Date No.

1. Task 1

① $X = (I - A^T) \cdot A^{-1}$

② $X^T C = [2A(X+B)]^T = I$

$X^T C = 2(X+B)^T \cdot A^T = 2X^T \cdot A^T + 2B^T \cdot A^T$

$\therefore X^T (C - 2A^T) = 2B^T \cdot A^T$

$\therefore X^T = 2(A \cdot B)^T \cdot (C - 2A^T)^{-1}$

$\therefore X = [2(A \cdot B)^T \cdot (C - 2A^T)^{-1}]^T$

③ if $(AX - y)^T \cdot A = 0$, $(AX - y)^T$ must equal 0

$\therefore X = A^{-1} y$, A must be invertible.

④ $(AX - y)^T A + X^T B = 0$

$(AX - y)^T = -X^T B \cdot A^{-1}$

$AX - y = -(A^{-1})^T \cdot B^T \cdot X$

$\therefore [A + (A^{-1})^T \cdot B^T] X = y$

$\therefore X = [A + (B \cdot A^{-1})^T]^{-1} \cdot y$

$\therefore$ A and $[A + (B \cdot A^{-1})^T]$ must be invertible.

2. Task 2

$\nabla f(x) \cdot \vec{v} = |\nabla f(x)| \cdot |\vec{v}| \cdot \cos\theta$

$\therefore$ when $\cos(\theta) = 1$, it has the fastest increasement.

In paticular, $\theta = 0$ which means $\vec{v}$ points in the same direction as $\nabla f(x)$