3 - Convolutional Networks

March 21, 2019

1 Convolutional Networks

So far we have worked with deep fully-connected networks, using them to explore different optimization strategies and network architectures. Fully-connected networks are a good testbed for experimentation because they are very computationally efficient, but in practice all state-of-the-art results use convolutional networks instead.

First you will implement several layer types that are used in convolutional networks. You will then use these layers to train a convolutional network on the CIFAR-10 dataset.

Acknowledgement: This exercise is adapted from Stanford CS231n.

```
In [1]: # As usual, a bit of setup
        import numpy as np
        import matplotlib.pyplot as plt
        from libs.classifiers.cnn import *
        from libs.data_utils import get_CIFAR10_data
        from libs.gradient_check import eval_numerical_gradient_array, eval_numerical_gradient
        from libs.layers import *
        from libs.fast_layers import *
        from libs.solver import Solver
        %matplotlib inline
        plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
        plt.rcParams['image.interpolation'] = 'nearest'
        plt.rcParams['image.cmap'] = 'gray'
        # for auto-reloading external modules
        \# see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-ipython
        %load ext autoreload
        %autoreload 2
        def rel_error(x, y):
            """ returns relative error """
            return np.max(np.abs(x - y) / (np.maximum(1e-8, np.abs(x) + np.abs(y))))
In [2]: # Load the (preprocessed) CIFAR10 data.
        data = get_CIFAR10_data()
```

2 Convolution: Naive forward pass

The core of a convolutional network is the convolution operation. In the file libs/layers.py, implement the forward pass for the convolution layer in the function conv_forward_naive.

You don't have to worry too much about efficiency at this point; just write the code in whatever way you find most clear.

You can test your implementation by running the following:

```
In [10]: x_shape = (2, 3, 4, 4)
         w \text{ shape} = (3, 3, 4, 4)
         x = np.linspace(-0.1, 0.5, num=np.prod(x_shape)).reshape(x_shape)
         w = np.linspace(-0.2, 0.3, num=np.prod(w_shape)).reshape(w_shape)
         b = np.linspace(-0.1, 0.2, num=3)
         conv_param = {'stride': 2, 'pad': 1}
         out, _ = conv_forward_naive(x, w, b, conv_param)
         correct_out = np.array([[[[[-0.08759809, -0.10987781],
                                    [-0.18387192, -0.2109216]],
                                   [[ 0.21027089, 0.21661097],
                                    [ 0.22847626, 0.23004637]],
                                   [[ 0.50813986, 0.54309974],
                                    [ 0.64082444, 0.67101435]]],
                                  [[[-0.98053589, -1.03143541],
                                    [-1.19128892, -1.24695841]],
                                   [[ 0.69108355, 0.66880383],
                                    [ 0.59480972, 0.56776003]],
                                   [[ 2.36270298, 2.36904306],
                                    [ 2.38090835, 2.38247847]]]])
         # Compare your output to ours; difference should be around 1e-8
         print('Testing conv_forward_naive')
         print('difference: ', rel_error(out, correct_out))
Testing conv_forward_naive
difference: 2.2121476417505994e-08
```

3 Aside: Image processing via convolutions

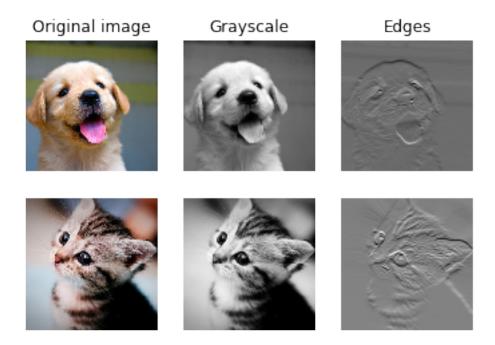
As fun way to both check your implementation and gain a better understanding of the type of operation that convolutional layers can perform, we will set up an input containing two images and manually set up filters that perform common image processing operations (grayscale conversion and edge detection). The convolution forward pass will apply these operations to each of the input images. We can then visualize the results as a sanity check.

```
In [9]: from scipy.misc import imread, imresize
       kitten, puppy = imread('kitten.jpg'), imread('puppy.jpg')
        # kitten is wide, and puppy is already square
       d = kitten.shape[1] - kitten.shape[0]
        kitten_cropped = kitten[:, int(d/2):int(-d/2), :]
        img_size = 200  # Make this smaller if it runs too slow
       x = np.zeros((2, 3, img_size, img_size))
        x[0, :, :, :] = imresize(puppy, (img_size, img_size)).transpose((2, 0, 1))
        x[1, :, :, :] = imresize(kitten_cropped, (img_size, img_size)).transpose((2, 0, 1))
        # Set up a convolutional weights holding 2 filters, each 3x3
        w = np.zeros((2, 3, 3, 3))
        # The first filter converts the image to grayscale.
        # Set up the red, green, and blue channels of the filter.
        w[0, 0, :, :] = [[0, 0, 0], [0, 0.3, 0], [0, 0, 0]]
        w[0, 1, :, :] = [[0, 0, 0], [0, 0.6, 0], [0, 0, 0]]
        w[0, 2, :, :] = [[0, 0, 0], [0, 0.1, 0], [0, 0, 0]]
        # Second filter detects horizontal edges in the blue channel.
        w[1, 2, :, :] = [[1, 2, 1], [0, 0, 0], [-1, -2, -1]]
        # Vector of biases. We don't need any bias for the grayscale
        # filter, but for the edge detection filter we want to add 128
        # to each output so that nothing is negative.
        b = np.array([0, 128])
        # Compute the result of convolving each input in x with each filter in w,
        # offsetting by b, and storing the results in out.
        out, _ = conv_forward_naive(x, w, b, {'stride': 1, 'pad': 1})
        def imshow_noax(img, normalize=True):
            """ Tiny helper to show images as uint8 and remove axis labels """
            if normalize:
                img_max, img_min = np.max(img), np.min(img)
                img = 255.0 * (img - img_min) / (img_max - img_min)
           plt.imshow(img.astype('uint8'))
```

plt.gca().axis('off')

```
plt.subplot(2, 3, 1)
        imshow_noax(puppy, normalize=False)
        plt.title('Original image')
        plt.subplot(2, 3, 2)
        imshow_noax(out[0, 0])
        plt.title('Grayscale')
        plt.subplot(2, 3, 3)
        imshow_noax(out[0, 1])
        plt.title('Edges')
        plt.subplot(2, 3, 4)
        imshow_noax(kitten_cropped, normalize=False)
        plt.subplot(2, 3, 5)
        imshow_noax(out[1, 0])
        plt.subplot(2, 3, 6)
        imshow_noax(out[1, 1])
        plt.show()
/Users/lixingxuan/anaconda3/envs/tesnorflow/lib/python3.6/site-packages/ipykernel_launcher.py:
'imread' is deprecated in SciPy 1.0.0, and will be removed in 1.2.0.
Use ``imageio.imread`` instead.
  This is separate from the ipykernel package so we can avoid doing imports until
/Users/lixingxuan/anaconda3/envs/tesnorflow/lib/python3.6/site-packages/ipykernel_launcher.py:
`imresize` is deprecated in SciPy 1.0.0, and will be removed in 1.2.0.
Use ``skimage.transform.resize`` instead.
  # Remove the CWD from sys.path while we load stuff.
/Users/lixingxuan/anaconda3/envs/tesnorflow/lib/python3.6/site-packages/ipykernel_launcher.py:
`imresize` is deprecated in SciPy 1.0.0, and will be removed in 1.2.0.
Use ``skimage.transform.resize`` instead.
  # This is added back by InteractiveShellApp.init_path()
200.0
200.0
```

Show the original images and the results of the conv operation



4 Convolution: Naive backward pass

Implement the backward pass for the convolution operation in the function conv_backward_naive in the file libs/layers.py. Again, you don't need to worry too much about computational efficiency.

When you are done, run the following to check your backward pass with a numeric gradient check.

```
In [15]: x = np.random.randn(4, 3, 5, 5)
    w = np.random.randn(2, 3, 3, 3)
    b = np.random.randn(2,)
    dout = np.random.randn(4, 2, 5, 5)
    conv_param = {'stride': 1, 'pad': 1}

    dx_num = eval_numerical_gradient_array(lambda x: conv_forward_naive(x, w, b, conv_paradw_num = eval_numerical_gradient_array(lambda w: conv_forward_naive(x, w, b, conv_paradb_num = eval_numerical_gradient_array(lambda b: conv_forward_naive(x, w, b, conv_paradb_num = eval_numerical_gradient_array(lambda b: conv_forward_naive(x, w, b, conv_paradb_numerical_gradient_array(lambda b: conv_forward_naive(x, w, b, conv_param)
    # print(w)
    dx, dw, db = conv_backward_naive(dout, cache)
```

Your errors should be around 1e-9'

print('Testing conv_backward_naive function')

```
print('db error: ', rel_error(db, db_num))
    print('dw error: ', rel_error(dw, dw_num))
    print('dx error: ', rel_error(dx, dx_num))

Testing conv_backward_naive function
db error: 1.2930519005346312e-11
dw error: 8.988567711821914e-10
dx error: 2.8224078751305698e-09
```

5 Max pooling: Naive forward

Implement the forward pass for the max-pooling operation in the function max_pool_forward_naive in the file libs/layers.py. Again, don't worry too much about computational efficiency.

Check your implementation by running the following:

```
In [21]: x_{shape} = (2, 3, 4, 4)
         x = np.linspace(-0.3, 0.4, num=np.prod(x shape)).reshape(x shape)
         pool_param = {'pool_width': 2, 'pool_height': 2, 'stride': 2}
         out, _ = max_pool_forward_naive(x, pool_param)
         correct_out = np.array([[[[-0.26315789, -0.24842105],
                                   [-0.20421053, -0.18947368]],
                                  [[-0.14526316, -0.13052632],
                                   [-0.08631579, -0.07157895]],
                                  [[-0.02736842, -0.01263158],
                                   [ 0.03157895, 0.04631579]]],
                                 [[[ 0.09052632, 0.10526316],
                                   [ 0.14947368, 0.16421053]],
                                  [[ 0.20842105, 0.22315789],
                                   [ 0.26736842, 0.28210526]],
                                  [[ 0.32631579, 0.34105263],
                                   [ 0.38526316, 0.4
         # Compare your output with ours. Difference should be around 1e-8.
         print('Testing max_pool_forward_naive function:')
         print('difference: ', rel_error(out, correct_out))
Testing max_pool_forward_naive function:
difference: 4.1666665157267834e-08
```

6 Max pooling: Naive backward

Implement the backward pass for the max-pooling operation in the function max_pool_backward_naive in the file libs/layers.py. You don't need to worry about

computational efficiency.

Check your implementation with numeric gradient checking by running the following:

7 Fast layers

Making convolution and pooling layers fast can be challenging. To spare you the pain, we've provided fast implementations of the forward and backward passes for convolution and pooling layers in the file libs/fast_layers.py.

The fast convolution implementation depends on a Cython extension; to compile it you need to run the following from the libs directory:

```
python setup.py build_ext --inplace
```

The API for the fast versions of the convolution and pooling layers is exactly the same as the naive versions that you implemented above: the forward pass receives data, weights, and parameters and produces outputs and a cache object; the backward pass recieves upstream derivatives and the cache object and produces gradients with respect to the data and weights.

NOTE: The fast implementation for pooling will only perform optimally if the pooling regions are non-overlapping and tile the input. If these conditions are not met then the fast pooling implementation will not be much faster than the naive implementation.

You can compare the performance of the naive and fast versions of these layers by running the following:

```
out_naive, cache_naive = conv_forward_naive(x, w, b, conv_param)
         t1 = time()
         out_fast, cache_fast = conv_forward_fast(x, w, b, conv_param)
         t2 = time()
         print('Testing conv_forward_fast:')
         print('Naive: %fs' % (t1 - t0))
         print('Fast: %fs' % (t2 - t1))
         print('Speedup: %fx' % ((t1 - t0) / (t2 - t1)))
         print('Difference: ', rel_error(out_naive, out_fast))
         t0 = time()
         dx_naive, dw_naive, db_naive = conv_backward_naive(dout, cache_naive)
         t1 = time()
         dx_fast, dw_fast, db_fast = conv_backward_fast(dout, cache_fast)
         t2 = time()
         print('\nTesting conv backward fast:')
         print('Naive: %fs' % (t1 - t0))
         print('Fast: %fs' % (t2 - t1))
         print('Speedup: %fx' % ((t1 - t0) / (t2 - t1)))
         print('dx difference: ', rel_error(dx_naive, dx_fast))
         print('dw difference: ', rel_error(dw_naive, dw_fast))
         print('db difference: ', rel_error(db_naive, db_fast))
Testing conv_forward_fast:
Naive: 3.675879s
Fast: 0.017478s
Speedup: 210.314741x
Difference: 1.5306441445248822e-11
Testing conv_backward_fast:
Naive: 6.287788s
Fast: 0.011723s
Speedup: 536.361501x
dx difference: 6.306264227232483e-10
dw difference: 1.8223529010104583e-12
db difference: 2.0457607498020912e-14
In [24]: from libs.fast_layers import max_pool_forward_fast, max_pool_backward_fast
         x = np.random.randn(100, 3, 32, 32)
         dout = np.random.randn(100, 3, 16, 16)
         pool_param = {'pool_height': 2, 'pool_width': 2, 'stride': 2}
```

t0 = time()

```
t0 = time()
         out_naive, cache_naive = max_pool_forward_naive(x, pool_param)
         t1 = time()
         out_fast, cache_fast = max_pool_forward_fast(x, pool_param)
         t2 = time()
         print('Testing pool forward fast:')
         print('Naive: %fs' % (t1 - t0))
         print('fast: %fs' % (t2 - t1))
         print('speedup: %fx' % ((t1 - t0) / (t2 - t1)))
         print('difference: ', rel_error(out_naive, out_fast))
         t0 = time()
         dx_naive = max_pool_backward_naive(dout, cache_naive)
         t1 = time()
         dx_fast = max_pool_backward_fast(dout, cache_fast)
         t2 = time()
         print('\nTesting pool_backward_fast:')
         print('Naive: %fs' % (t1 - t0))
         print('speedup: %fx' % ((t1 - t0) / (t2 - t1)))
         print('dx difference: ', rel_error(dx_naive, dx_fast))
Testing pool_forward_fast:
Naive: 0.335016s
fast: 0.002052s
speedup: 163.257581x
difference: 0.0
Testing pool backward fast:
Naive: 0.546594s
speedup: 49.332544x
dx difference: 0.0
```

8 Convolutional "sandwich" layers

Previously we introduced the concept of "sandwich" layers that combine multiple operations into commonly used patterns. In the file libs/layer_utils.py you will find sandwich layers that implement a few commonly used patterns for convolutional networks.

```
In [25]: from libs.layer_utils import conv_relu_pool_forward, conv_relu_pool_backward
    x = np.random.randn(2, 3, 16, 16)
    w = np.random.randn(3, 3, 3, 3)
    b = np.random.randn(3,)
    dout = np.random.randn(2, 3, 8, 8)
    conv_param = {'stride': 1, 'pad': 1}
```

```
pool_param = {'pool_height': 2, 'pool_width': 2, 'stride': 2}
         out, cache = conv_relu_pool_forward(x, w, b, conv_param, pool_param)
         dx, dw, db = conv_relu_pool_backward(dout, cache)
         dx_num = eval_numerical_gradient_array(lambda x: conv_relu_pool_forward(x, w, b, conv
         dw_num = eval_numerical_gradient_array(lambda w: conv_relu_pool_forward(x, w, b, conv
         db_num = eval_numerical_gradient_array(lambda b: conv_relu_pool_forward(x, w, b, conv
         print('Testing conv_relu_pool')
         print('dx error: ', rel_error(dx_num, dx))
         print('dw error: ', rel_error(dw_num, dw))
         print('db error: ', rel_error(db_num, db))
Testing conv_relu_pool
dx error: 3.690711055362238e-08
dw error: 1.360072785871424e-09
db error: 4.906317816753218e-11
In [26]: from libs.layer_utils import conv_relu_forward, conv_relu_backward
         x = np.random.randn(2, 3, 8, 8)
         w = np.random.randn(3, 3, 3, 3)
         b = np.random.randn(3,)
         dout = np.random.randn(2, 3, 8, 8)
         conv_param = {'stride': 1, 'pad': 1}
         out, cache = conv_relu_forward(x, w, b, conv_param)
         dx, dw, db = conv_relu_backward(dout, cache)
         dx_num = eval_numerical_gradient_array(lambda x: conv_relu_forward(x, w, b, conv_para)
         dw_num = eval_numerical_gradient_array(lambda w: conv_relu_forward(x, w, b, conv_para
         db_num = eval_numerical_gradient_array(lambda b: conv_relu_forward(x, w, b, conv_para)
         print('Testing conv_relu:')
         print('dx error: ', rel_error(dx_num, dx))
         print('dw error: ', rel_error(dw_num, dw))
         print('db error: ', rel_error(db_num, db))
Testing conv relu:
dx error: 6.4190515230260995e-09
dw error: 3.273630429609165e-09
db error: 8.311242203847743e-12
```

9 Three-layer ConvNet

Now that you have implemented all the necessary layers, we can put them together into a simple convolutional network.

Open the file libs/cnn.py and complete the implementation of the ThreeLayerConvNet class. Run the following cells to help you debug:

9.1 Sanity check loss

After you build a new network, one of the first things you should do is sanity check the loss. When we use the softmax loss, we expect the loss for random weights (and no regularization) to be about log(C) for C classes. When we add regularization this should go up.

```
In [27]: model = ThreeLayerConvNet()

N = 50
X = np.random.randn(N, 3, 32, 32)
y = np.random.randint(10, size=N)

loss, grads = model.loss(X, y)
print('Initial loss (no regularization): ', loss)
print('Should be near: ', np.log(10))

model.reg = 0.5
loss, grads = model.loss(X, y)
print('Initial loss (with regularization): ', loss)

Initial loss (no regularization): 2.3025848832323073
Should be near: 2.302585092994046
Initial loss (with regularization): 2.5091707236708
```

9.2 Gradient check

After the loss looks reasonable, use numeric gradient checking to make sure that your backward pass is correct. When you use numeric gradient checking you should use a small amount of artifical data and a small number of neurons at each layer.

```
for param_name in sorted(grads):
    f = lambda _: model.loss(X, y)[0]
    param_grad_num = eval_numerical_gradient(f, model.params[param_name], verbose=False = rel_error(param_grad_num, grads[param_name])
    print('%s max relative error: %e' % (param_name, rel_error(param_grad_num, grads[ymax relative error: 4.994076e-02
W2 max relative error: 5.465637e-03
W3 max relative error: 1.780337e-02
b1 max relative error: 5.484030e-06
b2 max relative error: 2.746745e-01
b3 max relative error: 1.530176e-09
```

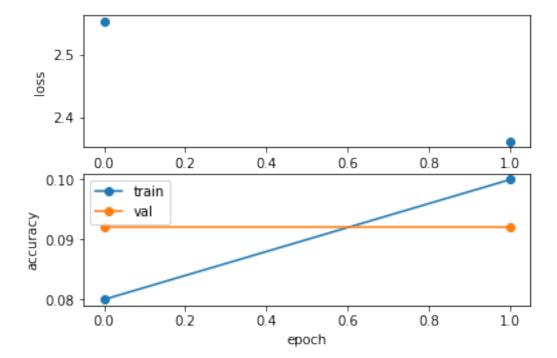
9.3 Overfit small data

A nice trick is to train your model with just a few training samples. You should be able to overfit small datasets, which will result in very high training accuracy and comparatively low validation accuracy.

```
In [33]: num_train = 100
         small_data = {
           'X_train': data['X_train'][:num_train],
           'y_train': data['y_train'][:num_train],
           'X_val': data['X_val'],
           'y_val': data['y_val'],
         }
         model = ThreeLayerConvNet(weight_scale=1e-2)
         solver = Solver(model, small_data,
                         num_epochs=1, batch_size=50,
                         update_rule='sgd',
                         optim_config={
                           'learning_rate': 5e-4,
                         verbose=True, print_every=1)
         solver.train()
(Epoch 0 / 1) (Iteration 1 / 2) loss: 2.553202 train acc: 0.080000 val_acc: 0.092000
(Epoch 1 / 1) (Iteration 2 / 2) loss: 2.362806 train acc: 0.100000 val_acc: 0.092000
```

Plotting the loss, training accuracy, and validation accuracy should show clear overfitting:

```
plt.subplot(2, 1, 2)
plt.plot(solver.train_acc_history, '-o')
plt.plot(solver.val_acc_history, '-o')
plt.legend(['train', 'val'], loc='upper left')
plt.xlabel('epoch')
plt.ylabel('accuracy')
plt.show()
```



9.4 Train the net

By training the three-layer convolutional network for **one epoch**, you should achieve greater than 40% accuracy on the training set:

```
(Epoch 0 / 1) (Iteration 1 / 490) loss: 2.304645 train acc: 0.107000 val_acc: 0.099000
(Epoch 0 / 1) (Iteration 21 / 490) loss: 2.304508 train acc: 0.107000 val_acc: 0.110000
(Epoch 0 / 1) (Iteration 41 / 490) loss: 2.304348 train acc: 0.107000 val_acc: 0.125000
(Epoch 0 / 1) (Iteration 61 / 490) loss: 2.304379 train acc: 0.107000 val_acc: 0.136000
(Epoch 0 / 1) (Iteration 81 / 490) loss: 2.304535 train acc: 0.107000 val acc: 0.146000
(Epoch 0 / 1) (Iteration 101 / 490) loss: 2.304096 train acc: 0.107000 val_acc: 0.156000
(Epoch 0 / 1) (Iteration 121 / 490) loss: 2.304263 train acc: 0.107000 val acc: 0.172000
(Epoch 0 / 1) (Iteration 141 / 490) loss: 2.304131 train acc: 0.107000 val_acc: 0.185000
(Epoch 0 / 1) (Iteration 161 / 490) loss: 2.303685 train acc: 0.107000 val acc: 0.183000
(Epoch 0 / 1) (Iteration 181 / 490) loss: 2.303669 train acc: 0.107000 val_acc: 0.188000
(Epoch 0 / 1) (Iteration 201 / 490) loss: 2.302984 train acc: 0.107000 val_acc: 0.185000
(Epoch 0 / 1) (Iteration 221 / 490) loss: 2.302657 train acc: 0.107000 val_acc: 0.180000
(Epoch 0 / 1) (Iteration 241 / 490) loss: 2.302858 train acc: 0.107000 val_acc: 0.176000
(Epoch 0 / 1) (Iteration 261 / 490) loss: 2.301963 train acc: 0.107000 val_acc: 0.170000
(Epoch 0 / 1) (Iteration 281 / 490) loss: 2.299265 train acc: 0.107000 val_acc: 0.175000
(Epoch 0 / 1) (Iteration 301 / 490) loss: 2.300273 train acc: 0.107000 val_acc: 0.174000
(Epoch 0 / 1) (Iteration 321 / 490) loss: 2.303351 train acc: 0.107000 val_acc: 0.170000
(Epoch 0 / 1) (Iteration 341 / 490) loss: 2.294920 train acc: 0.107000 val_acc: 0.152000
(Epoch 0 / 1) (Iteration 361 / 490) loss: 2.298207 train acc: 0.107000 val_acc: 0.136000
(Epoch 0 / 1) (Iteration 381 / 490) loss: 2.287176 train acc: 0.107000 val acc: 0.136000
(Epoch 0 / 1) (Iteration 401 / 490) loss: 2.281151 train acc: 0.107000 val_acc: 0.132000
(Epoch 0 / 1) (Iteration 421 / 490) loss: 2.263482 train acc: 0.107000 val acc: 0.124000
(Epoch 0 / 1) (Iteration 441 / 490) loss: 2.246952 train acc: 0.107000 val_acc: 0.118000
(Epoch 0 / 1) (Iteration 461 / 490) loss: 2.270600 train acc: 0.107000 val_acc: 0.131000
(Epoch 0 / 1) (Iteration 481 / 490) loss: 2.214126 train acc: 0.107000 val_acc: 0.157000
```

9.5 Visualize Filters

You can visualize the first-layer convolutional filters from the trained network by running the following:

```
In [32]: from libs.vis_utils import visualize_grid

    grid = visualize_grid(model.params['W1'].transpose(0, 2, 3, 1))
    plt.imshow(grid.astype('uint8'))
    plt.axis('off')
    plt.gcf().set_size_inches(5, 5)
    plt.show()
```

