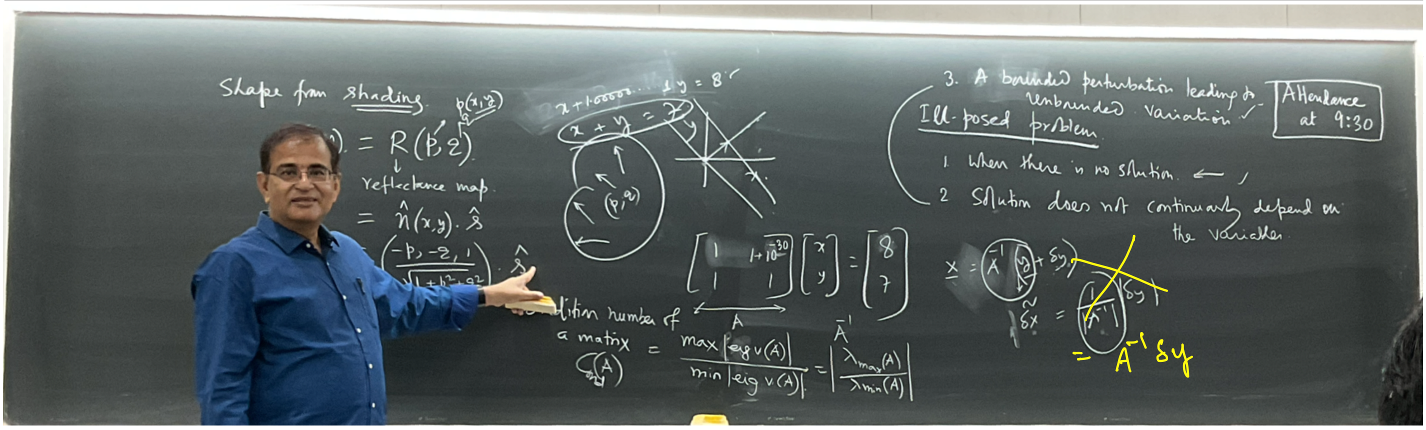
EE702: Lec-6 (29 Jan)



Okay so we need to convert ill-posed problem to a well-posed problem by taking assumptions.

Assumption: " Surface is smooth" $\Rightarrow$ Smoothness constraint regularisation

$$\min_{p,q} \iint \left(p_x^2 + p_y^2 + q_x^2 + q_y^2 \right) dx\,dy \qquad \text{subject to } E(x,y) = R(p,q)$$

$$\Downarrow$$

$$\min_{p,q} \iint \underbrace{\left[\underbrace{(E-R)^2}_{A(\theta)} + \lambda \left(p_x^2 + p_y^2 + q_x^2 + q_y^2 \right) \right]}_{F} dx\,dy$$

$\lambda \rightarrow$ regularisation parameter

$F \rightarrow$ functional

"Calculus of variation"

Soln: To min. F, use **Euler-Lagrange eqn!**

So....

$$\min_{p,q} \iint F(p, p_x, p_y, q, q_x, q_y)\, dx\,dy$$

$$F_p - \frac{\partial}{\partial x} F_{p_x} - \frac{\partial}{\partial y} F_{p_y} = 0 \Rightarrow -2(E-R)R_p - 2\lambda \underbrace{(p_{xx} + p_{yy})}_{\nabla^2 p} = 0$$

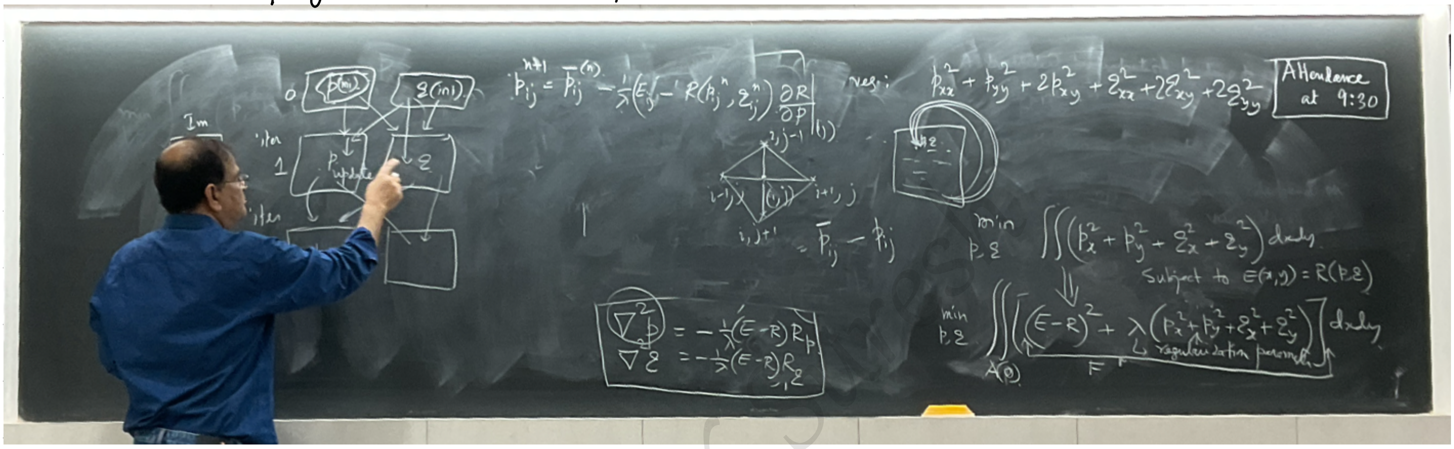
$$F_q - \frac{\partial}{\partial x} F_{q_x} - \frac{\partial}{\partial y} F_{q_y} = 0 \Rightarrow -2(E-R)R_q - 2\lambda \underbrace{(q_{xx} + q_{yy})}_{\nabla^2 q} = 0$$

$$\Rightarrow \boxed{\begin{aligned} \nabla^2 p &= \frac{-1}{\lambda}(E-R)R_p \\ \nabla^2 q &= \frac{-1}{\lambda}(E-R)R_q \end{aligned}}$$

So what we just did is "smoothening over 1^{st} derivative"
What about higher orders?
Pattern!!

Eg. 2^{nd} order: $F_p - \left(\frac{\partial}{\partial x} F_{p_x} + \frac{\partial}{\partial y} F_{p_y}\right) + \left(\frac{\partial^2}{\partial x^2} F_{p_{xx}} + \frac{\partial^2}{\partial x \partial y} F_{p_{xy}} + \frac{\partial^2}{\partial y^2} F_{p_{yy}}\right) = 0$ ✓

How to perform discrete Laplacian?



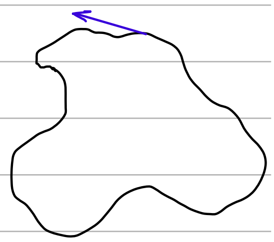
But... there is a MAJOR issue!!

$$\hat{n} = \frac{-p, -q, 1}{\sqrt{p^2+q^2+1}}$$

At occl. boundary,

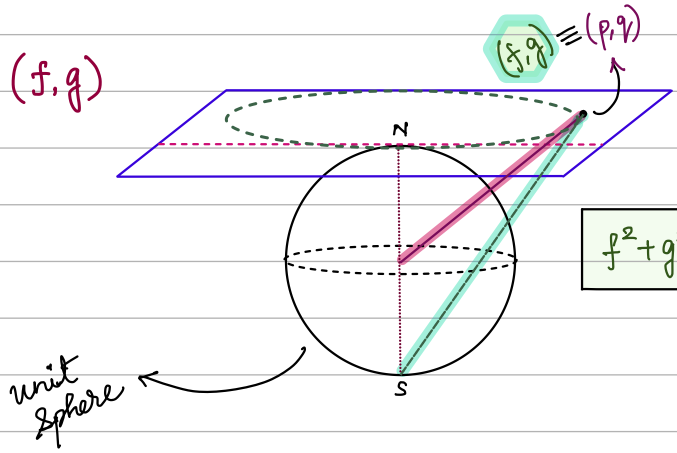
$$\hat{n} = \frac{(a, b, 0)}{\sqrt{a^2+b^2}}$$

cannot be equated unless p or q blows to infinity!



F. What's the roundabout now?

$(p,q) \rightarrow (f,g)$



$\boxed{f^2+g^2 \leq 4}$

$g = \frac{2q}{1+\sqrt{1+p^2+q^2}}$

$f = \frac{2p}{1+\sqrt{1+p^2+q^2}}$

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Another point, discretize the cost fn before getting into its solving!

$$L = \sum_i \sum_j \left[\left(E_{ij} - R_s(f_{ij}, g_{ij}) \right)^2 + \lambda \left[(f_{i+1,j} - f_{ij})^2 + (f_{i,j+1} - f_{i,j})^2 + g \ldots \right] \right]$$

$$\min_{p,q} \iint F(p, p_x, p_y, q, q_x, q_y)\, dx\, dy$$

$$\Rightarrow \frac{dL}{df_{k,l}} = 0 \quad \forall (k, l)$$

$$= -(E - R_s) \frac{dR_s}{df_{k,l}} - \lambda \left[(f_{k+1,l} - f_{k,l}) + (f_{k,l+1} - f_{k,l}) \right]$$