EE302, Section S2, Control Systems, Quiz 1, 10% 8th Feb 2024.

(Read Notes at the end : first read that.)

Q-1 A 2nd order transfer fn $G(s)$ without zeros is to be designed such that the step response has 2 or 3 of the following specifications $10\%OS$, 2% settling time $= 20$ seconds $= T_S$, Peak time $T_p = 3$ seconds.

(a) Design G_1 with all 3 properties.

(b) Design G_2, G_3, G_4 with 2 of the 3 properties, i.e.

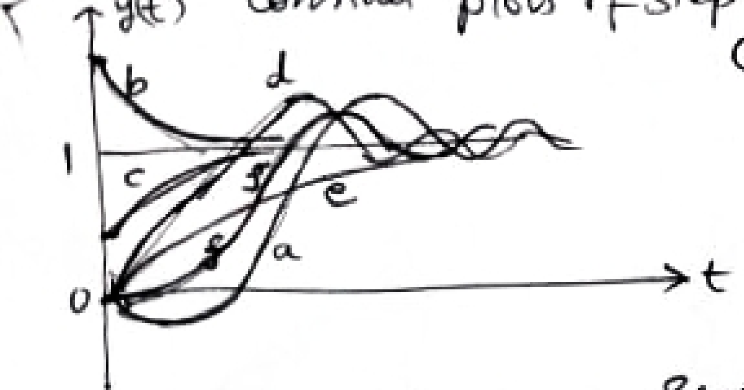
$G_2(s)$ having $\%OS$ & T_S as specified

$G_3(s)$ having $\%OS$ & T_p as specified.

$G_4(s)$ having T_p & T_S as specified.

(G_1, G_2, G_3 & G_4 should have no zero & DC-gain $= 1$).

Q-2 Consider plots of step responses 6 of them.
(2 of them are marked "f".)



Consider transfer functions: Seven of them.

$$\frac{+s+8}{s^2+2s+8}, \quad \frac{+s+8}{s^2+30s+8}, \quad \frac{3s+2}{2s+2}, \quad \frac{s+2}{2s+2}, \quad \frac{-s+8}{s^2+2s+8}$$

$$\frac{8}{s^2+2s+8}, \quad \frac{+s+8}{s-8}$$

(i) For each of the 6 plots, associate a transfer fn & give a brief reason.

(ii). For each of 7 transfer fns, find initial value, final value, initial rise rate: (find for all 7 even if not associated to a plot.)

$s-8+16$

Q-3: For analyzing the unit step response of $G(s) = \omega_n^2 / (s^2 + 2\zeta\omega_n s + \omega_n^2)$, $0 < \zeta < 1$

(a) plot in $\mathbb{C}$ (the s-plane), the curves corresponding to constant %OS (say C_1) and curve corresponding to constant T_s (settling time) say C_2.

Give direction to C_1 & C_2: going farther from origin

(b). Along C_1 & C_2, specify how the following change: increase or decrease or remain same: give brief reason.

T_p, T_s, T_r ← rise time (from 10% to 90%).

↑ peak time, ↑ settling (2%) time

Q-4: Give an RLC circuit ($R>0, L>0, C>0$) such that

(a) underdamped $\Leftrightarrow$ Resistance R is very small.

(b) ~~or~~ overdamped $\Leftrightarrow$ Resistance R is very large

(c) What should be input & output there? (introduce current/voltage source appropriately & suggest output variable.) Do this for each circuit (a) & (b).

(answer for (c) could be different for a & b)

Q-5: Consider $G(s) = \dfrac{1}{s^2 + 2\zeta s + 1}$, $0 < \zeta < 1$, & study the step response.

(a) under what condition can effect of an additional pole be neglected? Prove this.

(b) Under what condition can effect of additional zero be neglected? Prove this.

(Note: Effect of zero & additional pole is to be analyzed individually & not together. Effect on step response.)

Note: Read all questions fully first and ask queries within 10 minutes. NO clarification after that: assume appropriately yourself. Each question carries 5 marks. Attempt all questions

1] (a) $G_1 \Rightarrow$ Not possible as all the three conditions won't satisfy at the same time.

(2)

(b) 10% OS , $T_s = 20$

$$\exp\left(\frac{-\pi \xi_1}{\sqrt{1-\xi_1^2}}\right) = 0.1$$

$$\xi_1 = 0.5911$$

for 2% tolerence , $T_s = 20 = 4\tau$

$$\frac{4}{\omega_{n_1}\xi_1} = 20 \Rightarrow \omega_{n_1} = 0.3383$$

(1)

$$G_2 = \frac{\omega_{n_1}^2}{s^2 + 2\xi_1\omega_{n_1}s + \omega_{n_1}^2}$$

10% OS , $T_p = 3$ sec

$$\xi_2 = 0.5911$$

$$T_p = \frac{\pi}{\omega_{n_2}\sqrt{1-\xi_2^2}} = 3$$

$$\omega_{n_2} = 1.298$$

(1)

$$G_3 = \frac{\omega_{n_2}^2}{s^2 + 2\xi_2\omega_{n_2}s + \omega_{n_2}^2}$$

$T_p = 3 \text{ sec}$, $T_s = 20 \text{ sec}$

$$\frac{4}{\xi_3 \omega_{n3}} = 20 \rightarrow \omega_{n3} = \frac{1}{5 \xi_3}$$

$$T_p = \frac{\pi}{\omega_{n3}\sqrt{1-\xi_3^2}} = 3$$

$$\frac{5\pi \xi_3}{\sqrt{1-\xi_3^2}} = 3$$

1

$$\xi_3 = 0.1875$$

$$\omega_{n3} = 1.0661$$

$$G_4 = \frac{\omega_{n3}^2}{s^2 + 2\xi_3 \omega_{n3} s + \omega_{n3}^2}$$

2.(a)

① $\frac{8}{s^2+2s+8} \rightarrow \underline{\underline{f}} \rightarrow$ - system underdamped
- Simple 2nd Order system

② $\frac{s+8}{s^2+2s+8} \rightarrow \underline{\underline{d}} \rightarrow$ - system underdamped
- addition of zero with +ve coeff.
- rise time decreases

③ $\frac{-s+8}{s^2+2s+8} \rightarrow \underline{\underline{a}} \rightarrow$ - system underdamped
- addition of zero with -ve coeff.
- rise time increases initially goes below zero & comes up

④ $\frac{s+8}{s^2+30s+8} \rightarrow \underline{\underline{e}} \rightarrow$ - 2nd order system
- overdamped system (no peak)

⑤ $\frac{3s+2}{2s+2} \rightarrow \underline{\underline{b}} \rightarrow$ - 1st order system
- unit step + decaying exponential
$u(t) + \frac{1}{2}e^{-t}$

⑥ $\frac{s+2}{2s+2} \rightarrow \underline{\underline{c}} \rightarrow$ - 1st order system
- unit step - decaying exponential
$u(t) - \frac{1}{2}e^{-t}$

⑦ $\frac{s+8}{s-8} \rightarrow$ NA $\rightarrow$ - 1st order
- unstable system

a) $\frac{s+8}{s^2+2s+8}$ b) $\frac{s+8}{s^2+10s+8}$ c) $\frac{3s+9}{2s+2}$ d) $\frac{s+2}{2s+2}$ e) $\frac{-s+8}{s^2+2s+8}$

f) $\frac{8}{s^2+2s+8}$ g) $\frac{s+8}{s-8}$

IVT

$\lim_{s\to\infty} sF(s)$.

$= \lim_{s\to\infty} \frac{1}{s} \cdot s \cdot F(s)$.

a) 0

b) 0

c) $3/2$

d) $1/2$

e) 0

f) $= 0$

g) 1

FVT

$\lim_{s\to 0} F(s)$

a) 1

b) 1

c) 1

d) 1

e) 1

f) 1

g) NA

IRR.

$\lim_{s\to\infty} sF(s)$

a) 1

b) 1

c) $\frac{3}{2} + \frac{-1}{(2s+2)}$

$L^{-1}\left(\frac{1}{2s+2}\right)\Big|_{t=0} = \frac{-1}{2}$

d) $\frac{1}{2} + L^{-1}\left(\frac{1}{2(s+1)}\right)\Big|_{t=0} = \frac{1}{2}$

e) -1

f) 0

g) 16.

Mark

Q.2 (a)

Sub-parts	Marks
6	2
4	1.5
2	0.5
1	0.5
0	0

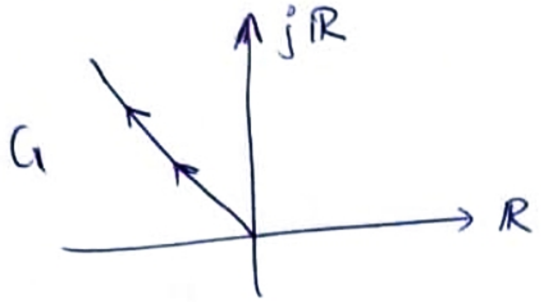
(b)

Subparts	Marks
7	3
4	2
2	1
1	0.5
0	0

if all subparts are correct

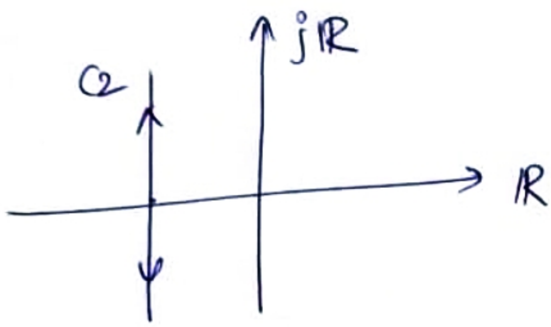
Q3) $G(s) = \dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$ $\quad 0 < \xi < 1$

a) constant % overshoot curve (C_1)



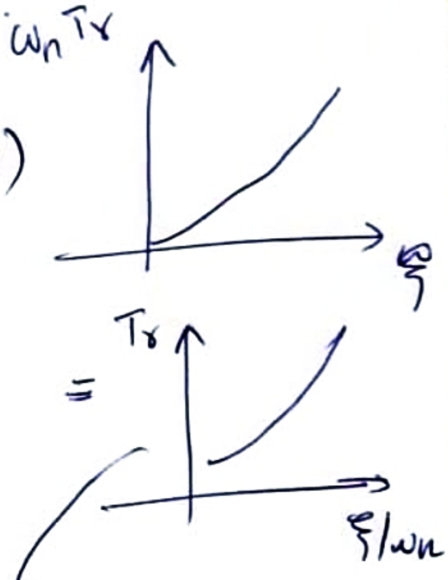
1 mark

constant T_s curve (C_2)



Real part of root is constant.

1 mark



b) Along c_2 ⇒ T_p decreases (since $\omega_d \uparrow$)

% OS increases

1.5 mark T_r decreases

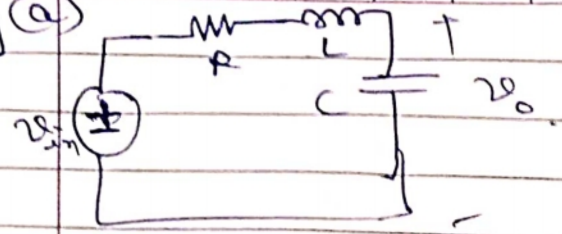
Along c_1 :⇒ T_p decreases

T_s decreases

1.5 mark T_r decreases

ξ decreases, ω_n increases ⇒ T_r decreases

Q04 (a)



$$\frac{v_o}{v_{in}} = \frac{1/sC}{1/sC + R + sL}$$

$$\frac{v_o}{v_{in}} = \frac{1}{s^2LC + RCs + 1}$$

$$\frac{v_o}{v_{in}} = \frac{1/LC}{s^2 + \frac{R}{L}s + 1/LC}$$

$$\omega_n = \frac{1}{\sqrt{LC}}, \quad 2\xi\omega_n = \frac{R}{L}$$

$$\xi = \frac{R\sqrt{LC}}{2L} \Rightarrow$$

$$\boxed{\xi = \frac{R}{2}\sqrt{\frac{C}{L}}}$$

taking $C = 10^{-8} F$, ~~R=10~~ $L = 10^{-6} H$

$R = 10\,\Omega$.

$$\xi = \frac{10}{2}\sqrt{\frac{10^{-8}}{10^{-6}}} \Rightarrow = 0.5$$

$\Rightarrow$ Underdamped.

(b) Same circuit with large R value can be an overdamped system.

Let take $R = 10K$.

$C = 10^{-8} F$, $L = 10^{-6} H$

$$\xi = \frac{10 \times 10^3}{2} 10^{-1}$$

$$\xi = 500$$

$\hookrightarrow$ hence overdamped for large R.

(c) In both the above cases (a) & (b) input is taken voltage source & output is ~~taken~~ voltage across capacitor (V_0).

Marking pattern

a) 1 marks → Circuit with transfer fn.
1 marks → choosing R, L, C s.t. it is ~~overdamped~~ Underdamped & R is small.

b) 1 marks → ckt with transfer fn.
1 marks → choosing R, L, C, s.t. it is overdamped & R is Large.

c) ~~1 marks~~ 0.5 mark for part (a) i/p & o/p
0.5 mark for part (b) i/p & o/p.

Q5

(a) $$G(s) = \frac{1}{s^2 + 2\zeta s + 1}$$

Transfer function after addition of pole and taking the step response will look like:→

$$C(s) = \frac{a}{s(s^2+2\zeta s+1)(s+a)}$$ ← to handle steady state value

taking partial fraction of C(s)

$$C(s) = \frac{A}{s} + \frac{Bs - C}{s^2+2\zeta s+1} + \frac{D}{(s+a)}$$

here we assumed that the nondominant pole a is located at $-c$ on the real axis

After calculation for A, B, C and D, we get:→

$$A = 1$$

$$B = \frac{2\zeta a - a^2}{a^2 - 2\zeta a + 1}$$

$$C = \frac{4\zeta^2 a - 2\zeta a^2 - a}{a^2 - 2\zeta a + 1}$$

$$D = \frac{-1}{a^2 - 2\zeta a + 1}$$

now as the nondominant pole $a \to \infty$;

$$\lim_{a\to\infty} D = \lim_{a\to\infty} \frac{-1}{a^2-2\zeta a+1} = 0$$

$$\therefore D = 0$$

$$\lim_{a\to\infty} C = \lim_{a\to\infty} \frac{4\zeta^2 a - 2\zeta a^2 - a}{a^2-2\zeta a+1} = -2\zeta$$

$$\therefore C = -2\zeta$$

$$\lim_{a\to\infty} B = \lim_{a\to\infty} \frac{2\zeta a - a^2}{a^2-2\zeta a+1} = -1$$

$$\therefore B = -1$$

$$\lim_{a\to\infty} A = \lim_{a\to\infty} (1) = 1 \qquad \therefore A = 1$$

Thus we observed the residue (i.e. D) of the nondominant pole and its response becomes zero as the nondominant pole approaches infinity.

(b) $$G(s) = \frac{1}{s^2+2\xi s+1}$$

adding a pole to $G(s)$ "such that steady state gain is not disturbed":→

But here two cases may happen zero can lie on left half plane or on right half plane too.

case 1:→ on left half plane

$$G(s) = \frac{(s/a+1)}{s^2+2\xi s+1}$$

step response:→

$$C(s) = \frac{(s/a+1)}{s(s^2+2\xi s+1)}$$

$$C(s) = \frac{1}{s(s^2+2\xi s+1)}\left(s/a\right) + \frac{1}{s(s^2+2\xi s+1)}$$

if $a \to \infty$ then

$$C(s) = \cancelto{0}{\frac{1}{s(s^2+2\zeta s+1)}\left(\frac{s}{a}\right)} + \frac{1}{s(s^2+2\zeta s+1)}$$

$$\therefore C(s) = \frac{1}{s(s^2+2\zeta s+1)}$$

<u>case 2:-</u> when zero added on right half plane

$$C(s) = \frac{(1 - s/a)}{s(s^2+2\zeta s+1)}$$

$$C(s) = \frac{1}{s(s^2+2\zeta s+1)} - \left(\frac{s}{a}\right)\left(\frac{1}{s(s^2+2\zeta s+1)}\right)$$

if $a \to \infty$

$$C(s) = \frac{1}{s(s^2+2\zeta s+1)} - \cancelto{0}{\left(\frac{s}{a}\right)\frac{1}{s(s^2+2\zeta s+1)}}$$

$$\therefore C(s) = \frac{1}{s(s^2+2\zeta s+1)}$$

method 2:→

For additional zero to be neglected

take zero as:→

$$s = -\frac{1}{\epsilon}$$

$$TF = \frac{\epsilon s + 1}{s^2 + 2\xi s + 1}$$

now proceed further.