EEN320 - Power Systems I (Συστήματα Ισχύος Ι)

Part 4: The per-phase and per-unit system representation

https://sps.cut.ac.cy/courses/een320/

Dr Petros Aristidou
Department of Electrical Engineering, Computer Engineering & Informatics

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Today's learning objectives

After this part of the lecture and additional reading, you should be able to ...

1. ...explain the use and advantages of the per-phase and per-unit system representations in power system computations;
2. ...convert physical quantities to their corresponding per unit values;
3. ...calculate stationary network conditions using the per unit system.

Outline

1. Per phase analysis of balanced three-phase AC systems
2. Principle and advantages of the per-unit system
3. Introduction of per unit quantities via an example
4. Conversion between different per unit systems
5. Choice of base values in power systems with several zones

1 Outline

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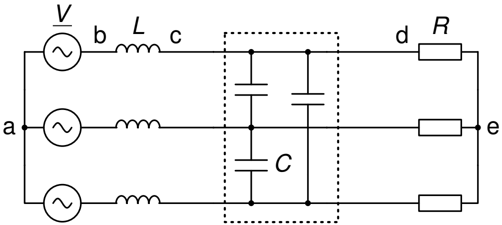
1 Per phase analysis (ανά φάση ανάλυση)

- Balanced three-phase system: phenomena in all 3 phases identical
- Voltages and currents in phases *b* and *c* merely shifted by $\pm 2\pi/3$ rad

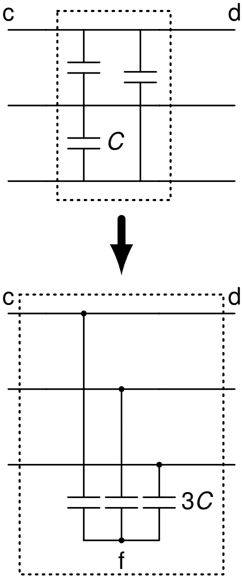
$\rightarrow$ Analyse three-phase circuit by equivalent single-phase circuit

- This requires the following steps:
  - Replace all Delta-connected elements by their equivalent Y-connected representation
  - Draw single-phase equivalent circuit for phase *a*
  - Conduct circuit analysis by using the equivalent single-phase circuit for phase *a*
  - Corresponding values for phases *b* and *c* obtained by adding $\pm 2\pi/3$ to values in phase *a*
- Note: under balanced conditions all neutrals have the same potential!

1 Per phase analysis - Example: Three-phase circuit



1 Per phase analysis - Example: Delta-Y-transformation



- Capacitors in Delta-connection need to be transformed into equivalent Y-connection

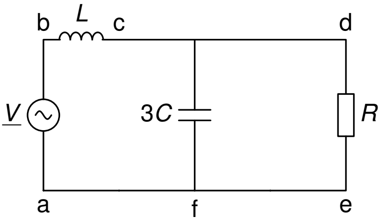
$$\underline{Z}_{\Delta} = 3\underline{Z}_{Y}$$

$$\underline{Z}_{C} = jX_{C} = \frac{1}{j\omega C}$$

$$\Rightarrow \underline{Z}_{C,Y} = \frac{1}{3}\underline{Z}_{C,\Delta} = \frac{1}{j\omega 3C}$$

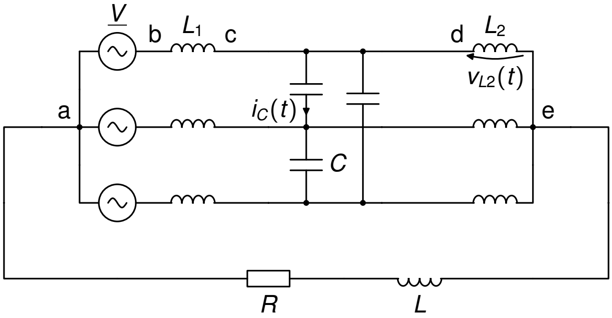
- Need 3 times higher capacitance in Y-connection!

1 Per phase analysis - Example: Single-phase equivalent circuit



- Neutrals a, f and e are all on same potential
  $\rightarrow$ can use single return conductor

1 Per phase analysis - Numerical example



In the diagram above, it is given that $v_a(t) = 350\cos(\omega t + 45^\circ)\ V$, with frequency $f = 50\ Hz$, $L_1 = 0.318\ mH$, $L_2 = 3.18\ mH$, $C = 1.592\ mF$, $R = 1\ \Omega$, $L = 0.0318\ mH$.

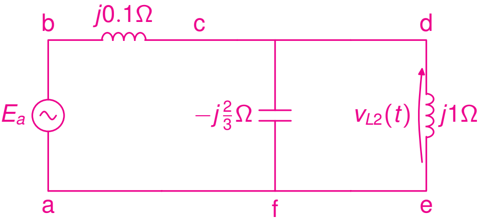
Task.
Find $v_{L2}(t)$ and $i_C(t)$.

Solution.
Since it's symmetrical we do per phase analysis.

$$E_a = \frac{350}{\sqrt{2}}\angle 45^\circ \qquad X_{L1} = j\omega L_1 = j314 \cdot 0.318 \cdot 10^{-3} \approx j0.1\Omega$$

$$X_{L2} = j\omega L_2 = j314 \cdot 3.18 \cdot 10^{-3} \approx j1\Omega \qquad X_L = j\omega L = j314 \cdot 0.0318 \cdot 10^{-3} \approx j0.01\Omega$$

$$X_C = \frac{-j}{\omega C} = \frac{-j}{314 \cdot 1.592 \cdot 10^{-3}} = -j2\Omega \qquad X_{CY} = -j\frac{2}{3}$$



1 Per phase analysis - Numerical example

$$Z_{tot} = \frac{j1 \cdot (-j2/3)}{j1 - j(2/3)} = -j2\Omega$$

$$V_2 = \frac{-j2}{-j2 + j0.1} E_a = 1.05 E_a = \frac{368}{\sqrt{2}} \angle 45^\circ \, V$$

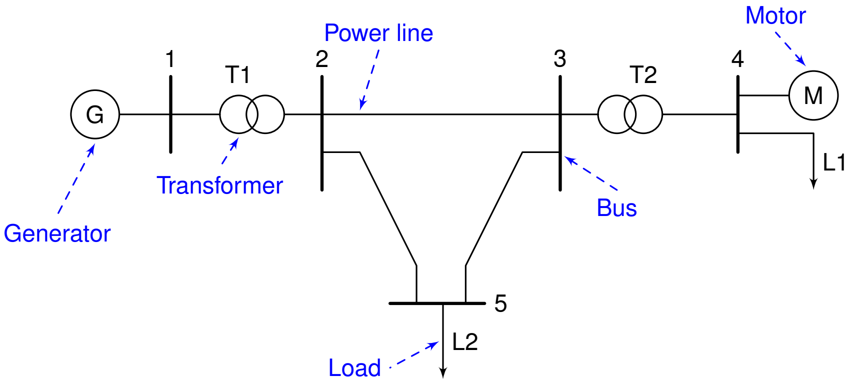
$$v_{L2}(t) = 368 \cos(\omega t + 45^\circ) \, V$$

$$I_{CY} = \frac{V_2}{X_{CY}} = \frac{\frac{368}{\sqrt{2}} \angle 45^\circ}{-j2/3} = \frac{\frac{368}{\sqrt{2}} \angle 45^\circ}{(2/3) \angle -90^\circ} = \frac{552}{\sqrt{2}} \angle 135^\circ \, A$$

$$I_C = \frac{I_{CY}}{\sqrt{3} \angle -30^\circ} = \frac{319}{\sqrt{2}} \angle 165^\circ \, A$$

$$i_c(t) = 319 \cos(\omega t + 165^\circ) A$$

1 Standard one-line diagram (μονογραμμικό διάγραμμα)



- In switching stations, power lines (γραμμές μεταφοράς), cables, transformers (μετασχηματιστές), generators (γεννήτριες), loads (φορτία), etc. are connected to each other via buses (or busbars)
- Bus (ζυγός) = equipotential metallic assembly

2 Outline

2 Principle of ”per unit” system

- Usual representation of physical quantities as product of numerical value and physical unit, e.g.

$$V = 400 \text{ kV}$$

- Alternative: representation of the quantity *relative* to another (base) quantity

$$\boxed{\text{value of quantity in pu} = \frac{\text{value of quantity in physical unit}}{\text{value of corresponding ”base” in same unit}}}$$

- Division by ”base” eliminates physical unit

→ per-unit (pu) system

- Example: base value for voltage $V_{\text{base}} = 400 \text{ kV}$

$$\frac{V}{V_{\text{base}}} = \frac{400 \text{ kV}}{400 \text{ kV}} = 1 \text{ pu}$$

2 Advantages (1)

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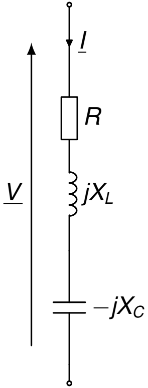
- Appropriate choice of base values gives pu-values very useful meaning
  - Example: express bus voltage V relative to nominal grid voltage V_{base} and suppose that
  $$v = 0.93 \text{ pu}$$
  - $\rightarrow$ We see immediately that value of v is 7% below nominal voltage
  - This is much easier to see than by looking at the absolute value $V = 372.03$ kV
- Better conditioning of numerical computations
  - Under normal operating conditions, voltage values in pu are close to 1
  - Networks of different dimensions and voltage levels can be represented in same order of magnitude
  - Example: 100 MVA $=$ 1 pu for large networks and 1 MVA$=$ 1 pu for small networks

2 Advantages (2)

- Easier comparison of components of different power ratings
  - Consider two transformers and suppose that their currents are indicated in pu with respect to their respective maximum currents
  - Suppose that $i_1 = 0.99$ pu and $i_2 = 0.35$ pu
  - We see immediately that transformer 1 is operating much closer to its limit than transformer 2
  - In general, parameters of similar devices have similar pu values, independently of their power rating (as long as the values are referred to that rating)

  $\rightarrow$ Can check quickly if data of a component/machine is within usual range
- Ideal transformer present in real transformer model is eliminated in the equivalent per-unit circuit (see example later)

3 Outline



- It holds that

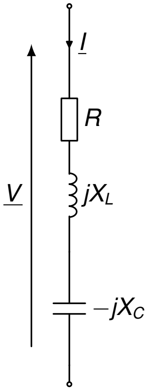
$$X_L = \omega L \qquad X_C = \frac{1}{\omega C} \qquad \frac{1}{j\omega C} = -j\frac{1}{\omega C} = -jX_C$$

- From KVL we have that

$$\underline{V} = R\underline{I} + jX_L\underline{I} - jX_C\underline{I}$$

- Goal: Represent variables in pu system
- First question: How to choose base quantities?

3 Exemplary circuit - Choice of base values



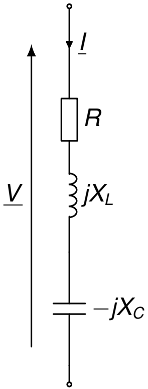
- Basic relations in stationary power systems

$$\underline{S} = \underline{V}\,\underline{I}^*, \quad \underline{V} = \underline{Z}\,\underline{I}$$

→ Can choose two independent base quantities

- Other base values are obtained using fundamental laws for electric circuits
- Typical choice of base quantities
  1) Base power S_B
  2) Base voltage V_B
- Note 1: base values are always real numbers!
- Note 2: usual numbers of base values correspond to nominal (power and voltage) ratings of the circuit

3 Exemplary circuit - Base voltage



- Introduce base voltage V_B
- Then the per unit representation of $\underline{V}$ is obtained as

$$\underline{v} = \frac{\underline{V}}{V_B}$$

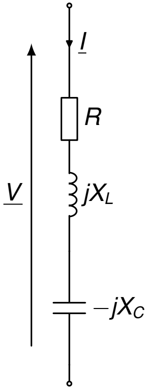
- Then

$$\underline{v} = \frac{\underline{V}}{V_B} = \frac{R\underline{I}}{V_B} + \frac{jX_L\underline{I}}{V_B} - \frac{jX_C\underline{I}}{V_B}$$

- Example: rated voltage of circuit is 110 kV

$\rightarrow$ Choose $V_B \approx 110$ kV

3 Exemplary circuit - Base power



- Introduce (single-phase) base power $S_{B1\phi}$

$$S_{B1\phi} = V_B I_B = \frac{V_B^2}{Z_B}$$

- Then the per unit representation of $\underline{S}$ is obtained as

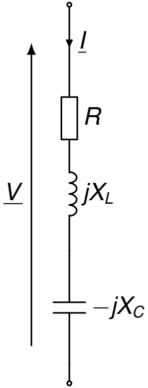
$$\underline{s} = \frac{\underline{S}}{S_{B1\phi}}$$

- And the base values for currents and impedances follow from the relations

$$I_B = \frac{S_{B1\phi}}{V_B} \qquad Z_B = \frac{V_B}{I_B} = \frac{V_B^2}{S_{B1\phi}}$$

- Per unit representation of current and impedance

$$\underline{i} = \frac{\underline{I}}{I_B} \qquad \underline{z} = \frac{\underline{Z}}{Z_B}$$



- For the voltage in per unit we had the relation

$$\underline{v} = \frac{\underline{V}}{V_B} = \frac{R\underline{I}}{V_B} + \frac{jX_L\underline{I}}{V_B} - \frac{jX_C\underline{I}}{V_B}$$

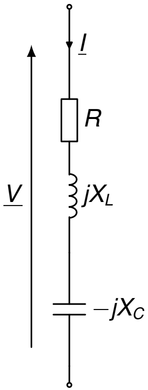
- By expressing the current in per unit via

$$\underline{i} = \frac{\underline{I}}{I_B}$$

we obtain

$$\underline{v} = \frac{\underline{V}}{V_B} = \frac{RI_B}{V_B}\underline{i} + \frac{jX_L I_B}{V_B}\underline{i} - \frac{jX_C I_B}{V_B}\underline{i}$$

3 Exemplary circuit - Per unit representation (2)



- Recall that base impedance is given by

$$Z_B = \frac{V_B}{I_B}$$

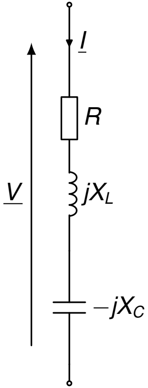
- Base impedance Z_B is base value for both real and complex impedances
- Hence

$$r = \frac{R}{Z_B} = \frac{RI_B}{V_B} \qquad x_L = \frac{X_L}{Z_B} = \frac{X_L I_B}{V_B} \qquad x_C = \frac{X_C}{Z_B} = \frac{X_C I_B}{V_B}$$

- We obtain

$$\begin{aligned}\underline{v} = \frac{\underline{V}}{V_B} &= \frac{RI_B}{V_B}\underline{i} + \frac{jX_L I_B}{V_B}\underline{i} - \frac{jX_C I_B}{V_B}\underline{i} \\ &= r\underline{i} + jx_L\underline{i} - jx_c\underline{i}\end{aligned}$$

3 Exemplary circuit - Per unit representation (3)



By introducing the overall impedance

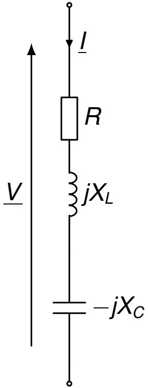
$$\underline{Z} = R + j(X_L - X_C)$$

and its per unit representation

$$\underline{z} = \frac{\underline{Z}}{Z_B} = r + j(x_L - x_C)$$

we obtain the compact representation in pu

$$\underline{v} = \underline{z}\,\underline{i}$$



1. Choose two base quantities, e.g. S_B and V_B

   S_B can be either single- or three-phase power

$$\boxed{S_{B3\phi} = 3S_{B1\phi}}$$

2. Other values obtained via electrical laws

   Base current $I_B = \frac{S_{B1\phi}}{V_B} = \frac{S_{B3\phi}}{3V_B} = \frac{S_{B3\phi}}{\sqrt{3}U_B}$ $\quad U_B = \sqrt{3}V_B$

   Base impedance $Z_B = \frac{V_B}{I_B} = \frac{V_B^2}{S_{B1\phi}} = \frac{3V_B^2}{S_{B3\phi}} = \frac{U_B^2}{S_{B3\phi}}$

   Base admittance $Y_B = G_B = B_B = \frac{1}{Z_B}$

- V_B and I_B are always RMS values per phase!
- In non-stationary conditions usually frequency and/or time are also normalised

4 Outline

4 Conversion between different per unit systems

- In practice, it is often necessary to convert values from one per unit system to another one
- Example: machine parameters are given in per unit values with respect to machine rating and we want to convert them into per unit values with respect to base values of power system to which machine is connected
- This can be done as follows

  Per unit value wrt first base: $x_1 = \frac{X}{X_{B,1}}$

  Per unit value wrt second base: $x_2 = \frac{X}{X_{B,2}}$

  Hence: $X = x_1 X_{B,1} = x_2 X_{B,2}$

$\rightarrow$ Conversion from base 1 to base 2:

$$\boxed{x_2 = x_1 \frac{X_{B,1}}{X_{B,2}}}$$

4 Conversion between different per unit systems - Example

Conversion of an impedance $\underline{Z}$ from base ”old” to base ”new”

- Per unit value wrt first base:

$$\underline{z}_{\text{old}} = \frac{\underline{Z}}{Z_B^{\text{old}}} = \frac{\underline{Z}\, S_{B1\phi}^{\text{old}}}{(V_B^{\text{old}})^2} \qquad Z_B^{\text{old}} = \frac{(V_B^{\text{old}})^2}{S_{B1\phi}^{\text{old}}}$$

- Per unit value wrt second base:

$$\underline{z}_{\text{new}} = \frac{\underline{Z}}{Z_B^{\text{new}}} = \frac{\underline{Z}\, S_{B1\phi}^{\text{new}}}{(V_B^{\text{new}})^2} \qquad Z_B^{\text{new}} = \frac{(V_B^{\text{new}})^2}{S_{B1\phi}^{\text{new}}}$$

- Conversion from base ”old” to base ”new”

$$\underline{z}_{\text{new}} = \underline{z}_{\text{old}} \frac{Z_B^{\text{old}}}{Z_B^{\text{new}}} = \underline{z}_{\text{old}} \frac{S_{B1\phi}^{\text{new}}}{S_{B1\phi}^{\text{old}}} \left(\frac{V_B^{\text{old}}}{V_B^{\text{new}}} \right)^2$$

4 Example: Conversion between per unit systems (1)

Task. A three-phase transformer is rated 400 MVA, 220Y/22Δ kV. The Y-equivalent short-circuit impedance measured on the low-voltage side of the transformer is 0.121 Ω. Due to the low resistance, this value can be considered to be equal to the leakage reactance of the transformer. Determine the per-unit reactance of the transformer by taking the secondary voltage as base voltage. Determine the per-unit reactance in a system with base values $S_{B3\phi} = 100$ MVA and $V_B = 230$ kV.

Solution. Per-unit transformer reactance on transformer base

$$S_{B3\phi} = 400 \text{ MVA}, \quad V_B = 22 \text{ kV} \rightarrow Z_B = \frac{3V_B^2}{S_{B3\phi}} = 3.63\ \Omega$$

$$x_T = \frac{X_T}{Z_B} = \frac{0.121}{3.63} = 0.033$$

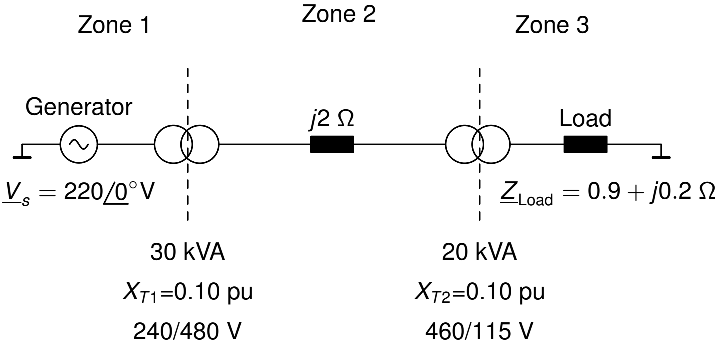
Conversion to system base

$$Z_B^{\text{new}} = \frac{3(V_B^{\text{new}})^2}{S_{B3\phi}^{\text{new}}} = 1587\ \Omega$$

$$x_T^{\text{new}} = x_T \frac{Z_B}{Z_B^{\text{new}}} = 7.62 \cdot 10^{-5} \text{ pu}$$

5 Outline

5 Example: 3-zone single-phase circuit



- $X_{Ti}\ldots$ leakage reactance of transformer, $i = 1, 2$
- Transformer winding resistors and shunt admittances are neglected

5 Choice of base values - General rules

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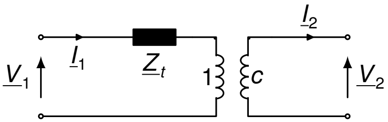
- Power base $S_{B1\phi}$ or $S_{B3\phi}$ is the same for the whole network
- Typical values: $S_{B3\phi} = 100$ MVA in HV networks; $S_{B3\phi} = 1$ MVA in MV networks
- Ratio of voltage bases V_{Bi} on either side of a transformer is chosen identical to ratio of transformer voltage rating

$$c = \frac{N_1}{N_2} \quad \rightarrow \quad \frac{V_{B1}}{V_{B2}} = c$$



- Since $\underline{V}_1 = c\underline{V}_2$, when following the previous rules, we have

$$\underline{v}_1 = \frac{\underline{V}_1}{V_{B1}} \qquad \underline{v}_2 = \frac{\underline{V}_2}{V_{B2}} = \frac{c\underline{V}_2}{V_{B1}} = \frac{\underline{V}_1}{V_{B1}} = \underline{v}_1$$

- Likewise, since $\underline{I}_2 = c\underline{I}_1$ and

$$I_{B1} = \frac{S_B}{V_{B1}} \qquad I_{B2} = \frac{S_B}{V_{B2}} = \frac{cS_B}{V_{B1}}$$

$$\underline{i}_1 = \frac{\underline{I}_1}{I_{B1}} \qquad \underline{i}_2 = \frac{\underline{I}_2}{I_{B2}} = \frac{\underline{I}_2}{cI_{B1}} = \frac{c\underline{I}_1}{cI_{B1}} = \underline{i}_1$$

5 Choice of base values - Consequences for transformer model (2)

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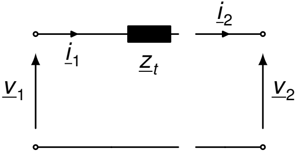
- Also, per unit impedance remains unchanged when referred to either side of a transformer

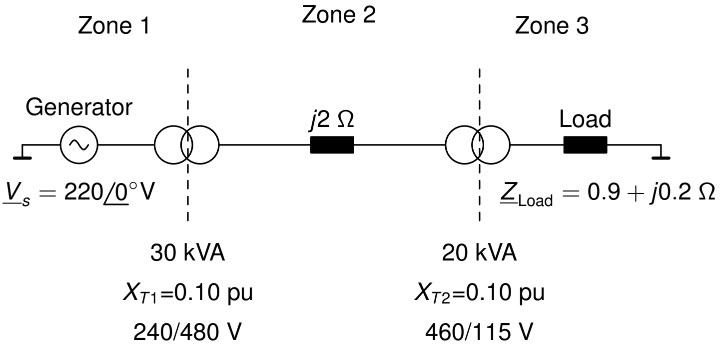
$$Z_{B1} = \frac{V_{B1}}{I_{B1}} = \frac{V_{B1}^2}{S_B} \qquad Z_{B2} = \frac{V_{B2}}{I_{B2}} = \frac{V_{B1}^2}{c^2 S_B}$$

$$\frac{\underline{Z}_1}{\underline{Z}_2} = c^2 \quad \rightarrow \quad \frac{\underline{z}_1}{\underline{z}_2}\frac{Z_{B1}}{Z_{B2}} = \frac{\underline{z}_1}{\underline{z}_2}c^2 = c^2 \quad \rightarrow \quad \frac{\underline{z}_1}{\underline{z}_2} = 1$$

→ Turns ratio c eliminated in equivalent per unit circuit (if base values chosen according to transformer voltage rating)!



5 Example - Three-zone system per-unit calculation



Task.

1. Choose $S_B = 30$ kVA and $V_{B1} = 240$ V and determine the per-unit impedances and per unit source voltage $\underline{v}_s$
2. Draw per-unit circuit
3. Calculate load current in per unit and Amperes

5 Example - Voltage bases for Zones 2 and 3

1)

- Base values for Zone 1 are given: $S_B = 30$ kVA and $V_{B1} = 240$ V
- Following our "general rules", choose base voltages for Zones 2 and 3 in proportion to transformer voltage ratings

$$V_{B2} = \frac{480\text{ V}}{240\text{ V}} V_{B1} = 480\text{ V}$$

$$V_{B3} = \frac{115\text{ V}}{460\text{ V}} V_{B2} = 120\text{ V}$$

- Base impedances in Zones 2 and 3 (single-phase system)

$$Z_{B2} = \frac{V_{B2}^2}{S_B} = \frac{480^2}{30,000} = 7.68\ \Omega \qquad Z_{B3} = \frac{V_{B3}^2}{S_B} = \frac{120^2}{30,000} = 0.48\ \Omega$$

- Base currents in Zones 2 and 3

$$I_{B2} = \frac{S_B}{V_{B2}} = \frac{30,000}{480} = 62.5\text{ A} \qquad I_{B3} = \frac{S_B}{V_{B2}} = \frac{30,000}{120} = 250\text{ A}$$

5 Example - Per unit impedances and source voltage (1)

1)

- Base values for transformer 1 are same as system base values in Zone 1 $\rightarrow x_{T1} = X_{T1} = 0.1$ pu
- Reactance of transformer 2 needs to be converted (use conversion rules)

$$x_{T2} = X_{T2} \frac{S_B}{S_{B2}} \left(\frac{460}{V_{B2}} \right)^2 = 0.1 \frac{30,000}{20,000} \left(\frac{460}{480} \right)^2 = 0.1378 \text{ pu}$$

Alternatively, use V_{B3}

$$x_{T2} = X_{T2} \frac{S_B}{S_{B3}} \left(\frac{115}{V_{B3}} \right)^2 = 0.1 \frac{30,000}{20,000} \left(\frac{115}{120} \right)^2 = 0.1378 \text{ pu}$$

5 Example - Per unit impedances and source voltage (2)

1)

- Line reactance in pu

$$x_l = \frac{X_l}{Z_{B2}} = \frac{2}{7.68} = 0.2604 \text{ pu}$$

- Load reactance in pu

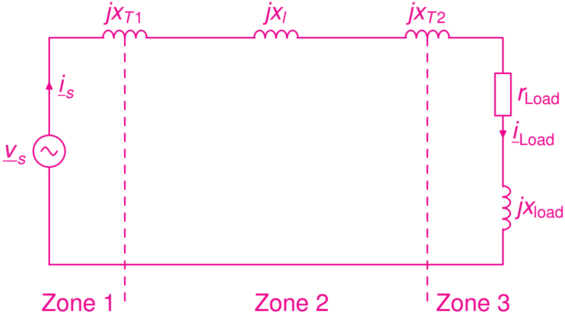
$$\underline{z}_{\text{Load}} = \frac{\underline{Z}_{\text{Load}}}{Z_{B3}} = \frac{0.9 + j0.2}{0.48} = 1.875 + j0.4167 \text{ pu}$$

- Source voltage in pu

$$\underline{v}_s = \frac{\underline{V}_s}{V_{B1}} = \frac{220\underline{/0^\circ}}{240} = 0.9167\underline{/0^\circ} \text{ pu}$$

5 Example - Equivalent per unit circuit

2)



5 Example - Load current

3)

- Per unit load current

$$\underline{i}_{\text{Load}} = \frac{\underline{v}_s}{\underline{z}_{\text{Load}} + j(x_{T1} + x_l + x_{T2})}$$
$$= \frac{0.9167\angle\underline{0^\circ}}{1.875 + j0.4167 + j(0.10 + 0.2604 + 0.1378)}$$
$$= \frac{0.9167\angle\underline{0^\circ}}{1.875 + j0.9149}$$
$$= \frac{0.9167\angle\underline{0^\circ}}{2.0863\angle\underline{26.01^\circ}} = 0.4395\angle\underline{-26.01^\circ} \text{ pu}$$

- Load current in Ampere

$$\underline{I}_{\text{Load}} = \underline{i}_{\text{Load}} \cdot I_{B3} = 0.4395\angle\underline{-26.01^\circ} \cdot 250 = 109.9\angle\underline{-26.01^\circ} \text{ A}$$

5 Summary

- Per unit system is frequently used in power system analysis
- Advantages
  - Easy evaluation of equipment status
  - Easy comparison of network status on different voltage levels
  - Better suited values for numerical calculations
  - Turns ratio of transformer eliminated in equivalent per unit circuit