

EE302-S2, Control Systems, Endsem (26th Apr 2024)

Notes & instructions:

- Attempt all five questions: each question carries 10 marks. Unnecessarily long, convoluted, redundant or irrelevant text could attract marks reduction.
 - Unless otherwise explicitly specified, k is real and positive.
 - If a graph-paper based question is asked, then use graph-paper judiciously after a rough sketch on your own answer-sheet (within 'rough-work', which will not be evaluated). Using judiciously will avoid carefully resketching (and save your own time) and also graph-paper.
 - Tie the graph paper within the answer-paper at the appropriate page. Write roll-number on the graph paper top.
 - Some questions might not have the sought answer. In such a case, give reasons why the sought answer is not possible.
 - If you feel a question has ambiguity and/or needs clarification, then assume yourself appropriately, state and justify your assumption and then proceed to solve the problem with that assumption.
- Do not call any TA or instructor for your query.

Ques 1: Consider the standard negative unity feedback configuration with $G(s) = \frac{1}{(s+1)(s+2)(s+2)}$.

- (a) On a graph-paper, sketch the root-locus for $k > 0$ and estimate, using your graph-paper's sketch,
- (i) the range of $k > 0$ that results in closed loop stability, and
 - (ii) the frequency ω_c at the highest value of k yielding closed loop stability.

(Use root-locus asymptotes to estimate: this is adequate accuracy.)

- (b) On (preferably same) graph-paper, sketch Bode gain/phase plots (asymptotic sketch only) to obtain approximate range of $k > 0$ that results in closed loop stability and frequency ω_c as in (a)(ii).
- (c) Sketch Nyquist plot and use Nyquist criteria for obtaining exact range of $k > 0$ that results in closed loop stability. (The sketch for (c) is on your plain answer sheet, and not on graph paper.)

Ques 2: Consider the transfer function $G(s) = \frac{2s^2 + 11s + 14}{s^2 + 5s + 6}$.

- (a) Obtain a controllable state space realization, prove its controllability.
- (b) Obtain an observable state space realization, prove its observability.
- (c) Obtain a state space realization which is both controllable and observable.

(In each of the cases above (with possibly different matrices A), each matrix A is required to be 2×2 .)

Ques 3: Answer any two out of the three below about a state-space system $\frac{d}{dt}x = Ax + Bu$, and $y = Cx + Du$.

- (a) Define the notion of state-controllability and give a test for checking controllability.
- (b) Define the notion of state-observability and give a test for checking observability.
- (c) State the pole-placement problem and the pole-placement theorem.

Ques 4: (a) For a 2×2 real matrix A , define e^{At} for $t \geq 0$.

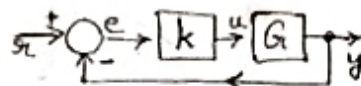
(b) For $A = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$, obtain e^{At} explicitly.

(c) Use the obtained e^{At} to find the impulse response of $G(s) = \frac{1}{s^2 - 3s + 2}$.

Ques 5: Consider the standard negative unity

feedback configuration as shown beside, with r the

unit step input, and $G(s) = \frac{1}{(s+1)(s+2)}$.



For the closed loop system's step response, it is desired to have 5% overshoot, together with a 2% settling time of at most 1 second. Using root-locus techniques,

(a) design a constant gain controller $u = k_p e$ that achieves both the requirements.

(b) design a PD controller $u = k_p e + k_d \frac{d}{dt} e$ that achieves both the requirements.

(Show intermediate steps/calculations using a root-locus sketch on your plain answer paper.)