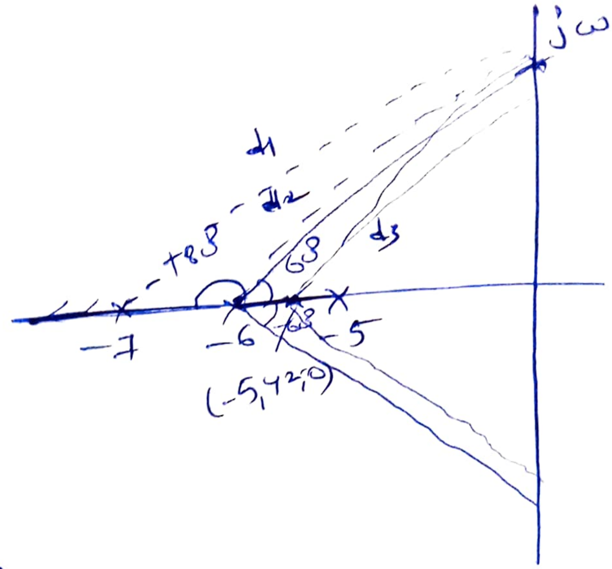
1) a)

$$G(s) = \frac{1}{(s+5)(s+6)(s+7)}$$



Poles = $-5, -6, -7$

Centroid $= \frac{-5-6-7}{3}$

$= -6$

$K = -(s+5)(s+6)(s+7)$

$= -(s^3 + 18s^2 + 107s + 210)$

$\frac{dK}{ds} = -3s^2 - 36s - 107 = 0$

$s = \frac{-18 \pm \sqrt{3}}{3} = -5.42$ ✓ (on Root locus), -6.57

$K = \frac{\pi \text{ poles (distance)}}{\pi \text{ zeroes}}$

$\boxed{K = d_1 \cdot d_2 \cdot d_3 \approx 1250}$ estimate using asymptotes

Asymptotes $= 60°, 180°, 300°$.

$\omega_c \approx 10.34$

b) $= s^3 + 13s^2 + 42s + 5s^2 + 65s + 42 \times 5 + K$

$\Rightarrow s^3 + 18s^2 + 107s + 210 + K$

s^3	1	107
s^2	18	$210+K$
s^1	$\frac{18 \times 107 - (210+K)}{18}$	0
s^0	$210+K$	

$K \in (1716, 0)$ for +ve range to make closed loop unstable.

1.(c) $\frac{k}{(j\omega+5)(j\omega+6)(j\omega+7)}$

$\frac{1}{2}$

$-18\omega^2 + 210 + 107j\omega - j\omega^3$

$107\omega - \omega^3 = 0$ $\quad \omega = 0$, $\omega^2 = 107$ $\quad \omega = \pm\sqrt{107}$

$\omega = 0$	$\omega^2 = 107$
$\frac{k}{210} > -1$	$\frac{k}{-18\times107+210} > -1$
$k > -210$	$k > -(-1716)$
	$k > 1716$

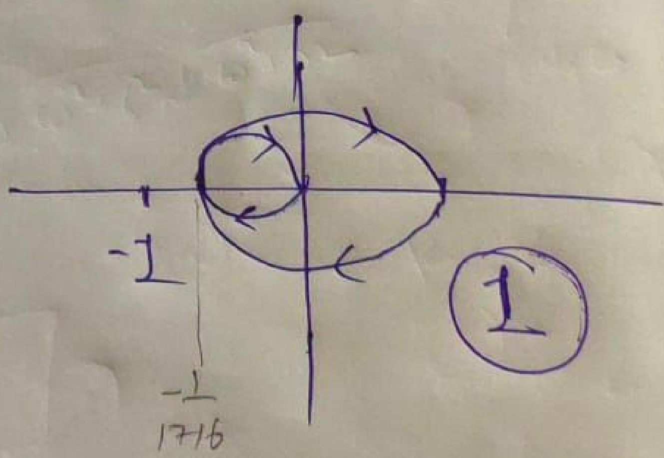
1

$k > 1716$

$P = 0$

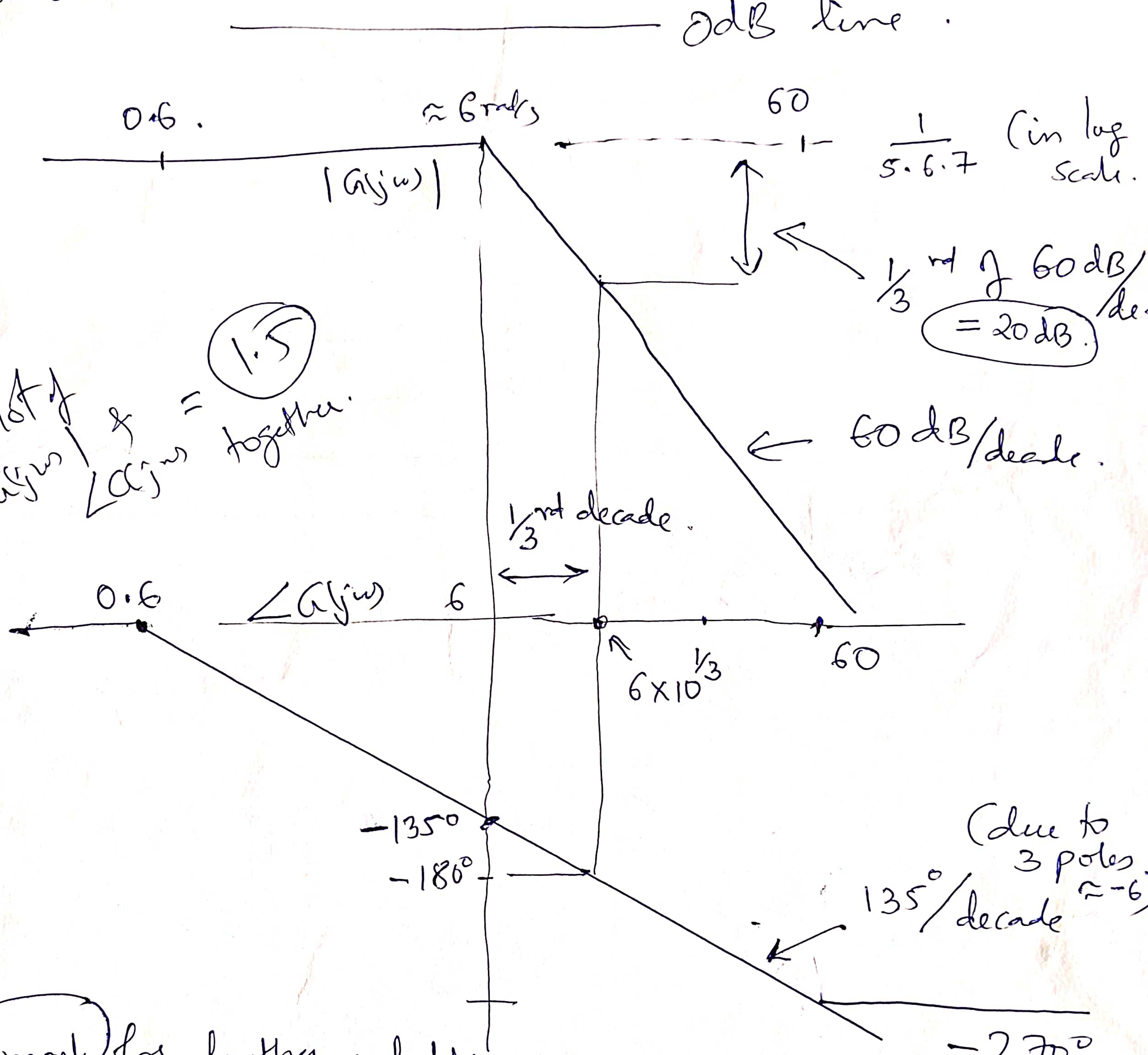
$Z = 0$

$N = P - Z$



1

Q-1 d Bode plot based estimation of k that makes closed loop unstable.



1 mark for further calculations

So gain margin = 20 dB ≡ 10

× 5×6×7 =

210 × 10 = 2100

So Bode plot asymptotes based estimate k = 2100

(assuming all 3 poles are at −6).

Any reasonable procedure to get k → full 2.5 marks.

(Q 2) given $p(s)$

(a) (i) No : presence of sign change in ~~both~~ the polynomial. ✓ 2

(ii) No. presence of sign change in the polynomial. ✓

2

(b) $p(s) = s^8 - 2s^7 - s^6 + s^5 + s^4 + s^3 + 3s + 2$.

s^8	1	-1	1	0	2
s^7	-2	1	1	3	
s^6	-0.5	1.5	1.5	2	
s^5	-5	-5	-5		
s^4	2	2	2		
s^3	~~0~~ 8	~~0~~ 4	~~0~~		
s^2	~~0~~ 1	2			
s^1	-12				
s^0	2				

4

Entire Row is zero;
$\Rightarrow$ Forming auxilliary equation :

$$A(s) = 2s^4 + 2s^2 + 2 = 0.$$

$$\frac{d\,A(s)}{ds} = 8s^3 + 4s = 0$$

(c) Total no. of poles from the eq^n = 8

Total no. of poles in open ~~b~~ Left half plane = [Total no. of sign changes in 1st column]

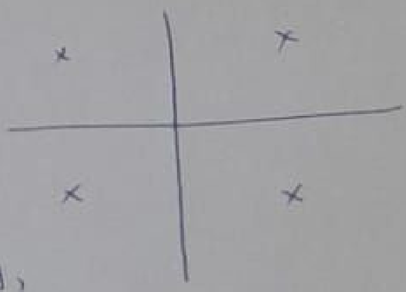
= 4 sign changes = 4 RHP poles.

4

Solving the auxiliary eq^n.

$2s^4 + 2s^2 + 2 = 0$ gives

$$\left[s = -\frac{1}{2} \pm i\frac{\sqrt{3}}{2} \quad ; \quad \frac{1}{2} \pm i\frac{\sqrt{3}}{2} \right]$$



$\Rightarrow$ '4' 2 of the poles ~~of~~ will lie in the RHP with symmetry, Rest 2 in the LHP.

$\therefore$ Poles on $j\omega$ - axis = 0.

$\Rightarrow$ 4 Poles in RHP
4 Poles in LHP
0 Pole in $j\omega$. (ans.

Q 3 (a)

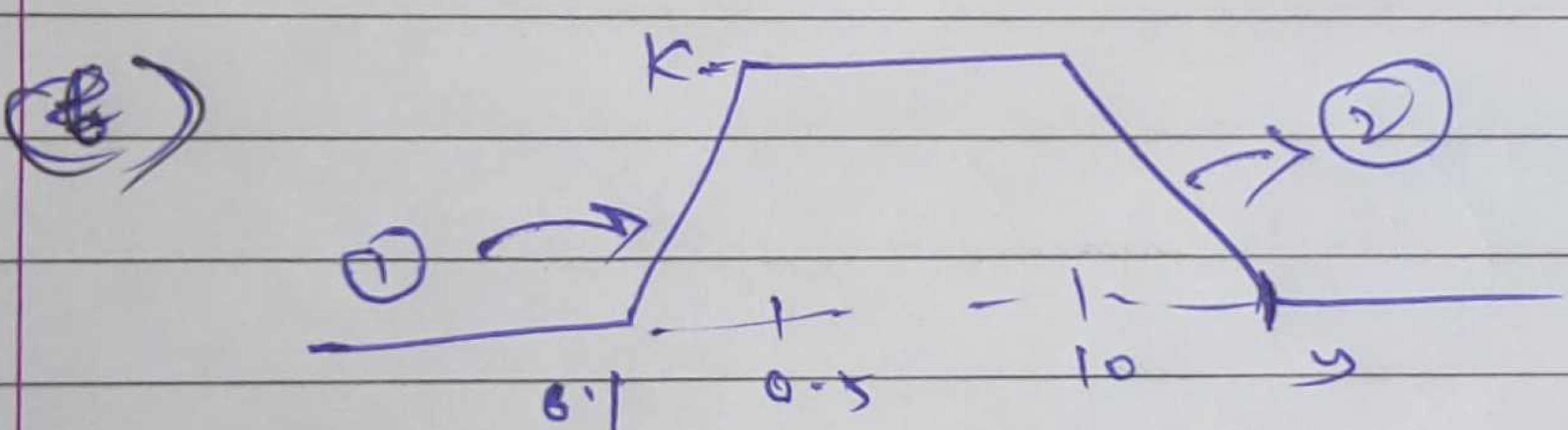
$$G_1(s) = \frac{5\left(\frac{s}{0.1}+1\right)\left(\frac{s}{50}+1\right)}{\left(\frac{s}{0.5}+1\right)\left(\frac{s}{10}+1\right)}$$

~~$G_2(s) =$~~

$$G_2(s) = \frac{25\left(\frac{s}{0.1}+1\right)^2\left(\frac{s}{50}+1\right)^2}{\left(\frac{s}{0.5}+1\right)^2\left(\frac{s}{10}+1\right)^2}$$

$G_1(s)$ & $G_2(s)$ are 2 possible transfer fn.

(b)



using the above asymptotic bode plot we can write general transfer fn. as

$$G(s) = \frac{K\left(\frac{s}{0.1}+1\right)\left(\frac{s}{y}+1\right)}{\left(\frac{s}{0.5}+1\right)\left(\frac{s}{10}+1\right)}$$

assuming slope 1 & slope 2 to be of 20dB/dec & -20dB/dec respectively.

we can choose $K = 5$ for slope 1 to be of 20dB/dec.

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& for slope ② to be -20 dB/dec we can choose
$y = 50$ rad/s^2.

Similarly we can assume general $G_{n_2}(s) = \dfrac{K\left(\frac{s}{0.1}+1\right)^2\left(\frac{s}{y}+1\right)^2}{\left(\frac{s}{0.1}+1\right)^2\left(\frac{s}{10}+1\right)^2}$

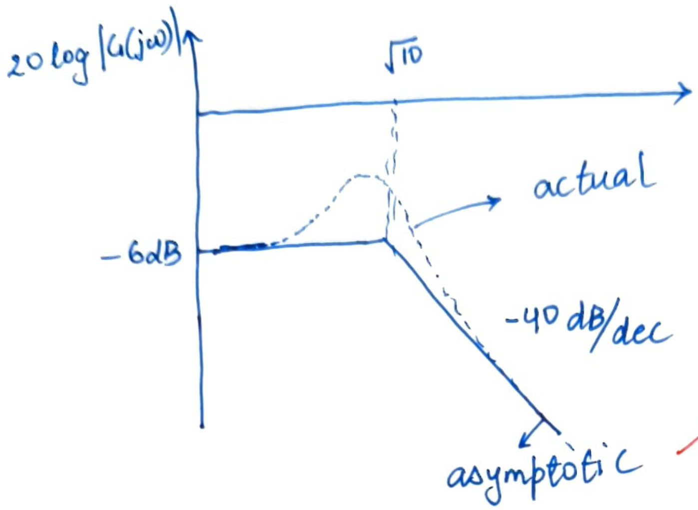
where if ~~we~~ we take slope ① to be of
40 dB/dec then $k = 25$ & if we take
slope ② to of -40 dB/dec then
$y = 50$

Marking Pattern

(a) $\dfrac{5}{s^2+0.1s+10} = \dfrac{1}{2}\left(\dfrac{10}{s^2+0.1s+10}\right) = \dfrac{1}{2}\left(\dfrac{10}{10-\omega^2+0.1j\omega}\right)$

$\omega_n = \sqrt{10}$, $\xi = \dfrac{0.1}{2\sqrt{10}} = 0.016$,

$20\log|G(j\omega)|_{\omega=0} = 20\log\left(\frac{1}{2}\right) = -6\,dB$

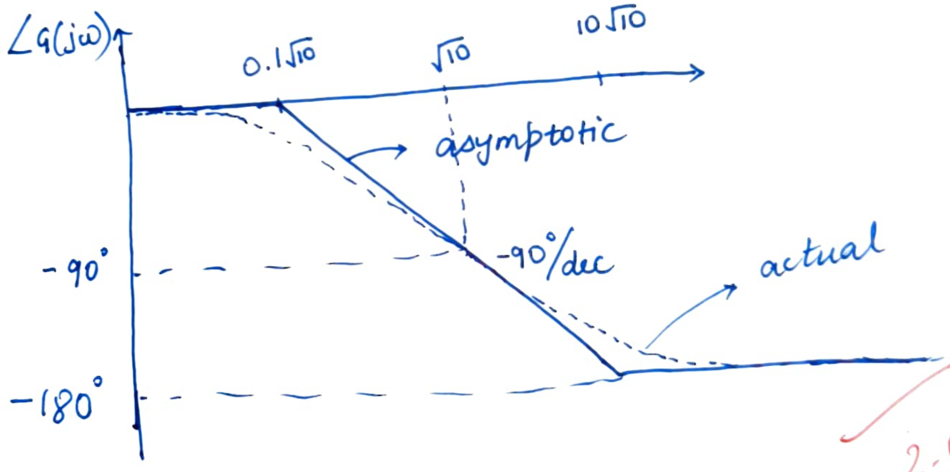


1.5 marks for asymptotic

1 mark for actual

2.5

$\angle G(j\omega) = -\tan^{-1}\left(\dfrac{0.1\omega}{10-\omega^2}\right)$



2.5

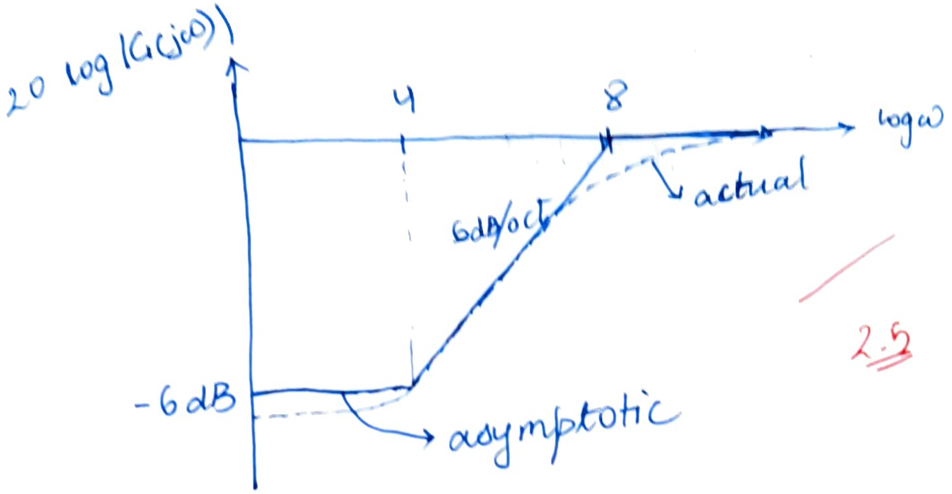
$\left.\angle G(j\omega)\right|_{\omega=0.1\sqrt{10}} = -0.183°$

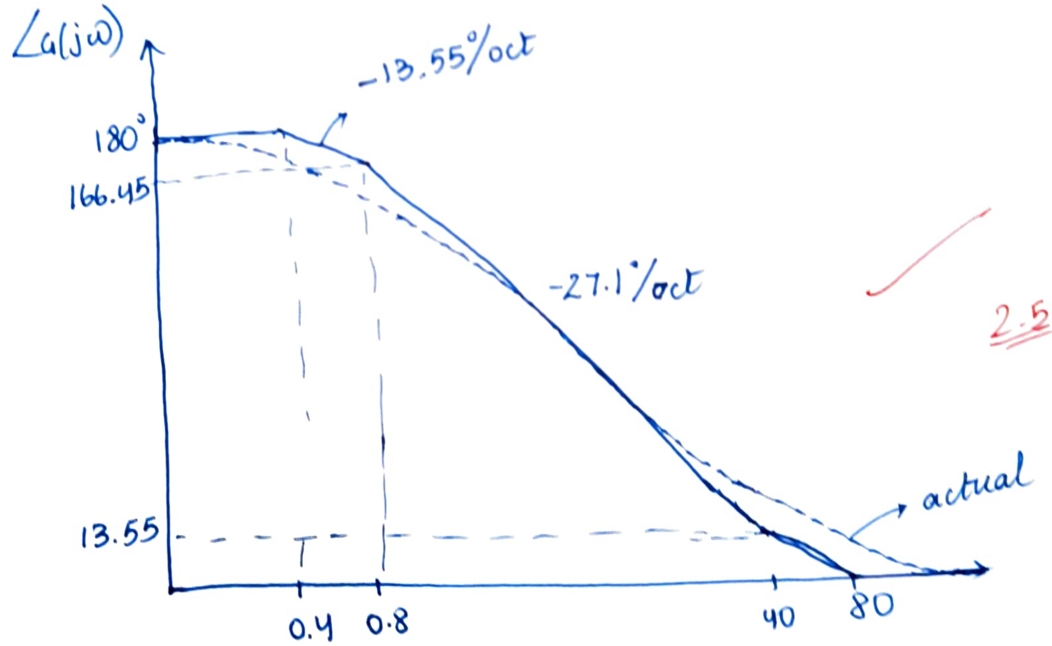
$\left.\angle G(j\omega)\right|_{\omega=10\sqrt{10}} = -179.82°$

Actual values

(b) $\dfrac{s-4}{s+8} = \dfrac{1}{2}\left(\dfrac{s/4-1}{1+s/8}\right) = \dfrac{j\omega-4}{j\omega+8}$

$$20\log|G(j\omega)|_{\omega=0} = 20\log\left(\frac{1}{2}\right) = -6\,dB$$





$$\angle G(j\omega) = \tan^{-1}\left(\frac{\omega}{-4}\right) - \tan^{-1}\left(\frac{\omega}{8}\right) = 180° - \tan^{-1}\left(\frac{\omega}{4}\right) - \tan^{-1}\left(\frac{\omega}{8}\right)$$

$$\left.\begin{aligned}
\angle G(j\omega)|_{\omega=0.4} &= 171.42° \\
\angle G(j\omega)|_{\omega=0.8} &= 162.98° \\
\angle G(j\omega)|_{\omega=40} &= 17.02° \\
\angle G(j\omega)|_{\omega=80} &= 8.57°
\end{aligned}\right\} \text{Actual values}$$

Q5)

a) A lead compensator is designed to increase the phase margin of a system. It essentially adds <u>positive phase</u> to the system over a certain range of frequencies.

2 marks

b) A lag compensator is used to improve steady-state error of the system. It adds <u>negative phase</u> to the system over a certain range of frequencies.

b) Lead compensator = High pass filter

2 marks Lag compensator = Low pass filter.

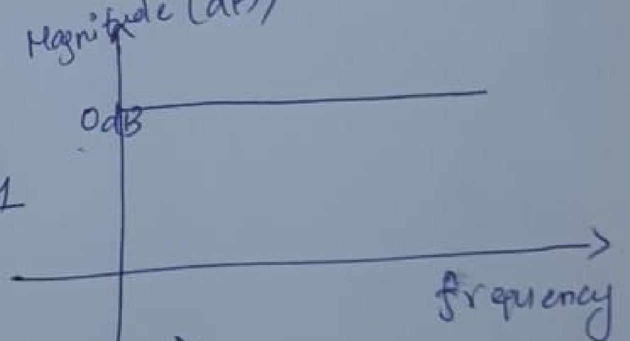
c) Transfer function of an All pass filter would be

6 marks

$$G_c(s) = \frac{s-a}{s+a} \quad (\text{first order})$$

<u>Magnitude response:</u>

$$G_c(j\omega) = \left|\frac{j\omega - a}{j\omega + a}\right| = \frac{\sqrt{\omega^2 + a^2}}{\sqrt{\omega^2 + a^2}} = 1$$



<u>Phase response:</u>

$$\angle G_c(j\omega) = \angle j\omega - a - \angle j\omega + a$$

$$= \tan^{-1}\left(\frac{\omega}{-a}\right) - \tan^{-1}\left(\frac{\omega}{a}\right)$$

$$= \pi - 2\tan^{-1}\left(\frac{\omega}{a}\right)$$