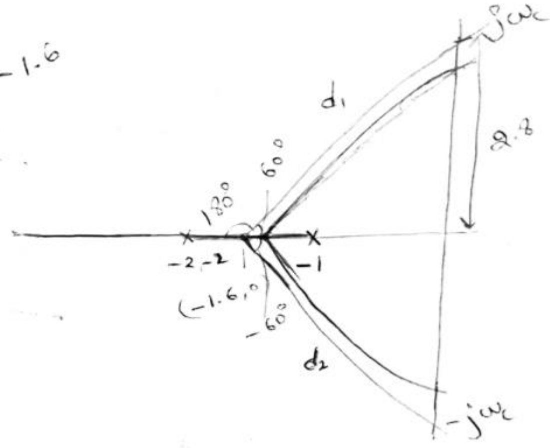
1) a) $G(s) = \dfrac{1}{(s+1)(s+2)(s+2)}$

Poles $= -1, -2, -2$

Centroid $= \dfrac{-1-2-2}{3} = -\dfrac{5}{3} \to -1.6$

Asymptotes $= 60°, 180°, 300°$




$K = -(s+1)(s+2)(s+2)$

$= -(s+1)(s^2+4s+4)$

$= -(s^3+4s^2+4s+s^2+4s+4)$

$= -(s^3+5s^2+8s+4)$

$\dfrac{dK}{ds} \Rightarrow -3s^2-10s-8 = 0$

$3s^2+10s+8 = 0.$

$s_{1,2} = \dfrac{-10 \pm \sqrt{10^2-4(3)(8)}}{2(3)} = \dfrac{-10 \pm 2}{6} = -\dfrac{4}{3}, -2.$

$= \underline{-1.3}, -2$

↳ Breakaway point.

(i) range of $k = \dfrac{\pi \text{ Poles (distance)}}{\pi \text{ zeroes}}$

$= d_1 \cdot d_2 \cdot d_3 \quad (|2.8j+1| \cdot |2.8j+2| \cdot |2.8j+2|)$

$\simeq 3\cancel{5}.20$

$K < 3\cancel{6}$ (closed loop stable)

(ii) From the graph $\omega_c \simeq 2.8$

(f) $\dfrac{k}{(s+1)(s+2)(s+2)}$ $\boxed{k' = 1.2.2. = 4}$

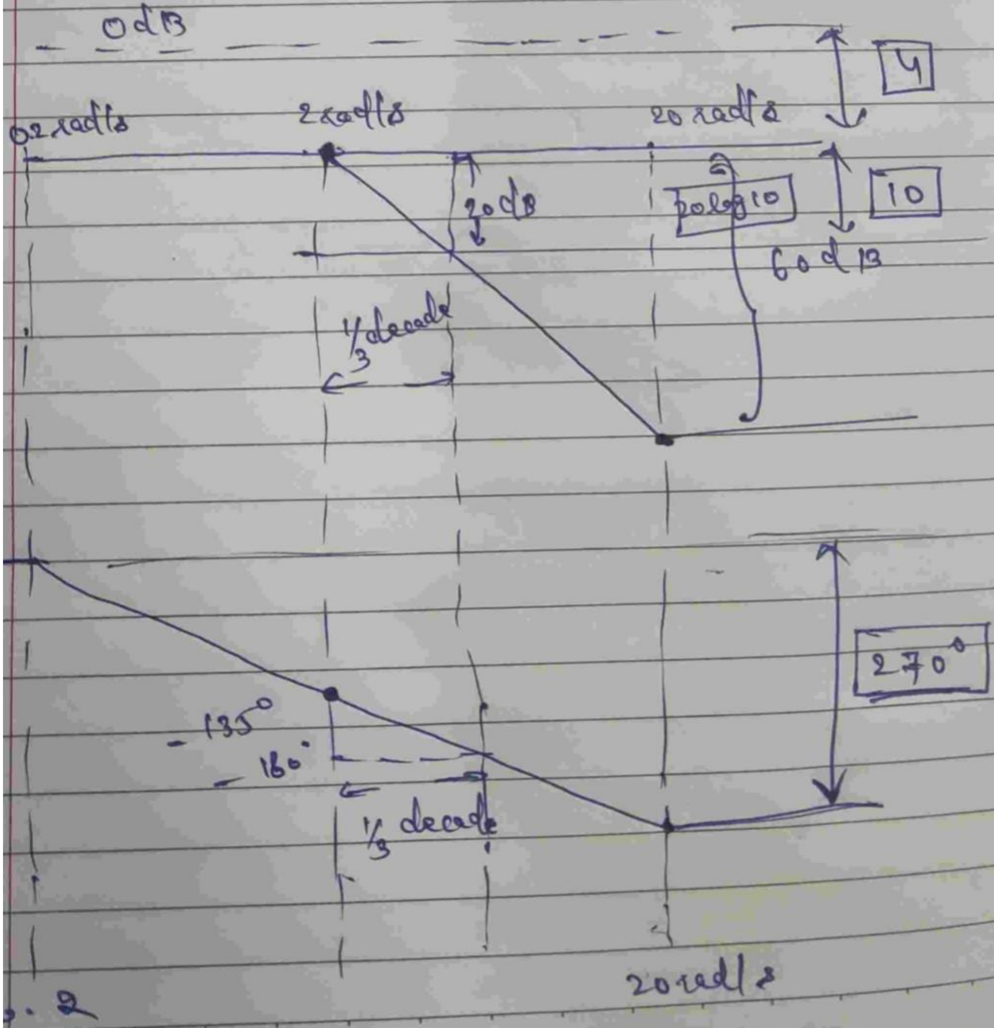
consider $\dfrac{1}{(s+2)^3}$ as approximat

we get 60 dB/dec slope in magnitude plot & in phase plot $-135°$/decade. slope starts at decade befor the cutoff freq. & end at decade after cutoff freq.

2 decade → 270° phase.

1 decade → 135° phase.

at 2 rad/s → $-135°$.



but we need to check at $-180^\circ$ to get GM
$-135^\circ - 45^\circ = -180^\circ$.

$\therefore$ we consider $\frac{1}{3}$ decade to compute GM
& total slope is 60dB/decade.

$$GM = \frac{1}{3}(60) = 20\,dB$$

$$20\log_{10} k'' = 20\,dB \Rightarrow k'' = 10$$

$$\boxed{k' \times k' = 40}$$

$$\boxed{0 < k < 40}$$

Q.1 (c) $\frac{k}{(j\omega+1)(j\omega+2)(j\omega+2)}$

$j(8\omega - \omega^3) + 4 - 5\omega^2$

$8\omega - \omega^3 = 0 \Rightarrow \omega = 0, \ \omega^2 = 8$

for $\omega = 0$

$\frac{k}{4} > -1 \Rightarrow k > -4$ — (1)

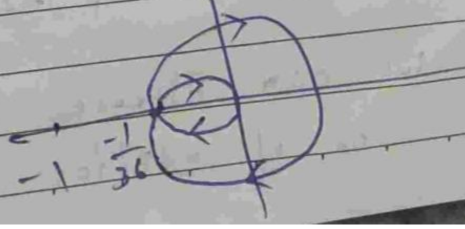
for $\omega^2 = 8$

$\frac{k}{4-5(8)} > -1$ — (2)

$\frac{k}{-36} > -1 \Rightarrow k < 36$

from (1) & (2) $\boxed{-4 < k < 36}$

Nyquist plot



Q.2 $\quad G(s) = \dfrac{2s^2 + 11s + 14}{s^2 + 5s + 6}$

(a) $\dfrac{Y(s)}{U(s)} = G(s) = \dfrac{Y(s)}{X(s)} \cdot \dfrac{X(s)}{U(s)}$

$\dfrac{Y(s)}{X(s)} = 2s^2 + 11s + 14 \qquad \dfrac{X(s)}{U(s)} = \dfrac{1}{s^2 + 5s + 6}$

$Y(s) = 2\ddot{x} + 11\dot{x} + 14x \qquad \ddot{x} + 5\dot{x} + 6x = U(s)$

$Y(s) = 2\dot{x}_2 + 11x_2 + 14x_1$

$= 2u - 10x_2 - 12x_1 + 11x_2 + 14x_1$

$= 2u + x_2 + 2x_1$

$x \rightarrow x_1$

$\dot{x} \rightarrow x_2 = \dot{x}_1$

$\dot{x}_2 = \ddot{x} = u - 5\dot{x} - 6x$

$\dot{x}_2 = u - 5x_2 - 6x_1$

$Y = \underbrace{\begin{bmatrix} 2 & 1 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{[2]}_{D} u$

$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B}$

(a) $\mathcal{C} = [B \quad AB] = \begin{bmatrix} 0 & 1 \\ 1 & -5 \end{bmatrix}$

$\text{rank}(\mathcal{C}) = 2 \therefore$ controllable

(b) $\mathcal{O} = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -5 \end{bmatrix}$

$\text{rank}(\mathcal{O}) = 2 \therefore$ observable

(b) $A_{ocf} = A_{ccf}^T$

$B_{ocf} = C_{ccf}^T$

$C_{ocf} = B_{ccf}^T$

$D_{ocf} = D_{ccf}^T$

**2)c) Does not exist**

**Q3**

Controllability

A given system $\dot{x} = Ax + Bu$ (or just pair $(A,B)$) is called controllable if for any initial condition $a \in \mathbb{R}^n$ & any final condition $b \in \mathbb{R}^n$, then exists an input $u$ & time interval $[0,T]$ such that solution of $\dot{x} = Ax + Bu$ satisfies $x(0) = a$ & $x(T) = b$.

A necessary & sufficient condition is that the matrix $[B \;\; AB \ldots A^{n-1}B]$ is rank $= n$.

Observability: A system $\dot{x} = Ax + Bu$, $y = Cx + Du$ (or just pair $(A,C)$) & initial condition $x(0)$ is called observable if for any $x(0)$, then exists $T > 0$ such that the measurement of $y(t)$ ( & $u(t)$) over the interval $[0,T]$ allows determination of $x(0)$.

In other words: observability is defined as....

If $x_1(0)$ & $x_2(0)$ : any two initial conditions result in same output trajectory $y(t)$ for some $T > 0$ & $t \in [0,T]$,

then $x_1(0) = x_2(0)$.

Rank test: system is observable if & only if $\begin{bmatrix} C \\ CA \\ CA^{n-1} \end{bmatrix}$ is full column rank (or rank $n$).

Pole placement problem: $A \in \mathbb{R}^{n \times n}$

Given $\dot{x} = Ax + Bu$, (find conditions on $(A,B)$) and arbitrary $d(s)$ of degree $n$, monic, real polynomial, find feedback law $u = Fx$ (or matrix $F$) such that $(A + BF)$ has characteristic polynomial $= d(s)$

( $\det[sI - A - BF] = d(s)$ )

Pole placement thm: Given $(A,B)$, arbitrary pole-placement is possible if & only if $(A,B)$ is controllable (or $[B \; AB \ldots A^{n-1}B]$ rank $= n$)

4]
$$A = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$$

(a) $e^{At} = L^{-1}[SI - A]^{-1}$

$$SI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 2 & s-3 \end{bmatrix}$$

$$[SI - A]^{-1} = \frac{1}{s(s-3)+2} \begin{bmatrix} s-3 & 1 \\ -2 & s \end{bmatrix} = \frac{1}{(s-2)(s-1)} \begin{bmatrix} s-3 & 1 \\ -2 & s \end{bmatrix}$$

$$L^{-1}[SI - A]^{-1} = \begin{bmatrix} 2e^{t} - e^{2t} & e^{2t} - e^{t} \\ -2(e^{2t} - e^{t}) & 2e^{2t} - e^{t} \end{bmatrix} u(t) = e^{At}$$

(b) $$G(s) = \frac{1}{s^2 - 3s + 2} = \frac{1}{(s-2)(s-1)}$$

Impulse response $h(t) = L^{-1}(G(s))$

Laplace inverse of $\frac{1}{(s-2)(s-1)}$ is already calculated in (a) part. The second element of $[SI-A]^{-1}$

$$h(t) = (e^{2t} - e^{t})u(t)$$

**Q5) a) Not Possible**

**b)Root locus has to be used to get the values of PD controller (ie the $K_p$, $K_d$ values in u = $K_p$e +$K_d$e_dot)**