EEN442 - Power Systems II (Συστήματα Ισχύος II)

Part 4: Unbalanced operation

https://sps.cut.ac.cy/courses/een442/

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This part's learning objectives

After this part of the lecture and additional reading, you should be able to ...

1. ...explain the use of symmetrical components to describe the unbalanced operation of three-phase power systems in steady-state;
2. ...perform simple computations and system analysis with symmetrical components;
3. ...explain the power balance in ABC and symmetrical components.

Outline

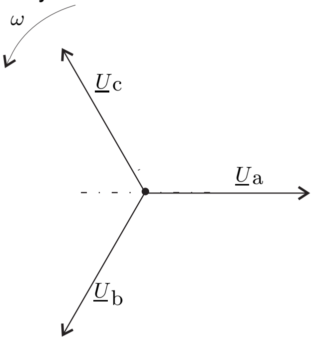
Unbalanced operation

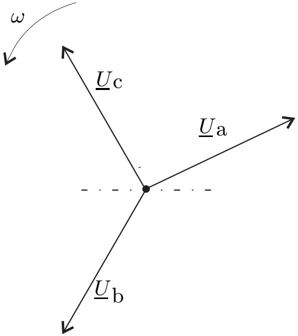
Voltage unbalance refers to a condition in a three-phase power system where the magnitudes of the three phase voltages are not equal or their phase angles are not exactly 120 degrees apart. This imbalance can lead to **inefficient operation of electrical equipment**, **increased heating in motors and transformers**, and **potential malfunction of sensitive electronics**.

Why would we have unbalanced operation?

- Unbalanced loads;
- unbalanced line parameters (e.g., untransposed and therefore, asymmetrical lines);
- unbalanced transformer parameters;
- ground faults or short circuits;

Why don't we use the same methods as in the previous parts of this course?





1 Outline

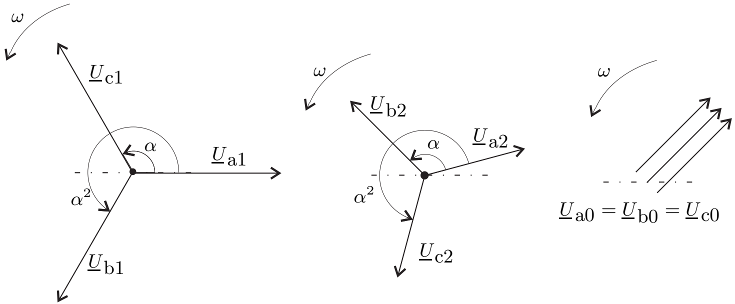
1 Main idea

A set of three unbalanced phasors can be decomposed into the sum of:

- three phasors making up a positive (or direct) sequence
- three phasors making up an negative sequence
- three phasors making up a zero sequence

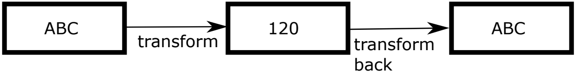
Thus, we end up with three 3-phase systems but each one of them is balanced (thus, easy to analyze). These are called the **symmetrical components**.

1 Symmetrical components



- **Positive sequence**: three rotating vectors, of same magnitude, shifted by 120° which an observer sees passing in the order a,b,c,a,b,c
- **Negative sequence**: three rotating vectors, of same magnitude, shifted by 120° which an observer sees passing in the order a,c,b,a,c,b
- **Zero sequence**: three rotating vectors, of same magnitude and in phase

1 Steps to perform analysis in symmetrical components



1. Transform ABC system to symmetrical components (this step is sometimes called "symmetrization").
2. Carry out all computations in that framework
3. Transform back to ABC to get actual currents and voltages (this step is sometimes called de-symmetrization).

Simplifications can be attributed to the fact that the symmetrical components are the eigenvectors of the admittance matrix

Projecting the ABC components to the 120 gives:

$$\underline{U}_{\mathrm{a}} = \underline{U}_{\mathrm{a1}} + \underline{U}_{\mathrm{a2}} + \underline{U}_{\mathrm{a0}} = \underline{U}_{1} + \underline{U}_{2} + \underline{U}_{0}$$
$$\underline{U}_{\mathrm{b}} = \underline{U}_{\mathrm{b1}} + \underline{U}_{\mathrm{b2}} + \underline{U}_{\mathrm{b0}} = \alpha^{2}\underline{U}_{1} + \alpha\underline{U}_{2} + \underline{U}_{0}$$
$$\underline{U}_{\mathrm{c}} = \underline{U}_{\mathrm{c1}} + \underline{U}_{\mathrm{c2}} + \underline{U}_{\mathrm{c0}} = \alpha\underline{U}_{1} + \alpha^{2}\underline{U}_{2} + \underline{U}_{0}$$

In matrix form:

$$\underbrace{\begin{pmatrix} \underline{U}_{\mathrm{a}} \\ \underline{U}_{\mathrm{b}} \\ \underline{U}_{\mathrm{c}} \end{pmatrix}}_{\underline{U}_{\mathrm{abc}}} = \underbrace{\begin{pmatrix} 1 & 1 & 1 \\ \alpha^{2} & \alpha & 1 \\ \alpha & \alpha^{2} & 1 \end{pmatrix}}_{\underline{\mathbf{T}}} \underbrace{\begin{pmatrix} \underline{U}_{1} \\ \underline{U}_{2} \\ \underline{U}_{0} \end{pmatrix}}_{\underline{U}_{120}}$$

1 Transformation matrix

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To get back to the ABC framework, we use the transformation matrix $\underline{\mathbf{S}}$, which is calculated as the inverse of $\underline{\mathbf{T}}$. The definition of the eigenvectors, having elements of magnitude 1, results in the fact that all elements of the matrix $\underline{\mathbf{S}}$ have a magnitude of $1/3$.

$$\underline{\mathbf{T}} = \begin{pmatrix} 1 & 1 & 1 \\ \alpha^2 & \alpha & 1 \\ \alpha & \alpha^2 & 1 \end{pmatrix} \qquad \underline{\mathbf{S}} = \underline{\mathbf{T}}^{-1} = \frac{1}{3} \cdot \begin{pmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{pmatrix}$$

where $\alpha = e^{j \cdot 120^\circ} = \frac{-1 + j \cdot \sqrt{3}}{2}$ and $\alpha^2 = e^{j \cdot 240^\circ} = \frac{-1 - j \cdot \sqrt{3}}{2}$.

It can be easily shown that $|\alpha| = 1$ and $1 + \alpha + \alpha^2 = 0$

1 ABC $\rightarrow$ 120

The inverse in matrix form is thus:

$$\underbrace{\begin{pmatrix} \underline{U}_1 \\ \underline{U}_2 \\ \underline{U}_0 \end{pmatrix}}_{\underline{U}_{120}} = \underbrace{\frac{1}{3}\begin{pmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{pmatrix}}_{\underline{\mathbf{S}}} \underbrace{\begin{pmatrix} \underline{U}_a \\ \underline{U}_b \\ \underline{U}_c \end{pmatrix}}_{\underline{U}_{abc}}$$

Which gives:

$$\underline{U}_1 = \frac{1}{3}\left(\underline{U}_a + \alpha \underline{U}_b + \alpha^2 \underline{U}_c\right)$$

$$\underline{U}_2 = \frac{1}{3}\left(\underline{U}_a + \alpha^2 \underline{U}_b + \alpha \underline{U}_c\right)$$

$$\underline{U}_0 = \frac{1}{3}\left(\underline{U}_a + \underline{U}_b + \underline{U}_c\right)$$

1 ABC $\to$ 120

Note that in a balanced system, we have:

$$\begin{pmatrix} \underline{U}_1 \\ \underline{U}_2 \\ \underline{U}_0 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \underline{U}_\mathrm{a} \\ \alpha^2 \underline{U}_\mathrm{a} \\ \alpha \underline{U}_\mathrm{a} \end{pmatrix} = \begin{pmatrix} \underline{U}_\mathrm{a} \\ 0 \\ 0 \end{pmatrix}$$

1 ABC → 120: Impedance

Assume a 3-phase coltage source feeding a 3-phase Y-connected load:

$$\begin{pmatrix} \underline{U}_a \\ \underline{U}_b \\ \underline{U}_c \end{pmatrix} = \begin{pmatrix} \underline{Z}_a & 0 & 0 \\ 0 & \underline{Z}_b & 0 \\ 0 & 0 & \underline{Z}_c \end{pmatrix} \begin{pmatrix} \underline{I}_a \\ \underline{I}_b \\ \underline{I}_c \end{pmatrix}$$

Converting to 120 sequence:

$$\underline{\mathbf{S}} \begin{pmatrix} \underline{U}_1 \\ \underline{U}_2 \\ \underline{U}_0 \end{pmatrix} = \begin{pmatrix} \underline{Z}_a & 0 & 0 \\ 0 & \underline{Z}_b & 0 \\ 0 & 0 & \underline{Z}_c \end{pmatrix} \underline{\mathbf{S}} \begin{pmatrix} \underline{I}_1 \\ \underline{I}_2 \\ \underline{I}_0 \end{pmatrix} \Leftrightarrow \begin{pmatrix} \underline{U}_1 \\ \underline{U}_2 \\ \underline{U}_0 \end{pmatrix} = \underbrace{\underline{\mathbf{T}} \begin{pmatrix} \underline{Z}_a & 0 & 0 \\ 0 & \underline{Z}_b & 0 \\ 0 & 0 & \underline{Z}_c \end{pmatrix} \underline{\mathbf{S}}}_{\underline{\mathbf{Z}}_{120}} \begin{pmatrix} \underline{I}_1 \\ \underline{I}_2 \\ \underline{I}_0 \end{pmatrix}$$

1 ABC $\rightarrow$ 120: Impedance

Assume a 3-phase coltage source feeding a 3-phase Y-connected load:

$$\begin{pmatrix} \underline{Z}_1 \\ \underline{Z}_2 \\ \underline{Z}_0 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} \underline{Z}_a + \underline{Z}_b + \underline{Z}_c & \underline{Z}_a + \alpha^2 \underline{Z}_b + \alpha \underline{Z}_c & \underline{Z}_a + \alpha \underline{Z}_b + \alpha^2 \underline{Z}_c \\ \underline{Z}_a + \alpha \underline{Z}_b + \alpha^2 \underline{Z}_c & \underline{Z}_a + \underline{Z}_b + \underline{Z}_c & \underline{Z}_a + \alpha^2 \underline{Z}_b + \alpha \underline{Z}_c \\ \underline{Z}_a + \alpha^2 \underline{Z}_b + \alpha \underline{Z}_c & \underline{Z}_a + \alpha \underline{Z}_b + \alpha^2 \underline{Z}_c & \underline{Z}_a + \underline{Z}_b + \underline{Z}_c \end{pmatrix}$$

If the impedances are the same ($\underline{Z}_a = \underline{Z}_b = \underline{Z}_c$):

$$\begin{pmatrix} \underline{Z}_1 \\ \underline{Z}_2 \\ \underline{Z}_0 \end{pmatrix} = \begin{pmatrix} \underline{Z}_a \\ \underline{Z}_b \\ \underline{Z}_c \end{pmatrix}$$

1 Comments

- It should be noted that the admittance matrices in the ABC and 120 systems incorporate the same information, but are not equal.
- No zero-sequence components exist if the sum of the unbalanced phasors is zero.
- Zero sequence components are never present in the **line voltages** regardless of the degree of unbalance. Can you explain this?

1 Summary

ABC	120
$\underline{\mathbf{I}}_{ABC} = \underline{\mathbf{T}}\,\underline{\mathbf{I}}_{120}$	$\underline{\mathbf{I}}_{120} = \underline{\mathbf{S}}\,\underline{\mathbf{I}}_{ABC}$
$\underline{\mathbf{U}}_{ABC} = \underline{\mathbf{T}}\,\underline{\mathbf{U}}_{120}$	$\underline{\mathbf{U}}_{120} = \underline{\mathbf{S}}\,\underline{\mathbf{U}}_{ABC}$
$\underline{\mathbf{E}}_{ABC} = \underline{\mathbf{T}}\,\underline{\mathbf{E}}_{120}$	$\underline{\mathbf{E}}_{120} = \underline{\mathbf{S}}\,\underline{\mathbf{E}}_{ABC}$
$\underline{\mathbf{Z}}_{ABC} = \underline{\mathbf{T}}\,\underline{\mathbf{Z}}_{120}\,\underline{\mathbf{S}}$	$\underline{\mathbf{Z}}_{120} = \underline{\mathbf{S}}\,\underline{\mathbf{Z}}_{ABC}\,\underline{\mathbf{T}}$
$\underline{\mathbf{Y}}_{ABC} = \underline{\mathbf{T}}\,\underline{\mathbf{Y}}_{120}\,\underline{\mathbf{S}}$	$\underline{\mathbf{Y}}_{120} = \underline{\mathbf{S}}\,\underline{\mathbf{Y}}_{ABC}\,\underline{\mathbf{T}}$
$\underline{\mathbf{I}}_{ABC} = \underline{\mathbf{Y}}_{ABC}\,\underline{\mathbf{E}}_{ABC}$	$\underline{\mathbf{I}}_{120} = \underline{\mathbf{Y}}_{120}\,\underline{\mathbf{E}}_{120}$

2 Outline

In ABC the three-phase apparent power is:

$$\underline{S}_{3\varphi} = \underline{U}_{\mathrm{a}}\underline{I}_{\mathrm{a}}^{*} + \underline{U}_{\mathrm{b}}\underline{I}_{\mathrm{b}}^{*} + \underline{U}_{\mathrm{c}}\underline{I}_{\mathrm{c}}^{*} = \underline{\mathbf{U}}\,\underline{\mathbf{I}}^{*}$$

Considering:

$$\begin{aligned} \underline{\mathbf{U}} &= \underline{\mathbf{T}}\,\underline{\mathbf{U}}_{120} \\ \underline{\mathbf{I}} &= \underline{\mathbf{T}}\,\underline{\mathbf{I}}_{120} \end{aligned}$$

We get:

$$\begin{aligned} \underline{S}_{3\varphi} &= \left(\underline{\mathbf{T}}\,\underline{\mathbf{U}}_{120}\right)^{T} \cdot \left(\underline{\mathbf{T}}\,\underline{\mathbf{I}}_{120}\right)^{*} = \\ &= \left(\underline{\mathbf{U}}_{120}\right)^{T} \cdot \underbrace{\underline{\mathbf{T}}^{T}\left(\underline{\mathbf{T}}\right)^{*}}_{3\,\mathbf{I_d}} \cdot \left(\underline{\mathbf{I}}_{120}\right)^{*} \end{aligned}$$

We can simplify this equation by calculating the middle part as:

$$\mathbf{T}^{T}\left(\underline{\mathbf{T}}\right)^{*} = \left(\left(\left(\underline{\mathbf{T}}\right)^{*}\right)^{T}\underline{\mathbf{T}}\right)^{T} = 3\left(\underline{\mathbf{T}}^{-1}\underline{\mathbf{T}}\right)^{T} = 3\,\mathbf{I_d}$$

where $\mathbf{I_d}$ represents the identity matrix.

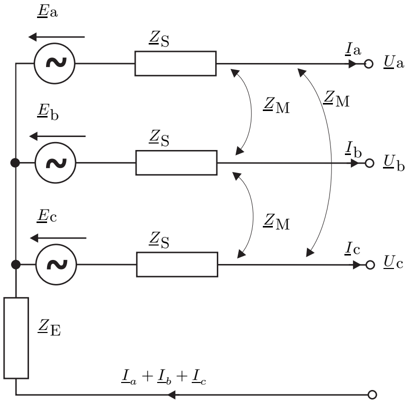
Leading to:

$$\begin{aligned} \underline{S}_{3\varphi} &= 3 \cdot \left(\underline{\mathbf{U}}_{120}\right)^T \cdot \left(\underline{\mathbf{I}}_{120}\right)^* = \\ &= 3 \cdot \left(\underline{U}_1 \underline{I}_1^* + \underline{U}_2 \underline{I}_2^* + \underline{U}_0 \underline{I}_0^*\right) \end{aligned}$$

- A factor of 3 arises between the expressions of the powers in the ABC and symmetrical component systems.
- Each element of the ABC system carries triple the power of its equivalent in the symmetrical component system

3 General system

Let's consider the following 3ϕ system:



where $\underline{Z}_\text{S}$ is the line impedance, $\underline{Z}_\text{M}$ is the mutual impedance between lines and $\underline{Z}_\text{E}$ in the neutral impedance.

3 General system

Using KCL and KVL, we can write the equations of the system for phase *a*:

$$\underline{U}_{\mathrm{a}} = -(\underline{I}_{\mathrm{a}} + \underline{I}_{\mathrm{b}} + \underline{I}_{\mathrm{c}})\underline{Z}_{\mathrm{E}} + \underline{E}_{\mathrm{a}} - \underline{Z}_{\mathrm{S}}\underline{I}_{\mathrm{a}} - \underline{Z}_{\mathrm{M}}\underline{I}_{\mathrm{b}} - \underline{Z}_{\mathrm{M}}\underline{I}_{\mathrm{c}}$$

Or, in matrix form for the entire system:

$$\begin{pmatrix} \underline{E}_{\mathrm{a}} \\ \underline{E}_{\mathrm{b}} \\ \underline{E}_{\mathrm{c}} \end{pmatrix} = \begin{pmatrix} \underline{Z}_{\mathrm{S}} + \underline{Z}_{\mathrm{E}} & \underline{Z}_{\mathrm{M}} + \underline{Z}_{\mathrm{E}} & \underline{Z}_{\mathrm{M}} + \underline{Z}_{\mathrm{E}} \\ \underline{Z}_{\mathrm{M}} + \underline{Z}_{\mathrm{E}} & \underline{Z}_{\mathrm{S}} + \underline{Z}_{\mathrm{E}} & \underline{Z}_{\mathrm{M}} + \underline{Z}_{\mathrm{E}} \\ \underline{Z}_{\mathrm{M}} + \underline{Z}_{\mathrm{E}} & \underline{Z}_{\mathrm{M}} + \underline{Z}_{\mathrm{E}} & \underline{Z}_{\mathrm{S}} + \underline{Z}_{\mathrm{E}} \end{pmatrix} \cdot \begin{pmatrix} \underline{I}_{\mathrm{a}} \\ \underline{I}_{\mathrm{b}} \\ \underline{I}_{\mathrm{c}} \end{pmatrix} + \begin{pmatrix} \underline{U}_{\mathrm{a}} \\ \underline{U}_{\mathrm{b}} \\ \underline{U}_{\mathrm{c}} \end{pmatrix}$$

3 Balanced system

In a balanced, properly transposed system, we can simplify:

$$
\begin{aligned}
\underline{U}_\mathrm{a} &= -(\underline{I}_\mathrm{a} + \underline{I}_\mathrm{b} + \underline{I}_\mathrm{c})\underline{Z}_\mathrm{E} + \underline{E}_\mathrm{a} - \underline{Z}_\mathrm{S}\underline{I}_\mathrm{a} - \underline{Z}_\mathrm{M}(\underline{I}_\mathrm{b} + \underline{I}_\mathrm{c}) \\
&= -\cancelto{0}{(\underline{I}_\mathrm{a} + \underline{I}_\mathrm{b} + \underline{I}_\mathrm{c})}\underline{Z}_\mathrm{E} + \underline{E}_\mathrm{a} - \underline{Z}_\mathrm{S}\underline{I}_\mathrm{a} - \underline{Z}_\mathrm{M}\cancelto{-\underline{I}_\mathrm{a}}{(\underline{I}_\mathrm{b} + \underline{I}_\mathrm{c})} \\
&= \underline{E}_\mathrm{a} - \underline{I}_\mathrm{a}(\underline{Z}_\mathrm{S} - \underline{Z}_\mathrm{M}) \\
&= \underline{E}_\mathrm{a} - \underline{I}_\mathrm{a}\underline{Z}_\mathrm{L}
\end{aligned}
$$

Leading to:

$$
\begin{pmatrix} \underline{E}_\mathrm{a} \\ \underline{E}_\mathrm{b} \\ \underline{E}_\mathrm{c} \end{pmatrix} = \begin{pmatrix} \underline{Z}_\mathrm{L} & 0 & 0 \\ 0 & \underline{Z}_\mathrm{L} & 0 \\ 0 & 0 & \underline{Z}_\mathrm{L} \end{pmatrix} \cdot \begin{pmatrix} \underline{I}_\mathrm{a} \\ \underline{I}_\mathrm{b} \\ \underline{I}_\mathrm{c} \end{pmatrix} + \begin{pmatrix} \underline{U}_\mathrm{a} \\ \underline{U}_\mathrm{b} \\ \underline{U}_\mathrm{c} \end{pmatrix}
$$

where $\underline{Z}_\mathrm{L} = \underline{Z}_\mathrm{S} - \underline{Z}_\mathrm{M}$.

Thus, we are able to analyze the system per-phase.

3 Unbalanced system

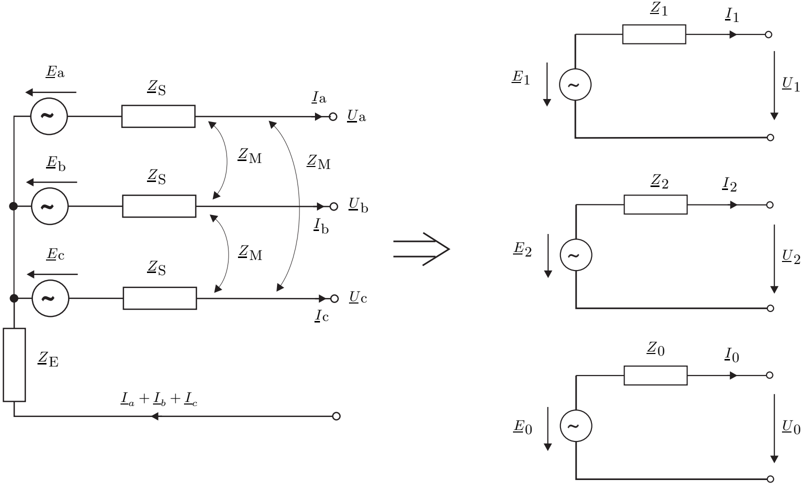
In an unbalanced system, the above approach does not work (why?). If we use the symmetrical components transformation (see slide 17) we can get:

$$\begin{pmatrix} \underline{E}_1 \\ \underline{E}_2 \\ \underline{E}_0 \end{pmatrix} = \begin{pmatrix} \underline{Z}_1 & 0 & 0 \\ 0 & \underline{Z}_2 & 0 \\ 0 & 0 & \underline{Z}_0 \end{pmatrix} \cdot \begin{pmatrix} \underline{I}_1 \\ \underline{I}_2 \\ \underline{I}_0 \end{pmatrix} + \begin{pmatrix} \underline{U}_1 \\ \underline{U}_2 \\ \underline{U}_0 \end{pmatrix} \tag{3.1}$$

where $\underline{Z}_1 = \underline{Z}_2 = \underline{Z}_S - \underline{Z}_M$ and $\underline{Z}_0 = \underline{Z}_S + 2\,\underline{Z}_M + 3\,\underline{Z}_E$.

✍ Even though the ABC system equations in an unbalanced system are not decoupled, the 120 equivalent equations are decoupled and we can analyze them per-phase.

Plotting the equivalent 120 circuits, gives:



3 Positive sequence system

- Corresponds to the single-phase equivalent circuit of the balanced three-phase system. The generator voltage feeds the circuit in the positive sequence system. This voltage is equal to the voltage $\underline{E}_{\mathrm{a}}$ of a symmetrically-operated generator.
- The impedances of the passive elements of the positive sequence system are included in the impedance $\underline{Z}_1$. It should be noted that $\underline{Z}_1$ is independent of the neutral-to-ground impedance $\underline{Z}_{\mathrm{E}}$.
- The grounding of the neutrals is irrelevant since the sum of the currents is zero. Delta-connected elements must be transformed into wye connections. In the symmetric, three-phase circuit, all neutral points have the same potential; it does not matter whether or not they are connected. Thus, all neutral points of the equivalent, positive sequence circuit can be thought of as connected.

3 Negative sequence system

- It is derived in a manner analogous to that of the positive sequence system. However, the voltage source component of the generator voltage is zero so that no supply voltage normally exists in the circuit.
- In the passive part of the network, the impedance $\underline{Z}_2$ is equal to $\underline{Z}_1$. This is due to the fact that the neutral point grounding has no effect on the negative sequence system (therefore, we can set $\underline{Z}_2 = \underline{Z}_1$).
- In the equivalent circuit of the negative sequence system, delta-wye transformations must be performed and all neutral points must be connected to one another.

3 Zero sequence system

- It is fed by the zero sequence component of the generator voltage, which is zero for symmetrical generators.

- The zero sequence impedance $\underline{Z}_0$ of the passive elements must be included. This impedance generally differs from the positive and negative sequence system impedances.

- The treatment of the neutral points is very important in the zero sequence system. As mentioned previously, zero sequence currents can **only** flow through neutral point connections. The connections in the zero sequence diagram correspond to the grounding conditions in the real physical system.

- Impedances at the neutral point connections must be included with **triple the value of the physical impedance**. The threefold value is necessary because triple the real zero sequence current actually flows through the neutral ground connection.

4 Outline

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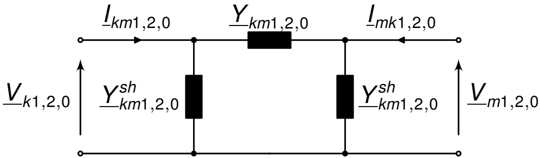
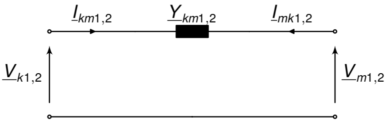


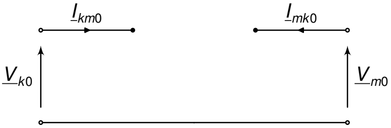
Positive, Negative, and Zero sequence model

$$\begin{bmatrix} \underline{I}_{km1,2,0} \\ \underline{I}_{mk1,2,0} \end{bmatrix} = \begin{bmatrix} \underline{Y}_{km1,2,0} + \underline{Y}^{sh}_{km1,2,0} & -\underline{Y}_{km1,2,0} \\ -\underline{Y}_{km1,2,0} & \underline{Y}_{km1,2,0} + \underline{Y}^{sh}_{km1,2,0} \end{bmatrix} \begin{bmatrix} \underline{V}_{k1,2,0} \\ \underline{V}_{m1,2,0} \end{bmatrix}$$

If the model is of type Yy, YNy, Yd, Dd:



Positive & Negative sequence

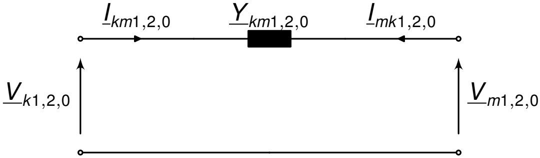


Zero sequence model

$$\begin{bmatrix} \underline{I}_{km1,2} \\ \underline{I}_{mk1,2} \end{bmatrix} = \begin{bmatrix} \underline{Y}_{km1,2} & -\underline{Y}_{km1,2} \\ -\underline{Y}_{km1,2} & \underline{Y}_{km1,2} \end{bmatrix} \begin{bmatrix} \underline{V}_{k1,2} \\ \underline{V}_{m1,2} \end{bmatrix}$$

$$\begin{bmatrix} \underline{I}_{km0} \\ \underline{I}_{mk0} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{V}_{k0} \\ \underline{V}_{m0} \end{bmatrix}$$

If the model is of type YNyn:

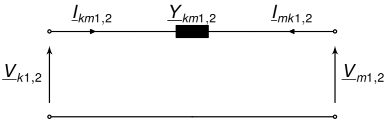


Positive & Negative & Zero sequence

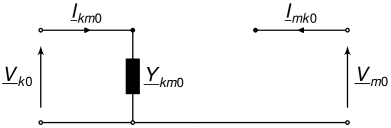
$$\begin{bmatrix} \underline{I}_{km1,2,0} \\ \underline{I}_{mk1,2,0} \end{bmatrix} = \begin{bmatrix} \underline{Y}_{km1,2,0} & -\underline{Y}_{km1,2,0} \\ -\underline{Y}_{km1,2,0} & \underline{Y}_{km1,2,0} \end{bmatrix} \begin{bmatrix} \underline{V}_{k1,2,0} \\ \underline{V}_{m1,2,0} \end{bmatrix}$$

4 Transformer model

If the model is of type YNd:



Positive & Negative sequence

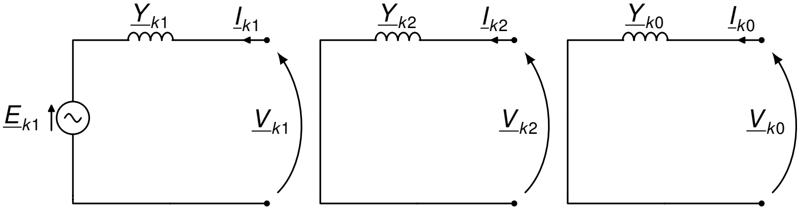


Zero sequence model

$$\begin{bmatrix} \underline{I}_{km1,2} \\ \underline{I}_{mk1,2} \end{bmatrix} = \begin{bmatrix} \underline{Y}_{km1,2} & -\underline{Y}_{km1,2} \\ -\underline{Y}_{km1,2} & \underline{Y}_{km1,2} \end{bmatrix} \begin{bmatrix} \underline{V}_{k1,2} \\ \underline{V}_{m1,2} \end{bmatrix}$$

$$\begin{bmatrix} \underline{I}_{km0} \\ \underline{I}_{mk0} \end{bmatrix} = \begin{bmatrix} Y_{km0} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{V}_{k0} \\ \underline{V}_{m0} \end{bmatrix}$$

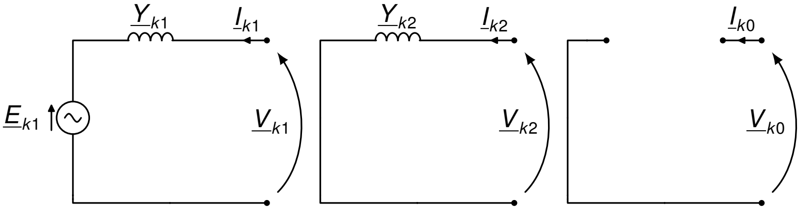
If the model is of type YN:



We use the load assumption for uniformity and the Norton equivalent:

$$\underline{I}_{k1} = \underline{Y}_{k1}\underline{V}_{k1} - \underline{Y}_{k1}\underline{E}_{k1}$$
$$\underline{I}_{k2,0} = \underline{Y}_{k2,0}\underline{V}_{k2,0}$$

If the model is of type Y or D:



We use the load assumption for uniformity and the Norton equivalent:

$$\underline{I}_{k1} = \underline{Y}_{k1}\underline{V}_{k1} - \underline{Y}_{k1}\underline{E}_{k1}$$
$$\underline{I}_{k2} = \underline{Y}_{k2}\underline{V}_{k2}$$
$$\underline{I}_{k0} = 0$$

5 Outline

5 Key Aspects of Grid Code Requirements

1. **Maximum Voltage Unbalance Limits**:
   - *IEC Standards*: According to IEC 61000-2-2, the recommended maximum voltage unbalance is typically 2% under normal operating conditions. Stricter limits, such as 1%, may apply for sensitive installations or specific equipment.
   - *IEEE Standards*: IEEE Std 1159 suggests similar limits, generally not exceeding 2% for standard industrial applications.
   - *Regional Variations*: Some countries may adopt different thresholds. For example, the UK's G98/G99 grid codes align with IEC standards, maintaining a 2% unbalance limit.
2. **Measurement and Reporting Requirements**: Continuous Monitoring: Grid-connected entities may be required to continuously monitor voltage quality, including unbalance levels. Reporting Frequency: Regular reporting (e.g., monthly or quarterly) may be mandated to ensure compliance and facilitate grid reliability assessments.
3. **Mitigation Obligations**: If voltage unbalance exceeds specified limits, grid codes often require the offending party to take corrective actions. This can include reconfiguring connections, installing power quality equipment, or adjusting operational parameters.

5 How Voltage Unbalance is Checked

1. **Measurement Techniques**:
   - *Power Quality Analyzers*: These devices measure voltage magnitude and phase angles across all three phases. They calculate the degree of unbalance by comparing the deviations from the ideal 1:1:1 ratio and 120-degree phase separation.
   - *Digital Relays and Monitoring Systems*: Advanced digital relays can incorporate power quality monitoring, providing real-time data on voltage unbalance alongside protection functions.
   - *Periodic Testing*: Utilities may perform scheduled tests using portable meters to verify compliance during routine maintenance or as part of periodic grid assessments.

2 Calculation Methods:

Voltage unbalance can be quantified using the **Voltage Unbalance Factor (VUF)**, which is calculated as:

$$\mathrm{VUF} = \frac{\sqrt{(V_a - V_{\text{avg}})^2 + (V_b - V_{\text{avg}})^2 + (V_c - V_{\text{avg}})^2}}{V_{\text{avg}}} \times 100\%$$

Where:

- V_a, V_b, V_c are the phase voltage magnitudes.
- $V_{\text{avg}} = \frac{V_a+V_b+V_c}{3}$ is the average voltage.

Alternatively, using symmetrical components:

$$\mathrm{VUF} = \frac{|V_2|}{|V_1|} \times 100\%$$

Where V_1 is the positive sequence voltage and V_2 is the negative sequence voltage.

5 Mitigation Strategies

1. **Balancing Loads**: Ensuring that the loads across all three phases are as equal as possible to minimize unbalance.
2. **Power Quality Equipment**:
   - *Phase Balancers*: Devices that redistribute unbalanced loads.
   - *Static Var Compensators (SVCs)*: Improve voltage stability and balance reactive power.
   - *Harmonic Filters*: Address harmonic distortions that can exacerbate voltage unbalance.
3. **System Configuration**: Optimizing the network layout and transformer connections to naturally mitigate unbalance.
4. **Regular Maintenance**: Periodic inspections and maintenance of electrical infrastructure to prevent conditions that could lead to voltage unbalance.