



Cyprus
University of
Technology

EEN452 - Control and Operation of Electric Power Systems

Part 2A: Synchronous machine model (simplified)

<https://sps.cut.ac.cy/courses/een452/>

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Today's learning objectives

After this part of the lecture and additional reading, you should be able to ...

- ① ... define the simplified synchronous machine model;
- ② ... understand the electromechanical interactions of a synchronous machine in steady-state.

Much of the material was adapted from the courses delivered by Prof. Thierry Van Cutsem at the University of Liege.

Why synchronous machines?

- produce the major part of the electric energy
 - range from a few kVA to a few hundred MVA
 - the biggest are rated 1500 MVA
- play an important role:
 - they impose the frequency of sinusoidal voltages and currents
 - they provide an "energy buffer" (through the kinetic energy stored in their rotating masses)
 - they can produce or consume reactive power (needed to regulate voltage).

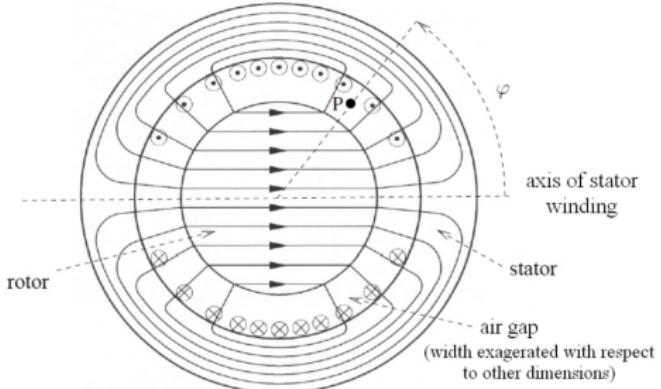
1 Outline

- 1 Principles of operation
- 2 Types of synchronous machines
- 3 Physical windings
- 4 Modelling of machine with magnetically coupled circuits
- 5 Machine in steady-state operation
- 6 Powers and motion in synchronous machine
- 7 Capability curves

1 Magnetic field created by the stator

- stator (or armature) = motionless, separated from the rotor by a small air gap
- subjected to varying magnetic flux → built up of thin laminations to decrease eddy (or Foucault) currents
- equipped with three windings, distributed 120 degrees apart in space.

Magnetic field created by a direct current flowing in one of the stator windings:

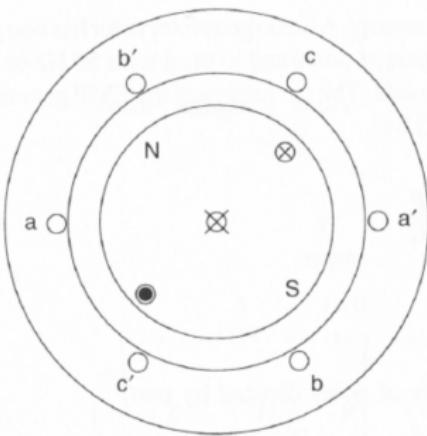


1 Magnetic field created by the stator

The magnetic field lines cross the air gap radially. The amplitude $B(\varphi)$ of the magnetic flux density at point P:

- is a periodic function of φ with period 2π
- this function has a "staircase" shape
- is made as close as possible to a sinusoid, by properly distributing the conductors along the air gap.

Layout of the three phases (each winding is represented by a single turn for clarity):



1 Magnetic field created by the stator

Total flux density created by the three phases at point P corresponding to angle φ :

$$B_{3\varphi}(\varphi) = k_i a \cos(\varphi) + k_i b \cos(\varphi - \frac{2\pi}{3}) + k_i c \cos(\varphi - \frac{4\pi}{3})$$

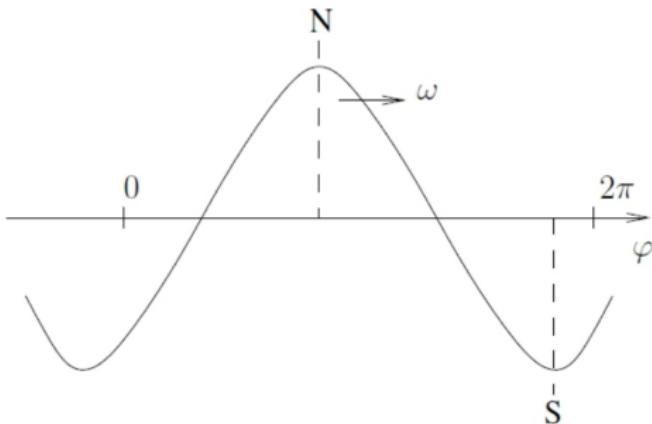
If three-phase alternating currents are flowing in the windings:

$$\begin{aligned} B_{3\varphi}(\varphi) &= \sqrt{2} k I \left[\cos(\omega t + \psi) \cos(\varphi) + \cos(\omega t + \psi - \frac{2\pi}{3}) \cos(\varphi - \frac{2\pi}{3}) \right. \\ &\quad \left. + \cos(\omega t + \psi - \frac{4\pi}{3}) \cos(\varphi - \frac{4\pi}{3}) \right] \\ &= \frac{\sqrt{2} k I}{2} \left[\cos(\omega t + \psi + \varphi) + \cos(\omega t + \psi - \varphi) + \cos(\omega t + \psi + \varphi - \frac{4\pi}{3}) \right. \\ &\quad \left. + \cos(\omega t + \psi - \varphi) + \cos(\omega t + \psi + \varphi - \frac{2\pi}{3}) + \cos(\omega t + \psi - \varphi) \right] \\ &= \frac{3\sqrt{2} k I}{2} \cos(\omega t + \psi - \varphi) \end{aligned}$$

This is the equation of a wave rotating in the air gap at the angular speed ω

1 Magnetic field created by the stator

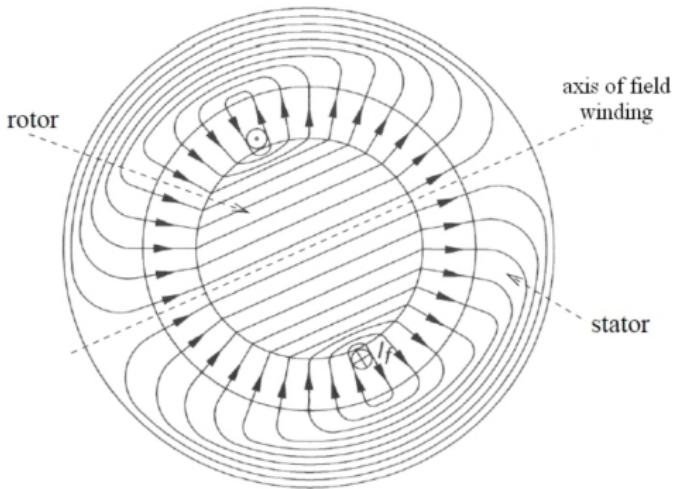
If we "unroll" the air-gap:



- The three-phase alternating currents all together produce the same magnetic field as a magnet (or a coil carrying a direct current) rotating at the angular speed ω
- North pole of magnet \rightarrow maximum of $B(\varphi)$
- South pole of magnet \rightarrow minimum of $B(\varphi)$

1 Magnetic field created by the rotor

Magnetic field created by this direct current (field winding represented by a single turn for clarity):

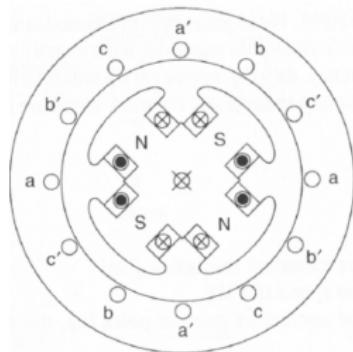


- rotor = rotating part, separated from the rotor by the air gap
- carries a winding in which a direct current flows, in steady-state operation
- referred to as field winding

1 Machines with multiple pairs of poles

Some turbines operate at a lower speed but AC voltages and currents at the stator must keep the same period $T = \frac{1}{f}$

- the rotor carries p **pairs of poles**
- during a period T , the rotor makes only $\frac{1}{p}$ of a whole revolution
- the stator carries p sets of (a, b, c) windings
- one winding spans an angle of π/p radians
- during a period T , each stator winding is still swept by one North and one South pole of rotor



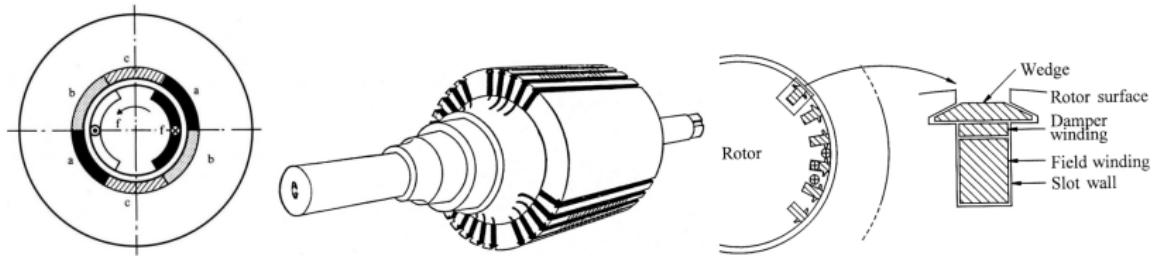
example for $p = 2$

- speed: $\frac{60 \cdot f}{p} \text{ rpm}$
- The various windings relative to a given phase are connected (in series or parallel) to end up with a three-phase machine.

2 Outline

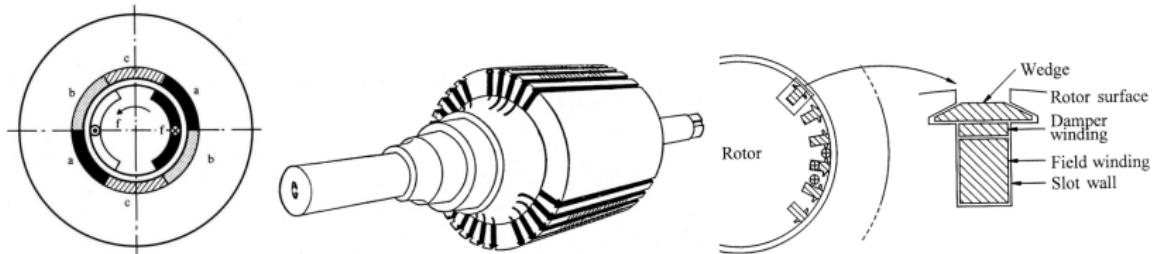
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2 Round-rotor generators (or turbo-alternators)



- Driven by steam or gas turbines, which rotate at high speed
- $p = 1$ (conventional thermal units) or $p = 2$ (nuclear units)
- cylindrical rotor made up of solid steel forging
- diameter << length (centrifugal force)

2 Round-rotor generators (or turbo-alternators)

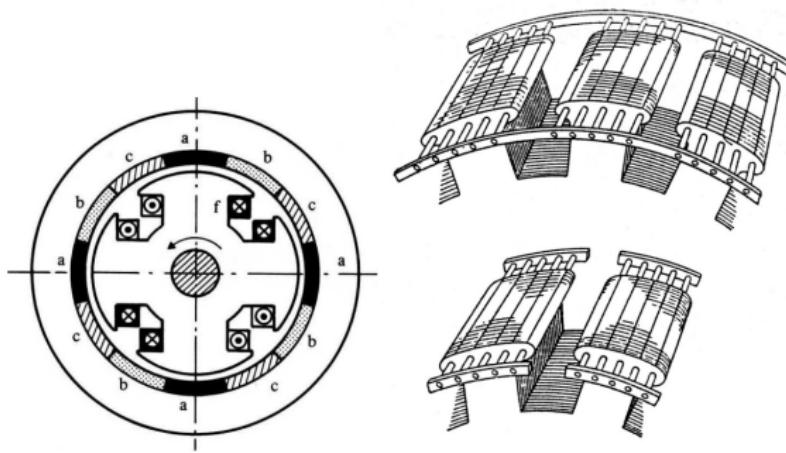


- field winding made up of conductors distributed on the rotor, in milled slots
- even if the generator efficiency is around 99%, the heat produced by Joule losses has to be evacuated.
- Large generators are cooled by hydrogen (heat evacuation 7 times better than air) or water (12 times better) flowing in the hollow stator conductors.

2 Round-rotor generators (or turbo-alternators)

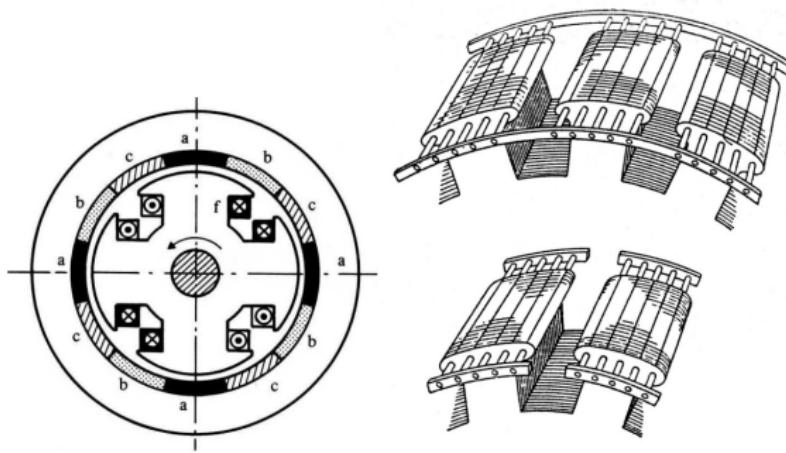


2 Salient-pole generators



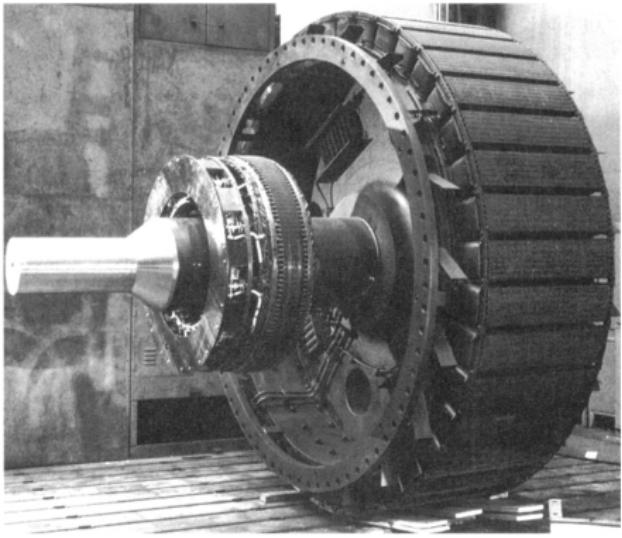
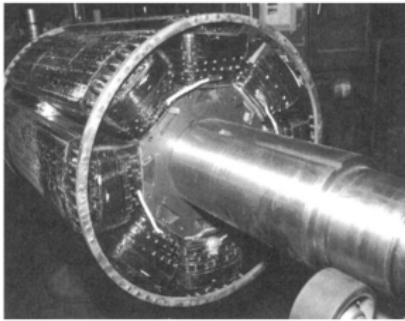
- Driven by hydraulic turbines (or diesel engines), which rotate at low speed
- p is much higher (at least 4) → it is more convenient to have field windings concentrated and placed on the poles
- air gap is not constant: min. in front of a pole, max. in between two poles

2 Salient-pole generators



- poles are shaped to also minimize space harmonics (see slide 6)
- diameter $>>$ length (to have space for the many poles)
- rotor is laminated (poles easier to construct)
- generators usually cooled by the flow of air around the rotor

2 Salient-pole generators



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3 Number of windings

There are typically 5 physical windings on a synchronous machine:

- 3 stator windings (a-phase, b-phase, and c-phase)
- 1 main field winding
- Damper or Amortisseur windings on the pole-faces
 - **round-rotor machines:** copper/brass bars placed in the same slots at the field winding, and interconnected to form a damper cage (similar to the squirrel cage of an induction motor)
 - **salient-pole machines:** copper/brass rods embedded in the poles and connected at their ends to rings or segments
 - They can be continuous or noncontinuous (see fig. in slide 15)

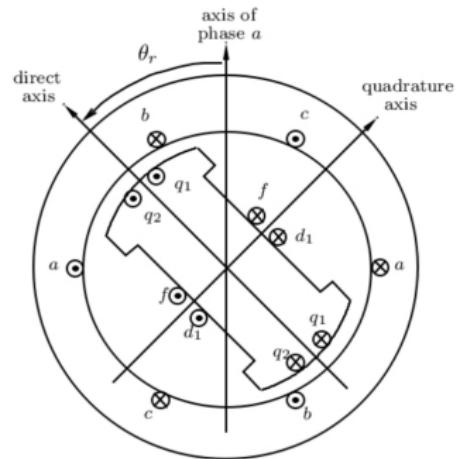
3 Damper windings

- **in perfect steady state:** the magnetic fields produced by both the stator and the rotor are fixed relative to the rotor → no current induced in dampers¹
- **after a disturbance:** the rotor moves with respect to stator magnetic field → currents are induced in the dampers...
... which, according to Lenz's law, create a damping torque helping the rotor to align on the stator magnetic field
- **Round-rotor generators:** the solid rotor offers a path for eddy currents, which produce an effect similar to those of amortisseurs.

¹Amortisseur means "dead"

3 Modeled windings

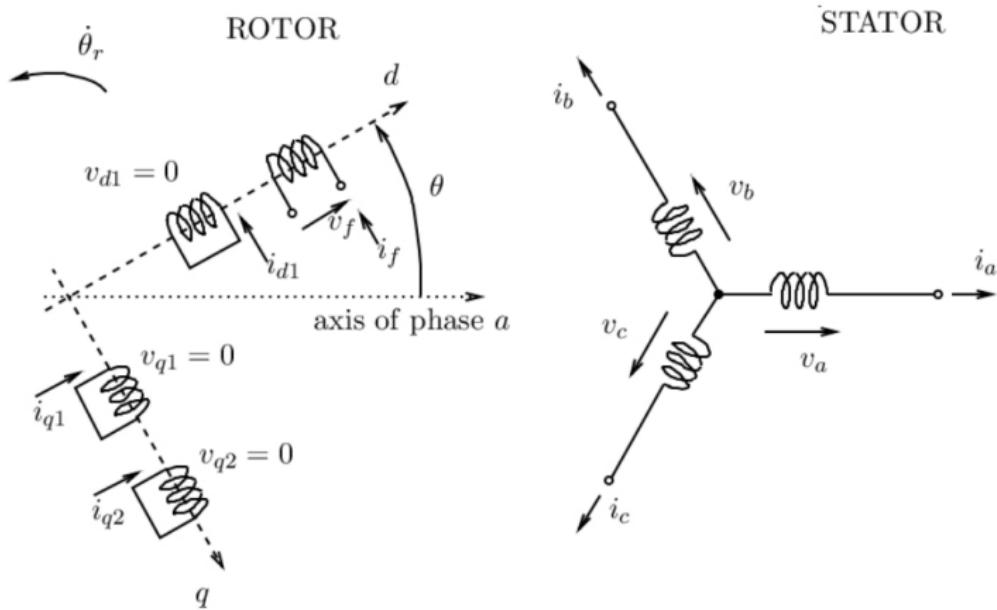
Number of rotor windings = degree of sophistication of model. But more detailed model → more data are needed while measurement devices can be connected only to the field winding.



- 3 stator windings
- Most widely used model: 3 or 4 rotor windings:
 - f : field winding, d_1, q_1 : damper windings
 - q_2 : accounts for eddy currents in rotor – not used in (laminated) salient-pole generators

3 Modeled windings

Detailed model (next lesson):

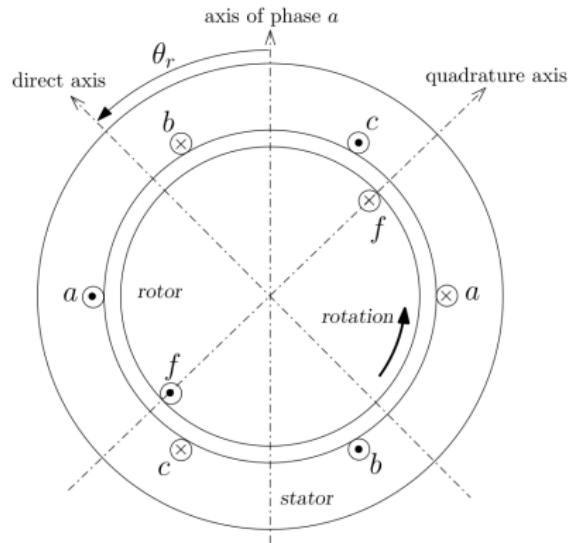


4 Outline

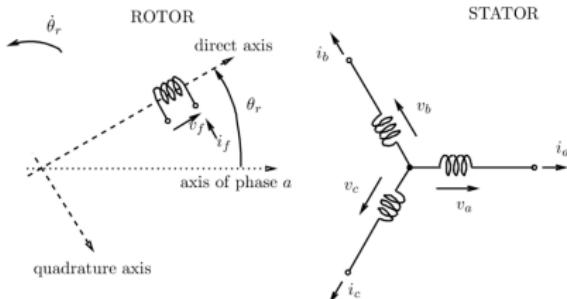
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4 Simplifying assumptions

- round rotor
- saturation of magnetic material neglected
- on the rotor: field winding only
(acceptable since focus is on
steady-state operation)
- single pair of poles (does not affect the
electrical behaviour)



4 Relations between voltages, fluxes and currents



Stator:

$$v_a = -R_a i_a - \frac{d\psi_a}{dt} \quad v_b = -R_b i_b - \frac{d\psi_b}{dt} \quad v_c = -R_c i_c - \frac{d\psi_c}{dt}$$

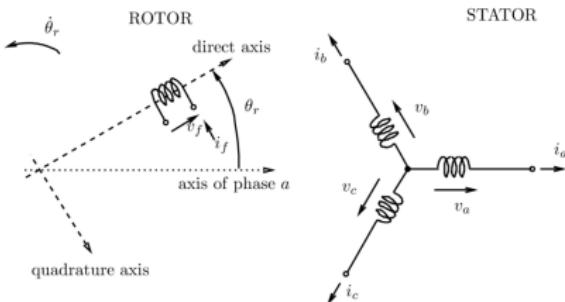
R_a : Resistance of each phase ψ_a , ψ_b and ψ_c : flux linkages in phases

Field winding:

$$v_f = -R_f i_f - \frac{d\psi_f}{dt}$$

R_f : Resistance of winding ψ_f : flux linkages in winding

4 Relations between voltages, fluxes and currents



- The magnetic circuit and all rotor winding circuits are symmetrical with respect to the polar and inter-polar (between-poles) axes
- We give these axes special names:
 - Polar axis: Direct, or d-axis
 - Interpolar axis: Quadrature, or q-axis
- q-axis is 90° from the d-axis but can be modeled both as *leading* or *lagging*. Both assumptions are correct and used by textbooks. → in this course, we assume **lagging**.

4 Inductance matrix

$$\begin{bmatrix} \psi_a \\ \psi_b \\ \psi_c \\ \psi_f \end{bmatrix} = \begin{bmatrix} L_o & -L_m & -L_m & L_{af} \cos \theta_r \\ -L_m & L_o & -L_m & L_{af} \cos(\theta_r - \frac{2\pi}{3}) \\ -L_m & -L_m & L_o & L_{af} \cos(\theta_r - \frac{4\pi}{3}) \\ L_{af} \cos \theta_r & L_{af} \cos(\theta_r - \frac{2\pi}{3}) & L_{af} \cos(\theta_r - \frac{4\pi}{3}) & L_{ff} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_f \end{bmatrix}$$

$\underbrace{\quad\quad\quad}_{\mathbf{L}(\theta_r)}$

where $L_o, L_m > 0$.

- Self-inductance of any stator winding is constant (due to round rotor)
- mutual inductance between any two phases is constant (due to round rotor)
- ... and negative since a positive current i_x in phase x creates a negative flux ψ_y in phase y ($x \neq y$)

4 Inductance matrix

$$\begin{bmatrix} \psi_a \\ \psi_b \\ \psi_c \\ \psi_f \end{bmatrix} = \begin{bmatrix} L_o & -L_m & -L_m & L_{af} \cos \theta_r \\ -L_m & L_o & -L_m & L_{af} \cos(\theta_r - \frac{2\pi}{3}) \\ -L_m & -L_m & L_o & L_{af} \cos(\theta_r - \frac{4\pi}{3}) \\ L_{af} \cos \theta_r & L_{af} \cos(\theta_r - \frac{2\pi}{3}) & L_{af} \cos(\theta_r - \frac{4\pi}{3}) & L_{ff} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_f \end{bmatrix}$$

$\underbrace{\quad\quad\quad}_{\mathbf{L}(\theta_r)}$

where $L_o, L_m > 0$.

- self-inductance of field winding is constant (path of magnetic field identical whatever the position of the rotor)
- mutual inductance between one phase and the field winding is maximum and positive when $\theta_r = 0$, zero when $\theta_r = \frac{\pi}{2}$, minimum and negative when $\theta_r = \pi$

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5 Fundamental equations

- rotation speed equal to nominal angular frequency:

$$\dot{\theta}_r = \omega_N \quad \theta_r = \theta_r^0 + \omega_N t$$

θ_r^0 : rotor position at $t = 0$

- constant direct current in field winding: $i_f = I_f$
- balanced three-phase voltages and currents in stator:

$$v_a(t) = \sqrt{2}V \cos(\omega_N t + \theta) \qquad i_a(t) = \sqrt{2}I \cos(\omega_N t + \psi)$$

$$v_b(t) = \sqrt{2}V \cos(\omega_N t + \theta - \frac{2\pi}{3}) \quad i_b(t) = \sqrt{2}I \cos(\omega_N t + \psi - \frac{2\pi}{3})$$

$$v_c(t) = \sqrt{2}V \cos(\omega_N t + \theta - \frac{4\pi}{3}) \quad i_c(t) = \sqrt{2}I \cos(\omega_N t + \psi - \frac{4\pi}{3})$$

with the corresponding phasors:

$$\underline{V} = V e^{j\theta} \qquad \underline{I} = I e^{j\psi}$$

5 Flux linkage in one stator winding (phase a)

$$\begin{aligned}\psi_a = & L_o \sqrt{2} I \cos(\omega_N t + \psi) - L_m \sqrt{2} I \cos(\omega_N t + \psi - \frac{2\pi}{3}) \\ & - L_m \sqrt{2} I \cos(\omega_N t + \psi - \frac{4\pi}{3}) + L_{af} \cos(\omega_N t + \theta_r^o) I_f\end{aligned}$$

Adding and subtracting $L_m \sqrt{2} I \cos(\omega_N t + \psi)$ yields:

$$\begin{aligned}\psi_a = & L_o \sqrt{2} I \cos(\omega_N t + \psi) + L_m \sqrt{2} I \cos(\omega_N t + \psi) \\ & - L_m \sqrt{2} I \underbrace{\left(\cos(\omega_N t + \psi) + \cos(\omega_N t + \psi - \frac{2\pi}{3}) + \cos(\omega_N t + \psi - \frac{4\pi}{3}) \right)}_0 \\ & + L_{af} I_f \cos(\omega_N t + \theta_r^o) \\ = & \underbrace{\sqrt{2}(L_o + L_m) I \cos(\omega_N t + \psi)}_{\psi_a^s} + \underbrace{L_{af} I_f \cos(\omega_N t + \theta_r^o)}_{\psi_a^r}\end{aligned}$$

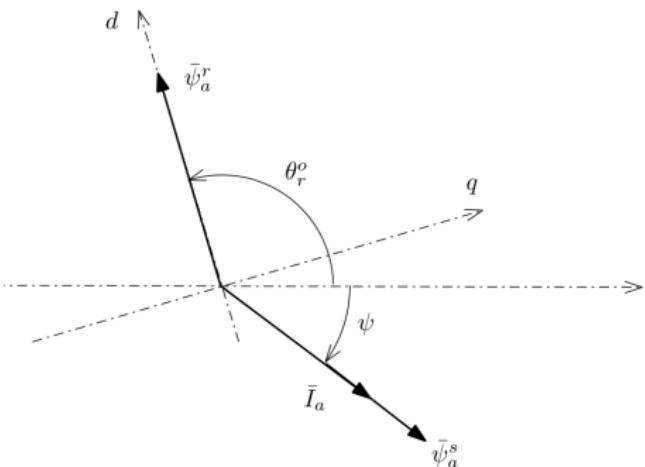
ψ_a^s : flux of the rotating field produced by the three stator currents

ψ_a^r : flux of the field created by the current i_f

5 Flux linkage in one stator winding (phase a)

Both flux components being sinusoidal functions of time (with angular frequency ω_N), they can be characterized by phasors:

$$\underline{\psi}_a^s = (L_o + L_m)I e^{j\psi} \quad \underline{\psi}_a^r = \frac{L_{af}}{\sqrt{2}} I_f e^{j\theta_r^o}$$



Phasor diagram:

Horizontal axis

= axis on which rotating vectors are projected

= axis to which the rotor position is referred, i.e. axis of phase a

5 Flux linkage in field winding

$$\begin{aligned}
 \psi_f &= L_{ff} I_f + L_{af} \cos(\omega_N t + \theta_r^o) \sqrt{2} I \cos(\omega_N t + \psi) \\
 &\quad + L_{af} \cos(\omega_N t + \theta_r^o - \frac{2\pi}{3}) \sqrt{2} I \cos(\omega_N t + \psi - \frac{2\pi}{3}) \\
 &\quad + L_{af} \cos(\omega_N t + \theta_r^o - \frac{4\pi}{3}) \sqrt{2} I \cos(\omega_N t + \psi - \frac{4\pi}{3}) \\
 &= L_{ff} I_f + \frac{\sqrt{2} L_{af}}{2} I [\cos(\theta_r^o - \psi) + \cos(2\omega_N t + \theta_r^o + \psi)] \\
 &\quad + \frac{\sqrt{2} L_{af}}{2} I \left[\cos(\theta_r^o - \psi) + \cos(2\omega_N t + \theta_r^o + \psi - \frac{4\pi}{3}) \right] \\
 &\quad + \frac{\sqrt{2} L_{af}}{2} I \left[\cos(\theta_r^o - \psi) + \cos(2\omega_N t + \theta_r^o + \psi + \frac{4\pi}{3}) \right] \\
 &= \underbrace{L_{ff} I_f}_{\psi_f^r} + \underbrace{\frac{3\sqrt{2} L_{af}}{2} I \cos(\theta_r^o - \psi)}_{\psi_f^s}
 \end{aligned}$$

ψ_f^s : flux of the rotating field produced by the three stator currents; constant magnitude; at an angle $\theta_r^o - \psi$ wrt to field winding

ψ_f^r : flux created by field current

5 Voltage-current relation at stator

Replacing v_a , i_a , and ψ_a by their expressions:

$$\begin{aligned}\sqrt{2}V \cos(\omega_N t + \theta) = & -R_a \sqrt{2}I \cos(\omega_N t + \psi) + \sqrt{2}\omega_N(L_o + L_m)I \sin(\omega_N t + \psi) \\ & + \sqrt{2} \frac{\omega_N L_{af}}{\sqrt{2}} I_f \sin(\omega_N t + \theta_r^o)\end{aligned}$$

Let's define:

- $X = \omega_N(L_o + L_m)$: the synchronous reactance of the machine
- $E_q = \frac{\omega_N L_{af}}{\sqrt{2}} I_f$: RMS value of an e.m.f. proportional to field current I_f

The above equation becomes:

$$\begin{aligned}\sqrt{2}V \cos(\omega_N t + \theta) = & -R_a \sqrt{2}I \cos(\omega_N t + \psi) + \sqrt{2}X I \cos(\omega_N t + \psi - \frac{\pi}{2}) \\ & + \sqrt{2}E_q \cos(\omega_N t + \theta_r^o - \frac{\pi}{2})\end{aligned}$$

5 Voltage-current relation at stator

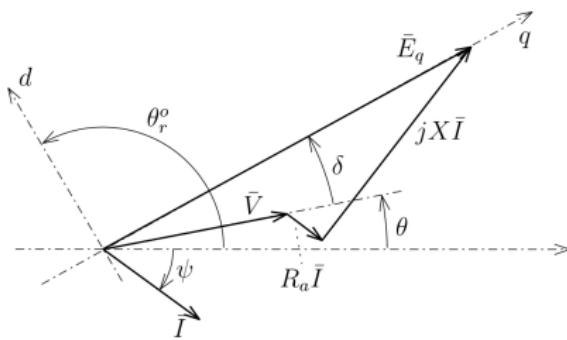
The corresponding phasor equation is:

$$V e^{j\theta} = -R_a I e^{j\psi} + X I e^{j\psi} e^{-j\frac{\pi}{2}} + E_q e^{j(\theta_r^o - \frac{\pi}{2})}$$

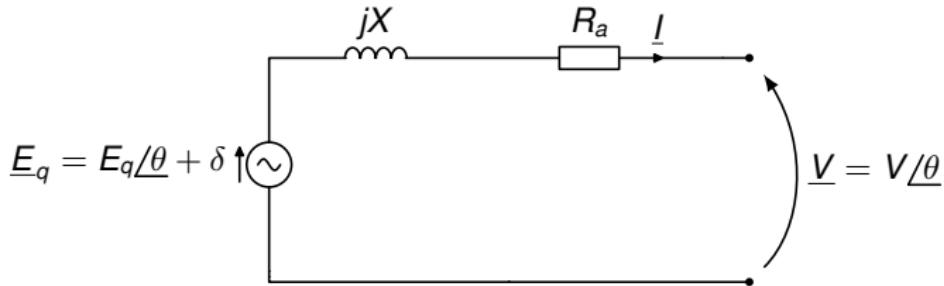
or, simply:

$$\underline{V} = -R_a \underline{I} - jX \underline{I} + \underline{E}_q$$

where $\underline{E}_q = E_q e^{j(\theta_r^o - \frac{\pi}{2})}$ is the phasor of the e.m.f. E_q , lying on the q axis



5 Per-phase equivalent circuit



- The synchronous reactance X characterizes the steady-state operation of the machine
- δ is the phase shift between the internal e.m.f. E_q and the terminal voltage V
- δ is called the internal angle, load angle, or power angle of the machine

- Nominal voltage V_N : voltage for which the machine has been designed (in particular its insulation).
The real voltage may deviate from this value by a few %
- nominal current I_N : current for which machine has been designed (in particular the cross-section of its conductors).
Maximum current that can be accepted without limit in time
- nominal apparent power $S_N = \sqrt{3}V_N I_N$

The machine parameters in per-unit on the base ($S_B = S_N$, $V_B = V_N/\sqrt{3}$):

- $R_a \approx 0.005$ pu
- $X \approx 1.5 - 2.5$ pu (for a round-rotor machine)

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6 Power balance of the stator

$$p_{r \rightarrow s} = p_T + p_{Js} + \frac{dW_{ms}}{dt}$$

where

- $p_{r \rightarrow s}$: power transfer from rotor to stator
- p_T : three-phase instantaneous power leaving the stator
- p_{Js} : Joule losses in stator windings
- W_{ms} : magnetic energy stored in the stator windings

The nature of $p_{r \rightarrow s}$

- mechanical power for sure (torque applied to rotating masses)
- is there some electromagnetic transfer of power (like in a transformer)?

6 Power balance of the rotor

$$p_f + P_m = p_{Jf} + \frac{dW_{mf}}{dt} + \frac{dW_c}{dt} + p_{r \rightarrow s}$$

where

- P_m : mechanical power provided by the turbine
- p_f : electrical power provided to the field winding by the excitation system
- p_{Jf} : Joule losses in the field winding
- W_{mf} : magnetic energy stored in the field winding
- W_c : kinetic energy of all rotating masses (generator and turbine)

Total electromagnetic energy stored in the machine:

$$\begin{aligned}
 W_{m,tot} &= \frac{1}{2} \begin{bmatrix} i_a & i_b & i_c & i_f \end{bmatrix} \mathbf{L}(\theta_r) \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_f \end{bmatrix} = \frac{1}{2} \begin{bmatrix} i_a & i_b & i_c & i_f \end{bmatrix} \begin{bmatrix} \psi_a \\ \psi_b \\ \psi_c \\ \psi_f \end{bmatrix} \\
 &= \underbrace{\frac{1}{2} (i_a \psi_a + i_b \psi_b + i_c \psi_c)}_{W_{ms}} + \underbrace{\frac{1}{2} i_f \psi_f}_{W_{mf}}
 \end{aligned}$$

6 Motion equation

$$J \frac{d^2\theta_r}{dt^2} = T_m - T_e$$

where

J : moment of inertia of all rotating masses

T_m : mechanical torque applied to the rotor by the turbine

T_e : electromagnetic torque applied to the rotor by the generator

Multiplying by the rotor speed $d\theta_r/dt$:

$$J \frac{d\theta_r}{dt} \frac{d^2\theta_r}{dt^2} = \frac{d\theta_r}{dt} T_m - \frac{d\theta_r}{dt} T_e$$

$$\Leftrightarrow \frac{dW_c}{dt} = P_m - \frac{d\theta_r}{dt} T_e$$

and the power balance of the rotor becomes:

$$p_f + \frac{d\theta_r}{dt} T_e = p_{Jf} + \frac{dW_{mf}}{dt} + p_{r \rightarrow s}$$

6 Power balance of stator

$$\begin{aligned}
 \frac{1}{2} i_a \psi_a &= (L_o + L_m) I^2 \cos^2(\omega_N t + \psi) + \frac{\sqrt{2}}{2} L_{af} I_f I \cos(\omega_N t + \theta_r^o) \cos(\omega_N t + \psi) \\
 &= \frac{1}{2} (L_o + L_m) I^2 + \frac{1}{2} (L_o + L_m) I^2 \cos(2\omega_N t + 2\psi) + \\
 &\quad \frac{\sqrt{2}}{4} L_{af} I_f I [\cos(\theta_r^o - \psi) + \cos(2\omega_N t + \theta_r^o + \psi)]
 \end{aligned}$$

By doing the same derivation for phases b and c, and adding all three results:

$$W_{ms} = \frac{1}{2} (i_a \psi_a + i_b \psi_b + i_c \psi_c) = \frac{3}{2} (L_o + L_m) I^2 + \frac{3\sqrt{2}}{4} L_{af} I_f I \cos(\theta_r^o - \psi)$$

$$W_{ms} \text{ is constant, i.e. } \frac{dW_{ms}}{dt} = 0$$

6 Power balance of stator

In three-phase balanced operation:

$$p_T = 3P$$

where P is the active power produced by one phase.

Hence, the power balance of the stator simply becomes :

$$p_{r \rightarrow s} = 3P + p_{Js}$$

6 Power balance of rotor

$$W_{mf} = \frac{1}{2} i_f \psi_f = \frac{1}{2} L_{ff} I_f^2 + \frac{3\sqrt{2}}{4} L_{af} I_f I_f \cos(\theta_r^o - \psi)$$

W_{mf} is constant, i.e. $\frac{dW_{mf}}{dt} = 0$

$$\frac{d\psi_f}{dt} = 0 \quad \Rightarrow \quad V_f = R_f I_f \quad \Rightarrow \quad p_f = R_f I_f^2 = p_{Jf}$$

In steady state, the power entering the field winding is dissipated in Joule losses!

The field current aims at “magnetizing” the rotor, allowing the torque T_e to be created, but the field winding does not exchange power with the other windings.

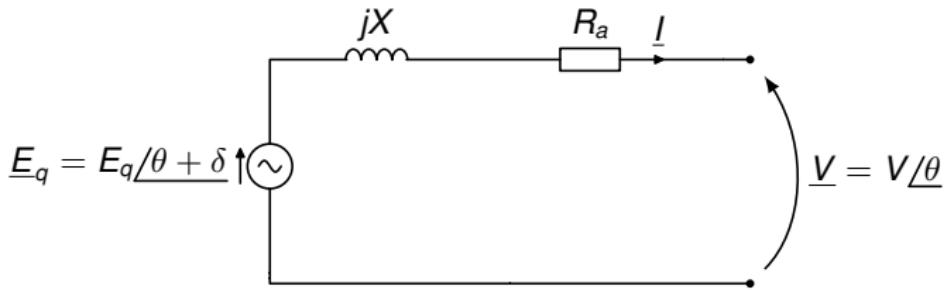
$$\frac{d\theta_r}{dt} = \omega_N \quad \frac{dW_c}{dt} = 0 \quad T_m = T_e \quad P_m = \omega_N T_e = \omega_N T_m$$

Hence, the power balance of the rotor simply becomes:

$$p_{r \rightarrow s} = \omega_N T_e = \omega_N T_m = P_m$$

where power $p_{r \rightarrow s}$ transferred from rotor to stator is purely mechanical!

6 Expression of active and reactive powers



Assuming $R_a \approx 0$, active and reactive power in per-unit can be given as:

$$P = -\frac{VE_q}{X} \sin(\theta - (\theta + \delta)) = \frac{VE_q}{X} \sin(\delta)$$

$$Q = -\frac{V^2}{X} + \frac{VE_q}{X} \cos(\theta - (\theta + \delta)) = -\frac{V^2}{X} + \frac{VE_q}{X} \cos(\delta)$$

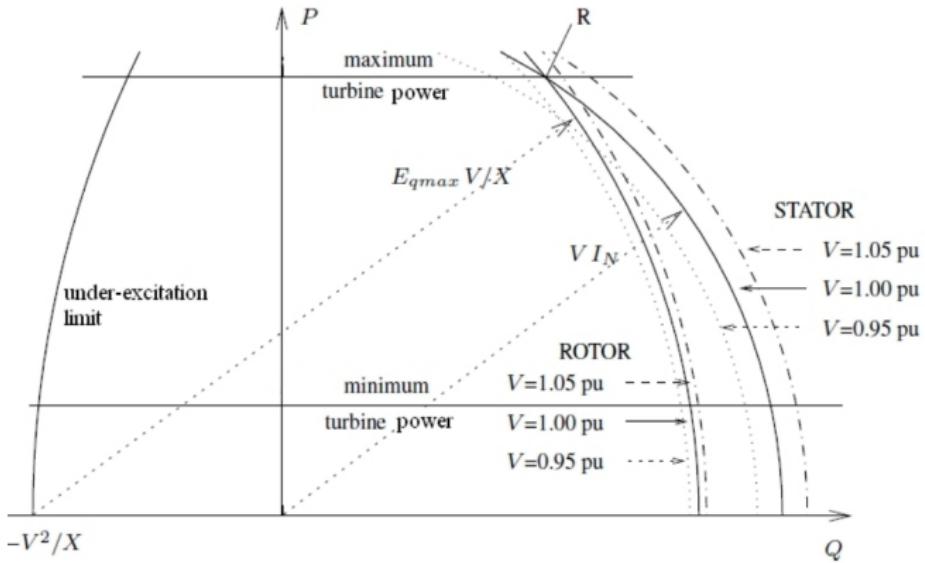
7 Outline

- 1 Principles of operation
- 2 Types of synchronous machines
- 3 Physical windings
- 4 Modelling of machine with magnetically coupled circuits
- 5 Machine in steady-state operation
- 6 Powers and motion in synchronous machine
- 7 Capability curves

7 Capability curves

Seen from the network, a generator is characterized by three variables: V , P and Q

The capability curves define the set of admissible operating points in the (P, Q) space, ***under constant voltage*** V (justified by automatic voltage regulator)



7 Capability curves

Stator (heating) limit

stator current $I = I_N$ in per-unit: $S^2 = P^2 + Q^2 = V^2 I_N^2$

Rotor (heating) limit

$$\text{field current } I_f = I_{fmax} \Rightarrow E_q = E_{qmax} = \frac{\omega_N L_{af}}{\sqrt{2}} I_{fmax}$$

With the same simplifying assumptions as before, and with $R_a = 0$:

$$P = \frac{E_{qmax} V}{X} \sin(\delta) \quad Q = \frac{E_{qmax} V}{X} \cos(\delta) - \frac{V^2}{X}$$

after eliminating δ :

$$\left(\frac{V E_{qmax}}{X} \right)^2 = \left(Q + \frac{V^2}{X} \right)^2 + P^2$$

7 Capability curves

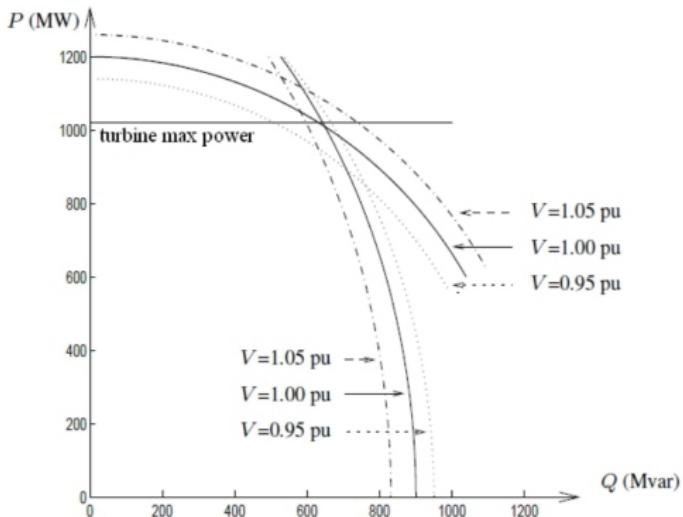
- Lower limit on active power caused by stability of combustion in thermal power plants
- maximum reactive power *increases* when the active power *decreases*
 - to relieve an overloaded machine, P can be decreased but this power has to be produced by some other generators
- for a given value of P , the maximum reactive power increases with V
 - this holds true under the simplifying assumption of a non saturated machine; see next slide for a case with saturation
- in practice, under $V = 1 \text{ pu}$, the two-by-two intersection points of respectively the turbine, the rotor and the stator limits are close to each other ("coherent" design of stator and rotor)
- the stator limits can be increased by a stronger cooling (e.g., higher hydrogen pressure in stator windings)

Under-excitation limit

Corresponds to a stability, not a thermal limit: absorbing more $Q \Rightarrow$ decreasing $E_q \Rightarrow$ decreasing $i_f \rightarrow$ maximum torque T_e decreases \Rightarrow risk of losing synchronism.

7 Capability curves

Capability curves ($Q > 0$ part only) of a real machine with saturation taken into account



- the overall shape of the curves is the same
- but the rotor limit becomes more constraining when V increases