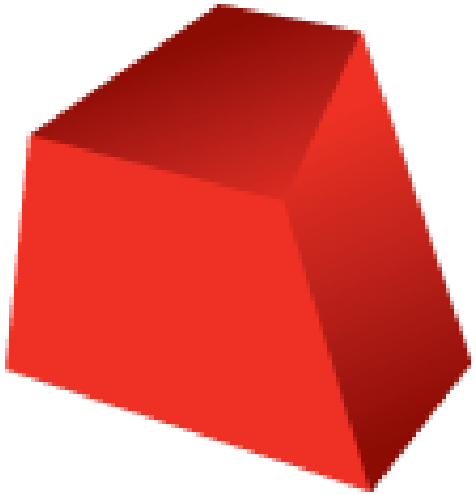
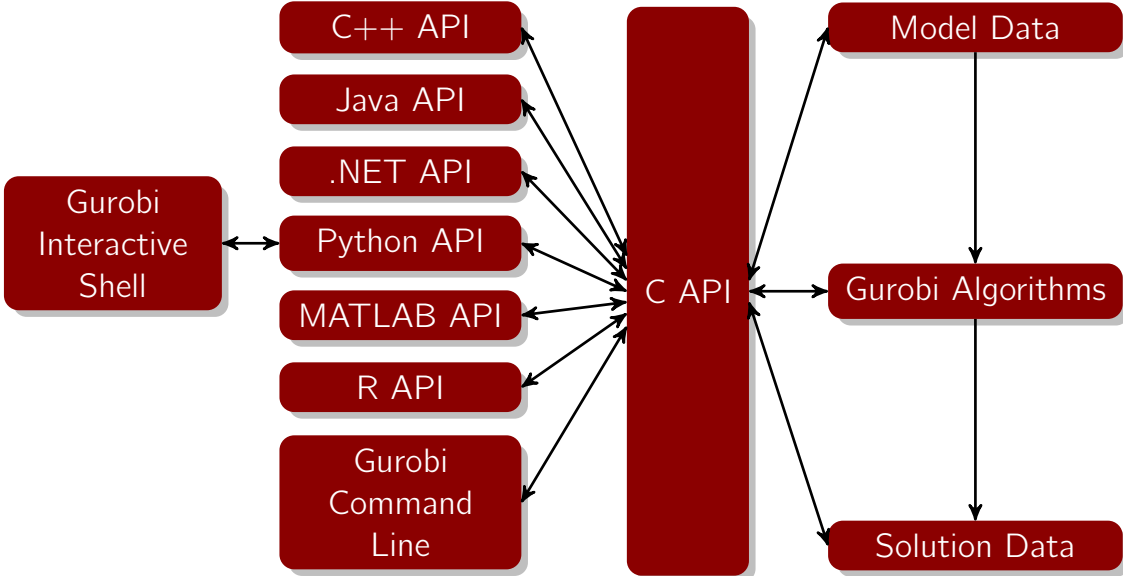


GUROBI

OPTIMIZATION


C++ API
Java API
.NET API
Gurobi Interactive Shell
Python API
MATLAB API
R API
Gurobi Command Line
C API
Model Data
Gurobi Algorithms
Solution Data

$$y = max(x_1, x_2, ..., c)$$

$$y = min(x_1, x_2, ..., c)$$

$$y = |x|$$

$y = x_1 \wedge x_2 \wedge x_3 ...$

$y = x_1 \vee x_2 \vee x_3 ...$

$$y = 1 \rightarrow a'x \leq b$$

$$y = pwl(x)$$

$$y = p_0 x^d + p_1 x^{d-1} + ... + p_{d-1} x + p_d$$

$y = e^x$

$$y = a^x$$

$y = x^a$

$$y = sin(x)$$

$$y = tan(x)$$

$r = \max\{x_1, \ldots, x_n, c\}$

γ

$x_1, \ldots, x_n$

c

n

x_j

$$r = \min\{x_1, \ldots, x_n, c\}$$

$$r = \mathrm{abs}\{x\}$$

x

$r = \mathrm{and}\{x_1, \ldots, x_n\}$

1

$r = \mathrm{or}\{x_1, \ldots, x_n\}$

$$z = f \rightarrow a^T x \leq b$$

z

$f \in \{0, 1\}$

$$a^T x \leq b$$

$z = 1 - f$

a_j

$$y = f(x)$$

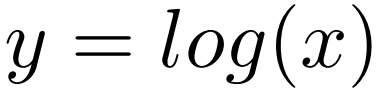
y

x^d

$d+1$

$$y = exp(x)$$

$a > 0$



a

$$x^T Q x + q^T x \leq b$$

$2x_0^2 + x_0x_1 + x_1^2$

$$Bx = b$$

B

b

$$B^T x = b$$

B^T

$$Bx = A_j$$

$B^{-1}A$

B^{-1}

$$\text{model.addConstr}(expr1 <= expr2)$$
$$\text{model.addConstr}(expr1 == 1)$$
$$\text{model.addConstr}(2 * x + 3 * y <= 4)$$

$\text{model.addQConstr}(qexpr1 <= qexpr2)$
$\text{model.addQConstr}(qexpr1 == 1)$
$\text{model.addQConstr}(2 * x * x + 3 * y * y <= 4)$

model.addConstr($expr1, GRB.LESS_EQUAL, expr2$)
model.addConstr($expr1, GRB.EQUAL, 1$)

$\text{model.addQConstr}(qexpr1, GRB.LESS_EQUAL, qexpr2)$
$\text{model.addQConstr}(qexpr1, GRB.EQUAL, 1)$

$\text{model.AddConstr}(expr1 <= expr2)$
$\text{model.AddConstr}(expr1 == 1)$
$\text{model.AddConstr}(2 * x + 3 * y <= 4)$

$$\text{model.AddQConstr}(qexpr1 <= qexpr2)$$
$$\text{model.AddQConstr}(qexpr1 == 1)$$
$$\text{model.AddQConstr}(2 * x * x + 3 * y * y <= 4)$$

$$Ax = b$$

$$x'_{Q_L} Q x_{Q_R} + c' x_c = \text{rhs}$$

$\text{model.addConstr}(qexpr1 <= qexpr2)$
$\text{model.addConstr}(qexpr1 == 1)$
$\text{model.addConstr}(2 * x * x + 3 * y * y <= 4)$

$Ax + b$

$2.0 * x$

$2.0 * var$

$x^T Q x + c^T x + \text{alpha}$

$$\ell \leq x \leq u$$

$$x^T Qc\, x + q^T x \leq \text{beta}$$

x_i

alpha

$$x[\text{resvar}] = \max\{\text{con}, x[j] : j \in \text{vars}\}$$

$$x[\text{resvar}] = \min\{\text{con}, x[j] : j \in \text{vars}\}$$

$$x[\text{resvar}] = |x[\text{argvar}]|$$

$$x[\text{resvar}] = \text{and}\{x[i] : i \in \text{vars}\}$$

$$x[\text{resvar}] = \text{or}\{x[i] : i \in \text{vars}\}$$

$x[\text{binvar}] = \text{binval} \Rightarrow \sum (x(j) \cdot \text{a}(j)) \;\; \text{sense rhs}$

$$\sum (x[\mathrm{vars}(j)] \cdot \mathrm{val}(j)) \;\; \mathrm{sense} \; \mathrm{rhs}$$

$$x[\mathrm{yvar}] = f(x[\mathrm{xvar}])$$

$$x[\text{yvar}] = p_0 x[\text{xvar}]^d + p_1 x[\text{xvar}]^{d-1} + \ldots + p_{d-1} x[\text{xvar}] + p_d$$

$x[\mathrm{yvar}] = \exp(x[\mathrm{xvar}])$

$$x[\mathrm{yvar}] = \mathrm{a}^{x[\mathrm{xvar}]}$$

$$x[\text{yvar}] = \log(x[\text{xvar}])$$

$$x[\mathrm{yvar}] = \log(x[\mathrm{xvar}]) \setminus \log(a)$$

x[yvar] = x[xvar]a

$x[\mathrm{yvar}] = \sin(x[\mathrm{xvar}])$

$$x[\mathrm{yvar}] = \cos(x[\mathrm{xvar}])$$

$x[\mathrm{yvar}] = \tan(x[\mathrm{xvar}])$

$$|ObjBound - ObjVal|/|ObjVal|$$

ObjBound

$ObjVal$

$3x + 4y \leq 5z$

$x > y$

$x - y$

$x <= Mb$

M

$$3x^2 + 4y^2 + 5z \leq 10$$

$$x^T x \leq y^2$$

$$x^T x \leq yz$$

$r = \max\{x_1, \ldots, x_k, c\}$

$x_1, \ldots, x_k$

$(r = 3, x_1 = 2, x_2 = 3, x_3 = 0)$

$$r = \max\{x_1, x_2, x_3, 1.7\}$$

3

2

1.7

$r = \min\{x_1, \ldots, x_k, c\}$

$(r = 3, x = -3)$

$r = \mathrm{and}\{x_1, \ldots, x_k\}$

$(r = 1, x_1 = 1, x_2 = 1, x_3 = 1)$

$r = \mathrm{and}\{x_1, x_2, x_3\}$

$r = \mathrm{or}\{x_1, \ldots, x_k\}$

$$y = f \rightarrow a^T x \leq b$$

$y = 1 - f$

(x, y)

$(x_1, y_1), (x_2, y_2), ..., (x_n, y_n)$

$$
\begin{aligned}
r &= x_j + s_j && \text{for all } j = 1, \ldots, k \\
r &= c + s_{k+1} && \\
z_1 + \ldots + z_{k+1} &= 1 && \\
SOS1(s_j, z_j) & && \text{for all } j = 1, \ldots, k+1 \\
s_j &\geq 0 && \text{for all } j = 1, \ldots, k+1 \\
z_j &\in \{0, 1\} && \text{for all } j = 1, \ldots, k+1
\end{aligned}
$$

$r \geq \max\{x_1, \ldots, x_k, c\}$

$s_j \geq 0$

$r \leq \max\{x_1, \ldots, x_k, c\}$

$z_j \in \{0, 1\}$

$$z_j = 1 \rightarrow s_j = 0$$

$r = x_j$

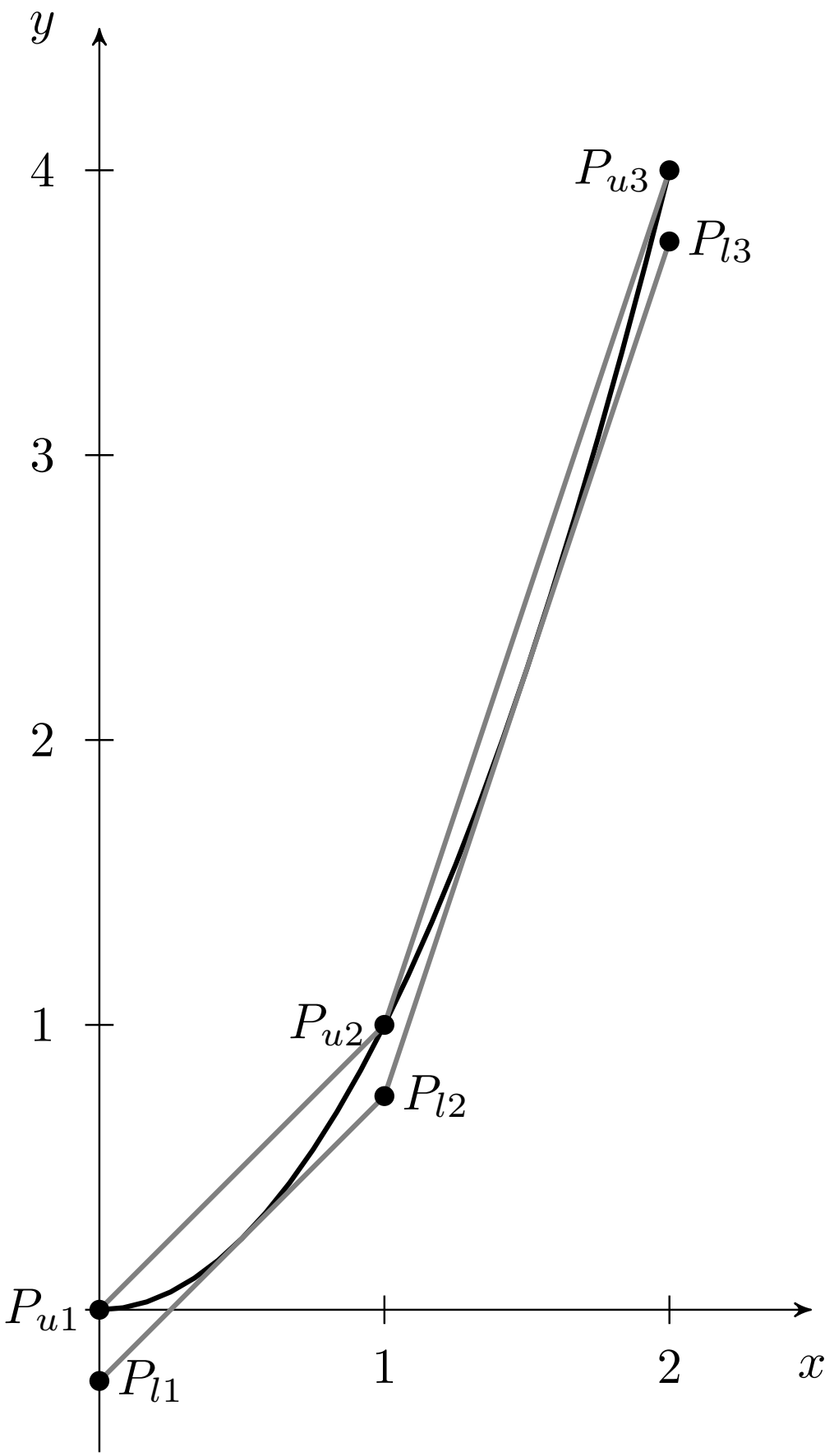
$r = c$

$$
\begin{array}{lcll}
r & \geq & x_j & \text{for all } j = 1, \ldots, k \\
r & \geq & c &
\end{array}
$$

$$y = p_0 x^n + p_1 x^{n-1} + ... + p_n x + p_{n+1}$$

$$y = ln(x)$$

$a >= 0$


y
4
3
2
1
P_{u3}
P_{l3}
P_{u2}
P_{l2}
P_{u1}
P_{l1}
1
2
x

$$y = x^2$$

1.0

$[0, 2]$

$P_{u1}(0,0), P_{u2}(1,1), P_{u3}(2,4)$

$P_{l1}(0, -0.25), P_{l2}(1, 0.75), P_{l3}(2, 3.75)$

P_{l1}, P_{l2}

P_{l3}

P_{u1}, P_{u2}

0.6

P_{ui}

0.4

P_{li}

$(0, -0.1), (1, 0.9)$

$(2, 3.9)$

-1

0.25

$y = 2x - 1$

$(x, y) = (1, 1)$

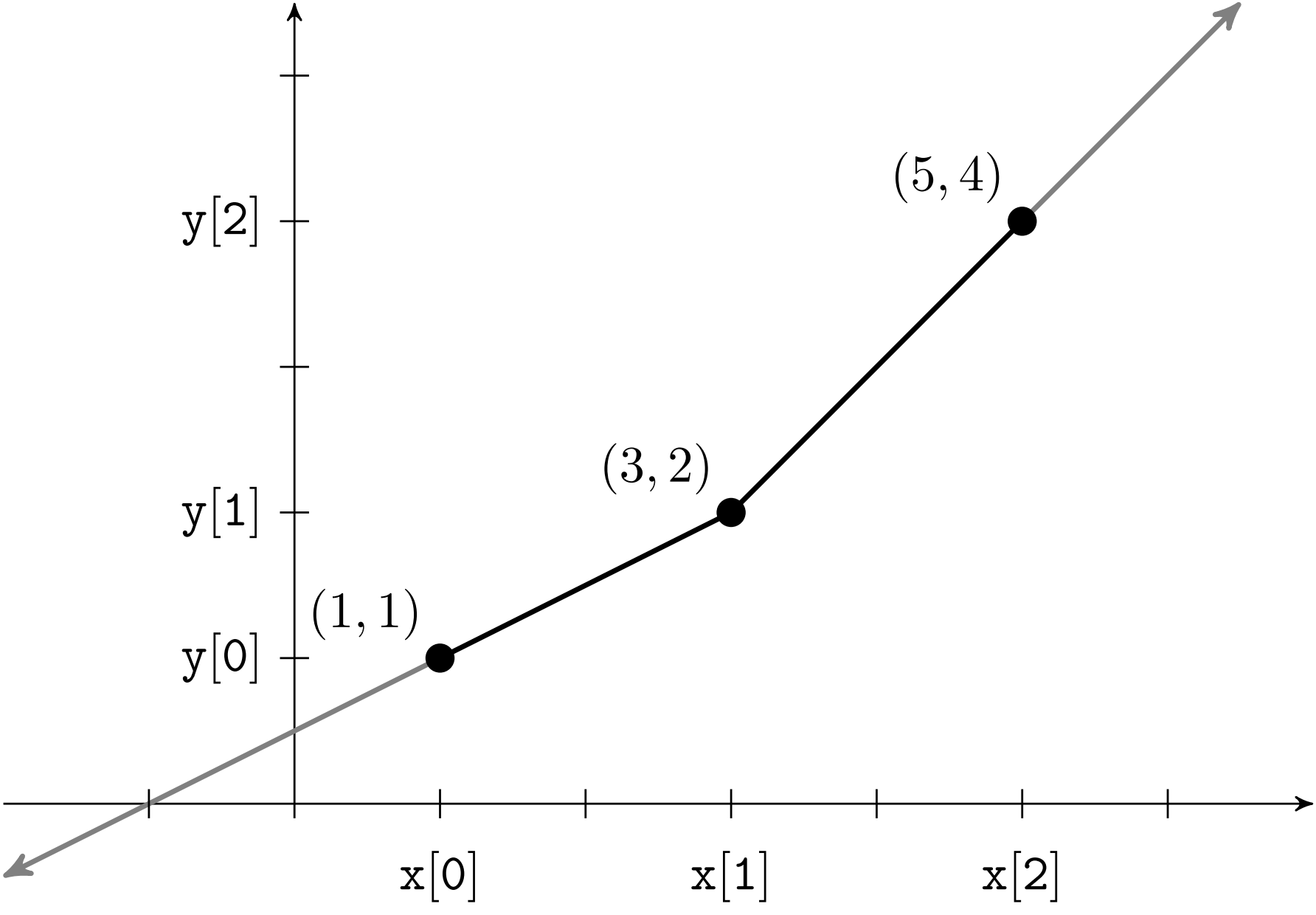
$x = 0.9$

$x = 1.1$

1.01

$x = 1$

$3x + 4y + 2$


(5, 4)
(3, 2)
(1, 1)
y[2]
y[1]
y[0]
x[0]
x[1]
x[2]

$(1, 1)$

$(3, 2)$

$(5, 4)$

$f(1) = 1$

$f(3) = 2$

$f(5) = 4$

$f(-1) = 0$

$f(6) = 5$

$$\mathrm{x} = [x_1, \ldots, x_n], \quad \mathrm{y} = [y_1, \ldots, y_n]$$

$$f(v) = \begin{cases} y_1 + \frac{y_2 - y_1}{x_2 - x_1}(v - x_1), & \text{if } v \leq x_1, \\ y_i + \frac{y_{i+1} - y_i}{x_{i+1} - x_i}(v - x_i), & \text{if } v \geq x_i \text{ and } v \leq x_{i+1}, \\ y_n + \frac{y_n - y_{n-1}}{x_n - x_{n-1}}(v - x_n), & \text{if } v \geq x_n. \end{cases}$$

$x = x_i$

$(x_{i-1}, y_{i-1}), (x_i, y_i), (x_{i+1}, y_{i+1}), (x_{i+2}, y_{i+2})$

$x_i = x_{i+1}$

$y_i \neq y_{i+1}$

$x_{i-1} \leq x < x_i$

$\left(x_{i-1}, y_{i-1}\right)$

(x_i, y_i)

$x_i \leq x < x_{i+2}$

$\left(x_{i+1}, y_{i+1}\right)$

$\left(x_{i+2}, y_{i+2}\right)$

y_i

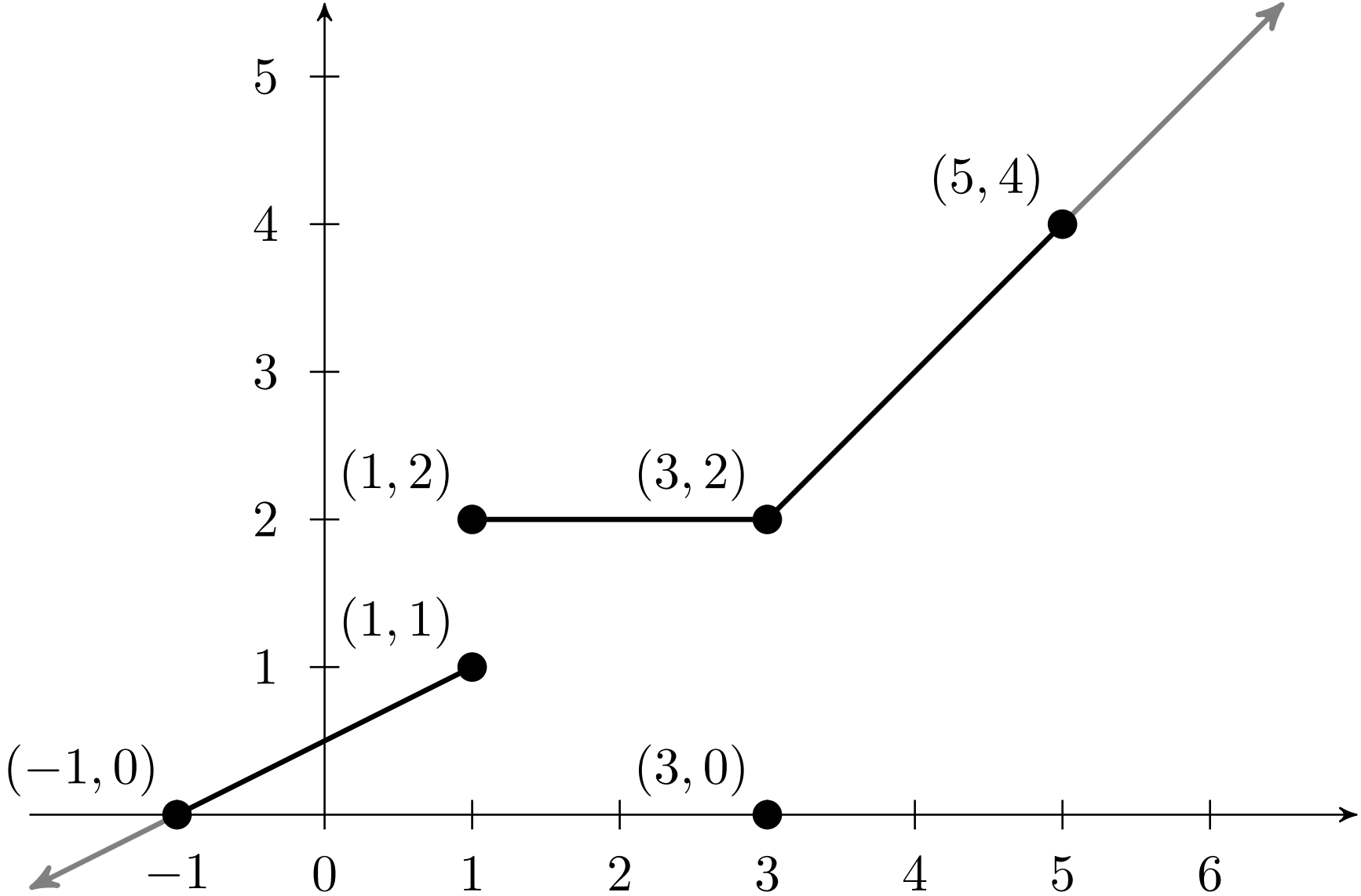
y_i+1

$(x_{i-2}, y_{i-2}), (x_{i-1}, y_{i-1}), (x_i, y_i), (x_{i+1}, y_{i+1}), (x_{i+2}, y_{i+2})$

$$x_{i-1} = x_i = x_{i+1}$$

$y_i \neq y_{i-1}$

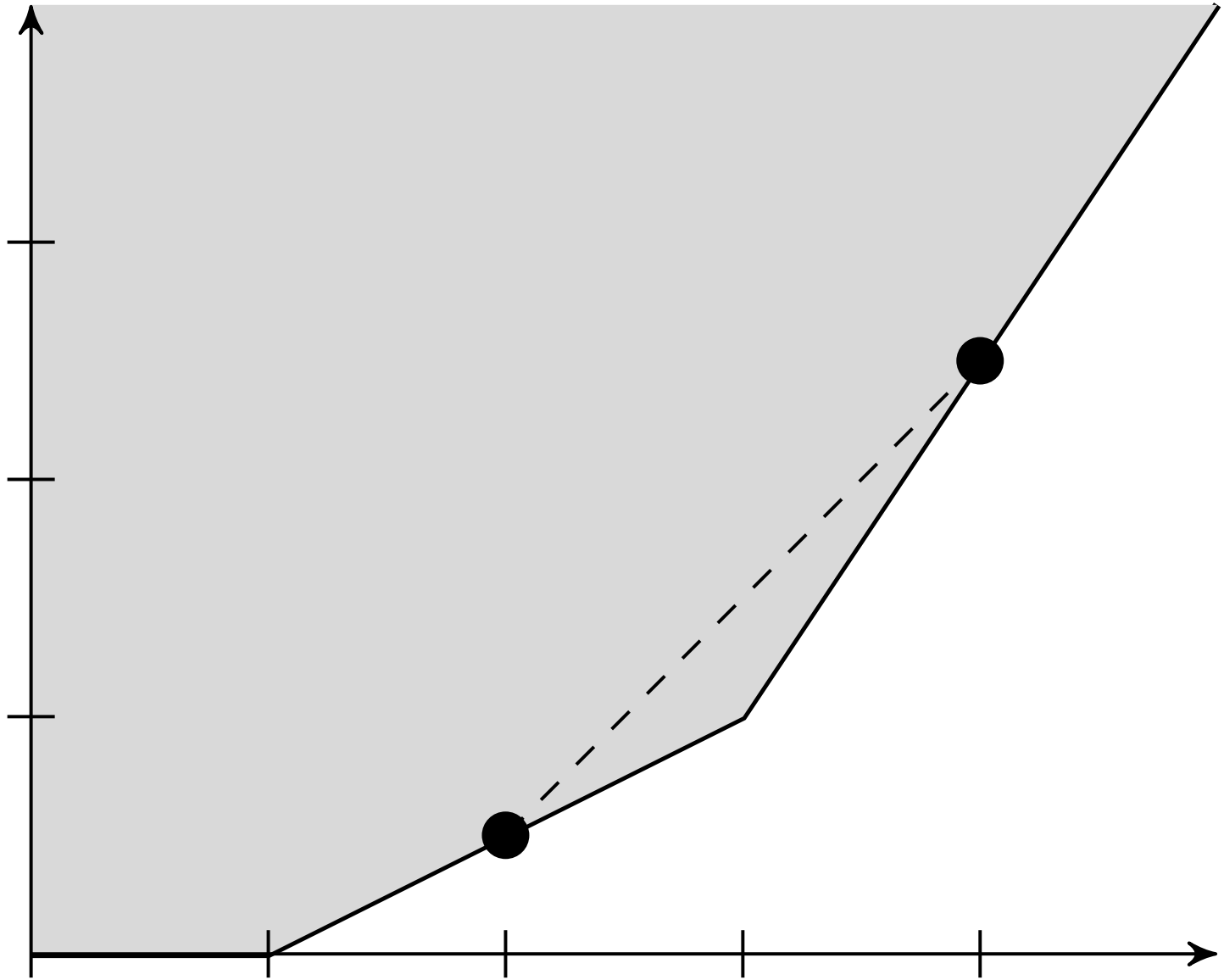
$y_i - 1$


5
4
3
2
1
−1
0
1
2
3
4
5
6
(5, 4)
(1, 2)
(3, 2)
(1, 1)
(−1, 0)
(3, 0)

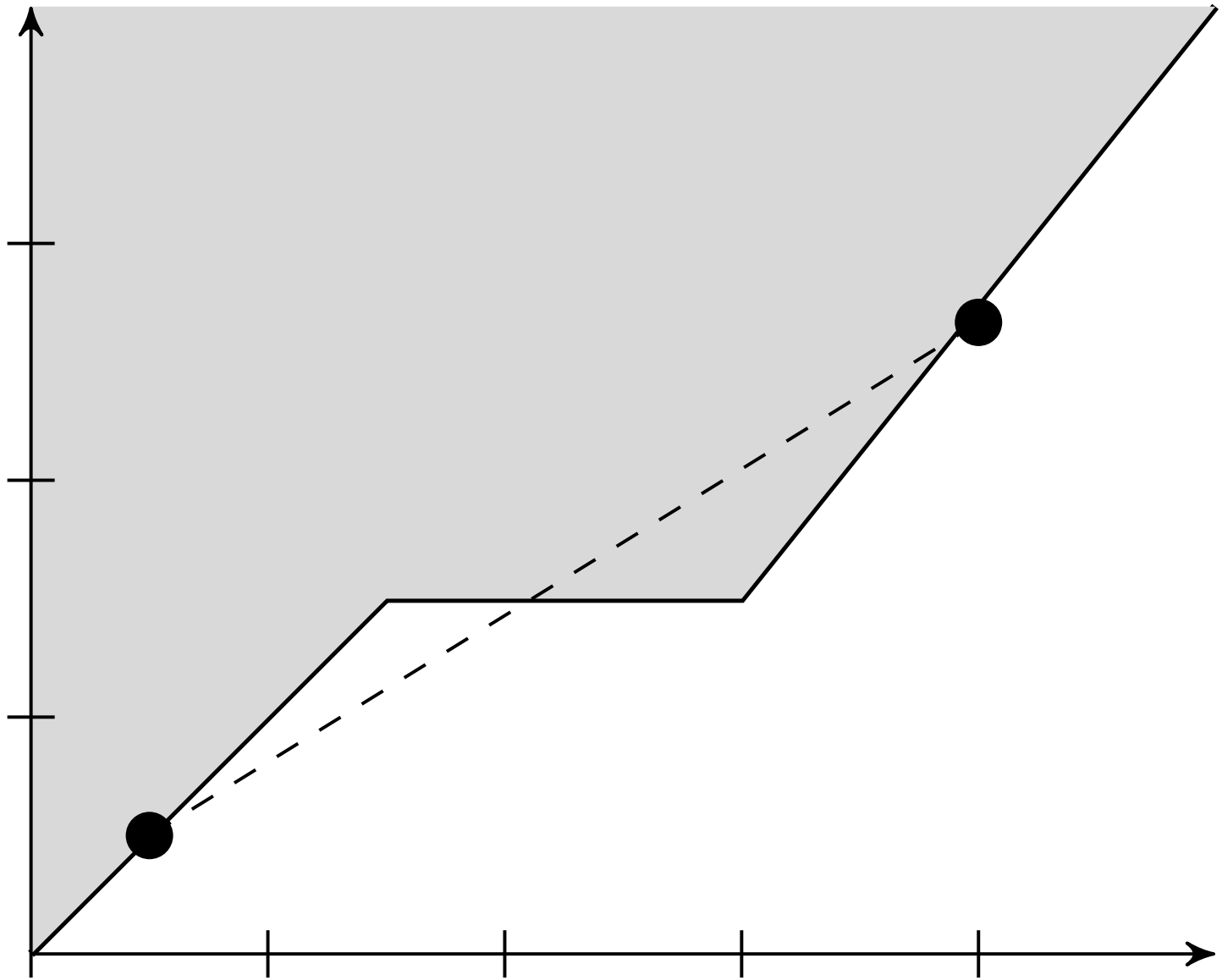
$(-1, 0), (1, 1), (1, 2), (3, 2), (3, 0), (3, 2)$

$(1, 2)$

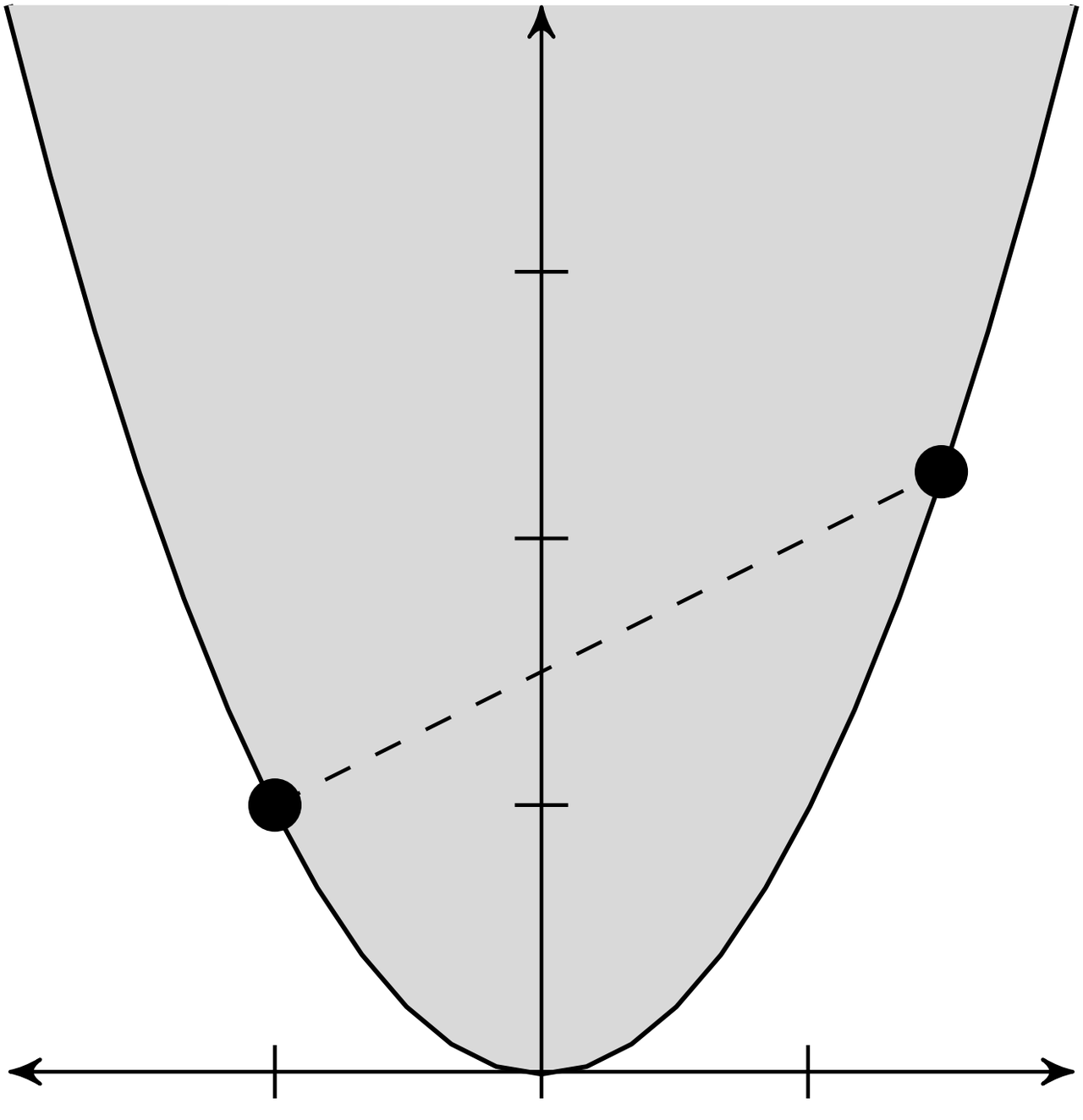
$(3, 0)$



$f(1)$



$$3x^2 + 4y^2 + 2xy + 2x + 3$$



$1e-6$

k

$$\bar{a}x = \lambda^t Ax \leq \lambda^t b = -\beta + \sum_{j:\bar{a}_j<0} \bar{a}_j U_j + \sum_{j:\bar{a}_j>0} \bar{a}_j L_j,$$

L_j

U_j

$\lambda_i \geq 0$

$\lambda_i \leq 0$

$\bar{a}_j \geq 0$

$U_j = \infty$

$\bar{a}_j \leq 0$

$L_j = -\infty$

λ

$$\begin{array}{ll} \text{minimize} & c'x \\ \text{subject to} & Ax \geq b \\ & x \geq 0 \end{array}$$

$$\begin{array}{ll} \text{maximize} & b'y \\ \text{subject to} & A'y \leq c \\ & y \geq 0 \end{array}$$

≥ 0

-2

$$a^T x \leq b$$

$$a^T x + s = b$$

$a^T x + s$

q

S

g

$l + q + s$

$l + q + s + g$

$i - l - q - s$

$a^T y \geq c$

$$a^T y - z = c$$

$a^T y - z$

m

$cols$

set

X

Set

$m.NumVars$

$m.numvars$

$2^{31} - 1$

$y \leq Mb$

$1e-5$

Mb

$$gap = |z_P - z_D| / |z_P|$$

$$z_P = z_D = 0$$

$z_P = 0$

$z_D \neq 0$

$$\left| z_P - z_D \right|$$

$$(10X01^2 + 2X01 * X02 + 2X02 * X01 + 2X02^2)/2$$

$10X01^2$

$4X01 * X02$

$$
\begin{array}{ll}
\text{minimize} & y - 1.3x(1-z) + (1-z) \\
\text{subject to} & 2y - 3x + 1.7w = 1.7 \\
& -y + x + xz(1-v) \geq 0 \\
& -y \leq 0, \\
& v, w, x, y, z \in \{0, 1\}.
\end{array}
$$

AA^T

$1 + x + 2y$

$y + 2z$

$$-1 \cdot (1 + x + 2y) + 2 \cdot (y + 2z) = -1 - x + 4z$$

10^6

10

5

100

$$\text{base_value} = \max\{bestsol, bestbound + |bestbound| * rgap, bestbound + agap\},$$

bestsol

bestbound

$rgap$

$agap$

20

120

$\{2, 2, 1\}$

$\{0.10, 0.05, 0.00\}$

$\{0, 1, 2\}$

$$\max\{10 \cdot 0.10, 10 \cdot 0.05, 0, 1\} \;=\; 1$$

$$\begin{aligned} x - 6y &= 1 \\ 0.333x - 2y &= .333 \end{aligned}$$

$$y := 0.1665x - 0.1665$$

$$
\begin{aligned}
x - 6 \cdot (0.1665x - 0.1665) &= 1 \\
\Leftrightarrow 0.001x &= 0.001
\end{aligned}
$$

$x = 1,\ y = 0$

$$\begin{aligned} x - 6y &= 1 \\ 0.33333333333333x - 2y &= 0.33333333333333 \end{aligned}$$

$y := 0.16666666666666667x - 0.16666666666666667$

$$
\begin{aligned}
x - 6 \cdot (0.1666666666666667x - 0.1666666666666667) &= 1 \\
\Leftrightarrow 2 \cdot 10^{-16}x + 1 + 2 \cdot 10^{-16} &\approx 1
\end{aligned}
$$

$x = 6y + 1$

$$1 \approx 1 + 2 \cdot 10^{-16}$$

$a \cdot x \leq b$

$a \cdot y \leq c$

$$\begin{array}{lll} \min & 0 & \\ s.t. & x \leq & 0 \\ & x \geq & 10^{-10} \end{array}$$

10^{-6}

10^{-5}

10^3

10^{-9}

$x \in [-10^{-6}, 10^{-6}]$

$x \in [-10^{10}, 10^{10}]$

10^{-16}

$$(P)\max\{cx : Ax = b, l \leq x \leq u\}$$

D

$D_{ii} > 0, \forall i$

$$(P_D)\max\{cDx' : ADx' = b, D^{-1}l \leq x' \leq D^{-1}u\}$$

10^4

10^{10}

P

$$\bar{f}(x) = M f_1(x) + f_2(x)$$

10^{-13}

$$\begin{aligned} 10^{-7}x + 10y &\leq 10 \\ x + 10^4 z &\leq 10^3 \\ x, y, z &\geq 0, \end{aligned}$$

$[10^{-7}, 10^{4}]$

10^5

$$\begin{aligned} 10^{-2}x' + 10y &\leq 10 \\ 10^{2}x' + 10z &\leq 1 \\ x', y, z &\geq 0, \end{aligned}$$

$x = 10^5 x'$

$[10^{-2}, 10^{2}]$

$[10^{-3}, 10^{6}]$

$$\begin{aligned} x - 10^6 y &\geq 0 \\ y &\in [0, 10] \end{aligned}$$

$$
\begin{aligned}
x - 10y_1 &\geq 0 \\
y_1 - 10y_2 &= 0 \\
y_2 - 10y_3 &= 0 \\
y_3 - 10y_4 &= 0 \\
y_4 - 10y_5 &= 0 \\
y_5 - 10y &= 0 \\
y &\in [0, 10]
\end{aligned}
$$

$y = -10^{-6},\ x = -1$

$$\begin{aligned} x - 10^3 y' &\geq 0 \\ y' &\in [0, 10^4] \end{aligned}$$

$10^{-3}y' = y$

-10^{-3}

-10^{-9}

$$
\begin{array}{lcl}
x & \leq & 10^6 y \\
x & \geq & 0 \\
y & \in & \{0,1\},
\end{array}
$$

$y = 0.0000099999$

9.999

$$
\begin{array}{lcl}
x & \leq & 10^3 y \\
x & \geq & 0 \\
y & \in & \{0, 1\}
\end{array}
$$

$x \leq 0.01$

$$y = 0 \Rightarrow x = 0$$

10^9

$$6 \cdot 10^6 / 0.00099 = 6.0606 \cdot 10^9.$$

10^{12}

10^{-8}

0.1

10^{-17}

$A^{-1}b$

$$A^{-1}(b+\varepsilon)$$

$$\eta(b,\varepsilon) := \frac{\|A^{-1}b\|}{\|A^{-1}(b+\varepsilon)\|} \Big/ \frac{\|b\|}{\|b+\varepsilon\|}.$$

$$\kappa(A) := \max_{b,\varepsilon} \eta(b, \varepsilon).$$

$$\kappa(A) = \frac{\lambda_{\max}}{\lambda_{\min}},$$

$$\kappa(A) = \|A\|\|A^{-1}\|.$$

$$\kappa(A) = 10^k$$

κ

$$\begin{array}{ll} \max & cx \\ s.t. & Ax \leq b. \end{array}$$

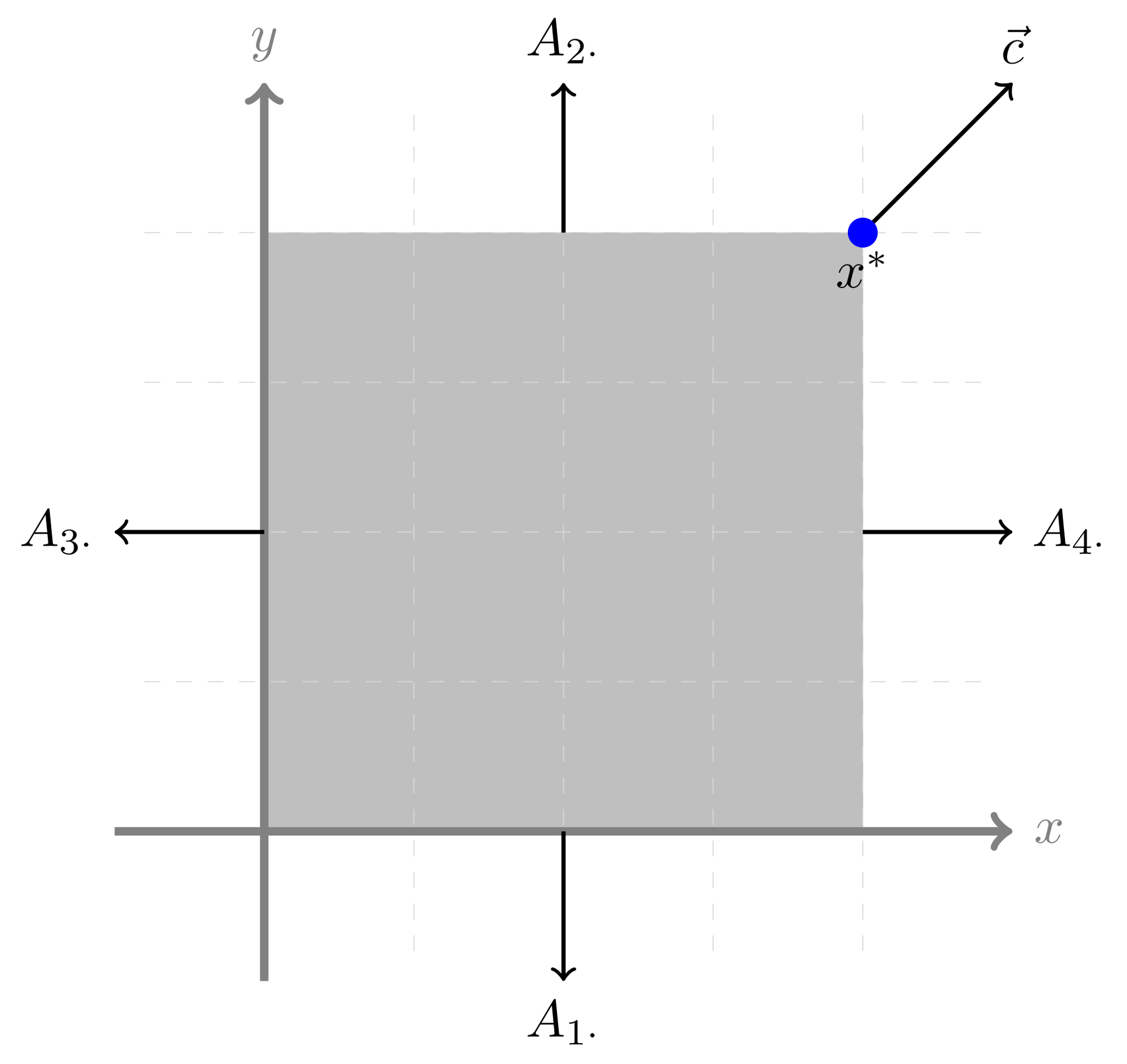
$$
\begin{array}{lrrl}
\max & x+y & \vec{c} = & (1,1) \\
s.t. & -x \leq 0 & A_{1\cdot} = & (-1,0) \\
& x \leq 1 & A_{2\cdot} = & (1,0) \\
& -y \leq 0 & A_{3\cdot} = & (0,-1) \\
& y \leq 1 & A_{4\cdot} = & (0,1).
\end{array}
$$

$b^t := (0, 1, 0, 1)$

$\max_{x \in \mathbb{R}^2} \{\vec{c}x : Ax \leq b\}.$

$\overline{c}$

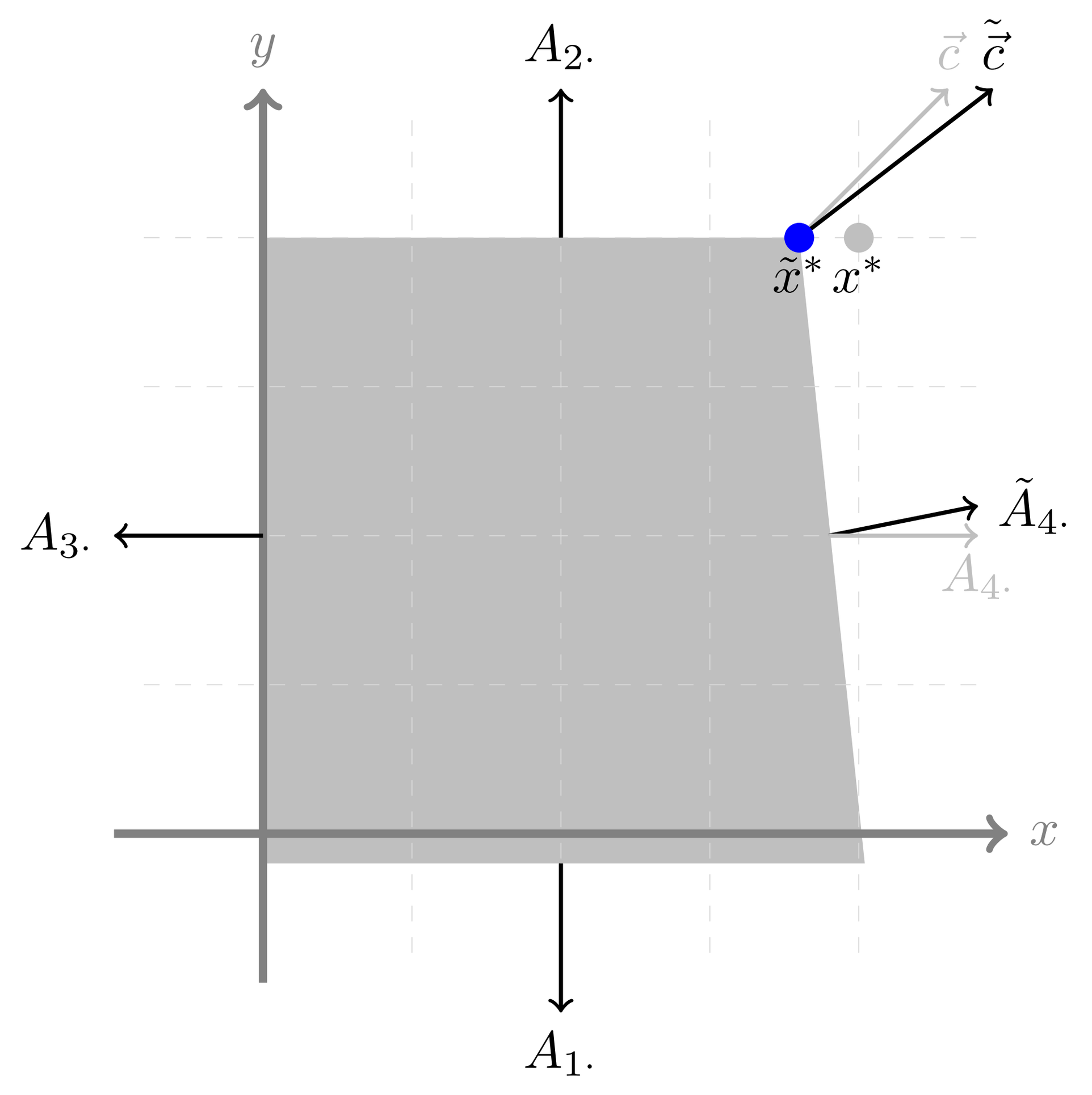
x^*


y
$A_2.$
$\vec{c}$
x^*
$A_3.$
$A_4.$
x
$A_1.$

$$\tilde{b}^t = (\varepsilon, 1, 0, 1)$$

$$\tilde{\vec{c}} = (1+\varepsilon, 1)$$

$\tilde{A}_{4.} = (\varepsilon, 1)$


y
$A_2.$
$\vec{c}$
$\tilde{\vec{c}}$
$\tilde{x}^*$
x^*
$\tilde{A}_4.$
$A_3.$
$A_4.$
x
$A_1.$

A_4.

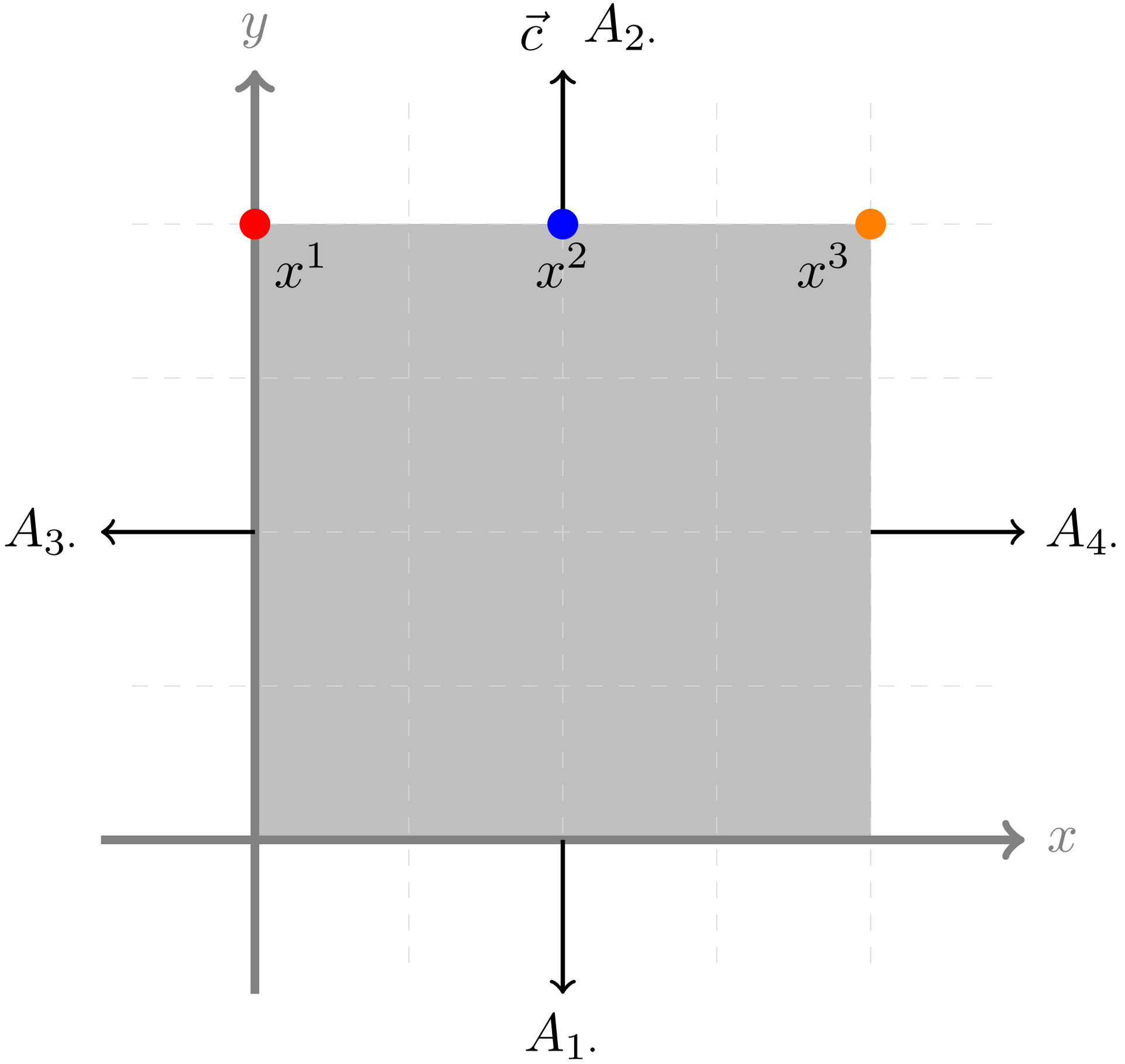
$x \leq 1$

$100x \leq 100$

$x + \varepsilon y \leq 1$

$$100x + \varepsilon y \leq 100$$

$$
\begin{array}{lrrl}
\max & y & \vec{c} = & (0,1) \\
s.t. & -x \leq 0 & A_{1\cdot} = & (-1,0) \\
& x \leq 1 & A_{2\cdot} = & (1,0) \\
& -y \leq 0 & A_{3\cdot} = & (0,-1) \\
& y \leq 1 & A_{4\cdot} = & (0,1).
\end{array}
$$

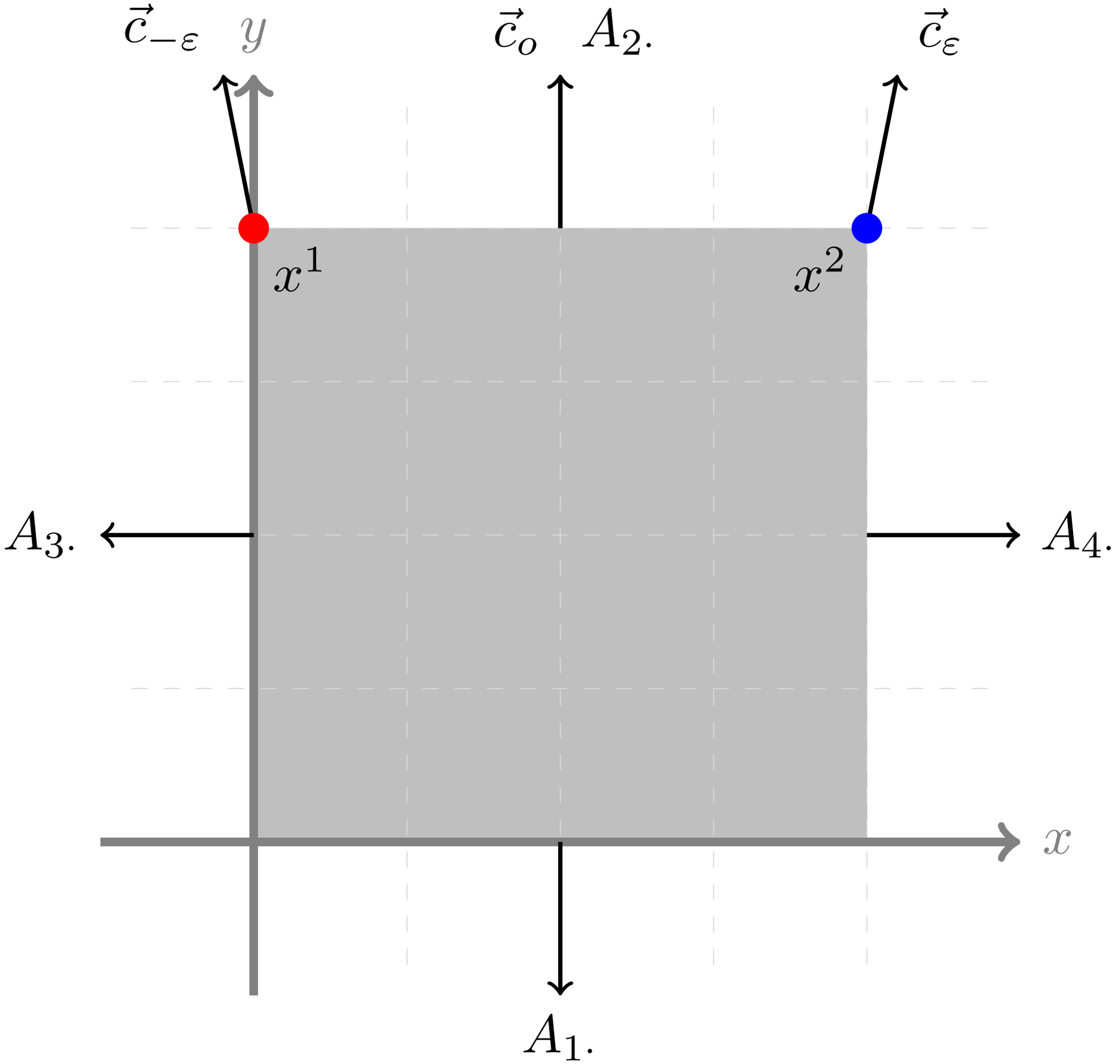

y
$\vec{c}$
$A_2.$
x^1
x^2
x^3
$A_3.$
$A_4.$
x
$A_1.$

x^1

x^3

x^2

$$
\begin{array}{lrrl}
\max & \varepsilon x + y & \vec{c} = & (\varepsilon, 1) \\
s.t. & -x \leq 0 & A_{1\cdot} = & (-1, 0) \\
& x \leq 1 & A_{2\cdot} = & (1, 0) \\
& -y \leq 0 & A_{3\cdot} = & (0, -1) \\
& y \leq 1 & A_{4\cdot} = & (0, 1).
\end{array}
$$


$\vec{c}_{-\varepsilon}$
y
$\vec{c}_o$
$A_2.$
$\vec{c}_{\varepsilon}$
x^1
x^2
$A_3.$
$A_4.$
x
$A_1.$

$(0, 0)$

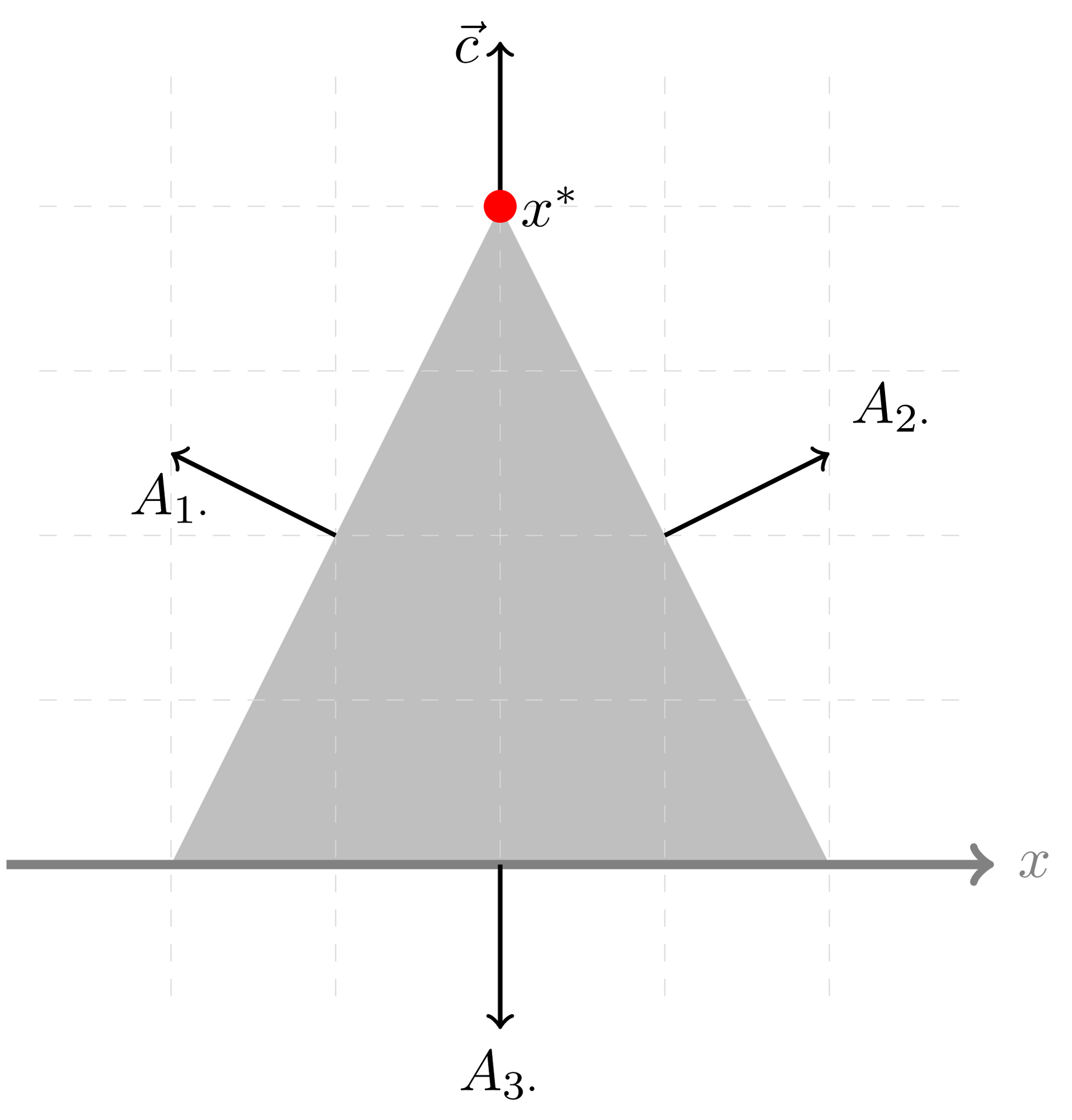
$(0, 1)$

$(10^6, 1)$

$(10^6, 0)$

$1 - 10^6 \varepsilon$

$$
\begin{array}{lrll}
\max & y & \vec{c} = & (0,1) \\
s.t. & -x + \varepsilon y \leq 1 & A_{1\cdot} = & (-1,\varepsilon) \\
& x + \varepsilon y \leq 1 & A_{2\cdot} = & (1,\varepsilon) \\
& -y \leq 0 & A_{3\cdot} = & (0,-1)
\end{array}
$$


$\vec{c}$
x^*
$A_2.$
$A_1.$
x
$A_3.$

A_1.

$A_2.$

$x^* = (0, \frac{1}{\varepsilon})$

$(1+\delta, 1)$

$\tilde{x}^* = (-\frac{\delta}{2}, \frac{2+\delta}{2\varepsilon})$

$$\|x^* - \tilde{x}^*\|_1 = \frac{|\delta|}{2} + \frac{|\delta|}{\varepsilon}$$

$(-1, \delta)$

$\tilde{x}^* = (1 - \frac{2\varepsilon}{\varepsilon+\delta}, \frac{2}{\varepsilon+\delta})$

$\delta = \varepsilon/2$

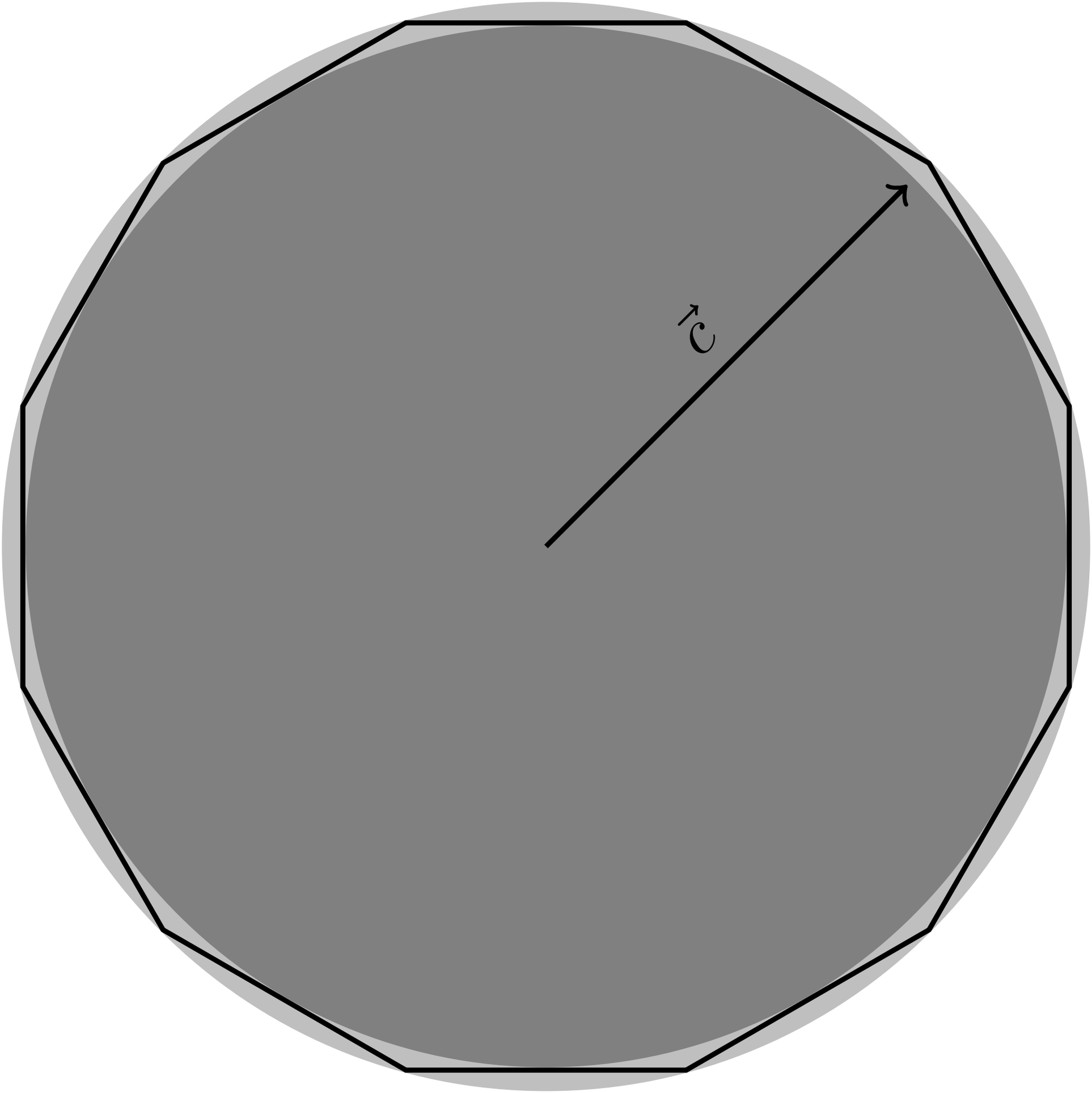
$$\frac{4}{3\varepsilon}$$

$\lim_{\varepsilon \to 0+} \left\| x^* \right\| = \infty$

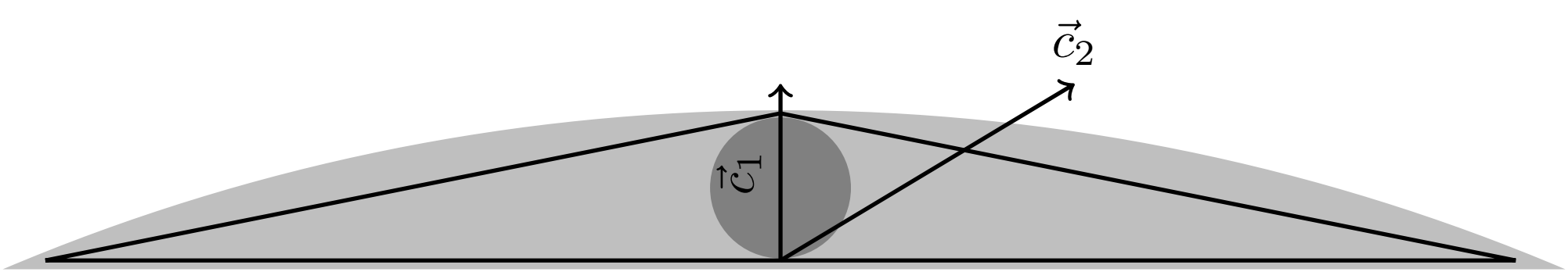
$y \leq 10^4$

10^{-4}

$$\sin(2\pi\tfrac{i}{10^6})x + \cos(2\pi\tfrac{i}{10^6})y \leq 1,\ \forall i \in \{1, \ldots, 10^6\},$$


$\vec{c}$

$[-1, 1]$


$\vec{c}_2$
$\vec{c}_1$

$\vec{c}_1 = (0, 1)$

$\vec{c}_2 = (1, 0)$