Transmit Precoding with Encoding using Zadoff-Chu Sequence for MIMO-OFDM System

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***Abstract*—Transmit precoding utilizes the channel state information (CSI) to precode the data so that the effect of fading is equalized. Three simple precoding schemes for Rayleigh fading multiple-input multiple-output (MIMO) system are explained. CSI is required only at the transmitter and not at the receiver. So the computational complexity is reduced at the receiver. The bit error rate (BER) at the receiver can be reduced by encoding the data using Zadoff-Chu sequence. Simulation results illustrate the obtained BER performance. A comparison with encoding using Gold sequence is also provided.**

***Index Terms*—Precoding, channel state information (CSI), time division duplex (TDD) system, Zadoff-Chu (ZC) sequence.**

I. INTRODUCTION

Multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) is a promising technology for 4G and 5G mobile communications [1]. By employing multiple antennas, multiple channels are created which provides diversity. OFDM divides a channel into a number of closely spaced sub-channels thereby increasing the spectral efficiency [2]. Cyclic prefix in OFDM reduces Inter Symbol Interference (ISI). So the combination of MIMO and OFDM can provide high data rate and high spectral efficiency along with reliable communication.

Fading refers to the distortion that a carrier-modulated signal experiences over a wireless communication path. Apart from the Line of Sight (LOS) path between the transmit antennas and the receive antennas, additional radio propagation paths result from reflection, diffraction and scattering of the wave. Hence multiple versions of the transmitted signal are obtained at the receiver at slightly different times. The different signals merge at the receiver and the combined signal varies widely in amplitude and phase. The effects of fading include rapid changes in signal strength, random frequency modulation and time dispersion [3].

The effect of fading can be equalized by applying precoding methods which use the channel state information at the transmitter (CSIT) [4]. To acquire the channel state information (CSI), the transmitter sends a pilot sequence to the receiver. The pilot sequence is known to both the transmitter and the receiver. CSI is acquired at the receiver from the received signal and then fed back to the transmitter. Forward and reverse links operate on the same frequency bands in time division duplex (TDD) systems [5] and the CSI can be acquired in a simpler way as the channel is assumed to be reciprocal. The receiver sends a known pilot sequence to the transmitter from which CSI for the reverse channel is estimated at the transmitter [6], [7]. Its conjugate transpose is taken as the CSI for the forward channel. So the need for first estimating the channel at the receiver is avoided.

The received data may contain errors as a result of noise, attenuation, etc. The performance of the system can be measured by computing the bit error rate (BER), as a function of SNR at the receiver. High BER causes increased packet loss and decreased throughput. So techniques which achieve low BER are required so that higher accuracy of data transmitted over wireless medium is ensured.

In Section II, three precoding schemes, namely, modified maximum ratio transmission (MRT) based precoding, real orthogonal space-time block code (OSTBC) based precoding and QR-based precoding are explained. In Section III, the generation and properties of Zadoff-Chu (ZC) sequences are explained. Results obtained from simulation are given in Section IV. It is followed by Section V which concludes the paper.

Notation: $\mathcal{N}(0, 1)$ represents real, zero mean and unit variance Gaussian random variable. $\mathcal{CN}(0, 1)$ represents complex circularly symmetric, zero mean and unit variance Gaussian random variable. Boldface small letters represent vectors. Boldface capital letters represent matrices. $\mathbf{A}^T$ indicates the transpose of $\mathbf{A}$. $\mathbf{A}^H$ represents the Hermitian of $\mathbf{A}$. $||a||$ represents the l_2 norm of a.

II. PRECODING METHODS

The block diagram of the system is given in Fig. 1. The binary data is modulated using QPSK. The modulated data symbols are transferred through a serial-to-parallel (S/P) converter. Each parallel bit stream is given to an inverse fast Fourier transform (IFFT) block after which cyclic prefix is appended. The bit streams are then passed through a parallel-to-serial (P/S) converter to form a single OFDM symbol. The OFDM symbol is then subjected to precoding. At the receiver the reverse process takes place to retrieve the original data stream.

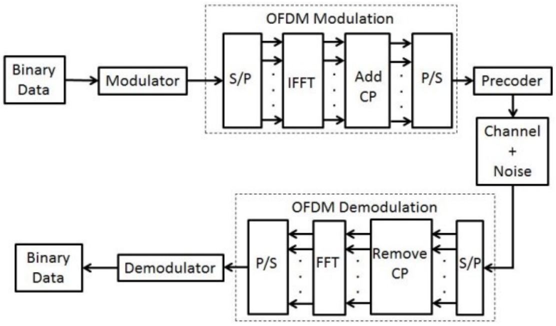

Fig. 1. MIMO-OFDM system with precoder.

A. Modified MRT-Based Precoding

Let N_t denote the number of antennas at the transmitter and N_r denote the number of antennas at the receiver. Consider $N_t \geq$ 2and $N_r = 1$.Let **h** be the N_t x 1 channel matrix with $\mathcal{CN}(0, 1)$ components which are independent and identically distributed (i.i.d). Let x be the data symbol to be transmitted. The precoding matrix is defined as $\mathbf{p} = \mathbf{h} \,/\, ||\mathbf{h}||^2$. The signal at the receiver, $y \in \mathbb{C}$, is given by

$$y = \sqrt{\frac{k\rho}{N_t}}\mathbf{h}^H\mathbf{p}x + n \quad (1)$$

This equation reduces to

$$y = \sqrt{\frac{k\rho}{N_t}}x + n \quad (2)$$

where $k = N_t(N_t - 1)$ is a power normalization constant, ρ represents the average transmit power across N_t transmit antennas per channel use and $n \in \mathbb{C}$ represents the noise at the receiver. The fading channel is fully equalized by this precoding scheme.

B. Real OSTBC-Based Precoding

In this precoding scheme real OSTBC is used. The real valued components of the received signal are considered. Consider $N_r = 1$. Let x be the data to be transmitted. The precoding matrix is defined as $\mathbf{P} = \widetilde{\mathbf{H}}^T/\alpha$ where $\alpha = h_1^2 + h_2^2 + \cdots + h_{N_t}^2$. The real OSTBC codeword **X** is generated from the precoded matrix **Px** using permutations and sign inversions of its entries. The channel matrix $\widetilde{\mathbf{H}}$ is orthogonal. The received signal can be written as

$$\mathbf{y} = \sqrt{\frac{k\rho}{N_t}}\widetilde{\mathbf{H}}\mathbf{P}\mathbf{x} + \mathbf{n} = \sqrt{\frac{k\rho}{N_t}}\mathbf{x} + \mathbf{n} \quad (3)$$

The noise vector has i.i.d. $\mathcal{N}(0, 1)$ entries and k can be written as

$$k = \frac{N_t(N_t-2)}{L} \quad (4)$$

The fading channel is fully equalized by this precoding scheme.

C. QR-Based Precoding

In this precoding scheme the QR decomposition of the channel matrix is used to obtain the precoding matrix. Consider a channel with N_r antennas at the receiver and $N_t \geq 2N_r$ antennas at the transmitter. Let **H** be the N_t x N_r Rayleigh fading channel matrix. The QR decomposition of **H** is given by **H** = **QR. Q** is an N_t X N_t unitary matrix and **R** is an N_t x N_r upper triangular matrix. The N_t X N_t precoding matrix is defined as **P** = **QU** where **U** is an N_t x N_t unitary matrix. The data vector can be written as $\mathbf{x} = [x_1 \quad x_2 \cdots \quad x_{N_r}]^T$ and the extended data vector $\tilde{\mathbf{x}}$ is obtained from the equation $\mathbf{R}^H\mathbf{U}\tilde{\mathbf{x}} = \mathbf{x}$. The received vector $\mathbf{y} \in \mathbb{C}^{N_r}$ can be written as

$$\mathbf{y} = \sqrt{\frac{k\rho}{N_t}}\mathbf{H}^H\mathbf{P}\tilde{\mathbf{x}} + \mathbf{n} = \sqrt{\frac{k\rho}{N_t}}\mathbf{x} + \mathbf{n} \quad (5)$$

The constant k can be written as

$$k = N_t\left[1 + \frac{1}{N_t - N_r}\right]^{-1} \quad (6)$$

III. ZADOFF-CHU (ZC) SEQUENCE

ZC sequences are poly phase sequences with optimum correlation properties [8]. ZC sequences are used in uplink physical control channel, primary synchronization signals, random access channel and uplink reference signals in long term evolution (LTE). The ZC sequence is defined as [9]

$$a_k = \begin{cases} e^{\frac{j2\pi r}{L}\left(\frac{k^2}{2}+qk\right)} & for\ L\ even \\ e^{\frac{j2\pi r}{L}\left(\frac{k(k+1)}{2}+qk\right)} & for\ L\ odd \end{cases} \quad (7)$$

where L is the length of the sequence, k=0,1,2,…,L-1, q is any integer and r is any integer relatively prime to L. The ZC sequences have constant amplitude and zero autocorrelation at non-zero lags [10]. Zero autocorrelation means the sequences are orthogonal to each other and orthogonality is desired to minimize interference between multiple users.

The modulated data symbols are multiplied using ZC sequence and then the precoding schemes are applied. At the receiver, the received symbols are multiplied with the inverse of the matrix representing the ZC sequence to decode the data. The precoding schemes along with encoding using ZC sequence considerably reduce the BER at the receiver. The modified block diagram is shown in Fig. 2.

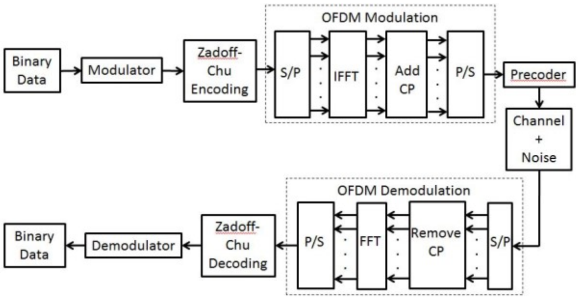

Fig. 2. MIMO-OFDM system with ZC sequence encoding and precoder.

IV. SIMULATION RESULTS

The performance of the precoding schemes with encoding using ZC sequence has been evaluated by considering a MIMO channel. The modulation technique used is quadrature phase shift keying (QPSK). The system parameters used are given in Table I.

TABLE I. SYSTEM PARAMETERS

Number of Transmit Antennas	4, 2
Number of Receive Antennas	2, 1
Modulation	QPSK
Number of bits generated	102400
Blocksize	2048
Length of Cyclic Prefix	160
Number of subcarriers	175
Encoding	ZC sequence, Gold sequence
L	7, 11
r	5, 3

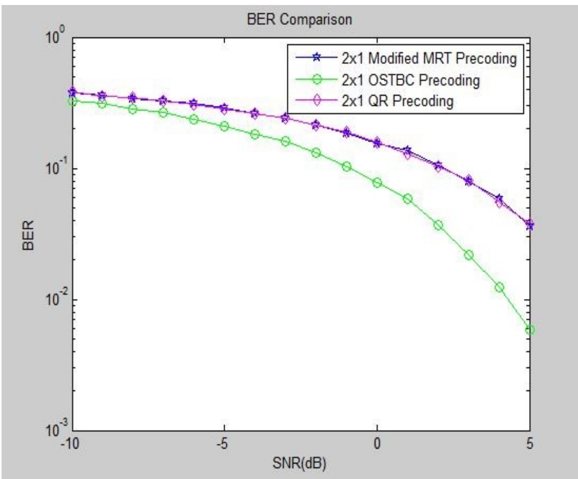

Fig. 3. BER performance of the three precoding schemes without encoding.

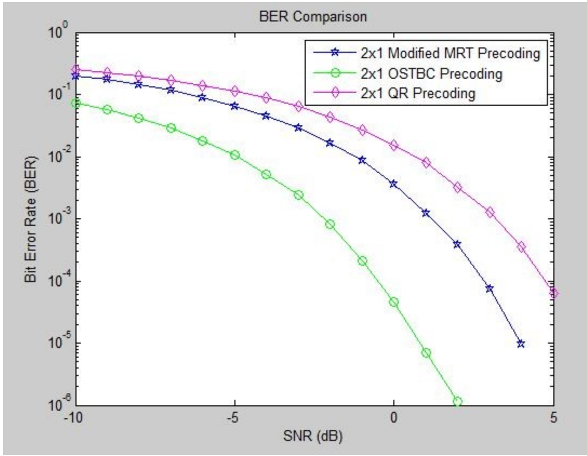

Fig. 4. BER performance of the three precoding schemes with ZC encoding (L=7).

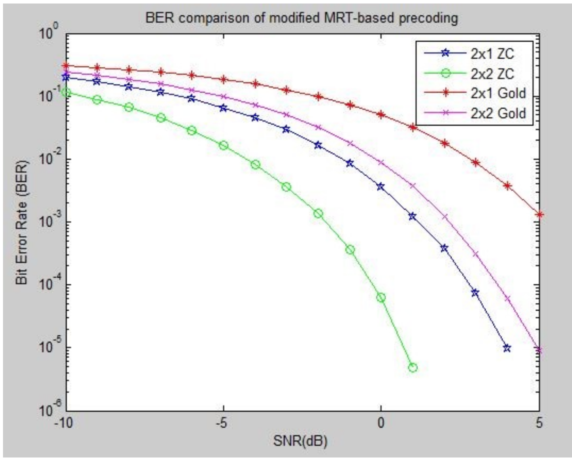

Fig. 5. BER performance of modified MRT-based precoding scheme with encoding using ZC and Gold sequences of length L=7 for 2x1 and 2x2 systems.

Figure 3 shows the BER performance of modified MRT, OSTBC and QR-based precoding schemes without encoding for a 2x1 system. The modified MRT-based and QR-based schemes have almost identical performance. OSTBC-based scheme has a 3-dB improvement compared to the other two schemes. All the three schemes exhibit waterfall-like behaviour. The modified MRT-based precoding scheme is the simplest one to implement. QR-based precoding scheme has the highest complexity among the three schemes.

Figure 4 shows the BER performance of the three schemes with encoding using ZC sequence of length L=7. The BER curves show significant improvement when encoding is applied. In Fig. 5 the performance of modified MRT-based precoding with encoding using ZC sequence is compared with encoding using Gold sequence for 2x1 and 2x2 systems. A higher transmit power of 4-dB is required for encoding using Gold sequence compared to ZC encoding to achieve the same BER.

Figure 6 shows the BER performance of OSTBC-based precoding scheme with encoding using ZC and Gold sequences for 2x1 and 4x1 systems. The difference between the curves is 2 dB and 3 dB for 2x1 and 4x1 systems, respectively.

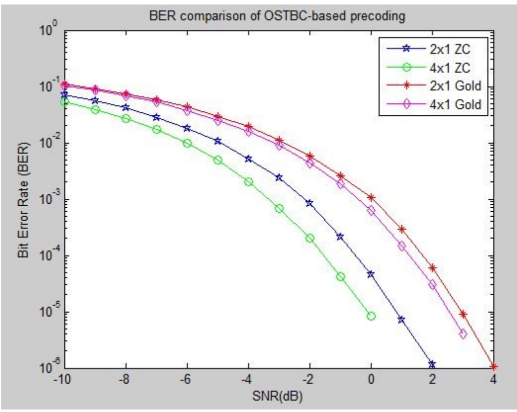

Fig. 6. BER performance of OSTBC-based precoding scheme with encoding using ZC and Gold sequences of length L=7 for 2x1 and 4x1 systems.

Figure 7 shows the BER comparison of QR-based precoding with encoding using ZC and Gold sequences for 2x1 and 4x2 systems. A 3-dB gain can be achieved for encoding using ZC sequence compared to Gold sequence encoding.

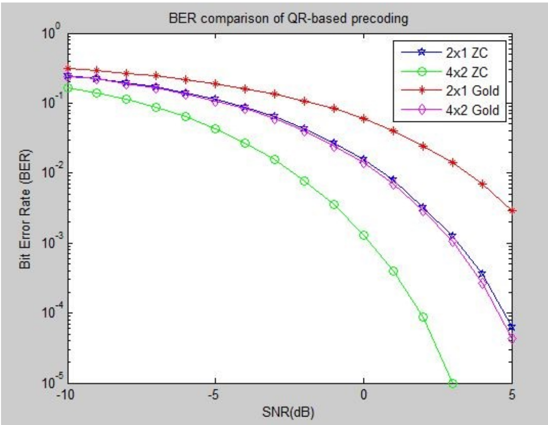

Fig. 7. BER performance of QR-based precoding scheme with encoding using ZC and Gold sequences of length L=7 for 2x1 and 4x2 systems.

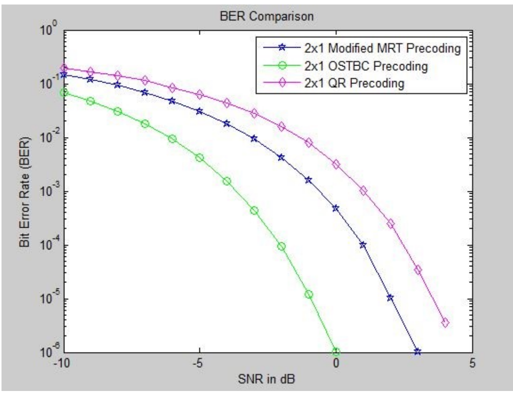

Fig 8. BER performance of the three precoding schemes with ZC encoding (L=11).

Figure 8 shows the BER performance of the three schemes with encoding using ZC sequence of length L=11. Figure 9 compares the BER performance of modified MRT-based precoding with ZC sequence encoding for 2x1 and 2x2 systems for L=11. Figure 10 shows the BER performance of OSTBC-based precoding with ZC sequence encoding for 2x1 and 4x1 systems for L=11. The performance improves as the length of the ZC sequence is increased.

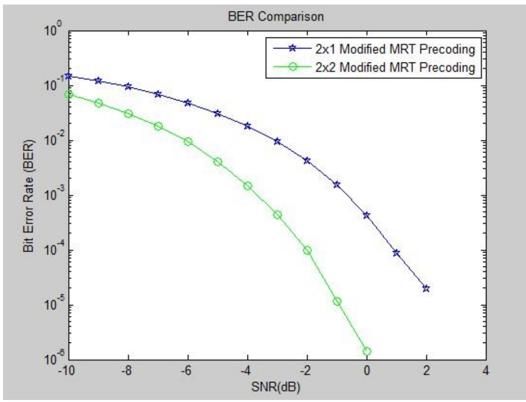

Fig. 9. BER performance of modified MRT-based precoding scheme with ZC encoding for 2x1 and 2x2 systems (L=11).

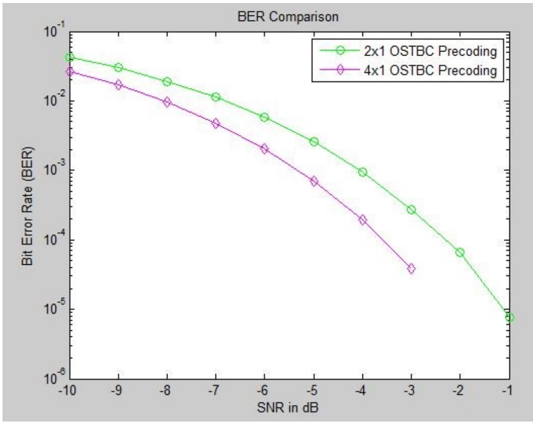

Fig. 10. BER performance of OSTBC-based precoding scheme with ZC encoding for 2x1 and 4x1 systems (L=11).

V. Conclusion

Three simple precoding schemes based on CSIT have been explained in this paper which can be used to equalize fading channel. The BER performance has been improved by encoding the data symbols using ZC sequence. Simulation results show the enhanced BER curves for the precoding schemes. The performance is further improved by increasing the number of transmit and receive antennas and by increasing the length of the ZC sequence.

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