International Pi Thunder Tournament Scoring System

Pi Tournament lasts 40 minutes, and is divided into 7 sets of questions. You will start with Set 1, and will receive Set $k+1$ after submitting Set k. You can take as much time as you wish for each set, but you cannot go back to a previous set after submitting it.

Sets 1 through 6 consist of short answer questions. Either your answer is correct or incorrect, and points are given accordingly. Tiebreaker question will only be taken into consideration when breaking ties.

Set 7 (the last set) consists of estimation questions. Your score is based on how close your answer is to the correct answer. Formula for score calculating will be provided in Set 7.

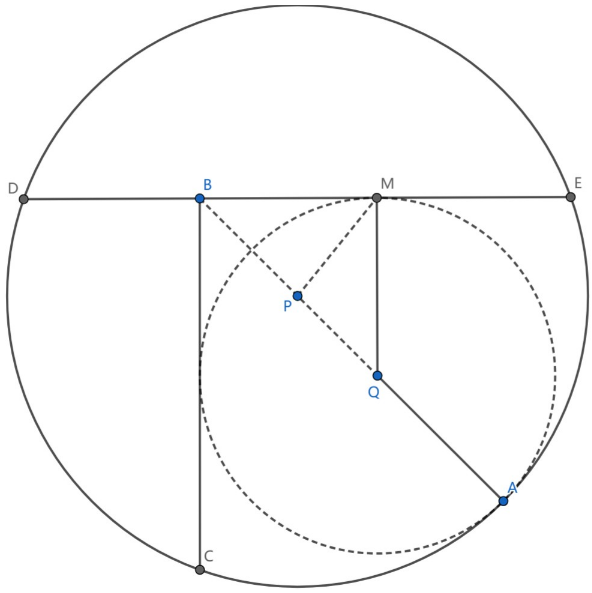
Each question within the set is equally weighted, but each set is weighted differently.

Set 1	10 marks each question;
Set 2	12 marks each question;
Set 3	15 marks each question;
Set 4	18 marks each question/1 addition mark for tiebreaker;
Set 5	23 marks each question/ 2 addition mark for tiebreaker;
Set 6	30 marks each question/ 3 addition mark for tiebreaker;
Set 7	20 marks(Maximum) each question

Set 1

1. Find the sum of the roots of equation $\pi x^2 - 2\pi^2 x + 3\pi^3 = e$

2. Given $\odot P$ and $\odot Q$, with radius 3 and 2 respectively, are internally tangent at A. If MP $\perp$ PQ, find the length of MP.



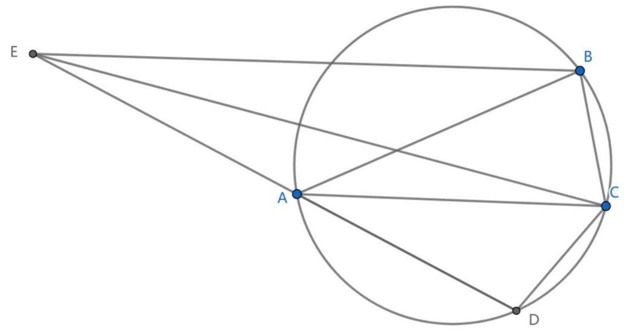
3. A group of 7 people of different heights can be arranged in any order, but they cannot be shorter than both the people on either side of them. How many ways are there to arrange them?

Set 2

4. Find the smallest positive integer k such that there is exactly one prime number of the form $kx + 60$ for the integers $0 \leq x \leq 10$.

5. How many ways are there to put mints(at least 1) in a 2x4 grid, but no two mints are adjacent to each other?

6. In cyclic quadrilateral ABCD, BC=CD, $\angle ADC = 2\angle BAD = 100°$. CE is the angle bisector of $\angle BCD$. Find $\angle BED$



Set 3

7. Let $s(n)$ be the sum of digits of n. Find,

$$\sum_{k=1}^{314} s(n).$$

8. The π Tournament committee starts with 2 apples on day 0. Each day, the committee will either eat 1 apple or buy 1 apple. The committee experiences 'panic' if their apple count ever hits 0. After 10 days, the committee has exactly 6 apples and did not experience 'panic' at any point during those 10 days. How many different ways could the committee have chosen to eat or buy apples over these 10 days?

9. The 314th multiple of 7 that only have digits 0,1 and 2 is n. Find $\lfloor \log_{10} n \rfloor$.

Set 4

10. Let $\omega = \frac{-1+\sqrt{3}i}{2}$. Find the sum of all real numbers t, $0 \le t < 2$, such that $z = \cos(\pi t) + i\sin(\pi t)$ and $|z^2 + \omega| = |z^{11} + \omega^2|$.

11. If a and b are selected uniformly from $\{0, 1, 2 \cdots 63\}$ with replacement, find the expected number of 1's in the binary representation of $a+b$.

12. Let I be the incenter of triangle ABC. AC and AB are tangent to $\odot I$ at E and F respectively. EF meets BI at G. M is the middle point of AC. If $\angle ABC = 70°$ and $\angle ACB = 35°$, find $\angle AMG$

13(Tiebreaker). $\mathcal{P}$ is a regular 314-gon in the coordinate plane. Let s be the number of distinct x-coordinates that vertices of $\mathcal{P}$ take. Compute the sum of all possible s.

Set 5

14. The value of,

$$\prod_{k=1}^{2025}\left(2^{2^k}-2^{2^{k-1}}+1\right)$$

Can be expressed as $\frac{aA^2+bA+c}{d}$. Find $a+b+c+d$.

15. Let AC and BD be two perpendicular diameters of $\odot O$. Point P is a moving point on the minor arc CD. Line PA intersects BD at point R, and line PB intersects AC at point Q. Let the areas of $\triangle OQR$ and $\triangle PQR$ be S_1 and S_2, respectively. Given that

$$\frac{1}{2S_1}-\frac{1}{S_2}=\frac{1}{2025}$$

Find the radius of $\odot O$.

16. Find the smallest positive integer n, such that for any positive odd number a,

$$2^{2025} \mid a^{2^n}-1.$$

17(Tiebreaker). What is the value of:

$$\int_0^1 \frac{e^x-e^{-x}}{2}\mathrm{d}x+\int_0^{\frac{e^2-1}{2e}} \ln(x+\sqrt{x^2+1})\mathrm{d}x.$$

Set 6

18. Suppose a, b, c are nonzero real numbers such that,

$$bc + \frac{1}{a} = ca + \frac{2}{b} = ab + \frac{7}{c} = \frac{1}{a+b+c}.$$

Find $a+b+c$.

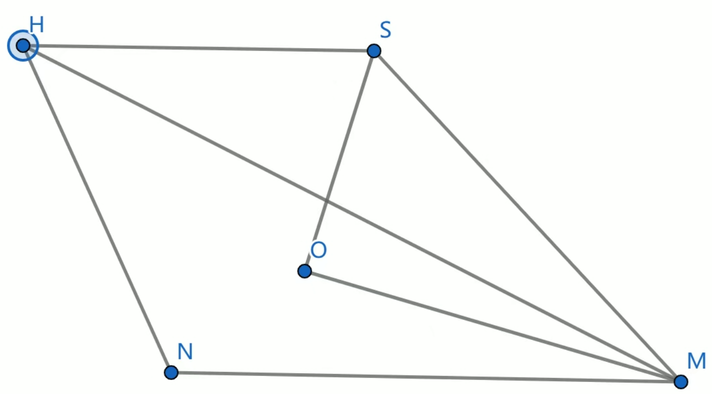
19. An 8×8 chessboard has alternating black and white squares. How many distinct rectangles, with sides on the grid lines of the checkerboard, that contains at least 3 black squares can be drawn on the chessboard?

20. Let $f(x) = 2^x + 3^x$. For how many integers $1 \leq n \leq 2025$ is $f(n)$ relatively prime to all of $f(0), f(1), \cdots, f(n-1)$?

21(Tiebreaker). In quadrilateral $HNMS$, let $HS \parallel NM$, and let O be the centroid of $\triangle HNM$. Denote the area of quadrilateral $HOMS$ as L, and let $X = HN \cdot MS$. Given that

$$\angle MOS = 90^\circ.$$

Find the minimum value of $\frac{X}{L}$.



Set 7

22. How many numbers less than 3,141,592 are the product of exactly 2 distinct primes? (You will receive $\max(0, \lfloor 20 - 40 \cdot |\frac{N}{A} - 1| \rfloor)$ points, if you submit N and the correct answer is A.)

23. What is the mean of the answers of all previous 21 questions from Set 1 to Set 6? (Submit a positive integer A. If the correct answer is C and your answer is A, you will receive $\lfloor 20(\min(\frac{A}{C}, \frac{C}{A}))^2 \rfloor$ points.)

24. Submit a positive integer $n < 10^5$. Let the product of all valid submission to this question be P_i. (You will receive $\max(0, \left\lfloor 20 - \frac{|(P_i')^\pi - n|}{10\pi} \right\rfloor)$ points, where P_i' is the number of different prime factors of P_i.)