Topic 5: Interdisciplinary Problems and Python Scripting

Lecture 5-4: Earthquake Dynamics

Wednesday, April 7, 2010

Contents

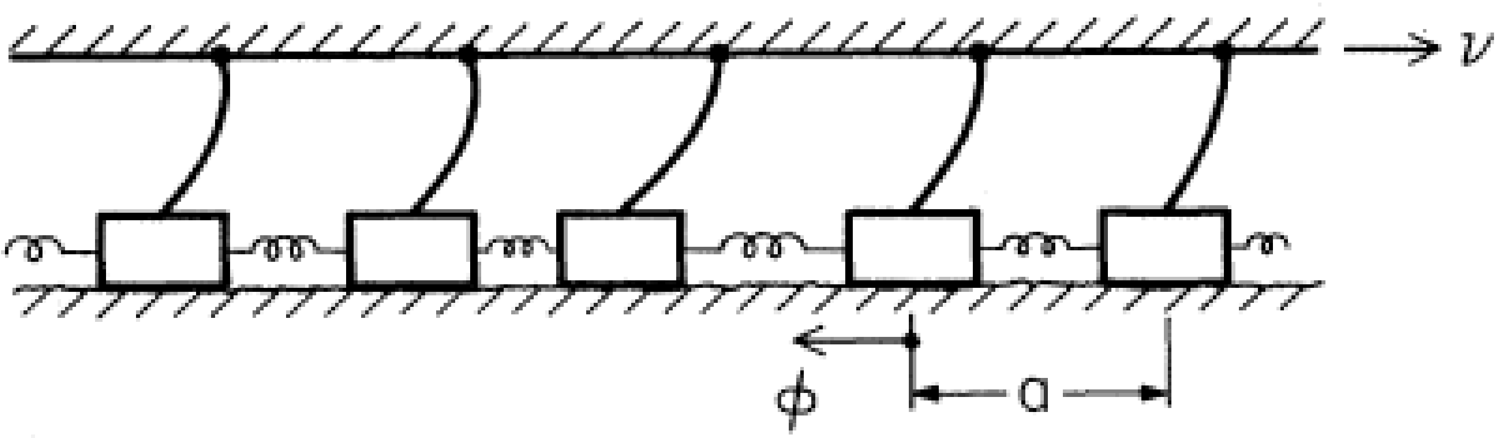
1 Earthquakes and Self-Organized Criticality

1.1 Earthquakes and the Gutenberg-Richter Law

- Every year, very large numbers of earthquakes occur around the world
- The magnitude of an earthquake is measured on the Richter Scale
- The distribution of earthquake magnitudes obeys the Gutenberg-Richter Law
- Carlson, Langer and Shaw, Rev. Mod. Phys. **66**, 657 (1994) review dynamics of earthquake faults using a simple physical model of masses and springs

2 Self-organized Criticality

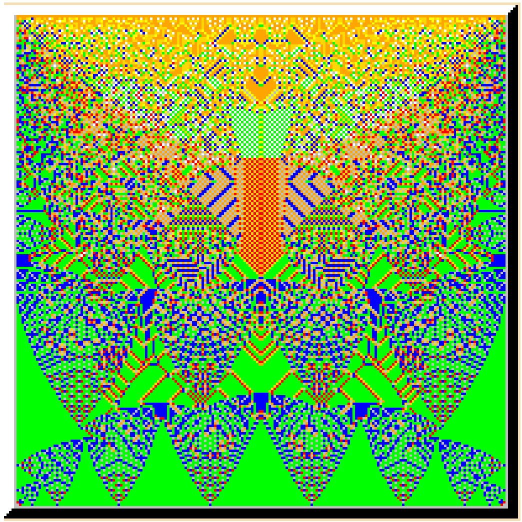
- Wikipedia Self-organized criticality
- Per Bak Wikipedia, Remembrance by Lee Smolin
- Bak, Tang and Wiesenfeld, Phys. Rev. Lett. **59**, 381 (1987) uses SOC to explain $1/f$ noise.

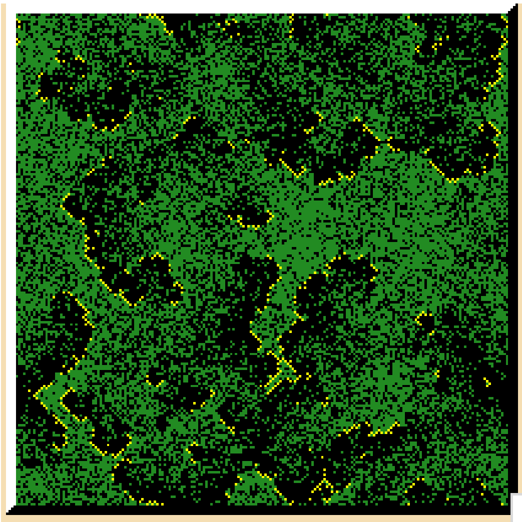


- Per Bak and Chao Tang, J. Geophy. Res. **94**, 15635 (1989) uses a simple probabilistic cellular automaton model to explain the Gutenberg-Richter law.
- Per Bak, Computers in Physics, **5**, 430 (1994), SOC in cellular automaton models of sandpiles, earthquakes, forest fires, etc.
- See Mike Creutz's Xtoys programs xsand.c and xtoys.c shown in the figure
- Simple 1-d sandpile automaton Java applet

2.1 Bak-Tang Earthquake Fault Model

- Physical picture
  - Earth's crust is periodic 2-D lattice





    * Blocks at each site
    * Connected by springs
    * Static and dynamic friction (stick-slip motion)
  - Blocks are pulled in a fixed direction by force due to tectonic plate motion

- The model
  - 2-D square lattice of side L with $N = L^2$ sites
  - Real variable $z(i, j)$ represents force on block at site (i, j)
    * Slip occurs at a critical threshold force $z_{\rm cr}$, e.g., $z_{\rm cr} = 4$
  - Initialize at time $t = 0$
    * Assign small random values to $z(i, j)$

– Repeat
  * Increase each $z(i, j)$ by small p, e.g., $p = 0.00001$ – (synchronous update), and let $t \rightarrow t + 1$
  * If *all* $z(i, j) \leq z_{\rm cr}$, fault is stable, go to Repeat
  * While any site (i, j) is unstable, decrease force by $z_{\rm cr} = 4$ at (i, j) and increase force at each neighbor by $z_{\rm cr}/4 = 1$
– The size of the event s is the total number of blocks that become unstable and slide
– Note that an instability can trigger a very large event!

Slip event sizes s as a function of time t

Distribution of sizes $N(s)$ versus s

Earthquake statistics for the New Madrid zone from Johnson and Nava, J. Geophys. Res. B**90**, 6737 (1985).

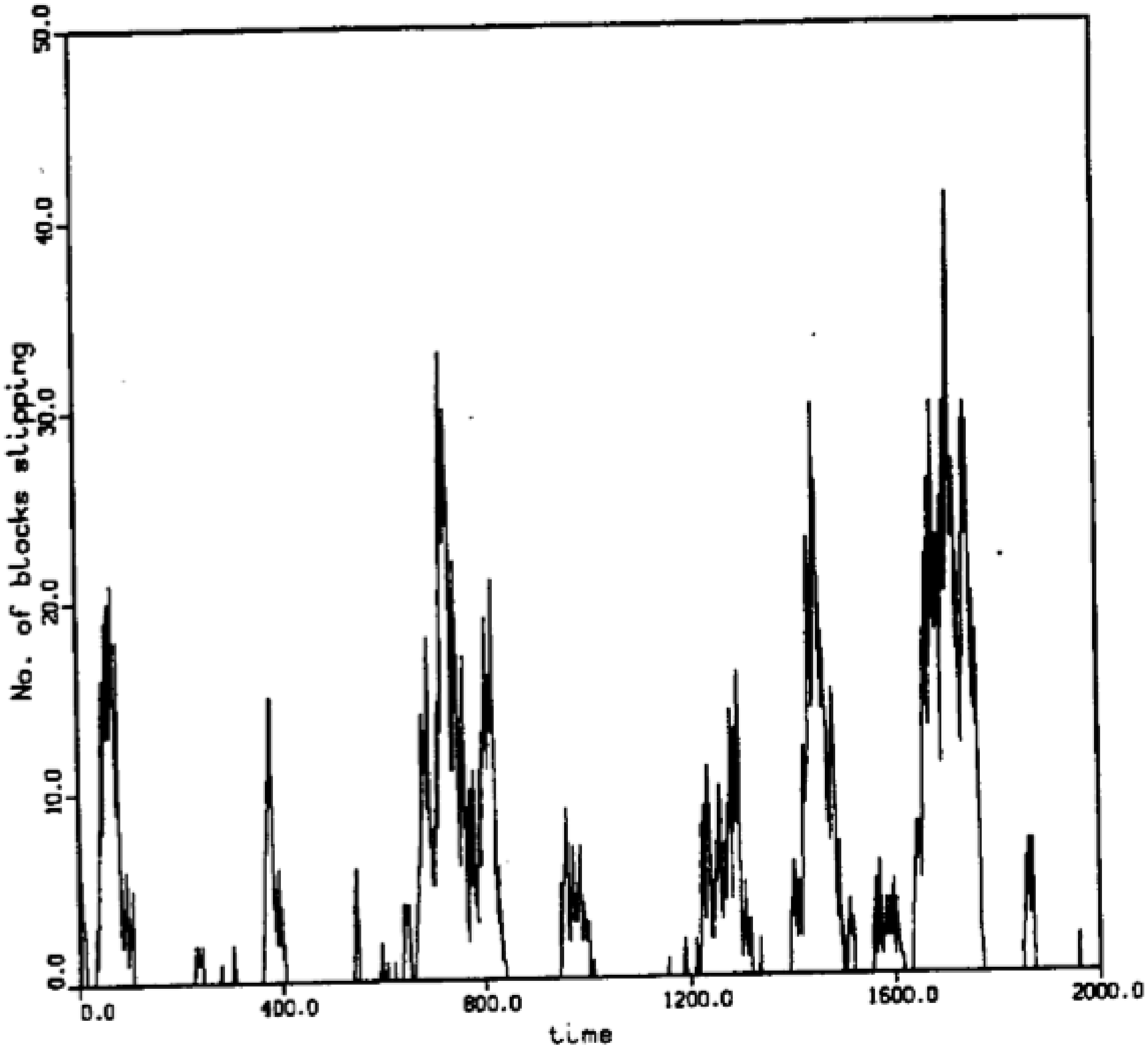
2.2 Gutenberg-Richter Law

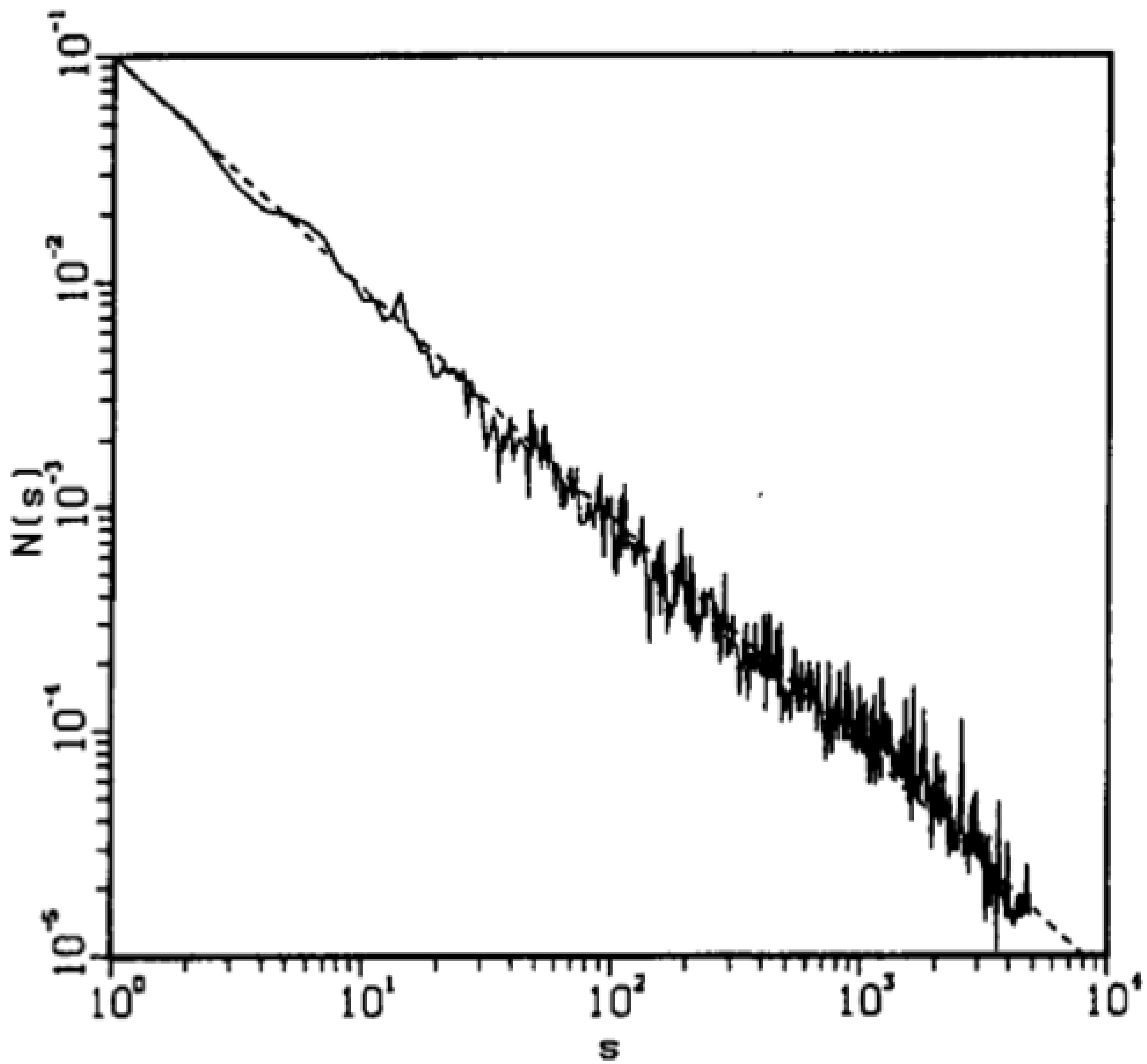
- Moment M of quake is the relative shift of fault
- Magnitude on the Richter scale

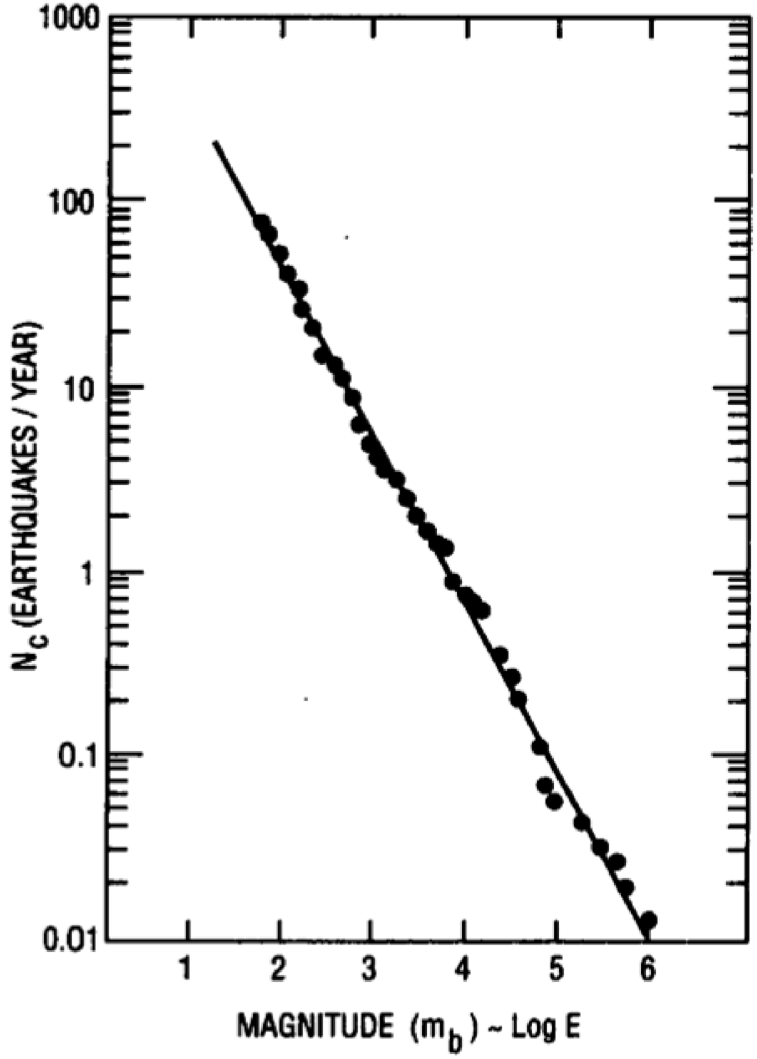
$$\mathcal{M} = \ln M$$

- Gutenberg-Richter law: probability of quake of magnitude $\mathcal{M}$

$$P(\mathcal{M}) = \frac{A}{M^b} = Ae^{-b\mathcal{M}}$$


No. of blocks slipping
time
0.0
10.0
20.0
30.0
40.0
50.0
0.0
400.0
800.0
1200.0
1600.0
2000.0


N(s)
s
10^-1
10^-2
10^-3
10^-4
10^-5
10^0
10^1
10^2
10^3
10^4


1000
100
10
1
0.1
0.01
N_c (EARTHQUAKES / YEAR)
1
2
3
4
5
6
MAGNITUDE (m_b) ~ Log E

where A is a constant and $b \simeq 0.8 - 1.5$ is a power-law exponent

- Energy released in event $E \propto M$
- Average energy released per event

$$E_{\text{average}} = \int_0^\infty EAe^{-b\mathcal{M}} \mathrm{d}\mathcal{M} \sim \int_0^\infty e^{(1-b)\mathcal{M}} \mathrm{d}\mathcal{M} = \frac{1}{1-b} e^{(1-b)\mathcal{M}} \Big|_0^\infty$$

diverges if $1 - b > 0$

- Maybe an indication that Earth is in a self-organized critical state!

2.3 Burridge-Knopoff Model

- Described in Section 12.2 of the textbook
- Top plate moves slowly with speed v_0 relative to bottom plate
- Crust between plates modeled by
  - Discrete masses m_i at positions x_i and velocities v_i
    - Simplest 1-d model has N blocks in a line with equal masses m
  - Attached to top plate with leaf-springs with constant k_p
  - Attached to neighboring masses with coil springs constant k_c
  - In frictional contact with bottom plate

* Static friction has maximum magnitude F_0
* Kinetic friction given by

$$F_f = -\frac{F_0 \text{sign}(v_i)}{1 + |v_i/v_f|}$$

  where v_f is a model parameter

- System undergoes stick-slip motion
- Newton's equations of motion for the blocks

$$\begin{aligned} m_i \frac{\mathrm{d}x_i}{\mathrm{d}t} &= v_i \\ m_i \frac{\mathrm{d}v_i}{\mathrm{d}t} &= k_c(x_{i+1} + x_{i-1} + 2x_i) + k_p(v_0 t - x_i) + F_f \end{aligned}$$

- Choose a time step Δt and use Euler's algorithm
- The frictional force is a little tricky!
  - If m_i is not moving at time step n, find the net spring force
    * If $|F_{i,\text{springs}}| \leq F_0$, static friction adjusts to make the net force on m_i exactly zero
    * If $|F_{i,\text{springs}}| > F_0$, static friction will have magnitude F_0 and oppose the net springs force
  - If m_i is moving at time step n, find the new velocity assuming kinetic friction
    * If the new velocity has same sign as the old velocity, the block continues slipping, so proceed

  * Else, set the new velocity to zero, the block sticks!

- For the simulations
  - Choose parameter, e.g., $N = 25, m = 1, k_p = 40, k_c = 250, F_0 = 50, v_0 = 100$
  - Choose a small enough time step, e.g., $\Delta t = 0.005$
  - Initialize the blocks at rest with random displacements from equilibrium of magnitude e.g., ± 0.001, otherwise the motion tends to be periodic!
  - Run the simulation for total times $t \sim 500$
- To measure the magnitude of an event:
  - The quake starts when any block begins to slip
  - The quake ends as soon as all blocks stick again
  - The moment of the quake is measured by

$$M = \sum_{\text{while slipping}} \left(\sum_i^N v_i \Delta t \right)$$

- To test the Gutenberg-Richter law:
  - Run a large number of simulations and make a histogram of magnitudes

$$\mathcal{M} = \ln M$$

  - Make a log-log plot of number of events versus $\mathcal{M}$

2.4 Cellular-Automaton Earthquake Model

- The Burridge-Knopoff model is computationally demanding
- Rundle, Jackson and Brown introduced a simplified cellular-automaton version, see Ferguson, Klein and Rundle, Computers in Physics, **12**, 34 (1998)
  - Use noninerital massless blocks with *slip deficit* variable $\phi_i(t)$, which measures the amount by which the block lags the upper plate
  - Each block has a static failure threshold $\sigma_{F,i}$ and a residual stress $\sigma_{R,i}$
  - The net stress on each block is

$$\sigma_i(t) = -K_L\phi_i(t) + K_C \sum_{q \text{ nearest neighbors } j} (\phi_j(t) - \phi_i(t))$$

    For a 2-d fault model $q = 4$

  - A block with $\sigma_i(t) > \sigma_{F,i}$ slips to a residual stress state $\sigma_{R,i}$ using

$$\phi_i(t+1) = \phi_i(t) + \frac{\sigma_i(t) - \sigma_{R,i}}{K_L + qK_C}\theta(\sigma_i(t) - \sigma_{F,i})$$

    where $\theta(x)$ is the unit step function $= 1$ if $x > 0$, zero otherwise. This is a deterministic slip model. Probabilistic slip rules can also be defined.

  - A quake is simulated by repeating the slippage rule on all blocks and and incrementing $t \to t + 1$ until all $\sigma_i(t) < \sigma_{F,i}$

– Simulate time between quakes by moving the top plate by $V\Delta T$ and decreasing each ϕ_i by this amount until the next quake occurs

References

[Bak-1987] P. Bak, C. Tang and K. Wiesenfeld, "Self-organized criticality: An explanation of the $1/f$ noise", Phys. Rev. Lett. **59**, 381 (1987), `http://doi.org/10.1103/PhysRevLett.59.381`.

[W-CA] Wikipedia: Cellular automaton, `http://en.wikipedia.org/wiki/Cellular_automaton`.

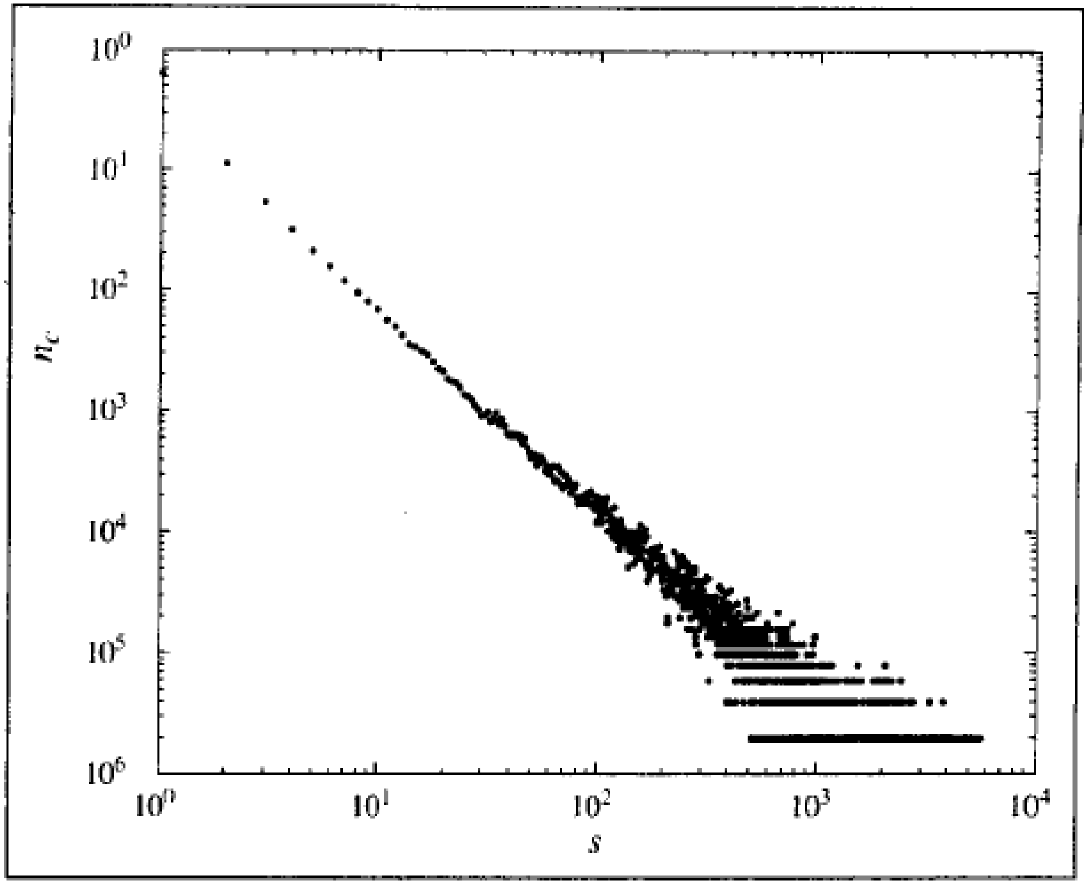


Figure 2. Log–log plot of the mean number of clusters $n_c(s)$ with s failed blocks for $L=128$, nearest-neighbor interactions, and closed boundary conditions. The parameters are $\sigma_F=50$, $\sigma_R=0$, $K_C=K_L=1$, and the zero-velocity-limit with the deterministic jump function. The negative of the slope corresponds to a cluster scaling exponent $\tau=1.6$ over the interval $1\leqslant s\leqslant 300$, where $n_c(s)\sim s^{-\tau}$, implying that $b=0.9$.