The homotopy category Det Let X. be a simplicial set. It x, y ∈ Xo, let  $Arr(x,y) = \{f: X^2 - x \times f(0) = x, f(1) = y\}$ We let  $icl_{x} \in Arr(x,x)$  be the 1-simplex V, ~ X. For I, ge Am(x, y) and a 2-simplex of 2 -> X5.1 x I sid we say or is a homotopy from I to g. Prop For Homotopy detines an equivalence relation on Am(x, y) il l'is an o-category Proof Let fixty be an arrow Than 1 - 1 - Re is a homotopy from I to itself. Now suppose I, q h e Arr(x, y) and that

8:12 - Xe o' 12 - X. e are homotopies from I to g and from I to h. Let 0": 12 -> 10 Y The have 13 (o",0,0',0) and ols (2) is homotopy from g to h. Il h=gl this shows reflect symmetry and transitivity tollows immediately from the previous argument. Prop II Elle How Let How (x, y) = Arr (x, y)/ and Clohe = lo The this has a structure a category by deting composition as ice let 1/1 -> C and extend to 12 -> C

Proof he son only for the object we know is final if the R (Y, X) is contractible for all y.

Def An int on eategory is pointed if it has

Det An int on- category is pointed if it has
an object which is both initial and of final. Call this zero
object. Zero morphism!

Examples (i) IT = example II

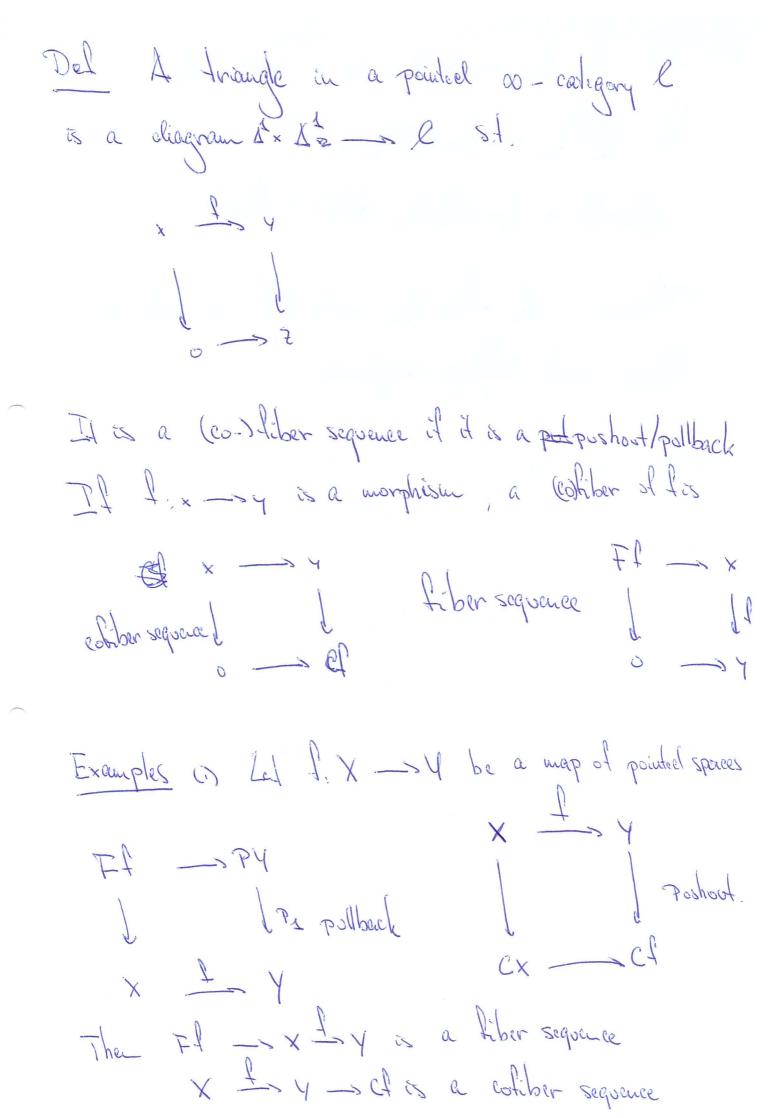
Examples (i) http is exactly the classical contectory of spaces modulo homotopy. Thus, & is initial and any contractible space is final in top

(ii) Top, is elevined similarly to Top:

n-simplices: (Xo, -, Xn) based spaces with

Painteel maps (hi, X: 1 Itap + - X)
satisfying coherence conclitions.
Then h Topo is the classical category of pointeel space modulo pointeel homotopy. Thus, any condractible

Space is a zero object. (iii) Let R be a suital ring. For neW, consider P(41, -, my) as a past and let Nucl (P(4s, -, nz)) denote the hon-elegenerate simplices of N(P({1, -, n})) Let (II mod) q = DR mod (Rt, -, wy)) with differentials induced by the maps on the nerve Then tex (R-mod) a consists of (R-moet) (K(0), -, K(9)) Logother with (hij King Third - Kil) Osicisa and compatibility constitions



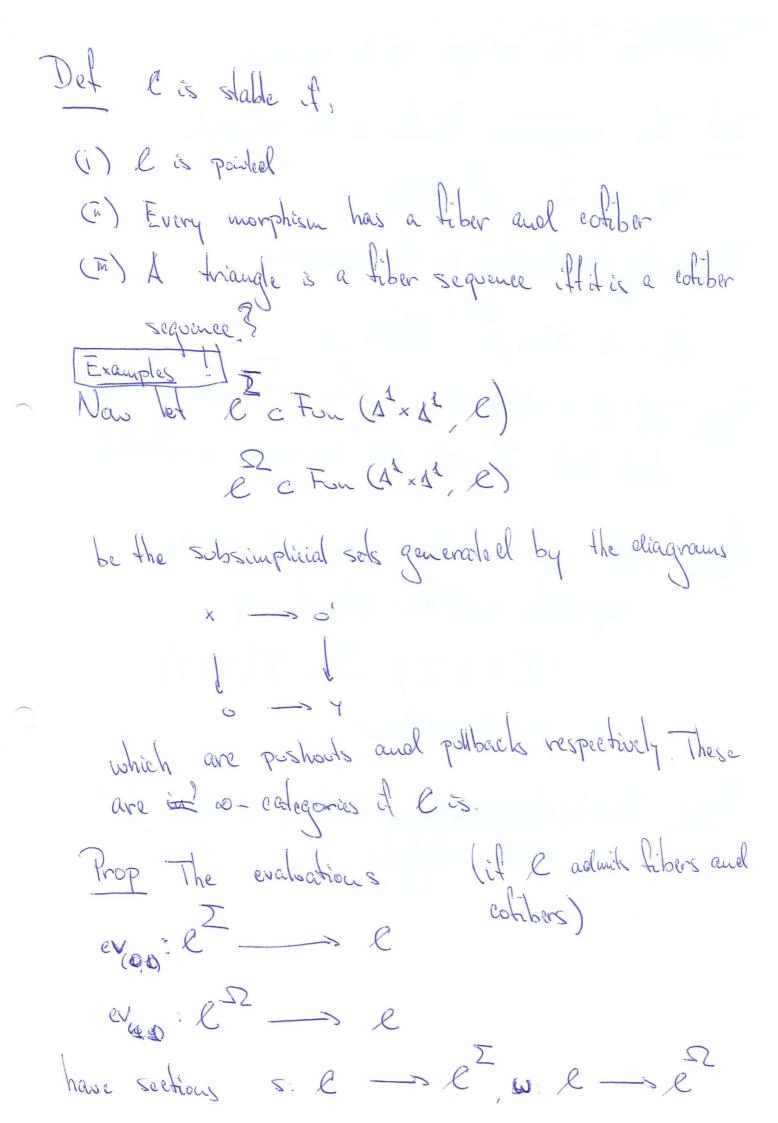
(i) II I. A. — B. is a map, then

let (Cf) = An (BBn-1 with

al(a,b) = (-al(a), al(b) - f(a))

The A. I. B. — Cf is both a

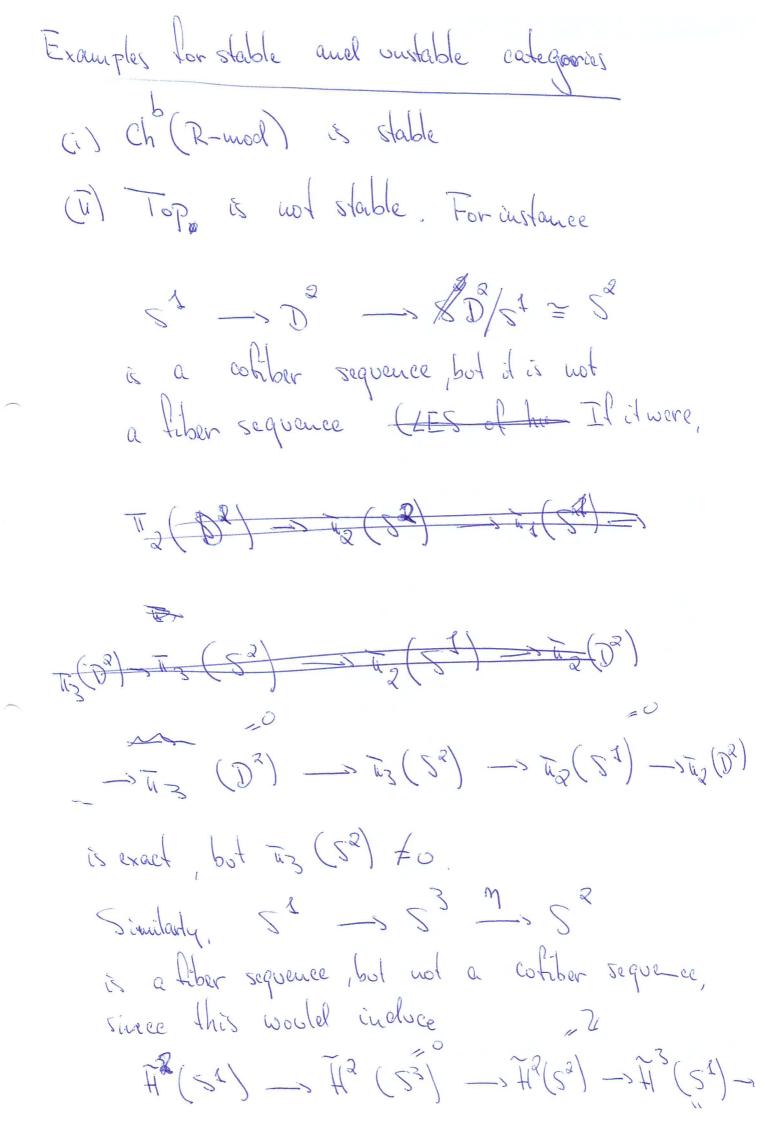
ther and cohiber sequence.

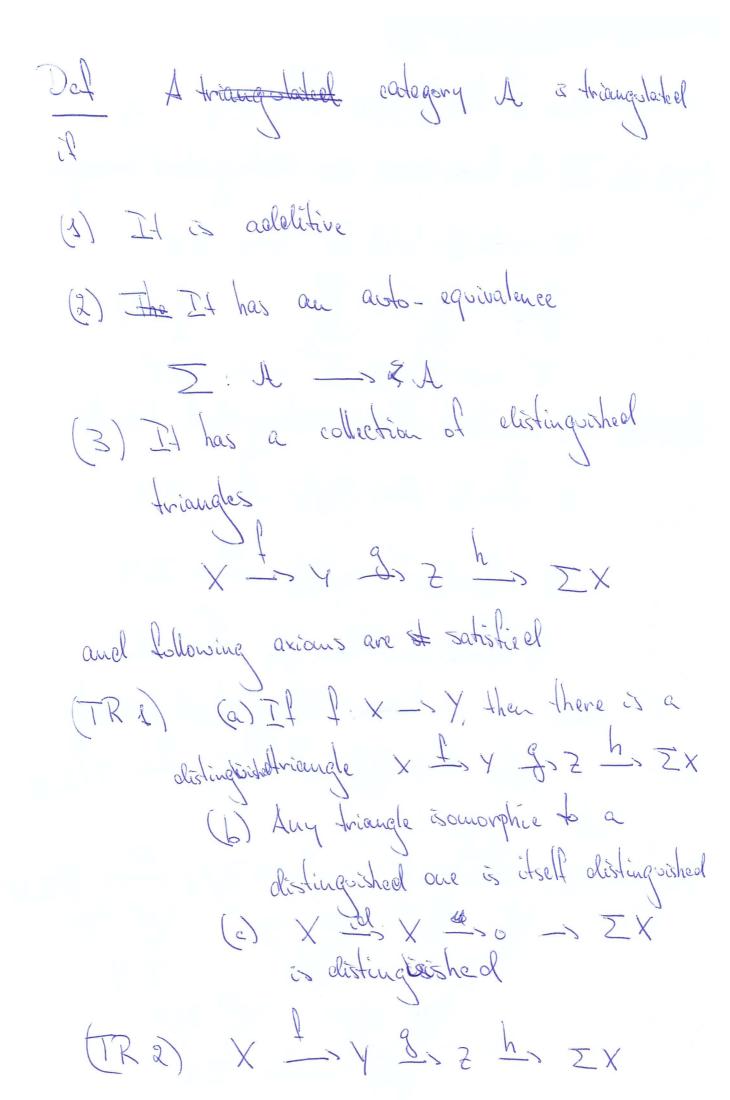


and these are unique up to homotopy.
Det The suspension functor is the composite
Zil se evan
Doally, the bop functor is the composite
ev <sub>(Q,0)</sub>
Rem If $e$ is stable, then $e^{z} = e^{z}$ and one can check that $z$ and $z$ are inverse to each other.
Lemma II R is stable, then h R admits coproclocks and the natural map
ZXUZY = Z(XUY)
is an isomorphism
Prop The homotopy category If e is stable, he is an additive category

Definition of the group shrocture on them (ZX, Y)
We can write ZX as the colimit of
$0 \leftarrow X \rightarrow X \leftarrow X \rightarrow X \rightarrow X \leftarrow X \rightarrow 0$ and $X \leftarrow X \rightarrow X \leftarrow X \rightarrow X \leftarrow X \rightarrow 0$
Oblain a chagram
0 < X -> X < X -> - X < X -> 0
$0 \leftarrow X \longrightarrow 0 \leftarrow X \longrightarrow 0 \leftarrow X \longrightarrow 0$
which yields map ZX — S ZXU — W ZX after passing to colimits. Then can define a group law by
How (ZX, Y) x - x How (ZX) = How (ZXL - UZX, Y) he he - How (ZX, Y)
If f. ZX -> 1 than it can be

represented by a diagram X to o Then - I is induced by X to o' and the zero map simply is hop With this This becomes abelien a for flow (ZX, Y). The can show this by eletining moltiplications by: 1) ZX - ZX 4 ZX 2) ZX ->> Z (ZXUZX) ZSZXUZX and showing they are compatible according to Echnique - Hilton





is distinguished off Y 2 = 2 h 2 2 x - 21 is (TR3) If the horis rows are distinguished triangles: x' - > 7' - > Zx' (TR4) Il we have three distinguished triangles X Ty Jon Y/X d ZX 4 952 - 524 d = 24 X gets Z ws Z/X ds ZX there is a trian distinguished triangle Y/x => 2/x => 2/y => 1/zx X got W W Y CP' ZOS X/ZX 2/x 21/2 Y/X -d IX