Cyprus
University of
Technology

EEN452 - Control and Operation of Electric Power Systems
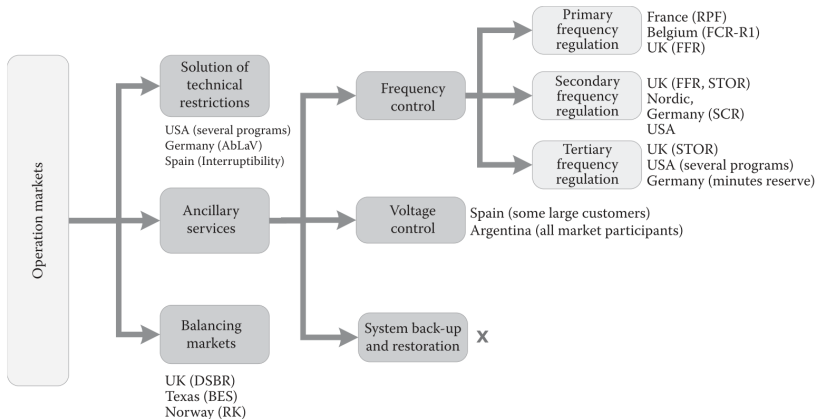Part 3: Frequency control
https://sps.cut.ac.cy/courses/een452/

Dr Petros Aristidou
Department of Electrical Engineering, Computer Engineering & Informatics
Last updated: March 17, 2025

Demand response for ancillary services

Source: Gómez-Expósito, A., Conejo, A. J., & Cañizares, C. A. (2018). Electric Energy Systems Analysis and Operation. CRC Press.

After this part of the lecture and additional reading, you should be able to ...

1   ... understand the fundamentals of frequency and power control;

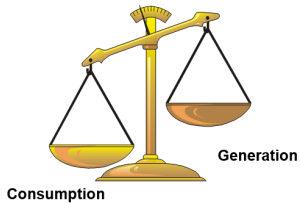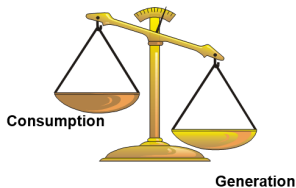2   ... perform calculations concerning primary and secondary frequency
control.

Cyprus
University of
Technology

Frequency must remain close to its nominal value:

- for the correct operation of loads (rotating machines)
- because it is an indication that the (active) power production balances the (active) power consumption (including network losses).

What is the link between frequency and active power balance?

- Electrical energy cannot be stored; it is produced when it is requested
- in the very first instants after a disturbance: the missing (resp. excess) amount of energy is taken from (resp. stored into) the rotating masses of the synchronous machines (see previous chapter)
- this causes a variation of their speed of rotation, and hence of frequency

Consumption

Generation

-250 -200 -150 -100 -50 0 50 100 150 200 250

The frequency increases

Generation

Consumption

-250 -200 -150 -100 -50 0 50 100 150 200 250

The frequency decreases

https://extranet.nationalgrid.com/RealTime (Data last updated on : 06/02/2021 19:37:00)
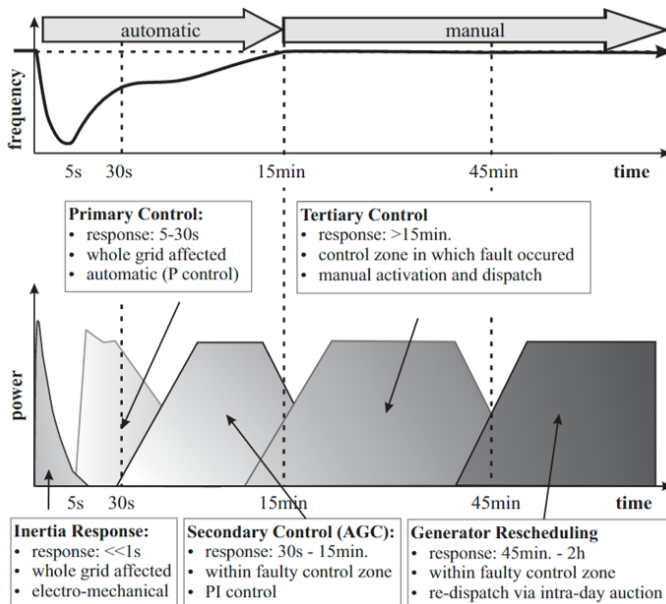
Cyprus
University of
Technology

The kinetic energy of the rotor is given by:

$$W_c = \frac{1}{2} J \omega_r^2 \cdot 10^{-6} \quad \text{MJ}$$

where
J: moment of inertia of all the rotating masses in kg-m^2
ω_r: synchronous speed of rotor in rad-mech/s

Converting to electrical speed with $\omega_s = (P/2)\omega_r$, gives:

$$W_c = \frac{1}{2} \left(J(2/P)^2 \omega_s \cdot 10^{-6} \right) \omega_s \cong \frac{1}{2} \left(J(2/P)^2 \omega_N \cdot 10^{-6} \right) \omega_s = \frac{1}{2} M \omega_s$$

where
$M = J(2/P)^2 \omega_N \cdot 10^{-6}$: moment of inertia in MJ-s/rad-elec
P: the number of poles
ω_s: the rotor speed in rad-elec/s
ω_N: the synchronous speed in rad-elec/s

The inertia constant of the machine, is defined as:

$$S_N H = W_c = \frac{1}{2} M \omega_s \quad \text{MJ}$$

where
S_N: the machine rating in MVA
$H = W_c/S_N$: the inertia constant in MJ/MVA or MW-s/MVA or pu-s

It follows:

$$M = \frac{2 S_N H}{\omega_s} \text{ MJ-s/rad-elec}$$

Setting $S_B = S_N$ (S_B the base power for the per-unit system):

$$M_{pu} = \frac{2H}{\omega_s} \ s^2/\text{rad-elec} \approx \frac{2H}{\omega_N} \ s^2/\text{rad-elec}$$

Example constants for various synchronous machines:

Table 12.1 Typical inertia constants of synchronous machines*

Type of Machine		Intertia Constant H Stored Energy in MW Sec per MVA**
Turbine Generator		
Condensing	1,800 rpm	9-6
	3,000 rpm	7-4
Non-Condensing	3,000 rpm	4-3
Water wheel Generator		
Slow-speed (< 200 rpm)		2-3
High-speed (> 200 rpm)		2-4
Synchronous Condenser***		
Large		1.25
Small		1.00
Synchronous Motor with load varying from		
1.0 to 5.0 and higher for heavy flywheels		2.00

From previous chapter, we know:

$$\frac{dW_c}{dt} = P_m - P_e$$

where
P_m: mechanical power applied to the rotor by the turbine
P_e: electrical power output of the stator (ignoring losses)

This leads to:

$$P_m - P_e = \frac{dW_c}{dt} = J\omega_r \frac{d^2\theta_r}{dt^2} = \left(J(2/P)^2 \omega_N \cdot 10^{-6}\right)\frac{d^2\theta_e}{dt^2} = M\frac{d^2\theta_e}{dt^2}$$

where
θ_e: rotor angle in rad-elec

Or, in per-unit:

$$P_m^{pu} - P_e^{pu} = M_{pu}\frac{d^2\theta_e}{dt^2} = M_{pu}\frac{d^2\delta}{dt^2} = M_{pu}\frac{d\omega_s}{dt} = \frac{2H}{\omega_N}\frac{d\omega_s}{dt}$$

where

$$\frac{d\delta}{dt} = \omega_s(t) - \omega_N$$

To account for mechanical rotational loss due to windage and friction, we usually add a frequency-dependent term D:

$$P_m^{pu} - P^{pu} - \frac{D}{\omega_N}\omega_s = M_{pu}\frac{d\omega_s}{dt} = \frac{2H}{\omega_N}\frac{d\omega_s}{dt}$$

The value of D/ω_N has typical values $[0, 2]$. The units of D are per unit power.

The Rate of Change of Frequency (ROCOF) defines the initial slope of the frequency after a power imbalance (ΔP_{pu}). Considering a generator connected to a load with $D = 0$:

$$\text{RoCoF} = \frac{d\omega_s}{dt} = \frac{\omega_N}{2H}\left(P_m^{pu} - P_e^{pu}\right) = \frac{\omega_N}{2H}\Delta P^{pu}$$

2 Multi-machine system

When we have multiple synchronous machines in the system, each one will have its own speed dictated by its own swing equation and parameters (H_i, $S_{N,i}$, $\omega_{s,i}$).
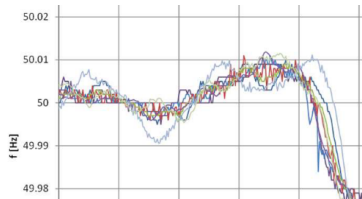

Which one do we use?

We can compute the inertia constant of a single machine referred to the common system as:

$$H_i' = \frac{H_i S_{N,i}}{\sum_{i=1}^{n} S_{N,i}}$$

Then, we can define the "center of inertia" (COI) frequency as a representative average frequency of the system:

$$\omega_{coi} = \frac{\sum_{i=1}^{n} H_i' \cdot \omega_{s,i}}{\sum_{i=1}^{n} H_i'}$$

The total system inertia is computed as:

$$H_{sys} = \frac{\sum_{i=1}^{n} H_i S_{N,i}}{S_{sys}} \qquad S_{sys} = \sum_{i=1}^{n} S_{N,i}$$

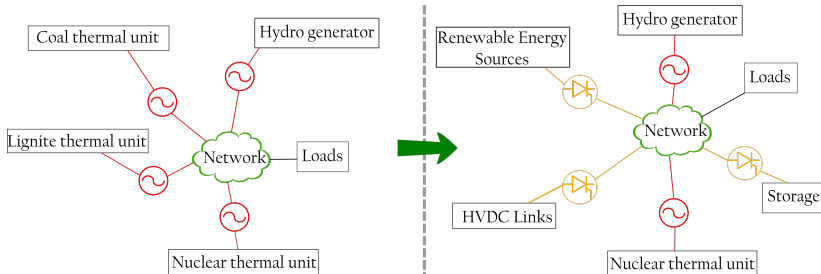This leads to the COI swing equation:

$$\text{RoCoF} = \frac{d\omega_{coi}}{dt} = \frac{\omega_N}{2H_{sys}} \Delta P_{sys}^{pu}$$

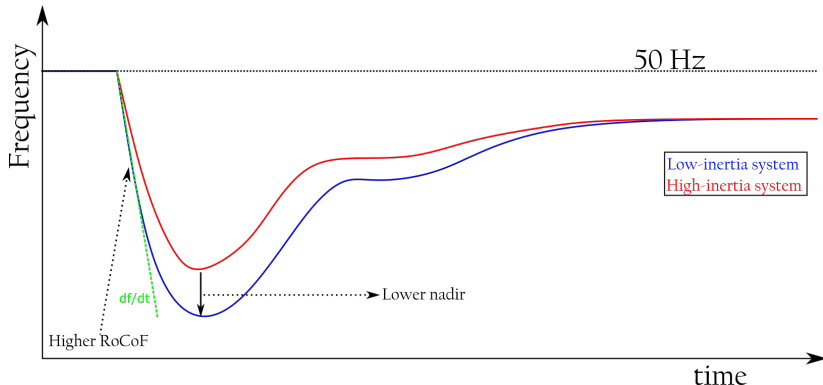With the frequency at time ΔT after the power imbalance (assuming the imbalance remains constant):

$$\omega_{coi}(t_0 + \Delta T) = \frac{\omega_N}{2H_{sys}} \Delta P_{sys}^{pu} \cdot \Delta T + \omega_{coi}(t_0)$$

- System inertia is crucial to retain the frequency initially after a power mismatch. It represents an energy storage buffer that stabilizes the system.

- Conventional units (synchronous-machine-based) are gradually displaced or switched off in favour of renewable generation units (power-electronic interfaced) with lower marginal costs.

- Power-electronic-interfaced generators do not inherently contribute to the system inertia as they lack kinetic inertia.

Cyprus
University of
Technology

- Lower system inertia leads to higher RoCoF (see slide 17) and higher frequency nadir.

- This can trigger frequency-based protections of generators and loads, damage generators due to vibrations, lead to motor disconnections, lead to sub-optimal operation of components, etc.

***We need to add inertia to the system!***

Solutions?

- Add power sources or energy buffers that can provide kinetic inertia inherently (flywheels, synchronous condensers, etc.).

- Program power-electronic interfaced generators and storage devices to ***emulate*** inertial response. This behaviour is called *virtual* or *synthetic inertia*.

- The power-electronic interfaced resources need to have enough energy stored (W_V) to provide virtual inertia (H_V) and they need to release energy in response to the RoCoF of energy.
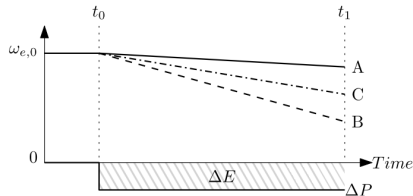
$$H_{sys} = \frac{\sum_{i=1}^{n_c} W_{c,i} + \sum_{i=1}^{n_V} W_{V,i}}{S_{sys}}$$

$$= \frac{\sum_{i=1}^{n_c} H_{c,i} S_{c,i} + \sum_{i=1}^{n_V} H_{V,i} S_{V,i}}{S_{sys}}$$

Virtual inertia
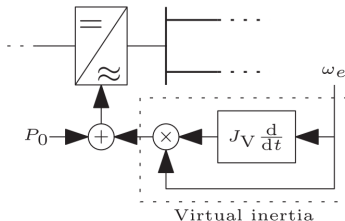
2 System inertia in future power systems
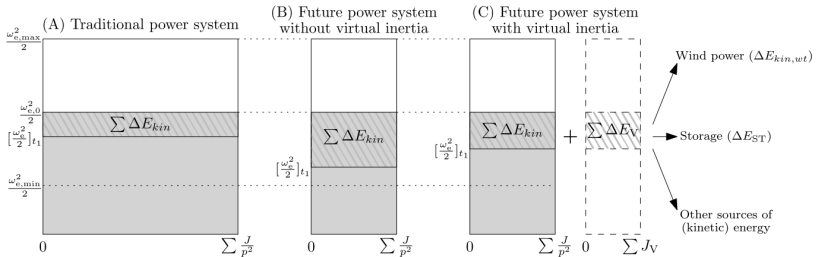
(a) Schematic representation of (kinetic) energy exchange

(b) Influence on power system frequency

Tielens, P. (2017). "Operation and control of power systems with low synchronous inertia", Doctoral thesis, KU Leuven.

Primary Control:
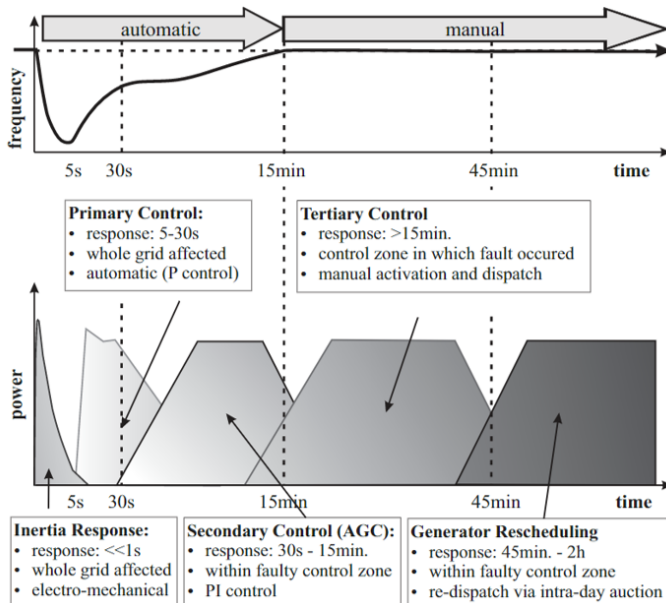- response: 5-30s
- whole grid affected
- automatic (P control)

Tertiary Control
- response: >15min.
- control zone in which fault occured
- manual activation and dispatch

Inertia Response:
- response: <<1s
- whole grid affected
- electro-mechanical

Secondary Control (AGC):
- response: 30s - 15min.
- within faulty control zone
- PI control

Generator Rescheduling
- response: 45min. - 2h
- within faulty control zone
- re-dispatch via intra-day auction

What will happen to the frequency if the imbalance is not "fixed"?

$$\frac{d\omega_s}{dt} = \frac{\omega_N}{2H} \left(P_m^{pu} - P_e^{pu} \right)$$

$\rightarrow$ The frequency will keep decreasing (or increasing) until it reaches the protection limits and the system will go into emergency mode (or blackout).
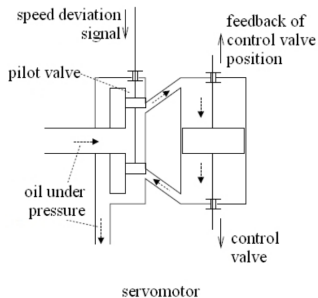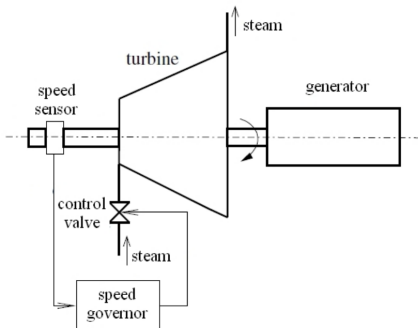
How do we "fix" the imbalance?

1. We can modify the mechanical power input P_m^{pu} by changing the setpoints of the prime mover (e.g., adjust the steam/water/gas flow in the turbines). This is automatically done by devices called ***speed governors*** (ρυθμιστές ταχύτητας).

2. We can modify the electrical power output P_e^{pu} by disconnecting (e.g., under-frequency load shedding schemes) or adjusting (e.g., flexible loads) electrical loads.

$\rightarrow$ Unscheduled disconnection of loads is not desirable because it affects the power quality of consumers and it's used only as a last resort for protection purposes.
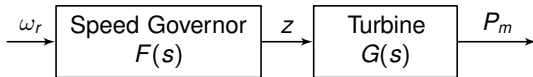
$\rightarrow$ The main solution mechanism are the speed governors in combination with flexible loads.
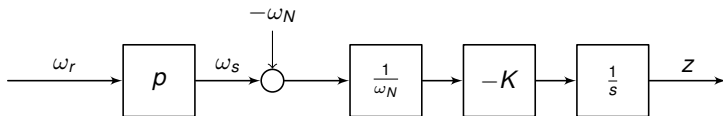
3.1 The speed governor: overview

where

ω_r: speed of rotation

z: fraction of opening of the turbine control valves ($0 \le z \le 1$)

P_m: mechanical power produced by the turbine

$G(s)$: transfer function between z and P_m

$F(s)$: transfer function between ω_r and z

where

$\omega_s = p\omega_m$: electrical speed

p: number of pairs of poles

ω_N: nominal frequency

Servomotor: represented by the gain $K > 0$ and the integrator

In steady state $\omega = p\omega_r = \omega_N \longrightarrow$ no frequency error

- a single generator can be equipped with an isochronous regulator
- that generator will alone ensure the active power balance of the whole system $\longrightarrow$ inappropriate for a large network!
- frequently used in gen-sets for backup generators supplying the entire system in case of islanding.
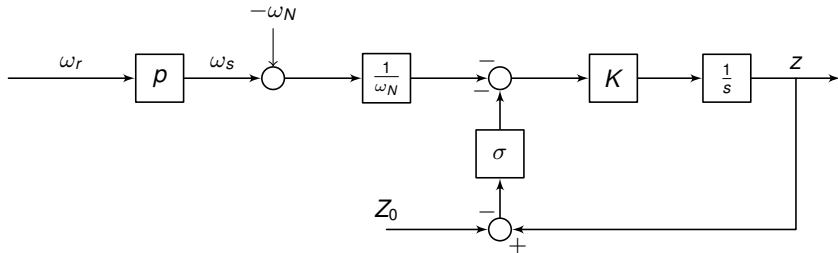
where
Z_0: valve opening setpoint (to modify the power production of the generator)

Equivalently:
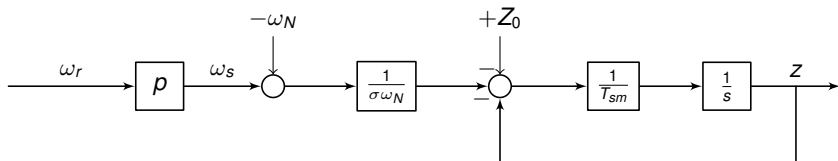
where

$$T_{sm} = \frac{1}{K\sigma} : \text{time constant of the servomotor}$$

$$z = \frac{1}{1 + sT_{sm}} \left(Z_0 - \frac{\omega_s - \omega_N}{\sigma\omega_N} \right)$$

What is the value of z in steady-state?

Cyprus
University of
Technology

- Steady-state characteristic of turbine-governor

$$P_m = P^0 - \frac{P_N}{\sigma} \frac{f - f_N}{f_N}$$

- P^0 is the power setpoint of generator

- P_N is the generator nominal power

- f_N is the system nominal frequency

- f is the measured frequency

- σ is the droop of the speed governor. It defines the ratio between the relative frequency deviation and the relative power deviation
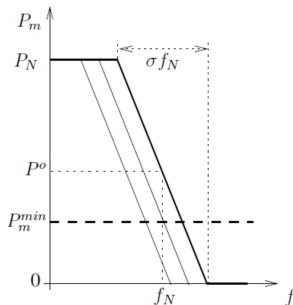
$$\left| \frac{\Delta\omega/\omega_N}{\Delta P_m/P_N} \right| = \left| \frac{\Delta f/f_N}{\Delta P_m/P_N} \right| = \sigma$$

(Typical values: 4-5%)

Cyprus
University of
Technology

- A frequency deviation $\Delta f = \sigma f_N = 0.04 \cdot 50 = 2$ Hz would result in a variation of mechanical power $\Delta P_m = P_N$

- **Infinite speed droop**: the machine operates at constant power, and does not participate in frequency control

- **Frequency regulation parameter**: Gives the power output change of the generator to a frequency change

$$R_f = \frac{\sigma f_N}{P_N} \text{ [Hz/MW]}$$

- The speed controller is of the ***proportional*** type
  - it leaves a steady-state frequency error, but . . .
  - this is **precisely** the signal allowing to share the effort over the various generators.

- system has come back to steady state $\Rightarrow$ all machines have the same electrical speed $= 2\pi f$

- the network is lossless

- the mechanical power produced by the turbines is completely converted into electrical power

- load is sensitive to frequency:

$$P_c = P_c^o p(f) \quad \text{with } p(f_N) = 1$$

$$= P_c^o \left(p(f_N) + \frac{dp}{df}\bigg|_{f=f_N} (f - f_N) \right) = P_c^o (1 + D(f - f_N))$$

D : sensitivity of load to frequency ($1/\mathrm{Hz}$)

- the system initially operates at the nominal frequency ($f = f_N$ for simplicity

The steady-state characteristics of the various generators can be combined into

$$P_m = \sum_{i=1}^{n} P_{mi} = \sum_{i=1}^{n} P_i^0 - \frac{f - f_N}{f_N} \sum_{i=1}^{n} \frac{P_{Ni}}{\sigma_i}$$

Expressing that load is balanced by generation:

$$\sum_{i=1}^{n} P_i^o - \frac{f - f_N}{f_N} \sum_{i=1}^{n} \frac{P_{Ni}}{\sigma_i} = P_c^o \left(1 + D\left(f - f_N\right)\right)$$

In particular, at the initial operating point: $\sum_{i=1}^{n} P_i^o = P_c^o$

Disturbance: increase ΔP_c of consumption, *the setpoints P_i^o being unchanged*

$$\sum_{i=1}^{n} P_i^o - \frac{f - f_N}{f_N} \sum_{i=1}^{n} \frac{P_{Ni}}{\sigma_i} = P_c^o \left(1 + D\left(f - f_N\right)\right) + \Delta P_c$$

$$\implies -\frac{f - f_N}{f_N} \sum_{i=1}^{n} \frac{P_{Ni}}{\sigma_i} = P_c^o D\left(f - f_N\right) + \Delta P_c$$

$$\Delta f = f - f_N = -\frac{\Delta P_c}{\beta} \quad \text{with} \quad \beta = DP_c^o + \frac{1}{f_N} \sum_{i=1}^{n} \frac{P_{Ni}}{\sigma_i}$$

- β is the composite frequency response characteristic ($\mathrm{MW/Hz}$). It characterizes the accuracy of primary frequency control. Also called "*network power frequency characteristic*" or "*stiffness of system*"

- Variation of power of j-th generator: $\Delta P_{mj} = -\frac{\Delta f \, P_{Nj}}{f_N \, \sigma_j} = \frac{\Delta P_c \, P_{Nj}}{f_N \, \beta \, \sigma_j}$

- The steady-state frequency error allows a ***predictable*** and ***adjustable*** sharing of the power variation over the various (participating) generators

- all speed droops being fixed, the larger the nominal power of a generator, the larger its participation

- all nominal powers being fixed, the smaller the speed droop of a generator, the larger its participation

- the larger the number of generators participating in frequency control, the smaller the frequency deviation

- only a fraction of the total number of generators participate in primary frequency control

- to participate, the generator must have a primary reserve, i.e. it must produce less than its maximum power

- this is not desirable:
  - for generators using a renewable energy source $\rightarrow$ Can only provide downward regulation (decrease active power output)
  - for units whose power cannot (easily) be varied: e.g. nuclear units

- primary reserve $=$ *service* offered by the producer on the corresponding dedicated market

- if it is selected, the generator is paid:
  - for making the reserve available (even if it is not activated)
  - as well as for the activation of the reserve:
    - amount paid to the producer for an increase of power
    - $\neq$ amount paid to the producer for a decrease of power.

- Inverters were set to halt production and disconnect from the grid if the grid frequency drifted above 50.2 Hz
- All were programmed to behave the same way !
  - collective shutdown would cause many gigawatts of generating capacity to leave the grid at the same time → lower frequency
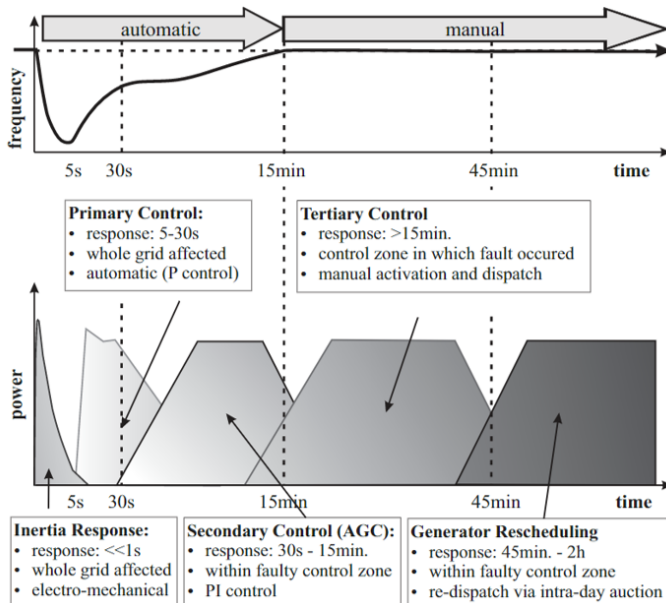 - Reconnection would cause frequency ramp up
    ⟶ yo-yo effect
- Solution?

Net load - March 31
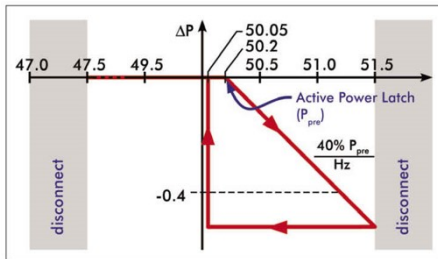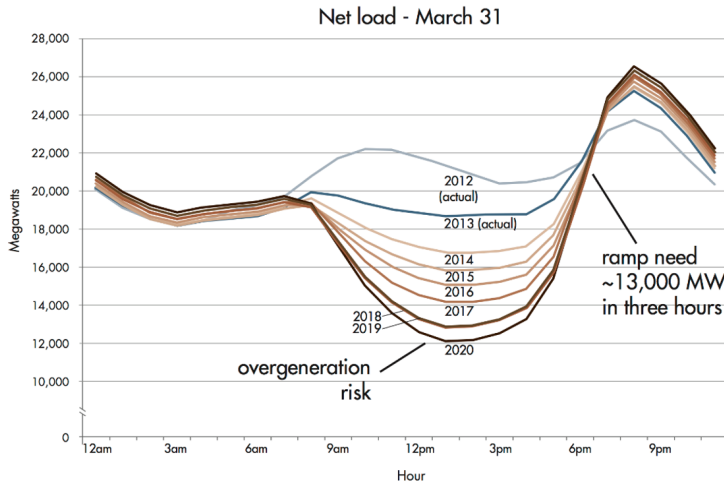
- In 2020 the grid operator were required to spin up 13 gigawatts of production in three hours time – an enormous change in capacity

- Solution?

1 **Fundamentals**

2 **Inertia response**

3 **Primary frequency control**

4 **Secondary frequency control**

Primary Control:
- response: 5-30s
- whole grid affected
- automatic (P control)

Tertiary Control
- response: >15min.
- control zone in which fault occured
- manual activation and dispatch

Inertia Response:
- response: <<1s
- whole grid affected
- electro-mechanical

Secondary Control (AGC):
- response: 30s - 15min.
- within faulty control zone
- PI control

Generator Rescheduling
- response: 45min. - 2h
- within faulty control zone
- re-dispatch via intra-day auction

What?

1. eliminate the frequency error inherent to primary frequency control

2. bring the power exchange between networks to the desired value (contracts)

3. restore the generator primary reserves

How?

- Split the interconnected system into multiple areas (corresponding to a country, to the network managed by a transmission operator, etc.)

- gather measurements in each control area:
  1. frequency
  2. sum of power flows in the tie-lines linking the area to the rest of the system

- sends set-point corrections ΔP_i^o are sent to dedicated generators in each area.
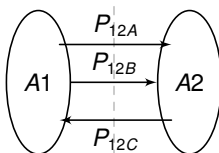
With the same modelling assumptions:

- generators of network 1: $P_{m1} = \sum_{i \in A1} P_i^o - \dfrac{f - f_N}{f_N} \sum_{i \in A1} \dfrac{P_{Ni}}{\sigma_i}$

- load of network 1: $P_{c1} = P_{c1}^o + D_1 P_{c1}^o \left(f - f_N \right)$

- power balance in network 1: $P_{m1} = P_{c1} + P_{12}$

- generators of network 2: $P_{m2} = \sum_{i \in A2} P_i^o - \dfrac{f - f_N}{f_N} \sum_{i \in A2} \dfrac{P_{Ni}}{\sigma_i}$

- load of network 2: $P_{c2} = P_{c2}^o + D_2 P_{c2}^o \left(f - f_N \right)$

- power balance in network 2: $P_{m2} = P_{c2} + P_{21} = P_{c2} - P_{12}$

- power balance of whole system : $P_{m1} + P_{m2} = P_{c1} + P_{c2}$

Scenario :

- the whole system operates initially at frequency f_N
- the load power in network 1 increases by ΔP_{c1}.

Applying the relations of primary frequency control:

$$\text{to network 1: } -\beta_1 \Delta f = \Delta P_{c1} + \Delta P_{12}$$
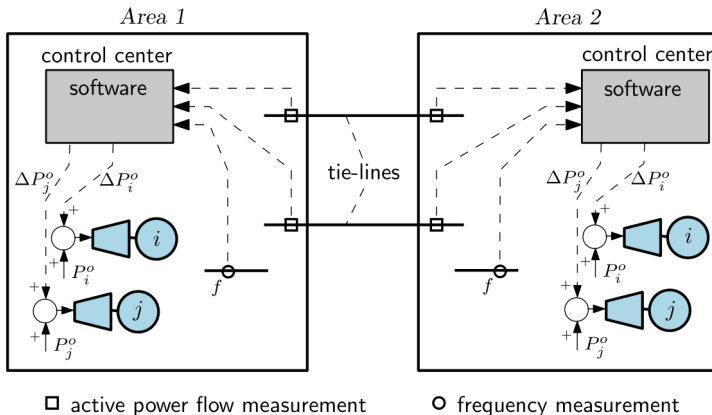$$\text{to network 2: } -\beta_2 \Delta f = -\Delta P_{12}$$

where β_1, β_2 : composite frequency response characteristics of networks 1 and 2

Hence the frequency changes by : $\Delta f = -\dfrac{\Delta P_{c1}}{\beta_1 + \beta_2}$ the tie-line power changes by:

$$\Delta P_{12} = -\frac{\beta_2}{\beta_1 + \beta_2} \Delta P_{c1} < 0$$

- the power flow from network 1 to network 2 decreases, due to the support provided to network 1 by the generators of network 2
- the larger β_2 with respect to β_1, the more pronounced this effect.

Control distributed in the various areas:

- measurements from one area gathered by the control center of that area
- no exchange of real-time measurements between areas

Area Control Error (ACE):

in area 1: $ACE_1 = P_{12} - P_{12}^o + \lambda_1 (f - f_N) = \Delta P_{12} + \lambda_1 \Delta f$

in area 2: $ACE_2 = P_{21} - P_{21}^o + \lambda_2 (f - f_N) = -\Delta P_{12} + \lambda_2 \Delta f$

λ_1, λ_2 : bias factors

Generator power correction (output of Proportional-Integral controller):

in area 1: $\Delta P_1^o = -K_{p1} ACE_1 - K_{i1} \int ACE_1 dt \quad K_{i1}, K_{p1} > 0$

in area 2: $\Delta P_2^o = -K_{p2} ACE_2 - K_{i2} \int ACE_2 dt \quad K_{i2}, K_{p2} > 0$

Distribution over the generators participating in secondary frequency control:

for the i-th generator of area 1: $P_i^o + \rho_i \Delta P_1^o \quad$ with $\quad \sum_i \rho_i = 1$

for the j-th generator of area 2: $P_j^0 + \rho_j \Delta P_2^o \quad$ with $\quad \sum_j \rho_j = 1$

When the system comes back to steady state, the integral control imposes:

$$ACE_1 = 0 \Rightarrow \Delta P_{12} + \lambda_1 \Delta f = 0$$
$$ACE_2 = 0 \Rightarrow -\Delta P_{12} + \lambda_2 \Delta f = 0$$

whose solution is: $\Delta f = 0$ and $\Delta P_{12} = 0$

$\longrightarrow$ **both objectives of secondary frequency control are met!**

Choosing the bias factors λ_i:

- They *do not impact the final system state* but the dynamics to reach it
- It is appropriate to choose: $\lambda_1 = \beta_1$ $\lambda_2 = \beta_2$
  Indeed, in the above example:

$$ACE_2|_{\lambda_2=\beta_2} = -\Delta P_{12} + \beta_2 \Delta f = 0 \Rightarrow \Delta P_2^o = 0$$

  no adjustment of the generators in zone 2 $\leftarrow$ *that's what we wanted!*

- the more λ_2 differs from β_2, the more the generators in zone 2 are *uselessly* adjusted by the secondary frequency controller.

Choosing the K_i and K_p gains of the PI controllers:

- They influence the dynamics, in particular the speed of action of secondary frequency control

- secondary frequency control must not act too promptly, in order not interfere with primary frequency control (which is the "first line of defense")

- quite often, $K_p = 0$ (integral control only).

**Choosing the participation factors ρ_i:**

- ρ_i coefficients: distribute the correction signal ΔP_1^o (or ΔP_2^o) on the participating generators, which must have secondary reserve

- for both primary and secondary frequency controls, the power variation that a participating unit commits to provide, in a given time interval, must be compatible with its maximum rate of change:

$$\simeq \text{a few } \% P_N \text{ / min for thermal units}$$
$$\simeq P_N \text{ / min for hydro units}$$

Three-area example:

- **1** wants to sell 1000 MW to **3**

- **2** does not want to sell nor to buy power.
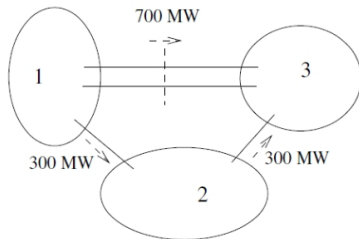
Settings of the secondary frequency controllers:

$$P_{12}^o + P_{13}^o = 1000\text{MW} \quad P_{21}^o + P_{23}^\circ = 0\text{MW} \quad P_{31}^o + P_{32}^\circ = -1000\text{MW}$$

After all secondary frequency controllers have acted:

$$ACE_1 = 0 \Rightarrow (P_{12} + P_{13}) - (P_{12}^\circ + P_{13}^\circ) + \lambda_1 \Delta f = P_{12} + P_{13} - 1000 + \lambda_1 \Delta f = 0$$

$$ACE_2 = 0 \Rightarrow (P_{21} + P_{23}) - \left(P_{21}^\circ + P_{23}^0\right) + \lambda_2 \Delta f = -P_{12} + P_{23} + \lambda_2 \Delta f = 0$$

$$ACE_3 = 0 \Rightarrow (P_{31} + P_{32}) - (P_{31}^o + P_{32}^o) + \lambda_3 \Delta f = -P_{13} - P_{23} + 1000 + \lambda_3 \Delta f = 0$$

$$\Rightarrow \Delta f = 0 \qquad P_{12} + P_{13} = 1000 \qquad P_{12} = P_{23} \qquad P_{13} + P_{23} = 1000$$

Secondary frequency control does not control individual power flows!