

1.2.1



1. Given $f(x) = \frac{2x+1}{3}$

(a) calculate the value of $f(7)$

$$f(7) = \frac{2(7)+1}{3} = \frac{15}{3} = 5$$

(b) $f^{-1}(x)$

x in terms of y

$$y = \frac{2x+1}{3}$$

$$\Rightarrow 3y = 2x+1$$

$$\Rightarrow \frac{3y-1}{2} = x$$

$$f^{-1}(x) = \frac{3x-1}{2}$$

2. $f(x) = 3x - 1$
 $g(x) = 2x + 4$

(a) $f(g(x)) = 3(2x + 4) - 1 = 6x + 12 - 1$
 $= 6x + 11$

$$\begin{aligned} f(g(2)) &= 6(2) + 11 \\ &= 12 + 11 \\ &= \underline{\underline{23}} \end{aligned}$$

(b) $ff(3) = 3(3x - 1) - 1 = 9x - 3 - 1 = 9x - 4$

$$\begin{aligned} f(f(3)) &= 9(3) - 4 \\ &= 27 - 4 \\ &= \underline{\underline{23}} \end{aligned}$$

(c) $gf(x) = 2(3x - 1) + 4 = 6x - 2 + 4$
 $= 6x + 2$

~~g(x)~~

3. $f(x)$, $g(x)$ and $h(x)$

$$f(x) = x^2 - 3$$

$$g(x) = 2x + 1$$

$$h(x) = \frac{x}{2}$$

$$\begin{aligned} \underline{\underline{(a)}} \quad f(g(x)) &= (2x+1)^2 - 3 \\ &= 4x^2 + 1 + 4x - 3 \\ &= 4x^2 + 4x - 2 \end{aligned}$$

$$\begin{aligned} \underline{\underline{(b)}} \quad g(h(x)) &= 2\left(\frac{x}{2}\right) + 1 \\ &= \underline{\underline{x+1}} \end{aligned}$$

(c) $h^{-1}(x)$

x in terms of y

$$y = \frac{x}{2} \Rightarrow x = 2y$$

$$\Rightarrow h^{-1}(x) = 2x$$

4. $f(x) = 4x - 7$

(a) Solve $f(x) = 17$

$$4x - 7 = 17$$

$$\Rightarrow 4x = 17 + 7$$

$$\Rightarrow 4x = 24$$

$$\Rightarrow x = \frac{24}{4}$$

$$\Rightarrow \underline{\underline{x = 6}}$$

(b) $f^{-1}(x)$

x in terms of y

$$4x - 7 = y$$

$$\Rightarrow 4x = y + 7$$

$$\Rightarrow x = \frac{y + 7}{4}$$

$$f^{-1}(x) = \underline{\underline{\frac{x + 7}{4}}}$$

5. $f(x) = x^2 + 2$ and $g(x) = x + 14$

given, $f(x) = g(x)$

$$\Rightarrow x^2 + 2 = x + 14$$

$$\Rightarrow x^2 - x - 12 = 0$$

$$\Rightarrow x^2 + 3x - 4x - 12 = 0$$

$$\Rightarrow x(x + 3) - 4(x + 3) = 0$$

$$\Rightarrow (x - 4)(x + 3) = 0$$

$$x = +4 \text{ or } -3$$

$$f(4) = (4)^2 + 2$$

$$= 18$$

$$g(4) = 4 + 14$$

$$= 18$$

$$f(-3) = 9 + 2$$

$$= 11$$

$$g(-3) = -3 + 14$$

$$= 11$$

$\therefore a = 4 \text{ and } -3$

6.

$$f(x) = 8 - 3x$$

$$g(x) = 4x$$

(a)

$$gf(3) = 4(8 - 3x)$$

$$= 32 - 12x$$

$$= 32 - 12(3)$$

$$= 32 - 36$$

$$= -4$$

(b)

$$gf(x) = 80$$

$$\Rightarrow 4(8 - 3x) = 80$$

$$\Rightarrow 32 - 12x = 80$$

$$\Rightarrow -12x = 80 - 32$$

$$\Rightarrow -12x = 48$$

$$\Rightarrow x = \frac{48}{-12} = -4$$

7. $f(x) = \frac{3x+1}{5}$

x in terms of y

$$y = \frac{3x+1}{5}$$

$$\Rightarrow \frac{3x}{5} = y - 1$$

$$\Rightarrow 3x = 5(y-1)$$

$$\Rightarrow x = \frac{5y-5}{3}$$

$$f'(x) = \frac{5x-5}{3}$$

8. $f(x) = x^2 + 3x - 5$

$$f(2x-1) = (2x-1)^2 + 3(2x-1)$$

$$= 4x^2 + 1 - 4x + 6x - 3 - 5$$

$$= \underline{\underline{4x^2 + 2x - 7}}$$

9. $f(x) = kx + 3$
 $g(x) = 2x - 4$

$$g(f(x)) = 2(kx + 3) - 4$$

$$= 2kx + 6 - 4$$

$$= 2kx + 2$$

$$g(f(2)) = 2k(2) + 2$$

$$= 4k + 2$$

$$4k + 2 = 34$$

$$\Rightarrow 4k = 32$$

$$\Rightarrow k = \frac{32}{4} = \underline{\underline{8}}$$

10. $f(x) = x^2 + 4$
 $g(x) = x - 9$

$$\begin{aligned} f(g(x)) &= (x-9)^2 + 4 \\ &= x^2 + 81 - 18x + 4 \\ &= x^2 - 18x + 85 \end{aligned}$$

$$\begin{aligned} g(f(x)) &= (x^2 + 4) - 9 \\ &= x^2 - 5 \end{aligned}$$

$$f(g(x)) = g(f(x))$$

$$\Rightarrow x^2 - 18x + 85 = x^2 - 5$$

$$\Rightarrow -18x = -5 - 85$$

$$\Rightarrow -18x = -90$$

$$\Rightarrow x = \frac{-90}{-18} = 5$$

11. $f(x) = x^2 + 2x + 1$

$$\begin{aligned} f(x+2) &= (x+2)^2 + 2(x+2) + 1 \\ &= x^2 + 4 + 4x + 2x + 4 + 1 \\ &= x^2 + 6x + 9 \end{aligned}$$

$$\begin{aligned} f(x+2) - f(x) &= x^2 + 6x + 9 - (x^2 + 2x + 1) \\ &= 6x + 9 - 2x - 1 = 4x + 8 \end{aligned}$$