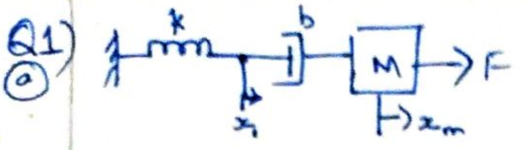
Q1) (a)



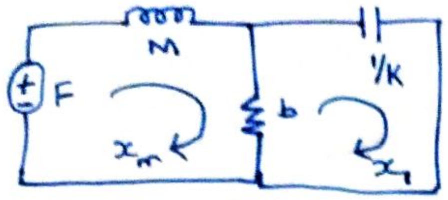
(i) $F \to V$

$x \to I$

$\because$ 2 displacement $\Rightarrow$ 2 currents

(ie) 2 loops

1 F $\Rightarrow$ one voltage source



Using Mesh analysis & taking L.T we get

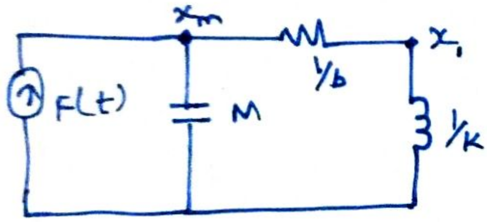
$F(s) = (Ms+b)X_m(s) - bX_1(s)$ —(1)

$0 = -bX_m(s) + \left(b + \frac{k}{s}\right)X_1(s)$ —(2) $\Rightarrow X_1(s) = bX_m(s)\left[\frac{s}{sb+k}\right]$

$\Rightarrow \frac{F(s)}{X_m(s)} = Ms + b - \frac{b^2 s}{k+bs}$ $\Rightarrow \frac{X_m(s)}{F(s)} = \frac{k+bs}{Mbs^2 + MKs + bk} = \frac{\frac{1}{m}\left(s+\frac{k}{b}\right)}{s^2 + \frac{k}{b}s + \frac{k}{m}}$

(ii) $F \to I$; one F $\Rightarrow$ one current source

$x \to V$; 2 disp $\Rightarrow$ 2 voltages (or) 2 diff nodes.



Using nodal analysis & taking L.T we get

$F(s) = X_m(s)[Ms+b] - bX_1(s)$

$0 = -bX_m(s) + X_1(s)\left[b + \frac{k}{s}\right]$

Same as (1) & (2) $\therefore$ yields same transfer function.

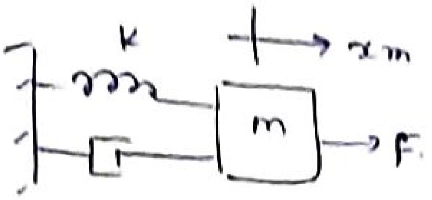
(b) Given K = 5 , M = 3

From the T.F of $\frac{X_m(s)}{F(s)}$ we have

$\omega_n = \sqrt{\frac{k}{M}} = \sqrt{\frac{5}{3}}$

$2\zeta\omega_n = \frac{5}{b} \Rightarrow \zeta = \frac{\sqrt{15}}{2b} > 0$ (when $b > 0$)

$\therefore$ For underdamped $0 < \zeta < 1 \Rightarrow b > \frac{\sqrt{15}}{2}$

2) 

given $k=5$, $m=3$;

Mechanical translational equation

$$\boxed{M\frac{d^2x_m(t)}{dt^2}+b\frac{dx(t)}{dt}+kx(t)=f(t)=F}$$

Taking Laplace transform. [assuming zero initial conditions.

$$Ms^2X(s)+bsX(s)+kX(s)=F(s)$$

$$G(s)=\frac{X(s)}{F(s)}=\frac{1}{Ms^2+bs+k}$$

$$s_{1,2}=\frac{-b}{2m}\pm\frac{\sqrt{b^2-4km}}{2m}$$

b). For system to be underdamped:

$$b^2-4km<0.$$

$$b^2<4km$$

$$b^2<4\times5\times3.$$

$$b<\sqrt{60}$$

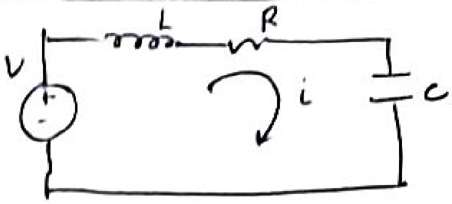
$$\boxed{b<7.7}$$

or

$Md(\ldots)/+$

Electrical System.

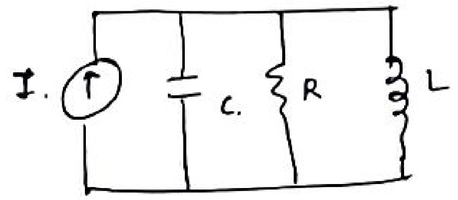
(I) Force voltage analogy.



$$V=L\frac{di}{dt}+iR+\frac{1}{C}\int i\,dt.$$

$$V=L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{C}$$

(II) Force - current analogy.



$$I=C\frac{dv}{dt}+\frac{V}{R}+\frac{1}{L}\int v\,dt$$

$$I=C\frac{d^2\phi}{dt^2}+\frac{1}{R}\frac{d\phi}{dt}+\frac{\phi}{L}.$$

∴ The Analogy from the given systems.

Electrical	Mechanical: Force voltage	Mechanical: Force current
(V)	Voltage	Force, f.
(I)	Force f.	velocity v.
Resistance R.	1/B	b.
Capacitance c	M	1/K.
L.	1/K	m.