Lake Phosphorus Model For the WSC Modeling Suite

Steve Carpenter, 2017-06-07

This handout explains the equations and parameter estimation process for the lake phosphorus model used to compute water quality in the WSC Modeling Suite (Motew et al. 2017). To limit confusion with other Yahara lakes water quality models, the model used for WSC will be called the "Stable 4 Lakes Model" or S4LM below.

The S4LM is similar to the mass-balance model of Carpenter and Lathrop (2014). The main differences are (1) The S4LM includes separate export terms for P standing stock and recently loaded P. Statistical model discrimination using AIC shows that the S4LM formulation fits data significantly better than the usual formulation with a single export term. (2) The S4LM is constructed to be dynamically stable for all four lakes under all possible inputs, whereas the model of Carpenter and Lathrop (2014) can be unstable for some sets of inputs. Dynamical stability over all possible input conditions was necessary to handle the extremely diverse set of scenario inputs used by WSC (Booth et al. 2016).

Over a short time interval dt, P mass dynamics for a given lake follow

$$P_{t+1} = P_t + \left[(1-w)L_t - sP_t - hP_t\right]dt \qquad [1]$$

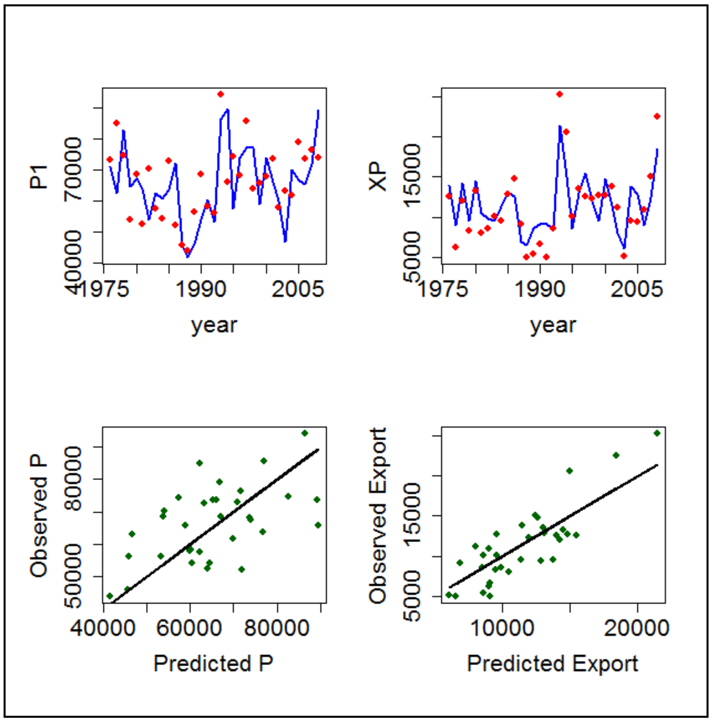
P is phosphorus mass in the lake, L is loading rate (mass/time), s is a sedimentation coefficient (1/t), h is an export coefficient for P mass (1/t), and w is an export fraction (dimensionless). The instantaneous rate of export from the lake is $X_t = \left(wL_t + hP_t\right)dt$ .Subscript t denotes the time step and dt is the length of a time step that is small relative to one year. To calculate one-year ahead predictions, equation [1] is solved numerically from the start to the end of the year.

For each lake, the model was fit numerically to observed annual time series of P_t, L_t, and X_t by maximizing the likelihood of one-year ahead projections. This process estimates values of s, h, w, σ_P, and σ_X where σ_P and σ_X are model errors for one-year projections of P and X, respectively. The normal distribution was used for likelihood calculations. Annual time series began on 1 November of year t-1 and ended 31 October of year t. These dates correspond with autumn turnover, when the lake is well mixed and estimates of P mass are relatively accurate. The time step dt was 1/30th of a year.

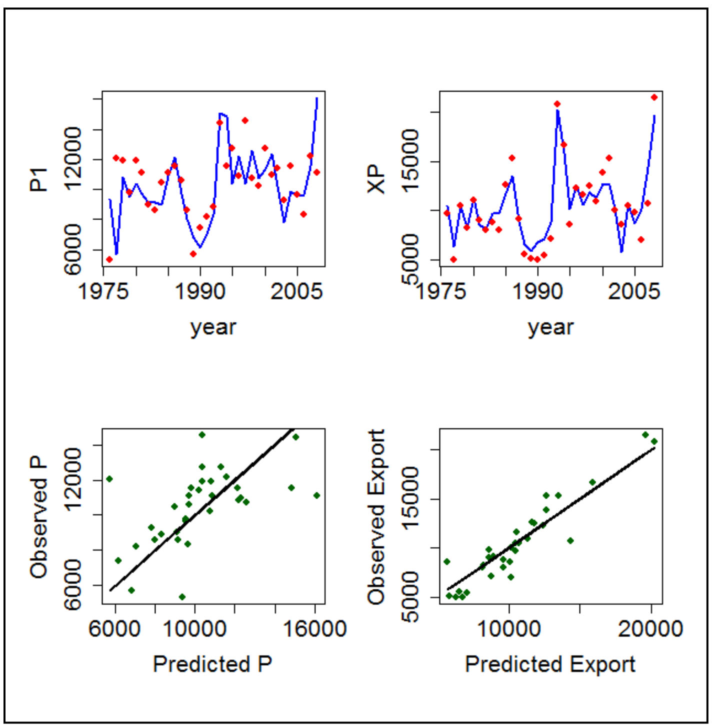
Parameter estimates with correlation coefficients for one-step predictions of P and X:

Lake	s	h	w	σ_P	σ_X	r for P	r for X
Mendota	0.342	0.0688	0.220	11,157	2565	0.547	0.840
Monona	0.893	0.0207	0.538	2119	1468	0.550	0.937
Waubesa	0.216	0.466	0.880	1109	2234	0.049	0.922
Kegonsa	0.557	0.76×10^{-5}	0.892	1911	3046	-0.023	0.910

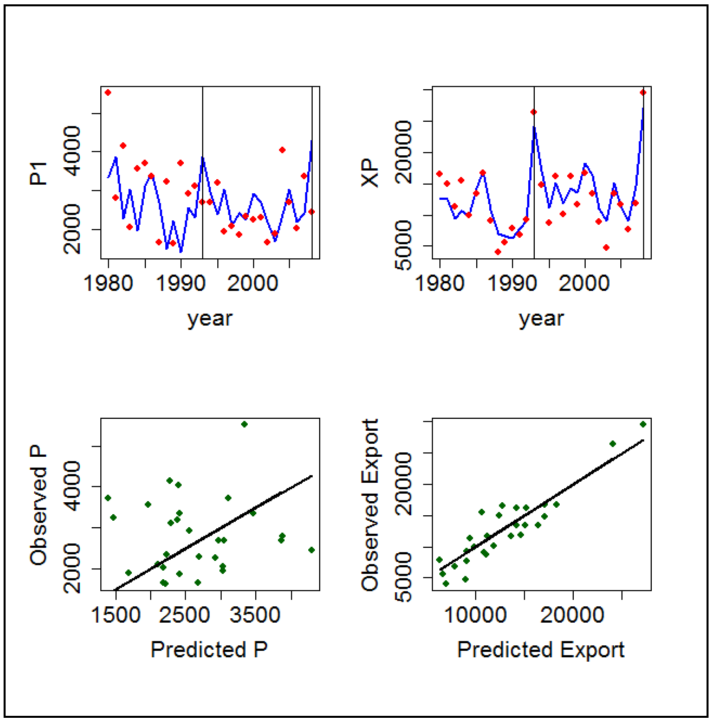
One-year-ahead predictions of exports are more accurate than those of P mass, and are relatively accurate for all lakes. Predictions of P mass are reasonably accurate for Mendota and Monona, but not for Waubesa and Kegonsa. Some details of the fits for each lake are presented below.



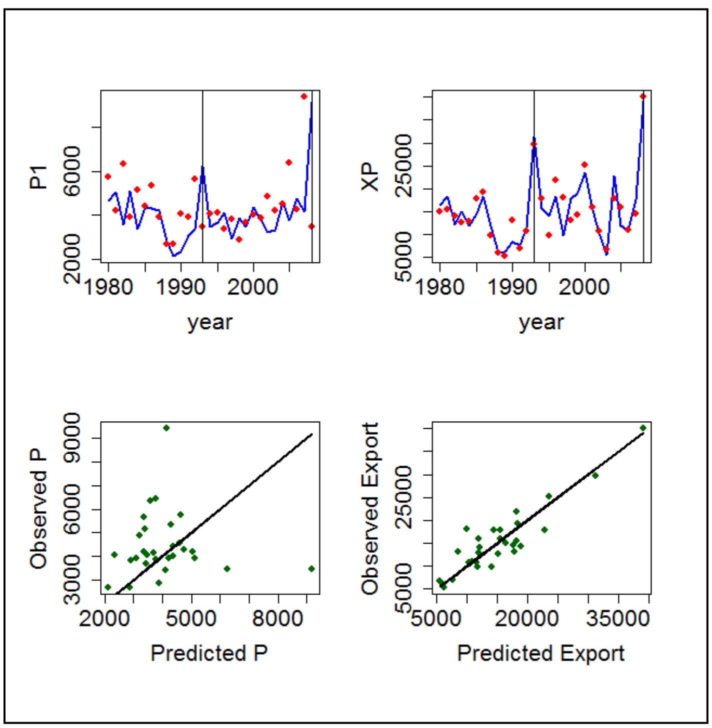
Lake Mendota: Upper row is one-year-ahead predictions (blue lines) and observations (red dots) of P mass (left) and export (right) versus year. Lower row shows one-year-ahead predicted versus observed for P mass (left) and export (right).



Lake Monona: Upper row is one-year-ahead predictions (blue lines) and observations (red dots) of P mass (left) and export (right) versus year. Lower row shows one-year-ahead predicted versus observed for P mass (left) and export (right).



Lake Waubesa: Upper row is one-year-ahead predictions (blue lines) and observations (red dots) of P mass (left) and export (right) versus year. Vertical lines show flood years of 1993 and 2008.
Lower row shows one-year-ahead predicted versus observed for P mass (left) and export (right).

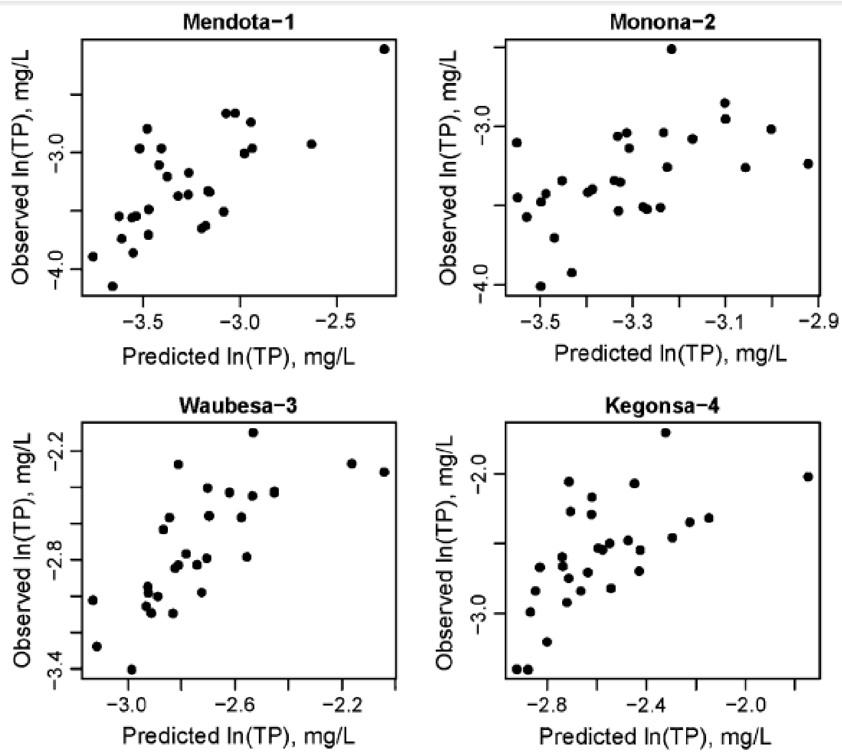


Lake Kegonsa: Upper row is one-year-ahead predictions (blue lines) and observations (red dots) of P mass (left) and export (right) versus year.Vertical lines show flood years of 1993 and 2008.
Lower row shows one-year-ahead predicted versus observed for P mass (left) and export (right).

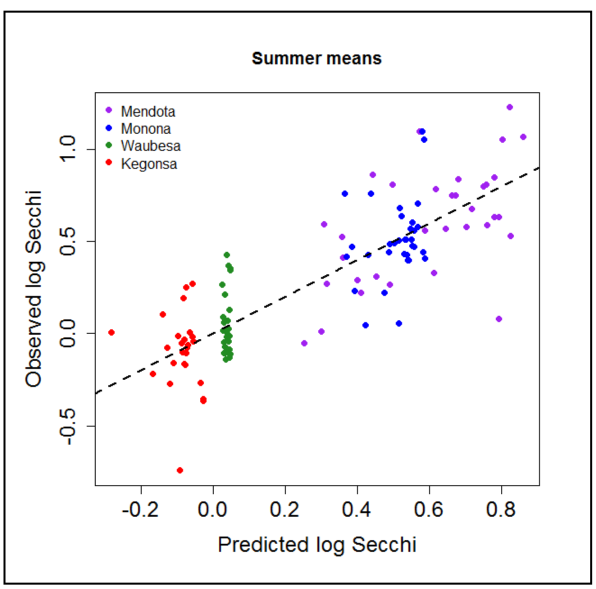
Model fits are successful in predicting large exports in the flood years of 1993 and 2008, but overshoot predictions of P mass in the flood years.

Summer epilimnetic [TP] in year t was predicted from 31 October lake P mass in year t using regressions (Carpenter and Lathrop 2014). These regressions appear optimal by the usual diagnostics. Nonetheless the prediction errors are not trivial, as shown below.

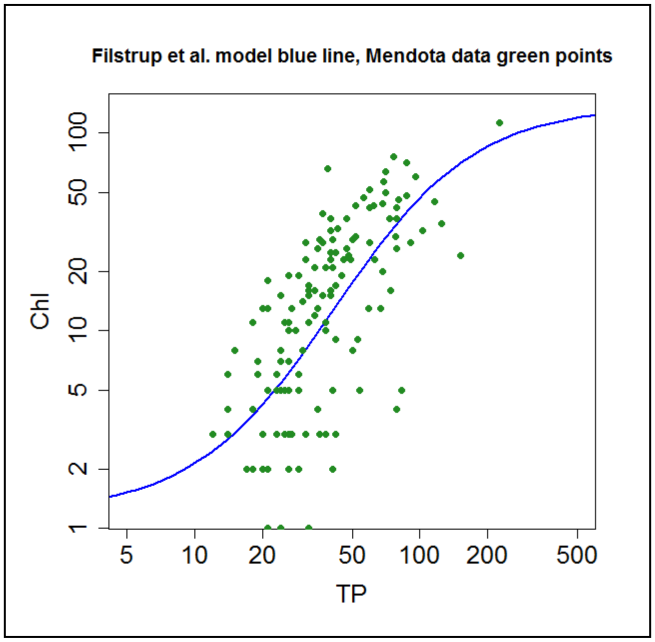
Fig. 3 Predicted log of summer TP versus observed log of summer TP (mg/L) for the four lakes. The number following each lake name indicates the lake's position in the chain. All regressions are significant with $p < 0.001$. Correlations of predictions and observations: Mendota 0.73, Monona 0.53, Waubesa 0.71, Kegonsa 0.64



Predictions versus observations for the best-fitting regression for summer Secchi depth are shown below. In log units, the s.d. of residuals is about 0.2, meaning that for a Secchi depth of 1.0 m the +/- s.d. range is 0.82 - 1.22 m. For a Secchi depth of 2.0 m the range is 1.64 - 2.44 m.



Chlorophyll data for the Madison Lakes are problematic. Multiple methods have been used, the methods do not inter-calibrate well, and in a few years the data do not make sense. Nonetheless chlorophyll is an important water quality indicator that we wanted to address. WSC used the regional multi-lake model of Filstrup et al (2014) to calculate chlorophyll from summer [TP]. The model for the Rock River drainage was used for WSC. For Lake Mendota we have enough high-quality chlorophyll data to test the model (below).



The mean error (bias) is 4.1 mg m^{-3} and the standard deviation of residuals around the blue curve in the figure is 13.4 mg m^{-3}. These seem acceptable considering that the range of observed chlorophyll is about 1-100 mg m^{-3}.

There is a mathematical ambiguity in the Filstrup paper. I discussed this with the authors and they agree with me. I saved the emails and a detailed explanation of the correction. The corrected version of the Filstrup model was used for all modeling in the Yahara2070 project.

References

Booth, E.G., J. Qiu, S.R. Carpenter, J. Schatz, X. Chen, K.J. Kucharik, S.P. Loheide, M.M. Motew, J.M. Siefert, and M.G. Turner. 2016. From qualitative to quantitative environmental scenarios: Translating storylines into biophysical modeling inputs at the watershed scale. Environmental Modeling and Software 85: 80-97. http://dx.doi.org/10.1016/j.envsoft.2016.08.008

Carpenter, S.R. and R.C. Lathrop. 2014. Phosphorus loading, transport and concentrations in a lake chain: a probabilistic model to compare management options. Aquatic Sciences 76: 145-154. DOI 10.1007/s00027-013-0324-5

Filstrup, C. T., T. Wagner, P. A. Soranno, E. H. Stanley, C. A. Stow, K. E. Webster, and J. A. Downing. 2014. Regional variability among nonlinear chlorophyll—phosphorus relationships in lakes. Limnology and Oceanography **59**:1691-1703.

Motew, M., Chen, X, Booth, E.G., Carpenter, S.R. Pinkas, P. Zipper, S., Loheide, S., Donner, S., Tsuruta, K., Vadas, P., Kucharik, C. 2017. The influence of legacy P on lake water quality in a Midwestern agricultural watershed. Ecosystems 20: in press. DOI: 10.1007/s10021-017-0125-0