① $A = \begin{bmatrix} 2 & -12 \\ 1 & -5 \end{bmatrix}$

$\lambda I = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

$|A - \lambda I| = \left| \begin{bmatrix} 2 & -12 \\ 1 & -5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right|$

$= \left| \begin{bmatrix} 2-\lambda & -12 \\ 1 & -5-\lambda \end{bmatrix} \right|$

$|ad - bc| = (2-\lambda)(-5-\lambda) - (-12)$

$\Rightarrow -10 - 2\lambda + 5\lambda + \lambda^2 + 12 = 0$

$\Rightarrow \lambda^2 + 3\lambda + 2 = 0$

$\Rightarrow \lambda^2 + \lambda + 2\lambda + 2 = 0$

$\Rightarrow \lambda(\lambda+1) + 2(\lambda+1) = 0$

$(\lambda+1)(\lambda+2)$

$\lambda_1 = -1 \qquad \lambda_2 = -2$ (Eigen values)

Eigen vectors

$$\begin{bmatrix} 2-\lambda & -12 \\ 1 & -5-\lambda \end{bmatrix} \cdot v_1 = 0$$

for $\lambda = -1$

$$\begin{bmatrix} 3 & -12 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} = 0$$

$$3v_{11} - 12v_{12} = 0 \Rightarrow 3v_{11} = 12v_{12} \Rightarrow \boxed{v_{11} = 4v_{12}}$$

$$\begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

for $\lambda = -2$

$$\begin{bmatrix} 2-\lambda & -12 \\ 1 & -5-\lambda \end{bmatrix} \cdot v_2 = 0$$

$$\begin{bmatrix} 4 & -12 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} = 0$$

$$4v_{21} - 12v_{22} = 0 \Rightarrow 4v_{21} = 12v_{22}$$

$$\Rightarrow \boxed{v_{21} = 3v_{22}}$$

$$\begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$