$\boxed{Q.1}$

$(s+1)(s+2)+k=0$

$s^2+3s+2+k=0$

$2+k=\omega_n^2$ — (1)

$2\xi\omega_n=3$ — (2)

$e^{-\pi\xi/\sqrt{1-\xi^2}}\times 100 = 2$

$50 = e^{\pi\xi/\sqrt{1-\xi^2}}$ — (3)

from (1), (2) & (3)

$\xi = \frac{3}{2\sqrt{2+k}}$

$50 = e^{\pi\frac{3}{2\sqrt{2+k}}/\sqrt{1-\frac{9}{4(2+k)}}}$

$\cancel{50 = e^{3\pi/\ldots}}$ exp

$\ln 50 = \frac{\pi 3}{\cancel{2\sqrt{2+k}}} \Big/ \sqrt{\frac{4(2+k)-9}{\cancel{4(2+k)}}}$

$\sqrt{8+4k-9} = \frac{3\pi}{\ln 50}$

$\sqrt{4k-1} = 3\pi/\ln 50$

$4k-1 = \left(\frac{3\pi}{\ln 50}\right)^2 \Rightarrow k = \frac{(2.4091)^2+1}{4}$

$\boxed{k = 1.7010}$

$s^2+3s+\cancel{2}+3.7010=0$

$\omega_n^2 = 3.7010$

$\omega_n = 1.924$

$\xi = 0.779$

settling time

$t_s = \frac{4}{\xi\omega_n}$

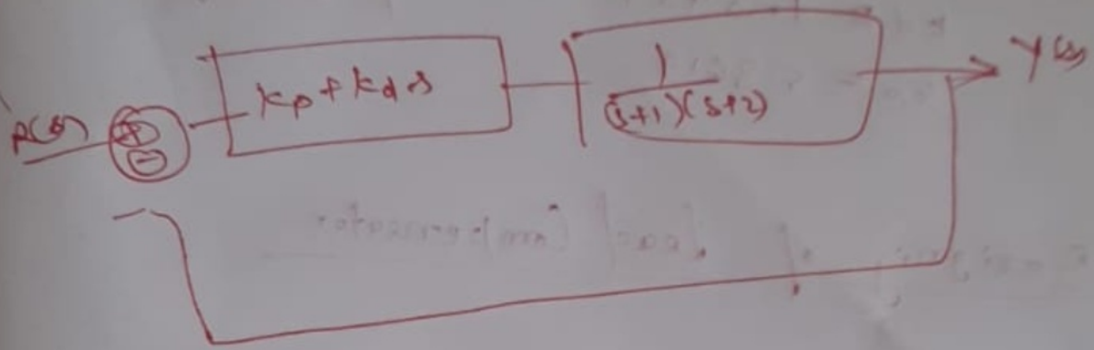
$= 2.668$ sec

→ Using graph paper ~~k value~~ $k = 1$.

→ 2% settling time using graph paper, $t_s = 2.67$ seconds.

→ Design of PD controller for

$t_s = 2.5$ seconds, overshoot 2%.



$$\frac{Y(s)}{R(s)} = \frac{k_p + k_d s}{s^2 + (3+k_d)s + (2+k_p)}$$

$\omega_n^2 = 2 + k_p;$

$2\xi\omega_n = 3 + k_d;$

Given $t_s = 2.5$ s

$t_s = \frac{4}{\xi\omega_n} \Rightarrow 2.5 \Rightarrow k_d = 1.2$

$\xi = \frac{3+k_d}{2\sqrt{2+k_p}} \Rightarrow 0.779 = \frac{3+1.2}{2\sqrt{2+k_p}}$

$\Rightarrow \sqrt{2+k_p} = \frac{4.2}{1.558}$

$= 2.7$

$k_p = 5.25$

$k_d = 1.2,$

$k_p = 5.25$

→ From graph paper method (for $t_s = 2.5$) & overshoot 6.6%.

$K_d = 0.24935$

$K_p = 1.0864$

→ From graph paper method for overshoot 2% & settling time 1.8.

$K_d = 1.3454$

$K_p = 5.8619$

→ Designing of Lead Compensator.

Q2) Let $G(s) = \dfrac{K}{(s+2)(s+20)(s+200)}$

Bode plot:

$$G(j\omega) = \frac{(K/8000)}{\left(1+\frac{j\omega}{2}\right)\left(1+\frac{j\omega}{20}\right)\left(1+\frac{j\omega}{200}\right)}$$

For simplicity assuming $K = 8000$. Then,

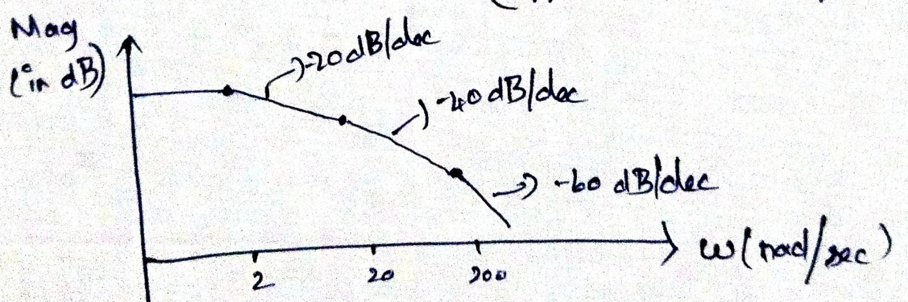
$$|G(j\omega)| = \frac{1}{\sqrt{1+\frac{\omega^2}{4}}\sqrt{1+\frac{\omega^2}{400}}\sqrt{1+\frac{\omega^2}{40000}}}$$

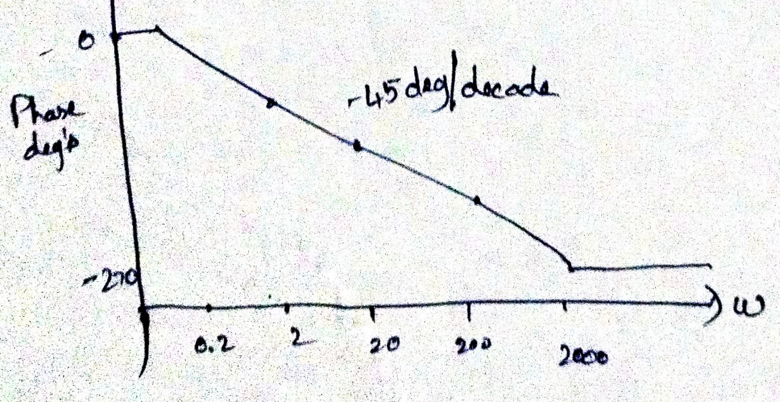
$$\phi = -\tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{20} - \tan^{-1}\frac{\omega}{200}$$

ω	0	∞
$\lvert G(j\omega)\rvert$	1	0
In dB	0	$-\infty$
ϕ	0	-270

Corner freq's $\rightarrow$ 2, 20, 200

(Approx bode plot)





To find phase crossover freq :

$$180 = \tan^{-1}\frac{\omega}{2} + \tan^{-1}\frac{\omega}{20} + \tan^{-1}\frac{\omega}{200}$$

$$\Rightarrow 0 = \frac{\omega}{2} + \frac{\omega}{20} + \frac{\omega}{200} - \frac{\omega^3}{8000}$$

$\because \tan(A+B+C) = \dfrac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$

$$\frac{\omega^3}{8000} = \frac{111\omega}{200}$$

$$\omega(\omega^2 - 4440) = 0$$

$$\boxed{\omega_{pc} = 66.6 \text{ rad/sec}}$$

$$G.M = \frac{1}{|G(j\omega_{pc})|} = \sqrt{1111}\,\sqrt{12.1}\,\sqrt{1.111} = 122.21$$

$\therefore$ If the gain is rised by a factor of greater than 122.21 then the system becomes unstable. (for $k = 8000$)

$\therefore$ For the given T.F $k > 8000 \times 122.21 = 9,77,680$ for unstable.

Routh array :

$$C.L\ T.F = \frac{k}{s^3 + 222s^2 + 4440s + (8000+k)}$$

s^3	1	4440
s^2	222	$8000+k$
s	$4404 - \frac{k}{222}$	0
s^0	$(8000+k)$	

For stability

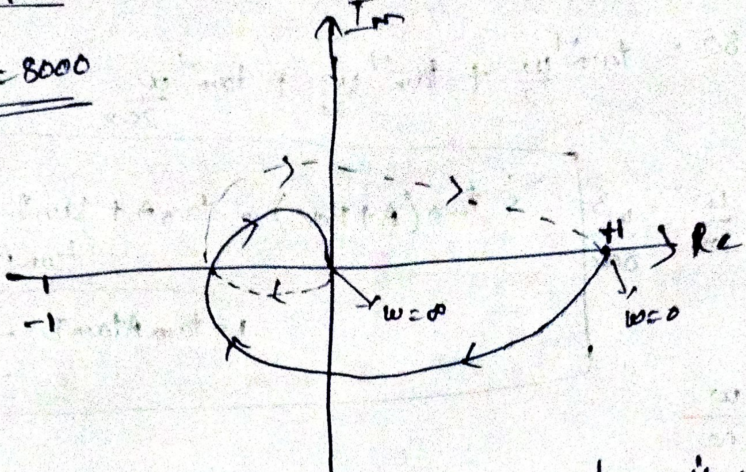
$$-8000 < k < 9,77,680$$

$\therefore$ For unstable

$$k > 9,77,680 \text{ or } k < -8000$$

Niquist plot:

For $k = 8000$



To find point of intersection on real axis,

$$G(j\omega) = \frac{1}{\left(1+\frac{j\omega}{2}\right)\left(1+\frac{j\omega}{20}\right)\left(1+\frac{j\omega}{200}\right)} = \frac{\left(1-\frac{j\omega}{2}\right)\left(1-\frac{j\omega}{20}\right)\left(1-\frac{j\omega}{200}\right)}{\left(1+\frac{\omega^2}{4}\right)\left(1+\frac{\omega^2}{400}\right)\left(1+\frac{\omega^2}{40000}\right)}$$

$$\text{Num} \rightarrow \left[\left(1-\frac{\omega^2}{40}\right) - \frac{11j\omega}{20}\right]\left(1-\frac{j\omega}{200}\right) = \left(1-\frac{\omega^2}{40}-\frac{11\omega^2}{4000}\right) - j\left[\frac{11\omega}{20} + \left(1-\frac{\omega^2}{40}\right)\frac{\omega}{200}\right]$$

$$\text{Im part} = 0 \Rightarrow \frac{11\omega}{20} + \left(1-\frac{\omega^2}{40}\right)\frac{\omega}{200} = 0$$

$$4400 + 40 - \omega^2 \Rightarrow \omega^2 = 4440$$

$$\boxed{\omega = 66.6 \text{ rad/sec}}$$

Corresponding Real part $\Rightarrow \dfrac{1-111-12.21}{1111 \times 12.1 \times 1.111} = -8.17 \times 10^{-3}$

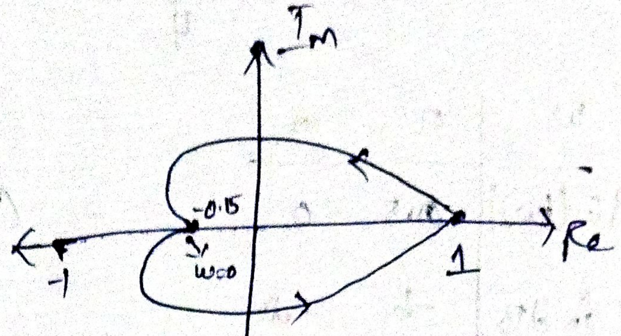
$\therefore$ If gain is raised higher than a factor of $\frac{1}{8.17 \times 10^{-3}} \approx 122.3$ the system becomes unstable. (at $k=8000$)

$\therefore$ $k > 8000 \times 122.3 \approx 977680$ for unstability.

Q3 $G(j\omega) = \dfrac{j\omega - 3}{20 + j\omega} = \dfrac{(-3 + j\omega)(20 - j\omega)}{400 + \omega^2}$

$= \dfrac{\omega^2 - 60 + 23j\omega}{400 + \omega^2}$

ω	0	∞
$G(j\omega)$	-0.15	1



$\therefore K > \dfrac{1}{0.15} = \dfrac{20}{3}$ for unstability.

Real part -->

Routh array:

C.L T.F $= \dfrac{s-3}{(s+20) + K(s-3)} = \dfrac{s+3}{s(1+K) + 20 - 3K}$

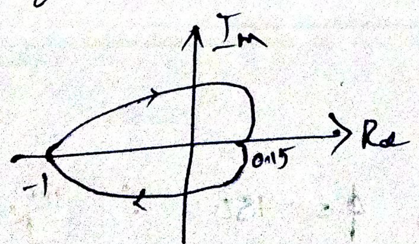
s	$1+K$
s^0	$20 - 3K$

For stability:

$-1 < K < \dfrac{20}{3}$

$\boxed{\text{For unstability } K > \frac{20}{3} \text{ or } K < -1}$

Also
Nyquist plot for $K = -1$



$\therefore$ when $K < -1$

the point $-1 + j0$ is encircled & leads to unstability.

Illy for Q2 from nyquist plot, using the same arguments, $K < -8000$ for unstable.

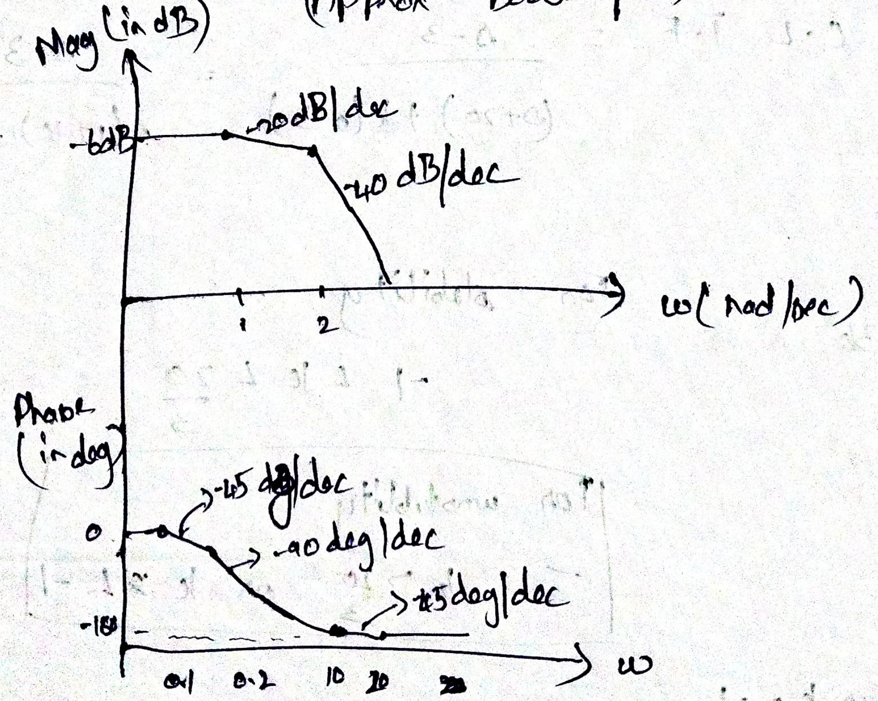
Q4 (a) $G(s) = \frac{0.5}{(1+s)(1+\frac{s}{2})}$ $G(j\omega) = \frac{0.5}{(1+j\omega)(1+\frac{j\omega}{2})}$

$|G(j\omega)| = \frac{0.5}{\sqrt{1+\omega^2}\sqrt{1+\frac{\omega^2}{4}}}$ $\phi = -\tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}$

ω	0	∞
$\lvert G(j\omega)\rvert$	0.5	0
in dB	-6	$-\infty$
ϕ	0	$-\pi$

Corner freq $\Rightarrow$ 1, 2

(Approx bode plot)



To find gain margin :

clearly at $\omega = \infty$; $\phi = -180$ $\therefore$ $\omega_{pc} = \infty$

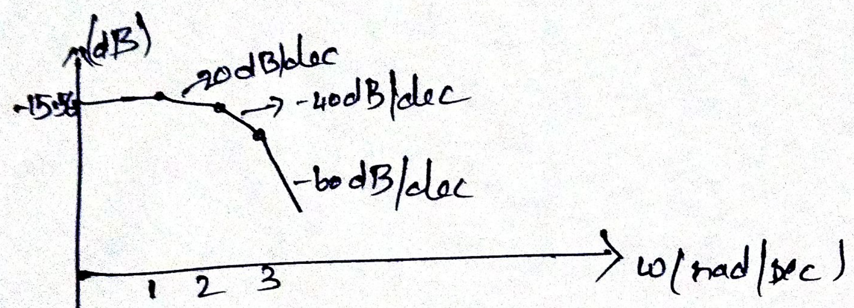
$\therefore$ G.M $= \frac{1}{|G(j\omega_{pc})|} = \frac{1}{0} = \infty$

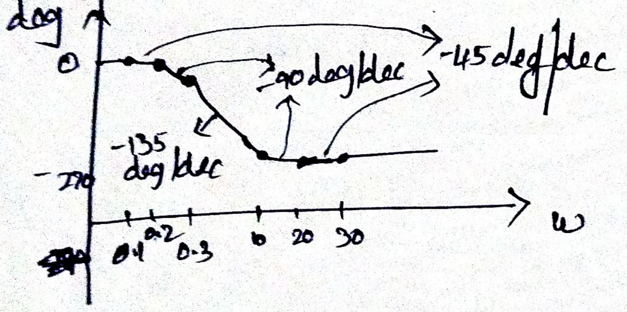
(3) $G(s) = \dfrac{1/6}{(1+s)(1+\frac{s}{2})(1+s/3)}$ $\qquad G(j\omega) = \dfrac{1/6}{(1+j\omega)(1+\frac{j\omega}{2})(1+\frac{j\omega}{3})}$

$|G(j\omega)| = \dfrac{1/6}{\sqrt{1+\omega^2}\sqrt{1+\frac{\omega^2}{4}}\sqrt{1+\frac{\omega^2}{9}}}$ $\qquad \phi = -\tan^{-1}\omega - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{3}$

ω	0	∞
$\lvert G(j\omega)\rvert$	$1/6$	0
in dB	-15.56	$-\infty$
ϕ	0	-270°

corner freq = 1, 2, 3





To find G.M:

$\omega_{pc} \Rightarrow \quad 180 = \tan^{-1}\omega + \tan^{-1}\frac{\omega}{2} + \tan^{-1}\frac{\omega}{3}$

$\Rightarrow 0 = \omega + \frac{\omega}{2} + \frac{\omega}{3} - \frac{\omega^3}{6} \quad \Rightarrow \quad \frac{\omega^3}{6} = \frac{11\omega}{6}$

$\omega(\omega^2 - 11) = 0 \qquad \Rightarrow \omega_{pc} = \sqrt{11}$ rad/sec

$G.M = \dfrac{1}{|G(j\omega_{pc})|} = 6 \times \sqrt{12} \times \sqrt{3.75} \times \sqrt{2.222} \approx 60$