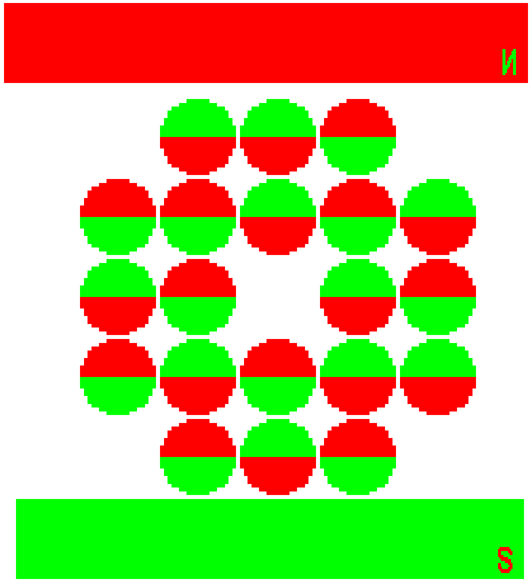
Lec 9
(MRI: the role of B_o field)

What decides how many spins will be up and how many will be down?

- Temperature
- The actual numbers are given by the Boltzmann distribution

Boltzmann distribution of spins



$$N^-/N^+ = \exp(-\Delta E/kT)$$

N^-: Number of spins at higher energy

N^+: Number of spins at lower energy

ΔE: Energy difference between two states

Signal ~ population difference between two states ($N^+ - N^-$).

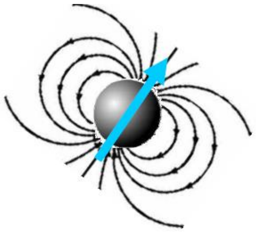
1. What will be the value of N^- when $T = 0$?

2. When do you think N^+ and N^- are likely to be equal?

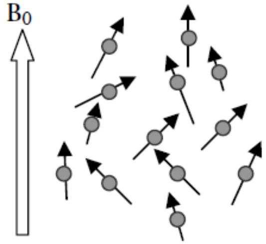
$$N^-/N^+ = \exp(-\Delta E/kT)$$

What happens during MRI? (1)

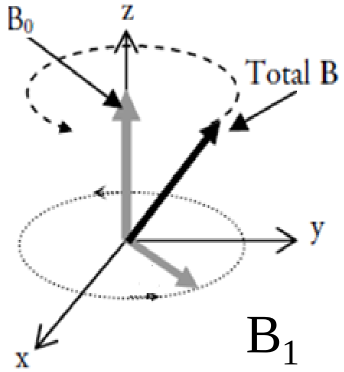
1. Hydrogen nuclei in tissue have “spin angular momentum” and associated magnetic moment.



2. In an external magnetic field (B_o), M_z lines up with B_o (along z-axis).

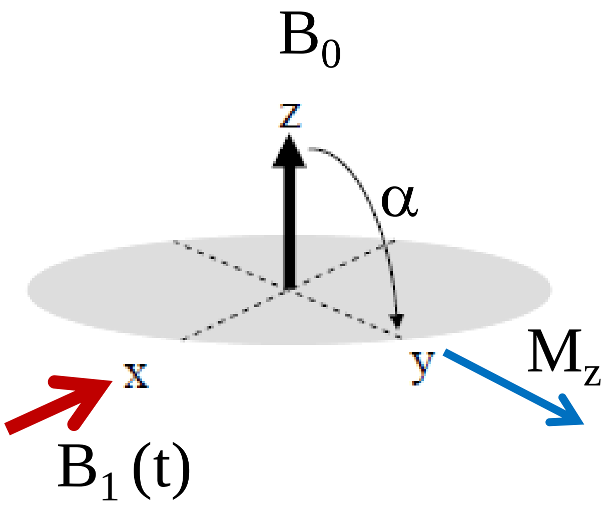


3. A rotating magnetic field (B_1) pulse is applied along x-axis.

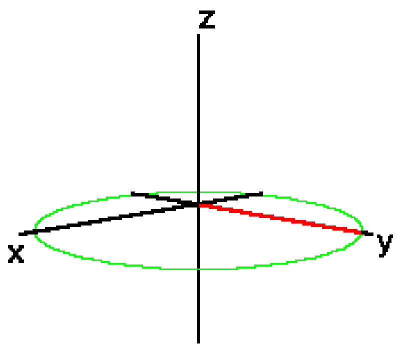


What happens during MRI? (2)

4. B_1 pulls away magnetization (M_z) from the z-axis with an angle α.

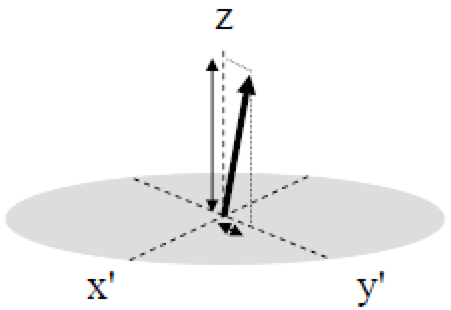


5. M_z rotates around z-axis at the "Larmor frequency".



What happens during MRI? (3)

6. B_1 is turned off. Only B_o remains.

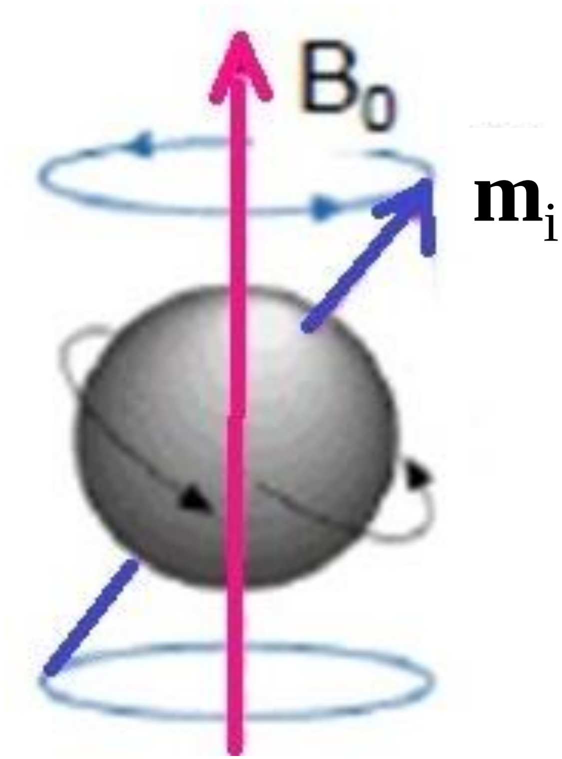


7. The XY-projection of M_z reduces with time, while Z-projection increases and returns to its equilibrium value ("relaxation").

8. Relaxation of M_z to its equilibrium value produces a voltage signal, which we measure.

Once we have grasped these concepts, we will bring on gradient field.

Larmor equation



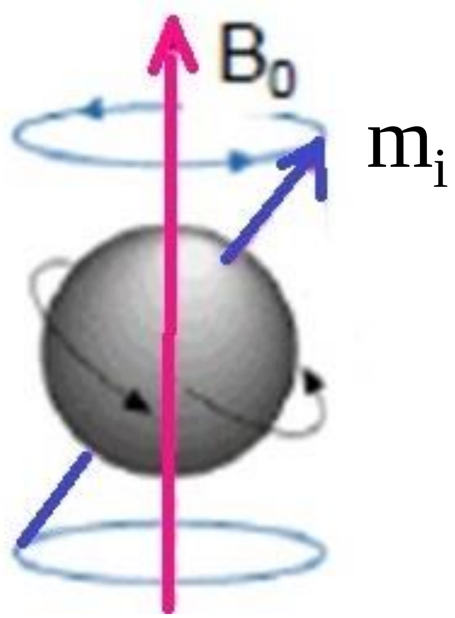
Torque ($\mathbf{T}$) = ($\mathbf{m}_i$ x $\mathbf{B_o}$) = d$\mathbf{L_i}$/dt

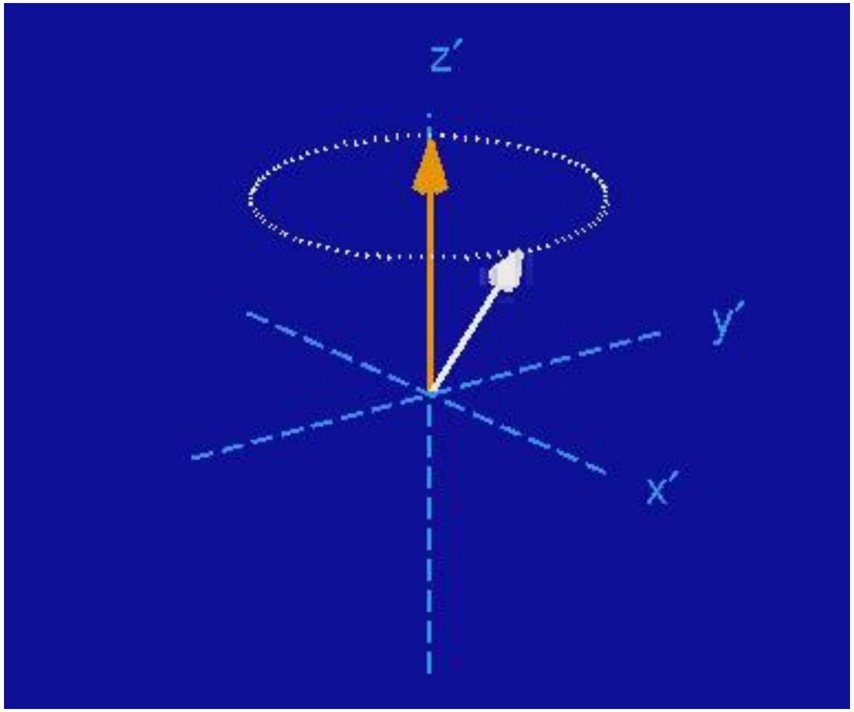
γ ($\mathbf{m}_i$ x $\mathbf{B_o}$) = d$\mathbf{m_i}$/dt

(since $\mathbf{m} = \gamma\mathbf{L}$)

For hydrogen, gyromagnetic ratio (γ)□= 42.58 MHz/Tesla

Larmor precession





Individual spin magnetic moments will precess about the magnetic field with Larmor frequency ($\nu = \gamma B_o$).

Gyromagnetic ratio (γ)

Individual spin magnetic moments will precess about the magnetic field with Larmor frequency ($\nu = \gamma B_o$).

For hydrogen, gyromagnetic ratio (γ)□= 42.58 MHz/Tesla

Nuclei with higher γ will precess faster in a given magnetic field.

Precession angle

- *Quantum mechanics* allows specific values of m_z. This makes only specific precession angles possible.

- Precession angle can have any value in *classical mechanics*.

Different nuclei precess with different Larmor frequencies (due to different g)

Element	**Biological Abundance**	**γ**
^{1}H	0.63	42.58
^{13}C	0.094	10.71
^{23}Na	0.00041	11.26
^{39}K	0.0024	1. 99

Need unpaired spin. Why?

Nuclear magnetic moment

- Both protons and neutrons can have magnetic moment. *This is why our current-carrying loop explanation of spin is oversimplified (i.e. this can't explain why neutrons have magnetic moment)*.

- The magnetic moments of a proton and a neutron <u>do not</u> exactly cancel each other.

- A nucleus with either an odd number of protons or odd number of neutrons will have a net magnetic moment.
- Why does ^{14}N have a net magnetic moment then?

Nuclide	*Number of Protons*	*Number of Neutrons*
^{1}H	1	0
^{2}H	1	1
^{13}C	6	7
^{14}N	7	7
^{17}O	8	9
^{19}F	9	10
^{23}Na	11	12
^{31}P	15	16
^{39}K	19	20

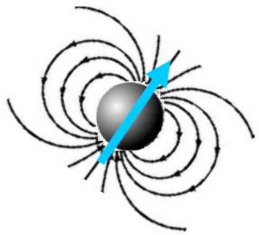
Net bulk magnetization is along B_o

Bulk magnetization: $\mathbf{M} = \sum_{i=1}^{N} \mathbf{m}_i$

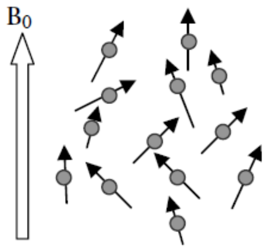
$\langle M_z \rangle \neq 0$, $\langle M_x \rangle = 0$, $\langle M_y \rangle = 0$

What happens during MRI? (1)

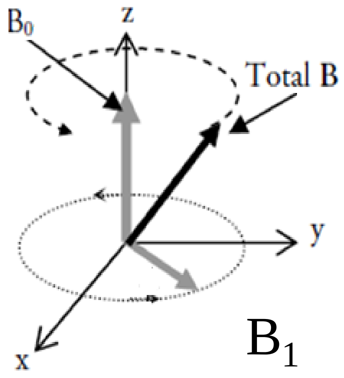
1. Hydrogen nuclei in tissue have “spin angular momentum” and associated magnetic moment.



2. In an external magnetic field (B_o), M_z lines up with B_o (along z-axis).

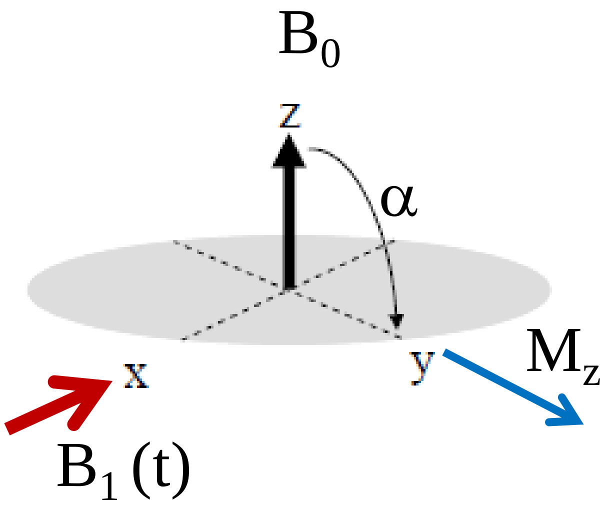


3. A rotating magnetic field (B_1) pulse is applied along x-axis.



What happens during MRI? (2)

4. B_1 pulls away magnetization (M_z) from the z-axis with an angle α.



5. M_z rotates around z-axis at the "Larmor frequency".