Lec 7
Introduction to MRI

Discovery of MRI led to Nobel prize in Physiology or Medicine in 2003

Press Release

6 October 2003

The Nobel Assembly at Karolinska Institutet has today decided to award
The Nobel Prize in Physiology or Medicine for 2003 jointly to

Paul C Lauterbur and **Peter Mansfield**

for their discoveries concerning **"magnetic resonance imaging"**

Can you guess how many more Nobel prizes are related to magnetic resonance?

1952 (Physics): Bloch and Purcell (*new methods for nuclear magnetic precision measurements)*

1991 (Chemistry): Ernst (*methodology for high precision nuclear magnetic resonance spectroscopy*)

2002 (Chemistry): Wuthrich (*nuclear magnetic resonance spectroscopy for determining the 3D structure of biological macromolecules in solution)*

Reference material for MRI

- Hendee: chapters 23 and 24
- Smith and Webb: pages 204 - 222
- There's an online book by Joseph Hornak at *http://www.cis.rit.edu/htbooks/mri/inside.htm*

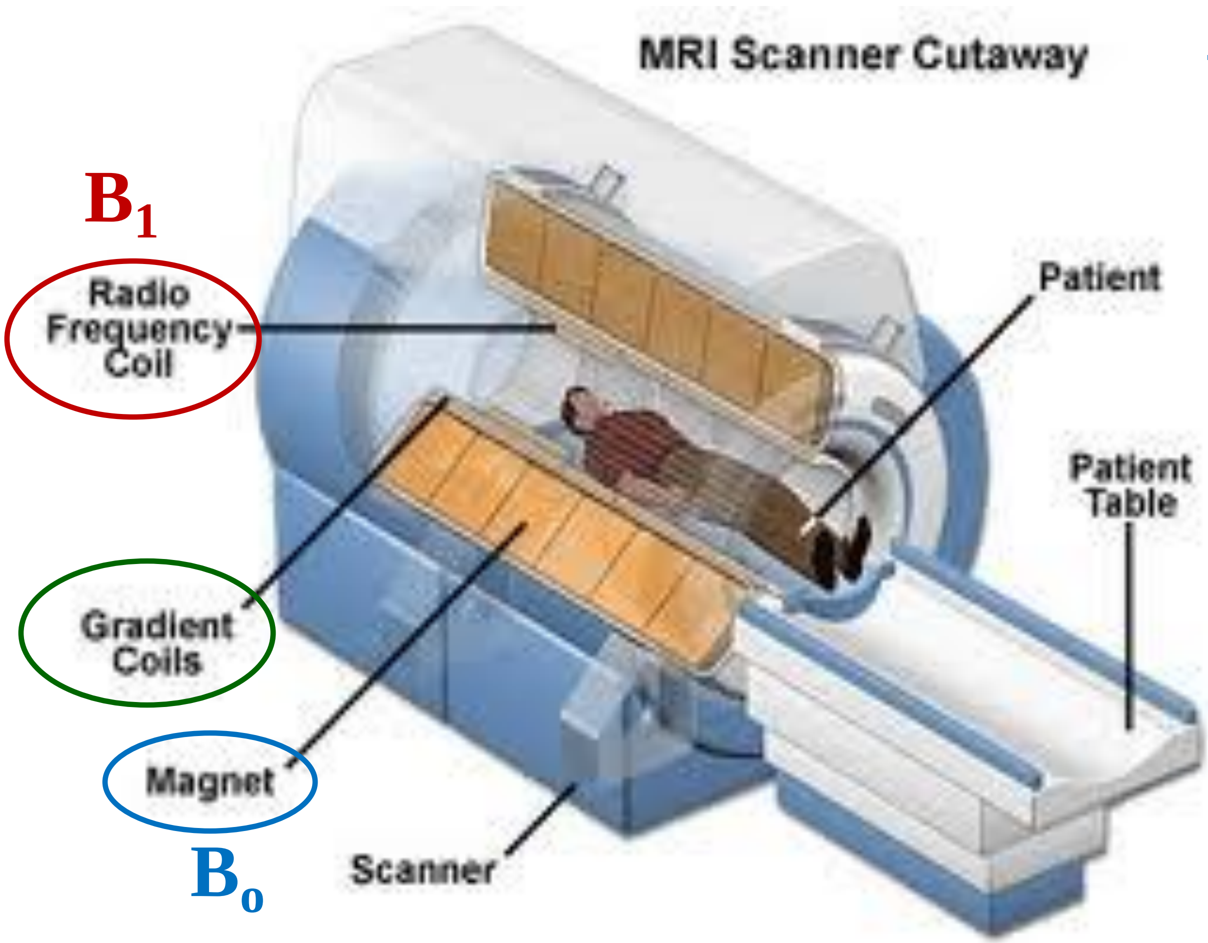
Magnetic Resonance Imaging

Interaction between spin magnetic moments in tissue and external magnetic fields.

Frequency (RF) of external magnetic field matches with "some" internal frequency in tissue.

Deals with behavior of atomic **nuclei** in magnetic field.

What magnetic fields do we have?



- Steady magnetic field, B_o (initially align spins)
- RF magnetic field, B_1 (excites aligned spins)
- Spatial modulation of B_o for image encoding

https://www.researchgate.net/figure/MRI-scanner-cutaway-thanks-for-the-image-from-34_fig1_280792219

Some pre-MRI checks for patients

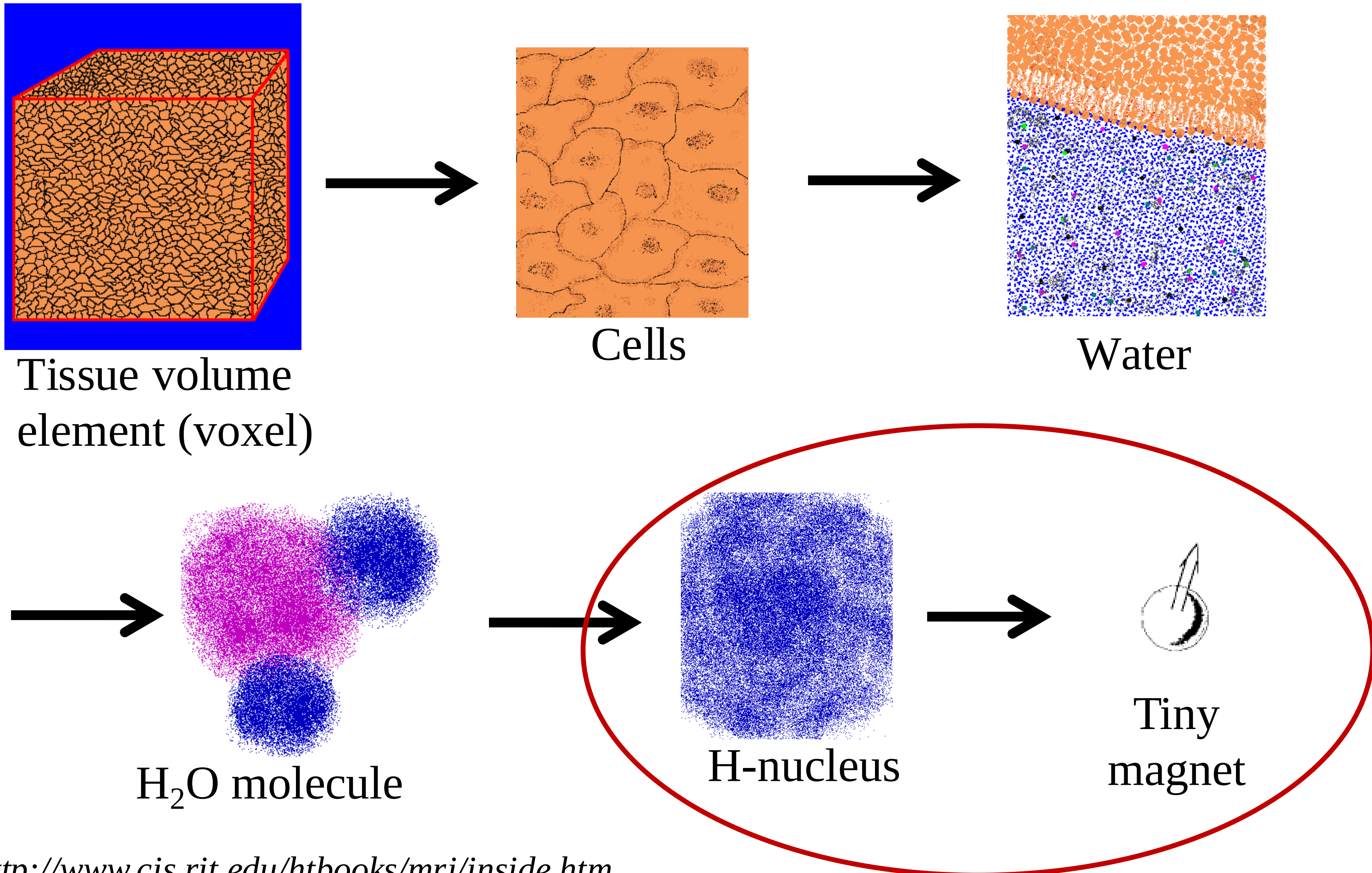
- Ferromagnetic objects
- Non-ferromagnetic objects: local image distortion
- Claustrophobia (~ 45 - 60 min)
- Movement (due to sneeze, cough, etc.)
- Acoustic protection (ear plugs for high noise levels)

Not all patients can undergo an MRI!

- Steady magnetic field ~ 1.5 - 3T
- Earth's magnetic field ~ 50μT

https://newcastleclinic.co.uk/mri-safety-week-precautions-to-take-before-an-mri-scan/

Origin of magnetic moment in tissue



http://www.cis.rit.edu/htbooks/mri/inside.htm

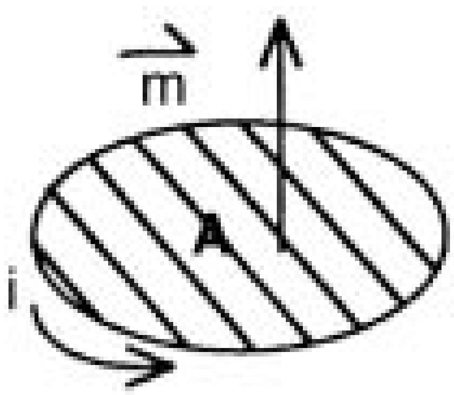
MRI measures signal from hydrogen nuclei

Element	Biological Abundance
Hydrogen	0.63
Carbon	0.094
Nitrogen	0.015
Sodium	0.00041
Phosphorus	0.0024
Calcium	0.0022
Oxygen	0.26

Frequency range

Clinical MRI: between 15 and 80 MHz for hydrogen imaging.

Classical magnetic moment (due to a current)

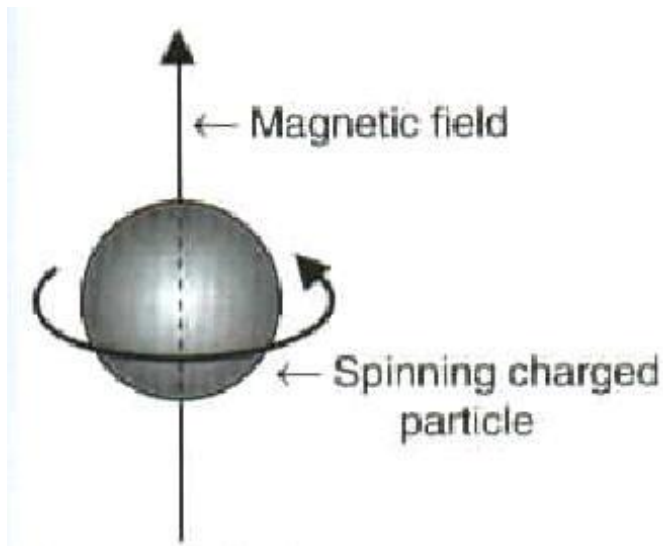


$$\vec{m} = i\vec{A}$$

Magnetic moment = (current) x (area of the current loop)

Magnetic moment of hydrogen nuclei

- Spin is actually a quantum mechanical concept. We give an oversimplified classical analogy in this course!
- Each spinning hydrogen nucleus (positive charge) has a “spin magnetic moment”.

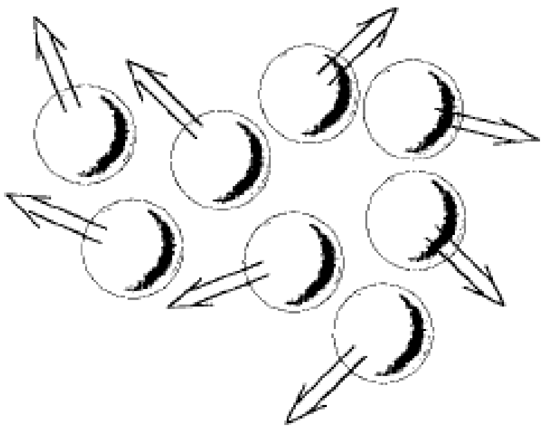


Calculate the number of spins inside a voxel.

- Take a voxel to be a cube of side 1 mm
- Hint: first find the number of water molecules in the voxel.

In absence of B_o

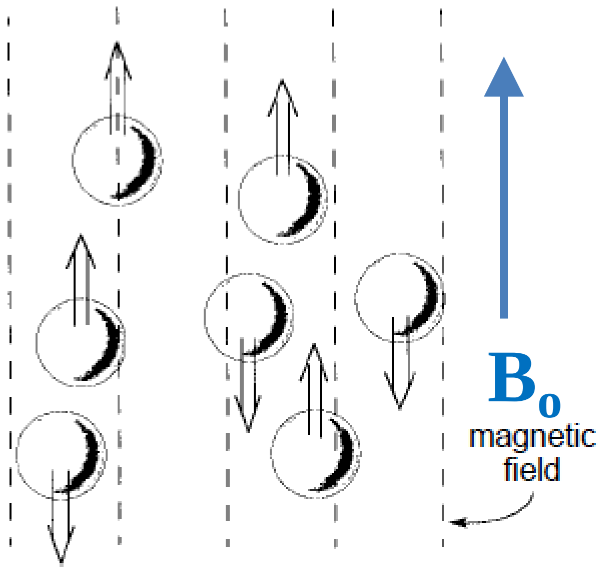
A single voxel (~ mm^3) has ~10^{19} spins.



Randomly oriented spins.
No net magnetization.

What happens in B_o field?

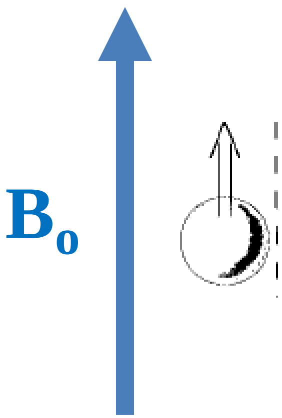
In presence of B_o

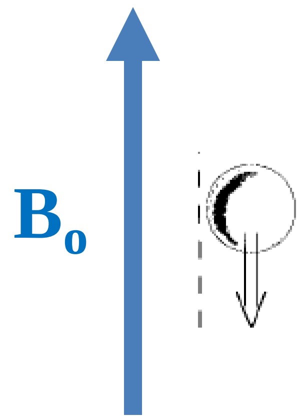


- Spins line up with $\mathbf{B_o}$
- Non-zero net magnetization

Some spins are parallel and some are anti-parallel to $\mathbf{B_o}$.

Spins are "quantized" in a magnetic field





Low energy

Parallel

High energy

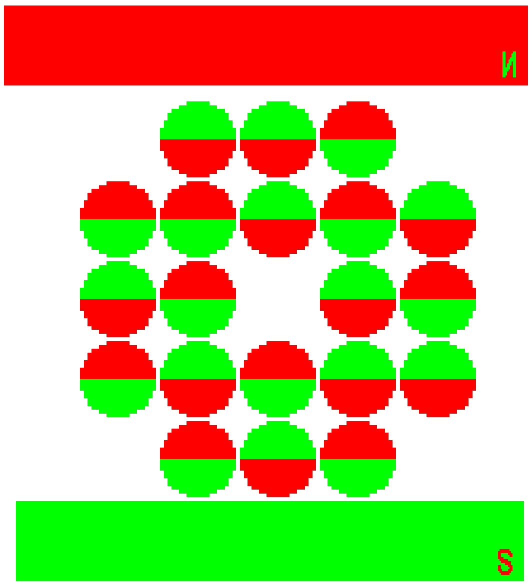
Anti-parallel

Lec 9
(MRI: the role of B_o field)

What decides how many spins will be up and how many will be down?

- Temperature
- The actual numbers are given by the Boltzmann distribution

Boltzmann distribution of spins



$N^-/N^+ = \exp(-\Delta E/kT)$

N^-: Number of spins at higher energy

N^+: Number of spins at lower energy

ΔE: Energy difference between two states

Signal ~ population difference between two states ($N^+ - N^-$).

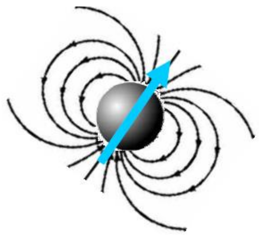
1. What will be the value of N^- when $T = 0$?

2. When do you think N^+ and N^- are likely to be equal?

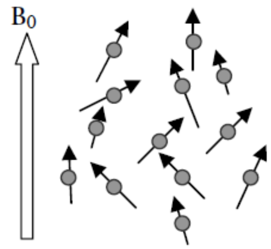
$$N^-/N^+ = \exp(-\Delta E/kT)$$

What happens during MRI? (1)

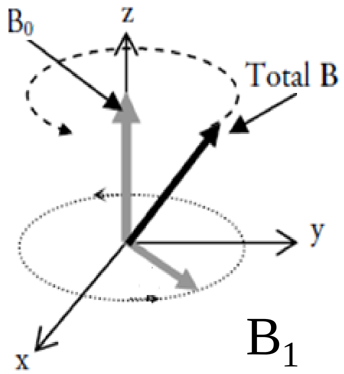
1. Hydrogen nuclei in tissue have “spin angular momentum” and associated magnetic moment.



2. In an external magnetic field (B_o), M_z lines up with B_o (along z-axis).

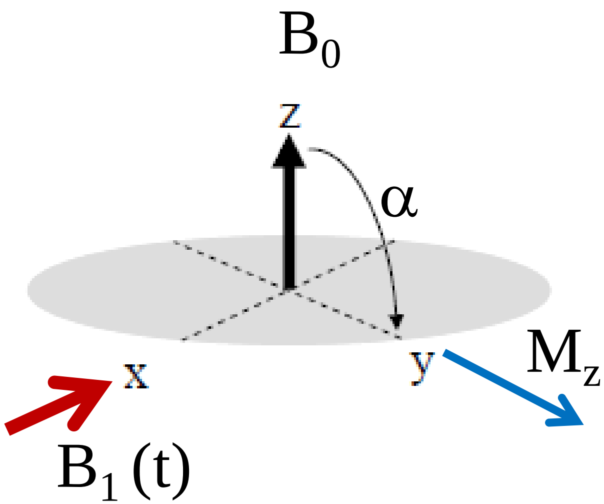


3. A rotating magnetic field (B_1) pulse is applied along x-axis.

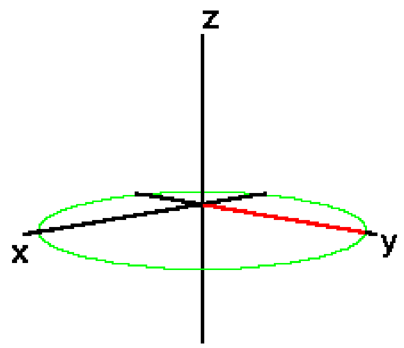


What happens during MRI? (2)

4. B_1 pulls away magnetization (M_z) from the z-axis with an angle α.



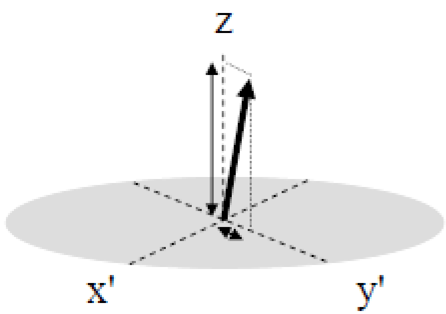
5. M_z rotates around z-axis at the "Larmor frequency".



What happens during MRI? (3)

6. B_1 is turned off. Only B_o remains.

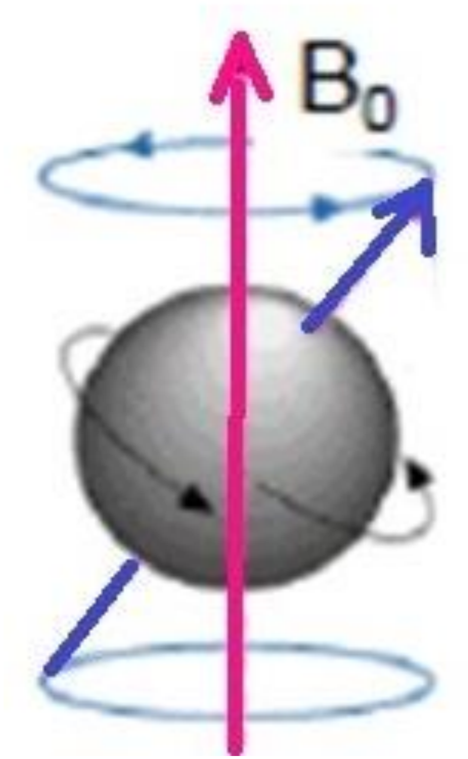
7. The XY-projection of M_z reduces with time, while Z-projection increases and returns to its equilibrium value ("relaxation").



8. Relaxation of M_z to its equilibrium value produces a voltage signal, which we measure.

Once we have grasped these concepts, we will bring on gradient field.

Larmor equation



$\mathbf{m}_i$

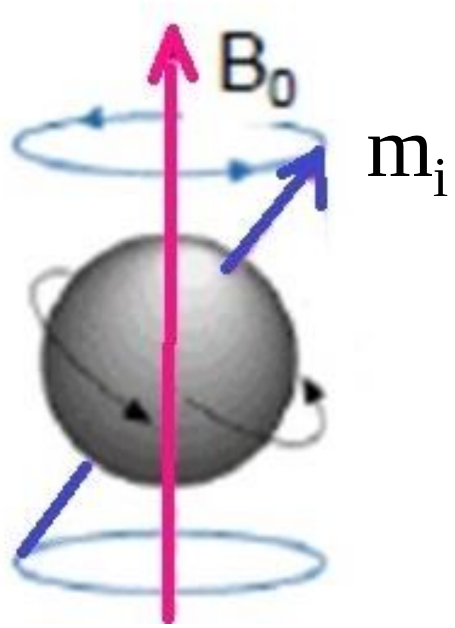
Torque ($\mathbf{T}$) = ($\mathbf{m}_i$ x $\mathbf{B_o}$) = d$\mathbf{L_i}$/dt

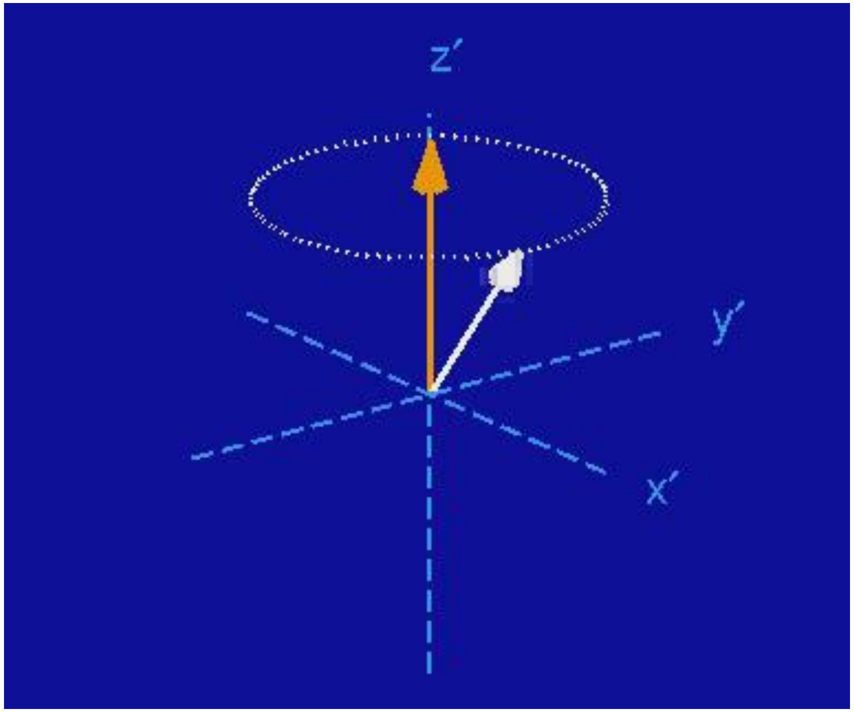
γ ($\mathbf{m}_i$ x $\mathbf{B_o}$) = d$\mathbf{m_i}$/dt

(since $\mathbf{m} = \gamma\mathbf{L}$)

For hydrogen, gyromagnetic ratio (γ)= 42.58 MHz/Tesla

Larmor precession





Individual spin magnetic moments will precess about the magnetic field with Larmor frequency ($\nu = \gamma B_o$).

Gyromagnetic ratio (γ)

Individual spin magnetic moments will precess about the magnetic field with Larmor frequency ($\nu = \gamma B_o$).

For hydrogen, gyromagnetic ratio (γ)□= 42.58 MHz/Tesla

Nuclei with higher γ will precess faster in a given magnetic field.

Precession angle

- *Quantum mechanics* allows specific values of m_z. This makes only specific precession angles possible.

- Precession angle can have any value in *classical mechanics*.

Different nuclei precess with different Larmor frequencies (due to different g)

Element	**Biological Abundance**	**γ**
^{1}H	0.63	42.58
^{13}C	0.094	10.71
^{23}Na	0.00041	11.26
^{39}K	0.0024	1. 99

Need unpaired spin. Why?

Nuclear magnetic moment

- Both protons and neutrons can have magnetic moment. *This is why our current-carrying loop explanation of spin is oversimplified (i.e. this can't explain why neutrons have magnetic moment)*.

- The magnetic moments of a proton and a neutron <u>do not</u> exactly cancel each other.

- A nucleus with either an odd number of protons or odd number of neutrons will have a net magnetic moment.
- Why does ^{14}N have a net magnetic moment then?

Nuclide	*Number of Protons*	*Number of Neutrons*
^{1}H	1	0
^{2}H	1	1
^{13}C	6	7
^{14}N	7	7
^{17}O	8	9
^{19}F	9	10
^{23}Na	11	12
^{31}P	15	16
^{39}K	19	20

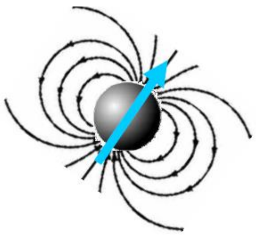
Net bulk magnetization is along B_o

Bulk magnetization: $\mathbf{M} = \sum_{i=1}^{N} \mathbf{m}_i$

$\langle M_z \rangle \neq 0$, $\langle M_x \rangle = 0$, $\langle M_y \rangle = 0$

What happens during MRI? (1)

1. Hydrogen nuclei in tissue have “spin angular momentum” and associated magnetic moment.



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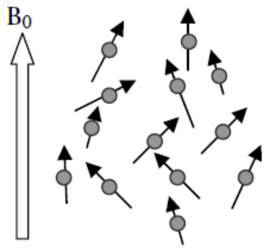

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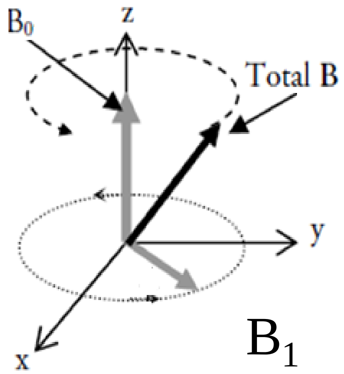

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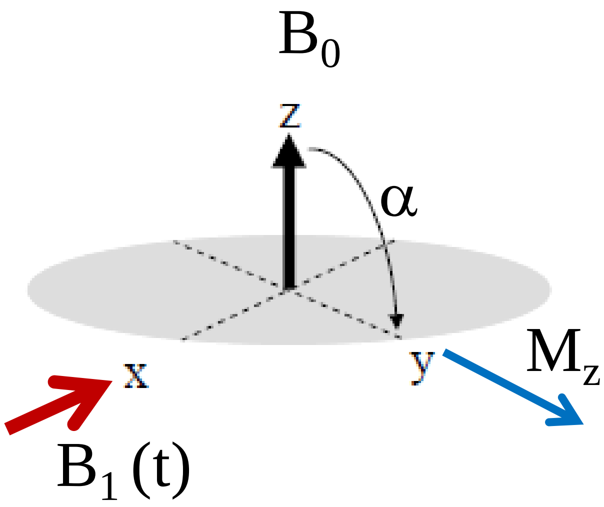

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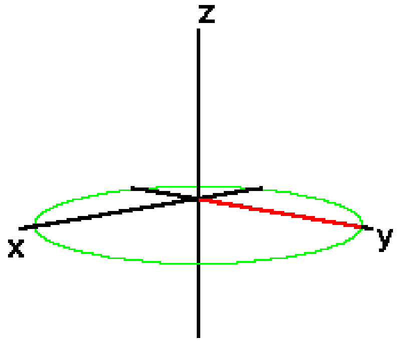

Lec 10
(MRI: the role of B_o and B_1 fields)

- A nucleus with either an odd number of protons or odd number of neutrons will have a net magnetic moment.
- Why does ^{14}N have a net magnetic moment then?

Nuclide	*Number of Protons*	*Number of Neutrons*
^{1}H	1	0
^{2}H	1	1
^{13}C	6	7
^{14}N	7	7
^{17}O	8	9
^{19}F	9	10
^{23}Na	11	12
^{31}P	15	16
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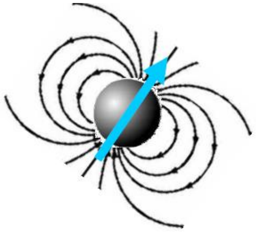

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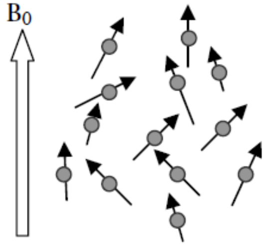

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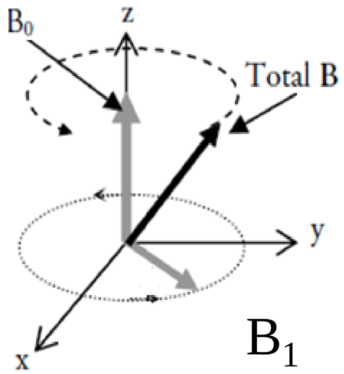

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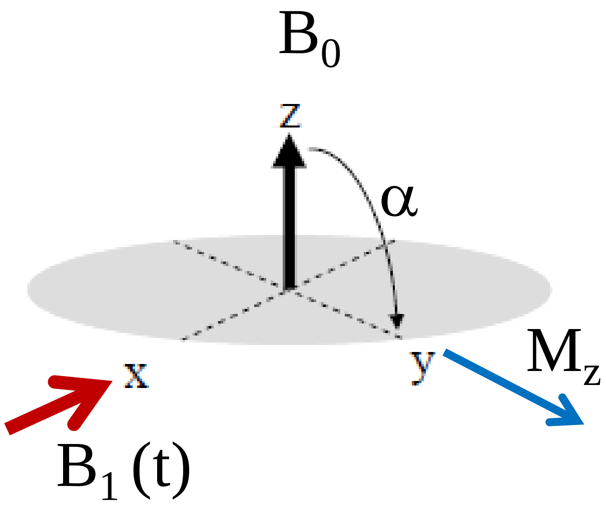

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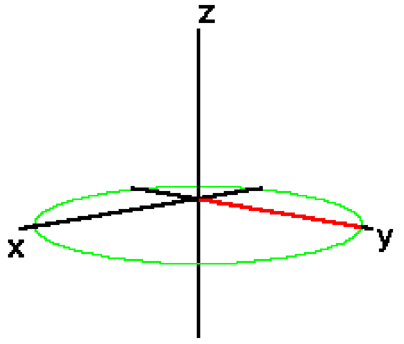
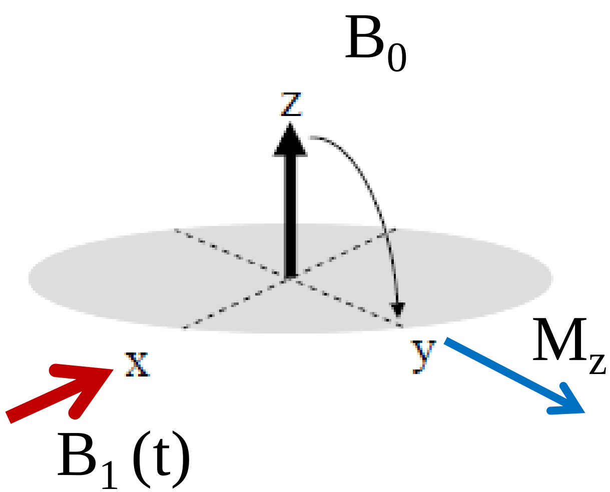


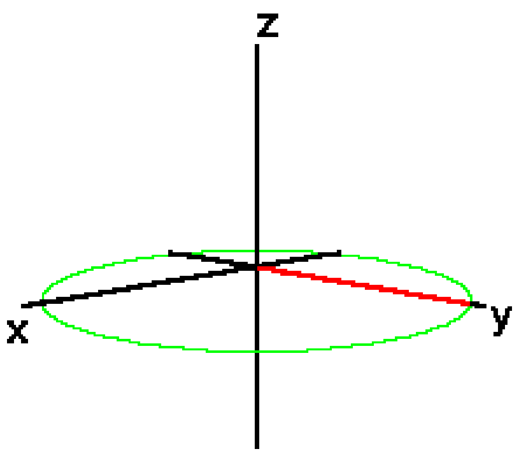
What happens in $B_1(t)$ field?

Magnetization in X-Y plane: RF field

- Apply RF pulse (10-100 MHz) B_1 along x-axis.
- B_1 (~ μT- mT) << B_o (~T).



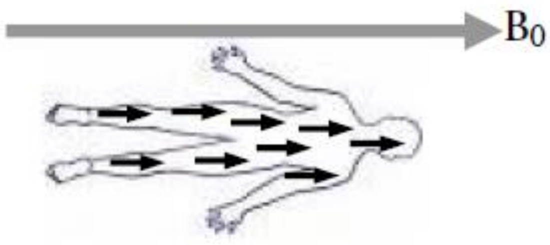
Net magnetization in xy plane (for π/2 pulse).



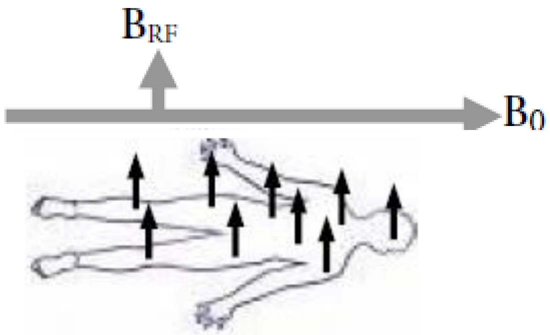
- M_z starts rotating about z-axis
- M_z returns to its equilibrium position along z-axis when RF field is turned off

If the magnitude of B_1 is much smaller (~ mT) than B_o (~ T), then how is it possible that B_1 can flip some spins?

How do we detect M_z?



Can't detect constant M_z (with B_o alone).

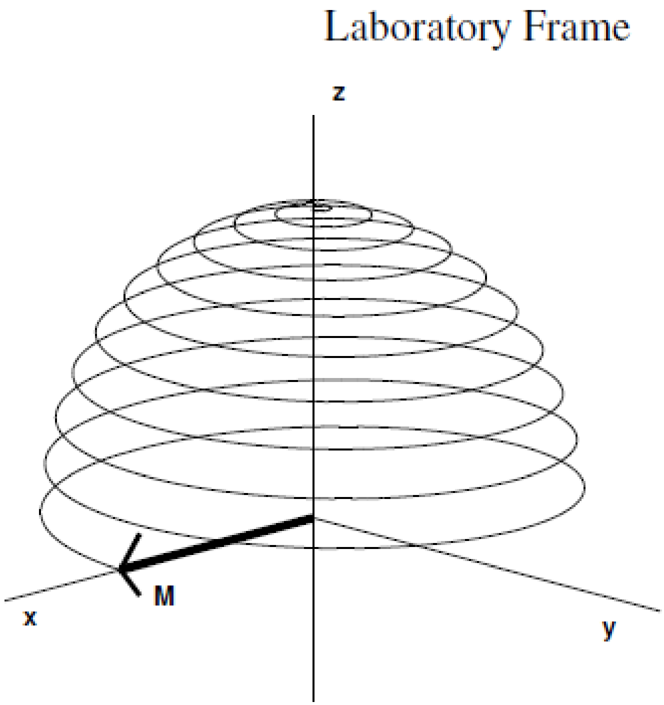


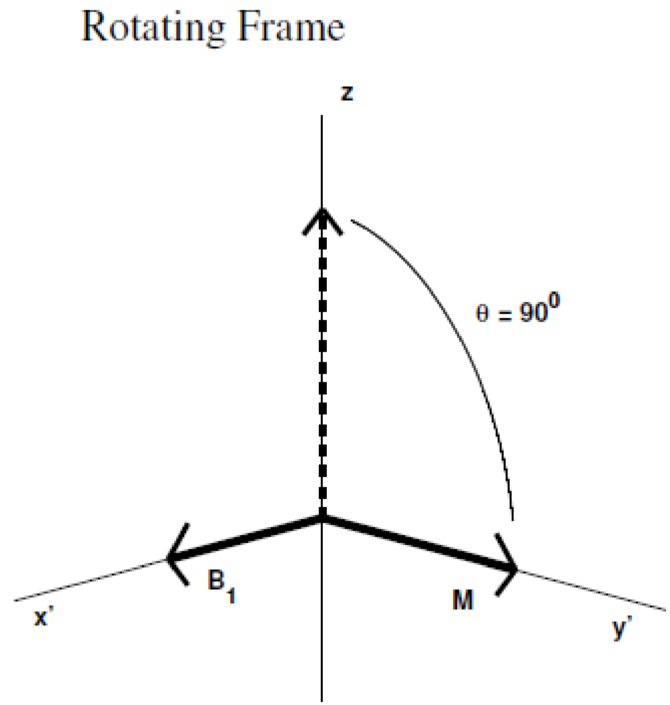
Can detect <u>time-varying</u> flux of M_z (generated using B_o and B_1)

RF coils for generating B_1 can also detect the voltage signal (Faraday induction).

$$V \sim - d\Phi/dt$$

We introduce a rotating reference frame. Why?



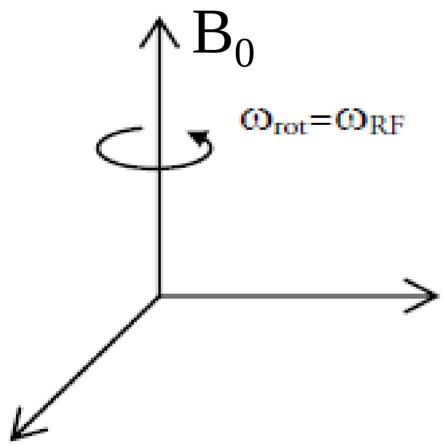


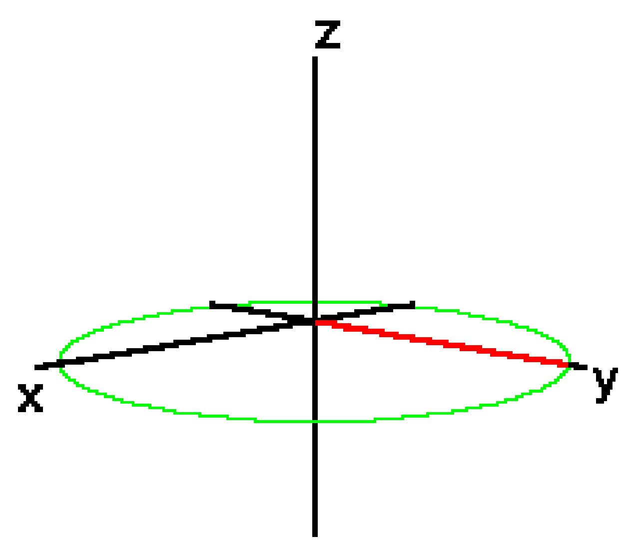
- In reality, each spin sees a slightly different magnetic field. Solving equations (that describe the time evolution of the magnetization vector, **M**) becomes a nightmare.
- All MRI hardware has been designed to work assuming a rotating reference frame.

Frame rotates about z-axis with Larmor frequency.

Lab frame coordinates: x, y, z

Rotating frame coordinates: x′, y′, z′

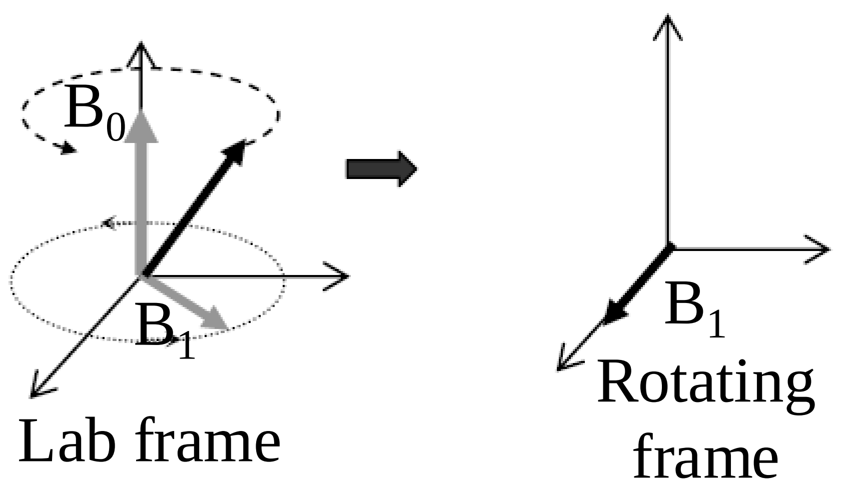




What do we see if we are sitting on the rotating frame?

B_1 is constant.

B_o vanishes.



Lec 11
(MRI: B_1 field)

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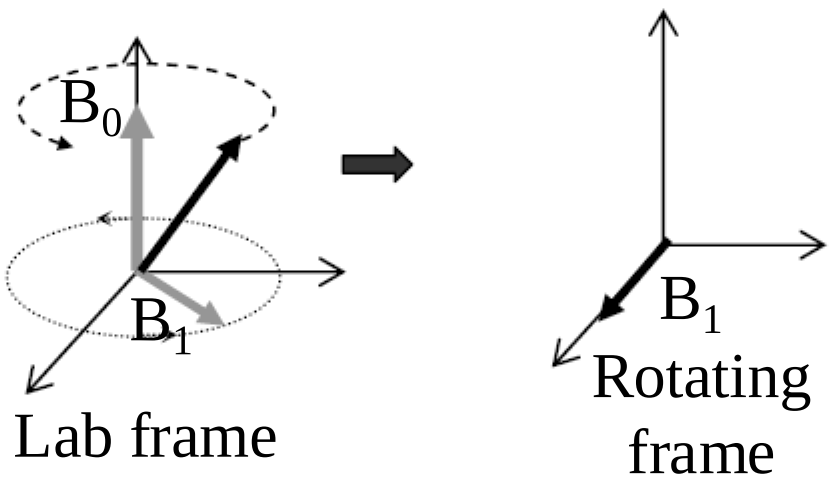


What happens to magnetization in the rotating frame?

Bloch equations

- Used describe the time evolution of “magnetization”
- Phenomenological equations

Time derivative of a vector ($\mathbf{A}$) in lab (fixed) frame

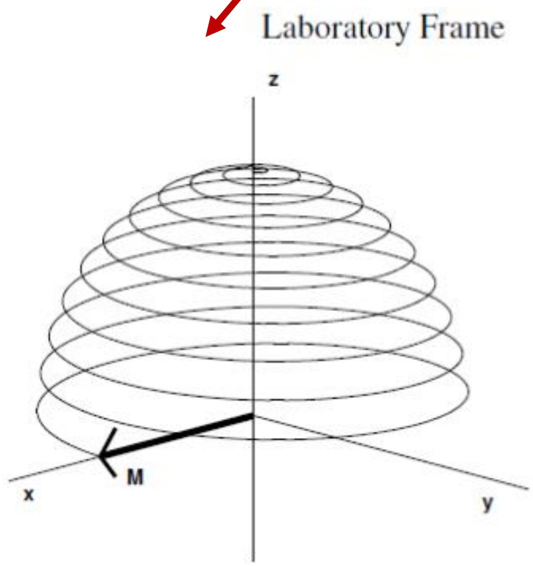
$$d\mathbf{A}/dt = D\mathbf{A}/Dt + (\boldsymbol{\omega} \times \mathbf{A})$$

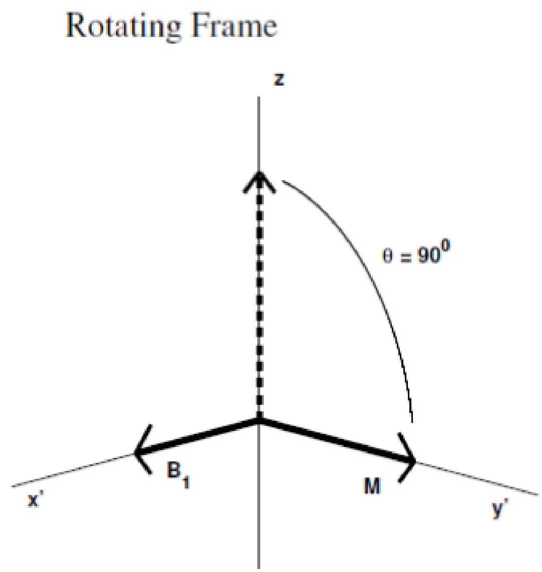
Describes motion of the rotating frame relative to lab frame.

Describes motion of vector in the rotating frame.

Time-derivative of magnetization

$$d\mathbf{M}/dt = D\mathbf{M}/Dt + (\omega \times \mathbf{M})$$





$$D\mathbf{M}/Dt = d\mathbf{M}/dt - (\omega \times \mathbf{M}) \text{ ----------- } (1)$$

We have already seen d**M**/dt = (γ**M** x **B**) --------- (2)

(Larmor equation)

From (1) and (2)

D**M**/Dt = γ(**M** x **B**) - (**ω** x **M**)

=> D**M**/Dt = γ(**M** x **B**) + (**M** x ω)

$$\boxed{\frac{D\vec{M}}{Dt} = \gamma\vec{M} \times \left(\vec{B} + \frac{\vec{\omega}}{\gamma}\right)} \text{ ------------- } (3)$$

$$\boxed{\frac{D\overrightarrow{M}}{Dt} = \gamma\overrightarrow{M} \times \left(\overrightarrow{B} + \frac{\overrightarrow{\omega}}{\gamma}\right)} \text{ ------------- } (3)$$

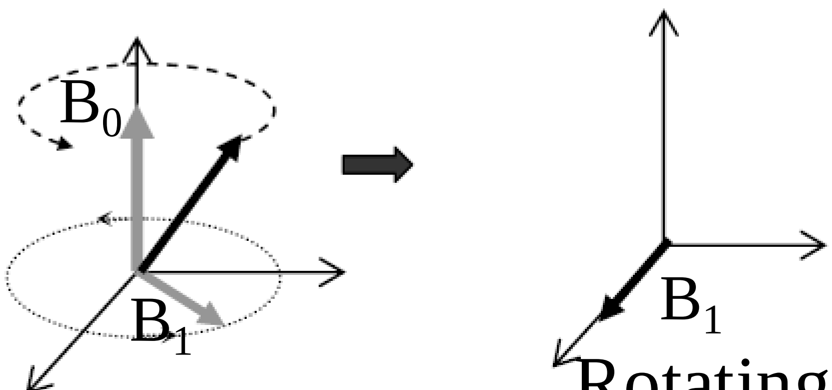
Here, $\overrightarrow{B} = B_1\hat{\boldsymbol{x}} + B_0\hat{\boldsymbol{z}}$ ------ (4)

We choose, $B_0 = -\frac{\omega}{\gamma}$

$$\boxed{\frac{D\overrightarrow{M}}{Dt} = \gamma\overrightarrow{M} \times B_1\hat{\boldsymbol{x}}} \text{ ------------- } (5)$$

What magnetic fields do we see if we are sitting on the rotating frame?

B_1 is constant.
B_o vanishes.

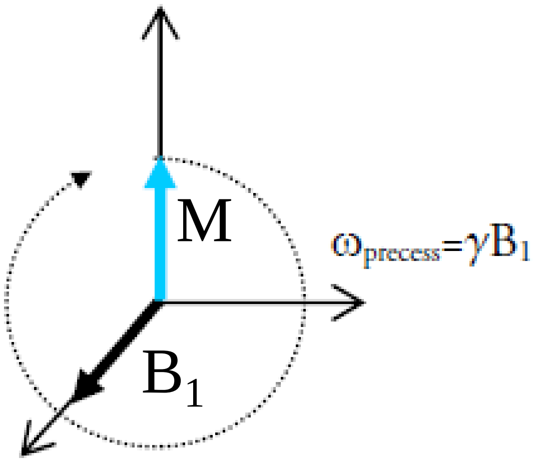


Lab frame

Rotating frame

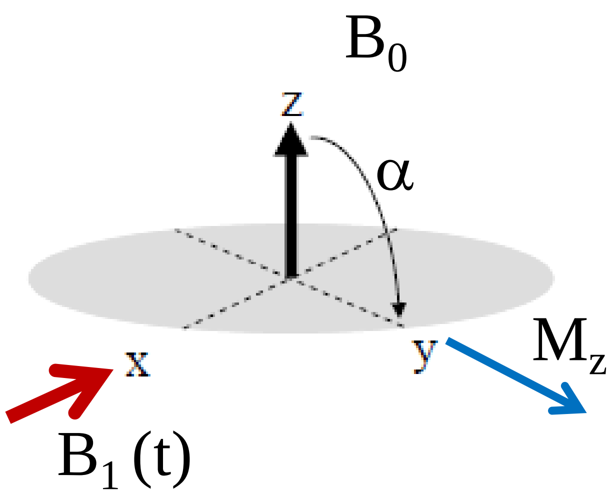
What happens to magnetization in rotating frame?

Magnetization (blue) precesses around B_1 with frequency $\gamma B_{1.}$



Tip angle (α)

- Tip angle (α): angle through which magnetization (M) is rotated by applying RF field.
- Depends on both B_1 and the duration of pulse (τ_{B1}).



$$\alpha = \omega\, \tau_{B1} = 2\pi\gamma\, B_1\, \tau_{B1}$$

$$\Rightarrow \tau_{B1} = \alpha / (2\pi\gamma\, B_1)$$

For a typical MRI scenario,

$B_1 = 10\mu T$, => $\tau \sim 0.5ms$

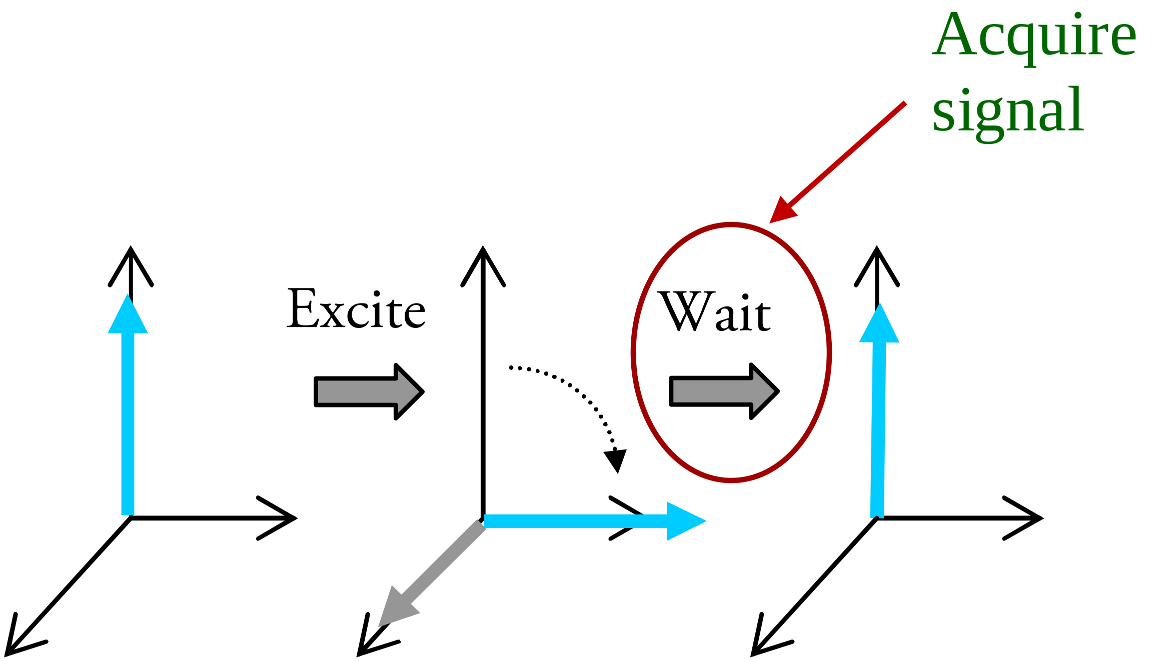
Typical pulses

- $\pi/2$ pulse leads to maximum transverse component of magnetization
- π pulse rotates magnetization from + z to –z. No transverse component.

What happens when RF-field is turned off?

- Magnetization returns to its equilibrium position along z-axis.
- We are interested in <u>how</u> it returns to equilibrium.
- This is when we measure the MRI signal (i.e. after RF field is turned off).

Relaxation



Relaxation of magnetization is measured after B_1 is turned off

Apply relaxation behaviour to magnetization components.

$$\frac{dM_z}{dt} = -\frac{(M_z - M_0)}{T_1}$$

$$\frac{dM_x}{dt} = -\frac{M_x}{T_2}$$

$$\frac{dM_y}{dt} = -\frac{M_y}{T_2}$$

- There are two relaxation times and they are different.
- Why is there no *Mo* in the equations for the transverse components?

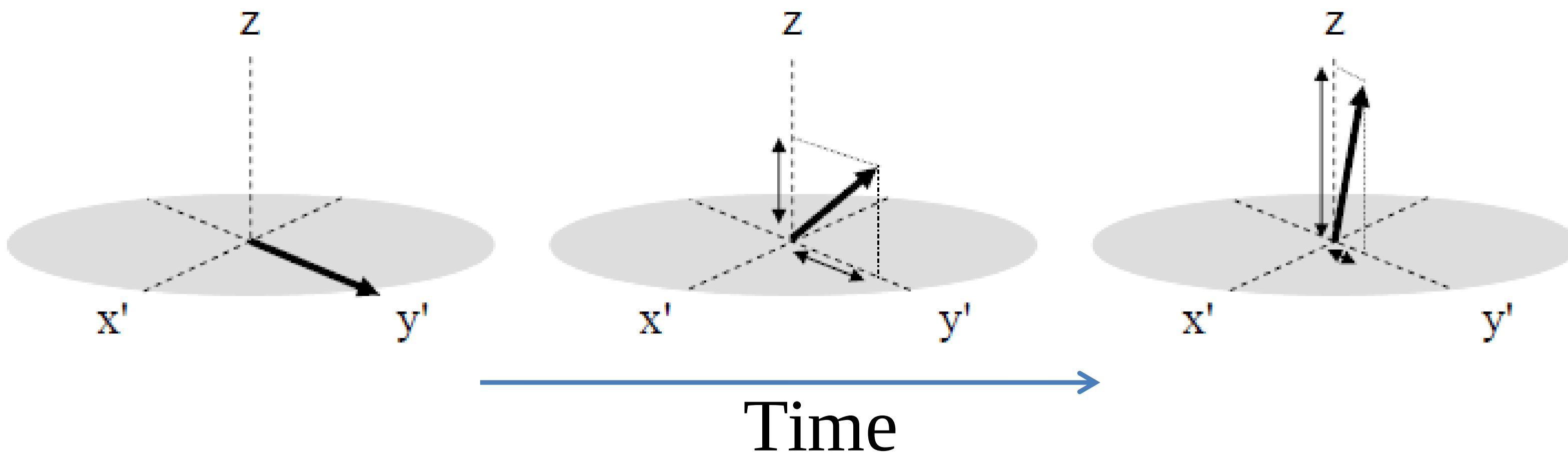
Lec 12
(MRI: relaxation)

Bloch equations with spin relaxation

$$\frac{d\vec{M}}{dt} = \gamma \vec{M} \times \vec{B_o} - \frac{M_x \hat{\boldsymbol{x}}}{T_2} - \frac{M_y \hat{\boldsymbol{y}}}{T_2} - \frac{(M_z - M_0)\hat{\boldsymbol{z}}}{T_1}$$

$$\begin{aligned} \frac{d}{dt} M_x &= \gamma M_y B_0 - \frac{M_x}{\tau_2} \\ \frac{d}{dt} M_y &= -\gamma M_x B_0 - \frac{M_y}{\tau_2} \\ \frac{d}{dt} M_z &= -\frac{M_z - M_0}{\tau_1} \end{aligned}$$

T_1-relaxation (longitudinal)



M_z returns to its equilibrium value (M_o) with time.

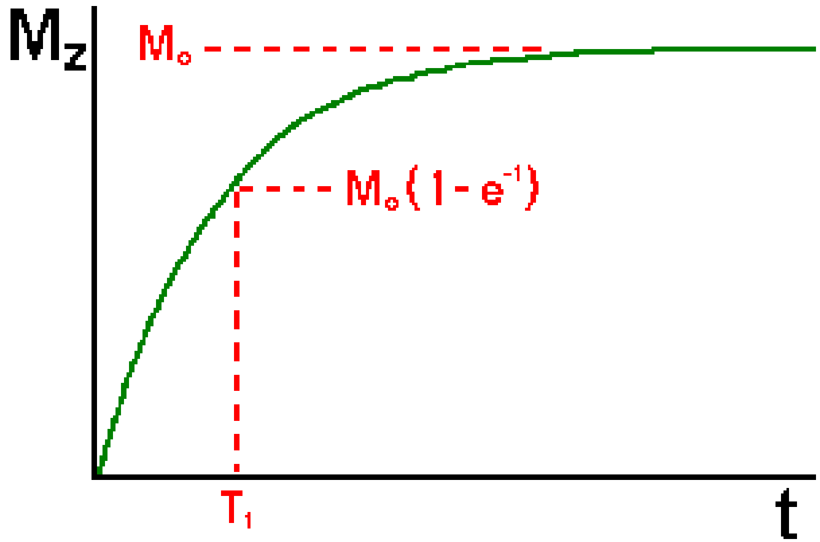
T_1: time constant describing how M_Z returns to its equilibrium value

Don't take this projection drawing from Smith and Webb too literally for understanding the origin of T_2.

What will the solution look like?

$$\frac{d}{dt}M_z = -\frac{M_z - M_0}{\tau_1}$$

What boundary conditions (for 90^o pulse) will you use to solve this equation?



$$M_z = M_o\,(1 - e^{\frac{-t}{T_1}}) \qquad \text{(for } \tfrac{\pi}{2} \text{ pulse)}$$

Solutions of Bloch equations

$$\begin{aligned}\frac{d}{dt}M_x &= \gamma M_y B_0 - \frac{M_x}{\tau_2}\\ \frac{d}{dt}M_y &= -\gamma M_x B_0 - \frac{M_y}{\tau_2}\\ \frac{d}{dt}M_z &= -\frac{M_z - M_0}{\tau_1}\end{aligned}$$

$$\boxed{M_{x,y}(t) = M_{x,y}(0)e^{-t/\tau_2}\cos\gamma B_0 t}$$

$$\boxed{M_z(t) = M_0(1 - e^{-t/\tau_1})}$$

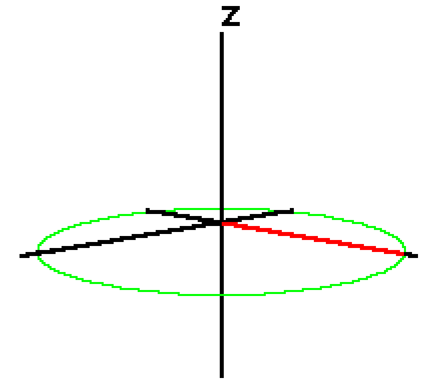
Measurement of relaxation times (T_1 and T_2)

- Saturation recovery (T_1)
- Inversion recovery (T_1)
- Spin echo (T_2)

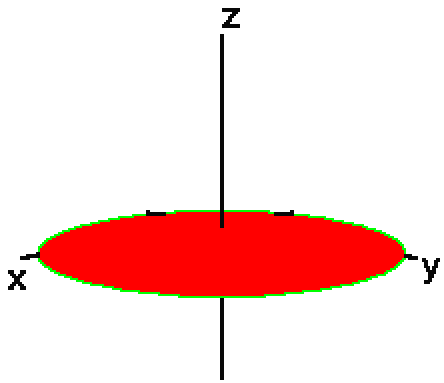
1. Saturation recovery

We use free induction decay to measure T_1 in an experiment

- Assume M to be along B_o.
- Switch on B_1 for time 't' such that, $\gamma \mathbf{B_1 t} = \frac{\pi}{2}$

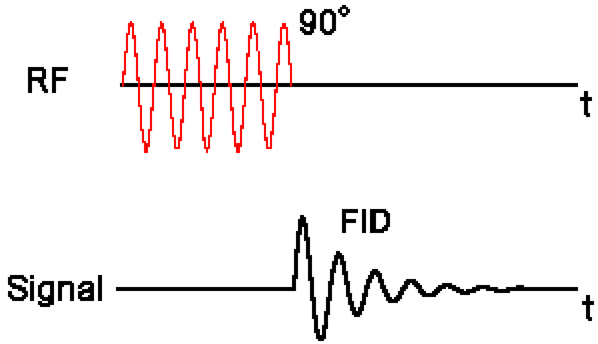


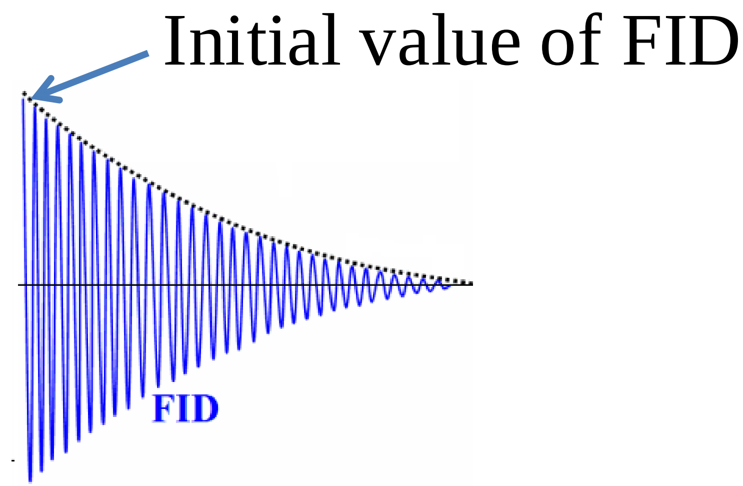
- Magnetization tips to x-y plane.



- Switch off B1 to allow magnetization to "relax" and reach its equilibrium value M_o.

- Changing magnetization induces signal in RF coil (free induction decay). But it is difficult to capture the FID signal in just one shot.





- Initial magnitude of FID voltage gives the initial "length" of the magnetization vector. It is this initial voltage that we measure, as we can't really capture the entire decay signal due to a limitation of measurement electronics.

So we tweak the experiment a bit.

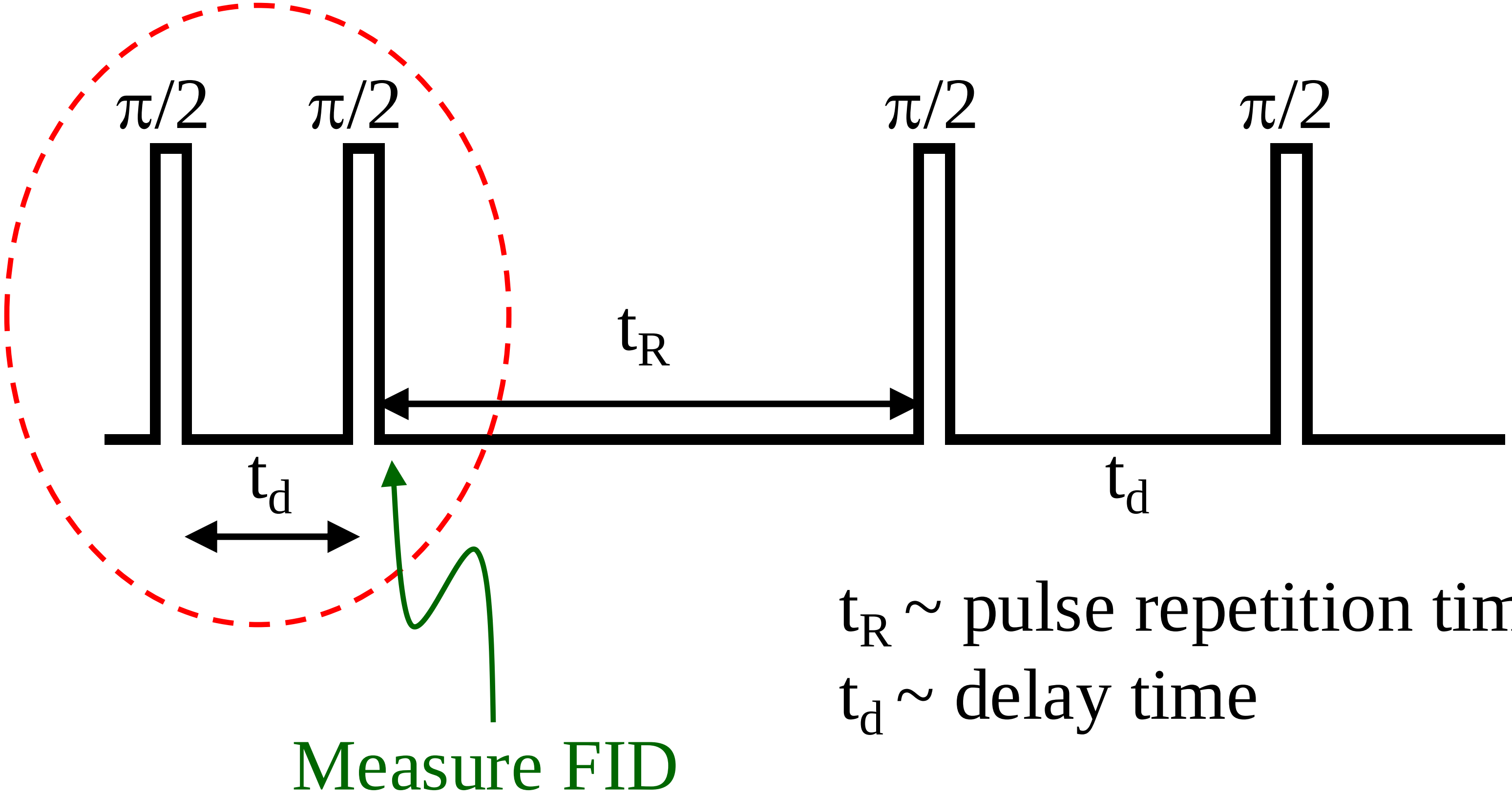
1. Apply one $\pi/2$ pulse.

2. Wait for time $\mathbf{t_d}$ for magnetization to relax partially without measuring anything. Will M_z be comparable to M_o after t_d?

3. The time $\mathbf{t_d}$ is set by you, and is, at first, a fraction of T_1. The relaxation of M_z over a period $\mathbf{t_d}$ is then incomplete.

4. The magnetization vector after $\mathbf{t_d}$ has a length that is much smaller than its equilibrium value M_o.

5. Now give a second $\pi/2$ pulse.
6. This forces M_z (**now with a value much smaller than M_o**) to get turned on to the x-y plane.
7. Allow this small M_z to relax completely. This time we will measure its relaxation.
8. Measure the FID signal during this relaxation step. This will give you the magnitude of M_z for the value of t_d you chose.
9. Repeat steps 1 – 8 by gradually increasing the value of t_d until it is at least ~ $5T_1$ - $10T_1$.

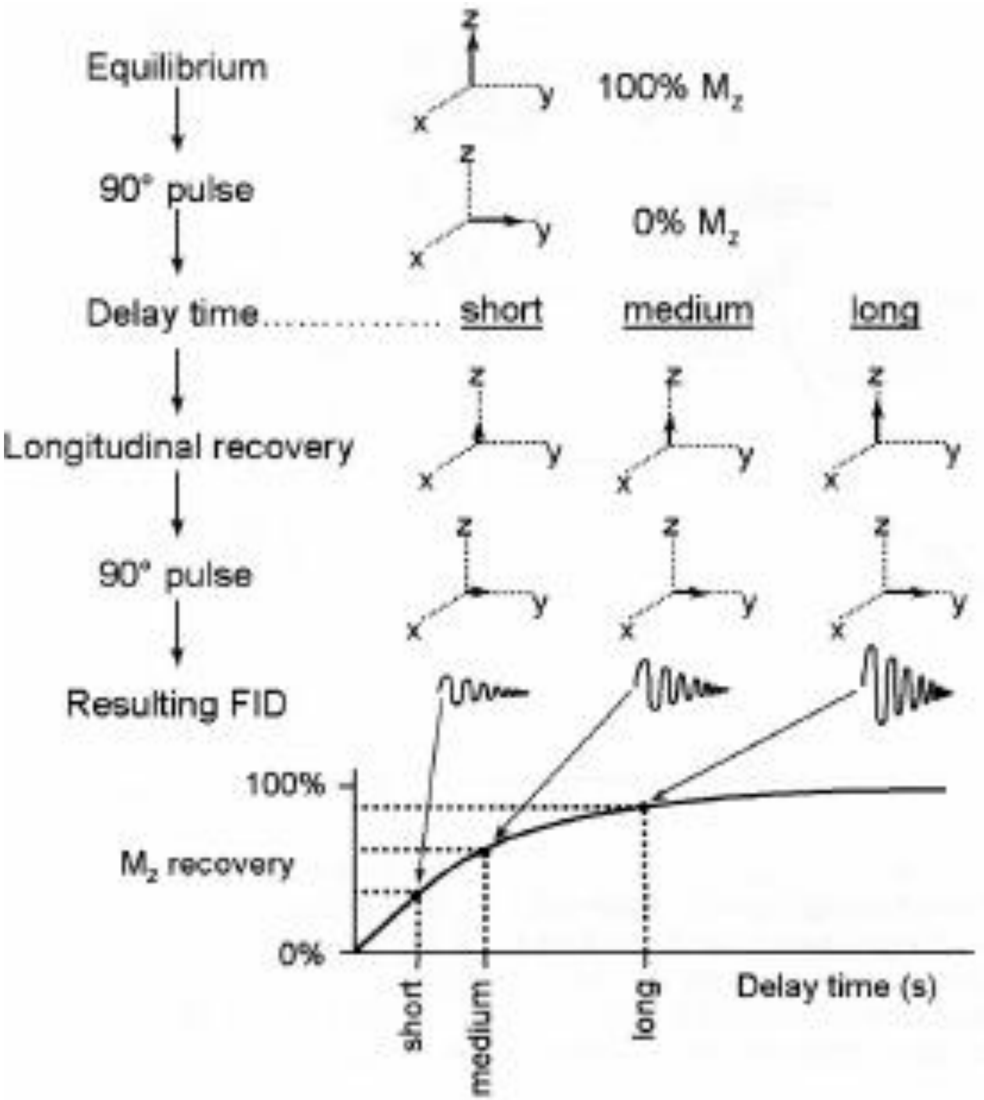


t_R ~ pulse repetition time

t_d ~ delay time

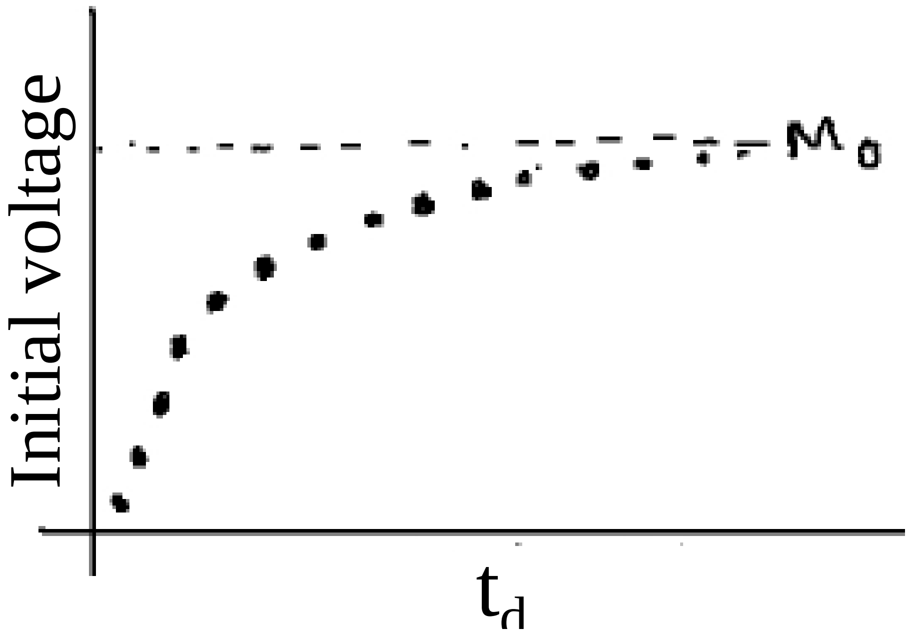
- Usually, $t_R \sim 5T_1$ => ensures full "<u>saturation</u>"
- Vary t_d (from a fraction of T_1 to a few times T_1).

Saturation recovery: $\pi/2 - t_d - \pi/2$ pulse



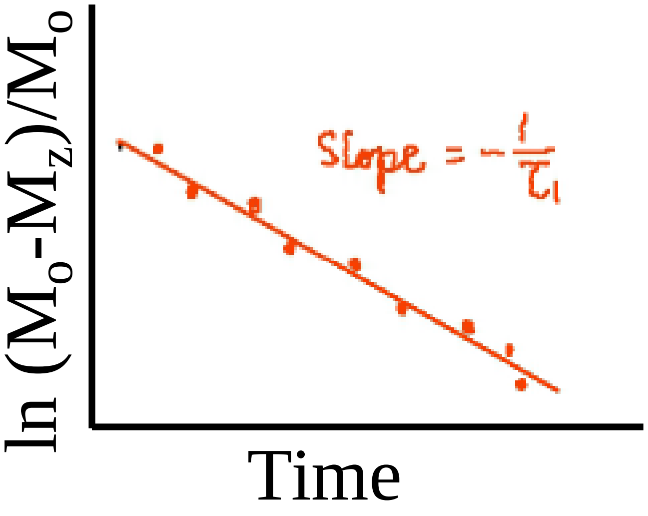
$$M_z(t) = M_0(1 - e^{-t/\tau_1})$$

At $t = \infty$, $M_z = M_o$



$$M_z(t) = M_0(1 - e^{-t/\tau_1})$$

$$\ln \frac{(Mo - Mz)}{M_o} = -\frac{t}{T_1}$$



Lec 14
(Spin echo and contrast in MRI images)

Recap: Measuring T_2 from FID is difficult

- FID decays with T_2^* time constant.

$$\frac{1}{T_2^{\ *}} = \frac{1}{T_2^{\ +}} + \frac{1}{T_2}$$

Inhomogeneous mag. field
(property of MR set-up)

spin-spin interaction

Recap: Dephasing of magnetization ("pure" T_2 effect)

- Each spin sees a slightly different magnetic field.
- Magnetization for each spin packet rotates <u>at its own Larmor frequency</u>.
- Net magnetization starts to dephase.
- Vector sum of transverse component is zero when totally dephased.

“Inhomogeneous” T_2-relaxation

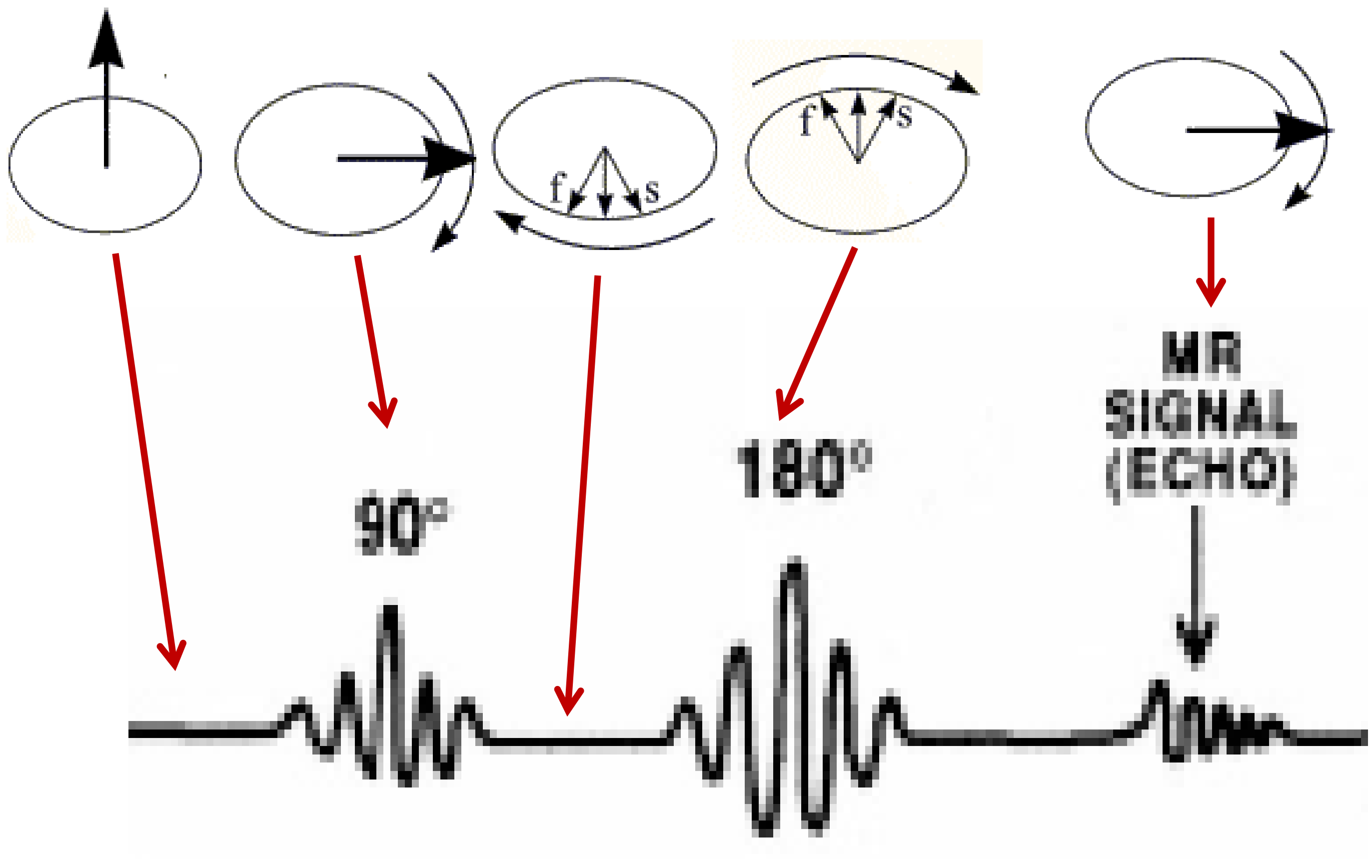
$$\frac{1}{T_2^{\,*}} = \frac{1}{T_2^{\,+}} + \frac{1}{T_2}$$

Inhomogeneous mag. field
(property of MR set-up)

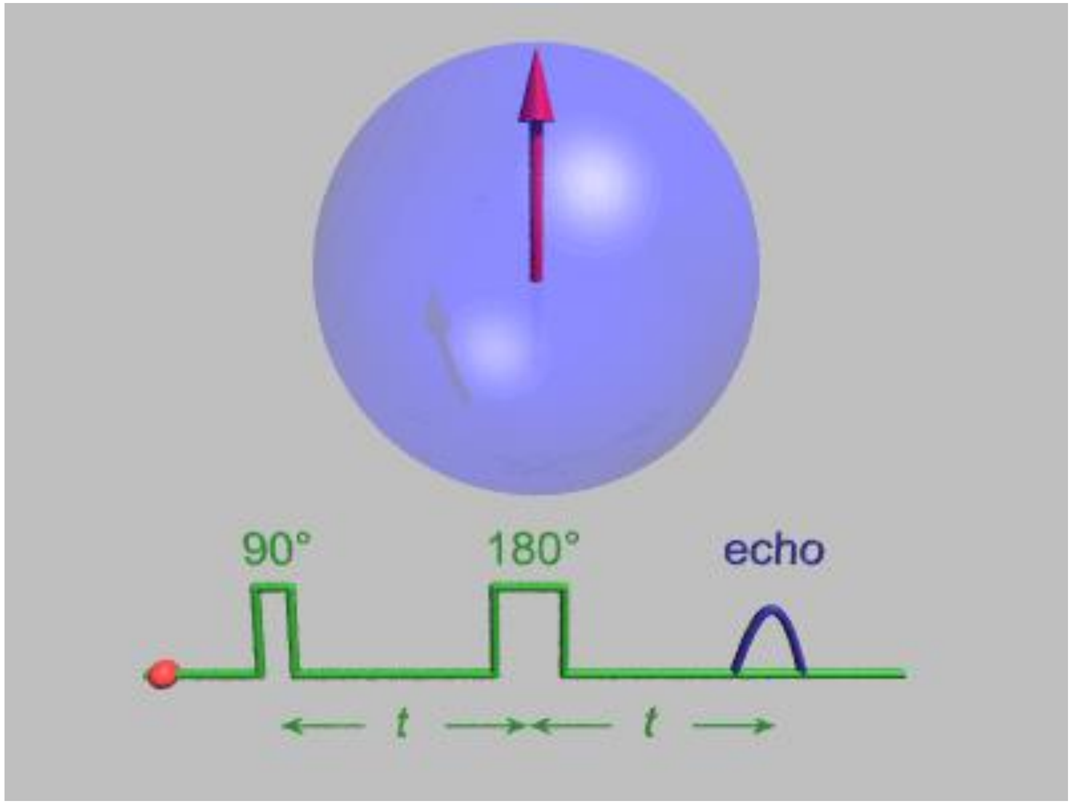
spin-spin interaction

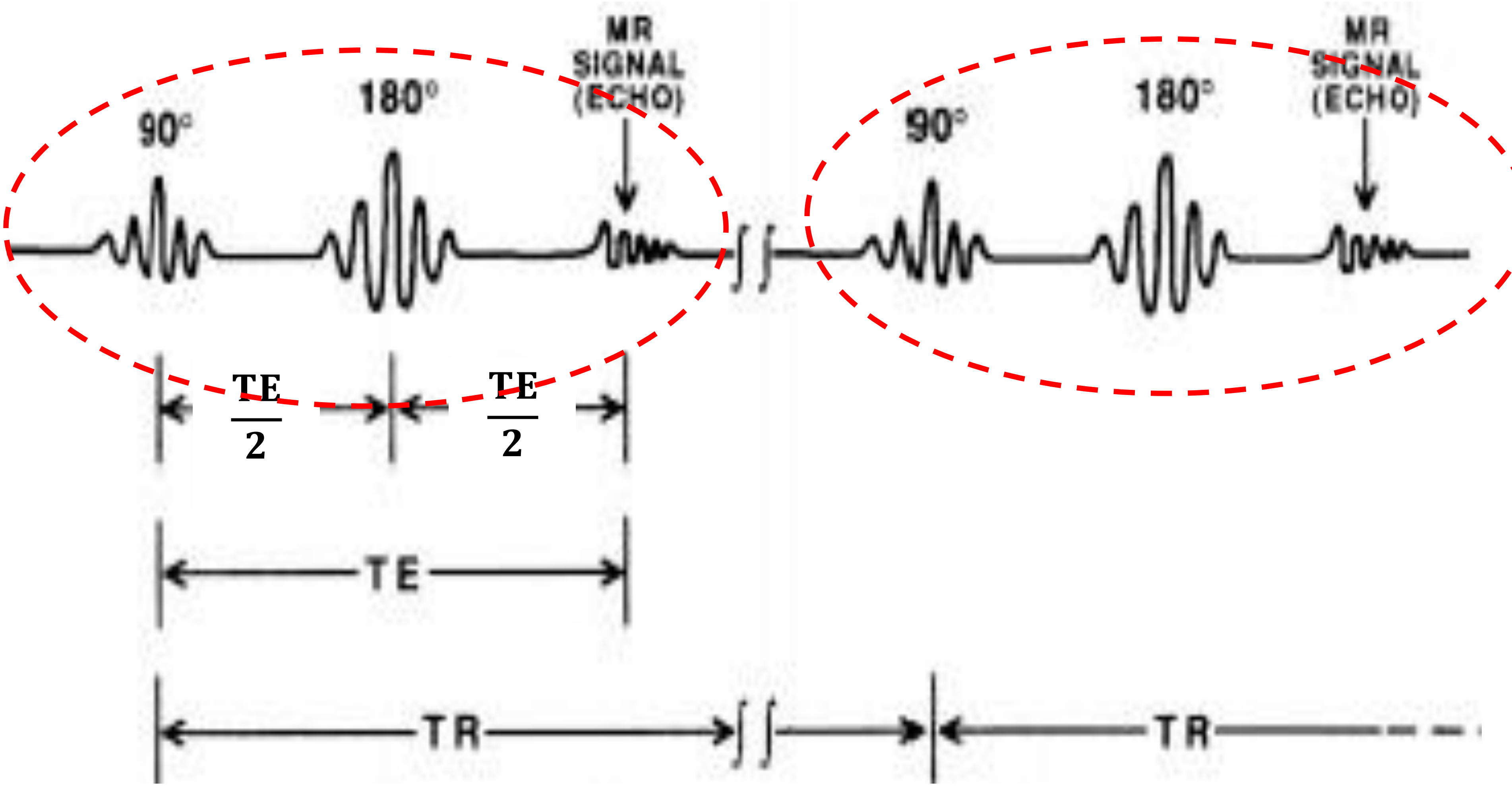
- Magnet design
- Different magnetic susceptibilities (e.g. near surgical implant, at tissue boundaries with different magnetic properties, etc.).

Spin echo measures T_2



Spin debunching happens due to T_2^+ processes (field inhomogeneities)


90°
180°
echo
t
t



- **TE** is echo time and **TR** is repetition time

Repetition time and echo time

These times are chosen by the experimenter.

- **TR** is the length of the relaxation time between two excitation ($\pi/2$) pulses.

- **TE** is the time interval between the excitation pulse ($\pi/2$) and measurement of MR signal.

T_1, T_2 are tissue properties. We do not choose them.

T_1 and T_2 of tissues

- Different tissues have different values of T_1 and T_2.
- Diseased tissues have different T_1 and T_2 compared to healthy tissues.
- T_1 and T_2 are not related.

T_1 , T_2 (milliseconds) of tissues

Tissue	**T_1 (@ 1.5T)**	**T_1 (@ 3T)**
Brain (white)	790	1100
Brain (grey)	920	1600
Liver	500	800
Skeletal muscle	870	1420
Lipid (subcutaneous)	290	360
Cartilage	1060	1240

T_2 (@ 1.5T)	**T_2 (@ 3T)**
90	60
100	80
50	40
60	30
160	130
42	37

What can you infer from this table?

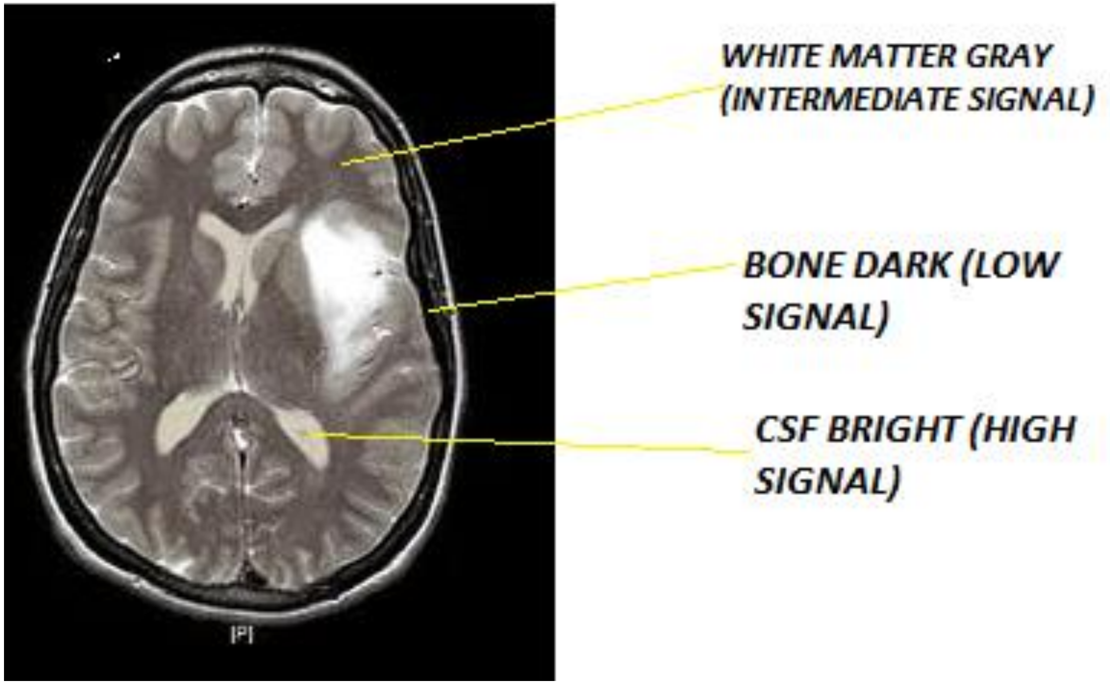
- $T_2 < T_{1.}$ for all tissues
- The values of T_1 and T_2 depend on the magnetic field (B_o).

$$\boxed{M_0 = \frac{N(\gamma\hbar)^2 B_0}{4kT}}$$

T_1, T_2 determine if we can measure signals from a particular tissue

- Can't measure MRI signals from **bone** .
- Extremely small T_2 (~ 0.01 ms).
- Signal disappears before measurement!

Image contrast



<u>High</u> signal intensity: bright

<u>Low</u> signal intensity: dark

<u>Intermediate</u> signal intensity: gray

Can we exploit T_1 and T_2 of different brain tissues to enhance image contrast?

Tissue	**T_1 (1.5T)**	**T_2 (1.5T)**
White matter	790 ms	90 ms
Grey matter	920 ms	100 ms
CSF	2400 ms	200 ms
Fat	270 ms	80 ms

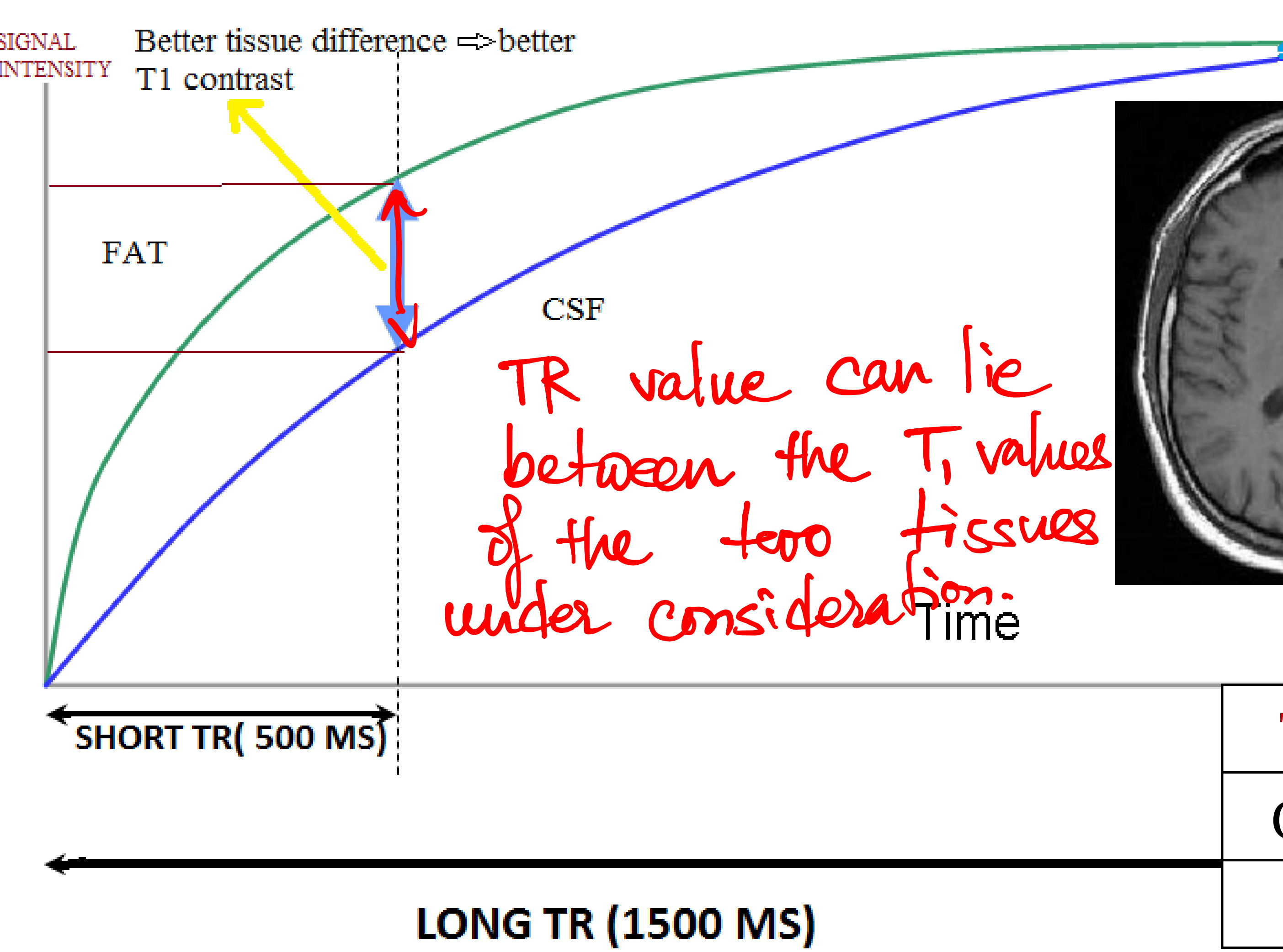
T_1 weighing of MRI images

- Short TR (appropriately chosen) will not allow some tissues to recover equilibrium magnetization (M_o).
- Long TR allows all tissues to recover completely.
- Keep TE short (~ 15ms) to neglect T_2 dependency.

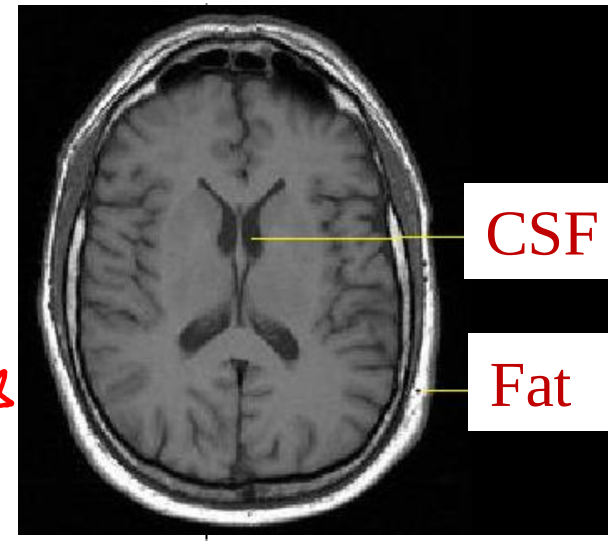
$$\boxed{M_0 = \frac{N(\gamma\hbar)^2 B_0}{4kT}}$$

How "short" should TR be?

T_1 weighed image



TR value can lie between the T_1 values of the two tissues under consideration.

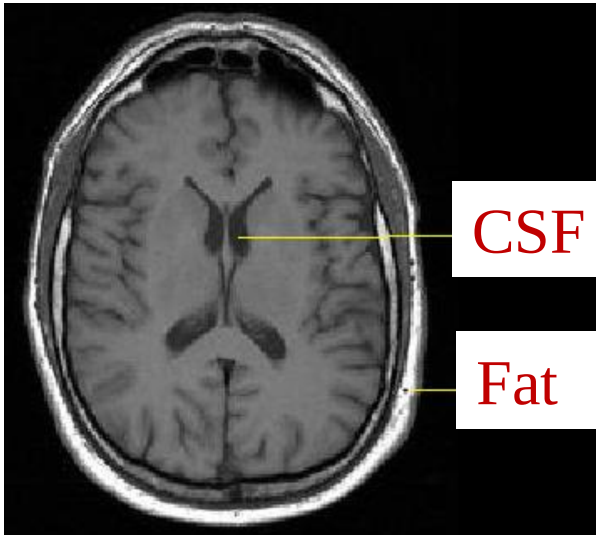


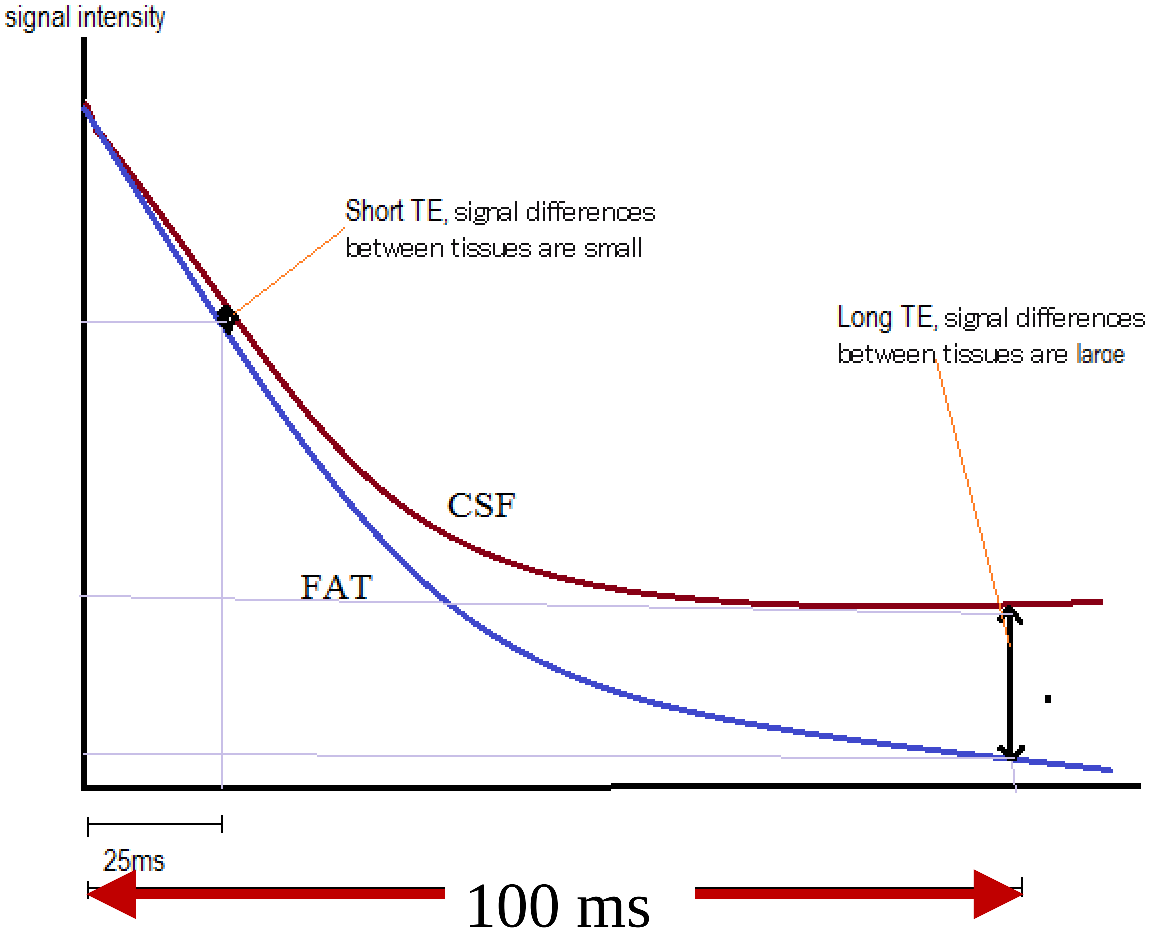
T_1 (1.5T) in ms	
CSF	2400
Fat	270

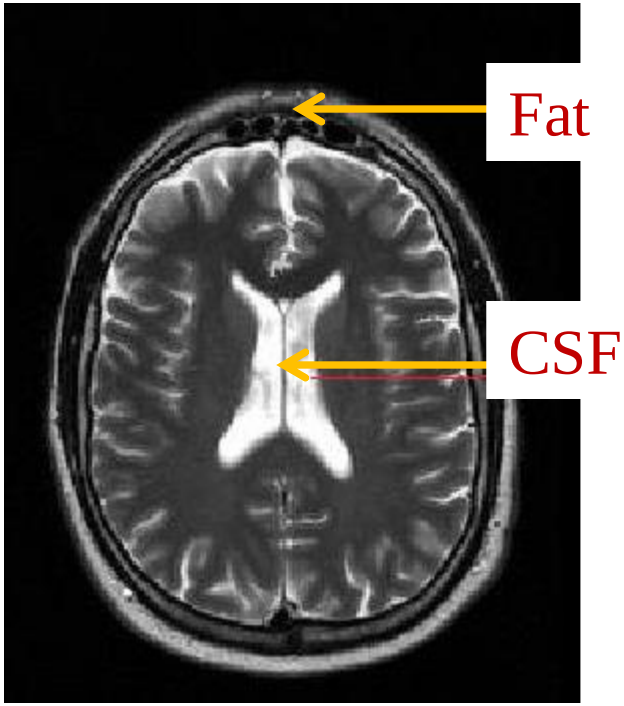
TR ~ 500 ms, TE ~ 15 ms

T_1- weighting gives strong signal for tissues with short relaxation times.

T_1 (1.5T) in ms	
CSF	2400
Fat	270



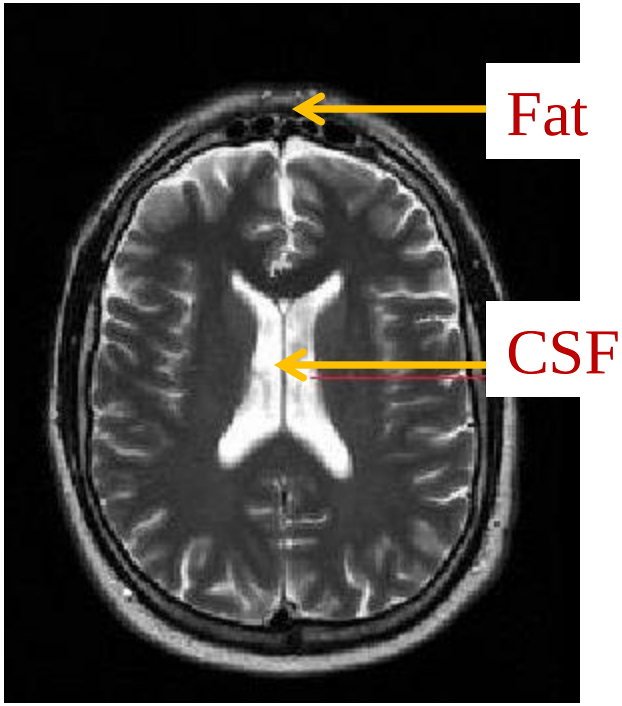




	T_1	T_2
CSF	2400	200
Fat	270	80

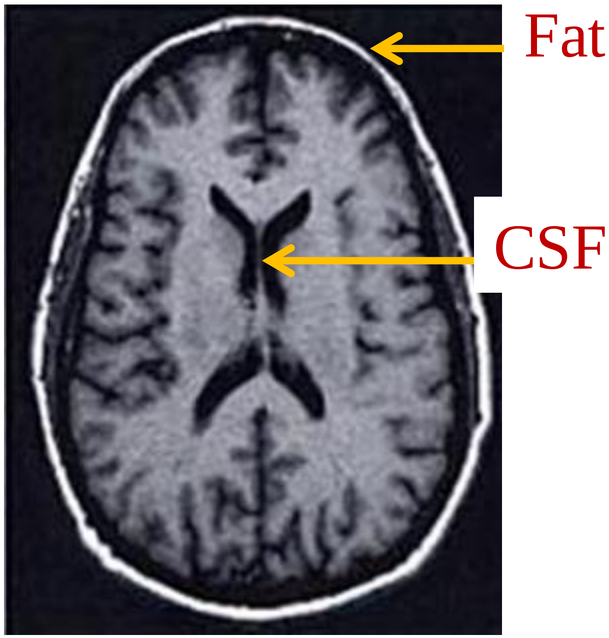
TR ~ 3000 ms, TE ~ 100 ms

T_2- weighting gives strong signal for tissues with long relaxation times.

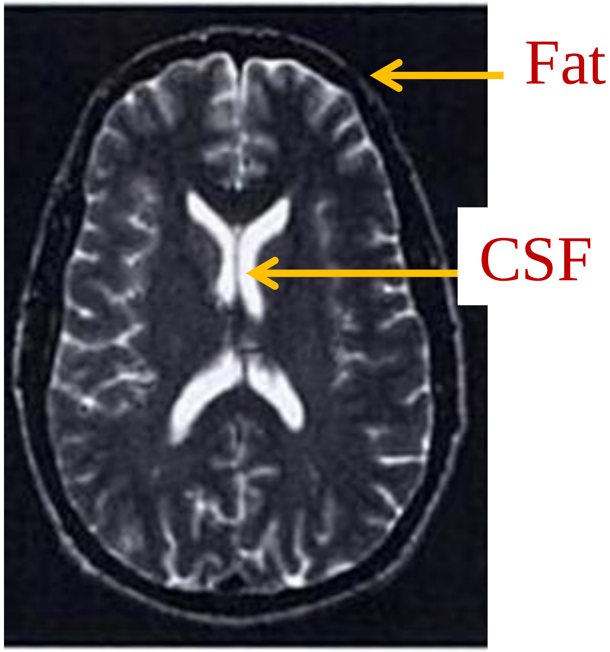


	T_1	T_2
CSF	2400	200
Fat	270	80

Are these images T_1 or T_2 weighed?



T_1-weighted
(TR = 600, TE = 11)



T_2-weighted
(TR = 3800, TE = 102)

	T_1	T_2
CSF	2400	200
Fat	270	80

Is it a good idea to exploit both T_1 and T_2 dependencies simultaneously to enhance the image contrast in MRI? Why or why not?

— No! T_1-weighing & T_2 weighing give opposite effects on the tissues. This would degrade the contrast.