Filter Design Assignment

EE338: Digital Signal Processing

Mid-Semester Take-Home Component

Sravan K Suresh

(22B3936)

Filter number: **63**

Reviewed by:

Neel Bhavesh Rambhia (22B1298)

Swarup Dasharath Patil (22B3953)

Under the guidance of:

Prof. Vikram M Gadre

Indian Institute of Technology Bombay

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1 Introduction

In digital signal processing (DSP), filter design is essential for isolating or removing certain frequency components from a signal. Filters are used in many fields, such as audio processing, telecommunications, and biomedical signal analysis. Designing a filter involves taking theoretical specifications and turning them into practical, working solutions. The challenge is to achieve the desired frequency response while sticking to requirements like passband and stopband tolerances, transition bandwidths, and filter types (e.g., IIR or FIR).

The process of designing the filter includes several important steps: first, deriving both the un-normalized and normalized specifications, then using the bilinear transformation to convert the analog filter specifications into a digital filter. Finally, we will check if the filter works as expected by analyzing its frequency response. Since we're restricted from using automatic design tools in MATLAB, we'll take a hands-on, step-by-step approach to the design process, which will help us better understand how IIR filters are built.
The report is organized as follows:
Section 2 discusses the un-normalized and normalized filter specifications.
Section 3 walks through the design steps, including the creation of the analog filter's transfer function and its conversion to a digital filter.
Section 4 shows how we verify the filter by analyzing its frequency response.

2 Specifications

The design problem involves the creation of a multi-bandpass Infinite Impulse Response (IIR) filter to process an analog signal bandlimited to 280 kHz, sampled at a rate of 630 kHz. The filter is designed to extract specific frequency bands while suppressing others, adhering to strict tolerances on the passband and stopband magnitudes. The following subsections outline the un-normalized and normalized filter specifications.

2.1 Un-Normalized Discrete Time Filter Specifications

The un-normalized discrete-time filter specifications are as follows:

- **Sampling Rate:** The analog signal is sampled at a rate of **630 kHz**.
- **Passband Tolerances:** The magnitude response in the passband must lie **between 1 and 0.85** (for IIR filters).
- **Stopband Tolerances:** The magnitude response in the stopband **must not exceed 0.15**.
- **Transition Bands:** For bandpass filters, the transition bands are **5 kHz** on either side of each passband.
- **Passband Frequencies:**
  - Group I: **65 kHz to 95 kHz**

- Group II: **210 kHz to 240 kHz**

This is because the Filter number assigned to me is $\mathbf{M = 63}$, thus $\mathbf{Q = 5}$ and $\mathbf{R = 8}$.

- **Stopband Frequencies:** All frequencies outside the specified passbands are considered stopbands.
- **Filter Type:** The passbands and stopbands are **monotonic** (non-oscillatory).
  - The Analog Transfer Function is defined by the two center frequencies and bandwidths as follows:
    - $\Omega_{c_1} = 1.0595$ and $B_1 = 0.1767$ for the first band.
    - $\Omega_{c_2} = 1.0627$ and $B_2 = 0.8159$ for the second band.

2.2 Normalized Digital Filter Specifications

The normalized digital filter specifications are derived by scaling the frequencies with respect to the Nyquist frequency, which is half the sampling rate (315 kHz). The normalized frequencies are calculated as follows:

$$f_{\text{normalized}} = \frac{f_{\text{actual}}}{f_{\text{Nyquist}}} = \frac{f_{\text{actual}}}{315\,\text{kHz}}$$

- **Normalized Passband Frequencies:**
  - Group I:
$$\frac{65\,\text{kHz} \times 2\pi}{630\,\text{kHz}} \approx 0.6483 \quad \text{to} \quad \frac{95\,\text{kHz} \times 2\pi}{630\,\text{kHz}} \approx 0.9475$$
  - Group II:
$$\frac{210\,\text{kHz} \times 2\pi}{630\,\text{kHz}} \approx 2.0944 \quad \text{to} \quad \frac{240\,\text{kHz} \times 2\pi}{630\,\text{kHz}} \approx 2.3936$$
- **Normalized Transition Bands:**
$$\frac{5\,\text{kHz} \times 2\pi}{630\,\text{kHz}} \approx 0.0159$$
- **Normalized Stopband Frequencies:** All frequencies outside the normalized passbands are considered stopbands.

These normalized specifications are used as the basis for the design of the analog and digital filters, ensuring that the filter meets the desired frequency response characteristics.

3 Design Process

The IIR multi-bandpass filter was designed using a Butterworth analog prototype filter with the bilinear transformation method. I have followed the protocol outlined in the problem statement, avoiding the use of MATLAB's complete filter design commands. The key steps are detailed below.

3.1 Analog Filter Specifications

The design began with the following critical specifications:

- Sampling frequency: $F_s = 630\,\text{kHz}$
- Passbands: Group I (65-95 kHz) and Group II (210-240 kHz)
- Transition bandwidth: 5 kHz on either side of passbands
- Passband tolerance: $\delta_1 = 0.15$ (magnitude between 1 and 0.85)
- Stopband tolerance: $\delta_2 = 0.15$ (magnitude ≤ 0.15)

3.2 Frequency Normalization

Frequencies were normalized to the Nyquist frequency (315 kHz):

$$\omega = \frac{2\pi f}{F_s}$$

Resulting in:

- Group I: $\omega_{p1} = [0.6483, 0.9475]$ rad/sample (MATLAB: `omega_p1`)
- Group II: $\omega_{p2} = [2.0944, 2.3936]$ rad/sample (MATLAB: `omega_p2`)

3.3 Bilinear Transformation and Analog Prototype

The bilinear transformation ($s = \frac{1-z^{-1}}{1+z^{-1}}$) was applied to map digital specifications to the analog domain. Critical parameters were calculated as:

- Transition band edges: $f_{stop_1} = [60, 100]\,\text{kHz}$, $f_{stop_2} = [205, 245]\,\text{kHz}$ (MATLAB: `f_stop1`, `f_stop2`)
- Analog frequency transformation:

$$\Omega = \tan\left(\frac{\omega}{2}\right) \quad \text{(MATLAB: Omega\_p1, Omega\_p2)}$$

- Bandwidths (B) and center frequencies (Ω_0):

$$B_1 = 0.1767, \quad \Omega_{0_1} = 1.0595 \quad \text{(Group I)} \quad \text{(MATLAB: B\_1, Omega0\_1)}$$

$$B_2 = 0.8159, \quad \Omega_{0_2} = 1.0627 \quad \text{(Group II)} \quad \text{(MATLAB: B\_2, Omega0\_2)}$$

3.4 Lowpass Prototype Design

The bandpass-to-lowpass transformation was implemented using:

$$\Omega_L \to \frac{\Omega^2 - \Omega_0^2}{B \cdot \Omega} \quad \text{(MATLAB: } \texttt{LP\_Omega1}, \texttt{LP\_Omega2})$$

and the lowpass-to-bandpass transformation was implemented using:

$$s_L \to \frac{s^2 + \Omega_0^2}{B \cdot s} \quad \text{(MATLAB: } \texttt{analog\_bpf1}, \texttt{analog\_bpf2})$$

where s_L is the prototype lowpass filter variable. The filter order N was determined by solving:

$$N \geq \left\lceil \frac{\log(D_2/D_1)}{2\log(\Omega_s/\Omega_p)} \right\rceil \quad \text{(MATLAB: } \texttt{N1}, \texttt{N2})$$

with $D_1 = 0.3841$ and $D_2 = 43.4444$. Both bands required $N = 10$.

3.5 Pole-Zero Plots

The pole-zero plots for the designed filters are shown in Figures 1 and 2. All the poles plotted here are the ones lying in the LHCP and the rest are discarded.

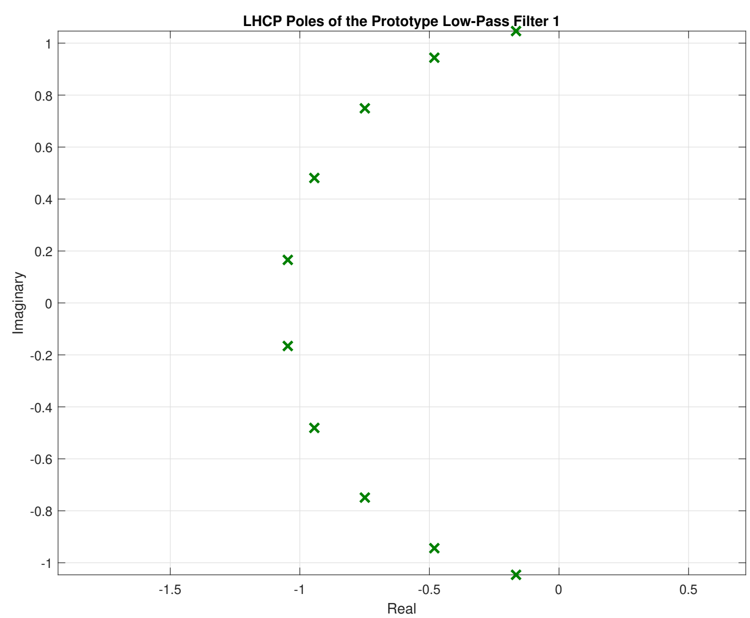


Figure 1: Pole-Zero Plot of the First Bandpass Filter

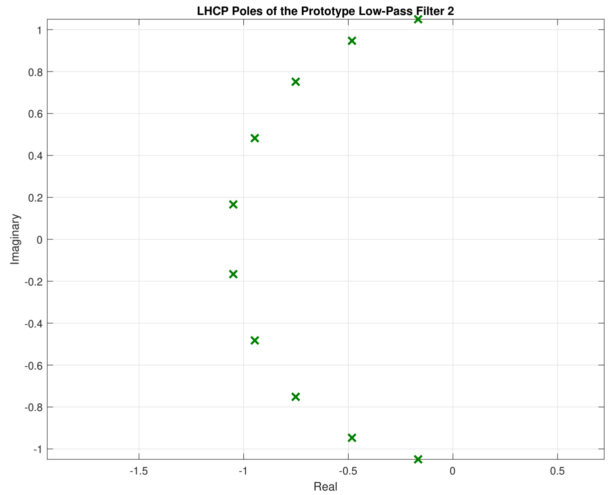


Figure 2: Pole-Zero Plot of the Second Bandpass Filter

3.6 Pole Locations and Transfer Functions

The 10th-order Butterworth prototype poles were calculated as:

$$s_{L_k} = \Omega_c \cdot e^{j\pi(2k+N-1)/2N}, \quad k = 1, 2, \ldots, 2N$$

Left-half-plane poles were selected for stability (Table 1). The analog transfer function for Group I was derived as:

$$H_{\text{Analog,LPF}}(s_L) = \frac{1}{(s_L/\Omega_c)^{10} + \cdots + 1}$$

where $\Omega_c = 1.0595$ (Group I) and $\Omega_c = 1.0627$ (Group II).

Table 1: Selected Left-Half-Plane Poles (Group I and Group II)

Group I		Group II	
Pole #	**Value (rad/s)**	**Pole #**	**Value (rad/s)**
1	$-0.1657 + 1.0464i$	1	$-0.1662 + 1.0496i$
2	$-0.4810 + 0.9440i$	2	$-0.4824 + 0.9468i$
3	$-0.7491 + 0.7491i$	3	$-0.7514 + 0.7514i$
4	$-0.9440 + 0.4810i$	4	$-0.9468 + 0.4824i$
5	$-1.0464 + 0.1657i$	5	$-1.0496 + 0.1662i$
6	$-1.0464 - 0.1657i$	6	$-1.0496 - 0.1662i$
7	$-0.9440 - 0.4810i$	7	$-0.9468 - 0.4824i$
8	$-0.7491 - 0.7491i$	8	$-0.7514 - 0.7514i$
9	$-0.4810 - 0.9440i$	9	$-0.4824 - 0.9468i$
10	$-0.1657 - 1.0464i$	10	$-0.1662 - 1.0496i$

3.7 Digital Filter Realization

The bilinear transformation was applied to both bandpass filters:

$$H_{\text{Digital}}(z) = H_{\text{Analog}}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right) \quad \text{(MATLAB: \texttt{digital\_bpf1}, \texttt{digital\_bpf2})}$$

Resulting in 20th-order IIR filters for each band. The final combined transfer function was obtained by parallel combination:

$$H_{\text{Total}}(z) = H_1(z) + H_2(z) \quad \text{(MATLAB: \texttt{num\_discrete\_total}, \texttt{den\_discrete\_total})}$$

3.8 Filter Coefficients

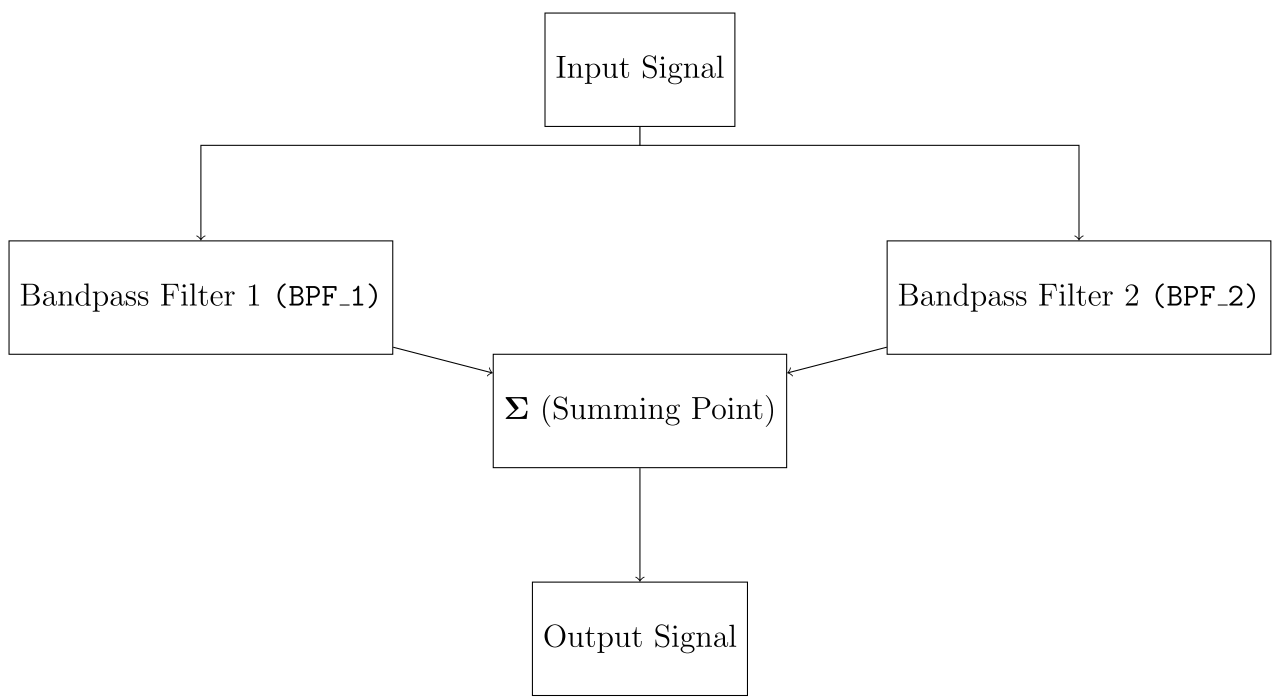
The normalized coefficients after bilinear transformation are shown below (first 5 terms shown for brevity). Full coefficient tables are provided in Appendix A.

- **Group I Numerator:** $b_1 = [2.98 \times 10^{-8}, 0, -2.98 \times 10^{-7}, ..., 2.98 \times 10^{-8}]$
- **Group I Denominator:** $a_1 = [1, -15.55, 120.98, ..., 7.66]$
- **Group II Numerator:** $b_2 = [3.08 \times 10^{-8}, 0, -3.08 \times 10^{-7}, ..., 3.08 \times 10^{-8}]$
- **Group II Denominator:** $a_2 = [1, 13.89, 98.95, ..., 7.70]$

3.9 Configuration of Combined Filter for Multi-bandpass

The magnitude functions of the filters add up, resulting in the overall transfer function:

$$H = H_1 + H_2$$



3.10 Final Transfer Function

The final combined transfer function after bilinear transformation is given by:

$$H(z) = \frac{N(z)}{D(z)}$$

where:

$$N(z) = 6.068242817 \times 10^{-8} z^0 - 6.525058231 \times 10^{-8} z^1 + \cdots + 4.660706921 \times 10^{-7} z^{40}$$
$$D(z) = 1z^0 - 1.66102557z^1 + 3.908601779z^2 - \cdots + 58.99535491z^{40}$$

The full numerator and denominator coefficients are provided in Appendix A.

4 Verification

The designed IIR multi-bandpass filter was verified by analyzing its frequency response and performance across the specified passbands and stopbands. The magnitude and phase responses were computed using MATLAB's `freqz` function, and the results are presented below.

4.1 Magnitude Response Analysis

The magnitude response of the filter was evaluated to ensure compliance with the design specifications. The following plots illustrate the filter's performance:

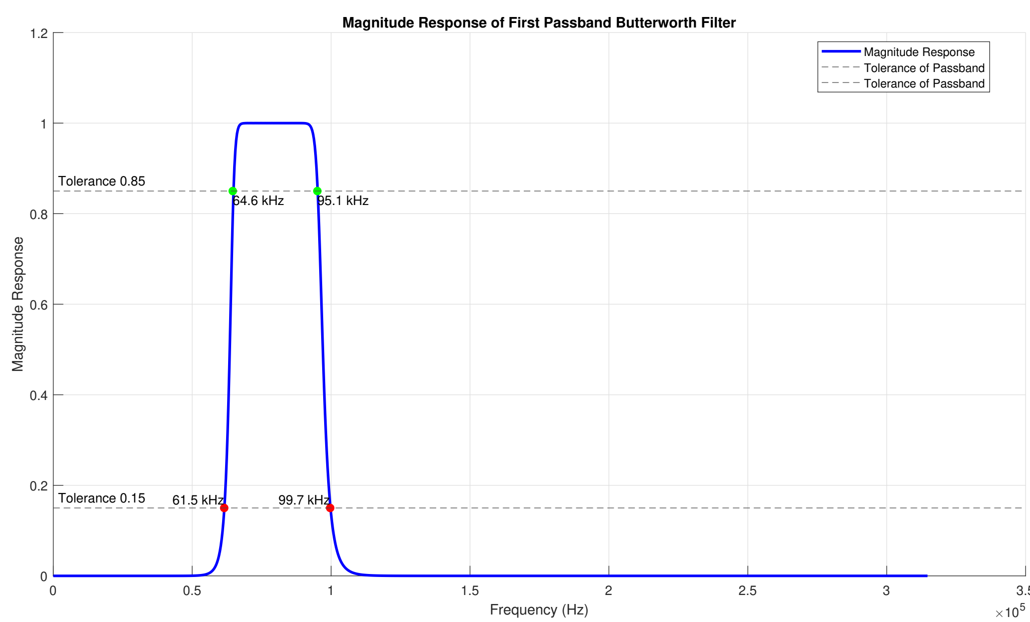


Figure 3: Magnitude Response in the First Passband (65-95 kHz)

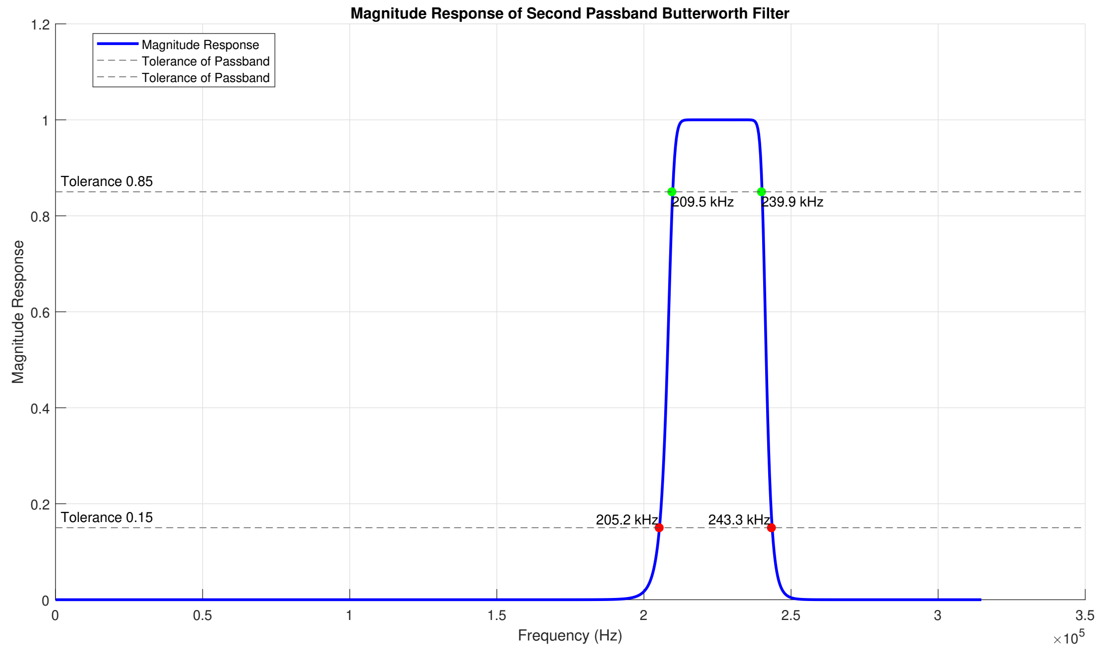


Figure 4: Magnitude Response in the Second Passband (210-240 kHz)

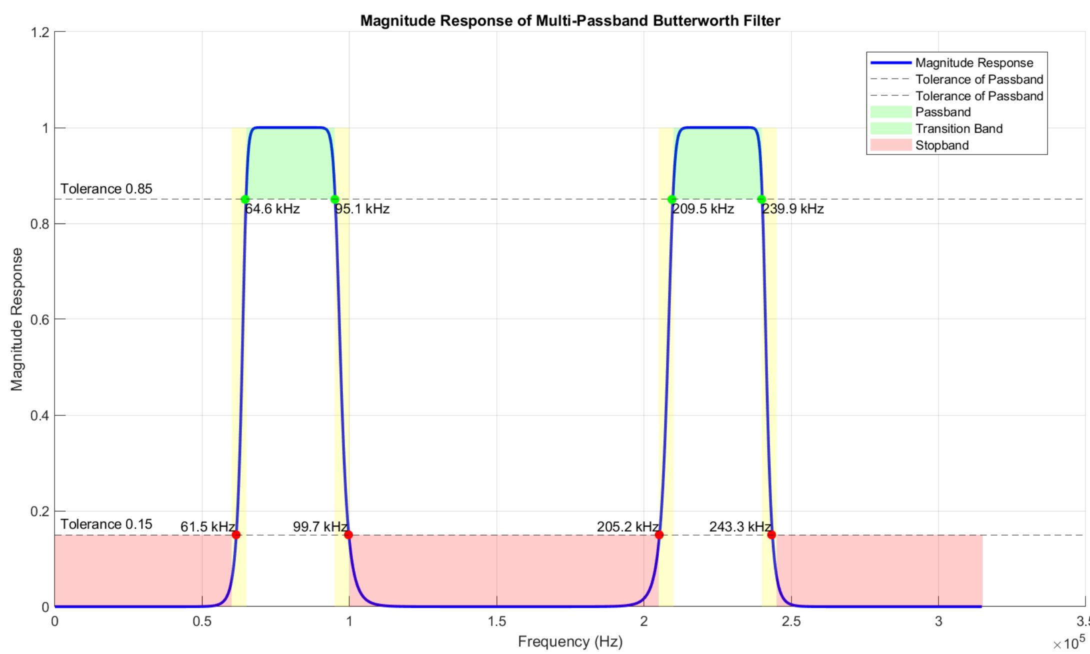


Figure 5: Magnitude Response Across Both Passbands

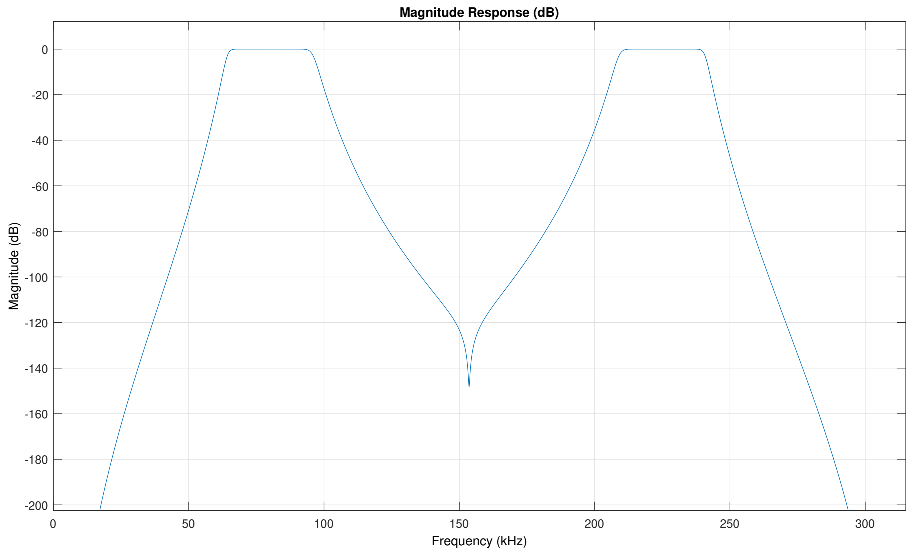


Figure 6: Magnitude Response in Decibels (dB) Across the Entire Frequency Range

4.1.1 Observations

- **First Passband (65-95 kHz):** As shown in Figure 3, the filter exhibits a flat magnitude response within the passband, with the magnitude lying between 1 and 0.85, as specified. The transition bands are sharp, with a width of approximately 5 kHz on either side.
- **Second Passband (210-240 kHz):** Figure 4 demonstrates that the filter meets the design requirements in the second passband, with the magnitude response adhering to the specified tolerances.
- **Multi-Passband Response:** Figure 5 shows the combined response of both passbands, confirming that the filter successfully isolates the desired frequency bands while suppressing others.
- **Decibel Response:** The dB-scale plot in Figure 6 provides a detailed view of the stopband attenuation, which is consistently below $-15\,\text{dB}$ (corresponding to a magnitude of 0.15), as required.

4.2 Conclusion

The verification process confirms that the designed IIR multi-bandpass filter meets all specified requirements, including:

- Passband magnitude response between 1 and 0.85.
- Stopband magnitude response ≤ 0.15.
- Transition bands of 5 kHz on either side of the passbands.

The filter's performance is further validated by the sharp transition bands and consistent stopband attenuation, as demonstrated in the provided plots.

A Filter Coefficients

A.1 Group I Filter Coefficients

Table 2: Numerator Coefficients for Group I (b_1)

Index	Value
0	2.98×10^{-8}
1	0
2	-2.98×10^{-7}
3	0
4	1.34×10^{-6}
5	0
6	-3.58×10^{-6}
7	0
8	6.27×10^{-6}
9	0
10	-7.52×10^{-6}
11	0
12	6.27×10^{-6}
13	0
14	-3.58×10^{-6}
15	0
16	1.34×10^{-6}
17	0
18	-2.98×10^{-7}
19	0
20	2.98×10^{-8}

Table 3: Denominator Coefficients for Group I (a_1)

Index	Value
0	1
1	-15.55
2	120.98
3	-621.48
4	2354.25
5	-6969.41
6	16693.92
7	-33080.34
8	55016.54
9	-77495.57
10	92920.25
11	-94994.34
12	82668.44
13	-60932.71
14	37694.99
15	-19291.98
16	7989.02
17	-2585.37
18	616.92
19	-97.20
20	7.66

A.2 Group II Filter Coefficients

Table 4: Numerator Coefficients for Group II (b_2)

Index	Value
0	3.08×10^{-8}
1	0
2	-3.08×10^{-7}
3	0
4	1.39×10^{-6}
5	0
6	-3.70×10^{-6}
7	0
8	6.48×10^{-6}
9	0
10	-7.77×10^{-6}
11	0
12	6.48×10^{-6}
13	0
14	-3.70×10^{-6}
15	0
16	1.39×10^{-6}
17	0
18	-3.08×10^{-7}
19	0
20	3.08×10^{-8}

Table 5: Denominator Coefficients for Group II (a_2)

Index	Value
0	1
1	13.89
2	98.95
3	473.39
4	1692.46
5	4781.93
6	11040.79
7	21278.92
8	34709.96
9	48338.63
10	57752.05
11	59285.40
12	52211.79
13	39259.02
14	24985.31
15	13274.07
16	5763.07
17	1977.45
18	507.03
19	87.30
20	7.70

A.3 Combined Filter Coefficients

Table 6: Numerator Coefficients for Combined Filter (b_{total})

Index	Value
0	$6.068242817 \times 10^{-8}$
1	$-6.525058231 \times 10^{-8}$
2	$6.07722918 \times 10^{-6}$
3	$-4.392875586 \times 10^{-6}$
4	$5.900591018 \times 10^{-5}$
5	$-2.478180075 \times 10^{-5}$
6	$-9.330021624 \times 10^{-5}$
7	0.0001182886844
8	−0.0009600459407
9	0.000245061049
10	0.001873287981
11	−0.001394238209
12	0.004043935107
13	0.0005724666391
14	−0.01288279119
15	0.004615549298
16	0.001603111649
17	−0.007088444285
18	0.02675404862
19	−0.001213499497
20	−0.02636311212
21	0.01083163788
22	−0.01050287711
23	−0.007590505362
24	0.0294802205
25	−0.002397850954
26	−0.01048841538
27	0.00516766831
28	−0.007945243792
29	−0.001577422897
30	0.006231013877
31	−0.0006286596819
32	−0.0001483011354
33	0.0003956516615
34	−0.0007944140288
35	$-9.257189089 \times 10^{-6}$
36	$9.777652713 \times 10^{-5}$
37	$-1.681202684 \times 10^{-5}$
38	$2.949675744 \times 10^{-5}$
39	$-3.934959167 \times 10^{-7}$
40	$4.660706921 \times 10^{-7}$

Table 7: Denominator Coefficients for Combined Filter (a_{total})

Index	Value
0	1
1	−1.66102557
2	3.908601779
3	−6.513055302
4	22.96089481
5	−33.03317886
6	61.6401329
7	−90.43718869
8	201.4867773
9	−257.9880092
10	409.6970326
11	−538.4943231
12	946.2231555
13	−1089.392548
14	1523.046544
15	−1809.636508
16	2704.549752
17	−2803.99755
18	3513.079398
19	−3785.637772
20	4979.003706
21	−4620.871963
22	5234.583234
23	−5106.289999
24	6009.097708
25	−4914.537877
26	5048.130133
27	−4423.982037
28	4689.513873
29	−3270.162673
30	3038.581055
31	−2354.370017
32	2245.672713
33	−1239.975721
34	1033.640543
35	−686.5463473
36	583.1044503
37	−204.7162194
38	150.5569062
39	−80.38865036
40	58.99535491