EE229 Cheatsheet

TABLE 4.1 PROPERTIES OF THE FOURIER TRANSFORM

Section	Property	Aperiodic signal	Fourier transform
		$x(t)$ $y(t)$	$X(j\omega)$ $Y(j\omega)$
4.3.1	Linearity	$ax(t)+by(t)$	$aX(j\omega)+bY(j\omega)$
4.3.2	Time Shifting	$x(t-t_0)$	$e^{-j\omega t_0}X(j\omega)$
4.3.6	Frequency Shifting	$e^{j\omega_0 t}x(t)$	$X(j(\omega-\omega_0))$
4.3.3	Conjugation	$x^*(t)$	$X^*(-j\omega)$
4.3.5	Time Reversal	$x(-t)$	$X(-j\omega)$
4.3.5	Time and Frequency Scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{j\omega}{a}\right)$
4.4	Convolution	$x(t)*y(t)$	$X(j\omega)Y(j\omega)$
4.5	Multiplication	$x(t)y(t)$	$\frac{1}{2\pi}\int_{-\infty}^{+\infty}X(j\theta)Y(j(\omega-\theta))d\theta$
4.3.4	Differentiation in Time	$\frac{d}{dt}x(t)$	$j\omega X(j\omega)$
4.3.4	Integration	$\int_{-\infty}^{t}x(t)dt$	$\frac{1}{j\omega}X(j\omega)+\pi X(0)\delta(\omega)$
4.3.6	Differentiation in Frequency	$tx(t)$	$j\frac{d}{d\omega}X(j\omega)$
4.3.3	Conjugate Symmetry for Real Signals	$x(t)$ real	$X(j\omega)=X^*(-j\omega)$ $\Re e\{X(j\omega)\}=\Re e\{X(-j\omega)\}$ $\Im m\{X(j\omega)\}=-\Im m\{X(-j\omega)\}$ $\lvert X(j\omega)\rvert=\lvert X(-j\omega)\rvert$ $\sphericalangle X(j\omega)=-\sphericalangle X(-j\omega)$
4.3.3	Symmetry for Real and Even Signals	$x(t)$ real and even	$X(j\omega)$ real and even
4.3.3	Symmetry for Real and Odd Signals	$x(t)$ real and odd	$X(j\omega)$ purely imaginary and odd
4.3.3	Even-Odd Decomposition for Real Signals	$x_e(t)=\mathcal{E}v\{x(t)\}$ [$x(t)$ real] $x_o(t)=\mathcal{O}d\{x(t)\}$ [$x(t)$ real]	$\Re e\{X(j\omega)\}$ $j\Im m\{X(j\omega)\}$

$$\boxed{x[n]-x[n-1]\stackrel{\mathcal{F}}{\longleftrightarrow}(1-e^{-j\omega})X(e^{j\omega}).}$$

$$\boxed{\sum_{m=-\infty}^{n}x[m]\stackrel{\mathcal{F}}{\longleftrightarrow}\frac{1}{1-e^{-j\omega}}X(e^{j\omega})+\pi X(e^{j0})\sum_{k=-\infty}^{+\infty}\delta(\omega-2\pi k).}$$

4.3.7 Parseval's Relation for Aperiodic Signals

$$\int_{-\infty}^{+\infty}|x(t)|^2dt=\frac{1}{2\pi}\int_{-\infty}^{+\infty}|X(j\omega)|^2d\omega$$

TABLE 4.2 BASIC FOURIER TRANSFORM PAIRS

Signal	Fourier transform	Fourier series coefficients (if periodic)
$\sum_{k=-\infty}^{+\infty}a_k e^{jk\omega_0 t}$	$2\pi\sum_{k=-\infty}^{+\infty}a_k\delta(\omega-k\omega_0)$	a_k
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	$a_1=1$ $a_k=0$, otherwise
$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	$a_1=a_{-1}=\frac{1}{2}$ $a_k=0$, otherwise
$\sin\omega_0 t$	$\frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$	$a_1=-a_{-1}=\frac{1}{2j}$ $a_k=0$, otherwise
$x(t)=1$	$2\pi\delta(\omega)$	$a_0=1,\ a_k=0,\ k\neq 0$ (this is the Fourier series representation for any choice of $T>0$)
Periodic square wave $x(t)=\begin{cases}1, & \lvert t\rvert<T_1\\ 0, & T_1<\lvert t\rvert\le\frac{T}{2}\end{cases}$ and $x(t+T)=x(t)$	$\sum_{k=-\infty}^{+\infty}\frac{2\sin k\omega_0T_1}{k}\delta(\omega-k\omega_0)$	$\frac{\omega_0T_1}{\pi}\text{sinc}\left(\frac{k\omega_0T_1}{\pi}\right)=\frac{\sin k\omega_0T_1}{k\pi}$
$\sum_{n=-\infty}^{+\infty}\delta(t-nT)$	$\frac{2\pi}{T}\sum_{k=-\infty}^{+\infty}\delta\left(\omega-\frac{2\pi k}{T}\right)$	$a_k=\frac{1}{T}$ for all k
$x(t)\begin{cases}1, & \lvert t\rvert<T_1\\ 0, & \lvert t\rvert>T_1\end{cases}$	$\frac{2\sin\omega T_1}{\omega}$	—
$\frac{\sin Wt}{\pi t}$	$X(j\omega)=\begin{cases}1, & \lvert\omega\rvert<W\\ 0, & \lvert\omega\rvert>W\end{cases}$	—
$\delta(t)$	1	—
$u(t)$	$\frac{1}{j\omega}+\pi\delta(\omega)$	—
$\delta(t-t_0)$	$e^{-j\omega t_0}$	—
$e^{-at}u(t)$, $\Re e\{a\}>0$	$\frac{1}{a+j\omega}$	—
$te^{-at}u(t)$, $\Re e\{a\}>0$	$\frac{1}{(a+j\omega)^2}$	—
$\frac{t^{n-1}}{(n-1)!}e^{-at}u(t)$, $\Re e\{a\}>0$	$\frac{1}{(a+j\omega)^n}$	—

TABLE 9.1 PROPERTIES OF THE LAPLACE TRANSFORM

Section	Property	Signal	Laplace Transform	ROC
		$x(t)$ $x_1(t)$ $x_2(t)$	$X(s)$ $X_1(s)$ $X_2(s)$	R R_1 R_2
9.5.1	Linearity	$ax_1(t)+bx_2(t)$	$aX_1(s)+bX_2(s)$	At least $R_1\cap R_2$
9.5.2	Time shifting	$x(t-t_0)$	$e^{-st_0}X(s)$	R
9.5.3	Shifting in the s-Domain	$e^{s_0t}x(t)$	$X(s-s_0)$	Shifted version of R (i.e., s is in the ROC if $s-s_0$ is in R)
9.5.4	Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	Scaled ROC (i.e., s is in the ROC if s/a is in R)
9.5.5	Conjugation	$x^*(t)$	$X^*(s^*)$	R
9.5.6	Convolution	$x_1(t)*x_2(t)$	$X_1(s)X_2(s)$	At least $R_1\cap R_2$
9.5.7	Differentiation in the Time Domain	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
9.5.8	Differentiation in the s-Domain	$-tx(t)$	$\frac{d}{ds}X(s)$	R
9.5.9	Integration in the Time Domain	$\int_{-\infty}^{t}x(\tau)d(\tau)$	$\frac{1}{s}X(s)$	At least $R\cap\{\Re e\{s\}>0\}$

Initial- and Final-Value Theorems

9.5.10 If $x(t)=0$ for $t<0$ and $x(t)$ contains no impulses or higher-order singularities at $t=0$, then

$$x(0^+)=\lim_{s\to\infty}sX(s)$$

If $x(t)=0$ for $t<0$ and $x(t)$ has a finite limit as $t\longrightarrow\infty$, then

$$\lim_{t\to\infty}x(t)=\lim_{s\to 0}sX(s)$$

TABLE 9.2 LAPLACE TRANSFORMS OF ELEMENTARY FUNCTIONS

Transform pair	Signal	Transform	ROC
1	$\delta(t)$	1	All s
2	$u(t)$	$\frac{1}{s}$	$\Re e\{s\}>0$
3	$-u(-t)$	$\frac{1}{s}$	$\Re e\{s\}<0$
4	$\frac{t^{n-1}}{(n-1)!}u(t)$	$\frac{1}{s^n}$	$\Re e\{s\}>0$
5	$-\frac{t^{n-1}}{(n-1)!}u(-t)$	$\frac{1}{s^n}$	$\Re e\{s\}<0$
6	$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$	$\Re e\{s\}>-\alpha$
7	$-e^{-\alpha t}u(-t)$	$\frac{1}{s+\alpha}$	$\Re e\{s\}<-\alpha$
8	$\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t)$	$\frac{1}{(s+\alpha)^n}$	$\Re e\{s\}>-\alpha$
9	$-\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(-t)$	$\frac{1}{(s+\alpha)^n}$	$\Re e\{s\}<-\alpha$
10	$\delta(t-T)$	e^{-sT}	All s
11	$[\cos\omega_0t]u(t)$	$\frac{s}{s^2+\omega_0^2}$	$\Re e\{s\}>0$
12	$[\sin\omega_0t]u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\Re e\{s\}>0$
13	$[e^{-\alpha t}\cos\omega_0t]u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega_0^2}$	$\Re e\{s\}>-\alpha$
14	$[e^{-\alpha t}\sin\omega_0t]u(t)$	$\frac{\omega_0}{(s+\alpha)^2+\omega_0^2}$	$\Re e\{s\}>-\alpha$
15	$u_n(t)=\frac{d^n\delta(t)}{dt^n}$	s^n	All s
16	$u_{-n}(t)=\underbrace{u(t)*\cdots*u(t)}_{n\text{ times}}$	$\frac{1}{s^n}$	$\Re e\{s\}>0$

TABLE 3.2 ***The Four Fourier Representations***

Time Domain	Periodic	Nonperiodic	
Continuous	Fourier Series $x(t)=\sum_{k=-\infty}^{\infty}X[k]e^{jk\omega_o t}$ $X[k]=\frac{1}{T}\int_{\langle T\rangle}x(t)e^{-jk\omega_o t}dt$ $x(t)$ has period T $\omega_o=\frac{2\pi}{T}$	Fourier Transform $x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(j\omega)e^{j\omega t}d\omega$ $X(j\omega)=\int_{-\infty}^{\infty}x(t)e^{-j\omega t}dt$	Nonperiodic
Discrete	Discrete-Time Fourier Series $x[n]=\sum_{k=\langle N\rangle}X[k]e^{jk\Omega_o n}$ $X[k]=\frac{1}{N}\sum_{n=\langle N\rangle}x[n]e^{-jk\Omega_o n}$ $x[n]$ and $X[k]$ have period N $\Omega_o=\frac{2\pi}{N}$	Discrete-Time Fourier Transform $x[n]=\frac{1}{2\pi}\int_{-\pi}^{\pi}X(e^{j\Omega})e^{j\Omega n}d\Omega$ $X(e^{j\Omega})=\sum_{n=-\infty}^{\infty}x[n]e^{-j\Omega n}$ $X(e^{j\Omega})$ has period 2π $\boxed{X(e^{j(\omega+2\pi)})=X(e^{j\omega}).}$	Periodic
	Discrete	Continuous	Frequency Domain

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TABLE 3.1 PROPERTIES OF CONTINUOUS-TIME FOURIER SERIES

Property	Section	Periodic Signal	Fourier Series Coefficients
		$\left.\begin{matrix} x(t) \\ y(t) \end{matrix}\right\}$ Periodic with period T and fundamental frequency $\omega_0 = 2\pi/T$	a_k b_k
Linearity	3.5.1	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time Shifting	3.5.2	$x(t - t_0)$	$a_k e^{-jk\omega_0 t_0} = a_k e^{-jk(2\pi/T)t_0}$
Frequency Shifting		$e^{jM\omega_0 t} = e^{jM(2\pi/T)t}x(t)$	a_{k-M}
Conjugation	3.5.6	$x^*(t)$	a^*_{-k}
Time Reversal	3.5.3	$x(-t)$	a_{-k}
Time Scaling	3.5.4	$x(\alpha t), \alpha > 0$ (periodic with period T/α)	a_k
Periodic Convolution		$\int_T x(\tau)y(t-\tau)d\tau$	$Ta_k b_k$
Multiplication	3.5.5	$x(t)y(t)$	$\sum_{l=-\infty}^{+\infty} a_l b_{k-l}$
Differentiation		$\frac{dx(t)}{dt}$	$jk\omega_0 a_k = jk\frac{2\pi}{T}a_k$
Integration		$\int_{-\infty}^{t} x(t)\,dt$ (finite valued and periodic only if $a_0 = 0$)	$\left(\frac{1}{jk\omega_0}\right)a_k = \left(\frac{1}{jk(2\pi/T)}\right)a_k$
Conjugate Symmetry for Real Signals	3.5.6	$x(t)$ real	$\begin{cases} a_k = a^*_{-k} \\ \Re e\{a_k\} = \Re e\{a_{-k}\} \\ \Im m\{a_k\} = -\Im m\{a_{-k}\} \\ \lvert a_k\rvert = \lvert a_{-k}\rvert \\ \sphericalangle a_k = -\sphericalangle a_{-k} \end{cases}$
Real and Even Signals	3.5.6	$x(t)$ real and even	a_k real and even
Real and Odd Signals	3.5.6	$x(t)$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals		$\begin{cases} x_e(t) = \mathcal{E}v\{x(t)\} & [x(t) \text{ real}] \\ x_o(t) = \mathcal{O}d\{x(t)\} & [x(t) \text{ real}] \end{cases}$	$\Re e\{a_k\}$ $j\Im m\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{T}\int_T \lvert x(t)\rvert^2 dt = \sum_{k=-\infty}^{+\infty} \lvert a_k\rvert^2$$

TABLE 3.2 PROPERTIES OF DISCRETE-TIME FOURIER SERIES

Property	Periodic Signal	Fourier Series Coefficients
	$\left.\begin{matrix} x[n] \\ y[n] \end{matrix}\right\}$ Periodic with period N and fundamental frequency $\omega_0 = 2\pi/N$	$\left.\begin{matrix} a_k \\ b_k \end{matrix}\right\}$ Periodic with period N
Linearity	$Ax[n] + By[n]$	$Aa_k + Bb_k$
Time Shifting	$x[n - n_0]$	$a_k e^{-jk(2\pi/N)n_0}$
Frequency Shifting	$e^{jM(2\pi/N)n}x[n]$	a_{k-M}
Conjugation	$x^*[n]$	a^*_{-k}
Time Reversal	$x[-n]$	a_{-k}
Time Scaling	$x_{(m)}[n] = \begin{cases} x[n/m], & \text{if } n \text{ is a multiple of } m \\ 0, & \text{if } n \text{ is not a multiple of } m \end{cases}$ (periodic with period mN)	$\frac{1}{m}a_k$ $\left(\begin{matrix}\text{viewed as periodic} \\ \text{with period } mN\end{matrix}\right)$
Periodic Convolution	$\sum_{r=\langle N\rangle} x[r]y[n-r]$	$Na_k b_k$
Multiplication	$x[n]y[n]$	$\sum_{l=\langle N\rangle} a_l b_{k-l}$
First Difference	$x[n] - x[n-1]$	$(1 - e^{-jk(2\pi/N)})a_k$
Running Sum	$\sum_{k=-\infty}^{n} x[k]$ $\left(\begin{matrix}\text{finite valued and periodic only} \\ \text{if } a_0 = 0\end{matrix}\right)$	$\left(\frac{1}{(1 - e^{-jk(2\pi/N)})}\right)a_k$
Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} a_k = a^*_{-k} \\ \Re e\{a_k\} = \Re e\{a_{-k}\} \\ \Im m\{a_k\} = -\Im m\{a_{-k}\} \\ \lvert a_k\rvert = \lvert a_{-k}\rvert \\ \sphericalangle a_k = -\sphericalangle a_{-k} \end{cases}$
Real and Even Signals	$x[n]$ real and even	a_k real and even
Real and Odd Signals	$x[n]$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals	$\begin{cases} x_e[n] = \mathcal{E}v\{x[n]\} & [x[n] \text{ real}] \\ x_o[n] = \mathcal{O}d\{x[n]\} & [x[n] \text{ real}] \end{cases}$	$\Re e\{a_k\}$ $j\Im m\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{N}\sum_{n=\langle N\rangle} \lvert x[n]\rvert^2 = \sum_{k=\langle N\rangle} \lvert a_k\rvert^2$$

5.1 PROPERTIES OF THE DISCRETE-TIME FOURIER TRANSFORM

Property	Aperiodic Signal	Fourier Transform
	$x[n]$ $y[n]$	$\left.\begin{matrix} X(e^{j\omega}) \\ Y(e^{j\omega}) \end{matrix}\right\}$ periodic with period 2π
Linearity	$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
Time Shifting	$x[n - n_0]$	$e^{-j\omega n_0}X(e^{j\omega})$
Frequency Shifting	$e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$
Conjugation	$x^*[n]$	$X^*(e^{-j\omega})$
Time Reversal	$x[-n]$	$X(e^{-j\omega})$
Time Expansion	$x_{(k)}[n] = \begin{cases} x[n/k], & \text{if } n = \text{multiple of } k \\ 0, & \text{if } n \neq \text{multiple of } k \end{cases}$	$X(e^{jk\omega})$
Convolution	$x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
Multiplication	$x[n]y[n]$	$\frac{1}{2\pi}\int_{2\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$
Differencing in Time	$x[n] - x[n-1]$	$(1 - e^{-j\omega})X(e^{j\omega})$
Accumulation	$\sum_{k=-\infty}^{n} x[k]$	$\frac{1}{1 - e^{-j\omega}}X(e^{j\omega}) + \pi X(e^{j0})\sum_{k=-\infty}^{+\infty}\delta(\omega - 2\pi k)$
Differentiation in Frequency	$nx[n]$	$j\frac{dX(e^{j\omega})}{d\omega}$
Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} X(e^{j\omega}) = X^*(e^{-j\omega}) \\ \Re e\{X(e^{j\omega})\} = \Re e\{X(e^{-j\omega})\} \\ \Im m\{X(e^{j\omega})\} = -\Im m\{X(e^{-j\omega})\} \\ \lvert X(e^{j\omega})\rvert = \lvert X(e^{-j\omega})\rvert \\ \sphericalangle X(e^{j\omega}) = -\sphericalangle X(e^{-j\omega}) \end{cases}$
Symmetry for Real, Even Signals	$x[n]$ real an even	$X(e^{j\omega})$ real and even
Symmetry for Real, Odd Signals	$x[n]$ real and odd	$X(e^{j\omega})$ purely imaginary and odd
Even-odd Decomposition of Real Signals	$x_e[n] = \mathcal{E}v\{x[n]\}$ $[x[n]$ real$]$ $x_o[n] = \mathcal{O}d\{x[n]\}$ $[x[n]$ real$]$	$\Re e\{X(e^{j\omega})\}$ $j\Im m\{X(e^{j\omega})\}$

Parseval's Relation for Aperiodic Signals

$$\sum_{n=-\infty}^{+\infty} \lvert x[n]\rvert^2 = \frac{1}{2\pi}\int_{2\pi} \lvert X(e^{j\omega})\rvert^2 d\omega$$

TABLE 5.2 BASIC DISCRETE-TIME FOURIER TRANSFORM PAIRS

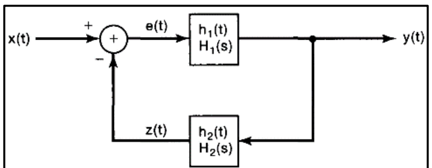
Signal	Fourier Transform	Fourier Series Coefficients (if periodic)
$\sum_{k=\langle N\rangle} a_k e^{jk(2\pi/N)n}$	$2\pi\sum_{k=-\infty}^{+\infty} a_k\delta\left(\omega - \frac{2\pi k}{N}\right)$	a_k
$e^{j\omega_0 n}$	$2\pi\sum_{l=-\infty}^{+\infty}\delta(\omega - \omega_0 - 2\pi l)$	(a) $\omega_0 = \frac{2\pi m}{N}$ $a_k = \begin{cases} 1, & k = m, m \pm N, m \pm 2N, \ldots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$\cos\omega_0 n$	$\pi\sum_{l=-\infty}^{+\infty}\{\delta(\omega - \omega_0 - 2\pi l) + \delta(\omega + \omega_0 - 2\pi l)\}$	(a) $\omega_0 = \frac{2\pi m}{N}$ $a_k = \begin{cases} \frac{1}{2}, & k = \pm m, \pm m \pm N, \pm m \pm 2N, \ldots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$\sin\omega_0 n$	$\frac{\pi}{j}\sum_{l=-\infty}^{+\infty}\{\delta(\omega - \omega_0 - 2\pi l) - \delta(\omega + \omega_0 - 2\pi l)\}$	(a) $\omega_0 = \frac{2\pi r}{N}$ $a_k = \begin{cases} \frac{1}{2j}, & k = r, r \pm N, r \pm 2N, \ldots \\ -\frac{1}{2j}, & k = -r, -r \pm N, -r \pm 2N, \ldots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$x[n] = 1$	$2\pi\sum_{l=-\infty}^{+\infty}\delta(\omega - 2\pi l)$	$a_k = \begin{cases} 1, & k = 0, \pm N, \pm 2N, \ldots \\ 0, & \text{otherwise} \end{cases}$
Periodic square wave $x[n] = \begin{cases} 1, & \lvert n\rvert \le N_1 \\ 0, & N_1 < \lvert n\rvert \le N/2 \end{cases}$ and $x[n + N] = x[n]$	$2\pi\sum_{k=-\infty}^{+\infty} a_k\delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k = \frac{\sin[(2\pi k/N)(N_1 + \frac{1}{2})]}{N\sin[2\pi k/2N]}, k \neq 0, \pm N, \pm 2N, \ldots$ $a_k = \frac{2N_1 + 1}{N}, k = 0, \pm N, \pm 2N, \ldots$
$\sum_{k=-\infty}^{+\infty}\delta[n - kN]$	$\frac{2\pi}{N}\sum_{k=-\infty}^{+\infty}\delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k = \frac{1}{N}$ for all k
$a^n u[n], \quad \lvert a\rvert < 1$	$\frac{1}{1 - ae^{-j\omega}}$	—
$x[n]\begin{cases} 1, & \lvert n\rvert \le N_1 \\ 0, & \lvert n\rvert > N_1 \end{cases}$	$\frac{\sin[\omega(N_1 + \frac{1}{2})]}{\sin(\omega/2)}$	—
$\frac{\sin Wn}{\pi n} = \frac{W}{\pi}\text{sinc}\left(\frac{Wn}{\pi}\right)$ $0 < W < \pi$	$X(\omega) = \begin{cases} 1, & 0 \le \lvert\omega\rvert \le W \\ 0, & W < \lvert\omega\rvert \le \pi \end{cases}$ $X(\omega)$ periodic with period 2π	
$\delta[n]$	1	
$u[n]$	$\frac{1}{1 - e^{-j\omega}} + \sum_{k=-\infty}^{+\infty}\pi\delta(\omega - 2\pi k)$	
$\delta[n - n_0]$	$e^{-j\omega n_0}$	
$(n + 1)a^n u[n], \quad \lvert a\rvert < 1$	$\frac{1}{(1 - ae^{-j\omega})^2}$	
$\frac{(n + r - 1)!}{n!(r - 1)!}a^n u[n], \quad \lvert a\rvert < 1$	$\frac{1}{(1 - ae^{-j\omega})^r}$	

$\int_{-\infty}^{t} f(\tau)d\tau$	$\frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$
$\delta(t)$	1
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
sgn (t)	$\frac{2}{j\omega}$
$j\frac{1}{\pi t}$	$\text{sgn}(\omega)$

TABLE 5.3 SUMMARY OF FOURIER SERIES AND TRANSFORM EXPRESSIONS

	Continuous time		Discrete time	
	Time domain	**Frequency domain**	**Time domain**	**Frequency domain**
Fourier Series	$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$ continuous time periodic in time	$a_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_0 t}$ discrete frequency aperiodic in frequency	$x[n] = \sum_{k=\langle N\rangle} a_k e^{jk(2\pi/N)n}$ discrete time periodic in time	duality $a_k = \frac{1}{N}\sum_{k=\langle N\rangle} x[n]e^{-jk(2\pi/N)n}$ discrete frequency periodic in frequency
Fourier Transform	$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)e^{j\omega t}d\omega$ continuous time aperiodic in time	duality $X(j\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}dt$ **continuous frequency** **aperiodic in frequency**	duality $x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}$ **discrete time** **aperiodic in time**	$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\omega n}$ continuous frequency periodic in frequency

We first must identify a continuous-time signal g(t) [=x(t) of our equation] with period T = 2π and Fourier coefficients a_k = x[k]. We then 'brand' g(t) as $X(e^{jw})$ and mould the equation to synthesis equation of DTFT!



$$Y(s) = H_1(s)[X(s) - H_2(s)Y(s)],$$

$$\frac{Y(s)}{X(s)} = H(s) = \frac{H_1(s)}{1 + H_1(s)H_2(s)}.$$

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EE229 Cheatsheet

4.12. Consider the Fourier transform pair

$$e^{-|t|} \overset{\mathfrak{F}}{\longleftrightarrow} \frac{2}{1+\omega^2}.$$

(a) Use the appropriate Fourier transform properties to find the Fourier transform of $te^{-|t|}$.

(b) Use the result from part (a), along with the duality property, to determine the Fourier transform of

$$\frac{4t}{(1+t^2)^2}.$$

4.12. (a) From Example 4.2 we know that

$$e^{-|t|} \overset{FT}{\longleftrightarrow} \frac{2}{1+\omega^2}.$$

Using the differentiation in frequency property, we have

$$te^{-|t|} \overset{FT}{\longleftrightarrow} j\frac{d}{d\omega}\left\{\frac{2}{1+\omega^2}\right\} = -\frac{4j\omega}{(1+\omega^2)^2}$$

(b) The duality property states that if

$$g(t) \overset{FT}{\longleftrightarrow} G(j\omega)$$

then

$$G(t) \overset{FT}{\longleftrightarrow} 2\pi g(j\omega).$$

Now, since

$$te^{-|t|} \overset{FT}{\longleftrightarrow} -\frac{4j\omega}{(1+\omega^2)^2}$$

we may use duality to write

$$-\frac{4jt}{(1+t^2)^2} \overset{FT}{\longleftrightarrow} 2\pi\omega e^{-|\omega|}$$

Multiplying both sides by j, we obtain

$$\frac{4t}{(1+t^2)^2} \overset{FT}{\longleftrightarrow} j2\pi\omega e^{-|\omega|}.$$

Example 4.12

Suppose that we wish to calculate the Fourier transform $X(j\omega)$ for the signal $x(t)$ displayed in Figure 4.16(a). Rather than applying the Fourier integral directly to $x(t)$, we instead consider the signal

$$g(t) = \frac{d}{dt}x(t).$$

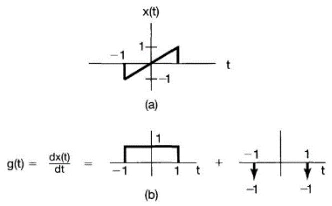

Figure 4.16 (a) A signal $x(t)$ for which the Fourier transform is to be evaluated; (b) representation of the derivative of $x(t)$ as the sum of two components.

As illustrated in Figure 4.16(b), $g(t)$ is the sum of a rectangular pulse and two impulses. The Fourier transforms of each of these component signals may be determined from Table 4.2:

$$G(j\omega) = \left(\frac{2\sin\omega}{\omega}\right) - e^{j\omega} - e^{-j\omega}.$$

Note that $G(0) = 0$. Using the integration property, we obtain

$$X(j\omega) = \frac{G(j\omega)}{j\omega} + \pi G(0)\delta(\omega).$$

With $G(0) = 0$ this becomes

$$X(j\omega) = \frac{2\sin\omega}{j\omega^2} - \frac{2\cos\omega}{j\omega}.$$

The expression for $X(j\omega)$ is purely imaginary and odd, which is consistent with the fact that $x(t)$ is real and odd.

We have the function

$$f(x) = \begin{cases} 1+x, & -1 \le x \le 0 \\ 1-x, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

we computed its Fourier transform:

$$\hat{f}(t) = \frac{\sin^2(\pi t)}{\pi^2 t^2}$$

we are asked to deduce the value of the integral

$$\int_0^\infty \frac{\sin^4 x}{x^4}dx$$

how to do this?

By Parseval's theorem we have

$$\begin{aligned}\int_{-\infty}^{\infty}\frac{\sin^4(x)}{x^4}dx &= \pi\int_{-\infty}^{\infty}\frac{\sin^4(\pi t)}{(\pi t)^4}dt \\ &= \pi\int_{-1}^{0}(1+x)^2dx + \pi\int_0^1(1-x)^2dx \\ &= \frac{2}{3}\pi\end{aligned}$$

and now observe that

$$\int_{-\infty}^{\infty}\frac{\sin^4(x)}{x^4}dx = 2\int_0^\infty\frac{\sin^4(x)}{x^4}dx$$

so

$$\int_0^\infty\frac{\sin^4(x)}{x^4}dx = \frac{\pi}{3}.$$

Example 3.14 ****

Suppose we are given the following facts about a sequence $x[n]$:

1. $x[n]$ is periodic with period $N = 6$.
2. $\sum_{n=0}^{5} x[n] = 2$.
3. $\sum_{n=2}^{7}(-1)^n x[n] = 1$.
4. $x[n]$ has the minimum power per period among the set of signals satisfying the preceding three conditions.

Let us determine the sequence $x[n]$. We denote the Fourier series coefficients of $x[n]$ by a_k. From Fact 2, we conclude that $a_0 = 1/3$. Noting that $(-1)^n = e^{-j\pi n} = e^{-j(2\pi/6)3n}$, we see from Fact 3 that $a_3 = 1/6$. From Parseval's relation (see Table 3.2), the average power in $x[n]$ is

$$P = \sum_{k=0}^{5}|a_k|^2. \tag{3.115}$$

Since each nonzero coefficient contributes a positive amount to P, and since the values of a_0 and a_3 are prespecified, the value of P is minimized by choosing $a_1 = a_2 = a_4 = a_5 = 0$. It then follows that

$$x[n] = a_0 + a_3 e^{j\pi n} = (1/3) + (1/6)(-1)^n, \tag{3.116}$$

which is sketched in Figure 3.20.

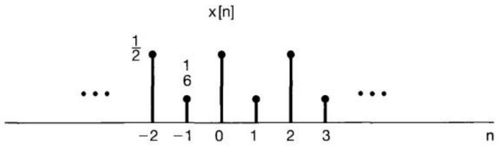

Figure 3.20 Sequence $x[n]$ that is consistent with the properties specified in Example 3.14.

4.15. Let $x(t)$ be a signal with Fourier transform $X(j\omega)$. Suppose we are given the following facts:

1. $x(t)$ is real.
2. $x(t) = 0$ for $t \le 0$.
3. $\frac{1}{2\pi}\int_{-\infty}^{\infty}\Re e\{X(j\omega)\}e^{j\omega t}\,d\omega = |t|e^{-|t|}$.

Determine a closed-form expression for $x(t)$.

4.15. Since $x(t)$ is real,

$$\mathcal{E}v\{x(t)\} = \frac{x(t)+x(-t)}{2} \overset{FT}{\longleftrightarrow} \mathcal{R}e\{X(j\omega)\}.$$

We are given that

$$\mathcal{IFT}\{\mathcal{R}e\{X(j\omega)\}\} = |t|e^{-|t|}.$$

Therefore,

$$\mathcal{E}v\{x(t)\} = \frac{x(t)+x(-t)}{2} = |t|e^{-|t|}.$$

We also know that $x(t) = 0$ for $t \le 0$. This implies that $x(-t)$ is zero for $t > 0$. We may conclude that

$$x(t) = 2|t|e^{-|t|} \quad \text{for } t \ge 0$$

Therefore,

$$x(t) = 2te^{-t}u(t)$$

4.37. Consider the signal $x(t)$ in Figure P4.37.

(a) Find the Fourier transform $X(j\omega)$ of $x(t)$.

(b) Sketch the signal

$$\tilde{x}(t) = x(t) * \sum_{k=-\infty}^{\infty}\delta(t-4k).$$

(c) Find another signal $g(t)$ such that $g(t)$ is not the same as $x(t)$ and

$$\tilde{x}(t) = g(t) * \sum_{k=-\infty}^{\infty}\delta(t-4k).$$

(d) Argue that, although $G(j\omega)$ is different from $X(j\omega)$, $G(j\frac{\pi k}{2}) = X(j\frac{\pi k}{2})$ for all integers k. You should not explicitly evaluate $G(j\omega)$ to answer this question.

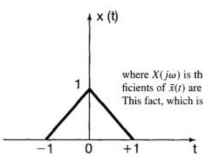

Figure P4.37

$$a_k = \frac{1}{T}X(j\omega)\Big|_{\omega=k\omega_0}, \tag{4.10}$$

where $X(j\omega)$ is the Fourier transform of $x(t)$. Equation 4.10 states that the Fourier coefficients of $\tilde{x}(t)$ are proportional to samples of the Fourier transform of one period of $\tilde{x}(t)$. This fact, which is often of use in practice, is examined further in Problem 4.37.

4.37. (a) Note that

$$x(t) = x_1(t) * x_1(t),$$

where

$$x_1(t) = \begin{cases}1, & |\omega| < \frac{1}{2} \\ 0, & \text{otherwise}\end{cases}$$

Also, the Fourier transform $X_1(j\omega)$ of $x_1(t)$ is

$$X_1(j\omega) = 2\frac{\sin(\omega/2)}{\omega}.$$

Using the convolution property we have

$$X(j\omega) = X_1(j\omega)X_1(j\omega) = \left[2\frac{\sin(\omega/2)}{\omega}\right]^2.$$

(b) The signal $\tilde{x}(t)$ is as shown in Figure S4.37

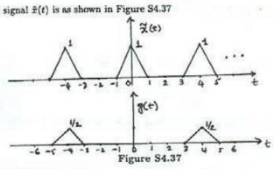

Figure S4.37

(c) One possible choice of $g(t)$ is as shown in Figure S4.37.

(d) Note that

$$\tilde{X}(j\omega) = X(j\omega)\frac{\pi}{2}\sum_{k=-\infty}^{\infty}\delta(j(\omega - k\tfrac{\pi}{2})) = G(j\omega)\frac{\pi}{2}\sum_{k=-\infty}^{\infty}\delta(j(\omega - k\tfrac{\pi}{2})).$$

This may also be written as

$$\tilde{X}(j\omega) = \frac{\pi}{2}\sum_{k=-\infty}^{\infty}X(j\pi k/2)\delta(j(\omega - k\tfrac{\pi}{2})) = \frac{\pi}{2}\sum_{k=-\infty}^{\infty}G(j\pi k/2)\delta(j(\omega - k\tfrac{\pi}{2}))$$

Clearly, this is possible only if

$$G(j\pi k/2) = X(j\pi k/2).$$

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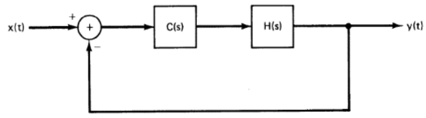


$$\frac{Y(s)}{X(s)} = \frac{C(s)H(s)}{1+C(s)H(s)}$$

Properties of Laplace Transform

Property 1: The ROC of X(s) consists of strips parallel to the jw-axis in the s-plane.

Property 2: For rational Laplace transforms, the ROC does not contain any poles.

Property 3: If x(t) is of finite duration and is absolutely integrable, then the ROC is the entire s-plane.

Property 4: If x(t) is right sided, and if the line Re{s} = σ_0 is in the ROC, then all values of s for which Re{s} > σ_0 will also be in the ROC.

Property 5: If x(t) is left sided, and if the line Re{s} = σ_0 is in the ROC, then all values of s for which Re{s} < σ_0 will also be in the ROC.

Property 6: If x(t) is two sided, and if the line Re{s} = σ_0 is in the ROC, then the ROC will consist of a strip in the s-plane that includes the line Re{s} = σ_0.

Property 7: If the Laplace transform X(s) of x(t) is rational, then its ROC is bounded by poles or extends to infinity. In addition, no poles of X(s) are contained in the ROC.

Property 8: If the Laplace transform X(s) of x(t) is rational, then if x(t) is right sided, the ROC is the region in the s-plane to the right of the rightmost pole. If x(t) is left sided, the ROC is the region in the s-plane to the left of the leftmost pole.

Property 9: The ROC associated with the system function for a causal system is a right-half plane.

Property 10: For a system with a rational system function, causality of the system is equivalent to the ROC being the right-half plane to the right of the rightmost pole.

Property 11: An LTI system is stable if and only if the ROC of its system function H(s) includes the entire jw-axis [i.e., Re{s} = 0].

Property 12: A causal system **with rational system function H(s)** is stable if and only if all of the poles of H(s) lie in the left-half of the s-plane-i.e., all of the poles have negative real parts.

Sampling

If we've a CT signal and
→ Equally spaced samples of $x(t)$, sampled at a sampling period of T: $x(nT) \longrightarrow n = 0, \pm 1, \pm 2, \ldots$

and if

$x(t)$ is Bandlimited : $X(\omega) = 0$, $|\omega| > \omega_m$ → Nyquist freq.

then under the condition:

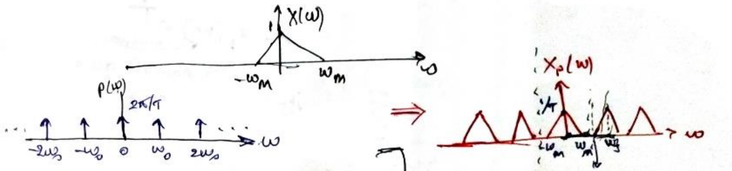
$$\boxed{\frac{2\pi}{T} \triangleq \omega_s > \underbrace{2\omega_M}_{\text{(Nyq. rate)}}}$$

then $x(t)$ is uniquely recoverable from the set of samples

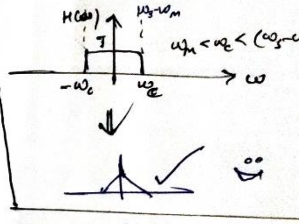
$$x(t) \longrightarrow \otimes \longrightarrow \cancel{y(t)}\ x_p(t), \quad p(t) \downarrow$$

$$x_p(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t-nT) = \sum_{n=-\infty}^{\infty} x(nT)\,\delta(t-nT)$$

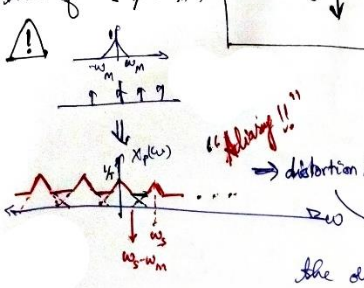
$$\therefore X_p(\omega) = \frac{1}{2\pi}\left(X(\omega) * P(\omega)\right).$$



Recovery



But then if $\omega_s \ngtr 2\omega_M$,



"Aliasing!!" → distortion

Now, if we pass this through a low-pass filter, we won't get back the original signal ∵ the individual replications have overlapped.

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