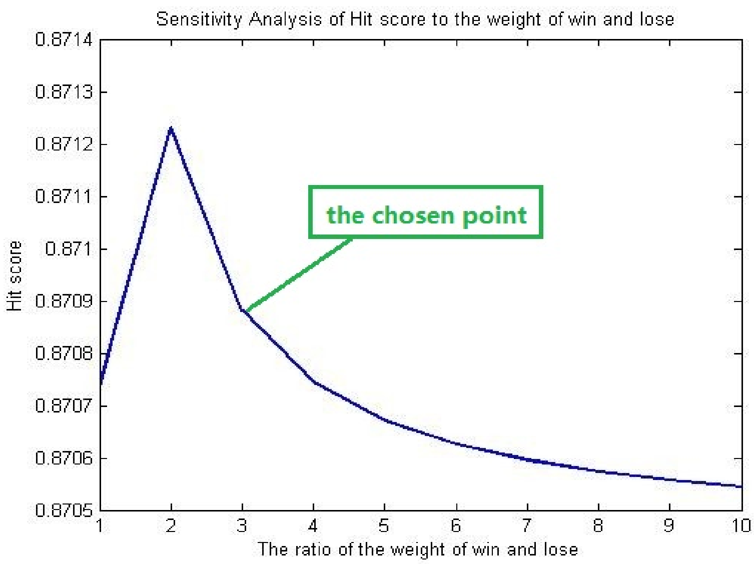


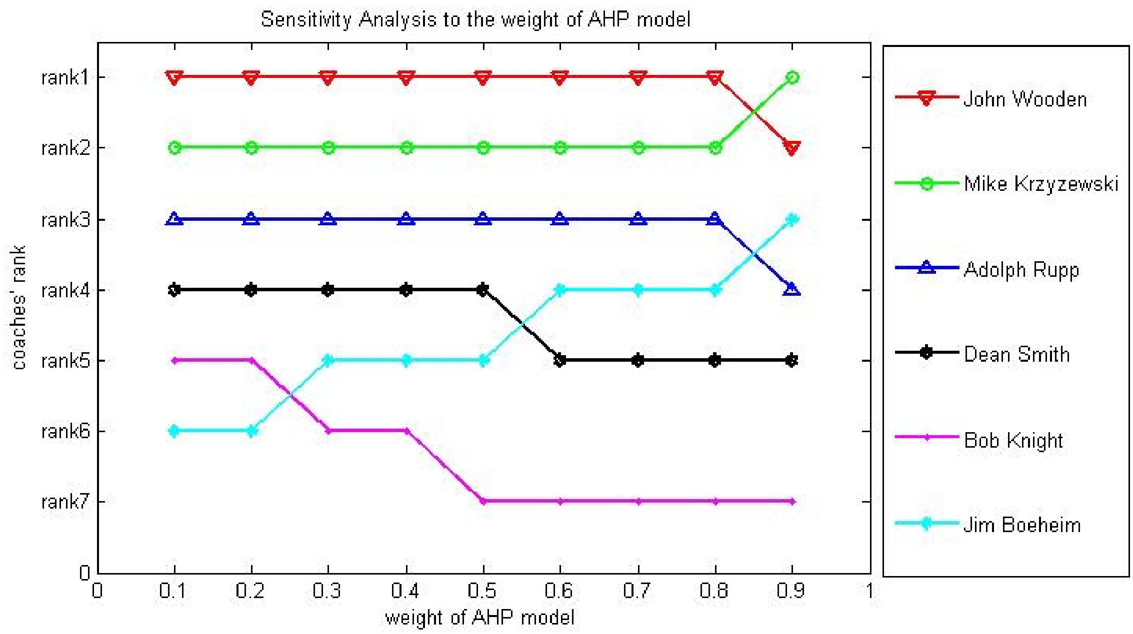
Figure 7.3 sensitivity analysis to the weight of win and lose

Conclusions:

- Model performs best when the weight of winning a game is two times as much as losing a game.
- If we attach the same weight to winning a game and losing a game, the model will have a poor hit score. And if the ratio of the weight of wins and loses is too high, it will also lead to a bad result.

7.2 Sensitivity Analysis on Aggregation weight

In this part, we will analyze how hit score (For AM) and rankings will change while varying the weight for AHP (W_1).



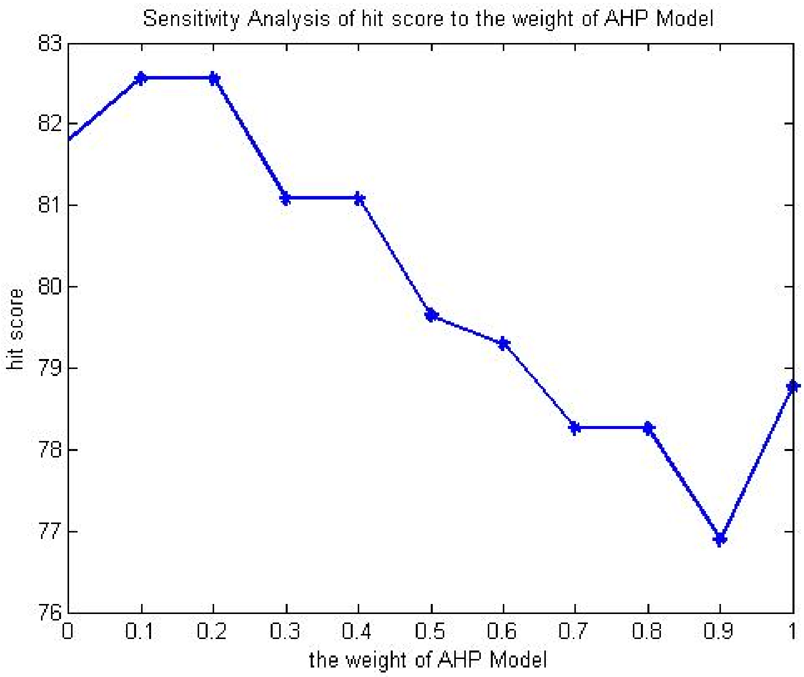


Figure 7.4 && 7.5

Conclusion:

- As the weight of AHP increases, the rank list will change. The rank of Dean Smith and Bob knight tend to decline and the rank of Jim Boeheim tend to rise.
- Since AHP is less accurate than FSE, hit score of AM would be optimal when the weight of AHP is small. But when the weight of AHP is zero, hit score doesn't reach the maximum. The maximum hit score is reached when the weight of AHP is 0.1-0.2.
- It proves that AM can perform well than either AHP or FSE.

7.3 Exploration: Evaluating Best President

Now we use our models to find the top ten presidents of the United States. We choose the 43 men who have been president of USA, and collect relative data from the internet [24].

A president can be evaluated also from following five aspects: *personal qualities, presidential achievements, leadership qualities, failures and faults, and popular opinion*:

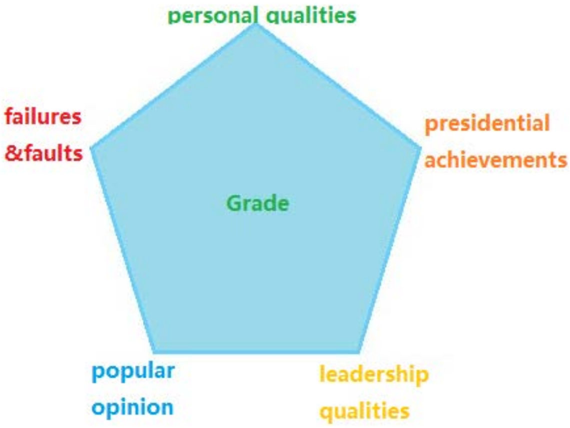


Figure 7.6 5-aspect norm of president of USA

The **personal qualities** includes imagination, intelligence and being willing to take risks, while the **presidential achievements** can be valued with ability of domestic accomplishments, executive appointments, foreign policy accomplishments, and ability of compromise. And the **leadership qualities** can be measured by party leadership ability and relations with congress. We also take **popular opinion** into consideration, and we gather votes from different polls, such as C-SPAN poll, ABC News poll, Washington College poll, Gallup poll, Rasmussen poll, and 2012 Gallup poll. The ranking of presidents of the United States is shown in following table.

Rank	Name	Rank	Name
1	Abraham Lincoln	6	Harry S. Truman
2	George Washington	7	Woodrow Wilson

3	Franklin D. Roosevelt	8	Dwight D. Eisenhower
4	Thomas Jefferson	9	James K. Polk
5	Theodore Roosevelt	10	Andrew Jackson

Table 7.2 the ranking of presidents of the United States

VIII. Strength and Weakness

Strength:

- When we articulate our own metrics for assessment, we try our best to include all the important elements of a coach to make the ranking more accurate. Time factor, gender, category are all discussed in the model.
- We evaluate the performance of a coach from 5 specific perspectives.
- We set up two different models to form an aggregation model. AHP includes more subjective factors while FSE appears to be more objective. The aggregation model is devoted to make clear the tradeoff between the two sides.
- We states a distinct quantification system which is expected to live up to the common sense.

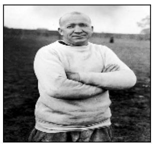
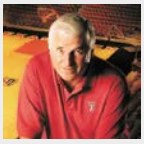
Weakness:

- We adopt totally eighteen indicators to evaluate a coach, namely not all elements is under consideration.
- Weights are everywhere in the model, but some weight assignments might not be the best scheme.

IX. Non-technical Explanation

For better or worse, coaches are often the faces of college sports programs. Different from players who stay only for a few years, coaches can exert longer influence in the college games. Here is a list of the top 5 coaches in the college basketball, college football, and college hockey.

Rank	college basketball	college football	college hockey
1	 John Wooden	 Bear Bryant	 Bob Johnson

2	 Mike Krzyzewski	 Knute Rockne	 Red Berenson
3	 Adolph Rupp	 Tom Osborne	 Jack Parker
4	 Dean Smith	 Joe Paterno	 Jerry York
5	 Bob Knight	 Bobby Bowden	 Ron Mason

The rankings proved to be a difficult task and job of a college coach is multifaceted. Firstly, we choose some coaches, who are in the list of Hall of Fame, or people who have established outstanding traditions and earned many awards, as our ranking candidates. Then, searching from the internet or other data sources, we try to collect relative data as detailed as possible. After choosing proper data, college coaches' rankings can be obtained. What's more, we search the existing rankings from the internet to serve as the evaluation criterion.

During the procedure of choosing data, we evaluate the coaches in our list of candidates from five aspects. As is known to all, the best college coaches tend to have good team's historical record, such as more wins and high win rate. What's more, SRS (Simple Rating System) and SOS (Strength of Schedule) can reflect the coaching ability. We also examined each coach's success in the postseason. Taking basketball as an example, the performance could be valued by counting the number of times for "Champion", "Runner-up", "Final Four", "Sweet Sixteen", "Second Round", and "First Round". In many case, we take into account coaches' contribution to the sports and honors, such as various awards in their field or putting their name in the "Hall of Fame".

After collecting and choosing coaches' detailed data, we define the importance of those aspects which can measure coaches' ability, and use the results to give each coach a score. The higher the coaches' scored on the relative aspects, the better their position on the ranking.

We use the data of the best college basketball coach–John Wooden to give some example. In his college basketball coach career, his team had won 826 games, and during his sixteen years NCAA tournament, he won ten championships and twelve straight trips to Final Fours. John Wooden has been recognized tremendous times for his achievements and created longer legacies in the college basketball games. Besides his fantastic and glory record, Wooden was recognized for his impact on college basketball as a member of the founding class of the National Collegiate Basketball Hall of Fame and was named The Sporting News "Greatest Coach of All Time" [23]. With so many honors and awards which can't be listed in detail there, John gets the highest score when we rank the coaches and is worthy the title of the best college basketball coach.

X. Future work

- Consider all possible sports coaches together, and make a college coaches rankings, regardless of what kind of sports coaches they are.
- Take other relevant coach information into consideration, because research suggests that characteristics of the coaches, such as the breadth of coaches' knowledge, authority, the ability of searching and cultivating talents, and attention to details, play a role in determining the best college coaches.
- Develop a general method to rank everything when there is one way to quantify.

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