



Cyprus
University of
Technology

EEN320 - Power Systems I (Συστήματα Ισχύος I)

Part 7: Induction machine

<https://sps.cut.ac.cy/courses/een320/>

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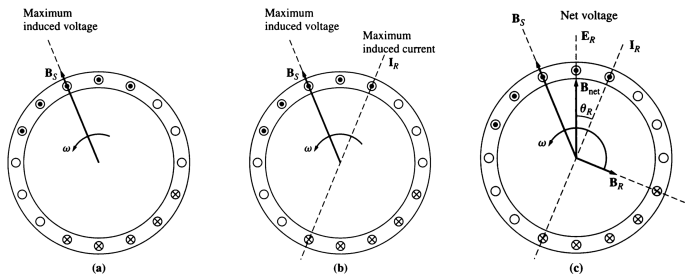
After this part of the lecture and additional reading, you should be able to . . .

- ① . . . Understand the key differences between a synchronous motor and an induction motor;
- ② . . . Understand the concept of rotor slip and its relationship to rotor frequency;
- ③ . . . Understand and know how to use the equivalent circuit of an induction motor; and,
- ④ . . . Be able to use the equation for the torque-speed characteristic curve.

- 1 **Basic concepts of induction machine**
- 2 **Induction motor equivalent model**
- 3 **Induction machine characteristics**

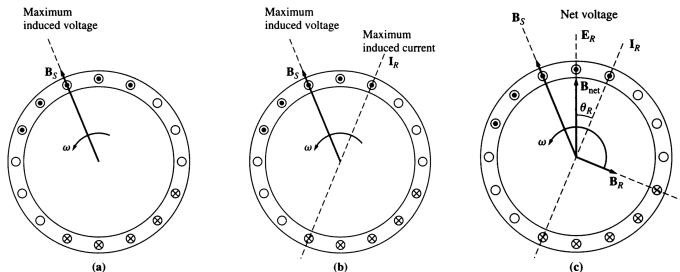
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1 Induced torque in induction machine



- Supplying three-phase voltage to the stator creates a rotating magnetic field \underline{B}_s with speed of rotation $n = (60 \cdot f_{se}) / (P/2)$
- The rotating magnetic field induces a voltage on the rotor (similar to a transformer). This is given by $e_{ind} = (\underline{v} \times \underline{B}) \cdot \underline{l}$ where \underline{v} is the velocity of the rotor *relative to the magnetic field*, \underline{B} the magnetic flux density and \underline{l} the length of the conductor.
- The induced voltage creates a current in the rotor \underline{I}_R (lagging the voltage due to the inductive nature of the rotor).

1 Induced torque in induction machine



- The induced current in the rotor I_R creates a rotor magnetic field B_R (lagging the current due to the inductive nature of the rotor).
- The induced torque is given by $\tau_{ind} = k \underline{B}_R \times \underline{B}_S$ (counter-clockwise).
- If the rotor was turning at synchronous speed, then the rotor bars would be stationary relative to the magnetic field and there would be no induced voltage $e_{ind} = 0$. Thus, no rotor current or magnetic field $\rightarrow \tau_{ind} = 0$

In normal operation both the rotor and stator magnetic fields rotate **together** at synchronous speed n_{sync} , while the rotor itself turns at a slower speed n_m . The **slip speed** is defined as:

$$n_{slip} = n_{sync} - n_m$$

The **slip** is then:

$$s = \frac{n_{sync} - n_m}{n_{sync}} \cdot 100\% = \frac{\omega_{sync} - \omega_m}{\omega_{sync}} \cdot 100\%$$

- At synchronous speed: $s = 0$
- At locked rotor speed: $s = 1$

The induction motor operates as a transformer but the secondary frequency is not necessarily the same as in the primary:

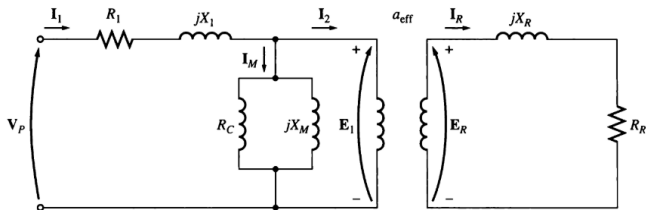
- If the rotor of a motor is locked so that it cannot move, then the rotor will have the same frequency as the stator
- If the rotor turns at synchronous speed, the frequency on the rotor will be zero.

The rotor current frequency can be expressed as:

$$f_{re} = s \cdot f_{se}$$

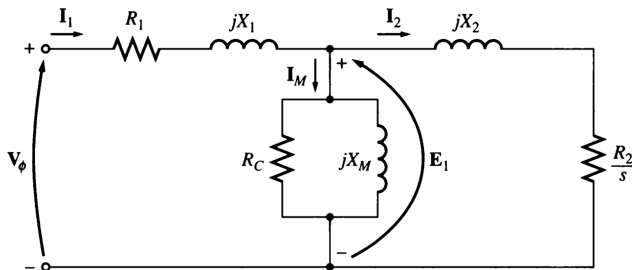
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- The induction machine is an electrical machine in which the stator windings are fed through a three- phase voltage source, while the rotor windings are short circuited and are circulated by currents induced by the stator.
- In balanced steady-state conditions, the induction machine has an analog behavior to that of a transformer and hence a transformer model can be used to represent this machine.
- It should be noted that the frequency on the secondary is different than the primary (unlike transformers).



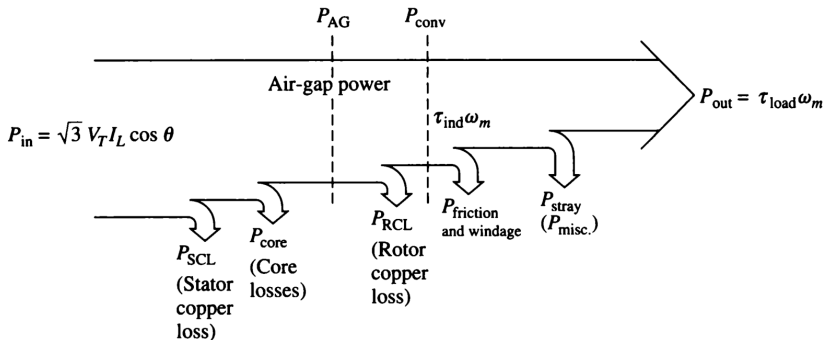
S. J. Chapman, Electric Machinery Fundamentals, 5th ed. McGraw-Hill, 2012.

Transferring at the primary, gives:



where R_2 and X_2 are estimated based on measurements.

2 Losses and the power-flow diagram



S. J. Chapman, Electric Machinery Fundamentals, 5th ed. McGraw-Hill, 2012.

2 Losses and the power-flow diagram

Based on the diagram of the induction motor:

- Stator copper losses

$$P_{SCL} = 3I_1^2 R_1$$

- Core losses

$$P_{core} = 3E_1^2 G_C$$

- Air-gap power

$$P_{AG} = P_{in} - P_{SCL} - P_{core} = 3I_2^2 \frac{R_2}{s}$$

- Rotor copper losses

$$P_{RCL} = 3I_2^2 R_2$$

- Developed mechanical power

$$P_{conv} = 3I_2^2 R_2 \left(\frac{1-s}{s} \right) = (1-s)P_{AG}$$

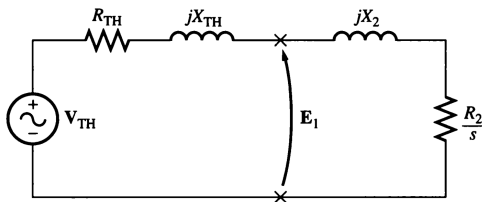
- Developed torque

$$\tau_{ind} = \frac{P_{conv}}{\omega_m} = \frac{(1-s)P_{AG}}{(1-s)\omega_{sync}} = \frac{P_{AG}}{\omega_{sync}}$$

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3 Thevenin equivalent

We can use the Thevenin equivalent for the primary side of the induction motor model (ignoring R_C):



where

$$V_{TH} = V_\phi \frac{X_M}{X_1 + X_M} \quad \text{and} \quad Z_{TH} = R_{TH} + jX_{TH} = \frac{Z_1 Z_M}{Z_1 + Z_M}$$

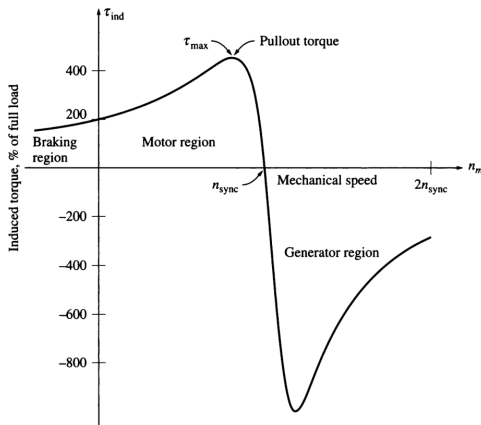
and

$$I_2 = \frac{V_{TH}}{Z_{TH} + Z_2}$$

3 Torque-speed characteristic

Using the Thevenin equivalent, we get:

$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}} = \frac{3 V_{TH}^2 R_2 / s}{\omega_{sync} \left[(R_{TH} + R_2 / s)^2 + (X_{TH} + X_2)^2 \right]}$$



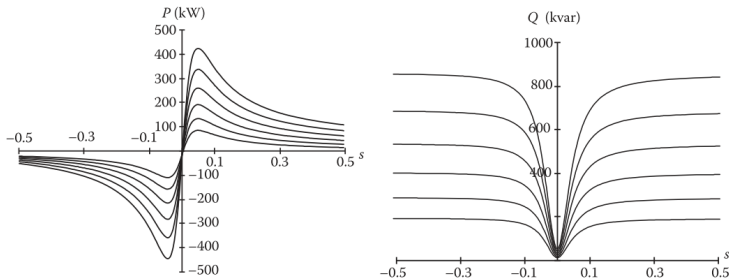
Using the *maximum power transfer theorem*, the slip at maximum power is given by:

$$\frac{R_2}{s} = \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2} \rightarrow s_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}$$

Leading to:

$$T_{ind-max} = \frac{3V_{TH}^2}{2\omega_{sync} \left[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2} \right]}$$

3 P-s and Q-s characteristics



- Special cases: $s = 1 \rightarrow$ locked-rotor, $s = 0 \rightarrow$ no-load
- Operating limits:
 - Stator thermal limit I_{max}
 - Dielectric insulation or maximum feeding voltage limit $V_{s,max}$
 - Stability or magnetizing limit (from curve)

A. Gomez-Exposito, A. J. Conejo, and C. A. Cañizares, Electric Energy Systems Analysis and Operation, 2018.

- Induction motors do not present the types of starting problems that synchronous motors do (check torque curve).
- However, the starting current required may cause an unacceptable dip in the power system voltage
- Starting apparent power is given

$$S_{start} = \frac{\text{rated power}}{\text{code letter factor}} \longrightarrow I_{start} = \frac{S_{start}}{\sqrt{3}V_T}$$

- To limit the starting current, different methods are used:
 - Autotransformer starter
 - Three-step resistive starter
 - Star-Delta method

3 Motor starting

Nominal code letter	Locked rotor, kVA/hp	Nominal code letter	Locked rotor, kVA/hp
A	0 – 3.15	L	9.00 – 10.00
B	3.15 – 3.55	M	10.00 – 11.00
C	3.55 – 4.00	N	11.20 – 12.50
D	4.00 – 4.50	P	12.50 – 14.00
E	4.50 – 5.00	R	14.00 – 16.00
F	5.00 – 5.60	S	16.00 – 18.00
G	5.60 – 6.30	T	18.00 – 20.00
H	6.30 – 7.10	U	20.00 – 22.40
J	7.10 – 8.00	V	22.40 and up
K	8.00 – 9.00		