Shearlet Transform Based Image Denoising Using Histogram Thresholding

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***Abstract*— This paper presents an efficient image denoising method by incorporating shearlet-based histogram thresholding. Nowadays, digital images are used in wide range of applications but most of these images are degraded during transmission and acquisition process. Removal of noise from images is still a challenging task for many researchers because there is always a trade-off between noise removal and fine edge preservation. This paper is based on image denoising using shearlet transform. Shearlets have excellent features for data analysis and processing, which overcomes the limitation of traditional methods. They are optimally sparse and have multi-scale and multi-directional properties which are optimal in representing image containing edges. In this paper, the proposed method is found to produce superior peak signal-to-noise ratio (PSNR) over the conventional denoising algorithms.**

***Keywords*— Discrete Shearlet Transform, Histogram Thresholding, Image Denoising, Multiresolution Analysis.**

I. Introduction

Nowadays, digital images are used in wide range of applications such as remote sensing, forensic, medical diagnosis, visual tracking, etc. But most of these images are deteriorated by noise during image transmission and acquisition process. The quality of the image must be preserved inorder to extract fine details from them. So the main aim of image denoising algorithms is to remove the noise while preserving fine details. Recently researchers had proposed many image denoising techniques to reduce the effect of noise.

Most of the conventional image denoising algorithms uses wavelet transform. Wavelet transform is effective in dealing with signals containing point singularities. Singularities such as lines or curves may or may not be present in higher dimensional signals and wavelets are unable to handle these distributed discontinuities very effectively [1]. Wavelets also have limited directional sensitivity, as a result of that edges in an image gets distorted.

Authors in [2] introduced curvelet transform which are optimal in representing image containing edges compared to wavelets. They are implemented by using laplacian pyramid and directional filter banks. But due to the lack of multiresolution analysis, curvelets are replaced by contourlets. Authors in [3] introduced contourlet transform which can capture intrinsic geometrical features of the image compared to curvelets. But the main limitation is that they have limited directional sensitivity compared to that of curvelets. From all of the above techniques, we see that conventional image denoising algorithms distorts edges in the image due to lack of directional sensitivity and multiresolution analysis.

In [4], authors introduced shearlet transform which provides sparse representation of multi-dimensional data. They have well localized waveforms and high directional sensitivity compared to other state-of-the art techniques. They are associated with multiscale and multidirectional decomposition, which enable them to capture intrinsic geometric features of image. In this paper, we had applied histogram thresholding to shearlet coefficients and output of different denoising algorithms is compared in terms of peak-signal-to-noise ratio (PSNR) in dB.

The paper is organized as follows. Section II gives an overview of Discrete Shearlet Transform and its n-term approximation error compared to conventional techniques. In Section III introduces the proposed image denoising method using histogram thresholding. Section IV gives the experimental results and comparison of different image denoising techniques. Conclusions are given in the final section.

II. The Discrete Shearlet Transform

Shearlet Transform combines multiscale and multi-directional representation and is very efficient to capture intrinsic geometric features of the multidimensional image. The shearlet decomposition of the image is shown in Figure 1. For a two dimensional image, the basis function of the shearlet transform is given by,

$$\mathcal{A}_{DS}(\psi) = \left\{ \psi_{j,k,l}(x) = |\det(D)|^{\frac{j}{2}}\, \psi(S^l D^j x - k) : j, l \in \mathbb{Z}, k \in \mathbb{Z}^2 \right\} \quad (1)$$

Where $\psi \in L^2(\mathbb{R}^2)$ forms a tight frame, D and S are 2×2 invertible matrices and $|\det(B)| = 1.$ Here D^j represents the dilation matrix and S^l represent the shearing matrix. From equation (1) we see that basis functions are not only limited to translation and scaling but also shearing along various orientations. As a result they provide better directional sensitivity compared to other techniques.

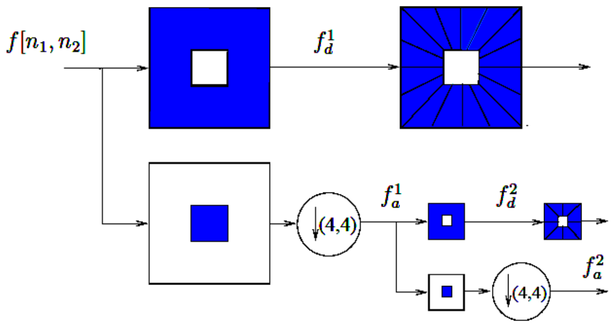


Figure 1: The shearlet decomposition of an image [4]

If $\mathcal{F} = \{\psi_{j,k,l}(x) : j,l \in \mathbb{Z}, k \in \mathbb{Z}^2\}$ represents dictionary of atoms or shearlet basis function such that every image can be represented using $\mathcal{F}$, then the approximation function is given by,

$$\mathcal{F}_N = \sum \langle \mathcal{F}, \psi_{j,k,l} \rangle \psi_{j,k,l} \quad (2)$$

The N-term approximation error which describes how well an image can be approximated by using dictionary of atoms or generally basis function is given by,

$$\mathcal{E}_N = \|\mathcal{F} - \mathcal{F}_N\| = \sum |\langle \mathcal{F}, \psi_{j,k,l} \rangle|^2 \quad (3)$$

As the value of N-term approximation decreases, we can say that algebraic sum of basis function is very close to that of original image. The N-term of approximation error of shearlet is given by,

$$\mathcal{E}_N \le CN^{-2}(\log N)^3 \quad (4)$$

which is optimal compared to that of wavelets (CN^{-1}) and Fourier transform ($CN^{-\frac{1}{2}}$).

III. Shearlet-Based Image Denoising

The block diagram of the shearlet based denoising method is shown in Figure 2.The major steps involved in the denoising algorithm are explained below:

1) Initially, Gaussian noise is added to original image with zero mean and variance σ^2
2) The noisy image is applied to a preprocessing filter; median filter is used in this method
3) Image is now decomposed into periodic and smooth components; Periodic component is applied to shearlet decomposition
4) Using Discrete Shearlet Transform, decompose the periodic image into four levels and at each level of decomposition few subband images are generated
5) For each subband, compute the histogram threshold. The threshold is applied to the noisy shearlet coefficients to get denoised coefficients
6) Apply Inverse Discrete Shearlet Transform to denoised coefficients to get the denoised image.

A. Periodic Plus Smooth Image Decomposition

Periodic plus smooth image decomposition is used to remove discontinuities across the boundaries of an image [5]. We decompose original image (U) into periodic component (P) and smooth component (S), which is given by,

$$U = P + S \quad (5)$$

Periodic image looks similar to that of original image, but its DFT is free from edge artifacts whereas smooth component represents slow variations inside the image. Since discontinuities across border of an image causes ringing artifacts across the boundaries, the periodic plus smooth image decomposition along with shearlet transform improves quality of denoised digital images. The steps can be summarized as: Let input image U(q,r) be a discrete M× N image:

1) Compute the boundary image V(q,r) of the original image U(q,r)
2) Compute DFT of the boundary image, i.e., $\hat{V}(q,r)$
3) The Poisson equation is given by

$$S(q,r) = \frac{\hat{V}(q,r)}{2\cos\left(\frac{2\pi q}{M}\right) + 2\cos\left(\frac{2\pi r}{N}\right) - 4} \quad (6)$$

4) The solution of Poisson's equation represents slow variations inside image i.e., smooth component
5) Periodic component can be obtained by subtracting smooth component from the original image

B. Shearlet Decomposition

The Figure 1 shows the shearlet decomposition of an image. The shearlet decomposition procedure is initiated by separating the periodic image into its high pass and low pass components, which is accomplished using Laplacian pyramid [6]. The high pass component is then applied to the directional filter bank, i.e., the shearing matrix. Directional filtering is not applied to the low pass component because at low frequencies would leak into the adjacent bands, which do not provide sparse representation.

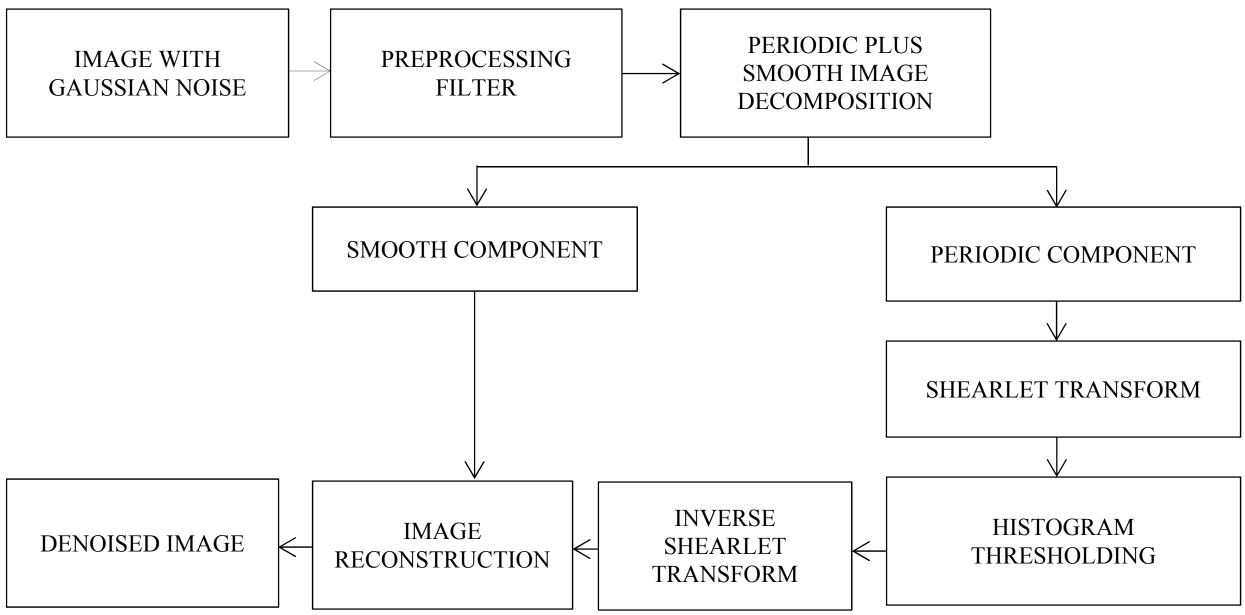


Figure 2: Block diagram of the shearlet-based denoising

C. Histogram Based Thresholding

Shearlet decomposition results in large number of shearlet coefficients and we need to separate noisy coefficients from original ones. Thresholding is very important because thresholding at large values result in loss of information whereas at low values result in background clutter. Let $S_K^j(x,y)$ represent the initial shearlet coefficient in the point (x,y) in each sub-band $K \epsilon \{K_1^j\ K_2^j \dots K_k^j\}$at scale j. The aim of this paper is to obtain denoised coefficient $D_k^j(\mathrm{x,y})$ at the point $S_K^j(x,y)$ by adjusting the pixel values i.e.,

$$D_k^j(\mathrm{x,y}) = \begin{cases} S_K^j(x,y) & if\ S_K^j(x,y) > T \\ 0 & if\ \ S_K^j(x,y) < T \end{cases} \tag{7}$$

where T is Histogram threshold.

Histogram is constructed by splitting the range of the data into equal-sized bins or classes. The numbers of points from the data set that fall into each class are counted. The procedure to obtain T is summarized below [7]:

1) Initially we select a threshold $\mathrm{T_0}$
2) Using threshold $\mathrm{T_0}$, we divide image into two groups of pixels L1 and L2
3) Compute mean for L1 and L2 i.e., $\mathrm{M_1}$ and $\mathrm{M_2}$
4) New threshold value is given by,

$$\mathrm{T} = \frac{1}{2}(\mathrm{M_1} + \mathrm{M_2}) \tag{8}$$

5) Repeat the above steps until the difference between successive iteration results in small value of T than $\mathrm{T_0}$. In this paper, we had performed 25 iterations.

IV. RESULTS AND DISCUSSION

Four different images of resolution 512×512 pixels shown in Figure 3 are used to evaluate the performance. The experiment is implemented in MATLAB and shearlet transform can be implemented using ShearLab Software Package. Let the image is represented by 'f ' and 'w' be the zero mean additive white gaussian noise with variance σ^2.Then the noisy image can be represented as,

$$f_n = f + w \tag{9}$$

The noisy image f_n is denoised by thresholding the shearlet coefficients within each subband. The performance of the system is evaluated and compared with other algorithms using peak signal to noise ratio (PSNR) in decibels, which is given by,

$$PSNR = 20\log_{10}\left(\frac{255}{MSE}\right) \tag{10}$$

where MSE is the mean square error. Given an image $f_r(i,j)$ and original image $f_o(i,j)$, then MSE is given by,

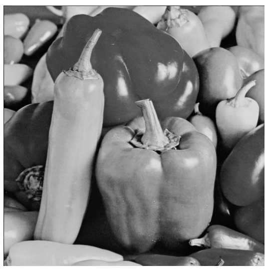
$$MSE = \sum_{i,j}\frac{[f_o(i,j) - f_r(i,j)]^2}{M \times N} \tag{11}$$

where M ×N is the size of image.

In our experiment, we used a four level shearlet decomposition wherein each level consisting of 3, 3, 4 and 4 numbers of shearing directions respectively. The number of directional sub-bands within each level $N_s = 2^s$where N_s is the number of shearing directions.

(a)

(b)



(c)

(d)

Figure 3: Test images (a) Lena, (b) Barbara, (c) Peppers, (d) Boat

Thus, the number of directional subbands within each level was obtained as 8, 8, 16 and 16 respectively. The Figure 4 shows an illustration of approximation and detail coefficients of four level shearlet transform of Boat image.

We tested the denoising schemes for the images having standard deviation $\sigma = 10$. The performance of proposed method compared to conventional methods is shown in TABLE I. It shows that image denoising using proposed method has high PSNR compared to other state-of-the art techniques.

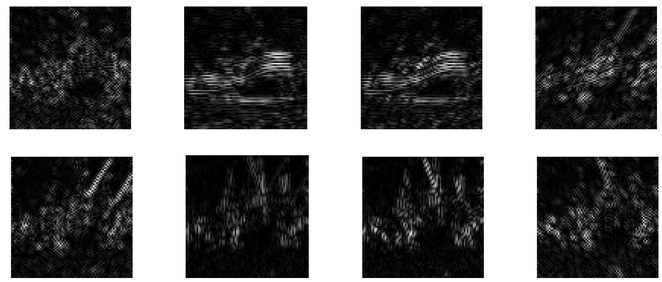
TABLE I
COMPARISON OF THE PERFORMANCE OF THE PROPOSED METHOD TO OTHER METHODS IN TERMS OF PSNR (dB) FOR A STANDARD DEVIATION OF $\sigma = 10$.

Techniques /Images	Lena	Barbara	Boat	Peppers
Bayes Estimate	29.5418	28.1903	28.9707	28.9950
Wavelet Hard Threshold	30.4534	29.6475	29.2039	30.0451
Wavelet Soft Threshold	31.2123	30.1041	30.0718	31.0081
The Proposed Method	**33.9307**	**32.5312**	**32.1247**	**33.1920**

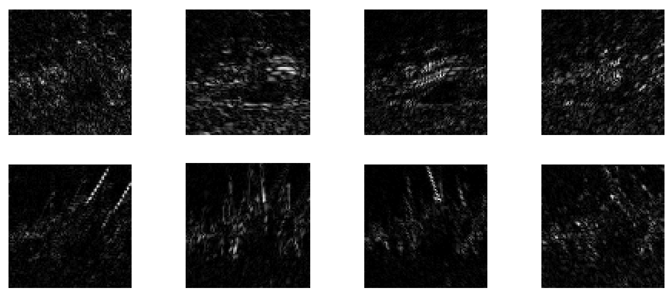
The original, noisy and restored version of the boat image is shown in Figure 5.

(a)

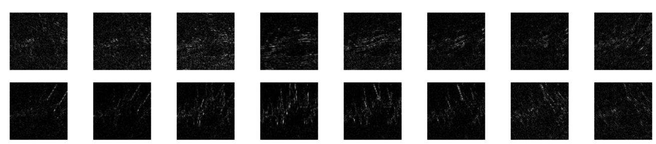
(b)



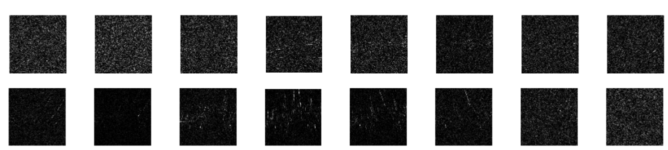
(c)



(d)

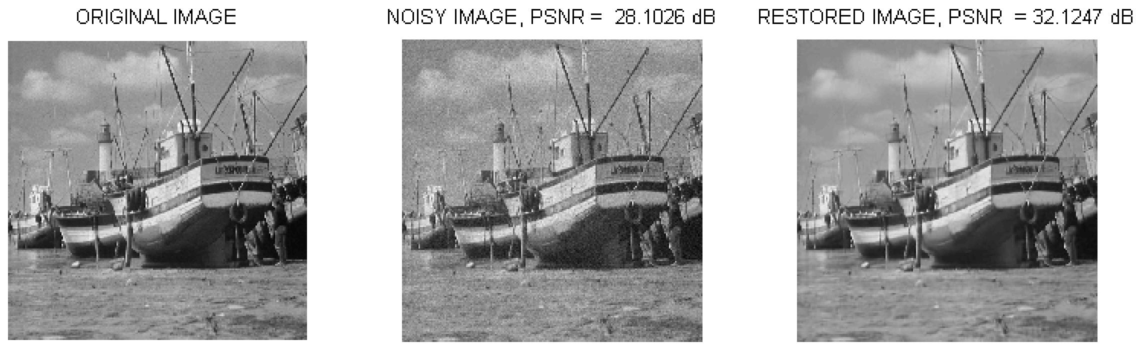


(e)



(f)

Figure 4: An illustration of shearlet decomposition (a) The noiseless input image (Boat image), (b) The approximate shearlet coefficients, (c) The detail shearlet coefficients of first level, (d) The detail shearlet coefficients of second level, (e) The detail shearlet coefficients of third level, and (f) The detail shearlet coefficients of fourth level.



(a) (b) (c)

Figure 5: The result of the proposed method (a) Original Boat image, (b) Noisy image (PSNR= 28.1026dB), (c) Restored image (PSNR= 32.1247dB)

V. CONCLUSION

In this paper, an efficient algorithm is proposed for removing noise from corrupted image by incorporating a shearlet-based histogram thresholding. This paper shows the comparison of different denoising methods that can be applied to the shearlet transform in order to obtain denoised image. The experimental result shows the proposed method provides high PSNR compared to other conventional methods. This is due to the better directional sensitivity and edge preservation ability of the shearlet transform compared to other algorithms.

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