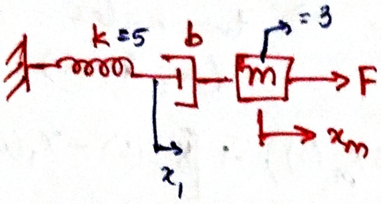
Tutorial - 4

Sol. 1]

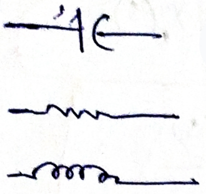


a) i) via $\begin{pmatrix}F\\x\end{pmatrix} \rightarrow \begin{pmatrix}v\\s\end{pmatrix}$

ii) via $\begin{pmatrix}F\\x\end{pmatrix} \rightarrow \begin{pmatrix}I\\v\end{pmatrix}$

i) Electrical

Z



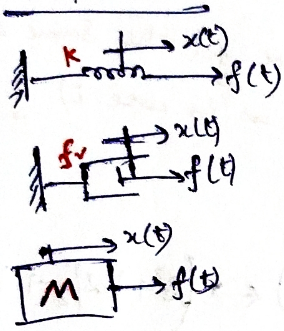
$\left(\frac{1}{cs}\right)$ $\qquad V(t) = \frac{1}{C}\int_0^t i(\tau)d\tau = \frac{q(t)}{c} \qquad i(t) = c\frac{dV(t)}{dt}$

(R) $\qquad V(t) = R\,i(t) = R\frac{dq(t)}{dt} \qquad i(t) = V(t)/R$

(LS) $\qquad V(t) = L\frac{di(t)}{dt} = L\frac{d^2q(t)}{dt^2} \qquad i(t) = \frac{1}{L}\int_0^t V(\tau)d\tau$

Mechanical

Z_m



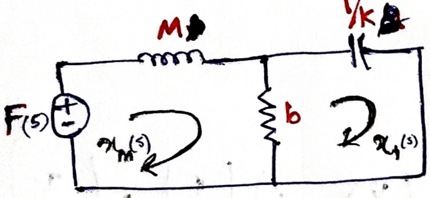
(k) $\qquad F(t) = k\int_0^t v(\tau)d\tau = k \cdot x(t)$

$(f_v s)$ $\qquad F(t) = \cancel{f_v}^{\,b}\, v(t) = f_v \frac{dx(t)}{dt}$

(Ms^2) $\qquad F(t) = M\frac{dv(t)}{dt} = M\frac{d^2x(t)}{dt^2}$

Now, i) F becomes —(V)— $\because$ there are two x (x_1 and x_m) $\therefore$ two "i" loops:

x becomes $\rightarrow i$



$F(s) = (b+Ms)X_m(s) - bX_1(s)$ ← isme thok do!

$0 = -bX_m(s) + \left(b+\frac{k}{s}\right)X_1(s)$

→ isme se $X_1(s)$ nikalke

$\therefore$ ~~$X_1(s) = \frac{F(s)}{-b} + \frac{(b+Ms)}{b}$~~

$X_1(s) = \frac{bX_m(s)}{\left(b+\frac{k}{s}\right)} = \frac{bs\,X_m(s)}{(sb+k)}$

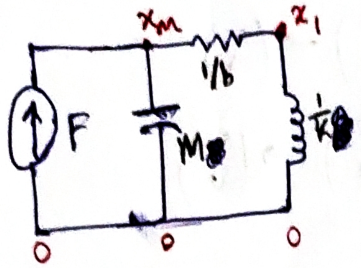
$\therefore \frac{X_M(s)}{F(s)} = \frac{1}{(b+Ms) - \frac{b^2s}{(sb+k)}} = \frac{sb+k}{s^2Mb+Mks+bk} = \boxed{\frac{\frac{1}{M}(s+k/b)}{s^2+ks/b+k/m}}$

cancelling off b from Nr & Dr and pulling out M from Dr

ii) $\begin{pmatrix} F \\ x \end{pmatrix} \rightarrow \begin{pmatrix} I \\ V \end{pmatrix}$ $\therefore$ F becomes $\uparrow i$ and x becomes V

$\Rightarrow$ two nodes (x_1 & x_m)

$\therefore$



$$F(s) = M\cdot(sX_m(s)) + \frac{[X_m(s) - X_1(s)]}{(1/b)}$$

$$\Rightarrow F(s) = X_m(s)\left[Ms + b\right] - b\,X_1(s)$$

Now, @ node x_1 : $\dfrac{X_m(s) - X_1(s)}{(1/b)} = \dfrac{1}{(1/k)} \cdot \dfrac{X_1(s)}{s}$

$$\Rightarrow 0 = -bX_m(s) + X_1(s)\left[b + \frac{k}{s}\right]$$

we've got the same eqns back as in case i) !

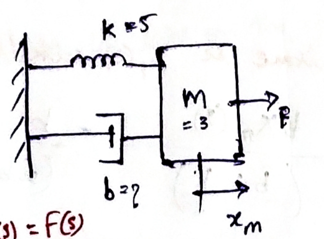
b) range of $b > 0$ for underdamped :

from TF, we compare : $(s^2 + 2\rho\omega_n s + \omega_n^2) \leftrightarrow (s^2 + ks/b + k/m)$.

$\therefore \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{5}{3}}$ and $2\rho\omega_n = \frac{k}{b}$ ($k = 5$) $\therefore \rho = \frac{5\sqrt{3}}{2\sqrt{5}\cdot b}$

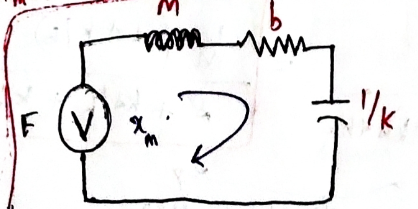
$\therefore$ for $\rho < 1$, $\boxed{b > \frac{\sqrt{15}}{2}}$

Sol. 2]



a] i) $\begin{pmatrix} F \\ x \end{pmatrix} \rightarrow \begin{pmatrix} V \\ I \end{pmatrix}$ ii) $\begin{pmatrix} F \\ x \end{pmatrix} \rightarrow \begin{pmatrix} I \\ V \end{pmatrix}$

$Ms^2X_m(s) + KX_m(s) + bsX_m(s) = F(s)$



$\Rightarrow \dfrac{X_m(s)}{F(s)} = \dfrac{1}{Ms^2 + bs + k}$

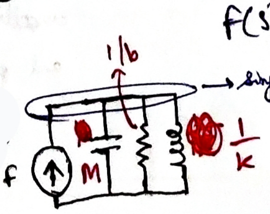
i) $\int_0^t i(\tau)d\tau$

$$F(s) = M\cdot[sX_m(s)] + bX_m(s) + \frac{1}{(1/k)}\frac{X_m(s)}{s}$$

$$\Rightarrow \frac{X_m(s)}{F(s)} = \frac{1}{Ms^2 + bs + K}$$

same eqn.

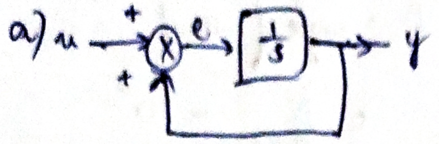
ii)



$$F(s) = M\left[sX_m(s)\right] + \frac{X_m(s)}{(1/b)} + \frac{1}{(1/k)}\frac{X_m(s)}{s}$$

b) $2\rho\omega_n = \frac{b}{M}$ ($\omega_n = \sqrt{k/m}$) $\Rightarrow \rho = \dfrac{b}{2\sqrt{mk}} < 1 \Rightarrow b < 2\sqrt{5\times3} \Rightarrow b < 7.7$

Sol. 3] a)

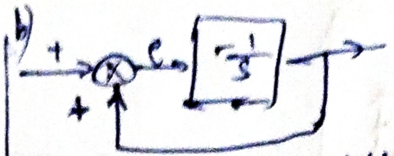


closed loop unstable

$$y = \frac{1}{s}\cdot e, \quad e = u + y \Rightarrow y = \frac{1}{s}(u+y)$$

$$\Rightarrow \frac{y(t)}{u(t)} = \frac{1/s}{1-1/s}$$

b)

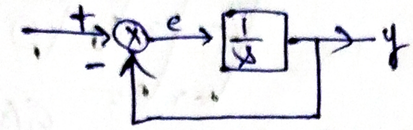


closed loop stable

$$y = -\frac{1}{s}(u+y)$$

$$\Rightarrow \frac{y(t)}{u(t)} = \frac{-1/s}{1+1/s}$$

Sol. 4] a)

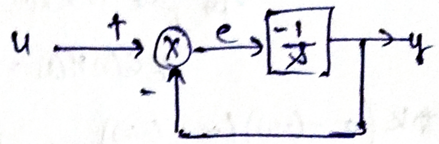


closed loop stable

$$y = \frac{1}{s}(u-y)$$

$$\Rightarrow \frac{y(t)}{u(t)} = \frac{1/s}{1+1/s}$$

b)



closed loop unstable

$$y = -\frac{1}{s}(u-y)$$

$$\Rightarrow \frac{y(t)}{u(t)} = \frac{-1/s}{1-1/s}$$

Sol. 5] a) $\frac{1}{s+2}$ TRUE

b) $\frac{s-1}{s+2}$ FALSE

Zero ∈ RHP

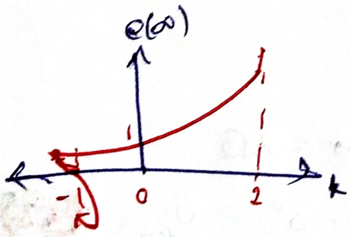
Statement: Higher gain leads to ~~causes faster transients~~ stability

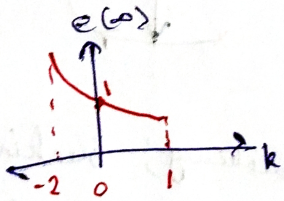
Sol. 6] Statement: Higher gain leads to faster transients (smaller τ)

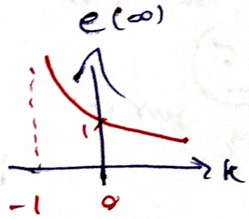
a) $\frac{1}{s+1}$ TRUE

b) $\frac{s+1}{s+5}$ FALSE

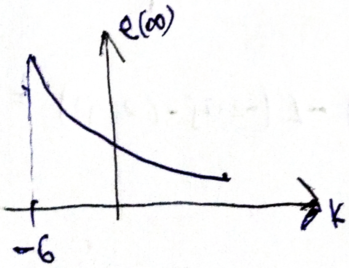
Sol. 7]





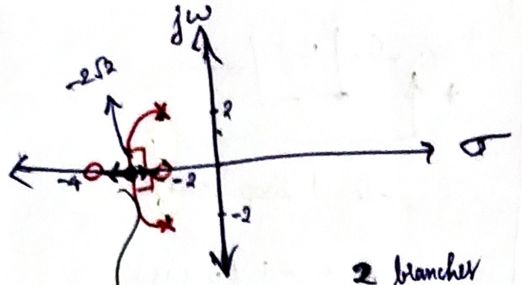


Sol. 8]



$T_{settling}$ is $\frac{4}{1.5}$ seconds whenever closed-loop poles are non-real.

Sol. 9]



θ = angle of arrival

$= \frac{\pi}{2} = 90^\circ$

Okay, but where exactly are they colliding and breaking off at?

2 branches

$\frac{-4 \pm \sqrt{16-32}}{2} = -2 \pm 2i$

real axis segment

$\frac{dK}{d\sigma} = 0$ ✓

For pole to exist as real,

$KG(s)H(s) = -1$

$\Rightarrow \frac{K(\sigma-(-4))(\sigma-(-2))}{\sigma^2+4s+8} = -1 \Rightarrow K = -\frac{(\sigma^2+4\sigma+8)}{(\sigma^2+6\sigma+8)}$

$G(s)H(s) = \frac{(\sigma+4)(\sigma+2)}{\sigma^2+4\sigma+8} = \frac{N(s)}{D(s)}$

$\therefore \frac{dk}{d\sigma} = 0 \Rightarrow N(s)D'(s) - N'(s)D(s) = 0$

$\Rightarrow (\sigma^2+6\sigma+8)(2\sigma+4) - (2\sigma+6)(\sigma^2+4\sigma+8) = 0$

$\Rightarrow \sigma^2(4+12-8-6) + 32 - 48 = 0 \Rightarrow 2\sigma^2 - 16 = 0$

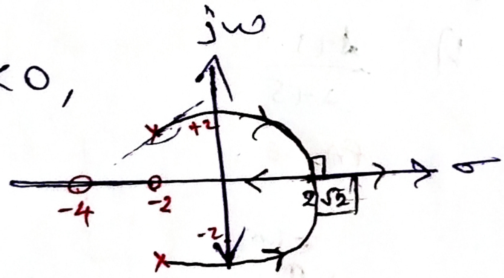
$\Rightarrow \sigma = \cancel{2\sqrt{2}}, -2\sqrt{2}$ ✓

(Trash)

$\therefore$ Nothing called break away here, $\theta_{arrival} = 0^\circ$

and break-in is @ $\underline{\sigma = -2\sqrt{2}}$. $\theta_{dep} =$ π

b) For $K < 0$,



$+2\sqrt{2}$ ✓

$\cancel{-2\sqrt{2}}$

Trash

As $k \to -\infty$, how's life?? or after breakin

very sus...

No breakaway; Breakin @ $\sigma = 2\sqrt{2}$

$\theta_{arrival} = 0^\circ$ $\theta_{dep} =$?

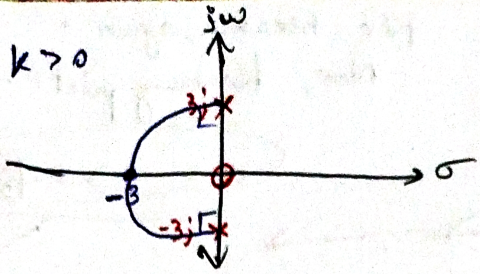
for $-2+2j$: $-\theta + \angle(-2+2j-(-4)) + \angle(-2+2j-(-2)) - \angle(-2+2j-(-2-2j)) = (2k+1)\pi$

$\Rightarrow -\theta + 45^\circ + \cancel{90^\circ - 90^\circ} = (2k+1)\pi$

$\boxed{\therefore \theta_1 = 45^\circ, \theta_2 = -315^\circ} \longrightarrow \underline{\theta_{dep}}$ ✓

Sol. 10] $K > 0$

$\frac{s}{s^2+9}$



$\theta = \frac{\pi}{n>2} = 90°$

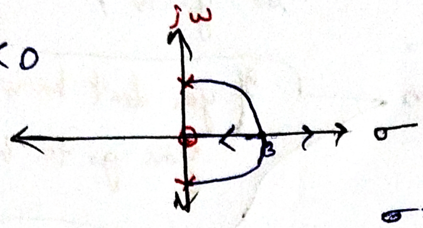
$\sigma = ?$

$\sigma(2\sigma) - \sigma^2 - 9 = 0 \Rightarrow \sigma = \pm 3$

$\theta_{arriv} = 0$, $\underline{\theta_{dep} = \pi}$ $\underline{\sigma = -3}$

$\cancel{+3}$ Trash -3 ✓

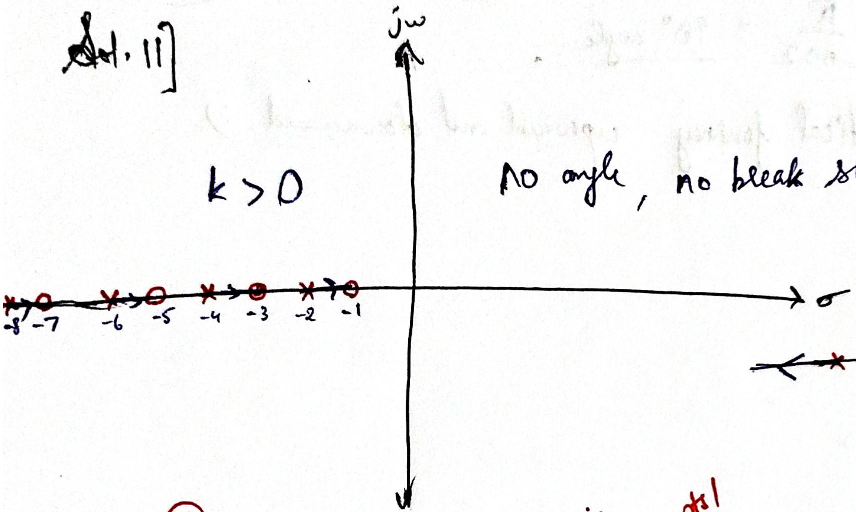
$K < 0$



$\sigma = 3$, others same

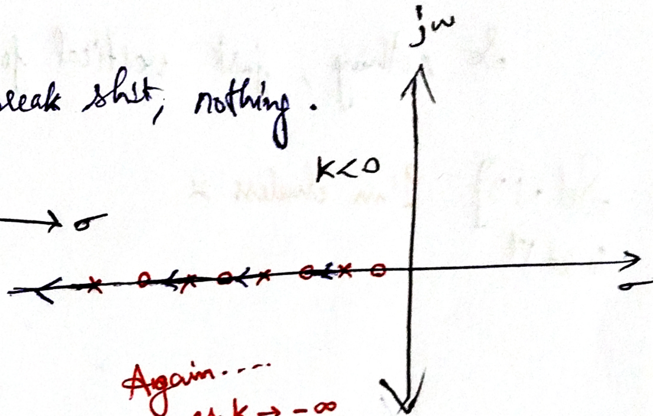
Sol. 11]

$K > 0$



No angle, no break shit, nothing.

$K < 0$



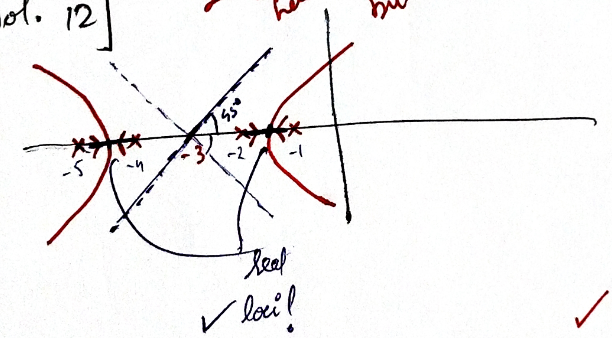
Again.... as $K \to -\infty$, very sus!

Nice Qn!

$\boxed{K > 0}$

Sol. 12]

Funny enough, here there's no breakin but 2 break-away pts!



$\frac{dk}{d\sigma} = 0 \Rightarrow 4\sigma^3 + 36\sigma^2 + 98\sigma + 78 = 0$

$\Rightarrow \sigma = -3$ Trash

✓ Break-aways $\left\{ \begin{array}{l} ✓ \ \sigma = -3 + \sqrt{\frac{5}{2}} \\ ✓ \ \sigma = -3 - \sqrt{\frac{5}{2}} \end{array} \right.$

No zeroes to "devour" the loci, so clearly 4 pitukdas of asymptotes

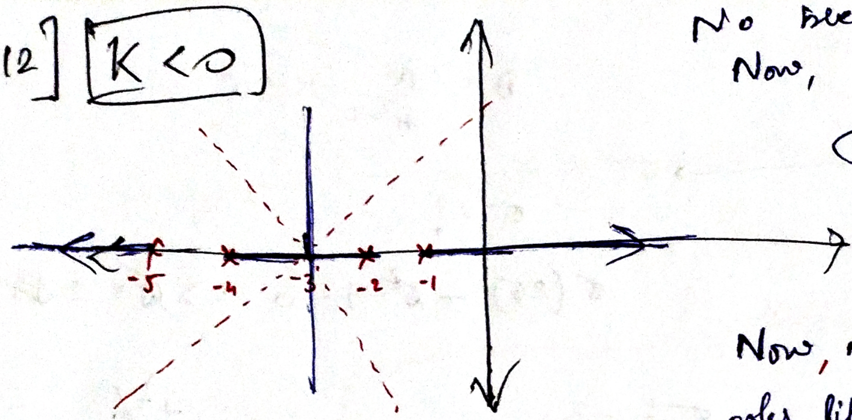
Finding loci now needs us to look out for asymptote...

$\sigma_a = \frac{(-1-2-4-5)}{4}$, $\theta_a = \frac{(2k+1)\pi}{4}$

$\Rightarrow \sigma_a = -3$, $\theta_a = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ Hurray!

Sol. 12] $\boxed{K<0}$

No breakin, again.
Now, breakaway point: -3

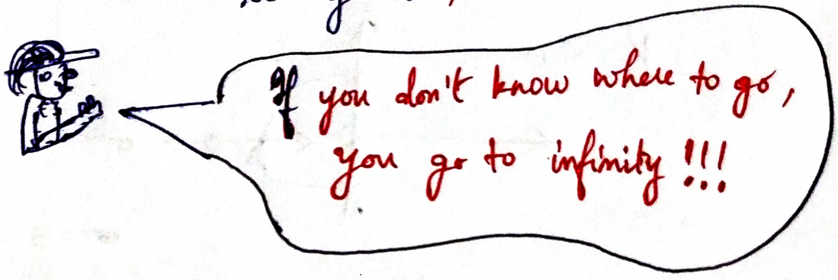


$\sigma = -3 \pm \sqrt{\frac{5}{2}}$ → Trash :(

Now, poles like -5 and -1 are worried :(

So yeah,

Prof' Behu



NOTE:

@ Breakaway pt

$\sigma = -3$, poles form $\frac{\pi}{\cancel{n}\,2} = \underline{90^\circ \text{ angle}}$.

So nothing, just vertical journey upward and downward :)