

# *Performance Analysis of Fast Convolution Based FBMC-OQAM System*

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**Abstract**—Filter Bank based Multicarrier (FBMC) system is a multicarrier scheme that has recently gained attention due to its better spectral properties compared to OFDM systems. FBMC system is one among the strong contenders for 5G implementations. While better designs of FBMC systems are evolving with time, it is imperative that the hardware complexity is not overlooked. It is important that filter bank designs with reduced computational resources are sought. Reducing computational complexity offers advantages like less power consumption, faster computations and reduced runtime resources. These advantages have more significance in 5G where ultra high speed and ultra low latency are the highlights. Filter bank designs using fast convolution employ less arithmetic resources compared to polyphase filter bank implementations. The fast convolution filter design makes use of only filter's frequency domain sample values for filtering since the filtering is restricted to frequency domain. So the conventional polyphase implementation of filter is completely done away with. In this paper, we present FBMC/OQAM system using Fast Convolution and the performance of the system is evaluated in a communication channel. The system is compared with conventional polyphase FBMC/OQAM system and it is found that fast convolution based system outperforms polyphase implementation of FBMC system in computational complexity and BER performance.

**Index Terms**—fast convolution, multicarrier communications, FBMC/OQAM, OFDM, cyclic prefix, spectral leakage

## I. INTRODUCTION

Multicarrier modulation is a technique for sending parallel streams of data through a channel at bit rates lower than the original data rate. This helps in mitigating fading effects introduced by the channel.

OFDM is a much popular multicarrier modulation technique for reasons that can be attributed to its reduced computation complexity, strong resistance to ISI and ICI by use of cyclic prefix and simple channel equalization to name a

few. These factors have played a crucial role in making OFDM a favourable choice for most of the wide communication applications like wireless local area standards, digital audio and video broadcasting standards, DSL internet access and 4G mobile systems. The whole idea of multicarrier modulation occurs in the IFFT and FFT operations in OFDM. Some of the major disadvantages of OFDM include the limited spectral containment and the requirement for a cyclic prefix. The large sidelobes of prototype filter in OFDM causes high interference with adjacent channels. The cyclic prefix used in OFDM must be at least the length of channel delay and helps to resist ISI and ICI among the OFDM symbols but leads to inefficiency due to redundancy of the transmitted data [1-5].

FBMC is a multicarrier system that has recently gained attention due to its better spectral properties compared to OFDM systems. FBMC system is one among the strong contenders for 5G implementations. FBMC works in transmultiplexer configuration with a set of synthesis filters and analysis filters along with FFT/IFFT block. The filters enable pulse shaping and are designed to meet the desired spectral properties. Orthogonality among carriers is achieved with the use of appropriate modulation. Cyclic prefix is not needed in FBMC data transmission [6-10]. However, the complexity shoots up while implementing the FBMC system due to the use of filters in the system.

There are a few works that have presented methods for complexity reduction of a filter. In [12] the authors suggest a Frequency Response Masking technique (FRM) for design of sharp transition band filters. In this technique sharp filters are derived from the filters with wider transition band by upsampling the impulse response by inserting zeros. This helps to create filters with very sparse coefficients leading to lower complexity. But the FRM techniques have the disadvantage that the arbitrary passband location of the

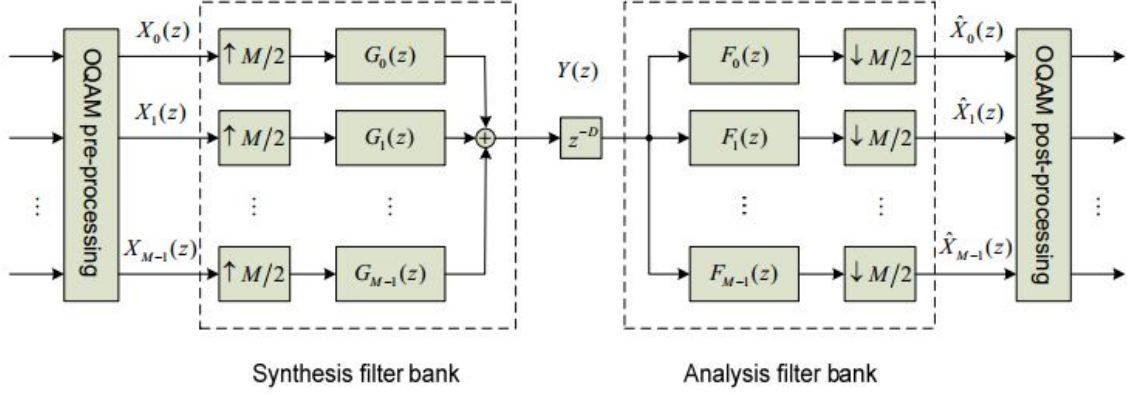


Fig 1. An FBMC-OQAM System

designed filters is not absolutely controllable [13].

In [14] the authors present a method to reduce the computations during synthesis of FBMC uplink waveforms in practical scenarios. The method relies on time domain interpolation to construct synthesis filter banks of size smaller than the analysis filter bank.

Another method with the potential to reduce computational complexity in a filter bank is the design using fast convolution [15-17,21]. This method acquires its importance when considering that the fast convolution becomes faster than time domain convolution when the number of FFT points is greater than 60 [18]. The fast convolution processing of filter design makes use of filter's frequency response samples; or rather FFT samples to accomplish the filtering. The sampling factor is controlled by selecting necessary input and output blocks during fast convolution.

The generation of FBMC/OQAM and FMT waveforms using Fast Convolution is discussed in [17] and the effectiveness of the FC filter design in terms of reduced spectral leakage, increased flexibility of filter bank and savings in computational complexity in FBMC/OQAM filter bank system is established. But the performance of the system in communication channels was not evaluated or studied by any of the authors.

In this paper, FBMC/OQAM system using Fast Convolution is presented. The BER performance of the system in an AWGN channel and the PAPR of the system is evaluated and is compared with that of conventional polyphase FBMC/OQAM system [19]. It is seen that BER performance and PAPR is better in the case of fast convolution based system

The paper is organized as follows: An FBMC/OQAM system is described in section II. Section III discusses filtering using fast convolution. In Section IV, FBMC/OQAM implementation using Fast convolution is discussed. Simulation results are presented in Section V and the paper is concluded in Section VI.

## II. FBMC-OQAM SYSTEM

FBMC works on the principle of transmultiplexer. The spectrum of the signal at each channel input is compressed and frequency translated to the corresponding channel center frequency. All the channel outputs are combined to form the transmitted signal effectively making it a frequency division multiplexed signal. At the receiver side the frequency division multiplexing is undone after filtering. The OQAM modulation brings the required orthogonality between adjacent channels.

The input signal undergoes OQAM preprocessing before entering the FBMC stage. The sampling factor is doubled as the signal is converted into OQAM symbols since each complex valued QAM symbol  $c_{k,n}$  is separated into its real and imaginary parts according to relation,

$$d_{k,n} = \begin{cases} \text{Re}(c_{k,n}), k \text{ even} \\ \text{Im}(c_{k,n}), k \text{ odd} \end{cases} \quad (1a)$$

$$d_{k,2n+1} = \begin{cases} \text{Im}(c_{k,n}), k \text{ even} \\ \text{Re}(c_{k,n}), k \text{ odd} \end{cases} \quad (1b)$$

These real valued symbols are then multiplied by  $\theta_{k,n} = j^{k+n}$  before proceeding to filtering [20]. OQAM post processing is the inverse operation that recovers the QAM signal by extracting the real part of each OQAM symbol processed by the analysis filter bank and combining two symbols at a time. OQAM post processing is the final stage of FBMC-OQAM system. The synthesis filters  $h_k(m)$  and analysis filters  $g_k(m)$ , respectively, are derived from the prototype filter  $h(m)$  of length N [19] as

$$h_k(m) = h(m) \exp(j \frac{2\pi k}{M} (m - \frac{N-1}{2})) \quad (2)$$

$$g_k(m) = h_k^*(N-1-m) \quad (3)$$

where  $m = 0, 1, \dots, N-1$ .

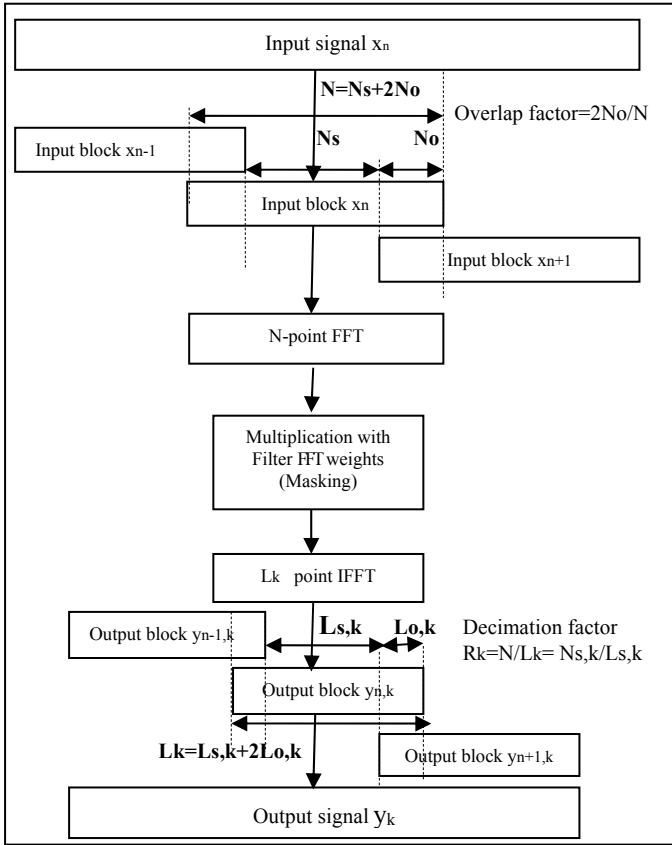


Fig 2. FC-FB channel of an analysis filter bank

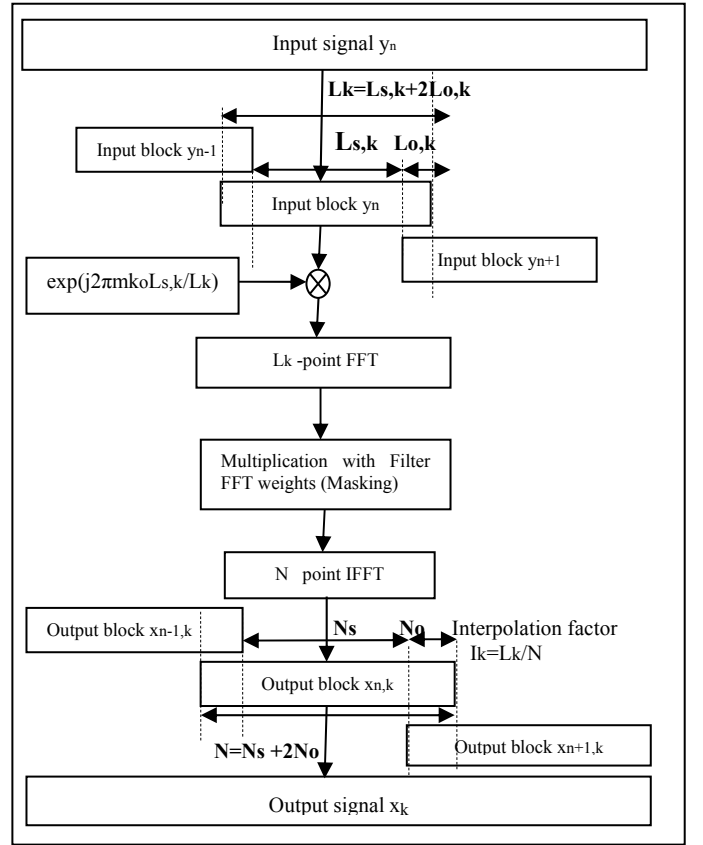


Fig 3. FC-FB channel of a synthesis filter bank

### III. FAST CONVOLUTION

Fast convolution processing is applied in case of linear convolution involving very long sequences. The incoming long sequence is split into several smaller length blocks and is convolved with the second sequence. But fast convolution process is in fact a circular convolution and not the linear convolution. Suppose the two sequences on which convolution is performed are  $x[n]$  and  $h[n]$  of length  $l_x$  and  $l_h$  respectively. Fast Convolution of the two sequences can be explained with the following equation [16]

$$h[n] * x[n] = \text{IFFT}(\text{FFT}(h[n]) * \text{FFT}(x[n])) \quad (4)$$

A linear convolution between  $x[n]$  and  $h[n]$  produces an output sequence of length  $l_x + l_h - 1$ . To make circular convolution equivalent to linear convolution the length of each sequence is made equal to  $l_x + l_h - 1$ .

Fast Convolution is done by using either overlap save processing or overlap add processing. Overlap-save processing method of fast convolution is preferred mostly for Fast Convolution since it is being faster compared to overlap add method due to omitting of any additions between consecutive blocks [21]. In overlap save processing, the incoming

sequence  $x[n]$  is split into smaller blocks and the last  $l_h - 1$  samples of previous block is appended to the beginning of each new block to make the total length  $l_x + l_h - 1$ . At the output, the first  $l_h - 1$  samples of each block are discarded and concatenated to the previous block to produce the final convolution output.

### IV. FAST CONVOLUTION BASED FBMC

A filter that makes use of fast convolution processing uses the frequency domain FFT weights of the filter instead of time domain impulse response for filtering. The FFT of the incoming signal block is multiplied with filter FFT weights and upon performing the IFFT, the time domain output signal is obtained [15,16,21]. The multirate structure of a fast convolution based filter is implemented by selecting appropriate length of input and output blocks [15].

Figure 2 gives detailed view of a FC-FB channel in an analysis filter bank [22]. The figure shows clearly the partitioning of a single block of input and output signals into overlapping and non overlapping portions. As discussed, each input block of length  $N$  has a non overlapping part and an overlapping part. The overlapping portion is appended on both sides of the non overlapping portion of the input block to maintain the symmetry. The input block  $N$  point FFT is masked with the  $L$  point FFT domain weights of the filter.

Performing IFFT operation on the result gives the output signal decimated by factor  $R=N/L=N_s/L_s,k$ . The first  $L_o$  and last  $L_o$  samples from the output block are discarded while the remaining  $L_s$  samples are retained and concatenated to the previous blocks of output samples.

The synthesis filter bank based on fast convolution is depicted in Fig 3 [23]. Each input block of  $k^{\text{th}}$  channel is multiplied with a signal  $c_{ph}(m) = \exp(j2\pi m k_o L_{s,k} / L_k)$ , where  $m$  is the block index and  $k_o$  is the center frequency of the  $k^{\text{th}}$  bandpass filter. This is done to maintain phase continuity among consecutive blocks [16].

The FFT domain weights of FBMC/OQAM waveform of adjacent channels are arranged as depicted in Fig 4. Since OQAM preprocessing causes an upsampling factor of 2, the adjacent channels overlap by half the channel bandwidth to accommodate all the subchannels together [17].

The FFT weights consist of only passband and transition band bins. The stopband bins are therefore considered zero. A filter with such a frequency response will be quite long, longer than the input block overlap length. Therefore the circular convolution through FC processing will not be equivalent to linear convolution and consequently the FC based filtering introduces a distortion called cyclic distortion. The consequence of cyclic distortion is that the impulse response becomes time varying. Thus the FC filter can be characterized as a linearly time varying system (LPTV) [16].

To minimize the effects of cyclic distortion, the filter's FFT weights are designed by an optimization problem for minimization of interference in stopband region. Assuming that we have the  $L$  frequency domain weights  $w_k$ , find the impulse response  $h(\eta)$  using  $N$  point IFFT of weight  $w_k$ . Now it is possible to model the time varying impulse response  $\tilde{h}_n(\eta)$  as explained in [16]. Calculate the frequency response  $\tilde{H}_n(\omega)$  for each  $n$ , using  $N$  point FFT of  $\tilde{h}_n(\eta)$ . The total interference for stopband region ( $\Omega_s$ ) is calculated as

$$I_s(\omega_i) = \sum_n \left| \tilde{H}_n(\omega_i) \right|^2 / L_s \quad \text{for } \omega_i \in \Omega_s \quad [16].$$

The computational complexity of a 128 channel FBMC/OQAM system was studied in [17]. It was observed that in case of 72 active subcarriers complexity of FC based FBMC/OQAM with design parameters  $N=1024$ ,  $L=16$  and  $L_s=10$  is 40 multiplications per detected symbol as against 43 and 55 multiplications involved in detection of a symbol at transmitter and receiver side of a polyphase FBMC/OQAM system respectively. The improvement in computational complexity of FC-based FBMC/OQAM over polyphase FBMC/OQAM system becomes more pronounced as the number of active subcarriers become less. The overlapping factor also plays an important role in controlling the computational complexity since it is seen in [17] that as the overlap factor is increased to 1/2 by selecting  $L_s=8$  the number of multiplications per detected symbol increases to 45 while it

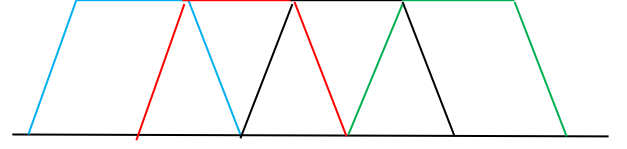


Fig 4. FBMC/OQAM multiplexing of 4 adjacent channels

was 40 multiplications/symbol in case of  $L_s=10$  with an overlap factor of 3/8.

## V. SIMULATION RESULTS

The simulations were carried out in MATLAB. A 64 channel FBMC-OQAM system is considered for implementation. A sampling factor of  $R=32$  was satisfied by selecting  $N=2048$  and  $L=64$ . Therefore filter FFT weights have to be designed such that cyclic distortion is minimized. Optimization was carried out with error due to interference as the cost function. Least squares algorithm with bounded constraints was used for optimization. The FFT weights were designed to follow a Root Raised Cosine (RRC) spectrum. So the weights were constrained to be within a lower bound of 0 and upper bound of 1. The obtained FFT weights are for low pass filter and are shown in Fig 5. The remaining subband filter weights were obtained by simply shifting the FFT weights of the low pass filter to be around the corresponding band pass center frequency. In this way the analysis filters and synthesis filters were designed. The FC processing is done for an overlap factor of 1/4. So, total overlapping length in each block will be  $1/4^{\text{th}}$  of the total length of the block.

BER of the implemented FBMC-OQAM system in an AWGN channel was evaluated and compared with that of polyphase FBMC-OQAM system in Fig 6. The FC based FBMC-OQAM exhibited improvement in terms of bit error rate when compared with the bit error rate of polyphase FBMC-OQAM.

Fig 7 shows CCDF vs PAPR plot of FC-FBMC/OQAM and polyphase FBMC/OQAM. It was found that the PAPR is improved significantly in case of FC based FBMC-OQAM system. A PAPR of upto only 12 db was observed in case of FC based FBMC-OQAM while it extends to more than 16 db in case of polyphase FBMC/OQAM system.

## VI. CONCLUSION

An FBMC/OQAM system is implemented using fast convolution processing structure. Savings in arithmetic complexity becomes more pronounced in cases of large number of subchannels where the number of active users is less. Such situations always occur in practical applications. The BER performance of FC-FBMC system is compared with that of conventional polyphase filter bank based system. comparison turns favourable towards the Fast Convolution based FBMC/OQAM system. So the system can be relied upon for better performance. The PAPR comparison also shows that the fast convolution based filter bank is a potential competitor to the conventional polyphase filter banks.

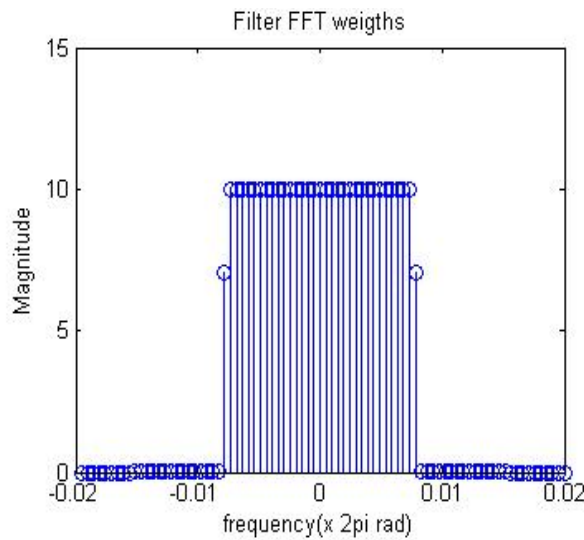


Fig 5 Filter FFT weights

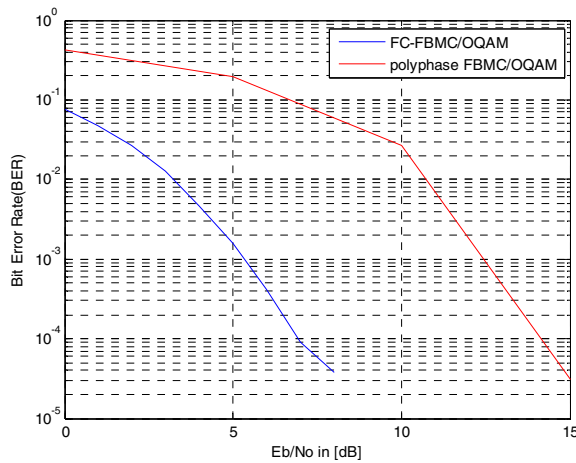


Fig 6. BER performance comparison between FC-FBMC/OQAM and conventional polyphase FBMC/OQAM

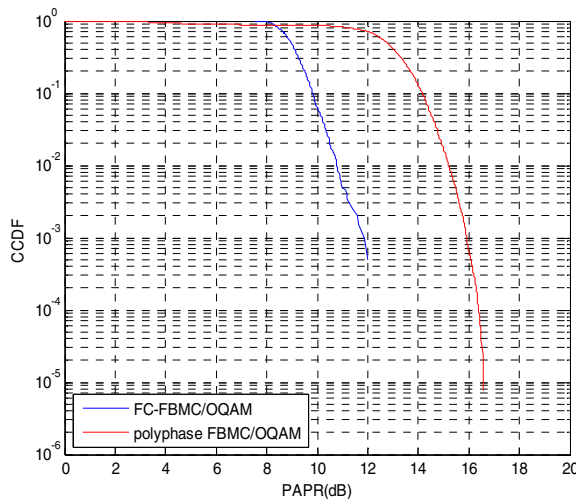


Fig 7. PAPR comparison between FC-FBMC/OQAM and conventional polyphase FBMC/OQAM

## REFERENCES

- [1] Wu, Yiyang, and William Y. Zou. "Orthogonal frequency division multiplexing: a multi-carrier modulation scheme." *IEEE Trans. Consumer Electronics*, vol.41, pp. 392-399, Aug. 1995.
- [2] Singh, Parul, and O. P. Sahu. "An Overview of ICI Self Cancellation Techniques in OFDM Systems." *IEEE Int. Conf. Computational Intelligence & Communication Technology (CICCT)*, pp. 299-302, Feb. 2015.
- [3] Singh, Shifali, Shobhana Shrivastava, and Vinay Rajpoot. "Advanced OFDM Systems for Terrestrial Multimedia Links."
- [4] Lim, Dae-Woon, Seok-Joong Heo, and Jong-Seon No. "An overview of peak-to-average power ratio reduction schemes for OFDM signals." *Journal of Communications and Networks*, pp.229-239, June 2009.
- [5] Bohge, Mathias, et al. "Dynamic resource allocation in OFDM systems: an overview of cross-layer optimization principles and techniques." *IEEE Network*, vol.21, pp. 53-59, Jan-Feb 2007.
- [6] Sriyananda, M. G. S., and Nandana Rajatheva. "Analysis of self interference in a basic FBMC system." *IEEE 78th Vehicular Technology Conf*, 2013.
- [7] Qu, Daiming, Shixian Lu, and Tao Jiang. "Multi-block joint optimization for the peak-to-average power ratio reduction of FBMC-OQAM signals." *IEEE Trans. Signal Processing*, vol 61, pp.1605-1613, 2013.
- [8] Farhang-Boroujeny, Behrouz. "OFDM versus filter bank multicarrier." *IEEE Signal Processing Magazine*, vol 28, pp.92-112, 2011.
- [9] Premnath, S., et al. "Beyond OFDM: best-effort dynamic spectrum access using filterbank multicarrier." *IEEE/ACM Trans. Networking*, vol 21, pp. 869-882, 2013.
- [10] Saeedi-Sourck, Hamid, et al. "Complexity and performance comparison of filter bank multicarrier and OFDM in uplink of multicarrier multiple access networks." *IEEE Trans. Signal Processing*, vol 59, 1907-1912, 2011.
- [11] M. Payaró, A. Pascual-Iserte, M. Nájara, "Performance Comparison between FBMC and OFDM in MIMO systems under Channel Uncertainty, *European Union Wireless Conf.*, pp.1023-1030, 2010.
- [12] Lim, Yong Ching. "Frequency-response masking approach for the synthesis of sharp linear phase digital filters." *IEEE Trans. Circuits and Systems*, vol.33, pp. 357-364.
- [13] A. Ambede, K. G. Smitha and A. P. Vinod, "A modified coefficient decimation method to realize low complexity FIR filters with enhanced frequency response flexibility and passband resolution", *Proc. 35th Int. Conf. TSP*, pp. 658-661.
- [14] T. Ihalainen; A. Viholainen; T. H. Stitz; M. Renfors "Generation of Filter Bank-Based Multicarrier Waveform Using Partial Synthesis and Time Domain Interpolation" *IEEE Trans. Circuits and Systems I*, vol. 57, no.7, 2010, pp. 1767 – 1778.
- [15] M. Renfors and F. Harris, "Highly adjustable multirate digital filters based on fast convolution," in *Proc. Eur. Conf. Circuit Theory Design*, Linköping, Sweden, Aug. 29–31, 2011, pp. 9–12.
- [16] Markku Renfors, Juha Yli-Kaakinen and Fredric J. Harris, "Analysis and Design of Efficient and Flexible Fast-Convolution Based Multirate Filter Banks", *IEEE Trans. Signal Processing*, vol.62, No.15, August 1 2014.
- [17] K. Shao; J. Alhava; J. Yli-Kaakinen; M. Renfors, "Fast Convolution Implementation of Filter Bank Multicarrier Waveform Processing", *IEEE Int. Symp. Circuits and Systems*, 2015, pp. 978-981.

- [18] Steven W. Smith, *The Scientist and Engineer's Guide to Digital Signal Processing*, 2<sup>nd</sup> ed, San Diego, USA: California Technical Publishing.
- [19] Varghese, Nisha, Job Chunkath, and V. S. Sheeba. "Peak-to-Average Power Ratio Reduction in FBMC-OQAM System." *Fourth IEEE Conf. Advances in Computing and Communications (ICACC)*, 2014.
- [20] T. H. Stitz, T. Ihalainen, A. Viholainen, and M. Renfors, "Pilot Based Synchronoization and Equalization in Filter Bank Multicarrier Communicatios, *EURASIP J. Adv. Signal Process.*, 2010, Article ID 741429.
- [21] M. Borgerding, "Turning overlap-save into a multiband mixing, downsampling filter bank," *IEEE Signal Process. Mag.*, pp. 158–162, Mar. 2006.
- [22] Markku Renfors and Juha Yli-Kaakinen "Channel Equalization in Fast-Convolution Filter Bank Based Receivers for Professional Mobile Radio" *Proc. European Wireless Conference* , 2014, pp. 1-5.
- [23] Juha Yli-Kaakinen and Markku Renfors, "Optimization of Flexible Filter Banks based on Fast-Convolution", *Proc. Int. Conf. Acoustics, Speech, Signal Process.*, Florence, Italy, May 4-9 2014, pp. 8342-8345.
- [24] C. Zhang and Z. Wang, "A fast frequency domain filter bank realization algorithm," in *Proc. Int. Conf. Signal Process.*, Beijing, China, Aug. 21–25, 2000, vol. 1, pp. 130–132.
- [25] P. Vaidyanathan, *Multirate Systems and Filter Banks*. Englewood Cliffs, NJ, USA: Prentice-Hall, 1993..
- [26] J. G. Proakis, *Digital Communications*, 4th ed. New York, NY, USA: McGraw-Hill, 2000.