Compressive beamforming using greedy algorithms.

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***Abstract*—Direction of Arrival (DOA) estimation is a topic of great interest in many fields like electromagnetic, seismic/ geophysical and acoustic sensing. High resolution algorithms for DOA estimation have been widely used in applications like SONAR, RADAR and wireless communications to resolve closely-situated sources. However these algorithms are very sensitive to the signal to noise ratio (SNR), the number of snapshots and correlation between sources. The DOA estimation problem can be viewed as a sparse representation problem as the signals impinging on an array are intrinsically sparse in the spatial domain. Hence compressed sensing techniques can be applied in DOA estimation. This paper explores the formulation of the DOA estimation as a sparse representation problem and compares the performance of different compressive beamforming techniques. Compressive beamforming is done using l_1 minimization and greedy algorithms. It is shown that greedy algorithms are faster than l_1 minimization. An improvement to greedy algorithm is proposed in this paper so that prior information about sparsity is not needed. Resolution is further improved using multiband signals. Thus the study shows that compressive beamforming can give 2 to 3 degree resolution from single snapshot which is better than existing methods.**

***Index Terms*—Compressed sensing, l_1 minimization, Orthogonal Matching Pursuit, Compressive beamforming**

I. Introduction

Direction of arrival means the direction from which a propagating wave arrives at a sensor array [1]. Signal source localization using sensor arrays is called DOA estimation. DOA has many applications like SONAR, RADAR and wireless communications. For example in defense applications, direction of a missile attack is to be determined accurately [2]. Different types of traditional methods exist for DOA estimation. Most common and simplest method used for DOA estimation is Conventional beamforming. It requires only single snapshot. But it has poor resolution. Other methods used for DOA estimation are MUltiple SIgnal Classification (MUSIC) and Minimum Variance Distortion less Response (MVDR). They overcome the resolution limit but require more number of snapshots. Estimation easily gets affected by low SNR and coherent sources [3].

Compressive beamforming refers to the application of different compressed sensing techniques in DOA estimation. Compressive beamforming offers super resolution from a single snapshot. It is robust to noise. Compressive beamforming offers better resolution even under worst SNR conditions [3].

Compressed sensing is an emerging field of technology. Compressed sensing utilizes sparsity of a signal to represent it with less number of measurements [4,5]. By compressed sensing a signal sampled even below the Nyquist rate can be perfectly reconstructed provided sparsity and incoherence of the system is satisfied [6-8]. Compressed sensing actually solves an underdetermined system of equations based on the assumption that the unknown variable is sparse. This can be done using l_1 minimization. l_1 minimization gives accurate results [9, 10]. But it consumes more time. Greedy algorithms like Orthogonal Matching pursuit (OMP), Matching Pursuit (MP) etc. can also be used for solving compressed sensing problems. They give faster results.

There are a few works that have presented the concept of compressive beam forming. In [11] it is shown that number of receivers can be reduced significantly for a given number of sensors using compressive beamforming. They use l_1 minimization for sparse recovery. But it consumes more time. In [3] authors present compressive beamforming using reweighted l_1 minimization. But it is also time consuming and it gives a biased estimate due to coherence of sensing matrix.

In [12] authors use greedy algorithms. It gives faster results. But number of sources required is to be provided as prior information. So we need to estimate number of sources before estimating direction.

In [13, 14] authors introduces performance enhancement using multiband signals. Super resolution can be achieved even in the presence of coherence.

In this paper we are using greedy algorithms like Orthogonal matching pursuit and Matching Pursuit for compressive beamforming. A disadvantage of greedy algorithms is that they require number of sources as prior information. So we are proposing an improvement to OMP by changing stopping criteria of the algorithm. Performance enhancement while using multiband signals is also analyzed.

The paper is organized as follows: Basics of compressed sensing are discussed in section II. Section III provides basics of DOA estimation. Section IV presents compressive beamforming and Section V discusses an improvement to OMP. Compressive beamforming using multiband signals are presented in Section VI. Simulation results are discussed in section VII and the paper is concluded in Section VI.

II. COMPRESSED SENSING

Compressed sensing is the innovative filed of technology which can bring revolutionary changes in signal processing. By Shannon's theorem a signal sampled at Nyquist rate which is equal to twice the signal frequency can be perfectly recovered from its samples. But compressed sensing techniques prove that a signal can be recovered from fewer measurements provided that signal is sparse. A sparse signal is a signal with fewer numbers of nonzero elements. Fewer measurements mean more processing speed and less storage space. Incoherence is another property which should be satisfied by a CS system. Incoherence extends the duality between time and frequency. According to this, signals having a sparse representation in a domain must be spread out in the other domain.

CS solves an underdetermined system of equations. In mathematical terms our aim is to find solution to the problem

. $$\mathbf{y} = \mathbf{A}\mathbf{x}\,. \tag{1}$$

Where $\mathbf{x}$ is an $\boldsymbol{N}\mathbf{x}\boldsymbol{1}$ vector with sparsity $\mathbf{k}$. $\mathbf{A}$ is an $\boldsymbol{M}\mathbf{x}\boldsymbol{N}$ matrix called sensing matrix. Sensing matrix should satisfy RIP property [7]. $\mathbf{y}$ is a $\boldsymbol{M}\mathbf{x}\boldsymbol{1}$ matrix. When $\mathbf{M} < \mathbf{N}$, the system becomes underdetermined. It will not have a unique solution. The system of equations can be solved by assuming that $\boldsymbol{x}$ is sparse. Under this condition solution becomes unique. We can formulate the problem as

$$\text{Minimize } \|\mathbf{x}\|_0 \quad \text{subject to } \mathbf{A}\mathbf{x} = \mathbf{y}\ . \tag{2}$$

But this is an NP hard problem; Nondeterministic polynomial problem which is hard to solve. So we go for l_1 minimization. The problem can be formulated as

$$\text{Minimize } \|\mathbf{x}\|_1 \text{ subject to } \mathbf{A}\mathbf{x} = \mathbf{y}\ . \tag{3}$$

Hence it becomes a convex optimization problem. This can be easily solved. While convex optimization techniques are powerful methods for computing sparse representations, there are also a variety of greedy methods for sparse recovery such as Matching pursuit algorithm, Orthogonal matching pursuit etc. Greedy methods iteratively calculate the support of $\mathbf{x}$.

a) Matching Pursuit

Matching pursuit iteratively estimates the sparse signal $\mathbf{x}$. In each iteration, correlation between residual and columns of sensing matrix is calculated. The position of the maximally correlated column is found. Using this information, $\mathbf{x}$ is updated accordingly. This process is repeated $\mathbf{k}$ times, where $\mathbf{k}$ is the sparsity [15].

b) Orthogonal Matching Pursuit

Orthogonal Matching pursuit is similar to matching pursuit. In OMP, sensing matrix should be normalized. In OMP, $\mathbf{x}$ is updated in each iteration by projecting $\mathbf{y}$ orthogonally onto columns of $\mathbf{A}$ associated with the current support. In OMP an element once picked will not be picked again[16].

III. DOA ESTIMATION METHODS

a) Conventional Beamforming (CBF)

CBF is the simplest DOA estimation method. In CBF, the outputs from sensor array elements are combined coherently. CBF is robust to noise. But it has poor resolution. It cannot distinguish closely situated resources. But it requires only a single snapshot. If $\mathbf{a}\ (\boldsymbol{\theta})$ is the steering vector in the direction $\boldsymbol{\theta}$, The CBF power spectrum is

$$\mathbf{P}_{CBF}(\theta) = \mathbf{a}(\theta)\mathbf{R}_{\mathbf{y}}\mathbf{a}(\theta)\,. \tag{4}$$

$\mathbf{R}_{\mathbf{y}}$ is the cross spectral matrix.

b) Minimum Variance Distortion less Response (MVDR)

MVDR method is very popular beamforming technique. In the case of this method, output power of the beam former is minimized provided that MVDR signal remains undistorted in the look direction. Based on this condition weight vectors are calculated. MVDR has better resolution compared to CBF. But it requires multiple snapshots.

$$\min_{\mathbf{w}}\ \mathbf{w}^{\mathbf{H}}\mathbf{R}_{\mathbf{y}}\mathbf{w} \text{ subject to } \quad \mathbf{w}\mathbf{a}(\theta) = 1 \tag{5}$$

$\mathbf{R}_{\mathbf{y}}$ denotes the cross spectral matrix.

*c) Multiple Signal Classification (*MUSIC)

In MUSIC the cross spectral matrix undergoes Eigen decomposition to separate signal and noise spaces. MUSIC utilizes orthogonality between signal and noise subspaces to locate the spectral maximum. MUSIC has high resolution, But just like MVDR MUSIC also requires multiple snapshots.

IV. COMPRESSIVE BEAMFORMING

DOA estimation can be done using compressed sensing. Sensing matrix $\mathbf{A}$ can be formed using steering vectors. Propagation delay from the signal source in the $\boldsymbol{\theta}_{\mathbf{i}}$ to each of the sensors is called a steering vector.

$$\mathbf{a}(\theta) = \mathbf{1}\big/\sqrt{\mathbf{M}}\,\mathbf{e}^{\mathbf{j}\left(2\pi/\lambda\right)\mathbf{r}\sin\theta} \tag{6}$$

$$\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2)\cdots\mathbf{a}(\theta_N)] \tag{7}$$

Where **r** denotes the sensor locations. **M** is the number of measurements and λ is the wavelength. **N** denotes the length of DOA grid [2]. Let **x** be the vector denoting source locations. Length of **x** is equal to length of DOA grid [-90:1:90]. **A** is the sensing matrix of the dimension ***M*x*N***. The vector y is a ***M*x*1*** matrix. **y** contains measurements from sensors and **n** is the noise, So mathematically the problem can be formulated as

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{n}\,. \tag{8}$$

This equation can be solved using compressed sensing signal recovery algorithms.

A limitation of compressive beamforming is the coherence of sensing matrix. The coherence of the sensing matrix affects the DOA estimation when sources are closely situated. Coherence may result a bias in estimation.

V. AN IMPROVEMENT TO OMP

OMP (Orthogonal Matching Pursuit) requires number of non-zero elements as prior information. By changing stopping criteria of OMP, this can be avoided. In the proposed method stopping criteria depends on statistical properties of signal residue. Usually OMP algorithm is repeated **k** times where **k** is the sparsity. In the case of OMP, in each loop column of sensing matrix which has highest correlation with residual vector is found. According to this support **T** and **x** is updated. After **k** times, norm of residual vector becomes a very small value. By using suitable threshold on norm of residue we can have better results. We have used noise level $\mathbf{n}_l$ as a threshold to get better results. Proposed algorithm is given below.

Algorithm

1. Input: $\mathbf{y} \in \mathbf{R}^{\mathbf{m}}$, $\mathbf{A}$, $\mathbf{n}_l$.
2. Initialize $\hat{\mathbf{x}}^{[0]} = \mathbf{0}$, $\mathbf{r}^{[0]} = \mathbf{y}$, $\mathbf{T}^{[0]} = \phi$.
3. For $\mathbf{i=1}, \mathbf{i} = \mathbf{i} + \mathbf{1}$, until $\left\|\mathbf{r}^{[\mathbf{i}]}\right\| \leq \mathbf{n}_l$ do
4. $\mathbf{g}^{[\mathbf{i}]} = \mathbf{A}^{\mathbf{T}}\mathbf{r}^{[\mathbf{i-1}]}$
5. $\boldsymbol{j}[\mathbf{i}] = \mathrm{argmax}_j \left|\mathbf{g}^{[\mathbf{i}]}\right| / \left\|\mathbf{A}_{\mathbf{j}}\right\|_2$
6. $\boldsymbol{T}^{[\mathbf{i}]} = \mathbf{T}^{[\mathbf{i-1}]} \cup \mathbf{j}^{[\mathbf{i}]}$
7. $\hat{\mathbf{x}}^{[i]}$
8. $\mathbf{r}^{[\mathbf{i}]} = \mathbf{y} - \mathbf{A}\hat{\mathbf{x}}^{[\mathbf{i}]}$
9. End for
10. Output: $\mathbf{r}^{[\mathbf{i}]}$ and $\hat{\mathbf{x}}^{[i]}$

VI. COMPRESSIVE BEAMFORMING USING MULTIBAND SIGNALS

In the case of multiband signals additional information in different bands can be utilized in the compressive beam forming, provided that amplitude of different bands is same. Hence high resolution can be achieved in spite of coherence.

Multiband signal can be decomposed into narrow band signals.

$$\mathbf{Y} = \mathbf{A}\mathbf{X} + \mathbf{N} \tag{9}$$

Where $\mathbf{Y} = [\mathbf{y}_1 \mathbf{y}_2 \cdots \mathbf{y}_L]^{\mathrm{T}}$, $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2 \cdots \mathbf{x}_L]^{\mathrm{T}}$, $\mathbf{A} = [\mathbf{A}_1, \mathbf{A}_2 \cdots \mathbf{A}_L]^T$ and $\mathbf{N} = [\mathbf{n}_1, \mathbf{n}_2 \cdots \mathbf{n}_L]^{\mathrm{T}}$. Where **L** is the number of bands. By applying compressed techniques equation (9) can be solved.

VII. SIMULATION RESULTS

a) DOA estimation using CBF, MVDR, MUSIC and CS.

DOA is estimated using Conventional beamforming, MVDR, MUSIC and compressed sensing (convex optimization). DOA grid used is [-90:0.1:90]. Number of sensors is fixed at 12. The separation between sensors is λ/ 2. Number of snapshots is 50. Sources are placed at 5 and 50 degrees.

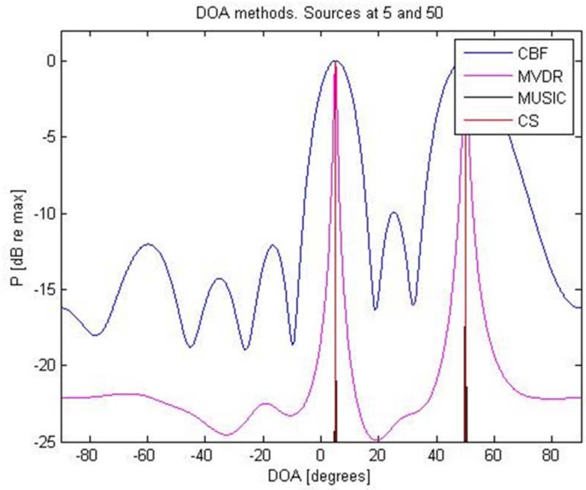

Fig 1. Different DOA estimation methods

In Fig.1, the output of compressive beam formers is spiked at 5 and 50 degrees. DOA estimation using compressed sensing has better resolution compared to traditional DOA methods.

b) Comparison of different Compressive beamforming techniques.

Compressive beamforming is done using different compressed sensing techniques like l_1 minimization, MP and OMP. The Algorithm proposed as an improvement to OMP is also used for DOA estimation. The performance of compressive beamforming techniques are compared with that of CBF. Percentage error in 500 iterations is taken as performance measure. Estimation within +2 or -2 difference from original angle is taken as correct estimation. DOA grid used is [–90:1:90]. Number of sensors is 12. The distance between sensors is λ/2. The sources are at 20 and 50 degrees. SNR varies from 5 to 25 dB.

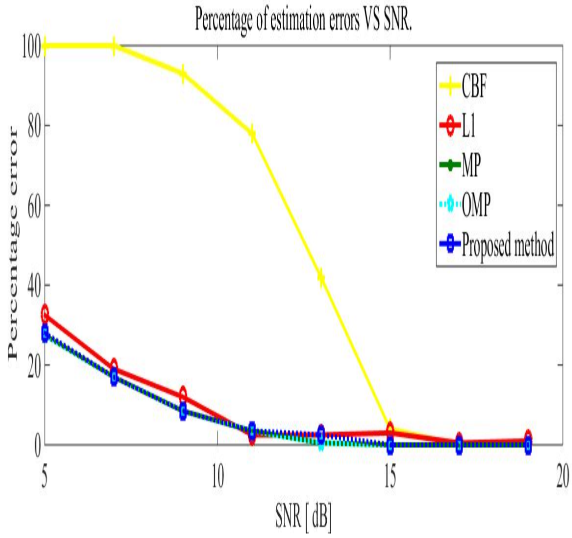

Fig2:Comparison of different compressive beamforming techniques.

Table1:Time taken by Compressive beamforming algorithms

Algorithm	Time
l_1 minimization	2.3245s
MP	0.0070s
OMP	0.0135s
Proposed	0.0077s

In the Fig 2, performance of conventional beam forming is very poor at low SNR. From 5 to 7 dB percentage of error is almost 100. But compressive beamforming techniques using $l_1$minimization, OMP and MP also give better results. The proposed algorithm also gives better results.

In the Table1, processing time of different compressive beamforming techniques is given. The greedy algorithms consumes less time. The performance of compressive beamforming techniques is almost similar. But greedy algorithms gives faster results.

c) Compressive beamforming using multiband signals.

Multiband Signal with frequencies 300MHz, 1300MHz, 2300MHz, 3300MHz and 4300MHz with same amplitude in all bands is considered in this paper. Compressive beamforming techniques using l_1 minimization, OMP and proposed algorithms are applied to the signal and results are compared with that of CBF. Sources are placed at 20 and 23 degrees. DOA grid is [-90:1:90]. Number of sensors are 12 and SNR is 15dB.

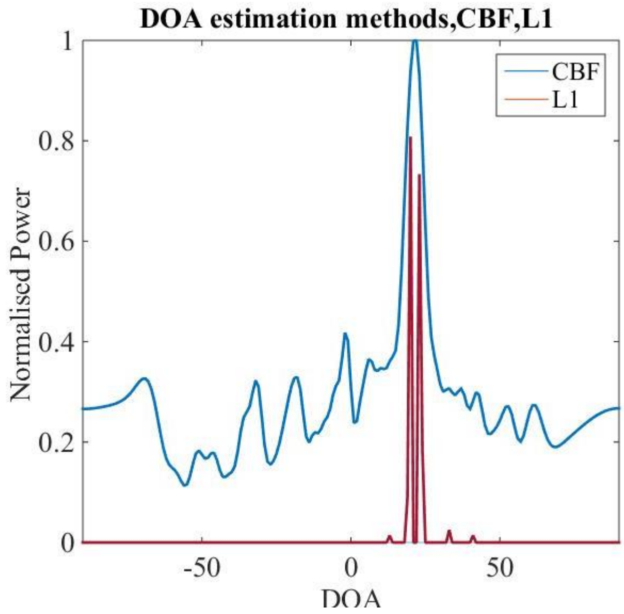

Fig3:DOA estimation of multiband signals using CBF and l_1.

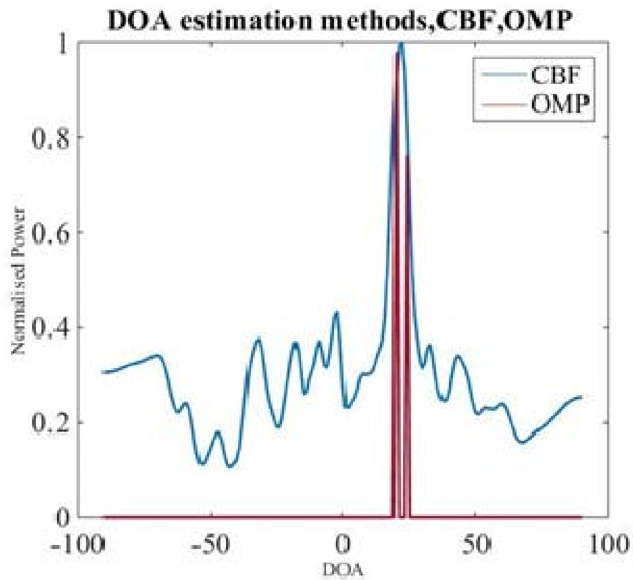

Fig 4:DOA estimation of multiband signals using CBF and OMP.

Fig 3 shows compressive beam forming using l_1 minimization and CBF in the case of multiband. Amplitude of compressive beamforming is accurately shows source positions even though they are separated by only 3 degrees. Compressive beam forming using OMP in Fig 4 and proposed method in Fig .5 has similar results.

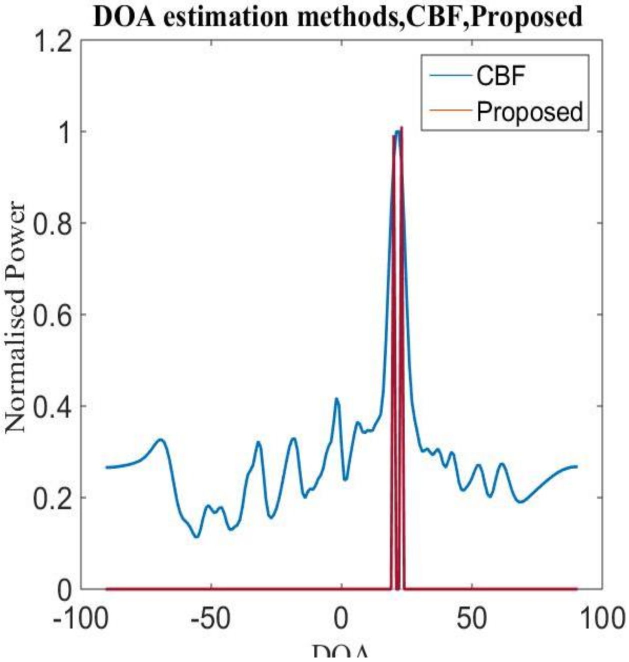

Fig5:DOA estimation of multiband signals using CBF and Proposed method..

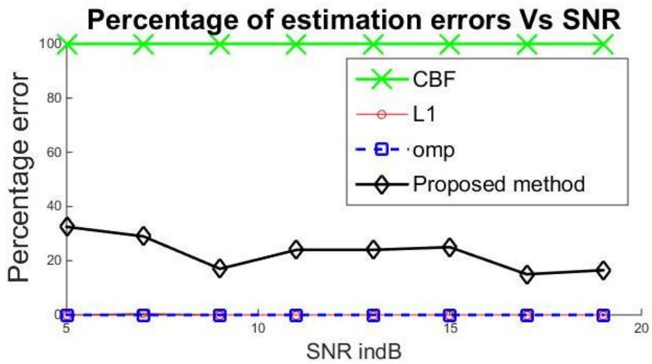

Fig6. Comparison of compressive beamforming techniques (Multiband)

Fig.6 shows comparison of performance of different beamforming techniques in the case of multiband signal. Performance measure is percentage error in 300 iterations. SNR varies from 5 to 25dB. Number of sensors is 12.Two targets at 20 and 23 degrees. In Fig.6 compressive beamforming techniques have very small percentage of error even when angle of separation is 3 degree where CBF shows 100 percentage errors.

VIII. CONCLUSION

DOA estimation using Conventional beamforming has low resolution. MVDR and MUSIC have better resolution but require multiple snapshots. CS methods have better resolution than Conventional Beamforming and requires single snapshot even in the noisy environment. Compressive beamforming can be done using convex optimization as well as greedy methods. Compressive beam forming using convex optimization has accurate results but consumes more time. Greedy algorithms are faster. Greedy algorithms like OMP and MP require number of targets to be specified beforehand. We have proposed an improvement to OMP. The proposed algorithm gives almost same performance as MP or OMP. But it does not require number of targets as prior information. Even though greedy algorithms are faster, the resolution is low due coherent sensing matrix. Coherent sensing matrix is a problem associated with estimation of closer targets. By using multiband signals performance can be improved even in the presence of coherence.

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