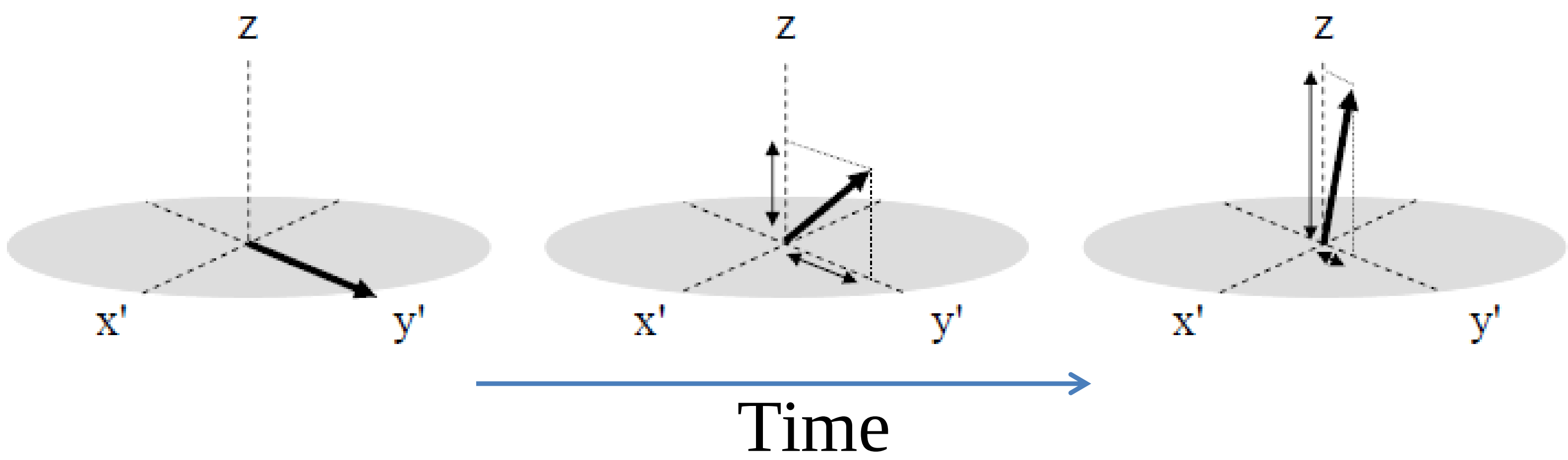
Lec 12
(MRI: relaxation)

Bloch equations with spin relaxation

$$\frac{d\vec{M}}{dt} = \gamma\vec{M} \times \vec{B_o} - \frac{M_x\hat{\boldsymbol{x}}}{T_2} - \frac{M_y\hat{\boldsymbol{y}}}{T_2} - \frac{(M_z - M_0)\hat{\boldsymbol{z}}}{T_1}$$

$$\begin{aligned}
\frac{d}{dt}M_x &= \gamma M_y B_0 - \frac{M_x}{\tau_2} \\
\frac{d}{dt}M_y &= -\gamma M_x B_0 - \frac{M_y}{\tau_2} \\
\frac{d}{dt}M_z &= -\frac{M_z - M_0}{\tau_1}
\end{aligned}$$

T_1-relaxation (longitudinal)



M_z returns to its equilibrium value (M_o) with time.

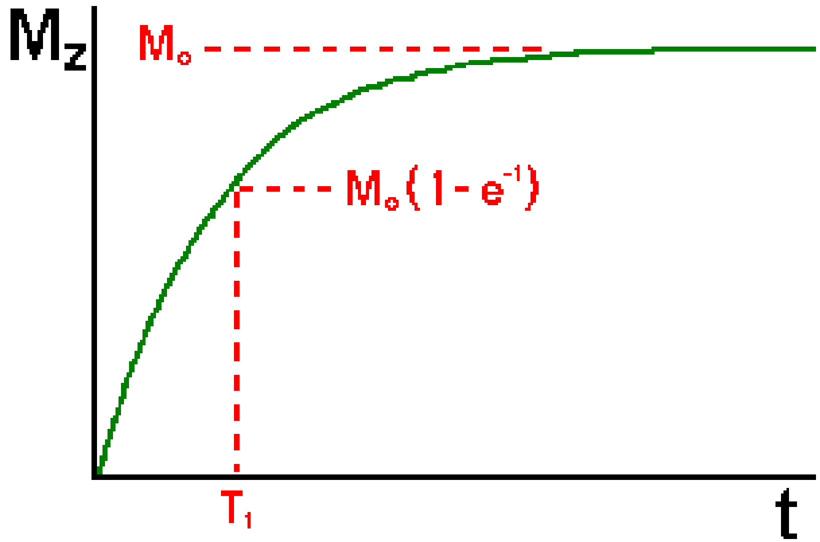
T_1: time constant describing how M_Z returns to its equilibrium value

Don't take this projection drawing from Smith and Webb too literally for understanding the origin of T_2.

What will the solution look like?

$$\frac{d}{dt}M_z = -\frac{M_z - M_0}{\tau_1}$$

What boundary conditions (for 90^o pulse) will you use to solve this equation?



$$M_z = M_o \left(1 - e^{\frac{-t}{T_1}} \right) \qquad \text{(for } \frac{\pi}{2} \text{ pulse)}$$

Solutions of Bloch equations

$$
\begin{aligned}
\frac{d}{dt}M_x &= \gamma M_y B_0 - \frac{M_x}{\tau_2} \\
\frac{d}{dt}M_y &= -\gamma M_x B_0 - \frac{M_y}{\tau_2} \\
\frac{d}{dt}M_z &= -\frac{M_z - M_0}{\tau_1}
\end{aligned}
$$

$$\boxed{M_{x,y}(t) = M_{x,y}(0)e^{-t/\tau_2}\cos\gamma B_0 t}$$

$$\boxed{M_z(t) = M_0(1 - e^{-t/\tau_1})}$$

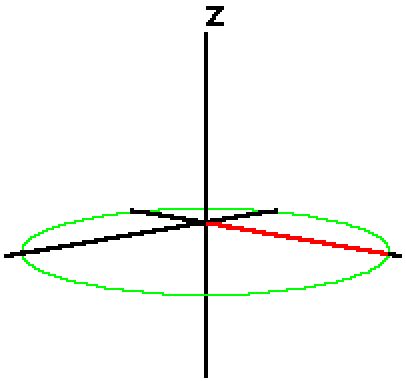
Measurement of relaxation times (T_1 and T_2)

- Saturation recovery (T_1)
- Inversion recovery (T_1)
- Spin echo (T_2)

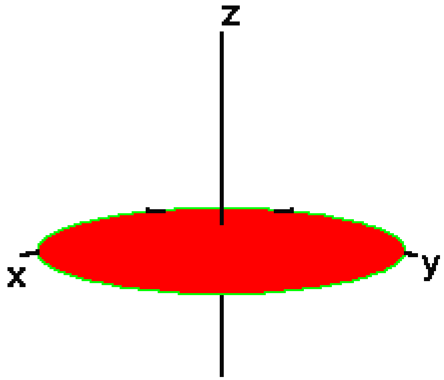
1. Saturation recovery

We use free induction decay to measure T_1 in an experiment

- Assume M to be along B_o.
- Switch on B_1 for time 't' such that, $\gamma B_1 t = \frac{\pi}{2}$

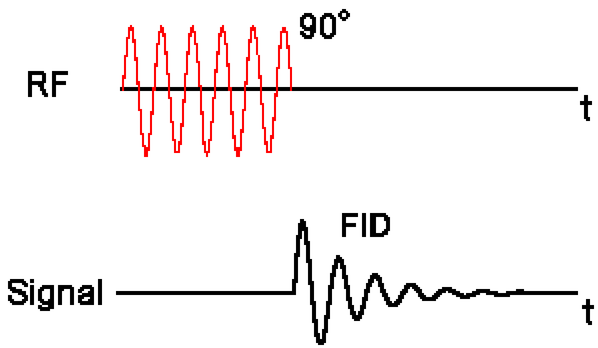


- Magnetization tips to x-y plane.

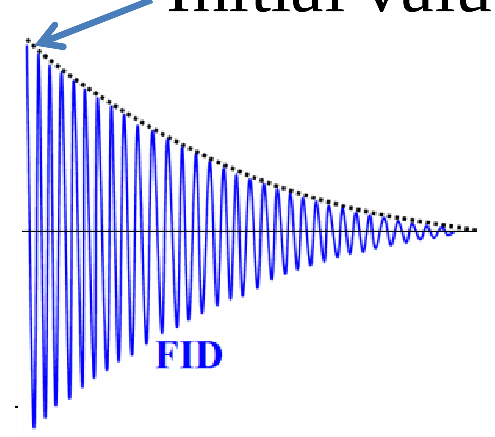


- Switch off B1 to allow magnetization to "relax" and reach its equilibrium value M_o.

- Changing magnetization induces signal in RF coil (free induction decay). But it is difficult to capture the FID signal in just one shot.



Initial value of FID



- Initial magnitude of FID voltage gives the initial “length” of the magnetization vector. It is this initial voltage that we measure, as we can’t really capture the entire decay signal due to a limitation of measurement electronics.

So we tweak the experiment a bit.

1. Apply one $\pi/2$ pulse.

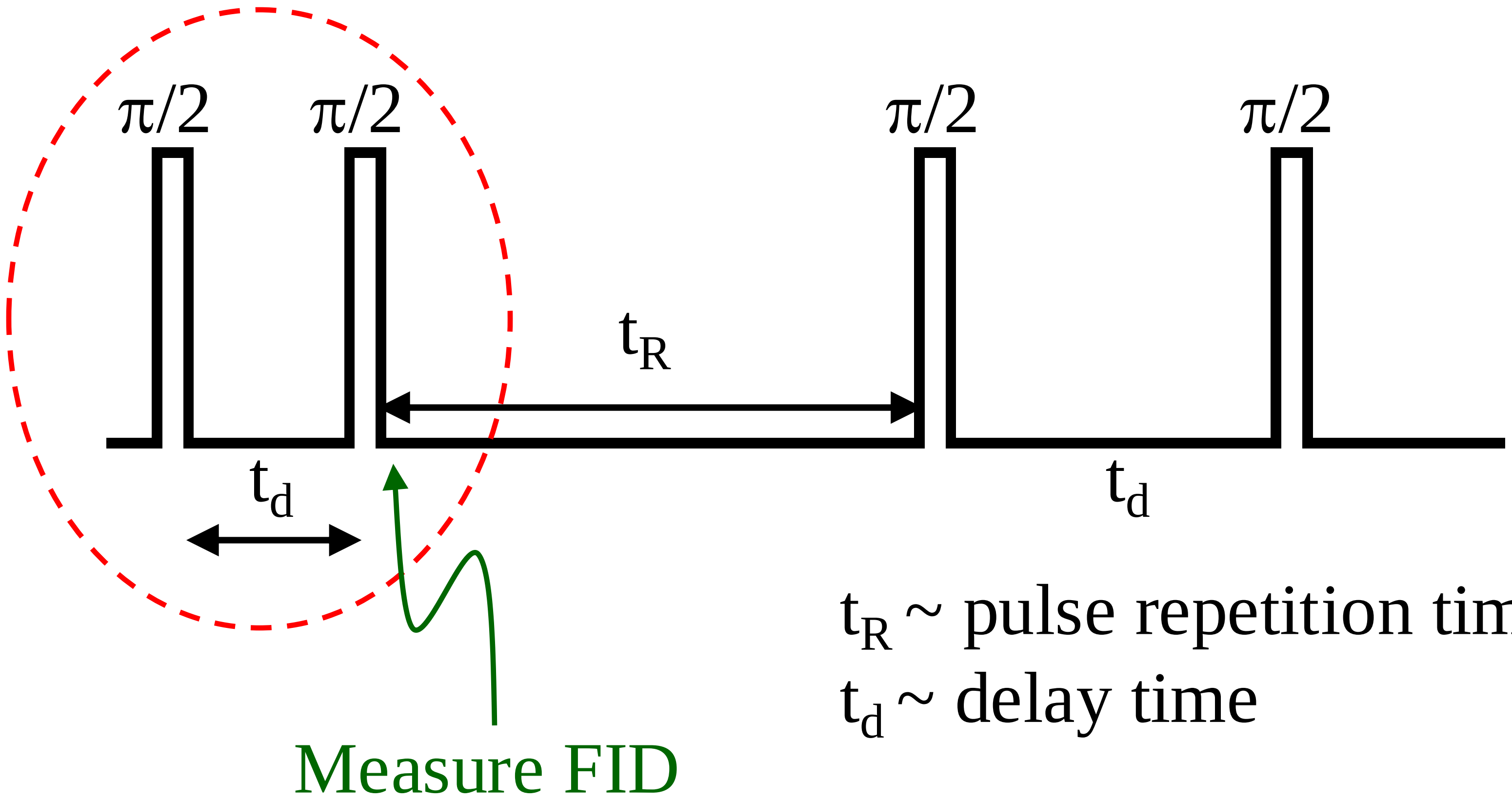
2. Wait for time $\mathbf{t_d}$ for magnetization to relax partially without measuring anything. Will M_z be comparable to M_o after t_d?

3. The time $\mathbf{t_d}$ is set by you, and is, at first, a fraction of T_1. The relaxation of M_z over a period $\mathbf{t_d}$ is then incomplete.

4. The magnetization vector after $\mathbf{t_d}$ has a length that is much smaller than its equilibrium value M_o.

5. Now give a second $\pi/2$ pulse.
6. This forces M_z (**now with a value much smaller than M_o**) to get turned on to the x-y plane.
7. Allow this small M_z to relax completely. This time we will measure its relaxation.
8. Measure the FID signal during this relaxation step. This will give you the magnitude of M_z for the value of t_d you chose.

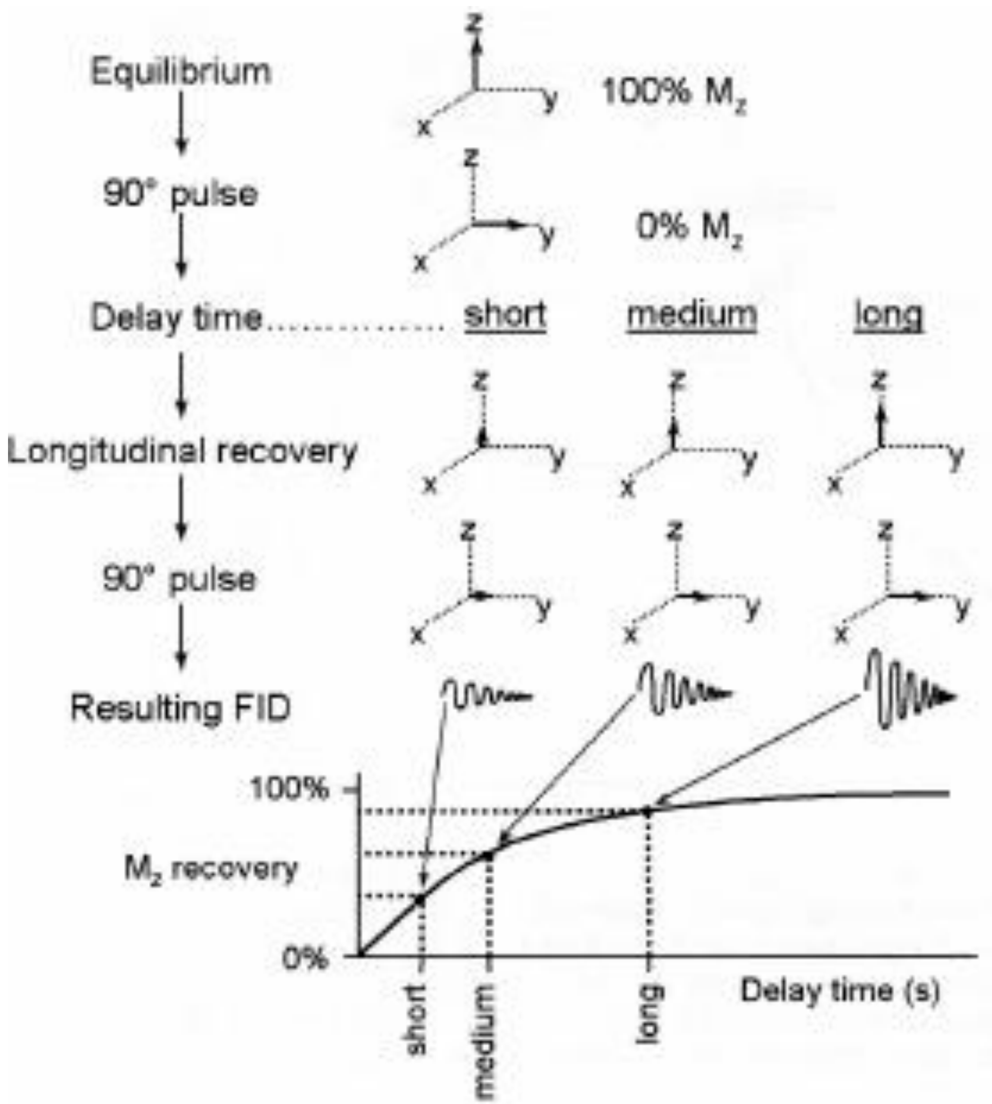
9. Repeat steps 1 – 8 by gradually increasing the value of t_d until it is at least ~ $5T_1$ - $10T_1$.



t_R ~ pulse repetition time
t_d ~ delay time

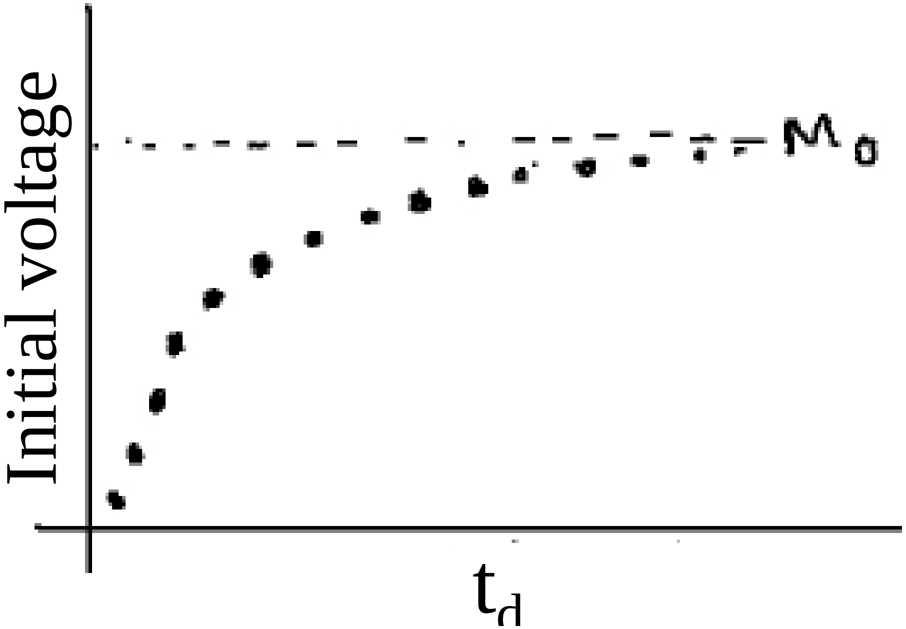
- Usually, $t_R \sim 5T_1$ => ensures full "<u>saturation</u>"
- Vary t_d (from a fraction of T_1 to a few times T_1).

Saturation recovery: $\pi/2 - t_d - \pi/2$ pulse



$$\boxed{M_z(t) = M_0(1 - e^{-t/\tau_1})}$$

At $t = \infty$, $M_z = M_o$



$$M_z(t) = M_0(1 - e^{-t/\tau_1})$$

$$\ln \frac{(Mo - Mz)}{M_o} = -\frac{t}{T_1}$$