Pseudocode for Mixing and Compression

Steve Carpenter, 2016-08-25

Start with a vector of annually deposited layers. There are T annually deposited layers indexed by subscript t which runs from 1 to T.

Assume that each layer contains the annually deposited mass of total organic plus inorganic material per unit of lake bottom.

Assume this deposited mass is constant. This assumption can be removed if we have data on annually deposited mass of total organic plus inorganic material per unit area of lake bottom.

Assume that we are interested in the concentration of a tracer per unit mass of this deposited material. The tracer could for example be a pigment, isotope of an element, frustules of a diatom species, carapaces of a zooplankton species, etc.

Mixing

If the core is varved (no mixing) then skip this step.

Assume that mixing occurs over a window of length -2σ to +2σ centered at the midpoint of a given layer t. Here σ is the standard deviation of mixing in units of annually deposited layers. For example σ could be the spread in years of the Cs-137 peak in the lake.

Compute a vector of weights using a Normal kernel. For example the weights could be areas of the unit normal distribution on the intervals bounded by

$$t + \left\{ -2\sigma, -n, -(n-1), \ldots, -1, -\frac{1}{2}, \frac{1}{2}, 1, \ldots, (n-1), n, 2\sigma \right\}$$

where n is the largest integer less than 2σ.

For more accuracy we could expand the weights to a wider range or insert more intermediate values but it will not have much effect.

Relativize weights so that they sum to one, to deal with the fact that the normal integral between ±2σ is less than 1.

Multiply the weights by the tracer concentrations in the annually-deposited layers between t–(n+1) and t+(n+1) of “target” layer, where the target layer is indexed zero.

Repeat for all possible target layers t.

Compression

If there is no compression, skip this step.

Fits to data (see Compression_Model+Fits_2016-08-25.doc) show that compression fits the following model. Let y = years per centimeter of core and x = age at the midpoint of a core section between 2 age measurements. Then y and x are linearly related:

$$y = a_0 + a_1 x \qquad [1]$$

where a_0 and a_1 are fitted coefficients that are greater than or equal to zero. Therefore centimeters per year of core (necessary for modeling compression) follows

$$\frac{1}{y} = \frac{1}{a_0 + a_1 x} \qquad [2]$$

Note that if a_1 is zero then there is no compression, and the following calculations can be skipped.

After the mixing calculation (previous section) is completed, we have a vector of T annual layers, with index t running from 0 to T.

Now we build a vector of S slices of the core, indexed by s from 1 to S. Here I assumed that slices are 1 cm thick, however it would be easy to alter the model for slices of other thicknesses.

If we are working with an actual observed core, then S is the length of the core.

If we are working with model simulations, then we assume reasonable values of a_0 and a_1 (see fits to data in Compression_Model+Fits_2016-08-25.doc) and integrate [2] over time to get S:

$$S = \int_0^T \frac{dt}{a_1 t + a_0} = \frac{1}{a_1}\left[\ln\left(a_1 T + a_0\right) - \ln a_0\right] \qquad [3]$$

Next we compute the average tracer concentration in each slice, working from the top of the core downward. For any 1 cm slice starting at t_0 the following identity is true (the left side is 1 cm):

$$1 = \int_{t_0}^{t_1} \frac{dt}{a_0 + a_1 t} \qquad [4]$$

Solve [4] for the time at the bottom of the slice t_1:

$$1=\frac{1}{a_1}\left[\ln(a_1t_1+a_0)-\ln(a_1t_0+a_0)\right]=\frac{1}{a_1}\ln\left[\frac{a_1t_1+a_0}{a_1t_0+a_0}\right]$$

$$e^{a_1}=\frac{a_1t_1+a_0}{a_1t_0+a_0} \qquad [5a,b,c]$$

$$t_1=\frac{1}{a_1}\left[\left(a_1t_0+a_0\right)e^{a_1}-a_0\right]$$

In summary, starting with the top of the core ($t_0 = 0$) solve for the time at the bottom of the first slice, t_1, using [5c]. Average the tracer concentrations over the annual layers between t_0 and t_1 to get the tracer concentration in the first slice. Fractions of layers are weighted by the fraction in computing the average tracer concentration.

Then, t_1 from the first slice becomes t_0 for the second slice. Compute t_1 for the second slice using [5c], and compute the average tracer concentration in the second slice.

Repeat for deeper slices until the core is completed.