



# AKTU DSTL



## DISCRETE MATHEMATICS

### ONE SHOT REVISION

**UNIT-II**

**FUNCTIONS & BOOLEAN ALGEBRA**



**17 MARCH 9:00PM**

**UNIT -2 : AKTU Syllabus****Functions**

- *Definition of Function*
- *Classification of functions*
- *Operations on functions*
- *Growth of function*

**Boolean Algebra:**

- *Introduction*
- *Axioms and Theorems of Boolean Algebra*
- *Algebraic manipulation of Boolean Expressions*
- *Simplification of Boolean functions*
- *Karnaugh maps*

# DISCRETE STRUCTURES & THEORY OF LOGICS (BCS303)

## UNIT -1 : Functions and Boolean Algebra

### Lecture - 01

#### Today's Target

- *Definition of Function*
- *Classification of functions*
- PYQ
- DPP

$R : A \rightarrow B$ **Function** $R \subseteq A \times B$ 

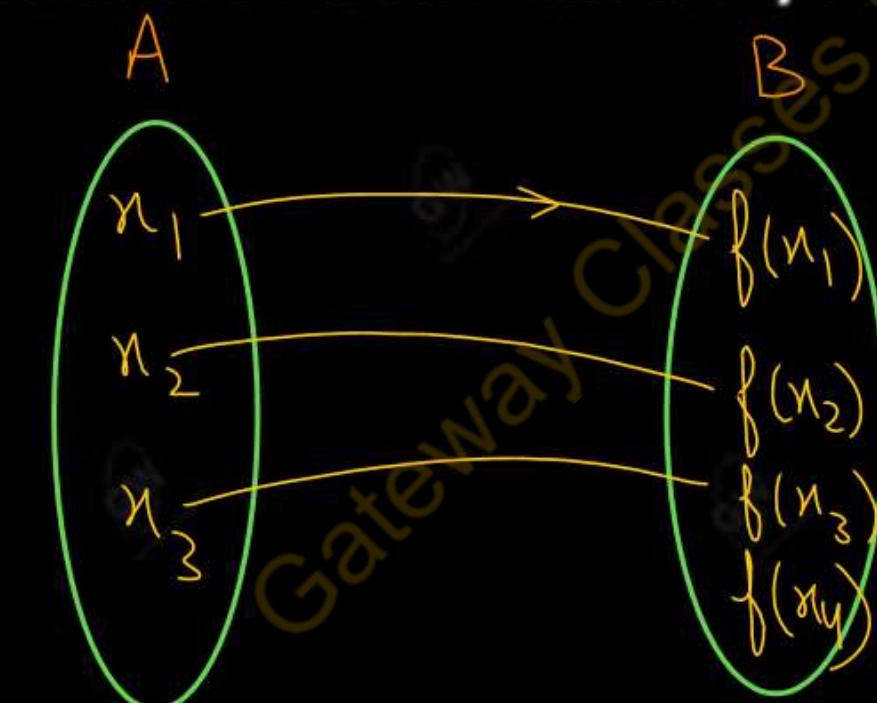
Let  $A$  and  $B$  are two non-empty sets

A function  $f : A \rightarrow B$  is a relation / Mapping from set  $A$  to set  $B$ , such that each and every element of Set  $A$  is related to exactly one element of set  $B$ .

**OR**

Those relations in which each and every element of set  $A$  has unique image in Set  $B$

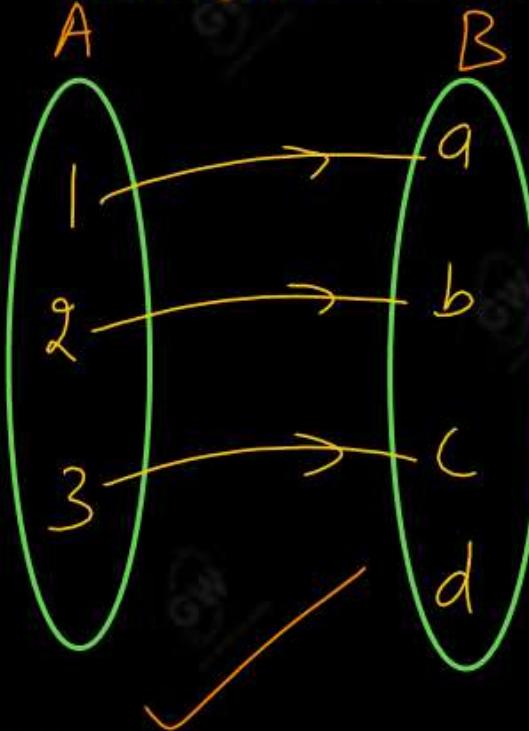
**Conclusion :** Every function is a relation but every relation is not a function .



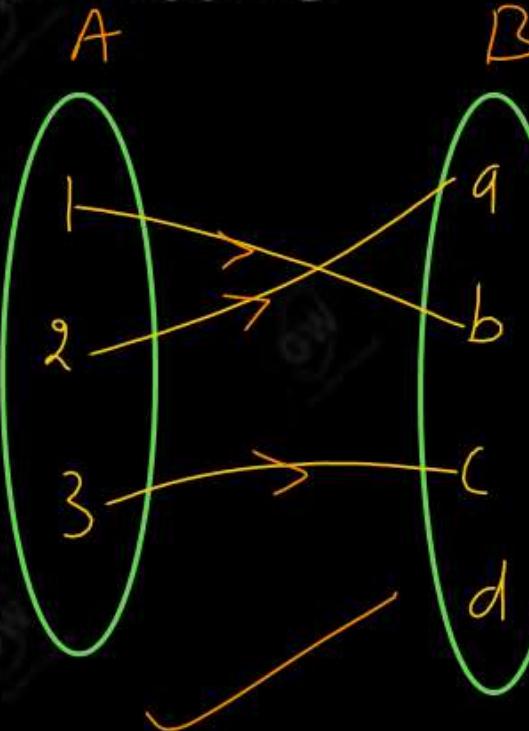
**Note:**  $f(x_1)$  is called image of  $x_1$  while  $x_1$  is called pre-image of  $f(x_1)$

**Example : Identify Function Mapping**

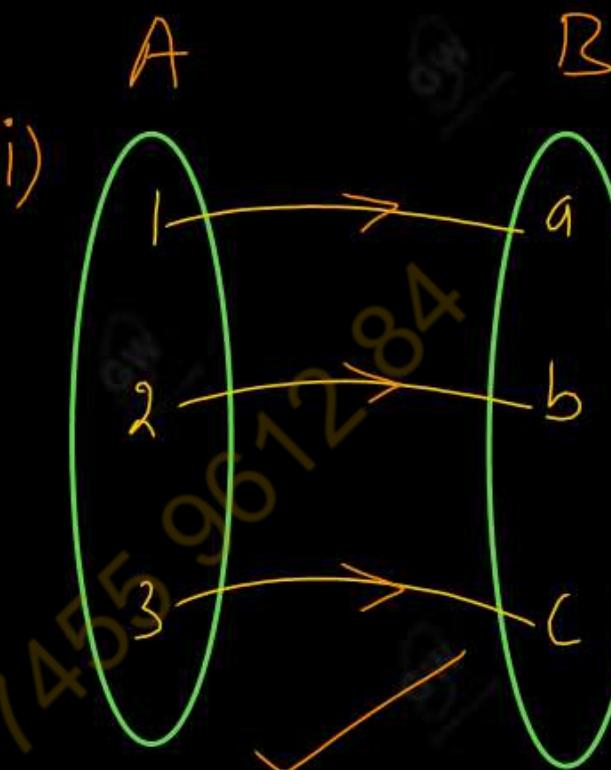
(i)



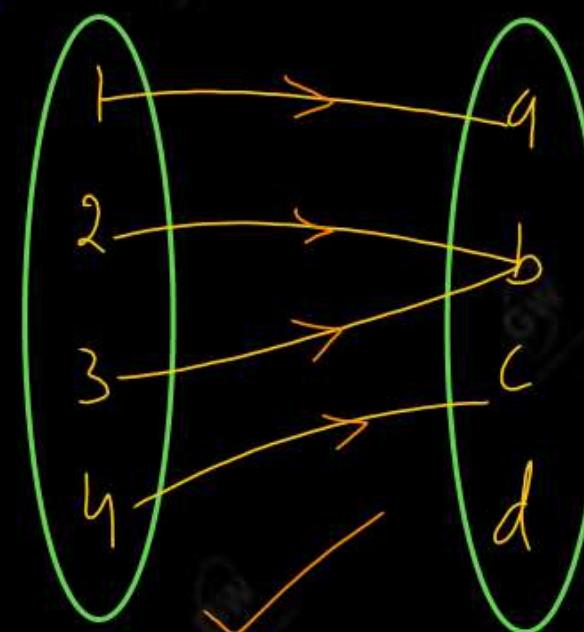
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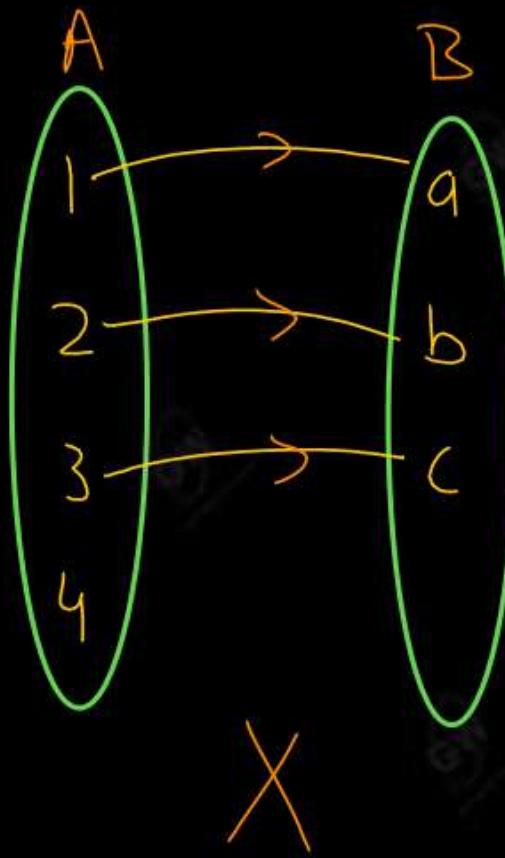
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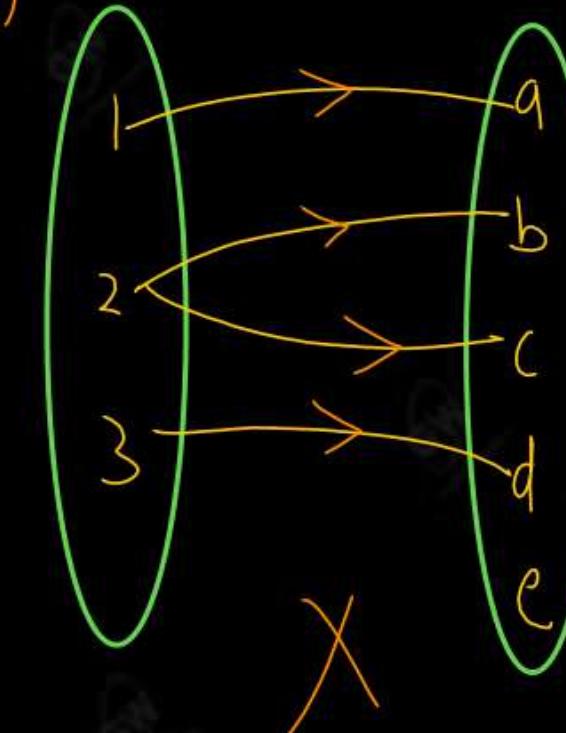
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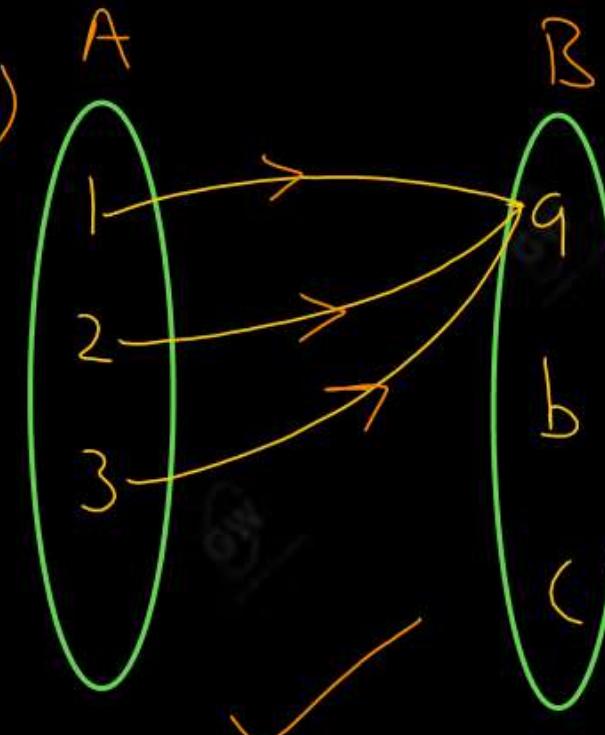
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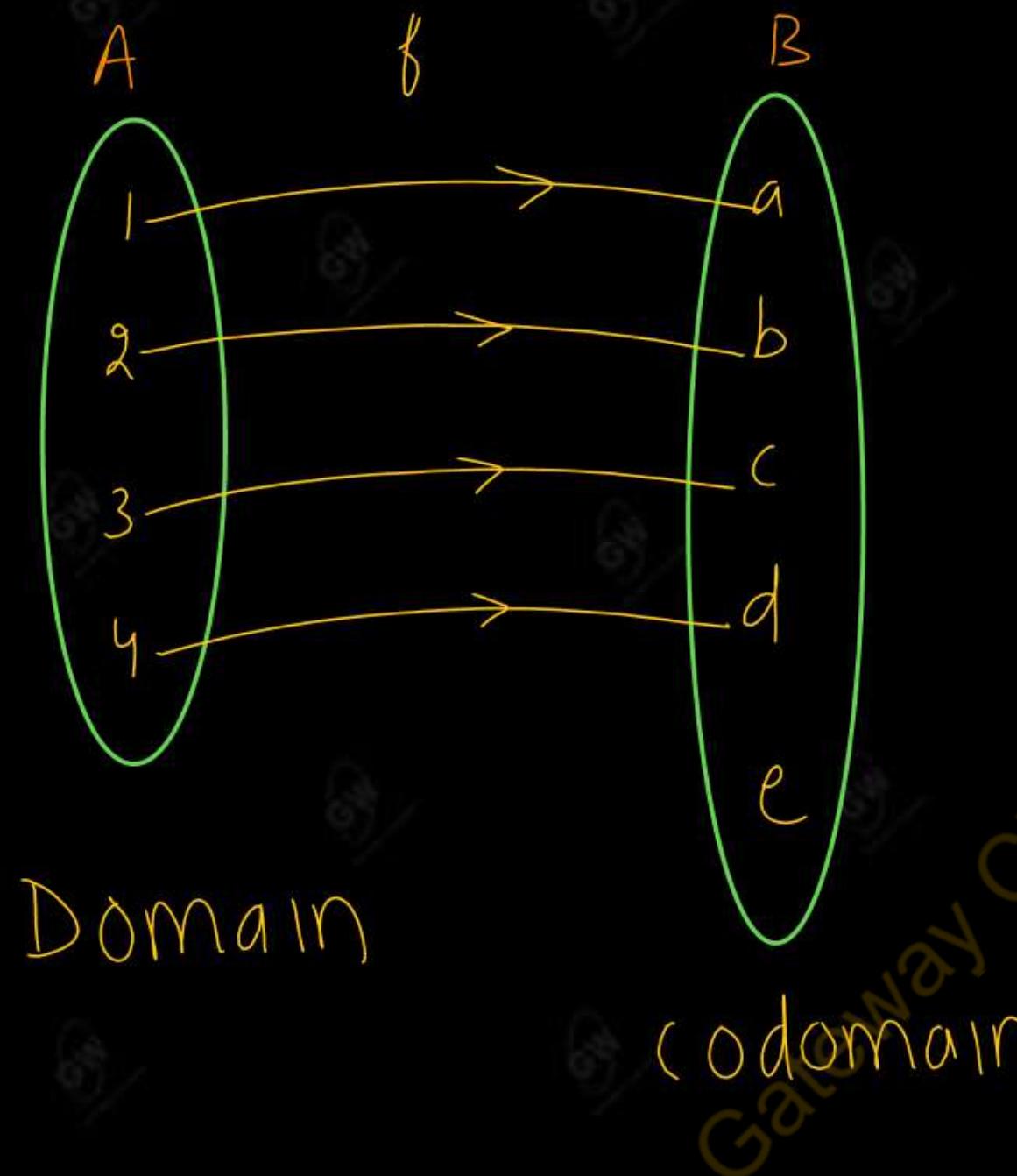


(vii)



(viii)



**Domain, Co-domain and Range of function**

$$\text{Dom}(f) = \{1, 2, 3, 4\}$$

$$\text{codomain} = \{a, b, c, d, e\}$$

$$\text{Range}(f) = \{a, b, c, d\}$$

$$f : R \rightarrow R$$

↓                      ↓

Domain                codomain

**Note:-** If  $n(A) = m$  and  $n(B) = n$ , then total number of possible function from  $A$  to  $B$

$$\text{number of functions} = (n)^m \quad (5)^4 =$$

# Classifications of function ( Mapping )

- (1) ONE –ONE FUNCTION (or injective function)
- (2) MANY-ONE FUNCTION
- (3) ONTO FUNCTION (or surjective function)
- (4) INTO FUNCTION
- ~~(5)~~ (5) ONE-ONE ONTO FUNCTION (Bijective function)
- (6) One-one into function
- (7) Many-one onto function
- (8) Many-one into function

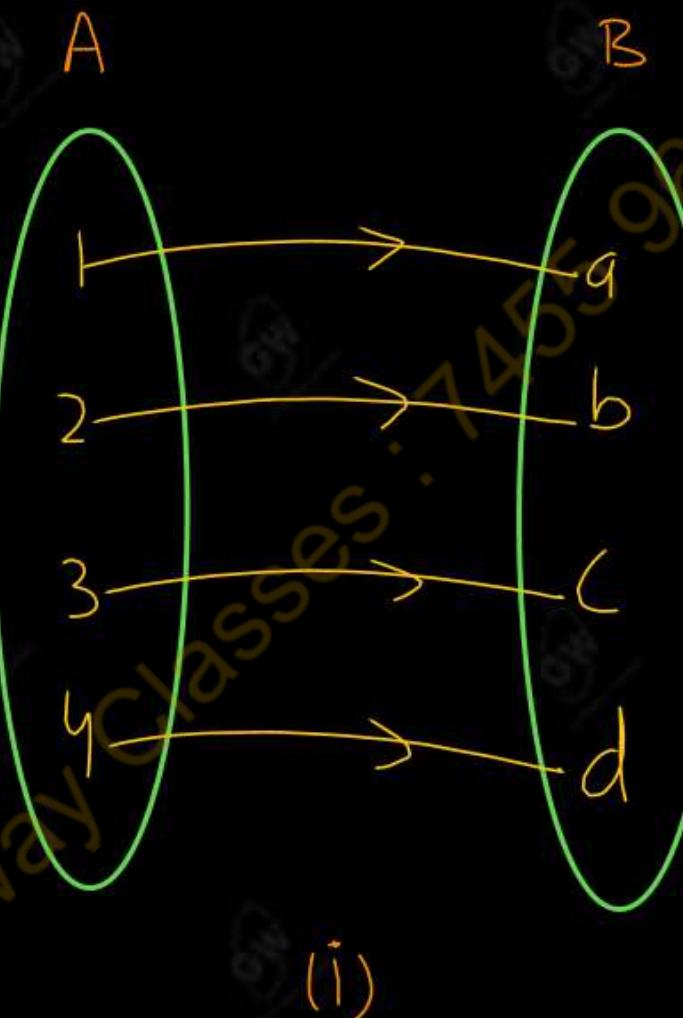
**ONE – ONE FUNCTION OR One to one function OR Injective function**

A function  $f : A \rightarrow B$  is said to be one-one function if different elements of  $A$  have different images in  $B$ .

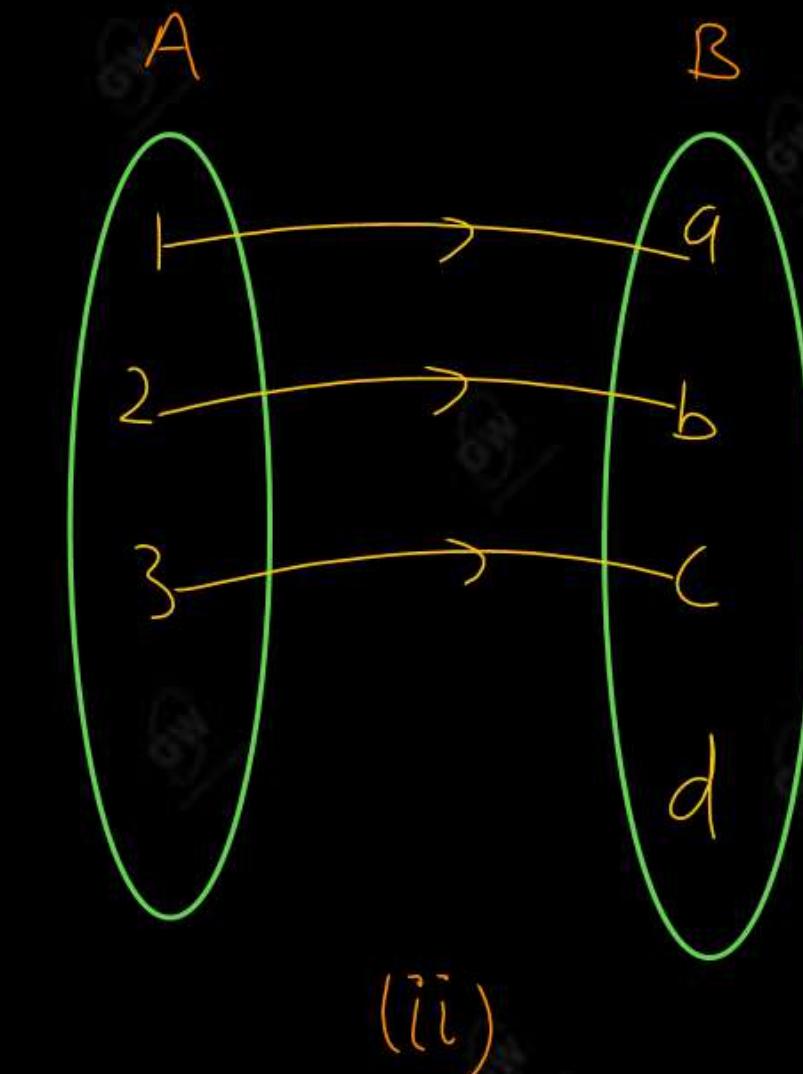
Thus,  $f : A \rightarrow B$  is one-one if and only if

$$f(x_1) = f(x_2)$$

$$\Rightarrow x_1 = x_2 \quad \forall x_1, x_2 \in A$$



(i)



(ii)



**Note:-** If  $n(A) = m$  and  $n(B) = n$ , then

Total number of one-one function =  ${}^n P_m = \frac{n!}{(n-m)!}$

Q. 1 :- Show that the function  $f: N \rightarrow N$ , given by  $f(x) = 2x + 3$  is one-one

$$f(n) = 2n + 3$$

Let  $n_1, n_2 \in N$

Put  $f(n_1) = f(n_2)$

$$2n_1 + 3 = 2n_2 + 3$$

$$\cancel{2n_1} = \cancel{2n_2}$$

$$n_1 = n_2$$

Hence,  $f(n)$  is a one-one function

Q.2:- Check whether the function  $f(x) = x^2 - 1$  is injective or not for  $f: R \rightarrow R$

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$$f(n) = n^2 - 1$$

Let  $n_1, n_2 \in R$

Put  $f(n_1) = f(n_2)$

$$n_1^2 - 1 = n_2^2 - 1$$

$$n_1^2 = n_2^2$$

$$n_1^2 - n_2^2 = 0$$

$$(n_1 - n_2)(n_1 + n_2) = 0$$

$$n_1 = n_2 \text{ or } n_1 = -n_2$$

Hence,  $f(n)$  is not injective

Q.3:- Check whether the function  $f(x) = x^3 + 2$  is one-one or not for  $f: R \rightarrow R$ .

$$f(n) = n^3 + 2$$

Let  $n_1, n_2 \in R$

Put  $f(n_1) = f(n_2)$

$$n_1^3 + 2 = n_2^3 + 2$$

$$n_1^3 = n_2^3$$

$$\boxed{n_1 = n_2}$$

Hence,  $f(n)$  is one-one function

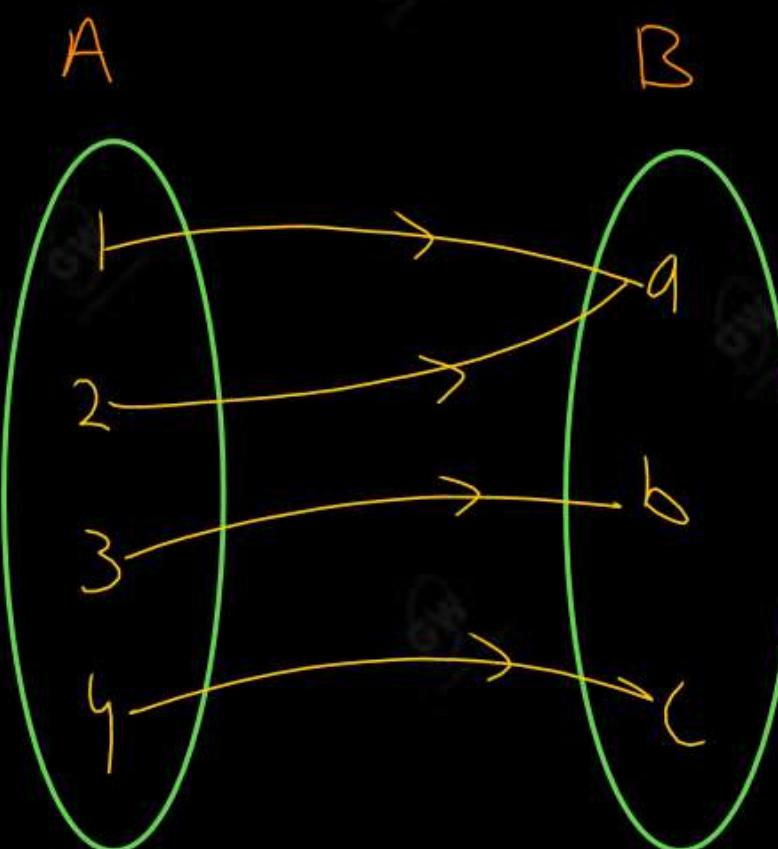
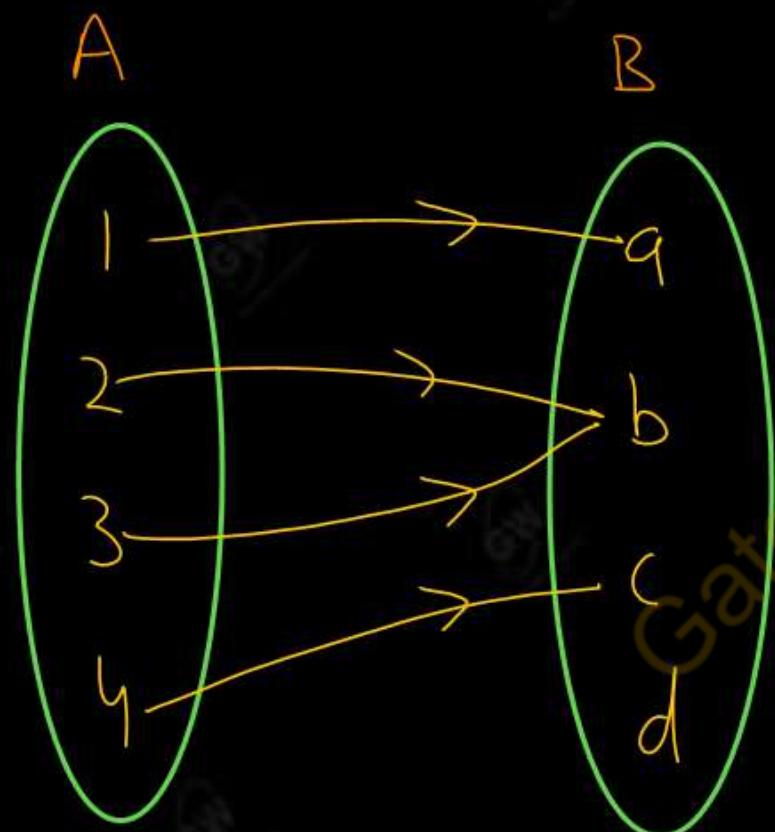
A function  $f: A \rightarrow B$  is said to be a many – one function if two or more elements of set A have same image in B.

Thus,  $f : A \rightarrow B$  is Many one if and only if

$$f(x_1) = f(x_2)$$

$$\Rightarrow x_1 \neq x_2 \quad \forall x_1, x_2 \in A$$

In other words,  $f : A \rightarrow B$  is Many one if it is not one – one function

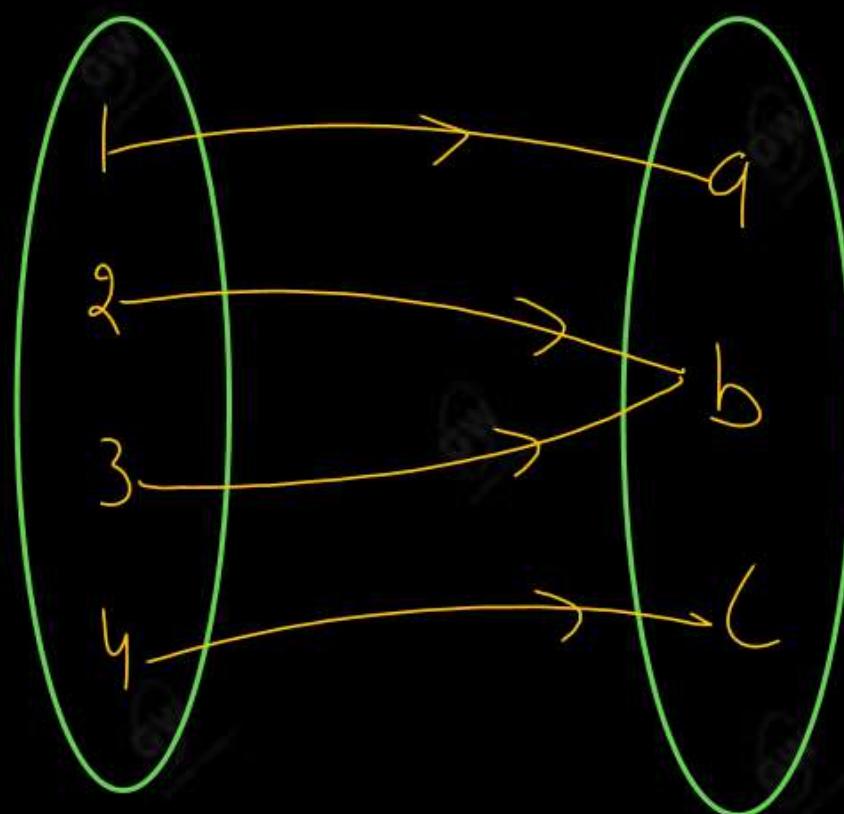
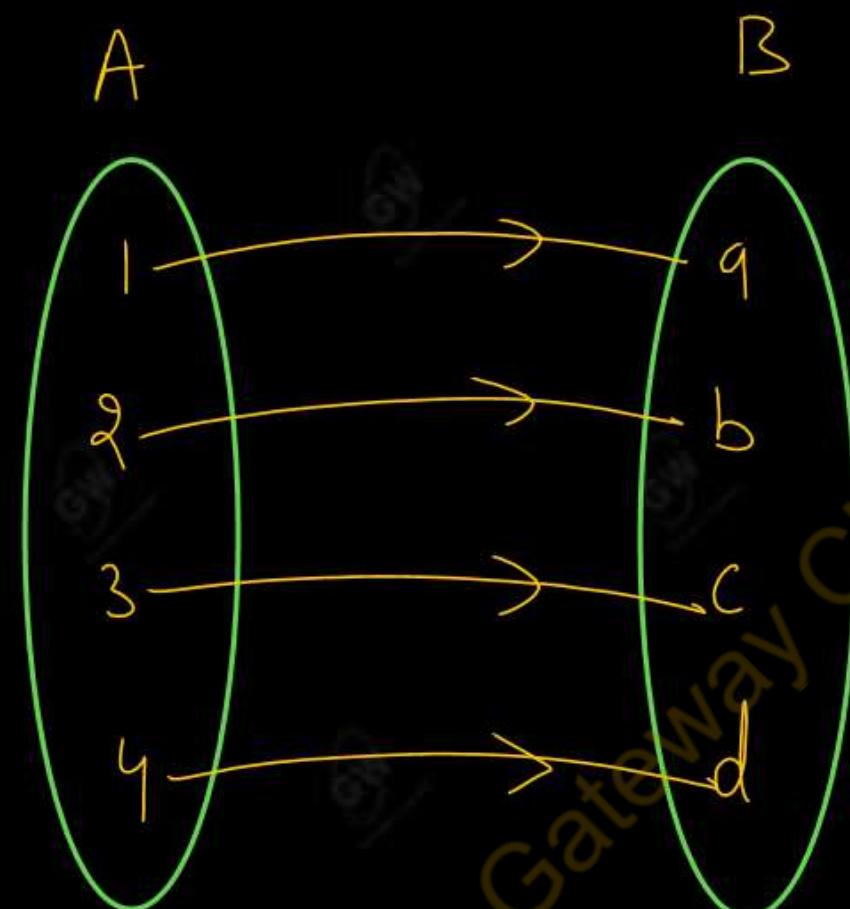


**ONTO FUNCTION (or Surjective function)**

A function  $f: A \rightarrow B$  is said to be an **ONTO** function if every element of set  $B$  have pre-image in set  $A$

i.e. Range of function = codomain of function

**Example**



Q.4:- Show that the function  $f : R \rightarrow R^+$  defined by  $f(x) = x^2 \quad \forall x \in R$  is an Onto function

$$f(n) = n^2$$

$$y = n^2$$

$$n^2 = y$$

$$n = \sqrt{y}$$

$$\Rightarrow y \in R^+$$

Hence

Range of function  $\Rightarrow$  codomain of function

$\Rightarrow f(n)$  is an ONTO Function

**Q.5:- Discuss the Surjectivity of the following functions**(i)  $f : R \rightarrow R$  given by  $f(x) = x^3 + 2$  for  $x \in R$ (ii)  $f : R \rightarrow R$  given by  $f(x) = x^2 + 2$  for  $x \in R$ 

(i)  $f(n) = n^3 + 2$

$y = n^3 + 2$

$y - 2 = n^3$

$n^3 = y - 2$

$n = (y - 2)^{1/3}$

$\Rightarrow y \in R$

Range of  $f$  = codomain of  $f$ Hence  $f(n)$  is a surjective function

(ii)  $f(n) = n^2 + 2$

$y = n^2 + 2$

$n^2 = y - 2$

$n = \sqrt{y - 2}$

$\Rightarrow y \geq 2$

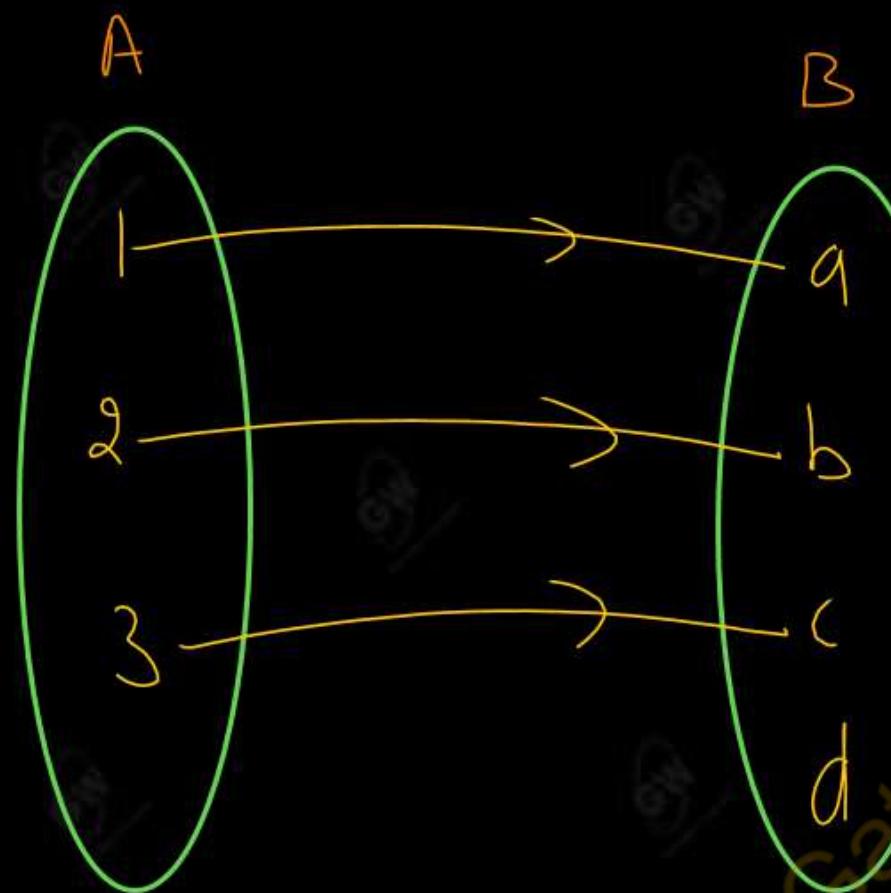
Range of  $f \neq$  codomain of  $f$ 

Hence, function is not surjective

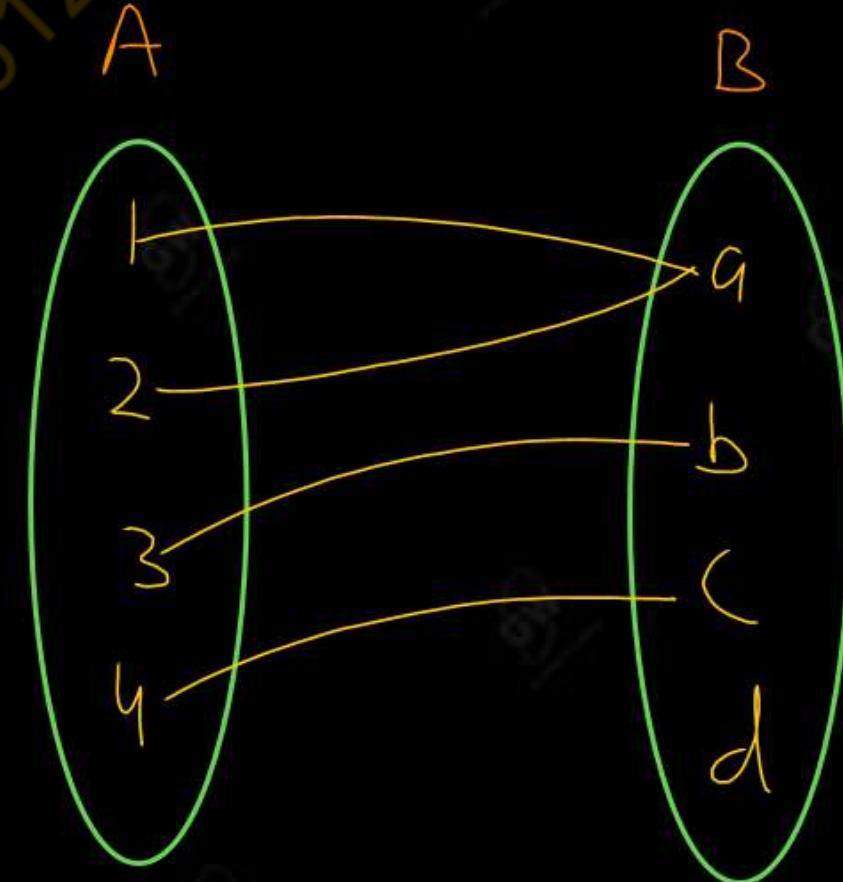
**INTO FUNCTION :**

A function  $f : A \rightarrow B$  is an **Into function** if there exist an element in  $B$  having no pre-image in  $A$ .

Thus,  $f : A \rightarrow B$  is an **Into function** if it is not **Onto function**

**Example**

(i)



(ii)

**ONE -ONE ONTO FUNCTION (Bijective function)**

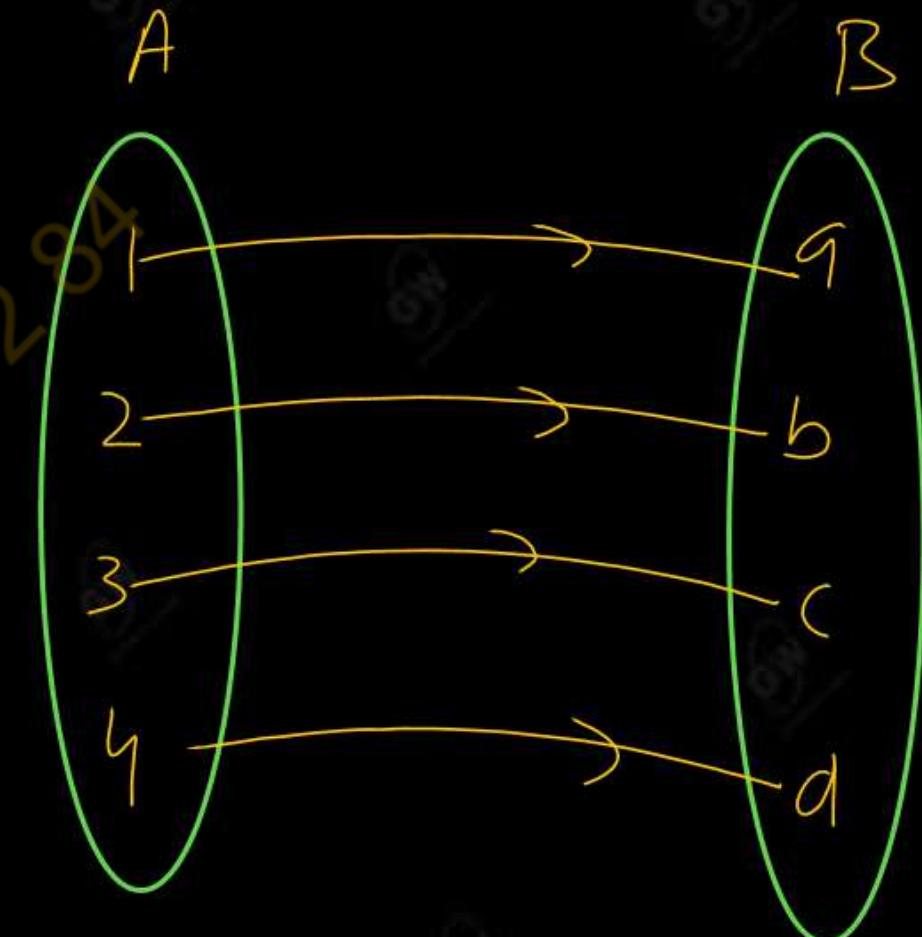
A function  $f : A \rightarrow B$  is Bijective if it is

(i) One – One Function

$$\begin{aligned}f(x_1) &= f(x_2) \\ \Rightarrow x_1 &= x_2\end{aligned}$$

(ii) Onto Function

For all  $y \in B$ , there exist  $x \in A$  such that  $f(x) = y$



**Q.6:- Check the Injectivity or Surjectivity of the following function****(i)  $f: N \rightarrow N$  given by  $f(x) = x^2$** **(ii)  $f: Z \rightarrow Z$  given by  $f(x) = x^2$** 

(i)  $f(n) = n^2$

Let  $n_1, n_2 \in N$ 

Put  $f(n_1) = f(n_2)$

$n_1^2 = n_2^2$

$$\boxed{n_1 = n_2}$$

Hence  $f(n)$  is injective

Again,

$f(n) = n^2$

$y = n^2$

$n^2 = y$

$n = \sqrt{y}$

 $\text{Range}(f) \neq \text{codomain}(f)$  $f(n)$  is surjective

**Q.7:- Show that the function  $f : \underbrace{R - \{3\}}_{\text{Domain}} \rightarrow \underbrace{R - \{1\}}_{\text{Range}}$  given by  $f(x) = \frac{x-2}{x-3}$  is a bijection**

$$f(n) = \frac{n-2}{n-3}$$

Let  $n_1, n_2 \in R - \{3\}$

Put  $f(n_1) = f(n_2)$

$$\frac{n_1-2}{n_1-3} = \frac{n_2-2}{n_2-3}$$

$$(n_1-2)(n_2-3) = (n_1-3)(n_2-2)$$

$$\cancel{n_1n_2 - 3n_1 - 2n_2 + 6} = \cancel{n_1n_2 - 2n_1 - 3n_2 + 6}$$

$$-3n_1 - 2n_2 = -2n_1 - 3n_2$$

$$+ n_1 = + n_2$$

$$n_1 = n_2$$

$f(n)$  is injective

Again

$$f(M) = \frac{M-2}{M-3}$$

$$y = \frac{M-2}{M-3}$$

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$$y(M-3) = M-2$$

$$My - 3y = M - 2$$

$$My - M = 3y - 2$$

$$M(y-1) = 3y - 2$$

$$M = \frac{3y-2}{y-1}$$

$\Rightarrow \text{Range} = R - \{1\}$

$$\Rightarrow \text{Range}(f) = \text{codomain } f$$

Hence

$f(n)$  is surjective

Since  $f(n)$  is injective as well as surjective

$\Rightarrow f(n)$  is bijective

## Topic : Classification of function

**Q.1.** Show that the function  $f : Z \rightarrow Z$  defined by  $f(x) = x^2 + 1$  for all  $x \in Z$  is a many – one function.

**Q.2.** Show that  $f : R \rightarrow R$ , given by  $f(x) = ax + b$  where  $a, b \in R, a \neq 0$  is a bijection

**Q.3.** Show that  $f : R \rightarrow R$ , if defined as  $f(x) = x^3$  is a bijection.

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Thank You

# DISCRETE STRUCTURES & THEORY OF LOGICS

## (Discrete Mathematics)

### UNIT - 2: Functions and Boolean Algebra

Lecture - 02

#### Today's Target

- ***Inverse of a Function***
- PYQ
- DPP

**ONE – ONE FUNCTION**

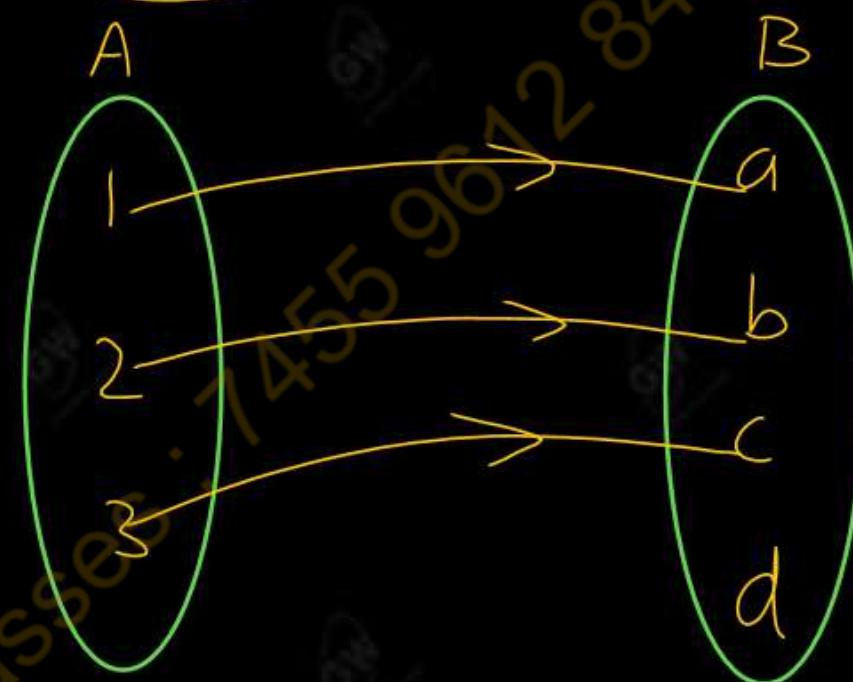
Each

A function  $f : A \rightarrow B$  is said to be one-one function if different elements of  $A$  have different images in  $B$ .

Thus,  $f : A \rightarrow B$  is one-one if and only if

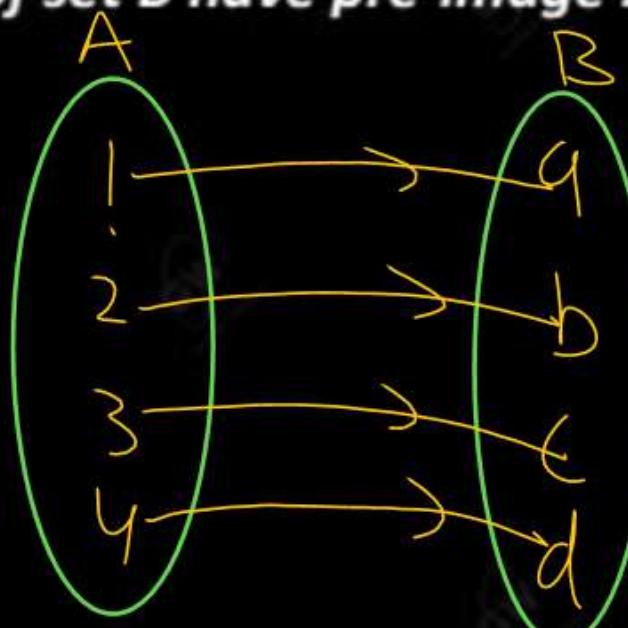
$$f(x_1) = f(x_2)$$

$$\Rightarrow x_1 = x_2 \quad \forall x_1, x_2 \in A$$

**ONTO FUNCTION (or Surjective function)**

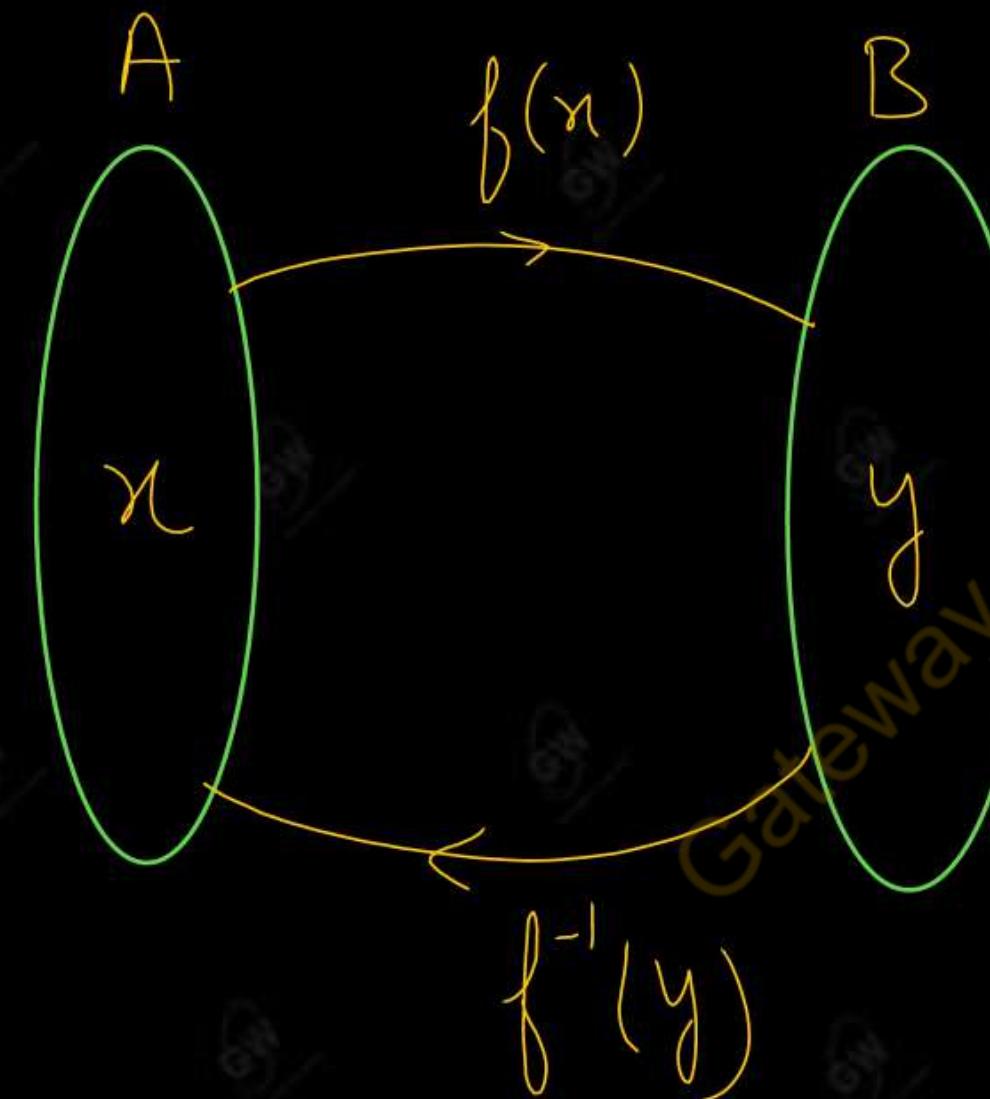
A function  $f : A \rightarrow B$  is said to be an ONTO function if every element of set B have pre-image in set A

i.e. Range of function = codomain of function



A function  $f : A \rightarrow B$  is called invertible function if it is one one onto function and its inverse is given as

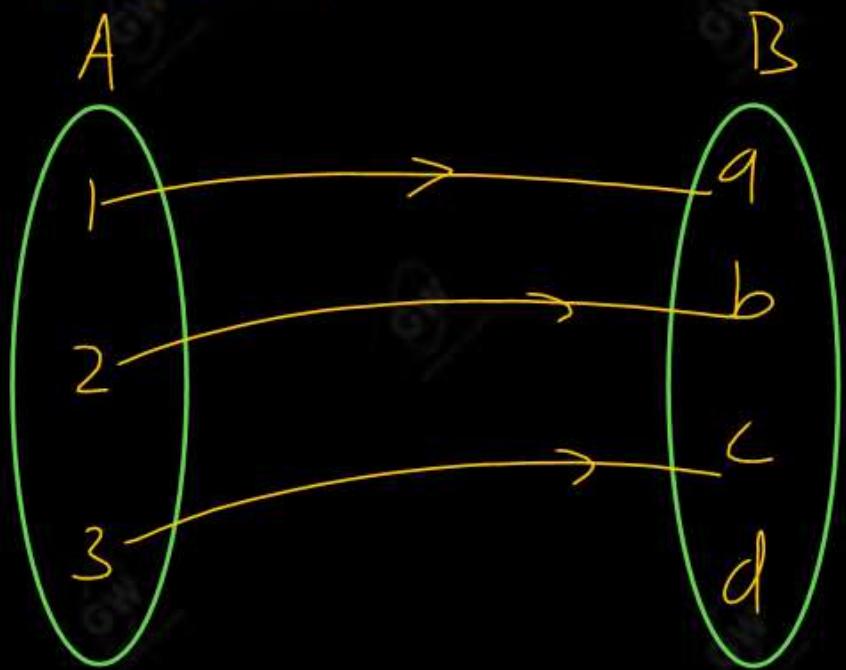
$$f^{-1} : B \rightarrow A$$



If  $f(x) = y$  then

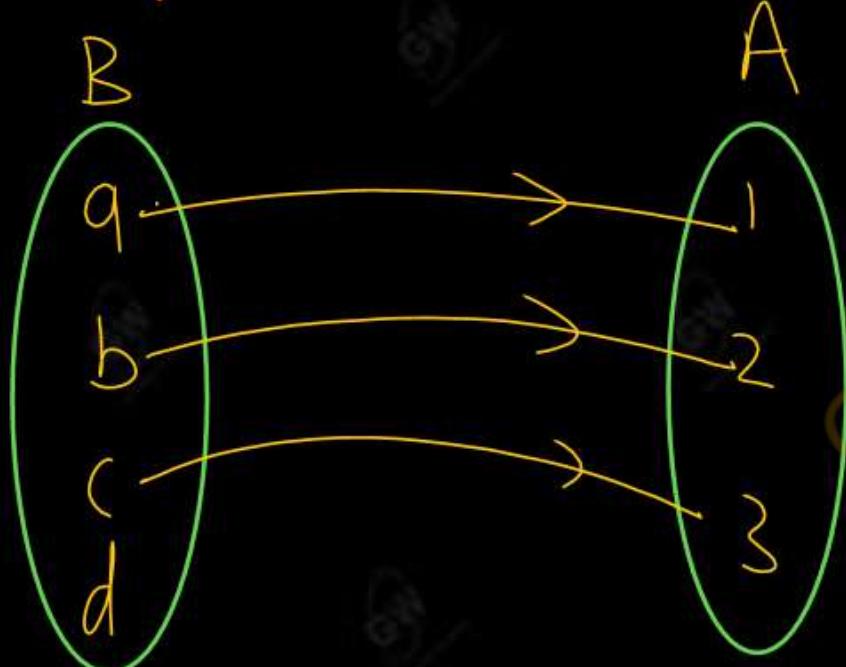
$$f^{-1}(y) = x$$

Example  $f: A \rightarrow B$



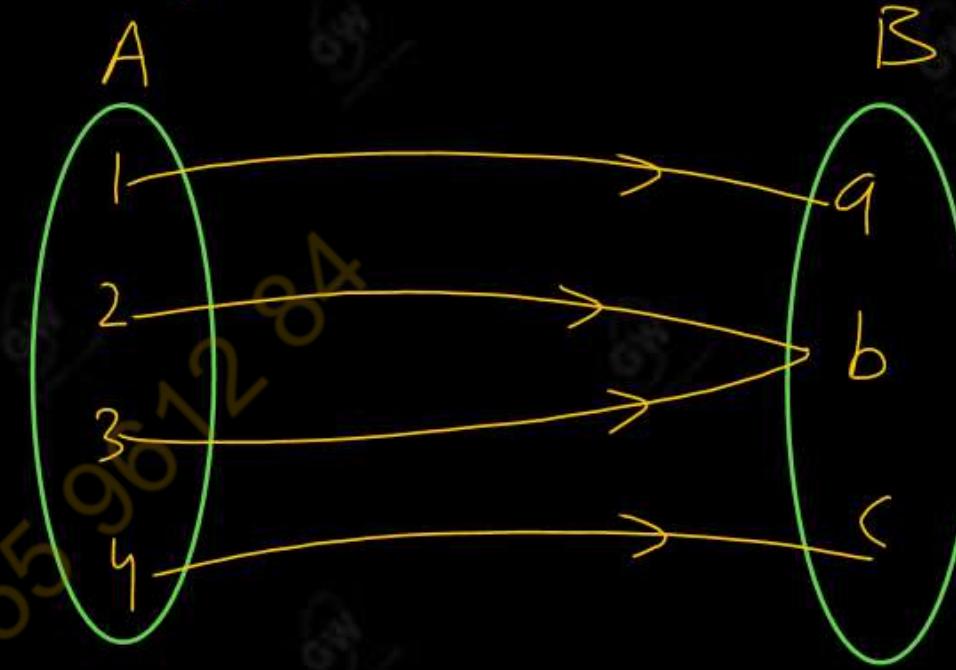
one-one  
into

$f^{-1}: B \rightarrow A$  X



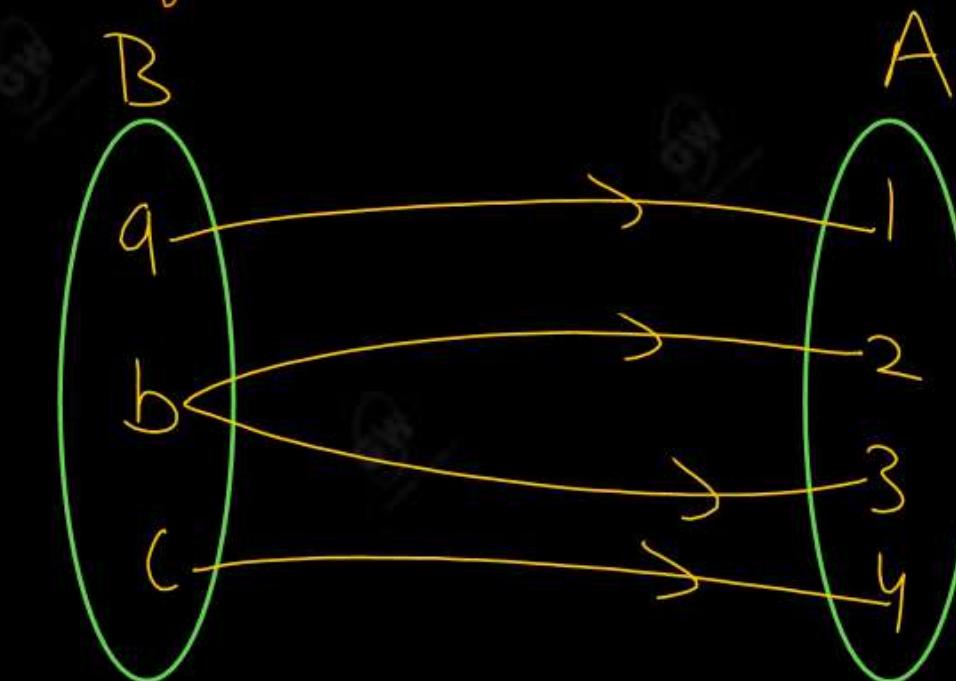
It is not  
a function

$f: A \rightarrow B$

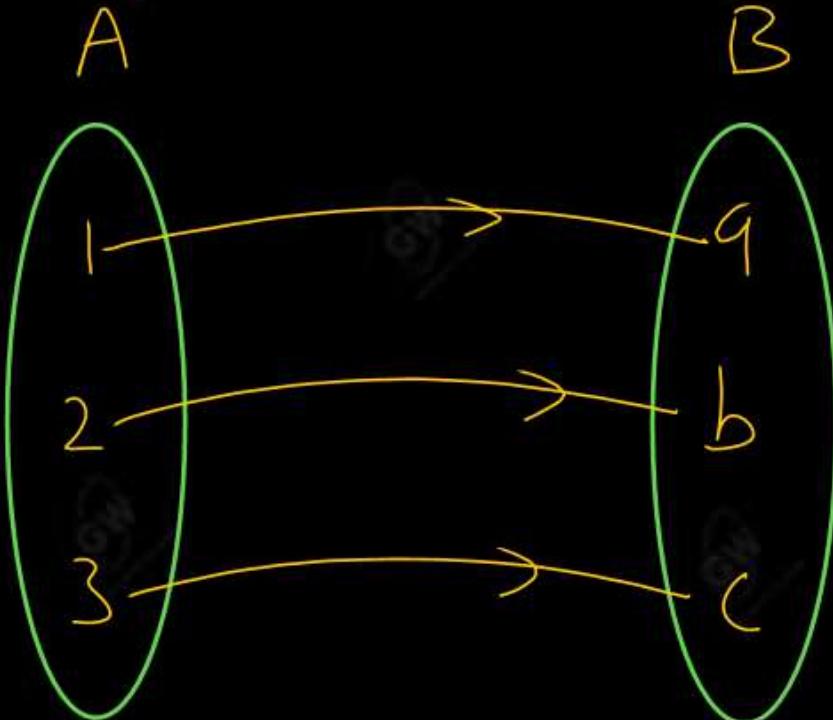


Many-one  
onto

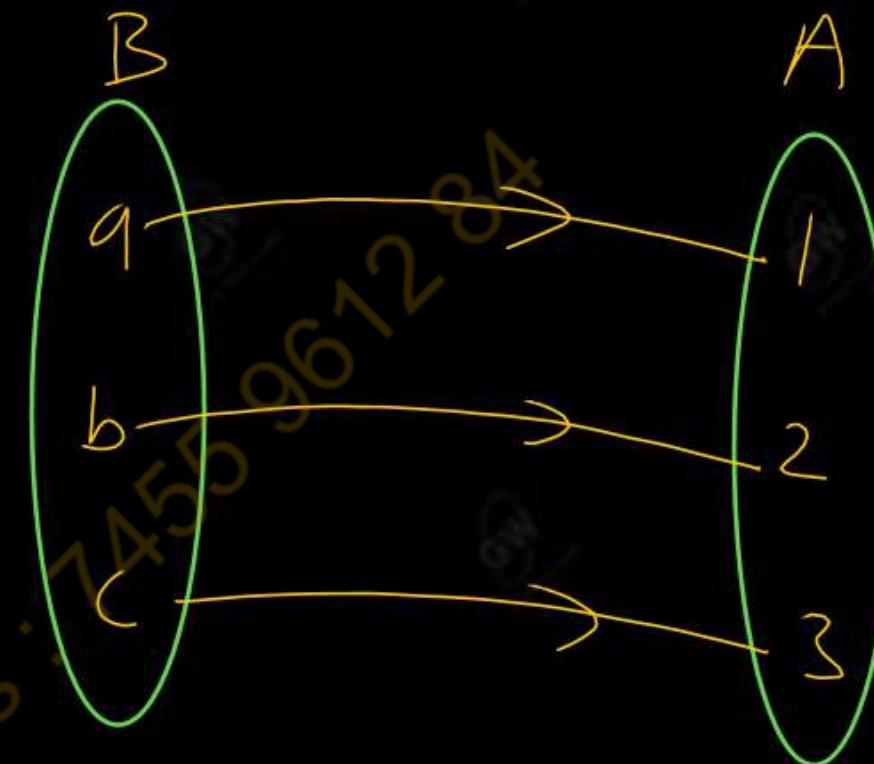
$f^{-1}: B \rightarrow A$  X



It is not  
a function

$f: A \rightarrow B$ 

one-one onto

 $f^{-1}: B \rightarrow A$ 

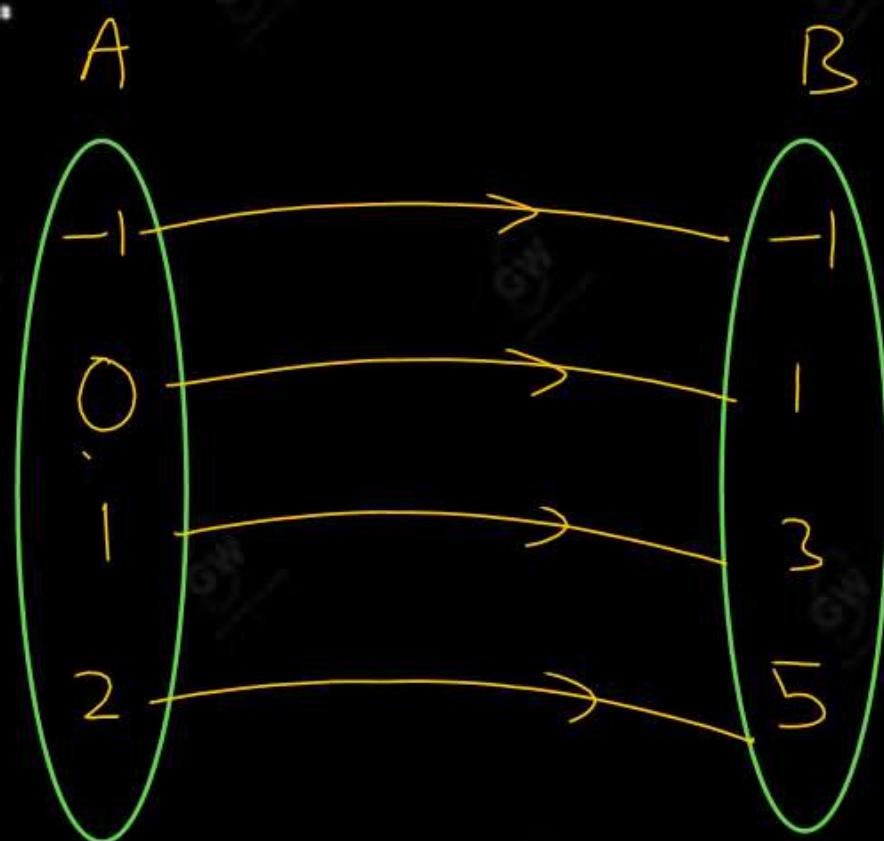
one-one onto

**Conclusion :** (i) A function  $f : A \rightarrow B$  is invertible if it is one one onto function

(ii) If  $f : A \rightarrow B$  is ONE-ONE ONTO, then  $f^{-1} : B \rightarrow A$  is also ONE-ONE ONTO

(iii) If  $f : A \rightarrow B$  is ONE-ONE ONTO function, then  $f^{-1} : B \rightarrow A$  is a unique function

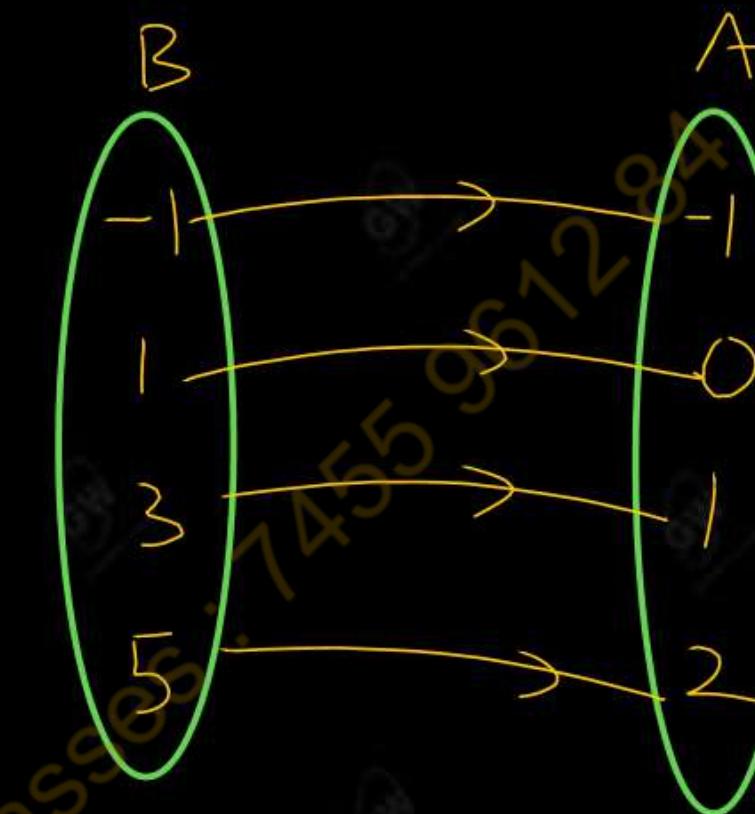
**Example:  $f(x) = 2x + 1$**



$$y = 2x + 1$$

$$2x = y - 1$$

$$x = \frac{y-1}{2}$$



$$f^{-1}(y) = \frac{y-1}{2}$$

### Steps to find INVERSE

- A Put  $f(x) = y$ , where  $y \in B$  and  $x \in A$ .
- Solve  $f(x) = y$  to obtain  $x$  in terms of  $y$ .
- Replace  $x$  by  $f^{-1}(y)$  to obtain the required inverse of  $f$ .

**Q.1:- Let  $f : R \rightarrow R$  is defined by  $f(x) = 3x - 7$ . Show that  $f$  is invertible and hence find the inverse of  $f$ .**

$$f(n) = 3n - 7$$

Let  $n_1, n_2 \in R$

$$\text{Put } f(n_1) = f(n_2)$$

$$3n_1 - 7 = 3n_2 - 7$$

$$3n_1 = 3n_2$$

$$n_1 = n_2$$

Hence  
 $f(n)$  is one-one function

Again

$$f(n) = 3n - 7$$

$$\text{Put } f(n) = y$$

$$y = 3n - 7$$

$$3n = y + 7$$

$$n = \frac{y+7}{3} - 0$$

$$\Rightarrow y \in R$$

For all  $y \in R$ ,  $n \in R$

$\Rightarrow$  Range = codomain

Hence  $f(n)$  is an onto function

$\Rightarrow f(n)$  is one-one onto

Replace  $n$  by  $f^{-1}(y)$  in ①

$$f^{-1}(y) = \frac{y+7}{3}$$

**Q.2:- Show that  $f : R - \{0\} \rightarrow R - \{0\}$  given by  $f(x) = \frac{1}{x}$ , is invertible and find  $f^{-1}$ .**

$$f(n) = \frac{1}{n}$$

Let  $n_1, n_2 \in R - \{0\}$

$$\text{Put } f(n_1) = f(n_2)$$

$$\frac{1}{n_1} = \frac{1}{n_2}$$

$$n_1 = n_2$$

$\Rightarrow f(n)$  is one-one

Again

$$f(n) = \frac{1}{n}$$

$$y = \frac{1}{n}$$

$$n = \frac{1}{y}$$

$$y \in R - \{0\}$$

For all  $y \in R - \{0\}$ ,  $n \in R - \{0\}$

Range  $f = \text{codomain } f$

$\Rightarrow f(n)$  is an onto function

$\Rightarrow f(n)$  is one-one onto

Replace  $n$  by  $f^{-1}(y)$  in ①

$$f^{-1}(y) = \frac{1}{y}$$

Q.3:- Is the function  $f : N \rightarrow N$  defined by  $f(x) = x^2$ ,  $x \in N$  is invertible

$$f(n) = n^2$$

Let  $n_1, n_2 \in N$

$$\text{Put } f(n_1) = f(n_2)$$

$$n_1^2 = n_2^2$$

$$\Rightarrow n_1 = n_2$$

$\Rightarrow f(n)$  is one-one

$$f(n) = n^2$$

$$\text{Put } f(n) = y$$

$$y = n^2$$

$$n^2 = y$$

$$n = \sqrt{y}$$

For all  $y \in N$

$$n \notin N$$

$\Rightarrow$

Range  $f = \text{co domain } f$

$$94$$

$\Rightarrow f(n)$  is not onto

Hence

$f(n)$  is not invertible

**Q.4:-** Let a function is defined as  $f: R - \{3\} \rightarrow R - \{1\}$ ,  $f(x) = (x-2)/(x-3)$ , then show that  $f$  is a bijective function and also compute the inverse of  $f$ . Where  $R$  is a set of real numbers.

$$f(n) = \frac{n-2}{n-3}$$

Let  $n_1, n_2 \in R - \{3\}$

$$\text{Put } f(n_1) = f(n_2)$$

$$\frac{n_1-2}{n_1-3} = \frac{n_2-2}{n_2-3}$$

$$(n_1-2)(n_2-3) = (n_2-2)(n_1-3)$$

~~$$n_1n_2 - 3n_1 - 2n_2 + 6 = n_1n_2 - 3n_2 - 2n_1 + 6$$~~

$$\begin{aligned} -3n_1 - 2n_2 &= -3n_2 - 2n_1 \\ -3n_1 + 2n_1 &= -3n_2 + 2n_2 \\ +n_1 &= +n_2 \\ n_1 &= n_2 \end{aligned}$$

$\Rightarrow f(n)$  is one-one

Again

$$f(n) = \frac{n-2}{n-3}$$

$$y = \frac{n-2}{n-3}$$

$$ny - 3y = n - 2$$

$$ny - n = 3y - 2$$

$$n(y-1) = 3y-2$$

$$n = \frac{3y-2}{y-1} \quad \text{--- (1)}$$

$$y \in R - \{1\}$$

For all  $y \in R - \{1\}$

$n \in R - \{3\}$

Range of function = codomain of function

$\Rightarrow f(n)$  is onto

$\Rightarrow f(n)$  is one-one onto function

Replace  $n$  by  $f^{-1}(y)$  in ①

$$f^{-1}(y) = \frac{3y-2}{y-1}$$

Q.5:- Find the inverse of function  $f(x) = \frac{2x}{x-1}$

$$f(n) = \frac{2n}{n-1}$$

Put  $f(n) = y$

$$y = \frac{2n}{n-1}$$

$$y(n-1) = 2n$$

$$ny - y = 2n$$

$$ny - 2n = y$$

$$n(y-2) = y$$

$$n = \frac{y}{y-2}$$

Replace  $n$  by  $f^{-1}(y)$

$$f^{-1}(y) = \frac{y}{y-2}$$

$$f: R - \{1\} \rightarrow R - \{2\}$$

**Topic : Inverse of a function**

**Q.1.** Let  $f : R \rightarrow R$  is defined by  $f(x) = ax + b$  where  $a, b \in R$  and  $a \neq 0$ . Show that  $f$  is invertible and find the inverse of  $f$ .

**Q.2.** Show that  $f : R - \{0\} \rightarrow R - \{0\}$ , given by  $f(x) = \frac{3}{x}$  is invertible and find  $f^{-1}$ .

**Q.3.** Let  $A = R - \{3\}$  and  $B = R - \{1\}$  Consider the function  $f: A \rightarrow B$ , defined by

$$f(x) = \frac{x-2}{x-3}$$

Show that  $f$  is invertible and find  $f^{-1}$ .

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# DISCRETE STRUCTURES & THEORY OF LOGICS

## (Discrete Mathematics)

UNIT - 2: **Functions** and Boolean Algebra

Lecture - 03

### Today's Target

- Composition of Functions
- PYQ
- DPP

# Composition of Functions

Let  $A, B$  and  $C$  be three non-empty sets and let

$$f : A \rightarrow B \text{ and } g : B \rightarrow C$$

be two functions

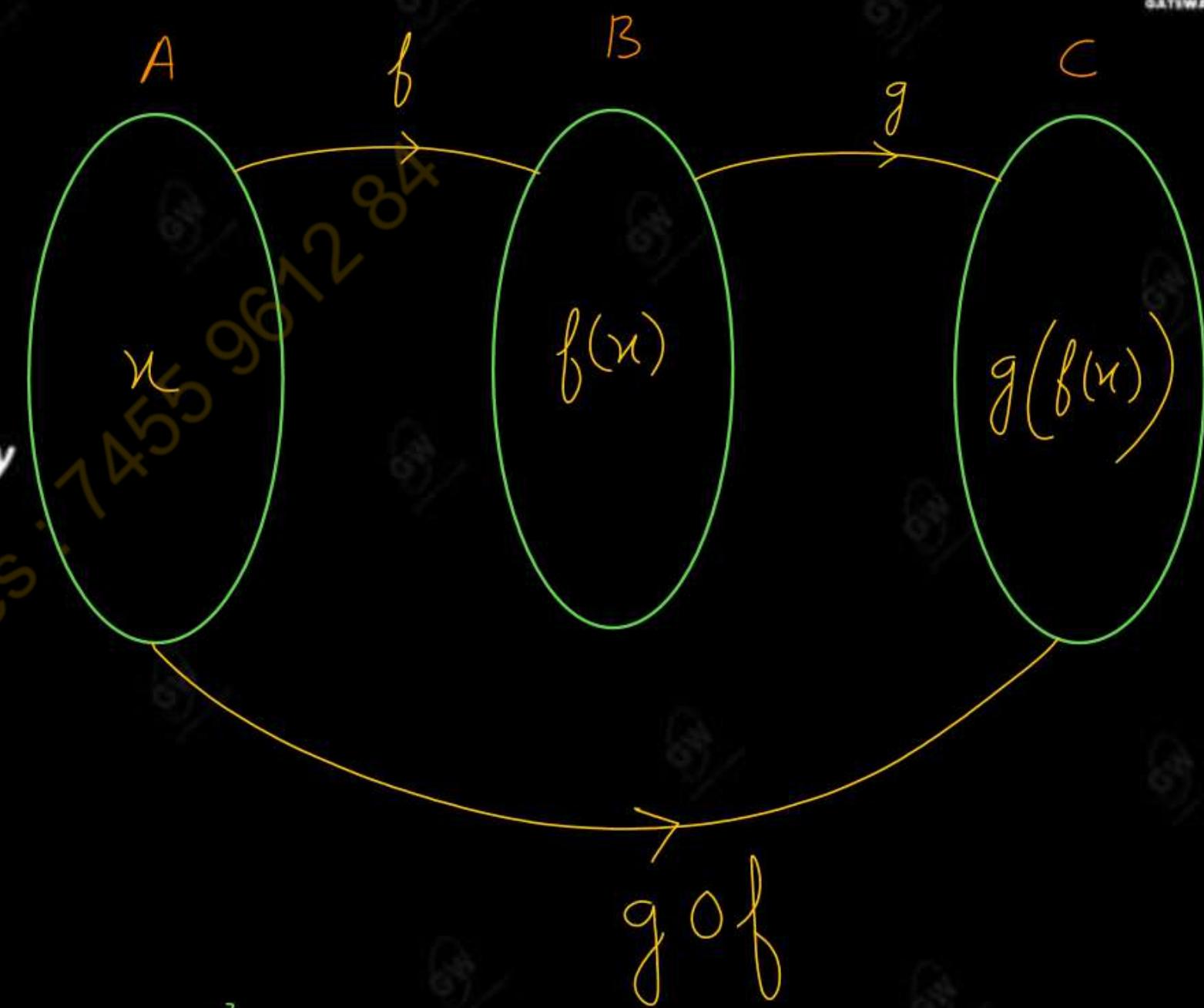
Then the composition of  $f$  and  $g$  is a function denoted by

$$gof : A \rightarrow C$$

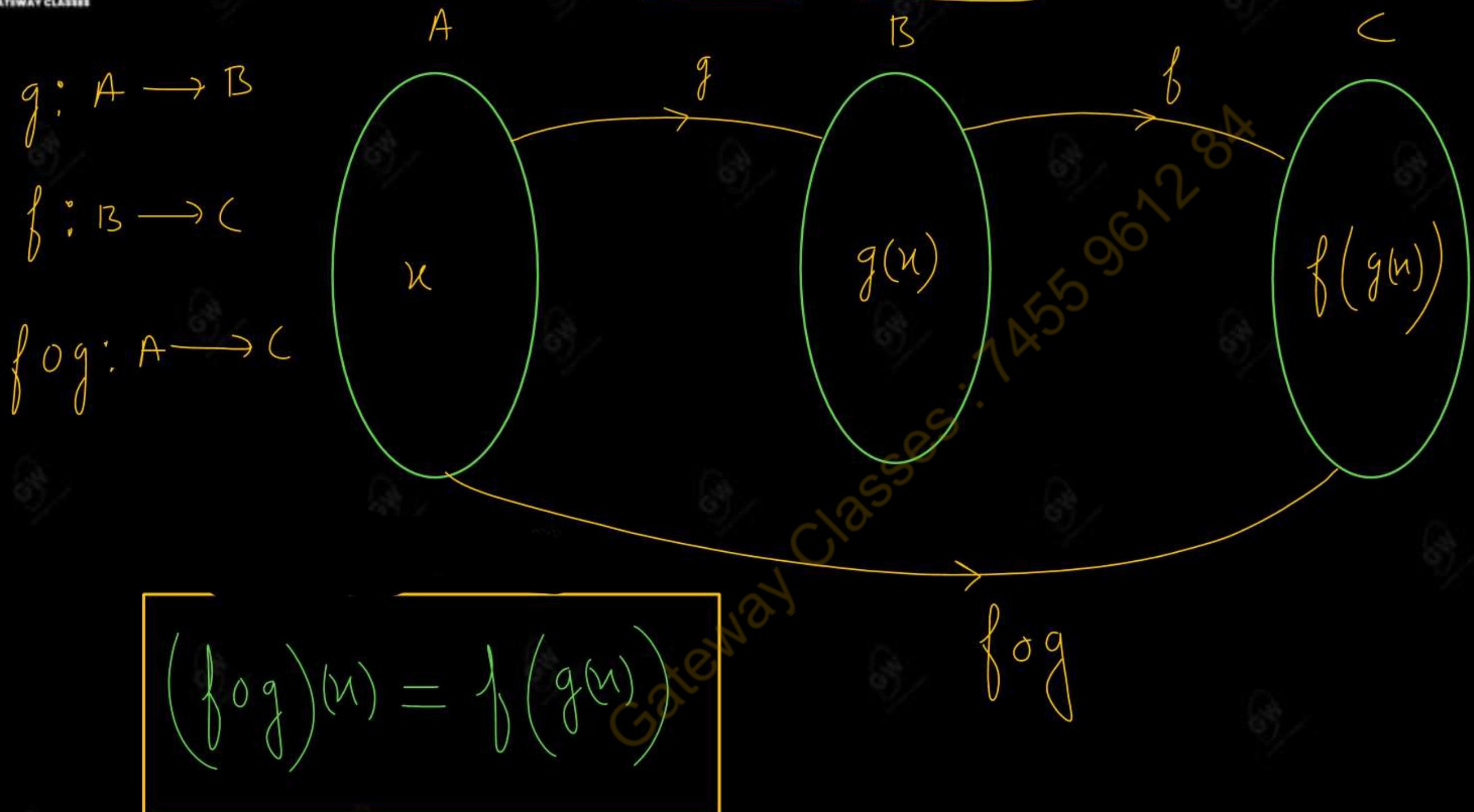
And is defined as

$$gof(x) = g\{f(x)\} \quad \forall x \in A$$

Note:-  $gof$  exists if and only if Range of  $f \subseteq$  domain of  $g$



Note:-  $f \circ g$  exists if and only if, Range of  $g \subseteq$  domain of  $f$



**Q.1:-** Let  $f = \{1, 3, 4\} \rightarrow \{1, 2, 5\}$  and  $g = \{1, 2, 5\} \rightarrow \{1, 3\}$  be defined as  $f = \{(1, 2)(3, 5)(4, 1)\}$  and  $g = \{(1, 3)(2, 3)(5, 1)\}$ , Find  $gof$  and  $fog$ .

Given

$$f = \{1, 3, 4\} \rightarrow \{1, 2, 5\}$$

$$g = \{1, 2, 5\} \rightarrow \{1, 3\}$$

$$f = \{(1, 2)(3, 5)(4, 1)\}$$

$$g = \{(1, 3)(2, 3)(5, 1)\}$$

$$(i) \text{ Range}(f) = \{1, 2, 5\}$$

$$\text{Dom}(g) = \{1, 2, 5\}$$

$$\Rightarrow \text{Range}(f) \subseteq \text{Dom}(g)$$

$\Rightarrow gof$  exists

$$\text{dom}(gof) = \text{dom}(f)$$

$$\text{dom}(gof) = \{1, 3, 4\}$$

$$(gof)(1) = g(f(1))$$

$$= g(2) = 3$$

$$(gof)(3) = g(f(3))$$

$$= g(5)$$

$$= 1$$

$$(gof)(4) = g(f(4))$$

$$= g(1)$$

$$= 3$$

$$g \circ f = \{(1, 3), (3, 1), (4, 3)\}$$

$$(11) \quad \text{range}(g) = \{1, 3\}$$

$$\text{dom}(\beta) = \{1, 3, 4\}$$

$$\Rightarrow \text{range}(g) \subseteq \text{dom}(\beta)$$

$\Rightarrow f \circ g$  exists

$$\text{dom}(f \circ g) = \text{dom}(g)$$

$$\text{dom}(f \circ g) = \{1, 2, 5\}$$

$$(f \circ g)(1) = f(g(1)) = f(3) = 5$$

$$(f \circ g)(2) = f(g(2)) = f(3) = 5$$

$$(f \circ g)(5) = f(g(5)) = f(1) = 2$$

$$f \circ g = \{(1, 5), (2, 5), (5, 2)\}$$

Q.2:- Find the composition mapping  $g \circ f$  if  $f: R \rightarrow R$  is given by  $f(x) = e^x$  and

(AKTU-2022-23)

$g: R \rightarrow R$  is given by  $g(x) = \sin x$ .

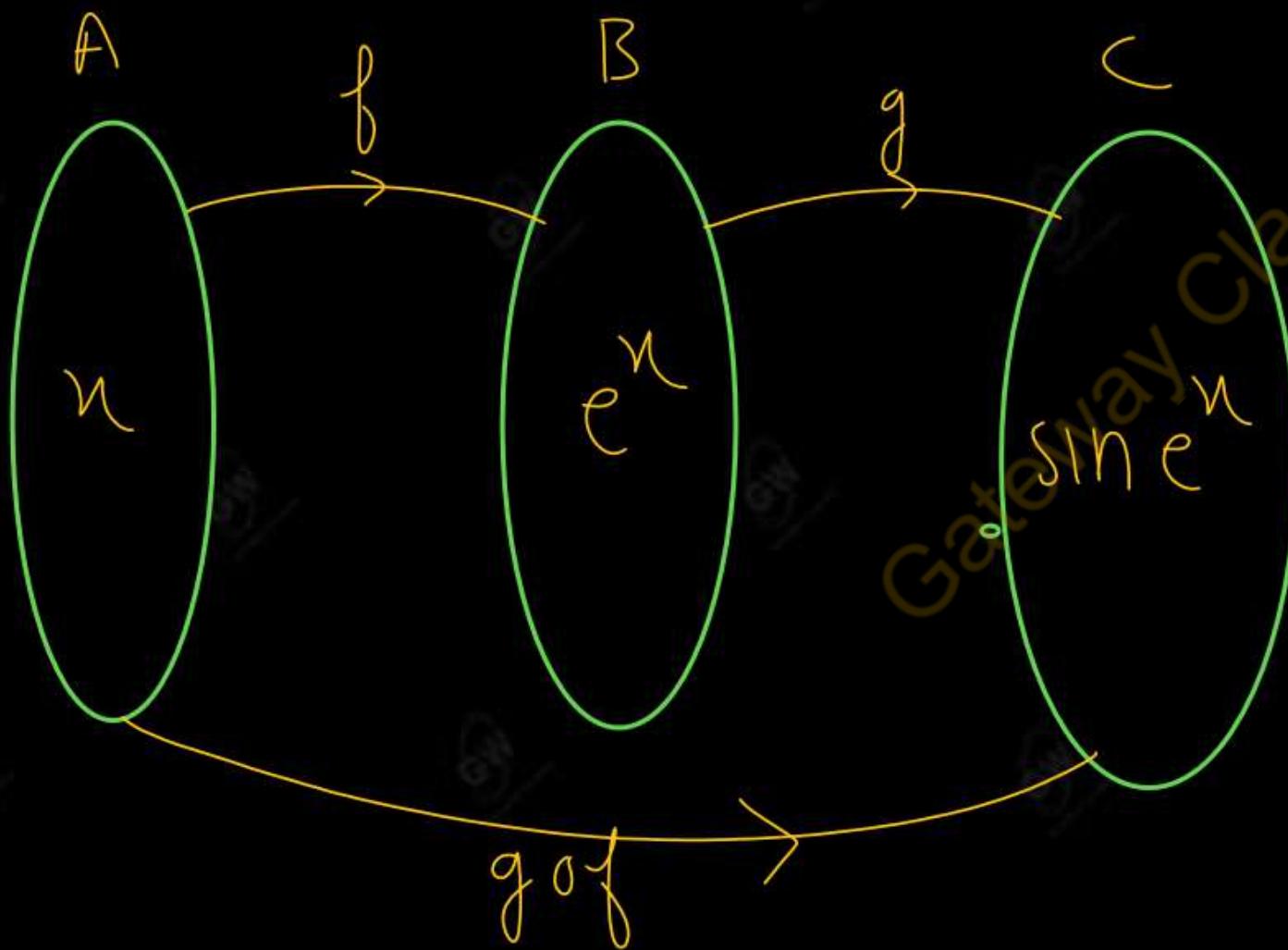
Given

$$f: R \rightarrow R, f(u) = e^u$$

$$g: R \rightarrow R, g(u) = \sin u$$

$$\begin{aligned} g \circ f(u) &= g(f(u)) \\ &= g(e^u) \end{aligned}$$

$$g \circ f(u) = \sin e^u$$



**Q.3:- Let  $A = \{1, 2, 3\}$**

$$B = \{a, b\}$$

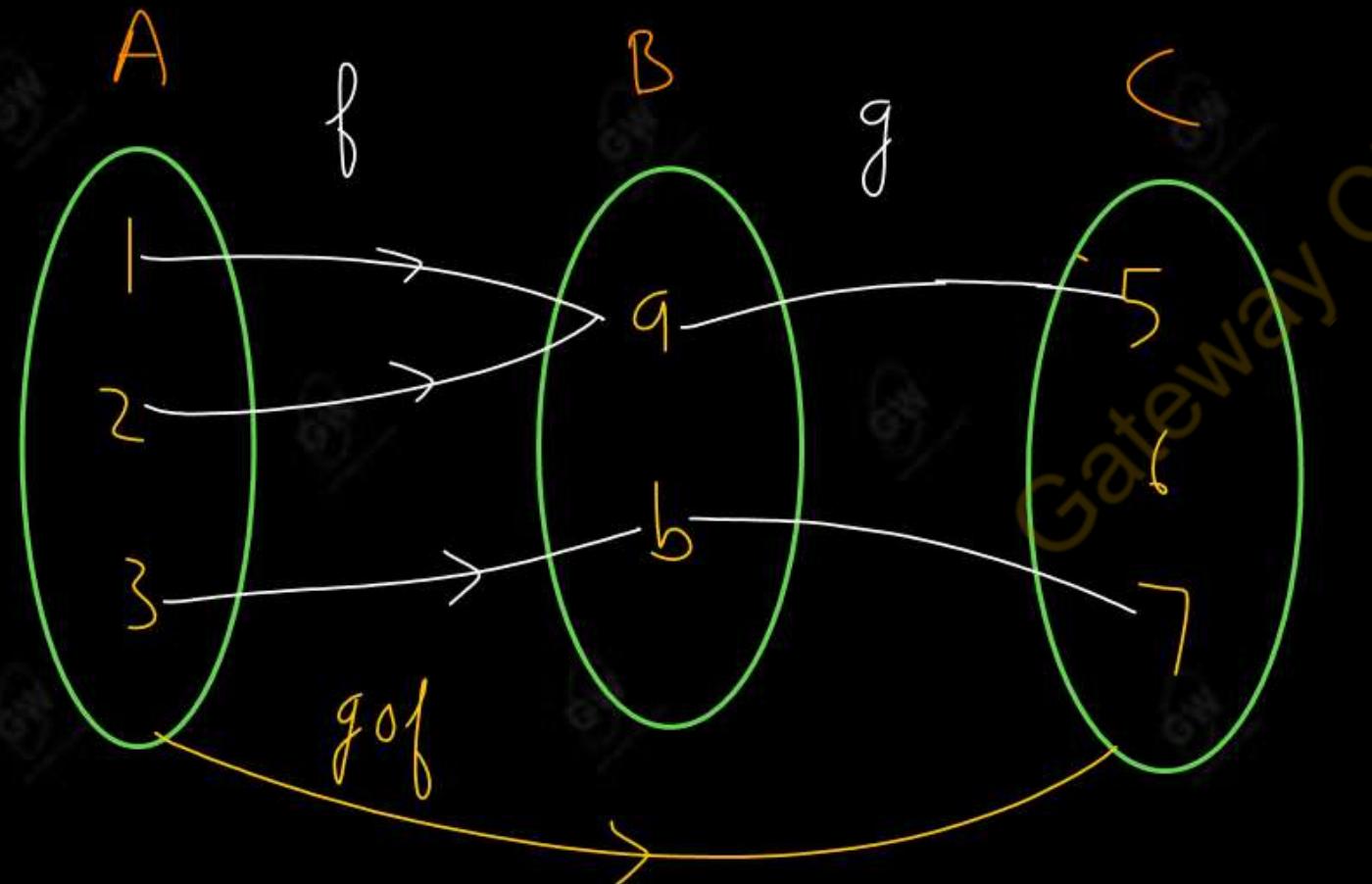
$$C = \{5, 6, 7\}$$

**Define  $f : A \rightarrow B$  where  $f = \{(1, a) (2, a)(3, b)\}$**

**and  $g : B \rightarrow C$  where  $g = \{(a, 5) (b, 7)\}$**

**Find the composition of  $f$  and  $g$ .**

I<sup>st</sup> Method



$$g \circ f = \{(1, 5) (2, 5) (3, 7)\}$$

2<sup>nd</sup> Method

$$\text{dom}(g \circ f) = \text{dom}(f)$$

$$= \{1, 2, 3\}$$

$$(g \circ f)(1) = g(f(1)) = g(a) = 5$$

$$(g \circ f)(2) = g(f(2)) = g(a) = 5$$

$$(g \circ f)(3) = g(f(3)) = g(b) = 7$$

$$g \circ f = \{(1, 5) (2, 5) (3, 7)\}$$

**Q.4:- Let  $f : R \rightarrow R$  and  $g : R \rightarrow R$  where  $R$  be the Set of real numbers.**

**Find (i)  $gof$  (ii)  $fog$  (iii)  $fof$  (iv)  $gog$ . Where  $f(x) = x^2$  and  $g(x) = x + 4$**

Given

$$f : R \rightarrow R, \quad f(n) = n^2$$

$$g : R \rightarrow R, \quad g(n) = n + 4$$

$$(i) \quad g \circ f = (g \circ f)(n)$$

$$= g(f(n))$$

$$= g(n^2)$$

$$= n^2 + 4$$

$$(ii) \quad f \circ g = (f \circ g)(n)$$

$$= f(g(n))$$

$$= f(n + 4)$$

$$= (n + 4)^2$$

$$(iii) \quad f \circ f = (f \circ f)(n)$$

$$= f(f(n))$$

$$= f(n^2)$$

$$f \circ f = (n^2)^2 = n^4$$

$$(iv) \quad g \circ g = g \circ g(n)$$

$$= g(g(n))$$

$$= g(n + 4)$$

$$= n + 4 + 4$$

$$= n + 8$$

Q.5:- If  $f : R \rightarrow R$  and  $g : R \rightarrow R$      $h : R \rightarrow R$  defined by

$$f(x) = 3x^2 + 2, \quad g(x) = 7x - 5 \quad \text{and} \quad h(x) = \frac{1}{x}.$$

Find the following composition functions.

(a)  $(fogoh)(x)$

(b)  $(hogof)(x)$

(c)  $(gog)(x)$

(d)  $(goh)(x)$

Given

$$f : R \rightarrow R, \quad f(n) = 3n^2 + 2$$

$$g : R \rightarrow R, \quad g(n) = 7n - 5$$

$$h : R \rightarrow R, \quad h(n) = \frac{1}{n}$$

(i)  $(fogoh)(n) = f(g(h(n)))$   
 $= f(g(\frac{1}{n})) = f(7 \times \frac{1}{n} - 5)$   
 $= 3\left(\frac{7}{n} - 5\right)^2 + 2$

$$(II) (h \circ g \circ f)(n) = h(g(f(n)))$$

$$= h(g(3n^2 + 2))$$

$$= h(7(3n^2 + 2) - 5)$$

$$= h(21n^2 + 14 - 5)$$

$$= h(21n^2 + 9)$$

$$= \frac{1}{21n^2 + 9}$$

$$(III) g \circ g(n) = g(g(n)) = g(7n - 5)$$

$$= 7(7n - 5) - 5$$

$$= 49n - 40$$

$$(IV) g \circ h(n) = g(h(n)) = g\left(\frac{1}{n}\right)$$

$$= \frac{7}{n} - 5$$

**Q.6:- Let  $X = \{a, b, c\}$ . Define  $f : X \rightarrow X$  such that**

$$f = \{(a, b), (b, a), (c, c)\}$$

**Find**    (i)  $f^2$                          (ii)  $f^3$                          (iii)  $f^4$

$$f^2 = f \circ f$$

Given

$$X = \{a, b, c\}$$

$$f = \{(a, a), (b, a), (c, c)\}$$

$$(i) \quad f^2(a) = (f \circ f)(a)$$

$$= f(f(a))$$

$$= f(a) = a$$

$$f^2(b) = (f \circ f)(b)$$

$$= f(f(b))$$

$$= f(a)$$

$$= b$$

$$f^2(c) = (f \circ f)(c)$$

$$= f(f(c))$$

$$= f(c)$$

**(UPTU-2014)**

$$f^2(c) = c$$

$$f^2 = \{(a, a), (b, b), (c, c)\}$$

$$(ii) \quad f^3(a) = (f \circ f)(a)$$

$$= f^2(f(a))$$

$$= f^2(a)$$

$$= a$$

$$f^3(b) = (f^2 \circ f)(b)$$

$$= f^2(f(b))$$

$$= f^2(a)$$

$$= a$$

$$f^3(c) = (f^2 \circ f)(c)$$

$$= f^2(f(c))$$

$$= f^2(c)$$

$$= c$$

$$f^3 = \{(a, a), (b, a), (c, c)\}$$

$$= f^3(a)$$

$$= a$$

$$f^3(c) = (f^2 \circ f)(c)$$

$$= f^3(f(c))$$

$$= f^3(c)$$

$$= c$$

$$f^3 = \{(a, a), (b, a), (c, c)\}$$

**Theorem:-** if  $f : A \rightarrow B$  and  $g : B \rightarrow C$  be one-one and onto function. Proof that

$gof : A \rightarrow C$  is one-one onto

And

$$(gof)^{-1} = f^{-1} \circ g^{-1}$$

Given

$f : A \rightarrow B$  is one-one onto

$g : B \rightarrow C$  is also one-one onto

Proof: Since  $f$  and  $g$  are one-one onto functions

$\therefore f^{-1}$  and  $g^{-1}$  both are exist

$$\text{If } y = f(u) \Rightarrow f^{-1}(y) = u \quad \text{--- (1)}$$

$$\text{If } z = g(y) \Rightarrow g^{-1}(z) = y \quad \text{--- (2)}$$

For one-one

Let  $u_1, u_2 \in A$

$$(gof)(u_1) = (gof)(u_2)$$

$$g(f(u_1)) = g(f(u_2))$$

$$g(f(n_1)) = g(f(n_2))$$

Since  $g$  is one-one

$$f(n_1) = f(n_2)$$

Since  $f$  is one-one

$$n_1 = n_2$$

$\Rightarrow g \circ f$  is one-one function

For onto

Let  $z \in C$

- Since  $g$  is an onto function, then

there exists an element  $y \in B$  such

that  $g(y) = z$

- Since  $f$  is an onto function, then

there exists an element  $n \in A$ , such

that  $f(n) = y$

$$\Rightarrow z = g(y) = g(f(n))$$

$$z = g \circ f(n)$$

Hence,  $g \circ f$  is an onto function

$\Rightarrow g \circ f$  is an one-one onto function

$\Rightarrow (g \circ f)^{-1}$  exist

$$(g \circ f)^{-1}(z) = n$$

— (3)

$$\begin{aligned} & \frac{\text{Now}}{(f^{-1} \circ g^{-1})(z)} = f^{-1}(g^{-1}(z)) \\ & = f^{-1}(y) \quad \{ \text{From (2)} \} \\ & = n \quad \{ \text{From (1)} \} \end{aligned}$$

$$(f^{-1} \circ g^{-1})(z) = n \quad — (4)$$

From (3) and (4)

$$(g \circ f)^{-1}(z) = (f^{-1} \circ g^{-1})(z)$$

$$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$$

## Topic : Composition of Functions

**Q.1** Let  $X = \{1, 2, 3\}$ ,  $Y = \{q, p\}$  and  $Z = \{a, b\}$ , let  $f: X \rightarrow Y$  be  $f = \{(1, p), (2, p), (3, q)\}$

$g: Y \rightarrow Z$  be  $g = \{(p, b), (q, b)\}$ . Find  $gof$  and show it pictorially.

(UPTU-2013)

$$g \circ f = \{(1, b), (2, b), (3, b)\}$$

**Q.2** If  $f: R \rightarrow R$  and  $g: R \rightarrow R$  defined by

$$f(x) = x^3 - 4x, g(x) = 1 / (x^2 + 1) \quad h(x) = x^4.$$

Find the following composition functions.

(a)  $(fogoh)(x)$

(b)  $(hogof)(x)$

(c)  $(gog)(x)$

(d)  $(goh)(x)$

(e)  $(n^2+1)^2 [1 + (n^2+1)^2]$

(d)  $(n^8+1)^{-1}$

Ans (a)  $(n^8+1)^{-3} - 4(n^8+1)^{-1}$

(b)  $\left[ (n^3 - 4n)^2 + 1 \right]^{-1}$

**Q.3** If  $f : \mathbb{Z} \rightarrow \mathbb{Z}$ ,  $g : \mathbb{Z} \rightarrow \mathbb{Z}$ ,  $h : \mathbb{Z} \rightarrow \mathbb{Z}$  defined by  $f(n) = n^2$ ,  $g(n) = n + 1$  and  $h(n) = n - 1$ . Find

(a)  $g \circ f \circ h$

(b)  $f \circ g \circ h$

(c)  $h \circ f \circ g$ .

Ans (a)  $n^2 - 2n + 2$

(b)  $n^2$

(c)  $n^2 + 2n$

**Q.4** Given  $f = \{(a, b), (b, a), (c, b)\}$  a function from  $X = \{a, b, c\}$  to  $X$ :

(a) Write  $f \circ f$  and  $f \circ f \circ f$  as sets of ordered pairs. Ans  $\{f \circ f\} = \{(a, a), (b, b), (c, c)\}$

(b) Define  $f^n = f \circ f \circ \dots \circ f$  to be the  $n$ -fold composition of  $f$  with itself. Find  $f^9$ .

$\{f \circ f \circ f\} = \{(a, b), (b, a), (c, d)\}$

(b)  $\{f^9\} = \{$

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# **DISCRETE STRUCTURES & THEORY OF LOGICS**

## **(Discrete Mathematics)**

### **UNIT -2 : Functions and Boolean Algebra**

**Lecture -04**

#### **Today's Target**

- Definitions of Boolean Algebra
- DPP

**Boolean Algebra is an algebraic structure which is based on the principle of logics.**

### **Algebraic Structure for Boolean Algebra**

(i)  $(B, +, \cdot, ', 0, 1)$

(ii)  $(B, \vee, \wedge, ', 0, 1)$

(iii)  $(B, \cup, \cap, ', 0, 1)$

#### **First Definition:**

A non-empty set  $B$  with two binary operations  $+$  and  $\cdot$ , a unary operation  $'$ , and two distinct elements  $0$  and  $1$  is called a Boolean algebra, denoted by  $(B, +, \cdot, ', 0, 1)$  if and only if the following properties are satisfied

1. Closure law :  $\forall a, b \in B$

(i)  $a + b \in B$

(ii)  $a \cdot b \in B$

2. Commutative law :  $\forall a, b \in B$

(i)  $a + b = b + a$

(ii)  $a \cdot b = b \cdot a$

3. Distributive law :  $\forall a, b, c \in B$

(i)  $a \cdot (b + c) = a \cdot b + a \cdot c$

(ii)  $a + (b \cdot c) = (a + b) \cdot (a + c)$

$(b + c) \cdot a = b \cdot a + c \cdot a$

$(b \cdot c) + a = (b + a) \cdot (c + a)$

**4. Identity law:**  $\forall a \in B$ 

(i)  $a + 0 = 0 + a = a$

Additive Identity

(ii)  $a \cdot 1 = 1 \cdot a = a$

Multiplicative Identity

**5. Complement law:**  $\forall a \in B$  there exist  $a' \in B$  such that

(i)  $a + a' = a' + a = 1$

(ii)  $a \cdot a' = a' \cdot a = 0$

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**Q1.** Show that the structure  $(B, +, \cdot, ', )$  is a Boolean algebra where  $B = \{0, 1\}$  and  $+$  and  $\cdot$  are two binary operation and  $'$  a unary operation on  $B$ , defined by the following table

$+$	0	1
0	0	1
1	1	1

(i)

$\cdot$	0	1
0	0	0
1	0	1

(ii)

$a$	$a'$
0	1
1	0

(iii)

Sol

① Closure law:  $B = \{0, 1\}$

(i)  $0+1 = 1 \in B$  Hence closure satisfied under  $+$  and  $\cdot$ .

(ii)  $0 \cdot 1 = 0 \in B$

② Commutative law:  $\forall a, b \in B$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$\Rightarrow 0 + 1 = 1 + 0$$

$$0 \cdot 1 = 0$$

$$1 \cdot 0 = 0$$

$$0 \cdot 1 = 1 \cdot 0$$

Hence commutative is satisfied  
under '+' and ' $\cdot$ '

③ Distributive law:  $\forall a, b, c \in B$

$$(i) a \cdot (b+c) = a \cdot b + a \cdot c$$

$$(ii) a + (b \cdot c) = (a+b) \cdot (a+c)$$

Let  $a = 1, b = 0, c = 1$

$$(i) a \cdot (b+c) = 1 \cdot (0+1)$$

$$= 1 \cdot 1$$

$$= 1$$

$$a \cdot b + a \cdot c = 1 \cdot 0 + 1 \cdot 1$$

$$= 0 + 1$$

$$= 1$$

$$\Rightarrow a \cdot (b+c) = a \cdot b + a \cdot c$$

$$(ii) a + (b \cdot c) = 1 + (0 \cdot 1)$$

$$= 1 + 0$$

$$= 1$$

$$\begin{aligned}(a+b).(a+c) &= (a+0).(a+1) \\&= a \cdot a \\&= a\end{aligned}$$

$$\Rightarrow a+(b \cdot c) = (a+b).(a+c)$$

Hence distributive law is satisfied

#### ④ Identity law

$$0 + 0 = 0$$

$$1 + 0 = 1$$

$$\begin{aligned}0 \cdot 1 &= 0 \\1 \cdot 1 &= 1\end{aligned}$$

Hence, 0 is additive identity

1 is multiplicative identity

#### ⑤ Complement law : For $0, 1 \in B$

$$1' = 0 \in B$$

$$0' = 1 \in B$$

$\Rightarrow$  complement law is satisfied

Hence  $(B, +, \cdot, ', 0, 1)$  is a Boolean Algebra

**Second Definition:**

A complemented distributive lattice is called a Boolean algebra.

OR

A non-empty set  $B$  with two binary operations  $\vee$  and  $\wedge$ , a unary operation ' $'$ , and two distinct elements  $0$  and  $1$  is called a Boolean algebra, denoted by  $(B, \vee, \wedge, ', 0, 1)$   
if and only if the following properties are satisfied

1. **Closure law :**  $\forall a, b \in B$

$$(i) a \vee b \in B$$

$$(ii) a \wedge b \in B$$

2. **Commutative law :**  $\forall a, b \in B$

$$(i) \underline{a \vee b = b \vee a}$$

$$(ii) \underline{a \wedge b = b \wedge a}$$

3. Distributive law:  $\forall a, b, c \in B$

(i)  $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

(ii)  $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

4. Identity law: There exist two elements  $o$  and  $1$  in  $B$  such that

(i)  $a \vee o = a$

$\textcircled{VI}$

(ii)  $a \wedge 1 = a$

5. Complement law:  $\forall a \in B$  there exist an element  $a' \in B$  such that

(i)  $a \vee a' = \textcircled{1}$

(ii)  $a \wedge a' = \textcircled{o}$

**Q. 2:** Let  $B$  be the set of all +ve divisors of 30 and the operations  $\vee$  and  $\wedge$  on  $B$  are defined as follows



$$a \vee b = LCM(a, b)$$

$$a \wedge b = HCF(a, b)$$

**Prove that  $(B, \vee, \wedge, ' )$  is a Boolean algebra**

## Solution

least element

, 10, 15, 30]   
 Gateway Class ↑  
 greatest element

composition table for  $\vee$

Composition table for  $\wedge$

$\wedge$	1	2	3	5	6	10	15	30
1	1	1	1	1	1	1	1	1
2	2	1	1	2	2	1	2	.
3	1	1	3	1	3	1	3	3
5	1	1	1	5	1	5	5	5
6	2	3	1	6	2	3	6	.
10	2	1	5	2	10	5	10	.
15	1	3	5	3	5	15	15	30
30	2	3	5	6	10	15	30	.

Composition table for complement

$a$	$a'$
1	30
2	15
3	10
5	6
6	5
10	3
15	2
30	1

least element = 1

greatest element = 30

$a \vee a' = 30$

$a \wedge a' = 1$

① Closure law:

since all the elements of  $\vee$  and  $\wedge$  table lie inside the set  $B$

Hence closure law is satisfied under  $\vee$  and  $\wedge$

$$\Rightarrow \forall a, b \in B$$

$$a \vee b \in B \text{ and } a \wedge b \in B$$

② Commutative law

since corresponding row and column<sup>34</sup> are identical in  $\vee$  and  $\wedge$  table

$\Rightarrow$  Commutative law is satisfied under  $\vee$  and  $\wedge$

$$\Rightarrow \forall a, b \in B$$

$$(i) a \vee b = b \vee a$$

$$(ii) a \wedge b = b \wedge a$$

### ③ Distributive law

$$(i) a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$$

$$(ii) a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$$

Let  $a = 2, b = 3, c = 6$

$$(i) a \vee (b \wedge c) = 2 \vee (3 \wedge 6)$$

$$= 2 \vee 3$$

$$= 6$$

$$(a \vee b) \wedge (a \vee c) = (2 \vee 3) \wedge (2 \vee 6)$$

$$= 6 \wedge 6$$

$$= 6$$

$$\Rightarrow a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$$

Similarly

$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$$

Hence distributive law is satisfied  
under  $\vee$  and  $\wedge$

④ Identity law:  $\forall a \in B$

From  $\vee$  table

$$a \vee 1 = a$$

From  $\wedge$  table

$$a \wedge 30 = a$$

Hence

$1$  is the identity element for  $\vee$

$30$  is       $\wedge$

⑤ Complement law

$\forall a \in B$  there exist

an element  $a' \in B$

such that

$$a \vee a' = 30$$

$$a \wedge a' = 1$$

Hence  $(B, \vee, \wedge, ', 0, 1)$

### Third Definition

A non-empty set  $B$  with two binary operations  $\cup$  and  $\cap$ , a unary operation ' $'$ , and two distinct elements  $o$  and  $1$  is called a Boolean algebra, denoted by  $(B, \cup, \cap, ', 0, 1)$   
if and only if the following properties are satisfied

1. **Closure law :**  $\forall A_1, A_2 \in B$

$$(i) A_1 \cup A_2 \in B$$

$$(ii) A_1 \cap A_2 \in B$$

2. **Commutative law :**  $\forall A_1, A_2 \in B$

$$(i) A_1 \cup A_2 = A_2 \cup A_1$$

$$(ii) A_1 \cap A_2 = A_2 \cap A_1$$

**3. Distributive law:**  $\forall A_1, A_2, A_3 \in B$

$$(i) A_1 \cup (A_2 \cap A_3) = (A_1 \cup A_2) \cap (A_1 \cup A_3)$$

$$(ii) A_1 \cap (A_2 \cup A_3) = (A_1 \cap A_2) \cup (A_1 \cap A_3)$$

**4. Identity law:**  $\forall A \in B$  there exist two element  $\phi$  and  $U$  in  $B$  such that

$$(i) A \cup \phi = A$$

$$(ii) A \cap U = A$$

**5. Complement law:**  $\forall A \in B$  there exist an element  $A' \in B$  such that

$$(i) A \cup A' = U \quad (ii) A \cap A' = \phi$$

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# DISCRETE STRUCTURES & THEORY OF LOGICS

## (Discrete Mathematics)

UNIT -2 : Functions and Boolean Algebra

Lecture - 05

### Today's Target

- Theorems of Boolean Algebra
- PYQ
- DPP

$(B, +, \cdot, ', 0, 1)$  is a Boolean algebra if and only if the following properties are satisfied

1. **Closure Laws:**  $\forall a, b \in B$

(i)  $a + b \in B$

(ii)  $a \cdot b \in B$

2. **Commutative Laws:**  $\forall a, b \in B$

(i)  $a + b = b + a$

(ii)  $a \cdot b = b \cdot a$

3. **Associative Laws:**  $\forall a, b, c \in B$

(i)  $a + (b + c) = (a + b) + c$

(ii)  $a \cdot (b \cdot c) = (a \cdot b) \cdot c$

4. **Distributive Laws:**  $\forall a, b, c \in B$

(i)  $a \cdot (b + c) = a \cdot b + a \cdot c$

(ii)  $a \cdot (b \cdot c) = (a + b) \cdot (a + c)$

$(b + c) \cdot a = b \cdot a + c \cdot a$

$(b \cdot c) + a = (b + a) \cdot (c + a)$

✓ 5. Identity Law:  $\forall a \in B$

(i)  $a + 0 = 0 + a = a$

Additive Identity

(ii)  $a \cdot 1 = 1 \cdot a = a$

Multiplicative Identity

✓ 6. Complement Law:  $\forall a \in B$  there exist  $a' \in B$  such that

(i)  $a + a' = a' + a = 1$

(ii)  $a \cdot a' = a' \cdot a = 0$

Theorems 1 - (Idempotent Law) : For every element of a Boolean algebra B

$$(i) a + a = a$$

(i) To Prove:  $a + a = a$

Proof: RHS

$$a = a + 0 \quad \text{by identity law}$$

$$= a + a \cdot a' \quad \text{by complement law}$$

$$= (a+a) \cdot (a+a') \quad \text{by distributive law}$$

$$= (a+a) \cdot 1 \quad \text{by complement law}$$

$$= a + a \quad \text{by identity law}$$

Hence proved

$$(ii) a \cdot a = a$$

(ii) To Prove :  $a \cdot a = a$

Proof: LHS

$$a \cdot a = a \cdot a + 0 \quad \text{by identity law}$$

$$= a \cdot a + a \cdot a' \quad \text{by complement law}$$

$$= a \cdot (a+a') \quad \text{by distributive law}$$

$$= a \cdot 1 \quad \text{by complement law}$$

$$= a \quad \text{by identity law}$$

$$= \text{RHS}$$

Hence proved

*Theorems 2 – (Bound<sup>ed</sup> laws) : For every element of a Boolean algebra B*(i)  $a + 1 = 1$ (i) To Prove:  $a + 1 = 1$ LHS:

$$a + 1 = a + (a + a') \text{ by complement law}$$

$$= (a + a) + a' \text{ by associative law}$$

$$= a + a' \text{ by idempotent law}$$

$$= 1 \text{ by complement}$$

= RHS

Hence proved

(ii)  $a \cdot 0 = 0$ To Prove :  $a \cdot 0 = 0$ Proof : LHS

$$a \cdot 0 = a \cdot (a \cdot a') \text{ by complement law}$$

$$= (a \cdot a) \cdot a' \text{ by Associative law}$$

$$= a \cdot a' \text{ by idempotent law}$$

$$= 0 \text{ by complement law}$$

= RHS

Hence proved

**Theorems 3 – (Absorption Laws) : For any two element  $a$  and  $b$  of a Boolean algebra  $B$ ,**

(i)  $a + a \cdot b = a$

(i) To Prove:  $a + a \cdot b = a$ Proof: LHS

$$a + a \cdot b = a \cdot 1 + a \cdot b \text{ By identity law}$$

$$= a \cdot (1+b) \text{ By distributive law}$$

$$= a \cdot 1 \text{ By Boundedness law}$$

$$= a \text{ By identity law}$$

= RHS

Hence proved

(ii)  $a \cdot (a + b) = a$

(ii) To Prove:  $a \cdot (a+b) = a$ Proof: LHS

$$a \cdot (a+b) = (a+0) \cdot (a+b) \text{ by identity law}$$

$$= a + (0 \cdot b) \text{ by distributive law}$$

$$= a + 0 \text{ by boundedness law}$$

$$= a \text{ by identity law}$$

= RHS

Hence proved

**Theorems 4 : In a Boolean algebra  $B$ , the identity elements are complementary to each other**i.e. for  $0, 1 \in B$  we have

(i) To Prove :  $0' = 1$

Proof :

$$\text{LHS} = 0'$$

$$= 0' + 0 \text{ by identity law}$$

$$= 1 \text{ by complement law}$$

$$= \text{RHS}$$

Hence proved

(i)  $0' = 1$

(ii)  $1' = 0$

(ii) To Prove :  $1' = 0$

Proof :

$$\text{LHS} = 1'$$

$$= 1 \cdot 1 \text{ by identity law}$$

$$= 0 \text{ by complement law}$$

$$= \text{RHS}$$

Hence proved

**Theorems 5:** The complement of each element of Boolean algebra B is unique**OR****For each  $a \in B$ ,  $a'$  is unique**Proof: Let  $a \in B$  be any elementSuppose  $x$  and  $y$  are two complements of  $a$ 

By complement law

$$a+x = x+a = 1 \quad \text{and} \quad a \cdot x = x \cdot a = 0 \quad \text{--- } ①$$

$$a+y = y+a = 1 \quad \text{and} \quad a \cdot y = y \cdot a = 0 \quad \text{--- } ②$$

Now

$$x = x \cdot 1$$

By identity law

$$x = x \cdot (a+y)$$

From ②

$$= x \cdot a + x \cdot y$$

By distributive law

$$= 0 + x \cdot y$$

From ①

$$= a \cdot y + x \cdot y$$

From ②

$$= (a+x) \cdot y \quad \text{By distributive law}$$

$$= 1 \cdot y$$

From ①

$$\boxed{x = y}$$

By identity law

Hence, complement of each element  
is unique

OR

**The elements 0 and 1 are unique in Boolean algebra.**

(i) To Prove Identity element 0 is unique for +  
Suppose 0 and  $0_1$  are two identity elements of Boolean algebra B for +

If 0 is the identity element

$$0_1 + 0 = 0_1 \quad \text{--- } ①$$

If  $0_1$  is the identity element

$$0 + 0_1 = 0 \quad \text{--- } ②$$

By commutative law

$$0_1 + 0 = 0 + 0_1$$

$$\Rightarrow 0 = 0_1 \quad (\text{From } ① \text{ and } ②)$$

Hence identity element 0 is unique

similarly, we can prove identity element 1 is unique for •

## Theorems 7 : Involution Law

For each element  $a$  of Boolean algebra  $B$

$$(a')' = a$$

**Theorems 8 – (De Morgan's Law) : For any two elements  $a$  and  $b$  of Boolean algebra  $B$** 

DPP-1

(i)  $(a + b)' = a' \cdot b'$

To Prove :  $(a + b)' = a' \cdot b'$ Proof : To show the complement of  $(a + b)$ is  $a' \cdot b'$ , it is sufficient to prove

$$(a + b) + a' \cdot b' = 1 \quad \text{--- } ①$$

$$(a + b) \cdot (a' \cdot b') = 0 \quad \text{--- } ②$$

LHS of ①

$$(a + b) + a' \cdot b' = (a + b + a') \cdot (a + b + b')$$

By Distributive law

(ii)  $(a \cdot b)' = a' + b' \quad \forall a, b \in B$

$$= (b + a + a') \cdot (a + b + b')$$

By commutative law

$$= (b + 1) \cdot (a + 1)$$

By complement law

$$= 1 \cdot 1 \quad \text{By boundedness law}$$

$$= 1 \quad \text{By identity law}$$

= RHS

LHS of ②

$$(a + b) \cdot (a' \cdot b')$$

$$= a \cdot (a' \cdot b') + b \cdot (a' \cdot b')$$

By distributive law

$$= a \cdot (a' \cdot b') + b \cdot (b' \cdot a')$$

By commutative law

$$= (a \cdot a') \cdot b' + (b \cdot b') \cdot a'$$

By Associative law

$$= 0 \cdot b' + 0 \cdot a'$$

By Complement law

$$= 0 + 0 \quad \text{By Boundedness law}$$

$$= 0 \quad \text{By Idempotent law}$$

= RHS

$$\Rightarrow (a + b)' = a' \cdot b'$$

Hence proved

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# DISCRETE STRUCTURES & THEORY OF LOGICS (BCS303)

## UNIT -2 : Functions and Boolean Algebra

### Lecture - 06

#### Today's Target

- Truth table
- Boolean Function
- Minterm
- Maxterm
- Disjunction Normal Form (D N Form)  
OR  
Sum of Products Form (SOP Form)

- The basic operations used in Boolean algebra are

(i) Logical Addition

(ii) Logical Multiplication

(iii) Complementation

0	False	OFF	LOW	OPEN	NO
1	True	ON	HIGH	CLOSED	YES

- Each variable in Boolean algebra has either 0 or 1 (False or True)

- Here 1 or 0 do not represent actual numbers

Logical Addition : Logical Addition is denoted by + or by OR

Truth Table for logical addition

Total input combination =  $2^n$

$0 + 0 = 0$  otherwise 1

INPUT		OUTPUT
A	B	$C = A + B$
1	1	1
1	0	1
0	1	1
0	0	0

**Logical Multiplication :** Logical Multiplication is denoted by  $\bullet$  or by AND

**Truth Table for logical Multiplication**

$$1 \cdot 1 = 1 \text{ otherwise } 0$$

INPUT		
A	B	C = A $\cdot$ B
1	1	1
1	0	0
0	1	0
0	0	0

**Logical Complementation :** Logical Complementation is denoted by  $\neg$  or by ' $(\bar{A}, A')$ '

**Truth Table for logical Complementation**

INPUT	
A	B = $\bar{A}$
1	0
0	1

## Boolean Function (or Boolean Polynomial)

An expression obtained by the application of binary operation (+ or  $\vee$ ) and ( $\bullet$  or  $\wedge$ ) and unary operation (' ) on finite number of elements of Boolean algebra  $(B, +, \bullet, ')$  or  $(B, \vee, \wedge, ')$  is called a Boolean function or Boolean Polynomial.

A variable  $x$  is called a Boolean variable if it assumes values either 0 or 1.

Note:- (i)  $\wedge$  or  $\bullet$   $\longrightarrow$  Meet or Conjunction  
(ii)  $\vee$  or  $+$   $\longrightarrow$  Join or Disjunction

### Examples:

#### 1. Function of one Variables

$$f(x) = x + x'$$

OR

$$f(x) = x \vee x'$$

**2. Function of two variables  $x$  and  $y$** 

$$f(x, y) = x \cdot y' + x' \cdot y$$

**OR**

$$f(x, y) = (x \wedge y') \vee (x' \wedge y)$$

**3. Function of three variables  $x, y$  and  $z$** 

$$f(x, y, z) = x \cdot y \cdot z + x' \cdot y \cdot z + x \cdot y' \cdot z'$$

**OR**

$$f(x, y, z) = (x \wedge y \wedge z) \vee (x' \wedge y \wedge z) \vee (x \wedge y' \wedge z')$$

**Minterm (OR Minimal Boolean Function)**

A minterm of n-variables is a **product** of n literals in which each variable appears exactly once in either true or complemented form, but not both.

**For Examples:**

1. The list of all minterms of two variables  $x$  and  $y$  are

$xy, x'y, xy', x'y'$

3. The list of all minterms of three variables  $x, y$  and  $z$  are

$xyz, xyz', xy'z, x'yz, x'y'z', x'y'z, x'yz', xy'z'$

**Note:**

(1) **n-variables can be combined to form  $2^n$  minterm**

(2) **The output value of a minterm is 1**

## Maxterm (or Maximal Boolean Function)

A maxterm of  $n$  variables is a sum of  $n$  literals in which each variable appears exactly once in either true or complimented form, but not both.

**For Examples:**

1. All The maxterm of two variables  $x$  and  $y$  are

$$x + y, x' + y, x + y', x' + y'$$

2. All The maxterm of three variables  $x$ ,  $y$  and  $z$  are

$$(x + y + z), (x + y + z'), (x + y' + z), (x' + y + z), (x' + y' + z'), (x' + y' + z), (x' + y + z'), (x + y' + z')$$

**Note:**

- (1) The number of maxterm in  $n$ -variables are  $2^n$
- (2) The output value of a maxterm is 0

**Note:****For Minterm:-**1 → **Use same variable**0 → **Use Complement of variable****For Maxterm:-**0 → **Use same variable**1 → **Use Complement****Truth Table:****1. For two variable  $x, y$** 

$x$	$y$	Minterms ( $m$ )	Maxterms ( $M$ )
1	1	$m_1 = x \cdot y$	$M_1 = x' + y'$
1	0	$m_2 = x \cdot y'$	$M_2 = x' + y$
0	1	$m_3 = x' \cdot y$	$M_3 = x + y'$
0	0	$m_4 = x' \cdot y'$	$M_4 = x + y$

## 3. For three variable

$x$	$y$	$z$	Minterm( $m$ )	Maxterm( $M$ )
1	1	1	$m_1 = n \cdot y \cdot z$	$M_1 = n' + y' + z'$
1	1	0	$m_2 = n \cdot y \cdot z'$	$M_2 = n' + y' + z$
1	0	1	$m_3 = n \cdot y' \cdot z$	$M_3 = n' + y + z'$
1	0	0	$m_4 = n \cdot y' \cdot z'$	$M_4 = n' + y + z$
0	1	1	$m_5 = n' \cdot y \cdot z$	$M_5 = n + y' + z'$
0	1	0	$m_6 = n' \cdot y \cdot z'$	$M_6 = n + y' + z$
0	0	1	$m_7 = n' \cdot y' \cdot z$	$M_7 = n + y + z'$
0	0	0	$m_8 = n' \cdot y' \cdot z'$	$M_8 = n + y + z$

↓  
**Disjunction Normal Form**

**Disjunction Normal Form (D N Form)**

**OR**

**Sum of Product Form (SOP Form) / sum of minterms**

**OR**

**Canonical Form**

*A Boolean function which can be written as the sum of Minterms, is called disjunctive normal form or D N Form*

**Note:- As Minterm is product of variables, so D N form is also called sum of minterm or sum of product form (SOP)**

Conjunction Normal Form

Next Lecture

**Step to find D N form**

**STEP-1:** Make a truth table for given Boolean function

$\wedge$ or •	<i>Conjunction Meet Product</i>	$T \quad T \Rightarrow T$ otherwise F $1 \quad 1 \Rightarrow 1$ otherwise 0
$\vee$ or +	<i>Disjunction Join Addition</i>	$F \quad F \Rightarrow F$ otherwise T $0 \quad 0 \Rightarrow 0$ otherwise 1
' or -	<i>Complement Prime Negation</i>	$T \rightarrow F$ $F \rightarrow T$ $1 \rightarrow 0$ $0 \rightarrow 1$

**STEP-2:** See the entries 1 in the last column of truth table and write corresponding minterms.

**STEP-3:** By adding the all minterms, we will get required D N form or SOP form

**Q. 1. Express the Boolean function in to D N form**  $f(x, y) = x + x' \cdot y$

$$f(n, y) = n + n' \cdot y$$

$x$	$y$	$x'$	$x' \cdot y$	$x + x' \cdot y$	Minterm
1	1	0	0	1	$m_1 = xy$
1	0	0	0	1	$m_2 = x y'$
0	1	1	1	1	$m_3 = x'y$
0	0	1	0	0	

✓

✓

Required D N Form

$$f(n, y) = ny + ny' + n'y$$

2nd Method

$$f(n, y) = x + x' \cdot y$$

$$f(n, y) = x \cdot 1 + x' \cdot y$$

$$f(n, y) = x(y + y') + x' \cdot y$$

$$f(n, y) = ny + ny' + n'y$$

(AKTU-2020)

$x$	$y$	$z$	$(x+y)$	$(x+z)$	$(x+y) \cdot (x+z)$	$z'$	$y+z'$	$(x+y) \cdot (x+z) + y + z'$	Minterm
1	1	1	1	1	1	0	1	1	$m_1 = xyz$
1	1	0	1	1	1	1	1	1	$m_2 = xyz'$
1	0	1	1	1	1	0	0	1	$m_3 = xy'z$
1	0	0	1	1	1	1	1	1	$m_4 = xy'z'$
0	1	1	1	1	1	0	1	1	$m_5 = x'yz$
0	1	0	1	0	0	1	1	1	$m_6 = x'yz'$
0	0	1	0	1	0	0	0	0	$m_7 = x'y'z$
0	0	0	0	0	0	1	1	1	$m_8 = x'y'z'$

Required DN Form

$$f(n, y, z) = nyz + nyz' + ny'z + ny'z' + n'y'z + n'y'z' + n'yz'$$

2nd Method

$$\begin{aligned}
 f(n, y, z) &= (n+y)(n+z) + y + z' \\
 &= n + yz + y + z' \\
 &= n \cdot 1 \cdot 1 + 1 \cdot yz + 1 \cdot y \cdot 1 + 1 \cdot 1 \cdot z' \\
 &= n(y+y')(z+z') + (n+n)yz + (n+n')y(z+z') + (n+n')(y+y')(z+z') \\
 &= (ny+ny')(z+z') + nyz + n'yz + (ny+ny')(z+z') + (n+n')(yz+y'z') \\
 &= nyz + nyz' + ny'z + ny'z' + \cancel{nyz} + \cancel{nyz'} + \cancel{n'yz} + \cancel{n'yz'} + \cancel{nyz} + \cancel{nyz'} \\
 &\quad + \cancel{ny'z'} + \cancel{n'yz} + \cancel{n'y'z'}
 \end{aligned}$$

**Q. 3.** Let  $E(x, y, z) = [(x+y)' + (x' \cdot z)]'$  be a Boolean expression. Write  $E(x, y, z)$  in disjunctive normal form.

$x$	$y$	$z$	$x+y$	$(x+y)'$	$x'$	$x' \cdot z$	$(x+y)' + (x' \cdot z)$	$[(x+y)'+(x' \cdot z)]'$	Minterms
1	1	1	1	0	0	0	0	1	$m_1 = xyz$
1	1	0	1	0	0	0	0	1	$m_2 = xyz'$
1	0	1	1	0	0	0	0	1	$m_3 = xy'z$
1	0	0	1	0	0	0	0	1	$m_4 = xy'z'$
0	1	1	1	0	1	1	1	0	$m_5 = x'y'z$
0	1	0	1	0	1	0	1	1	
0	0	1	0	1	1	1	1	0	
0	0	0	0	1	1	0	1	0	

Required DN Form

$$E(n, y, z) = nyz + nyz' + ny'z + ny'z' + n'y'z'$$

2<sup>nd</sup> Method

$$\begin{aligned} E(n, y, z) &= [(n+y)' + (n', z)]' \\ &= (n+y)' \cdot (n', z)' \\ &= (n+y) \cdot (n + z') \\ &= n + yz' \end{aligned}$$

$$\begin{aligned} E(n, y, z) &= n + yz' \\ &= n(y + y')(z + z') + (n + n')yz' \\ &= (ny + ny')(z + z') + nyz' + n'y'z' \\ &= nyz + nyz' + ny'z + ny'z' + \cancel{nyz'} \\ &\quad + \cancel{n'y'z'} \end{aligned}$$

$$E(n, y, z) = nyz + nyz' + ny'z + ny'z' + n'y'z'$$

## Topic : Disjunction Normal Form (D N Form)

**Q.1.** Represent each of the following Boolean function in disjunctive normal form.

(i)  $f(x_1, x_2) = (x_1 + x_2)(x'_1 + x'_2)$

(ii)  $f(x, y) = x + xy$

(iii)  $f(x_1, x_2, x_3) = x_1x'_2 + x_3$

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# DISCRETE STRUCTURES & THEORY OF LOGICS (BCS303)

## UNIT -2 : Functions and Boolean Algebra

### Lecture - 07

#### Today's Target

➤ *Conjunctive Normal Form (CN Form)*

OR

*Product of Sum Form (POS Form)*

➤ *Conversion of disjunctive normal form(DNF) of a boolean function to its conjunctive normal form and vice versa*

➤ *AKTU PYQs*

➤ *DPPs*

**Boolean Function****Disjunction Normal Form**

Sum of minterm

**Conjunctive Normal Form (CN Form)****OR****Product of Sum Form (POS Form)****OR****Canonical Form**

A Boolean function which can be written as the product of Maxterm is called conjunctive normal form or CN Form

**Note:-** As Maxterm is the Sum of variables, so CN form is also called product of sum of form.

**Conjunction Normal Form**

Product of Maxterm

**Step to find CN form****STEP-1: Make a truth table for given Boolean function**

$\wedge$ or •	<i>Conjunction Meet Product</i>	$T \quad T \Rightarrow T$ otherwise F $1 \quad 1 \Rightarrow 1$ otherwise 0
$\vee$ or +	<i>Disjunction Join Addition</i>	$F \quad F \Rightarrow F$ otherwise T $0 \quad 0 \Rightarrow 0$ otherwise 1
' or -	<i>Complement Prime Negation</i>	$T \rightarrow F$ $1 \rightarrow 0$ $F \rightarrow T$ $0 \rightarrow 1$

**STEP-2: See the entries 0 in the last column of truth table and write corresponding maxterms.****STEP-3: By product of all maxterm, we will get required CN form or POS form**

**Note:**

**For Minterm:-**

1 → *Use same variable*

0 → *Use Complement of variable*

**For Maxterm:-**

0 → *Use same variable*

1 → *Use Complement*

Q. 1:- Express the Boolean function  $f(a,b,c) = ab + a'c$  as a product of maxterm.

$$f(a, b, c) = ab + a'c$$

$a$	$b$	$c$	$ab$	$a'$	$a'c$	$f(a, b, c) = ab + a'c$	Maxterm ( $M$ )
1	1	1	1	0	0		
1	1	0	0	0	0		
→	1	0	0	0	0	0	$M_1 = a' + b + c'$
→	1	0	0	0	0	0	$M_2 = a' + b + c$
→	0	1	0	0	1	1	$M_3 = a + b' + c$
→	0	1	0	0	1	0	$M_4 = a + b + c$
→	0	0	0	1	1	0	
→	0	0	0	0	1	1	
→	0	0	0	0	0	0	

Required CNF form

$$f(a, b, c) = (a' + b + c')(a' + b + c)(a + b' + c)(a + b + c)$$

Find Method

$$f(a, b, c) = (ab + a'c)$$



$$\begin{aligned} \textcircled{1} \quad (a+b)' &= a'b' \\ \textcircled{2} \quad (ab)' &= a'+b' \end{aligned}$$

De Morgan's law

$$= (ab + a')(ab + c)$$

$$= (a' + ab)(c + ab)$$

$$= (a' + a)(a' + b)(c + a)(c + b)$$

$$= 1 (a' + b)(a + c)(b + c)$$

$$= (a' + b)(a + c)(b + c)$$

$$a + b \cdot c = (a + b) \cdot (a + c)$$

Distributive law



**Q. 2:- Let  $f(x, y, z) = [(x+y)' + \cdot (x' \cdot z)]'$  be a Boolean expression. Write  $f(x, y, z)$  in conjunctive normal form**

(AKTU-2016)

$$f(x, y, z) = \overbrace{[(x+y)' + (x' \cdot z)]'}^{\text{Conjunctive Normal Form}}$$

$x$	$y$	$z$	$x+y$	$(x+y)'$	$x'$	$x' \cdot z$	$(x+y)'+(x' \cdot z)$	$f(x, y, z) = ((x+y)'+(x' \cdot z))'$
1	1	1	1	0	0	0	0	1
1	1	0	1	0	0	0	0	1
1	0	1	1	0	0	0	0	1
1	0	0	1	0	0	0	0	1
→ 0	1	1	1	0	1	0	1	0 ✓
→ 0	1	0	1	0	1	0	0	1
→ 0	0	1	0	1	1	1	1	0 ✓
→ 0	0	0	0	1	1	1	1	0 ✓

$$M_1 = (n + y^1 + z^1)$$

$$M_2 = (n + y + z^1)$$

$$M_3 = (n + y + z)$$

Required CN Form

$$\boxed{f(n, y, z) = (n + y^1 + z^1) \\ (n + y + z^1) \\ (n + y + z)}$$

Find Method

$$f(n, y, z) = [(n + y)^1 + (n^1 \cdot z)]^1$$

$$= (n + y) \cdot (n^1 \cdot z)^1$$

$$= (n + y) \cdot (n + z^1)$$

$$= (n + y + z^1)(n + y^1 + z^1)$$

$$= (n + y + z)(n + y + z^1)(n + y + z^1)(n + y^1 + z^1)$$

$$\boxed{f(n, y, z) = (n + y + z)(n + y + z^1)(n + y^1 + z^1)}$$

# Conversion of disjunctive normal form (DNF) of a boolean function to its conjunctive normal form and vice - versa

$$\text{DN} \rightarrow \text{CN} \quad \text{OR} \quad \text{CN} \rightarrow \text{DN}$$

Q.3 Convert the boolean function  $f(x, y) = xy' + x'y + x'y'$  to its conjunctive normal form

$$f(n, y) = ny' + n'y + n'y'$$

Complete DNF Form for 2 variables  $n$  and  $y$

$$f(n, y) = ny + ny' + n'y + n'y'$$

$$f'(n, y) = ny$$

$$f''(n, y) = (ny)'$$

$$f''(n, y) = n' + y'$$

Required  
CNF Form

**Q.4** Convert the boolean function  $f(x, y, z) = (x' + y + z')(x' + y + z)(x + y' + z)$  to its disjunctive normal form.

$$f(x, y, z) = (x' + y + z')(x' + y + z)(x + y' + z)$$

complete CNF form for three variables  $x, y$  and  $z$

$$\begin{aligned} f(x, y, z) &= (x + y + z)(x + y + z') \cancel{(x + y' + z)} (x' + y + z)(x' + y' + z') \\ &\quad (x' + y' + z) (x' + y + z') (x + y' + z) \end{aligned}$$

$$f(x, y, z) = (x + y + z)(x + y + z')(x' + y' + z') (x' + y + z) (x + y' + z')$$

$$f''(x, y, z) = \left[ (x + y + z)(x + y + z')(x' + y' + z') (x' + y + z) (x + y' + z') \right]'$$

$$f''(n, y, z) = \underbrace{[(n+y+z)(n+y+z')(n'+y'+z')(n'+y'+z')]}_{} + (n+y+z)^4$$

$$f''(n, y, z) = \underbrace{[(n+y+z)(n+y+z')(n'+y'+z')]}_{} + (n'+y'+z')^4 + n'y'z$$

$$f''(n, y, z) = [(n+y+z)(n+y+z')]^4 + (n'+y'+z')^4 + nyz^4 + n'y'z$$

$$f''(n, y, z) = (n+y+z)^4 + (n+y+z')^4 + nyz^4 + nyz'^4 + n'y'z$$

$$f''(n, y, z) = ny^4 z^4 + ny^4 z'^4 + ny^4 z^4 + ny^4 z'^4 + n'y^4 z^4$$

Required DN Form

**Q.1.** Express the Boolean function in to CN form  $f(x, y) = x + x' \cdot y$

**Q.2.** Find the product of sum expansion of each of the following

(i)  $f(x, y, z) = (x + z) y$

(ii)  $f(x, y, z) = xy'$

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# DISCRETE STRUCTURES & THEORY OF LOGICS

## (Discrete Mathematics)

UNIT - 2 : Functions and Boolean Algebra

Lecture - 08

### Today's Target

- **K – Map for SOP form**
- PYQ
- DPP

1. By Karnaugh Map OR By K-Map

2. By Boolean algebra

### K - Map :

The K - Map is a pictorial representation of truth table of the Boolean functions and used to minimize the Boolean functions

#### Note :

K - Map Matrix contain  $2^n$  cell / square, where  $n$  is the total number of variables

### K-Map

For SOP form

For POS form

Next Lecture

✓ 1. For two variables  $x$  and  $y$ :

- No. of cells in K - Map =  $2^2 = 4$
- Each cell contain a minterm

$x'$	$y'$	$y'$	$y$
$x'$	$x'y'$	$x'y$	$xy$
$x$	$xy'$	$xy$	$yy$
	0	1	2
	2	3	3

$x'$	$y'$	$0$	$1$
$x'$	$0$	$0$	$1$
$x$	$1$	$0$	$1$
	$1$	$2$	$3$

## 2. For three variables $x$ , $y$ and $z$

- No. of cells in K-Map =  $2^3 = 8$
- Each cell represent a minterm

$x \backslash yz$	00	01	11	10
0	000	001	011	010
1	100	101	111	110
	4	5	7	6

$x \backslash yz$	00	01	10	11
0	$x'y'z'$	$x'y'z$	$x'yz$	$xyz'$
1	$xy'z'$	$xy'z$	$xyz$	$xyz'$
	0	1	3	2

### 3. For Four variables $x$ , $y$ , $z$ and $w$

► No. of cells in K - Map =  $2^4 = 16$

► Each cell represent a Minterm

$\bar{x}\bar{y}$	$\bar{x}w$	$\bar{z}w$	$zw$	$z\bar{w}'$
$\bar{x}'y'$	$\bar{x}'y'z'w'$	$\bar{x}'y'z'w$	$\bar{x}'y'z w$	$\bar{x}'y'z w'$
0	1	3	2	
$\bar{x}'y$	$\bar{x}'y z'w'$	$\bar{x}'y z'w$	$\bar{x}'y z w$	$\bar{x}'y z w'$
4	5	7	6	
$\bar{x}y$	$\bar{x}yz'w'$	$\bar{x}yz'w$	$\bar{x}yz w$	$\bar{x}yz w'$
12	13	15	14	
$xy'$	$xy'z'w'$	$xy'z'w$	$xy'z w$	$xy'z w'$
8	9	11	10	

$\bar{x}\bar{y}$	$\bar{x}w$	$\bar{z}w$	$zw$	$z\bar{w}'$
$\bar{x}'y'$	$\bar{x}'y'z'w'$	$\bar{x}'y'z'w$	$\bar{x}'y'z w$	$\bar{x}'y'z w'$
00	0000	0001	0011	0010
01	0100	0101	0111	0110
11	1100	1101	1111	1110
10	1000	1001	1011	1010
01	0000	0001	0011	0010
11	1100	1101	1111	1110

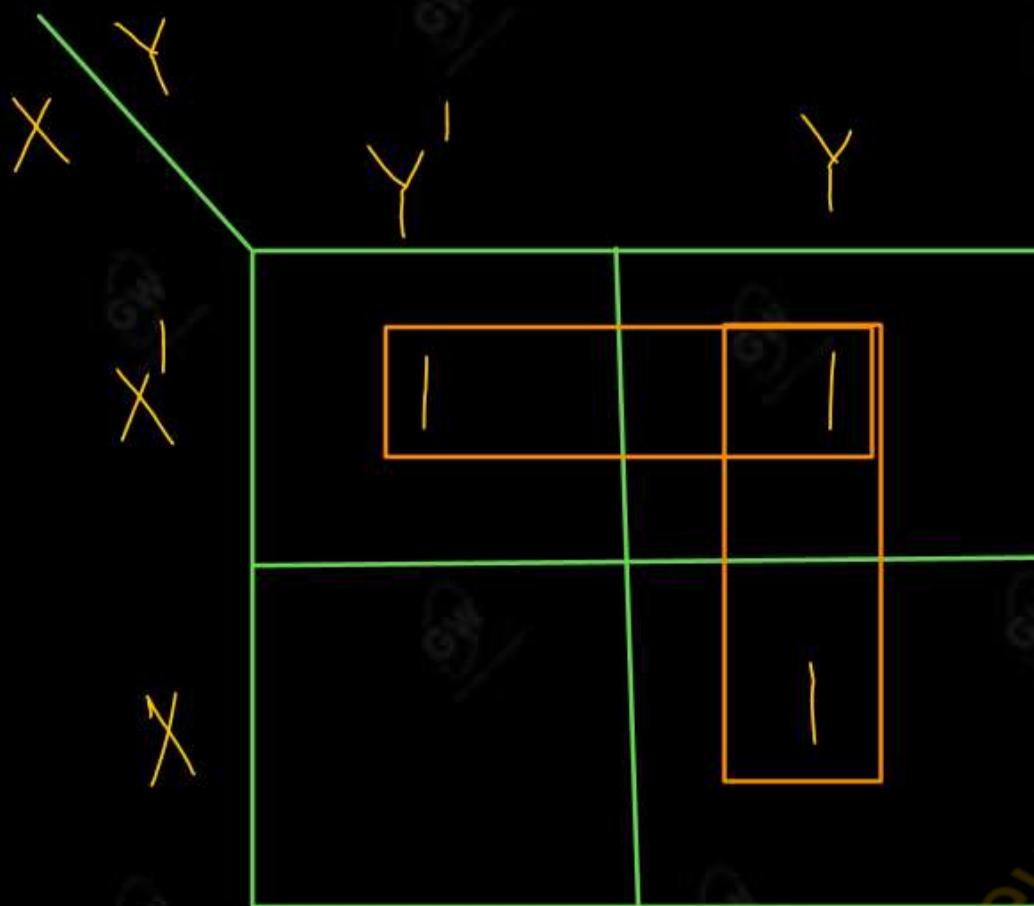
## **Working Rule**

- 1. Construct the K-Map**
- 2. Put 1 in the cell corresponding to each Minterm given in the Boolean function**
- 3. Making Groups of 1**
  - (i) Group of 1 contain 1, 2, 4, 8 ..... Cells
  - (ii) Diagonal groups are not allowed
  - (iii) Overlapping allowed in groups
  - (iv) Group should be as large as possible
  - (v) No 1 should be as left without grouping
- 4. Write the Minimised term for all the groups keeping in mind**
  - (i) Write only unchanged variable for each group
  - (ii) Check rows and column values for each group
- 5. Add all the minimised term to get required result**

Q. 1. Using the K - Map to find minimal form of the Boolean functions  $f(X, Y) = XY + X'Y + X'Y'$

K-map for two variables

$$= \sum(0, 1, 3)$$



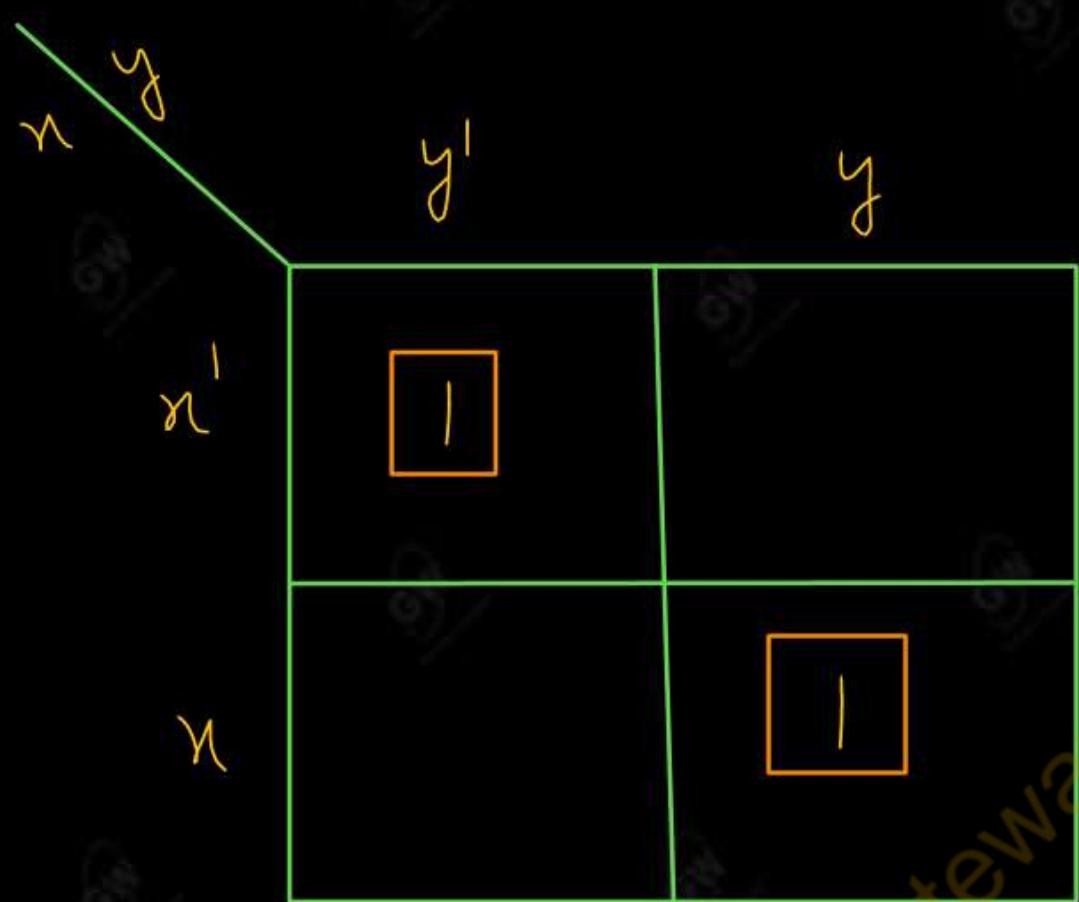
Hence, the required minimal form

$$f(X, Y) = X' + Y$$

Q. 2. Using the K-Map to find minimal form of the Boolean functions  $f(x, y) = xy + x'y'$

K-map for two variables

$$= \sum(0, 3)$$



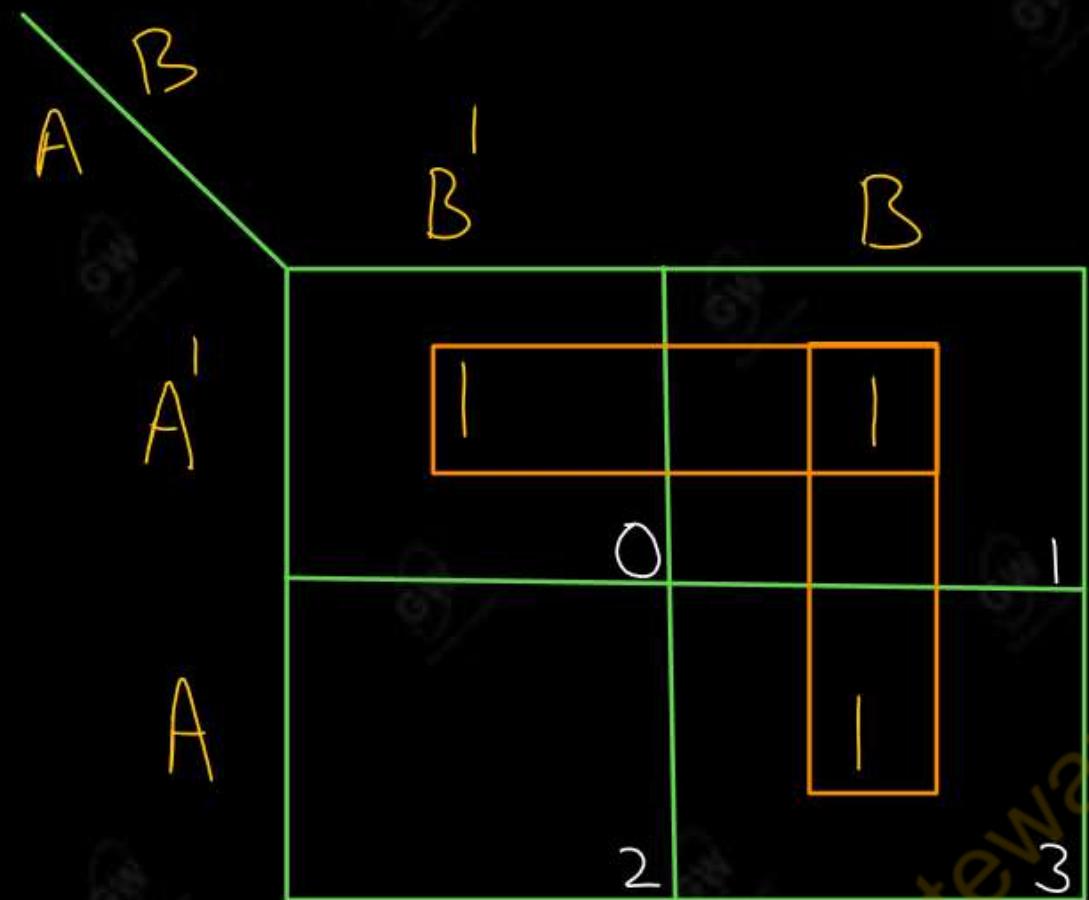
Required Minimal form

$$f(x, y) = x'y' + xy$$

Q. 3. Using the Karnaugh Map to find minimal form of the Boolean functions  $f(A, B) = \Sigma(0, 1, 3)$

K-map for 2 variables

$$= A'B' + A'B + AB$$



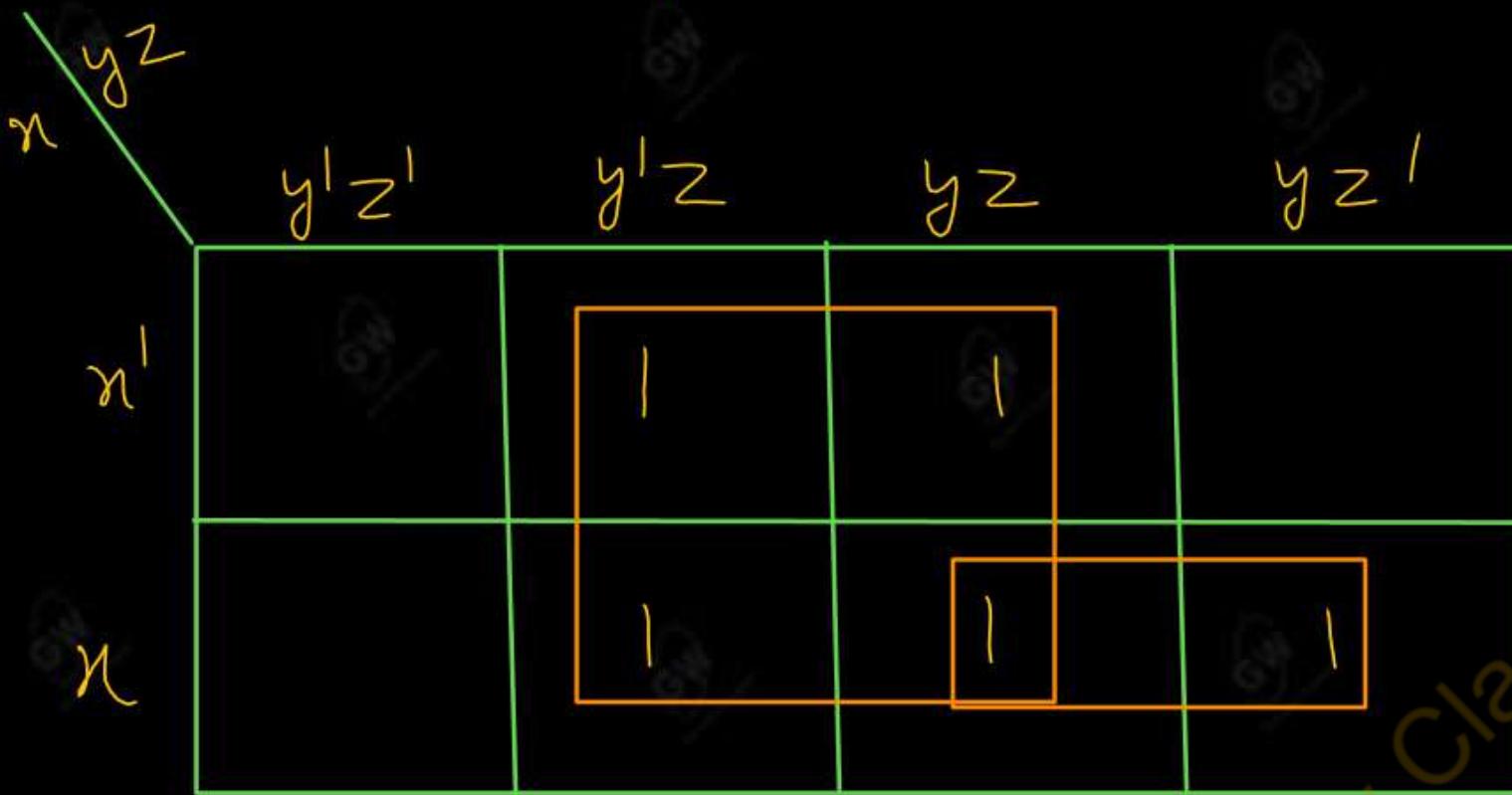
Required minimal form

$$f(A, B) = A' + B$$

Q. 4. Find using the K - Map a minimal form of the Boolean functions

K-map for 3 variables

$$f(x, y, z) = xyz + xyz' + xy'z + x'yz + x'y'z$$



Required Minimal form

$$f(x, y, z) = z + yz$$

Q. 5. Find using the K-Map a minimal form of the Boolean functions

K-map for 3 variables

	$B'C$	$B'C'$	$BC$	$BC'$
A	00	01	11	10
$A'$	0	1	1	1
A	1			

$$f(A, B, C) = ABC + ABC' + A'BC' + A'B'C$$
$$= \sum (1, 2, 6, 7)$$

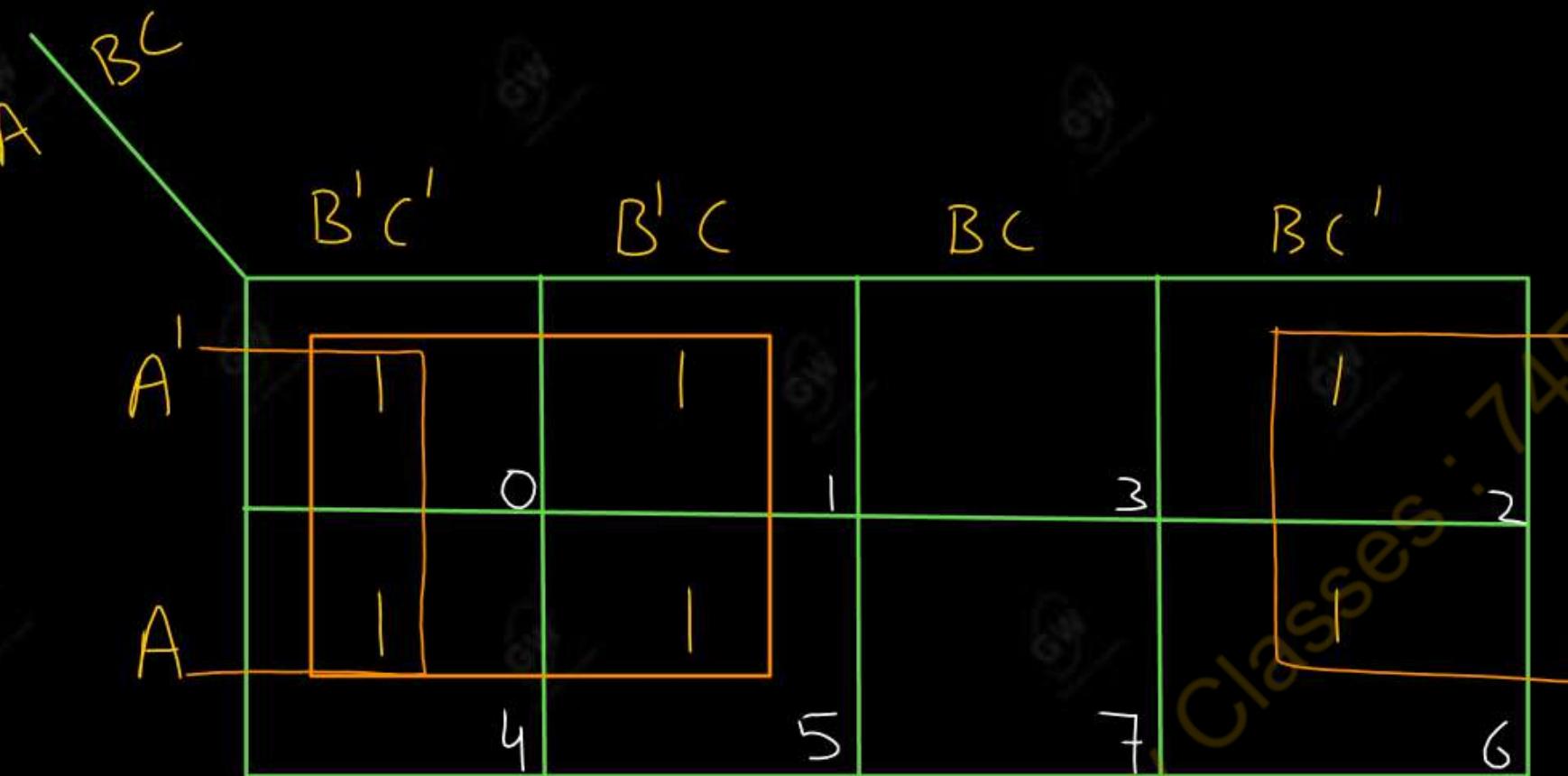
Required minimal form

$$f(A, B, C) = A\bar{B} + \bar{B}C' + A'\bar{B}C$$

Q. 6. Find using the K - Map a minimal form of the Boolean functions

K-map for 3 variables

$$F(A, B, C) = \Sigma (0, 1, 2, 4, 5, 6)$$



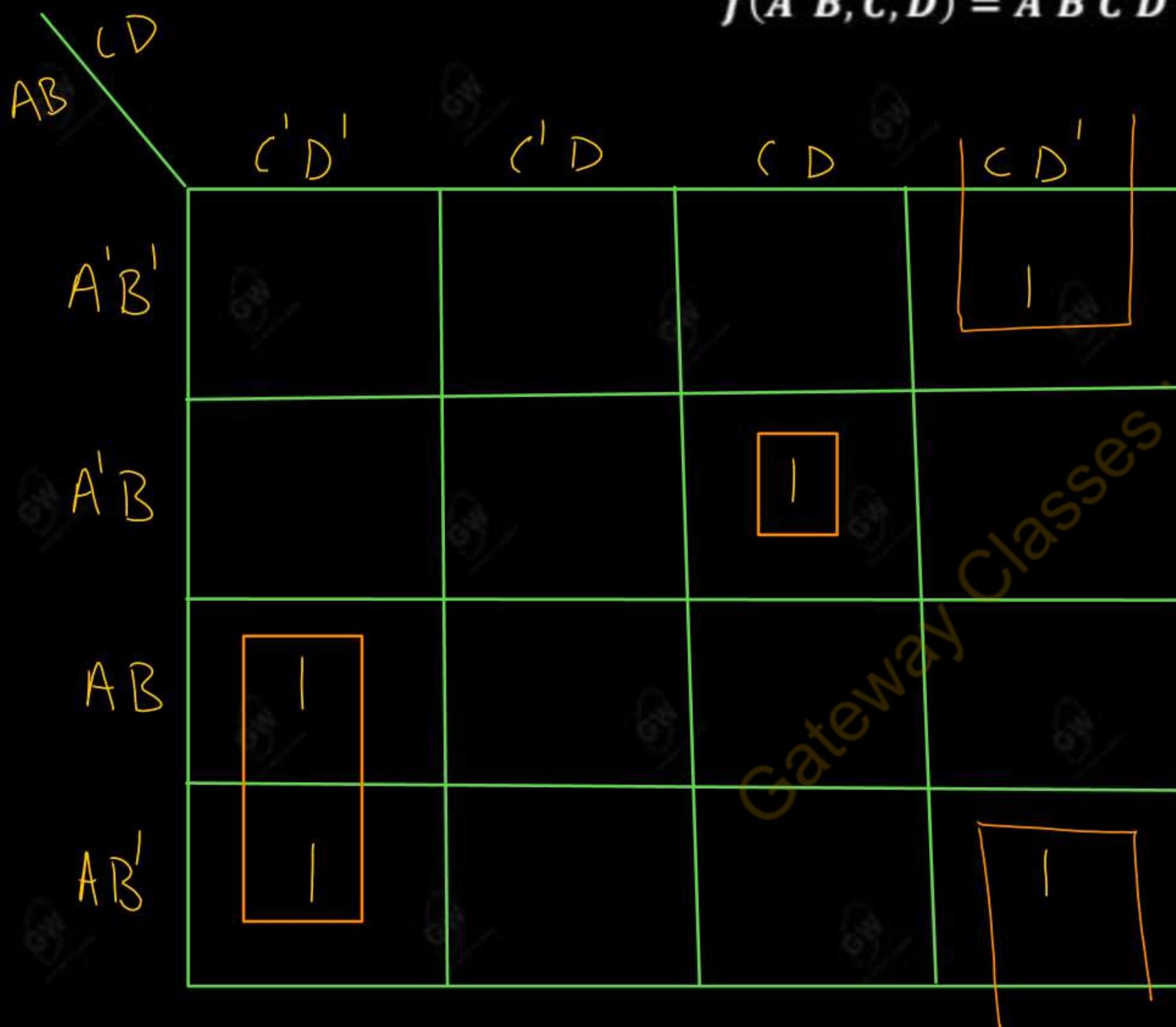
Required minimal form

$$F(A, B, C) = B' + C'$$

Q. 7:- Use K - Map to find a minimal form

$$= \sum_m(2, 7, 8, 10, 12)$$

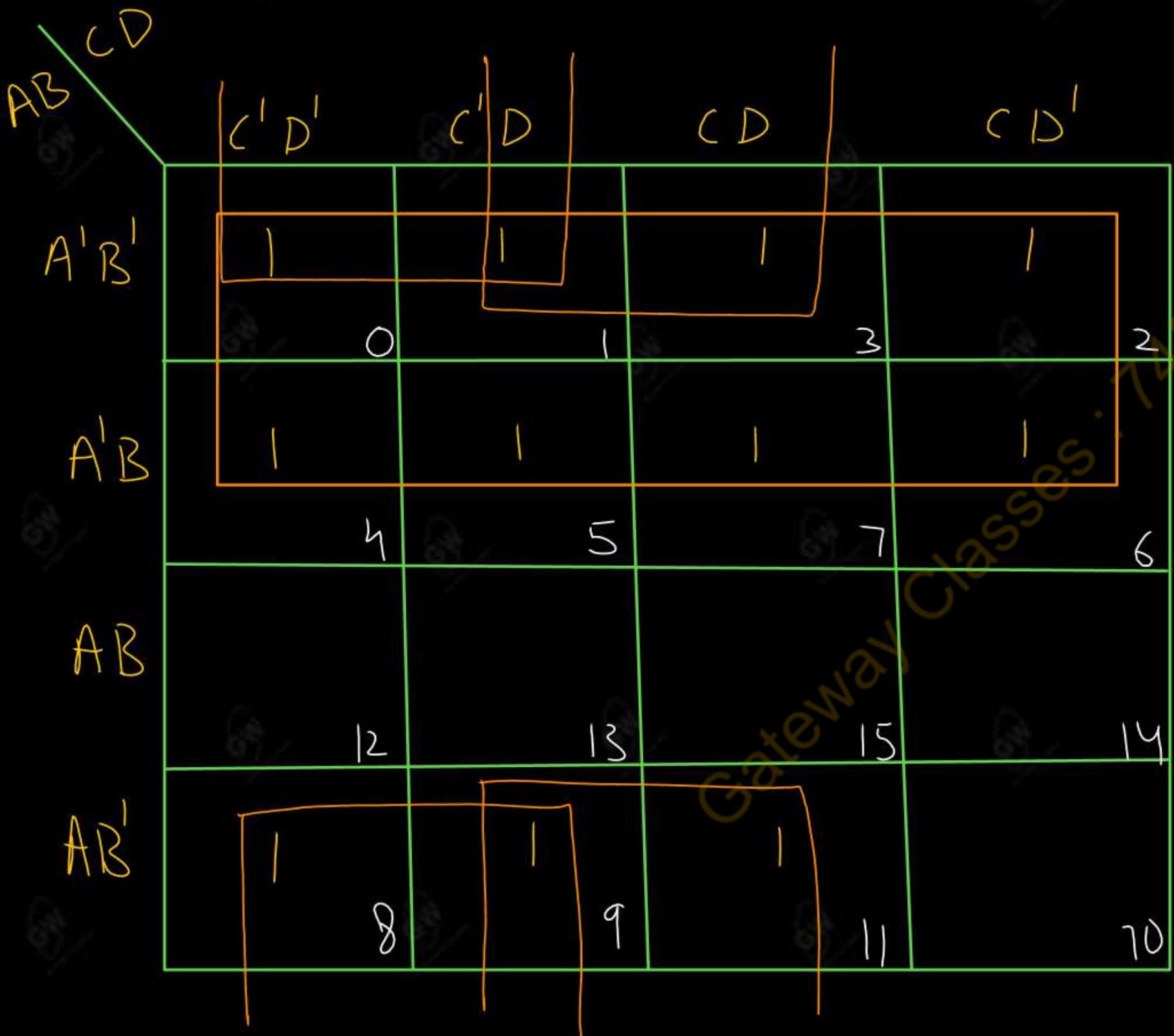
$$f(A, B, C, D) = A'B'C'D + AB'C'D' + A'B'C'D' + AB'C'D' + AB'C'D'$$



Required minimal form

$$\begin{aligned} f(A, B, C, D) &= A(C'D) + B'(C'D' \\ &\quad + A'B(CD) \end{aligned}$$

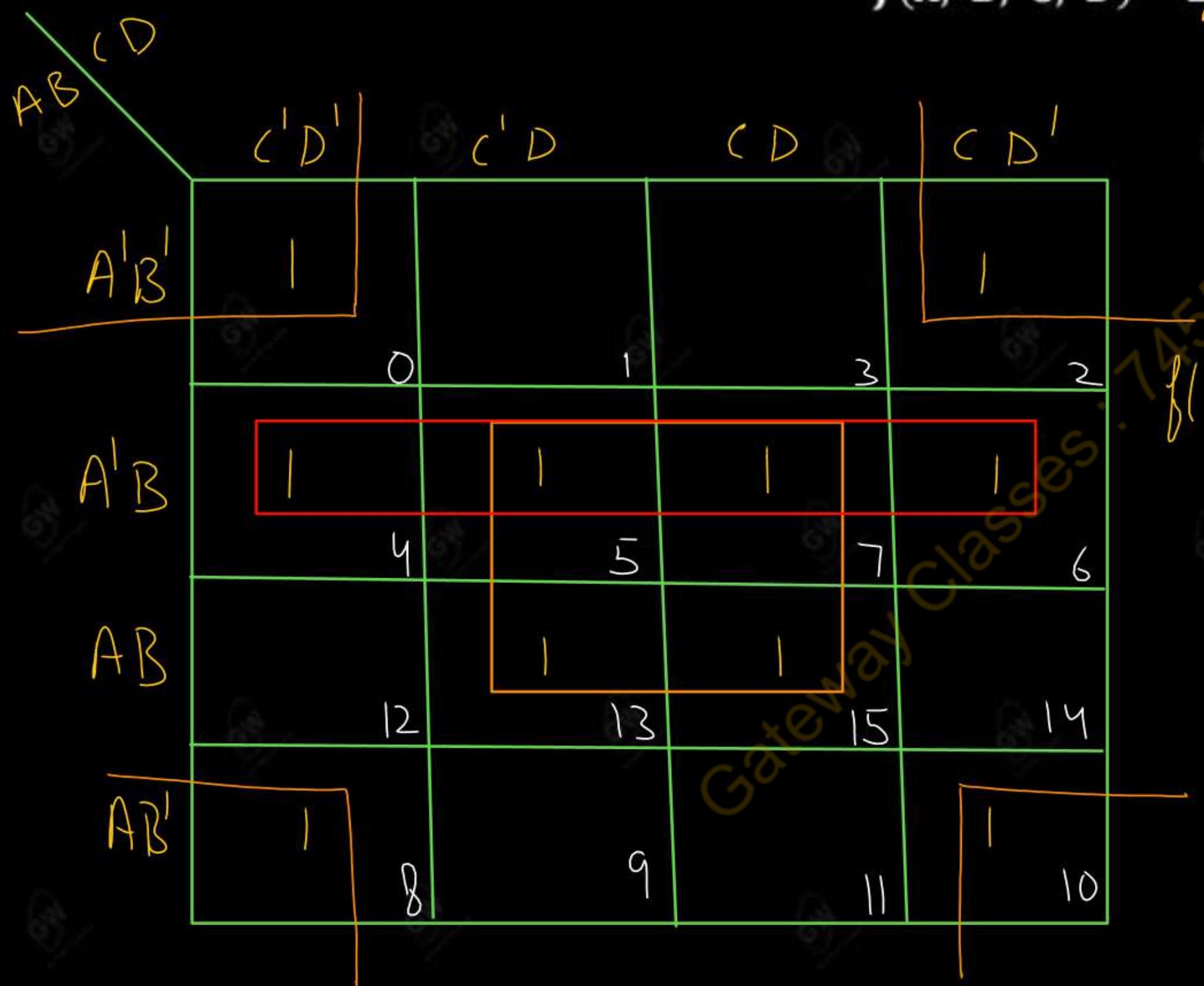
Q. 8:- Simplify the Boolean function  $F(A, B, C, D) = \Sigma(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 11)$  ✓ IMP



Simplified form  
 $F(A, B, C, D) = A' + B'C' + B'D$

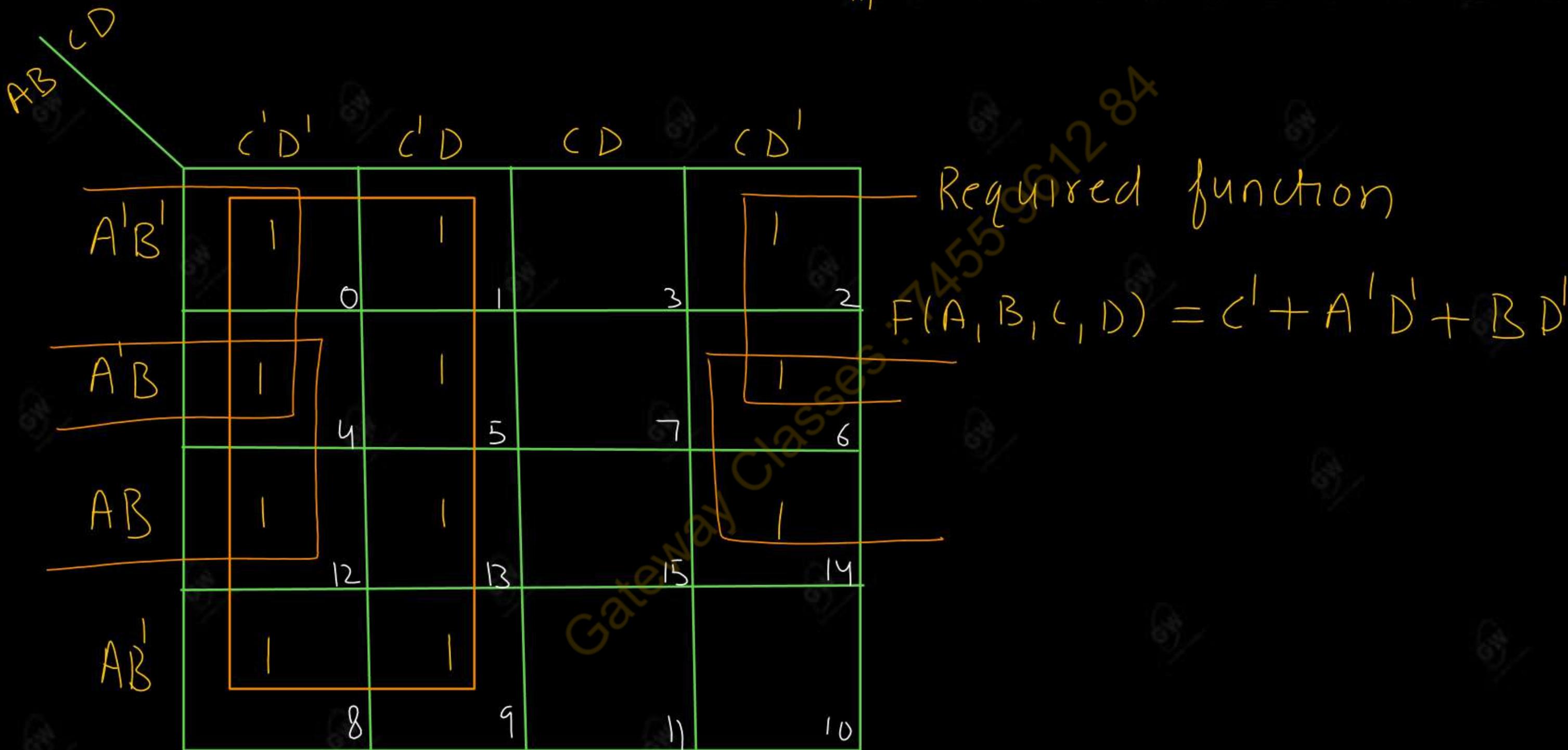
Q. 9:- Solve the following Boolean function using K – Map

$$f(A, B, C, D) = \sum_m(0, 2, 4, 5, 6, 7, 8, 10, 13, 15)$$



Required minimal form  
 $f(A, B, C, D) = BD + A'B + B'D'$

$$F(A, B, C, D) = \Sigma_m (m_0, m_1, m_2, m_4, m_5, m_6, m_8, m_9, m_{12}, m_{13}, m_{14})$$



## Topic : K – Map for SOP form

Q.1. Using the K – Map to find minimal form of the Boolean functions  $f(x, y) = x'y + xy$

Q. 2. Using the Karnaugh Map to find minimal form of the Boolean functions  $f(A, B) = \Sigma(0, 2)$

Q.3. Find using the K – Map a minimal form of the Boolean functions  $f(X, Y, Z) = XYZ + XYZ' + X'YZ$

Q. 4. Find using the K – Map a minimal form of the Boolean functions  $f(A, B, C) = \Sigma_m (0, 2, 3, 4, 7)$

Q. 5. Use K – Map to find a minimal form  $f(x, y, z, w) = xyzw + xyzw' + xy'zw' + x'y'zw + x'y'zw'$

Q.6 Solve  $E(x, y, z, t) = \Sigma (0, 2, 6, 8, 10, 12, 14, 15)$  using K – Map.

Q.7. Solve the following Boolean functions using K – Map  $F(A, B, C, D) = \Sigma (0, 2, 5, 7, 8, 10, 13, 15)$

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# DISCRETE STRUCTURES & THEORY OF LOGICS

## (Discrete Mathematics)

### UNIT - 2 : Functions and Boolean Algebra

Lecture - 09

#### Today's Target

- *K – Map for POS form*
- *Simplification of Boolean function by Boolean algebra*
- PYQ
- DPP

**K-Map**

For SOP form

OR

(Sum of Minterm )

OR

(DN Form)

For POS form

OR

(Product of Maxterm)

OR

(CN Form)

1. For two variables  $x$  and  $y$ :

- No. of cells in K - Map = 4
- Each cell contain a Maxterm

$x$	$y$	$x'$	$y'$
$x$	$y$	$x'$	$y'$
$x'$	$y'$	$x$	$y$

Maxterms:

Cell	Maxterm
0	$x + y$
1	$x + y'$
2	$x' + y$
3	$x' + y'$

$x$	$y$	$x'$	$y'$
$x$	$y$	$x'$	$y'$
$x'$	$y'$	$x$	$y$

Maxterms:

Cell	Maxterm
0	00
1	01
2	10
3	11

2. For three variables  $x$ ,  $y$  and  $z$

No. of cells in K - Map =  $2^3 = 8$

Each cell represent a Maxterm

$\bar{x}\bar{y}z$	$\bar{x}yz$	$\bar{x}y\bar{z}$	$\bar{x}y\bar{z}$	$x\bar{y}z$	$x\bar{y}z$	$x\bar{y}\bar{z}$	$x\bar{y}\bar{z}$
				0	1	3	2
				4	5	7	6

$\bar{x}\bar{y}z$	$\bar{x}y\bar{z}$	$x\bar{y}z$	$x\bar{y}\bar{z}$
00	01	11	10
0	1	3	2

### 3. For Four variables $n, y, z$ and $w$

No. of cells in K - Map =  $2^4 = 16$

Each cell represent a Maxterm

		$\bar{w}z'y$			
		$z + w$	$z + w'$	$z' + w'$	$z' + w$
$\bar{n} + y$	0	0	1	3	2
	4	5	7	6	
$\bar{n} + y'$	12	13	15	14	
	8	9	11	10	

		$\bar{w}z'y$			
		$z + w$	$z + w'$	$z' + w'$	$z' + w$
$\bar{n} + y$	0	0	1	3	2
	4	5	7	6	
$\bar{n} + y'$	12	13	15	14	
	8	9	11	10	

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**Working Rule**

1. Construct Karnaugh Map

2. Put 0 in the cell corresponding to each *Maxterm* given in the Boolean function

3. Making Groups of 0

(i) Group of 0 contain 1, 2, 4, 8 ..... Cells

(ii) Diagonal groups are not allowed

(iii) Overlapping allowed in groups

(iv) Group should be as large as possible

(v) No 0 should be as left without grouping

4. Write the Minimised term for all the groups keeping in mind

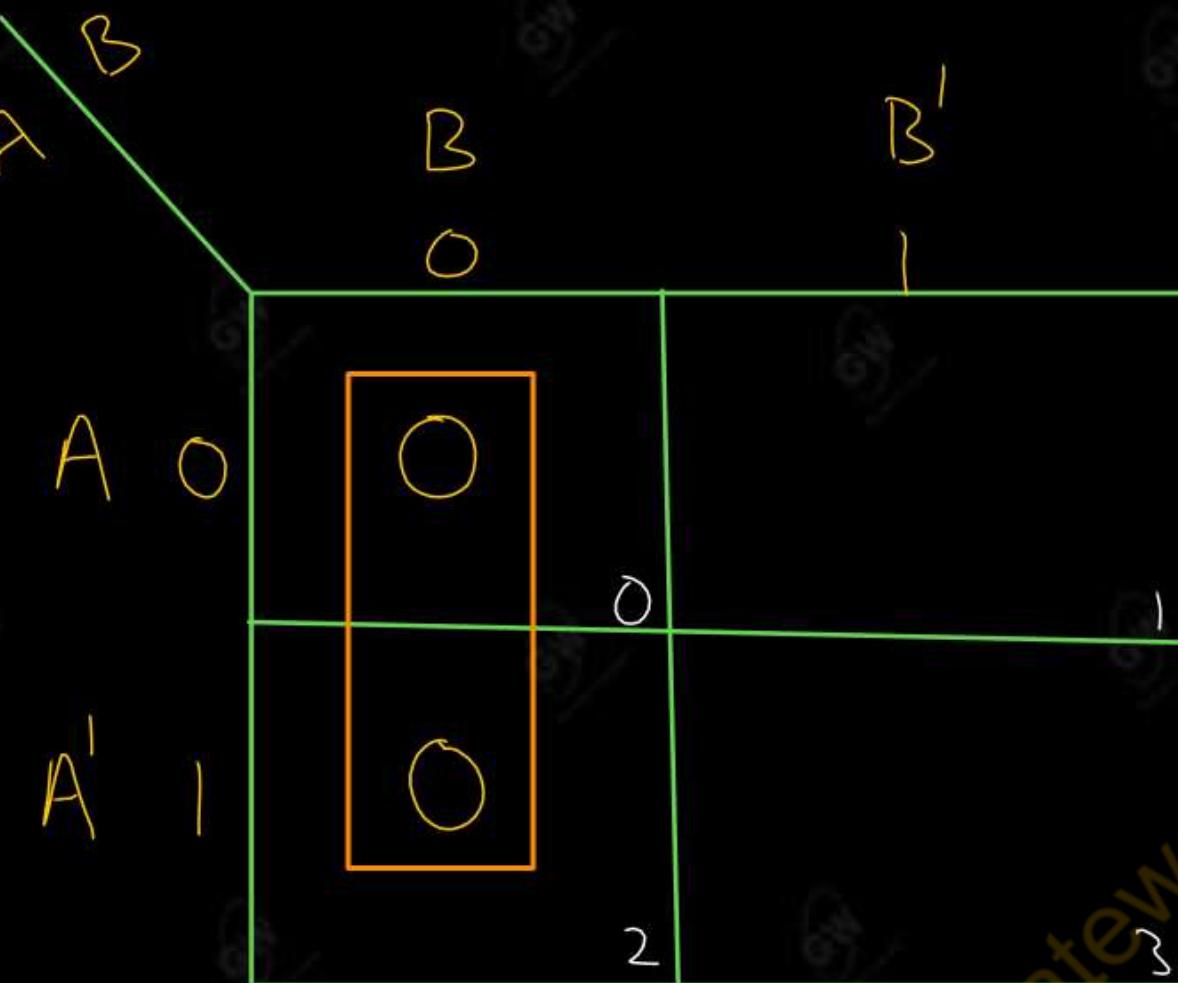
(i) Write only unchanged variable for each group

(ii) Check rows and column values for each group

5. Add all the minimised term to get required result

Q. 1:- Minimise the Boolean expression  $f(A, B) = \overline{\pi} \cup (0, 2)$

$$= (A + B)(A' + B')$$

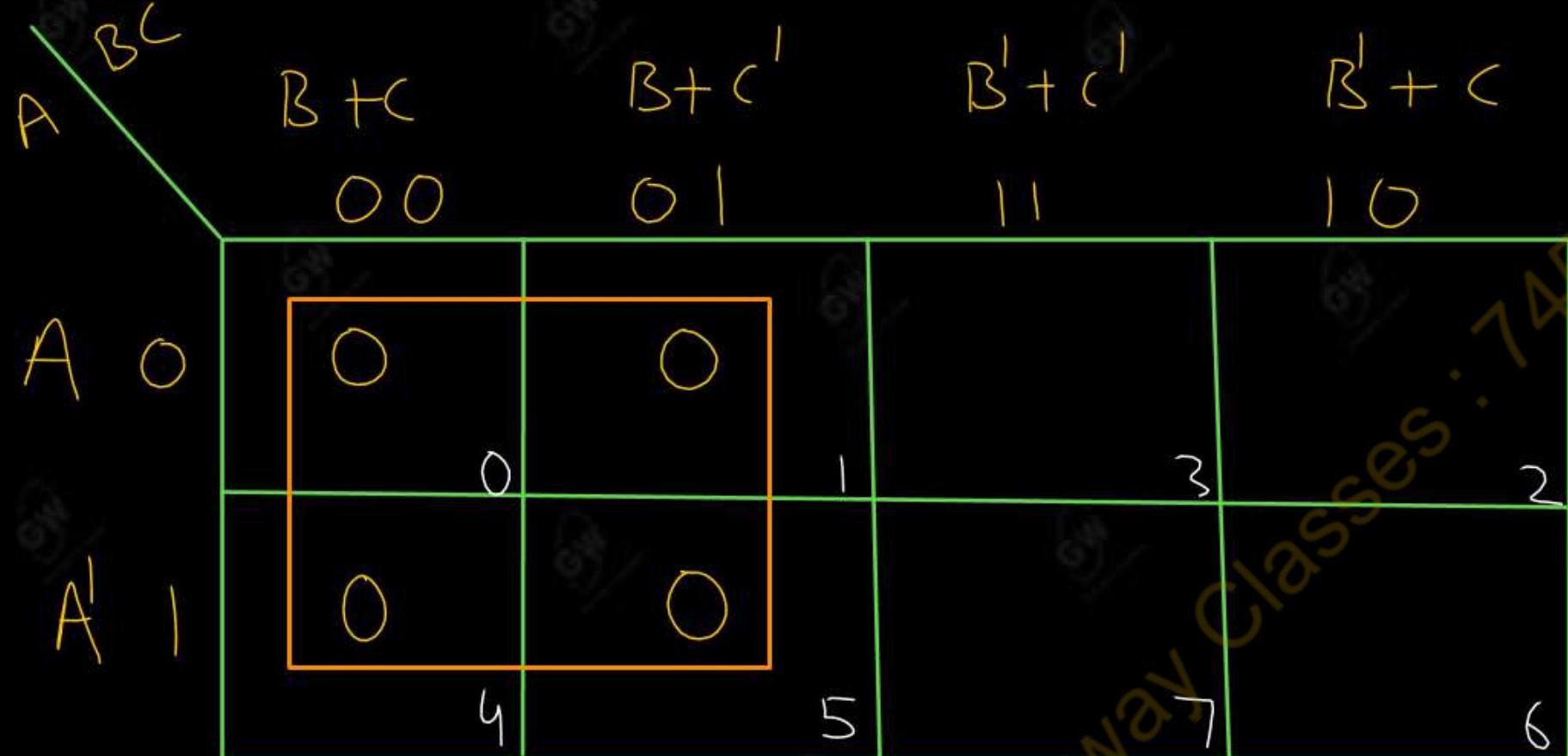


Required minimal form

$$f(A, B) = B$$

Q. 2:- Draw the K-map and simplify the Boolean expression  $F(A, B, C) = \overline{\prod M(0, 1, 4, 5)}$

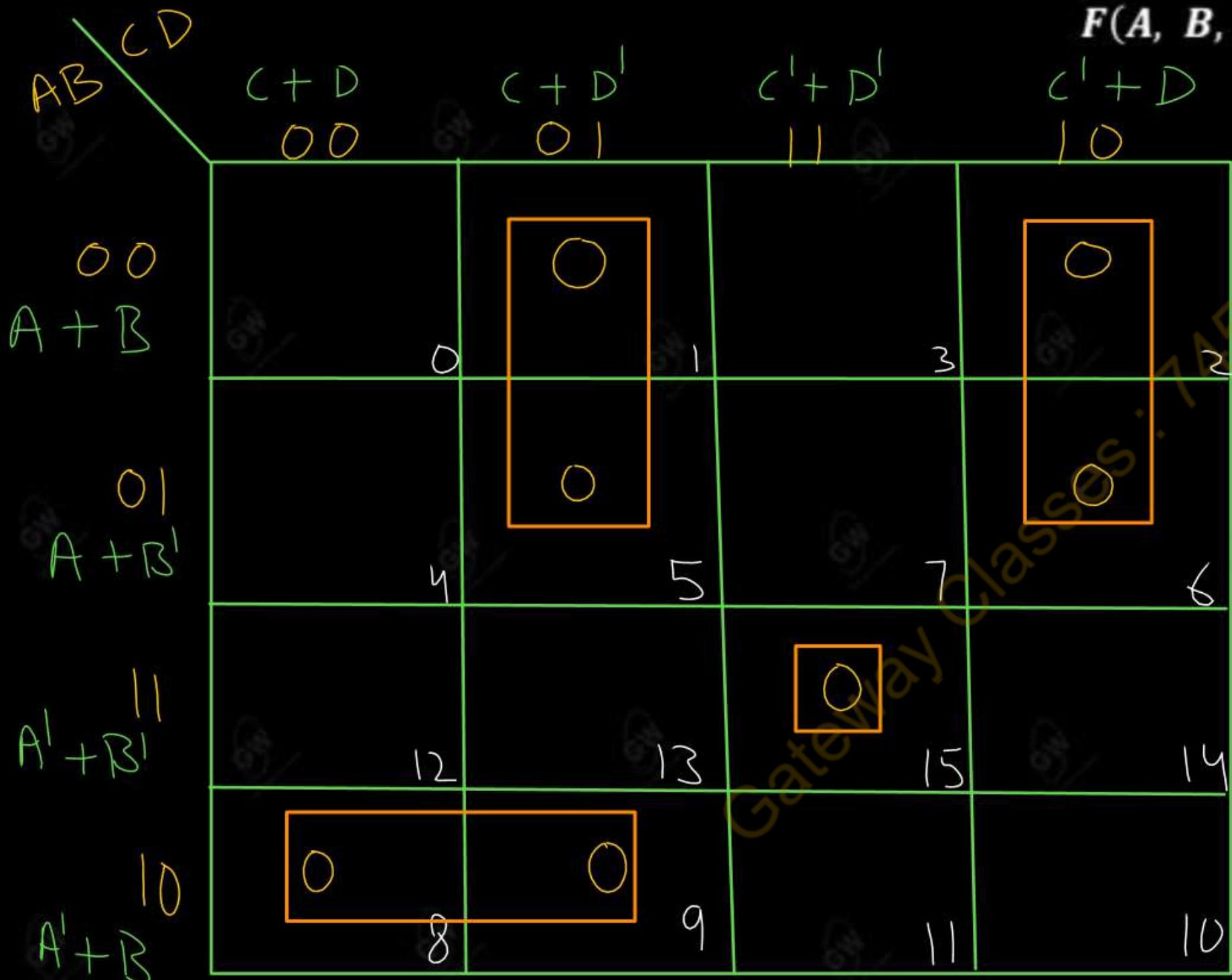
$$= (A + B + C)(A + B + C')(A' + B + C)(A' + B + C')$$



Required simplified form

$$F(A, B, C) = B$$

Q. 3:- Draw the K-map and simplify the Boolean function



$$F(A, B, C, D) = \prod M(1, 2, 5, 6, 8, 9, 15)$$

Simplified Boolean function

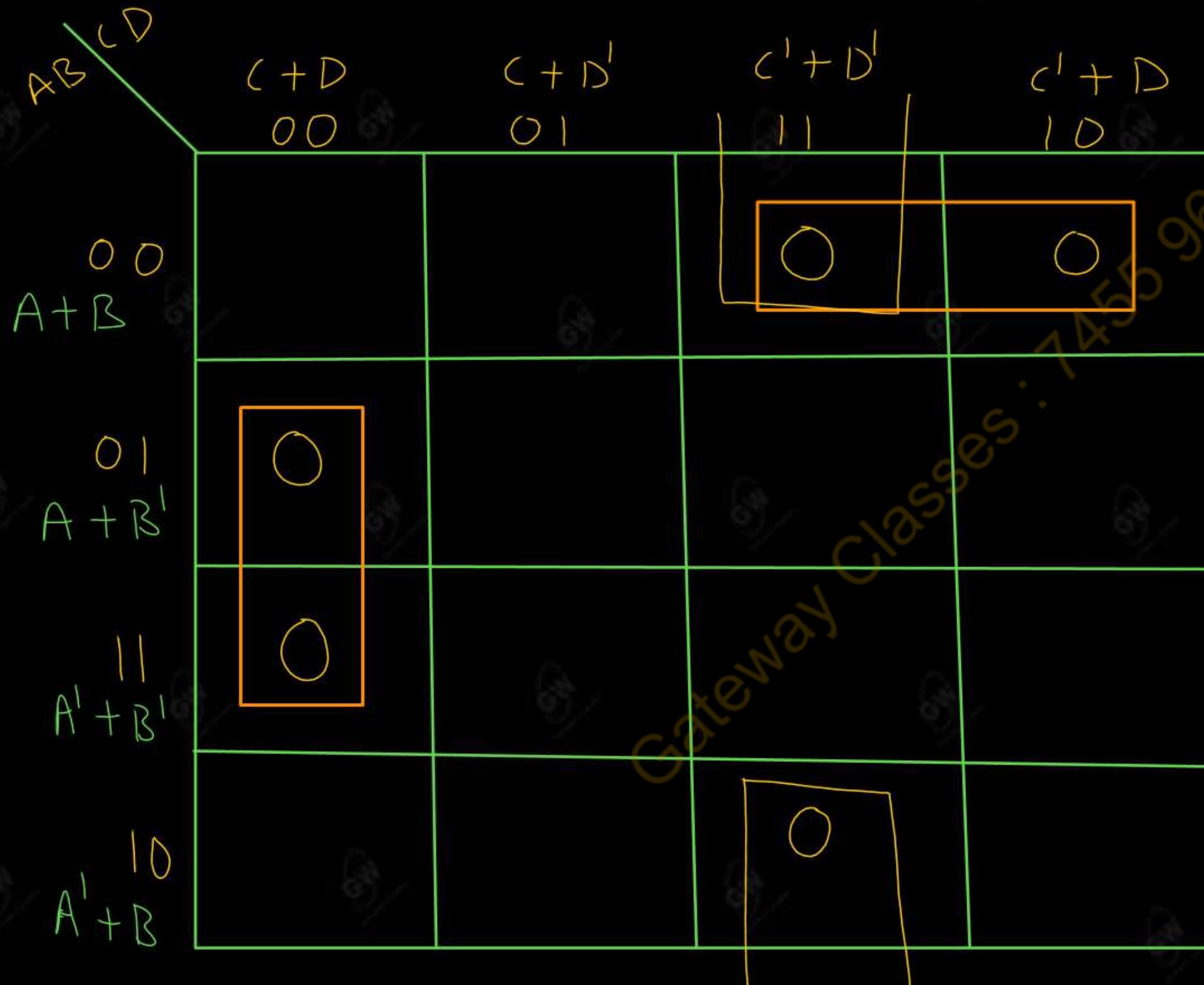
$$F(A, B, C, D) = (A + C + D')$$

$$(A + C' + D)(A' + B + C)$$

$$(A' + B' + C' + D')$$

Q. 4:- Simplify using K-map to obtain a minimum POS expression:

$$F(A, B, C, D) = (A' + B' + C + D)(A + B' + C + D)(A + B + C' + D')(A' + B + C' + D')(A + B + C' + D)$$



Simplified Boolean function

$$F(A, B, C, D) = (B' + C + D)$$

$$(A + B + C')(B + C' + D')$$

## Q.5 Simplify the Boolean expressions

(a)  $C(B + C)(A + B + C)$

(b)  $XY + X'Z + YZ$

(c)  $A + B(A + B) + A(A' + B)$

(a)  $C(B + C)(A + B + C)$

$= C(A + B + C) \left\{ \because a + a = a \right\}$

$\therefore (A + (B + C))$

$$\begin{aligned}
 &= CA + CB + CC \\
 &= CA + C \quad \left\{ a + a = a \right\}
 \end{aligned}$$

(b)  $XY + X'Z + YZ$

$= \underline{XY} + \underline{X'Z} + \underline{YZ} + \underline{X'YZ}$

$$= XY + XYZ + X'Z + X'ZY$$

$$= XY(1+Z) + X'Z(1+Y)$$

$$= XY + X'Z \quad \left\{ \because a+1 = 1 \right\}$$

$$(c) A + B(A+B) + A(A'+B)$$

$$= A + AB + BB + AA' + AB$$

$$= A + AB + AB + B + 0 \quad \left\{ \begin{array}{l} \because a \cdot a' = 0 \\ a \cdot a = a \end{array} \right.$$

$$= A + AB + B \quad \left\{ \begin{array}{l} \because a+a = a \end{array} \right\}$$

$$= A + B \quad \left\{ a+a' = a \right\}$$

## DPP- 9

## Topic : K – Map for POS form

**Q.1.** Draw the K – map and simplify the Boolean function  $F(A, B, C, D) = \overline{\overline{M}}(1, 2, 4, 6, 8, 9)$

**Q.2.** Draw the K – map and simplify the Boolean function  $F(A, B, C, D) = \overline{\overline{M}}(3, 4, 6, 7, 11, 12, 13, 14, 15)$

**Q. 3.** Simplify the following using Boolean algebra

(a)  $X = (A + B + AB)(A + C)$

(b)  $A = XY + (XZ)' + XY'Z (XY + Z)$

(c)  $A = XY + XYZ + X'Y + XY'Z$

(d)  $A = X[Y + Z (XY + XZ)']$

(e)  $A = XY + X'YZ' + YZ$

(f)  $A = (XY' + Z)(X + Y')Z$

(g)  $A = XY' + Z + (X' + Y)Z'$

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