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Embedded Image Coding Using Wavelet Difference Reduction

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ABSTRACT We present an embedded image coding method, which basically consists of three steps, *Discrete Wavelet Transform*, *Differential Coding*, and Binary *Reduction*. Both J. Shapiro's embedded zerotree wavelet algorithm, and A. Said and W. A. Pearlman's codetree algorithm use spatial orientation tree structures to implicitly locate the significant wavelet transform coefficients. Here a direct approach to find the positions of these significant coefficients is presented. The encoding can be stopped at any point, which allows a target rate or distortion metric to be met exactly. The bits in the bit stream are generated in the order of importance, yielding a fully embedded code to successively approximate the original image source; thus it's well suited for progressive image transmission. The decoder can also terminate the decoding at any point, and produce a lower (bit) rate reconstruction image. Our algorithm is very simple in its form (which will make the encoding and decoding very fast), requires no training of any kind or prior knowledge of image sources, and has a clear geometric structure. The image coding results of it are quite competitive with almost all previous reported image compression algorithms on standard test images.

1 Introduction

Wavelet theory and applications have grown explosively in the last decade. It has become a cutting-edge technology in applied mathematics, neural networks, numerical computation, and signal processing, especially in the area of image compression. Due to its good localization property in both the spatial domain and spectral domain, a wavelet transform can handle well transient signals, and hence significant compression ratios may be obtained. Current research on wavelet based image compression (see for example [Sha93, SP96, XRO97], etc) has shown the high promise of this relatively new yet almost mature technology.

In this paper, we present a lossy image codec based on index coding. It contains the following features:

- A discrete wavelet transform which removes the spatial and spectral redundancies of digital images to a large extends.

- Index coding (differential coding and binary reduction) which represents the positions of significant wavelet transform coefficients very efficiently.

- Ordered bit plane transmission which provides a successive approximation of image sources and facilitates progressive image transmission.
- Adaptive arithmetic coding which requires no training of any kind or prior knowledge of image sources.

This paper is organized as follows. Section 2, 3, and 4 explain the discrete wavelet transform, differential coding, and binary reduction, respectively. In Section 5 we combine these three methods together and present the Wavelet Difference Reduction algorithm. Section 6 contains the experimental results of this algorithm. As a result of the algorithm, we discuss synthetic aperture radar (SAR) image compression in Section 7. The paper is concluded in Section 8.

2 Discrete Wavelet Transform

The terminology "wavelet" was first introduced in 1984 by A. Grossmann and J. Morlet [GM84]. An $L^2(\mathbf{R})$ function $\psi(x)$ induces an *orthonormal wavelet system* of $L^2(\mathbf{R})$ if the dilations and translations of $\psi(x)$, $\{\psi_{j,k}(x) = 2^{j/2}\psi(2^j x - k) : j, k \in \mathbf{Z}\}$ constitute an orthonormal basis of $L^2(\mathbf{R})$. We call $\psi(x)$ the *wavelet function.* If $\psi(x)$ is compactly supported, then this wavelet system is associated with a *multiresolution analysis.* A multiresolution analysis consists of a sequence of embedded closed subspaces $V_j \subset L^2(\mathbf{R}), j \in \mathbf{Z}$, satisfying $\overline{\bigcup_{j\in\mathbf{Z}} V_j} = L^2(\mathbf{R})$ and $\bigcap_{j\in\mathbf{Z}} V_j = \{0\}$.Moreover, there should be an $L^2(\mathbf{R})$function $\phi(x)$such that $\{\phi_{j,k}(x) = 2^{j/2}\phi(2^j x - k) : k \in \mathbf{Z}\}$ is an orthonormal basis for V_j for all $j \in \mathbf{Z}$. Since $\phi(x) \in V_0 \subset V_1$, we have

$$\phi(x) = \sum_{k\in\mathbf{Z}} a_k \phi(2x - k), \tag{17.1}$$

and $\psi(x)$ and $\phi(x)$ are related by

$$\psi(x) = \sum_{k\in\mathbf{Z}} (-1)^k a_{-k+1} \phi(2x - k). \tag{17.2}$$

We call $\{a_k : k \in \mathbf{Z}\}$ and $\phi(x)$ the *scaling filter* and the *scaling function* of the wavelet system, respectively. Be defining $b_k := (-1)^k a_{-k+1}$, we call $\{b_k : k \in \mathbf{Z}\}$ the *wavelet filter.* As one knows, it is straightforward to define an orthonormal wavelet system by (17.2) from a multiresolution analysis.

For a discrete signal x, the *discrete wavelet transform* (DWT) of x consists of two part, the low frequency part L and the high frequency part H. They can be computed by

$$L_k = 2^{-1/2} \sum_{n\in\mathbf{Z}} a_{n-2k} x_n,$$

$$H_k = 2^{-1/2} \sum_{n\in\mathbf{Z}} b_{n-2k} x_n.$$

And the *inverse discrete wavelet transform* (IDWT) will give back x from L and H,

$$x_n = 2^{-1/2} \sum_{k\in\mathbf{Z}} (a_{n-2k} L_k + b_{n-2k} H_k).$$

In the case of a *biorthogonal wavelet system* where synthesis filters are different from analysis filters (and consequently synthesis scaling/wavelet functions are different from analysis scaling/wavelet functions), the IDWT is given by

$$x_n = 2^{-1/2} \sum_{k \in \mathbb{Z}} \left(\tilde{a}_{n-2k} L_k + \tilde{b}_{n-2k} H_k \right) ,$$

where $\tilde{a}$ and $\tilde{b}$ are synthesis scaling filter and wavelet filter, respectively.

For more details about discrete wavelet transform and wavelet analysis, we refer to [Chu92, Dau92, Mal89, Mey92, RW97, SN95, VK95, Wic93], etc. The discrete wavelet transform can remove the redundancies of image sources very successfully, and we will take it as the first step in our image compression algorithm.

3 Differential Coding

Differential Coding [GHLR75] takes the difference of adjacent values. This is a quite useful coding scheme when we have a set of integers with monotonically increasing order. For example, if we have an integer set $\mathcal{S}$,

$$\mathcal{S} = \{1, 2, 5, 36, 42\} ,$$

then its *difference set* $\mathcal{S}'$ is

$$\mathcal{S}' = \{1, 1, 3, 31, 6\} .$$

And it's straightforward to get back $\mathcal{S}$ from the difference set $\mathcal{S}'$ by taking the partial sum of $\mathcal{S}'$.

4 Binary Reduction

Binary Reduction is one of the representations of positive binary integers, with the shortest representation length, as described in [Eli75]. It's the binary coding of an integer, with the most significant bit removed. For example, since the binary representation of 19 is 10011, then the binary reduction of 19 is 0011. For the example in Section 3, the binary reduction of $\mathcal{S}'$ will be

$$\mathcal{S}'' = \{, , 1, 1111, 10\} .$$

And we call $\mathcal{S}''$ the *reduced set* of $\mathcal{S}$. Note that there are no coded symbols before the first two commas "," in $\mathcal{S}''$. In practice, one will need some end of message symbol to separate different elements when the reduced set is used, like the comma "," above.

The binary reduction is a reversible procedure by adding a "1" as the most significant bit in the binary representation.

5 Description of the Algorithm

After taking the discrete wavelet transform of an image data, all wavelet transform coefficients will be ordered in such a way that the coefficients at coarser scale will come before the coefficients at finer scale. A typical scanning pattern [Sha93] is indicated in Figure 1. In a wavelet decomposition domain with N scales, the scan begins at LL_N, then HL_N, LH_N, HH_N, at which point it moves on to scale N–1, etc. Because of the nature of the discrete wavelet transform, inside each high-low subregion $HL_N, HL_{N-1}, \cdots, HL_1$, the scanning order goes column by column, and inside each low-high subregion $LH_N, LH_{N-1}, \cdots, LH_1$, the scanning order goes row by row. A wavelet transform coefficient x is defined as *significant* with respect to a threshold T if $|x| \geq T$, otherwise x is said to be *insignificant.*

The wavelet transform coefficients are stored in three ordered sets, the set of significant coefficients (**SCS**), the temporary set containing significant coefficients found in a given round (**TPS**), and the set of insignificant coefficients (**ICS**). The initial **ICS** contains all the wavelet transform coefficients with the order shown in Figure 1, while **SCS** and **TPS** are empty. And the initial threshold T is chosen such that $|x_j| < 2T$ for all the wavelet transform coefficients x_j, and for some j_0, $|x_{j_0}| \geq T$. The encoder of this Wavelet Difference Reduction algorithm will output the initial threshold T.

First we have a sorting pass. In the sorting pass, all significant coefficients in **ICS** with respect to T will be moved out and put into **TPS**. Let $\mathcal{S}$ be the indices (in **ICS**) of these significant coefficients. The encoder outputs the reduced set $\mathcal{S}''$ of $\mathcal{S}$. Instead of using "," as the end of message symbol to separate different elements in $\mathcal{S}''$, we will take the signs (either "+" or "–") of these significant coefficients as the end of message symbol. For example, if $\mathcal{S} = \{1, 2, 5, 36, 42\}$, and the signs of these five significant coefficients are "+ – + + –", then the encoding output $\mathcal{S}''$ will be "+ – 1 + 1111 + 10–". Then update the indexing in **ICS**. For example, if x_3 is moved to **TPS**, then all coefficients after x_3 in **ICS** will have their indices subtracted by 1, and so on.

Right after the sorting pass, we have a refinement pass. In the refinement pass, the magnitudes of coefficients in **SCS** will have an additional bit of precision. For example, if the magnitude of a coefficient in **SCS** is known to be in [32, 64), then it will be decided at this stage whether it is in (32,48) or [48,64). And a "0" symbol will indicate it is in the lower half [32, 48), while a "1" symbol will indicate it is in the upper half [48, 64). Output all these refinement values "0" and "1". Then append **TPS** to the end of **SCS**, reset **TPS** to the empty set, and divide T by 2. Another round begins with the sorting pass.

The resulting symbol stream in the sorting pass and refinement pass will be further coded by adaptive arithmetic coding [WNC87]. And the encoding can be stopped at any point, which allows a target rate or distortion metric to be met exactly.

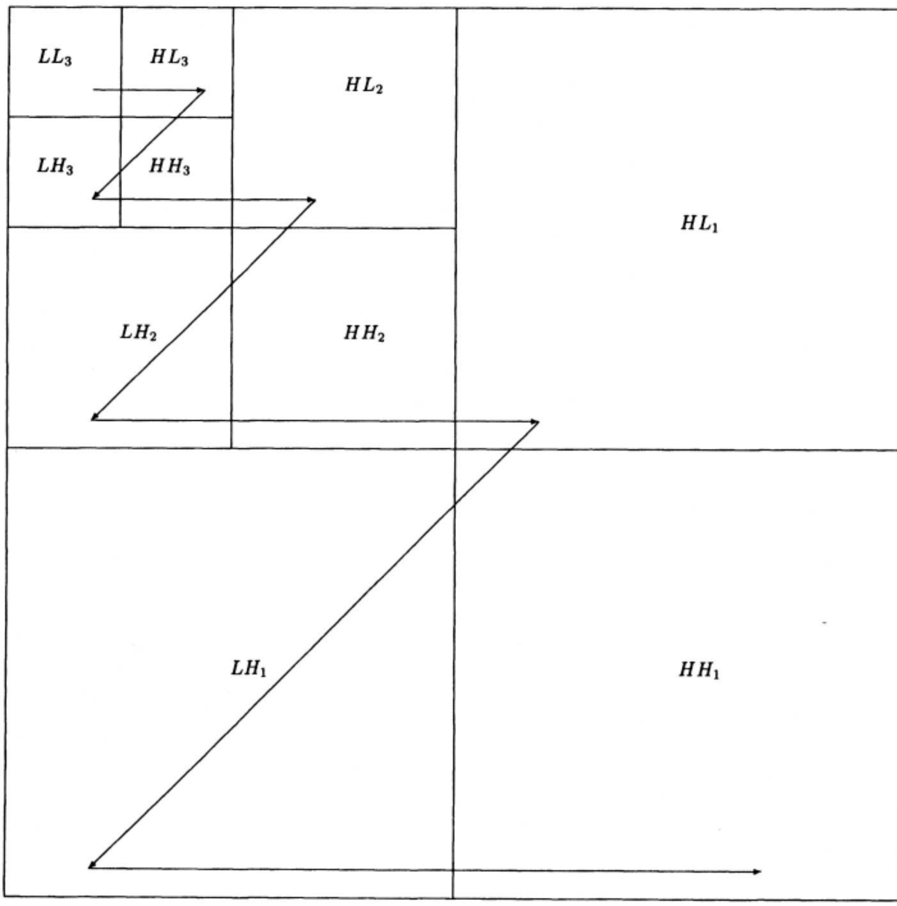

FIGURE 1. Scanning order of the wavelet transform coefficients with 3 scales.

FIGURE 2. Original

6 Experimental Results

Experiments have been done on all the 8 bits per pixel (bpp), grayscale test images, available from ftp://links.uwaterloo.ca:/pub/BragZone/, which include "Barbara", "Goldhill", "Lena" and others. And we used the Cohen-Daubechies-Feauveau 9/7-tap filters (CDF-97) [CDF92] with six scales. The symmetry of CDF-97 allows the "reflection" extension at the images edges. For our purpose, the compression performance is measured by the peak signal to noise ratio

$$\mathrm{PSNR} = 10 \log_{10} \left(\frac{255^2}{\mathrm{MSE}} \right) \mathrm{dB},$$

where MSE is the mean square error between the original image and the reconstructed one. Some other criterion might have been more preferable. However, to make a direct comparison with other coders, PSNR is chosen. And the bit rate is calculated from the actual size of the compressed file.

Our experimental results show that the coding performance of the current implementation of this Wavelet Difference Reduction algorithm is quite competitive with other previous reported image compression algorithms on standard test images. Some early results were reported in [TW96]. Here we include some coding results for the 8 bpp, 512×512 grayscale "Lena" image. Figure 2 is the original "Lena" image, and Figure 3 is the one with compression ratio 8:1, using the Wavelet Difference Reduction algorithm, and having $\mathrm{PSNR} = 40.03$ dB. Because the Wavelet Difference Reduction algorithm is an embedded coding scheme, one can actually achieve any given compression ratio.

FIGURE 3. 8:1 Compression, PSNR = 40.03 dB

7 SAR Image Compression

Real-time transmission of sensor data such as synthetic aperture radar (SAR) images is of great importance for both time critical applications such as military search and destroy missions as well as in scientific survey applications. Furthermore, since post processing of the collected data in either application involves search, classification and tracking of targets, the requirements for a "good" compression algorithm is typically very different from that of lossy image compression algorithms developed for compressing still-images. The definition of targets are application dependent and could be military vehicles, trees in the rain forest, oil spills etc.

To compress SAR images, first we take the logarithm of each pixel value in the SAR image data. Then apply the discrete wavelet transform on these numbers. And the following steps are just alternatively index coding and getting refinement values on the wavelet transform coefficients, as described in the Wavelet Difference Reduction algorithm. Adaptive arithmetic coding will compress the symbol stream further.

Since the Wavelet Difference Reduction algorithm locates the position of significant wavelet transform coefficients directly and contains a clear geometric structure, we may process the SAR image data directly in the compressed wavelet domain, for example, speckle reduction. For more details, we refer to [TGW+96].

The test data we used here is a fully polarimetric SAR image of the Lincoln north building in Lincoln, MA, collected by the Lincoln Laboratory MMW SAR. It was preprocessed with a technique known as the polarimetric whitening filer (PWF) [NBCO90]. We apply the Wavelet Difference Reduction algorithm on this PWFed SAR image. In practice the compression ratio can be set to any real number greater

FIGURE 4. PWFed SAR Image of a Building

than 1. Figure 4, 5, 6 and 7 show the SAR image and a sequence of images obtained by compressing the SAR image using the Wavelet Difference Reduction algorithm at the compression ratios 20:1, 80:1, and 400:1. Visually the image quality is still well preserved at the ratio 80:1, which indicates substantial advantage over JPEG compression [Wal91].

8 Conclusions

In this paper we presented an embedded image coding method, the Wavelet Difference Reduction algorithm. Compared with Shapiro's EZW [Sha93], and Said and Pearlman's SPC [SP96] algorithm, one can see that all these three embedded compression algorithm share the same basic structure. More specifically, a generic

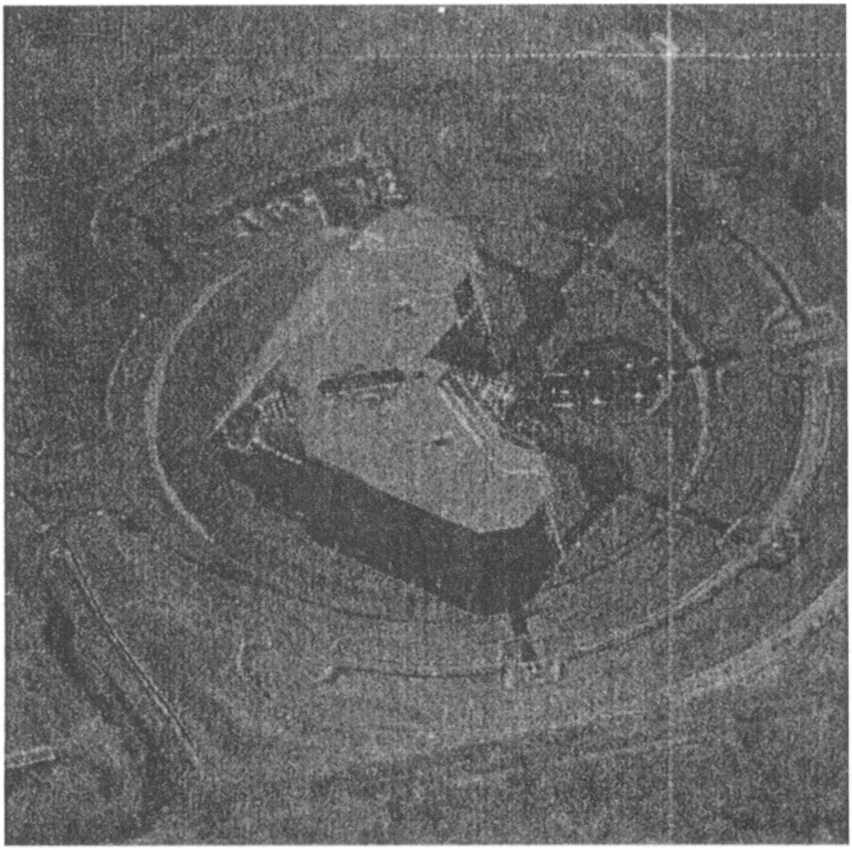

FIGURE 5. Decompressed Image at the Compression Ratio 20:1

model including all these three algorithms (and some others) consists of five steps:

1. Take the discrete wavelet transform of the original image.
2. Order the wavelet transform coefficients from coarser scale to finer scale, as in Figure 1. Set the initial threshold T.
3. (Sorting Pass) Find the positions of significant coefficients with respect to T, and move these significant coefficients out.
4. (Refinement Pass) Get the refinement values of all significant coefficients, except those just found in the sorting pass of this round.
5. Divide T by 2 and go to step **3**.

The resulting symbol stream in step 3 and 4 will be further encoded by a lossless data compression algorithm.

FIGURE 6. Decompressed Image at the Compression Ratio 80:1

The only difference among these three algorithms is in Step 3, the sorting pass. In EZW, Shapiro employs the self similarity tree structure. In SPC, a set partitioning algorithm is presented which provides a better tree structure. In our algorithm, we combines the differential coding and binary reduction. The concept of combining the differential coding and binary reduction is actually a fairly general concept and not specific to the wavelet decomposition domain. For example, it can be applied to the Partition Priority Coding (PPC) [HDG92], and one would expect some possible improvement in the image coding results.

Acknowledgments: We would like to thank our colleagues in the Computational Mathematics Laboratory, Rice University, especially C. Sidney Burrus, Haitao Guo and Markus Lang, for fruitful discussion, exchange of code, and lots of feedback. Amir Said explained their SPC algorithm in details to us and made their code

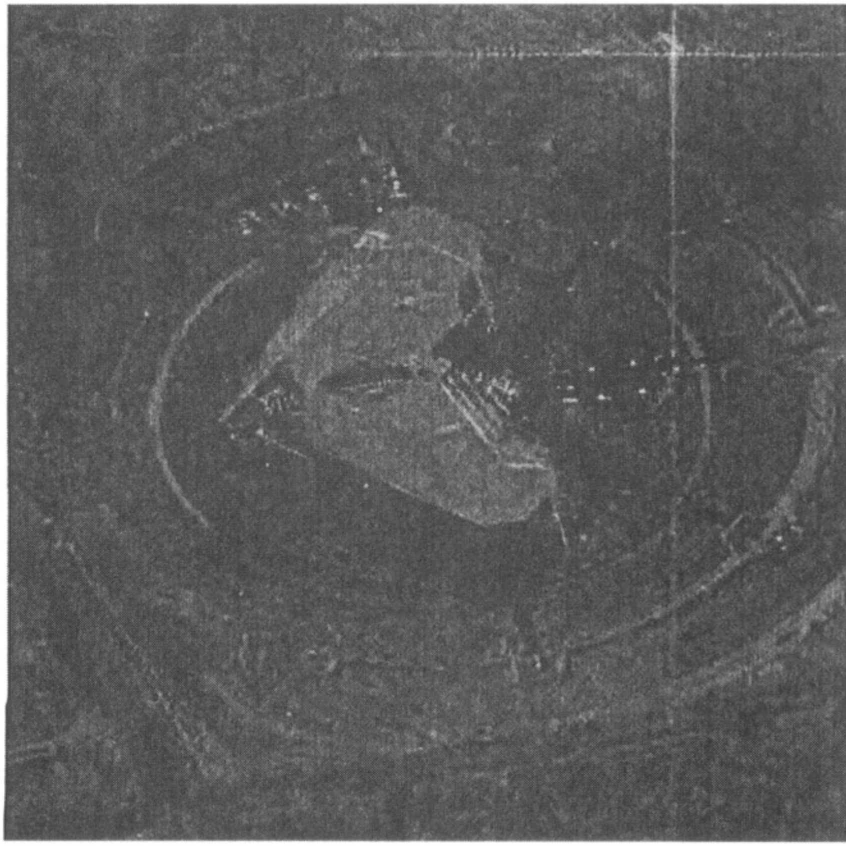

FIGURE 7. Decompressed Image at the Compression Ratio 400:1

available. We would like to take this opportunity to thank him. A special thanks goes to Alistair Moffat for valuable help. Part of the work for this paper was done during our visit at the Center for Medical Visualization and Diagnostic Systems (MeVis), University of Bremen. We would like to thank Heinz-Otto Peitgen, Carl Evertsz, Hartmut Juergens, and all others at Mevis, for their hospitality. This work was supported in part by ARPA and Texas ATP. Part of this work was presented at the IEEE Data Compression Conference, Snowbird, Utah, April 1996 [TW96] and the SPIE's 10th Annual International Symposium on Aerospace/Defense Sensing, Simulation, and Controls, Orlando, Florida, April 1996 [TGW+96].

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