

Mid-semester examination

Max marks: 100

Time: 2 hours

You will not require a calculator. Begin the answer to each question on a fresh page. Your solutions should be neat and coherent.

Throughout, \star denotes the convolution operation, u denotes the unit step signal. Sravan K Suresh
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1. [20 marks] Consider the following discrete-time system, where x denotes the input signal, and y the output signal:

$$y[n] = x[2n].$$

(This is called a *decimation* or *down-sampling* operation; we will have more to say about this operation later in the course.)

- (a) Check if this system is memoryless, linear, time-invariant, causal, and stable. Give reasons for your answers.
 - (b) Suppose that the input signal is periodic with an even-valued fundamental period N . Is the output signal periodic? If so, what can you say about its fundamental period?
 - (c) Suppose that the input signal is periodic with an odd-valued fundamental period N . Is the output signal periodic? If so, what can you say about its fundamental period?
2. [20 marks] Define the signal $x(t)$ as follows.

$$x(t) = \begin{cases} 0 & t < 0 \\ t & t \in [0, 1] \\ 2 & t \in (1, 2] \\ 0 & t > 2. \end{cases}$$

Sketch carefully the following signals.

- (a) $x(-2t - 1)$
- (b) $x(t) * u(t)$
- (c) $x'(t) * u(t)$
- (d) $x''(t) * u(t)$

3. [15 marks] Consider the LTI system defined by the following input-output relationship:

$$y[n] = x[n] - 2x[n - 1] + x[n - 2]$$

- (a) Sketch the impulse response $h[n]$ of this system.
- (b) Sketch the output signal corresponding to the input signal $x[n] = nu[n]$.
- (c) Obtain the frequency response $H(\omega)$ corresponding to this system. Sketch the amplitude response $|H(\omega)|$ over $\omega \in [-\pi, \pi]$.

4. [15 marks] Consider the causal LTI system defined by the linear constant coefficient equation

$$y(t) + 2y'(t) = x(t).$$

- (a) How might you realise this system in practice?
- (b) Obtain the impulse response of this system.
- (c) Obtain the output of this system corresponding to the input signal $x(t) = \cos(\omega_0 t)$.

5. [10 marks] Consider a periodic signal $x[n]$ having fundamental period 5, and discrete time Fourier series coefficients $\{a_k\}$.

Express the discrete time Fourier series coefficients $\{b_k\}$ corresponding to $y[n] = (-1)^n x[n]$ in terms of $\{a_k\}$.

6. [20 marks] Consider the continuous time periodic signals $x(t)$ and $y(t)$ with period 1, defined as:

$$\begin{aligned}x(t) &= t^2 \quad (t \in [0, 1]), \\y(t) &= t \quad (t \in [0, 1]).\end{aligned}$$

- (a) Compute the Fourier series coefficients $\{a_k\}$ corresponding to $x(t)$.
- (b) Compute the Fourier series coefficients $\{b_k\}$ corresponding to $y(t)$.

Answers

Ques