In[1]:=
```
fn = 4.3 * x1 + 31.8 * x2 + 63.3 * x3 + 15.8 * x4 + 68.5 * x5 + 4.7 * x6;
b1 = 32.97; b2 = 25.12; b3 = -29.08; b4 = -78.02;
g1 = 17.1 * x1 + 38.2 * x2 + 204.2 * x3 + 212.3 * x4 + 623.4 * x5 + 1495.5 * x6 - 169.0 * x1 * x3 -
    3580.0 * x3 * x5 - 3810.0 * x4 * x5 - 18500.0 * x4 * x6 - 24300.0 * x5 * x6 - b1;
g2 = 17.9 * x1 + 36.8 * x2 + 113.9 * x3 + 169.7 * x4 + 337.8 * x5 + 1385.2 * x6 -
    139.0 * x1 * x3 - 2450.0 * x4 * x5 - 16600.0 * x4 * x6 - 17200.0 * x5 * x6 - b2;
g3 = -273.0 * x2 - 70.0 * x4 - 819.0 * x5 + 26000.0 * x4 * x5 - b3;
g4 = 159.9 * x1 - 311.0 * x2 + 587.0 * x4 + 391.0 * x5 + 2198.0 * x6 - 14000.0 * x1 * x6 - b4;
```

In[7]:=
```
ν1 = 0.02; ν2 = 0.000; ν3 = 0.0000; ν4 = 0.000; ν5 = 0.020; ν6 = 0.01;
Δ1 = 0.31; Δ2 = 0.046;
Δ3 = 0.068;
Δ4 = 0.042;
Δ5 = 0.028; Δ6 = 0.0134;
δ1 = Δ1 / ndiv;
δ2 = Δ2 / (ndiv);
δ3 = Δ3 / ndiv;
δ4 = Δ4 / ndiv;
δ5 = Δ5 / (ndiv);
δ6 = Δ6 / ndiv; SeedRandom[3];
```

In[9]:=
```
Cons = {x1 - ν1 ≥ 0, x2 - ν2 ≥ 0, x3 - ν3 ≥ 0, x4 - ν4 ≥ 0, x5 - ν5 ≥ 0, x6 - ν6 ≥ 0,
    Δ4 - x4 ≥ 0.0, Δ3 - x3 ≥ 0, Δ1 - x1 ≥ 0, Δ2 - x2 ≥ 0, Δ5 - x5 ≥ 0.0, Δ6 - x6 ≥ 0.0};
```

In[10]:=
```
NMinimize[{fn, g1 ≥ 0, g2 ≥ 0, g3 ≥ 0, g4 ≥ 0, Cons}, {x1, x2, x3, x4, x5, x6}]
{g1, g2} /. Part[%, 2]
```

Out[10]=
$$\left\{3.13581,\ \left\{x_1 \to 0.268564,\ x_2 \to -9.9991 \times 10^{-9},\ x_3 \to -9.99959 \times 10^{-9},\ x_4 \to -9.99961 \times 10^{-9},\ x_5 \to 0.028,\ x_6 \to 0.0134\right\}\right\}$$

Out[11]=
$$\left\{-9.96023 \times 10^{-9},\ 1.25395\right\}$$

In[12]:=
```
x0 = 0;
Constraints = Cons;
```

In[14]:=
```
xz = {x1 → Δ1 - ν1, x2 → Δ2 - ν2, x3 → Δ3 - ν3, x4 → Δ4 - ν4, x5 → Δ5 - ν5, x6 → Δ6 - ν6}
```

Out[14]=
$$\{x_1 \to 0.29,\ x_2 \to 0.046,\ x_3 \to 0.068,\ x_4 \to 0.042,\ x_5 \to 0.008,\ x_6 \to 0.0034\}$$

In[15]:=
```
f = (fn /. xz) + (∂x1 fn /. xz) * (x1 - (x1 /. xz)) + (∂x2 fn /. xz) * (x2 - (x2 /. xz)) +
   (∂x3 fn /. xz) * (x3 - (x3 /. xz)) + (∂x4 fn /. xz) * (x4 - (x4 /. xz)) +
   (∂x5 fn /. xz) * (x5 - (x5 /. xz)) + (∂x6 fn /. xz) * (x6 - (x6 /. xz))
```

Out[15]=
$$8.24178 + 4.3\ (-0.29 + x_1) + 31.8\ (-0.046 + x_2) + 63.3\ (-0.068 + x_3) + 15.8\ (-0.042 + x_4) + 68.5\ (-0.008 + x_5) + 4.7\ (-0.0034 + x_6)$$

In[16]:=
```
Off[Join::heads]; Off[Set::write];
lbnd = 0.0001;
```

In[18]:=
```
Constraints = Cons
```

Out[18]=
$$\{-0.02 + x_1 \geq 0,\ 0. + x_2 \geq 0,\ 0. + x_3 \geq 0,\ 0. + x_4 \geq 0,\ -0.02 + x_5 \geq 0,\ -0.01 + x_6 \geq 0,$$
$$0.042 - x_4 \geq 0.,\ 0.068 - x_3 \geq 0,\ 0.31 - x_1 \geq 0,\ 0.046 - x_2 \geq 0,\ 0.028 - x_5 \geq 0.,\ 0.0134 - x_6 \geq 0.\}$$

In[19]:=
```
(* Cut the first slice. *)
```

In[20]:=
```
ndiv = 5;
η1 = ndiv; η2 = ndiv; η3 = ndiv;
η4 = ndiv;
η5 = ndiv;
η6 = ndiv;
η7 = ndiv;
η8 = ndiv;
η9 = ndiv;
pos = 1; mfo = 20.0;
ialg = 1;
σ = 0.98; γ1 = 1.0; γ2 = 1.0;
τ1 = 0.905; β = 1.0; Pos = pos = 1; ialg = 1;
γ1 = 1.0; γ2 = 1.0;
τ = 0.905;
β = 1.0; Pos = 1;
```

In[25]:=
```
lbnd_i2_ := ν_i2
```

In[26]:=
```
Clear[jval, i7];
jval[i7_] := Which[i7 == 1, 1, i7 == 2, 1, i7 == 3,
  1, i7 == 4, 0, i7 == 5, 0, i7 == 6, 0, i7 == 7, 0, i7 == 8, 0, i7 == 9, 0]
```

In[27]:=
```
For[i = 0, i ≤ 5,
 {For[j = 0, j ≤ jval[i], { lw[i, j] = lbnd_j + (j - 1) * δ_i; ur[i, j] = lbnd_j + (j) * δ_i;
    Print["i,j=" i, j, " ", lw[i, j], " @ ", ur[i, j]];}, j++]}, i++]
```

```
i,j=1 0.02 @ 0.082
i,j=2 0.062 @ 0.124
2 i,j=1 0.02 @ 0.0292
2 i,j=2 0.0092 @ 0.0184
3 i,j=1 0.02 @ 0.0336
3 i,j=2 0.0136 @ 0.0272
4 i,j=1 0.02 @ 0.0284
5 i,j=1 0.02 @ 0.0256
6 i,j=1 0.02 @ 0.02268
```

In[28]:=
```
Constraints = Cons
```

Out[28]=
$$\{-0.02 + x_1 \geq 0,\ 0. + x_2 \geq 0,\ 0. + x_3 \geq 0,\ 0. + x_4 \geq 0,\ -0.02 + x_5 \geq 0,\ -0.01 + x_6 \geq 0,$$
$$0.042 - x_4 \geq 0.,\ 0.068 - x_3 \geq 0,\ 0.31 - x_1 \geq 0,\ 0.046 - x_2 \geq 0,\ 0.028 - x_5 \geq 0.,\ 0.0134 - x_6 \geq 0.\}$$

In[29]:=
```
scut = {};
```

In[30]:=
```
cnt = 0; Off[General::munfl];
For[j1 = 0, j1 ≤ jval[1], {For[j2 = 0, j2 ≤ jval[2],
    {For[j3 = 0, j3 ≤ jval[3], {For[j4 = 0, j4 ≤ jval[4], {For[j5 = 0, j5 ≤ jval[5],
              {For[j6 = 0, j6 ≤ jval[6], {cnt = cnt + 1; data[j1, j2, j3, j4, j5, j6] = {};
                  data[j1, j2, j3, j4, j5, j6] = Flatten[Table[{x1, x2, x3, x4, x5, x6,
                      -gpos, f}, {x1, lw[1, j1], ur[1, j1], (ur[1, j1] - lw[1, j1]) / η1},
                    {x2, lw[2, j2], ur[2, j2], (ur[2, j2] - lw[2, j2]) / η2},
                    {x3, lw[3, j3], ur[3, j3], (ur[3, j3] - lw[3, j3]) / η3},
                    {x4, lw[4, j4], ur[4, j4], (ur[4, j4] - lw[4, j4]) / η4},
                    {x5, lw[5, j5], ur[5, j5], (ur[5, j5] - lw[5, j5]) / η5},
                    {x6, lw[6, j6], ur[6, j6], (ur[6, j6] - lw[6, j6]) / η6}], 5];},
                j6++];}, j5++];}, j4++];}, j3++];}, j2++];}, j1++]
```

In[31]:=
```
data[j1, j2, j3, j4, j5, j6]
```

Out[31]=
```
{{0.062, 0.0092, 0.0136, 0.02, 0.02, 0.02, 1.9175, 3.20004},
 {0.062, 0.0092, 0.0136, 0.02, 0.02, 0.020536, 1.57473, 3.20256},
 {0.062, 0.0092, 0.0136, 0.02, 0.02, 0.021072, 1.23196, 3.20508}, ··· 46 651 ···,
 {0.124, 0.0184, 0.0272, 0.0284, 0.0256, 0.022144, 0.730416, 5.14648},
 {0.124, 0.0184, 0.0272, 0.0284, 0.0256, 0.02268, 0.543878, 5.149}}
```

Size in memory: 11.2 MB | + Show more | Show all | Iconize | Store full expression in notebook

In[32]:=
```
Print[cnt];
```

```
8
```

In[33]:=
```
Constraints = Cons
```

Out[33]=
$$\{-0.02 + x_1 \geq 0,\ 0. + x_2 \geq 0,\ 0. + x_3 \geq 0,\ 0. + x_4 \geq 0,\ -0.02 + x_5 \geq 0,\ -0.01 + x_6 \geq 0,\ 0.042 - x_4 \geq 0.,\ 0.068 - x_3 \geq 0,\ 0.31 - x_1 \geq 0,\ 0.046 - x_2 \geq 0,\ 0.028 - x_5 \geq 0.,\ 0.0134 - x_6 \geq 0.\}$$

In[34]:=
```
InitialSolution = Minimize[f, Constraints, {x1, x2, x3, x4, x5, x6}]
```

Out[34]=
$$\{1.503,\ \{x_1 \to 0.02,\ x_2 \to 0.,\ x_3 \to 0.,\ x_4 \to 0.,\ x_5 \to 0.02,\ x_6 \to 0.01\}\}$$

In[35]:=
```
(*{x1,s[1],5.54},{x2,s[2],4.0},{x3,s[3],4.0},
{x4,s[4],12.02},{x5,s[5],0.702},{x6,s[6],0.852}*)
```

In[36]:=
```
s[i_] := Part[Part[Part[InitialSolution, 2], i], 2]
```

In[37]:=
```
{s[1], s[2], s[3], s[4], s[5], s[6]}
```

Out[37]=
```
{0.02, 0., 0., 0., 0.02, 0.01}
```

In[38]:=
```
xz = Part[InitialSolution, 2]
f = (fn /. xz) + (∂x1 fn /. xz) * (x1 - (x1 /. xz)) + (∂x2 fn /. xz) * (x2 - (x2 /. xz)) +
  (∂x3 fn /. xz) * (x3 - (x3 /. xz)) + (∂x4 fn /. xz) * (x4 - (x4 /. xz)) +
  (∂x5 fn /. xz) * (x5 - (x5 /. xz)) + (∂x6 fn /. xz) * (x6 - (x6 /. xz))
```

Out[38]=
```
{x1 → 0.02, x2 → 0., x3 → 0., x4 → 0., x5 → 0.02, x6 → 0.01}
```

Out[39]=
```
1.503 + 4.3 (-0.02 + x1) + 31.8 (0. + x2) +
 63.3 (0. + x3) + 15.8 (0. + x4) + 68.5 (-0.02 + x5) + 4.7 (-0.01 + x6)
```

In[40]:=
```
Values = {(g1 /. Part[InitialSolution, 2]), (g2 /. Part[InitialSolution, 2]),
    (g3 /. Part[InitialSolution, 2]), (g4 /. Part[InitialSolution, 2])};
xx = -Max[-Values, x0]
gCut = 1/(1.0 * 10^4) *
    (ExpandAll[xx + (∂x1 gPos /. Part[InitialSolution, 2]) * (x1 - (x1 /. Part[InitialSolution, 2])) +
       (∂x2 gPos /. Part[InitialSolution, 2]) * (x2 - (x2 /. Part[InitialSolution, 2])) +
       (∂x3 gPos /. Part[InitialSolution, 2]) * (x3 - (x3 /. Part[InitialSolution, 2])) +
       (∂x4 gPos /. Part[InitialSolution, 2]) * (x4 - (x4 /. Part[InitialSolution, 2])) +
       (∂x5 gPos /. Part[InitialSolution, 2]) * (x5 - (x5 /. Part[InitialSolution, 2])) +
       (∂x6 gPos /. Part[InitialSolution, 2]) * (x6 - (x6 /. Part[InitialSolution, 2]))]);
Cons = Join[Cons, {gCut ≥ 0}]
```

Out[41]=
```
-10.065
```

Out[43]=
```
{-0.02 + x1 ≥ 0, 0. + x2 ≥ 0, 0. + x3 ≥ 0, 0. + x4 ≥ 0, -0.02 + x5 ≥ 0, -0.01 + x6 ≥ 0,
 0.042 - x4 ≥ 0., 0.068 - x3 ≥ 0, 0.31 - x1 ≥ 0, 0.046 - x2 ≥ 0, 0.028 - x5 ≥ 0., 0.0134 - x6 ≥ 0.,
 0.0001 (-28.11 + 17.1 x1 + 38.2 x2 + 129.22 x3 - 48.9 x4 + 380.4 x5 + 1009.5 x6) ≥ 0}
```

In[44]:=
```
For[ii = 0, ii ≤ 67, {
ialg = ialg + 1; (* If[ialg>4,{ψ1=Part[Part[xz,1],2];
      ψ2=Part[Part[xz,2],2];
      ψ3=Part[Part[xz,3],2];
      ψ4=Part[Part[xz,4],2];
      ψ5=Part[Part[xz,5],2];
      ψ6=Part[Part[xz,6],2];
      ur[1,j1]=ψ1;
      ur[2,j2]=ψ2;
      ur[3,j3]=ψ3;ur[4,j3]=ψ4;ur[5,j3]=ψ5;ur[6,j3]=ψ6;}];*)
   pos = Pos;
cnt = 0; Off[General::munfl];
   For[j1 = 0, j1 ≤ jval[1], {For[j2 = 0, j2 ≤ jval[2], {For[j3 = 0, j3 ≤ jval[3],
           {For[j4 = 0, j4 ≤ jval[4], {For[j5 = 0, j5 ≤ jval[5], {For[j6 = 0,
                    j6 ≤ jval[6], {cnt = cnt + 1; data[j1, j2, j3, j4, j5, j6] = {};
```

```
                    data[j1, j2, j3, j4, j5, j6] = Flatten[Table[{x1, x2, x3, x4, x5, x6,
                        -gpos, f}, {x1, lw[1, j1], ur[1, j1], (ur[1, j1] - lw[1, j1])/η1},
                      {x2, lw[2, j2], ur[2, j2], (ur[2, j2] - lw[2, j2])/η2},
                      {x3, lw[3, j3], ur[3, j3], (ur[3, j3] - lw[3, j3])/η3},
                      {x4, lw[4, j4], ur[4, j4], (ur[4, j4] - lw[4, j4])/η4},
                      {x5, lw[5, j5], ur[5, j5], (ur[5, j5] - lw[5, j5])/η5},
                      {x6, lw[6, j6], ur[6, j6], (ur[6, j6] - lw[6, j6])/η6}], 5];},
                  j6++];}, j5++];}, j4++];}, j3++];}, j2++];}, j1++] ×
Print[cnt];
cnt = 0;
   For[j1 = 0, j1 ≤ jval[1], {For[j2 = 0, j2 ≤ jval[2], {For[j3 = 0, j3 ≤ jval[3], {For[
            j4 = 0, j4 ≤ jval[4], {For[j5 = 0, j5 ≤ jval[5], {For[j6 = 0, j6 ≤ jval[6], {
 cnt = cnt + 1;
fo[j1, j2, j3, j4, j5, j6] =
                    Table[Part[Part[data[j1, j2, j3, j4, j5, j6], i], 8],
                     {i, 1, Length[data[j1, j2, j3, j4, j5, j6]]}];
ev[j1, j2, j3, j4, j5, j6] = Table[{Abs[τ * Min[fo[j1, j2, j3, j4, j5, j6]] -
                       Part[Part[data[j1, j2, j3, j4, j5, j6], i], 8]]},
                     {i, 1, Length[data[j1, j2, j3, j4, j5, j6]]}];
eo3[j1, j2, j3, j4, j5, j6] = Part[Flatten[Position[ev[j1, j2, j3, j4, j5, j6],
                       Min[ev[j1, j2, j3, j4, j5, j6]]]], 1];
kdat[j1, j2, j3, j4, j5, j6] =
                     Part[data[j1, j2, j3, j4, j5, j6], eo3[j1, j2, j3, j4, j5, j6]];
                    (*Print["kdat=",kdat[j1,j2,j3,j4,j5,j6]];*)
kxo[j1, j2, j3, j4, j5, j6] = Part[kdat[j1, j2, j3, j4, j5, j6], 7];
kx1[j1, j2, j3, j4, j5, j6] = Part[kdat[j1, j2, j3, j4, j5, j6], 1];
kx2[j1, j2, j3, j4, j5, j6] = Part[kdat[j1, j2, j3, j4, j5, j6], 2];
kx3[j1, j2, j3, j4, j5, j6] = Part[kdat[j1, j2, j3, j4, j5, j6], 3];
                    kx4[j1, j2, j3, j4, j5, j6] = Part[kdat[j1, j2, j3, j4, j5, j6], 4];
                    kx5[j1, j2, j3, j4, j5, j6] = Part[kdat[j1, j2, j3, j4, j5, j6], 5];
                    kx6[j1, j2, j3, j4, j5, j6] = Part[kdat[j1, j2, j3, j4, j5, j6], 6];
rr = {x1 → kx1[j1, j2, j3, j4, j5, j6], x2 → kx2[j1, j2, j3, j4, j5, j6], x3 →
                       kx3[j1, j2, j3, j4, j5, j6], x4 → kx4[j1, j2, j3, j4, j5, j6], x5 →
                       kx5[j1, j2, j3, j4, j5, j6], x6 → kx6[j1, j2, j3, j4, j5, j6]};
Θ1 = (∂x1 gpos) /. rr;
Θ2 = (∂x2 gpos) /. rr;
Θ3 = (∂x3 gpos) /. rr;
                    Θ4 = (∂x4 gpos) /. rr;
```

```
                        Θ5 = (∂x5 gpos) /. rr;
Θ6 = (∂x6 gpos) /. rr;
CuttingHyperplane2[cnt] =
Simplify[γ1 * kxo[j1, j2, j3, j4, j5, j6] +
                            (Θ1 * (x1 - kx1[j1, j2, j3, j4, j5, j6])) +
(Θ2 * (x2 - kx2[j1, j2, j3, j4, j5, j6])) +
Θ3 * (x3 - kx3[j1, j2, j3, j4, j5, j6]) +
Θ4 * (x4 - kx4[j1, j2, j3, j4, j5, j6]) +
Θ5 * (x5 - kx5[j1, j2, j3, j4, j5, j6]) +
Θ6 * (x6 - kx6[j1, j2, j3, j4, j5, j6])];
(*Print[cnt, " ",CuttingHyperplane2[cnt]];*)
}, j6++];}, j5++];}, j4++];}, j3++];}, j2++];}, j1++];
pi_[a_] := Coefficient[a, xi, 1];
p[i_] :=
    If[Part[ans, i] < 0, Rescale[Part[ans, i], {-20, 100}, {0, Δi}], Part[ans, i]];
For[cnt = 1, cnt ≤ 7, {
aa1 = CuttingHyperplane2[1];
aa2 = CuttingHyperplane2[cnt]; (*Print[cnt,aa1,aa2];*)
a1 = {p1[aa1], p2[aa1], p3[aa1], p4[aa1], p5[aa1], p6[aa1]};
a2 = {p1[aa2], p2[aa2], p3[aa2], p4[aa1], p5[aa1], p6[aa1]};
b1 = Coefficient[Coefficient[Coefficient[Coefficient[
          Coefficient[Coefficient[aa1, x1, 0], x2, 0], x3, 0], x4, 0], x5, 0], x6, 0];
b2 = Coefficient[Coefficient[Coefficient[Coefficient[
          Coefficient[Coefficient[aa2, x1, 0], x2, 0], x3, 0], x4, 0], x5, 0], x6, 0];
     ax1 = (a1) / a1.a2;
     ax2 = (a2) / a1.a2;
bx1 = (b1) / (b1 * b2);
     bx2 = (b2) / (b1 * b2);
ω = ArcCos[Mod[ax1.ax2, 1]];
a1 = ax1; a2 = ax2; b1 = bx1; b2 = bx2;
x1 = ((b1 - b2 * Cos[ω]) / Sin[ω]^2) * {{Part[a1, 1]}, {Part[a1, 2]},
          {Part[a1, 3]}, {Part[a1, 4]}, {Part[a1, 5]}, {Part[a1, 6]}} +
        ((b2 - b1 * Cos[ω]) / Sin[ω]^2) * {{Part[a2, 1]}, {Part[a2, 2]},
          {Part[a1, 3]}, {Part[a1, 4]}, {Part[a1, 5]}, {Part[a1, 6]}};
A = {a1, a2};
P = IdentityMatrix[6];
G = A.P;
     u := RandomReal[{0.6, 0.98}];
Off[RowReduce::luc]; Z = RowReduce[G];(*Print["Z=",Z]*);
F1 = Transpose[Z]; F = Transpose[Part[F1, {3, 4, 5, 6}]];(*Print["F=",F]*);
     ℱ = Join[-F, IdentityMatrix[4]];(*Print[ℱ]*);
     ς = 1 * {u, u, u, u}; (*Print[ς]*);
cx = P.ℱ.ς; (*Print["cx=",cx]*);
ans = Flatten[x1 + cx];
pnt[cnt] = {p[1], p[2], p[3], p[4], p[5], p[6]};
(* Print["*** ",cnt,pnt[cnt]];*)
```

```
}, cnt++];
cnt = 0; ca = {};
For[ cnt = 1, cnt ≤ 7, {
rh = {x1 → Part[pnt[cnt], 1], x2 → Part[pnt[cnt], 2], x3 → Part[pnt[cnt], 3],
      x4 → Part[pnt[cnt], 4], x5 → Part[pnt[cnt], 5], x6 → Part[pnt[cnt], 6]};
pdat = {x1, x2, x3, x4, x5, x6, gpos} /. rh; (*Print[pdat];*)
    x1 = 1.0 * (x1) /. rh;
    x2 = 1.0 * (x2) /. rh;
    x3 = 1.0 * (x3) /. rh;
    x4 = 1.0 * (x4) /. rh;
    x5 = 1.0 * (x5) /. rh;
    x6 = 1.0 * (x6) /. rh;
    rh2 = {x1 → x1, x2 → x2, x3 → x3, x4 → x4, x5 → x5, x6 → x6};
    pdat = {x1, x2, x3, x4, x5, x6, gpos} /. rh;
ca = Join[ca, {pdat}];
pxo = Part[pdat, 7];
px1 = Part[pdat, 1];
px2 = Part[pdat, 2];
px3 = Part[pdat, 3];
    px4 = Part[pdat, 4];
    px5 = Part[pdat, 5];
    px6 = Part[pdat, 6];
θ1 = (∂x1 gpos) /. {x1 → px1, x2 → px2, x3 → px3, x4 → px4, x5 → px5, x6 → px6};
θ2 = (∂x2 gpos) /. {x1 → px1, x2 → px2, x3 → px3, x4 → px4, x5 → px5, x6 → px6};
    θ3 = (∂x3 gpos) /. {x1 → px1, x2 → px2, x3 → px3, x4 → px4, x5 → px5, x6 → px6};
    θ4 = (∂x4 gpos) /. {x1 → px1, x2 → px2, x3 → px3, x4 → px4, x5 → px5, x6 → px6};
    θ5 = (∂x5 gpos) /. {x1 → px1, x2 → px2, x3 → px3, x4 → px4, x5 → px5, x6 → px6};
    θ6 = (∂x6 gpos) /. {x1 → px1, x2 → px2, x3 → px3, x4 → px4, x5 → px5, x6 → px6};
Cut1[cnt] = (*1/(1.0*10^4)**)(Simplify[γ1 * pxo + (θ1 * (x1 - px1) + θ2 * (x2 - px2) +
          θ3 * (x3 - px3) + θ4 * (x4 - px4) + θ5 * (x5 - px5) + θ6 * (x6 - px6))]);
(*Print[Cut1[cnt]];*)
    Label[Hi];}, cnt++]; (*Print[ca];*)
rhs2 = Expand[FindFit[ca, 0.98 * pxo +
      ((ϕ1 * (x1 - px1) + ϕ2 * (x2 - px2) + ϕ3 * (x3 - px3) + ϕ4 * (x4 - px4) + ϕ5 * (x5 - px5) +
        ϕ6 * (x6 - px6))), {ϕ1, ϕ2, ϕ3, ϕ4, ϕ5, ϕ6}, {x1, x2, x3, x4, x5, x6}]];
aa1 = Simplify[0.98 * pxo + ((ϕ1 * (x1 - px1) + ϕ2 * (x2 - px2) + ϕ3 * (x3 - px3) +
        ϕ4 * (x4 - px4) + ϕ5 * (x5 - px5) + ϕ6 * (x6 - px6))) /. rhs2];
  a1 = {p1[aa1], p2[aa1], p3[aa1], p4[aa1], p5[aa1], p6[aa1]};
  GCH = Simplify[ (aa1) / (√(a1.a1))];
  Print["GCH=", GCH];
Constr1 = {}; For[cnt = 1, cnt ≤ 7,
   {Constr1 = Join[Constr1, {CuttingHyperplane2[cnt] ≥ 0}];}, cnt++];
Constr2 = {}; For[cnt = 1, cnt ≤ 7, {Constr2 = Join[Constr2, {Cut1[cnt] ≥ 0}];}, cnt++];
ConstrT = Join[Constr1, Constr2];
Cons = Join[Cons, {GCH ≥ 0}];
temp = Join[Cons, Constr2]; Constraints = Join[Constraints, temp];
```

```
(*Print["**1**",xz,f]*);
   InitialSolution = Minimize[f, Constraints, {x1, x2, x3, x4, x5, x6}];
   Print[InitialSolution];
xz = Part[InitialSolution, 2]; f =
    Simplify[(fn /. xz) + (∂x1 fn /. xz) * (x1 - (x1 /. xz)) + (∂x2 fn /. xz) * (x2 - (x2 /. xz)) +
      (∂x3 fn /. xz) * (x3 - (x3 /. xz)) + (∂x4 fn /. xz) * (x4 - (x4 /. xz)) + (∂x5 fn /. xz) *
       (x5 - (x5 /. xz)) + (∂x6 fn /. xz) * (x6 - (x6 /. xz))];(*Print["**2**",xz,f]*);
Values = {(g1 /. Part[InitialSolution, 2]), (g2 /. Part[InitialSolution, 2]),
     (g3 /. Part[InitialSolution, 2]), (g4 /. Part[InitialSolution, 2])};
   xx = -Max[-Values];
   Print[ialg, " ** xx=", xx];
   x0 = xx;
   Print["***   ", {Values}, xx]; Print[xx]; If[Abs[xx] < 1.0 × 10^-11, {Print[xx];
     Break[];}];
Pos = Part[Flatten[Position[Values, x0]], 1];
   Print["Pos=", Pos, "  ", xx];
gCut = 1/(1.0 * 10^4) * ExpandAll[
      xx + (∂x1 gPos /. Part[InitialSolution, 2]) * (x1 - (x1 /. Part[InitialSolution, 2])) +
       (∂x2 gPos /. Part[InitialSolution, 2]) * (x2 - (x2 /. Part[InitialSolution, 2])) +
       (∂x3 gPos /. Part[InitialSolution, 2]) * (x3 - (x3 /. Part[InitialSolution, 2])) +
       (∂x4 gPos /. Part[InitialSolution, 2]) * (x4 - (x4 /. Part[InitialSolution, 2])) +
       (∂x5 gPos /. Part[InitialSolution, 2]) * (x5 - (x5 /. Part[InitialSolution, 2])) +
       (∂x6 gPos /. Part[InitialSolution, 2]) * (x6 - (x6 /. Part[InitialSolution, 2]))];
Cons = Join[Cons, {gCut ≥ 0}];
}, ii++];
```

```
8
GCH= 0.197152 + 0.0027076 x1 + 0.923497 x2 + 0.0461467 x3 - 0.0303208 x4 - 0.298863 x5 - 0.234047 x6
{2.9696, {x1 → 0.229912, x2 → 0., x3 → 0., x4 → 0., x5 → 0.028, x6 → 0.0134}}
2 ** xx=-0.66096
***   {{-0.66096, 0.56207, 6.148, 112.053}}-0.66096
-0.66096
Pos=1  -0.66096
8
GCH= 0.204747 - 0.000400998 x1 + 0.965032 x2 - 0.0389289 x3 - 0.148285 x4 - 0.133219 x5 - 0.165716 x6
{3.13581, {x1 → 0.268565, x2 → 0., x3 → 0., x4 → 0., x5 → 0.028, x6 → 0.0134}}
3 ** xx=-3.55271×10^-15
***   {{-3.55271×10^-15, 1.25395, 6.148, 110.982}}-3.55271×10^-15
-3.55271×10^-15
-3.55271×10^-15
```