EE 229 **Autumn 2023**

End-semester examination

Max marks: 90 Time: 3 hours

Each question carries 10 marks. You will not require a calculator. Begin the answer to each question on a fresh page. Your solutions should be neat and coherent.

Throughout, δ denotes the unit impulse signal, and u denotes the unit step signal.

1. Compute the Fourier transform $X(\omega)$ corresponding to the following signals:

   (a) $x_1(t) = e^{-|t|}\cos(t)$.

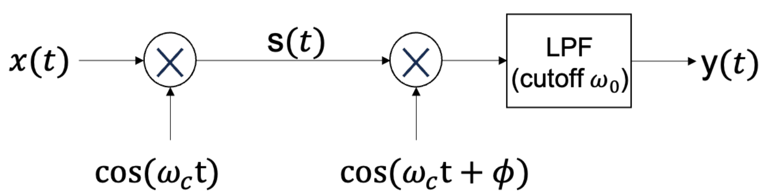
   (b) $x_2(t) = \dfrac{1}{t^2 + 2t + 2}$

2. Consider the following scenario. Jai wishes to send a bandlimited message $x(t)$, having bandwidth less than ω_0, to his friend Veeru. Jai modulates his message with a sinusoidal carrier, and transmits the signal

$$s(t) = m(t)\cos(\omega_c t),$$

   where $\omega_c \gg \omega_0$.

   On receiving $s(t)$, Veeru multiplies this received signal with $\cos(\omega_c t + \phi)$ (note the phase offset between the carrier signals used by Jai and Veeru), and then passes this product signal though an ideal low pass filter with cutoff frequency ω_0 to obtain $y(t)$. This pipeline is illustrated as follows.



   Express Veeru's output $y(t)$ in terms of $x(t)$.

3. Consider an LTI system with impulse response $h(t) = e^{-t}u(t)$. Find the output $y(t)$ of this system corresponding to the input

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t - 2k).$$

   Note: You have to provide an explicit, closed form, time domain description of $y(t)$.

4. Recall that $\mathrm{sinc}(t) := \frac{\sin(\pi x)}{\pi x}$. Compute the values of the following integrals:

   (a) $\int_{-\infty}^{\infty} \mathrm{sinc}(t)dt$

   (b) $\int_{-\infty}^{\infty} \mathrm{sinc}^2(t)dt$

5. Consider the causal LSI system with transfer function $H(s) = \frac{1}{(s+1)(s+2)}$. Obtain the output signal $y(t)$ from this system, corresponding to the input signal $x(t) = \cos(t)$.

   Note: You have to provide an explicit, closed form, time domain description of $y(t)$.

6. Two discrete time signals $x_1[n]$ and $x_2[n]$ are *interleaved* into a single signal $x[\cdot]$ as follows:

$$x[2n] = x_1[n], \quad x[2n+1] = x_2[n]$$

   for $n \in \mathbb{N}$.

   Express the DTFT corresponding to $x[\cdot]$ in terms of the DTFT of $x_1[\cdot]$ and $x_2[\cdot]$.

7. The signal $x_c(t)$, having bandwidth $\omega_c < \pi$, is sampled with sampling interval $T_s = 1$. The resulting discrete time signal $x_d[n]$ is passed through an LTI system with impulse response $h_d[n]$. The output $y_d[n]$ of this system is interpolated to obtain the continuous time output signal $y_c(t)$ as follows.

$$y_c(t) = \sum_{k=-\infty}^{\infty} y_d[k]\mathrm{sinc}\,(t-k)$$

   Obtain the frequency response $H_d(\cdot)$ of the discrete time filter such that $y_c(t) = x_c''(t)$.
   (Here, $x''(t)$ denotes the second derivative of $x(t)$.)

8. The signal $y(t) = e^{-2t}u(t)$ is the output of a causal LSI system for which the system function is

$$H(s) = \frac{s-1}{s+1}.$$

   Identify two possible inputs to this system that would produce the above output.

9. Consider a discrete-time signal $x[n]$ with DTFT $X(\omega)$. For a given $k \in \mathbb{N}$, we *downsample* x to obtain the signal $x_{\downarrow k}[n]$, defined as

$$x_{\downarrow k}[n] = x[kn].$$

   (a) Express the spectrum $X_{\downarrow 2}(\omega)$ of $x_{\downarrow 2}[n]$ in terms of $X(\omega)$.

   (b) Under what conditions can $x[n]$ be recovered from $x_{\downarrow 2}[n]$?