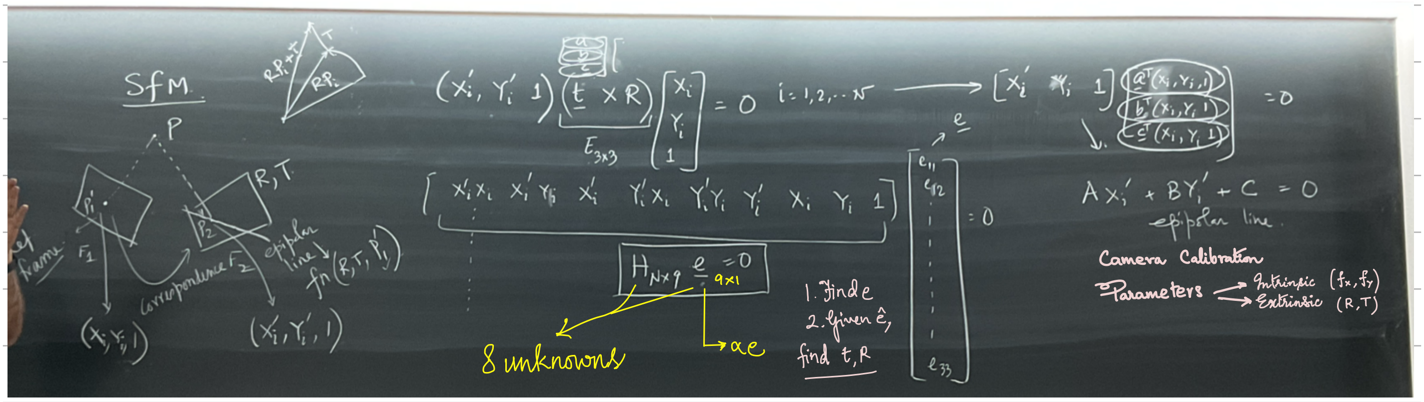
EE702: Lec-12 (19 Feb)



- Calibrated Cameras – Essential Matrix
- Un-Calibrated Cameras – Fundamental matrix
- There are five parameters in the essential matrix—three for rotation and two for the direction of translation (scale is not set)—along with two other constraints. The two additional constraints on the essential matrix are: (1) the determinant is 0 because it is rank-deficient (a 3-by-3 matrix of rank 2); and (2) its two nonzero singular values are equal because the matrix $\hat{T}$ is skewsymmetric and R is a rotation matrix.
- The fundamental matrix F is just like the essential matrix E, except that F operates in image pixel coordinates whereas E operates in physical coordinates. The fundamental matrix F is of rank 2. The fundamental matrix has seven parameters, two for each epipole and three for the homography that relates the two image planes (the scale aspect is missing from the usual four parameters).

(Credits: Robotic Vision Lab, Brigham Young University)

1 $\boldsymbol{x}_m'^{\top} \mathbf{F} \boldsymbol{x}_m = 0$

$$\begin{bmatrix} x_m' & y_m' & 1 \end{bmatrix} \begin{bmatrix} f_1 & f_2 & f_3 \\ f_4 & f_5 & f_6 \\ f_7 & f_8 & f_9 \end{bmatrix} \begin{bmatrix} x_m \\ y_m \\ 1 \end{bmatrix} = 0$$

2

$$x_m x_m' f_1 + x_m y_m' f_2 + x_m f_3 + y_m x_m' f_4 + y_m y_m' f_5 + y_m f_6 + x_m' f_7 + y_m' f_8 + f_9 = 0$$

3 $\boldsymbol{Af} = \mathbf{0}$ minimize $||Af||^2$ subject to $||f||^2 = 1$

$$\begin{bmatrix} x_1x_1' & x_1y_1' & x_1 & y_1x_1' & y_1y_1' & y_1 & x_1' & y_1' & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_Mx_M' & x_My_M' & x_M & y_Mx_M' & y_My_M' & y_M & x_M' & y_M' & 1 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \\ f_8 \\ f_9 \end{bmatrix} = \mathbf{0}$$

Problem: Given a matrix F, find the matrix F' of rank k that is closest to F,

$$\min_{\substack{F' \\ \text{rank}(F') = k}} \|F - F'\|^2$$

Solution: Compute the singular value decomposition of F,

$$F = U\Sigma V^T$$

Form a matrix Σ' by replacing all but the k largest singular values in Σ with 0.

Then the problem solution is the matrix F' formed as,

$$F' = U\Sigma' V^T$$

Given eight or more point correspondences, $\mathbf{x}$ can be determined as follows. Each point correspondence yields one epipolar equation (Eq. 1). With N ($N \geq 8$) point correspondences, we can stack them together into a vector equation as follows:

$$\mathbf{A}^T\mathbf{x} = \mathbf{0}, \tag{5}$$

with $\mathbf{A} = [\mathbf{a}_1, \ldots, \mathbf{a}_N]$. The solution to $\mathbf{x}$ can be obtained by minimizing the least squares, subject to $\|\mathbf{x}\| = 1$. With Lagrange multiplier, this is equivalent to minimizing

$$\|\mathbf{A}^T\mathbf{x}\|^2 + \lambda\left(1 - \|\mathbf{x}\|^2\right). \tag{6}$$

With simple algebra, it can be found that the solution to $\mathbf{x}$ is the eigenvector of the 9×9 matrix $\mathbf{AA}^T$ associated with the smallest eigenvalue.

When the image coordinates (u_i, v_i) are in pixels, the elements of matrix $\mathbf{A}$ have orders of difference in value, and matrix $\mathbf{AA}^T$ may not be well conditioned. One remedy is to pre-normalize the image points, and several solutions are examined in [1]. One simplest approach is to perform a scaling and translation such that all image coordinates are within $[-1, 1]$. Compared with using directly the pixel coordinates, significant improvement in accuracy has been observed.

The matrix $\mathbf{M}$ estimated above is obtained by ignoring its property. For example, the estimated $\mathbf{M}$ is usually not rank-2. To obtain the closest rank-2 matrix, "closest" in terms of Fronobius norm, we perform a singular value decomposition on $\mathbf{M}$, i.e.,

$$\mathbf{M} = \mathbf{USV}, \tag{7}$$

where $\mathbf{S} = \text{diag}(s_1, s_2, s_3)$ with $s_1 \geq s_2 \geq s_3 \geq 0$. Replacing the smallest singular value by zero, i.e., $\hat{\mathbf{S}} = \text{diag}(s_1, s_2, 0)$, then

$$\mathbf{U\hat{S}V} \equiv \hat{\mathbf{M}} \tag{8}$$

is the optimal rank-2 matrix.

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8-point algorithm

$$\underbrace{t \times R}_{\text{5 indep. var}} = \underbrace{\begin{bmatrix} 0 & t_x & -t_z \\ -t_x & 0 & t_y \\ t_z & -t_y & 0 \end{bmatrix}}_{\cancel{3}\ 2 \text{ d.o.f}} \underbrace{\begin{bmatrix} R \end{bmatrix}}_{3 \text{ d.o.f}}$$

$He = 0$ → Eigenvector corresponding to the smallest e. value

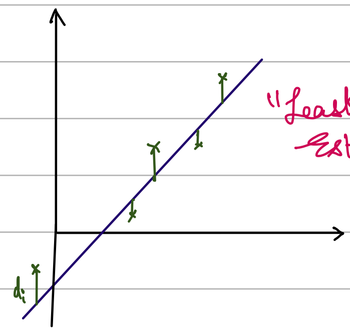
Orthonormal

$|e| = 1$

$$\xrightarrow{SVD} = U \Sigma V^T e$$

diag.

$H_8 e'_{8\times 1} = -\underline{1} = \underline{y}$ $\Rightarrow$ $\boxed{y = H\theta + \eta}$ → Perturbation



$\min\limits_{\substack{\theta \\ (m,c)}} \Sigma d_i^2$

"Least squares Estimate"

$y_i = m x_i + c$

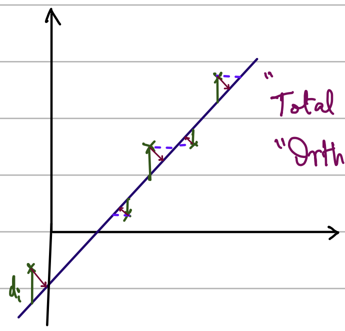
$$= \underbrace{\begin{bmatrix} x_i & 1 \end{bmatrix}}_{H} \underbrace{\begin{bmatrix} m \\ c \end{bmatrix}}_{\theta} + \eta$$

$$\min_{\theta} \| (y - H\theta)^T (y - H\theta) \|$$

$$\Rightarrow H^T (y - H\theta) = 0 \qquad \boxed{\hat{\theta} = (H^T H)^{-1} H^T y}$$

"Weighted least squares"

$$\min \|y - H\theta\|^2 + \lambda |\theta|^2 \xrightarrow{soln} (H^T H + \lambda I^{-1}) H^T y$$



$\min\limits_{\substack{\theta \\ (m,c)}} \Sigma d_i^2$

"Total Least sq" $\Rightarrow$ $y + \delta y = (H + \delta H)\theta$

"Orthogonal LS"

$$\Rightarrow \min |\eta|^2 + \|\delta H\|_F^2 \longrightarrow \begin{bmatrix} y + \delta y & \vdots & H + \delta H \end{bmatrix} \begin{bmatrix} I \\ \theta \end{bmatrix} = 0$$

$H_8 e_8 = -1$

$He = 0$

$y = H\theta + \eta$ → $P_\eta \sim \mathcal{N}(0, \sigma_n^2)$

$$\Rightarrow p(y - H\theta) \sim \mathcal{N}(0, \sigma_n^2)$$

$$\max_{\theta} \underline{p(\eta | \theta)} = \max_{\theta} \prod_i \frac{1}{\sqrt{2\pi\sigma_n^2}} \exp\left(\frac{-\eta_i^2}{2\sigma^2}\right)$$

max likelyhood of $\theta \ni \eta$ is like that

$p(\eta_1|\theta) \cdot p(\eta_2|\theta) \cdots$

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