Tutorial 5

Monday, 18 March 2024 4:28 PM

1. a) $(s+1)(s+2)(s+3) + k = 0$

$\therefore s^3 + 6s^2 + 11s + 6 + k = 0$

$$\begin{array}{llll} s^3 & 1 & 11 \\ s^2 & 6 & 6+k \\ s & 60-k/6 & 0 \\ 1 & 6+k & 0 \end{array}$$

$\therefore \frac{60-k}{6} > 0 \;;\; 6+k > 0$

$\therefore k > -6 \;;\; k < 60$

$\therefore k \in (-6, 60)$

b) $s^2 + 3s + 2 + ks - k = 0$

$$\begin{array}{lll} s^2 & 1 & 2-k \\ s & k+3 & 0 \\ 1 & 2-k & \end{array}$$

$\therefore k+3 > 0 \;;\; 2-k > 0$

$\therefore k \in (-3, 2)$

c) $\left(s^2 + \frac{s}{2}\right)(s+1)(s+20) + k(s+3)(s+7) = 0$

$\therefore s^4 + \frac{43s^3}{2} + \left(\frac{61}{2} + k\right)s^2 + 10(1+k)s + 21k = 0$

$$\begin{array}{llll} s^4 & 1 & \frac{61}{2}+k & 21k \\ s^3 & 43/2 & 10(1+k) & 0 \\ s^2 & \frac{2583+46k}{86} & 21k & 0 \\ s & * & 0 & 0 \\ 1 & 21k & 0 & 0 \end{array}$$

$* \; 460k^2 - 12539k + 25830 > 0$

$\therefore k \in (0, 2.245) \cup (25.014, \infty)$

d) $\frac{s+1}{(s+2)(s+3)} \Rightarrow s^2 + 5s + 6 + ks + k$

$$\begin{array}{lll} s^2 & 1 & 6+k \\ s & k+5 & 0 \\ 1 & 6+k & 0 \end{array}$$

$\therefore k > -5$

e) $\frac{(s+2)}{s(s+1)(s+p)} \Rightarrow s^3 + (p+1)s^2 + ps + ks + 2k$

$$\begin{array}{lll} s^3 & 1 & p+k \\ s^2 & p+1 & 2k \\ s & x & 0 \\ 1 & 2k & \end{array}$$

$x = \frac{1}{p+1}\left(p^2 + p(k+1) - k\right)$

$\therefore p^2 + p(k+1) - k > 0$

$\therefore k(p-1) + p^2 + p > 0$

$\therefore k > \frac{-p^2 - p}{(p-1)}$ $\quad \therefore k > 0$

f) $s^4 + 5s^2 + 4 + k$

$$\begin{array}{llll} s^4 & 1 & 5 & 4+k \\ s^3 & 4 & 10 & 0 \\ s^2 & 10/4 & 4+k & 0 \\ s & 9-4k & 0 & 0 \\ 1 & 4+k & 0 & 0 \end{array}$$

$\therefore k \in \left(-4, \frac{9}{4}\right)$

2. $s = z - 1$

a) $\frac{z+3}{(z-1)(z+1)}$ $\quad \therefore z^2 - 1 + kz + 3k = 0$

$$\begin{array}{lll} z^2 & 1 & 3k-1 \\ z & k & 0 \\ 1 & 3k-1 & \end{array}$$

$\therefore k > 0 \;,\; 3k-1 > 0$

$\therefore k \in \left(\frac{1}{3}, \infty\right)$

b) $\frac{1}{(z-1)z(z+1)}$ $\quad \therefore z^3 - z + k = 0$

$$\begin{array}{llll} z^3 & 1 & -1 & \\ z^2 & \varepsilon & k & \\ z & -(k+\varepsilon)/\varepsilon & 0 & \\ 1 & k & & \end{array}$$

$\varepsilon = +$

$+$

$+$

$\therefore -(k+\varepsilon) > 0 \;,\; k > 0$

$\therefore k + \varepsilon < 0$

$\therefore k < -\varepsilon \;,\; k > 0$

$\therefore$ No such k exists

3. i) $s^5 - 7s^4 + 16s^3 - 16s^2 + 15s - 9$

$$\begin{array}{llll} s^5 & 1 & 16 & 15 \\ s^4 & -7 & -16 & -9 \\ s^3 & 96/7 & 96/7 & 0 \\ s^2 & -9 & -9 & 0 \\ \hline s & 2 & 0 & 0 \\ 1 & -9 & 0 & 0 \end{array} \qquad \begin{array}{lll} 1 & 1 & 0 \\ -1 & -1 & 0 \end{array}$$

$\therefore -s^2 - 1 = 0$

$\therefore 2s$

	$-s^2-1$	Other
OLHP	1	0
$j\mathbb{R}$	0	0
ORHP	1	3

ii) $s^6 + s^5 + 2s^4 + 2s^3 + 9s^2 + 9s = s\left(s^5 + s^4 + 2s^3 + 2s^2 + 9s + 9\right)$

$$\begin{array}{llll} s^5 & 1 & 2 & 9 \\ s^4 & 1 & 2 & 9 \\ \hline s^3 & \cancel{4}\,1 & \cancel{4}\,1 & 0 \\ s^2 & 1 & 9 & 0 \\ s & -8 & 0 & 0 \\ 1 & 9 & 0 & 0 \end{array}$$

$\therefore s^4 + 2s^2 + 9$

$\therefore 4s^3 + 4s$

	s^4+2s^2+9	Other
OLHP	2	1
$j\mathbb{R}$	0	1
ORHP	2	0

iii) $2s^3 - 24s + 32$ $\qquad 32s^3 - 24s^2 + 2$

$$\begin{array}{lll} s^3 & 32 & 0 \\ s^2 & -24 & 2 \\ s & 8/3 & 0 \\ 1 & 2 & 0 \end{array}$$

2 ORHP , 1 OLHP

iv) 2 ORHP , 2 OLHP

4. $4s^2 + cs + (2-c) = 0$

$\therefore 2 - c > 0 \;,\; -c < 0$ $\quad \therefore c > 0,\; c < 2$

$\therefore c \in (0, 2)$

5. a) $y = CGe$ $\quad \therefore e = r - y = r - CGe$

$\therefore r = e(1 + CG)$

$\therefore \frac{r}{y} = \frac{1+CG}{CG}$ $\quad \therefore \frac{y}{r} = \frac{CG}{1+CG} = \frac{n_c n_p}{d_c d_p + n_c n_p}$

$e = Gy$ $\quad \therefore e = r - Cy$ $\quad \therefore Gy = r - Cy$

$\therefore \frac{y}{r} = \frac{1}{G+C} = \frac{d_c d_p}{n_c d_p + n_p d_c}$

b) Zeros $\Rightarrow n_c n_p \;,\; d_c d_p$

c) $\frac{k n_p}{d_p + k n_p} \;,\; \frac{d_p}{k d_p + n_p}$ $\longrightarrow$ Yes it will also happen here

d) No, this will not be happening

e) Yes

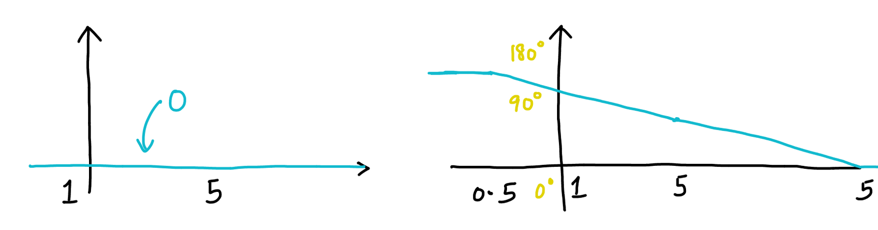
6. a) $R_2 - \frac{j}{\omega C}$ $\;;\;$ $R_2 + \left(\frac{\omega^2 LC - 1}{\omega L}\right) j$

Let the plant region have impedance $= Z(s)$

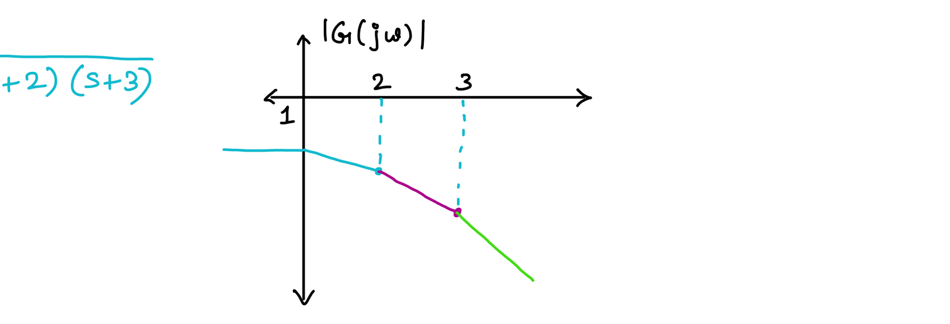
Forward path $\Rightarrow \frac{R_1 Z}{1 + R_1 Z}$ $\;;\;$ Back path $\Rightarrow \frac{Z}{1 + R_1 Z}$

7.

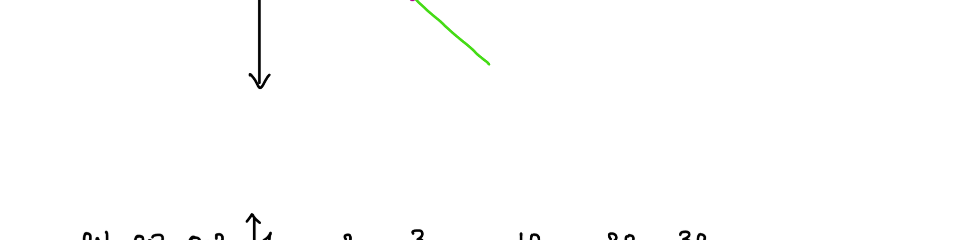
a) $s + 2$



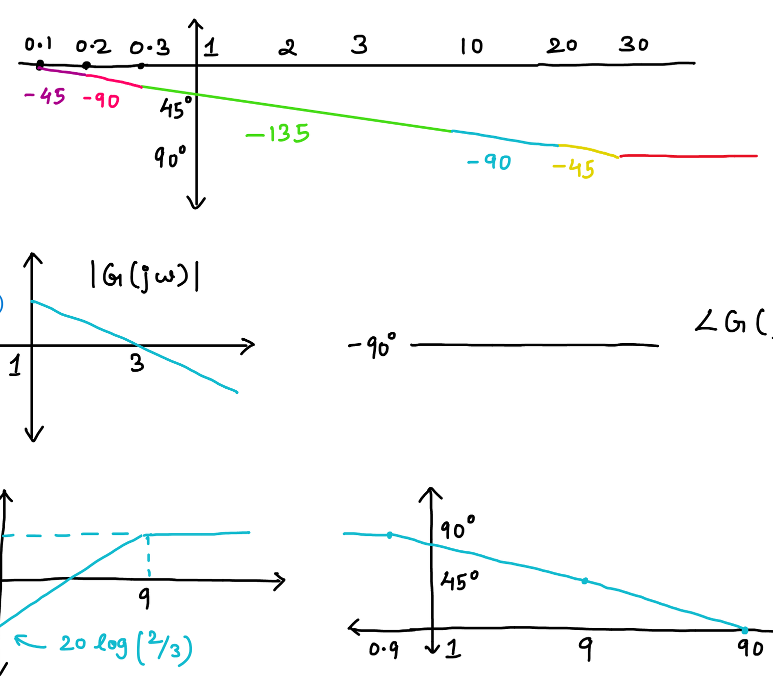
c) $\frac{s+5}{s-5}$



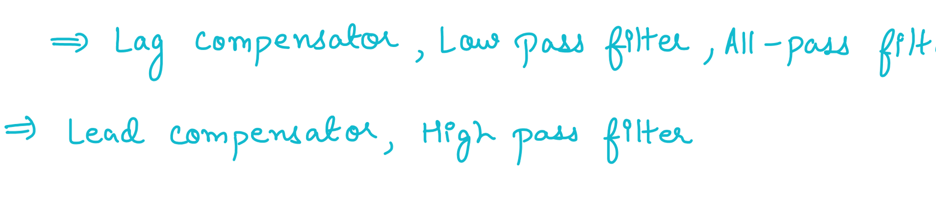
d) $\frac{s-5}{s+5}$



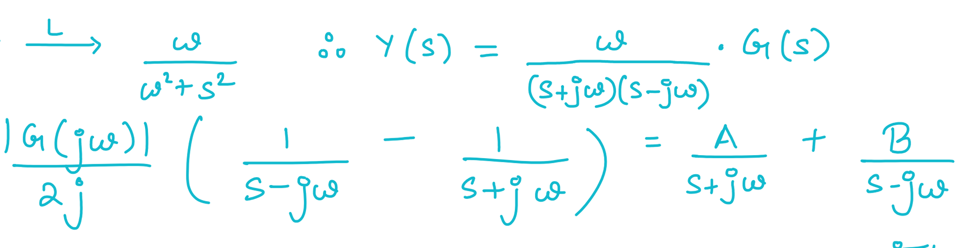
e) $\frac{1}{(s+1)(s+2)(s+3)}$



f) $\frac{3}{s}$



g) $\frac{6s}{s+9}$



8. a) $\frac{s + 0.1}{s + 0.05}$ $\Rightarrow$ Lag compensator, Low pass filter, All-pass filter

b) $\frac{s+8}{s+20}$ $\Rightarrow$ Lead compensator, High pass filter

9. $\sin \omega t \xrightarrow{L} \frac{\omega}{\omega^2 + s^2}$ $\quad \therefore Y(s) = \frac{\omega}{(s+j\omega)(s-j\omega)} \cdot G(s)$

$\therefore \frac{|G(j\omega)|}{2j}\left(\frac{1}{s - j\omega} - \frac{1}{s + j\omega}\right) = \frac{A}{s+j\omega} + \frac{B}{s-j\omega}$

$\Rightarrow A = -\frac{|G(j\omega)|}{2j} = \frac{|G(j\omega)|\,j}{2} = \frac{|G(j\omega)|\,e^{-j\pi/2}}{2}$

$\Rightarrow B = \frac{|G(j\omega)|}{2j} = -\frac{|G(j\omega)|\,j}{2} = \frac{|G(j\omega)|\,e^{j\pi/2}}{2}$

$\therefore \frac{|G(j\omega)|}{2}\left(e^{-j(\omega t + \pi/2)} + e^{j(\omega t + \pi/2)}\right)$

$\Rightarrow |G(j\omega)| \cos(\omega t + \pi/2) = -|G(j\omega)| \sin \omega t$

10. $I(s) \Rightarrow \frac{\omega}{s^2 + \omega^2}\, Z(s)$

$\therefore y(t) = |Z(j\omega)| \sin(\omega t + \angle Z(j\omega))$