Page No.:
Date:

Calculus Assignment

1. Academical Questions

1.1. Purely academical

1. $f(x) = 3x^2 + 5x + 7$,

Ans $f'(x) = 6x + 5$ $\left[\because \frac{d(x^n)}{dx} = nx^{n-1}\right.$

2. $g(x) = 2x^3 - 4x$

Ans $g'(x) = 6x^2 - 4$

1.2. Academical with Data Science background

1.2.1: $f(x) = (2x^3)(5x^2 + 3x)$

$$= \frac{d}{dx}(2x^3).(5x^2+3x) + (2x^3)\frac{d}{dx}(5x^2+3x)$$

$\left[\because (uv)' = u'v + uv'\right.$

$$= 6x^2[5x^2+3x] + 2x^3(10x+3)$$

$$= 30x^4 + 18x^3 + 20x^4 + 6x^3$$
$$= 50x^4 + 24x^3$$

1.2.2 $$g(x) = \frac{3x^2+2x+1}{x^2+1}$$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

$$= \frac{\frac{d}{dx}(3x^2+2x+1) \times (x^2+1) - (3x^2+2x+1)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$= \frac{[6x+2+0][x^2+1] - [3x^2+2x+1][2x]}{(x^2+1)^2}$$

$$= \frac{(\cancel{6x^3}+6x+2x^2+2) - (\cancel{6x^3}+4x^2+2x)}{(x^2+1)^2}$$

$$= \frac{4x-2x^2+2}{(x^2+1)(x^2+1)}$$

Page No.:
Date:

1.2.3:

$$f(x) = \sin(3x^2+2x)$$

$$f'(x) = \cos(3x^2+2x)\,\frac{d}{dx}(3x^2+2x)$$

$$= \cos(3x^2+2x)(6x+2)$$

$$= 6x\cos(3x^2+2x) + 2\cos(3x^2+2x)$$

1.2.4:

$$h(x) = e^{5x-3x^2}$$

$$h'(x) = e^{u}\,\frac{du}{dx}$$

$$= e^{5x-3x^2}\,\frac{d}{dx}(5x-3x^2)$$

$$= e^{5x-3x^2}(5-6x)$$

Date

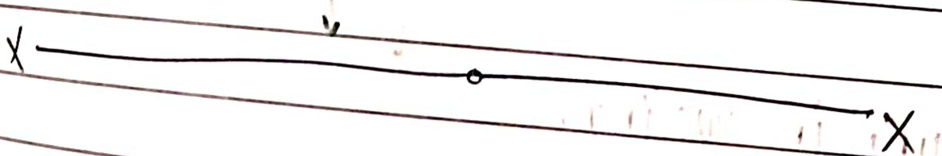
1.2.5 : $x^2+y^2=25$

$$\frac{dy}{dx}(x^2+y^2)=\frac{dy}{dx}(25)$$

$$\Rightarrow 2x+2y\frac{dy}{dx}=0$$

$$\Rightarrow \cancel{2}y\frac{dy}{dx}=-\cancel{2}x$$

$$\Rightarrow \frac{dy}{dx}=-\frac{x}{y}$$



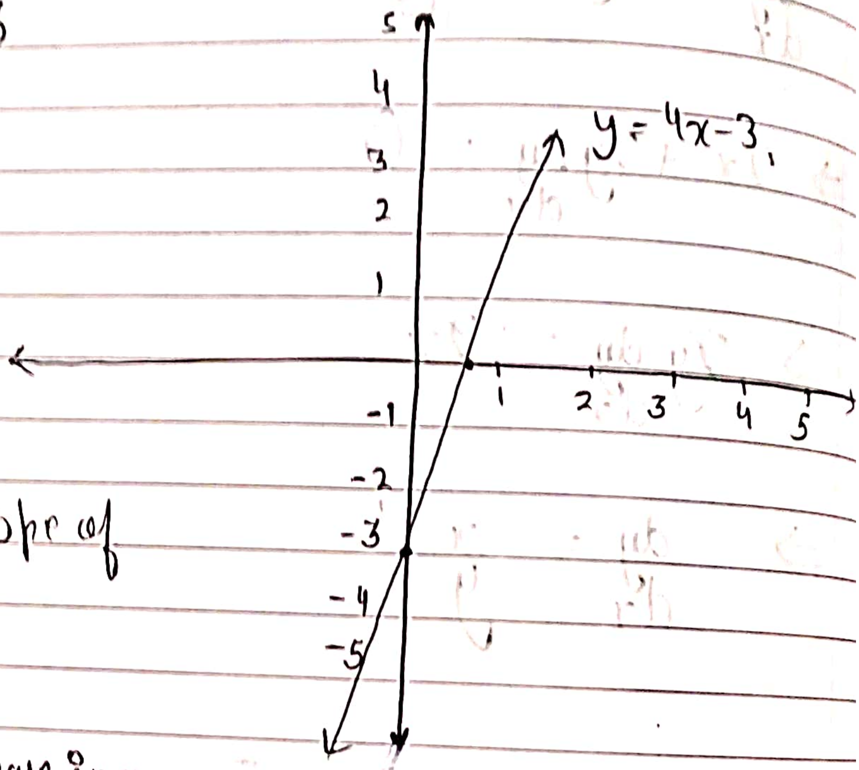
2. Data Science Questions

2.1 Slope in linear Regression

1. $h(x) = 4x - 3$

$h'(x) = 4$

$y = 4x - 3$



- It represents slope of the line

- every unit increase in x, h(x) increases by 4 units.

- the slope indicates the rate of change of the dependent variable h(x) w.r.t. independent variable x.

2.2 Chain rule in logistic regression

(2) $f(x) = \frac{1}{1+e^{-x}}$

Let $1+e^{-x} = u(x)$

$\frac{1}{u} = v(u)$

$u'(x) = -e^{-x}$

$v'(x) = -\frac{1}{u^2}$

$y = v(u(x))$

$y' = \frac{-1}{(1+e^{-x})^2} \times (-e^{(-x)})$

$= \frac{e^{-x}}{(1+e^{-x})^2}$

• when rate of change is very high, it becomes difficult to understand probability.

③ $g(x) = e^{2x}$

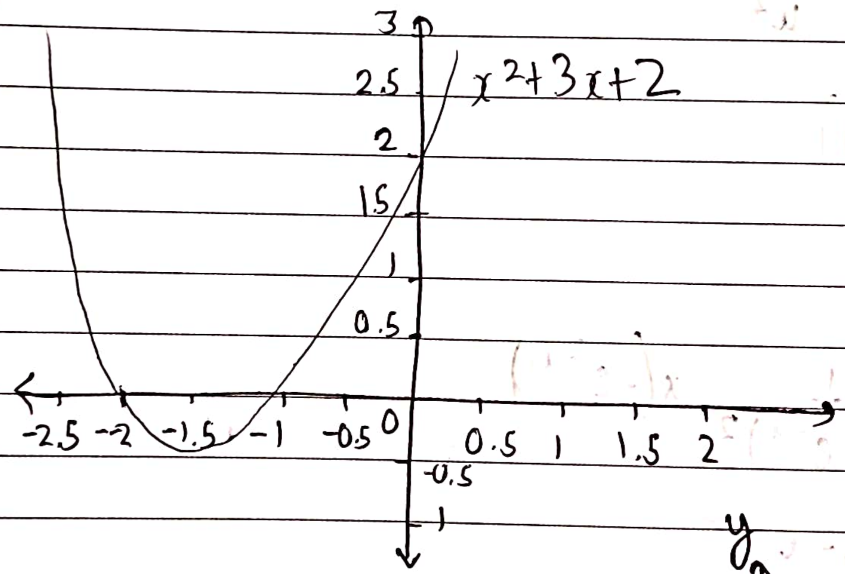
$g'(x) = e^{2x} \dfrac{d}{dx}(2x)$ $\quad [\because \dfrac{d(e^x)}{dx} = e^x$

$= e^{2x} . 2$

$= 2 . e^{2x}$

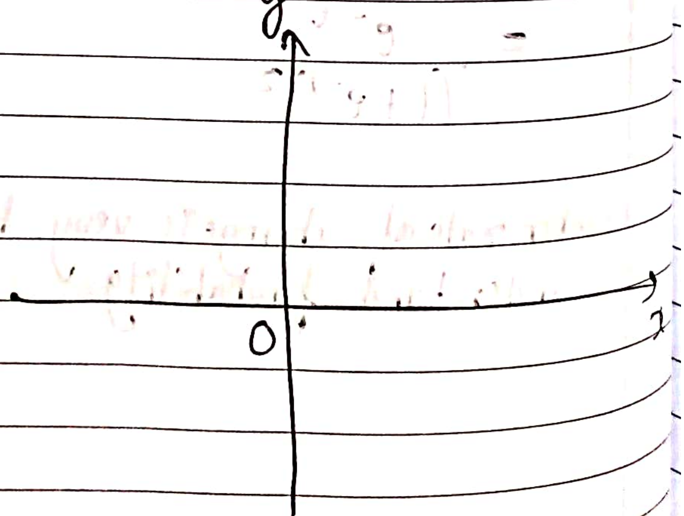
2.3 Gradient descent Basics -

④ $f(x) = x^2 + 3x + 2$



$f'(x) = 2x + 3$

finding critical points



$2x + 3 = 0$

$x = -\dfrac{3}{2}$

Page No.:
Date:

$f''(x) = 2$

$f\left(\frac{-3}{2}\right) = \left(\frac{-3}{2}\right)^2 + 3\left(\frac{-3}{2}\right) + 2 = \frac{-1}{4}$

minimum value of function $f(x) = \frac{-1}{4}$ and it occurs at

$x = \frac{-3}{2}$ (after that is steep ascent)

(5.) $J(m) = (mx - y)^2$

$g(m) = mx - y$
$f(x) = x^2$

$J(m) = f(g(m))$

$f'(x) = 2x$
$g'(m) = x$

$J'(m) = f'(g(m))\, g'(m)$

$= 2(mx - y) * x$

$= 2x(mx - y)$

Page No.:
Date:

2.4 Activation Functions & Derivatives

⑥ **ReLU (Rectified Linear Unit)**

$$f(x) = \max(0, x)$$

for $x > 0$, $f(x) = x$
for $x \leq 0$, $f(x) = 0$

$f'(x)$: for

- for, $x > 0$,
  $f(x) = x$,

  $f'(x) = 1$

- for, $x < 0$,
  $f(x) = 0$,

  $f'(x) = 0$

$$\boxed{f'(x) = \begin{cases} 1, & x > 0 \\ 0, & x \leq 0 \end{cases}}$$

Page No.:

Date:

(7) Tanh(x)

activation function is defined as

$$h(x) = \tanh(x) = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$

Derivative of tanh(x):

$$h(x) = \tanh(x)$$

$$h'(x) = 1 - \tanh^2(x)$$

(i) $h'(x)$ at $x = 0$

$$\tanh(0) = 0$$

$$h'(0) = 1 - \tanh^2(0) = 1 - 0^2 = 1$$

$$\therefore h'(0) = 1$$

Key Insights:

1. the derivative $h'(x) = 1 - \tanh^2(x)$