Energy Detection of Unknown Signals with Diversity Reception in $\lambda - \mu$ Fading Channel

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***Abstract*—In this paper, we obtained the closed form analytical expression for the average detection probability of the energy detector in terms of series form expression over $\lambda - \mu$ fading distribution. We start with the no-diversity case, then further results are extended to two diversity reception cases such as square-law-selection (SLS) and collaborative detection case. Since arriving analytical expression is used for the obtaining the different fading distributions of $\lambda - \mu$ fading channel. The performance analysis of energy detection is also discussed with the receiver operating characteristics (ROC) curves. Our closed form expression is validated by numerical results.**

***Index Terms*—Spectrum sensing, Diversity, Average detection probability, $\lambda - \mu$ Fading.**

I. Introduction

The detection of unknown signal is one of the key challenges in wireless communications. The cognitive radio (CR) can solve this problem by allowing the secondary users (SU) unscrupulously access the licensed band when the primary users (PU) are absent. This kind of functionality can be realized in the form of spectrum sensing with a matched filter, a cyclostationary feature detection and an energy detection techniques [1]. In this work, we used the energy detection (ED) technique, which is a non-coherent type spectrum sensing technique that evaluate the received signal energy level at the unlicensed users over an observation time window and after comparison with a predefined threshold value to decide the presence or absence of the unknown signal.

In conventional studies, quite a lot predictable statistical representations were well defined the analysis of ED-based spectrum sensing for distinct communication and fading scenario due to low implementation complexity and no requirements for knowledge of the signal. In [2], author addressed the problem of unknown deterministic signal detection over a flat band-limited additive white Gaussian noise (AWGN) channel by ROC curves and that decision statistics follows the central chi-square and non-central chi-square distribution due to absence or presence of PU. The closed form expressions for the average probability of detection under Rayleigh, Rician and Nakagami distributions for single and different diversity combining scenarios are derived in [3]. Whereas the performance in cooperative-MIMO spectrum sensing and in relay-based cognitive radio networks has been analyzed by [4]–[8]. The effect of multipath fading and shadowing for signal detection were analyzed by K and KG channel models in [9]. The performance of ED over Weibull fading channels was reported in [10]. A semi-analytic approach for analyzing the performance of detection capability of unknown deterministic signals was derived in [11], by the moment-generating function (MGF) method. This method was employed in the case of maximal-ratio combining (MRC) over Rayleigh, Rice and Nakagami-m fading in [11] as well as in the useful case of correlated Rayleigh and Rician fading channels in [12]. A novel expression for average probability of detection for no-diversity and for different diversity scenario in low signal-to-noise-ratio (SNR) over flexible $\eta - \mu$, $\kappa - \mu$ and $\kappa - \mu$ extreme fading channels have been analyzed in [13]–[15].

The $\lambda - \mu$ distribution is a generalized fading model that can be used to represent the better small-scale variation of the fading signal [16]. The $\lambda - \mu$ fading channel includes the Hoyt, the Nakagami-m, the Rayleigh, and the One-sided Gaussian fading channel as special cases. It has been observed that it provides better fitting to experimental data outperforms by the other fading distributions such as Rayleigh, Nakagami-m, and Weibull. On spite usefulness of the model, no related research work for ED over this channel is reported in the literature. In this work, we investigate the performance of ED over $\lambda - \mu$ Distribution. The closed form expression of average probability of detection is derived for no diversity case and then extended to the square-law-selection (SLS) and collaborative detection diversity cases.

The remaining of the paper is organized as, section II describes the considered system and channel model. Section III describes the analysis of the average detection probabilities over generalized $\lambda - \mu$ fading channel. Section IV discusses the numerical results and finally the Section V concludes the paper.

II. System and Channel Model

A. Energy Detection (ED)

In general, The received signal waveform at the secondary user can be represented by using the binary hypothesis model as [3] given below

$$y(t)=\begin{cases} n(t) & ;H_0 \\ h.s(t)+n(t) & ;H_1 \end{cases} \tag{1}$$

Where $s(t)$ is an unknown deterministic signal, h denotes the channel coefficient amplitude and $n(t)$ is an additive white Gaussian noise (AWGN) process. The hypotheses H_0 and H_1 represents the absence and presence of the primary signal. The received signal $y(t)$ is passed to band pass filter (BPF) for filtering then the output of the filter squared, integrated and multiplied by $2/N_0$, which is expressed as $y=(2/N_0)\int_0^T |r(t)|^2 dt$, where N_0 and T are the one sided power spectral density and the time interval. Under AWGN, the test statistic follows the central chi-square distribution under H_0 and noncentral chi-square distribution under H_1 with $2u$ degrees of freedom, where $u = TW$ denotes the time-bandwidth product. Finally the energy detector determines the status of primary signal by comparing the measured energy with predefined threshold λ. if $y > \lambda$ then signal is present, otherwise the signal is absent. The two probabilities, the probability of detection P_d and probability of false alarm P_f can be obtained as [3]

$$P_{\mathrm{f}} = \Pr(y>\lambda|\mathcal{H}_0)=\frac{\Gamma(u,\lambda/2)}{\Gamma(u)} \tag{2}$$

$$P_{\mathrm{d}} = \Pr(y>\lambda|\mathcal{H}_1)=Q_u\left(\sqrt{2\gamma},\sqrt{\lambda}\right) \tag{3}$$

Where $\Gamma(.,.)$ indicates the upper incomplete gamma function, $\gamma = h^2 E_s/N_0$ is the instantaneous SNR of the received signal at the energy detector, λ is the decision threshold and $Q_m(a,b)$ is the generalized Marcum Q-function i.e. $Q_m(a,b)=\frac{1}{a^{m-1}}\int_b^\infty x^m e^{-(x^2+a^2)/2} I_{m-1}(ax)dx$.

B. $\lambda-\mu$ Fading Channel

The $\lambda-\mu$ channel is a general fading channel that can be used to better represent the small-scale variation of the fading signal. The probability density function (PDF) $f_\gamma(\gamma)$ of instantaneous SNR γ under $\lambda-\mu$ Fading is written as [16]

$$f_\gamma(\gamma)=\frac{2\sqrt(\Pi)\mu^{\mu+\frac{1}{2}}}{\Gamma(\mu)\bar{\gamma}\lambda^{\mu-\frac{1}{2}}\sqrt{1-\lambda^2}}\left(\frac{\gamma}{\bar{\gamma}}\right)^{\mu-\frac{1}{2}} exp\left(\frac{-2\mu\gamma}{(1-\lambda^2)\bar{\gamma}}\right) \times I_{\mu-\frac{1}{2}}\left(\frac{2\mu\lambda\gamma}{\bar{\gamma}(1-\lambda^2)}\right) \tag{4}$$

Where $\bar{\gamma}$ denotes the average SNR, $I_v(.)$ is the modified Bessel function of the first kind with arbitrary order v. The $0 \le \lambda \le 1$ and $\mu \ge 0$ are arbitrary fading parameters, where λ denotes the correlation between the in-phase and quadrature components and μ represents the number of multipath clusters, which can be calculated analytically by $\mu=\frac{E^2(R^2)}{2V(R^2)}(1+\lambda^2)$ where $R=\sum_{l=1}^n ({X_l}^2+{Y_l}^2)$ is the envelope of the fading signal. E(.) and V(.) are the expectation and the variance operators, respectively.

III. Average detection probability over λ - μ fading channels

A. No Diversity

For communication systems, the average detection probability over fading channel is obtained by averaging P_d in (3) over the PDF of IID $\lambda-\mu$ fading channel in (4). Hence $\overline{P_d}$ can be written as

$$\overline{P_d}=\int_0^\infty Q_u(\sqrt{2\gamma},\sqrt{\lambda})f_\gamma(\gamma)d\gamma \tag{5}$$

The infinite series representation of the generalized Marcum Q-function $Q_u(.)$ is given by [19]

$$Q_u(\sqrt{2\gamma},\sqrt{\lambda})=\sum_{l=0}^\infty \frac{e^{-\gamma}(\gamma)^l\Gamma(l+u,\lambda/2)}{l!\Gamma(l+u)} \tag{6}$$

Substituting (4) and (6) in (5), the $\overline{P_d}$ can be written as

$$\overline{P_d}=\int_0^\infty \sum_{l=0}^\infty \frac{e^{-\gamma}(\gamma)^l\Gamma(l+u,\lambda/2)}{l!\Gamma(l+u)} \frac{2\sqrt(\Pi)\mu^{\mu+\frac{1}{2}}}{\Gamma(\mu)\bar{\gamma}\lambda^{\mu-\frac{1}{2}}\sqrt{1-\lambda^2}}\left(\frac{\gamma}{\bar{\gamma}}\right)^{\mu-\frac{1}{2}} exp\left(\frac{-2\mu\gamma}{(1-\lambda^2)\bar{\gamma}}\right) I_{\mu-\frac{1}{2}}\left(\frac{2\mu\lambda\gamma}{\bar{\gamma}(1-\lambda^2)}\right)d\gamma \tag{7}$$

By collating constant terms and by performing some mathematical simplification (7) can be written as

$$\overline{P_d}=\sum_{l=0}^\infty \frac{2\sqrt(\Pi)\mu^{\mu+\frac{1}{2}}\Gamma(l+u,\lambda/2)}{l!\Gamma(l+u)\Gamma(\mu)\bar{\gamma}^{\mu+\frac{1}{2}}\lambda^{\mu-\frac{1}{2}}\sqrt{1-\lambda^2}} \underbrace{\int_0^\infty \gamma^{l+\mu-\frac{1}{2}} e^{-\left(1+\frac{2\mu}{(1-\lambda^2)\bar{\gamma}}\right)\gamma} I_{\mu-\frac{1}{2}}\left(\frac{2\mu\lambda\gamma}{\bar{\gamma}(1-\lambda^2)}\right)d\gamma}_{I_1} \tag{8}$$

The modified Bessel function term $I_v(.)$ in (8) can be written in terms of infinite series function as in (9) using [[20], Eqn.(8.845)].

$$I_{\mu-\frac{1}{2}}\left(\frac{2\mu\lambda\gamma}{\bar{\gamma}(1-\lambda^2)}\right)=\sum_{m=0}^{\infty}\frac{1}{m!2^{2m+\mu-\frac{1}{2}}\Gamma(m+\mu+\frac{1}{2})} \times\left(\frac{2\mu\lambda\gamma}{\bar{\gamma}(1-\lambda^2)}\right)^{2m+\mu-\frac{1}{2}} \quad (9)$$

Now (8) can be written as

$$\overline{P_d}=\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\frac{2\sqrt{(\Pi)}\lambda^{2m}\mu^{2m+2\mu}\Gamma(l+u,\lambda/2)}{l!m!\bar{\gamma}^{2m+2\mu}(1-\lambda^2)^{2m+\mu}\Gamma(l+u)\Gamma(\mu)\Gamma(m+\mu+\frac{1}{2})}\underbrace{\int_0^{\infty}\gamma^{l+2m+2\mu-1}e^{-\left(1+\frac{2\mu}{(1-\lambda^2)\bar{\gamma}}\right)\gamma}d\gamma}_{I_1} \quad (10)$$

The integral I_1 in (10) can be solved using [[20], Eqn.(3.381.4)]. Hence the closed form expression of average detection probability $\overline{P_d}$ can be obtained as

$$\overline{P_d}=\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\frac{2\sqrt{(\Pi)}\lambda^{2m}\mu^{2m+2\mu}\Gamma(l+u,\lambda/2)\Gamma(l+2m+2\mu)}{l!m!\bar{\gamma}^{2m+2\mu}(1-\lambda^2)^{2m+\mu}\Gamma(l+u)\Gamma(\mu)\Gamma(m+\mu+\frac{1}{2})}\left(1+\frac{2\mu}{(1-\lambda^2)\bar{\gamma}}\right)^{-(l+2m+2\mu)} \quad (11)$$

B. Square Law Selection (SLS)

The SLS diversity scheme principle is based on the selection of the branch with maximum decision statistics $y_{SLS}=max(y_1,y_2...y_L)$, where y_{SLS} and L represents the selected diversity branch and number of diversity branches, respectively. Based on this diversity scheme, the probability of miss detection ${P_m}^{SLS}$ under the AWGN channel can be obtained as [17]

$${P_m}^{SLS}=\prod_{i=1}^{L}\left[1-Q_u(\sqrt{2\gamma_i},\sqrt{\lambda})\right] \quad (12)$$

Similarly, using this diversity scheme the $\bar{P_m}^{SLS}$ over $\lambda-\mu$ fading channel can be written as

$$\bar{P_m}^{SLS}=\int_0^{\infty}\prod_{i=1}^{L}\left[1-Q_u(\sqrt{2\gamma_i},\sqrt{\lambda})\right]f_{\gamma_i}(\gamma_i)d\gamma_i \quad (13)$$

And the probability of detection $P_d=1-P_m$ with SLS diversity can be calculated as

$$\bar{P_d}^{SLS}=1-\prod_{i=1}^{L}\left[1-\underbrace{\int_0^{\infty}Q_u(\sqrt{2\gamma_i},\sqrt{\lambda})f_{\gamma_i}(\gamma_i)d\gamma_i}_{I_2}\right] \quad (14)$$

The integral I_2 in (14) can be evaluated is same as that of non-diversity case. Hence $\bar{P_d}^{SLS}$ can be written as

$$\bar{P_d}^{SLS}=1-\prod_{i=1}^{L}\left[1-\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\frac{2\sqrt{(\Pi)}\lambda^{2m}\mu^{2m+2\mu}\Gamma(l+u,\lambda/2)\Gamma(l+2m+2\mu)}{l!m!\bar{\gamma_i}^{2m+2\mu}(1-\lambda^2)^{2m+\mu}\Gamma(l+u)\Gamma(\mu)\Gamma(m+\mu+\frac{1}{2})}\left(1+\frac{2\mu}{(1-\lambda^2)\bar{\gamma_i}}\right)^{-(l+2m+2\mu)}\right] \quad (15)$$

The corresponding probability of false alarm P_f with SLS scheme is independent of SNR and is given by [[3],Eqn.(14)]

$${P_f}^{SLS}=1-\left[1-\frac{\Gamma(u,\lambda/2)}{\Gamma(u)}\right]^L \quad (16)$$

C. Collaborative Detection

In this case, the performance of energy detection can be improved by collaborating secondary users when they share their information. For the communication systems, the probability of detection and probability of false alarm with n collaborating users are given by [18]

$$Q_d \triangleq 1-(1-P_d)^n \tag{17}$$

and

$$Q_f \triangleq 1-(1-P_f)^n \tag{18}$$

In this scenario, the closed form representation of average probability of detection with n collaborating users is deduced by substituting (11) into (17)

$$Q_d = 1-\left[1-\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\frac{2\sqrt{(\Pi)}\lambda^{2m}\mu^{2m+2\mu}\Gamma(l+u,\lambda/2)\Gamma(l+2m+2\mu)}{l!m!\bar{\gamma}^{2m+2\mu}(1-\lambda^2)^{2m+\mu}\Gamma(l+u)\Gamma(\mu)\Gamma(m+\mu+\frac{1}{2})}\left(1+\frac{2\mu}{(1-\lambda^2)\bar{\gamma}}\right)^{-(l+2m+2\mu)}\right]^n \tag{19}$$

and Q_f be remains same as P_f and it can be obtained as

$$Q_f = 1-\left[1-\frac{\Gamma(u,\lambda/2)}{\Gamma(u)}\right]^n \tag{20}$$

IV. Numerical Results

In this section, we analysis the behavior of ED under $\lambda-\mu$ fading channel through average detection probability $\overline{P_d}$ versus average SNR $\bar{\gamma}$ curve and complementary receiver operating characteristic (ROC) curve ($\overline{P_m}$ versus P_f). We discussed the effect of λ and μ fading parameters on $\overline{P_d}$ and it also provides the different fading channels as special cases.

Fig. 1 depicts the $\overline{P_d}$ versus $\bar{\gamma}$ curve for no diversity case over $\lambda-\mu$ fading channel with a fixed value of μ ($\mu = 0.5$) and varying λ and fig. 2 shows the $\overline{P_d}$ versus $\bar{\gamma}$ curve for a fixed value of λ ($\lambda = 0.33$) and varying μ. In both figures, we fixed $P_f = 0.1$ and $u = 2$. In fig. 1, the ED shows the better detection performance for lower λ due to low interference between inphase and quadrature components. In fig. 2, the ED shows the better detection performance for higher μ due to the advantage of multipath effect. The different combination of $\lambda-\mu$ values through which the conventional fading channels were obtained as, for Rayleigh distribution $\lambda \to 0, \mu = 0.5$, for One-sided Gaussian distribution $\lambda \to 1, \mu = 0.5$ and to Nakagami-q (Hoyt) distribution $\lambda = 0.33, \mu = 0.5$ [16].

Fig. 3 demonstrate the effect of λ and μ fading parameters on $\overline{P_d}$. We observe that with the increase in parameter λ the detection performance degrades and with an increase in parameter μ the detection performance improves. Here result also displays the impact of average SNR $\bar{\gamma}$ on the energy detector, the higher average SNR improves the detection capability.

Fig 4 shows the curve of complementary receiver operating characteristic (ROC) ($\bar{P_m}^{SLS}$ versus P_f^{SLS}) for the number of diversity branches from $L = 1$ to $L = 5$ using $\lambda = 0.5$, $\mu = 1$ and $u = 2$ and the average SNR for each branch is set to $\bar{\gamma}_1 = 0\ dB$, $\bar{\gamma}_2 = 1\ dB$, $\bar{\gamma}_3 = 2\ dB$, $\bar{\gamma}_4 = 3\ dB$ and $\bar{\gamma}_5 = 4\ dB$. The performance of ED improves, when number of diversity branches increase.

Fig 5 demonstrates the complementary ROC curve for ED with up to six collaborating users ($n = 6$) by setting $\lambda = 0.5$, $\mu = 1$, $u = 2$ and $\bar{\gamma} = 3\ dB$. The performance of detection capability improves as the number of collaborating users increase.

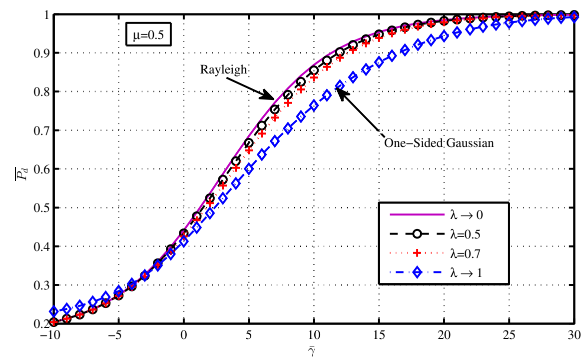

Fig. 1. $\overline{P_d}$ vs $\overline{\gamma}$ (dB) for no diversity scheme with $P_f = 0.1$,$u = 2$ and fixed μ ($\mu = 0.5$).

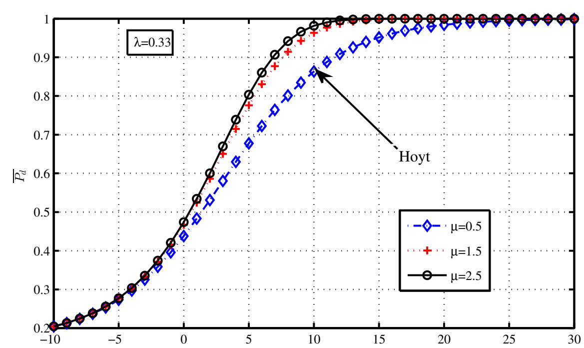

Fig. 2. $\overline{P_d}$ vs $\overline{\gamma}$ (dB) for no diversity scheme with $P_f = 0.1$,$u = 2$ and fixed λ ($\lambda = 0.33$).

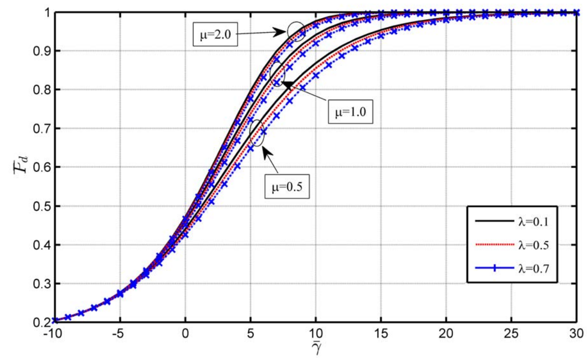

Fig. 3. $\overline{P_d}$ vs $\bar{\gamma}$ (dB) for no diversity scheme with $P_f = 0.1$,$u = 2$ and different values for λ and μ.

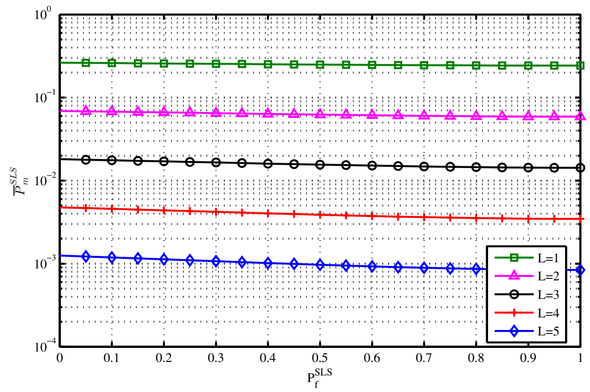

Fig. 4. Complementary ROC curve ($\overline{P_m}$ versus P_f) for SLS diversity scheme in $\lambda - \mu$ Fading Channel with $\lambda = 0.5$, $\mu = 1$, $u = 2$, $\bar{\gamma}_1 = 0\ dB$, $\bar{\gamma}_2 = 1\ dB$, $\bar{\gamma}_3 = 2\ dB$, $\bar{\gamma}_4 = 3\ dB$ and $\bar{\gamma}_5 = 4\ dB$ for different L branches.

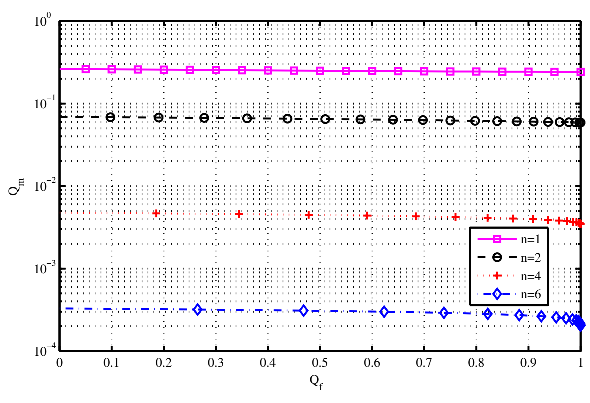

Fig. 5. Complementary ROC curve in $\lambda - \mu$ Fading Channel with $\lambda = 0.5$, $\mu = 1$, $u = 2$ and $\bar{\gamma} = 3\ dB$ for n collaborating users.

V. Conclusion

This paper explores the performance of ED over $\lambda - \mu$ Fading Channel. Novel closed form expression is derived for the average probability of detection. We also discussed the different fading distributions as special cases which is validated by various $\lambda - \mu$ values. The obtained results for no diversity case, then extends to the square law selection (SLS) and collaborative detection cases, which improves the performance of energy detector. Furthermore, it was shown that the effect of $\lambda - \mu$ fading parameters improves the energy detection performance at low SNR values, which is useful for future analysis of energy-efficient based cognitive radio systems.

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