softmax

February 25, 2019

1 Softmax exercise

Complete and hand in this completed worksheet (including its outputs and any supporting code outside of the worksheet) with your assignment submission.

In this exercise, you will:

- implement a fully-vectorized **loss function** for the Softmax classifier
- implement the fully-vectorized expression for its **analytic gradient**
- **check your implementation** with numerical gradient
- use a validation set to **tune the learning rate and regularization** strength
- **optimize** the loss function with **SGD**
- **visualize** the final learned weights

```
In [1]: import random
        import numpy as np
        from data_utils import load_CIFAR10
        import matplotlib.pyplot as plt

        %matplotlib inline
        plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
        plt.rcParams['image.interpolation'] = 'nearest'
        plt.rcParams['image.cmap'] = 'gray'

        # for auto-reloading extenrnal modules
        # see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-ipython
        %load_ext autoreload
        %autoreload 2
```

```
In [2]: def rel_error(out, correct_out):
            return np.sum(abs(out - correct_out) / (abs(out) + abs(correct_out)))
```

```
In [3]: def get_CIFAR10_data(num_training=49000, num_validation=1000, num_test=1000, num_dev=50
            """
            Load the CIFAR-10 dataset from disk and perform preprocessing to prepare
            it for the linear classifier. These are the same steps as we used for the
            Softmax, but condensed to a single function.
            """
            # Load the raw CIFAR-10 data
```

```
    cifar10_dir = 'datasets/cifar-10-batches-py'
    X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

    # subsample the data
    mask = range(num_training, num_training + num_validation)
    X_val = X_train[mask]
    y_val = y_train[mask]
    mask = range(num_training)
    X_train = X_train[mask]
    y_train = y_train[mask]
    mask = range(num_test)
    X_test = X_test[mask]
    y_test = y_test[mask]

    # # We will also make a development set, which is a small subset of
    # the training set.
    mask = np.random.choice(num_training, num_dev, replace=False)
    X_dev = X_train[mask]
    y_dev = y_train[mask]

    # Preprocessing: reshape the image data into rows
    X_train = np.reshape(X_train, (X_train.shape[0], -1))
    X_val = np.reshape(X_val, (X_val.shape[0], -1))
    X_test = np.reshape(X_test, (X_test.shape[0], -1))
    X_dev = np.reshape(X_dev, (X_dev.shape[0], -1))

    # Normalize the data: subtract the mean image
    mean_image = np.mean(X_train, axis = 0)
    X_train -= mean_image
    X_val -= mean_image
    X_test -= mean_image
    X_dev -= mean_image

    # add bias dimension and transform into columns
    X_train = np.hstack([X_train, np.ones((X_train.shape[0], 1))])
    X_val = np.hstack([X_val, np.ones((X_val.shape[0], 1))])
    X_test = np.hstack([X_test, np.ones((X_test.shape[0], 1))])
    X_dev = np.hstack([X_dev, np.ones((X_dev.shape[0], 1))])

    return X_train, y_train, X_val, y_val, X_test, y_test, X_dev, y_dev


# Invoke the above function to get our data.
X_train, y_train, X_val, y_val, X_test, y_test, X_dev, y_dev = get_CIFAR10_data()
print('Train data shape: ', X_train.shape)
print('Train labels shape: ', y_train.shape)
print('Validation data shape: ', X_val.shape)
print('Validation labels shape: ', y_val.shape)
```

```
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
print('dev data shape: ', X_dev.shape)
print('dev labels shape: ', y_dev.shape)
```

```
Train data shape:  (49000, 3073)
Train labels shape:  (49000,)
Validation data shape:  (1000, 3073)
Validation labels shape:  (1000,)
Test data shape:  (1000, 3073)
Test labels shape:  (1000,)
dev data shape:  (500, 3073)
dev labels shape:  (500,)
```

In [4]:
```
# Create one-hot vectors for label
num_class = 10
y_train_oh = np.zeros((y_train.shape[0], 10))
y_train_oh[np.arange(y_train.shape[0]), y_train] = 1
y_val_oh = np.zeros((y_val.shape[0], 10))
y_val_oh[np.arange(y_val.shape[0]), y_val] = 1
y_test_oh = np.zeros((y_test.shape[0], 10))
y_test_oh[np.arange(y_test.shape[0]), y_test] = 1

y_dev_oh = np.zeros((y_dev.shape[0], 10))
y_dev_oh[np.arange(y_dev.shape[0]), y_dev] = 1
```

2 Regression as classifier

The most simple and straightforward approach to learn a classifier is to map the input data (raw image values) to class label (one-hot vector). The loss function is defined as following:

$$\mathcal{L} = \frac{1}{n}\|\mathbf{X}\mathbf{W} - \mathbf{y}\|_F^2 \qquad (1)$$

Where: * $\mathbf{W} \in \mathbb{R}^{(d+1)\times C}$: Classifier weight * $\mathbf{X} \in \mathbb{R}^{n\times(d+1)}$: Dataset * $\mathbf{y} \in \mathbb{R}^{n\times C}$: Class label (one-hot vector)

3 Optimization

Given the loss function (1), the next problem is how to solve the weight **W**. We now discuss 2 approaches: * Random search * Closed-form solution

3.1 Random search

In [5]:
```
bestloss = float('inf')
for num in range(100):
    W = np.random.randn(3073, 10) * 0.0001
```

```python
        loss = np.linalg.norm(X_dev.dot(W) - y_dev_oh)
        if (loss < bestloss):
            bestloss = loss
            bestW = W
        print('in attempt %d the loss was %f, best %f' % (num, loss, bestloss))
```

```
in attempt 0 the loss was 31.097852, best 31.097852
in attempt 1 the loss was 35.986907, best 31.097852
in attempt 2 the loss was 32.936750, best 31.097852
in attempt 3 the loss was 32.263297, best 31.097852
in attempt 4 the loss was 33.316720, best 31.097852
in attempt 5 the loss was 33.530687, best 31.097852
in attempt 6 the loss was 33.089863, best 31.097852
in attempt 7 the loss was 33.797979, best 31.097852
in attempt 8 the loss was 31.849634, best 31.097852
in attempt 9 the loss was 31.078356, best 31.078356
in attempt 10 the loss was 32.901067, best 31.078356
in attempt 11 the loss was 32.724453, best 31.078356
in attempt 12 the loss was 34.693179, best 31.078356
in attempt 13 the loss was 33.269654, best 31.078356
in attempt 14 the loss was 33.896164, best 31.078356
in attempt 15 the loss was 31.727043, best 31.078356
in attempt 16 the loss was 31.291954, best 31.078356
in attempt 17 the loss was 34.024641, best 31.078356
in attempt 18 the loss was 31.856058, best 31.078356
in attempt 19 the loss was 34.706234, best 31.078356
in attempt 20 the loss was 32.552190, best 31.078356
in attempt 21 the loss was 30.909683, best 30.909683
in attempt 22 the loss was 33.826905, best 30.909683
in attempt 23 the loss was 31.748100, best 30.909683
in attempt 24 the loss was 33.683554, best 30.909683
in attempt 25 the loss was 33.235776, best 30.909683
in attempt 26 the loss was 33.766795, best 30.909683
in attempt 27 the loss was 32.458615, best 30.909683
in attempt 28 the loss was 32.793571, best 30.909683
in attempt 29 the loss was 32.980274, best 30.909683
in attempt 30 the loss was 33.646236, best 30.909683
in attempt 31 the loss was 32.000726, best 30.909683
in attempt 32 the loss was 34.343205, best 30.909683
in attempt 33 the loss was 32.809490, best 30.909683
in attempt 34 the loss was 30.753257, best 30.753257
in attempt 35 the loss was 33.363264, best 30.753257
in attempt 36 the loss was 32.521487, best 30.753257
in attempt 37 the loss was 32.286729, best 30.753257
in attempt 38 the loss was 33.466272, best 30.753257
in attempt 39 the loss was 33.767820, best 30.753257
in attempt 40 the loss was 34.014519, best 30.753257
in attempt 41 the loss was 33.604679, best 30.753257
```

```
in attempt 42 the loss was 33.021665, best 30.753257
in attempt 43 the loss was 34.238782, best 30.753257
in attempt 44 the loss was 31.868839, best 30.753257
in attempt 45 the loss was 34.560101, best 30.753257
in attempt 46 the loss was 33.859088, best 30.753257
in attempt 47 the loss was 33.872607, best 30.753257
in attempt 48 the loss was 35.401592, best 30.753257
in attempt 49 the loss was 32.297212, best 30.753257
in attempt 50 the loss was 33.731449, best 30.753257
in attempt 51 the loss was 32.974246, best 30.753257
in attempt 52 the loss was 33.360920, best 30.753257
in attempt 53 the loss was 36.814551, best 30.753257
in attempt 54 the loss was 32.383545, best 30.753257
in attempt 55 the loss was 33.313233, best 30.753257
in attempt 56 the loss was 32.943898, best 30.753257
in attempt 57 the loss was 35.537028, best 30.753257
in attempt 58 the loss was 31.539891, best 30.753257
in attempt 59 the loss was 31.464733, best 30.753257
in attempt 60 the loss was 32.306981, best 30.753257
in attempt 61 the loss was 31.490039, best 30.753257
in attempt 62 the loss was 32.139464, best 30.753257
in attempt 63 the loss was 33.191718, best 30.753257
in attempt 64 the loss was 32.380865, best 30.753257
in attempt 65 the loss was 32.790204, best 30.753257
in attempt 66 the loss was 30.227760, best 30.227760
in attempt 67 the loss was 30.776805, best 30.227760
in attempt 68 the loss was 32.365543, best 30.227760
in attempt 69 the loss was 36.965447, best 30.227760
in attempt 70 the loss was 32.575221, best 30.227760
in attempt 71 the loss was 30.665061, best 30.227760
in attempt 72 the loss was 33.697618, best 30.227760
in attempt 73 the loss was 35.596273, best 30.227760
in attempt 74 the loss was 32.185674, best 30.227760
in attempt 75 the loss was 32.197692, best 30.227760
in attempt 76 the loss was 32.185795, best 30.227760
in attempt 77 the loss was 33.575504, best 30.227760
in attempt 78 the loss was 32.962141, best 30.227760
in attempt 79 the loss was 34.287886, best 30.227760
in attempt 80 the loss was 32.794918, best 30.227760
in attempt 81 the loss was 33.452308, best 30.227760
in attempt 82 the loss was 33.735422, best 30.227760
in attempt 83 the loss was 32.112291, best 30.227760
in attempt 84 the loss was 34.023853, best 30.227760
in attempt 85 the loss was 32.760048, best 30.227760
in attempt 86 the loss was 32.747929, best 30.227760
in attempt 87 the loss was 32.216412, best 30.227760
in attempt 88 the loss was 33.294907, best 30.227760
in attempt 89 the loss was 31.712067, best 30.227760
```

```
in attempt 90 the loss was 31.398513, best 30.227760
in attempt 91 the loss was 34.414248, best 30.227760
in attempt 92 the loss was 32.986523, best 30.227760
in attempt 93 the loss was 35.948089, best 30.227760
in attempt 94 the loss was 33.894141, best 30.227760
in attempt 95 the loss was 32.095476, best 30.227760
in attempt 96 the loss was 32.895736, best 30.227760
in attempt 97 the loss was 32.178781, best 30.227760
in attempt 98 the loss was 33.690184, best 30.227760
in attempt 99 the loss was 32.605644, best 30.227760
```

```
In [6]: # How bestW perform:
        print('Accuracy on train set: ', np.sum(np.argmin(np.abs(1 - X_dev.dot(bestW)), axis=1
        print('Accuracy on test set: ', np.sum(np.argmin(np.abs(1 - X_test.dot(bestW)), axis=1
```

```
Accuracy on train set:  15.8
Accuracy on test set:  16.900000000000002
```

You can clearly see that the performance is very low, almost at the random level.

3.2 Closed-form solution

The closed-form solution is achieved by:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{W}} = \frac{2}{n}\mathbf{X}^T(\mathbf{X}\mathbf{W} - \mathbf{y}) = 0$$

$$\Leftrightarrow \mathbf{W}^* = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}$$

```
In [7]: ################################################################################
        # TODO:                                                                        #
        # Implement the closed-form solution of the weight W.                          #
        ################################################################################
        X_train_trans = np.transpose(X_train)
        pre = np.matmul(X_train_trans, X_train)
        pre_inverse = np.linalg.inv(pre)
        pre_pre = np.matmul(pre_inverse, X_train_trans)
        W = np.matmul(pre_pre, y_train_oh)



        ################################################################################
        #                              END OF YOUR CODE                                #
        ################################################################################
```

```
In [8]: # Check accuracy:
        print('Train set accuracy: ', np.sum(np.argmin(np.abs(1 - X_train.dot(W)), axis=1) == y
        print('Test set accuracy: ', np.sum(np.argmin(np.abs(1 - X_test.dot(W)), axis=1) == y_t
```

```
Train set accuracy:  51.163265306122454
Test set accuracy:  36.199999999999996
```

Now, you can see that the performance is much better.

3.3 Regularization

A simple way to improve performance is to include the L2-regularization penalty.

$$\mathcal{L} = \frac{1}{n}\|\mathbf{X}\mathbf{W} - \mathbf{y}\|_F^2 + \lambda\|\mathbf{W}\|_F^2 \tag{2}$$

The closed-form solution now is:

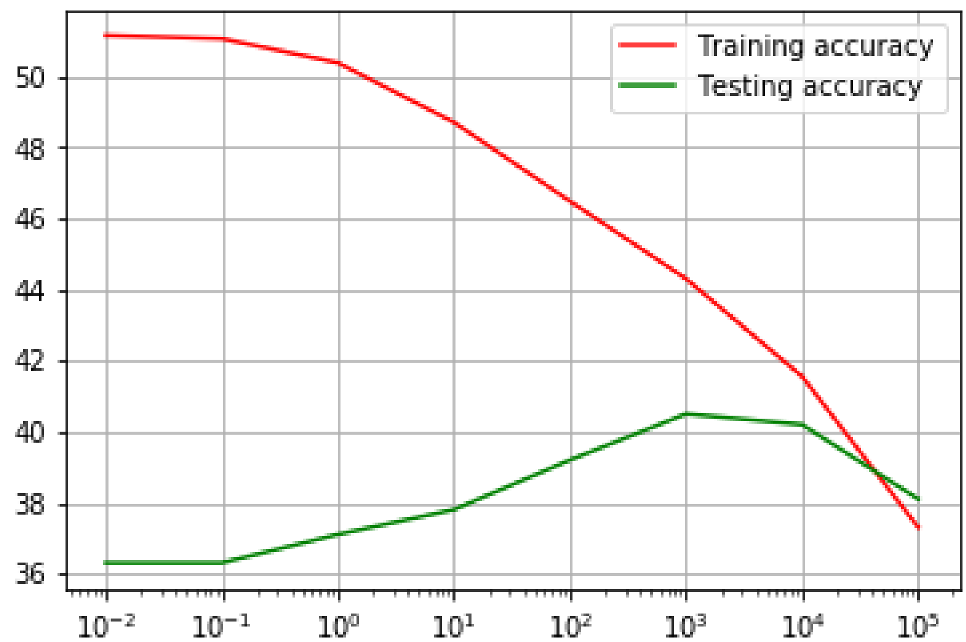
$$\Leftrightarrow \mathbf{W}^* = (\mathbf{X}^T\mathbf{X} + \lambda n\mathbf{I})^{-1}\mathbf{X}^T\mathbf{y}$$

```
In [9]: # try several values of lambda to see how it helps:
        lambdas = [0.01, 0.1, 1, 10, 100, 1000, 10000, 100000]
        train_acc = np.zeros((len(lambdas)))
        test_acc = np.zeros((len(lambdas)))
        for i in range(len(lambdas)):
            l = lambdas[i]
            n,d = X_train.shape[0], X_train.shape[1]
            ################################################################################
            # TODO:                                                                        #
            # Implement the closed-form solution of the weight W with regularization.      #
            ################################################################################
            X_train_trans = np.transpose(X_train)
            pre_1 = np.matmul(X_train_trans, X_train)
            pre = pre_1 + l * n * np.identity(pre_1.shape[0])
            pre_inverse = np.linalg.inv(pre)
            pre_pre = np.matmul(pre_inverse, X_train_trans)
            W = np.matmul(pre_pre, y_train_oh)

            ################################################################################
            #                              END OF YOUR CODE                                #
            ################################################################################
            train_acc[i] = np.sum(np.argmin(np.abs(1 - X_train.dot(W)), axis=1) == y_train).as
            print('Train set accuracy of ', l, ' : ', train_acc[i])
            test_acc[i]  = np.sum(np.argmin(np.abs(1 - X_te
                                               st.dot(W)), axis=1) == y_test).astype(np.fl
            print('Test set accuracy of ', l, ' : ', test_acc[i])
```

```
Train set accuracy of  0.01  :  51.159183673469386
Test set accuracy of  0.01  :  36.3
Train set accuracy of  0.1  :  51.069387755102035
Test set accuracy of  0.1  :  36.3
Train set accuracy of  1  :  50.39591836734694
Test set accuracy of  1  :  37.1
```

```
Train set accuracy of  10  :  48.71632653061225
Test set accuracy of  10  :  37.8
Train set accuracy of  100  :  46.49183673469388
Test set accuracy of  100  :  39.2
Train set accuracy of  1000  :  44.304081632653066
Test set accuracy of  1000  :  40.5
Train set accuracy of  10000  :  41.55714285714286
Test set accuracy of  10000  :  40.2
Train set accuracy of  100000  :  37.30204081632653
Test set accuracy of  100000  :  38.1
```

```
In [10]: plt.semilogx(lambdas, train_acc, 'r', label="Training accuracy")
         plt.semilogx(lambdas, test_acc, 'g', label="Testing accuracy")

         plt.legend()
         plt.grid(True)
         plt.show()
```

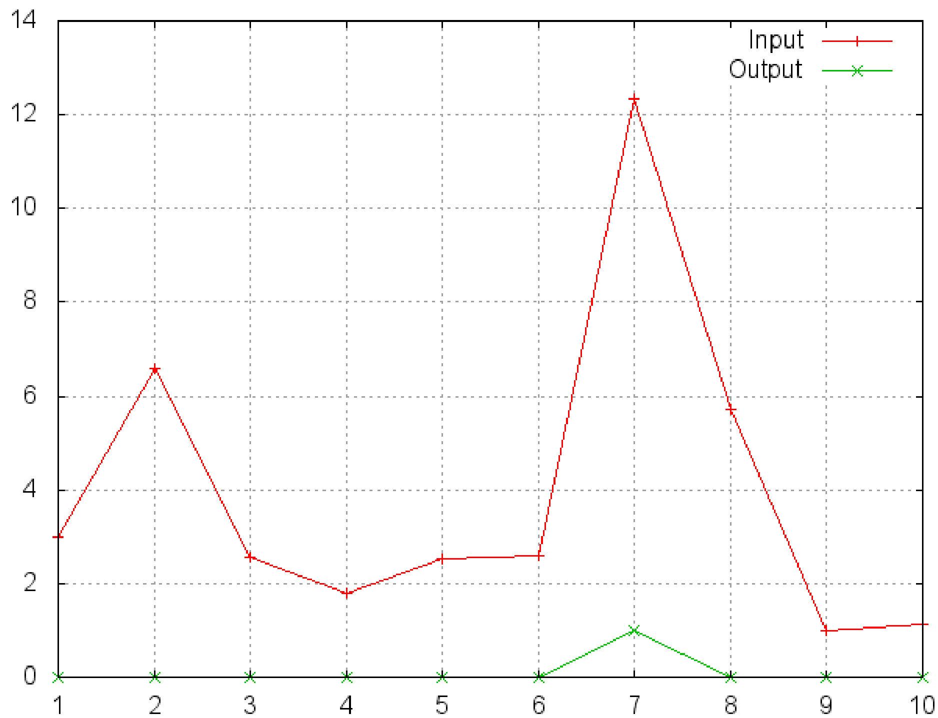


Question: Try to explain why the performances on the training and test set have such behaviors as we change the value of λ.

Your answer: When λ is small, W tends to be overfitting. When λ is 1000, it has the most optimized result for testing set.

3.4 Softmax Classifier

The predicted probability for the j-th class given a sample vector x and a weight W is:



softmax

$$P(y = j \mid x) = \frac{e^{-xw_j}}{\sum\limits_{c=1}^{C} e^{-xw_c}}$$

Your code for this section will all be written inside **classifiers/softmax.py**.

```
In [14]: # First implement the naive softmax loss function with nested loops.
         # Open the file classifiers/softmax.py and implement the
         # softmax_loss_naive function.

         from classifiers.softmax import softmax_loss_naive
         import time

         # Generate a random softmax weight matrix and use it to compute the loss.
         W = np.random.randn(3073, 10) * 0.0001
         loss, grad = softmax_loss_naive(W, X_dev, y_dev, 0.0)

         # As a rough sanity check, our loss should be something close to -log(0.1).
         print('loss: %f' % loss)
         print('sanity check: %f' % (-np.log(0.1)))
```

```
loss: 2.332788
sanity check: 2.302585
```

Question: Why do we expect our loss to be close to -log(0.1)? Explain briefly.**

Your answer: Because we have in total 10 classes. So we compare our result with random guessing.

4 Optimization

4.1 Random search

```
In [15]: bestloss = float('inf')
         for num in range(100):
             W = np.random.randn(3073, 10) * 0.0001
             loss, _ = softmax_loss_naive(W, X_dev, y_dev, 0.0)
             if (loss < bestloss):
                 bestloss = loss
                 bestW = W
             print('in attempt %d the loss was %f, best %f' % (num, loss, bestloss))
```

```
in attempt 0 the loss was 2.315287, best 2.315287
in attempt 1 the loss was 2.315728, best 2.315287
in attempt 2 the loss was 2.279245, best 2.279245
in attempt 3 the loss was 2.338965, best 2.279245
in attempt 4 the loss was 2.364009, best 2.279245
in attempt 5 the loss was 2.399972, best 2.279245
in attempt 6 the loss was 2.364766, best 2.279245
in attempt 7 the loss was 2.354514, best 2.279245
in attempt 8 the loss was 2.409205, best 2.279245
in attempt 9 the loss was 2.323656, best 2.279245
in attempt 10 the loss was 2.397302, best 2.279245
in attempt 11 the loss was 2.339812, best 2.279245
in attempt 12 the loss was 2.379946, best 2.279245
in attempt 13 the loss was 2.315949, best 2.279245
in attempt 14 the loss was 2.361623, best 2.279245
in attempt 15 the loss was 2.318217, best 2.279245
in attempt 16 the loss was 2.368780, best 2.279245
in attempt 17 the loss was 2.363134, best 2.279245
in attempt 18 the loss was 2.366291, best 2.279245
in attempt 19 the loss was 2.339898, best 2.279245
in attempt 20 the loss was 2.355486, best 2.279245
in attempt 21 the loss was 2.334065, best 2.279245
in attempt 22 the loss was 2.315165, best 2.279245
in attempt 23 the loss was 2.351390, best 2.279245
in attempt 24 the loss was 2.364131, best 2.279245
in attempt 25 the loss was 2.384957, best 2.279245
in attempt 26 the loss was 2.358619, best 2.279245
in attempt 27 the loss was 2.376561, best 2.279245
in attempt 28 the loss was 2.329522, best 2.279245
in attempt 29 the loss was 2.337146, best 2.279245
in attempt 30 the loss was 2.353836, best 2.279245
```

```
in attempt 31 the loss was 2.321441, best 2.279245
in attempt 32 the loss was 2.333559, best 2.279245
in attempt 33 the loss was 2.330200, best 2.279245
in attempt 34 the loss was 2.307366, best 2.279245
in attempt 35 the loss was 2.361179, best 2.279245
in attempt 36 the loss was 2.323411, best 2.279245
in attempt 37 the loss was 2.337375, best 2.279245
in attempt 38 the loss was 2.381666, best 2.279245
in attempt 39 the loss was 2.351327, best 2.279245
in attempt 40 the loss was 2.355621, best 2.279245
in attempt 41 the loss was 2.401918, best 2.279245
in attempt 42 the loss was 2.371483, best 2.279245
in attempt 43 the loss was 2.397893, best 2.279245
in attempt 44 the loss was 2.336500, best 2.279245
in attempt 45 the loss was 2.306861, best 2.279245
in attempt 46 the loss was 2.343061, best 2.279245
in attempt 47 the loss was 2.362867, best 2.279245
in attempt 48 the loss was 2.391230, best 2.279245
in attempt 49 the loss was 2.374461, best 2.279245
in attempt 50 the loss was 2.383866, best 2.279245
in attempt 51 the loss was 2.369601, best 2.279245
in attempt 52 the loss was 2.399752, best 2.279245
in attempt 53 the loss was 2.346172, best 2.279245
in attempt 54 the loss was 2.309303, best 2.279245
in attempt 55 the loss was 2.366009, best 2.279245
in attempt 56 the loss was 2.312034, best 2.279245
in attempt 57 the loss was 2.399871, best 2.279245
in attempt 58 the loss was 2.369382, best 2.279245
in attempt 59 the loss was 2.353133, best 2.279245
in attempt 60 the loss was 2.318029, best 2.279245
in attempt 61 the loss was 2.375471, best 2.279245
in attempt 62 the loss was 2.335294, best 2.279245
in attempt 63 the loss was 2.388040, best 2.279245
in attempt 64 the loss was 2.296572, best 2.279245
in attempt 65 the loss was 2.327079, best 2.279245
in attempt 66 the loss was 2.376346, best 2.279245
in attempt 67 the loss was 2.371548, best 2.279245
in attempt 68 the loss was 2.328372, best 2.279245
in attempt 69 the loss was 2.379915, best 2.279245
in attempt 70 the loss was 2.377497, best 2.279245
in attempt 71 the loss was 2.297419, best 2.279245
in attempt 72 the loss was 2.311660, best 2.279245
in attempt 73 the loss was 2.366952, best 2.279245
in attempt 74 the loss was 2.342677, best 2.279245
in attempt 75 the loss was 2.351796, best 2.279245
in attempt 76 the loss was 2.318753, best 2.279245
in attempt 77 the loss was 2.342505, best 2.279245
in attempt 78 the loss was 2.333173, best 2.279245
```

```
in attempt 79 the loss was 2.351038, best 2.279245
in attempt 80 the loss was 2.388554, best 2.279245
in attempt 81 the loss was 2.326636, best 2.279245
in attempt 82 the loss was 2.343161, best 2.279245
in attempt 83 the loss was 2.345845, best 2.279245
in attempt 84 the loss was 2.378235, best 2.279245
in attempt 85 the loss was 2.323155, best 2.279245
in attempt 86 the loss was 2.418805, best 2.279245
in attempt 87 the loss was 2.382597, best 2.279245
in attempt 88 the loss was 2.369079, best 2.279245
in attempt 89 the loss was 2.340579, best 2.279245
in attempt 90 the loss was 2.364311, best 2.279245
in attempt 91 the loss was 2.331434, best 2.279245
in attempt 92 the loss was 2.293285, best 2.279245
in attempt 93 the loss was 2.394173, best 2.279245
in attempt 94 the loss was 2.362335, best 2.279245
in attempt 95 the loss was 2.370387, best 2.279245
in attempt 96 the loss was 2.323099, best 2.279245
in attempt 97 the loss was 2.334008, best 2.279245
in attempt 98 the loss was 2.317522, best 2.279245
in attempt 99 the loss was 2.355111, best 2.279245
```

```
In [16]: # How bestW perform on trainset
         scores = X_train.dot(bestW)
         y_pred = np.argmax(scores, axis=1)
         print('Accuracy on train set %f' % np.mean(y_pred == y_train))

         # evaluate performance of test set
         scores = X_test.dot(bestW)
         y_pred = np.argmax(scores, axis=1)
         print('Accuracy on test set %f' % np.mean(y_pred == y_test))
```

```
Accuracy on train set 0.131245
Accuracy on test set 0.124000
```

Compare the performance when using random search with *regression classifier* and *softmax classifier*. You can see how much useful the softmax classifier is.

4.2 Stochastic Gradient descent

Even though it is possible to achieve closed-form solution with softmax classifier, it would be more complicated. In fact, we could achieve very good results with gradient descent approach. Additionally, in case of very large dataset, it is impossible to load the whole dataset into the memory. Gradient descent can help to optimize the loss function in batch.

$$\mathbf{W}^{t+1} = \mathbf{W}^{t}\alpha\frac{\partial\mathcal{L}(\mathbf{x};\mathbf{W}^{\sqcup})}{\partial\mathbf{W}^{t}}$$

Where α is the learning rate, $\mathcal{L}$ is a loss function, and $\mathbf{x}$ is a batch of training dataset.

In [17]:
```
# Complete the implementation of softmax_loss_naive and implement a (naive)
# version of the gradient that uses nested loops.
loss, grad = softmax_loss_naive(W, X_dev, y_dev, 0.0)

# Use numeric gradient checking as a debugging tool.
# The numeric gradient should be close to the analytic gradient.
from gradient_check import grad_check_sparse
f = lambda w: softmax_loss_naive(w, X_dev, y_dev, 0.0)[0]
grad_numerical = grad_check_sparse(f, W, grad, 10)

# gradient check with regularization
loss, grad = softmax_loss_naive(W, X_dev, y_dev, 1e2)
f = lambda w: softmax_loss_naive(w, X_dev, y_dev, 1e2)[0]
grad_numerical = grad_check_sparse(f, W, grad, 10)
```

```
numerical: -0.863772 analytic: -0.863772, relative error: 3.286460e-08
numerical: -0.830557 analytic: -0.830557, relative error: 9.243450e-08
numerical: 2.159302 analytic: 2.159302, relative error: 3.404888e-08
numerical: -0.075499 analytic: -0.075499, relative error: 2.376996e-07
numerical: -5.666792 analytic: -5.666792, relative error: 2.276835e-09
numerical: -2.117591 analytic: -2.117591, relative error: 5.739073e-09
numerical: -0.995018 analytic: -0.995018, relative error: 1.605112e-08
numerical: 0.199430 analytic: 0.199430, relative error: 1.770890e-07
numerical: -0.060309 analytic: -0.060309, relative error: 9.127597e-08
numerical: -2.069928 analytic: -2.069928, relative error: 1.044419e-08
numerical: 0.176114 analytic: 0.176114, relative error: 2.150958e-07
numerical: -4.152779 analytic: -4.152779, relative error: 4.241791e-09
numerical: 0.782594 analytic: 0.782594, relative error: 1.654405e-08
numerical: 0.193260 analytic: 0.193260, relative error: 1.609427e-07
numerical: -2.683427 analytic: -2.683427, relative error: 1.281667e-09
numerical: -0.846214 analytic: -0.846214, relative error: 1.180543e-08
numerical: -0.289115 analytic: -0.289115, relative error: 2.914200e-08
numerical: 0.035788 analytic: 0.035788, relative error: 1.307111e-06
numerical: -3.161925 analytic: -3.161925, relative error: 5.695438e-09
numerical: 0.322899 analytic: 0.322899, relative error: 1.329794e-07
```

In [24]:
```
# Now that we have a naive implementation of the softmax loss function and its gradie
# implement a vectorized version in softmax_loss_vectorized.
# The two versions should compute the same results, but the vectorized version should
# much faster.
tic = time.time()
loss_naive, grad_naive = softmax_loss_naive(W, X_dev, y_dev, 0.00001)
toc = time.time()
print('naive loss: %e computed in %fs' % (loss_naive, toc - tic))
```

```
from classifiers.softmax import softmax_loss_vectorized
tic = time.time()
loss_vectorized, grad_vectorized = softmax_loss_vectorized(W, X_dev, y_dev, 0.00001)
toc = time.time()
print('vectorized loss: %e computed in %fs' % (loss_vectorized, toc - tic))

# We use the Frobenius norm to compare the two versions
# of the gradient.
grad_difference = np.linalg.norm(grad_naive - grad_vectorized, ord='fro')
print('Loss difference: %f' % np.abs(loss_naive - loss_vectorized))
print('Gradient difference: %f' % grad_difference)
```

```
naive loss: 2.355111e+00 computed in 11.699731s
vectorized loss: 8.587773e+00 computed in 0.009061s
Loss difference: 6.232662
Gradient difference: 141.386003
```

In [27]:
```
from classifiers.linear_classifier import *
# from classifiers import linear_classifier

classifier = Softmax()
tic = time.time()
loss_hist = classifier.train(X_train, y_train, learning_rate=1e-7, reg=5e4,
                             num_iters=1500, verbose=True)
toc = time.time()
print('That took %fs' % (toc - tic))
```

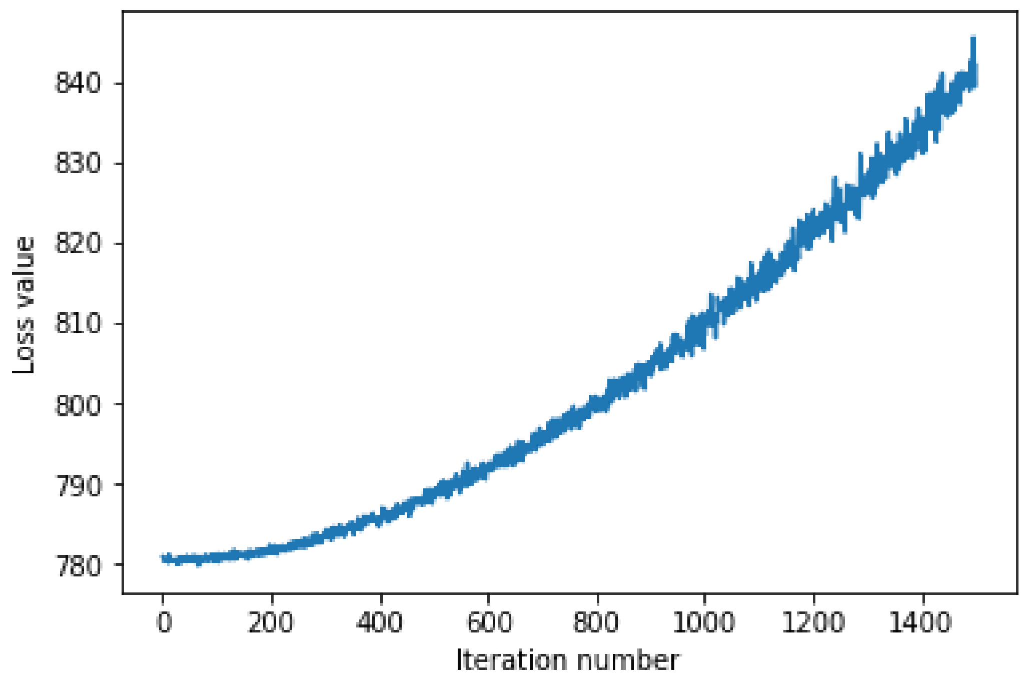
```
iteration 0 / 1500: loss 780.933791
iteration 100 / 1500: loss 781.235478
iteration 200 / 1500: loss 781.834763
iteration 300 / 1500: loss 784.080822
iteration 400 / 1500: loss 785.783550
iteration 500 / 1500: loss 788.199196
iteration 600 / 1500: loss 792.060046
iteration 700 / 1500: loss 795.235677
iteration 800 / 1500: loss 799.411085
iteration 900 / 1500: loss 804.025054
iteration 1000 / 1500: loss 808.421515
iteration 1100 / 1500: loss 815.391185
iteration 1200 / 1500: loss 820.561515
iteration 1300 / 1500: loss 828.572252
iteration 1400 / 1500: loss 834.540254
That took 4.650405s
```

In [28]:
```
# Write the Softmax.predict function and evaluate the performance on both the
# training and validation set
y_train_pred = classifier.predict(X_train)
```

```
print('training accuracy: %f' % (np.mean(y_train == y_train_pred), ))
y_val_pred = classifier.predict(X_val)
print('validation accuracy: %f' % (np.mean(y_val == y_val_pred), ))
```

```
training accuracy: 0.248061
validation accuracy: 0.263000
```

In [29]:
```
# A useful debugging strategy is to plot the loss as a function of
# iteration number:
plt.plot(loss_hist)
plt.xlabel('Iteration number')
plt.ylabel('Loss value')
plt.show()
```



In [30]:
```
# evaluate on test set
# Evaluate the best softmax on test set
y_test_pred = classifier.predict(X_test)
test_accuracy = np.mean(y_test == y_test_pred)
print('softmax on raw pixels final test set accuracy: %.2f' % (100*test_accuracy, ))
```

```
softmax on raw pixels final test set accuracy: 26.70
```

In [33]:
```
# Use the validation set to tune hyperparameters (regularization strength and
# learning rate). You should experiment with different ranges for the learning
```

```python
# rates and regularization strengths; if you are careful you should be able to
# get a classification accuracy of over 35% on the validation set.

import copy
best_val_acc = -1
best_softmax = None        # You may need to use copy.deepcopy(object)
learning_rates = [1e-4,1e-5, 1e-6,5e-7]
regularization_strengths = [1e-4, 1e-3, 1e-2, 1e-1, 1e0]
results = {}
################################################################################
# TODO:                                                                        #
# Use the validation set to set the learning rate and regularization strength. #
# Save the best trained softmax classifer in best_softmax.                     #
################################################################################
for lr in learning_rates:
    for rs in regularization_strengths:
        classifier = Softmax()
        classifier.train(X_train, y_train, learning_rate=lr, reg=rs,
                          num_iters=1500, verbose=True)
        y_val_pred = classifier.predict(X_val)
        val_acc = np.mean(y_val == y_val_pred)
        if val_acc > best_val_acc:
            best_softmax = classifier
            best_val_acc = val_acc


################################################################################
#                              END OF YOUR CODE                                #
################################################################################
print('best cross-validation accuracy: %.2f' % best_val_acc)
```

```
iteration 0 / 1500: loss 10.833682


/Users/lixingxuan/Desktop/Computer Vision/Week-01/forStudents/classifiers/softmax.py:90: Runtin
  loss = np.sum(-np.log(correct))


iteration 100 / 1500: loss inf
iteration 200 / 1500: loss inf
iteration 300 / 1500: loss inf
iteration 400 / 1500: loss inf
iteration 500 / 1500: loss inf
iteration 600 / 1500: loss inf
iteration 700 / 1500: loss inf
iteration 800 / 1500: loss inf
iteration 900 / 1500: loss inf
iteration 1000 / 1500: loss inf
```

```
iteration 1100 / 1500: loss inf
iteration 1200 / 1500: loss inf
iteration 1300 / 1500: loss inf
iteration 1400 / 1500: loss inf
iteration 0 / 1500: loss 10.256135
iteration 100 / 1500: loss inf
iteration 200 / 1500: loss inf
iteration 300 / 1500: loss inf
iteration 400 / 1500: loss inf
iteration 500 / 1500: loss inf
iteration 600 / 1500: loss inf
iteration 700 / 1500: loss inf
iteration 800 / 1500: loss inf
iteration 900 / 1500: loss inf
iteration 1000 / 1500: loss inf
iteration 1100 / 1500: loss inf
iteration 1200 / 1500: loss inf
iteration 1300 / 1500: loss inf
iteration 1400 / 1500: loss inf
iteration 0 / 1500: loss 11.253733
iteration 100 / 1500: loss inf
iteration 200 / 1500: loss inf
iteration 300 / 1500: loss inf
iteration 400 / 1500: loss inf
iteration 500 / 1500: loss inf
iteration 600 / 1500: loss inf
iteration 700 / 1500: loss inf
iteration 800 / 1500: loss inf
iteration 900 / 1500: loss inf
iteration 1000 / 1500: loss inf
iteration 1100 / 1500: loss inf
iteration 1200 / 1500: loss inf
iteration 1300 / 1500: loss inf
iteration 1400 / 1500: loss inf
iteration 0 / 1500: loss 11.130422
iteration 100 / 1500: loss inf
iteration 200 / 1500: loss inf
iteration 300 / 1500: loss inf
iteration 400 / 1500: loss inf
iteration 500 / 1500: loss inf
iteration 600 / 1500: loss inf
iteration 700 / 1500: loss inf
iteration 800 / 1500: loss inf
iteration 900 / 1500: loss inf
iteration 1000 / 1500: loss inf
iteration 1100 / 1500: loss inf
iteration 1200 / 1500: loss inf
iteration 1300 / 1500: loss inf
```

```
iteration 1400 / 1500: loss inf
iteration 0 / 1500: loss 11.434804
iteration 100 / 1500: loss inf
iteration 200 / 1500: loss inf
iteration 300 / 1500: loss inf
iteration 400 / 1500: loss inf
iteration 500 / 1500: loss inf
iteration 600 / 1500: loss inf
iteration 700 / 1500: loss inf
iteration 800 / 1500: loss inf
iteration 900 / 1500: loss inf
iteration 1000 / 1500: loss inf
iteration 1100 / 1500: loss inf
iteration 1200 / 1500: loss inf
iteration 1300 / 1500: loss inf
iteration 1400 / 1500: loss inf
iteration 0 / 1500: loss 10.616550
iteration 100 / 1500: loss inf
iteration 200 / 1500: loss inf
iteration 300 / 1500: loss inf
iteration 400 / 1500: loss inf
iteration 500 / 1500: loss inf
iteration 600 / 1500: loss inf
iteration 700 / 1500: loss inf
iteration 800 / 1500: loss inf
iteration 900 / 1500: loss inf
iteration 1000 / 1500: loss inf
iteration 1100 / 1500: loss inf
iteration 1200 / 1500: loss inf
iteration 1300 / 1500: loss inf
iteration 1400 / 1500: loss inf
iteration 0 / 1500: loss 11.057112
iteration 100 / 1500: loss inf
iteration 200 / 1500: loss inf
iteration 300 / 1500: loss inf
iteration 400 / 1500: loss inf
iteration 500 / 1500: loss inf
iteration 600 / 1500: loss inf
iteration 700 / 1500: loss inf
iteration 800 / 1500: loss inf
iteration 900 / 1500: loss inf
iteration 1000 / 1500: loss inf
iteration 1100 / 1500: loss inf
iteration 1200 / 1500: loss inf
iteration 1300 / 1500: loss inf
iteration 1400 / 1500: loss inf
iteration 0 / 1500: loss 10.969724
iteration 100 / 1500: loss 100.659225
```

```
iteration 200 / 1500: loss inf
iteration 300 / 1500: loss inf
iteration 400 / 1500: loss inf
iteration 500 / 1500: loss inf
iteration 600 / 1500: loss inf
iteration 700 / 1500: loss inf
iteration 800 / 1500: loss inf
iteration 900 / 1500: loss inf
iteration 1000 / 1500: loss inf
iteration 1100 / 1500: loss inf
iteration 1200 / 1500: loss inf
iteration 1300 / 1500: loss inf
iteration 1400 / 1500: loss inf
iteration 0 / 1500: loss 10.538474
iteration 100 / 1500: loss inf
iteration 200 / 1500: loss inf
iteration 300 / 1500: loss inf
iteration 400 / 1500: loss inf
iteration 500 / 1500: loss inf
iteration 600 / 1500: loss inf
iteration 700 / 1500: loss inf
iteration 800 / 1500: loss inf
iteration 900 / 1500: loss inf
iteration 1000 / 1500: loss inf
iteration 1100 / 1500: loss inf
iteration 1200 / 1500: loss inf
iteration 1300 / 1500: loss inf
iteration 1400 / 1500: loss inf
iteration 0 / 1500: loss 11.880431
iteration 100 / 1500: loss inf
iteration 200 / 1500: loss inf
iteration 300 / 1500: loss inf
iteration 400 / 1500: loss inf
iteration 500 / 1500: loss inf
iteration 600 / 1500: loss inf
iteration 700 / 1500: loss inf
iteration 800 / 1500: loss inf
iteration 900 / 1500: loss inf
iteration 1000 / 1500: loss inf
iteration 1100 / 1500: loss inf
iteration 1200 / 1500: loss inf
iteration 1300 / 1500: loss inf
iteration 1400 / 1500: loss inf
iteration 0 / 1500: loss 11.461731
iteration 100 / 1500: loss 17.508315
iteration 200 / 1500: loss 31.471587
iteration 300 / 1500: loss 40.589631
iteration 400 / 1500: loss 49.922190
```

```
iteration 500 / 1500: loss 73.070554
iteration 600 / 1500: loss 74.045998
iteration 700 / 1500: loss inf
iteration 800 / 1500: loss 81.753876
iteration 900 / 1500: loss inf
iteration 1000 / 1500: loss 119.025187
iteration 1100 / 1500: loss inf
iteration 1200 / 1500: loss inf
iteration 1300 / 1500: loss inf
iteration 1400 / 1500: loss inf
iteration 0 / 1500: loss 10.812399
iteration 100 / 1500: loss 18.555991
iteration 200 / 1500: loss 26.984466
iteration 300 / 1500: loss 34.214327
iteration 400 / 1500: loss 42.535130
iteration 500 / 1500: loss 65.451920
iteration 600 / 1500: loss 63.369286
iteration 700 / 1500: loss inf
iteration 800 / 1500: loss 82.947996
iteration 900 / 1500: loss 102.974315
iteration 1000 / 1500: loss inf
iteration 1100 / 1500: loss inf
iteration 1200 / 1500: loss inf
iteration 1300 / 1500: loss inf
iteration 1400 / 1500: loss inf
iteration 0 / 1500: loss 10.637520
iteration 100 / 1500: loss 17.337011
iteration 200 / 1500: loss 25.298642
iteration 300 / 1500: loss 38.467463
iteration 400 / 1500: loss 54.939574
iteration 500 / 1500: loss 60.414140
iteration 600 / 1500: loss 72.455802
iteration 700 / 1500: loss inf
iteration 800 / 1500: loss 90.164946
iteration 900 / 1500: loss 100.187124
iteration 1000 / 1500: loss inf
iteration 1100 / 1500: loss inf
iteration 1200 / 1500: loss inf
iteration 1300 / 1500: loss inf
iteration 1400 / 1500: loss inf
iteration 0 / 1500: loss 10.639573
iteration 100 / 1500: loss 17.354602
iteration 200 / 1500: loss 27.982728
iteration 300 / 1500: loss 34.465859
iteration 400 / 1500: loss 53.257845
iteration 500 / 1500: loss 57.105250
iteration 600 / 1500: loss 70.661566
iteration 700 / 1500: loss 85.868442
```

```
iteration 800 / 1500: loss inf
iteration 900 / 1500: loss 91.792896
iteration 1000 / 1500: loss inf
iteration 1100 / 1500: loss inf
iteration 1200 / 1500: loss inf
iteration 1300 / 1500: loss inf
iteration 1400 / 1500: loss inf
iteration 0 / 1500: loss 10.599205
iteration 100 / 1500: loss 16.179100
iteration 200 / 1500: loss 25.230719
iteration 300 / 1500: loss 37.032863
iteration 400 / 1500: loss 51.087448
iteration 500 / 1500: loss 55.806179
iteration 600 / 1500: loss 84.189182
iteration 700 / 1500: loss inf
iteration 800 / 1500: loss 98.865651
iteration 900 / 1500: loss inf
iteration 1000 / 1500: loss inf
iteration 1100 / 1500: loss inf
iteration 1200 / 1500: loss inf
iteration 1300 / 1500: loss inf
iteration 1400 / 1500: loss inf
iteration 0 / 1500: loss 11.035841
iteration 100 / 1500: loss 14.950523
iteration 200 / 1500: loss 21.025990
iteration 300 / 1500: loss 26.354624
iteration 400 / 1500: loss 34.132120
iteration 500 / 1500: loss 38.264403
iteration 600 / 1500: loss 43.352790
iteration 700 / 1500: loss 46.031743
iteration 800 / 1500: loss 51.374813
iteration 900 / 1500: loss 50.505969
iteration 1000 / 1500: loss 61.946527
iteration 1100 / 1500: loss 72.432727
iteration 1200 / 1500: loss 81.072738
iteration 1300 / 1500: loss 90.872571
iteration 1400 / 1500: loss 71.133581
iteration 0 / 1500: loss 10.704950
iteration 100 / 1500: loss 13.705619
iteration 200 / 1500: loss 17.180218
iteration 300 / 1500: loss 22.420049
iteration 400 / 1500: loss 28.359984
iteration 500 / 1500: loss 34.363021
iteration 600 / 1500: loss 40.114517
iteration 700 / 1500: loss 44.531037
iteration 800 / 1500: loss 52.417248
iteration 900 / 1500: loss 57.963793
iteration 1000 / 1500: loss 62.803546
```

```
iteration 1100 / 1500: loss 72.991386
iteration 1200 / 1500: loss 79.633485
iteration 1300 / 1500: loss 80.579326
iteration 1400 / 1500: loss 93.773303
iteration 0 / 1500: loss 10.588889
iteration 100 / 1500: loss 14.113477
iteration 200 / 1500: loss 16.690732
iteration 300 / 1500: loss 20.885429
iteration 400 / 1500: loss 27.943624
iteration 500 / 1500: loss 35.677015
iteration 600 / 1500: loss 41.977149
iteration 700 / 1500: loss 43.387496
iteration 800 / 1500: loss 56.239834
iteration 900 / 1500: loss 53.695806
iteration 1000 / 1500: loss 65.048353
iteration 1100 / 1500: loss 56.544349
iteration 1200 / 1500: loss 74.650788
iteration 1300 / 1500: loss 67.251700
iteration 1400 / 1500: loss 88.379485
iteration 0 / 1500: loss 9.969727
iteration 100 / 1500: loss 12.915702
iteration 200 / 1500: loss 16.621113
iteration 300 / 1500: loss 20.708916
iteration 400 / 1500: loss 25.566202
iteration 500 / 1500: loss 33.287885
iteration 600 / 1500: loss 36.368469
iteration 700 / 1500: loss 40.637098
iteration 800 / 1500: loss 49.242602
iteration 900 / 1500: loss 50.145033
iteration 1000 / 1500: loss 51.928346
iteration 1100 / 1500: loss 80.383780
iteration 1200 / 1500: loss 72.144918
iteration 1300 / 1500: loss 76.597436
iteration 1400 / 1500: loss 69.619972
iteration 0 / 1500: loss 10.484946
iteration 100 / 1500: loss 12.304672
iteration 200 / 1500: loss 19.542368
iteration 300 / 1500: loss 20.984010
iteration 400 / 1500: loss 29.869059
iteration 500 / 1500: loss 34.094379
iteration 600 / 1500: loss 36.014864
iteration 700 / 1500: loss 46.991440
iteration 800 / 1500: loss 49.816525
iteration 900 / 1500: loss 51.478987
iteration 1000 / 1500: loss 62.890280
iteration 1100 / 1500: loss 66.512474
iteration 1200 / 1500: loss 71.642227
iteration 1300 / 1500: loss 69.214154
```

```
iteration 1400 / 1500: loss inf
best cross-validation accuracy: 0.27
```

```
In [34]: # evaluate on test set
         # Evaluate the best softmax on test set
         y_test_pred = best_softmax.predict(X_test)
         test_accuracy = np.mean(y_test == y_test_pred)
         print('softmax on raw pixels final test set accuracy: %.2f' % (100*test_accuracy, ))
```

```
softmax on raw pixels final test set accuracy: 26.40
```