Lecture 1
Introduction to medical imaging

We will discuss the following questions in this module

1. Why do we need medical imaging?
2. Why are there so many different imaging modalities?
3. What are the shortcomings of current imaging techniques?

Why do we need medical imaging?

Why do we need medical imaging?

- Diagnosing diseases

www.bbc.com

- Structural and functional information about healthy tissues

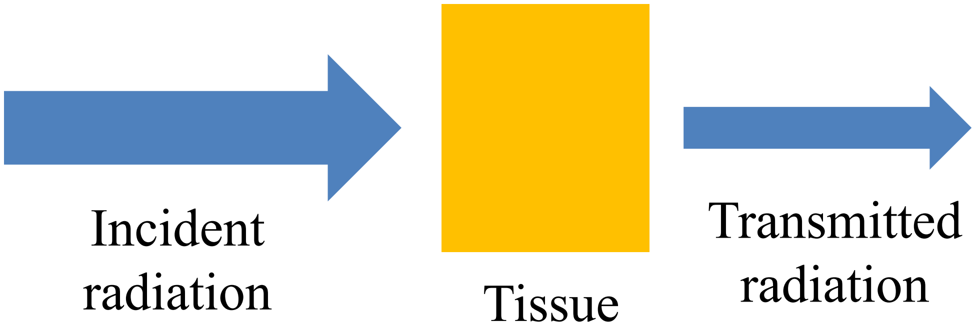
www.mayoclinic.org

- Usually non-invasive

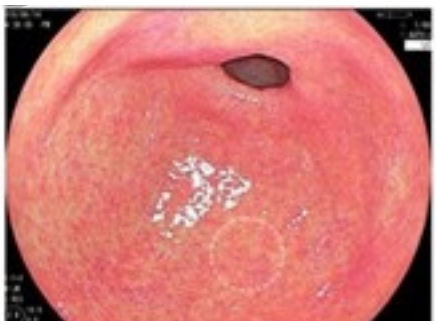
What is the principle of medical imaging?

- Interaction of some form of energy (e.g. electromagnetic wave for x-rays, sound waves for ultrasound imaging, etc.) with biological tissues.

- Tissue needs to be **semi-transparent** to the radiation. Neither completely transparent, nor completely opaque to the radiation.

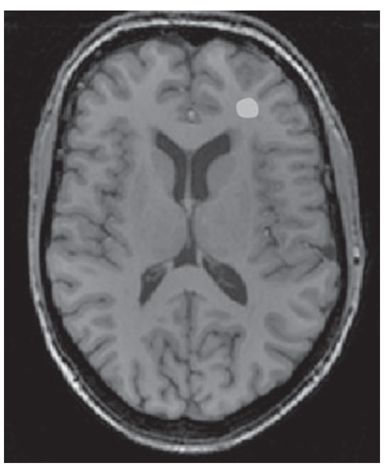


Can you identify these imaging techniques?



Endoscopy
(Sun, *et. al.*, Sci. Rep., 2017)

X-ray
(Smith and Webb)



MRI
(Smith and Webb)

Ultrasound
(www.wikipedia.org)

Why are there so many different medical imaging modalities?

Why are there so many different medical imaging modalities?

- Each technique involves a separate physical interaction of energy with the biological tissue.
- Each technique measures different physical properties of tissue.
- Two tissues may be very similar in one property, but differ in another.

Are there any problems with current imaging techniques?

Are there any problems with current imaging techniques?

- Spatial resolution ~ 1 mm. Early detection of cancer is difficult without molecular diagnostic tests!
- Hazards (ionizing x-rays, high magnetic fields)
- Bulky, expensive equipment. Needs trained people to operate.
- Slow

Lecture 2
Introduction to x-rays

Reading material for x-ray production

1. Smith and Webb: Chap. 2, pages 34 - 42.
2. Hendee: Chap. 2 (pages 12 - 16) for atomic physics concepts; Chap. 5 for x-ray production

Attempt the worked out problems in each chapter.

X-rays

www.wikipedia.org

- The oldest diagnostic imaging technique (image taken in 1895).
- Led to the first Nobel prize in physics in 1901

For the interesting story behind the discovery, read: https://www.nobelprize.org/prizes/physics/1901/perspectives/

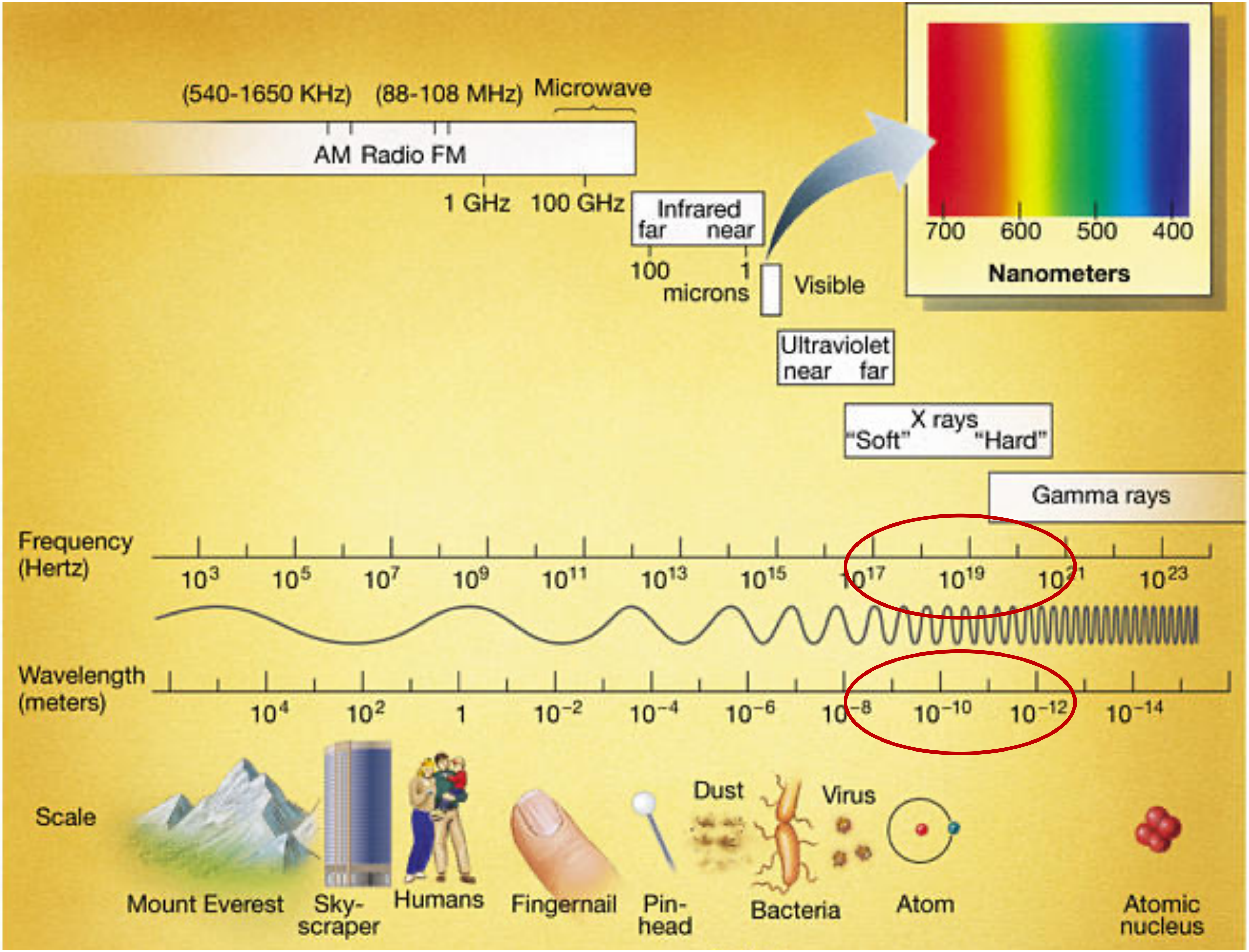
What are x-rays?

- Made of photons with no mass or charge. Can't be deflected by electric or magnetic fields.

- Travels in vacuum with a speed of ~ 3×10^8 m/s.

- Energy (E) = $h\nu = hc/\lambda$;

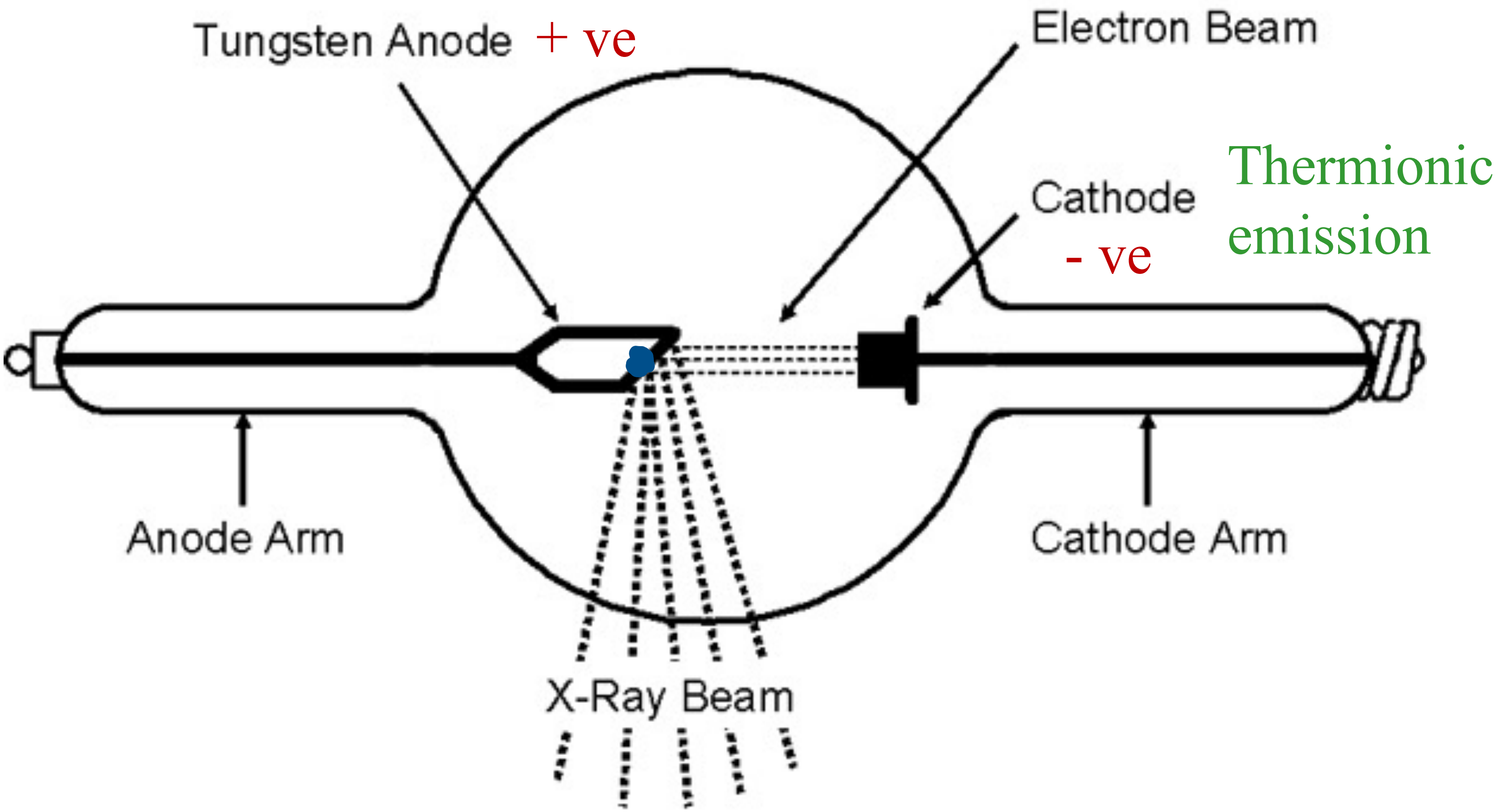
  h: Planck's constant (6.626×10^{-34} Joule-sec)


(540-1650 KHz)
(88-108 MHz)
Microwave
AM Radio FM
1 GHz
100 GHz
Infrared
far near
100
1
microns
Visible
700
600
500
400
Nanometers
Ultraviolet
near far
X rays
"Soft" "Hard"
Gamma rays
Frequency (Hertz)
10^3 10^5 10^7 10^9 10^{11} 10^{13} 10^{15} 10^{17} 10^{19} 10^{21} 10^{23}
Wavelength (meters)
10^4 10^2 1 10^{-2} 10^{-4} 10^{-6} 10^{-8} 10^{-10} 10^{-12} 10^{-14}
Scale
Mount Everest
Sky-scraper
Humans
Fingernail
Pin-head
Dust
Bacteria
Virus
Atom
Atomic nucleus

How are x-rays generated?

X-ray production: Coolidge tube

X-ray production: Coolidge tube



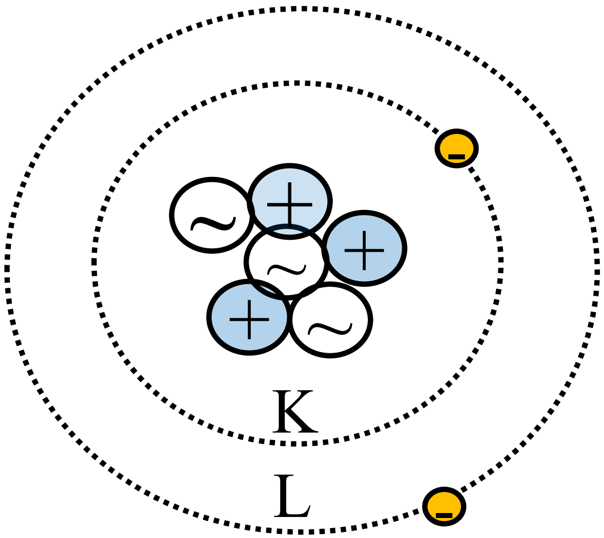
Some of the kinetic energy of electrons hitting the target is converted into x-ray photons; the rest is dissipated as heat.

X-rays are of two kinds

1. Characteristic X-rays
2. Bremsstrahlung (translates in English as "braking radiation")

What happens inside the target (anode)?

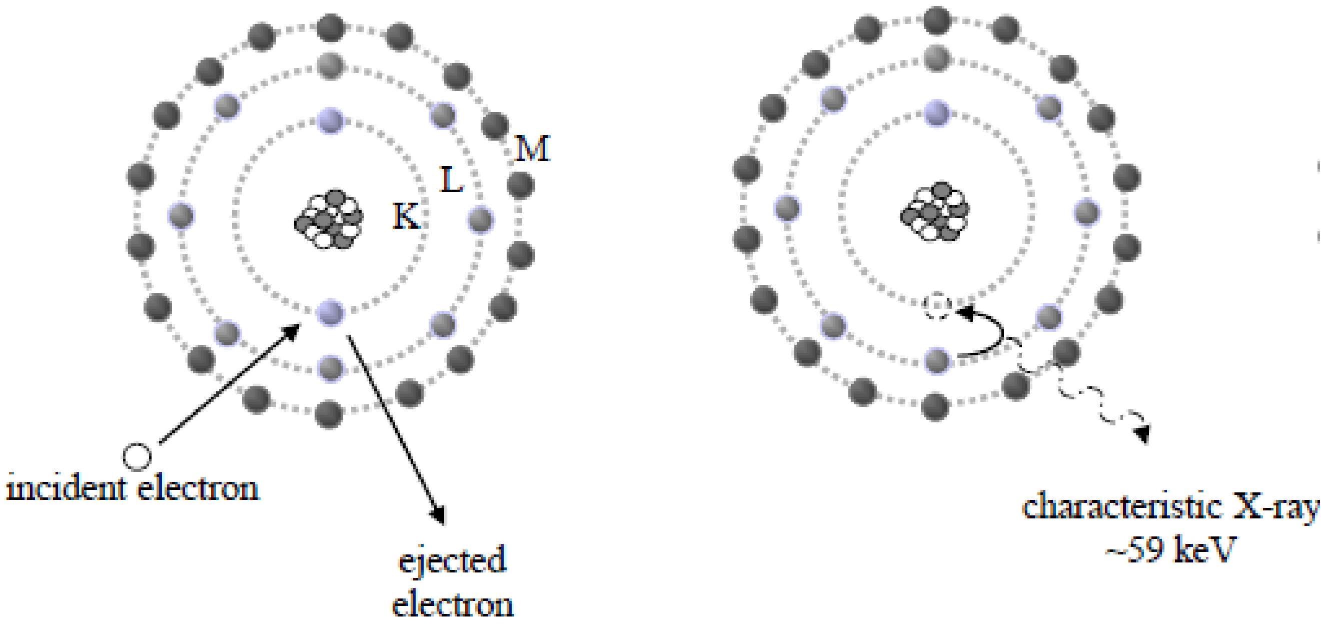
- Electrons have discrete energy levels
- Binding energy: energy input needed to remove electron from atom. Higher binding energy for electrons in inner shells.
- Electron will release energy when it moves from higher to lower energy level.



Shells: K, L,...

Lec 3: Generation of x-rays and X-ray tube

Characteristic x-rays: have specific energies



1. Electron from cathode knocks out an inner shell (K) electron from the anode
2. Another electron from a higher energy shell (L) in anode fills the vacancy
3. Energy given up by L $\rightarrow$ K transition is emitted as x-rays.

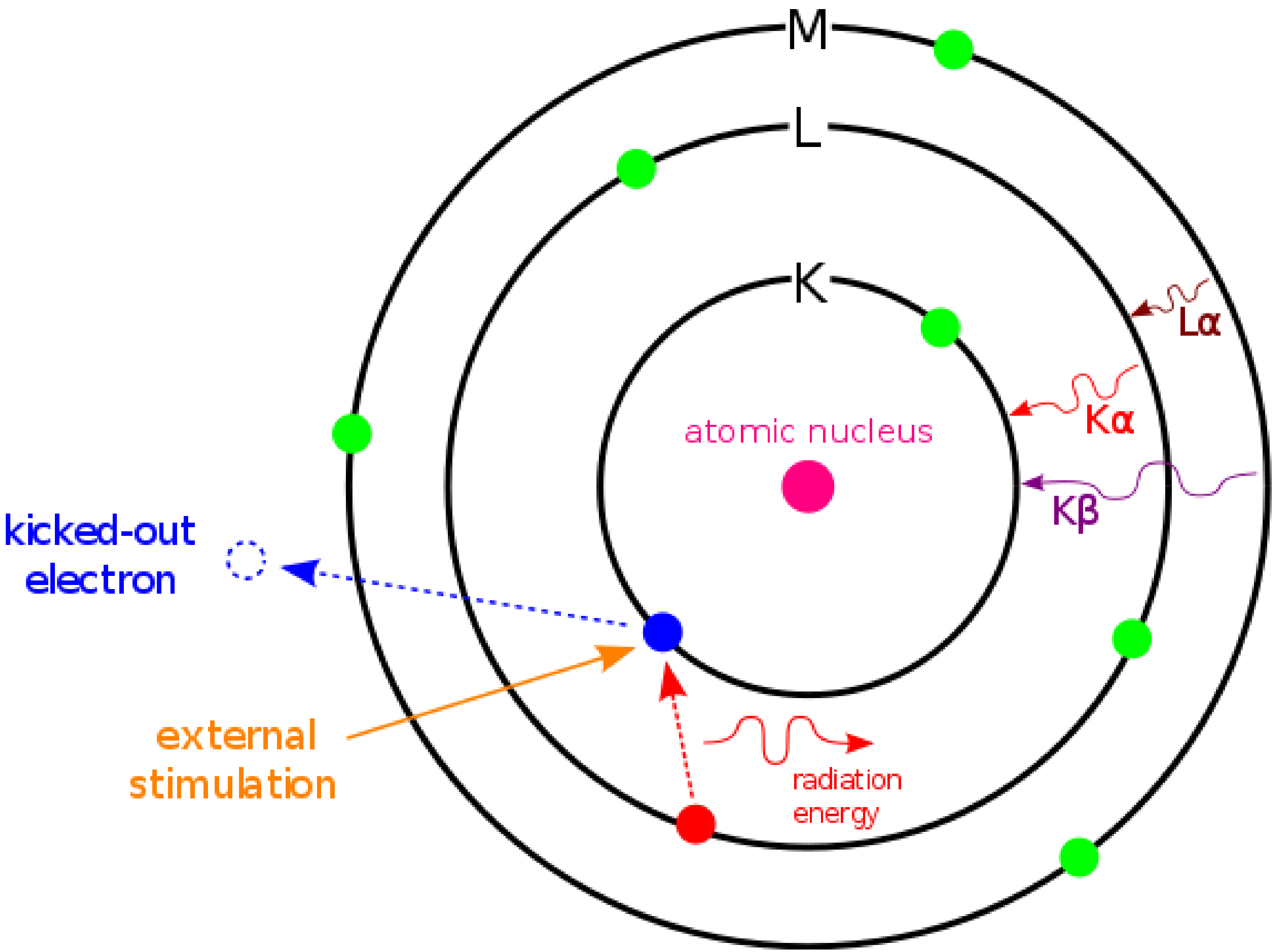
Wavelength and energy of X-rays

Shell	Tungsten (keV)	Molybdenum (keV)
K	69.5	20
L	10.2-12.1	2.5-2.8
M	1.9-2.8	0.4-0.5

Example: Calculate the wavelength range of characteristic x-rays emitted during a transition from M level to K level in Tungsten.

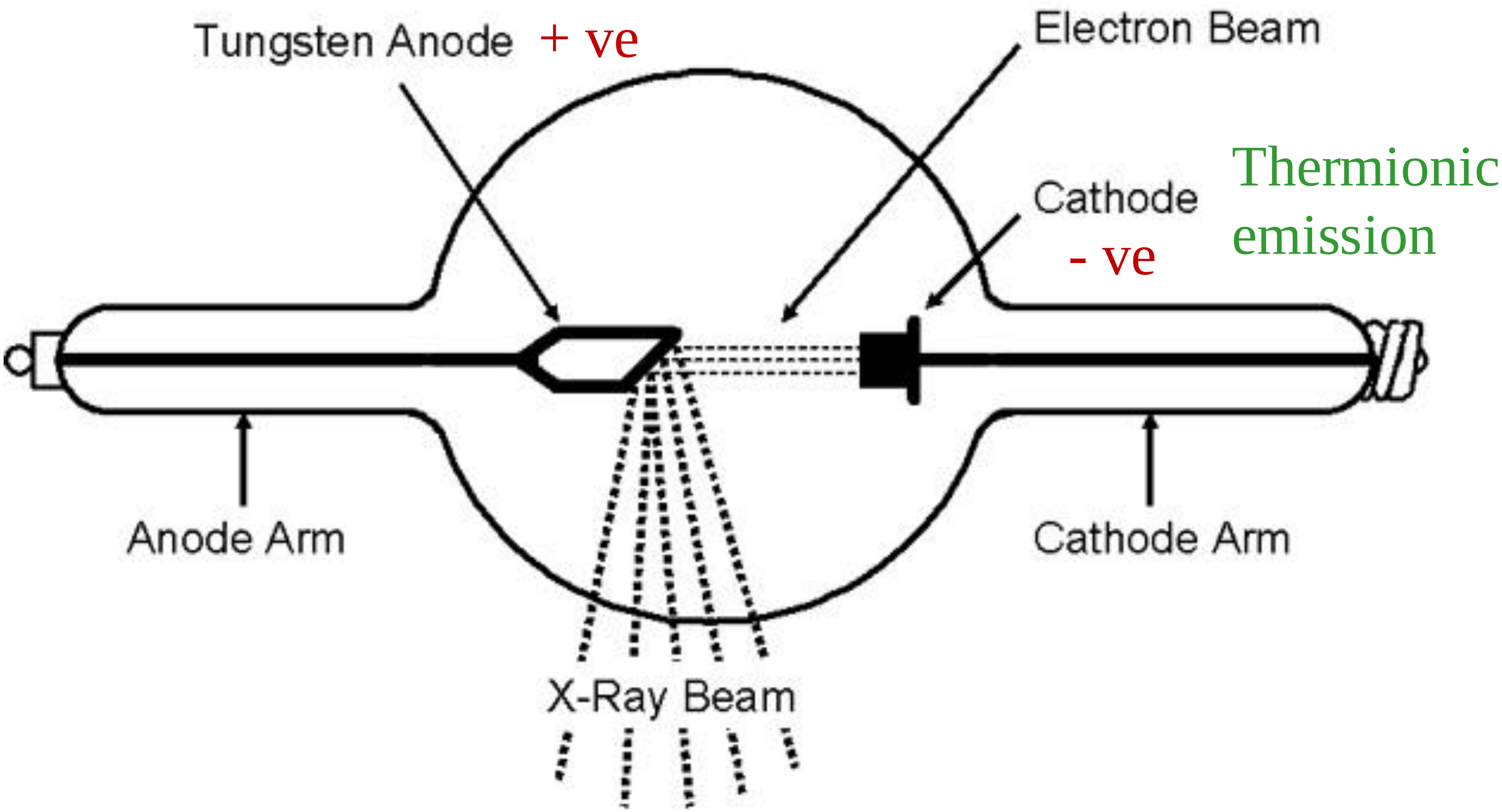
If, E in keV and λ in nm, then E (keV) = 1.24/ λ (nm)

Nomenclature of characteristic lines



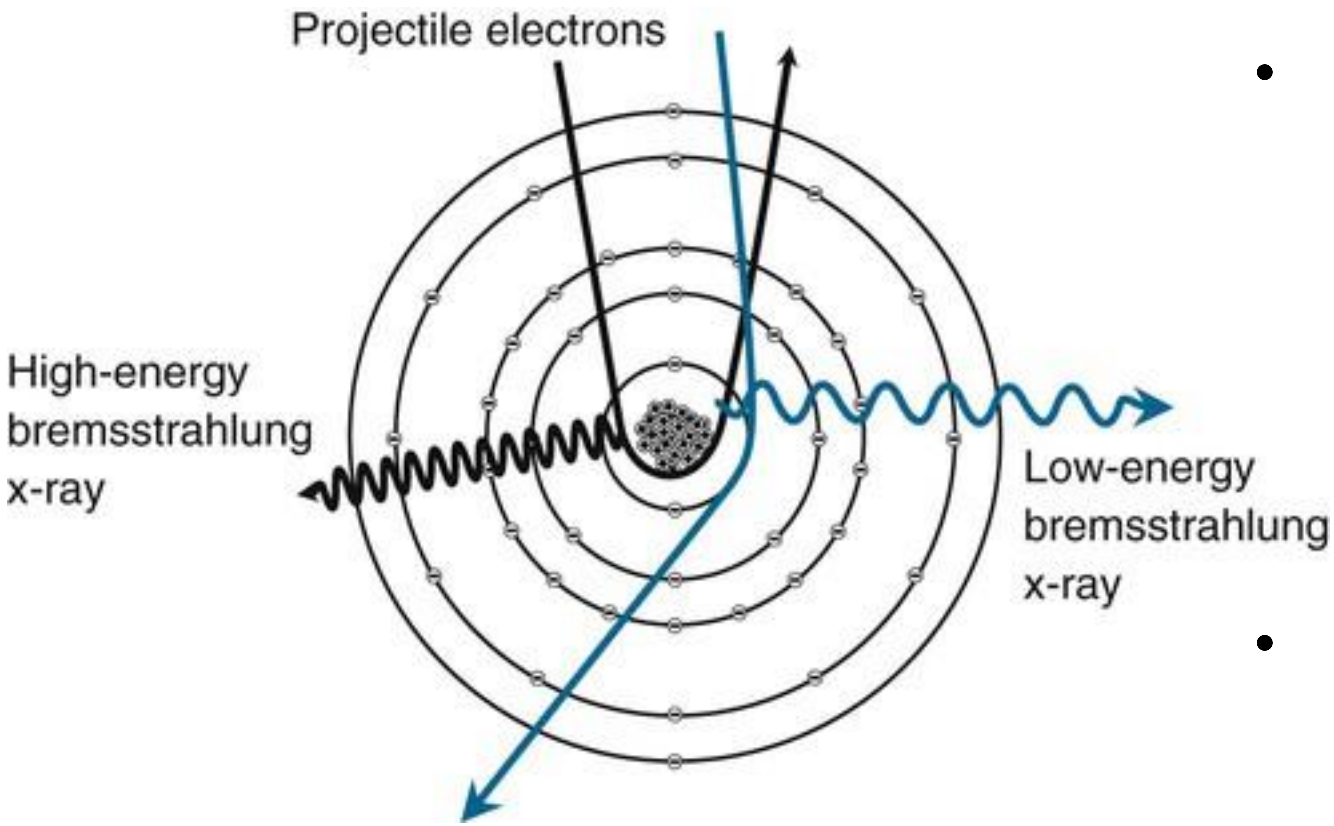
http://www.soest.hawaii.edu/HIGP/Faculty/sksharma/GG711/GG711Lec14EDS.pdf

X-ray production: Coolidge tube



Some of the kinetic energy of electrons hitting the target is converted into x-ray photons; the rest is dissipated as heat.

Continuous energy x-rays: Bremsstrahlung



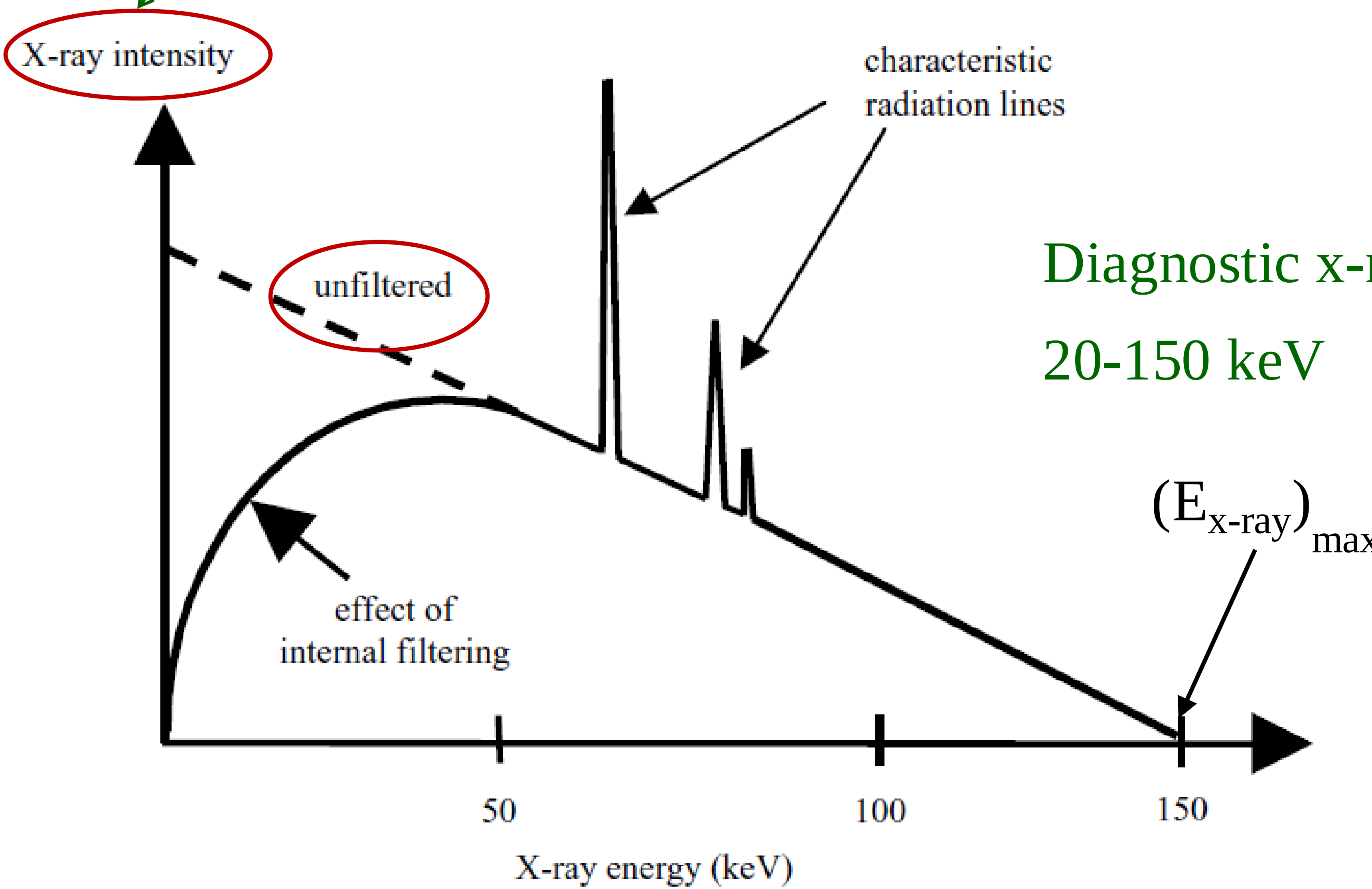
http://physicsopenlab.org/2017/08/02/bremsstrahlung-radiation/

- Instead of knocking off an electron in the anode, electron from cathode changes its direction
- X-rays are emitted with <u>continuous</u> energies.

$$E_{x\text{-}ray} = E_{incident} - E_{final}$$

X-ray spectrum

of photons in the beam



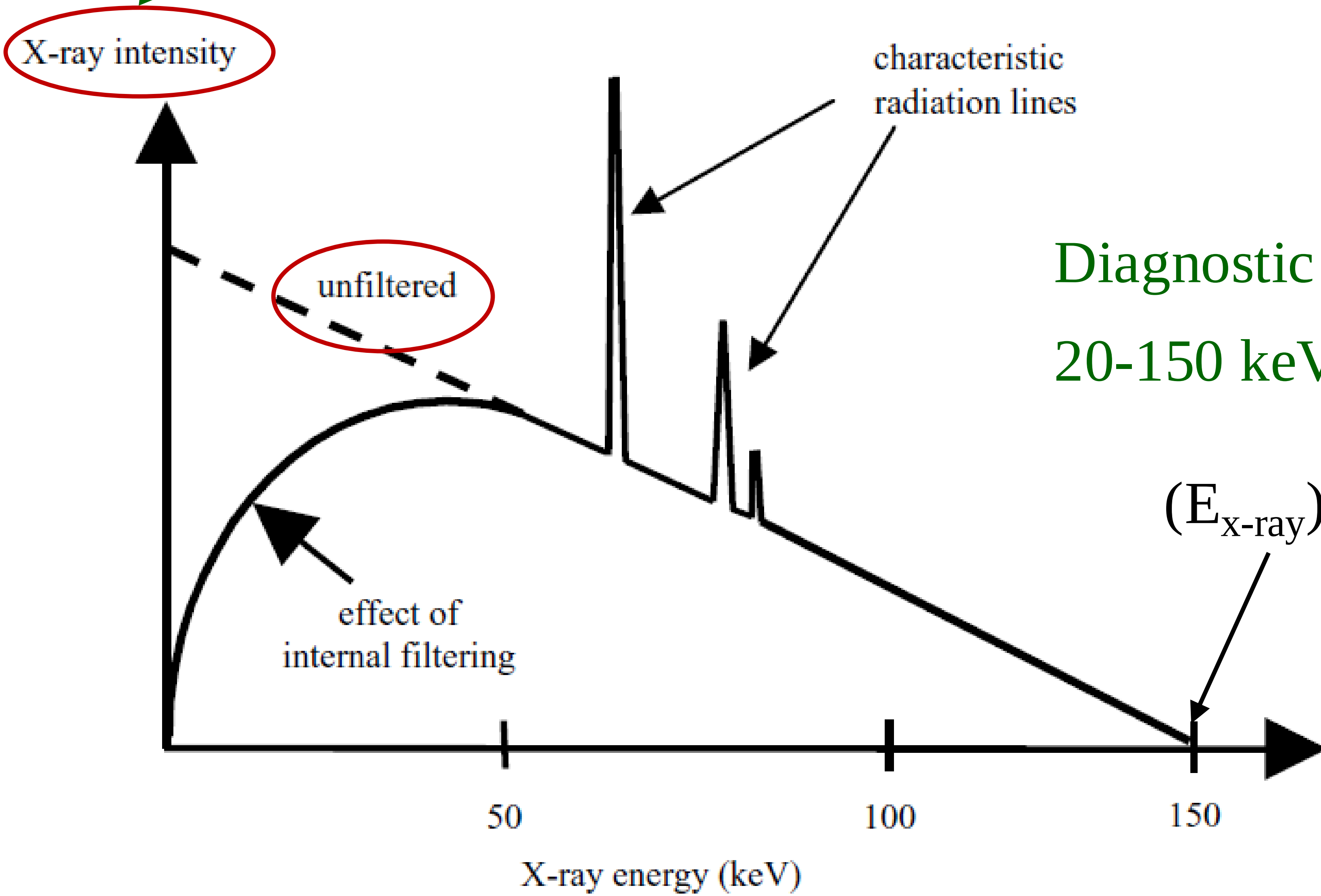
Diagnostic x-rays:

20-150 keV

Lec 4: X-ray tube

Recap: X-ray spectrum

of photons in the beam



$(E_{x\text{-}ray})_{max}$

Diagnostic x-rays:

20-150 keV

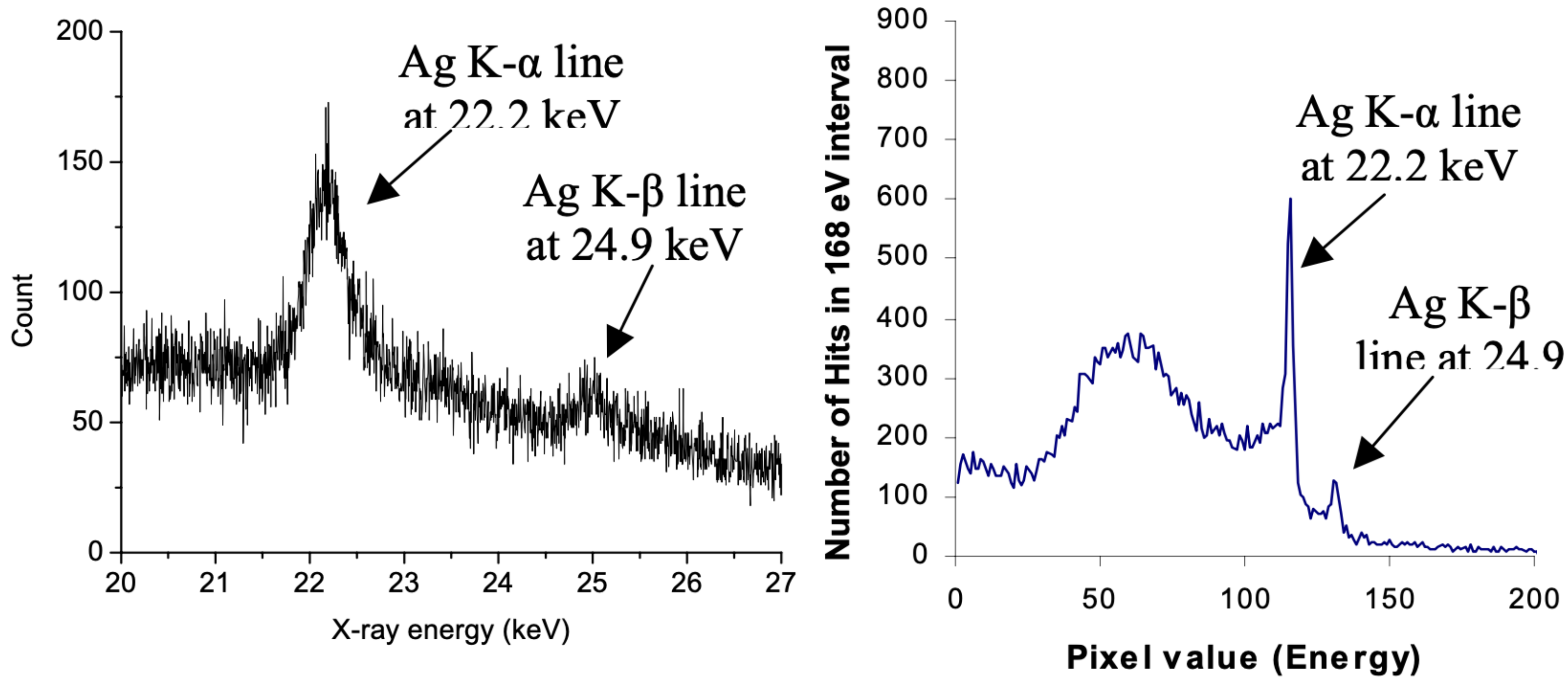
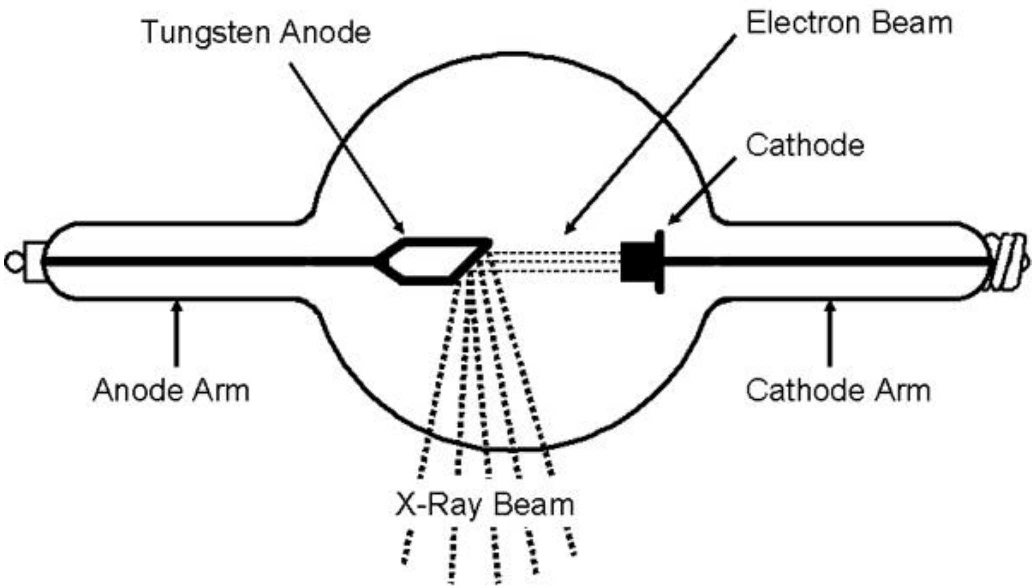


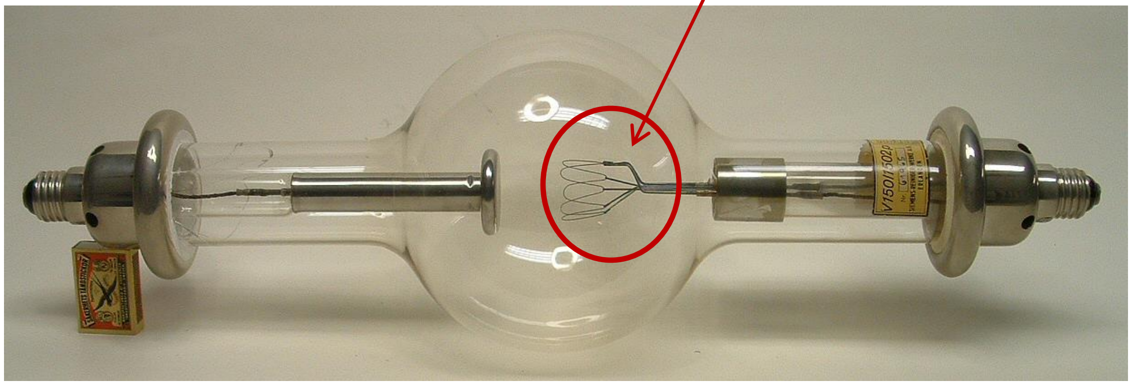
Figure 1: Typical K-alpha/K-beta spectra taken by the two single hit CCD cameras show clearly measurable K-alpha and K-beta peaks.

X-ray tube: basic parts



- Filament (or cathode): held at –ve voltage, temp. ~ 2200^{o}C for thermionic emission
- Target (or anode): held at +ve voltage (~ 150 kV)
- Housing: Vacuum tube, surrounded by oil. Lead shield, with a glass window.

Filament



www.crtsite.com

Filament current is used to heat up the cathode.

Thermionic emission: electrons leave the filament surface <u>due to thermal energy</u> (~ 2200^{o}C) .

Work function

Work function (ϕ): Energy needed to free a loosely-bound valence electron from the surface of the cathode.

$$E_{input} > \phi$$

Thermal

Filament current

Filament current density: $J = AT^2 e^{-\varphi/kT}$

(Richardson-Dushman equation)

$$A = \frac{4\pi e m k^2}{h^3} = 1.2 \times 10^6 \mathrm{A/m^2K^2}$$

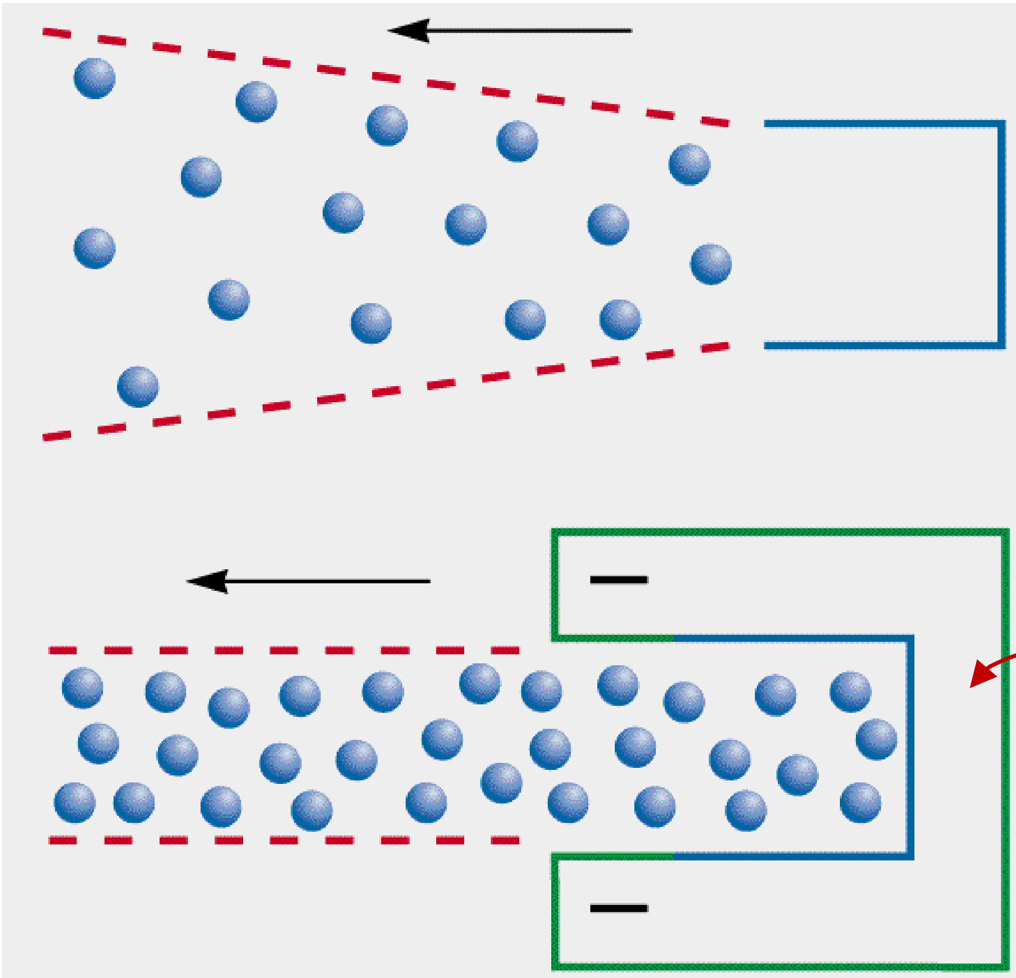
Nobel prize to Richardson in 1928 for thermionic emission

Space charge

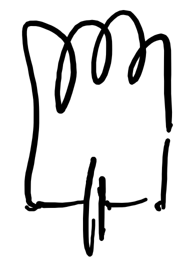
Cloud of emitted electrons around the filament.

Makes it difficult for further electrons to be emitted.

Cathode



Electrons without focusing



In presence of a focusing cup

Filament + focusing cup = cathode (-ve charge)

Bushong, chapter 7

"Sun burn" of the filament

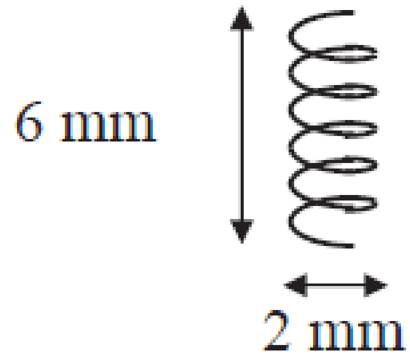
- Particles vaporize under high heat and solidify on the glass.
- Destroys the vacuum integrity of the tube.

New filament materials:

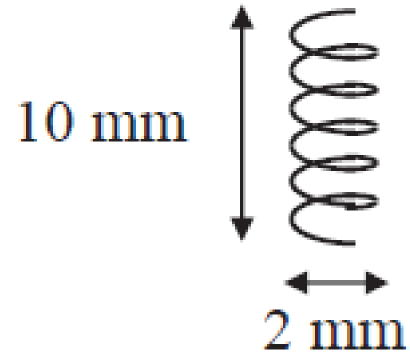
1. Reduced work function (certain oxide coatings)
2. Low sun-burn effect (add thorium to tungsten)

Dual filament

Small filament: Resolve fine features due to tight focus.

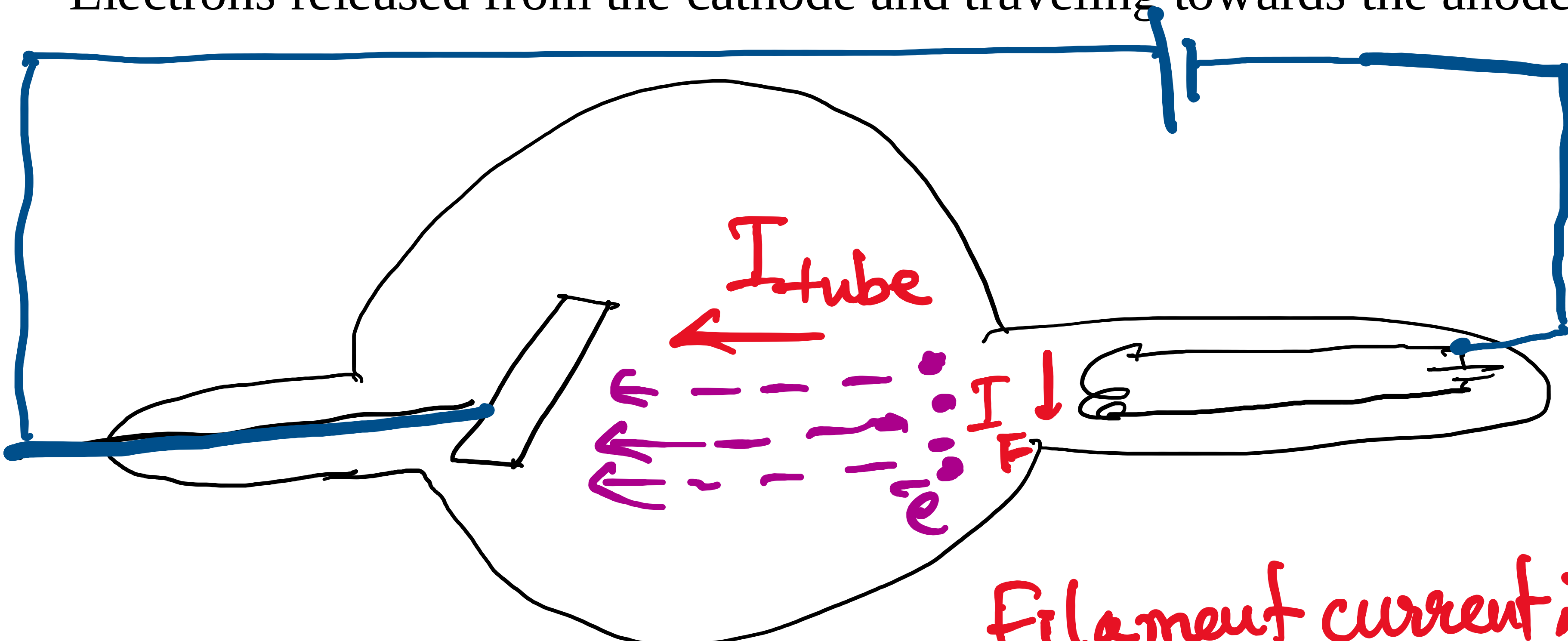


Big filament: Gives short, intense exposure (high electron emission). Useful to avoid motion blurring.



Tube current

Electrons released from the cathode and traveling towards the anode.

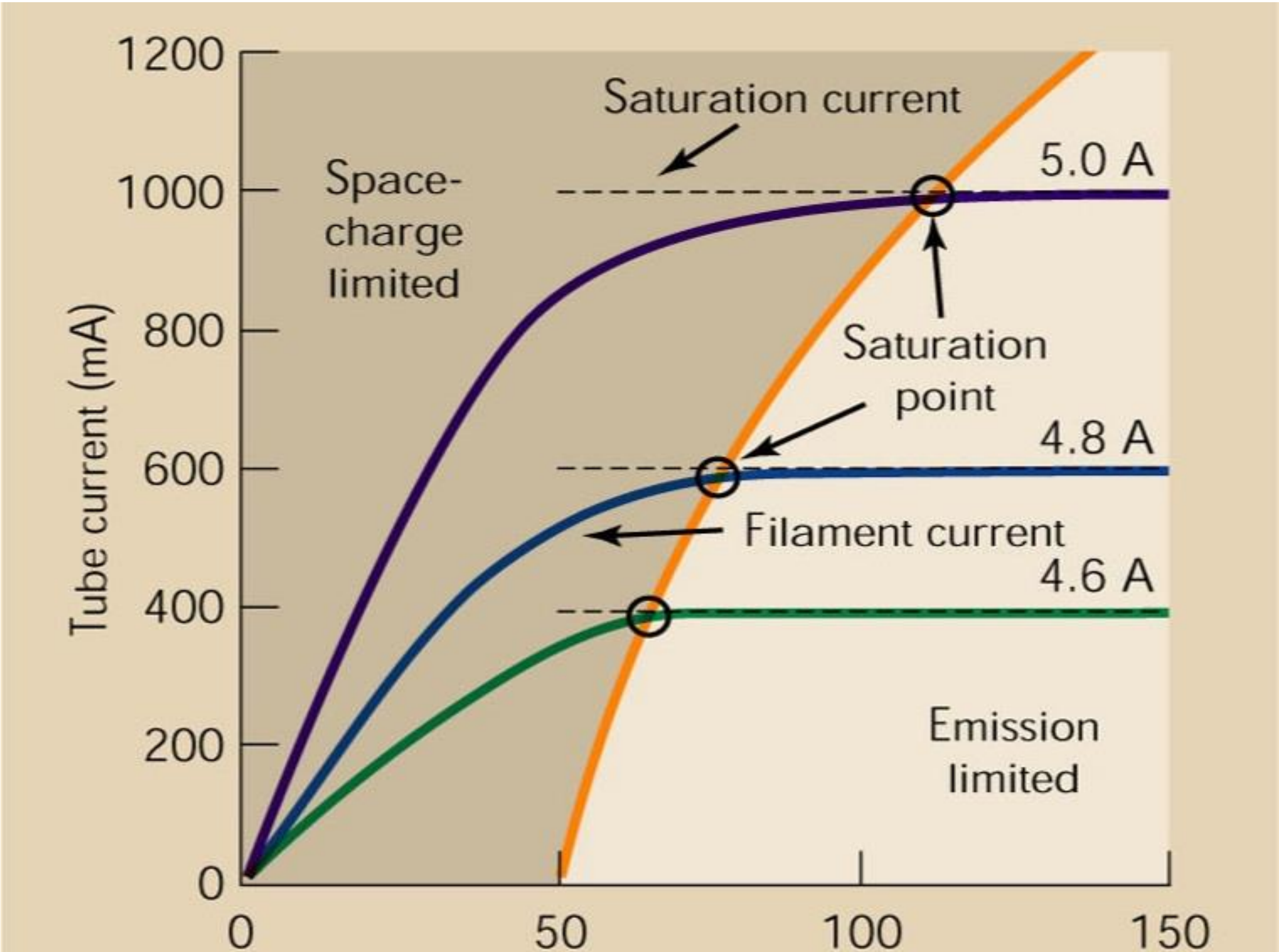


Filament current: leads to thermionic emission.

Tube current: leads to x-ray generation by impinging on anode

Lec 5: X-ray tube

Operating the tube



Tube voltage

The space-charge limited regime occurs when the current of electrons emitted by the cathode is constrained by the repulsive forces between the electrons themselves. This repulsion creates a "space charge" around the cathode, which inhibits the flow of additional electrons.

The emission-limited regime occurs when the current of electrons is constrained by the rate at which electrons can be emitted from the cathode. This situation typically arises when the cathode cannot provide enough electrons to match the current demand set by the applied voltage.

Heat dissipation in the anode

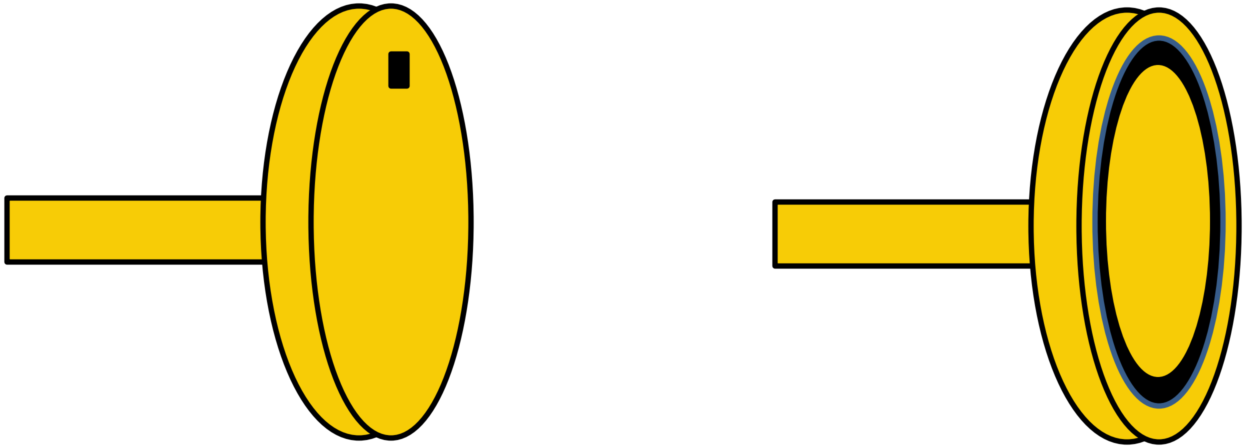
~ 99% of energy is dissipated as heat in the anode. Can reduce its lifetime. What can we do to improve the situation?

What can we do?

1. Anode material (adding rhenium to tungsten improves strength)

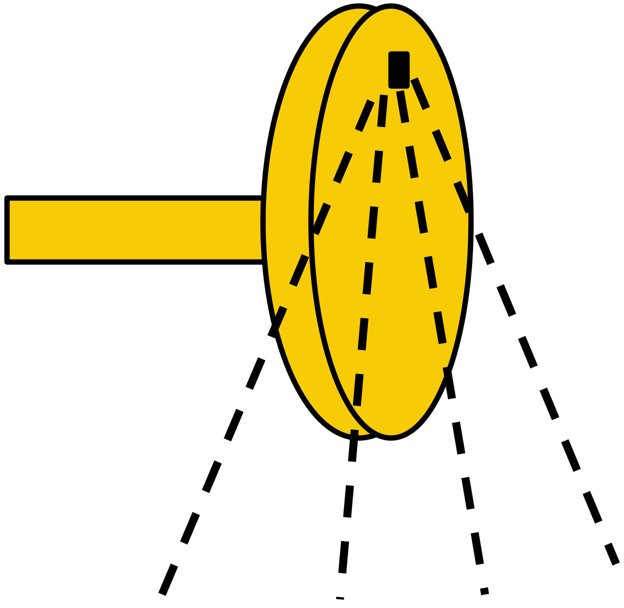
2. Thin layer of anode material (e.g. tungsten) embedded in a thick copper block

3. Rotation of anode (~3000 rpm)



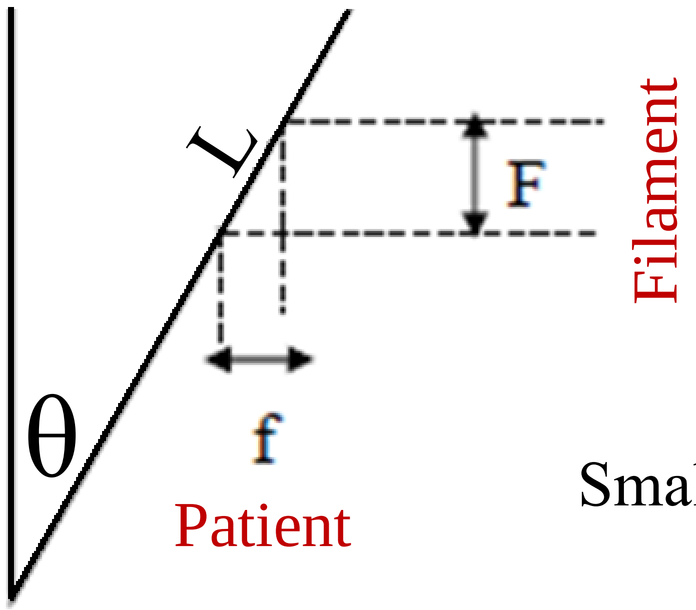
Focal spot in anode

Volume of the anode within which electrons are absorbed and x-rays are emitted



Anode bevel: line focus

Bevel angle (θ) = 12 - 15^o

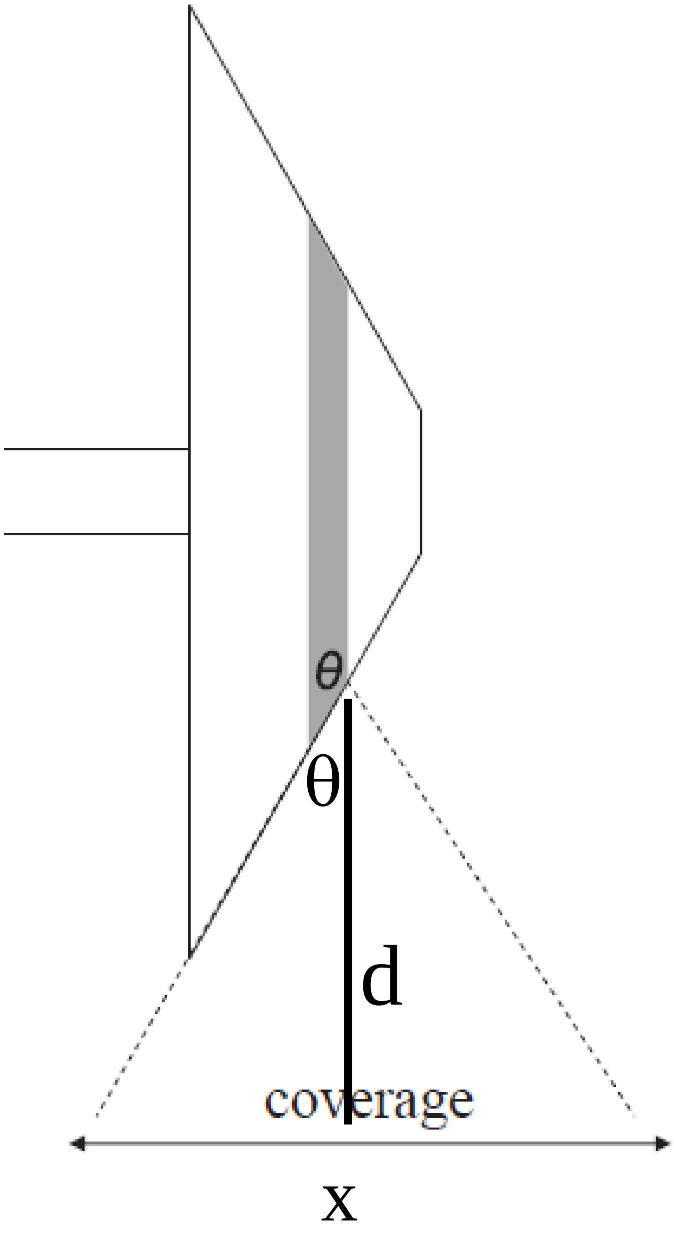


$f = L \sin \theta$
$F = L \cos \theta$

Small focal spot from patient's side

Effective focal spot size ranges between 0.6 and 1.2 mm

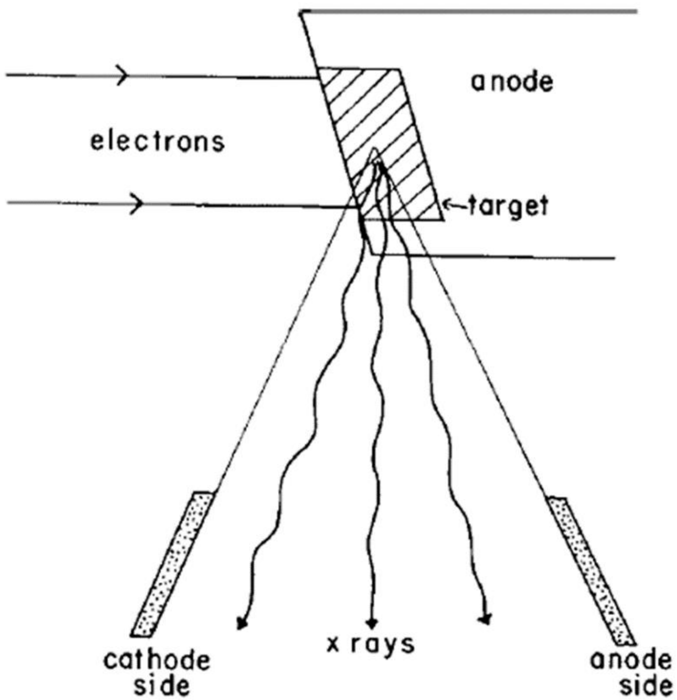
Coverage



Calculate the coverage (x) of the X-rays in terms of source-patient distance (d) and bevel angle (θ).

$$\frac{x}{2d} = \tan\theta$$

Anode heel effect



Hendee, pg. 81

- X-rays travel a longer distance to reach the anode side => more attenuation
- Signal intensity varies from one side to the other of an x-ray image.

Higher intensity x-rays on cathode side than anode side

Discussion point:

What kind of measures can we put in the x-ray tube to improve the image focus? (I am not talking about image processing here).

Filters

- Inherent filtration in tube (we have already seen in x-ray spectrum). <u>Inherent filtration is equivalent to 1 mm Al filter.</u>

- Additional filters (e.g. aluminium metal sheets) are added in the beam path to remove low energy x-rays.

- Why do we need filters?

Soft X-rays: X-ray spectra produced by an X-ray tube include a range of X-ray energies. Some of these X-rays have relatively low energy, often referred to as "soft" X-rays.

Ineffectiveness in Imaging: Soft X-rays do not penetrate tissues or objects as effectively as higher-energy X-rays. Instead, they are more likely to be absorbed by the body or the X-ray equipment, which doesn't contribute to image formation but increases patient dose unnecessarily.

Enhanced Contrast: Removing soft X-rays helps in achieving better image contrast by ensuring that only the more penetrating X-rays reach the detector or film. This improves the overall diagnostic quality of the image.

Reduce Scattering: Soft X-rays can contribute to scattering, which can reduce image clarity and affect the performance of the X-ray detector or film. Filters help minimize this scattering effect.

X-ray output

Proportional to the product of

tube current (mA) and exposure time (sec)

Lec 6: X-ray tube, attenuation photoelectric effect

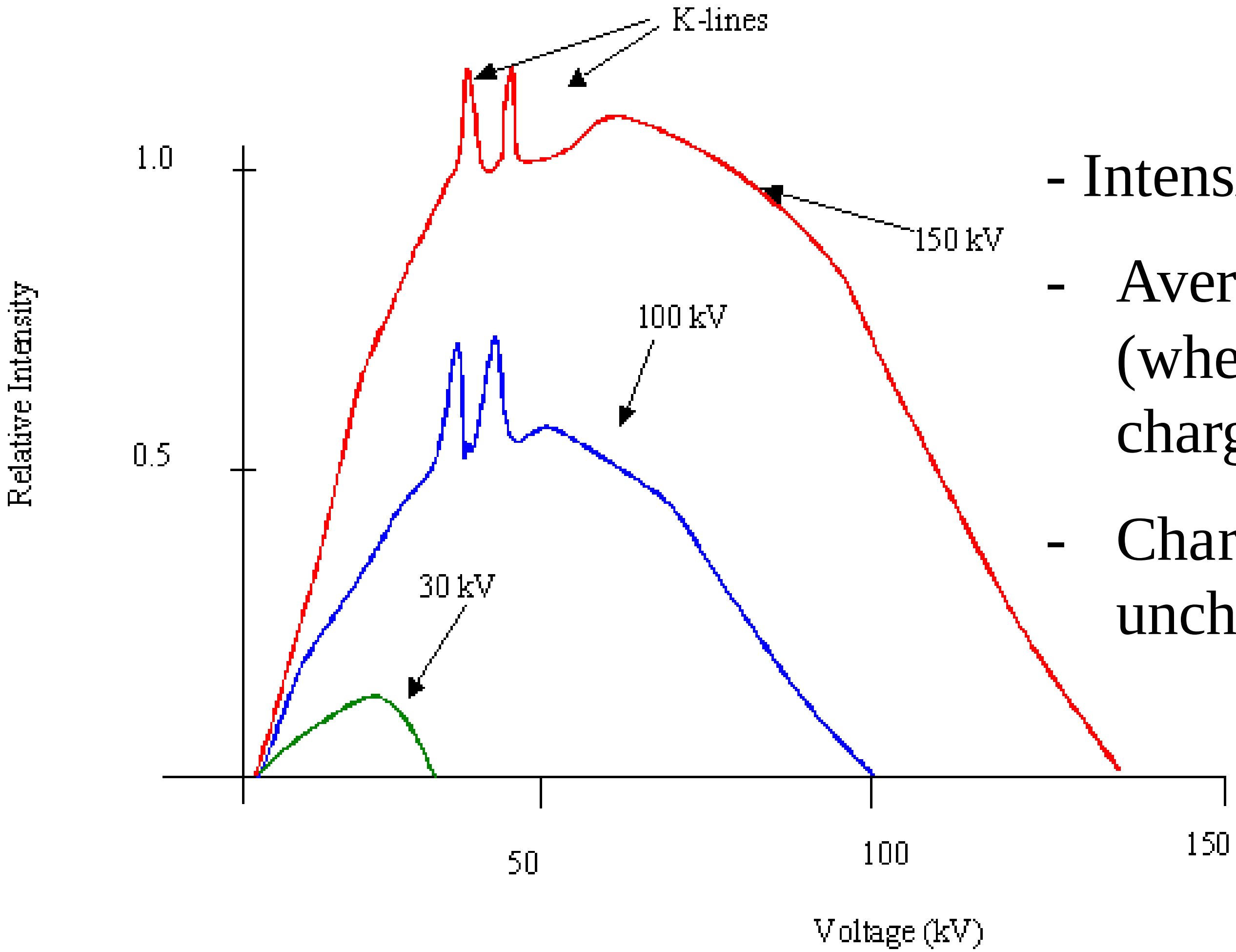
How does the x-ray spectrum change when the tube parameters are changed?

Think about what will happen to

1. Intensity
2. Average energy
3. Characteristic peaks

Increase in tube voltage

Increase in tube voltage



- Intensity goes up.
- Average energy goes up (when we operate in space charge limited region).
- Characteristic lines are unchanged

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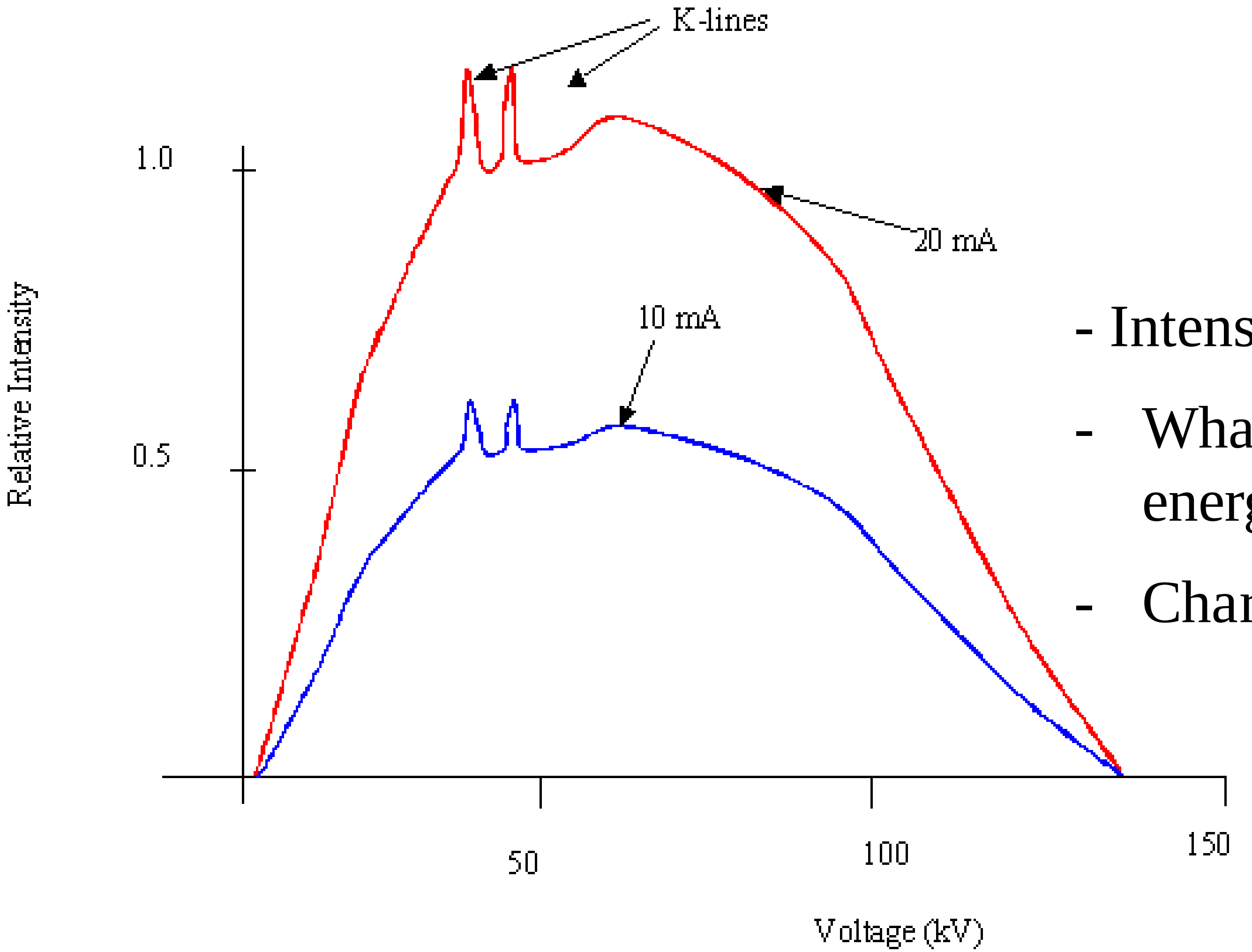
Increase in tube current

Increase in tube current



- Intensity goes up.
- What about average energy?
- Characteristic peaks?

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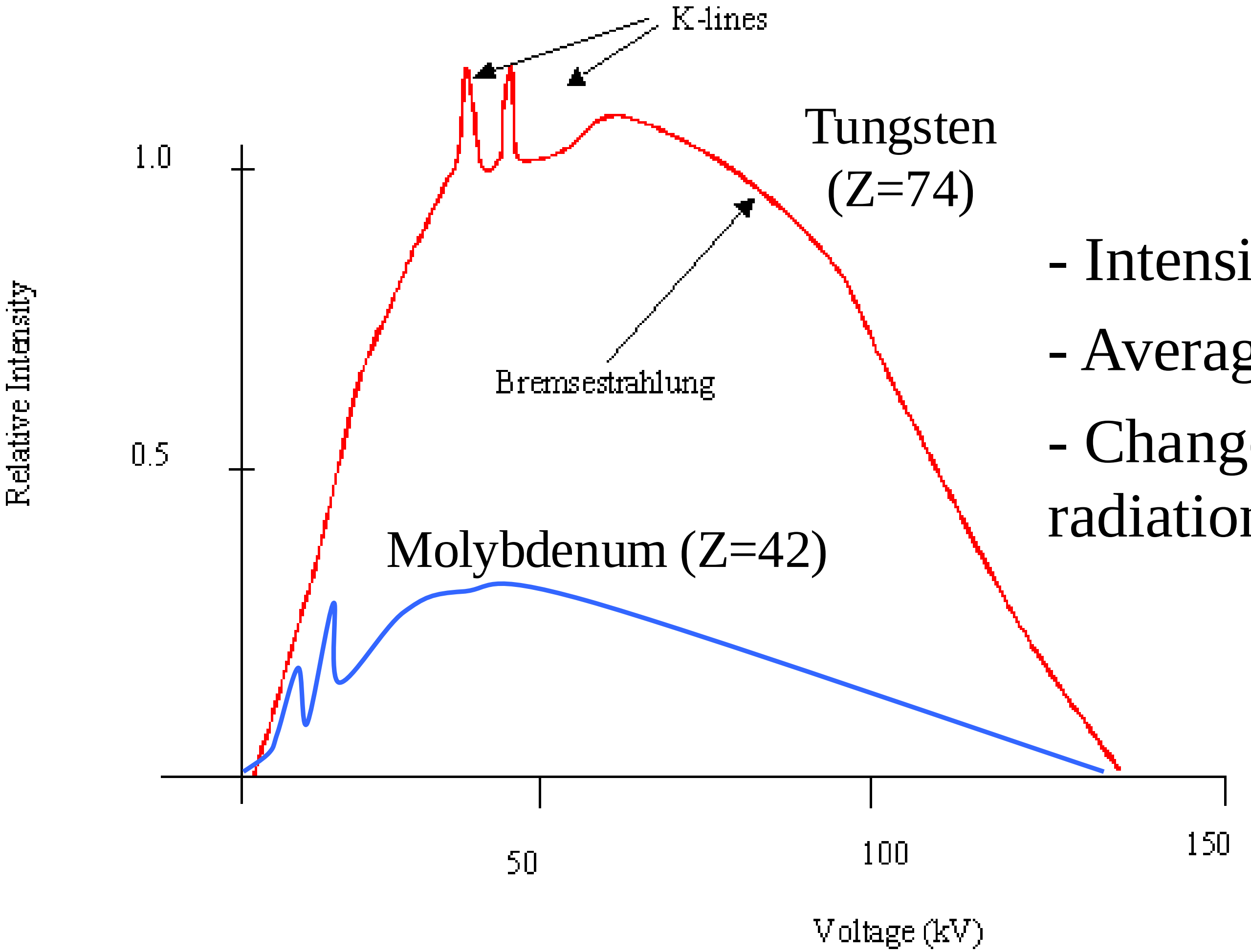
Increase in target (anode) material Z

Increase in target (anode) material Z



- Intensity goes up.
- Average energy goes up.
- Change in characteristic radiation.

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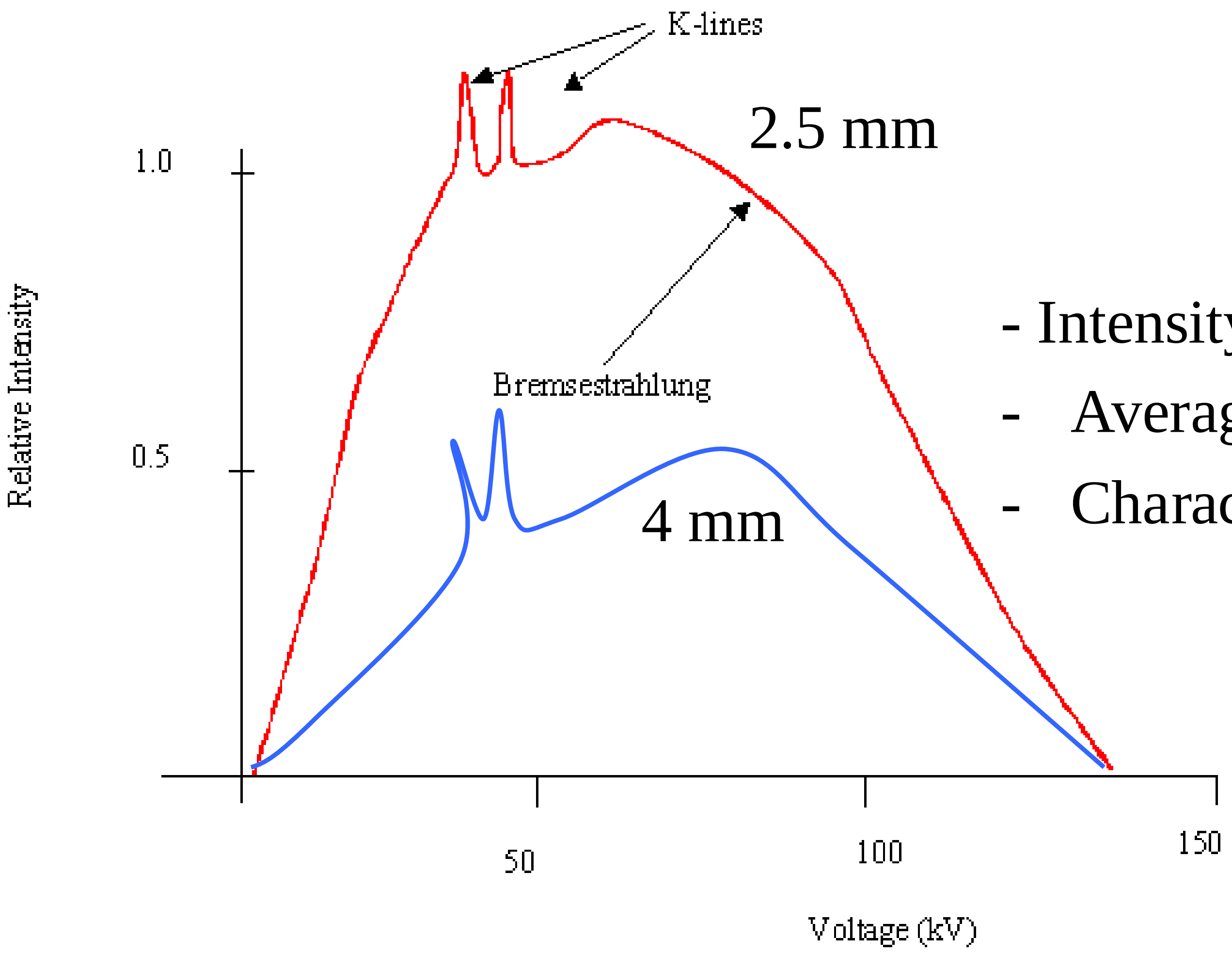
Increase in filter thickness

Increase in filter thickness



- Intensity goes down.
- Average energy goes up.
- Characteristic peaks?

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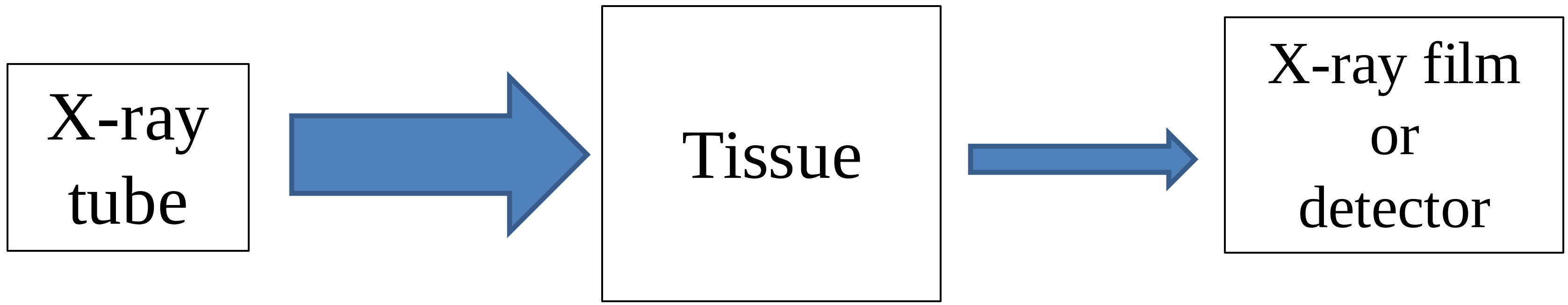
Interaction of X-rays with tissues

Interaction with tissues leads to x-ray attenuation



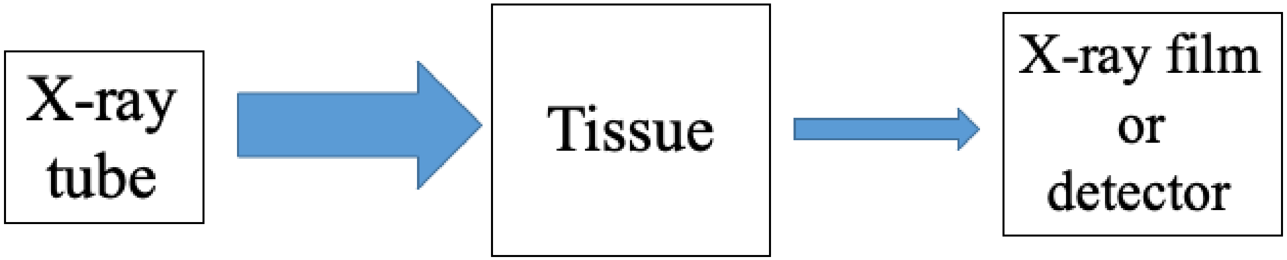
Hendee, chapter 7

Smith and Webb, chapter 2

Attenuation mechanisms: absorption and scattering

Absorption: removal of x-ray photons from the beam

Scattering: change in direction of the photon, usually with reduced energy



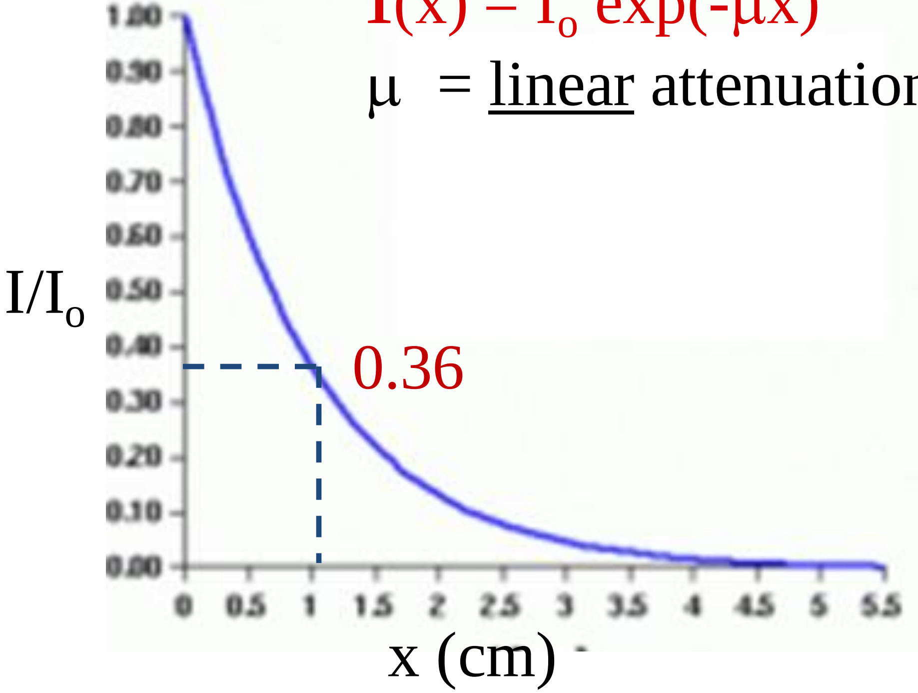
Attenuation and contrast?

High contrast needs sufficiently high **differential attenuation** of x-rays in various tissues.

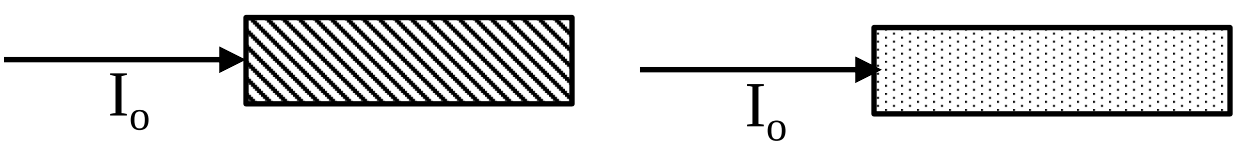
X-ray attenuation

$I(x) = I_o \exp(-\mu x)$

μ = <u>linear</u> attenuation coefficient; unit of μ : 1/distance



How would you compare attenuation in two tissues?



Linear and mass attenuation coefficients

Linear (μ) μ: 1/cm

Mass ($\mu_m = \mu/\rho$) μ_m: cm^2/gm

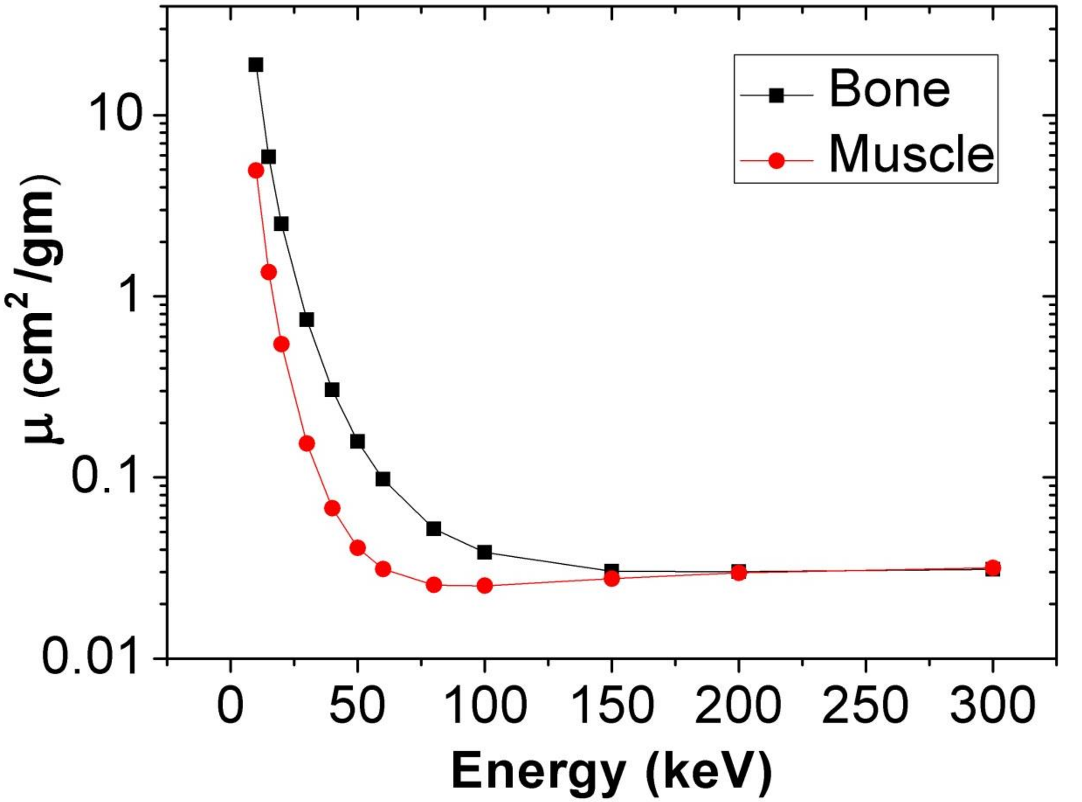
$$I(x) = I_o \exp(-\mu x)$$

$$I(x) = I_o \exp(-\mu_m \rho x)$$

Watch out when solving problems!

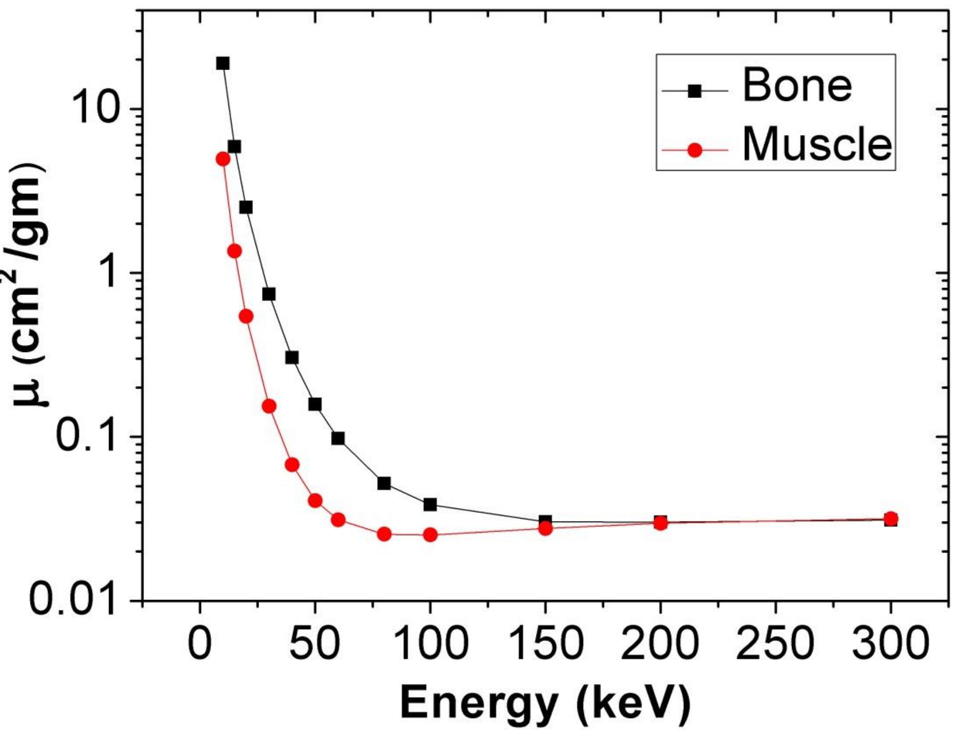
Value of μ depends on x-ray energy

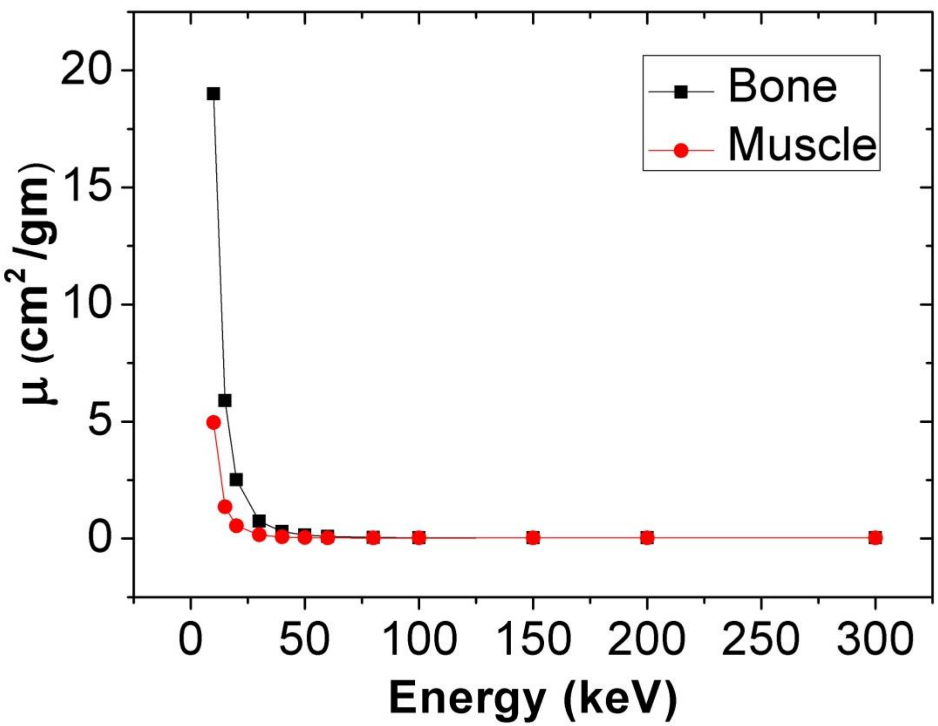
X-ray photon energy (keV)	Mass attenuation coefficient (cm^2/g)	
	Compact bone	Muscle
10	19.0	4.96
15	5.89	1.36
20	2.51	0.544
30	0.743	0.154
40	0.305	0.0677
50	0.158	0.0409
60	0.0979	0.0312
80	0.0520	0.0255
100	0.0386	0.0252
150	0.0304	0.0276
200	0.0302	0.0297
300	0.0311	0.0317



Which energy range should we choose for better contrast?

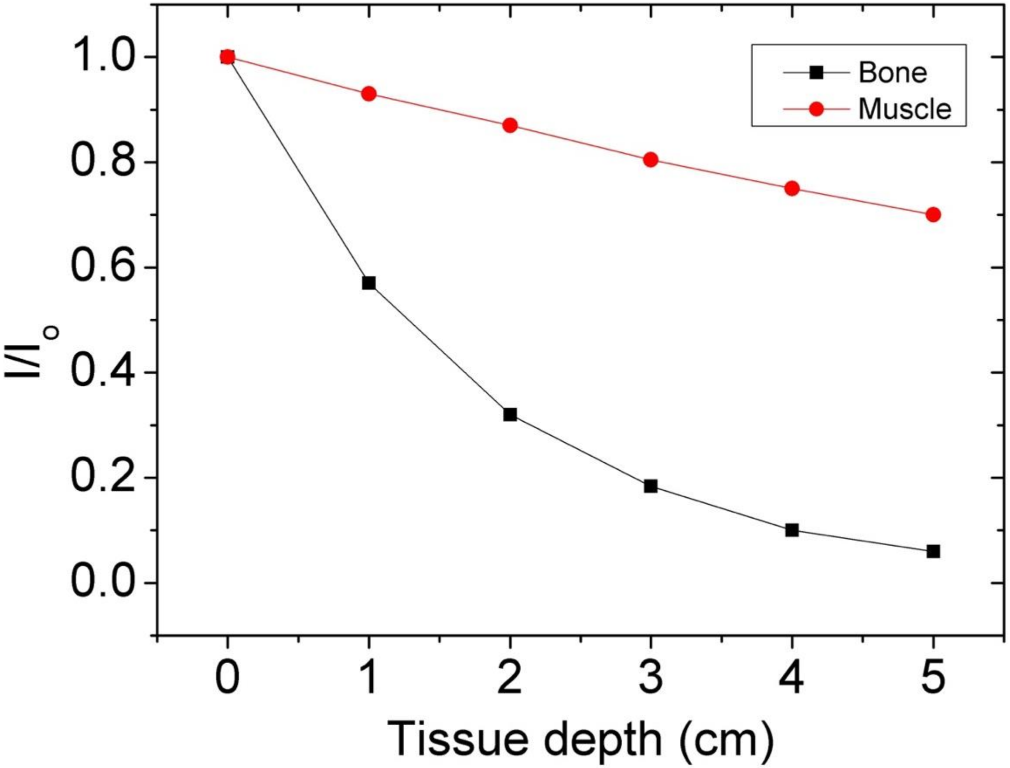
Let's take a look at the y-axis scales…





Plot x-ray intensity in bone and muscle as it progresses in x direction. Assume a tissue depth of 5cm.

Tissue	Density (g/cc)	Effective atomic number
Muscle	1.06	7.4
Fat	0.91	6.9
Bone	1.85	13.8



Recap: Value of μ depends on x-ray energy

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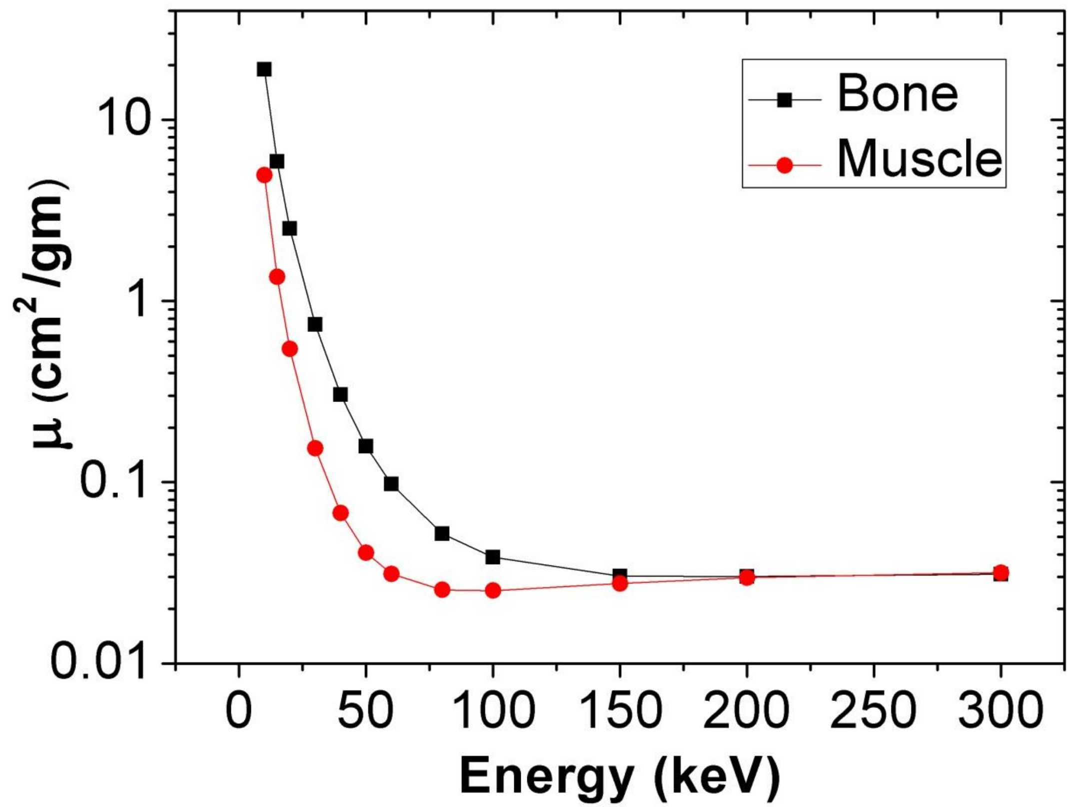
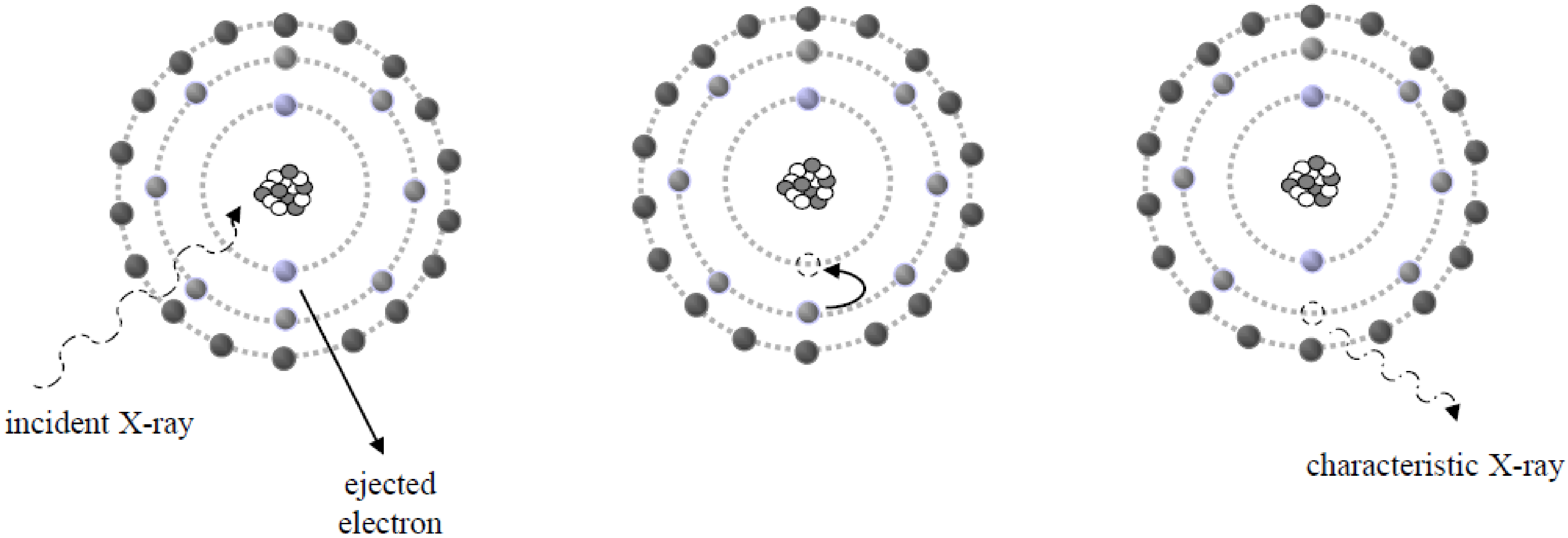


Which energy range should we choose for better contrast?

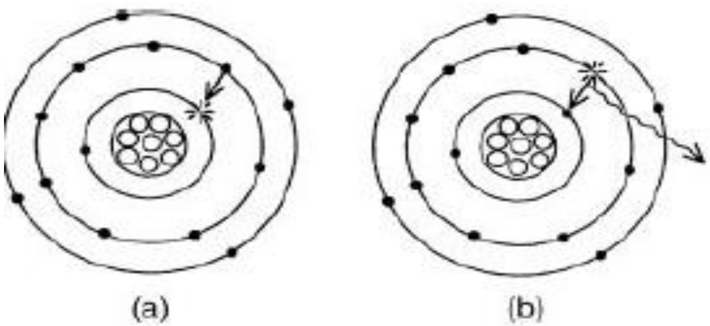
Photoelectric effect



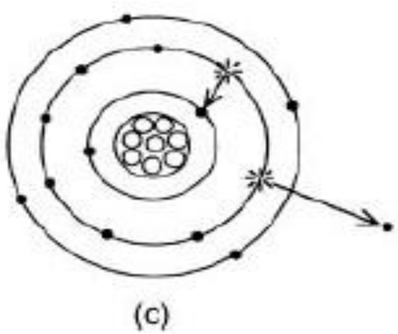
- Photon loses <u>all</u> its energy in one interaction with the tissue
- Watch out- the schematic diagram looks very similar to generation of characteristic x-rays.
- Inner shells

Auger electrons in photoelectric effect

X-ray photon ejects electron



Transition of electron from higher to lower level emits another photon



Auger transition ejects another electron instead of photon

- Transition from higher (say, L) to lower energy shell (say, K) ejects another electron (usually from the same higher energy shell).
- Causes ionization of tissue.

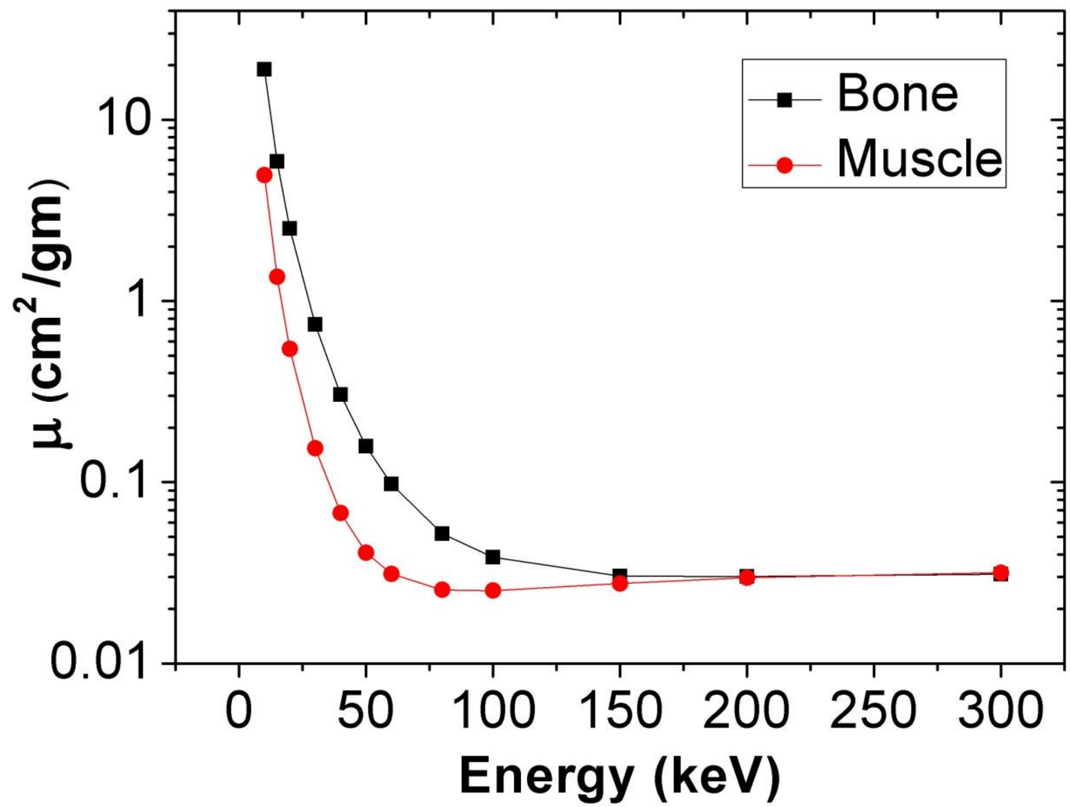
$$K.E._{\text{Auger}} = E_{Bi} - 2E_{Bo}$$

Hendee, Pg. 16

Lec 7: photoelectric and Compton effects; contrast agents

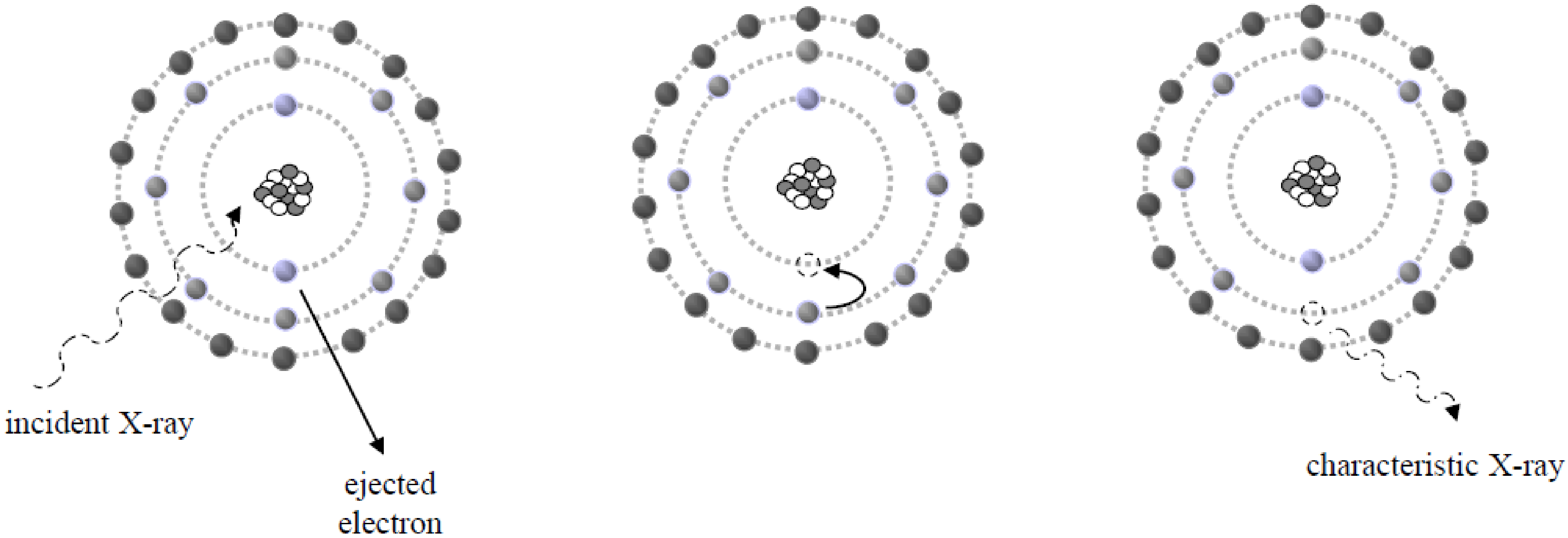
Recap: Value of μ depends on x-ray energy

X-ray photon energy (keV)	Mass attenuation coefficient (cm^2/g)	
	Compact bone	Muscle
10	19.0	4.96
15	5.89	1.36
20	2.51	0.544
30	0.743	0.154
40	0.305	0.0677
50	0.158	0.0409
60	0.0979	0.0312
80	0.0520	0.0255
100	0.0386	0.0252
150	0.0304	0.0276
200	0.0302	0.0297
300	0.0311	0.0317



Which energy range should we choose for better contrast?

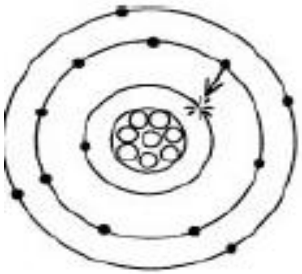
Photoelectric effect



- Photon loses <u>all</u> its energy in one interaction with the tissue
- Watch out- the schematic diagram looks very similar to generation of characteristic x-rays.
- Inner shells

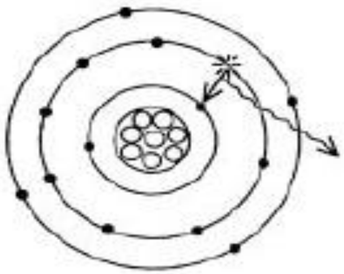
Auger electrons in photoelectric effect

X-ray photon ejects electron

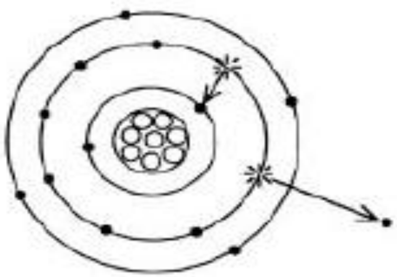


(a)

Transition of electron from higher to lower level emits another photon



(b)



(c)

Auger transition ejects another electron instead of photon

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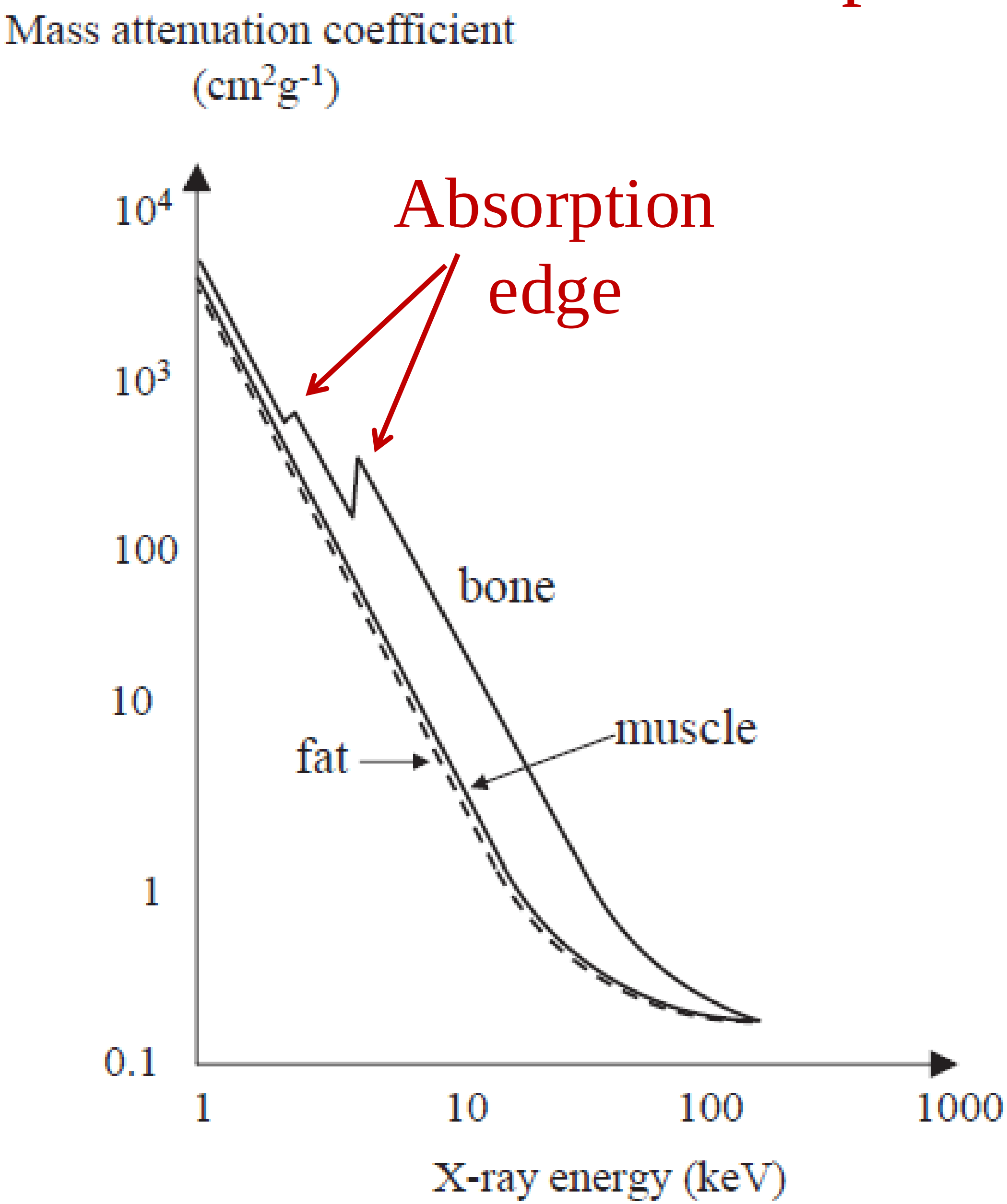
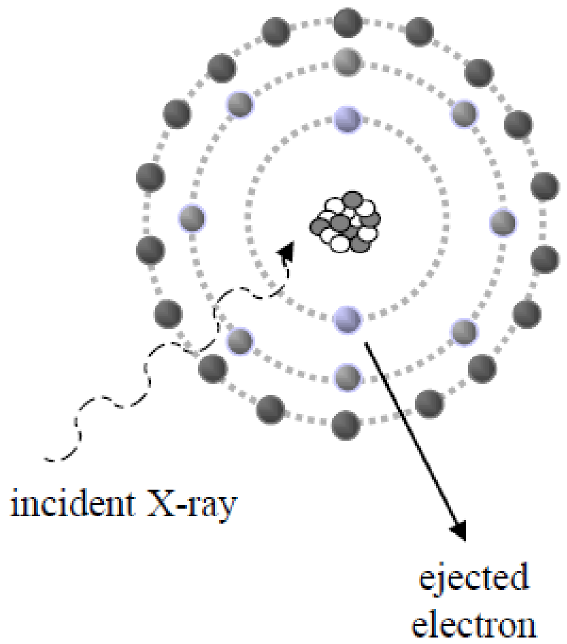
Hendee, Pg. 16

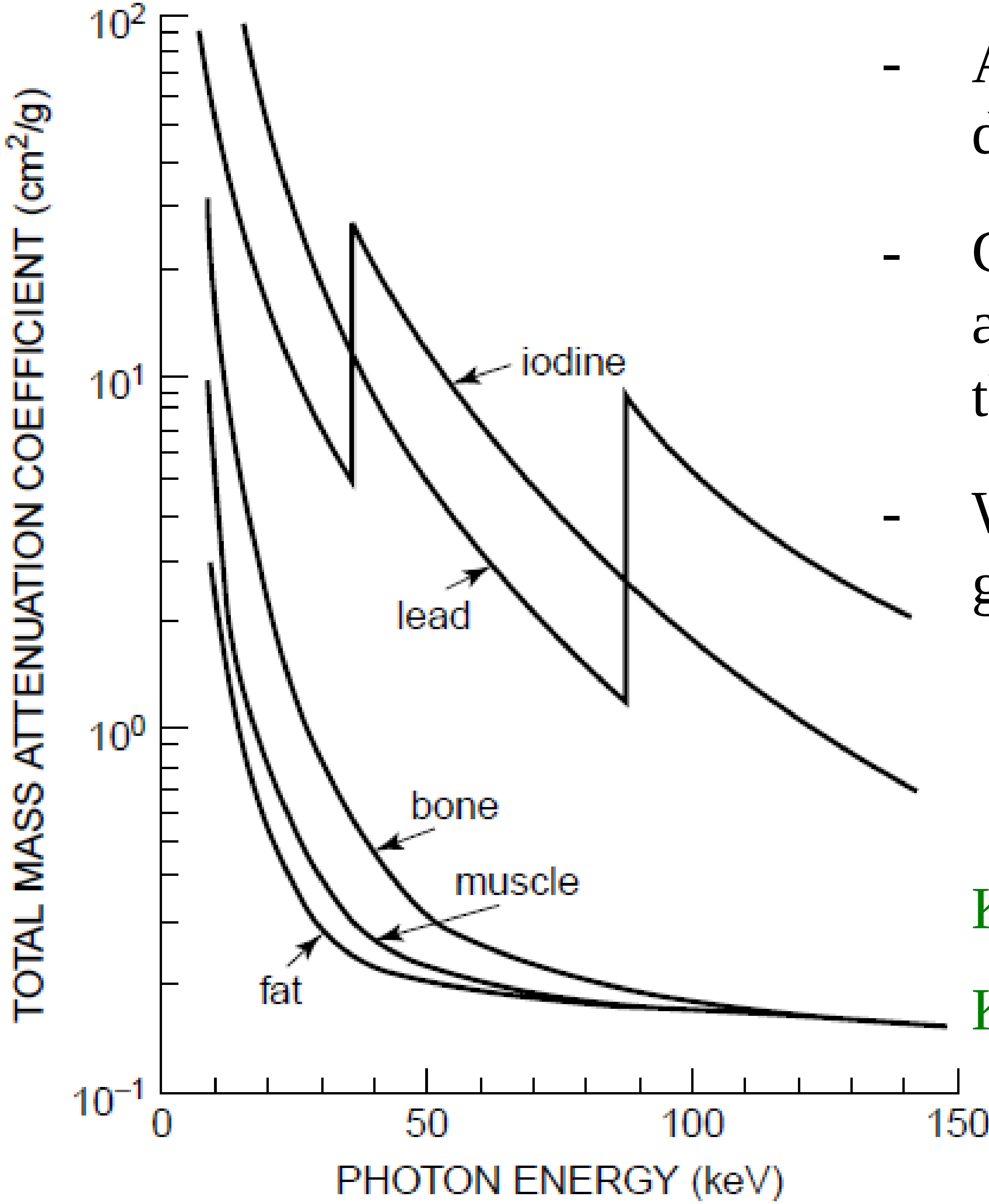
Absorption edge





Without absorption edges, attenuation coefficient of bone would have been similar to soft tissues.

Contrast agents



- Absorption edge helps in design of contrast agents
- Can we use lead as a contrast agent? Ignore its toxicity for the moment.
- Why are barium and iodine good contrast agents?

K-edge of iodine: 33keV

K-edge of lead: 88 keV

Angiograms with and without a contrast agent.

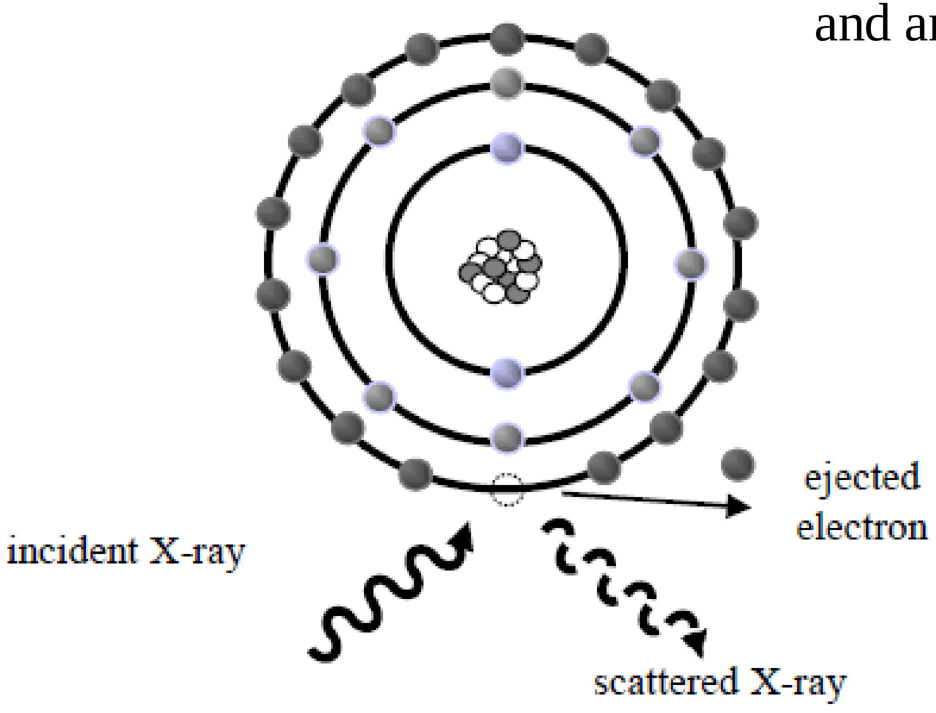
Effective atomic numbers of air (Z_{eff} = 7.65) and muscle (Z_{eff} = 7.4) are very similar. Air is often used to displace fluids that interfere with imaging. Why?

Compton scatter

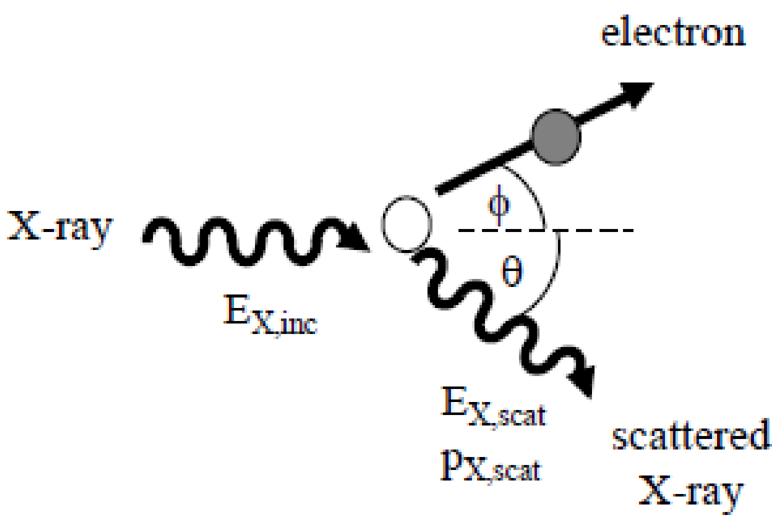
- X-ray photon loses a <u>part</u> of its energy in one interaction

- Outer shells

- Leads to a scattered x-ray photon and an ejected electron.



Scattered x-ray wavelength



- Initial: incident x-ray photon + electron at rest
- Final: scattered x-ray photon + ejected electron
- Solve both energy balance and momentum balance equations.

$$\Delta\lambda = h/m_o c\,(1 - \text{Cos}\theta)$$

Electron rest mass (m_o) = 9.11 x 10^{-31} kg

θ = x-ray scatter angle

Compton wavelength

$$\lambda c = h/m_o c$$

Compton wavelength ~ 2.5 pm

Change in wavelength

$$\Delta\lambda = (h/m_o c)\,(1 - \mathrm{Cos}\theta)$$

When does the maximum change in wavelength occur?

Change in energy in Compton scatter

$\Delta\lambda = h/m_oc\ (1 - \text{Cos}\theta)$ ----- (1)

$1/E_1 = 1/E_o + (1/\ 0.5\ \text{MeV})(1 - \text{Cos}\theta)$ ----- (2)

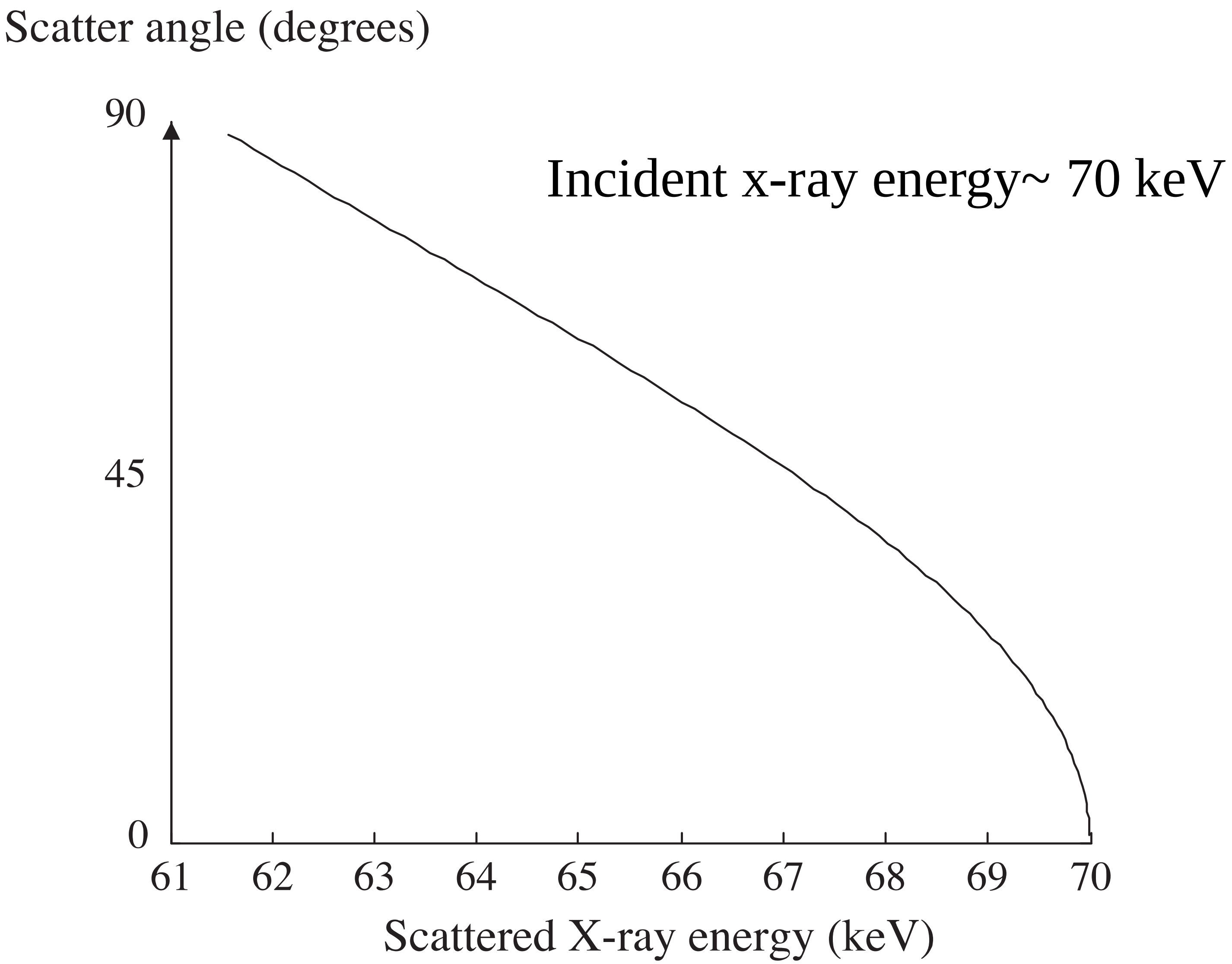
(Put $\lambda = hc/E$ and use rest mass of electron as 0.5 MeV)

Compton scatter is a problem in diagnostic imaging

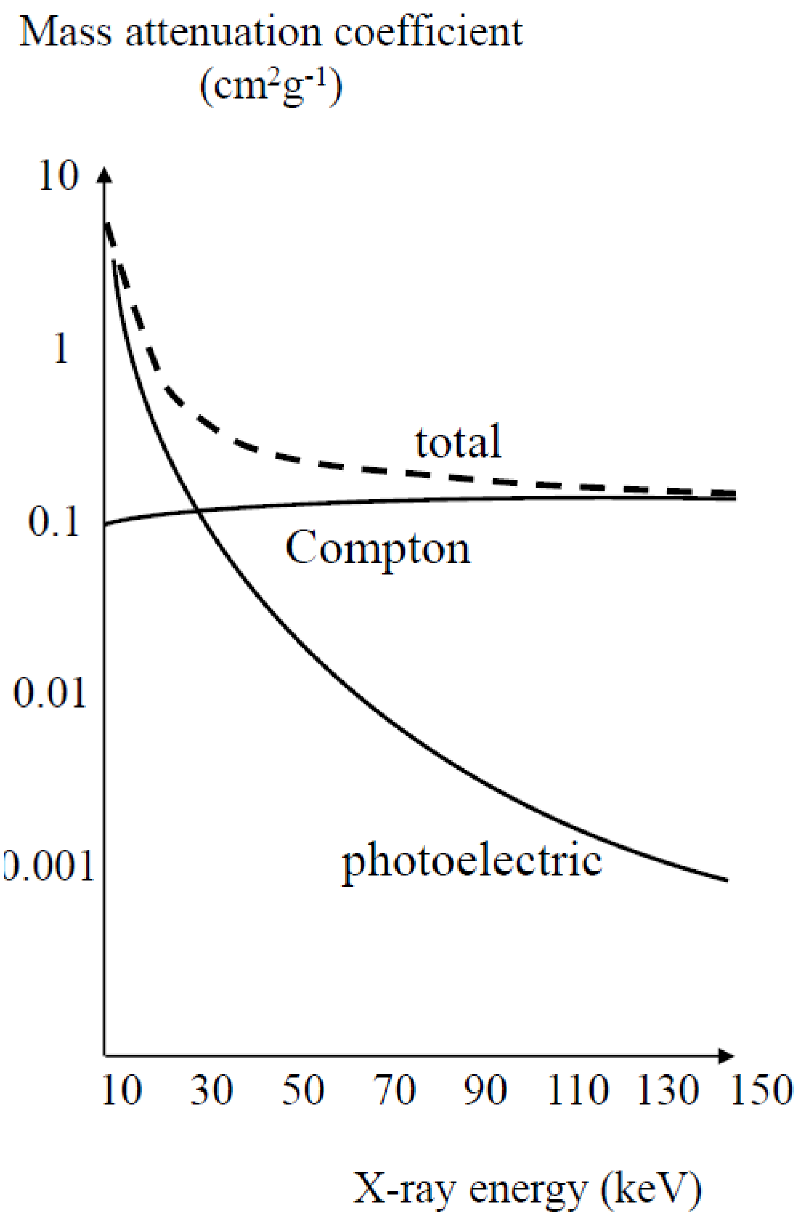
- Scattered x-ray photons have the same energy range as incident photons (for diagnostic x-ray energies)

- Does not contribute to good contrast

Problem with scatter



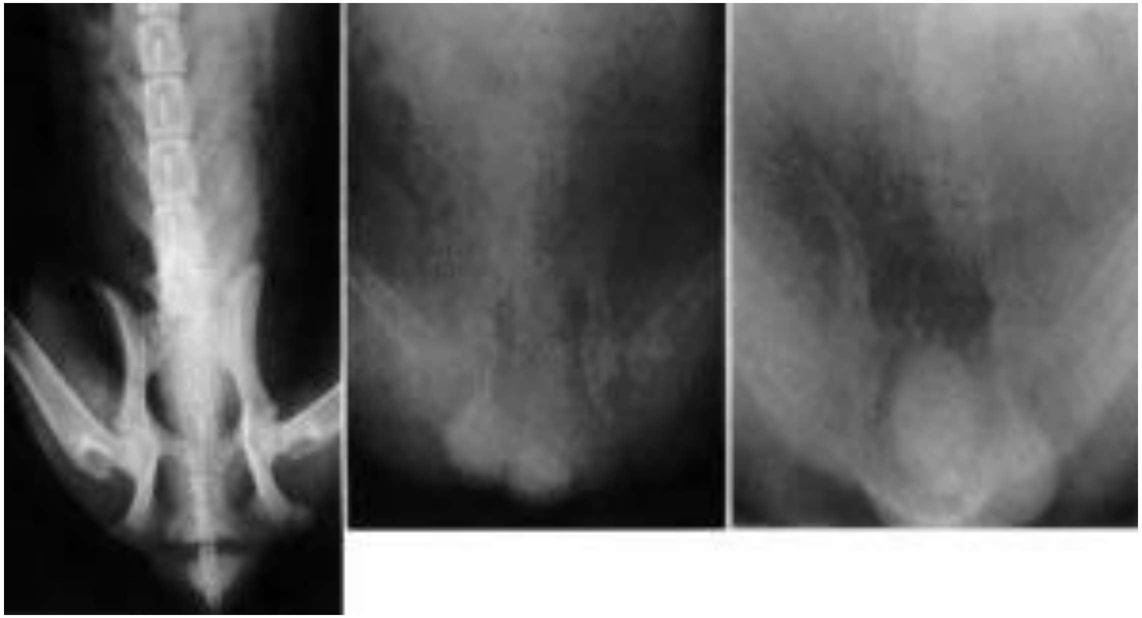
Attenuation mechanisms vs. energy



Low energies: Photoelectric interaction dominates

High energies: Compton effect dominates

Incident energy and contrast



MARGIN FIGURE 4-19

Radiographs taken at 70 kVp, 250 kVp, and 1.25 MeV (^{60}Co). These films illustrate the loss of radiographic contrast as the energy of the incident photons increases.

Lec 7
Introduction to MRI

Discovery of MRI led to Nobel prize in Physiology or Medicine in 2003

Press Release

6 October 2003

The Nobel Assembly at Karolinska Institutet has today decided to award
The Nobel Prize in Physiology or Medicine for 2003 jointly to

Paul C Lauterbur and **Peter Mansfield**

for their discoveries concerning **"magnetic resonance imaging"**

Can you guess how many more Nobel prizes are related to magnetic resonance?

1952 (Physics): Bloch and Purcell (*new methods for nuclear magnetic precision measurements)*

1991 (Chemistry): Ernst (*methodology for high precision nuclear magnetic resonance spectroscopy*)

2002 (Chemistry): Wuthrich (*nuclear magnetic resonance spectroscopy for determining the 3D structure of biological macromolecules in solution)*

Reference material for MRI

- Hendee: chapters 23 and 24
- Smith and Webb: pages 204 - 222
- There’s an online book by Joseph Hornak at *http://www.cis.rit.edu/htbooks/mri/inside.htm*

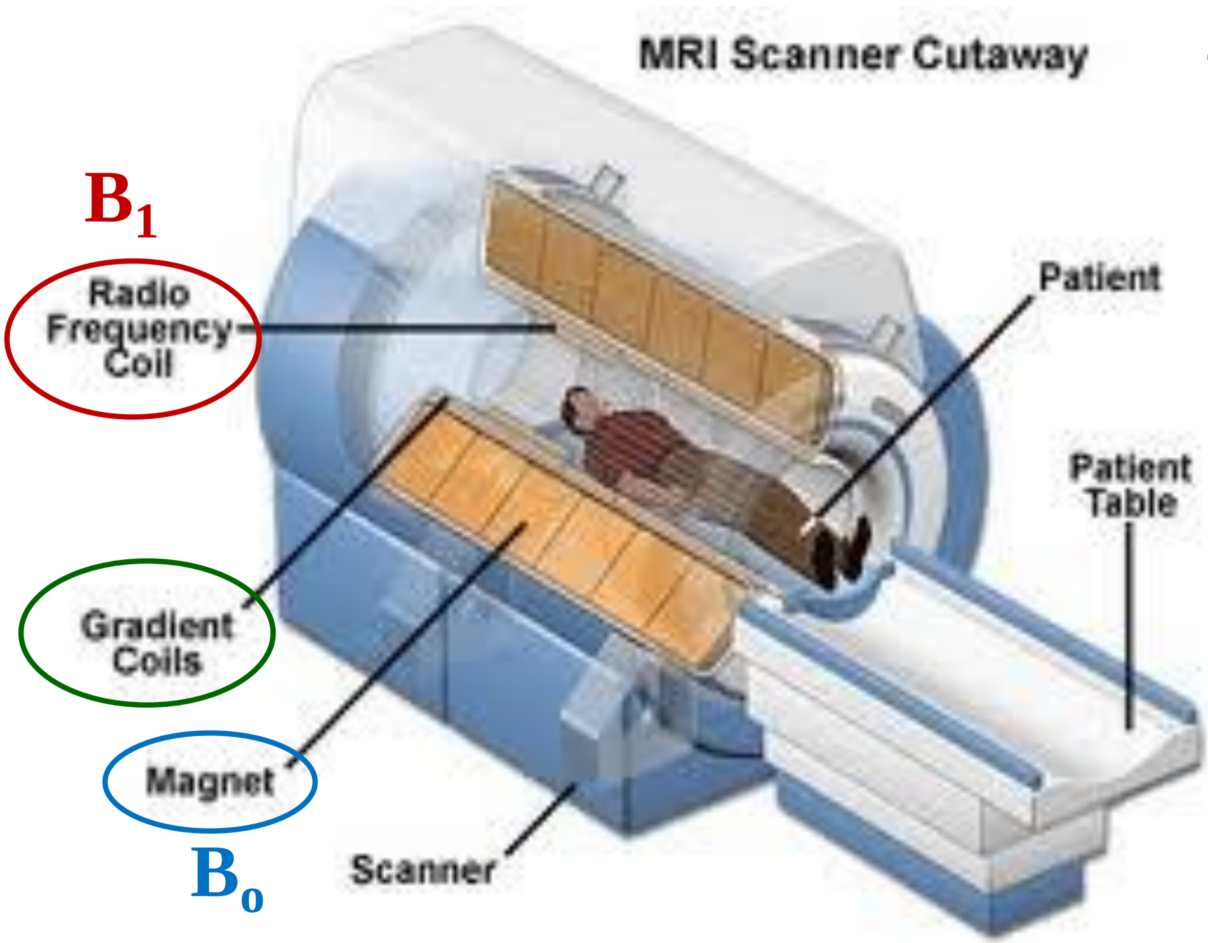
Magnetic Resonance Imaging

Interaction between spin magnetic moments in tissue and external magnetic fields.

Frequency (RF) of external magnetic field matches with “some” internal frequency in tissue.

Deals with behavior of atomic **nuclei** in magnetic field.

What magnetic fields do we have?



- Steady magnetic field, B_o (initially align spins)
- RF magnetic field, B_1 (excites aligned spins)
- Spatial modulation of B_o for image encoding

https://www.researchgate.net/figure/MRI-scanner-cutaway-thanks-for-the-image-from-34_fig1_280792219

Some pre-MRI checks for patients

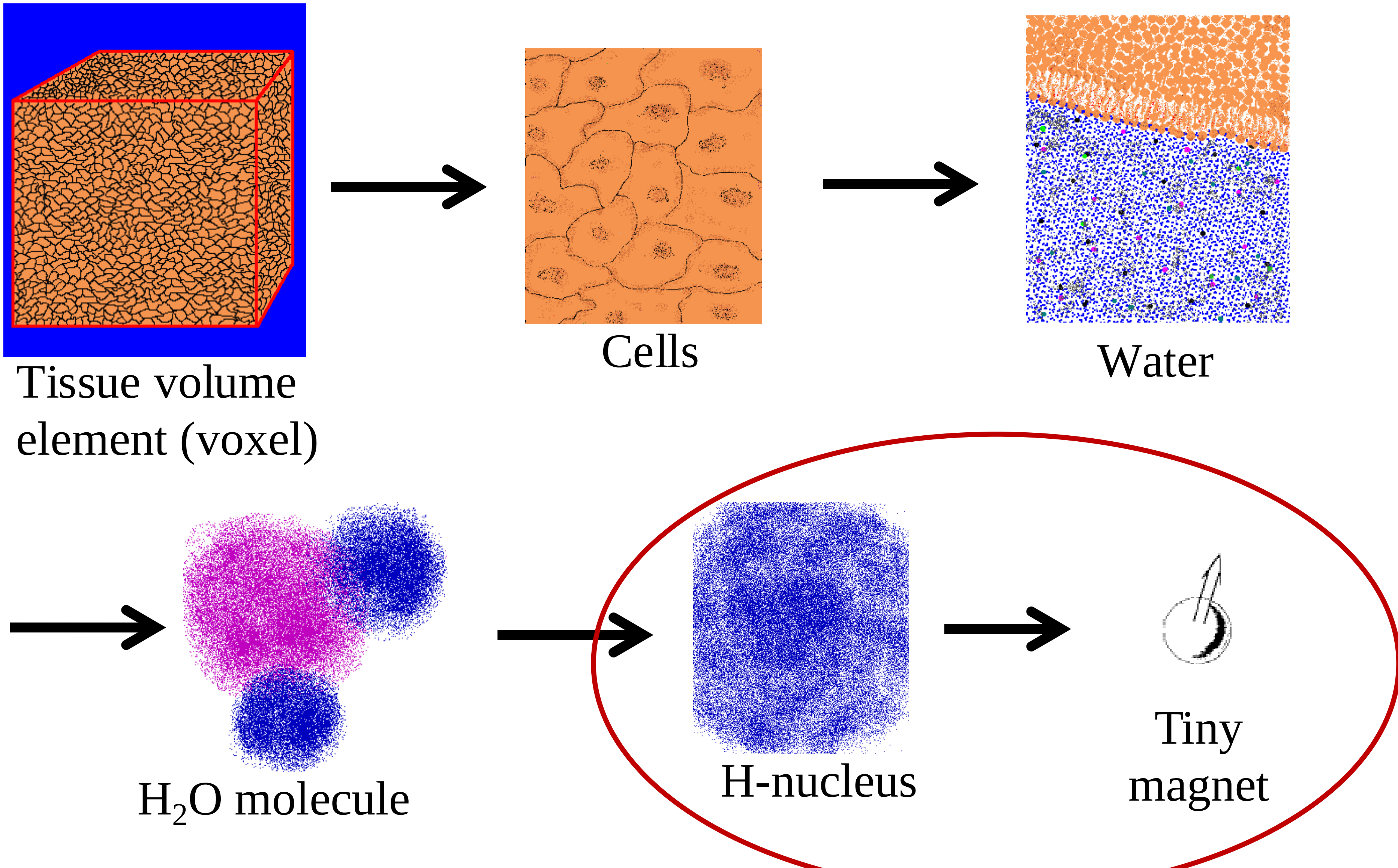
- Ferromagnetic objects
- Non-ferromagnetic objects: local image distortion
- Claustrophobia (~ 45 - 60 min)
- Movement (due to sneeze, cough, etc.)
- Acoustic protection (ear plugs for high noise levels)

Not all patients can undergo an MRI!

- Steady magnetic field ~ 1.5 - 3T
- Earth's magnetic field ~ 50μT

https://newcastleclinic.co.uk/mri-safety-week-precautions-to-take-before-an-mri-scan/

Origin of magnetic moment in tissue



http://www.cis.rit.edu/htbooks/mri/inside.htm

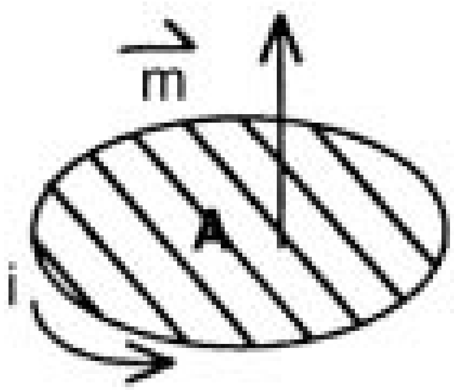
MRI measures signal from hydrogen nuclei

Element	Biological Abundance
Hydrogen	0.63
Carbon	0.094
Nitrogen	0.015
Sodium	0.00041
Phosphorus	0.0024
Calcium	0.0022
Oxygen	0.26

Frequency range

Clinical MRI: between 15 and 80 MHz for hydrogen imaging.

Classical magnetic moment (due to a current)



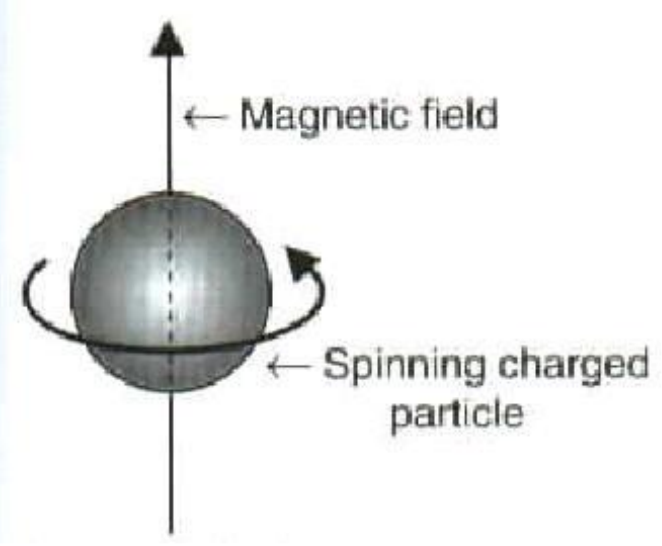
$$\vec{m} = i\vec{A}$$

Magnetic moment = (current) x (area of the current loop)

Magnetic moment of hydrogen nuclei

- Spin is actually a quantum mechanical concept. We give an oversimplified classical analogy in this course!

- Each spinning hydrogen nucleus (positive charge) has a "spin magnetic moment".

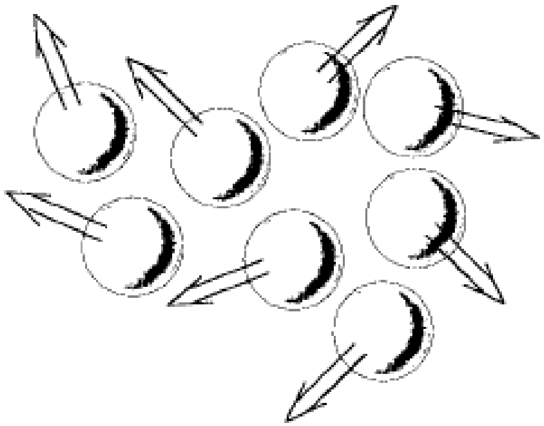


Calculate the number of spins inside a voxel.

- Take a voxel to be a cube of side 1 mm
- Hint: first find the number of water molecules in the voxel.

In absence of B_o

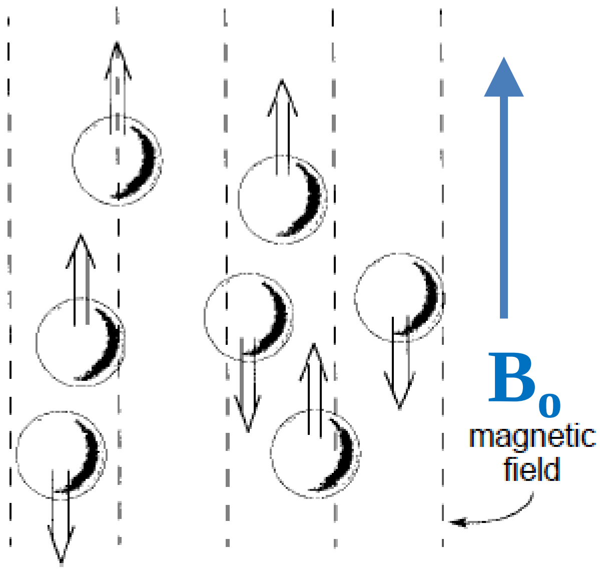
A single voxel (~ mm^3) has ~10^{19} spins.



Randomly oriented spins.
No net magnetization.

What happens in B_o field?

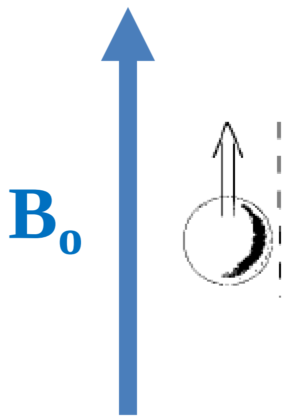
In presence of B_o



- Spins line up with $\mathbf{B_o}$
- Non-zero net magnetization

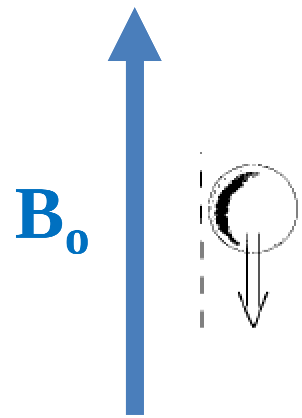
Some spins are parallel and some are anti-parallel to $\mathbf{B_o}$.

Spins are “quantized” in a magnetic field



Low energy

Parallel



High energy

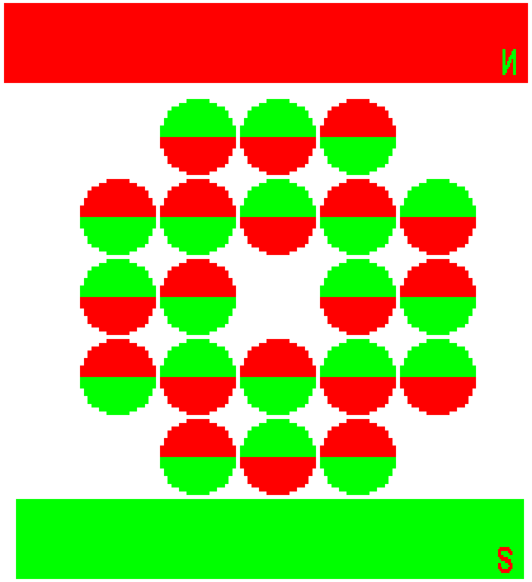
Anti-parallel

Lec 9
(MRI: the role of B_o field)

What decides how many spins will be up and how many will be down?

- Temperature
- The actual numbers are given by the Boltzmann distribution

Boltzmann distribution of spins



$$N^-/N^+ = \exp(-\Delta E/kT)$$

N^-: Number of spins at higher energy

N^+: Number of spins at lower energy

ΔE: Energy difference between two states

Signal ~ population difference between two states (N^+ - N^-).

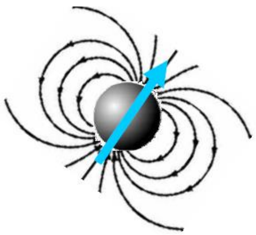
1. What will be the value of N^- when $T = 0$?

2. When do you think N^+ and N^- are likely to be equal?

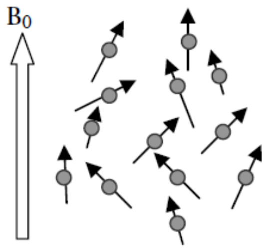
$$N^-/N^+ = \exp(-\Delta E/kT)$$

What happens during MRI? (1)

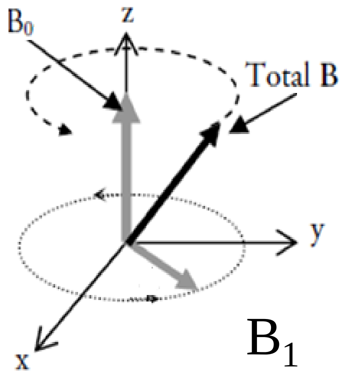
1. Hydrogen nuclei in tissue have "spin angular momentum" and associated magnetic moment.



2. In an external magnetic field (B_o), M_z lines up with B_o (along z-axis).

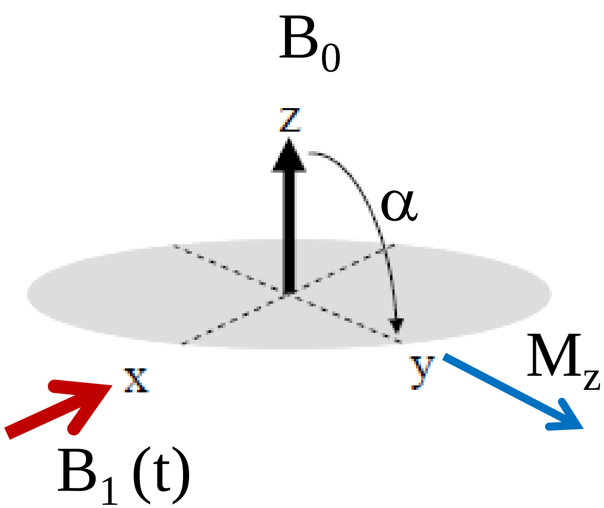


3. A rotating magnetic field (B_1) pulse is applied along x-axis.

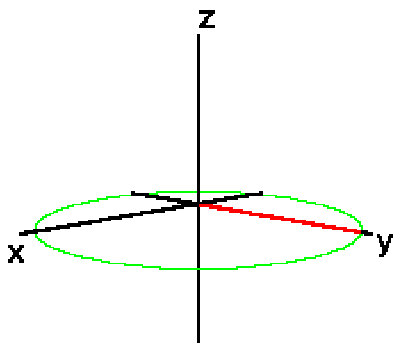


What happens during MRI? (2)

4. B_1 pulls away magnetization (M_z) from the z-axis with an angle α.



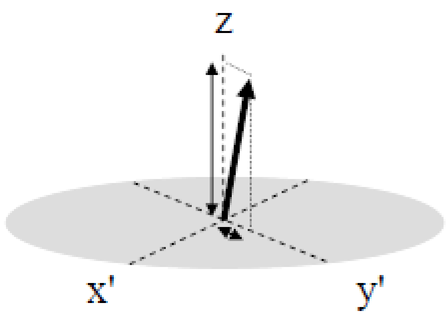
5. M_z rotates around z-axis at the "Larmor frequency".



What happens during MRI? (3)

6. B_1 is turned off. Only B_o remains.

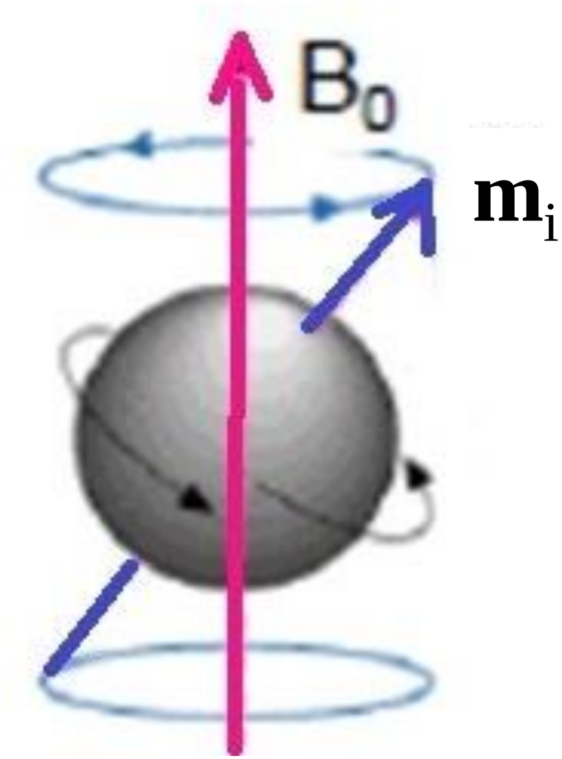
7. The XY-projection of M_z reduces with time, while Z-projection increases and returns to its equilibrium value ("relaxation").



8. Relaxation of M_z to its equilibrium value produces a voltage signal, which we measure.

Once we have grasped these concepts, we will bring on gradient field.

Larmor equation



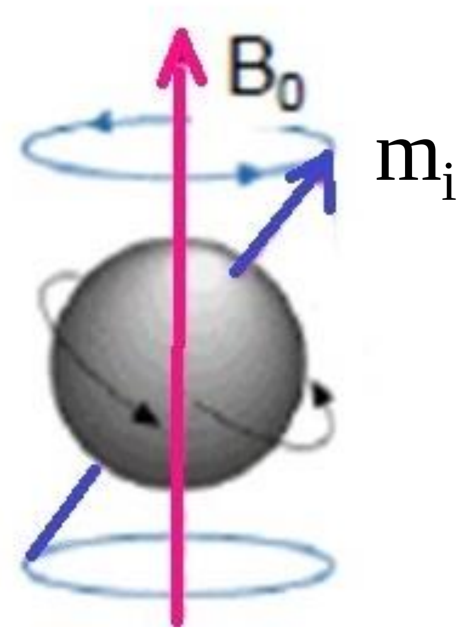
$$\text{Torque } (\mathbf{T}) = (\mathbf{m}_i \times \mathbf{B_o}) = d\mathbf{L_i}/dt$$

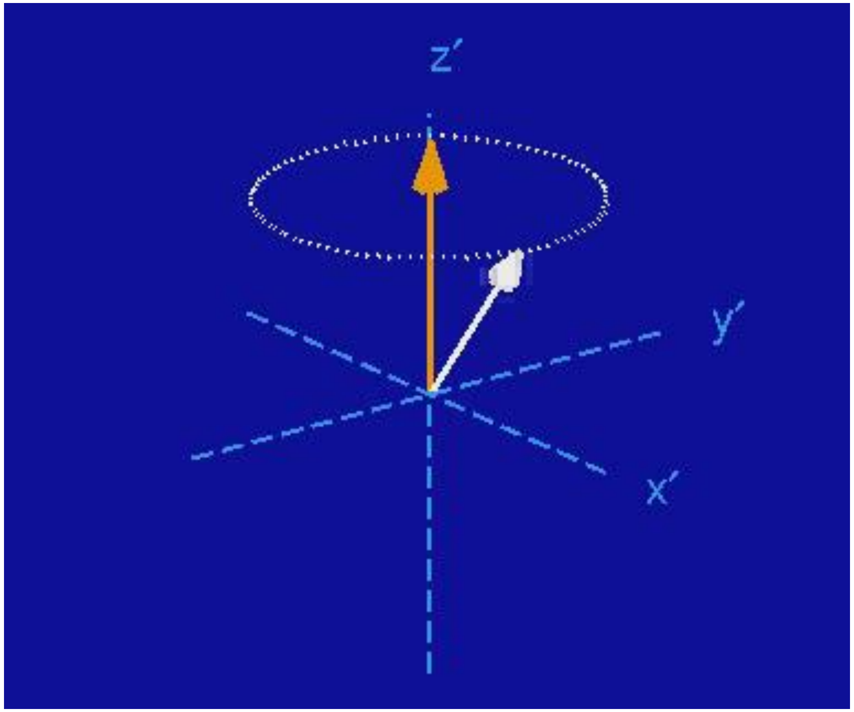
$$\gamma\ (\mathbf{m}_i \times \mathbf{B_o}) = d\mathbf{m_i}/dt$$

$$(\text{since } \mathbf{m} = \gamma \mathbf{L})$$

For hydrogen, gyromagnetic ratio (γ) = 42.58 MHz/Tesla

Larmor precession





Individual spin magnetic moments will precess about the magnetic field with Larmor frequency ($\nu = \gamma\ B_o$).

Gyromagnetic ratio (γ)

Individual spin magnetic moments will precess about the magnetic field with Larmor frequency ($\nu = \gamma B_o$).

For hydrogen, gyromagnetic ratio (γ)□= 42.58 MHz/Tesla

Nuclei with higher γ will precess faster in a given magnetic field.

Precession angle

- *Quantum mechanics* allows specific values of m_z. This makes only specific precession angles possible.

- Precession angle can have any value in *classical mechanics*.

Different nuclei precess with different Larmor frequencies (due to different g)

Element	**Biological Abundance**	**γ**
^{1}H	0.63	42.58
^{13}C	0.094	10.71
^{23}Na	0.00041	11.26
^{39}K	0.0024	1. 99

Need unpaired spin. Why?

Nuclear magnetic moment

- Both protons and neutrons can have magnetic moment. *This is why our current-carrying loop explanation of spin is oversimplified (i.e. this can't explain why neutrons have magnetic moment)*.

- The magnetic moments of a proton and a neutron <u>do not</u> exactly cancel each other.

- A nucleus with either an odd number of protons or odd number of neutrons will have a net magnetic moment.
- Why does ^{14}N have a net magnetic moment then?

Nuclide	*Number of Protons*	*Number of Neutrons*
^{1}H	1	0
^{2}H	1	1
^{13}C	6	7
^{14}N	7	7
^{17}O	8	9
^{19}F	9	10
^{23}Na	11	12
^{31}P	15	16
^{39}K	19	20

Net bulk magnetization is along B_o

Bulk magnetization: $\mathbf{M} = \sum_{i=1}^{N} \mathbf{m}_i$

$\langle M_z \rangle \neq 0$, $\langle M_x \rangle = 0$, $\langle M_y \rangle = 0$

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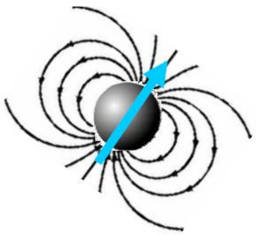

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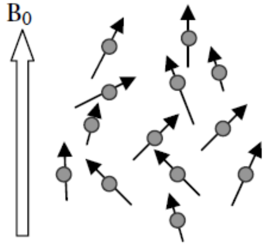

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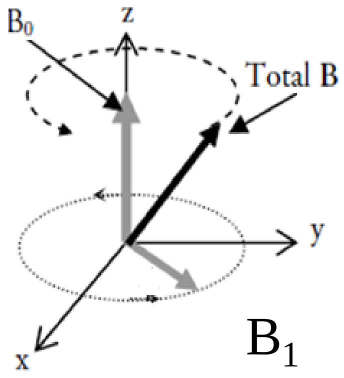

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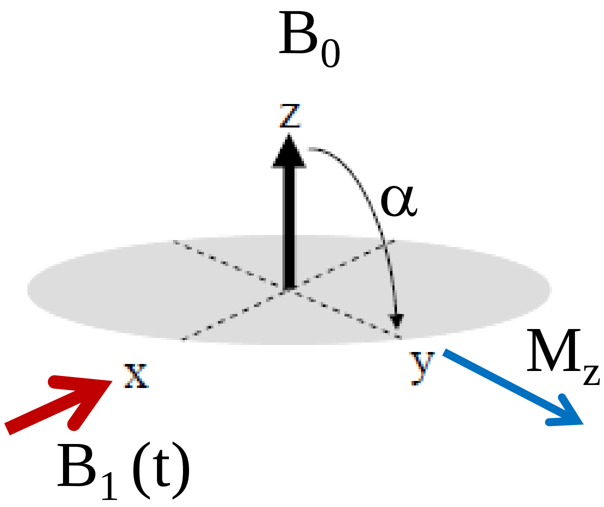

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