

Quiz 3

Name: SRAVAN K SURESH

Roll number: 22B3936

Write your name and roll number above. Write complete solutions (not just the final answer) in the space provided after each question. You are permitted to refer to your notes while taking the quiz.

1. [5 marks] Obtain the Fourier Transform of the signal $x(t) = te^{-|t|}$.

Your answer:

$$\text{Consider } x_1(t) = e^{-at} \xrightarrow{\text{FT}} X_1(\omega) = \frac{1}{(a+j\omega)}$$

∴ from Fourier property,

$$\mathcal{F}(x(t) * y(t)) \xrightarrow{\quad} X(\omega) Y(\omega)$$

$$\text{let } x(t) = y(t) = x_1(t)$$

$$\therefore X(\omega) = Y(\omega) = \frac{1}{a+j\omega}$$

$$x_1(t) * x_1(t)$$

$$\text{and } X(\omega) Y(\omega)$$

$$= e^{-at} u(t) * e^{-at} u(t)$$

$$= \frac{1}{a+j\omega} \cdot \frac{1}{a+j\omega}$$

$$= te^{-|t|}$$

$$= \underline{\underline{\frac{1}{(a+j\omega)^2}}}$$

$$\mathcal{F}(te^{-|t|}) = \underline{\underline{\frac{1}{(a+j\omega)^2}}}$$

2. [5 marks] Obtain the Fourier Transform of the following signal.

$$x(t) = \frac{t}{(1+t^2)^2}$$

Your answer:

$$\therefore e^{-at} \xleftrightarrow{\text{FT}} \frac{2a}{a^2+\omega^2} \quad (\text{FT pair})$$

$$a=1 \Rightarrow e^{-t} \xrightarrow{\text{FT}} \frac{2}{1+\omega^2}$$

Now, exploiting duality: $g(t) \xrightarrow{\text{FT}} G(\omega)$

$$\text{then } G(t) \xrightarrow{\text{FT}} 2\pi \cdot g(\omega)$$

$$\therefore \frac{2}{1+t^2} \xrightarrow{\text{FT}} 2\pi \cdot e^{-|w|}$$

Now, property of differentiation: $\frac{d}{dt} x(t) \xrightarrow{\text{FT}} (j\omega) X(j\omega)$

$$\therefore 2 \times \frac{(-2)t}{(1+t^2)^2} \xrightarrow{\text{FT}} (j\omega) \cdot 2\pi \cdot e^{-|w|}$$

Using Linearity,

$$\Rightarrow \mathcal{F}\left(\frac{t}{(1+t^2)^2}\right) = \frac{(j\omega) 2\pi e^{-|w|}}{(-4)}$$

$$= \boxed{-\frac{\pi}{2} j\omega e^{-|w|}}$$