EE302-S2, Control Systems, Endsem (26th Apr 2024)

Notes & instructions:

- Attempt all five questions: each question carries 10 marks. Unnecessarily long, convolved, redundant or irrelevant text could attract marks-reduction.
- Unless otherwise explicitly specified, k is real and positive.
- If a graph-paper based question is asked, then use graph-paper judiciously after a rough sketch on your own answer-sheet (within 'rough-work', which will **not be evaluated**). Using judiciously will avoid carefully resketching (and save your own time) and also graph-paper.
- Tie the graph paper **within** the answer-paper at the appropriate page. Write roll-number on the graph paper top.
- Some questions might not have the sought answer. In such a case, give reasons why the sought answer is not possible.
- If you feel a question has ambiguity and/or needs clarification, then assume yourself appropriately, state and justify your assumption and then proceed to solve the problem with that assumption.

  Do not call any TA or instructor for your query.

Ques 1: Consider the standard negative unity feedback configuration with $G(s) = \frac{1}{(s+1)(s+2)(s+2)}$.
(a) On a graph-paper, sketch the root-locus for $k > 0$ and **estimate**, using your graph-paper's sketch,

(i) the range of $k > 0$ that results in closed loop stability, and

(ii) the frequency ω_c at the highest value of k yielding closed loop stability.

(Use root-locus asymptotes to estimate: this is adequate accuracy.)

(b) On (preferably same) graph-paper, sketch Bode gain/phase plots (asymptotic sketch only) to obtain **approximate** range of $k > 0$ that results in closed loop stability and frequency ω_c as in (a)(ii).

(c) Sketch Nyquist plot and use Nyquist criteria for obtaining exact range of $k > 0$ that results in closed loop stability. (The sketch for (c) is on your **plain** answer sheet, and not on graph paper.)

Ques 2: Consider the transfer function $G(s) = \frac{2s^2 + 11s + 14}{s^2 + 5s + 6}$.

(a) Obtain a controllable state space realization, prove its controllability.

(b) Obtain an observable state space realization, prove its observability.

(c) Obtain a state space realization which is both controllable and observable.

(In each of the cases above (with possibly different matrices A), each matrix A is required to be $2\times\mathbf{2}$.)

Ques 3: Answer **any two out of the three** below about a state-space system $\frac{d}{dt}x = Ax + Bu$, and $y = Cx + Du$.

(a) Define the notion of state-controllability and give a test for checking controllability.

(b) Define the notion of state-observability and give a test for checking observability.

(c) State the pole-placement problem and the pole-placement theorem.

Ques 4: (a) For a 2×2 real matrix A, define e^{At} for $t \geqslant 0$.

(b) For $A = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$, obtain e^{At} explicitly.

(c) Use the obtained e^{At} to find the impulse response of $G(s) = \frac{1}{s^2 - 3s + 2}$.

Ques 5: Consider the standard negative unity

feedback configuration as shown beside, with r the unit step input, and $G(s) = \frac{1}{(s+1)(s+2)}$.

For the closed loop system's step response, it is desired to have 5% overshoot, together with a 2% settling time of at most 1 second. Using **root-locus techniques**,

(a) design a constant gain controller $u = k_p e$ that achieves both the requirements.

(b) design a PD controller $u = k_p e + k_d \frac{d}{dt} e$ that achieves both the requirements.

(Show intermediate steps/calculations using a root-locus sketch on your plain answer paper.)