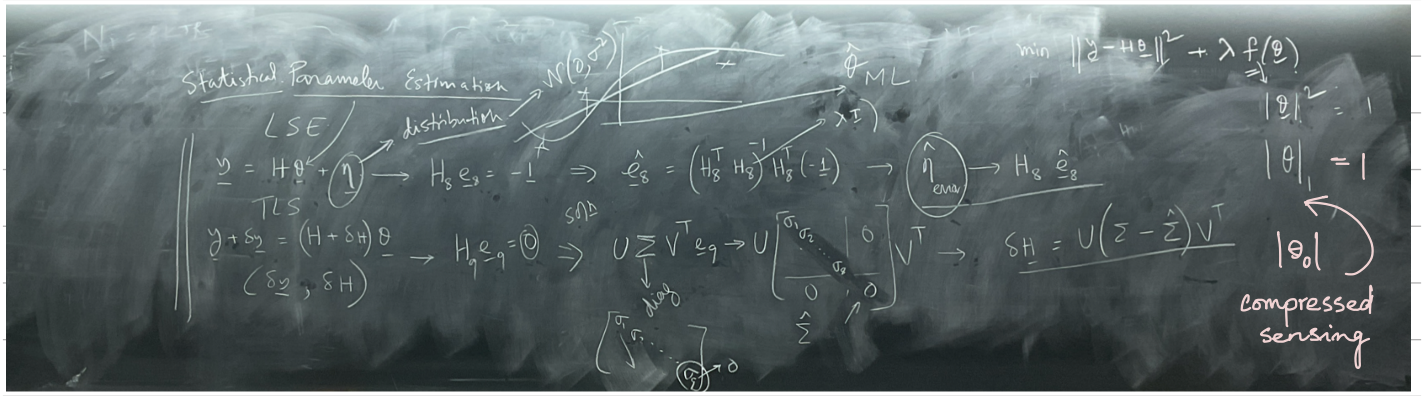
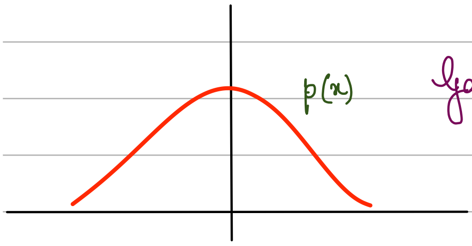
EE702: Lec-13 (21 Feb)





Gaussian → Symmetric
→ Unimodal

What if $\int_{\text{mode}\pm\epsilon} p(x) \geq \inf()$? Think.

Consider $p(x) = \alpha \mathcal{N}(m_1, \sigma_1^2) + (1-\alpha)\mathcal{N}(m_2, \sigma_2^2)$

mixture density fn

Model: $y_i = h + \eta_i$ [Additive]

$\theta \sim \mathcal{N}(m_\theta, \sigma_\theta^2)$

η_i (measurement error) $\sim \mathcal{N}(0, \sigma_n^2)$

} mAP (maximum Aposteriori estimate)

ML ⟶ $\max \ln(p(y|\theta))$

mAP ⟶ $\max p(\theta|y) = \dfrac{\max p(\theta, y)}{p(y)}$

$$= \max\left(\frac{p(y|\theta)\,p(\theta)}{p(y)}\right) \Rightarrow \max p(y|\theta)\,p(\theta)$$

$$= \max\Big[\underbrace{\ln(p(y|\theta))}_{ML} + \underbrace{\ln p(\theta)}_{prior}\Big]$$

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$$= \max\left[\ln\left(\exp\left(-\sum_{i=1}^{M}(y_i-\theta)^2/2\sigma_n^2\right)\right) + \ln\frac{}{\sqrt{}}\left(\exp\left(-(\theta-m_\theta)^2/2\sigma_\theta^2\right)\right)\right]$$

$$= \min_{\theta}\left[\sum_{i=1}^{M}(y_i-\theta)^2/2\sigma_n^2 + (\theta-m\theta)^2/2\sigma_\theta^2\right]$$

using $\sum y_c = M\bar{m}$,

$$\boxed{\hat{\theta}_{mAP} = \frac{\frac{M\bar{m}}{\sigma_n^2} + \frac{m_\theta}{\sigma_\theta^2}}{\frac{M}{\sigma_n^2} + \frac{1}{\sigma_\theta^2}}}$$

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* Sorenson

Properties of Estimator

$T(\theta)$ $\quad\quad$ $\hat{\theta} \to$ r.v or not? ✓ r.v

Biased? or unbiased?

$p(\hat{\theta}) = ?$

$\mathbb{E}[\hat{\theta}] = ?$

* $\hat{\theta}_m \sim \mathcal{N}\left(0, \frac{\sigma^2}{N}\right)$ or $\hat{\theta}_{10} \sim \mathcal{N}(0, \sigma^2)$? What would you prefer ?

✓ $\hat{\theta}_m \sim \mathcal{N}\left(0, \frac{\sigma^2}{N}\right)$.

$E[\hat{\theta}] = \theta$ "unbiased"

As $N \to \infty$, bias $\to 0$

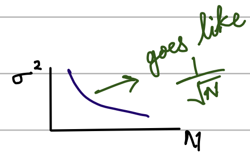
"Asymptotically unbiased"

bias $\to E(\hat{\theta}) - \theta$

$\Rightarrow \mathbb{E}\left[(\hat{\theta}-\theta)\right]^2 \to 0$ as $N \to \infty$

variance $\quad$ "unbiased" $\quad$ "Consistent"

$\sigma = \sqrt{\frac{\Sigma(x_i-\bar{x})^2}{N-1}}$ or $\sqrt{\frac{\Sigma(x_i-\bar{x})^2}{N}}$?



Ans :

correct if $\bar{x}$ calculated from given x_is

correct only when $\bar{x}$ is separately given.

* variance of median goes like $N^{-1/3}$.