

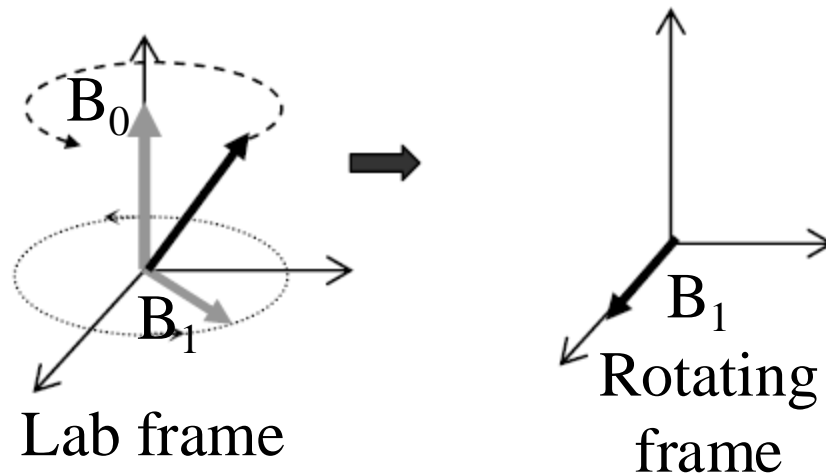
# Lec 11

## (MRI: $B_1$ field)

What do we see if we are sitting on the rotating frame?

$B_1$  is constant.

$B_0$  vanishes.



What happens to magnetization  
in the rotating frame?

# Bloch equations

- Used describe the time evolution of “magnetization”
- Phenomenological equations

# Time derivative of a vector ( $\mathbf{A}$ ) in lab (fixed) frame

$$d\mathbf{A}/dt = \underbrace{D\mathbf{A}/Dt}_{\text{Describes motion of vector in the rotating frame.}} + \underbrace{(\boldsymbol{\omega} \times \mathbf{A})}_{\text{Describes motion of the rotating frame relative to lab frame.}}$$

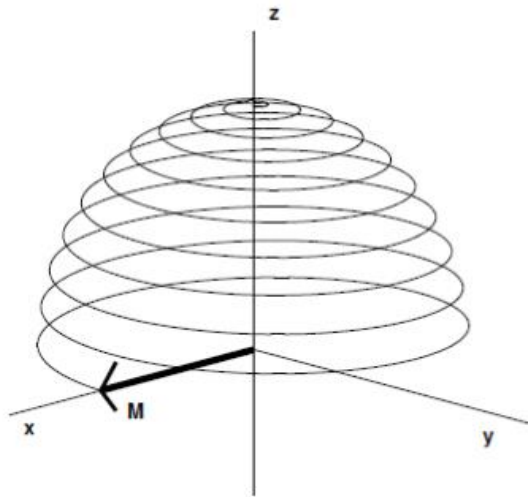
Describes motion of the rotating frame relative to lab frame.

Describes motion of vector in the rotating frame.

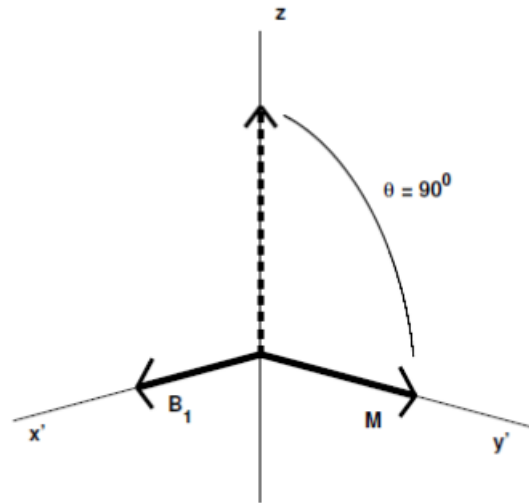
# Time-derivative of magnetization

$$d\mathbf{M}/dt = D\mathbf{M}/Dt + (\boldsymbol{\omega} \times \mathbf{M})$$

Laboratory Frame



Rotating Frame



$$D\mathbf{M}/Dt = d\mathbf{M}/dt - (\boldsymbol{\omega} \times \mathbf{M}) \text{ ----- (1)}$$

We have already seen  $d\mathbf{M}/dt = (\gamma\mathbf{M} \times \mathbf{B})$  ----- (2)  
(Larmor equation)

From (1) and (2)

$$D\mathbf{M}/Dt = \gamma(\mathbf{M} \times \mathbf{B}) - (\boldsymbol{\omega} \times \mathbf{M})$$

$$\Rightarrow D\mathbf{M}/Dt = \gamma(\mathbf{M} \times \mathbf{B}) + (\mathbf{M} \times \boldsymbol{\omega})$$

$$\boxed{\frac{D\vec{M}}{Dt} = \gamma\vec{M} \times \left( \vec{B} + \frac{\vec{\omega}}{\gamma} \right)} \text{-----} (3)$$

$$\boxed{\frac{D\vec{M}}{Dt} = \gamma \vec{M} \times \left( \vec{B} + \frac{\vec{\omega}}{\gamma} \right)} \text{-----} (3)$$

Here,  $\vec{B} = B_1 \hat{x} + B_0 \hat{z}$  ----- (4)

We choose,  $B_0 = -\frac{\omega}{\gamma}$

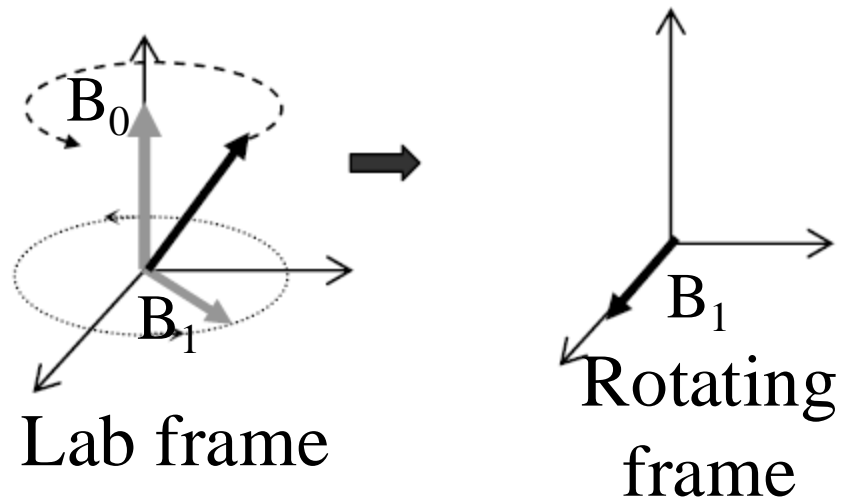
$$\boxed{\frac{D\vec{M}}{Dt} = \gamma \vec{M} \times B_1 \hat{x}} \text{-----} (5)$$



What magnetic fields do we see if we are sitting on the rotating frame?

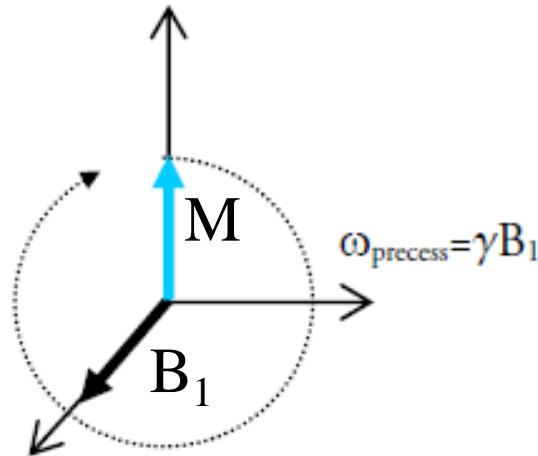
$B_1$  is constant.

$B_0$  vanishes.



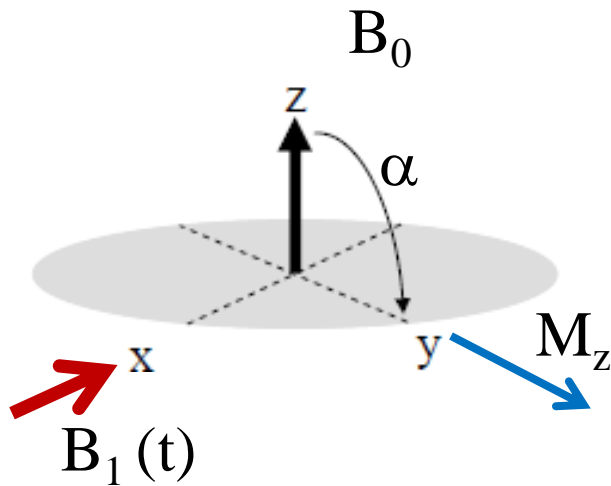
# What happens to magnetization in rotating frame?

Magnetization (blue) precesses around  $B_1$  with frequency  $\gamma B_1$ .



# Tip angle ( $\alpha$ )

- **Tip angle ( $\alpha$ )**: angle through which magnetization ( $M$ ) is rotated by applying RF field.
- Depends on both  $B_1$  and the duration of pulse ( $\tau_{B1}$ ).



$$\alpha = \omega \tau_{B1} = 2\pi\gamma B_1 \tau_{B1}$$
$$\Rightarrow \tau_{B1} = \alpha / (2\pi\gamma B_1)$$

For a typical MRI scenario,  
 $B_1 = 10\mu\text{T}$ ,  $\Rightarrow \tau \sim 0.5\text{ms}$

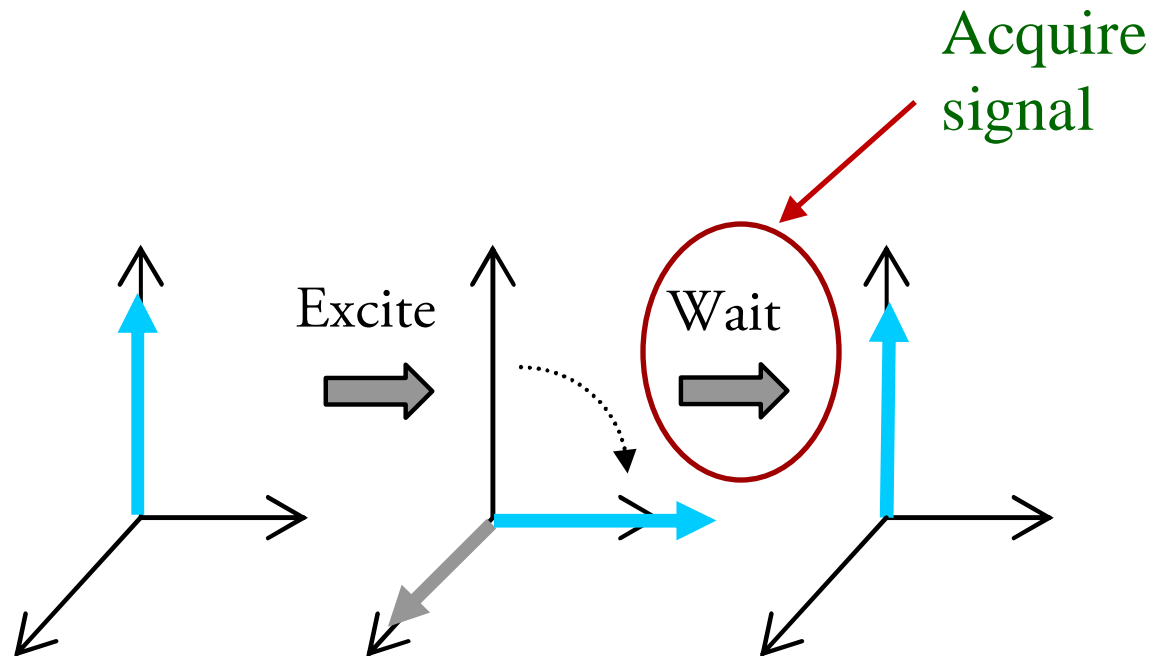
# Typical pulses

- $\pi/2$  pulse leads to maximum transverse component of magnetization
- $\pi$  pulse rotates magnetization from  $+z$  to  $-z$ . No transverse component.

# What happens when RF-field is turned off?

- Magnetization returns to its equilibrium position along z-axis.
- We are interested in how it returns to equilibrium.
- This is when we measure the MRI signal (i.e. after RF field is turned off).

# Relaxation



# Relaxation of magnetization is measured after $B_1$ is turned off

Apply relaxation behaviour to magnetization components.

$$\frac{dM_z}{dt} = - \frac{(M_z - M_0)}{T_1}$$

$$\frac{dM_x}{dt} = - \frac{M_x}{T_2}$$

$$\frac{dM_y}{dt} = - \frac{M_y}{T_2}$$

- There are **two** relaxation times and they are different.
- Why is there no  $M_0$  in the equations for the transverse components?