In[1]:=
```
fn = -3.0 * x3 * x1^2 + 7.5 * x2^2 - 5.0 * x1^2;
```

In[2]:=
```
g1 = Sqrt[x1] - 4. * x2^2 - 2 * x3;
g2 = 6.0 * x3^2 - 1.2 * x1^2 + 3.0 * x3 * x2;
```

In[4]:=
```
SeedRandom[4];
```

In[5]:=
```
ndiv = 10; q1 = 0.7`;
zz1 = 1.9; zz2 = 0.3; zz3 = 0.9;
```

In[7]:=
```
Δi1_ := q1 * zzi1
```

In[8]:=
```
{Δ1, Δ2, Δ3}
```

Out[8]=
```
{1.33, 0.21, 0.63}
```

In[9]:=
```
δ1 = Δ1/ndiv; δ2 = Δ2/ndiv; δ3 = Δ3/ndiv;
```

In[10]:=
```
Cons = {Δ1 - x1 ≥ 0, Δ2 - x2 ≥ 0, Δ3 - x3 ≥ 0, x1 ≥ 0, x2 ≥ 0, x3 ≥ 0}; x0 = 0;
```

In[11]:=
```
Constraints = Cons
```

Out[11]=
```
{1.33 - x1 ≥ 0, 0.21 - x2 ≥ 0, 0.63 - x3 ≥ 0, x1 ≥ 0, x2 ≥ 0, x3 ≥ 0}
```

In[12]:=
```
scut = {};
```

In[13]:=
```
Clear[linearCut, iter];
linearCut[scut_, iter_] := Module[{tt}, gCut = scut; iter = iter + 1;
InitialSolution = Minimize[f, Constraints, {x1, x2, x3}];
Values = {(g1 /. Part[InitialSolution, 2]), (g2 /. Part[InitialSolution, 2])};
xx = -Max[-Values, x0]; x0 = xx;
Pos = Part[Flatten[Position[Values, xx]], 1];
gCut =
    ExpandAll[xx + (∂x1 gPos /. Part[InitialSolution, 2]) * (x1 - (x1 /. Part[InitialSolution, 2])) +
      (∂x2 gPos /. Part[InitialSolution, 2]) * (x2 - (x2 /. Part[InitialSolution, 2])) +
      (∂x3 gPos /. Part[InitialSolution, 2]) * (x3 - (x3 /. Part[InitialSolution, 2]))];
Constraints = Join[Constraints, {gCut ≥ 0}];
{{gCut}, Pos, InitialSolution}]
mxo = 0.7; mfo = 20.0; ialg = 1;
σ = 0.6;
γ = 0.92;
τ = 0.905;
β = 0.98; scut = Constraints; TOL = 1.0 * 10^-7;
```

In[17]:=
```
xz = {x1 → Δ1 - 0., x2 → Δ2 - 0.0, x3 → Δ3 - 0.0}
```

Out[17]=
```
{x1 → 1.33, x2 → 0.21, x3 → 0.63}
```

In[18]:=
```
f = Simplify[(fn /. xz) + (∂x1 fn /. xz) * (x1 - (x1 /. xz)) +
    (∂x2 fn /. xz) * (x2 - (x2 /. xz)) + (∂x3 fn /. xz) * (x3 - (x3 /. xz))]
```

Out[18]=
$$15.2002 - 18.3274\, x_1 + 3.15\, x_2 - 5.3067\, x_3$$

In[19]:=
```
aa = linearCut[scut, 1];
InitialSolution = Part[aa, 3]
```

Out[20]=
$$\{-12.5185, \{x_1 \to 1.33, x_2 \to 0., x_3 \to 0.63\}\}$$

In[21]:=
```
xz = Part[InitialSolution, 2];
f = (fn /. xz) + (∂x1 fn /. xz) * (x1 - (x1 /. xz)) +
    (∂x2 fn /. xz) * (x2 - (x2 /. xz)) + (∂x3 fn /. xz) * (x3 - (x3 /. xz));
```

In[23]:=
```
For[ii = 0, ii < 29, {
scut = Constraints; ialg = ialg + 1; pos = Pos;
If[ialg ≤ 2, data = Flatten[Table[{x1, x2, x3, -gpos, f},
      {x1, 0.0, Δ1, δ1}, {x2, 0.0, Δ2, δ2}, {x3, 0.0, Δ3, δ3}], 2];
ca = Flatten[
     Table[{x1, x2, x3, -gpos}, {x1, 0.0, Δ1, δ1}, {x2, 0.0, Δ2, δ2}, {x3, 0.0, Δ3, δ3}], 2];
cc = {}; If[ialg == 1, σ = 0.6, σ = 0.8];
   For[i = 0, i < Length[data], {If[Part[Part[data, i], 4] > 0 && Part[Part[data, i], 4] <
         σ * mxo, cc = Join[cc, {Part[data, i]}];];}, i++]; data = Flatten[Table[
       {x1, x2, x3, -gpos, f}, {x1, 0.0, Δ1, δ1}, {x2, 0.0, Δ2, δ2}, {x3, 0.0, Δ3, δ3}], 2];];
If[ialg > 2,
{data = Flatten[Table[{x1, x2, x3, -gpos, f},
       {x1, 0.0, Δ1, δ1}, {x2, 0.0, Δ2, δ2}, {x3, 0.0, Δ3, δ3}], 2];
ca = Flatten[
      Table[{x1, x2, x3, -gpos}, {x1, 0.0, Δ1, δ1}, {x2, 0.0, Δ2, δ2}, {x3, 0.0, Δ3, δ3}], 2];
cc = {}; If[ialg == 1, σ = 0.6, σ = 0.96];
For[i = 0, i < Length[data],
     {If[Part[Part[data, i], 4] > 0 && Part[Part[data, i], 4] < σ * mxo &&
         Part[Part[data, i], 4] > 0 , cc = Join[cc, {Part[data, i]}]];}, i++];}];
fg = Table[Part[Part[cc, i], 4], {i, 1, Length[cc]}];
fo = Table[Part[Part[cc, i], 5], {i, 1, Length[cc]}];
mxo = Max[fg];
ev = Table[{Abs[τ * Min[fo] - Part[Part[cc, i], 5]]}, {i, 1, Length[cc]}];
eo3 = Part[Flatten[Position[ev, Min[ev]]], 1];
kdat = Part[cc, eo3];
kxo = Part[kdat, 4];
kx1 = Part[kdat, 1];
kx2 = Part[kdat, 2];
kx3 = Part[kdat, 3];
Θ1 = (∂x1 gpos) /. {x1 → kx1, x2 → kx2, x3 → kx3};
Θ2 = (∂x2 gpos) /. {x1 → kx1, x2 → kx2, x3 → kx3};
Θ3 = (∂x3 gpos) /. {x1 → kx1, x2 → kx2, x3 → kx3};
kxo = Part[kdat, 4];
```

```
  kx1 = Part[kdat, 1];
kx2 = Part[kdat, 2];
kx3 = Part[kdat, 3];
SharpCut1 = Simplify[γ * kxo + (Θ1 * (x1 - kx1) + Θ2 * (x2 - kx2) + Θ3 * (x3 - kx3))];
rhs2 = Expand[FindFit[ca,
      β * kxo - ((ϕ1 * (x1 - kx1) + ϕ2 * (x2 - kx2) + ϕ3 * (x3 - kx3))), {ϕ1, ϕ2, ϕ3}, {x1, x2, x3}]];
SharpCut2 = Simplify[β * kxo - ((ϕ1 * (x1 - kx1) + ϕ2 * (x2 - kx2) + ϕ3 * (x3 - kx3))) /. rhs2];
coefs = {Coefficient[rhs2, x1], Coefficient[rhs2, x2], Coefficient[rhs2, x3]};
Constraints = Flatten[Join[Constraints, {SharpCut1 ≥ 0, SharpCut2 ≥ 0}]];
InitialSolution = Minimize[f, Constraints, {x1, x2, x3}];
xz = Part[InitialSolution, 2];
f = (fn /. xz) + (∂x1 fn /. xz) * (x1 - (x1 /. xz)) +
     (∂x2 fn /. xz) * (x2 - (x2 /. xz)) + (∂x3 fn /. xz) * (x3 - (x3 /. xz));
Values = {(g1 /. Part[InitialSolution, 2]), (g2 /. Part[InitialSolution, 2])};
xx = -Max[-Values];
  x0 = xx; Print[-Values, x0];
  If[Abs[xx] < 1.0 × 10^-9, {Print[xx];
    Print[{ialg - 1, Length[cc], InitialSolution}]; Break[];}];
Pos = Part[Flatten[Position[Values, xx]], 1];
  Print["Pos=", Pos, "  ", xx];
If[ialg > 0, {gCut =
      ExpandAll[xx + (∂x1 gPos /. Part[InitialSolution, 2]) * (x1 - (x1 /. Part[InitialSolution, 2])) +
        (∂x2 gPos /. Part[InitialSolution, 2]) * (x2 - (x2 /. Part[InitialSolution, 2])) +
        (∂x3 gPos /. Part[InitialSolution, 2]) * (x3 - (x3 /. Part[InitialSolution, 2]))];
Constraints = Join[Constraints, {gCut ≥ 0}];}];
Print[{ialg - 1, Length[cc], InitialSolution}];
  Print[ialg, " ** xx=", xx];
  x0 = xx; If[Abs[xx] < TOL, {Print[xx]; Break[];}];
}, ii++]
```

```
{0.0169732, 0.0149944}-0.0169732

Pos=1  -0.0169732

{1, 454, {-11.9045, {x1 → 1.33, x2 → 0.0651405, x3 → 0.576628}}}

2 ** xx=-0.0169732

{0.00246369, 0.0782253}-0.0782253

Pos=2  -0.0782253

{2, 441, {-11.7691, {x1 → 1.33, x2 → 0.0899583, x3 → 0.561675}}}

3 ** xx=-0.0782253

{0.00142092, 0.00198511}-0.00198511

Pos=2  -0.00198511

{3, 249, {-10.91, {x1 → 1.28383, x2 → 0.0827767, x3 → 0.553538}}}

4 ** xx=-0.00198511

{0.00160409, 2.15616 × 10^-6}-0.00160409
```

```
Pos=1   -0.00160409
{4, 239, {-10.8993, {x1 → 1.28248, x2 → 0.0839602, x3 → 0.552937}}}
5 ** xx=-0.00160409
{1.08315×10^-6, 7.85212×10^-6}-7.85212×10^-6
Pos=2   -7.85212×10^-6
{5, 400, {-10.8309, {x1 → 1.27894, x2 → 0.0839654, x3 → 0.551351}}}
6 ** xx=-7.85212×10^-6
{1.08774×10^-6, 4.47036×10^-11}-1.08774×10^-6
Pos=1   -1.08774×10^-6
{6, 224, {-10.8309, {x1 → 1.27893, x2 → 0.0839701, x3 → 0.551348}}}
7 ** xx=-1.08774×10^-6
{5.02043×10^-13, 6.0127×10^-11}-6.0127×10^-11
-6.0127×10^-11
{7, 377, {-10.8308, {x1 → 1.27893, x2 → 0.0839701, x3 → 0.551347}}}
```

In[24]:=
```
Values
```
Out[24]=
$$\{-5.02043\times10^{-13},\ -6.0127\times10^{-11}\}$$

In[25]:=
```
q1
```
Out[25]=
```
0.7
```

In[26]:=
```
Minimize[
 {fn, g1 ≥ 0, g2 ≥ 0, Δ1 - x1 ≥ 0, Δ2 - x2 ≥ 0, Δ3 - x3 ≥ 0, x1 ≥ 0, x2 ≥ 0, x3 ≥ 0}, {x1, x2, x3}]
```
Out[26]=
```
{-10.9836, {x1 → 1.28346, x2 → 0.0546964, x3 → 0.560466}}
```