



Cyprus  
University of  
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## EEN320 - Power Systems I (Συστήματα Ισχύος Ι)

Part 3: The power transformer

<https://sps.cut.ac.cy/courses/een320/>

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After this part of the lecture and additional reading, you should be able to . . .

- ① . . . explain the role of power transformers in power systems;
- ② . . . explain the fundamental principles of single-phase and three-phase power transformers;
- ③ . . . derive idealised and practical models for power transformers;
- ④ . . . determine stationary operating conditions of power transformers via circuit calculations.

## 1 Why do we need power transformers?

## 2 Single-phase transformer

- Principle of a transformer
- Ideal transformer model
- Real transformer model

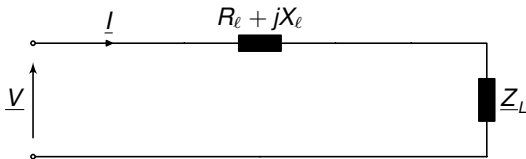
## 3 Three-phase transformer

- Schematic representation of a three-phase transformer
- Types of three-phase transformers
- Configuration of three-phase transformers
- Transformation ratio and equivalent single-phase circuit of three-phase transformers

- 1 **Why do we need power transformers?**
- 2 Single-phase transformer
- 3 Three-phase transformer

- Power transformers are essential pieces of equipment in power systems
- They are used to
  - Step-up voltage, e.g. at terminals of generators (step-up transformer)
  - Step-down voltage, e.g. to distribute power at low voltage to end-user (step-down transformer)
  - Control voltages at some busbars (in sub-transmission and distribution networks)
  - Control power flows in some parts of meshed networks

# 1 Why do we need high voltage levels? (1)



1) Total transmitted apparent power:  $\underline{S} = \underline{V} \underline{I}^*$

Active power losses due to line resistance:  $P_{\text{loss}} = I^2 R_\ell$

⇒ Lower current to transmit same power at higher voltage

- Less transmission losses
- Fewer conductors and paralleled systems needed
- But: higher costs for insulators and power poles

⇒ Economic optimum  $V_{\text{economic}} \sim \sqrt{|\underline{S}|}$  ( $\underline{S}$  is apparent power)

Voltage magnitude (in kV) should increase approximately with square root of magnitude of transmitted apparent power  $S$  (in MVA)

# 1 Why do we need high voltage levels? (2)

## 2) Surge impedance loading (SIL) or natural loading (*details in the last chapter of this course!*)

- SIL: loading of a line at which reactive power is neither produced nor absorbed

$$SIL = \frac{V^2}{Z_W}$$

- Characteristic impedance of transmission line  $Z_W \approx \sqrt{\frac{L'}{C'}}$
- $L'$  is line inductance per unit length,  $C'$  is line capacitance per unit length
- Loaded below SIL: line "supplies" reactive power to system; consequence: voltage raises (Ferranti effect)
- Loaded above SIL: line "consumes" reactive power; consequence: voltage reduces
- For typical transmission line  $Z_W \approx 250 \dots 380 \Omega$

$$\Rightarrow V = \sqrt{SIL \cdot Z_W}$$

$$V \approx 16 \dots 19 \sqrt{SIL}$$

- Note: this is a similar relation to 1)

# 1 Why do we need high voltage levels? (3)

## 3) Provision of short circuit current (*details in EEN442!*)

- Usual overcurrent protection devices require certain min. short circuit current to trigger
- Short circuit current

$$I_{SC} \approx \frac{V}{\sqrt{3}X} \quad (1/\sqrt{3} \text{ since we need to express } V \text{ in line-ground}),$$

where  $X$  is short circuit reactance of considered circuit

- Seems to indicate  $I_{SC} \uparrow$  if  $V \uparrow$
- But: for synchronous machines and transformers, usually *relative* reactance  $x$  instead of absolute reactance  $X$  given
- Relation between  $x$  and  $X$  defined via nominal impedance

$$Z_N = \frac{V_N}{\sqrt{3}I_N}$$

of machine/transformer, i.e.,

$$X = xZ_N = x \frac{V_N}{\sqrt{3}I_N} = x \frac{V_N^2}{S_N}$$



## 3) Provision of short circuit current (ctd.)

- Hence, short circuit current (with  $V = V_N$ )

$$I_{SC} = \frac{V_N}{\sqrt{3}X} = \frac{V_N}{\sqrt{3}} \frac{S_N}{xV_N^2} = \frac{S_N}{x\sqrt{3}V_N} \approx \frac{1}{V_N}$$

⇒ Actually, for given nominal (rated) power  $S_N$ ,  $I_{SC} \downarrow$  if  $V_N \uparrow$

- Likewise, to keep  $I_{SC}$  fixed, one needs that  $V_N \uparrow$  if  $S_N \uparrow$
- Note: these considerations are only valid for one single machine/transformer
- Transmission line impedances are not severely modified by voltage level
- But: higher voltage level ⇒ longer lines ⇒ higher absolute lumped line impedance ⇒ lower  $I_{SC}$  for fixed  $V$

## 4) Interconnection of several grids

### 1 Why do we need power transformers?

### 2 **Single-phase transformer**

- Principle of a transformer
- Ideal transformer model
- Real transformer model

### 3 Three-phase transformer

- From electromagnetism<sup>1</sup>, we know that Maxwell's equations connect the magnetic field quantities with the electric field and the electric current intensity.
- More specifically, Ampère's law states that:

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial(\epsilon \mathbf{E})}{\partial t}$$

where:

- $\mathbf{H}$  is the magnetic field intensity vector (ένταση μαγνητικού πεδίου) and is measured in Ampere per meter (A/m)
- $\mathbf{J}$  is the electric current density (πυκνότητα ηλεκτρικού ρεύματος) and is measured in ampere per square meter (A/m<sup>2</sup>)
- $\mathbf{E}$  is the electric field density (ένταση ηλεκτρικού πεδίου)
- $\epsilon$  is the dielectric constant of the material (διηλεκτρική σταθερά υλικού)

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<sup>1</sup> Check Physics III and Electromagnetism course

- Under the assumption that our system size is much smaller than the electromagnetic wavelength  $\frac{c}{f} = \frac{2\pi c}{\omega}$ , or equivalently, when:

$$\frac{\omega \ell}{c} \ll 1$$

where  $c$  is the speed of light,  $f$  the frequency,  $\omega = 2\pi f$  is the angular frequency, and  $\ell$  is the largest distance in our system.

- Then, the electromagnetic waves propagate instantaneously and the electric and magnetic fields are decoupled. Thus, we can analyse the permanent magnetic field (μόνιμο μαγνητικό πεδίο) which simplifies to:

$$\nabla \times \mathbf{H} = \mathbf{J}$$

- Note that the magnetic flux density (μαγνητική επαγωγή) is given by:

$$\mathbf{B} = \mu \mathbf{H}$$

- $\mu$  is the magnetic permeability of the material (μαγνητική διαπερατότητα υλικού), which can be defined also as  $\mu = \mu_r \mu_0$  with  $\mu_0 = 4\pi \cdot 10^{-7}$  the vacuum permeability and  $\mu_r$  the relative permeability. The SI unit is Henry per meter (H/m).
- $\mathbf{B}$  is measured in Tesla (T) which is equal to Weber per meter squared ( $\text{Wb}/\text{m}^2$ ).
- Using Stokes theorem, we can get the integral form of Ampère's law:

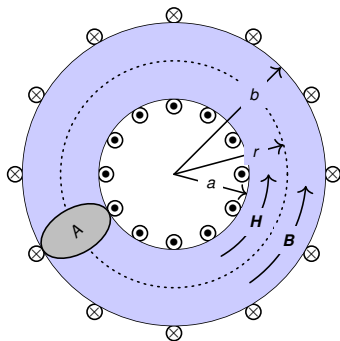
$$\oint_C \mathbf{H}_{tan} d\mathbf{l} = I_{enclosed}$$

showing that the tangential component of the magnetic field intensity vector ( $\mathbf{H}_{tan}$ ) integrated along a closed path ( $C$ ) equals the net current enclosed by that path ( $I_{enclosed}$ ).

- Faraday's law: A changing magnetic field (example: magnet moving through a conducting coil) generates an electric field and vice-versa
- Fundamental principle for many electrical appliances, such as motors, generators or *transformers*
- Mathematically:

$$\mathcal{E}(t) = \frac{d\Phi}{dt}$$

- $\mathcal{E}(t)$  is electromotive (ηλεκτρεγερτική) force (EMF) = voltage induced
- $\Phi = \mathbf{A}\mathbf{B}_{tan}$  is the magnetic flux (πεπλεγμένη ροή) = cross-sectional area exposed to magnetic field  $\times$  component of magnetic field vector normal to that area.  $\Phi$  is measured in Webers (Wb).



Using Ampère's law, we get:

- For  $r > b \rightarrow \oint_C \mathbf{H}_{tan} d\mathbf{l} = NI - NI = 0$
- For  $r < a \rightarrow \oint_C \mathbf{H}_{tan} d\mathbf{l} = 0$
- For  $a < r < b \rightarrow \oint_C \mathbf{H}_{tan} d\mathbf{l} = 2\pi r H = NI = \mathcal{F}$   
where  $\mathcal{F}$  is called magnetomotive force (mmf) (μαγνητογερτική δύναμη)  
and the SI units are *ampere-turn* (αμπερελίγματα).

- Due to symmetry, we get:

$$H = \frac{\mathcal{F}}{2\pi r}$$

- Which leads to:

$$B = \frac{\mu \mathcal{F}}{2\pi r}$$

- If the cross-section is small, we can assume uniform magnetic field equal to:

$$B_\mu = \frac{\mu \mathcal{F}}{2\pi r_\mu}$$

where  $r_\mu = (a + b)/2$

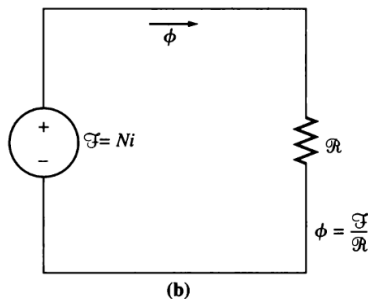
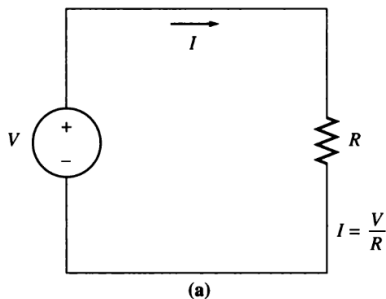
- Leading to magnetic flux of:

$$\Phi = B_\mu A = \mathcal{F} \frac{\mu A}{\ell_\mu} = \frac{\mathcal{F}}{\mathcal{R}_m} \quad \Rightarrow \quad \mathcal{F} = \Phi \mathcal{R}_m$$

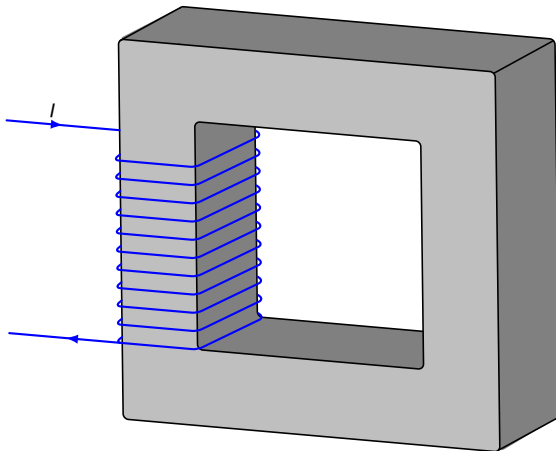
where  $\ell_\mu = 2\pi r_\mu$  is the mean length of the magnetic circuit and  $\mathcal{R}_m = \frac{\ell_\mu}{\mu A}$  the reluctance of the magnetic circuit.



This way, we can transform the problem into an equivalent electric circuit:



## 2.1 Magnetic circuits - Example



**Task.** In the above magnetic circuit, compute the current  $I$  at the coil with  $N = 500$  so that the magnetic field is  $B = 1$  T. It is given that the outer dimensions of the magnetic core are  $20\text{cm} \times 20\text{cm}$  and the cross-section area is  $4\text{cm} \times 4\text{cm}$ . The relative magnetic permeability is  $\mu_r = 3980$  and can be considered constant.

**Solution.**

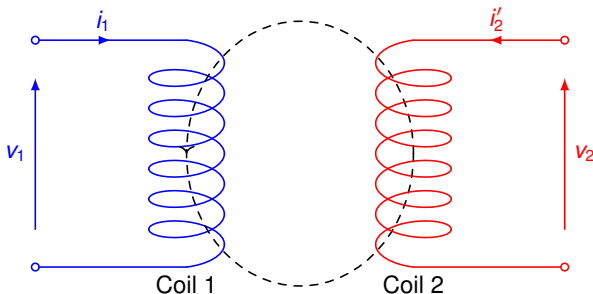
$$\ell_{\mu} = 4 \cdot 16 = 64 \text{ cm} = 0.64 \text{ m}$$

$$A = 4 \cdot 4 = 16 \text{ cm}^2 = 16 \cdot 10^{-4} \text{ m}^2$$

$$\mathcal{F} = \Phi \frac{\ell_{\mu}}{\mu_0 \mu_r A} = BA \frac{\ell_{\mu}}{\mu_0 \mu_r A} = B \frac{\ell_{\mu}}{\mu_0 \mu_r} = 127.95 \text{ ampere-turns}$$

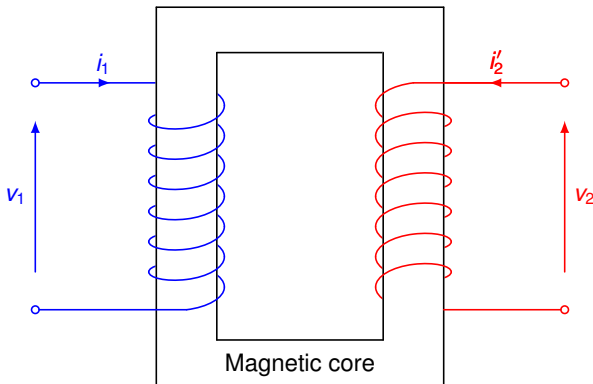
$$I = \frac{\mathcal{F}}{N} = 0.256 \text{ A}$$

## 2.1 Principle of a transformer - Coupled coils (1)



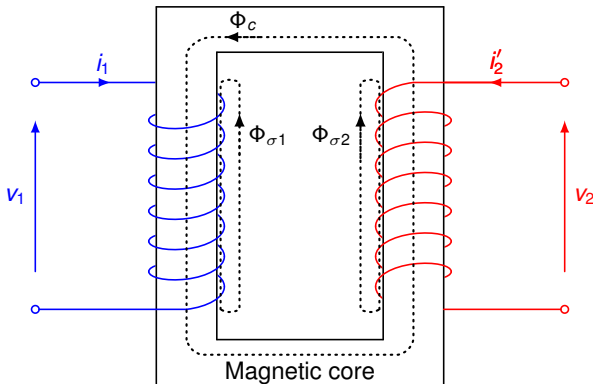
- Alternating current in coil produces alternating magnetic field (Faraday's law)
  - Two coils located in a common magnetic field influence each other
- Magnetic coupling: any variation of current in one coil causes variation of the flux linked with adjacent coil

## 2.1 Principle of a transformer - Coupled coils (2)



- Two coils mounted on common magnetic core (usually iron) → magnetic flux in both coils almost identical (strong magnetic coupling)

## 2.1 Principle of a transformer - Coupled coils (3)



- AC voltage  $v_1$  at terminals of coil 1  $\rightarrow$  AC current  $i_1$  in coil 1  $\rightarrow$  alternating magnetic field  $\rightarrow$  voltage induced in coil 2  $\rightarrow$  current  $i'_2$  in coil 2  $\rightarrow$  magnetic field superposed to field caused by  $i_1$
- $\Phi_c$  mutual flux in magnetic core flowing through both coils;  $\Phi_{\sigma 1}$  and  $\Phi_{\sigma 2}$  are leakage fluxes; note: above drawing assumes  $|i_1| > |i'_2|$

- Mutual flux (ροή μαγνητίσεως)  $\Phi_c$ : flux crossing both coils and causes magnetic coupling
- In each coil a *leakage flux* (ροή σκέδασης)  $\Phi_{\sigma 1}$  and  $\Phi_{\sigma 2}$  is developed in addition to mutual flux  $\Phi_c$ .
- Leakage flux: components of magnetic field crossing coil 1 or 2 but not passing through magnetic core.
- Usually:  $\Phi_{\sigma 1} \ll \Phi_c$ ,  $\Phi_{\sigma 2} \ll \Phi_c$
- Flux linkages (συνολική ροή)  $\psi_1$  and  $\psi_2$  in coils given by

$$\psi_1 = N_1 \Phi_c + \psi_{\sigma 1}$$

$$\psi_2 = N_2 \Phi_c + \psi_{\sigma 2}$$

- If we apply Ampère's law on the figure of slide 22, we get:

$$N_1 i_1 - N_2 i_2' = H_c \ell_c = \frac{\ell_c B_c}{\mu_c} = \left( \frac{\ell_c}{\mu_c A_c} \right) \Phi_c = \mathcal{R}_m \Phi_c$$

- $\ell_c$ : the length of the path in the core
- $A_c$ : the cross-section area of the magnetic core
- $\mu_c$ : the magnetic permeability of the magnetic core
- $\mathcal{R}_m$ : core reluctance (μαγνητική αντίσταση) of magnetic circuit
- Direction of magnetic fluxes dependent on sense of winding of coils (here is a minus)



Ideal transformer model is based on following assumptions

- A1: There are no leakage fluxes  $\rightarrow$  ideal coupling between coils
- A2: There are no losses in the transformer (neither in core nor in windings)
- A3: The permeability of the core material is infinite (= core has ideal magnetic conductivity)

- A1: No leakage fluxes, i.e.  $\psi_{\sigma 1} = \psi_{\sigma 2} = 0$

→ Faraday's law of induction (Slide 14) implies that

$$v_1 = \frac{d\psi_1}{dt} = N_1 \frac{d\Phi_c}{dt}$$
$$v_2 = \frac{d\psi_2}{dt} = N_2 \frac{d\Phi_c}{dt}$$

- Rearranging terms

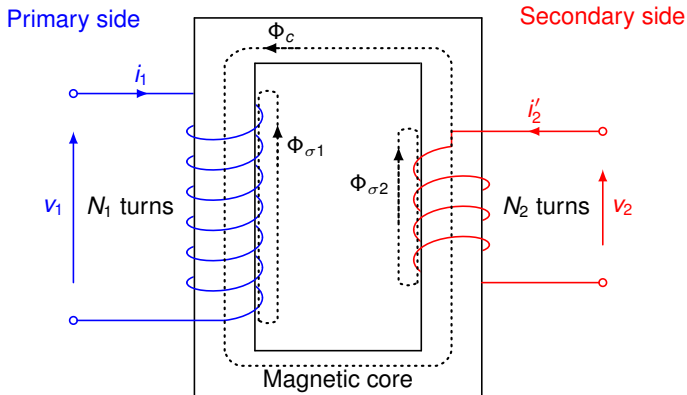
$$\frac{v_1}{N_1} = \frac{d\Phi_c}{dt} = \frac{v_2}{N_2}$$

→ Turns ratio between primary and secondary voltage

$$\boxed{\frac{v_1}{v_2} = \frac{N_1}{N_2} = c \in \mathbb{R}}$$

(Note: in three-phase transformers  $c$  can also be a complex number)

## 2.2 Basic single-phase transformer - Relation between primary and secondary voltages



- Number of turns can be different on primary and secondary windings
- Step-up transformer: secondary voltage  $v_2 >$  primary voltage  $v_1$
- Step-down transformer: secondary voltage  $v_2 <$  primary voltage  $v_1$

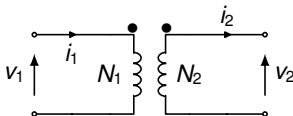
- A2: No losses (the coils have no resistance)
- A3: ideally conducting magnetic core (the permeability of the core material is infinite)  $\mu_c \rightarrow \infty \rightarrow \mathcal{R}_m = 0$

→ We obtain from Slide 24

$$N_1 i_1 - N_2 i_2' = 0$$

- Define  $i_2 = -i_2'$  to obtain

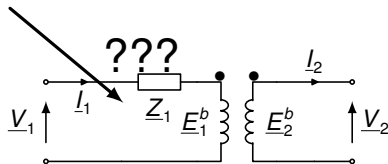
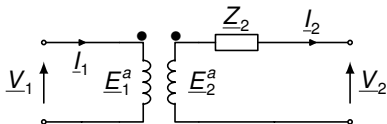
$$\boxed{\frac{i_2}{i_1} = \frac{N_1}{N_2} = c \in \mathbb{R}}$$

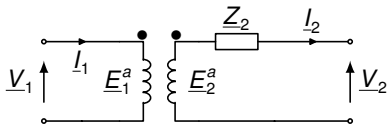


$$\frac{v_1}{v_2} = \frac{N_1}{N_2} = c, \quad \frac{i_2}{i_1} = \frac{N_1}{N_2} = c$$

- Single-phase two-winding transformer
- $c$  is the turns ratio
- "●" indicate sense of winding of coils:
  - "●" opposite of each other  $\rightarrow$  identical direction

## 2.2 Ideal transformer model - Impedance transformation





- Apply KVL

$$\underline{V}_1 = \underline{E}_1^a = c \underline{E}_2^a = c(\underline{V}_2 + \underline{I}_2 \underline{Z}_2)$$

$$\underline{V}_2 = \underline{E}_2^b = \frac{1}{c} \underline{E}_1^b = \frac{1}{c}(\underline{V}_1 - \underline{I}_1 \underline{Z}_1)$$

- Solve equations for  $\underline{V}_2$

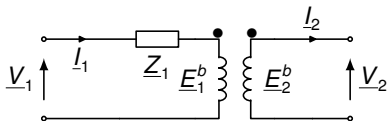
$$c \underline{V}_2 = \underline{V}_1 - c \underline{I}_2 \underline{Z}_2$$

$$c \underline{V}_2 = \underline{V}_1 - \underline{I}_1 \underline{Z}_1$$

- Subtract equations from each other

$$0 = -c \underline{I}_2 \underline{Z}_2 + \underline{I}_1 \underline{Z}_1$$

$$\boxed{\frac{\underline{Z}_1}{\underline{Z}_2} = c \frac{\underline{I}_2}{\underline{I}_1} = c^2}$$



- Relation of primary and secondary voltages  $\underline{E}_1$  and  $\underline{E}_2$

$$\frac{\underline{E}_1}{\underline{E}_2} = c$$

- Relation of primary and secondary currents  $\underline{I}_1$  and  $\underline{I}_2$

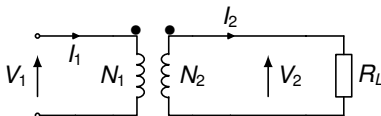
$$\frac{\underline{I}_1}{\underline{I}_2} = \frac{1}{c}$$

- Relation of primary and secondary impedances  $\underline{Z}_1$  and  $\underline{Z}_2$

$$\frac{\underline{Z}_1}{\underline{Z}_2} = c^2$$

- There are no losses in an idealised transformer!





**Task.** A single-phase transformer has 2500 turns on the primary winding and 500 turns on the secondary winding. The resistive load on the secondary side is  $\underline{Z}_L = R_L = 12 \, \Omega$ . The voltage magnitude (RMS) at the terminals of the primary winding is  $V_1 = 6 \, \text{kV}$ . Determine the voltage magnitude (RMS) at the secondary side and the load current.

**Solution.**

$$c = \frac{N_1}{N_2} = \frac{2500}{500} = 5$$

$$V_2 = \frac{V_1}{c} = 1.2 \, \text{kV}$$

$$I_2 = \frac{V_2}{R_L} = 0.1 \, \text{kA}$$

- Thus far: Discussion and derivation of idealised transformer model under *idealised* conditions and assumptions
- In the following, we will drop some of these assumptions to obtain a more realistic transformer model with
  - Non-zero leakage fluxes
  - Winding losses
  - Finite core permeability  $\mu_c$

- Each linkage flux  $\psi_{\sigma 1}$  and  $\psi_{\sigma 2}$  only interacts with one coil
- Can consider them independently of each other and as being caused by additional separate *leakage inductors* with inductances  $L_{\sigma 1}$  and  $L_{\sigma 2}$  in primary, respectively secondary, circuit
- Flux linkage between leakage inductors and leakage fields happens over air
- Linear relationship

$$\psi_{\sigma 1} = L_{\sigma 1} i_1$$

$$\psi_{\sigma 2} = L_{\sigma 2} i_2'$$

- Leakage fluxes diminish coupling between coils 1 and 2
- They represent a *non-ideal* property of a transformer

- Real windings possess ohmic (resistive) component
- $R_1$  resistance of coil 1;  $R_2$  resistance of coil 2
- KVL for primary circuit

$$\begin{aligned}v_1 &= R_1 i_1 + \frac{d\psi_1}{dt} = R_1 i_1 + \frac{d}{dt} (\psi_{\sigma 1} + N_1 \Phi_c) \\&= R_1 i_1 + L_{\sigma 1} \frac{di_1}{dt} + N_1 \frac{d\Phi_c}{dt}\end{aligned}$$

- KVL for secondary circuit

$$\begin{aligned}v_2 &= R_2 i'_2 + \frac{d\psi_2}{dt} = R_2 i'_2 + \frac{d}{dt} (\psi_{\sigma 2} + N_2 \Phi_c) \\&= R_2 i'_2 + L_{\sigma 2} \frac{di'_2}{dt} + N_2 \frac{d\Phi_c}{dt} \\&= -R_2 i_2 - L_{\sigma 2} \frac{di_2}{dt} + N_2 \frac{d\Phi_c}{dt}\end{aligned}$$

- Ohmic law of magnetic circuit from Slide 24 with  $\mathcal{R}_m \neq 0$

$$\mathcal{R}_m \Phi_c = N_1 i_1 - N_2 i_2'$$

- Calculation of **no load losses** → open-circuit the secondary → set  $i_2' = 0$

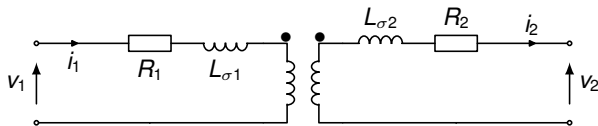
→ Magnetising current

$$i_1|_{i_2'=0} = i_m = \frac{\mathcal{R}_m \Phi_c}{N_1}$$

- Magnetising current  $i_m$  can be considered in transformer model via magnetising inductance  $L_h$
- Value of  $L_h$  can be derived from equation for induced voltage

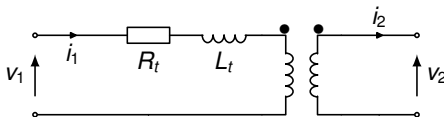
$$N_1 \frac{d\Phi_c}{dt} = L_h \frac{di_m}{dt}$$

## 2.3 Real transformer model with leakage inductors and winding resistors



- Real windings possess ohmic (resistive) component
  - $R_1$  resistance of coil 1;  $R_2$  resistance of coil 2
  - Real windings possess a component of leakage flux that only interacts with one coil, but does not link to the other coil
- Can consider this by additional separate *leakage inductors* with inductances  $L_{\sigma 1}$  and  $L_{\sigma 2}$  in primary, respectively secondary, circuit
- $L_{\sigma 1}$  leakage inductance coil 1;  $L_{\sigma 2}$  leakage inductance coil 2

## 2.3 Real transformer model with leakage inductors and winding resistors transformed to primary side



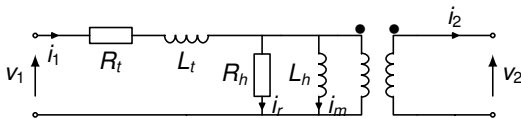
- Transform resistances and inductances to primary side

$$\begin{aligned} R_t &= R_1 + c^2 R_2 \\ L_t &= L_{\sigma 1} + c^2 L_{\sigma 2} \end{aligned}$$

→ Practical transformer model with flux leakages and ohmic losses in windings

- In reality, magnetic core has voltage- and frequency-dependent losses
- Magnetic conductivity of core also not ideal
- Power delivered on secondary side is lower than power injected on primary side!
- Transformer also consumes power if there is no current flowing on the secondary side ( $i_2 = 0$ ) to compensate for losses in magnetic core
- These losses are called *no load losses*
- No load losses = losses on primary coil + losses in magnetic core
- Reactive component of losses can be modelled by *magnetising inductance*
- Magnetising inductance then absorbs *magnetising current*

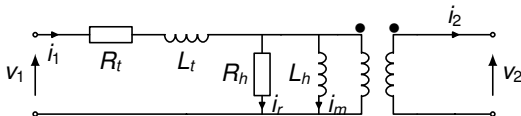




- In addition to reactive power losses there are also active power losses in core
- Active power losses represented by resistance  $R_h$  in parallel to  $L_h$
- Primary current under load (i.e.  $i_2 \neq 0$ )

$$i_1 = i_m + i_r + \frac{i_2}{c}$$

- In practice,  $i_m + i_r < 0.01 i_{2,\text{rated}}$



- Complete transformer model with
  - Leakage losses (represented by  $L_t$ )
  - Coil losses (represented by  $R_t$ )
  - Core losses (represented by  $L_h$  (inductive) and  $R_h$  (ohmic))
- Further phenomena to improve model
  - Core saturation → non-linear behaviour
  - Inrush current (energising current: non-sinusoidal and large DC component)
  - Non-sinusoidal exciting current

- Usual relation between leakage inductance  $L_t$  and magnetising inductance  $L_h$

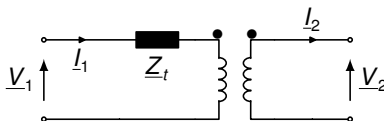
$$L_h \gg L_t$$

- Usual relation between coil resistance  $R_t$  and core resistance  $R_h$

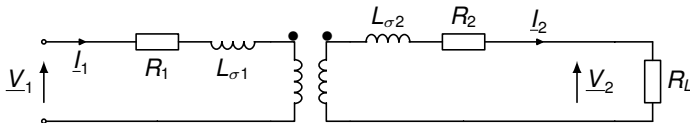
$$R_h \gg R_t$$

→ Neglect  $R_h$  and  $L_h$  in model

- Be careful: this simplification is only valid under load!



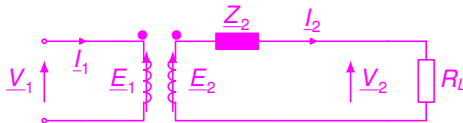
$$\underline{Z}_t = R_t + j\omega L_t$$



**Task.** A single-phase transformer has 2000 turns on the primary winding and 500 turns on the secondary winding. The winding resistances are  $R_1 = 2.0 \, \Omega$  and  $R_2 = 0.125 \, \Omega$ . The leakage reactances are  $X_1 = \omega L_{\sigma 1} = 8.0 \, \Omega$  and  $X_2 = \omega L_{\sigma 2} = 0.5 \, \Omega$ . The resistive load on the secondary side is  $Z_L = R_L = 12 \, \Omega$ . The voltage magnitude (RMS) at the terminals of the primary winding is  $V_1 = 6 \, \text{kV}$ . Determine the voltage at the load and the load current.

### Solution.

Equivalent circuit with all quantities referred to secondary side



Express all quantities with respect to secondary side

$$c = \frac{N_1}{N_2} = \frac{2000}{500} = 4 \quad \underline{Z}_2 = R_2 + jX_2 + \frac{R_1 + jX_1}{c^2} = 0.25 + j1.0 \, \Omega$$

$$\underline{E}_1 = \underline{V}_1 = 6\angle 0^\circ \text{ kV} \quad \underline{E}_2 = \frac{\underline{E}_1}{c} = \frac{\underline{V}_1}{c} = 1.25 \text{ kV}$$

$$\underline{I}_2 = \frac{\underline{E}_2}{\underline{Z}_2 + R_L} = 0.1022 - j0.0084 = 0.1\angle -4.7^\circ \text{ kA}$$

$$\underline{V}_2 = R_L \underline{I}_2 = 1.2\angle -4.7^\circ \text{ kV}$$

A single-phase two-winding transformer is rated 20 kVA, 480/120 V, 60 Hz.

- During a short-circuit test, where rated current at rated frequency is applied to the 480-volt winding (denoted winding 1), with the 120-volt winding (winding 2) shorted, the following readings are obtained:  
 $V_1 = 35 \text{ V}$ ,  $P_1 = 300 \text{ W}$ .
- During an open-circuit test, where rated voltage is applied to winding 2, with winding 1 open, the following readings are obtained:  $I_2 = 12 \text{ A}$ ,  $P_2 = 200 \text{ W}$ .

### Task.

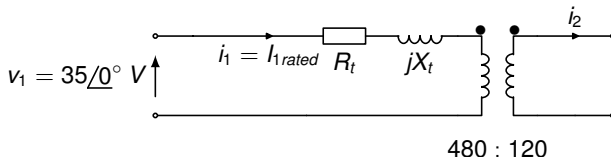
- 1 From the short-circuit test, determine the equivalent series impedance  $Z_t = R_t + jX_t$  referred to winding 1. Neglect the shunt admittance.
- 2 From the open-circuit test, determine the shunt admittance  $Y_m = G_c + jB_m = \frac{1}{Z_h} = \frac{1}{R_h || jX_h}$  referred to winding 1. Neglect the series impedance.

---

Source: J. D. Glover, M. S. Sarma and T. Overbye, "Power System Analysis & Design", 6th edition, Cengage Learning, 2017

## 2.3 Transformer short-circuit and open-circuit tests - Example

**Solution.** 1) Short-circuit neglecting admittance



$$I_{1rated} = \frac{S_{rated}}{V_{1rated}} = \frac{20 \cdot 10^3}{480} = 41.667 \text{ A}$$

$$R_t = \frac{P_1}{I_{1rated}^2} = \frac{300}{41.667^2} = 0.1728 \Omega$$

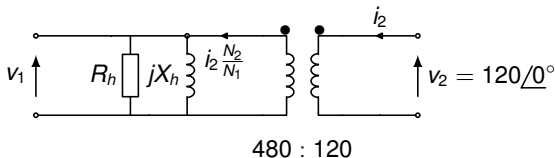
$$|Z_t| = \frac{v_1}{I_{1rated}} = \frac{35}{41.667} = 0.84 \Omega$$

$$X_t = \sqrt{Z_t^2 - R_t^2} = 0.822 \Omega$$

$$Z_t = R_t + jX_t = 0.1728 + j0.822 = 0.84\angle 78.13^\circ \Omega$$

## 2.3 Transformer short-circuit and open-circuit tests - Example

**Solution.** 2) Open-circuit neglecting impedance



$$v_1 = \frac{N_1}{N_2} v_2 = \frac{480}{120} 120 = 480 \text{ V}$$

$$G_c = \frac{P_2}{v_1^2} = \frac{200}{480^2} = 0.000868 \text{ S}$$

$$|Y_m| = \frac{i_1}{v_1} = \frac{i_2 \frac{N_2}{N_1}}{v_1} = \frac{12 \frac{120}{480}}{480} = 0.00625 \text{ S}$$

$$B_m = \sqrt{Y_m^2 - G_c^2} = 0.00619 \text{ S}$$

$$Y_m = 0.000868 - j0.00619 = 0.00625 \angle -82.02^\circ \text{ S}$$

---

Why is  $B_m$  negative?



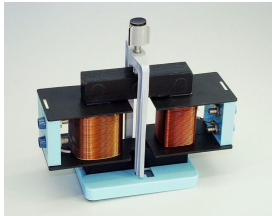
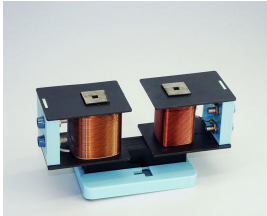
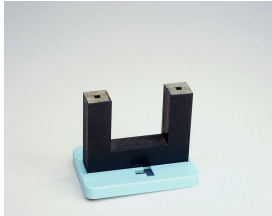
### 1 Why do we need power transformers?

### 2 Single-phase transformer

### 3 Three-phase transformer

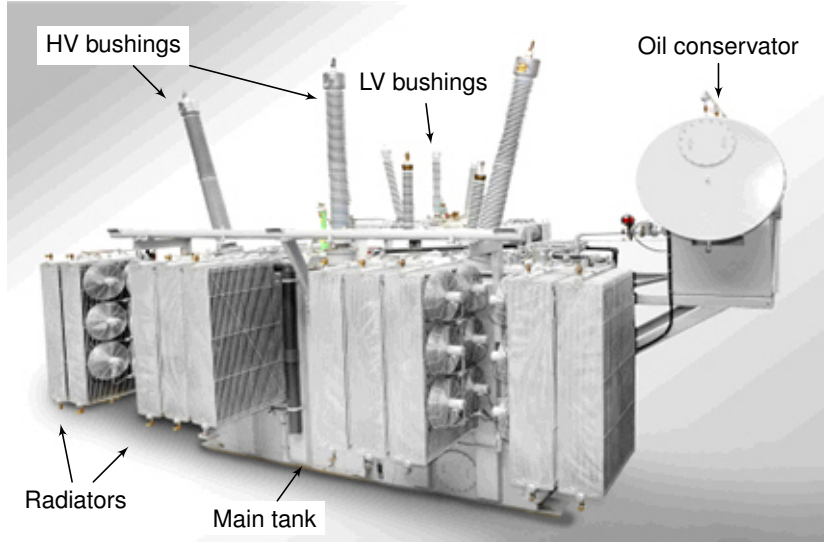
- Schematic representation of a three-phase transformer
- Types of three-phase transformers
- Configuration of three-phase transformers
- Transformation ratio and equivalent single-phase circuit of three-phase transformers

### 3 Basic single-phase transformer



Single-phase transformer

©Zátonyi Sándor



- HV and LV bushings make electrical connections from external transmission or distribution circuits into transformer tank
- Primary and secondary coils are normally made of copper, with paper insulation and are mounted on the outside of magnetic steel core
- Magnetic core is formed from laminated sheets of steel to minimise electrical losses, which would occur if a solid steel core was used
- Core and windings are mounted within a steel tank, which is sealed against atmospheric pollution, particularly moisture
- Tank is oil-filled, with oil acting as both high voltage insulation material and cooling medium
- Conservator tank provides a "head" of oil to ensure that tank is full, even under expansion and contraction of oil when thermally cycled during service
- External radiators are connected to tank to cool oil

## 3.1 Example of a three-phase transformer (1)



Large generator or transmission transformer can have mass of 300 – 500 tonnes → can represent significant logistical challenge in transporting unit from factory, via road, rail or ship, to its final destination at (potentially remote) substation

©Philippe Mertens

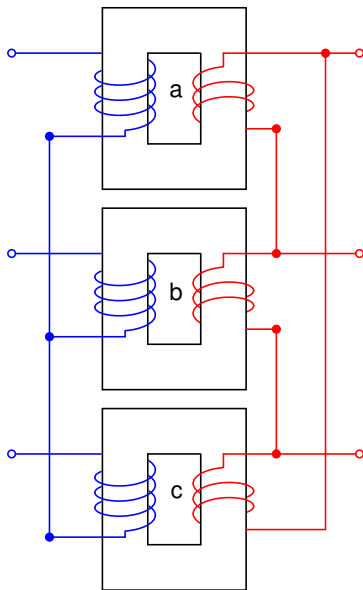
## 3.1 Example of a three-phase transformer (2)



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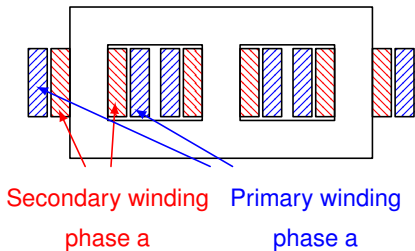
©Chris Lyles

## 3.2 Three-phase transformer - First type

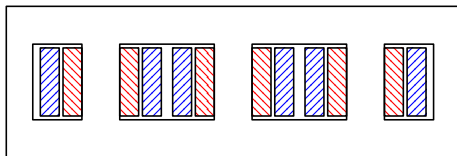


- Use three single-phase transformers
- If one transformer fails, only that one needs to be replaced
- Easier to carry
- Usually used for transformers of very large nominal power
- Phases are not coupled magnetically

### Core-type construction (3 legs/limbs)



### Shell-type construction (5 legs/limbs)



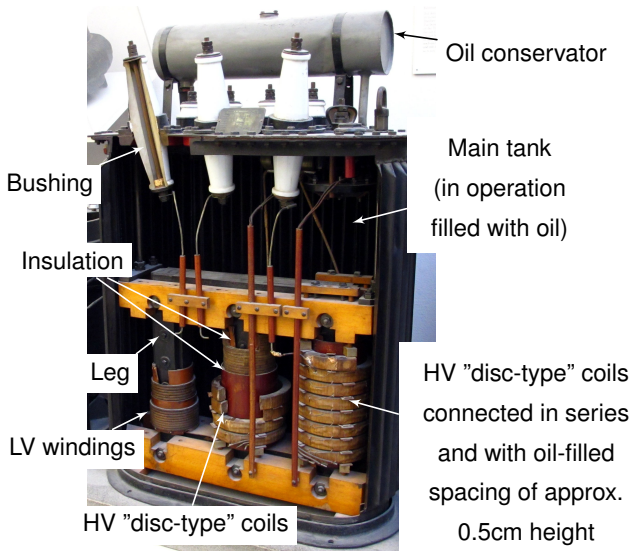
- Three phases mounted on single magnetic (iron) core
- Lighter and more compact than first type; less material and space needed
- Phases are magnetically coupled
- In balanced operation: fluxes in legs sum up to zero

$$\Phi_a + \Phi_b + \Phi_c = 0$$

- Core configuration or shell configuration



## 3.2 Cross-sectional scheme of three-phase transformer



©Dmm2va7

## 3.2 Example of a core-type three-phase transformer



©Faruku

## 3.2 Example of a three-phase transformer



©Nikitas Lamprou

## 3.2 Example of a three-phase transformer



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©Nikitas Lamprou

## 3.2 Example of a three-phase transformer



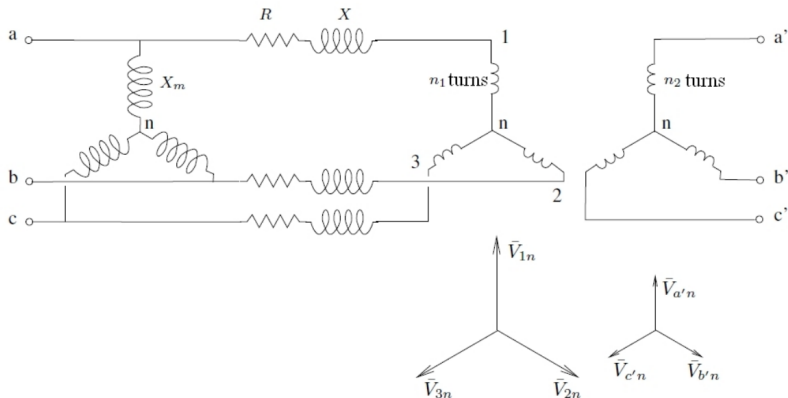
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©Nikitas Lamprou

### 3.3 Three-phase transformer - Configuration of primary and secondary side

Four main different configuration possibilities: Y-Y, Y-Delta, Delta-Y, Delta-Delta

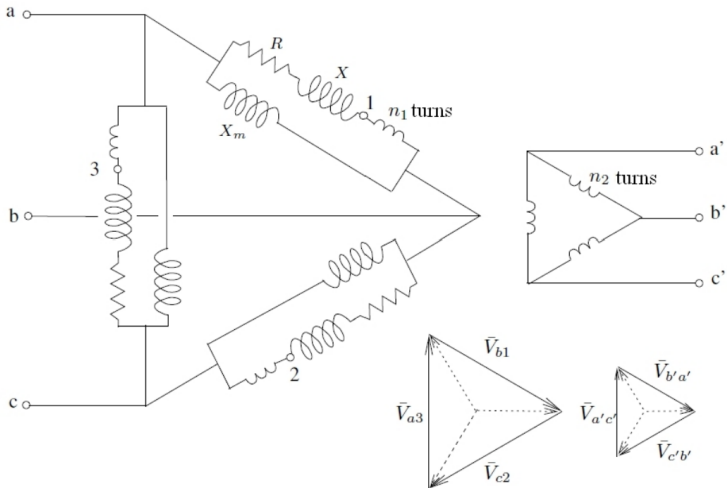
#### 1. Y-Y-configuration



- Preferred at very high voltage level since voltage across each coil is  $\sqrt{3}$  lower
- Possibility to connect neutral to ground (safety protection)

## 3.3 Configuration of three-phase transformers (2)

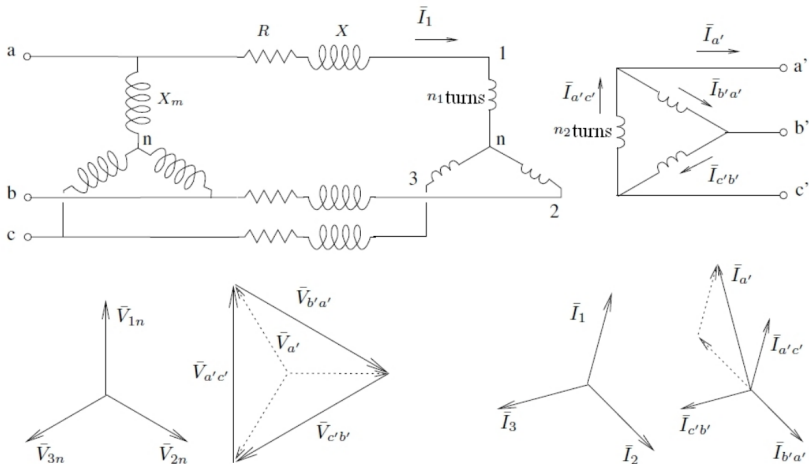
### 2. Delta-Delta-configuration



- Preferred for high currents, as currents in phases  $\sqrt{3}$  lower
- Also used to eliminate harmonics

## 3.3 Configuration of three-phase transformers (3)

### 3. Y-Delta-configuration



- Frequent transformer configuration connecting generator to grid
- On high-voltage side, neutral point is grounded (for protection)



- Standardized abbreviation of I.E.C. (International Electrotechnical Commission)
- Classification of transformer configuration consists of 2 letters and 1 integer
- First letter: Configuration of high-voltage side; upper-case letter used (**Y** or **D**)
- Second letter: Configuration of low-voltage side; lower-case letter used (**y** or **d**)
- Integer: Phase shift between voltages at primary and secondary winding of the same phase as multiple of  $30^\circ$  ( $\pi/6$ ) assuming the transformer is ideal
- Additionally in Y-connection: **n** after **Y** or **y** to indicate that neutral is grounded

- Example 1: Yd5
    - High-voltage side in Y-connection; neutral not grounded
    - Low-voltage side in Delta-connection
    - Phase-shift between high- and low-voltage:  $5 \cdot 30^\circ = 150^\circ$
  - Example 2: Yny0
    - High-voltage side in Y-connection; neutral grounded
    - Low-voltage side in y-connection
    - Phase-shift between high- and low-voltage:  $0^\circ$
- If same configuration on high- and low-voltage side, then no phase displacement between voltages or displacement of  $180^\circ$  (depending on sense of winding coils)

- In balanced operation, in principle can use single-phase equivalent circuit for analysis
- But if analysing Yd or Dy connections, need to consider
  - ① Not same voltages on primary and secondary side: one has phase voltages the other line voltages
    - Transformation ratio also affected!
  - ② Phase and line voltages also differ in phase
    - Additional phase displacement Phase displacement is integer multiple of  $\pi/6 = 30^\circ$

- Impact on amplitude considered by introducing additional scalar  $k$ 
  - Yy or Dd:  $k = 1$
  - Yd-configuration:  $k = \sqrt{3}$
  - Dy-configuration:  $k = \frac{1}{\sqrt{3}}$
- Impact on phase displacement considered by introducing additional phase shift element

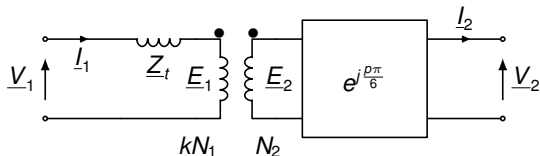
$$e^{j\frac{p\pi}{6}}$$

where  $p = \{0, 1 \dots, 11\}$  is an integer

→ Complex transformation ratio

$$\underline{c} = k \frac{N_1}{N_2} e^{j\frac{p\pi}{6}}$$

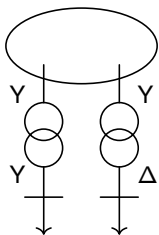
- Note:  $|e^{j\frac{p\pi}{6}}| \rightarrow$  amplitude of transformation not influenced by  $p$



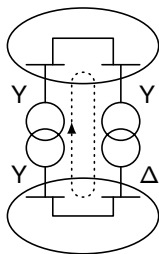
- Combine single-phase transformer model with complex transformation ratio
- Then

$$\underline{E}_1 = \underline{c} \underline{V}_2$$
$$\underline{I}_2 = \underline{c}^* \underline{I}_1$$

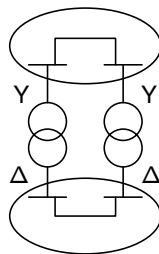
### 3.4 Three-phase transformer - Caution with phase displacements



allowed



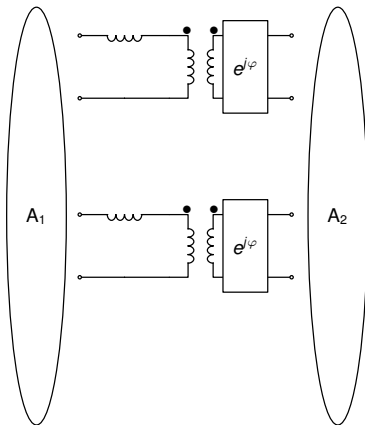
forbidden



allowed

- Many transformers present in electrical network
- If they work in parallel (i.e. in one same loop), they have to have the same phase displacement  $p$
- If this is not taken into account, there might be very large power flows between transformers

### 3.4 Three-phase transformer - Simplification of computations



- Two transformers in parallel with same phase displacement  
 $\varphi = \varphi_A = \varphi_B$
  - Then, phase displacement elements can be removed without changing
    - Magnitude of branch currents and bus voltages
    - Complex power flows in branches (as  $\underline{V} \underline{I}^* = \underline{V} e^{j\varphi} \underline{I}^* e^{-j\varphi}$ )
- Usually, phase displacement elements ignored in analysis and computations under balanced steady-state conditions

- Power systems work at different voltage levels
- Transformers needed to connect different voltage levels
- Models for single- and three-phase transformer
- Idealised and practical (more complicated) model
- In practice, core losses (shunt impedances) can often be neglected (under load!)
- Three-phase transformers have in general a complex transformation ratio
- Voltage ratio of transformer can be modified whilst it is in service by adding device known as an On-line Tap-changer (OLTC); this allows different numbers of turns to be connected on one of the coils (on each phase) of the transformer; typical adjustment in range  $\pm 10\%$