Determination of Propagation Constant Using 1D-FDTD with MATLAB

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***Abstract*— this report mainly concentrates on determination of propagation constant of a planar transmission lines such as Microstrip line and coplanar wave guide (CPW) by 1D- FDTD using conventional MATLAB code. Apart from that, this article presents study of basic phenomenon like reflection at an interface between two media, and design of material absorbers. The fields Ey and Hz are simulated along the line Y=Z=0 i.e. propagation along X-axis. Implementation of source (Gaussian pulse, sinusoidal) and effects of various boundaries such as Mur ABC, PML on incident/scattered/total fields are investigated. Also presented a comparison plot of phase constant by using CST microwave studio and MATLAB.**

***Index Terms*—1D- FDTD, Microstrip line, CPW, Mur ABC, PML, Gaussian pulse, Sinusoidal.**

I. INTRODUCTION

The Finite Difference Time Domain (FDTD), Numerical method of a differential type time domain approach is a versatile method requiring almost no preprocessing of Maxwell's equations to arrive at governing equations [1]. It directly approximates the differential operators in the Maxwell's curl equations, on a grid staggered in space and time. The impetus for the progress in FDTD method is provided by the unique contribution of Kane Yee [2].

We choose MATLAB for coding because of comprehensive library of graphic routines. This often critical to understand the working of a FDTD algorithm.

E-and H- fields are computed on a grid, with a marching on-in-time, discretization of time, with field components are offset by $\frac{\Delta x}{2}$ relative to each other and E & H fields are evaluated at $\frac{\Delta t}{2}$ apart in time, where Δx and Δt are spatial and temporal discretization respectively [3].

This paper explains the above procedure in step by step process: Section II describes the reduction of 3D Maxwell's equations to 1D form by considering the propagation in X-direction. Section III describes and verifies the Absorbing Boundary Conditions (ABC). Section IV describes the transmission and reflection of E-field between two media. Section V describes and verifies the result, propagation constant (β) with CST Microwave Studio.

II. FORMULATION

A. *Reduction of 3D Maxwell's equations to 1D.*

The source free (J=0), 3D Maxwell's equations are

$$\nabla \times \mathrm{E} = -\frac{\partial B}{\partial t} \tag{1}$$

$$\nabla \times \mathrm{H} = \frac{\partial D}{\partial t} \tag{2}$$

In the case of 1D FDTD, we assume TEM wave propagation along X-direction i.e.

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial z} = 0$$

From pointing theorem, wave propagation along X- direction then E-field is in Y-direction and H-field is in Z-direction. On solving equations (1) and (2) with above assumption,

$$\epsilon \frac{\partial E_y}{\partial t} = -\frac{\partial H_z}{\partial x} \tag{3}$$

$$\mu \frac{\partial H_z}{\partial t} = -\frac{\partial E_y}{\partial x} \tag{4}$$

Substituting partial space derivative of (3) in (4) or vice versa produces 1D wave equation.

$$\left[\frac{\partial^2}{\partial x^2} - \mu\epsilon \frac{\partial^2}{\partial t^2} \right] \varphi = 0 \tag{5}$$

Where φ represents either E_y or H_z. In free space $\mu = \mu_0,\ \epsilon = \epsilon_0$ and (5) takes in the familiar form,

$$\left[\frac{\partial^2}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right] \varphi = 0 \tag{6}$$

Where $c = \frac{1}{\sqrt{\mu_0\ \varepsilon_0}}$ velocity of light in vacuum [4].

B. *1D time update equations.*

In FDTD analysis, equations (3) and (4) has to represent in difference approximations. The central difference equation is used for highest accuracy and is defined by [5],

$$\frac{\partial f}{\partial x} = \frac{f\left(x_0+\frac{\Delta x}{2}\right)-f\left(x_0-\frac{\Delta x}{2}\right)}{\Delta x} + O(\Delta x^2) \quad (7)$$

So, having second order accuracy.

Representing equations (3) and (4) in the form of (7),

$$\frac{Ey_i^{n+1/2}-Ey_i^{n-1/2}}{\Delta t} = \frac{-1}{\epsilon}\,\frac{Hz_{i+1/2}^{n}-Hz_{i-1/2}^{n}}{\Delta x} \quad (8)$$

$$\frac{Hz_{i+1/2}^{n+1}-Hz_{i+1/2}^{n}}{\Delta t} = \frac{-1}{\mu}\,\frac{Ey_{i+1}^{n+1/2}-Ey_i^{n+1/2}}{\Delta x} \quad (9)$$

Where i and n are integers representing space and temporal respectively. The time advanced equations are,

$$E_{y_i}^{\,n+1/2} = E_{y_i}^{\,n-1/2} - \frac{\Delta t}{\epsilon}\,\frac{Hz_{i+1/2}^{n}-Hz_{i-1/2}^{n}}{\Delta x} \quad (10)$$

$$H_{z_{i+1/2}}^{\,n+1} = H_{z_{i+1/2}}^{\,n} - \frac{\Delta t}{\mu}\,\frac{Ey_{i+1}^{n+1/2}-Ey_i^{n+1/2}}{\Delta x} \quad (11)$$

The above expressions can be coded in MATLAB as follows,

```
for   n= 1: nsteps
        for i= 1: Imax
                ey(i) = ey(i) + (Δt/Δx ∗ ϵ)*(hz(i-1)-hz(i));
                hz(i) = hz(i) + (Δt/Δx ∗ μ)*(ey(i)-ey(i-1));
        end
end
```

C. *Excitation (source) of grid*

A Gaussian pulse [6] is often used in FDTD because of its smooth amplitude variations and large frequency content.

$$\text{Ey}(t) = \exp\left[\frac{-(t-to)^2}{T^2}\right] \quad (12)$$

Where t_0 = Centre of pulse, T= width of pulse.

```
for   n= 1: nsteps
   time loop Pulse in time (Gaussian, sinusoidal)
        for i= 1: Imax      % main fdtd loop
                ey(i) = ey(i) + (Δt/Δx ∗ ϵ)*(hz(i-1)-hz(i));
                hz(i) = hz(i) + (Δt/Δx ∗ μ)*(ey(i)-ey(i-1));
        end
end
```

if it is sinusoidal source then

Ey (t) = sin (2*pi*f*n*Δt). (13)

III. BOUNDARY CONDITIONS

A. Mur's 1st order Absorbing Boundary Condition.

If a plane wave traveling along the $-x$ direction then this wave will not suffer reflection at x = constant plane if

$$\varphi(x, t) = \varphi(x - \Delta x, t + \Delta t)$$

The Taylor's series expansion of

$$\varphi(x - \Delta x, t + \Delta t) = \varphi(x, t) - (\partial \varphi/\partial x)\,\Delta x + (\partial \varphi/\partial t)\,\Delta t \ldots$$

And impose $\Delta x = c\,\Delta t$ (courant stability limit) and on solving, the Mur's boundary at node N+1 or at the last E-field node at right is [7]

$$E_{y_{N+1}}^{\,n+1} = E_{y_N}^{\,n} + \frac{C\,\Delta t-\Delta x}{C\,\Delta t+\Delta x}\left(E_{y_N}^{\,n+1} - E_{y_{N+1}}^{\,n}\right)$$

To place a Mur's boundary at node1 or at the first E-field node at left is

$$E_{y_1}^{\,n+1} = E_{y_2}^{\,n} + \frac{C\,\Delta t-\Delta x}{C\,\Delta t+\Delta x}\left(E_{y_2}^{\,n+1} - E_{y_1}^{\,n}\right)$$

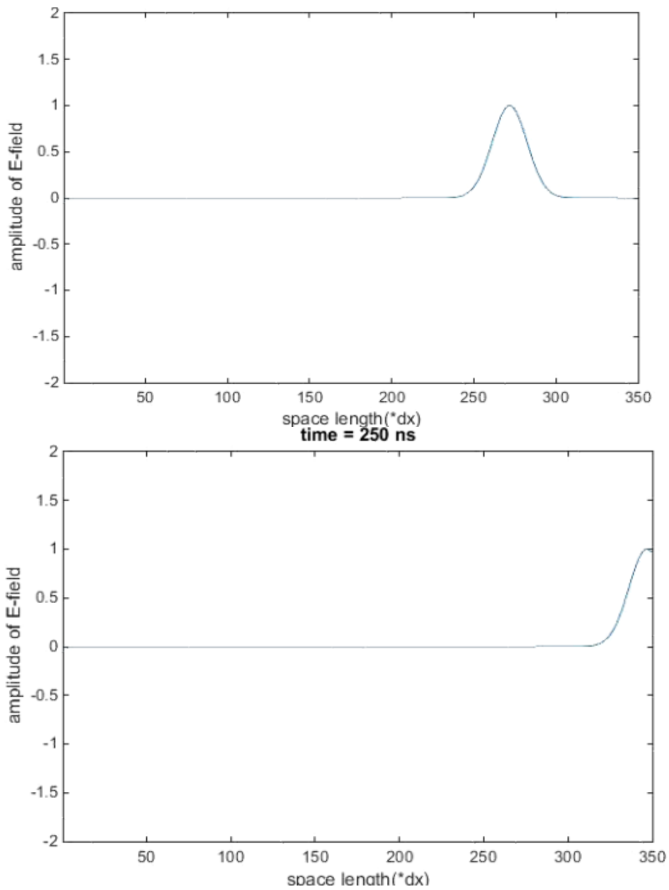

Fig (1). Gaussian pulse excitation at left most node and pulse is absorbed by right most node due to Mur ABC

B. Perfectly matched layer (PML) or Material absorbing boundary conditions

An artificial dielectric for the lossy medium with parameters (μ', ε', σ^*) where σ^* is the fictitious magnetic conductivity of the medium.

The material parameters are selected such that when a wave is incident normal to the interface separating free space and lossy medium, there should be no reflection.

This is possible only if the intrinsic impedance of the two media is same; that is, $Z_0 = Z_m$ where Z_0 is the free space impedance = $\sqrt{\mu_0/\varepsilon_0}$ and Z_m is the impedance of the lossy medium defined as $Z_m = \sqrt{\frac{\mu - j\sigma *}{\epsilon - j\sigma}}$ on solving, the matching condition gives $\frac{\sigma *}{\mu 0} = \frac{\sigma}{\varepsilon 0}$. σ^* can be calculated from proper range of σ. Thus the update equations with PML are [8]

hz(i) = e* hz(i) – b*(ey(i+1)–ey(i))

$$\% \ b = \frac{dt}{dx} \frac{1}{\mu_0 \mu_r + \frac{\sigma * \Delta t}{2}} \qquad \% \ \mathrm{e} = \frac{\mu_0 \mu_r - \frac{\sigma * \Delta t}{2}}{\mu_0 \mu_r + \frac{\sigma * \Delta t}{2}}$$

ey(i) = d*ey(i)-a*(hz(i) -hz (i-1))

$$\% \ a = \frac{dt}{dx} \frac{1}{\varepsilon_0 \varepsilon_r + \frac{\sigma \Delta t}{2}} \ , \% \mathrm{d} = \frac{\varepsilon_0 \varepsilon_r - \frac{\sigma \Delta t}{2}}{\varepsilon_0 \varepsilon_r + \frac{\sigma \Delta t}{2}}$$

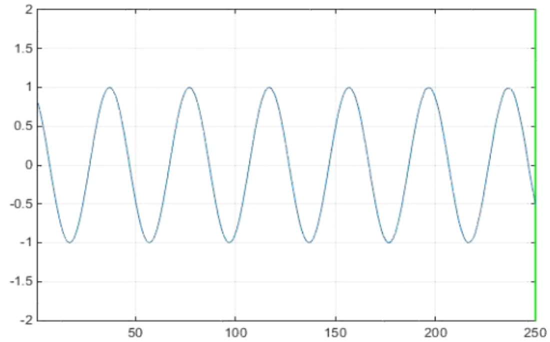

Fig (2). Propagation of CW sine wave and is absorbed by material absorber

IV. REFLECTION AT AN INTERFACE

By including the conductivity and dielectric constant of medium, the update equations are given by

hz(i) = hz(i) – b*(ey(i+1)–ey(i)) % $b = dt/(\mu_0 \ dx)$
ey(i) = d*ey(i)-a*(hz(i) -hz (i-1))

$$\% a = \frac{dt}{dx} \frac{1}{\varepsilon_0 \varepsilon_r + \frac{\sigma \Delta t}{2}} \ , \% \ \mathrm{d} = \frac{\varepsilon_0 \varepsilon_r - \frac{\sigma \Delta t}{2}}{\varepsilon_0 \varepsilon_r + \frac{\sigma \Delta t}{2}}$$

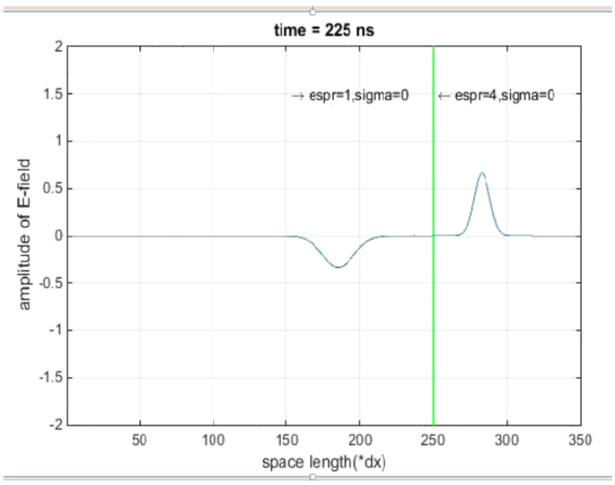

Fig (3) Unity amplitude pulse incident normally on dielectric medium ($\epsilon_r = 4$) from free space, reflected pulse magnitude=0.33 & transmission pulse=0.66.

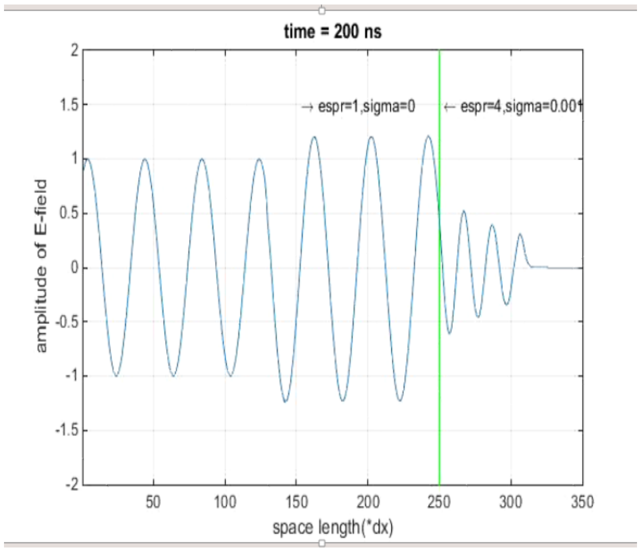

Fig (4) Propagation of CW sine wave (f=50MHz) through layered medium. The parameters Δx =15cm, α =0.5. The node at i=250, separates vacuum ($\epsilon_r = 1, \sigma = 0$) and lossy dielectric ($\epsilon_r = 4$, $\sigma = 0.001$S). The phase constant can be determined from guided wavelength λ_g and it is $\beta = \frac{2\pi}{\lambda_g}$. From the above figure, the distance between two successive peaks gives λ_g in respective medium.

V. RESULTS

A. β (Rad/m) versus frequency plot of Microstrip line

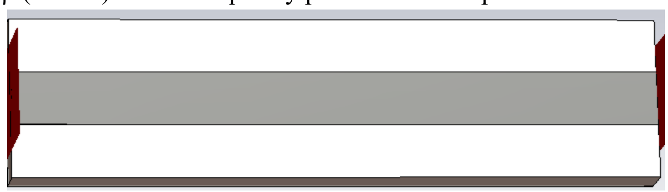

Fig (5) Microstrip line having trace width W=2mm, thickness of substrate=1.6mm, $\epsilon_r = 2.2$

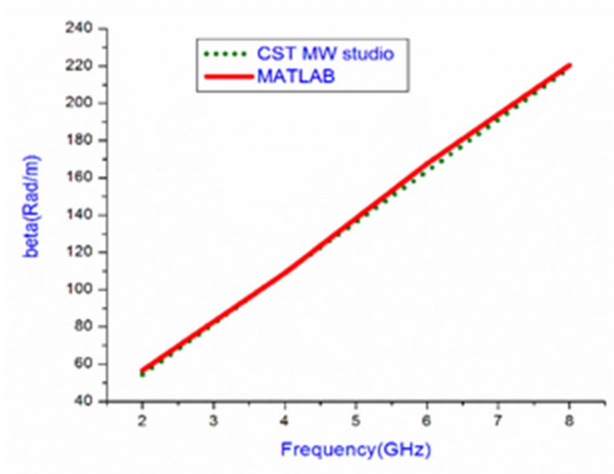

Fig (6) Comparison of β of above Microstrip line using FDTD and CST Microwave Studio

B. β (Rad/m) versus frequency plot of CPW

Fig(7) Coplanar Waveguide (CPW) having trace width=1.6mm, gap=2mm, ϵ_r=2.2, thickness of substrate=1.6mm

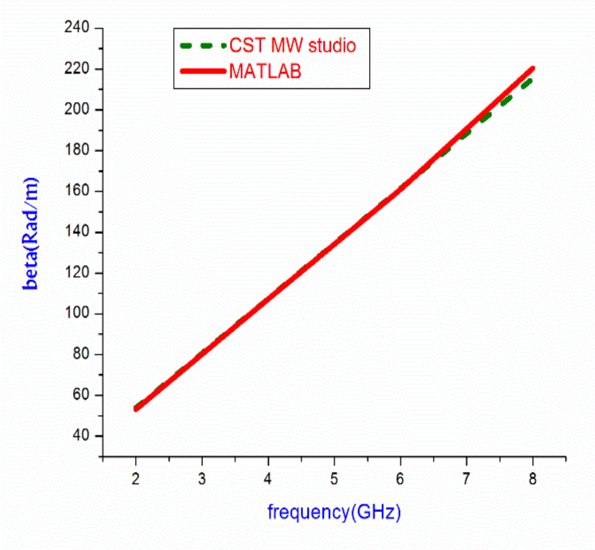

Fig (8) Comparison of β of above CPW using FDTD and CST Microwave Studio

VI. CONCLUSION

This paper successfully demonstrated the determination of propagation constant (β) by using FDTD algorithm with the help of MATLAB. Also reflection & transmission phenomenon of electromagnetic wave at an interface between two media

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