EE 229 Midsem

1.

a) System is not memoryless, and is not causal.

$(y[1] = x[2])$.

System is stable & not time invariant.

System is linear.

(b) $y[n + \frac{N}{2}] = x[2n + N]$

$= x[2n] = y[n]$.

y is periodic with period $\frac{N}{2}$

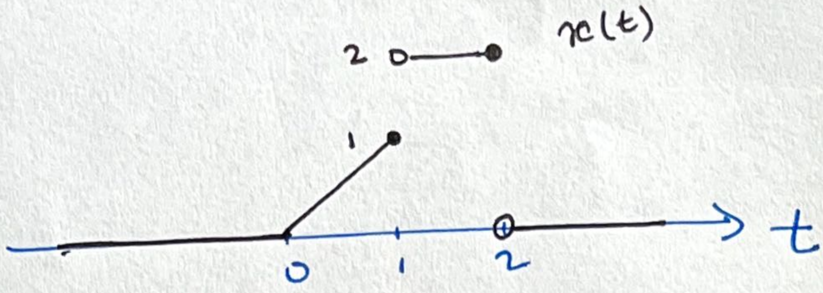
$\Rightarrow$ fundamental period $\leq \frac{N}{2}$.

(c) $y[n+N] = x[2n+2N] = x[2n] = y[n]$.

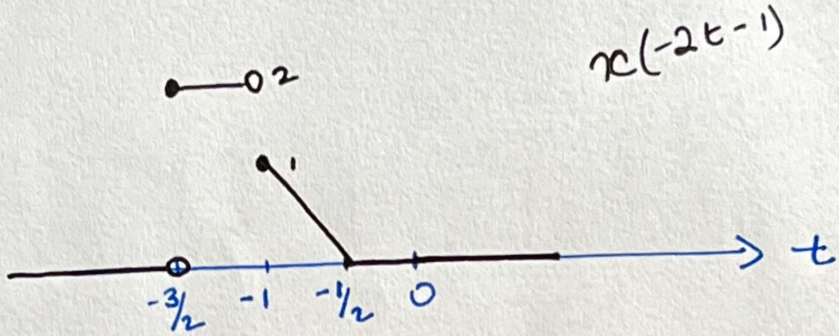
y is periodic with period N

$\Rightarrow$ fundamental period $\leq N$.

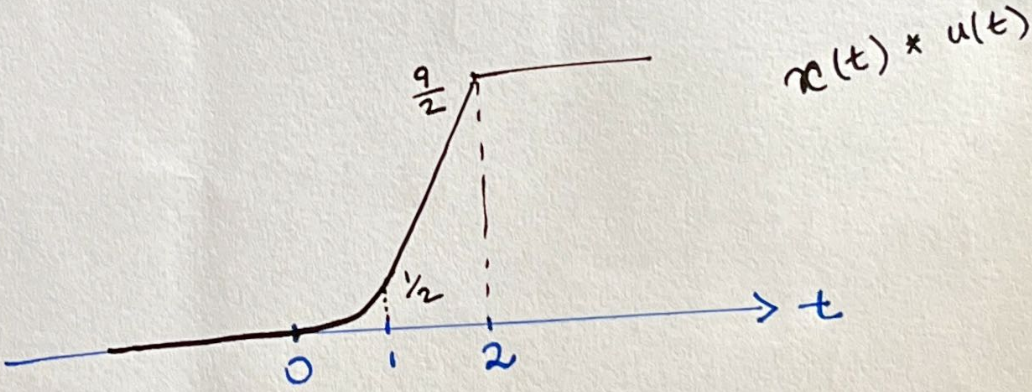
2.



(a)

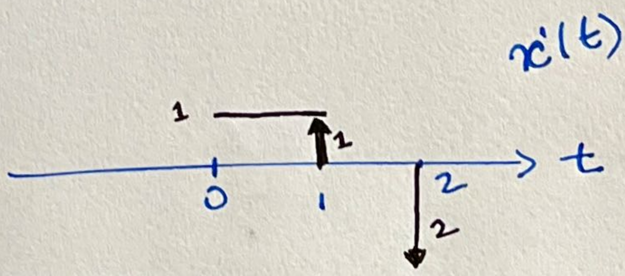


(b)



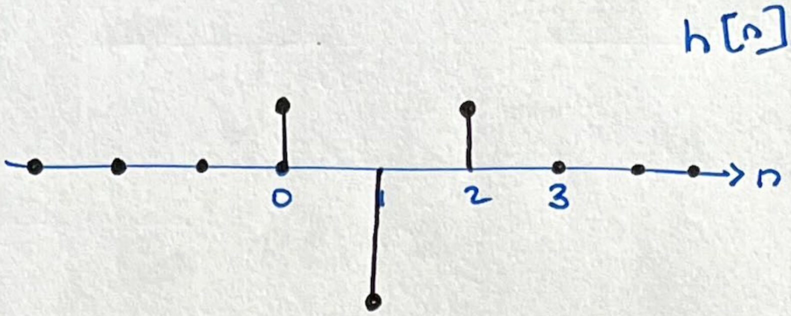
(c) $x' * u = x$
↳ sketched above

(d) $x'' * u = x'$

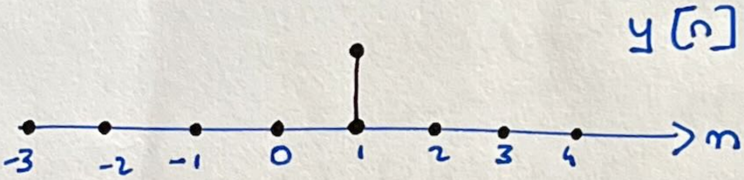


$\boxed{3.}$

a) $h[n] = \delta[n] - 2\delta[n-1] + \delta[n-2]$

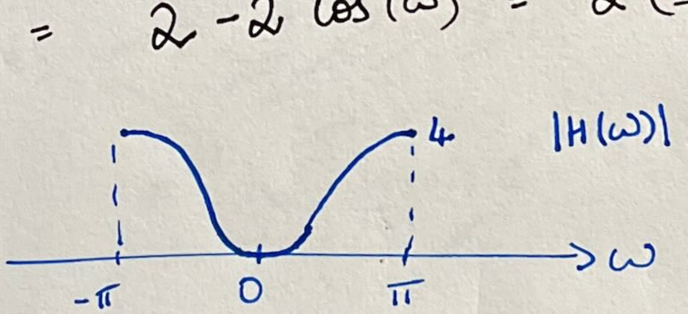


b) $$y[n] = \begin{cases} 0 & n \le 0 \\ 1 & n = 1 \\ 0 & n \geq 2 \end{cases}$$



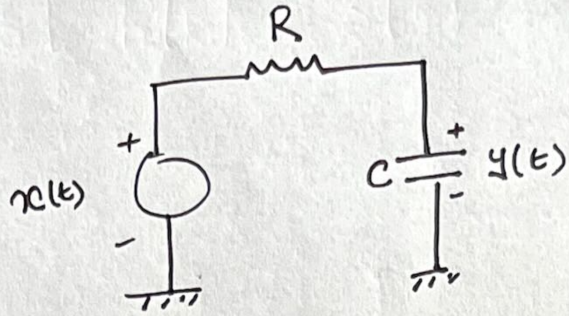
c) $$H(\omega) = 1 - 2e^{-j\omega} + e^{-2j\omega}$$
$$= \left(1 - e^{-j\omega}\right)^2$$

$$|H(\omega)| = \left(1 - e^{-j\omega}\right)\left(1 - e^{j\omega}\right)$$
$$= 2 - 2\cos(\omega) = 2\left(1 - \cos(\omega)\right)$$



4.

(a)



Set $RC = 2$

(b) Let $S(t)$ denote the step response.

Clearly, $S(t) = 0 \quad \forall t < 0$,

$$S(t) + 2S'(t) = 1 \quad \forall t \geq 0,$$

$$S(0) = 0 \quad \text{(initial rest)}$$

$$\Rightarrow S(t) = \left(1 - e^{-t/2}\right) u(t)$$

$$\Rightarrow \boxed{h(t) = \frac{e^{-t/2}}{2} u(t)}$$

(c) $$H(\omega) = \frac{1}{1 + 2j\omega}$$

$$\Rightarrow \cos(\omega_0 t) \to \cancel{\tfrac{1}{2}\left(H(\omega_0) + H(-\omega_0)\right)}$$

$$\to \frac{1}{2}\left\{ H(\omega_0)\, e^{j\omega_0 t} + H(-\omega_0)\, e^{-j\omega_0 t} \right\}$$

$\boxed{5.}$ $y[n]$ is periodic with period 10.

Let $\{c_n\}$ denote the FS co-efficients of $x[n]$ with period 10.

$$\Rightarrow \quad c_n = \begin{cases} a_{n/2} & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

$$(-1)^n x[n] = e^{j\pi n} x[n]$$

$$= e^{j\left(\frac{2\pi}{10}\right).5n} x[n]$$

$$\Rightarrow \quad b_k = C_{k-5}$$

$$\Rightarrow \quad b_k = \begin{cases} 0 & k = 0 \\ a_3 & k = 1 \\ 0 & k = 2 \\ a_4 & k = 3 \\ 0 & k = 4 \\ a_5 & k = 5 \\ 0 & k = 6 \\ a_6 = a_1 & k = 7 \\ 0 & k = 8 \\ a_2 & k = 9 \end{cases}$$

$\boxed{6.}$ Say $y'(t) \xrightarrow{FS} \{c_k\}$

$$C_0 = 0$$
$$C_k = -1 \quad k \neq 0$$
$$\because \; y'(t) = 1 - \sum_{k=-\infty}^{\infty} \delta(t-k)$$

$$\Rightarrow \; b_k = \begin{cases} 1/2 & k = 0 \\ \dfrac{C_k}{jk\omega_0} & k \neq 0 \end{cases}$$

$$= \begin{cases} 1/2 & k = 0 \\ \dfrac{-1}{jk\omega_0} & k \neq 0 \end{cases}$$

$$\Rightarrow \; a_k = \begin{cases} 1/3 & k = 0 \\ \dfrac{-1}{(jk\omega_0)^2} & k \neq 0 \end{cases}$$

Here, $\omega_0 = \dfrac{2\pi}{1} = 2\pi.$