# Comparative Analysis of Offset Estimation Capabilities in Mathematical Sequences for WLAN

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Abstract — In this paper, the performance of different mathematical sequences used for data aided offset estimation in Multiple Input Multiple Output Orthogonal Frequency Division Multiplexing (MIMO OFDM) is analyzed. If the synchronization system uses multiple preambles, side lobes of preamble can degrade timing synchronization performance and care to be taken to minimize them. The synchronization performance of binary sequences PN and Barker are compared with complex exponential CAZAC and Zadoff Chu sequences. The parameters used for evaluation includes timing metric based on Schmidl and Cox algorithm, Minn algorithm and integrated side lobe levels of preambles. As the accurate estimation and correction of timing and frequency offset is a crucial factor in Wireless Local Area Networks (WLAN) using MIMO OFDM, the selection of apt sequences in preamble structure have great importance for maintaining synchronization.

Keywords—MIMO OFDM, Timing Metric, Schmidl and Cox algorthm, Minn's Algorthm, Barker, Zadoff Chu,ISL value, side lobes.

# I. INTRODUCTION

The rapid growth in wireless communication demands enhanced data rate, capacity and high quality multimedia services. The data throughput can be improved significantly without additional bandwidth or transmit power by placing multiple antenna elements on transmitter / receiver or on both, the technology referred as Multiple Input Multiple Output (MIMO). Increased spectral efficiency through spatial multiplexing, improved link reliability through antenna diversity and high channel capacity are the main attractions of MIMO. Sufficient robustness to radio channel impairments can be provided by selecting Orthogonal Frequency Division Multiplexing modulation (OFDM) scheme. OFDM is a multicarrier system where the modulation is attained through Inverse Fast Fourier Transform (IFFT) and simultaneous transmission of data through closely packed orthogonal carriers is allowed utilizing parallel processing techniques. Dividing the available spectrum into several orthogonal subchannels whose bandwidth is less than coherence bandwidth of the channel, a frequency selective fading channel can be converted into a collection of flat fading subchannels.

Combining the above two technologies, MIMO OFDM systems increases data rate as well as improve quality

of service against fading which is one of the main challenges in wireless communication.

OFDM have been adopted in wireless LAN standards since 1999. IEEE802.11a first used OFDM for a data rate upto 54 Mbps at a radiofrequency band 5-6MHz and IEEE802.11n first used MIMO OFDM in 2009 to achieve a rate upto 600 Mbps in 40 MHz mode with four antennas.

The benefits of MIMO OFDM strongly depend on perfect synchronization and all the transceiver systems based on OFDM / MIMO OFDM are highly sensitive to timing and frequency offsets. It is required that receiver must be synchronized to both the time frame and the transmitted frequency. But in the case of all practical wireless communication systems, frequency and timing offsets are created due to discrepancies between transmitter and receiver oscillators and Doppler shifts introduced by nonlinear channels. The destructive effects caused by the offsets are reduction in signal amplitude and the introduction of Inter Carrier Interference (ICI) caused by loss of orthogonality between subcarriers. As the presence of these offsets degrades the system performance in a non-graceful way, the estimation and correction of offsets in MIMO OFDM have been a topic of active research since last decade.

Different approaches for offset estimation [1-4] include data-aided synchronization which uses preambles before OFDM signals, blind synchronization methods which exploit the inherent characteristics in OFDM signals like redundancy in the cyclic prefix and semi blind synchronization that includes pilot symbols in addition to cyclic prefix. This paper considers offset estimation using repeated preamble sequences for comparing the performance of different mathematical sequences because of its accuracy and reduced complexity in estimating the errors [9-11]. Eventhough the blind and semi blind techniques are easier to apply and save bandwidth, they achieve the target at the expense of increased computational complexity and degraded performance.

This paper is organized as follows. The offset estimation methods in OFDM and conventional methods like Schmidl &Cox Algorithm and Minn's algorithm are reviewed in Section II. Different mathematical sequences used in offset estimation are briefly introduced in Section III. The performance of different mathematical sequences is evaluated

using MATLAB simulation and it is given in Section IV. Finally, the paper is concluded in Section V.

### II. OFFSET ESTIMATION IN OFDM

Paul H. Moose evaluated the degradations in the OFDM performance due to timing and frequency offsets and proposed a frequency synchronization algorithm using two identical repeated training symbols and maximum likelihood principles [4]. Eventhough the algorithm estimated frequency offsets, the range of estimation was limited to half the subcarrier spacing of repeated symbols. To achieve timing synchronization, the accurate identification of frame starting point at the receiver is crucial. Keller [8] suggested a maximum correlation criterion for frame detection. But the peaks due to the presence of high Peak to Average Power Ratio (PAPR) values in OFDM signals was sometimes wrongly counted as peak in correlation metric which resulted in wrong detection of frames. Schmidl and Cox [1] suggested a time synchronization algorithm based on a metric after normalising the squared value of correlation by the power of second training sequence. This method eliminated the above mentioned problem caused by high PAPR and is regarded as reference for frame detection and coarse timing. The procedure in [4] was extended by Schmidl and Cox algorithm (SCA) to achieve a higher acquisition range. If the time synchronization is not correct, there occurs loss of information. It is very important that the OFDM receiver should know where to start the starting of the OFDM symbol from. So determining the starting of OFDM symbol must be given as priority as finding the initial position of data packet. The main idea in the SCA is the use of coarse and fine timing metric functions to perform timing synchronization. Coarse timing synchronization detects the beginning of frame where the fine timing synchronization detects the starting of useful data in the frame. Here a preamble structure of type [S S] is selected where S represents the sequence under evaluation. The acquisition is achieved in two steps as given in figure 1 through the use of a two symbol training sequence.

Frame detection is carried out by searching for a symbol in which the first half is identical to the second half in time domain. After partially correcting the carrier frequency offset, the correlation between the two repeated preambles is computed. The two preamble sequences remain almost identical even after passing through the channel. Anyway there will be a phase difference between them which is induced by the carrier frequency offset. The conjugate of a sample from the first half is multiplied bythe corresponding sample of the second half and phase angle of the product is calculated which will be approximately  $\pi\Delta f$  [5]. The correlation window is then slided through the length of the symbols and the timingmetric corresponding to the index corresponding to the maximum of timing metric is considered as the best timing point. The phase of the correlation metric at the best timing point is estimated as the frequency offset. If there are L samples in one half of the training sequence, the sum of product of conjugate of sample with corresponding pair in the next half can be represented by following equation.

$$P(d) = \sum_{m=0}^{L-1} (r_{d+m}^* r_{d+m+L})$$
 (1)

The energy in the received second half-symbol is given by R(d).

$$R(d) = \sum_{m=0}^{L-1} |r_{d+m+L}|^2$$
 (2)

A timing metric can be defined as M(d).

$$M(d) = \frac{|P(d)|^2}{(R(d))^2}$$
 (3)

The metric reaches a plateau when the window of L samples completely overlaps the first training symbol and the start of the OFDM frame can be taken anywhere in the spread of that plateau.

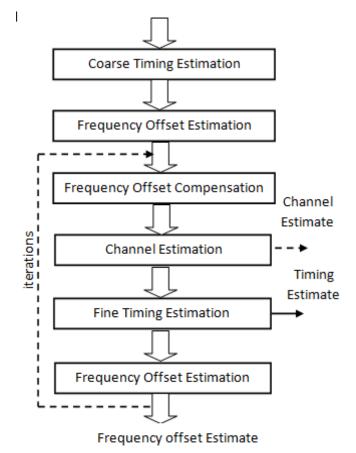


Fig. 1. Offset estimation procedure

Minn's method [13] overcomes the broad plateau issue by selecting a structure [ S S –S –S]. The timing metric is same as that of equation 3.

$$P(d) = \sum_{k=0}^{1} \sum_{m=0}^{L-1} r^* (d + 2Lk + m) \cdot r(d + 2Lk + m + L)$$
(4)

$$R(d) = \sum_{k=0}^{1} \sum_{m=0}^{L-1} |r(d+2Lk+m+L)|^2$$
 (5)

The correlation and synchronization properties of different mathematical sequences are analyzed using both SCA and Minn's algorithm in this paper.

### III. MATHEMATICAL SEQUENCES AIDING SYNCHRONIZATION

The preamble based synchronization techniques are equipped with a preamble sequence in the header of each OFDM frame and this sequence indicates the starting position of the frame at the receiver. Selection of this mathematical sequence is important so as to derive optimised offset estimation. They are desired to posses good correlation properties, computational complexity and low Peak to Average Power Ratio (PAPR). The preamble sequences must be selected such that their Fourier transforms are of constant magnitude and auto correlation of sequences from various transmitters to be an impulse. Orthogonality of preamble sequence to be maintained to ensure timing synchronization. Zero autocorrelation property implies that sequence is orthogonal at nonzero lags. Sequences with low aperiodic autocorrelation side lobes shows to have optimised Cramer Rao bound since the fluctuations in the received energy is found minimum for them in frequency selective Rayleigh fading channels. Also sequences with low power in side lobes have more immunity to inter symbol interference. So Peak Side Lobe level (PSL) and Integrated Side Lobe level (ISL) can be used as metrics to measure the goodness of autocorrelation properties of the sequence [12].

For a sequence  $x = [x_1 \ x_2, \dots, x_n]$ , aperiodic autocorrelation  $C_k$ , PSL and ISL are defined as follows.

$$C_K = \sum_{n=1}^{N-k} x_n^* x_{n+k}, \qquad k = 0, 1, \dots, N-1$$
 (6)

$$PSL = \max\{|C_k|\}_k^{N-1} \tag{7}$$

$$ISL = \sum_{k=1}^{N-1} |C_k|^2$$
 (8)

Fredrick Tufvesson [6] have shown that Pseudo Noise (PN) based preamble gives better detection properties, sharp synchronization peaks and lower false detection probability.

In 1990, M Grayson and M Darnell reported that bipolar sequences developed by R H Barker posses excellent autocorrelation functions which are either zero or negative at all time shifts except zero and can be considered as optimum preambles. Barker Sequence is a subset of PN sequence of N values of +1 and -1 and can be used for effective frame synchronization [7].

Preamble sequences of constant amplitude can be generated by Constant Amplitude Zero Autocorrelation waveforms (CAZAC) which is a periodic complex valued signal with modulus one and out of phase periodic autocorrelation equal to zero.

Zadoff Chu Sequence or Chu sequence is a special class of CAZAC with special properties like orthogonality and zero autocorrelation of root sequence with its cyclic shifted versions.

Preamble using Chu sequence is expressed as

$$x(n) = \begin{cases} e^{\frac{j\pi Mn(n+1)}{N}}, & \text{if } N \text{ is odd} \\ e^{\frac{j\pi Mn^2}{N}}, & \text{if } N \text{ is even} \end{cases}$$
(9)

where N is the length of CAZAC sequence, M is a natural number relatively prime to N and  $0 \le n \le N-1$ .

Long Term Evolution (LTE) standard make use of Chu signals as Primary Synchronization Signals (PSS), Reference Signal (RS) in both uplink and downlink and also in Physical Uplink Control Channel (PUCCH). It enhances power amplification properties of LTE modem signal as the time variations are less. Also channel estimation at the receiver become more flexible due to small frequency variation.

### IV. RESULTS

Simulations have been run in MATLAB to evaluate metric performance of different mathematical sequences at both high and low signal to noise ratio levels. Preamble structure proposed by Schmidl and Cox was initially selected to test the frame detection capabilities of different signals. Selecting the structure [S S] as the platform, the ratio of correlations was selected as timing metric as given by equation 3. Results shows that at both high and low SNR levels Chu, CAZAC and Barker sequences are exhibiting satisfactory synchronisation properties.

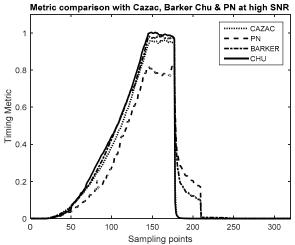


Fig. 2. Performance comparison of CAZAC, Chu, PN and Barker sequences at high SNR

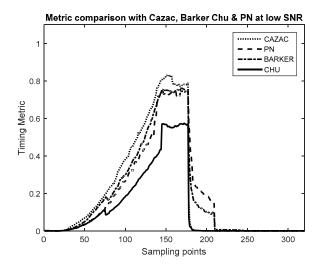


Fig. 3. Performance comparison of CAZAC, Chu, PN and Barker sequences at low SNR

To overcome the deleterious effects due to timing offsets, the starting of the frame to be identified accurately at the receiver which can be achieved by sequences with excellent correlation properties. As the above mentioned structure suffers from plateau problem, there is ambiguity in determining right timing position. So in the next simulation a Minn's structure is used as the platform to evaluate the sequences. Here also the result shows that Barker sequence is showing a high performance similar to that of complex exponential Chu sequence.

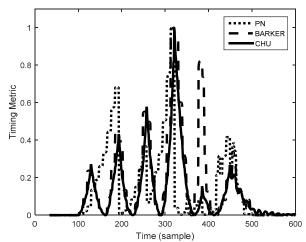


Fig. 4. Comparison of sequences using Minn's algorithm

Next the performance of different mathematical sequences is compared using a metric success measure. The timing metric based on correlation ratio is detecting the arrival of data packets at even low signal to noise ratios using CAZAC, Chu and Barker. But the PN sequence is giving satisfactory detection performance only at high signal to noise ratios.

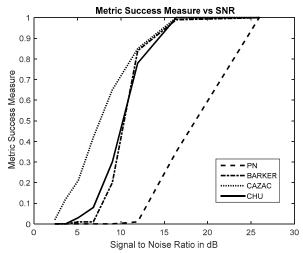


Fig. 5. Analysis of sequences using Metric Success Measure

Whether binary or complex signal, an important criterion to select a sequence for synchronisation is its autocorrelation property. The autocorrelation properties of different sequences are compared using the parameter integrated side lobe level (ISL) and sequences with minimum preamble is preferred to attain perfect synchronization. Integrated side lobe level is mathematically computed for Barker, Chu and CAZAC sequences.

TABLE 1.COMPARISON OF ISL VALUES FOR DIFFERENT SEQUENCES AT DIFFERENT LENGTH

Length of sequence	Name of sequence		
	Barker	Chu	CAZAC
32 bits	2300	1135	1079
64 bits	16808	4572	4256
128 bits	120032	18467	16841
256 bits	928140	77132	66834

The table1 shows that side lobe energy is increasing with number of bits and optimum ISL values are obtained for CAZAC sequence.

### V. CONCLUSION

This paper analyzed and compared the performance of different mathematical sequences and explored the parameters to be tested before considering them as candidates for preambles in offset estimation systems. The sequences are analyzed using Schmidl and Cox algorithm, Minn algorithm and ISL values. Barker sequence and PN sequence with relatively less computational complexity is compared with complicated exponential sequences like CAZAC and Chu. Eventhough ISL value is comparatively higher for Barker sequence it is showing good performance as CAZAC and Chu in Schmidl and Minn timing metrics. Anyway overall performance is found to be better for CAZAC sequence.

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