EEN442 - Power Systems II (Συστήματα Ισχύος II)

Part 2: Fundamentals of power system operation

https://sps.cut.ac.cy/courses/een442/

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Today's learning objectives

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After this part of the lecture and additional reading, you should be able to ...

1. ...describe and analyse the behaviour of a transmission line under different operating conditions;
2. ...explain the Ferranti effect;
3. ...use the *PV* and $P - \delta$ characteristica to determine the steady-state voltage and angle stability of a power system.

Fundamentals of power system operation - Overview

- In this part of the lecture, we investigate the stationary current and voltage relations as well as the resulting active and reactive power flows on an AC power line
- For this purpose, we use the wave equation discussed in EEN320
- Thereby, we focus on a series of practically relevant scenarios
- The analysis is performed under two assumptions:
  1) The operating conditions are balanced $\rightarrow$ analysis is performed via single-phase equivalent circuits
  2) The network is in steady-state (for assessment of dynamic phenomena other models are required)
- Furthermore, we consider all powers *per phase*. The corresponding three-phase power can be calculated using the conventions introduced in Part 1.

Outline

1 Outline

1 Reminder: Π-equivalent circuit of homogeneous power line

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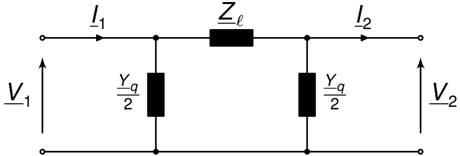


where:

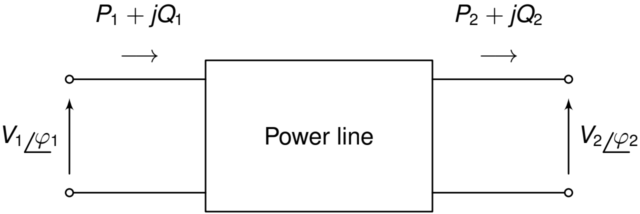
$$\underline{Z}_\ell = \underline{Z}_W \sinh(\underline{\gamma}\ell)$$

$$\frac{\underline{Y}_q}{2} = \frac{\cosh(\underline{\gamma}\ell) - 1}{\underline{Z}_W \sinh(\underline{\gamma}\ell)} = \frac{1}{\underline{Z}_W} \tanh\left(\frac{\underline{\gamma}\ell}{2}\right)$$

$$\boxed{\begin{aligned} \underline{\gamma} &= \sqrt{(R' + j\omega L')(G' + j\omega C')} \\ \underline{Z}_w &= \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} \end{aligned}}$$

These parameters correspond to exact relations between currents and voltages according to wave equation for $x = 0$ and $x = \ell$

1 Decoupled quantities - Power flow on a power line

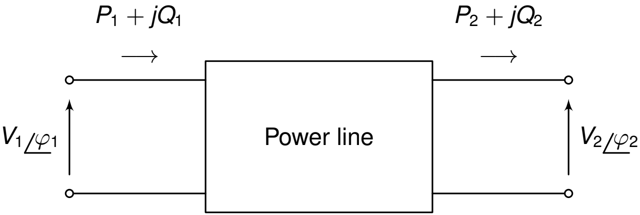


- Several ways to mathematically describe power flow over a power line
- Usually, we use complex voltage together with active and reactive powers at each end of line
- This yields 8 real quantities

$$V_1, \varphi_1, P_1, Q_1, V_2, \varphi_2, P_2, Q_2$$

- Which of the above quantities are decoupled (i.e. independent) of each other and which are not?

1 Decoupled quantities - Examples

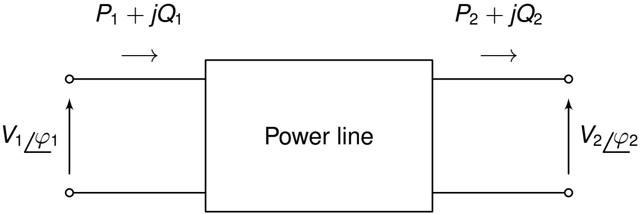


- Not all quantities in above graphic are independent of each other
- Examples:
  - $\underline{V}_1$ and $\underline{V}_2$ are coupled via line characteristics (see previous lectures)

  → Therefore it is customary to take one angle, e.g. φ_2, as reference; hence, one "loses" one quantity in the formulas

  - Power flows are also coupled; if P_1 and Q_1 are fixed, then P_2 and Q_2 can be computed if $\underline{V}_1$ *or* $\underline{V}_2$ is fixed, too
  - If $\underline{V}_1$ and $\underline{V}_2$ are fixed, P_1, P_2, Q_1 and Q_2 are also fixed and can *not* be adjusted independently

1 Decoupled quantities - Common triples



- $\mathbf{V_1}, \varphi_1, \mathbf{V_2}$: powers result from line characteristics and given quantities; practical example: power line connects two bulk ”stiff” power networks
- $\mathbf{V_1}, \mathbf{P_2}, \mathbf{Q_2}$ (or $\mathbf{P_1}, \mathbf{Q_1}, \mathbf{V_2}$): By fixing voltage on one end of line and power on other end, remaining quantities follow; practical example: consumer with fixed power demand connected via power line to network
- $\mathbf{V_1}, \mathbf{P_1}, \mathbf{Q_1}$: By fixing quantities at sending end of line, voltage and powers at receiving end follow; practical example: power plant that feeds network over power line

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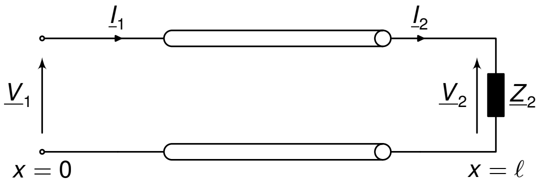
2 Surge impedance loading - Meaning



- Surge impedance loading (SIL) = power delivered when line is loaded with its surge impedance, i.e.

$$\underline{Z}_2 = \underline{Z}_w = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

- SIL also called natural loading
- In the following, we consider two cases
  - Lossless line ($R' = G' = 0$)
  - Lossy line ($R' \neq 0$, $G' \neq 0$)

2.1 SIL of lossless power line - Receiving end

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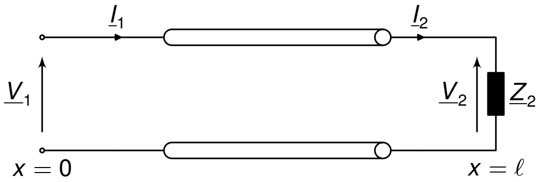


- Lossless power line: $R' = G' = 0 \rightarrow$ surge impedance $Z_w = \sqrt{\frac{L'}{C'}}$
- Active power delivered at end of line

$$P_2 = \frac{|\underline{V}_2|^2}{Z_2} = \frac{|\underline{V}_2|^2}{Z_w}$$

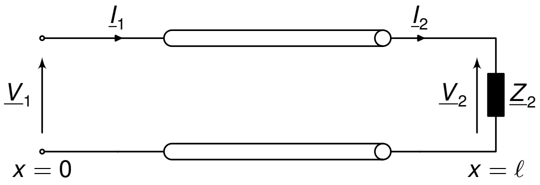
- Reactive power delivered at end of line ($Z_2 = Z_w$ is real in lossless case)

$$Q_2 = 0$$

- Current at end of line

$$\underline{I}_2 = \frac{\underline{V}_2}{Z_2} = \frac{\underline{V}_2}{Z_w}$$

2.1 SIL of lossless power line - Sending end (1)



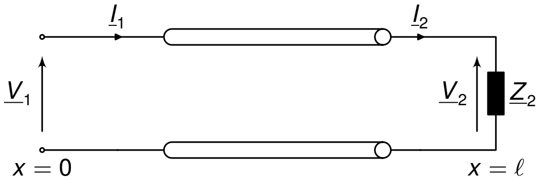
- From full line model equations with $x = 0$ and $\underline{\gamma} = j\omega\sqrt{L'C'} = j\beta$

$$\underline{V}_1 = \cosh(j\beta\ell)\underline{V}_2 + Z_W \sinh(j\beta\ell)\underline{I}_2$$
$$\underline{I}_1 = \frac{\underline{V}_2}{Z_W}\sinh(j\beta\ell) + \cosh(j\beta\ell)\underline{I}_2$$

- With $\cosh(j\beta) = \cos(\beta)$ and $\sinh(j\beta) = j\sin(\beta)$ we obtain

$$\underline{V}_1 = \cos(\beta\ell)\underline{V}_2 + jZ_W \sin(\beta\ell)\underline{I}_2$$
$$\underline{I}_1 = j\frac{\underline{V}_2}{Z_W}\sin(\beta\ell) + \cos(\beta\ell)\underline{I}_2$$

2.1 SIL of lossless power line - Sending end (2)

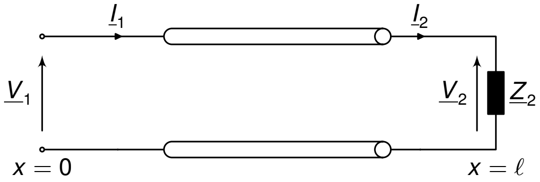


- Using $\underline{I}_2 = \frac{\underline{V}_2}{\underline{Z}_w}$ yields

$$\begin{aligned}
\underline{V}_1 &= \cos(\beta\ell)\underline{V}_2 + jZ_W \sin(\beta\ell)\frac{\underline{V}_2}{\underline{Z}_w} \\
&= \underline{V}_2(\cos(\beta\ell) + j\sin(\beta\ell) = \underline{V}_2 e^{j\beta\ell} \\
\underline{I}_1 &= j\frac{\underline{V}_2}{\underline{Z}_W}\sin(\beta\ell) + \cos(\beta\ell)\frac{\underline{V}_2}{\underline{Z}_w} \\
&= \underline{I}_2(\cos(\beta\ell) + j\sin(\beta\ell) = \underline{I}_2 e^{j\beta\ell}
\end{aligned}$$

$\rightarrow$ Voltage and current are shifted by angle $\beta\ell$ at end of line
Thereby, their amplitudes remain unchanged

2.1 SIL of lossless power line - Active power



- For active power at both end of lines, we have that (as line is lossless)

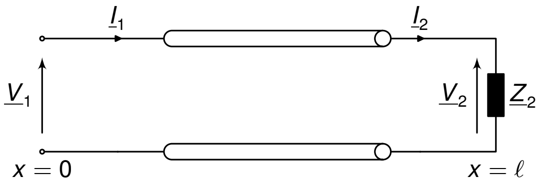
$$P_1 = \underline{V}_1 \underline{I}_1^* = \underline{V}_2 \underline{I}_2^* = P_2 = \frac{|\underline{V}_1|^2}{Z_w}$$

- This particular loading of line is called *surge impedance loading (SIL)*

$$\boxed{P_{SIL} = \frac{|\underline{V}|^2}{Z_w}}$$

- For this loading we achieve optimal transmission conditions (amplitudes of voltage and current remain constant along whole line)
- In practice, loading usually differs from SIL

2.1 SIL of lossless power line - Reactive power



- For SIL, *reactive power flow on line is zero*

→ At each point on line, reactive power ”absorption” of line inductance *equals* reactive power ”production” of line capacitance

$$Q'_C = Q'_L \quad \Rightarrow \quad V^2 \omega C' = I^2 \omega L' \quad \Rightarrow \quad \frac{V^2}{I^2} = \frac{L'}{C'} = Z_w^2$$

2.1 SIL of lossless power line - Comments on reactive power

- Surge impedance of overhead lines (OHLs) between $200 - 400\ \Omega$
- OHL inductance significantly larger than OHL capacitance

$\rightarrow$ Reactive power "absorbed" by OHL inductance exceeds reactive power "produced" by OHL capacitance even for small currents

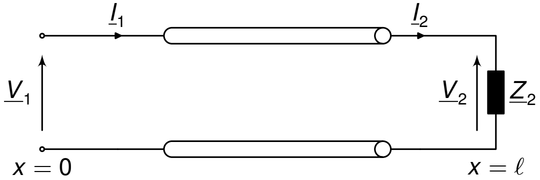
$\rightarrow$ OHLs often operated above their SIL; then they "absorb" reactive power

- Compared to OHLs, cables have very low surge impedance ($\approx 30 - 50\ \Omega$)

$\rightarrow$ SIL usually above thermal limit of cable

$\rightarrow$ Cables usually "produce" reactive power

2.2 SIL of lossy power line - Sending end (1)



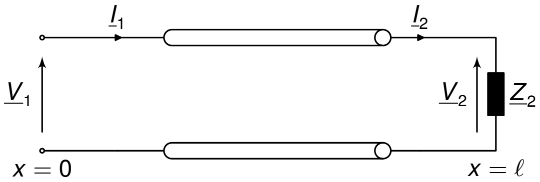
- Lossy line $\rightarrow$ $\underline{Z}_w$ is complex
- As before, we consider the case $\underline{Z}_2 = \underline{Z}_w$
- Current at receiving end of line

$$\underline{I}_2 = \frac{\underline{V}_2}{\underline{Z}_2} = \frac{\underline{V}_2}{\underline{Z}_w}$$

- Apparent power at receiving end of line

$$\underline{S}_2 = P_2 + jQ_2 = \underline{V}_2 \underline{I}_2^* = \frac{|\underline{V}_2|^2}{\underline{Z}_w^*}$$

2.2 SIL of lossy power line - Sending end (2)

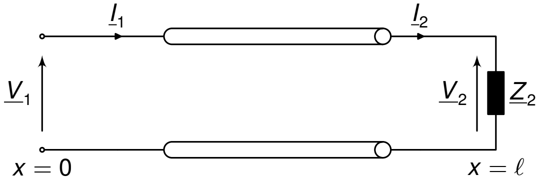


- From solution of wave equation with $x = 0$ and $\underline{\gamma} = \alpha + j\beta$ (see EEN320, transmission line characteristics)

$$\begin{aligned}
\underline{V}_1 &= \cosh(\underline{\gamma}\ell)\underline{V}_2 + \underline{Z}_W \sinh(\underline{\gamma}\ell)\underline{I}_2 \\
&= \cosh(\underline{\gamma}\ell)\underline{V}_2 + \underline{Z}_W \sinh(\underline{\gamma}\ell)\frac{\underline{V}_2}{\underline{Z}_w} \\
&= \underline{V}_2\left(\cosh(\underline{\gamma}\ell) + \sinh(\underline{\gamma}\ell)\right) = \underline{V}_2 e^{\underline{\gamma}\ell} \\
\underline{I}_1 &= \frac{\underline{V}_2}{\underline{Z}_W}\sinh(\underline{\gamma}\ell) + \cosh(\underline{\gamma}\ell)\underline{I}_2 \\
&= \underline{I}_2\left(\cosh(\underline{\gamma}\ell) + \sinh(\underline{\gamma}\ell)\right) = \underline{I}_2 e^{\underline{\gamma}\ell}
\end{aligned}$$

Note: To obtain the last equality, we have used $\cosh(x) + \sinh(x) = e^x$

2.2 SIL of lossy power line - Sending end (3)



- Apparent power at sending end

$$\underline{S}_1 = P_1 + jQ_1 = \underline{V}_1 \underline{I}_1^* = \underline{V}_2 \frac{\underline{V}_2^*}{\underline{Z}_w^*} e^{2\alpha\ell} = \underline{S}_2 e^{2\alpha\ell}$$

→ As in lossless case, phase angle between voltage and current remains constant along line; phase shift is proportional to βx

→ But now, active and reactive power decrease with line length; same applies to voltage and current

2.2 Typical values for SI and SIL of lossy power line

Table: Typical values for OHLs[1]

Rated voltage in kV	132	275	400
$\underline{Z}_w$ [Ω]	373	302	296
P_{SIL} [MW]	47	250	540

Table: Typical values for cables[2]

Rated voltage in kV	115	230	500
$\underline{Z}_w$ [Ω]	36.2	37.1	50.4
P_{SIL} [MW]	365	1426	4960

[1] Source: B. M. Weedy et al., "Electric Power Systems", John Wiley & Sons, 2012

[2] Source: P. Kundur, "Power System Stability", McGraw-Hill, 1994

3 Outline

3 The two extrema - Overview

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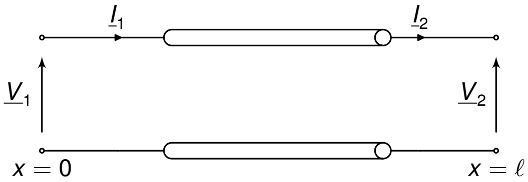
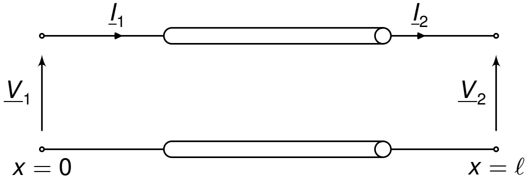
- Next, we analyse the behaviour of a power line in two special cases
  - No load
  - Short circuit
- To simplify our calculations, we restrict ourselves to the lossless case

3.1 No load conditions - Setup



- No load condition can occur if
  - Voltage is applied to unloaded line
  - Load at end of line is disconnected
- Main characteristic: $\underline{I}_2 = 0$

3.1 No load conditions - Current and voltage at sending end



- Solution of wave equation with $x = 0$ and $\underline{\gamma} = j\omega\sqrt{L'C'} = j\beta$ yields (see Part 5, Sect. 4.2)

$$\underline{V}_1 = \cos(\beta\ell)\underline{V}_2$$

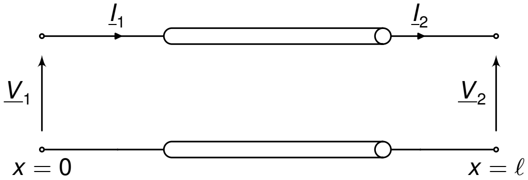
$$\underline{I}_1 = j\frac{\underline{V}_2}{Z_W}\sin(\beta\ell)$$

- Recall that we may fix one of two voltage angles. Setting $\varphi_1 = 0$, we have

$$V_1 = \cos(\beta\ell)V_2$$

$$\underline{I}_1 = j\frac{V_2}{Z_W}\sin(\beta\ell)$$

3.1 No load conditions - Ferranti effect



- Keeping V_1 constant, we get

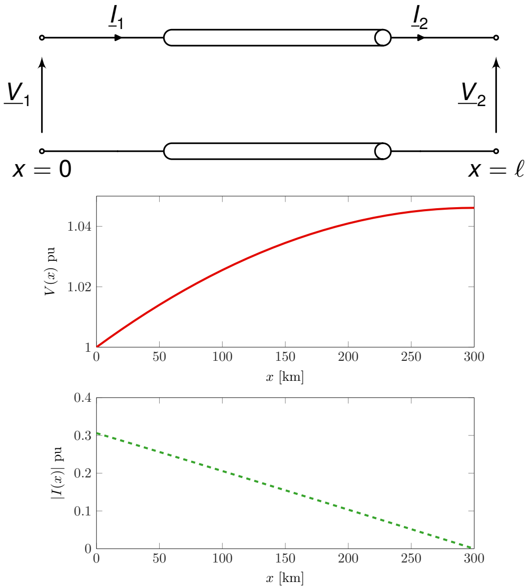
$$V_2 = \frac{V_1}{\cos(\beta\ell)}$$

$$\underline{I}_1 = \frac{jV_2}{Z_w}\sin(\beta\ell) = \frac{jV_1\tan(\beta\ell)}{Z_w}$$

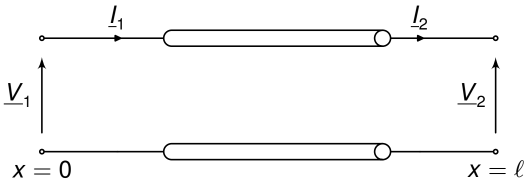
→ Voltage amplitude increases along line, while that of current decreases ($\underline{I}_2 = 0$)

- This phenomenon is called *Ferranti effect* (because it was first observed by the British engineer Sebastian Ziani de Ferranti in 1887)

3.1 No load conditions - Ferranti effect illustration



3.1 No load conditions - Ferranti effect resonance



- It holds that (ε_0 is electric constant and μ_0 magnetic constant)

$$\beta = \omega\sqrt{L'C'} \approx \omega\sqrt{\varepsilon\varepsilon_0\mu_0}$$

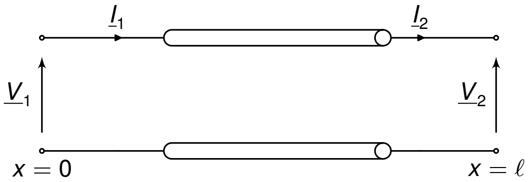
- Permittivity of air $\varepsilon = 1$
- For $\omega = 2\pi 50$ [rad/s], we have that $\beta \approx \frac{6^\circ}{100\ \text{km}}$

→ Extreme scenario: resonance; achieved for 50 Hz at $\ell = 1500$ km

$$\beta\ell = \frac{6^\circ \times 1500\ \text{km}}{100\ \text{km}} = 90^\circ = \frac{\pi}{2}$$

- Then $\cos(\beta\ell) = 0$ and $V_2 \to \infty$

3.1 No load conditions - Impedance



- Impedance at sending end

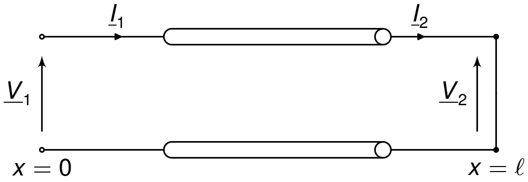
$$\underline{Z}_1 = \frac{\underline{V}_1}{\underline{I}_1} = -j\frac{Z_w}{\tan(\beta\ell)}$$

- We can see that impedance has *capacitive* character

$\rightarrow$ High loading currents required!

- In practice: amplitude of $\underline{V}_1$ not stiff, but additionally increased by loading currents $\rightarrow$ need to be careful with voltage rise already for line lengths of 300km

3.2 Short circuit conditions - Voltage and current



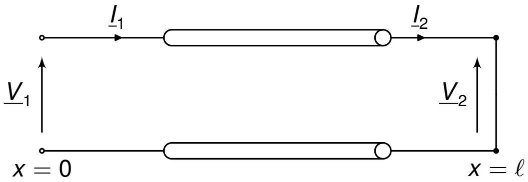
- Short circuit $\rightarrow \underline{V}_2 = 0$
- Solution of wave equation with $x = 0$ and $\underline{\gamma} = j\omega\sqrt{L'C'} = j\beta$ yields (see EEN320 for more details)

$$\begin{aligned} \underline{V}_1 &= j\underline{I}_2 Z_w \sin(\beta\ell) \\ \underline{I}_1 &= \underline{I}_2 \cos(\beta\ell) \end{aligned}$$

- In analogy to voltage in no load condition, now current increases along line

$$\underline{I}_2 = \frac{\underline{I}_1}{\cos(\beta\ell)}$$

3.2 Short circuit conditions - Impedance

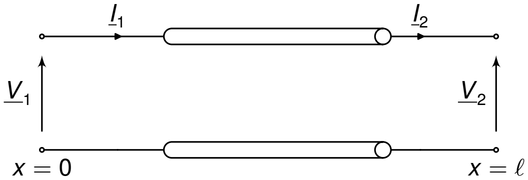


- Short circuit impedance

$$\frac{\underline{V}_1}{\underline{I}_1} = \underline{Z}_1 = jZ_w \tan(\beta\ell)$$

- For $\omega = 2\pi 50$ [rad/s], short circuit impedance is *inductive* for line lengths < 1500 km
- As before, resonance $|\underline{I}_2| \to \infty$ for $\beta\ell = \pi/2$

4 Outline



- Power transmission over a power line causes losses:
  - Ohmic components of line (resistance; conductance) cause active power losses
  - Reactive components of line (inductance; capacitance) influence reactive power flow

→ Apparent power at receiving end of line differs from apparent power at sending end!

- For voltage relation along line, reactive power is most important as discussed hereafter for *lossless* line

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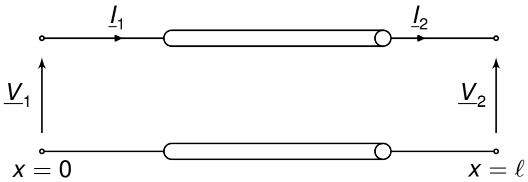
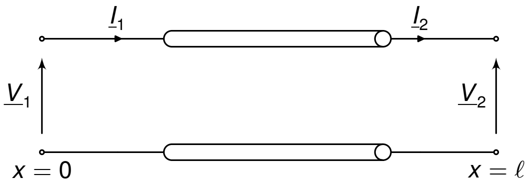


- Wave equation for *lossless* line

$$\begin{bmatrix} \underline{V}_1 \\ \underline{I}_1 \end{bmatrix} = \begin{bmatrix} \cos(\beta\ell) & jZ_w \sin(\beta\ell) \\ j\frac{\sin(\beta\ell)}{Z_w} & \cos(\beta\ell) \end{bmatrix} \begin{bmatrix} \underline{V}_2 \\ \underline{I}_2 \end{bmatrix}$$

$\rightarrow$ Apparent power $\underline{S}_1 = \underline{V}_1 \underline{I}_1^* = P_1 + jQ_1$ at sending end is dependent on apparent power $\underline{S}_2 = \underline{V}_2 \underline{I}_2^* = P_2 + jQ_2$ at receiving end of line



- Hence, we have

$$\underline{S}_1 = \underline{V}_1 \underline{I}_1^* = P_1 + jQ_1 = j\cos(\beta\ell)\sin(\beta\ell)\left(|\underline{I}_2|^2 - |\underline{V}_2|^2 \frac{1}{Z_w}\right) + \sin^2(\beta\ell)\underline{I}_2\underline{V}_2^* + \cos^2(\beta\ell)\underline{V}_2\underline{I}_2^*$$

- For our analysis, it is convenient to fix $\underline{V}_2$ and express $\underline{S}_1$ in terms of SIL

$$P_{SIL} = \frac{|\underline{V}_2|^2}{Z_w}$$

- Using the relations

$$|\underline{V}_2|^2 = P_{SIL} Z_w$$

$$\underline{I}_2^* = \frac{\underline{S}_2}{\underline{V}_2} \quad \rightarrow \quad |\underline{I}_2|^2 = \frac{|\underline{S}_2|^2}{|\underline{V}_2|^2} = \frac{|\underline{S}_2|^2}{P_{SIL} Z_w}$$

$$\underline{I}_2 \underline{V}_2^* = (\underline{V}_2 \underline{I}_2^*)^* = \underline{S}_2^* = P_2 - jQ_2$$

$$\cos^2(\beta\ell) - \sin^2(\beta\ell) = \cos(2\beta\ell)$$

$$\cos(\beta\ell)\sin(\beta\ell) = \frac{1}{2}\sin(2\beta\ell)$$

we can rewrite the equation for $\underline{S}_1$ as follows

$$\underline{S}_1 = P_1 + jQ_1 = P_2 + j\left(Q_2\cos(2\beta\ell) + \frac{1}{2}\sin(2\beta\ell)\left(\frac{|\underline{S}_2|^2}{P_{SIL}} - P_{SIL}\right)\right)$$

- For lossless line

$$P_1 = P_2 \quad \rightarrow \quad \text{can focus further analysis on reactive power flows}$$

4 Reactive power demand of a power line - Reactive power flows

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- Relation of reactive power flows

$$Q_1 = Q_2 \cos(2\beta\ell) + \frac{1}{2}\sin(2\beta\ell)\left(\frac{|\underline{S}_2|^2}{P_{SIL}} - P_{SIL}\right)$$

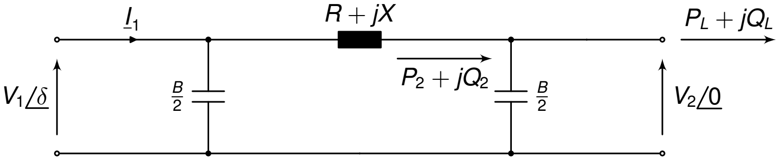
- Q_1 dependent on Q_2 ("load demand") and reactive power demand of line
- With simplifying approximation $\cos(2\beta\ell) \approx 1$, reactive power demand of line given by

$$\Delta Q = Q_1 - Q_2 \approx \underbrace{\frac{1}{2}\sin(2\beta\ell)\frac{|\underline{S}_2|^2}{P_{SIL}}}_{\text{inductive component } Q_L} - \underbrace{\frac{1}{2}\sin(2\beta\ell)P_{SIL}}_{\text{capacitive component } Q_C}$$

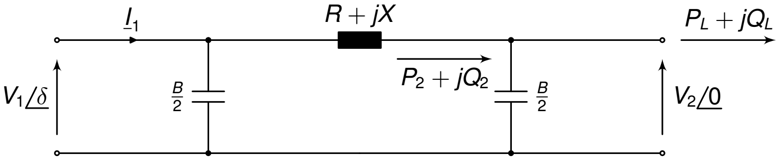
- $\underline{S}_2 = P_{SIL} \rightarrow \Delta Q = 0$
- $|\underline{S}_2| = 0 \rightarrow \Delta Q <$ line produces reactive power ($Q_L = 0$, $Q_C > 0$)
- $|\underline{S}_2| > P_{SIL} \rightarrow \Delta Q > 0$ line absorbs reactive power ($Q_L > Q_C$)
- $|\underline{S}_2| < P_{SIL} \rightarrow \Delta Q < 0$ line produces reactive power ($Q_L < Q_C$)

5 Outline

5 Voltage drop across a power line - Setup



- Π-model of power line of length ℓ and $G' = 0$, $R = R'\ell$, $X = \omega L'\ell$ and $B = \omega C'\ell$
- Load at end of line: $P_L + jQ_L$
- We want to derive a formula for voltage drop across line
- For this purpose it is convenient to define V_2 on real line and denote angle between V_2 and $\underline{V}_1$ by δ



- Shunt elements B produce reactive power

→ We can obtain ”net” reactive power flow Q_2 on line by subtracting reactive power Q_C produced by B from Q_L, i.e.,

$$P_2 = P_L$$
$$Q_2 = Q_L - Q_C$$

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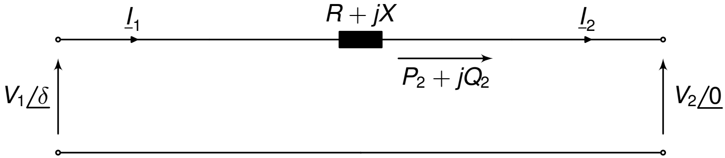
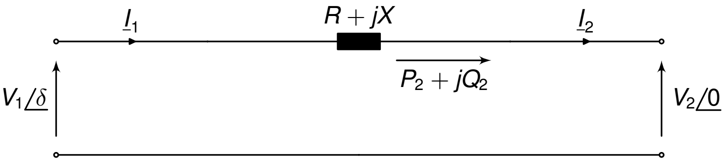


- By using $Q_2 = Q_L - Q_C$ we can simplify considered circuit as shown above

5 Voltage drop across a power line - Current and voltage



- Current $\underline{I}_2$ as function of apparent power $\underline{S}_2 = P_2 + jQ_2$ and $\underline{V}_2 = V_2$

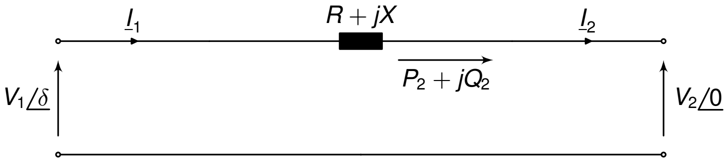
$$\underline{I}_1 = \underline{I}_2 = \frac{\underline{S}_2^*}{V_2} = \frac{P_2 - jQ_2}{V_2}$$

- Voltage $\underline{V}_1$

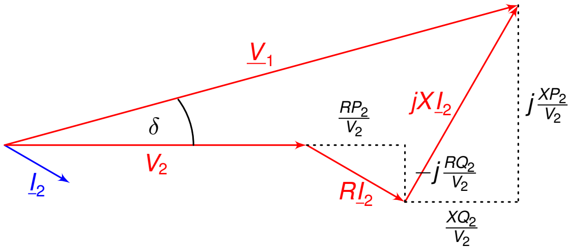
$$\begin{aligned}\underline{V}_1 &= V_2 + (R + jX)\underline{I}_2 = V_2 + (R + jX)\frac{P_2 - jQ_2}{V_2} \\ &= \left(V_2 + \frac{RP_2 + XQ_2}{V_2}\right) + j\left(\frac{XP_2 - RQ_2}{V_2}\right)\end{aligned}$$

$$|\underline{V}_1| = V_1 = \sqrt{\left(V_2 + \frac{RP_2 + XQ_2}{V_2}\right)^2 + \left(\frac{XP_2 - RQ_2}{V_2}\right)^2}$$

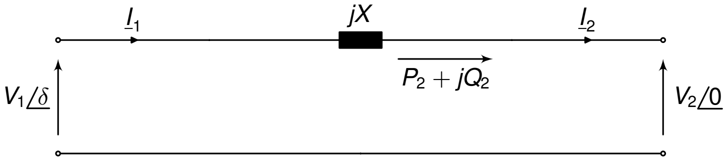
5 Voltage drop across a power line - Phasor diagram



$X \gg R$



5 Voltage drop across a power line - Lossless line



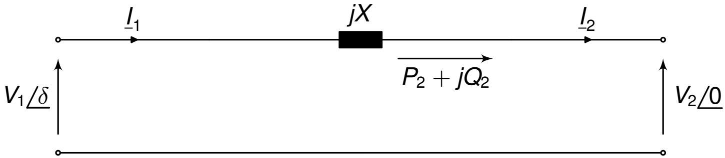
- Lossless line $\rightarrow R = 0$
- Expression for V_1 simplifies to

$$\underline{V}_1 = V_1\cos(\delta) + jV_1\sin(\delta) = \left(V_2 + \frac{XQ_2}{V_2}\right) + j\left(\frac{XP_2}{V_2}\right)$$

- Separating the real with the imaginary parts, gives

$$\boxed{P_2 = \frac{V_1 V_2 \sin(\delta)}{X}} \qquad \boxed{Q_2 = \frac{V_1 V_2 \cos(\delta) - V_2^2}{X}}$$

5 Voltage drop across a power line - Lossless line



- The magnitude of V_1 is given by

$$|\underline{V}_1| = V_1 = \sqrt{\left(V_2 + \frac{XQ_2}{V_2}\right)^2 + \left(\frac{XP_2}{V_2}\right)^2}$$

- In most scenarios $|XP_2/V_2| \ll V_2$ and expression for V_1 can be further simplified to

$$|\underline{V}_1| = V_1 \approx V_2 + \frac{XQ_2}{V_2}$$

$\rightarrow$ $\Delta V = V_1 - V_2$ mainly influenced by reactive power Q_2!

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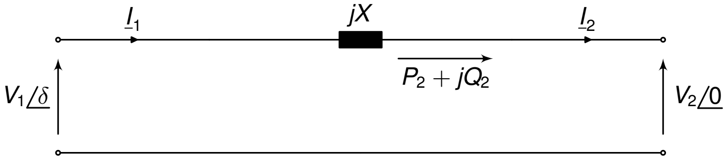
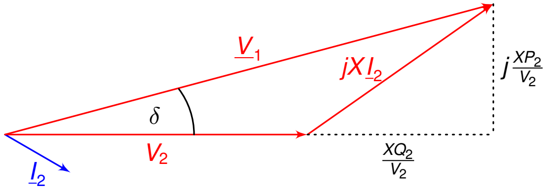
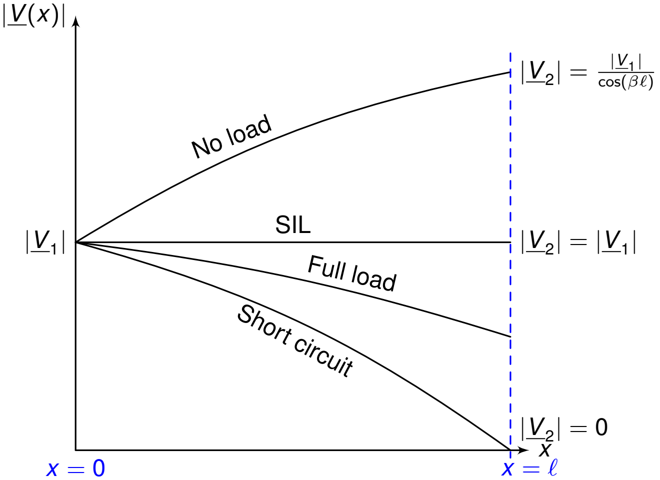




$\rightarrow$ Phase angle δ mainly influenced by active power P_2!

- We have assumed V_2 and $\underline{S}_2$ are known and we want to calculate $\underline{V}_1$; often also V_1 and $\underline{S}_2$ given and we seek to compute $\underline{V}_2$; this can be done in an equivalent manner

5 Summary - Voltage characteristics of a power line



- The discussed scenarios mainly apply to OHLs; cables typically have different properties

6 Outline

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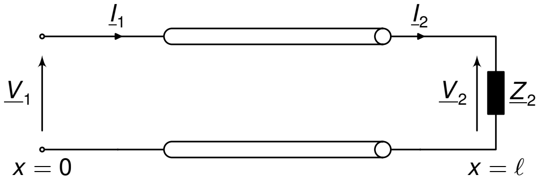
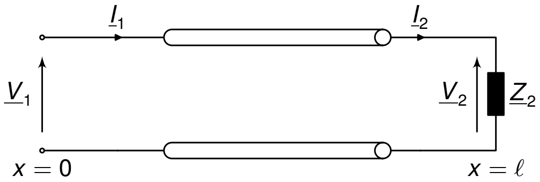


- Consider exemplary 200 km/420 kV ($= V_{LL} = \sqrt{3}V_2$) power line with following characteristics

$$R' = 0.031\ \Omega/\text{km},\ L' = 1.06\ \text{mH/km},\ C' = 11.9\ \text{nF/km},\ G' = 0,\ f = 50\ \text{Hz}$$

- Assume line is loaded with surge impedance $\underline{Z}_2 = \underline{Z}_w$
- Propagation constant

$$\underline{\gamma} = \sqrt{(0.031 + j0.333)j3.74 \cdot 10^{-6}} = (0.052 + j1.117)10^{-3}$$
$$\underline{\gamma}\ell = \alpha\ell + j\beta\ell = 0.0104 + j0.2234$$



- Characteristic impedance (neglecting imaginary part)

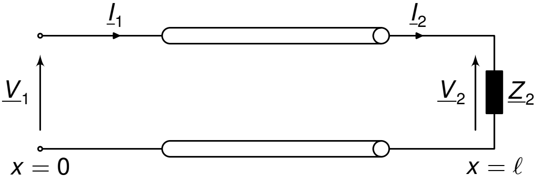
$$Z_w = 298.5\ \Omega$$

- Active power drawn by load at receiving end of line

$$P_2 = \frac{V_{LL}^2}{Z_w} \approx 591\ \text{MW}$$

- Current RMS magnitude (per phase)

$$I_2 = \frac{\frac{V_{LL}}{\sqrt{3}}}{Z_w} = \frac{420\ \text{kV}}{\sqrt{3} \cdot 298.5\ \Omega} = 812.4\ \text{A}$$



- Line losses can be approximated by

$$\Delta P = P_1 - P_2 \approx 3R'\ell I_2^2 = 3 \cdot 0.031 \cdot 200 \cdot 812.4^2 = 12.3 \text{ MW}$$

- $P_1 = P_2 + \Delta P \approx 591 + 12.3 = 603.3$ MW
- **Alternative**: We can calculate exact value for P_1 from wave equation (see Part 6 Section 2.2)

$$P_1 = P_2 e^{2\alpha\ell} = 603.6 \text{ MW}$$

$\rightarrow$ Our approximation is fairly accurate

$\rightarrow$ Very high efficiency for power transmission!

$$e^{-2\alpha\ell} = 0.979 \quad \leftrightarrow \quad 97.9\%$$

7 Outline

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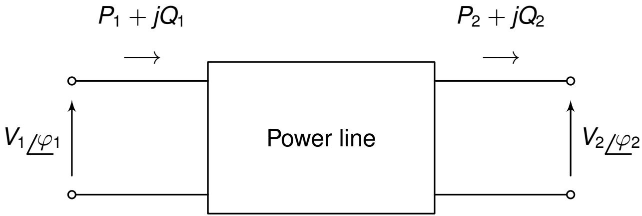
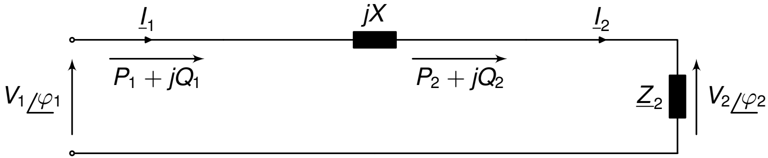
7 P-V characteristic - Motivation



- We have seen that power flows over a line affect voltages at sending and receiving end
- If load demand at receiving end is too large, then voltage drop can be significant and even lead to *voltage instability*
- Therefore, it is important to understand relation between voltage drop and load demand



- Suppose that $V_1 = 1$ pu, $\varphi_1 = 0$ and that power line is lossless $R' = G' = 0$

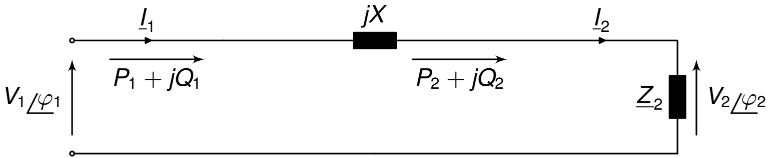
- We have that $\underline{I}_1 = \underline{I}_2$,

$$\underline{V}_2 = V_1 - jX\underline{I}$$

and with $\delta = \varphi_1 - \varphi_2 = -\varphi_2$

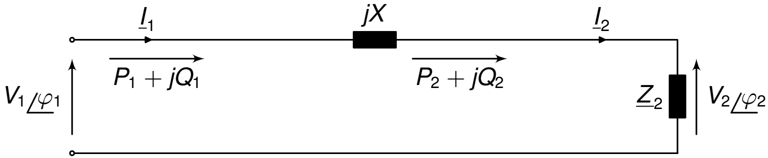
$$\underline{S}_2 = \underline{V}_2 \underline{I}^* = \underline{V}_2 \frac{V_1 - \underline{V}_2^*}{-jX}$$

$$= \frac{j}{X} \left(V_1 V_2 \cos(\delta) + jV_1 V_2 \sin(\delta) - V_2^2 \right)$$



- Decomposing $\underline{S}_2$ in real and imaginary parts yields

$$P_2 = -\frac{V_1 V_2}{X} \sin(\delta)$$
$$Q_2 = \frac{1}{X}\left(V_1 V_2 \cos(\delta) - V_2^2\right)$$



- Same procedure as for receiving end

$$\underline{S}_1 = \underline{V}_1 \underline{I}^* = \underline{V}_1 \frac{V_1 - \underline{V}_2^*}{-jX}$$

- Decomposing $\underline{S}_1$ in real and imaginary parts yields

$$P_1 = \frac{V_1 V_2}{X} \sin(\delta)$$

$$Q_1 = \frac{1}{X} \left(V_1^2 - V_1 V_2 \cos(\delta) \right)$$

- These equations are called *power flow* or *load flow* equations of the lossless system

7 P-V characteristic - A solution to power flow equations (1)

- When does a solution to power flow equations exist?

$$X^2 P_2^2 = (V_1 V_2 \sin(\delta))^2$$
$$\left(XQ_2 + V_2^2\right)^2 = (V_1 V_2 \cos(\delta))^2$$
$$\Rightarrow 0 = (V_2^2)^2 + (2Q_2X - V_1^2)V_2^2 + X^2(P_2^2 + Q_2^2)$$

- The above is a quadratic equation in $y = V_2^2$, i.e.,

$$cy^2 + by + a = 0 \qquad c = 1 \qquad b = (2Q_2X - V_1^2) \qquad a = X^2(P_2^2 + Q_2^2)$$

- Condition for existence of at least one real solution (see quadratic formula)

$$b^2 - 4ac \geq 0$$
$$\Leftrightarrow \quad (2Q_2X - V_1^2)^2 - 4X^2(P_2^2 + Q_2^2) = V_1^4 - 4X^2P_2^2 - 4Q_2XV_1^2 \geq 0$$

- Solution to quadratic equation

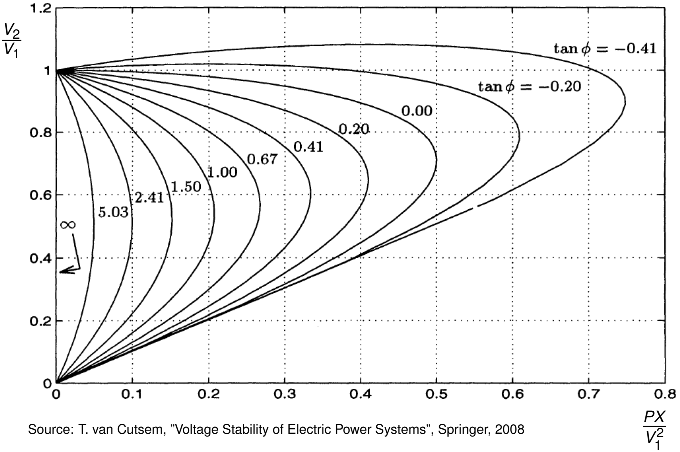
$$y_{1,2} = \frac{V_1^2}{2} - Q_2 X \pm \sqrt{\frac{V_1^4}{4} - X^2 P_2^2 - X Q_2 V_1^2}$$

- Voltage magnitude at receiving end

$$V_2 = \sqrt{y_{1,2}} = \sqrt{\frac{V_1^2}{2} - Q_2 X \pm \sqrt{\frac{V_1^4}{4} - X^2 P_2^2 - X Q_2 V_1^2}}$$

→ To each loading $\underline{S}_2 = P_2 + jQ_2$ there exist, in general, two real solutions for V_2!

7 P-V characteristic - $P - V$ curve



Source: T. van Cutsem, "Voltage Stability of Electric Power Systems", Springer, 2008

- Due to its shape, $P - V$ curve is also called *nose curve*
- Power factor: $\cos(\phi) = \frac{P}{|S|} \rightarrow Q = P\tan(\phi)$

7 P-V characteristic - Comments on $P-V$ curve

- Power transfer is limited by power factor
- Power factor has significant influence on voltage magnitude at receiving end of line
- Inductive load ($\mathtt{tan}(\varphi) > 0$): lower transmittible active power and lower voltage magnitude V_2
- Capacitive load ($\mathtt{tan}(\varphi) < 0$): higher transmittible active power and flatter voltage profile in upper part of nose curve

$\rightarrow$ Voltage V_2 can be regulated ("parallel compensation") with additional capacitances (compare Part 2 of lecture)

- Specific active power value can be achieved at two different voltage magnitudes
- Usually, higher value chosen ($\approx$ 1 pu) as results in lower current and also low voltage values can lead to instability

7 P-V characteristic - Comments on $P-V$ curve (1)

- Power transfer is limited by power factor
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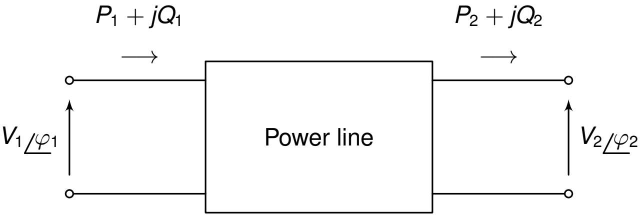
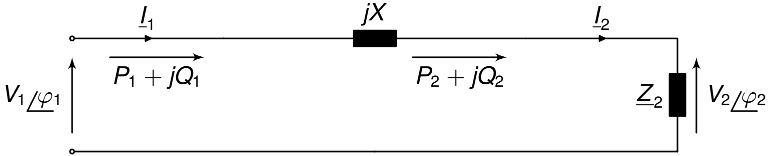
8 Outline

8 P-δ characteristic - Setup



- Another important relation is that between the transmitted active power flow and the phase angle difference between the voltages at sending and receiving end of line (i.e., $\underline{V}_1 = V_1\underline{/\varphi_1}$ and $\underline{V}_2 = V_2\underline{/\varphi_2}$)
- We conduct our analysis under the following assumptions
  - The voltage magnitudes V_1 and V_2 are constant
  - The line impedance is purely inductive, i.e., $\underline{Z}_\ell = jX = j\omega L'\ell$
- However, the phase angle difference $\delta = \varphi_1 - \varphi_2$ may vary

8 P-δ characteristic - Current flow



- Voltages in polar form

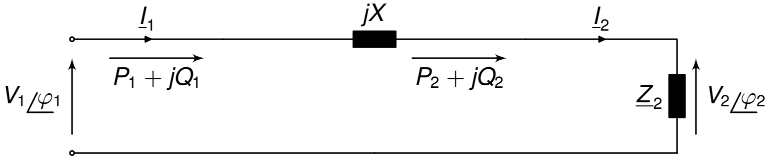
$$\underline{V}_1 = V_1 e^{j\varphi_1} \qquad \underline{V}_2 = V_2 e^{j\varphi_2}$$

- Lossless line $\rightarrow P_1 = P_2 = \Re(\underline{V}_1 \underline{I}_1^*)$

$$\underline{I}_1 = \underline{I}_2 = \frac{\underline{V}_1 - \underline{V}_2}{jX} = \frac{V_1 e^{j\varphi_1} - V_2 e^{j\varphi_2}}{jX}$$

$$\underline{I}_1^* = \frac{V_1 e^{-j\varphi_1} - V_2 e^{-j\varphi_2}}{-jX}$$

8 P-δ characteristic - Active power flow (1)



- Without loss of generality, we set $\varphi_2 = 0$
- Then with *transmission angle* $\delta = \varphi_1 - \varphi_2$

$$P = P_1 = P_2 = \Re\left(V_1 e^{j\varphi_1} \frac{j}{X}\left(V_1 e^{-j\varphi_1} - V_2\right)\right) = \Re\left(jV_1^2 \frac{1}{X} - j\frac{1}{X} V_1 V_2 e^{j\varphi_1}\right)$$

$$= \Re\left(jV_1^2 \frac{1}{X} - jV_1 V_2 \frac{1}{X}\cos(\varphi_1) + V_1 V_2 \frac{1}{X}\sin(\varphi_1)\right)$$

$$= V_1 V_2 \frac{1}{X}\sin(\varphi_1) = V_1 V_2 \frac{1}{X}\sin(\delta)$$

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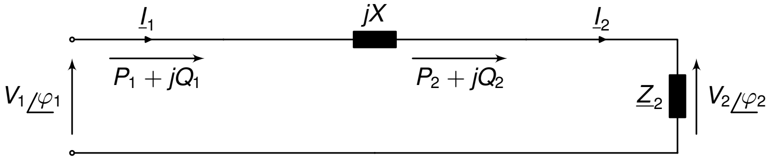


- We can rewrite active power in terms of charateristic impedance Z_w, phase constant β and line length ℓ (see Part 5 Section 4.4)

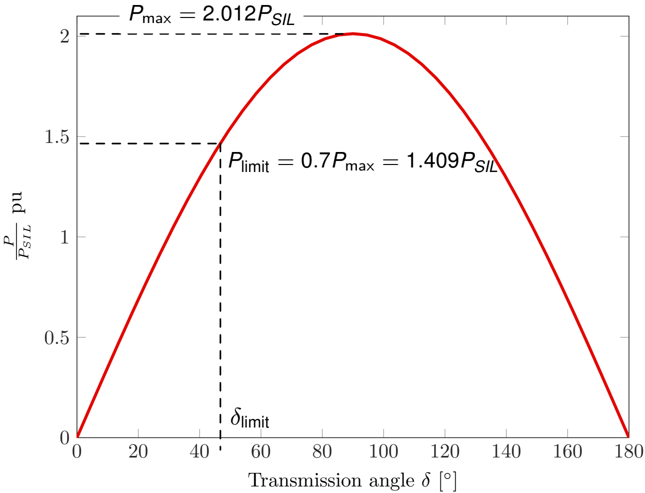
$$P = P_1 = P_2 = V_1 V_2 \frac{1}{X}\sin(\delta) = V_1 V_2 \frac{\sin(\delta)}{Z_w \sin(\beta\ell)} \qquad P_{SIL} = \frac{|\underline{V}_B|^2}{Z_w}$$

$$\rightarrow \quad \frac{P}{P_{SIL}} = v_1 v_2 \frac{\sin(\delta)}{\sin(\beta\ell)}$$

- Example: $\ell = 400$ km, $\beta\ell = 0.52$ rad, $v_1 = v_2 = 1$ pu

$$P_{\max} = \frac{1}{\sin(\beta\ell)} = 2.012 P_{SIL}$$

8 P-δ characteristic - Diagram



- $\delta_{\text{limit}} \approx 40^\circ$

8 Loading characteristics and limits

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- Power lines can not be loaded arbitrarily heavily
- In particular, the following restrictions apply
  - **Thermal limit**: Too high loading can lead to line sag (or rapid ageing in cables)
  - **Voltage drop**: For power quality and stability reasons, voltage magnitude at any bus in a network should not deviate by more than 10% from nominal value
  - **Transmission angle**: For stability reasons, the transmission angle should not exceed a certain maximum value, but a *steady-state stability margin* should be maintained

$$\text{steady-state stability margin} = \frac{P_{\text{max}} - P_{\text{limit}}}{P_{\text{max}}} \cdot 100\% \geq 30\%$$