

1] (a) $G_1 \Rightarrow$ Not possible as all the three conditions won't satisfy at the same time. (2)

(b) 10% OS, $T_s = 20$

$$\exp\left(\frac{-\pi \xi_1}{\sqrt{1-\xi_1^2}}\right) = 0.1$$

$$\xi_1 = 0.5911$$

for 2% tolerance, $T_s = 20 = 4\tau$

$$\frac{4}{\omega_{n1} \xi_1} = 20 \Rightarrow \omega_{n1} = 0.3383$$

$$G_2 = \frac{\omega_{n1}^2}{s^2 + 2\xi_1 \omega_{n1} s + \omega_{n1}^2}$$

10% OS, $T_p = 3 \text{ sec}$

$$\xi_2 = 0.5911$$

$$T_p = \frac{\pi}{\omega_{n2} \sqrt{1-\xi_2^2}} = 3$$

$$\omega_{n2} = 1.298$$

$$G_3 = \frac{\omega_{n2}^2}{s^2 + 2\xi_2 \omega_{n2} s + \omega_{n2}^2}$$

$$T_p = 3 \text{ sec} , T_s = 20 \text{ sec}$$

$$\frac{4}{\xi_3 \omega_{n3}} = 20 \Rightarrow \omega_{n3} = \frac{1}{5 \xi_3}$$

$$T_p = \frac{\pi}{\omega_{n3} \sqrt{1 - \xi_3^2}} = 3$$

$$\frac{5\pi \xi_3}{\sqrt{1 - \xi_3^2}} = 3$$

$$\xi_3 = 0.1875$$

$$\omega_{n3} = 1.0661$$

$$G_y = \frac{\omega_{n3}^2}{s^2 + 2\xi_3 \omega_{n3} s + \omega_{n3}^2}$$

2. (a)
- ① $\frac{8}{s^2+2s+8} \rightarrow \underline{\underline{f}} \rightarrow$
- system underdamped
 - Simple 2nd Order system
- ② $\frac{s+8}{s^2+2s+8} \rightarrow \underline{\underline{d}} \rightarrow$
- system underdamped
 - addition of zero with +ve coeff.
 - rise time decreases
- ③ $\frac{-s+8}{s^2+2s+8} \rightarrow \underline{\underline{a}} \rightarrow$
- system underdamped
 - addition of zero with -ve coeff.
 - rise time increases initially goes below zero & comes up
- ④ $\frac{s+8}{s^2+30s+8} \rightarrow \underline{\underline{e}} \rightarrow$
- 2nd order system
 - overdamped system (no peak)
- ⑤ $\frac{3s+2}{2s+2} \rightarrow \underline{\underline{b}} \rightarrow$
- 1st order system
 - unit step + decaying exponential
 - $u(t) + \frac{1}{2}e^{-t}$
- ⑥ $\frac{s+2}{2s+2} \rightarrow \underline{\underline{c}} \rightarrow$
- 1st order system
 - unit step - decaying exponential
 - $u(t) - \frac{1}{2}e^{-t}$
- ⑦ $\frac{s+8}{s-8} \rightarrow \text{NA} \rightarrow$
- 1st order
 - unstable system

$$a) \frac{s+8}{s^2+2s+8}$$

$$b) \frac{s+8}{s^2+70s+8}$$

$$c) \frac{3s+9}{2s+2}$$

$$d) \frac{s+2}{2s+2}$$

$$e) \frac{-s+8}{s^2+2s+8}$$

$$f) \frac{s}{s^2+2s+8}$$

$$g) \frac{s+8}{s-8}$$

TVT

$$\lim_{s \rightarrow \infty} s f(s)$$

$$= \lim_{s \rightarrow \infty} \frac{1}{s} \cdot (s \cdot f(s))$$

$$a) 0$$

$$b) 0$$

$$c) 3/2$$

$$d) 1/2$$

$$e) 0$$

$$f) = 0$$

$$g) 1$$

FVT

$$\lim_{s \rightarrow 0} f(s)$$

$$a) 1$$

$$b) 1$$

$$c) 1$$

$$d) 1$$

$$e) 1$$

$$f) 1$$

$$g) NA$$

IRK

$$\lim_{s \rightarrow \infty} s f(s)$$

$$a) 1$$

$$b) 1$$

$$c) \frac{3}{2} + \frac{-1}{(2s+2)}$$

$$\lim_{s \rightarrow \infty} \left(\frac{3}{2} + \frac{-1}{(2s+2)} \right) = \frac{3}{2}$$

$$d) \frac{1}{2} + \frac{1}{2(s+1)} \Big|_{s=0} = \frac{1}{2}$$

$$e) -1$$

$$f) 0$$

$$g) 16$$

Mark

Q.2

(a)

sub- parts	Marks
6	2
4	1.5
2	0.5
1	0.5
0	0

(b)

Subparts	Marks
7	3
4	2
2	1
1	0.5
0	0

if all subparts are correct

Q3)

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$0 < \xi < 1$$

a) constant % overshoot curve (C_1)



① mark

b) constant T_s curve (C_2)



Real part of root is constant.

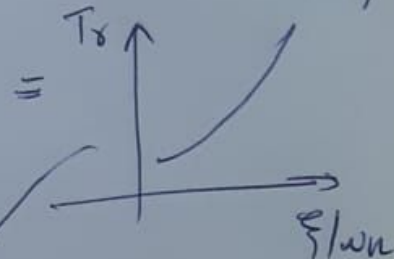
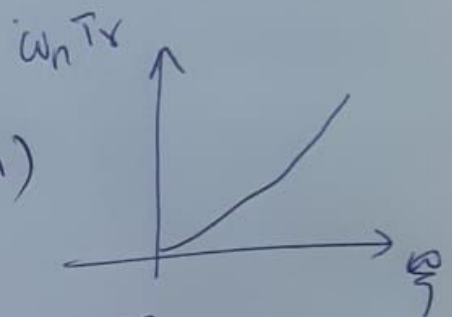
① mark

b) Along $C_2 \Rightarrow T_p$ decreases (since $\omega_d \uparrow$)

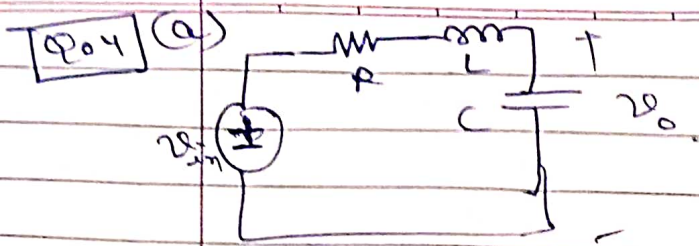
①.5 mark % OS increases
mark T_r decreases

Along $C_1 \Rightarrow T_p$ decreases

①.5 mark T_s decreases
mark T_r decreases



ξ decreases
 ω_n increases $\Rightarrow T_r$ decreases



$$\frac{v_o}{v_{in}} = \frac{Y_{sc}}{Y_{sc} + R + sL}$$

$$\frac{v_o}{v_{in}} = \frac{1}{s^2 LC + RCs + 1}$$

$$\frac{v_o}{v_{in}} = \frac{1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$\omega_n = \frac{1}{\sqrt{LC}}, \quad 2\zeta\omega_n = \frac{R}{L}$$

$$\zeta = \frac{R\sqrt{LC}}{2L} \Rightarrow$$

$$\boxed{\zeta = \frac{R}{2} \sqrt{\frac{C}{L}}}$$

taking $C = 10^{-8} \text{ F}$, $L = 10^{-6} \text{ H}$
 $R = 10 \Omega$

$$\zeta = \frac{10}{2} \sqrt{\frac{10^{-8}}{10^{-6}}} \Rightarrow = 0.5$$

\Rightarrow Underdamped.

(b) Same circuit with large R value can be an overdamped system.

Let take $R = 10 \text{ K}$
 $C = 10^{-8} \text{ F}$, $L = 10^{-6} \text{ H}$

$$\zeta = \frac{10 \times 10^3}{2} \sqrt{\frac{10^{-8}}{10^{-6}}}$$

$$\zeta = 500$$

\Rightarrow Hence overdamped for large R .

(c) In both the above cases (a) & (b)
input is taken voltage source &
output is taken voltage across
capacitor (V_o).

Marking pattern

a) 1 marks \rightarrow Circuit with transfer fn.
1 marks \rightarrow Choosing R, L, C s.t. it is
~~overdamped~~ & R is small.
Underdamped.

b) 1 marks \rightarrow Ckt with transfer fn.
1 marks \rightarrow Choosing R, L, C s.t. it is
overdamped & R is large.

c) 1 marks: 0.5 mark for part (a) i/p & o/p
0.5 mark for part (b) i/p & o/p.

Q5

(a) $G(s) = \frac{1}{s^2 + 2\zeta s + 1}$

Transfer function after addition of pole and taking the step response will look like \rightarrow

$$C(s) = \frac{a}{s(s^2 + 2\zeta s + 1)(s+a)}$$

$a \leftarrow$ to handle steady state value

taking partial fraction of $C(s)$

$$C(s) = \frac{A}{s} + \frac{Bs + C}{s^2 + 2\zeta s + 1} + \frac{D}{(s+a)}$$

here we assumed that the non dominant pole a is located at $-a$ on the real axis

After calculation for A, B, C and D , we get \rightarrow

$$A = 1$$

$$B = \frac{2\zeta a - a^2}{a^2 - 2\zeta a + 1}$$

$$C = \frac{4\zeta^2 a - 2\zeta a^2 - a}{a^2 - 2\zeta a + 1}$$

$$D = \frac{-1}{a^2 - 2\zeta a + 1}$$

now as the nondominant pole $a \rightarrow \infty$,

$$\lim_{a \rightarrow \infty} D = \lim_{a \rightarrow \infty} \frac{-1}{a^2 - 2fa + 1} = 0$$

$$\therefore D = 0$$

$$\lim_{a \rightarrow \infty} C = \lim_{a \rightarrow \infty} \frac{4f^2a - 2fa^2 - a}{a^2 - 2fa + 1} = -2f$$
$$\therefore C = -2f$$

$$\lim_{a \rightarrow \infty} B = \lim_{a \rightarrow \infty} \frac{2fa - a^2}{a^2 - 2fa + 1} = -1$$
$$\therefore B = -1$$

$$\lim_{a \rightarrow \infty} A = \lim_{a \rightarrow \infty} (1) = 1$$
$$\therefore A = 1$$

Thus we observed the residue (i.e. D) of the nondominant pole and its response becomes zero as the nondominant pole approaches infinity.

(b)

$$G(s) = \frac{1}{s^2 + 2\zeta s + 1}$$

adding a pole to $G(s)$ "Such that steady state gain is not disturbed" \Rightarrow

But here two cases may happen zero can lie on left half plane or on right half plane too.

Case 1: \Rightarrow on left half plane

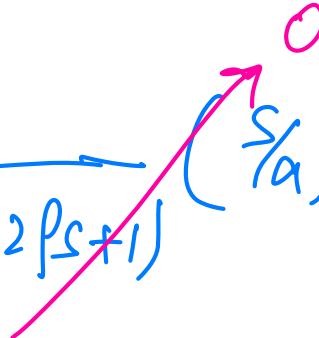
$$G(s) = \frac{\left(\frac{s}{a} + 1\right)}{s^2 + 2\zeta s + 1}$$

step response \Rightarrow

$$C(s) = \frac{\left(\frac{s}{a} + 1\right)}{s(s^2 + 2\zeta s + 1)}$$

$$C(s) = \frac{1}{s(s^2 + 2\zeta s + 1)} \left(\frac{s}{a}\right) + \frac{1}{s(s^2 + 2\zeta s + 1)}$$

if $a \rightarrow \infty$ then

$$C(s) = \frac{1}{s(s^2 + 2\zeta s + 1)} \left(\frac{s}{a} \right) + \frac{1}{s(s^2 + 2\zeta s + 1)}$$


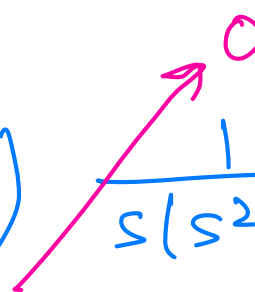
$$\therefore C(s) = \frac{1}{s(s^2 + 2\zeta s + 1)}$$

case 2: when zero added on right half plane

$$C(s) = \frac{(1 - s/a)}{s(s^2 + 2\zeta s + 1)}$$

$$C(s) = \frac{1}{s(s^2 + 2\zeta s + 1)} - \left(\frac{s}{a} \right) \left(\frac{1}{s(s^2 + 2\zeta s + 1)} \right)$$

if $a \rightarrow \infty$

$$C(s) = \frac{1}{s(s^2 + 2\zeta s + 1)} - \left(\frac{s}{a} \right) \frac{1}{s(s^2 + 2\zeta s + 1)}$$


$$\therefore C(s) = \frac{1}{s(s^2 + 2\zeta s + 1)}$$

method 2:->

For additional zero to be neglected
take zero as:->

$$s = -\frac{1}{\epsilon}$$

$$Tf = \frac{\epsilon s + 1}{s^2 + 2\zeta s + 1}$$

now proceed further.