



Applied Linear Algebra

(Course Code: EE 635)

Department of Electrical Engineering
Indian Institute of Technology Bombay

Instructor:
Dwaipayan
Mukherjee
Date: 30th
August,
2023

Time: 1.25 Hours

Quiz-1

Total Points: 40

Instructions

Sravan K Suresh
22B3936

- Standard symbols and notations have their usual meanings.
- Answer all questions. You may attempt the questions in any order.
- If some information is missing, make suitable assumptions and state them.

1. Suppose S is a set of matrices of the form $\begin{bmatrix} a & a \\ a & a \end{bmatrix}$, where $a \in \mathbb{Q}$ (set of rational numbers).

It is known that an integral domain has the property that *product of two non-zero elements is non-zero*.

- Does the aforesaid property hold for S ? Justify your answer. *yes*
- Determine the multiplicative identity of S , if it exists. *I^{2x2}*
- Does every element in S have a multiplicative inverse? Obtain the same for those elements that may have such an inverse. *No, RREF + I^{2x2}*

[2+1.5+1.5]

2. Provide an example of each of the following:

- ★ (a) A non-empty subset of \mathbb{R}^2 , say $V \subseteq \mathbb{R}^2$, which is closed under vector addition and under taking additive inverse, but is not a subspace of \mathbb{R}^2 . *$\mathbb{Z} \setminus \{0,0\}$*
- (b) A vector space that has exactly 81 elements. *$M = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix} \right), (a, b, c, d \in \mathbb{Z}_3)$*
- (c) A non-empty subset of \mathbb{R}^2 , say $U \subseteq \mathbb{R}^2$, which is closed under scalar multiplication, but is not a subspace of \mathbb{R}^2 . *$U = \{(a, b) : a, b \geq 0, a, b \in \mathbb{R}\}$*

[2 × 3]

3. Suppose $X = \mathbb{Z}^2 := \{(x, y) : x, y \in \mathbb{Z}\}$. Define addition, \oplus , and multiplication, \otimes in the following manner:

$$(x, y) \oplus (a, b) = (x + a, y + b), \text{ and}$$

$$(x, y) \otimes (a, b) = (ax + 2by, bx + ay).$$

- (a) Assert through proper justification whether (X, \oplus, \otimes) is an integral domain. Hence, obtain the '0' and '1' of this integral domain (or even if it is not an integral domain).
- (b) Solve for an element $p = (p_x, p_y) \in X$ such that $p^2 = 2$ (if such a p exists). [Hint: For the '1' obtained in (a), define $n := \underbrace{1 \oplus 1 \oplus \dots \oplus 1}_{n \text{ times}}$

[4+5]

4. Prove or disprove the following assertion: For a matrix $A \in \mathbb{R}^{n \times n}$ it follows that $\ker(A) \cap \text{im}(A) = 0_n$.

**Sravan K Suresh
22B3936**

[6]

5. Obtain all solutions of $x^2 - 10x + 16 = 0$ over \mathbb{Z}_2 , and \mathbb{Z}_8 . [4]
6. (Traffic flow) Consider the traffic flow depicted in the map shown in Fig. 1 over a

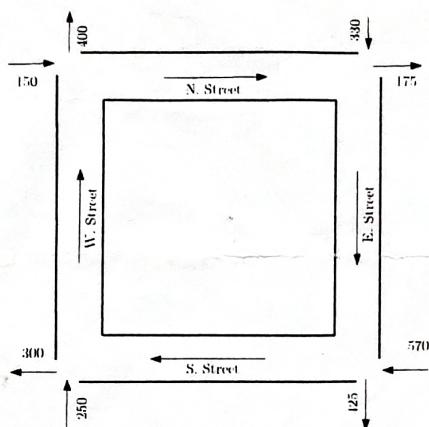


Figure 1: Traffic flow over a 1-hour period

one-hour open window while there is no flow of traffic during the remainder of the day. It is known that the streets are one-way, there are no residences in the map, and the entire area is a 'No Parking' zone.

- (a) Does there exist a solution to the problem of determining the number of vehicles passing through each of the four main roads? Justify your answer mathematically. If it is solvable, is the solution unique? In any case outline possible solution(s).
- (b) Suppose the number of vehicles entering the north-east junction were increased by 50 and the number of vehicles leaving through the south-west junction increased by 30. Can you solve for the problem in (a)? Justify.
- (c) For the problem in (a), does a solution exist if the total number of vehicles through W. Street was 200 during the one-hour period? Justify.

[5+3+2]