Analysis of Carrier Frequency Offset in WFRFT-OFDM Systems using MLE

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Abstract— **Traditional Orthogonal Frequency Division Multiplexing (OFDM) system is a multi carrier transmission technique that splits data stream into a number of data subcarriers in parallel. In this method, a 4-Weighted Fractional Fourier Transform (4-WFRFT) is proposed for modulation / demodulation of the subcarriers. The method uses Maximum Likelihood Estimation (MLE) for Carrier Frequency Offset estimation and correction in MIMO-OFDM systems. The proposed method makes efficient use of the frequency spectrum by unifying the two competitive carrier schemes – single carrier (SC) and multi-carrier (MC). The distortion resistance capability of the communication system is improved. The mean square error performance (MSE) of WFRFT-MIMO-OFDM systems is analyzed here.**

Keywords—4-Weighted Fractional Fourier Transform (4-WFRFT), Maximum Likelihood Estimation(MLE),Single Carrier(SC), Multi Carrier(MC), Mean Square Error (MSE).

I.INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a modulation scheme used for many of the latest wireless and telecommunication standards. OFDM has been chosen for the Wi-Fi standards like 802.11a, 802.11n, 802.11ac and more [1]. It has been also chosen for the telecommunications standards like LTE / LTE-A and other standards like WiMAX etc. OFDM is also used in many broadcast standards including DAB Digital Radio, Digital Video Broadcast Standards - DVB, Digital Radio Mondi ale used for the long, medium and short wave bands [1].

The advantages of OFDM make it a suitable choice. OFDM provides immunity to selective fading, resilience to interference, ISI and narrow band effects. It makes optimum use of the available spectrum, thereby providing spectral efficiency. The technique makes channel equalization simpler. The disadvantages include frequency offsets which are often caused by Doppler shifts or mismatch between the transmitter and receiver oscillators. The frequency offset should be maintained within a fraction of sub-carrier spacing to avoid the bit error degradations. Hence the design of synchronization part becomes more critical. OFDM systems are very sensitive to frequency offset and multipath delay spread. The system performance is affected when orthogonality is lost. Orthogonality can be achieved either by choosing a carrier spacing equal to the reciprocal of useful symbol period or by error-free frequency and time synchronization at the receiver side [2].

Combination of MIMO with OFDM systems enables use of more antennas with larger bandwidths for high data rate transmission. This technique enables indoor wireless systems to achieve a data rate of the order of several hundreds of Mbps and spectrum efficiency of the order of several tens of bits/Hz/s. The high data-rate and spectral efficiency is attained by the parallel transmission of data in space and frequency domain. A throughput of 1 Gbit/sec and beyond can be achieved using MIMO-OFDM. Also, link reliability can be improved [3].

The method proposed involves OFDM modulation/demodulation using 4-WFRFT [4] in a MIMO-OFDM system. The carrier frequency offset estimation and correction [5] is done using MLE method.

WFRFT is a co-modulation process known as hybrid-carrier (HC) scheme, as it can achieve a smooth transition between Single-carrier (SC) and Multi-carrier (MC). Thus, a hybrid carrier scheme is compatible with single as well as multi – carrier. WFRFT has been proposed to reduce ISI and ICI. It is defined in terms of weighted superposition of Eigen functions [6].

In this paper, MSE performance of WFRFT-MIMO OFDM system with MLE estimation is simulated and evaluated. Using WFRFT, the distortion resistance capability of the communication system seems to be improved [7].The proposed method employs WFRFT modulation /demodulation instead of DFT/IDFT that suppresses the ICI and ISI and performs better in the presence of CFO. Due to frequency offset, orthogonality of subcarriers is lost thereby causing ICI. As a result, ICI lowers the signal to noise ratio (SNR) and increases the error probability. A modulation/demodulation using 4-WFRFT is chosen, as the signals are difficult to intercept and recognize for the non-destination receivers and makes efficient use of the spectrum.

Synchronization is achieved using MLE algorithm. MLE is preferred as it doesn't require any training symbols or pilots. The rest of the paper is organized as follows: Section II gives the definition of 4-Weighted Fractional Fourier Transform. Section III describes the system model of 4-WFRFT in the presence of CFO. Section IV includes the simulations involved and Section V gives the conclusion.

II. SYSTEM MODEL

A. Definition of 4-Weighted Fractional Fourier Transform

For an integrable function *g(x),* its Fourier Transform *G (k)* is

$$G(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} g(x)e^{-jkx}\,dx \qquad (1)$$

The Fourier Transform is a periodic transform with period four having eigen values : 1,-1,i and –i. On swapping x by k, we get four functions of the integer order FT [8].

$$\begin{aligned} F^1[g(x)] = F^1[g_0(x)] &\xrightarrow{x=k} g_1(x) = G(x) \\ F^2[g_0(x)] = F^1[g_1(x)] &\xrightarrow{x=k} g_2(x) = g(-x) \\ F^3[g_0(x)] = F^1[g_2(x)] &\xrightarrow{x=k} g_3(x) = G(-x) \\ F^4[g_0(x)] = F^1[g_3(x)] &\xrightarrow{x=k} g_4(x) = g(x) \qquad (2) \end{aligned}$$

Any form of FRFT F^{α}, where α is a real number and represents transform order should obey the law of conservation of energy. In Weighted Fractional Fourier Transform (WFRFT), the weighting coefficients w_l are expressed as in eq. (2).

$$F^{\alpha}[g(x)] = w_0(\alpha)g(x) + w_1(\alpha)G(x) + w_2(\alpha)g(-x) + w_3(\alpha)G(-x), \qquad (3)$$

$$w_l(\alpha) = \cos\left(\frac{(\alpha-1)\pi}{4}\right)\cos\left(\frac{2(\alpha-1)\pi}{4}\right)\exp\left(\frac{3(\alpha-1)\pi i}{4}\right)$$

$$(l\ /\ 0,1,2,3) \qquad (4)$$

$w_l(\alpha)$ represent periodic functions, that makes WFRFT exhibit same periodicity property as that of FT. Hence, α is defined over the range [-2, 2] or [0, 4].

B. 4-WFRFT Digital Communication System

Here the DFT/IDFT is replaced by WFRFT/IWFRFT[9]. The 4-WFRFT is realized as shown in fig.1 . The data symbols are divided to four sub-channels using a serial to parallel converter. The Sub channels- "Sub channel 1" and "Sub channel 3" uses DFT/IDFT for the OFDM modulation process. The "Subchannel0" and "Subchannel2" act as "Inverse Module " without DFT/IDFT module which accomplish a Single Carrier modulation process. Choosing optimal value of α helps the transition between SC and MC smooth.

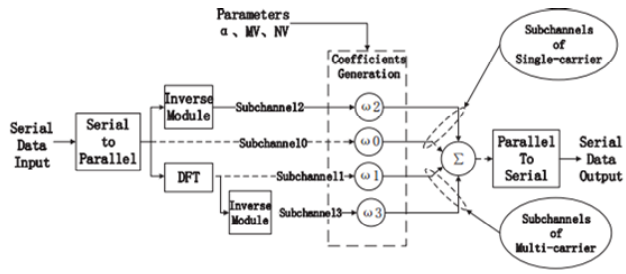

Fig.1 Realization of W-FRFT

α=1 transforms the system into an OFDM system whereas α=0 transforms the system into a single carrier system.

III. PROPOSED METHOD

A. Transmitted Signal

N-point Fast Fourier Transform (FFT) is used for multi-carrier modulation in WFRFT systems. The transformed baseband signal after IWFRFT and cyclic prefix (CP) insertion is given as :

$$S_0(n) = \omega_0 X_0(n) + \omega_1 \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} X_0(k)e^{-j\frac{2\pi}{N}kn} + \omega_2 X_0(-n) + \omega_3 \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} X_0(k)e^{j\frac{2\pi}{N}kn} \qquad (5)$$

Where n ϵ {0,..., N-1}, $X_0(n)$ is the signal without subcarrier modulation. While $X_0(k)$ is the k^{th} modulated subcarrier signal, w_l is determined by (3), and α is the fractional factor ranging from 0 to 2.

B. MIMO-OFDM System Model

A MIMO-OFDM system with Q transmit antennas and L receive antennas , denoted as Q*L system.

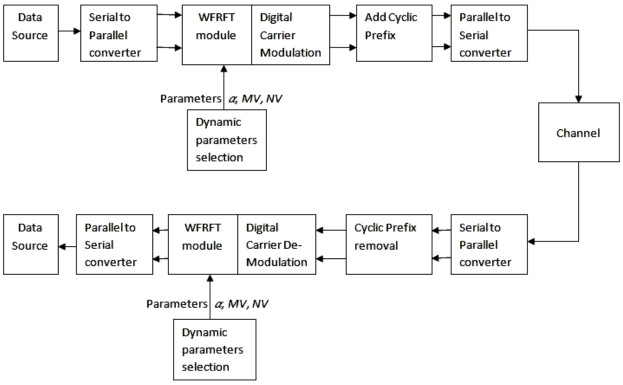

Fig.2 A WFRFT - OFDM System for a SISO system

For a single-antenna system, i.e., SISO, the transmitted signal s(k) is converted from serial to parallel stream. The signals are WFRFT modulated after digital modulation using some modulation schemes like Binary Phase Shift Keying (BPSK) or Quadrature Amplitude Modulation (QAM). An OFDM symbol s_n, of length N+L is generated by adding the last L samples to the data sample of length N as cyclic prefix(CP). Frames are built and is sent through the channel.

The received signal with carrier frequency offset and AWGN can be given as in eq.(6):

$$r(k)=s(k)\ \exp(j2\pi k\mathcal{E}/N)+w(k) \tag{6}$$

Where $w_p(k)$ is the p^{th} receive antenna with zero mean and σ^2 , variance. $\mathcal{E}$ represents the CFO normalized to the inter-carrier spacing.

CFO estimation involves two steps namely: Acquisition and Tracking. Acquisition can be used to achieve better estimation range, where as tracking can be used to attain better estimation accuracy.

C. Maximum Likelihood Estimator

The method does frequency estimation and correction using MLE Algorithm.

The ML estimation for θ and $\mathcal{E}$ can be estimated by maximizing the argument Λ (θ, $\mathcal{E}$) in eq. (7) [10]. Here we assume that r is a jointly Gaussian random vector and the log-likelihood function can be written as eq. (8-10):

$$\Lambda\,(\theta\,,\varepsilon)=|\gamma(\theta)|\cos\big(2\pi\varepsilon+<\gamma(\theta)\big)\,\text{-}\rho\emptyset(\theta)$$

(7)

$$\gamma(m)\underline{\Delta}\sum_{k=0}^{N-1} r(k)\,r^*(k+N)$$

(8)

$$\emptyset(m)\underline{\Delta}\frac{1}{2}\sum_{k=m}^{m+L-1}|r(k)|^2|r(k+N)|^2$$

(9)

$$\rho\underline{\Delta}\left|\frac{E\{r(k)r^*(k+N)\}}{\sqrt{E\{|r(k)|^2\}E\{|r(k+N)|^2\}}}\right|$$

(10)

Where ρ is the magnitude of the correlation coefficient between r(k) and r(k+N) eq. (11).

$$\rho=\frac{\sigma_s^2}{\sigma_s^2+\sigma_n^2}=\frac{SNR}{SNR+1} \tag{11}$$

The joint Estimation of θ and ε is given by Equation (12) and (13)

$$\widehat{\theta_{ML}}=\arg\max\{|\gamma(\theta)|\text{-}\rho\Phi(\theta)\} \tag{12}$$

$$\widehat{\varepsilon_{ML}}=\text{-}\frac{1}{2\pi}\angle\gamma(\widehat{\theta_{ML}}) \tag{13}$$

The received signal r_n is sent to the ML estimator and then both the integer frequency offset θ and fractional frequency offset $\mathcal{E}$ are given [11]. After the timing and frequency correction, CP is removed from r_n and the remaining signal of length N is given to the WFRFT block as input. The WFRFT output y_n is multiplied by the complex signal p_n to generate the signal q_n for BPSK demodulation and q_n is divided into the real and imaginary parts and then both are sent to the phase difference generator which has the transfer function of f(x).

After that the output of imaginary part is multiplied by j and adds to the output of real part to form the signal u_n eq. (14)

$$u_n=real(f(q_n))+j.\,imag(f(q_n)) \tag{14}$$

The phase difference φ_n eq. (15) is given as:

$$\varphi_n=angle\ (q_n)\ \text{-}\ angle\ (u_n) \tag{15}$$

A low- pass filter is used to eliminate the noise in φ_n and output the signal φ_n, which is the phase difference between the received signal and its most proximate real value. φ_n could not be used to fix the phase error eq. (16) directly and it needs an angle to complex conversion:

$$p_n=\cos(\varphi_n')+j.\sin(\varphi_n') \tag{16}$$

IV. RESULTS

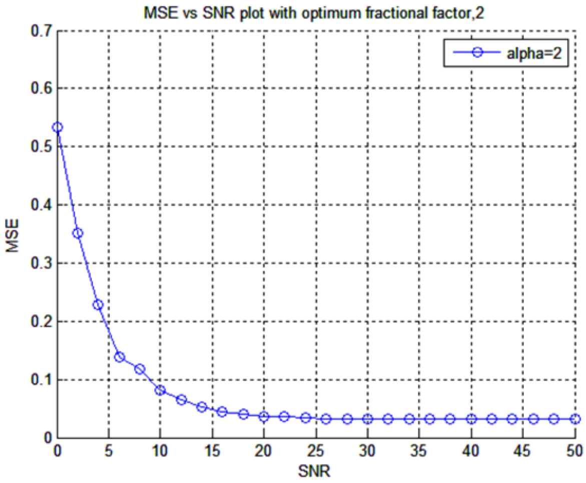

Fig. 3. MSE against SNR plot for fractional factor, alpha=2

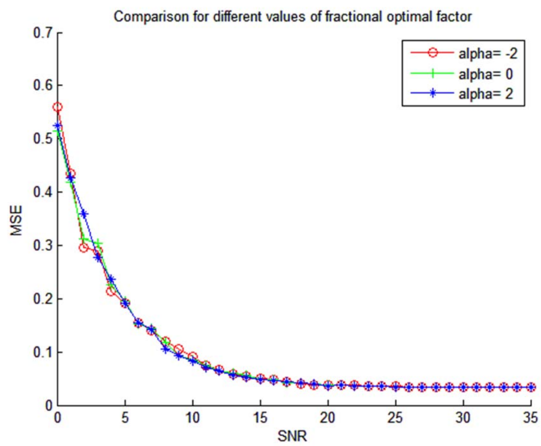

Fig. 4. Comparison showing different alpha values. The red line shows MSE vs. SNR plot for alpha=-2. The green line shows MSE vs. SNR plot for alpha=0. The blue line shows MSE vs. SNR plot for alpha=2.

The figures Fig. 3. and Fig. 4. show the simulated results. Fig. 3. shows MSE vs. SNR plot for an optimal fractional factor, α =2. The mean square error goes on decreasing, as the value of SNR increases. From the decreasing exponential function, it is clear that the proposed method provides a better estimate of frequency offset.

The figure Fig. 4. shows MSE vs. SNR plot for different values of α, say -2, 0 and 2 respectively. From the figure, it can be seen that MSE is lowest for α=0 initially and it is highest for α=-2. As SNR increases, the mean square error decreases to a lower value.

The figure Fig. 5. shows the CDF plot of Peak Average Power Ratio (PAPR). By choosing an optimum value for fractional factor, the method can be used to obtain more accurate estimates and results. The method has been simulated for 2x2 MIMO-OFDM systems with AWGN channel conditions.

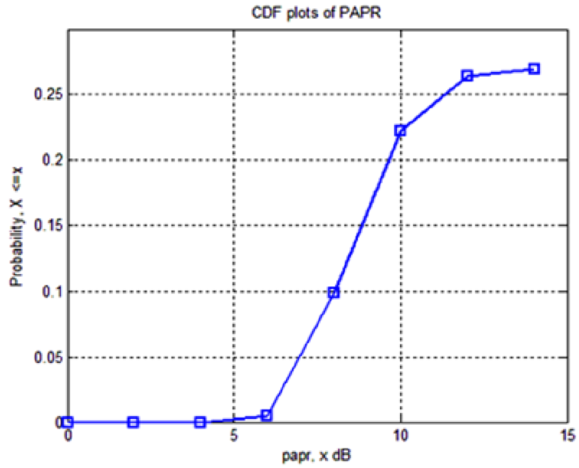

Fig. 5. CDF plot of PAPR

V. CONCLUSION

The paper proposes a method using WFRFT for MIMO-OFDM systems for CFO estimation. The method uses MLE algorithm for offset estimation and correction. It can be used to suppress ICI. The cost of computing α^{th} order WFRFT is given by O (Mlog2M+4M) [13]. The improvement in performance accounts for the complexity of WFRFT systems for CFO estimation.

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