EE-229 (Signal Processing-I)

Homework-1

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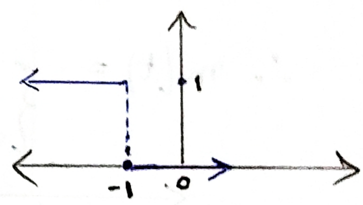
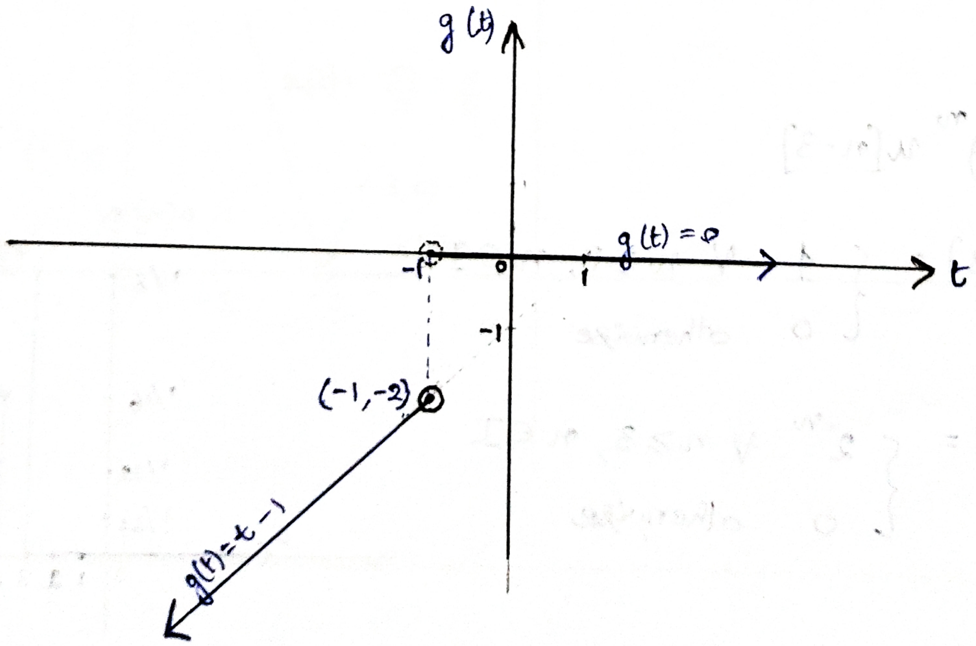
1] a) $g(t) = t\,u(-t-1) - u(-t-1)$

Sol- Observing that $u(-t-1) = u(-(t+1))$ $\longrightarrow$



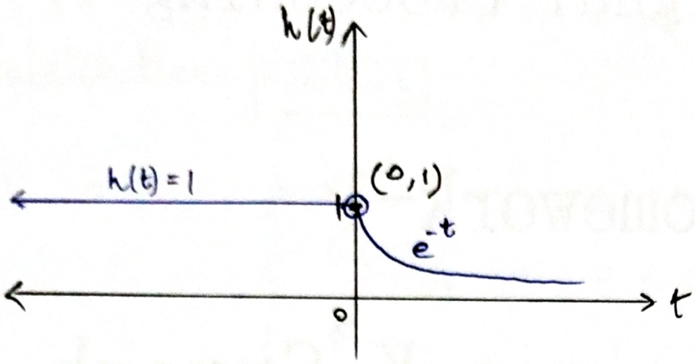
$\therefore$ $g(t)$ simplifies to $(t-1)\,u(-(t+1))$

$$\Rightarrow g(t) = (t-1)\,u(-(t-(-1))) = \begin{cases} (t-1), & -\infty < t \leq -1 \\ 0, & -1 < t < \infty \end{cases}$$



1] b) $h(t) = e^{-t\,u(t)}, \; -1 \le t \le 1$

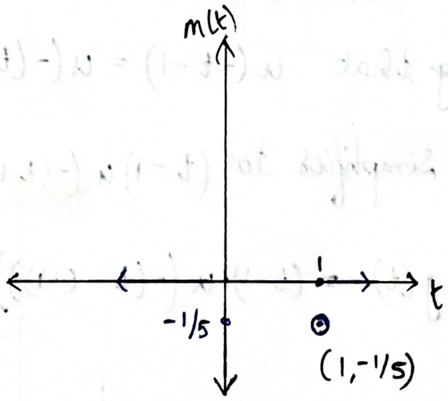
Sol. $u(t) = \begin{cases} 1 & 0 \le t < \infty \\ 0 & -\infty < t < 0 \end{cases}$ $\therefore\; h(t) = \begin{cases} e^{-t} & , 0 \le t < \infty \\ e^{0} = 1 & , -\infty < t < 0 \end{cases}$



c) $m(t) = \left(\dfrac{\sin\left[\frac{\pi}{2}(t-2)\right]}{t^2+4} \right) \delta(1-t)$

Sol. $\because\; \delta(1-t) = \begin{cases} 1 & , t = 1 \\ 0 & , t \neq 1 \end{cases}$

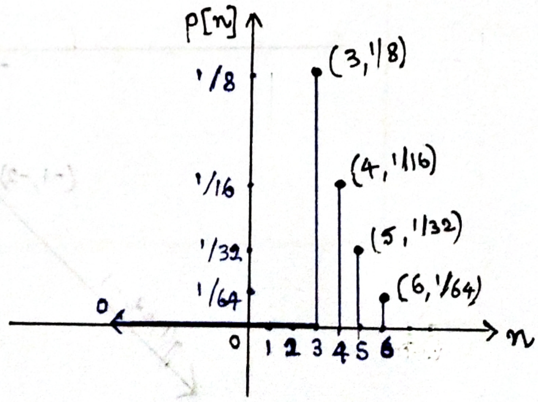
$\therefore\; m(t) = \begin{cases} -1/5 & , t = 1 \\ 0 & , t \neq 1 \end{cases}$



d) $p[n] = \left(\frac{1}{2}\right)^n u[n-3]$

Sol. $\because\; u[n-3] = \begin{cases} 1 & \forall\, n \ge 3,\, n \in I \\ 0 & \text{otherwise} \end{cases}$

$\therefore\; p[n] = \begin{cases} 2^{-n} & \forall\, n \ge 3,\, n \in I \\ 0 & \text{otherwise} \end{cases}$



4] Given, $y(t) = \left(\frac{1}{5}\right) x(-2t-3)$

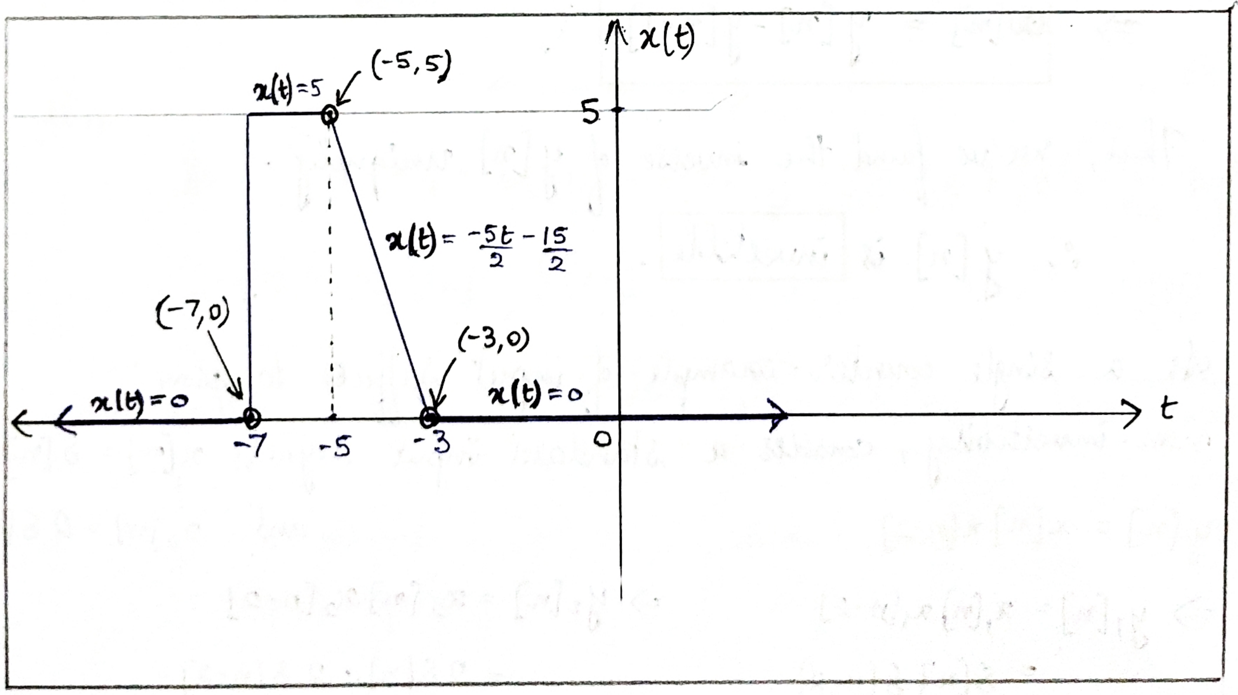
$$\therefore \frac{x}{5}(-2t-3) = \begin{cases} 0 & , -\infty < t < 0 \\ t & , 0 \leq t < 1 \\ 1 & , 1 \leq t < 2 \\ 0 & , 2 \leq t < \infty \end{cases}$$

Substitution : $\boxed{-2t-3 = k}$

$$\therefore x(k) = \begin{cases} 0 & , -\infty < t < 0 \\ 5\frac{(k+3)}{(-2)} & , 0 \leq t < 1 \\ 5 & , 1 \leq t < 2 \\ 0 & , 2 \leq t < \infty \end{cases}$$

$$\boxed{x(k) = \begin{cases} 0 & , \infty > k > -3 \\ -\frac{5k}{2} - \frac{15}{2} & , -3 \geq k > -5 \\ 5 & , -5 \geq k > -7 \\ 0 & , -7 \geq k > -\infty \end{cases}}$$

We've just obtained the original $x(t)$.



5] a) $y[n] = \sum_{k=-\infty}^{n} x[k]$

"INVERTIBILITY": $\forall n_1, n_2 \in \mathbb{Z}$, if $n_1 \neq n_2 \Leftrightarrow y[n_1] \neq y[n_2]$.

Now, to perform operations (so as to check if we can arrive at a fn that uniquely determines input from output) we may temporarily switch to the continuous time-system:

$$y(t) = \int_{-\infty}^{t} x(k)\,dk \, .$$

Using Newton-Leibnitz theorem, $\frac{dy(t)}{dt} = x(t)$ (we see the input being retrieved back!)

Using fundamental principle of derivative:

$$\lim_{\delta \to 0} \frac{y(t+\delta) - y(t)}{(t+\delta) - (t)} = x(t)$$

From here on, we get back to our initial discrete-time system by tuning δ to be one unit interval in discrete-time:

$$\frac{y[n] - y[n-1]}{[n] - [n-1]} = x[n] \Rightarrow x[n] = y[n] - y[n-1]$$

$$\Rightarrow \boxed{w[n] = y[n] - y[n-1]} \, .$$

Thus, we've found the inverse of $y[n]$ uniquely.

$\therefore$ $y[n]$ is $\boxed{\text{invertible}}$.

b) As a single counter-example of input suffices to prove non-invertibility, consider a standard input signals $x_1[n] = \delta[n]$ and $x_2[n] = 2\delta[n]$

$y[n] = x[n]\,x[n-2]$

$\Rightarrow y_1[n] = x_1[n]\,x_1[n-2]$

$= \delta[n]\,\delta[n-2]$

$= 0$

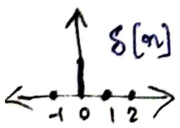
(as evident from their plots because pointwise multiplication gives nothing but 0 throughout the sequence)

$\Rightarrow y_2[n] = x_2[n]\,x_2[n-2]$

$= 2\delta[n] \cdot 2\delta[n-2]$

$= 4\left(\delta[n] \cdot \delta[n-2]\right)$

$= 0$



$\delta[n]$

$\delta[n-1]$

$\because \boxed{y_1[n] = y_2[n]}$ but $\boxed{x_1[n] \neq x_2[n]}$

thus distinct outputs do NOT imply distinct inputs in this case.

$\therefore$ $\boxed{\text{NON-INVERTIBLE}}$

c) $y[n] = n\,x[n]$

Sol. Counter-example for proving non-invertibility:

$$x_1[n] = \delta[n] \Rightarrow y_1[n] = n\,\delta[n] = n\,\delta(n-0) = 0\cdot\delta(n-0) = 0.$$

$$x_2[n] = 2\delta[n] \Rightarrow y_2[n] = n\cdot 2\delta[n] = 2(n\delta(n-0)) = 2(0\cdot\delta(n-0)) = 0.$$

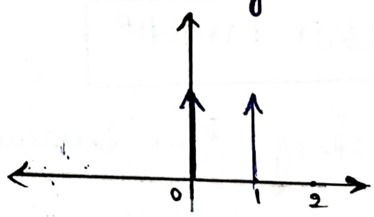
Property used: $\boxed{x[n]\cdot\delta(n-n_0) = x[n_0]\cdot\delta(n-n_0)}$

$\because$ $\boxed{y_1[n] = y_2[n] \text{ but } x_1[n] \neq x_2[n]}$ $\therefore$ $\boxed{\text{NON-INVERTIBLE}}$

d) $y[n] = x[2n]$

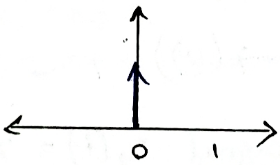
Sol. Counter-example for proving non-invertibility:

Consider $x_1[n] = \delta[n] + \delta[n-1]$



and

$x_2[n] = \delta[n]$



then

$$y_1[n] = x_1[2n] = \delta[2n] + \delta[2n-1] = \delta[2n] + \cancel{\delta\left[2\left(n-\tfrac{1}{2}\right)\right]}$$

DOES NOT EXIST $\because$ we have discrete time-intervals!

$$y_2[n] = x_2[2n] = \delta[2n]$$

$\because$ $\boxed{\underbrace{y_1[n] = y_2[n]\,(= \delta[2n])}_{\text{non-distinct output signals}} \text{ but } \underbrace{x_1[n] \neq x_2[n]}_{\text{distinct input signals}}}$ $\therefore$ $\boxed{\text{NON-INVERTIBLE}}$

6] a) $y(t) = \sin(x(t) - x(0))$

Sol. Clearly NOT ~~time invariant~~ CAUSAL, since $x(0)$ is a value "in future" for time coordinate less than 0, eg.: $x(-1)$ where $t=-1$. So from this point of view of $t=-1$ instance, the system shows dependence on future

$\therefore$ $\boxed{\text{NOT CAUSAL}}$.

Checking linearity:

$$x_1(t) \rightarrow y_1(t) = \sin(x_1(t) - x(0))$$

$$x_2(t) \rightarrow y_2(t) = \sin(x_2(t) - x(0))$$

$$x_3(t) \rightarrow y_3(t) = \sin(x_3(t) - x(0))$$

$$\downarrow \qquad = \sin(ax_1(t) + bx_2(t) - x(0))$$

$$= \underline{ax_1(t) + bx_2(t)} \qquad \boxed{y_3(t) \neq a y_1(t) + b y_2(t)}$$

$\therefore$ $\boxed{\text{NOT LINEAR}}$

Checking time-invariance:

$$y_1(t) = \sin(x_1(t) - x(0))$$

Consider the time-shifted signal $x_2(t) = x_1(t-t_0)$

then $y_2(t) = \sin(x_1(t-t_0) - x(0))$

Checking with:

$$y_1(t-t_0) = \sin(x_1(t-t_0) - x(0))$$

$\because$ $\boxed{y_1(t-t_0) = y_2(t)}$, $\therefore$ $\boxed{\text{it is TIME-INVARIANT.}}$

b) $y(t) = x(\sin(t))$

Sol. This system is $\boxed{\text{NOT causal}}$ as $y(t)$ at some time may depend on future values of $x(t)$. Eg. $y(-2\pi) = 0 = x(0)$.

Linearity:

$$x_1(t) \rightarrow y_1(t) = x_1(\sin(t))$$

$$x_2(t) \rightarrow y_2(t) = x_2(\sin(t))$$

let $x_3(t) = ax_1(t) + bx_2(t) \longrightarrow y_3(t) = x_3(\sin(t))$

$$= ax_1(\sin(t)) + bx_2(\sin(t))$$

$$\boxed{y_3(t) = ay_1(t) + by_2(t)}$$

$\therefore$ IT IS $\boxed{\text{LINEAR}}$

Time-invariance: $y_1(t) = \cancel{\sin(x_1}\ x_1(\sin(t))$

let $x_2(t) = x_1(t-t_0)$

$\Rightarrow y_2(t) = x_1(\sin(t-t_0))$

Checking with:

$$y_1(t-t_0) = x_1(\sin(t-t_0))$$

$\boxed{y_1(t-t_0) = y_2(t)}$ $\therefore$ it is $\boxed{\text{TIME-INVARIANT}}$.

c) $y(t) = \max_{s\in[t-1,t]} \{x(s)\}$

Sol. Clearly, $y(t)$ depends only on present and past as the MAX value of signals to be computed have their time instance varying between t and $t-1$ $\therefore$ no dependence on future.

$\therefore$ it is $\boxed{\text{CAUSAL}}$

For the other properties, let's consider a sample plot of input signal $x(s)$:

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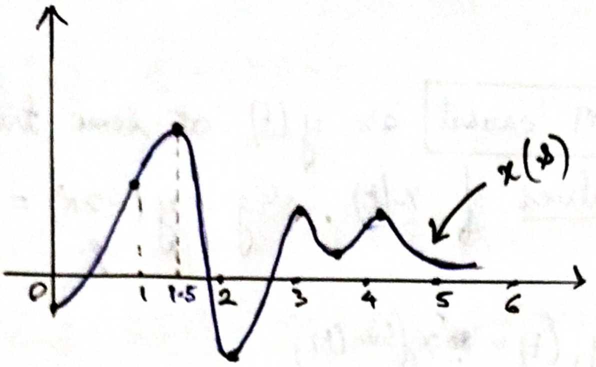


Here,

$y(1) = x(1)$,

$y(2) = x(1.5)$

$y(3) = x(3)$

$y(4) = x(3)$ again.

Thus, the system like $\mathrm{MAX}_{s \in [t-1, t]} \{x(s)\}$ need NOT be ~~necessarily~~ possessing linearity and time-invariance for each and every input signal.

$\therefore$ $\boxed{\text{NOT LINEAR, NOT TIME-INVARIANT}}$.

d) $y(t) = x(t/3)$

Sol. $\boxed{\text{NOT CAUSAL}}$, because for time instances like $t = -3$, $x(t/3) = x(-1)$ which is futuristic value for signal at time $t = -3$.

Linearity: $x_1(t) \rightarrow y_1(t) = x_1(t/3)$

$x_2(t) \rightarrow y_2(t) = x_2(t/3)$

let $x_3(t) = a x_1(t) + b x_2(t) \longrightarrow y_3(t) = x_3(t/3)$

$= a x_1(t/3) + b x_2(t/3)$

$\boxed{y_3(t) = a y_1(t) + b y_2(t)}$ $\therefore$ $\boxed{\text{LINEAR}}$.

Now this system is clearly $\boxed{\text{NOT time-invariant}}$ as the input signal gets broadened by a factor of 3, thus NOT varying with shift.

$y_1(t) = x_1(t/3)$

let $x_2(t) = x_1(t - t_0) \Rightarrow y_2(t) = x_2(t/3)$

$= x_1(t/3 - t_0)$

but $y_1(t - t_0) = x_1\left(\frac{t - t_0}{3}\right) = x_1(t/3 - t_0/3)$ $\therefore$ $\boxed{y_1(t - t_0) \neq y_2(t)}$.

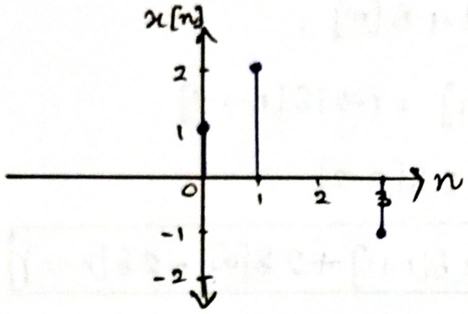
7] a) $y[n] = x^2[n-1]$

b) $y[n] = t\, x(2t-1)$

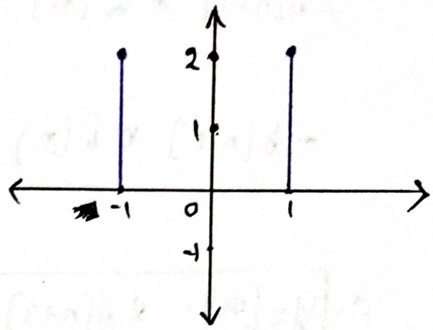
8] Given: $x[n] = \delta[n] + 2\delta[n-1] - \delta[n-3]$

and $h[n] = 2\delta[n+1] + 2\delta[n-1]$

$x[n]$:



$h[n]$:



a) $y_1[n] = x[n] * h[n]$

Convolving $h[n]$ with $x[n]$ term by term and then adding all of them to obtain complete eqn of convolved plot:

(happens when one of the fns is flipped and slid along the n-axis to sum the plots.)

$h[n] * \delta[n] = h[n] = 2\delta[n+1] + 2\delta[n-1]$

$h[n] * 2\delta[n-1] = (2\times2)\,\delta[n+1-1] + (2\times2)\,\delta[n-1-1]$

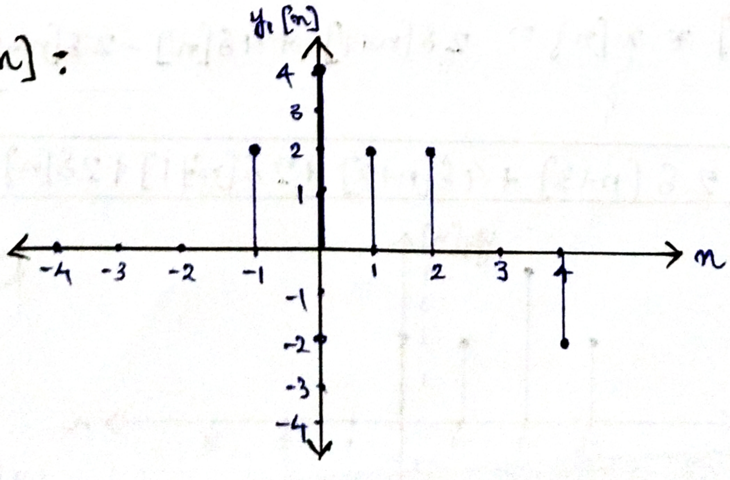
$= 4\delta[n] + 4\delta[n-2]$.

$h[n] * (-\delta[n-3]) = (2\times-1)\,\delta[n+1-3] + (2\times-1)\,\delta[n-1-3]$

$= -2\delta[n-2] - 2\delta[n-4]$.

Thus, $\boxed{x[n] * h[n] = 2\delta[n+1] + 4\delta[n] + 2\delta[n-1] + 2\delta[n-2] - 2\delta[n-4]}$.

$\therefore\ y_1[n]$:



8] b) $x[n+2] = \delta[n+2] + 2\delta[n+1] - \delta[n-1]$

$h[n] = 2\delta[n+1] + 2\delta[n-1]$.

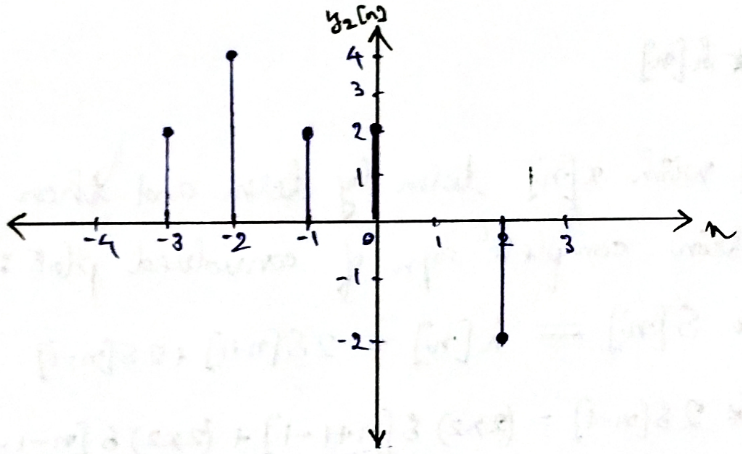
Sol. $y_2[n] = x[n+2] * h[n]$

$$\delta[n+2] * h[n] = 2\delta[n+1+2] + 2\delta[n-1+2]$$
$$= 2\delta[n+3] + 2\delta[n+1]$$

$$2\delta[n+1] * h[n] = 2\times 2\,\delta[n+1+1] + 2\times 2\,\delta[n-1+1]$$
$$= 4\delta[n+2] + 4\delta[n] .$$

$$-\delta[n-1] * h[n] = -2\delta[n+1-1] + (-2)\delta[n-1-1]$$
$$= -2\delta[n] - 2\delta[n-2]$$

$$\therefore \boxed{y_2[n] = 2\delta[n+3] + 4\delta[n+2] + 2\delta[n+1] + 2\delta[n] - 2\delta[n-2]} .$$



c) $x[n] = \delta[n] + 2\delta[n-1] + -\delta[n-3]$

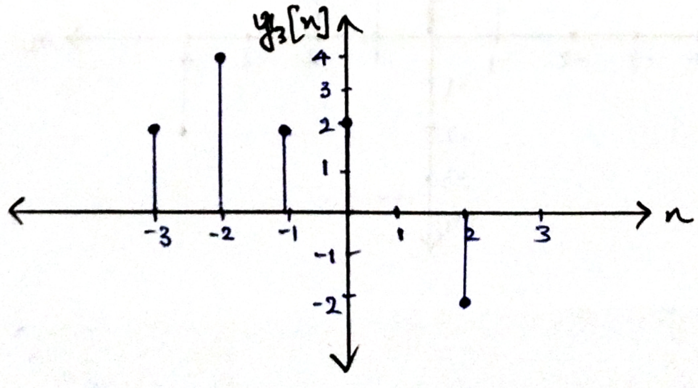
$h[n+2] = 2\delta[n+3] + 2\delta[n+1]$.

Sol. $y_3[n] = x[n] * h[n+2]$

$$2\delta[n+3] * x[n] = 2\delta[n+3] + 4\delta[n+2] - 2\delta[n]$$

$$2\delta[n+1] * x[n] = 2\delta[n+1] + 4\delta[n] - 2\delta[n-2]$$

$$\therefore \boxed{y_3[n] = 2\delta[n+3] + 4\delta[n+2] + 2\delta[n+1] + 2\delta[n] - 2\delta[n-2]}$$



OBSERVATION:

$y_2[n] = y_3[n]$

i.e.,

$x[n+2] * h[n] = x[n] * h[n+2]$

(10)

9] Given $x[n] = \begin{cases} 1, & 0 \le n \le 9 \\ 0, & \text{elsewhere} \end{cases}$ and $h[n] = \begin{cases} 1, & 0 \le n \le N \\ 0, & \text{elsewhere} \end{cases}$

$y[n] = x[n] * h[n]$.

where $N \geq 9$, $N \in \mathbb{Z}$. $y[4] = 5$, $y[14] = 0$

Sol.

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$\because$ before 0 and after 9 the value of $x[n] = 0$,

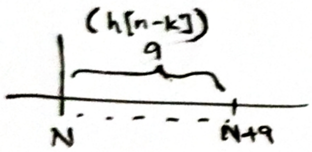
$\therefore$ summation reduces to $y[n] = \sum_{k=0}^{9} \cancel{x[k]}^{\,1}\, h[n-k]$

$$= \sum_{k=0}^{9} h[n-k]$$

$\therefore$ $y[n]$ is the summation of shifted replicas of $h[n]$.

As the last replica begins at $n=9$ and $h[n] = 0 \ \forall n > N$,

$\therefore$ $y[n] = 0 \ \forall n > (N+9)$.



With this, given that $y[14] = 0$, $\therefore N \leq 4$.

~~(this~~

Also, $\because y[4] = 5$, $\therefore$ $h[n]$ has atleast 5 times the convolved pts of (1×1).

But since we've concluded that $\boxed{N \leq 4}$,

$\therefore$ only possible way for this to happen is by having $(1 \times 1) = 1$ at $\underbrace{n=0,\ n=1,\ n=2,\ n=3,\ n=4}_{5 \text{ non-zero points}}$

and nothing more than this.

$\therefore$ N can't help but to be equal to $\boxed{4}$.