Some Useful Mathematical Formulas

a. Trigonometric identities

$e^{j\theta} = \cos(\theta) + j\sin(\theta)$ where $j = \sqrt{-1}$

$e^{-j\theta} = \cos(\theta) - j\sin(\theta)$

$$\sin(\alpha) = \frac{e^{j\alpha} - e^{-j\alpha}}{2j} = \alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \cdots$$

$$\cos(\alpha) = \frac{e^{j\alpha} + e^{-j\alpha}}{2} = 1 - \frac{\alpha^2}{2!} + \frac{\alpha^4}{4!} - \frac{\alpha^6}{6!} + \cdots$$

$\sin(-\alpha) = -\sin(\alpha)$

$\cos(-\alpha) = \cos(\alpha)$

$\sin(\alpha + 90^{o}) = \cos(\alpha)$

$\sin(\alpha - 90^{o}) = -\cos(\alpha)$

$\cos(\alpha + 90^{o}) = -\sin(\alpha)$

$\cos(\alpha - 90^{o}) = \sin(\alpha)$

$\sin(\alpha \pm 180^{o}) = -\sin(\alpha)$

$\cos(\alpha \pm 180^{o}) = -\cos(\alpha)$

$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$

$\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$

$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$

$\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$

$2\sin(\alpha)\cos(\beta) = \sin(\alpha + \beta) + \sin(\alpha - \beta)$

$2\cos(\alpha)\sin(\beta) = \sin(\alpha + \beta) - \sin(\alpha - \beta)$

$$2\cos(\alpha)\cos(\beta) = \cos(\alpha+\beta) + \cos(\alpha-\beta)$$

$$2\sin\alpha \sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta)$$

$$\tan(\alpha+\beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}$$

$$\tan(\alpha-\beta) = \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha)\tan(\beta)}$$

$$\sin^2(\alpha) + \cos^2(\alpha) = 1$$

$$\sec^2(\alpha) = \tan^2(\alpha) + 1$$

$$\csc^2(\alpha) = \cot^2(\alpha) + 1$$

$$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$$

$$\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha) = 2\cos^2(\alpha) - 1 = 1 - 2\sin^2(\alpha)$$

$$\cos(3\alpha) = 4\cos^3(\alpha) - 3\cos(\alpha)$$

$$\sin(3\alpha) = 3\sin(\alpha) - 4\sin^3(\alpha)$$

$$\sin^2(\alpha) = \frac{1}{2}\left[1 - \cos(2\alpha)\right]$$

$$\cos^2(\alpha) = \frac{1}{2}\left[1 + \cos(2\alpha)\right]$$

$$\sin^3(\alpha) = \frac{1}{4}\left[3\sin(\alpha) - \sin(3\alpha)\right]$$

$$\cos^3(\alpha) = \frac{1}{4}\left[3\cos(\alpha) + \cos(3\alpha)\right]$$

b. Hyperbolic functions

$$e^{\alpha} = 1 + \alpha + \frac{\alpha^2}{2!} + \frac{\alpha^3}{3!} + \frac{\alpha^4}{4!} + \cdots$$

$$e^{\alpha} = \cosh(\alpha) + \sinh(\alpha)$$

$$e^{-\alpha} = \cosh(\alpha) - \sinh(\alpha)$$

$$\sinh(\alpha) = \frac{e^{\alpha} - e^{-\alpha}}{2} = \alpha + \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} + \cdots$$

$$\cosh(\alpha) = \frac{e^{\alpha} + e^{-\alpha}}{2} = 1 + \frac{\alpha^2}{2!} + \frac{\alpha^4}{4!} + \cdots$$

$$\cosh^2(\alpha) - \sinh^2(\alpha) = 1$$

$$\tanh^2(\alpha) + \mathrm{sech}^2(\alpha) = 1$$

$$\sinh(-\alpha) = -\sinh(\alpha)$$

$$\cosh(-\alpha) = \cosh(\alpha)$$

$$\sinh(j\alpha) = j\sin(\alpha)$$

$$\cosh(j\alpha) = \cos(\alpha)$$

$$\cos(j\alpha) = \cosh(\alpha)$$

$$\sin(j\alpha) = j\sinh(\alpha)$$

$$\sinh(2\alpha) = 2\sinh(\alpha)\cosh(\alpha)$$

$$\cosh(2\alpha) = \cosh^2(\alpha) + \sinh^2(\alpha) = 2\cosh^2(\alpha) - 1 = 1 + 2\sinh^2(\alpha)$$

$$\sinh(3\alpha) = 3\sinh(\alpha) + 4\sinh^3(\alpha)$$

$$\cosh(3\alpha) = 4\sinh^3(\alpha) - 3\cosh(\alpha)$$

$$\cosh(\alpha + \beta) = \cosh(\alpha)\cosh(\beta) + \sinh(\alpha)\sinh(\beta)$$

$$\cosh(\alpha - \beta) = \cosh(\alpha)\cosh(\beta) - \sinh(\alpha)\sinh(\beta)$$

$$\sinh(\alpha+\beta)=\sinh(\alpha)\cosh(\beta)+\cosh(\alpha)\sinh(\beta)$$

$$\sinh(\alpha-\beta)=\sinh(\alpha)\cosh(\beta)-\cosh(\alpha)\sinh(\beta)$$

$$\cosh(\alpha+j\beta)=\cosh(\alpha)\cos(\beta)+j\sinh(\alpha)\sin(\beta)$$

$$\cosh(\alpha-j\beta)=\cosh(\alpha)\cos(\beta)-j\sinh(\alpha)\sin(\beta)$$

$$\sinh(\alpha+j\beta)=\sinh(\alpha)\cos(\beta)+j\cosh(\alpha)\sin(\beta)$$

$$\sinh(\alpha-j\beta)=\sinh(\alpha)\cos(\beta)-j\cosh(\alpha)\sin(\beta)$$

$$\tanh(\alpha+\beta)=\frac{\tanh(\alpha)+\tanh(\beta)}{1+\tanh(\alpha)\tanh(\beta)}$$

$$\sin(\alpha+j\beta)=\sin(\alpha)\cosh(\beta)+j\cos(\alpha)\sinh(\beta)$$

$$\sin(\alpha-j\beta)=\sin(\alpha)\cosh(\beta)-j\cos(\alpha)\sinh(\beta)$$

$$\cos(\alpha+j\beta)=\cos(\alpha)\cosh(\beta)-j\sin(\alpha)\sinh(\beta)$$

$$\cos(\alpha-j\beta)=\cos(\alpha)\cosh(\beta)+j\sin(\alpha)\sinh(\beta)$$

c. Logarithmic functions

$$\log_b MN=\log_b M+\log_b N$$

$$\log_b (M/N)=\log_b M-\mathrm{lob}_b N$$

$$\log_b (1/N)=-\log_b N$$

$$\log_b (M^n)=n\log_b M$$

$$\log_a N=\log_b N\log_a b=\frac{\log_b N}{\log_b a}$$

$$\ln N=\log_{10} N\ln 10=2.302585\log_{10} N$$

$$\log_{10} N=\ln N\log_{10} e=0.434294\ln N$$

Differential Calculus

$$\frac{d}{dx}\left(c\,x^{n}\right)=c\,n\,x^{n-1}$$

$$\frac{d}{dx}\left[u+v+w\right]=\frac{du}{dx}+\frac{dv}{dx}+\frac{dw}{dx}$$

$$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^{2}}$$

$$\frac{d}{dx}\left[v^{u}\right]=u\,v^{u-1}\frac{dv}{dx}+\ln(v)\,v^{u}\frac{du}{dx}$$

$$\frac{d}{d\theta}\left[\sin(\alpha)\right]=\cos(\alpha)\frac{d\alpha}{d\theta}$$

$$\frac{d}{d\theta}\left[\cos(\alpha)\right]=-\sin(\alpha)\frac{d\alpha}{d\theta}$$

$$\frac{d}{d\theta}\left[\tan(\alpha)\right]=\sec^{2}(\alpha)\frac{d\alpha}{d\theta}$$

$$\frac{d}{d\theta}\left[\cot(\alpha)\right]=-\csc^{2}(\alpha)\frac{d\alpha}{d\theta}$$

$$\frac{d}{dx}\left[\sin^{-1}(u)\right]=\frac{1}{\sqrt{1-u^{2}}}\frac{du}{dx}$$

$$\frac{d}{dx}\left[\cos^{-1}(u)\right]=-\frac{1}{\sqrt{1-u^{2}}}\frac{du}{dx}$$

$$\frac{d}{dx}\left[\tan^{-1}(u)\right]=\frac{1}{1+u^{2}}\frac{du}{dx}$$

$$\frac{d}{dx}\left[\cot^{-1}(u)\right]=-\frac{1}{1+u^{2}}\frac{du}{dx}$$

$$\frac{d}{dx}\left[\ln(u)\right]=\frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}\left(e^{u}\right)=e^{u}\frac{du}{dx}$$

e. Integral Calculus, Indefinite Integrals

The constant of integration is to be understood with all indefinite integrals

$$\int dx = x$$

$$\int x\,dx = \frac{x^2}{2}$$

$$\int x^2\,dx = \frac{x^3}{3}$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \qquad n \neq -1$$

$$\int \frac{dx}{x} = \ln|x|$$

$$\int \frac{dx}{x^2} = -\frac{1}{x}$$

$$\int \frac{dx}{x^3} = -\frac{1}{2x^2}$$

$$\int \frac{dx}{x^n} = -\frac{1}{(n-1)\,x^{n-1}} \qquad n \neq 1$$

$$\int (a+bx)^n\,dx = \frac{(a+bx)^{n+1}}{b(n+1)} \qquad n \neq -1$$

$$\int \frac{dx}{a^2+b^2x^2} = \frac{1}{ab}\tan^{-1}\left(\frac{bx}{a}\right)$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{dx}{\left(a^2+x^2\right)^2} = \frac{x}{2a^2(a^2+x^2)} + \frac{1}{2a^3}\tan^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{x\,dx}{a^2+x^2} = \frac{1}{2}\ln\left|a^2+x^2\right|$$

$$\int \frac{x\,dx}{(a^2+x^2)^2} = -\frac{1}{2\left(a^2+x^2\right)}$$

$$\int \frac{x\,dx}{\left(a^2+x^2\right)^3} = -\frac{1}{4\left(a^2+x^2\right)^2}$$

$$\int \frac{x\,dx}{\left(a^2+x^2\right)^{n+1}} = -\frac{1}{2n\left(a^2+x^2\right)^n}$$

$$\int \frac{dx}{a^2-b^2x^2} = \frac{1}{2ab}\ln\left|\frac{a+bx}{a-bx}\right|$$

$$\int \frac{x\,dx}{a^2-x^2} = -\frac{1}{2}\ln\left|a^2-x^2\right|$$

$$\int x^{p/2}dx = \frac{2}{p+2}x^{(p+2)/2}$$

$$\int x^{1/2}dx = \frac{2}{3}x^{3/2}$$

$$\int x^{3/2}dx = \frac{2}{5}x^{5/2}$$

$$\int \frac{dx}{x^{p/2}} = -\frac{2}{(p-2)x^{(p-2)/2}}$$

$$\int \frac{dx}{x^{1/2}} = 2\,x^{1/2}$$

$$\int \frac{dx}{x^{3/2}} = -\frac{2}{x^{1/2}} = -2\,x^{-1/2}$$

$$\int \frac{dx}{\sqrt{a^2+x^2}} = \ln\left|x+\sqrt{a^2+x^2}\right|$$

$$\int \frac{x\,dx}{\sqrt{a^2+x^2}} = \sqrt{a^2+x^2}$$

$$\int \frac{x^2\,dx}{\sqrt{a^2+x^2}} = \frac{x\sqrt{a^2+x^2}}{2} - \frac{a^2}{2}\ln\left|x+\sqrt{a^2+x^2}\right|$$

$$\int \frac{dx}{\left(a^2+x^2\right)^{3/2}} = \frac{1}{a^2}\frac{x}{\sqrt{a^2+x^2}}$$

$$\int \frac{x\,dx}{\left(a^2+x^2\right)^{3/2}} = -\frac{1}{\sqrt{a^2+x^2}}$$

$$\int \frac{dx}{\left(a^2+x^2\right)^{5/2}} = \frac{1}{a^4}\left[\frac{x}{\sqrt{a^2+x^2}} - \frac{1}{3}\left(\frac{x}{\sqrt{a^2+x^2}}\right)^3\right]$$

$$\int \frac{x^2\,dx}{\left(a^2+x^2\right)^{3/2}} = -\frac{x}{\sqrt{a^2+x^2}} + \ln\left|x+\sqrt{a^2+x^2}\right|$$

$$\int \frac{x\,dx}{\left(a^2+x^2\right)^{5/2}} = -\frac{1}{3}\frac{1}{\left(a^2+x^2\right)^{3/2}}$$

$$\int \frac{x^2\,dx}{\left(a^2+x^2\right)^{5/2}} = \frac{1}{3a^2}\left(\frac{x}{\sqrt{a^2+x^2}}\right)^3$$

$$\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\ln\left|x+\sqrt{a^2+x^2}\right|$$

$$\int x\sqrt{a^2+x^2}\,dx = \frac{1}{3}\left(a^2+x^2\right)^{3/2}$$

$$\int \cos(ax)\,dx = \frac{\sin(ax)}{a}$$

$$\int \sin(ax)\,dx = -\frac{\cos(ax)}{a}$$

$$\int \tan(ax)\,dx = -\frac{1}{a}\ln\left|\cos(ax)\right|$$

$$\int \cot(ax)\,dx = \frac{1}{a}\ln\left|\sin(ax)\right|$$

$$\int \sec(ax)\,dx = \int \frac{dx}{\cos(ax)} = \frac{1}{a}\ln\left|\tan\left(\frac{ax}{2}+\frac{\pi}{4}\right)\right|$$

$$\int \csc(ax)\,dx = \int \frac{dx}{\sin(ax)} = \frac{1}{a}\ln\left|\tan\left(\frac{ax}{2}\right)\right|$$

$$\int x\ \sin(ax)\ dx = \frac{1}{a^2}\left(\sin(ax) - a\,x\ \cos(ax)\right)$$

$$\int x\ \cos(ax)\ dx = \frac{1}{a^2}\left(\cos(ax) + ax\sin(ax)\right)$$

$$\int x^2\ \sin(ax)\ dx = \frac{1}{a^3}\left[2ax\ \sin(ax) + 2\cos(ax) - a^2x^2\ \cos(ax)\right]$$

$$\int x^2 \cos(ax)\ dx = \frac{1}{a^3}\left[2ax\ \cos(ax) - 2\sin(ax) + a^2x^2\ \sin(ax)\right]$$

$$\int e^{ax}\,dx = \frac{e^{ax}}{a}$$

$$\int b^{ax}\ dx = \frac{b^{ax}}{a\ln|b|}$$

$$\int x\ e^{ax}\,dx = \frac{e^{ax}}{a^2}(ax-1)$$

$$\int x^2\ e^{ax}\ dx = \frac{e^{ax}}{a^3}\left[a^2x^2 - 2ax + 2\right]$$

$$\int e^{ax}\ \sin(bx)\ dx = \frac{e^{ax}}{a^2+b^2}\left[a\ \sin(bx) - b\ \cos(bx)\right]$$

$$\int e^{ax}\ \cos(bx)\ dx = \frac{e^{ax}}{a^2+b^2}\left[a\ \cos(bx) + b\ \sin(bx)\right]$$

$$\int \sin^{-1}(ax)\,dx = x\ \sin^{-1}(ax) + \frac{\sqrt{1-a^2x^2}}{a}$$

$$\int \cos^{-1}(ax)\,dx = x\ \cos^{-1}(ax) - \frac{\sqrt{1-a^2x^2}}{a}$$

$$\int \tan^{-1}(ax)\,dx = x\ \tan^{-1}(ax) - \frac{\ln|1+a^2x^2|}{2a}$$

$$\int \sin(n\theta)\,d\theta = -\frac{\cos(n\theta)}{n}$$

$$\int \cos(n\theta)\,d\theta = \frac{\sin(n\theta)}{n}$$

$$\int \sin^2(n\theta)\,d\theta = \frac{\theta}{2} - \frac{\sin(2n\theta)}{4n}$$

$$\int \cos^2(n\theta)\,d\theta = \frac{\theta}{2} + \frac{\sin(2n\theta)}{4n}$$

$$\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax)$$

$$\int \csc^2(ax)\,dx = -\frac{1}{a}\cot(ax)$$

$$\int \tan^2(ax)\,dx = \frac{1}{a}\tan(ax) - x$$

$$\int \cot^2(ax)\,dx = -\frac{1}{a}\cot(ax) - x$$

$$\int \sin(m\theta)\cos(n\theta)\,d\theta = -\frac{1}{2}\left[\frac{\cos(m-n)\theta}{m-n} + \frac{\cos(m+n)\theta}{m+n}\right] \qquad \text{when } m^2 \neq n^2$$

$$\int \sin(m\theta)\sin(n\theta)\,d\theta = \frac{1}{2}\left[\frac{\sin(m-n)\theta}{m-n} - \frac{\sin(m+n)\theta}{m+n}\right] \qquad \text{when } m^2 \neq n^2$$

$$\int \cos(m\theta)\cos(n\theta)\,d\theta = \frac{1}{2}\left[\frac{\sin(m-n)\theta}{m-n} + \frac{\sin(m+n)\theta}{m+n}\right] \qquad \text{when } m^2 \neq n^2$$

$$\int \sin(n\theta)\cos(n\theta)\,d\theta = \frac{\sin^2(n\theta)}{2n} \;= -\frac{\cos(2n\theta)}{4n}$$

$$\int \sin^n(ax)\cos(ax)\,dx = \frac{\sin^{n+1}(ax)}{(n+1)a} \qquad \text{when} \quad n \neq -1$$

$$\int \ln|ax|\,dx = x\ln|ax| - x$$

$$\int x^n \ln|ax)|\,dx = \frac{x^{n+1}}{n+1}\ln|ax| - \frac{x^{n+1}}{(n+1)^2} \qquad n \neq -1$$

$$\int \frac{\ln|ax|}{x}\,dx = \frac{1}{2}\left[\ln|ax|\right]^2$$

$$\int \sinh(n\theta)\,d\theta = \frac{1}{n}\cosh(n\theta)$$

$$\int \cosh(n\theta)\,d\theta = \frac{1}{n}\sinh(n\theta)$$

$$\int \tanh(n\theta)\,d\theta = \frac{1}{n}\ln\left|\cosh(n\theta)\right|$$

$$\int \coth(n\theta)\,d\theta = \frac{1}{n}\ln\left|\sinh(n\theta)\right|$$

$$\int \sec h(n\theta)\,d\theta = \frac{1}{n}\sin^{-1}\left[\tanh(n\theta)\right]$$

$$\int \csc h(n\theta)\,d\theta = \frac{1}{n}\ln\left|\tanh\left(\frac{n\theta}{2}\right)\right|$$

$$\int \sinh^2(n\theta)\,d\theta = \frac{\sinh(2n\theta)}{4n} - \frac{\theta}{2}$$

$$\int \cosh^2(n\theta)\,d\theta = \frac{\sinh(2n\theta)}{4n} + \frac{\theta}{2}$$

$$\int \tanh^2(n\theta)\,d\theta = \theta - \frac{1}{n}\tanh(n\theta)$$

$$\int \coth^2(n\theta)\,d\theta = \theta - \frac{1}{n}\coth(n\theta)$$

$$\int \operatorname{sech}^2(n\theta)\,d\theta = \frac{1}{n}\tanh(n\theta)$$

$$\int \operatorname{csch}^2(n\theta)\,d\theta = -\frac{1}{n}\coth(n\theta)$$

f. Integral Calculus, Indefinite Integrals

$$\int_0^\infty e^{-ax}\,dx = \frac{1}{a} \qquad a > 0$$

$$\int_0^\infty x\,e^{-ax}\,dx = \frac{1}{a^2} \qquad a > 0$$

$$\int_0^\infty x^2\,e^{-ax}\,dx = \frac{2}{a^3} \qquad a > 0$$

$$\int_0^\infty x^n\,e^{-ax}\,dx = \frac{n!}{a^{n+1}} \qquad a > 0 \text{ and } n \geq 0$$

$$\int_0^\infty x^{1/2}\,e^{-ax}\,dx = \frac{1}{2a}\sqrt{\frac{\pi}{a}} \qquad a > 0$$

$$\int_0^\infty x^{-1/2}\,e^{-ax}\,dx = \sqrt{\frac{\pi}{a}} \qquad a > 0$$

$$\int_0^\infty e^{-ax}\sin(bx)\,dx = \frac{b}{a^2+b^2} \qquad a > 0$$

$$\int_0^\infty e^{-ax}\cos(bx)\,dx = \frac{a}{a^2+b^2} \qquad a > 0$$

$$\int_0^\infty x\,e^{-ax}\sin(bx)\,dx = \frac{2ab}{\left(a^2+b^2\right)^2} \qquad a > 0$$

$$\int_0^\infty x\,e^{-ax}\cos(bx)\,dx = \frac{a^2 - b^2}{\left(a^2 + b^2\right)^2} \qquad a > 0$$

$$\int_0^{2\pi} \sin(n\theta)\,d\theta = 0 \qquad n \neq 0$$

$$\int_0^{2\pi} \cos(n\theta)\,d\theta = 0 \qquad n \neq 0$$

$$\int_0^{2\pi} \sin^2(n\theta)\,d\theta = \pi \qquad n \neq 0$$

$$\int_0^{2\pi} \cos^2(n\theta)\,d\theta = \pi \qquad n \neq 0$$

$$\int_0^{2\pi} \sin(m\theta)\cos(n\theta)\,d\theta = 0 \qquad m \neq n\ (m \text{ and } n \text{ are integers})$$

$$\int_0^{2\pi} \sin(m\theta)\sin(n\theta)\,d\theta = 0 \qquad m \neq n\ (m \text{ and } n \text{ are integers})$$

$$\int_0^{2\pi} \cos(m\theta)\cos(n\theta)\,d\theta = 0 \qquad m \neq n\ (m \text{ and } n \text{ are integers})$$

$$\int_0^{2\pi} \sin(n\theta)\cos(n\theta)\,d\theta = 0 \qquad n \neq 0$$

$$\int_0^{\pi} \sin(n\theta)\,d\theta = \frac{1}{n}\left[1 - \cos(n\pi)\right] \qquad n \neq 0$$

$$\int_0^{\pi} \cos(n\theta)\,d\theta = 0 \qquad n \neq 0$$

$$\int_0^{\pi} \sin^2(n\theta)\,d\theta = \frac{\pi}{2} \qquad n \neq 0$$

$$\int_0^{\pi} \cos^2(n\theta)\, d\theta = \frac{\pi}{2} \qquad n \neq 0$$

$$\int_0^{\pi} \sin(m\theta)\cos(n\theta)\, d\theta = 0 \qquad m = n \text{ (m and n are integers)}$$

$$= 0 \qquad m \neq n \text{ but } m+n = \text{even}$$

$$= \frac{2m}{m^2 - n^2} \qquad m \neq n \text{ but } m+n = \text{odd}$$

$$\int_0^{\pi} \sin(m\theta)\sin(n\theta)\, d\theta = 0 \qquad m \neq n \text{ (m and n are integers)}$$

$$\int_0^{\pi} \cos(m\theta)\cos(n\theta)\, d\theta = 0 \qquad m \neq n \text{ (m and n are integers)}$$

$$\int_0^{\pi} \sin(n\theta)\cos(n\theta)\, d\theta = 0 \qquad n \neq 0$$

$$\int_0^{\pi/2} \sin(n\theta)\, d\theta = \frac{1}{n}\left[1 - \cos\left(\frac{n\pi}{2}\right)\right] \qquad n \neq 0$$

$$\int_0^{\pi/2} \cos(n\theta)\, d\theta = \frac{1}{n}\sin\left(\frac{n\pi}{2}\right) \qquad n \neq 0$$

$$\int_0^{\pi/2} \sin^2(n\theta)\, d\theta = \frac{\pi}{4} \qquad n \neq 0$$

$$\int_0^{\pi/2} \cos^2(n\theta)\, d\theta = \frac{\pi}{4} \qquad n \neq 0$$

g. Series

$$(1+x)^n = 1 + n\,x + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots \qquad |x| < 1$$

$$(1-x)^n = 1 - n\,x + \frac{n(n-1)}{2!}x^2 - \frac{n(n-1)(n-2)}{3!}x^3 + \cdots \qquad |x| < 1$$

$$(1+x)^{-n} = 1 - n\,x + \frac{n(n+1)}{2!}x^2 - \frac{n(n+1)(n+2)}{3!}x^3 + \cdots \qquad |x| < 1$$

$$(1-x)^{-n} = 1 + n\,x + \frac{n(n+1)}{2!}x^2 + \frac{n(n+1)(n+2)}{3!}x^3 + \cdots \qquad |x| < 1$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots \qquad |x| < 1$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \cdots = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n} \qquad \text{for all } x$$