# EEN452 - Control and Operation of Electric Power Systems

## Part 2B: Synchronous machine model (detailed)

https://sps.cut.ac.cy/courses/een452/

Dr Petros Aristidou
Department of Electrical Engineering, Computer Engineering & Informatics
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# Today's learning objectives

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Extend the model of the synchronous machine considered in the previous lesson to . . .

1. . . . add more details appropriate for dynamic studies;
2. . . . include the effect of damper windings;
3. . . . be applicable to machines with salient-pole rotors (hydro power plants);

Much of the material was adapted from the courses delivered by Prof. Thierry Van Cutsem at the University of Liege.

## Remarks

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In this lesson, we consider:

- a machine with a single pair of poles, for simplicity. This does not affect the electrical behaviour of the generator (it affects the moment of inertia and the kinetic energy of rotating masses)
- the general case of a ***salient-pole*** machine. For a round-rotor machine: set some parameters to the same value in the $d$ and $q$ axes (to account for the equal air gap width)
- the configuration with ***four*** rotor windings ($f$ , $d_1$, $q_1$, $q_2$). For a salient-pole generator: remove the $q_2$ winding.

# 1 Outline

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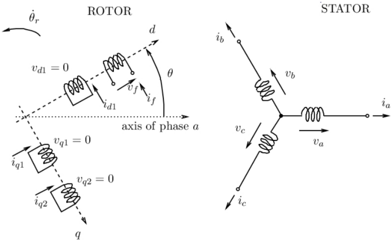
# 1 Relations between voltages, fluxes and currents




Stator windings (generator convention):

$$v_a(t) = -R_a i_a(t) - \frac{d\psi_a}{dt} \quad v_b(t) = -R_b i_b(t) - \frac{d\psi_b}{dt} \quad v_c(t) = -R_c i_c(t) - \frac{d\psi_c}{dt}$$

$R_?$ : Resistance of (a,b,c) phase $\quad \psi_?$ : flux linkage in (a,b,c) phase

In matrix form:

$$\boldsymbol{v}_T = -\boldsymbol{R}_T \boldsymbol{i}_T - \frac{d\boldsymbol{\psi}_T}{dt}$$

$$\boldsymbol{R}_T = diag(R_a\ R_a\ R_a)$$

## 1 Relations between voltages, fluxes and currents

Field windings (motor convention):

$$v_f(t) = R_f i_f(t) + \frac{d\psi_f}{dt}$$
$$0 = R_{d1} i_{d1}(t) + \frac{d\psi_{d1}}{dt}$$
$$0 = R_{q1} i_{q1}(t) + \frac{d\psi_{q1}}{dt}$$
$$0 = R_{q2} i_{q2}(t) + \frac{d\psi_{q2}}{dt}$$

$R_?$ : Resistance of (f, d1, q1, q2) winding $\psi_?$ : flux linkages in (f, d1, q1, q2) winding

In matrix form:

$$\boldsymbol{v}_r = -\boldsymbol{R}_r \boldsymbol{i}_r - \frac{d\boldsymbol{\psi}_r}{dt}$$
$$\boldsymbol{R}_r = diag(R_f \ R_{d1} \ R_{q1} \ R_{q2})$$

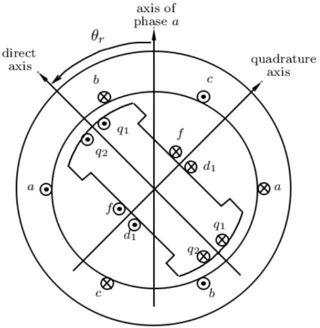
# 1 Inductances

Saturation being neglected, the fluxes vary linearly with the currents according to:

$$\begin{bmatrix} \psi_T \\ \psi_r \end{bmatrix} = \begin{bmatrix} \boldsymbol{L}_{TT}(\theta_r) & \boldsymbol{L}_{Tr}(\theta_r) \\ \boldsymbol{L}_{Tr}^T(\theta_r) & \boldsymbol{L}_{rr} \end{bmatrix} \begin{bmatrix} \boldsymbol{i}_T \\ \boldsymbol{i}_r \end{bmatrix}$$

- $\boldsymbol{L}_{TT}$ and $\boldsymbol{L}_{Tr}$ vary with the position $\theta_r$ of the rotor but $\boldsymbol{L}_{rr}$ does not
- the components of $\boldsymbol{L}_{TT}$ and $\boldsymbol{L}_{Tr}$ are periodic functions of $\theta_r$
- the space harmonics in $\theta_r$ are assumed negligible = sinusoidal machine assumption.

# 1 Inductances




$$\boldsymbol{L}_{TT}(\theta_r) =$$

$$\begin{bmatrix} L_0 + L_1\cos(2\theta_r) & -L_m - L_1\cos\left(2(\theta_r + \frac{\pi}{6})\right) & -L_m - L_1\cos\left(2(\theta_r - \frac{\pi}{6})\right) \\ -L_m - L_1\cos\left(2(\theta_r + \frac{\pi}{6})\right) & L_0 + L_1\cos\left(2(\theta_r - \frac{2\pi}{3})\right) & -L_m - L_1\cos\left(2(\theta_r + \frac{\pi}{2})\right) \\ -L_m - L_1\cos\left(2(\theta_r - \frac{\pi}{6})\right) & -L_m - L_1\cos\left(2(\theta_r + \frac{\pi}{2})\right) & L_0 + L_1\cos\left(2(\theta_r + \frac{2\pi}{3})\right) \end{bmatrix}$$

$$L_0,\ L_1,\ L_m > 0$$

# 1 Inductances

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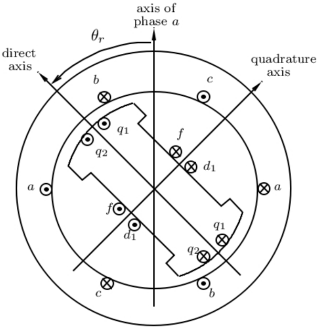


$$\boldsymbol{L}_{Tr}(\theta_r) =$$

$$\begin{bmatrix} L_{af}\cos(\theta_r) & L_{ad1}\cos(\theta_r) & L_{aq1}\sin(\theta_r) & L_{aq2}\sin(\theta_r) \\ L_{af}\cos(\theta_r - \frac{2\pi}{3}) & L_{ad1}\cos(\theta_r - \frac{2\pi}{3}) & L_{aq1}\sin(\theta_r - \frac{2\pi}{3}) & L_{aq2}\sin(\theta_r - \frac{2\pi}{3}) \\ L_{af}\cos(\theta_r + \frac{2\pi}{3}) & L_{ad1}\cos(\theta_r + \frac{2\pi}{3}) & L_{aq1}\sin(\theta_r + \frac{2\pi}{3}) & L_{aq2}\sin(\theta_r + \frac{2\pi}{3}) \end{bmatrix}$$

$$L_{af},\ L_{ad1},\ L_{aq1},\ L_{aq2} > 0$$

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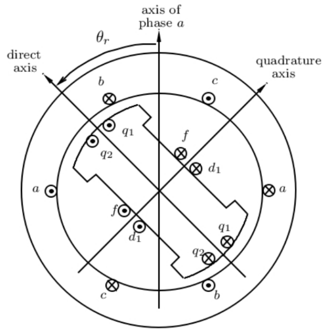


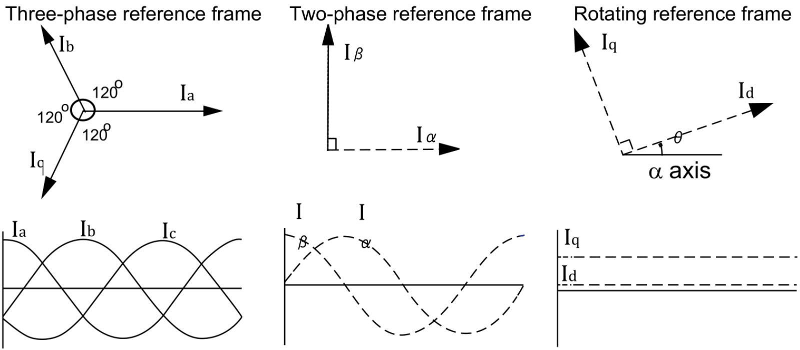

$$\boldsymbol{L}_{rr} = \begin{bmatrix} L_{ff} & L_{fd1} & 0 & 0 \\ L_{fd1} & L_{d1d1} & 0 & 0 \\ 0 & 0 & L_{q1q1} & L_{q1q2} \\ 0 & 0 & L_{q1q2} & L_{q2q2} \end{bmatrix}$$

# 2 Outline

# 2 Park and Clarke transformations

- Flux linkages, induced voltages, and currents change continuously as the electric circuit is in relative motion – **very difficult to model and solve!**
- Mathematical transformations are often used to decouple variables and to solve equations involving time varying quantities by referring all variables to a common frame of reference
- Among the various transformation methods, the most well-known are:
  - Clarke Transformation
  - Park Transformation

- ***Clarke Transformation***: This transformation converts balanced three-phase quantities into balanced two-phase quadrature quantities.
- ***Park Transformation***: This transformation converts vectors in balanced two-phase orthogonal stationary system into orthogonal rotating reference frame.

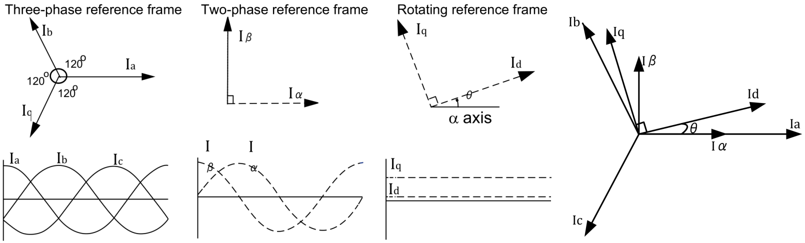


# 2 Park and Clarke transformations



The three reference frames considered in this implementation are:

- Three-phase reference frame, in which $I_a$, $I_b$, and $I_c$ are co-planar three-phase quantities at an angle of 120 degrees to each other.
- Orthogonal stationary reference frame, in which $I_\alpha$ (along $\alpha$ axis) and $I_\beta$ (along $\beta$ axis) are perpendicular to each other, but in the same plane as the three-phase reference frame.
- Orthogonal rotating reference frame, in which $I_d$ is at an angle $\theta$ (rotation angle) to the $\alpha$ axis and $I_q$ is perpendicular to $I_d$ along the q axis.



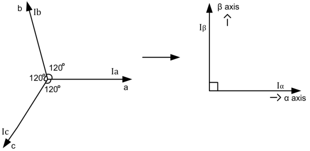
## 2 Clarke transformation

The three-phase quantities are translated from the three-phase reference frame to the two-axis orthogonal stationary reference frame using Clarke transformation[1]:

$$\begin{bmatrix} \alpha \\ \beta \\ 0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

where:

- *a*, *b*, and *c* are three-phase quantities
- $\alpha$ and $\beta$ are stationary orthogonal reference frame quantities
- 0 is the zero component of the two-axis system in the stationary reference frame




[1] We use a power invariant version that preserves active and reactive power

## 2 Park transformation

The two-axis orthogonal stationary reference frame quantities are transformed into rotating reference frame quantities using Park transformation[2]:

$$\begin{bmatrix} d \\ q \\ 0 \end{bmatrix} = \underbrace{\sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\theta_r) & \cos(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r + \frac{2\pi}{3}) \\ \sin(\theta_r) & \sin(\theta_r - \frac{2\pi}{3}) & \sin(\theta_r + \frac{2\pi}{3}) \\ \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{bmatrix}}_{\mathcal{P}} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

where:

- $I_a$, $I_b$, and $I_c$ are three-phase quantities
- $I_d$ and $I_q$ are the components of the two-axis system in the rotating reference frame
- 0 is the zero component of the two-axis system in the stationary reference frame

[2] We use a power invariant version that preserves active and reactive power

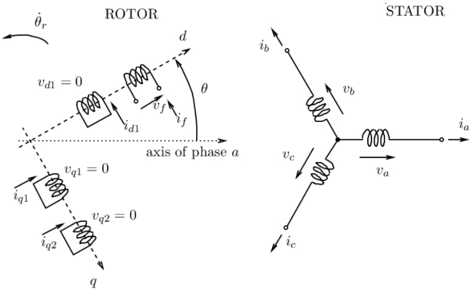
## 2 Park transformation

Transforming the stator quantities gives:

$$\boldsymbol{v}_{dq0} = \mathcal{P}\boldsymbol{v}_{abc} \qquad \boldsymbol{i}_{dq0} = \mathcal{P}\boldsymbol{i}_{abc} \qquad \psi_{dq0} = \mathcal{P}\psi_{abc}$$

We can also see that:

$$\mathcal{P}\mathcal{P}^T = \boldsymbol{I} \Leftrightarrow \mathcal{P}^{-1} = \mathcal{P}^T$$



## 2 Park transformation

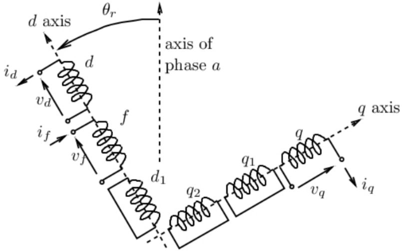
Total magnetic field created by the currents $I_a$, $I_b$, and $I_c$:

$$\text{projected on d axis: } k\left(\cos(\theta_r)i_a + \cos(\theta_r - \frac{2\pi}{3})i_b + \cos(\theta_r - \frac{4\pi}{3})i_c\right) = k\sqrt{\frac{3}{2}}i_d$$

$$\text{projected on q axis: } k\left(\sin(\theta_r)i_a + \sin(\theta_r - \frac{2\pi}{3})i_b + \sin(\theta_r - \frac{4\pi}{3})i_c\right) = k\sqrt{\frac{3}{2}}i_q$$

The Park transformation consists of replacing the $(a, b, c)$ stator windings by three equivalent windings $(d, q, 0)$:

- the $d$ winding is attached to the $d$ axis
- the $q$ winding is attached to the $q$ axis
- the currents $i_d$ and $i_q$ produce together the same magnetic field, to the multiplicative constant $\sqrt{\frac{3}{2}}$



# 2 Park equations of the synchronous machine

Applying the Park transformation to the equations of slide 5, we get:

$$v_d = -R_a i_d - \dot{\theta}_r \psi_q - \frac{d\psi_d}{dt}$$

$$v_q = -R_a i_q + \dot{\theta}_r \psi_d - \frac{d\psi_q}{dt}$$

$$v_0 = -R_a i_0 - \frac{d\psi_0}{dt}$$

where:

$\dot{\theta}_r \psi_q,\ \dot{\theta}_r \psi_d$: speed voltages

$\frac{d\psi_d}{dt},\ \frac{d\psi_q}{dt}$: transformer voltages

# 2 Park equations of the synchronous machine

Applying the Park transformation to the equations of slide 7, we get:

$$\begin{bmatrix} \boldsymbol{\psi}_T \\ \boldsymbol{\psi}_r \end{bmatrix} = \begin{bmatrix} \boldsymbol{L}_{TT} & \boldsymbol{L}_{Tr} \\ \boldsymbol{L}_{Tr}^T & \boldsymbol{L}_{rr} \end{bmatrix} \begin{bmatrix} \boldsymbol{i}_T \\ \boldsymbol{i}_r \end{bmatrix} \Leftrightarrow \begin{bmatrix} \mathcal{P}^{-1}\boldsymbol{\psi}_P \\ \boldsymbol{\psi}_r \end{bmatrix} = \begin{bmatrix} \boldsymbol{L}_{TT} & \boldsymbol{L}_{Tr} \\ \boldsymbol{L}_{Tr}^T & \boldsymbol{L}_{rr} \end{bmatrix} \begin{bmatrix} \mathcal{P}^{-1}\boldsymbol{i}_P \\ \boldsymbol{i}_r \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{\psi}_P \\ \boldsymbol{\psi}_r \end{bmatrix} = \begin{bmatrix} \mathcal{P}\boldsymbol{L}_{TT}\mathcal{P}^{-1} & \mathcal{P}\boldsymbol{L}_{Tr} \\ \boldsymbol{L}_{Tr}^T\mathcal{P}^{-1} & \boldsymbol{L}_{rr} \end{bmatrix} \begin{bmatrix} \boldsymbol{i}_P \\ \boldsymbol{i}_r \end{bmatrix} = \begin{bmatrix} \boldsymbol{L}_{PP} & \boldsymbol{L}_{Pr} \\ \boldsymbol{L}_{rP}^T & \boldsymbol{L}_{rr} \end{bmatrix} \begin{bmatrix} \boldsymbol{i}_P \\ \boldsymbol{i}_r \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{L}_{PP} & \boldsymbol{L}_{Pr} \\ \boldsymbol{L}_{rP}^T & \boldsymbol{L}_{rr} \end{bmatrix} = \begin{bmatrix} L_{dd} & & & L_{df} & L_{dd1} & & \\ & L_{qq} & & & & L_{qq1} & L_{qq2} \\ & & L_{00} & & & & \\ L_{df} & & & L_{ff} & L_{fd1} & & \\ L_{dd1} & & & L_{fd1} & L_{d1d1} & & \\ & L_{qq1} & & & & L_{q1q1} & L_{q1q2} \\ & L_{qq2} & & & & L_{q1q2} & L_{q2q2} \end{bmatrix}$$

*zero entries have been left empty for legibility

# 2 Park equations of the synchronous machine

where:

$$L_{dd} = L_0 L_m + \frac{3}{2} L_1$$

$$L_{qq} = L_0 L_m - \frac{3}{2} L_1$$

$$L_{df} = \sqrt{\frac{3}{2}} L_{af}$$

$$L_{dd1} = \sqrt{\frac{3}{2}} L_{ad1}$$

$$L_{qq1} = \sqrt{\frac{3}{2}} L_{aq1}$$

$$L_{qq2} = \sqrt{\frac{3}{2}} L_{aq2}$$

$$L_{00} = L_0 - 2L_m$$

- As expected, a ll components are independent of the rotor position $\theta_r$!
- There is no magnetic coupling between d and q axes (this was already assumed in $\boldsymbol{L}_{Tr}$ and $\boldsymbol{L}_{rr}$: zero mutual inductances between coils with orthogonal axes).

# 2 Park equations of the synchronous machine

If we ignore the 0 component (is this a valid simplification?) and we group (d, f, d1) and (q, q1, q2), we get:

$$\begin{bmatrix} v_d \\ -v_f \\ 0 \end{bmatrix} = - \begin{bmatrix} R_a & & \\ & R_f & \\ & & R_{d1} \end{bmatrix} \begin{bmatrix} i_d \\ i_f \\ i_{d1} \end{bmatrix} - \begin{bmatrix} \dot{\theta}_r \psi_q \\ 0 \\ 0 \end{bmatrix} - \frac{d}{dt} \begin{bmatrix} \psi_d \\ \psi_f \\ \psi_{d1} \end{bmatrix}$$

$$\begin{bmatrix} v_q \\ 0 \\ 0 \end{bmatrix} = - \begin{bmatrix} R_a & & \\ & R_{q1} & \\ & & R_{q2} \end{bmatrix} \begin{bmatrix} i_q \\ i_{q1} \\ i_{q2} \end{bmatrix} + \begin{bmatrix} \dot{\theta}_r \psi_d \\ 0 \\ 0 \end{bmatrix} - \frac{d}{dt} \begin{bmatrix} \psi_q \\ \psi_{q1} \\ \psi_{q2} \end{bmatrix}$$

with the following flux-current relations:

$$\begin{bmatrix} \psi_d \\ \psi_f \\ \psi_{d1} \end{bmatrix} = \begin{bmatrix} L_{dd} & L_{df} & L_{dd1} \\ L_{df} & L_{ff} & L_{fd1} \\ L_{dd1} & L_{fd1} & L_{d1d1} \end{bmatrix} \begin{bmatrix} i_d \\ i_f \\ i_{d1} \end{bmatrix}$$

$$\begin{bmatrix} \psi_q \\ \psi_{q1} \\ \psi_{q2} \end{bmatrix} = \begin{bmatrix} L_{qq} & L_{qq1} & L_{qq2} \\ L_{qq1} & L_{q1q1} & L_{q1q2} \\ L_{qq2} & L_{q1q2} & L_{q2q2} \end{bmatrix} \begin{bmatrix} i_q \\ i_{q1} \\ i_{q2} \end{bmatrix}$$

## 3 Balance of power at stator

$$p_T + p_{Js} + \frac{dW_{ms}}{dt} = p_{r\to s}$$

where
$p_T$: three-phase instantaneous power leaving the stator
$p_{Js}$: Joule losses in stator windings
$W_{ms}$: magnetic energy stored in the stator windings
$p_{r\to s}$: power transfer from rotor to stator (mechanical? electrical?)

Three-phase instantaneous power leaving the stator:

$$p_T(t) = v_a i_a + v_b i_b + v_c i_c = v_d i_d + v_q i_q + v_0 i_0$$
$$= -\underbrace{\left(R_a i_d^2 + R_a i_q^2 + R_a i_0^2\right)}_{p_{Js}} - \underbrace{\left(i_d \frac{d\psi_d}{dt} + i_q \frac{d\psi_q}{dt} + i_0 \frac{d\psi_0}{dt}\right)}_{dW_{ms}/dt} + \dot{\theta}_r(\psi_d i_q - \psi_q i_d)$$

$$\Rightarrow p_{r\to s} = \dot{\theta}_r(\psi_d i_q - \psi_q i_d)$$

## 3 Balance of power at rotor

$$P_m + p_f = p_{Jr} + \frac{dW_{mr}}{dt} + p_{r\to s} + \frac{dW_c}{dt}$$

where
$P_m$: mechanical power provided by the turbine
$p_f$: electrical power provided to the field winding (by the excitation system)
$p_{Jr}$: Joule losses in the rotor windings
$W_{mr}$: magnetic energy stored in the rotor windings
$W_c$: kinetic energy of all rotating masses

Instantaneous power provided to field winding:

$$\begin{aligned} p_f &= v_f i_f = v_f i_f + v_{d1} i_{d1} + v_{q1} i_{q1} + v_{q2} i_{q2} \\ &= \underbrace{\left(R_f i_f^2 + R_{d1} i_{d1}^2 + R_{q1} i_{q1}^2 + R_{q2} i_{q2}^2\right)}_{p_{Jr}} + \underbrace{\left(i_f \frac{d\psi_f}{dt} + i_{d1}\frac{d\psi_{d1}}{dt} + i_{q1}\frac{d\psi_{q1}}{dt} + i_{q2}\frac{d\psi_{q2}}{dt}\right)}_{dW_{mr}/dt} \end{aligned}$$

$$\Rightarrow P_m - \frac{dW_c}{dt} = \dot{\theta}_r(\psi_d i_q - \psi_q i_d)$$

# 3 Equation of rotor motion

$$J\frac{d^2\theta_r}{dt^2} = T_m - T_e$$

where
$J$: moment of inertia of all the rotating masses
$T_m$: mechanical torque applied to the rotor by the turbine
$T_e$: electromagnetic torque applied to the rotor by the generator

Multiplying by $\dot{\theta}_r$:

$$J\dot{\theta}_r\ddot{\theta}_r = \dot{\theta}_r(T_m - T_e)$$

$$\frac{dW_c}{dt} = P_m - \dot{\theta}_r T_e$$

where
$P_m$: mechanical power applied to the rotor by the turbine

Hence, the (compact and elegant!) expression of the electromagnetic torque is:

$$T_e = \psi_d i_q - \psi_q i_d$$

# 3 Components of the torque $T_e$



$$T_e = L_{dd} i_d i_q + L_{df} i_f i_q + L_{dd1} i_{d1} i_q - L_{qq} i_q i_d - L_{qq1} i_{q1} i_d - L_{qq2} i_{q2} i_d$$

$(L_{dd} - L_{qq}) i_d i_q$: synchronous torque due to rotor saliency

- exists in salient-pole machines only
- even without excitation ($i_f = 0$), the rotor tends to align its direct axis with the axis of the rotating magnetic field created by the stator currents, offering to the latter a longer path in iron
- a significant fraction of the total torque in a salient-pole generator

$L_{dd1} i_{d1} i_q - L_{qq1} i_{q1} i_d - L_{qq2} i_{q2} i_d$: damping torque

- due to currents induced in the damper windings
- zero in steady-state operation

$L_{df} i_f i_q$: only component involving the field current $i_f$

- the main part of the total torque in steady-state operation
- in steady state, it is the synchronous torque due to excitation
- during transients, the field winding also contributes to the damping torque

# 4 Outline

# 4 Remarks

In steady-state we have:

- Balanced three-phase currents of angular frequency $\omega_N$ flow in the stator windings
- a direct current flows in the field winding subjected to a constant excitation voltage:

$$i_f = \frac{V_f}{R_f}$$

- the rotor rotates at the synchronous speed:

$$\theta_r = \theta_r^0 + \omega_N t$$

- no current is induced in the other rotor circuits:

$$i_{d1} = i_{q1} = i_{q2} = 0$$

# 4 Operation with stator opened

$$
\begin{aligned}
& i_a = i_b = i_c = 0 \\
\Rightarrow \quad & i_d = i_q = i_0 = 0 \\
\Rightarrow \quad & \psi_d = L_{df} i_f \quad \text{and} \quad \psi_q = 0
\end{aligned}
$$

Park equations:

$$v_d = 0, \qquad v_q = \omega_N \psi_d = \omega_N L_{df} i_f$$

Getting back to the stator voltages, e.g. in phase *a*:

$$v_a(t) = \sqrt{\frac{2}{3}} \omega_N L_{df} i_f \sin(\theta_r^0 + \omega_N t) = \sqrt{2} E_q \sin(\theta_r^0 + \omega_N t)$$

where:
$E_q = \frac{\omega_N L_{df} i_f}{\sqrt{3}}$: e.m.f. proportional to excitation current = RMS voltage at the terminal of the opened machine.

# 4 Operation under load



$$v_a(t) = \sqrt{2}V\cos(\omega_N t + \theta) \qquad i_a(t) = \sqrt{2}I\cos(\omega_N t + \psi)$$

$$v_b(t) = \sqrt{2}V\cos(\omega_N t + \theta - \frac{2\pi}{3}) \quad i_b(t) = \sqrt{2}I\cos(\omega_N t + \psi - \frac{2\pi}{3})$$

$$v_c(t) = \sqrt{2}V\cos(\omega_N t + \theta + \frac{2\pi}{3}) \quad i_c(t) = \sqrt{2}I\cos(\omega_N t + \psi + \frac{2\pi}{3})$$

$$\begin{aligned}
i_d =& \sqrt{\frac{2}{3}}\sqrt{2}I\Big[\cos(\theta_r^0 + \omega_N t)\cos(\omega_N t + \psi) + \cos(\theta_r^0 + \omega_N t - \frac{2\pi}{3})\cos(\omega_N t + \psi - \frac{2\pi}{3}) \\
&+ \cos(\theta_r^0 + \omega_N t + \frac{2\pi}{3})\cos(\omega_N t + \psi + \frac{2\pi}{3})\,\Big] \\
=& \frac{I}{\sqrt{3}}\Big[\cos(\theta_r^0 + 2\omega_N t + \psi) + \cos(\theta_r^0 + 2\omega_N t + \psi - \frac{4\pi}{3}) + \cos(\theta_r^0 + 2\omega_N t + \psi - \frac{4\pi}{3}) \\
&+ 3\cos(\theta_r^0 - \psi)\,\Big] = \sqrt{3}I\cos(\theta_r^0 - \psi)
\end{aligned}$$

Similarly:

$$i_q = \sqrt{3}I\sin(\theta_r^0 - \psi) \qquad i_0 = 0$$

$$v_d = \sqrt{3}V\cos(\theta_r^0 - \theta) \quad v_q = \sqrt{3}V\sin(\theta_r^0 - \theta) \quad v_0 = 0$$

In steady-state, $i_d$ and $i_q$ are constant. This was expected!

# 4 Magnetic flux in the d and q windings

$$\psi_d = L_{dd} i_d + L_{df} i_f$$
$$\psi_q = L_{qq} i_q$$

The electromagnetic torque:

$$T_e = \psi_d i_q - \psi_q i_d$$

is constant. This is important from mechanical viewpoint (no vibrations!).

Park equations:

$$v_d = -R_a i_d - \omega_N L_{qq} i_q = -R_a i_d - X_q i_q$$
$$v_q = -R_a i_q - \omega_N L_{dd} i_d = -R_a i_q + X_d i_d + \sqrt{3} E_q$$
$$v_0 = 0$$

where
$X_d = \omega_N L_{dd}$: direct-axis synchronous reactance
$X_q = \omega_N L_{qq}$: quadrature-axis synchronous reactance

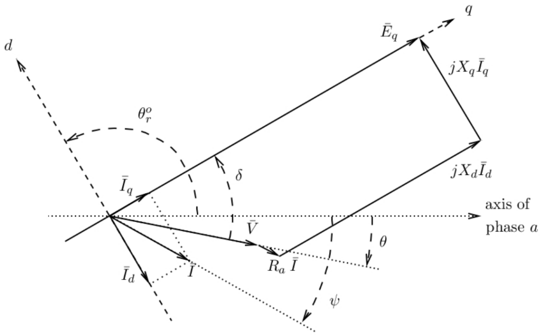
# 4 Phasor diagram

The Park equations become:

$$V\cos(\theta_r^0-\theta) = -R_a I\cos(\theta_r^0-\psi) - X_q I\sin(\theta_r^0-\psi)$$
$$V\sin(\theta_r^0-\theta) = -R_a I\sin(\theta_r^0-\psi) + X_d I\cos(\theta_r^0-\psi) + E_q$$

which are the projections on the d and q axes of the complex equation:

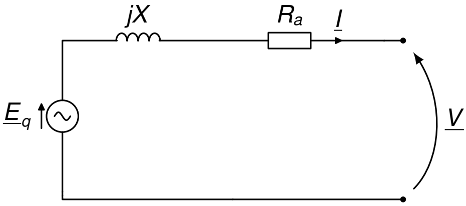
$$\underline{E}_q = \underline{V} + R_a\underline{I} + jX_d\underline{I}_d + jX_q\underline{I}_q$$



# 4 Equivalent diagram

Round-rotor machine ($X_d = X_q = X$):

$$\underline{E}_q = \underline{V} + R_a\underline{I} + jX(\underline{I}_d + \underline{I}_q) = \underline{V} + R_a\underline{I} + jX\underline{I}$$




***Not valid for a salient-pole generator!***

$$\underline{E}_q = E_q e^{j(\theta_r^0 - \frac{\pi}{2})}$$

$$\underline{I}_d = I\cos(\theta_r^0 - \psi)e^{j\theta_r^0} = \frac{i_d}{\sqrt{3}}e^{j\theta_r^0}$$

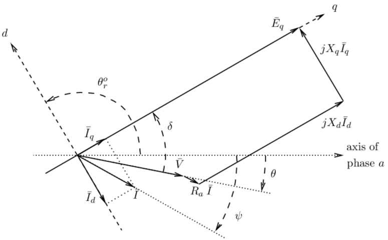
$$\underline{I}_q = I\sin(\theta_r^0 - \psi)e^{j(\theta_r^0 - \frac{\pi}{2})} = -j\frac{i_q}{\sqrt{3}}e^{j\theta_r^0}$$

$$\underline{I} = \underline{I}_d + \underline{I}_q = \left(\frac{i_d}{\sqrt{3}} - j\frac{i_q}{\sqrt{3}}\right)e^{j\theta_r^0}$$

$$\underline{V}_d = V\cos(\theta_r^0 - \theta)e^{j\theta_r^0} = \frac{v_d}{\sqrt{3}}e^{j\theta_r^0}$$

$$\underline{V}_q = V\sin(\theta_r^0 - \theta)e^{j(\theta_r^0 - \frac{\pi}{2})} = -j\frac{v_q}{\sqrt{3}}e^{j\theta_r^0}$$

$$\underline{V} = \underline{V}_d + \underline{V}_q = \left(\frac{v_d}{\sqrt{3}} - j\frac{v_q}{\sqrt{3}}\right)e^{j\theta_r^0}$$



## 4 Powers

Three-phase complex power produced by the machine:

$$\underline{S} = 3\underline{V}\,\underline{I}^* = 3\left(\frac{v_d}{\sqrt{3}} - j\frac{v_q}{\sqrt{3}}\right)\left(\frac{i_d}{\sqrt{3}} + j\frac{i_q}{\sqrt{3}}\right) = (v_d - jv_q)(i_d + ji_q)$$

$$\Rightarrow \quad P = v_d i_d + v_q i_q \qquad Q = v_d i_q - v_q i_d$$

$P$ and $Q$ as functions of $V$, $E_q$ and the internal angle $\delta$. Assuming $R_a \approxeq 0$:

$$v_d = -X_q i_q \qquad \Rightarrow \quad i_q = -\frac{v_d}{X_q}$$

$$v_q = -X_d i_d + \sqrt{3}E_q \qquad \Rightarrow \quad i_d = -\frac{v_q - \sqrt{3}E_q}{X_d}$$

$$v_d = \sqrt{3}V\cos(\theta_r^0 - \theta) = -\sqrt{3}V\sin(\delta), \qquad v_q = \sqrt{3}V\sin(\theta_r^0 - \theta) = \sqrt{3}V\cos(\delta)$$

$$P = 3\frac{E_q V}{X_d}\sin(\delta) + \frac{3V^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin(2\delta) \xrightarrow{round-rotor} P = 3\frac{E_q V}{X}\sin(\delta)$$

$$Q = 3\frac{E_q V}{X_d}\cos(\delta) - \frac{3V^2}{2}\left(\frac{\sin^2(\delta)}{X_q} + \frac{\cos^2(\delta)}{X_d}\right) \xrightarrow{round-rotor} Q = 3\frac{E_q V}{X}\cos(\delta) - 3\frac{V^2}{X}$$

# 5 Outline

## 5 Stator

- nominal line voltage $V_N$: voltage for which the machine has been designed (in particular its insulation). The real voltage may deviate from this value by a few %
- nominal current $I_N$: current for which machine has been designed (in particular the cross-section of its conductors). Maximum current that can be accepted without limit in time
- nominal apparent power: $S_N = \sqrt{3} V_N I_N$

Conversion of parameters in per unit values:

- base power: $S_B = S_N$
- base voltage: $V_B = \frac{V_N}{\sqrt{3}}$
- base current: $I_B = \frac{S_N}{3V_B}$
- base impedance: $Z_B = \frac{3V_B^2}{S_B}$

# 5 Orders of magnitude

(more typical of machines with a nominal power above 100 MVA)
(pu values on the machine base)

| | round-rotor | salient-pole |
|---|---|---|
| resistance $R_a$ | 0.005 pu | |
| direct-axis reactance $X_d$ | 1.5 - 2.5 pu | 0.9 - 1.5 pu |
| quadrature-axis reactance $X_q$ | 1.5 - 2.5 pu | 0.5 - 1.1 pu |

## 5 Park (equivalent) windings

- base power: $S_N$
- base voltage: $\sqrt{3}V_B$
- base current: $\sqrt{3}I_B = \frac{S_N}{\sqrt{3}V_B}$ (single-phase formula!)

Thus:

$$i_{dpu} = \frac{i_d}{\sqrt{3}I_b} = \frac{\sqrt{3}}{\sqrt{3}}\frac{I}{I_B}\cos(\theta_r^0 - \psi) = I_{pu}\cos(\theta_r^0 - \psi)$$

$$i_{qpu} = I_{pu}\sin(\theta_r^0 - \psi), \quad v_{dpu} = V_{pu}\cos(\theta_r^0 - \theta), \quad v_{qpu} = V_{pu}\sin(\theta_r^0 - \theta)$$

$$\underline{I} = \underline{I}_d + \underline{I}_q = (i_d - ji_q)e^{j\theta_r^0} \qquad \underline{V} = \underline{V}_d + \underline{V}_q = (v_d - jv_q)e^{j\theta_r^0}$$

- All coefficients $\sqrt{3}$ have disappeared
- hence, the Park currents (resp. voltages) are exactly the projections on the machine $d$ and $q$ axes of the phasor $\underline{I}$ (resp. $\underline{V}$)