Topic 5: Interdisciplinary Problems and Python Scripting

Lecture 5-1: Cellular Automata

Wednesday, March 31, 2010

Contents

1 Cellular Automaton Models

One of the first *Cellular Automaton* models was John von Neumann's *self-reproducing automaton* which he constructed in 1948 in an attempt to explain biological reproduction. The model had many interesting features, including the fact that it was equivalent to a universal Turing Machine. The most famous cellular automaton is probably Conway's Game of Life [W-Game-of-Life].

1.1 Basic ingredients of cellular automaton models

Cellular automata model dynamical systems using discrete approximations including

1. Continuous space x, y, z is replaced by a finite number of *cells* fixed in space, usually in a regular array or lattice.
2. Continuous dynamical functions are also approximated by a discrete set of values at each cell site.
3. Continuous time t is made discrete.
4. The dynamical equation of motion is replaced by a *local rule*: at each time step, cell values are given new values which depend on the cell values in a small *local neighborhood*.

5. The cell values are updated *simultaneously* or *synchronously*

1.2 One-dimensional Cellular Automaton

A simple 1-d cellular automaton consists of cells arranged in a line. Each cell is assumed to have two values which can be labeled with the binary digits 0 and 1. This is therefore called a *Boolean* cellular automaton.

A simple choice for the neighborhood of a cell is the cell itself and its two nearest neighbors.

The next value of a cell depends on the values of its neighboring cells. Since each neighbor can take 2 values, the total number of values of the neighbors is $2 \times 2 \times 2 = 8$. An example of a local rule is

t:	111	110	101	100	011	010	001	000
t + 1:	0	1	0	1	1	0	1	0

The total number of such rules is $2^8 = 256$, which is the number of different 8-digit binary numbers, i.e., the number of different possibilities on the second line. The above rule is called rule 90 from the decimal representation of the second line

$$\begin{aligned} 01011010 &= 0 \times 2^7 + 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 \\ &\quad +0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\ &= 64 + 16 + 8 + 2 = 90\ . \end{aligned}$$

Wolfram studied analyzed cellular automaton models like this one and concluded that there are 4 possible types of behavior for such automata:

- **Limit point behavior:** The cell values tend to a unique fixed state independent of the initial state.
- **Limit cycle behavior:** Stable periodic structures emerge.
- **Chaotic behavior:** The time evolution is non-periodic.
- **Complex behavior:** Complex and localized propagating structures are formed.

2 Introduction to Python Scripting

2.1 Python: Batteries Included

A recent issue [CiSE-V9I3] of Computers in Science and Engineering describes the use of Python in computational science and engineering.

- Python is primarily an interpreted language, similar to Mathematica, in which statements can be evaluated immediately.
- Python has a very simple and clean syntax, and Python code looks very much like pseudocode.
- M.I.T. 6 has switched to Python `http://www.youtube.com/watch?v=k6U-i4gXkLM`

- Python is free and opensource, and many high-quality libraries have been developed for various applications. The following libraries are recommended in one of the CiSE articles:

  SciPy: Scientific Tools for Python – `http://www.scipy.org/`

  iPython: an Interactive Computing Environment – `http://ipython.scipy.org/`

  Matplotlib: 2-D Plotting Library – `http://matplotlib.sourceforge.net/`

  MayaVi: a 3-D Data Visualizer – `http://mayavi.sourceforge.net/`

  Swig: Simplified Wrapper and Interface Generator – connect to C/C++ – `http://www.swig.org/`

  Python Cheese Shop: list of Python packages – `http://pypi.python.org/pypi`

2.2 Python code for 1-d cellular automaton

- Python is very popular with programmers
- Google finds this `http://code.activestate.com/recipes/343386-wolfram-style-cellular-aut`
- Need to install Python Imaging Library `http://www.pythonware.com/products/pil/`

References

[W-CA] Wikipedia: Cellular automaton, `http://en.wikipedia.org/wiki/Cellular_automaton`.

[W-Game-of-Life] Wikipedia: Conway's Game of Life, `http://en.wikipedia.org/wiki/Conway's_Game_of_Life`.

[Wolfram-Atlas] The Wolfram Atlas: One-dimensional cellular automata: `http://atlas.wolfram.com/TOC/TOC_200.html`.

[CiSE-V9I3] Computing in Science and Engineering, Volume 9, Issue 3, May-June 2007, `http://ieeexplore.ieee.org/xpl/tocresult.jsp?isnumber=4160244`.