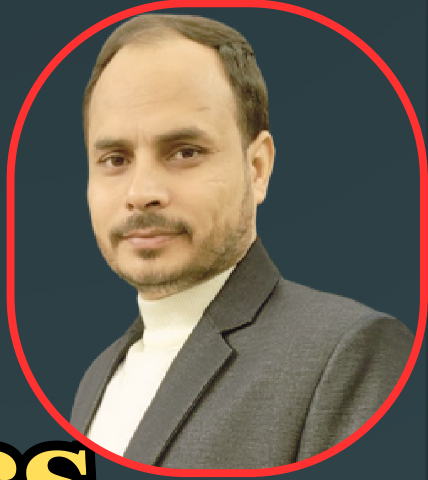
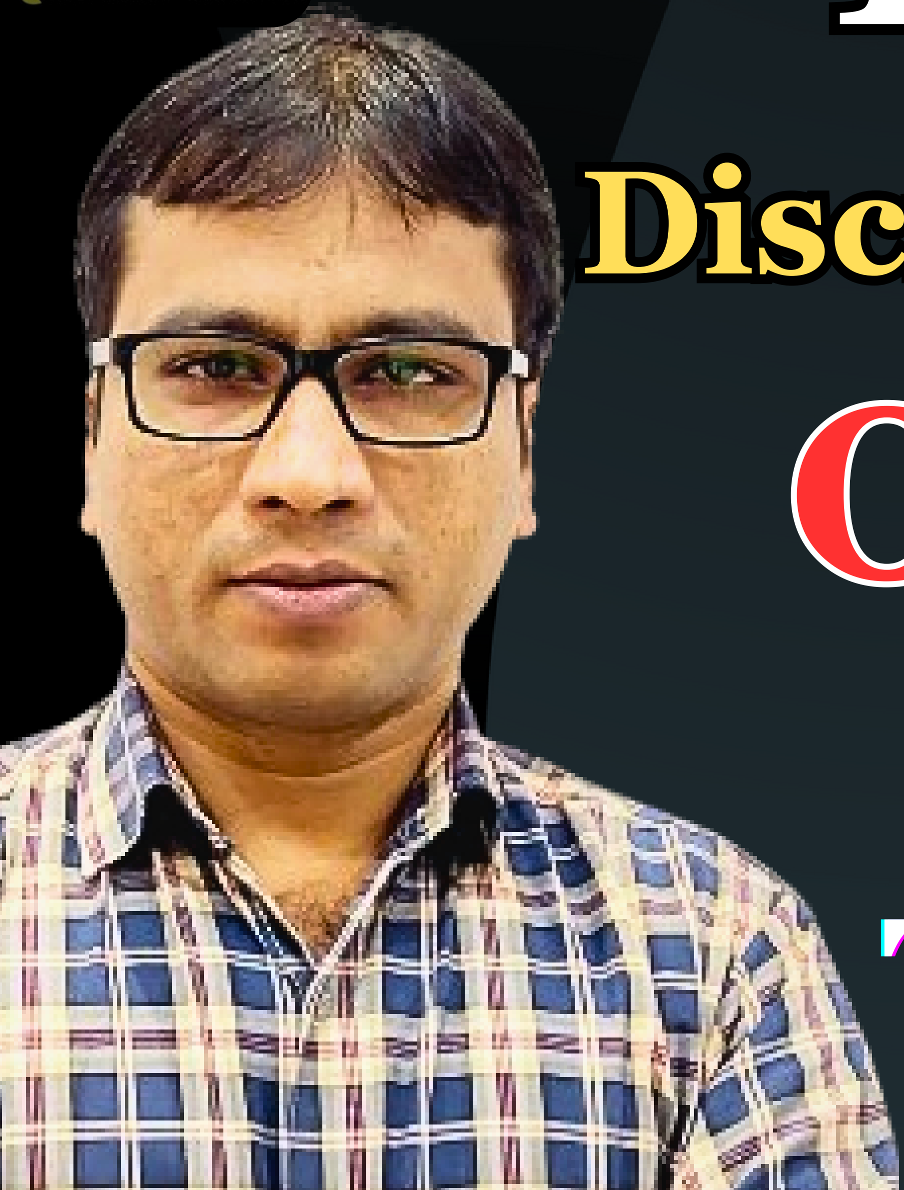
GATEWAY CLASSES

DSTL



Discrete Mathematics



ONE SHOT

Unit-III

Theory of Logics

Unit – 3 III Semester Theory Examination – 2022 – 23

GW GATEWAY CLASSES

Section – A

j. Identify whether $(p \wedge q) \rightarrow (p \vee q)$ is tautology or contradiction with using Truth table.

Section – B

c. (i) Express Converse, Inverse and contrapositive of the following statement "If $x + 5 = 8$ then $x = 3$"

(ii) Show that the statements $P \leftrightarrow Q$ and $(P \wedge Q) \vee (\neg P \wedge \neg Q)$ are equivalent

Section – C

3. Attempt any one part of the following

(a) Prove the validity of the following argument. If Mary runs for office, She will be elected. If Mary attends the meeting, she will run for office. Either Mary will attend the meeting or she will go to India. But Mary cannot go to India. "Thus Mary will be elected".

(b) Convert the following two statements in quantified expressions of predicate logic

(i) For every number there is a number greater than that number.

(ii) Sum of every two integer is an integer.

(iii) Not Every man is perfect.

(iv) There is no student in the class who knows Spanish and German

Unit – 3 III Semester Theory Examination – 2021 – 22

Section – A

g. *Translate the conditional statement* If it rains, then I will stay at home" *into contrapositive, converse and inverse statement* .

h. *State Universal Modus Ponens and Universal Modus Tollens laws.*

Section – B

d. *Construct the truth table for the following statements*:

(i) $(P \to Q') \to P'$ (ii) $P \leftrightarrow (P' \vee Q')$.

Section – C

6. *Attempt any one part of the following*

(a) *Use rules of inference to Justify that the three hypotheses* (i) "*If it does not rain or if it is not foggy, then the sailing race will be held and the life saving demonstration will go on.* (ii) *If the sailing race is held then the trophy will be awarded.* " (iii) The trophy was not awarded." *imply the conclusion* (iv) "*It rained.*

(b) *Justify that the following premises are inconsistent.* (i) *If Nirmala misses many classes through illness then he fails high school.* (ii) *If Nirmala fails high school, then he is uneducated.* (iii) *If Nirmala reads a lot of books then he is not uneducated.* (iv) *Nirmala misses many classes through illness and reads a lot of books.*

Section – A

g. Write the negation of th following statement:
"If I wake up early in th morning, then I will be healthy."

h. Express the following statement in symbolic form: "All flowers are beautiful."

Section – B

d. Prove the validity of the following argument "If I get the job and work hard, then I will get promotd, then I will be happy. I will not be happy.
Therefore, either I will not get the job, or I will not work hard.".

Section – C

6. Attempt any one part of the following

(a) Define tautology, contradiction and contingency? Check whether $(p \vee q) \wedge (\sim p \vee r) \rightarrow (q \vee r)$ is a tautology, contradiction or contingency.

(b) Translate the following statements in symbolic form

(i) The sum of two positive integers is always positive.
(ii) Everyone is loved by someone.
(iii) Some people are not admired by everone.
(iv) If a person is female and is a parent, then this person is someone's mother.

Section – A

f. Write the contra positive of the implication: "If it is Sunday then it is a holiday"

g. Show that the propositions $p \rightarrow q$ and $\neg p \vee q$ are logically equivalent.

Section – B

d. Show that $((P \vee Q) \wedge \neg(\neg Q \vee \neg R)) \vee (\neg P \vee \neg Q) \vee (\neg P \vee \neg R)$ is a tautology by using equivalences.

Section – C

6. Attempt any one part of the following

(b) Obtain the principle disjunctive and conjunctive normal forms of the formula $(\boxminus p \rightarrow r) \wedge (p \leftrightarrow q)$.

6. Attempt any one part of the following

(a) Explain various Rules of Inference for Propositional Logic.

(b) Prove the validity of the following argument "if the races are fixed so the casinos are crooked, then the tourist trade will decline. If the tourist trade decreases, then the police will be happy. The police force is never happy. Therefore, the races are not fixed.

Unit – 3 III Semester Theory Examination – 2018 – 19

GW GATEWAY CLASSES

Section – A

d. What are the contrapositive, converse, and the inverse of the conditional statement "The home team wins whenever it is raining?"

g. Show that $\neg(p \lor q)$ and $\neg p \land \neg q$ are logically equivalent.

Section – B

d. Show that $((p \lor q) \land \sim(\sim p \land (\sim q \lor \sim r))) \lor (\sim p \land \sim q) \lor (\sim p \lor \sim r)$ is a tautology without using truth table.

Section – C

6. Attempt any one part of the following

(a) What is a tautology, contradiction and contingency? Show that $(p \lor q) \land (\neg p \lor r) \rightarrow (q \lor r)$ is a tautology, contradiction or contingency.

(b) Show that the premises "It is not sunny this afternoon and it is colder than yesterday", "We will go swimming only if it is sunny"," If we do not go swimming, then we will take a canoe trip", and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset."

UNIT -3 : Theory of Logics

- *Propositions*
- *Truth tables*
- *Tautology*
- *Satisfiability*
- *Contradiction*
- *Algebra of Proposition*
- *Theory of inference*
- *Predicate logic*
- *First order Predicate*
- *Well Formed Formula of predicate*
- *Quantifiers*
- *Inference Theory of predicate logic*

Propositions (Statement):

A proposition or statement is a declarative sentence that is either true or false, but not both simultaneously.

- If any proposition is true then its truth value is denoted by T or 1 and if the proposition is false then its truth value is denoted by F or 0.
- Questions, commands, order, exclamations, wish etc. are not proposition
- T , F are called truth value
- T , F are also called propositional constant

Following are the propositions

Propositions	Truth Value
(1) Two Plus Two is Four	T
(2) Two Plus Two is Six	F
(3) Paris is in India	F
(4) $x = 2$ is a solution of $x^2 = 4$	T
(5) The Sun rises in the west	F
(6) $6 < 7$	T
(7) $3 > 5$	F

Following are not propositions

(1) Where are you going ? (Question)

(2) Close the door (Command)

(3) What a hot day ! (Exclamations)

(4) May god held you (Wish)

(5) $4 - x = 8$

Propositions Variables

A proposition or statement is denoted by the letters p, q, r ..... called propositional variables.

Example

$p =$ Two plus Two is four

$q =$ Delhi is the capital of India.

Proposition

Simple Proposition
OR
Proposition
OR
Atomic Proposition

Compound Proposition
OR
Molecular Proposition
OR
Composite Proposition

Simple Proposition :

A proposition consisting of only a single propositional Variable or single propositional constant is called an simple proposition or proposition

Example

1. The sun rises in the west

   $p =$ The sun rises in the west

2. Delhi is in India

   $q =$ Delhi is in India

Compound Proposition

A proposition obtained from the combinations of two or more propositions by means of logical connectives or negating a single proposition.

Example

p q

1. John is intelligent or studies every night

$p \lor q$ = John is intelligent or studies every night

p q

2. The earth is round and revolves around the sun

$p \land q$ = The earth is round and revolves around the sun

Logical Connectives or Logical operators

The words or symbols used to form compound Statement (or propositions) are called logical connectives.

Table of Logical connectives with their symbols

S.No.	Name	Symbol	Connective Word
1	Negation	$\sim or \neg$	Not
2	Conjunction	$\wedge$	And
3	Disjunction	$\vee$	Or
4	Conditional	$\rightarrow or \Rightarrow$	If ...... then
5	Bi-conditional	$\leftrightarrow or \Leftrightarrow$	Iff or if and only if

Truth Table

A truth table is a table that shows the truth value of a compound Statement (or proposition) for all possible cases.

(1) Negation ($\sim$ *or* $\neg$)

If p is any proposition, the negation of p, denoted by $\sim p$ and read as not p, is a proposition which is false when p is true and true when p is false.

Truth table for negation

p	$\sim p$
T	F
F	T

Note

① All = some ... + Not

② All + Not | No | None = Some | someone

Example: What is the negation of each of the following propositions ?

(i) p : *Paris is in france.*

(ii) q : *All students are intelligent.*

(iii) r : *Today is tuesday.*

(iv) p : Ramesh is a good player of Hockey

Solution

(i) $\sim p$: *Paris is not in france.*

(ii) $\sim q$: *Some students are not intelligent.*

(iii) $\sim r$: *Today is not tuesday.*

(iv) $\sim p$: Ramesh is not a good player of Hockey

(2) Conjunction ($\wedge$)

If p and q are two statements, then conjunction of p and q is the compound statement denoted by $p \wedge q$ and read as "p and q"

The compound statement $p \wedge q$ is true when both p and q are true otherwise it is false.

Truth table for Conjunction

T T $\longrightarrow$ T otherwise F

(i) When compound statement have 2 components p and q

No. of rows $= 2^2 = 4$.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

(ii) When compound statement have 3 components p , q and r

No. of rows $= 2^3 = 8.$

p	q	r	$p \wedge q$	$p \wedge q \wedge r$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	F
T	F	F	F	F
F	T	T	F	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

Example : From the Conjunction of p and q for each of the following.

(i) p : *Ram is healthy* q : *He has blue eyes*

(ii) p : *It is cold* q : *It is raining*

(iii) $p : 5x + 6 = 26$ q : $x > 3$

Solution

(i) $p \wedge q$: *Ram is healthy and he has blue eyes.*

(ii) $p \wedge q$: *It is cold and raining.*

(iii) $p \wedge q : 5x + 6 = 26$ *and* $x > 3$.

(3) Disjunction ($\vee$)

If p and q is are two statement, then disjunction of p and q is the compound statement denoted by $p \vee q$ and read as "p or q"

The compound statement $p \vee q$ is false when both p and q are false otherwise it is true

Truth table for Disjunction F F $\longrightarrow$ F otherwise T

(i) When compound statement have 2 components p and q

No. of rows $= 2^2 = 4.$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

(ii) When compound statement have 3 components p, q, r

No. of rows $= 2^3 = 8$.

p	q	r	$p \vee q$	$p \vee q \vee r$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	F

Example : From the Disjunction of p and q for each of the following.

(i) p : *It is cold* q : *It is raining*

(ii) p : *He will go to Delhi* q : *He will go to Kolkata*

Solution

(i) $p \vee q$: *It is cold or raining.*

(ii) $p \vee q$: *He will go to Delhi or kolkata.*

Q.1: Find the truth table for the statement $(q \wedge r) \wedge (p \vee \sim r)$

p	q	r	$\sim r$	$q \wedge r$	$p \vee \sim r$	$(q \wedge r) \wedge (p \vee \sim r)$
T	T	T	F	T	T	T
T	T	F	T	F	T	F
T	F	T	F	F	T	F
T	F	F	T	F	T	F
F	T	T	F	T	F	F
F	T	F	T	F	T	F
F	F	T	F	F	F	F
F	F	F	T	F	T	F

Q.2:- Find the truth table of the compound proposition $(\sim p \wedge (\sim q \wedge r)) \vee (q \wedge r) \vee (q \wedge r)$ $= A$

p	q	r	$\sim p$	$\sim q$	$(\sim q \wedge r)$	$(\sim p \wedge (\sim q \wedge r))$	$q \wedge r$	$p \wedge r$	A
T	T	T	F	F	F	F	T	T	T
T	T	F	F	F	F	F	F	F	F
T	F	T	F	T	T	F	F	T	T
T	F	F	F	T	F	F	F	F	F
F	T	T	T	F	F	F	T	F	T
F	T	F	T	F	F	F	F	F	F
F	F	T	T	T	T	T	F	F	T
F	F	F	T	T	F	F	F	F	F

Q.3:- Find the truth table of the statement $(p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$ = A

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim p \wedge q$	$p \wedge \sim q$	$\sim p \wedge \sim q$	A
T	T	F	F	T	F	F	F	T
T	F	F	T	F	F	T	F	T
F	T	T	F	F	T	F	F	T
F	F	T	T	F	F	F	T	T

Discrete Mathematics by Gulshan Sir

DPP

Topic : Truth table for compound statement

Q.1. Make a truth table for the statement $(p \wedge q) \vee (\sim p)$

Q.2. Construct a truth table for the compound proposition $p^\wedge(\sim q \vee q)$

Q.3. Construct a truth table for the compound Statement $\sim (p \vee q) \vee (\sim p^\wedge \sim q)$

(4) Conditional statement or Direct implication or implication ($\Rightarrow$ *or* $\rightarrow$)

If p *and* q are statements, then the compound statement "if p *then* q ", denoted by $p \rightarrow q$ or $p \Rightarrow q$ is called conditional statement

T F $\longrightarrow$ F otherwise T

The compound statement $p \rightarrow q$ is false when p is true and q is false otherwise it is true.

$p \rightarrow q$

Antecedent or hypothesis

Consequent or Conclusion

Truth Table

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Example :

(i) p : $Tomorrow\ is\ sunday$

q : $Today\ is\ saturday$

$p \rightarrow q$: If tomorrow is Sunday then today is Saturday.

(ii) p : $It\ rains.$

q : $I\ will\ carry\ an\ umbrella$

$p \rightarrow q$: If it rains then I will carry an umbrella .

Kinds of conditional

If $p \to q$ is a conditional statement or Direct implication or implication.

(1) *Converse Implication* : $q \to p$

(2) *Inverse Implication* : $\sim p \to \sim q$

(3) *Contrapositive Implication* : $\sim q \to \sim p$

NOTE -

Negation of conditional Statement

$$\boxed{\begin{array}{l} p \to q \equiv \sim p \vee q \\ \sim(p \to q) \equiv p \wedge \sim q \end{array}}$$

Truth Table for kinds of conditional

p	q	$\sim p$	$\sim q$	$p \to q$	$q \to p$	$\sim p \to \sim q$	$\sim q \to \sim p$	$\sim p \vee q$
T	T	F	F	T	T	T	T	T
T	F	F	T	F	T	T	F	F
F	T	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T	T

Example :

p : *It rains*

q : *The crops will grow*

$p \rightarrow q$: If it rains then the crops will grow .

$q \rightarrow p$: If the crops grow then there has been rain.

$\sim p \rightarrow \sim q$: If it does not rain then the crop will not grow.

$\sim q \rightarrow \sim p$: If the crops do not grow then there has been no rain rains.

Q.1 *What are the contrapositive, converse, and the inverse of the conditional statement "The home team wins whenever it is raining?"*

AKTU-2019

q (The home team wins) — p (it is raining)

p : It is raining

q : The home team wins

$p \rightarrow q$: If it is raining then the home team wins

Converse

$q \rightarrow p$: If the home team wins then it is raining

Inverse

$\sim p \rightarrow \sim q$: If it is not raining then the home team does not wins

Contrapositive

$\sim q \rightarrow \sim p$: If the home team does not win then it is not raining

Q.2 *Write the contra positive of the implication:*

"If $\underbrace{\text{it is Sunday}}_{p}$ then $\underbrace{\text{it is a holiday}}_{q}$"

AKTU-2020

p: It is sunday

q: It is a holiday

$p \to q$: If it is sunday then it is holiday

contrapositive

$\sim q \to \sim p$: If it is not holiday then it is not Sunday

Q.3 *Write the negation of the following statement:*

"If $\underbrace{\text{I wake up early in the morning}}_{p}$, then $\underbrace{\text{I will be healthy}}_{q}$" **AKTU-2021**

p : I wake up early in the morning

q : I will be healthy

$p \to q$: "If I wake up early in the morning, then I will be healthy"

Negation

$$\sim(p \to q) = \sim(\sim p \vee q)$$

$= (p \wedge \sim q)$: I wake up early in the morning and I will not be healthy

Q.4 *Translate the conditional statement* "If it rains, then I will stay at home" *into contrapositive, converse and inverse statement .* AKTU-2022

p   q

p : It rains

q : I will stay at home

$p \rightarrow q$: If it rains, then I will stay at home

converse

$q \rightarrow p$: If I will stay at home then it rains

Inverse

$\sim p \rightarrow \sim q$: If it does not rain then I will not stay at home

contrapositive

$\sim q \rightarrow \sim p$: If I will not stay at home then it does not rain.

(5) Bi-conditional Statement ($\leftrightarrow$ *or* $\Leftrightarrow$)

If p *and* q are statements, then the compound statement 'p if and only if q', denoted by $p \leftrightarrow q$ is called a bi-conditional statement .

The compound statement $p \leftrightarrow q$ is true when both components have save values.

T T $\longrightarrow$ T

F F $\longrightarrow$ T

Truth Table

p	q	$p \longleftrightarrow q$	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \wedge (q \rightarrow p)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

Note : $p \longleftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Example :

(i) p : *Sales of houses fall*

q : *Interes rate rises*

$p \leftrightarrow q$: Sales of houses fall if and only if interest rate rises .

(ii) $p : 7 > 5$

$q : 7 - 5 > 0$

$p \leftrightarrow q : 7 > 5 \; iff \; 7 - 5 > 0$

SUMMARY

Negation (NOT)	$\sim$	T F		F T		
Conjunction (AND)	$\wedge$	T	T	T	Otherwise	F
Disjunction (OR)	$\vee$	F	F	F	Otherwise	T
Conditional (IF....THEN)	$\rightarrow$ or $\Rightarrow$	T	F	F	Otherwise	T
Bi-conditional (IFF)	$\leftrightarrow$ or $\Leftrightarrow$	T F	T F	T T	Otherwise	F

Q.1 :- Construct a truth table for the following statement $(p \vee q \Rightarrow r) \Leftrightarrow \left((p \Rightarrow r) \vee (q \Rightarrow r)\right) = A$ **(2015, 13)**

p	q	r	$p \vee q$	$(p \vee q \Rightarrow r)$	$p \Rightarrow r$	$q \Rightarrow r$	$(p \Rightarrow r) \vee (q \Rightarrow r)$	A
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	T	T	T	T	T	T
T	F	F	T	F	F	T	T	F
F	T	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T	F
F	F	T	F	T	T	T	T	T
F	F	F	F	T	T	T	T	T

Q.2 :- Find the truth table for the statement $(p \Leftrightarrow q \wedge r) \Rightarrow (\sim r \Rightarrow \sim p) = A$ (2006, 13)

p	q	r	$\sim p$	$\sim r$	$q \wedge r$	$(p \Leftrightarrow q \wedge r)$	$(\sim r \Rightarrow \sim p)$	A
T	T	T	F	F	T	T	T	T
T	T	F	F	T	F	F	F	T
T	F	T	F	F	F	F	T	T
T	F	F	F	T	F	F	F	T
F	T	T	T	F	T	F	T	T
F	T	F	T	T	F	T	T	T
F	F	T	T	F	F	T	T	T
F	F	F	T	T	F	T	T	T

Q.3 :- Given that the value of $p \leftrightarrow q$ is true. Can you determine the value of $\sim p \vee (p \leftrightarrow q)$? (2007)

p	q	$p \leftrightarrow q$	$\sim p$	$\sim p \vee (p \leftrightarrow q)$
T	T	$\boxed{T}$	F	$\boxed{T}$
T	F	F	F	F
F	T	F	T	T
F	F	$\boxed{T}$	T	$\boxed{T}$

When $p \leftrightarrow q$ is true then $\sim p \vee (p \leftrightarrow q)$ is true

Q.4 :- Given that the value of $p \longrightarrow q$ is false, determine the value of $(\sim p \vee \sim q) \Rightarrow q = A$ (2009)

p	q	$p \to q$	$\sim p$	$\sim q$	$(\sim p \vee \sim q)$	A
T	T	T	F	F	F	T
T	F	$\boxed{F}$	F	T	T	$\boxed{F}$
F	T	T	T	F	T	T
F	F	T	T	T	T	F

Value of $(\sim p \vee \sim q) \Rightarrow q$ is False

Discrete Mathematics by Gulshan Sir

DPP

Topic : Truth table for compound statement

Q.1. Construct a truth table for the following statement : $(p \Rightarrow q) \wedge (q \Rightarrow r) \Rightarrow (p \Rightarrow r)$ [AKTU]

Q.2 Prepare a table for the following statement : $(p \Leftrightarrow q) \wedge (r \vee q)$

Q.3 Prepare a table for the following statement : $\{(p \vee q) \wedge r\} \Rightarrow q)$ [AKTU]

Q.4 Construct the truth tables for the following : $(\sim p \rightarrow r) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$

Tautology:

A compound Statement (or proposition) which contain only T for all cases in the last column of its truth table is called a Tautology.

A tautology is also called Logically valid or Logically true statement

Contradiction:

A compound Statement (or proposition) which contain only F for all cases in the last column of its truth table is called a contradiction.

A contradiction is also called Logically invalid or Logically false statement

Contingency:

A Statement (or proposition) that is neither a Tautology nor a contradiction is called a contingency.

A contingency is also called Logically valid or Logically true statement

Note - Last Colum contain both T and F

Satisfiability

A compound statement is satisfiability if there is at least one true result in its truth table

Statement

Tautology
(Always true)

Contingency
(Either true or False)

Contradiction
(Always False)

Satisfiable

Logically valid

Unsatisfiable

Logically in valid

Q.1 Show that the given statement is a tautology: $((P \lor Q) \land (P \to R) \land (Q \to R)) \to R$

Let $(P \lor Q) \land (P \to R) = A$ and $((P \lor Q) \land (P \to R) \land (Q \to R)) = B$

P	Q	R	$P \lor Q$	$P \to R$	A	$Q \to R$	B	$B \to R$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	T	T	T	T	T	T
T	F	F	T	F	F	T	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	F	F	T
F	F	T	F	T	F	T	F	T
F	F	F	F	T	F	T	F	T

Since, all entries in the last column are T, then the given statement is a Tautology

Q.2 Prove that the statement $(p \vee q) \wedge (\sim p) \wedge (\sim q)$ is a contradiction.

Let $(p \vee q) \wedge (\sim p) = A$

p	q	$\sim p$	$\sim q$	$p \vee q$	A	$A \wedge \sim q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	T	F
F	F	T	T	F	F	F

Since, all the entries in the last column of truth table are F only

$\Rightarrow$ The given statement is a contradiction

Q.3 Prove that the statement $(p \Rightarrow \sim q) \Leftrightarrow (q \Rightarrow p)$ is a contingency

p	q	$\sim q$	$p \Rightarrow \sim q$	$q \Rightarrow p$	$(p \Rightarrow \sim q) \Leftrightarrow (q \Rightarrow p)$
T	T	F	F	T	F
T	F	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T

The given statement is neither a Tautology nor a contradiction

$\Rightarrow$ It is a contingency

Q.4 Show that the given formula is a tautology: Let $\sim p \wedge (\sim q \vee \sim r) = A$

$$\{(p \vee q) \wedge \sim(\sim p \wedge (\sim q \vee \sim r))\} \vee (\sim p \wedge \sim q) \vee (\sim p \wedge \sim r)$$ (UPTU-2007)

Let $\{(p \vee q) \wedge \sim A\} = B$, Let $(\sim p \wedge \sim q) = C$ Let $(\sim p \wedge \sim r) = D$

p	q	r	$\sim p$	$\sim q$	$\sim r$	$p \vee q$	$\sim q \vee \sim r$	A	$\sim A$	B	C	$B \vee C$	D	$B \vee C \vee D$
T	T	T	F	F	F	T	F	F	T	T	F	T	F	T
T	T	F	F	F	T	T	T	F	T	T	F	T	F	T
T	F	T	F	T	F	T	T	F	T	T	F	T	F	T
T	F	F	F	T	T	T	T	F	T	T	F	T	F	T
F	T	T	T	F	F	T	F	F	T	T	F	T	F	T
F	T	F	T	F	T	T	T	T	F	F	F	F	T	T
F	F	T	T	T	F	F	T	T	F	F	T	T	F	T
F	F	F	T	T	T	F	T	T	F	F	T	T	T	T

Q.5 Prove that the given statement is a tautology: $(p \Rightarrow q) \vee r \Leftrightarrow [(p \vee r) \Rightarrow (q \vee r)]$

Let $(p \Rightarrow q) \vee r = A$

OR

Let $(p \vee r) \Rightarrow (q \vee r) = B$

Prove that the truth value of the following are independent of their components : $(p \Rightarrow q) \vee r \Leftrightarrow [(p \vee r) \Rightarrow (q \vee r)]$

p	q	r	$p \Rightarrow q$	A	$p \vee r$	$q \vee r$	B	$A \Leftrightarrow B$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T	T
T	F	T	F	T	T	T	T	T
T	F	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	F	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	F	F	T	T

$\Rightarrow$ Tautology

Hence, the truth values of the given statement are independent of their components

DPP

Topic : Truth table for compound statement

Q.1 Prove that the given statement is a tautology: $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$

OR

Prove that the truth value of the following are independent of their components : $((p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$

Q.2 *Prove that* $(p \Leftrightarrow q) \wedge (q \Leftrightarrow r) \Rightarrow (p \Leftrightarrow r)$ *is a tautology.* (UPTU-2013)

Q.3 Prove that $[(p \wedge r) \vee (q \wedge \sim r)] \Leftrightarrow [(\sim p \wedge r) \vee (\sim q \wedge \sim r)]$ is a contradiction.

Q.4 Determine whether the following statements are tautology, contradiction or statisfiable:

(i) $(p \rightarrow q) \wedge (q \rightarrow r) \wedge (p \wedge \sim r)$ (UPTU-2016)

(ii) $(p \vee \sim q) \wedge (\sim p \wedge \sim q) \vee q$ (UPTU-2017)

(iii) $[p \wedge (p \rightarrow q)] \rightarrow \sim q$

Logical Equivalence

Two statement (or proposition) are called logically equivalent if the truth values of both the statements (or proposition) are always identical.

If two statement $Pand\ Q$ are logically equivalent, then these are represented by

$$\boxed{P \equiv Q}$$

Note: The necessary and sufficient condition for two statements P and Q to be logically equivalent is that

$P \Leftrightarrow Q$ is a Tautology.

Q.1 Show that $\sim(p \Rightarrow q) \equiv \{p \wedge (\sim q)\}$

Let $P = \sim(p \Rightarrow q)$

$Q = p \wedge (\sim q)$

To Prove

$P \equiv Q$

p	q	$\sim q$	$p \Rightarrow q$	P	Q
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	F	F

since truth value of P and Q are identical

$\Rightarrow P \equiv Q$

Q.2 Show that the statement $\overbrace{p \rightarrow (q \vee r)}$ *and* $\overbrace{[(p \rightarrow q) \vee (p \rightarrow r)]}$ *are equivalent.*

Let $P = p \rightarrow (q \vee r)$ and $Q = [(p \rightarrow q) \vee (p \rightarrow r)]$

To Prove

$P \equiv Q$

p	q	r	$q \vee r$	P	$p \rightarrow q$	$p \rightarrow r$	Q	$P \Leftrightarrow Q$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T	T
T	F	T	T	T	F	T	T	T
T	F	F	F	F	F	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	F	T	T	T	T	T

since

$P \Leftrightarrow Q$ is a Tautology

$\Rightarrow P \equiv Q$

Hence proved

Q.3 If p & q are two statements, show that the implication $p \Rightarrow q$ and its contra-positive $(\sim q) \Rightarrow (\sim p)$ are logically equivalent. **[U.P.T.U 2015]**

To Prove

$$p \Rightarrow q \equiv (\sim q) \Rightarrow (\sim p)$$

p	q	$\sim p$	$\sim q$	$p \Rightarrow q$	$(\sim q) \Rightarrow (\sim p)$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Since, truth values of both statement are idendical

$\therefore$ $\boxed{p \Rightarrow q \equiv (\sim q) \Rightarrow (\sim p)}$

Discrete Mathematics by Gulshan Sir

DPP

Topic : Truth table for compound statement

Q.1. Show that $(p \vee q) \Longrightarrow (p \wedge q) \equiv p \Leftrightarrow q$ (UPTU-2019)

Q.2 Show that $q \Rightarrow p$ converse of $p \Rightarrow q$ and its inverse $(\sim p) \Rightarrow (\sim q)$ are logically equivalent.

Laws of Proposition / Algebra of Proposition / Logical Equivalence

1. Idempotent Law
2. Commutative Law
3. Associative Law
4. Distributive Law
5. De-Morgan's Law
6. Identity Law / Dominance law
7. Complement Law
8. Absorption Law
9. Double Negation law

$A \cap A = A$

$A \cup A = A$

1. Idempotent Law

$(i)\ p \wedge p = p$

p	p	$p \wedge p$
T	T	T
F	F	F

Since column under p and $p \wedge p$ are identical

$\therefore \boxed{p \wedge p = p}$

$(ii)\ p \vee p = p$

p	p	$p \vee p$
T	T	T
F	F	F

2. Commutative Law

(i) $p \wedge q = q \wedge p$

$A \cap B = B \cap A$

p	q	$p \wedge q$	$q \wedge p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

Since column under $p \wedge q$ and $q \wedge p$ are identical

(ii) $p \vee q = q \vee p$

$A \cup B = B \cup A$

p	q	$p \vee q$	$q \vee p$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

3. Associative Law $(A \cup B) \cup C = A \cup (B \cup C)$ **(2005 & 2010)**

(i) $(p \vee q) \vee r = p \vee (q \vee r)$

p	q	r	$(p \vee q)$	$(q \vee r)$	$(p \vee q) \vee r$ ✓	$p \vee (q \vee r)$ ✓
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	T	F	T	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F

(ii) $(p \wedge q) \wedge r = p \wedge (q \wedge r)$

4. Distributive Law

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(i) $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

p	q	r	$(q \wedge r)$	$(p \vee q)$	$(p \vee r)$	$p \vee (q \wedge r)$ ✓	$(p \vee q) \wedge (p \vee r)$ ✓
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	F	F	F
F	F	T	F	F	T	F	F
F	F	F	F	F	F	F	F

(ii) $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

(5) - De-Morgan's Law

$(A \cap B)^c = A^c \cup B^c$

(i) $\sim(p \wedge q) = (\sim p) \vee (\sim q)$

p	q	$(p \wedge q)$	$\sim p$	$\sim q$	$\sim(p \wedge q)$	$(\sim p) \vee (\sim q)$
T	T	T	F	F	F	F
T	F	F	F	T	T	T
F	T	F	T	F	T	T
F	F	F	T	T	T	T

(ii) $\sim (p \vee q) = (\sim p) \wedge (\sim q)$ $\quad (A \cup B)^c = A^c \cap B^c$

6. Identity Law (i) $A \cap U = A$ (ii) $A \cup \phi = A$

(i) The identity element for conjunction is Tautology (t)

$p \wedge t = t \wedge p = p$ OR $\boxed{P \wedge T = P}$

p	t	$p \wedge t$	$t \wedge p$
T	T	T	T
F	T	F	F

Dominance Law

(i) $p \vee T = T$

(ii) $p \wedge F = F$

(ii) The identity element for disjunction is Contradiction (f)

$p \vee f = f \vee p = p$ OR $\boxed{P \vee F = P}$

p	f	$p \vee f$	$f \vee p$
T	F	T	T
F	F	F	F

$A \cup A^c = U$ $\qquad$ $A \cap A^c = \phi$

7. Complement Law

For every statement p there exist its negation$(\sim p)$ such that

(i) $p \vee (\sim p) = t$ $\qquad$ (ii) $p \wedge (\sim p) = f$

p	$\sim p$	$p \vee (\sim p)$	t
T	F	T	T
F	T	T	T

p	$\sim p$	$p \wedge (\sim p)$	f
T	F	F	F
F	T	F	F

8. Absorption Law

$A \cup (A \cap B) = A$

(i) $p \vee (p \wedge q) = p$

p	q	$p \wedge q$	$p \vee (p \wedge q)$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

(ii) $p \wedge (p \vee q) = p$

9. Double Negation law

$\sim(\sim p) \equiv p$

Laws of Proposition/Logical equivalences

1. Identity Law

$p \wedge T = p$

$p \vee F = p$

2. Dominance Law

$p \vee T = T$

$p \wedge F = F$

3. Idempotent Law

(i) $p \wedge p = p$

(ii) $p \vee p = p$

4. Commutative Law

(i) $p \wedge q = q \wedge p$

(ii) $p \vee q = q \vee p$

5. Double Negation Law

$\sim (\sim p) = p$

6. Associative Law

(i) $(p \vee q) \vee r = p \vee (q \vee r)$

(ii) $(p \wedge q) \wedge r = p \wedge (q \wedge r)$

7. Distributive Law

(i) $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

(ii) $p \wedge (q \vee r) = (p \wedge q) \vee (q \wedge r)$

8. De-Morgan's Law

(i) $\sim (p \wedge q) = (\sim p) \vee (\sim q)$

(ii) $\sim (p \vee q) = (\sim p) \wedge (\sim q)$

9. Complement Law

(i) $p \vee (\sim p) = T$

(ii) $p \wedge (\sim p) = F$

10. Absorption Law

(i) $p \vee (p \wedge q) = p$

(ii) $p \wedge (p \vee q) = p$

NOTE :

(i) $p \rightarrow q = \sim p \vee q$

(ii) $p \rightarrow q = (p \rightarrow q) \wedge (q \rightarrow p)$

$= (p \wedge q) \vee (\sim p \wedge \sim q)$

1. To Prove Tautology without using truth table

Q1 . Show that $p \wedge q \to p \vee q$ is tautology.

$(p \wedge q) \to (p \vee q)$

$\equiv \sim(p \wedge q) \vee (p \vee q)$ $\quad (\because p \to q \equiv \sim p \vee q)$

$\equiv (\sim p \vee \sim q) \vee (p \vee q)$ $\quad$ (By Demorgan's law)

$\equiv (\sim p \vee \sim q) \vee (q \vee p)$ $\quad$ (By Commutative law)

$\equiv \sim p \vee (\sim q \vee q) \vee p$ $\quad$ (By Associative law)

$\equiv \sim p \vee T \vee p$ $\quad$ (By Negation law)

$\equiv T$ $\quad$ (By Dominance law)

Hence $p \wedge q \to p \vee q$ is a Tautology

Normal Form

DNF CNF

Disjunctive Normal form (DNF)

A logical expression is said to be in Disjunctive Normal form if it is the sum of elementary products.

Examples

$(i)\ p \vee (q \wedge r)$

$(ii) (p \wedge q) \vee (q \wedge r)$

Conjunctive Normal Form (CNF)

A logical expression is said to be in conjunctive normal form if it consists of a product of elementary sums.

Examples

(i) $p \wedge (q \vee r)$

(ii) $(p \vee q) \wedge (q \vee r)$

Steps to obtain DNF and CNF of a given logical expression

1. Remove all $\rightarrow$ and $\leftrightarrow$ by an equivalent expression containing the connectives $\wedge, \vee, \sim$ only.
2. Eliminate $\sim$ before sums and products by using the double negation law or by using De – Morgan's laws.
3. Apply the distributive law until a sum of elementary product is obtained.

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Q.3 *Obtain the disjunctive normal forms of the followings*

(i) $p \wedge (p \rightarrow q)$

$p \wedge (p \rightarrow q)$

$\equiv p \wedge (\sim p \vee q)$ $(\because p \rightarrow q \equiv \sim p \vee q)$

By Distributive law

$\equiv (p \wedge \sim p) \vee (p \wedge q)$

$\equiv F \vee (p \wedge q)$ (By Negation Law)

Required Disjunction Form

(ii) $p \vee (\sim p \rightarrow (q \vee (q \rightarrow \sim r)))$

$p \vee (\sim p \rightarrow (q \vee (q \rightarrow \sim r)))$

$\equiv p \vee (\sim p \rightarrow (q \vee (\sim q \vee \sim r)))$ $(\because p \rightarrow q \equiv \sim p \vee q)$

$\equiv p \vee (p \vee (q \vee (\sim q \vee \sim r)))$ (")

$\equiv p \vee (p \vee (q \vee \sim q) \vee \sim r))$

$\equiv p \vee p \vee q \vee \sim q \vee \sim r$

$\equiv p \vee q \vee \sim q \vee \sim r$ (By Idempotent law)

Required Disjunction Normal Form

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Q.4 *Obtain a conjunctive normal form of thte followings :*

(a) $p \wedge (p \rightarrow q)$

$p \wedge (p \rightarrow q)$

$(\because p \rightarrow q \equiv \sim p \vee q)$

$p \wedge (\sim p \vee q)$

Which is required Conjunctive Normal Form

(b) $[q \vee (p \wedge r)] \wedge \sim [(p \vee r) \wedge q]$

$[q \vee (p \wedge r)] \wedge \sim [(p \vee r) \wedge q]$

$\equiv [q \vee (p \wedge r)] \wedge [\sim (p \vee r) \vee \sim q]$ (By Demorgan's Law)

$\equiv [q \vee (p \wedge r)] \wedge [(\sim p \wedge \sim r) \vee \sim q]$ (")

$\equiv [(q \vee p) \wedge (q \vee r)] \wedge (\sim p \vee \sim q) \wedge (\sim q \vee \sim r)$ (By Associative Law)

It is required conjunctive normal Form

Principle Disjunctive Normal Form (PDNF)

PDNF = sum of minterms

If p and q are proposition variables

Minterms : $p \wedge q$, $\quad p \wedge \sim q$, $\quad \sim p \wedge q$, $\quad \sim p \wedge \sim q$

Methods to find PDNF

1. By using truth table

(i) Write minterms corresponding to truth value T

(ii) PDNF = sum of minterms

2. Without using truth table

(i) find DNF

(ii) introduce missing terms

(ii) remove duplication

Principle Conjunctive Normal Form (PCNF)

PCNF = Product of maxterms

If p and q are proposition variables

Maxterms : $p \vee q$, $\quad p \vee \sim q$, $\quad \sim p \vee q$, $\quad \sim p \vee \sim q$

Methods to find PCNF

1. By using truth table

(i) Write maxterms corresponding to truth value F

(ii) PCNF = Product of maxterms

2. Without using truth table

(i) find CNF

(ii) introduce missing terms

(ii) remove duplication

Q.5 *Obtain the principle disjunctive and conjunctive normal forms of the formula*

$(p \to r) \wedge (p \leftrightarrow q)$.

By Truth table

p	q	r	$p \to r$	$p \leftrightarrow q$	$(p \to r) \wedge (p \leftrightarrow q)$
T	T	T	T	T	T
T	T	F	F	T	F
T	F	T	T	F	F
T	F	F	F	F	F
F	T	T	T	F	F
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

PDNF

$$= (p \wedge q \wedge r) \vee (\sim p \wedge \sim q \wedge r) \vee (\sim p \wedge \sim q \wedge \sim r)$$

PCNF

$$(\sim p \vee \sim q \vee r) \wedge (\sim p \vee q \vee \sim r) \wedge (\sim p \vee q \vee r) \wedge (p \vee \sim q \vee \sim r) \wedge (p \vee \sim q \vee r)$$

SUMMARY

Negation (NOT)	$\sim$	T F		F T		
Conjunction (AND)	$\wedge$	T	T	T	Otherwise	F
Disjunction (OR)	$\vee$	F	F	F	Otherwise	T
Conditional (IF....THEN)	$\longrightarrow$ or $\Rightarrow$	T	F	F	Otherwise	T
Bi-conditional (IFF)	$\leftrightarrow$ or $\Leftrightarrow$	T F	T F	T T	Otherwise	F

Kinds of conditional

If $p \to q$ is a conditional statement or Direct implication.

(1) *Converse Implication* : $q \to p$

(2) *Inverse Implication* : $\sim p \to \sim q$

(3) *Contrapositive Implication* : $\sim q \to \sim p$

Laws of Proposition

1. Idempotent Law

$(i)\ p \wedge p = p$

$(ii)\ p \vee p = p$

2. Commutative Law

$(i)\ p \wedge q = q \wedge p$

$(ii)\ p \vee q = q \vee p$

5. De-Morgan's Law

$(i)\ \sim (p \wedge q) = (\sim p) \vee (\sim q)$

$(ii)\ \sim (p \vee q) = (\sim p) \wedge (\sim q)$

Argument

- An argument is a process by which a conclusion is drawn from a given set of porpositions
- the given set of porpositions are called premises.
- The final proposition derived from given proposition is called conclusion.

An Argument which yield a conclusion c from the premises $p_1, p_2, p_3 \ldots \ldots, p_n$ denoted by

$$\boxed{p_1, p_2, p_3 \ldots \ldots \ldots, p_n \vdash C}$$

Valid Argument

- An argument $p_1, p_2, \ldots., p_n \vdash C$ is said to be Valid argument if the conclusion is true whenever all the premisesare true.
- An argument $p_1, p_2, p_3 \ldots \ldots \ldots, p_n \vdash C$ is said to be Valid if and only if the statement $\boxed{(p_1 \wedge p_2 \wedge \cdots \wedge p_n) \rightarrow C}$ is tautology.
- Any conclusion which is arrived by following rules of inferences is called a valid conclusion and the argument is called Valid argument.

Representation of an Argument

An argument $p_1, p_2, \dots, p_n \vdash p$ is written as

$$\begin{array}{c} \left.\begin{array}{c} p_1 \\ p_2 \\ \vdots \\ p_n \end{array}\right\} \text{premises} \\ \hline c \ (\text{conclusion}) \end{array}$$

In the above representation premises are listed above the horizontal line and the conclusion below the horizontal line.

Method to determine whether a given argument is valid or not

(i) By truth table

(ii) By Rules of inference

Rules of inference

T F → F

- The Rules of inference are criteria for determining the validity of an argument.
- *Any conclusion which is arrived by following rules of inferences is called a valid conclusion and the argument is called Valid argument.*

1. *Addition*

(i) $\dfrac{p}{\therefore p \vee q}$ (ii) $\dfrac{q}{\therefore p \vee q}$

p ✓	q	$p \vee q$ ✓	$p \to (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

2. *Simplification*

(i) $\dfrac{p \wedge q}{p}$ (ii) $\dfrac{p \wedge q}{q}$

3. Conjunction

$$\begin{array}{c} p \\ \underline{q} \\ \therefore p \wedge q \end{array}$$

4. Modus ponens

Rule of Detachment

$$\begin{array}{c} p \to q \\ \underline{p} \\ \therefore q \end{array}$$

p	q	$p \to q$	$(p \to q) \wedge q$	$[(p \to q) \wedge q] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	F	T

5. Modus tollens

$$\begin{array}{c} p \to q \\ \underline{\sim q} \\ \sim p \end{array}$$

6. Hypothetical syllogism

$$\begin{array}{l} p \to q \\ \underline{q \to r} \\ \therefore p \to r \end{array}$$

p	q	r	$p \to q$	$q \to r$	$p \to r$	$(p\to q)\wedge(q \to r)$ A	$A \to p \to r$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

7. Disjunction syllogism

$$\begin{array}{c} p \vee q \\ \underline{\sim p} \\ q \end{array}$$

8. Absorption

$$\begin{array}{c} \underline{p \to q} \\ p \to (p \wedge q) \end{array}$$

9. Constructive dilemma

$$\frac{\begin{array}{c}(p \to q) \wedge (r \to s) \\ p \vee r\end{array}}{q \vee s}$$

10. Destructive dilemma

$$\frac{\begin{array}{c}(p \to q) \wedge (r \to s) \\ \sim q \vee \sim s\end{array}}{\sim p \vee \sim r}$$

Q.1 ***Show that the following argument is valid :***

$$\begin{array}{c} p \vee q \\ \sim p \\ \hline q \end{array}$$

By Truth table

Above Argument is valid if

$[(p \vee q) \wedge \sim p] \longrightarrow q$ is a Tautology

p	q	$\sim p$	$p \vee q$	$(p \vee q) \wedge \sim p$	$[(p \vee q) \wedge \sim p] \to q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T
	C	P	P		

Hence given Argument is valid

By Rules of inferences

$$\begin{array}{c} p \vee q \\ \sim p \\ \hline q \end{array}$$

Above Argument is valid by Disjunction Syllogism

Q.2 ***Show that the argument*** $\overset{1}{p}, \overset{2}{p} \to q, \overset{3}{q} \to r \vdash r$ ***is valid:***

By Truth table → Above Argument is valid if $[p \wedge (p \to q) \wedge (q \to r)] \longrightarrow r$ is tautology

p ✓	q	r ✓	$p \to q$ ✓	$q \to r$ ✓			
T	T	T	T	T			
T	T	F	T	F			
T	F	T	F	T			
T	F	F	F	T			
F	T	T	T	T			
F	T	F	T	F			
F	F	T	T	T			
F	F	F	T	T			

By the rules of inferences

(1) $p \to q$

p

q (By Modus ponens

(2) $q \longrightarrow r$

q

r (By Modus ponen)

Hence given Argument is valid

Q.3 *Test the validity of the following argument: If a man is a bachelor, he is worried, If a man is a worried, he dies young. Therefore Bachelors die young*

$$\begin{array}{l} p \to q \\ q \to r \\ \hline p \to r \end{array}$$

Let

p = Man is a bachelor

q = He is worried

r = He dies young

$$\begin{array}{c} p \to q \\ q \to r \\ \hline p \to r \end{array}$$

Above Statement is valid By Hypothetical Syllogism

By Truth table

p	q	r	$p \to q$ (P)	$q \to r$ (P)	$p \to r$ (C)	$(p \to q) \wedge (q \to r)$ (A)	$A \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Q.4 Prove that the argument $p \overset{1}{\rightarrow} q,\ q \overset{2}{\rightarrow} r,\ r \overset{3}{\rightarrow} s,\ \overset{4}{\sim s},\ \overset{5}{p \vee t} \vdash \overset{c}{t}$ is valid without using truth table.

Solution

1.	$p \to q$	*Premise (Given)*
2.	$q \to r$	*Premise (Given)*
3.	$r \to s$	*Premise (Given)*
~~4.~~	$\cancel{p \to r}$	~~*Hypothetical syllogism using 1 and 2*~~
~~5.~~	$\cancel{p \to s}$	~~*Hypothetical syllogism using 3 and 4*~~
4.	$\sim s$	*Premise (Given)*
~~7.~~	$\cancel{\sim p}$	~~*Modus tollens using 5 and 6*~~
5.	$p \vee t$	*Premise (Given)*
~~9.~~	$\cancel{t}$	~~*disjunctive syllogism using 7 and 8*~~

(1) $$\begin{array}{l} p \to q \\ q \to r \\ \hline p \to r \end{array}$$ (By Hypothetical syllogism)

(2) $$\begin{array}{l} p \to r \\ r \to s \\ \hline p \to s \end{array}$$ (")

(3) $$\begin{array}{l} p \to s \\ \sim s \\ \hline \sim p \end{array}$$ (By Modus tollens)

(4) $p \vee t$

$\sim p$

---

t (By Disjunction Syllogism)

Hence, given Argument is valid

Q.5 Show that s is a valid conclusion from the premises $p \to q$, $p \to r$, $\sim(q \wedge r)$ and $s \vee p$.

Solution

1.	$p \to q$	Premise (Given)
2.	$p \to r$	Premise (Given)
3.	$(p \to q) \wedge (p \to r)$	Conjunction using 1 and 2
4.	$\sim(q \wedge r)$	Premise (Given)
5.	$\sim q \vee \sim r$	Demorgan's law using 4
6.	$\sim p \vee \sim p$	Destructive dilema using 3 and 5
7.	$\sim p$	Idempotent law using 6
8.	$s \vee p$	Premise (Given)
9.	s	Disjunctive syllogism using 7 and 8

Q.6 *Prove the validity of the following argument "If I get the job and work hard, then I will get promoted. If I get promoted, then I will be happy. I will not be happy. therefore, either I will not get the job or I will not work hard"*

Solution: Let

p = I get the job

q = Work hard

r = I will get promoted

s = I will be happy

$$\begin{array}{c} (p \wedge q) \to r \\ r \to s \\ \sim s \\ \hline \sim p \vee \sim q \end{array}$$

1.	$(p \wedge q) \to r$	*Premise (Given)*
2.	$r \to s$	*Premise (Given)*
~~3.~~	~~$(p \wedge q) \to \cancel{r}$~~ s	~~*Hypothetical syllogism using 1 and 2*~~
4.	$\sim s$	*Premise (Given)*
5.	$\sim (p \wedge q)$	*Modus tollens using 3 and 4*
6.	$\sim p \vee \sim q$	*Demorgan's law using 5 (conclusion)*

Hence the argument is valid

$$\begin{array}{c} (p \wedge q) \to r \\ r \to s \\ \hline (p \wedge q) \to s \end{array}$$

② $$\begin{array}{c} (p \wedge q) \to s \\ \sim s \\ \hline \sim (p \wedge q) \end{array}$$

③ $\sim (p \wedge q)$

$\sim p \vee \sim q$

Hence given argument is valid'

DPP

Topic : Theory of inference (Part -1)

Q.1 *Show that the argument* $p,\ p \to q \vdash q$ *is valid*

Q.2 *Test the validity of the following argument: "If it rains then it will be cold, If it is cold then I shall stay at home. Since it rains therefore I shall stay at home".*

Q.3 *Test the validity of the argument:*

If two side of a triangle are equal, then the opposite angles are equal.

Two side of a triangle are not equal.

The opposite angles are not equal

Q.1 *Test the validity of following argument. If I will select in IAS examination, then I will not be able to go to London. Since, I am going to London, I will not select in IAS examination*

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Let

p : I will select in IAS Examination

q : I am going to london.

1. $p \longrightarrow \sim q$
2. q

$\sim p$

By Truth table – First Method

p	q	$\sim p$	$\sim q$	$p \rightarrow \sim q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Whenever premises are conclusion also true

Hence, Argument is valid

By Truth table - Second Method

Above Argument is valid if

$[(p \to \sim q) \wedge q] \longrightarrow \sim p$ is a Tautology

p	q	$\sim p$	$\sim q$	$p \to \sim q$	$(p \to \sim q) \wedge q$	$[(p \to \sim q) \wedge q] \to \sim p$
T	T	F	F	F	F	T
T	F	F	T	T	F	T
F	T	T	F	T	T	T
F	F	T	T	T	F	T

Hence, given Argument is valid

By Rules of inferences

$$\begin{array}{l} p \to \sim q \\ \quad q \\ \hline \sim p \end{array}$$

(By Modus tollens)

Hence, Above Argument is a valid argument

Q.2 Consider the following argument and determine whether it is valid.

Either *I will get good marks or I will not graduate*. *If I did not graduate I will go to Canada*. *I get good marks*. Thus, *I would not go to Canada*.

Let

p : I will get good marks

q : I will graduate

r : I will to to canada

1. $p \vee \sim q$

2. $\sim q \longrightarrow r$

3. p

$\sim r$

p	q	r	$\sim q$	$\sim r$	$p \vee \sim q$ ✓	$\sim q \to r$ ✓
$\boxed{T}$	$\boxed{T}$	$\boxed{T}$	$\boxed{F}$	$\boxed{F}$	$\boxed{T}$	$\boxed{T}$
T	T	F	F	T	T	T
T	F	T	T	F	T	T
T	F	F	T	T	T	F
F	T	T	F	F	F	T
F	T	F	F	T	F	T
F	F	T	T	F	T	T
F	F	F	T	T	T	F

Whenever primes are true, conclusion are not true. Hence Arument is not valid

Q.3 ***Check the validity of the following arguments. Show with the use of symbolic notation.*** $x^2 = y^2$ ***only if*** $x = y$

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$$\frac{x = y}{x^2 = y^2}$$

Let $p : x^2 = y^2$

$q : x = y$

1. $p \rightarrow q$

2. q

c p

Above Argument is valid if

$[(p \rightarrow q) \wedge q] \rightarrow p$ is a tautology

p	q	$p \rightarrow q$	$(p \rightarrow q) \wedge q$	$[(p \rightarrow q) \wedge q] \rightarrow p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

$[(p \rightarrow q) \wedge q] \rightarrow p$ is not a Tautology

Hence, it is not a Tautology

DPP

Topic : Theory of inference (P

Q.1. *"If the labour market is perfect then the wages of all persons in a particular employment will be equal. But it is always the case that wages for such persons are not equal therefore the balour market is not perfect". Test the validity of the argument.* AKTU

Q.2 *Prove the validity of the following argument "if the races are fixed so the casinos are crooked, then the tourist trade will decline. If the tourist trade decreases, then the police be happy. The police force is never happy. Therefore, the races are not fixed.* AKTU

Q.3 *Use rules of inference to Justify that the three hypotheses (i)"If it does not rain or if it is not fogy, then the sailing race will be held and the lifesaving demonstrationwill go on." (ii) If the sailing race is held, then the trophy will be awarded. (iii) "The trophy was not awarded." imply the conclusion (iv) It rained."* AKTU

Q. 4 *Prove the validity of the followng argument.*
If Mary runs for office, She will be elected. If Mary attends the meeting, she will run for office. Either Mary will attend the meeting or she will go to India. But Mary cannot go to India. AKTU

Q.5 *Test the validity of the argument:*

"If Ashok wins then Ram will be happy. If Kamal wins Raju will be happy. Either Ashol will win or Kamal will win. However if Ashok win, Raju will not be happy and if Kamal wins Ram will not be happy. So Ram will be happy if and only if Raju is not happy".

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7 + 7 + 2

UNIT -3 : *THEORY OF LOGICS*

$\rightarrow$ $\boxed{\text{Proposition Logic}}$

$\rightarrow$ $\boxed{\text{Predicate Logic}}$

7 or 2

Lecture - 09

Today's Target

- *Predicate Logic*
- PYQ
- DPP

Predicate

A Predicate $P(x)$ is a sentence that contains a finite number of variables and become a proposition when specific values are substituted for the variables, Where

$P(x)$ is a Propositional function

and x is a predicate variable

UD

Domain (Universe of Discourse or Simply Universe)

The domain of a predicate variable is the set of all possible values that may be substituted in place of variables.

Examples

(1) Ram is a student

Shyam is a student

Mohan is a student

$P(x)$: x is a student

The domain for $P(x)$: "x is a student" can be taken as the set of all human names.

(2) $P(x) : x > 3$

$P(4) : 4 > 3, Which\ is\ true$

$P(1) : 1 > 3, Which\ is\ false$

(3) $P(x, y) : x = y + 2$

$P(1, 2) : 1 = 2 + 2, \quad which\ is\ false$

$P(2, 0) : 2 = 0 + 2, \quad Which\ is\ true$

Quantifiers

Quantifires are the words that refer to quantities such as some, few, many, all, none and indicate how frequantly a certain statement is true.

There are two types of Quantifiers

(i) *Universal Quantifier*

(ii) *Existential Quantifier*

$\forall$ $\quad$ $\forall x$

Universal Quantifier

The Phrase " for all " denoted by ∀ is called the Universal Quantifier

$\forall x$ *represemt " for all x "*

" for every x "

" for each x "

" for any x "

Examples

(1) *All human beings are mortal*

$P(x)$: *x is mortal*

The above sentence can be written as

$$(\forall\, x \in U)P(x)$$

OR

$$\forall\, x\, P(x)$$

Where U is the domain denoting the set of all human beings.

2nd Method

For every x, if x is a human beings then x is mortal

H(x) : x is human beings

M(x) : x is mortal

$$\boxed{\forall x\, [H(x) \longrightarrow M(x)]}$$

(2) *All Students are smart*

$P(x)$: *x is smart*

$$\forall\, x\, P(x)$$

Existential Quantifier

- The Phrase " there exist " denoted by ∃ is called existential quantifier
- $\exists x$ represemt " there exists " x

  " there is an x "

  " for some x "

  " there is atleast one x "

Examples

(1) Some human beings are mortal

$P(x)$: x is mortal

The above sentence can be written as

$$(\exists x \in U)P(x)$$

Or

$$\exists x\, P(x)$$

Where U is the domain denoting the set of all human beings.

Ther exist an x, such that x is human beings and x is mortal

$H(x)$: x is human beings

$M(x)$: x is mortal

$$\boxed{\exists x\,[H(x) \wedge M(x)]}$$

(2) Some Students are smart

$P(x)$: x is smart

$\exists\, x\, P(x)$

Q.1 Let Z, the set of integer, be the universe of discourse and consider the statements

(i) $(\forall x \in Z)\; x^2 = x$

(ii) $(\exists x \in Z)\; x^2 = x.$

Find the truth values of each of the statements

Sol.

(i) $P(x)$: $x^2 = x$

$P(3): 3^2 = 3$

$\therefore\ (\forall x \in Z) x^2 = x$, is False

(ii) $P(x)$: $x^2 = x$

$P(0): 0^2 = 0$

$P(1): 1^2 = 1$

$\therefore\ (\exists x \in Z) x^2 = x$, is True

Translate english sentences into logical expression

The logical operators and quantifiers can be used to express english sentences into logical expression.

1. *Every person is precious*

For every x, if x is person then x is precious

$P(x)$: x is a person

$Q(x)$: x is precious

$\forall x(M(x) \rightarrow P(x))$

2. *Every student is clever*

For every x, if x is a student then x is clever

$S(x)$: x is a student

$C(x)$: x is clever

$\forall x(S(x) \rightarrow C(x))$

3. *All men are mortal*

$M(x)$: *x is men*

$A(x)$: *x is mortul*

$\forall x(M(x) \rightarrow A(x))$

4. *Any integer is either positive or negative*

$I(x)$: *x is an integer*

$P(x)$: *x is either positive or negative*

$\forall x(I(x) \rightarrow P(x))$

5 *There exist a student*

$P(x)$: *x is a student*

$\exists x\, P(x)$

6 *Some students are clever*

There exist an x, such that x is a student and x is clever

$S(x)$: *x is a student*

$C(x)$: *x is a clever*

$\exists x\, (S(x) \wedge C(x))$

7 *Some students are not successsful*

There exist an x, such that x is a student and x is not successful

$S(x)$: *x is a student*

$P(x)$: *x is a Successful*

$\exists\, x\ (S(x)\ \wedge \sim P(x))$

9 *Not All birds can fly*

$B(x)$: *x is a brid*

$F(x)$: *x can fly*

$\sim \forall\, x\big(B(x) \rightarrow F(x)\big)$

8 *All birds can fly*

$B(x)$: *x is a brid*

$F(x)$: *x can fly*

$\forall\, x\big(B(x) \rightarrow F(x)\big)$

10 *There is a student who likes mathematics but not geography*

$P(x)$: *x is a student*

$Q(x)$: *x like Mathematics*

$R(x)$: *x like Geography*

$\exists\, x\ (P(x) \wedge Q(x) \wedge \sim R(x))$

11 *Some men are genious*

$M(x)$: x is a men

$G(x)$: x is genious

$\exists x\,(M(x) \land G(x))$

12 *Some numbers are rational*

$N(x)$: x is a number

$R(x)$: x is Rational

$\exists x\,(N(x) \land R(x))$

1. $\boxed{p \rightarrow q \equiv \sim p \lor q}$ ✓

2. Demorgan's law

$$\boxed{\begin{array}{l} \text{(i) } \sim(p \lor q) \equiv \sim p \land \sim q \\ \text{(ii) } \sim(p \land q) \equiv \sim p \lor \sim q \end{array}}$$

3. $\sim(\sim p) \equiv p$

Negation of Quantified Statements

Statements	*Negation*
All true : $\forall x\, P(x)$	At least one false : $\exists x\,(\sim P(x)$
At least one false : $\exists x\,(\sim P(x)$	All true : $\forall x\, P(x)$
All false : $\forall x\,(\sim P(x))$	At least one true : $\exists x\, P(x)$
At least one true : $\exists x\, P(x)$	All false : $\forall x\,(\sim P(x))$

Examples :- *Write the Negation of the following statements*

1. *All integers are greater than 8*

$P(x)$: *x is a integer*

$Q(x)$: *x is greater than 8*

$\forall x\,(P(x) \rightarrow Q(x))$

$\forall x\,(P(x) \rightarrow Q(x)) \equiv \forall x\,(\sim P(x) \vee Q(x))$

Negation :

Demorgan's law $\longrightarrow$ $\sim [\forall x\,(\sim P(x) \vee Q(x))]$

$\exists x\,(P(x) \wedge \sim Q(x))$

2. For all real number x, if $x > 3$ then $x^2 > 9$

$P(x) : x > 3$

$Q(x) : x^2 > 9$

$\forall x\,(P(x) \to Q(x))$

$\equiv \forall x\,(\sim P(x) \vee Q(x))$

Negation :

$\equiv \exists x[\sim(\sim P(x) \vee Q(x))]$

$\sim[\forall x\,(\sim P(x) \vee Q(x))] \equiv \exists x\,(P(x) \wedge \sim Q(x))$

By Demorgan's law

3. Some students are intelligent

$P(x) : x$ is a Student

$Q(x) : x$ is intelligent

$\exists x\,(P(x) \wedge Q(x))$

Negation :

$\sim \exists x\,(P(x) \wedge Q(x)) \equiv \forall x \sim (P(x) \wedge Q(x))$

$\equiv \forall x\,(\sim P(x) \vee \sim Q(x))$

By Demorgan's law

4. *Any integer is either positive or negative*

$P(x)$: x is an integer

$Q(x)$: x is either positive or negative

$$\forall x\,(P(x) \to Q(x)) \equiv \forall x\,(\sim P(x) \vee Q(x))$$

Negation :

$$\sim [\forall x\,(\sim P(x) \vee Q(x))]$$

By Demorgan's law

$$\exists x\,(P(x) \wedge \sim Q(x))$$

Q.2 Express the following statement in symbolic form: "All flowers are beautiful." AKTU 2021

$F(x)$: x is a Flower

$B(x)$: x is beautiful

$$\forall x\,(F(x) \to B(x))$$

Q.3 Write the Negation using quantifier, variables and predicate symbols

(i) All flowers are red

(ii) Some men are genius

(iii) Some numbers are not rational

(iv) Every girl is not clever

1. All flowers are red

$P(x)$: x is a Flower

$Q(x)$: x is red

$\forall x\,(P(x) \to Q(x)) \equiv \forall x\,(\sim P(x) \vee Q(x))$

$\sim [\forall x\,(\sim P(x) \vee Q(x))] \equiv \exists x\,(P(x) \wedge \sim Q(x))$

By Demorgan's law

2. Some men are genius

$P(x)$: x is a men

$Q(x)$: x is genius

$\exists x\,(P(x) \wedge Q(x))$

$\sim [\exists x\,(P(x) \wedge Q(x))] \equiv \forall x\,(\sim P(x) \wedge \sim Q(x))$

By Demorgan's law

3. *Some numbers are not rational*

$P(x) : x \text{ is a number}$

$Q(x) : x \text{ is rational}$

$\exists x\, (P(x) \land \sim Q(x))$

$\equiv \sim [\exists x\, (P(x) \land \sim Q(x))]$

$\equiv \forall x\, (\sim P(x) \lor \quad Q(x))$

4. *Every girl is not clever*

$P(x) : x \text{ is a girl}$

$Q(x) : x \text{ is clever}$

$\forall x\, (P(x) \to \sim Q(x)) \equiv \forall x\, (\sim P(x) \lor \sim Q(x))$

$\sim [\forall x\, (\sim P(x) \lor \sim Q(x))] \equiv \exists x\, (P(x) \land Q(x))$

Q.4 *Convert the following two statements in quantified expressions of predicate logic*

(i) *For every number there is a number greater than that number.* (AKTU-2023)

(ii) *Sum of every two integer is an integer.*

(iii) *Not Every man is perfect.*

(iv) *There is no student in the class who knows Spanish and German*

(i) $P(x)$: x is a number

$Q(y)$: y is a number greater than x

$\forall x \forall y [P(x) \rightarrow Q(y)]$

OR

$G(x, y)$: x is greater than y

$\forall x \forall y\, G(x, y)$

(iii) $M(x)$: x is a Man

$P(x)$: x is perfect

$\forall x [M(x) \rightarrow P(x)]$

$\sim [\forall x (M(x) \rightarrow P(x))]$

(ii) $P(x)$: x is an integer

$Q(y)$: y is an integer

$S(x, y)$: $x + y$ is an integer

$\forall x \forall y [P(x) \wedge Q(Y) \rightarrow S(x, y)]$

(iv) $P(x)$: x is a student

$Q(x)$: x know spanish and German

$\boxed{\forall x [P(x) \rightarrow \sim Q(x)]}$ Ans

Negation

$\sim \forall x [P(x) \rightarrow \sim Q(x)]$

$\exists x \sim [\sim P(x) \vee \sim Q(x)]$

$\exists x [P(x) \wedge Q(x)]$

Q.5 *Translate the following statements in symbolic form* (AKTU-2021)

(i) *The sum of two positive integers is always positive.*

(ii) *Every one is loved by someone.*

(iii) *Some people are not admired by everone.*

(iv) *If a person is female and is a parent, then this person is someone's mother.*

(i) *"For all x and y, if x and y are positive, then x + y is positive"*

$P(x)$: *x is a positive integer*

$Q(y)$: *y is a positive integer*

$S(x, y)$: *x + y is a positive integer*

$\forall x \forall y [P(x) \wedge Q(y) \rightarrow S(x, y)]$

(ii) *Let x and y are persons*

$P(x, y)$: *x is loved by y*

$\forall x \exists y \; P(x, y)$

(iii) *Let x and y are persons*

$A(x, y)$: *x admires y*

$(\exists x) \sim (\forall y)[A(x, y)]$

$(\exists x)(\exists y)[\sim A(x, y)]$

(iv) For every person x "If a person x is female and person x is a parent, then there exists a person y such that person x is mother of person y"

$F(x)$: *x is a Female*

$P(x)$: *x is Parent*

$M(x, y)$: *x is the mother of y*

$\forall x [F(x) \wedge P(x) \rightarrow \exists y \, M(x, y)]$

Rules of inference in Predicate Logic:

(i) *Universal specification*

By this rule if the premise $\forall\, xP(x)$ *is true then* $P(c)$ *is true where* c *is particular memeber of Domain.*

$$\frac{P(c)}{\therefore \forall\, xP(x)}$$

(ii) *Universal Generalization*

By this rule if $P(c)$ *is true for all* c *in Domain then* $\forall\, x\, P(x)$ *is true.*

$$\frac{P(c)}{\therefore (\forall\, x) P(x)}$$

(iii) *Existential Specification*

By this rule if $\exists\, x\;\; P(x)$ *is true then* $P(x)$ *is true for some particaular member* c *of Domain.*

$$\frac{\exists\, x\, P(x)}{\therefore P(c)}$$

(iv) Existential generalization

By this rule if $P(c)$ is ture for some particular memeber c in Domain, then $(\exists x)\, P(x)$ is true.

$$\frac{P(c)}{\therefore (\exists x)\, P(x)}$$

(v) Universal modus ponens :

(AKTU-2022)

By this rule if $P(x) \rightarrow Q(x)$ is ture for every x and $P(a)$ is true for some particular member a in Domain then $Q(a)$ is true.

$$\forall x\, (P(x) \rightarrow Q(x))$$

$$\frac{P(a)}{\therefore Q(a)}$$

(vi) Universal modus tollens

(AKTU-2022)

By this rule if $P(x) \rightarrow Q(x)$ is ture for every x and $\sim Q(a)$ is true for some particaular a in Domain then $Q(a)$ is true.

$$(\forall x)\, P(x) \rightarrow Q(x)$$

$$\frac{\sim Q(a)}{\therefore \sim P(a)}$$

Q.6 Rewrite the following argument using quantifiers, variables and predicate symbols. Prove the validity of the argument.

All healthy people eat an apple a day.

Ram does not eat an apple a day.

Ram is not a healthy person.

Sol Let $P(x)$: x is a healthy person

$Q(x)$: x eat an apple a day

Let C Stands for Ram

$$\forall x\,[P(x) \to Q(x)]$$

$$\frac{\sim Q(c)}{\therefore \sim P(c)}$$

By universal Modus tollens this argument is valid.

Discrete Mathematics by Gulshan Sir

DPP- 9

Topic : *Predicate Logic*

Q. 1 $\boldsymbol{Let\ D = \{1, 2, 3 \ldots \ldots .9\}, determine\ the\ truth\ value\ of\ each\ of\ the\ following\ statement.}$

1. $(\forall x \in D)x + 4 < 15$
2. $(\exists x \in D)x + 4 = 10$
3. $(\forall x \in D)x + 4 \leq 10$
4. $(\exists x \in D)x + 4 > 15$

Q. 2 *Rewrite the following argument using quantifiers, variables and predicate symbols. Prove thevalidity of the argument.*

$If\ a\ number\ is\ odd\ then\ its\ square\ is\ odd$

$K\ is\ a\ perticular\ number\ that\ is\ odd$

$K^2\ is\ odd$

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Thank You