Extended Gaussian Function based Adaptive Filter Design for Filter Bank Multicarrier Systems

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***Abstract*—Filter bank multicarrier (FBMC) systems have been adopted as the most promising physical layer candidate for next generation wireless communication standards. The prototype filter in the FBMC system plays a key role in determining the system performance. An enhancement in transmission performance is possible, if a flexible pulse shape adaptive scheme is utilized in FBMC systems rather than going for a compromised choice of a fixed prototype filter. A relationship between the parameter of the filter with which the filter is adapted and the ratio of filter spread in time and frequency is also established. The dominance in performance of the proposed filter over the state-of-the-art filters is demonstrated through bit error rate (BER) and signal to interference (SIR) plots under doubly dispersive channel conditions.**

***Index Terms*—FBMC, 5G, OFDM, Prototype filter, extended Gaussian function.**

I. Introduction

The research interests on fifth generation (5G) multicarrier technologies have tremendously increased over the last decade. The 4G long term evolution advanced (LTE-A) based wireless communication system offers gigabit internet experience, high definition multimedia services at low cost, flexibility in switching between different wireless standards and so on. 5G systems, which are expected to be deployed in 2020, offers giga-bit data-rate experience under high mobility and increasing data traffic demand, low latency, high throughput etc. at reduced cost and low power consumption [1]. Orthogonal frequency division multiplexing has been the most widely adopted candidate as the physical layer waveform over the last two decades. The OFDM has various advantages like ease of generation, per-subcarrier equalization, high spectral efficiency and so on. Most of the advantages of OFDM are because of the orthogonality of the subcarriers in OFDM. Under doubly dispersive channels which involve both time and frequency spreading, the orthogonality of the subcarriers is lost and the claimed advantages of OFDM become insignificant. In OFDM, the presence of cyclic prefix reduces the spectral efficiency and increases the latency. The prototype filter used in OFDM is the time limited rectangular filter which has high side lobes in frequency domain. Under doubly dispersive channels, this results in spectral leakage between adjacent subcarriers, which ultimately leads to inter carrier interference. The presence of large side lobes also makes the application of OFDM difficult in cognitive radio scenario. Due to these drawbacks of OFDM, another multicarrier scheme called filter bank multicarrier (FBMC) is presently being considered and studied as a better physical layer candidate waveform than OFDM.

In FBMC, an array of filters are used at both transmitter and receiver sides. A set of serial data is converted to parallel data and passed through a set of filters modulated at a particular subcarrier frequency. The signals are summed up and passed through the channel. At the receiver, the transmitted symbols are demodulated by passing through the same set of filters, modulated at the corresponding subcarrier frequency. In the FBMC systems the vital role is played by the prototype filter used. If the prototype filter of the FBMC system satisfies certain properties, the disadvantages of OFDM can be avoided and hence FBMC has become the promising candidate for next generation wireless communication systems. The basic conditions to be satisfied by the prototype filters are that, it must be well localized in both time and frequency domains and also it should satisfy the Nyquist criterion in both the domains [2]. The minimum time-frequency dispersion product is shown by the Gaussian pulse but does not satisfy the Nyquist criteria in both the domains.

Various studies have been carried out for the design of prototype filters of FBMC systems. The two prevalent conventional isotropic prototype filter designs are isotropic orthogonal transform algorithm (IOTA) filter proposed by Alard and the weighted Hermite pulse (WHP) proposed by Haas and Belfore. The IOTA filter is designed by performing a double orthogonalization procedure on the Gaussian filter in both time and frequency domains [3]. The WHP filter is designed as a weighted sum of isotropic Hermite pulses [4]. The weights are obtained by optimization procedure such that the filter satisfies Nyquist criterion in both time and frequency domains. In order to improve the performance under doubly dispersive channels, a modified approach for the WHP design was proposed which concentrated on finding the weights such that the Nyquist criterion is satisfied not only at the required time frequency in stances, but also at adjacent lattice points around the neighbourhood of the prescribed points [5,6].

Even though these filters show satisfactory performance under the absence of channel, their performance is not optimum under the doubly dispersive channels. In the presence of the channel, a gain in transmission performance, in terms of signal to interference ratio (SIR), can be achieved if the spreads of the prototype pulse in time and frequency domains matches with the spreads of the channel in both the domains [7]. A closed form expression for the IOTA filter, called extended Gaussian function (EGF), is proposed in [8]. The possibility of changing the spread of the EGF in time and frequency domains by varying a parameter was identified in [9, 10]. The pulse adaptivity using the EGF for three different values of the parameter was investigated in [9].

In this paper, a channel adaptive prototype filter design procedure based on the EGF is proposed. Assuming that the channel spreads in time and frequency domains are known, a relationship between the spreads of the pulse in both the domains and the channel dispersion is established and the adaptive pulse is used as the prototype filter in FBMC systems. The performance analysis of the proposed filter is carried out in terms of SIR plots and bit error rate (BER) plots.

The paper is organized as follows. The FBMC system model and the conditions for perfect reconstruction are explained in section II. The proposed EGF based adaptive filter design method is explained in section III. The simulation results and analysis is presented in section IV and the paper is concluded in section V.

II. FBMC SYSTEM MODEL

The transceiver block diagram for OFDM can also be extended to FBMC. The prototype filter in the case of OFDM is the time limited rectangular filter whereas for the various prototype filters for FBMC are isotropic filters that are well localized in time and frequency domains. Let $s_k(n)$ be the symbol transmitted at k-th subcarrier frequency and n-th time instant. The transmitted signal is given as [2]

$$x(t) = \sum_{n} \sum_{k=0}^{K-1} s_k(n) p(t - nT) e^{j2\pi(t-nT)kF} \quad (1)$$

The symbol sampling period and subcarrier spacing are denoted by T and F respectively. The impulse response of the prototype filter is represented as $p(t)$. The number of subcarriers is denoted as K. At the receiver, a set of matched filter bank is used to demodulate the transmitted symbols. The transmitted symbols are correctly reconstructed at the receiver if the prototype filter satisfies Nyquist criteria in both time and frequency domains simultaneously. The modulated filters at k-th transmitter and l-th receiver sides are given as

$$p_k(t) = p(t)e^{j2\pi kFt} \quad (4)$$

$$p_l(t) = p(t)e^{j2\pi lFt} \quad (5)$$

The Nyquist criteria in time and frequency is given as [2]

$$\int p_k(t - aT) p_l^*(t - bT) dt = \delta_{ab}\delta_{kl} \quad (6)$$

where $\delta_{ab}\delta_{kl} = \begin{cases} 1, & if a = b \text{ and } k = l \\ 0, & otherwise \end{cases}$.

As the dual Nyquist criterion involves two variables of time and frequency, the conditions of perfect reconstruction can be mapped to a two dimensional function of time and frequency called the ambiguity function. The ambiguity function is given as

$$A_p(\tau, \nu) = \int_{-\infty}^{\infty} p\left(t + \frac{\tau}{2}\right) p^*\left(t - \frac{\tau}{2}\right) e^{-j2\pi\nu t} dt \quad (7)$$

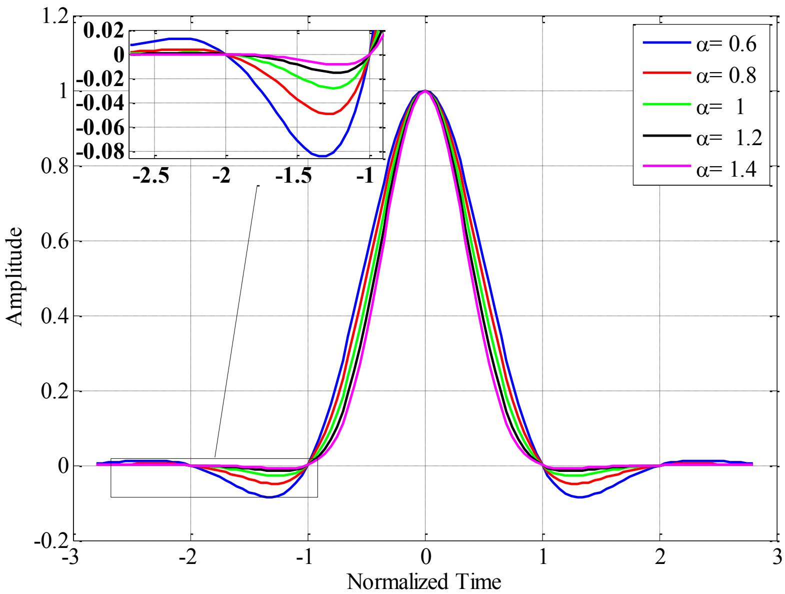

Fig. 1. Time domain response of EGF for different α values.

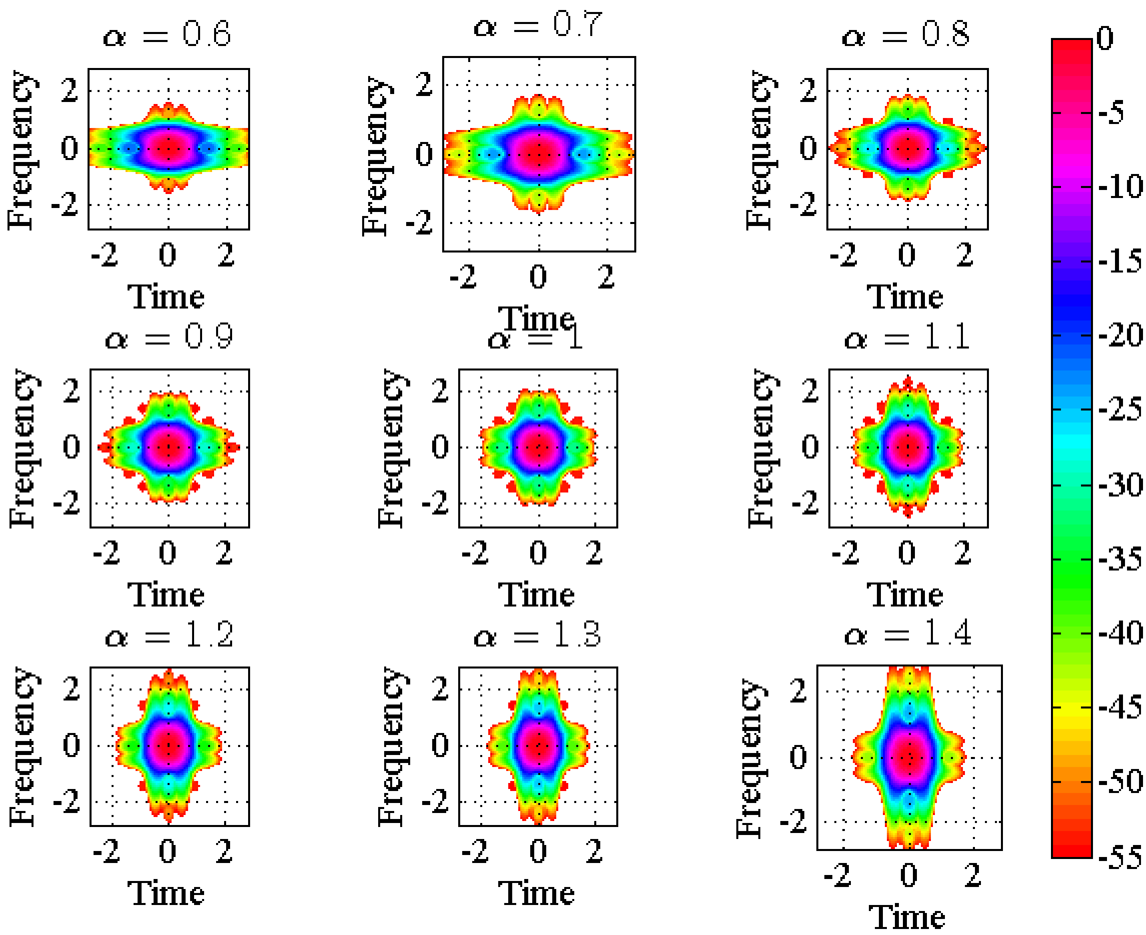

Fig. 2. Ambiguity function variation for different α values.

where the time and frequency variables are represented by τ and ν respectively. The importance of ambiguity function of the filter and its relationship with the transmitted and received symbols is studied in [11]. The discrete ambiguity function corresponding to (7) is given as

$$A_p(mT_s, qF_s) = \sum_{k=-\infty}^{\infty} p\left(k + \frac{mT_s}{2}\right) p^*\left(k - \frac{mT_s}{2}\right) e^{-j2\pi kqF_s} \quad (8)$$

The generalized Nyquist conditions are mapped to discrete ambiguity function as,

$$A_p(mT, nF) = \begin{cases} 1, & m = n = 0 \\ 0, & otherwise \end{cases} \quad (9)$$

III. Proposed Adaptive Filter Design

Time-frequency localization of the prototype filter is an important aspect which influences multicarrier transmission under doubly dispersive channels. The time-frequency localization is quantified using the Heisenberg parameter in [15] as

$$\mathcal{H} = \frac{1}{4\pi\sigma_t\sigma_f} \leq 1 \quad (10)$$

where σ_t and σ_f are time and frequency dispersions of the prototype filter respectively. If the Heisenberg parameter is large, the time-frequency spread product of the filter will be small. If the time and frequency dispersions are almost equal, the filter will be isotropic and the ambiguity function energy will be more concentrated in its main-lobe. Minimum the delay-Doppler product and more isotropic the filter , better will be the immunity of the FBMC system against doubly dispersive channels. Even though optimum value of (10) is obtained for a Gaussian pulse, it fails to satisfy the orthogonality conditions (6) and hence leads to considerable reconstruction error. Various isotropic filters like IOTA, WHP etc., which are derived from the Gaussian filter, are perfectly isotropic and are chosen as a compromised choice for the prototype filter used in FBMC systems.

In order to maximize the robustness under doubly dispersive channels, the time-frequency dispersions of the prototype filter must match with the delay and Doppler dispersions of the channel [12]. The immunity against ISI and ICI can also be improved if the symbol and sub-carrier spacing are appropriately chosen to match the delay spread and Doppler spread respectively. These two constraints can be expressed mathematically as

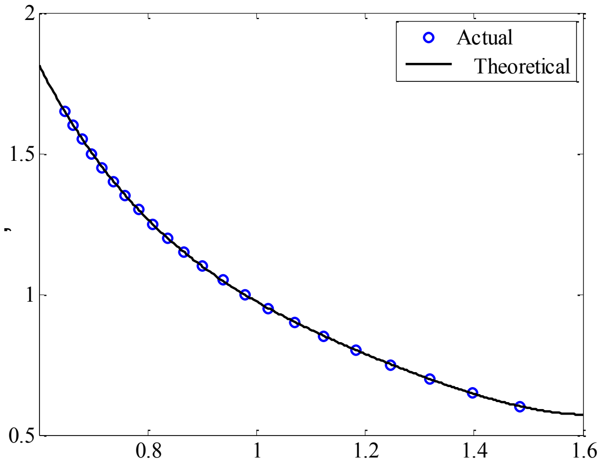

Fig. 3. α versus η.

$$\frac{\sigma_t}{\sigma_f} = \frac{T}{F} \approx \frac{\tau_{rms}}{f_D} \qquad (11)$$

where τ_{rms} represent the maximum delay spread and f_D represents the maximum Doppler spread of the channel respectively. The ratio $\eta = \frac{\sigma_t}{\sigma_f}$ is called the direction parameter which decides the spread of the pulse in time-frequency domain. This equality will pave the foundation for further explorations and findings in the formulation of an adaptive filter by adopting either pulse shape adaptation or time-frequency grid adaptation or both.

The equation for EGF is given in [8] as

$$h_{\alpha,\nu_0,\tau_0} = \frac{1}{2}\left[\sum_{k=0}^{K_s} \bar{d}_{k,\alpha,\nu_0}\left[g_\alpha\left(t+\frac{k}{\tau_0}\right) g_\alpha\left(t + \frac{k}{\tau_0}\right)\right]\right] \sum_{l=0}^{K_s} d_{l,\frac{1}{\alpha},\tau_0} \cos\left(\frac{2\pi l t}{\tau_0}\right) \qquad (12)$$

The coefficients $\bar{d}_{k,\alpha,\nu_0}$ can be found out using the procedure given in [8] and $g_\alpha(t)$ represents the Gaussian filter. The parameter K_s is to account for the number of coefficients to be considered so that the EGF is approximated closely to the IOTA filter. K_s is chosen as 14 in the present study. The shape of the filter can be varied in time and frequency domains by varying the parameter α. It would be tedious to find a relationship between the time-frequency dispersions, σ_t and σ_f , and the parameter α. Instead, the σ_t and σ_f values for the pulse can be found out numerically for different values of α. A relationship between α and the ratio η can be found out graphically using curve-fitting method or by using an artificial neural network. The graphical method is preferred in the present study and the relationship is obtained as

$$\alpha = 2.3\eta^4 - 11\eta^3 + 21\eta^2 - 19\eta + 7.8 \qquad (13)$$

The explanation and graphical proof of (13) is given in Section IV.

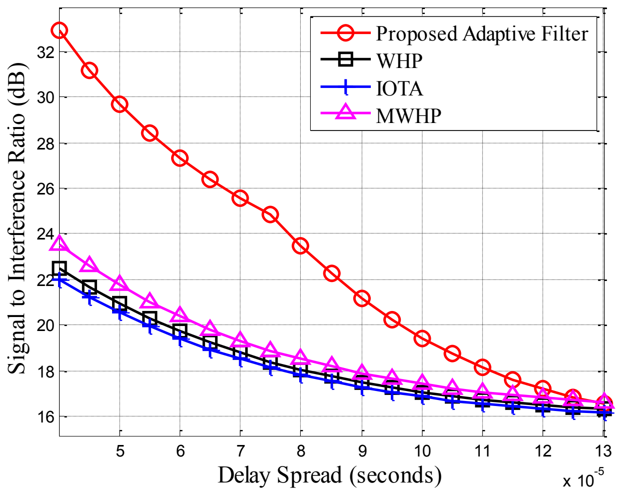

Fig. 4. SIR versus maximum delay spread

IV. Simulation Results and Analysis

In this section, various aspects of the proposed adaptive pulse design problem is demonstrated through numerical simulations. The impulse response of EGF for various α values is shown in Fig. 1. It can be seen that as the value of α increases, the sidelobe level in time domain decreases. Hence the side lobe level in frequency domain increases with an increase in α . The variation of the shape ambiguity function of the prototype filter for various value of α is shown in Fig. 2. For low values of α the spread of the ambiguity function is more along the horizontal direction, i.e. along the time domain. As the value of α reaches 1, the ambiguity function is symmetric, i.e. the pulse is the isotropic IOTA pulse. As the α value increases further, the ambiguity function spreads more towards the frequency axis. The variation of the α values versus the ratio of channel spreads ,η is shown in Fig. 3. The theoretical variation and the variation of α obtained by curve

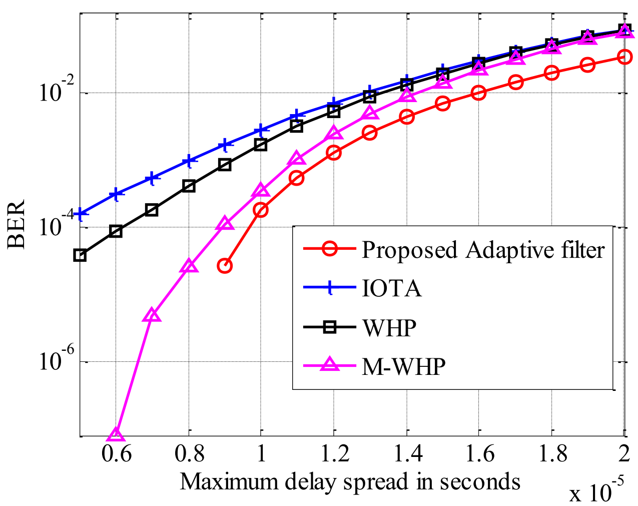

Fig. 5. BER versus maximum delay spread

fitting using (13) is also shown. The SIR and BER performance of the proposed adaptive filter is compared with the state of the art IOTA, WHP and MWHP filters. The SIR versus maximum delay spread values for a maximum Doppler spread of 3 kHz using 16 PSK is shown in Fig. 4. The channel scatter function is generated as that having a uniform power delay profile and Jake's Doppler profile. The proposed adaptive filter shows a significant improvement in SIR than the conventional filters. The BER versus maximum delay spread plot is shown in Fig. 5. The proposed filter shows improved performance in terms of BER for the entire range of delay spreads.

V. CONCLUSION

An adaptive filter which varies its time domain and frequency domain characteristics according to channel dispersions in both the domains is proposed. The variation of the shape of the pulse with respect to one parameter is analyzed using ambiguity function plots. A relationship between the parameter and the ratio of channel dispersions are also provided based on which the filter is adapted. The proposed filter shows significant improvement in performance under doubly dispersive channels in terms of SIR and BER.

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