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6. Constraint-Based Optimization - Multiple Constraints

Solve a constraint-based optimization problem using Lagrange multipliers to minimize blood glucose levels while maintaining a healthy BMI and blood pressure.

Dataset

- Pregnancies: no. of times pregnant
- Glucose: Plasma Glucose concentration
- BloodPressure: Diastolic blood pressure
- SkinThickness: Triceps skinfold thickness
- Insulin: 2-hour-serum insulin
- BMI: Body mass index
- Diabetes Pedigree Function: A diabetes pedigree function
- Age: years → integer
- Outcome: target variable (1 = diabetes, 0 = no diabetes)

DESCRIPTIVE STATISTICS

	Pregnancies	Gulucose	B.P.	S.T.	Insulin	BMI	Age
mean	3.845	120.89	69.105	20.53	79.79	31.99	33.24
std	3.36	31.97	19.35	15.59	115.24	7.88	11.76
median	3.0	117.0	72.0	23.0	30.5	32.0	29.0
mode	1	99	70	0	0	32.0	22

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IQR	5.00	41.25	18.00	32.00	121.25	9.30	17
SKEWNESS	0.90	0.17	-1.84	0.11	2.27	-0.43	1.13
Kurtosis	0.16	0.64	5.18	-0.52	7.21	3.29	0.64

- avg. age of data is 33.24
- avg. BMI is 31.99

- for BMI mean, median, mode = 32.0 (approx)
  suggests a normal distribution.

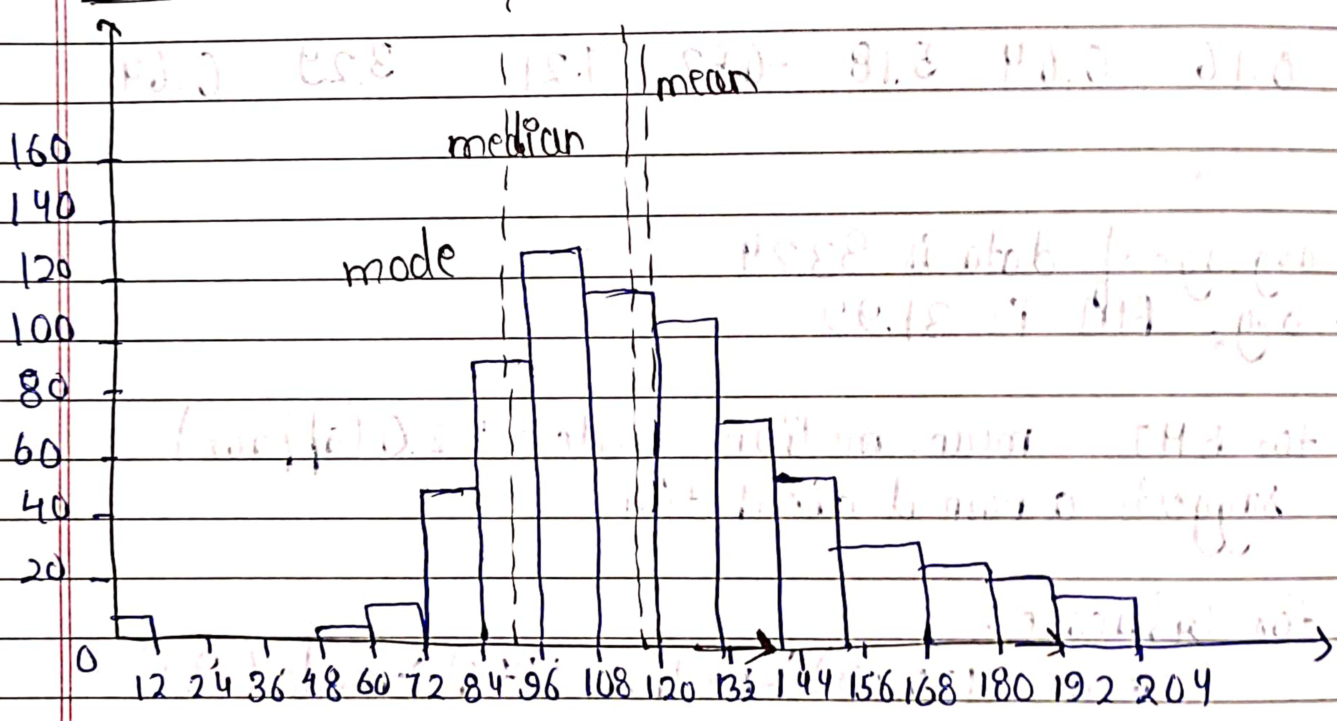
- for Glucose:
  mean (120.89) > median (117.0) > mode (99)

  ⇒ suggests a skewness on the -ve side.

- for features B.P. and Insulin we see extreme skewness in data

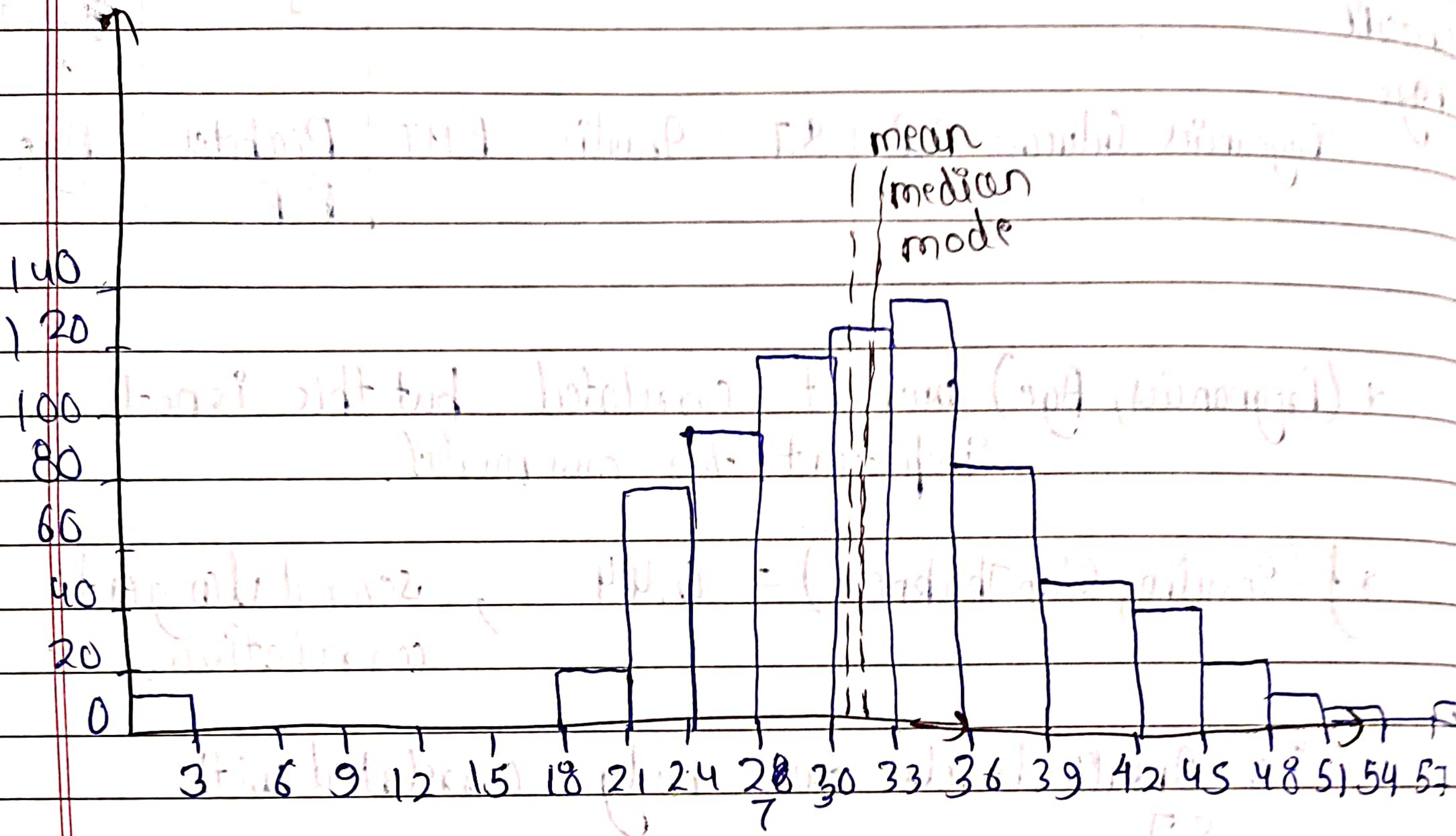
Using histograms to find the distributions

• Glucose :



* dataset is a little positively skewed.
* glucose levels are on the higher values.
* (0-12) range may indicate outliers.

(BMI histogram):



we see that mean ≈ median = mode

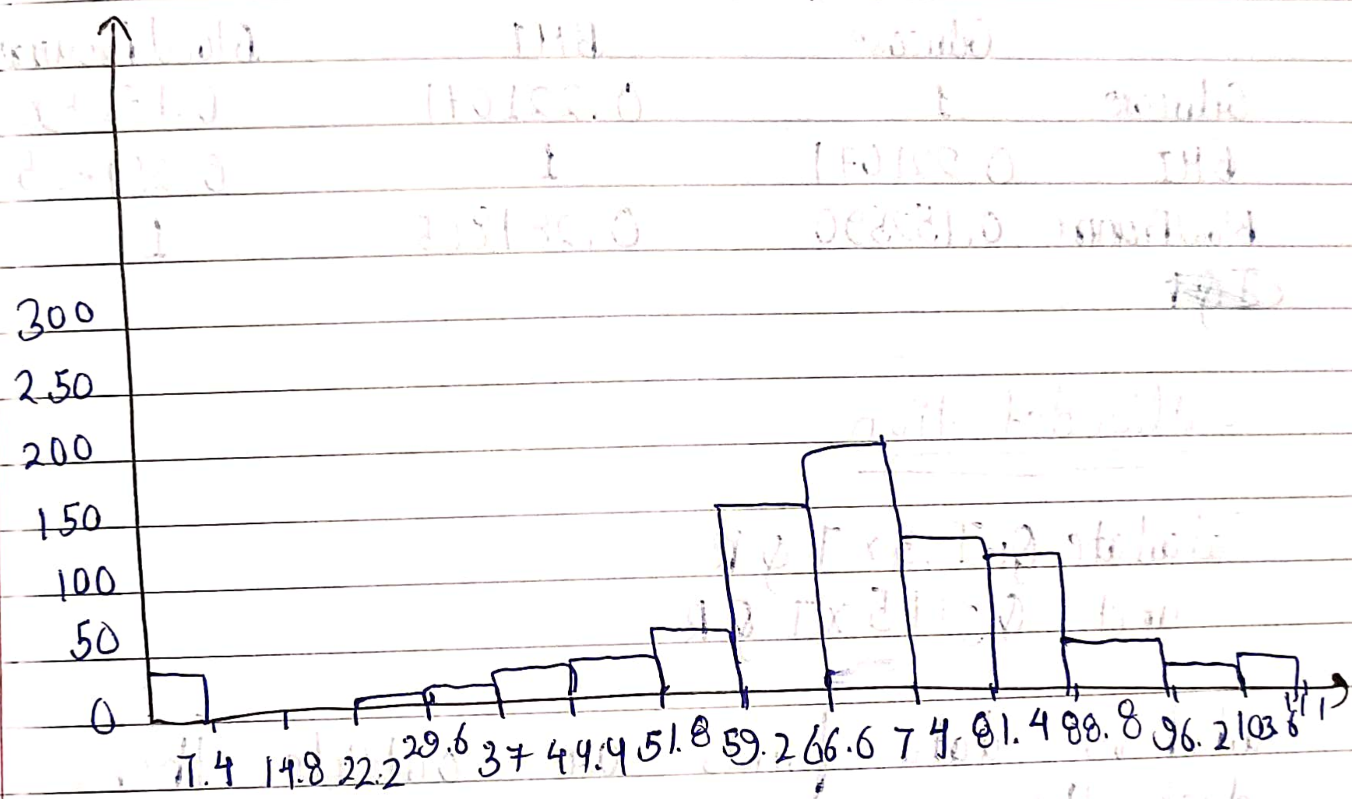
⇒ data follows almost normal distributions

⇒ there are some outliers also (at extremeties)

⇒ extreme datapoints may indicate outliers (∵ BMI cannot be zero)

(Blood Pressure)

* extreme points indicate outliers.



Correlation Matrix -

	Glucose	BMI	Blood Pressure
Glucose	1	0.221071	0.152590
BMI	0.221071	1	0.281805
Blood Pressure	0.152590	0.281805	1

Outlier detection

Calculate $Q_1 - 1.5 \times IQR$
and $Q_3 + 1.5 \times IQR$

whichever data points are outside them drop them

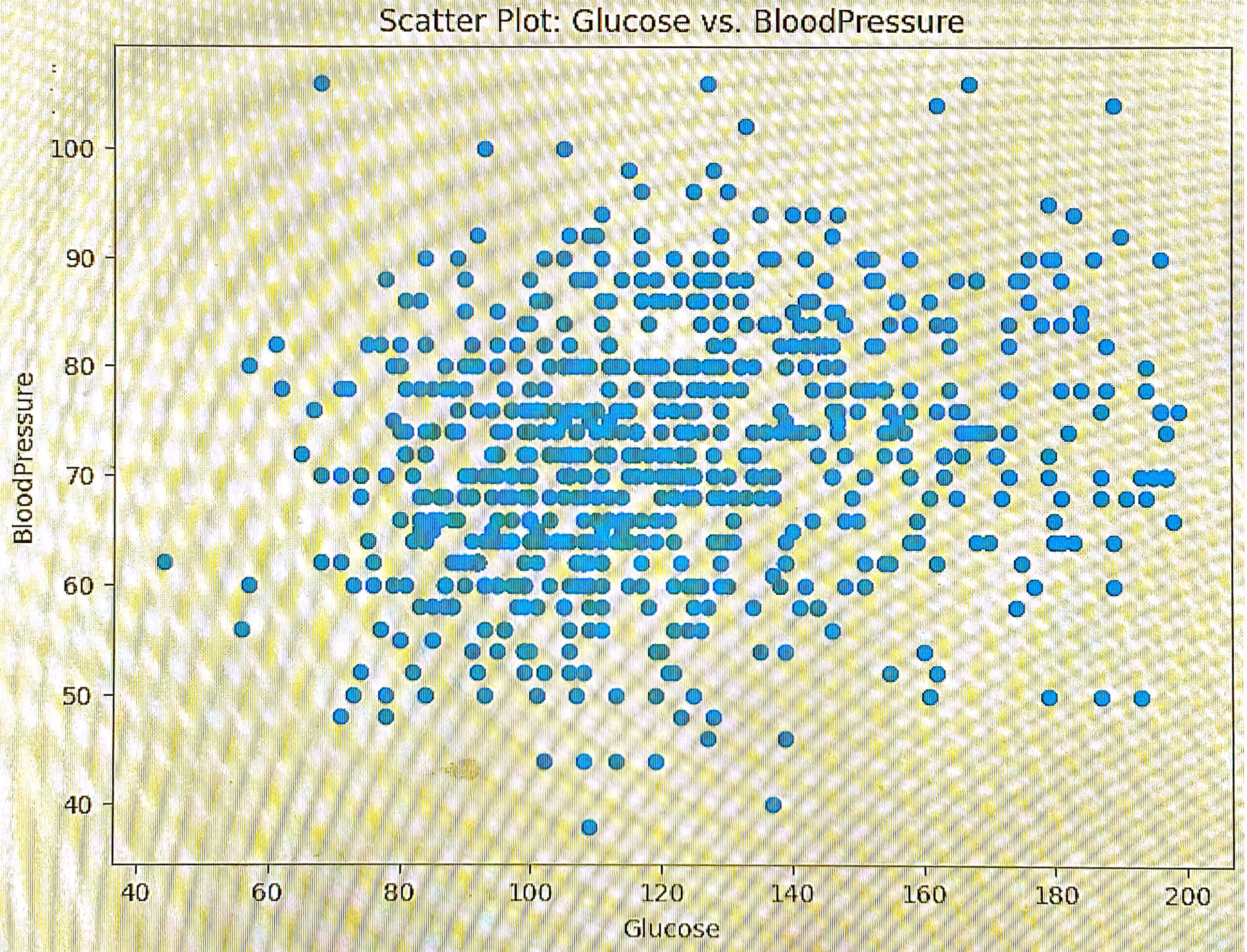
∴ there are outliers.

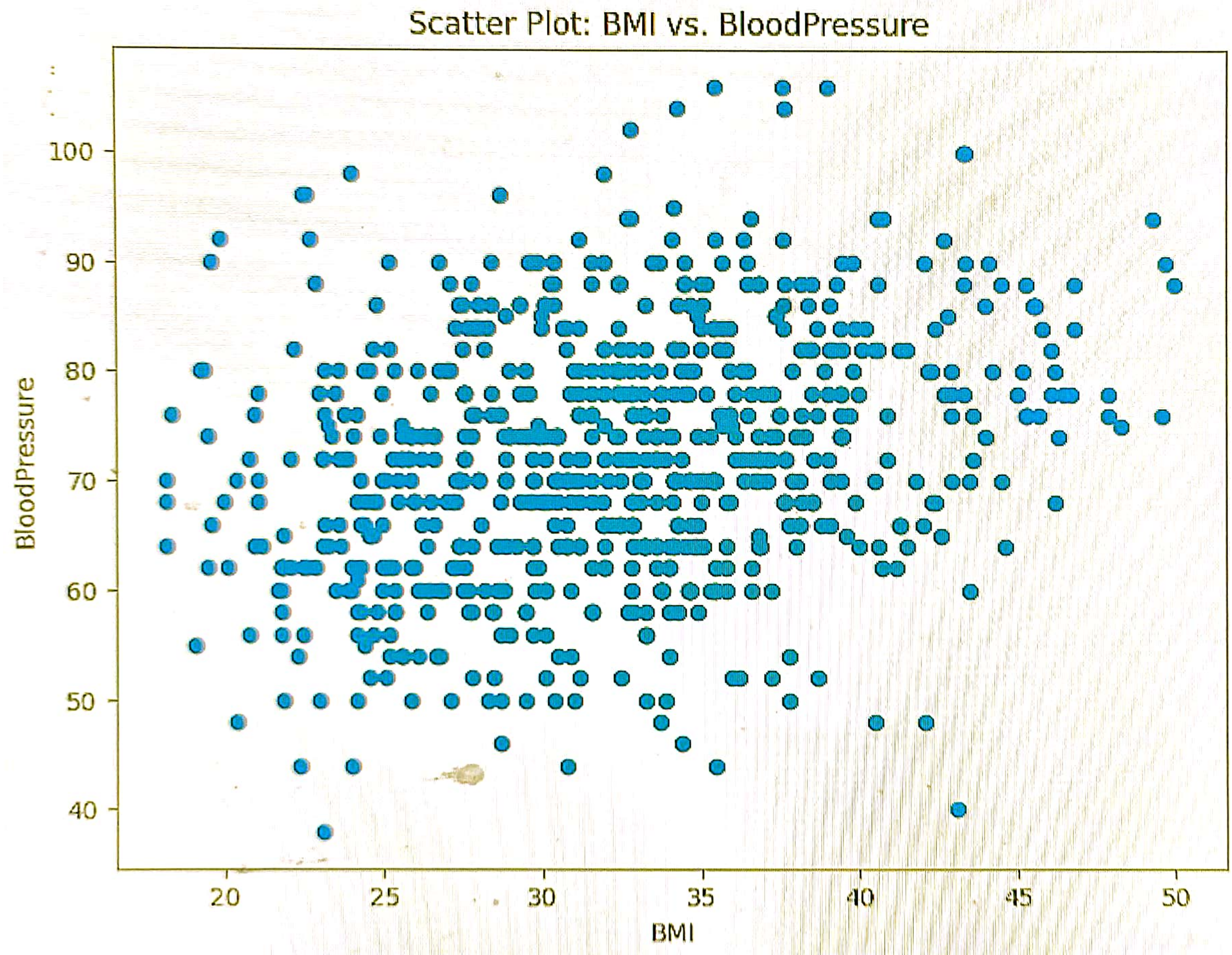
After updated dataset calculate correlation

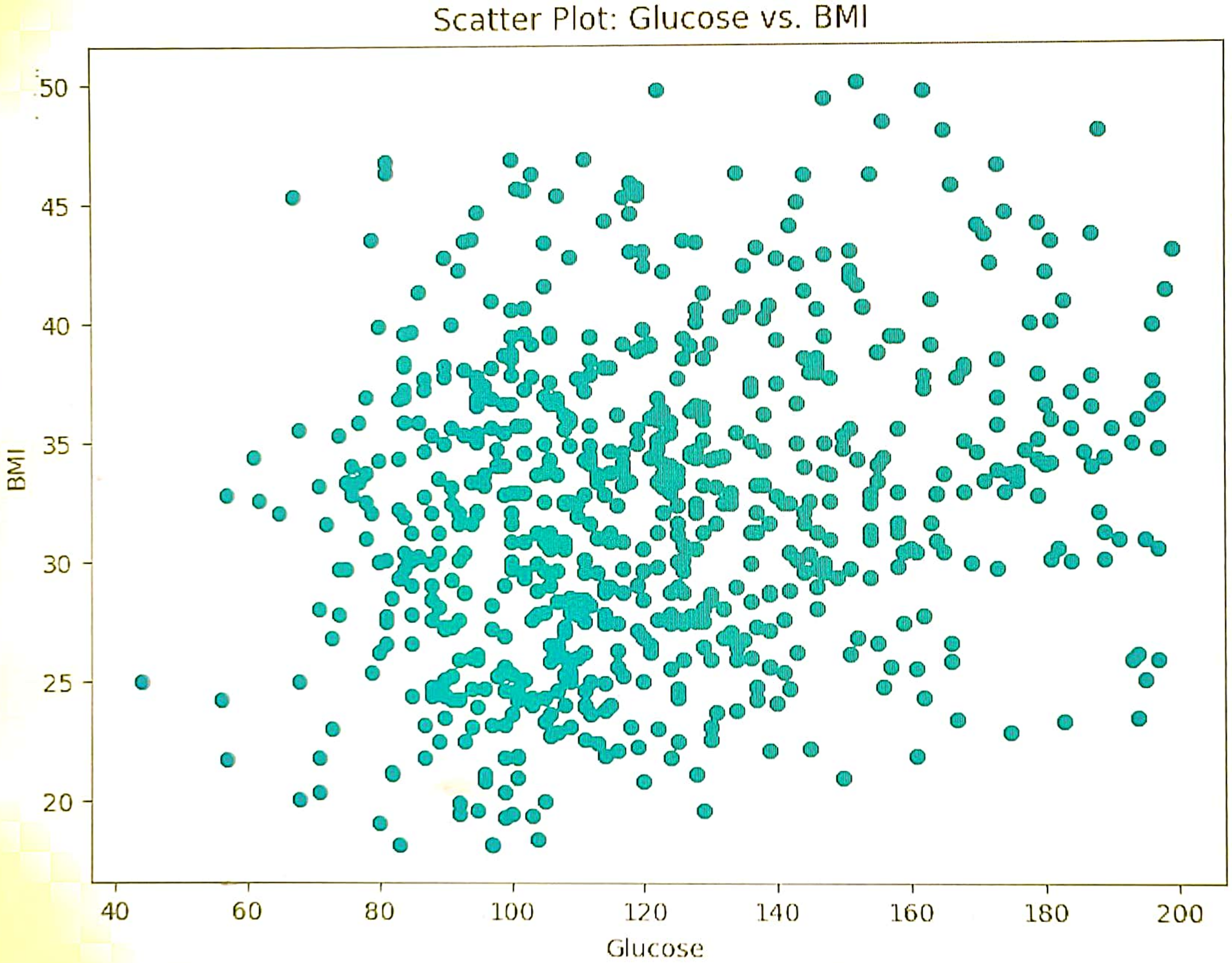
matrix again:

	Glucose	BMI	B.P.
Glucose	1	0.21	0.20
BMI	0.21	1	0.28
BP	0.21	0.28	1

- we see that every feature has a weak positive correlation with each other.


Scatter Plot: Glucose vs. BloodPressure
BloodPressure
40
50
60
70
80
90
100
Glucose
40
60
80
100
120
140
160
180
200


Scatter Plot: BMI vs. BloodPressure
BloodPressure
100
90
80
70
60
50
40
20
25
30
35
40
45
50
BMI


Scatter Plot: Glucose vs. BMI
BMI
50
45
40
35
30
25
20
40
60
80
100
120
140
160
180
200
Glucose

from the scatterplots we see

- (Glucose, BMI) has no strong correlation, but some clustering around mid-range values.
- (Glucose, B.P.): no clear trend
- (BMI, B.P.): slight positive trend.

6.3 Define the Optimization Problem

Objective function:-

minimize blood glucose levels (x_1);

$$\min f(x_1) = x_1$$

$x_1 \rightarrow$ Glucose levels

Define Constraints -

- BMI Constraint: $17.5 < x_2 < 21.5$
- Blood Pressure: $80 < x_3 < 120$

- Formulate langrangian with Inequality constraints:

$$\mathcal{L}(x_1, x_2, x_3, \lambda_1, \lambda_2, \mu_1, \mu_2) = x_1 + \lambda_1(x_2 - 17.5) + \lambda_2(21.5 - x_2)$$

$$+ \mu_1(x_3 - 80) + \mu_2(120 - x_3);$$

$\lambda_1 \geq 0$ and $\lambda_2 \geq 0$ enforce lower and upper bounds of BMI.

$\mu_1 \geq 0$ and $\mu_2 \geq 0$ enforce the lower and upper bounds of blood pressure.

- Compute Partial derivatives

F.O.D.s

1. w.r.t. x_1 (glucose level):

$$\frac{\partial \mathcal{L}}{\partial x_1} = 1$$

not possible, meaning x_1 must be minimized directly

$$\frac{\partial L}{\partial x_2} = \lambda_1 - \lambda_2 = 0$$

$$\Rightarrow \lambda_1 = \lambda_2$$

3. w.r.t. x_3 (Blood Pressure):

$$\frac{\partial L}{\partial x_3} = \mu_1 - \mu_2 = 0$$

$$\Rightarrow \mu_1 = \mu_2$$

$$\frac{\partial L}{\partial \lambda_1} = 17.5 - x_2$$

$$\frac{\partial L}{\partial \lambda_2} = x_2 - 21.5$$

$$\frac{\partial L}{\partial \mu_1} = 80 - x_3$$

$$\frac{\partial L}{\partial \mu_2} = x_3 - 120$$

- **Interpret the Solution:**

1. Minimizing x_1 (Glucose level)

$\because$ x_1 is minimized independently; we assume that it is minimized at lowest feasible level according to data.

2. BMI Condition

optimal solution satisfies $17.5 \leq x_2 \leq 21.5$

- if $x_2 = 17.5$, $\lambda_1 > 0$, meaning lower bound is active
- if $x_2 = 21.5$, $\lambda_2 > 0$, meaning upper bound is active.

- if neither is active, BMI is the interior range, λ_1 and λ_2 are zero.

3. Blood Pressure Condition

$80 \le x_3 \le 120$

- if $x_3 = 80$, $\mu_1 > 0$, meaning lower bound active.
- if $x_3 = 120$, $\mu_2 > 0$, meaning upper bound active.

Example iteration

Initialize Values

$x_1 = 130$ $x_2 = 19.5$ $x_3 = 100$

$\lambda_1 = 0.1$, $\lambda_2 = 0.1$, $\mu_1 = 0.1$, $\mu_2 = 0.1$

Learning rate:

$\alpha = 0.01$

Compute Gradients (to update variables)

$x_1 = 130 - (0.01 \times 1) = 129.99$

$x_2 = 19.5 - (0.01 \times (-0.1 + 0.1)) = 19.5$

(No change $\Rightarrow$ gradient zero)

$x_3 = 100 - (0.01 \times (-0.1 + 0.1)) = 100$

(No change $\Rightarrow$ gradient zero)

Update multipliers

$\lambda_1 = \lambda_1 + \alpha(17.5 - x_2)$

$\lambda_2 = \lambda_2 + \alpha(x_2 - 21.5)$

$\mu_1 = \mu_1 + \alpha(80 - x_3)$

$\mu_2 = \mu_2 + \alpha(x_3 - 120)$

$\lambda_1 = 0.1 + (0.01 \times (17.5 - 19.5)) = 0.08$

$\lambda_2 = 0.1 + (0.01 \times (19.5 - 21.5)) = 0.08$

$\mu_1 = 0.1 + (0.01 \times (80 - 100)) = 0.1 + (0.01 \times -20) = -0.1$

$\mu_2 = 0.1 + (0.01 \times (100 - 120)) = 0.1 + (0.01 \times -20)$

$= -0.1$

Final updated values after 1st iteration

$x_1 = 120.99$, $x_2 = 19.5$; $x_3 = 100$

$\lambda_1 = 0.08$, $\lambda_2 = 0.08$, $\mu_1 = -0.1$, $\mu_2 = -0.1$

we will repeat these steps till convergence

Interpretation

* x_1 (Blood Glucose Level) decreased slightly.
* x_2, x_3 did not change because they were already within the range.
* If x_2 is above 21.5, λ_2 would increase to push it down.

If x_2 is below 17.5, λ_1 would increase to push it up.

- for x_3, $80 \le x_3 \le 120$

- If x_3 was above 120, μ_2 would increase to push it down.

- If x_3 was below 80, μ_1 would increase to push it up.

- Lagrange Multipliers: $(\lambda_1, \lambda_2, \mu_1, \mu_2)$

- these values represent penalties for violating constraints.

After evaluation

final results are

x_1 (Glucose) $= 115.99$
x_2 (BMI) $= 21.5$
x_3 (B.P.) $= 80$

$\lambda_1 = 0$
$\lambda_2 = 0.1$
$\mu_1 = 0.1$
$\mu_2 = 0$

minimum value of $f(x_1) = \underline{115.99}$