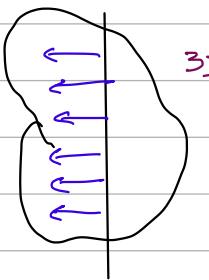


22/01/2025

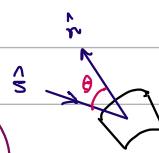
Just a heads up: We are still on Lambertian surface only!



3D

Assumption:  $\hat{s}$  known

$$E(x, y) = \text{const.} \cdot \cos\theta \\ = \hat{n}(x, y) \cdot \hat{s}$$



$$\hat{s} = \begin{pmatrix} 1, 0, p_s \\ 0, 1, q_s \end{pmatrix}$$

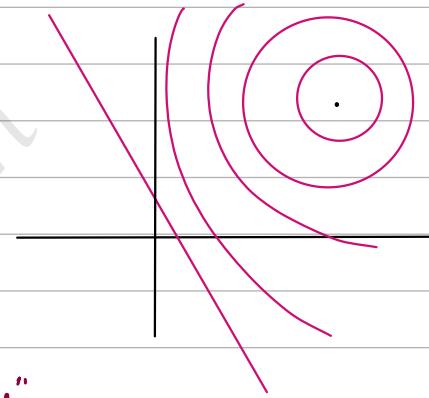
$$= \left( \frac{-p_s, -q_s, 1}{\sqrt{1 + p_s^2 + q_s^2}} \right)$$

Assume

$$c = 1$$

$$c = \frac{(-p, -q, 1)}{\sqrt{p^2 + q^2 + 1}} \cdot \frac{(-p_s, -q_s, 1)}{\sqrt{p_s^2 + q_s^2 + 1}}$$

$$\Rightarrow c^2 (1 + p_s^2 + q_s^2) (1 + p^2 + q^2) = (1 + pp_s + qq_s)^2$$

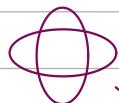


~~Some inherent, obvious constraints~~

"Photogrammetry"

$\hat{z}$  = viewing direction

$$\hat{s}_1, \hat{s}_2$$



→ MAX = 4  
min = 0

$$\hat{s}_1, \hat{s}_2, \hat{s}_3$$

$$f(\hat{n}, \hat{s}_1) = c_1$$

$$f(\hat{n}, \hat{s}_2) = c_2$$

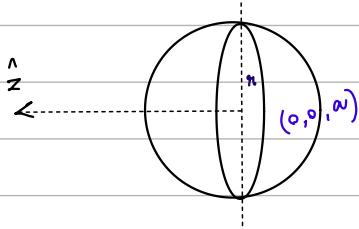
$$f(\hat{n}, \hat{s}_3) = c_3$$

$f(x, y) \rightarrow$  Albedo

$$\underbrace{\begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix}}_S \cdot \underbrace{\begin{bmatrix} f(\hat{n}) \\ c_1 \\ c_2 \\ c_3 \end{bmatrix}}_C = \underbrace{\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}}_C$$

"Photometric stereo"

$$\Rightarrow f(\hat{n}) = S^{-1} C = \frac{1}{[s_1 \ s_2 \ s_3]} \left[ c_1 (\hat{s}_2 \times \hat{s}_3) + c_2 (\hat{s}_3 \times \hat{s}_1) + c_3 (\hat{s}_1 \times \hat{s}_2) \right]$$



$$\hat{s} = (0, 0, 1)$$

$$(z-a)^2 + x^2 + y^2 = r^2$$

$$z = \sqrt{r^2 - x^2 - y^2} + a$$

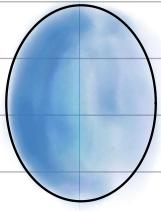
Now,

$$p = \frac{\partial z}{\partial x} = \frac{-x}{z-a}$$

$$q = \frac{\partial z}{\partial y} = \frac{-y}{z-a}$$

$$E(x, y) = \hat{n} \cdot \hat{s} = \frac{1}{\sqrt{1+p^2+q^2}}$$

$$= \frac{z-a}{r} = \frac{1}{2} \sqrt{\frac{x^2+y^2}{r^2}} = \sqrt{1 - \frac{x^2+y^2}{r^2}}$$



"Shaded image"

Sravan K Suresh