

10. Optimizing Ad Campaigns

| Platform | CPC | CTR | Conversion Rate |
|-----------|--------|------|-----------------|
| FB | \$0.50 | 2% | 10% |
| Google | \$0.80 | 3% | 15% |
| Instagram | \$0.30 | 1.5% | 8% |

Goal: allocated budget of \$10,000
(max. no. of conversions possible)

Define variables and constraints-

$x_1 = \text{F.B}$

$x_2 = \text{Google}$

$x_3 = \text{Instagram}$

$$x_1 + x_2 + x_3 = 10,000, \quad x_1, x_2, x_3 \geq 0$$

$$\text{Conversion} = \frac{\text{Budget}}{\text{CPC}} \times \text{CTR} \times \text{Conversion rate}$$

#F.B.

$$\text{CPC} = 0.50$$

$$\text{CTR} = 0.02$$

$$\text{Conversion rate} = 0.10$$

$$C_1 = \frac{x_1}{0.50} \times 0.02 \times 0.10 = x_1 \times \left(\frac{1}{0.5} \times 0.02 \times 0.10 \right)$$

$$= x_1 \times 0.004$$

#Google

$$\text{CPC} = 0.80$$

$$\text{CTR} = 0.03$$

$$\text{Conversion rate} = 0.15$$

$$C_2 = \frac{x_2}{0.80} \times 0.03 \times 0.15 = x_2 \times 0.005625$$

Instagram

$$CPC = 0.30$$

$$CTR = 0.015$$

$$\text{Conversion rate} = 0.08$$

$$C_3 = \frac{x_3}{0.30} \times 0.015 \times 0.08 = x_3 \times \left(\frac{1}{0.30} \times 0.015 \times 0.08 \right)$$

$$= x_3 \times 0.004$$

• Objective Function

$$C_{\text{Total}} = C_1 + C_2 + C_3 = 0.004x_1 + 0.005625x_2 + 0.004x_3$$

$$\max. (C_{\text{Total}}), x_1 + x_2 + x_3 = 10,000, x_i \geq 0$$

∴ x_2 has the largest coefficient, allocating all budget to google yields highest total conversion.

$$x_1 = 0, x_2 = 10,000, x_3 = 0$$

$$C_{\text{max}} = 56.25 \text{ conversions (expected)}$$

Interpretation and Business recommendation

- suggested optimal allocation into Google Ads
- Cost per conversion:

$$\frac{\$10,000}{56.25} = \$177.78 \text{ per conversion}$$

- In real world practice, it may show diminishing returns.
- Different platforms might reach different demographics, therefore, still a diverse allocation budget should be considered.

✕

$$x_1 + x_2 + x_3 = 10,000, \quad x_1, x_2, x_3 \geq 0$$

~~$$x_1 = 1, x_2 = 8, x_3 = 1$$~~

Now,

$$x_1 = 1$$

$$x_3 = 1$$

$$x_2 = 9998$$

$$x_3 = 10000 - x_1 - x_2$$

$$\begin{aligned} C(x_1, x_2) &= 0.004x_1 + 0.005625x_2 + 0.004(10000 - x_1 - x_2) \\ &= 40 + 0.001625x_2 \end{aligned}$$

$$\bullet x_1 \geq 1$$

$$\bullet x_2 \geq 1$$

$$x_3 = 10,000 - x_1 - x_2 \geq 1, \quad x_1 + x_2 \leq 9,999$$

feasible region (x_1, x_2) is:

$$\{(x_1, x_2) \mid x_1 \geq 1, x_2 \geq 1, x_1 + x_2 \leq 9,999\}$$

still best thing to do is maximize x_2 as large as possible.

$$1 + x_2 \leq 9,999$$

$$\Rightarrow x_2 \leq 9,998$$

Optimum allocation is still

$$x_1 = 1, x_2 = 9998, x_3 = 1$$

Interpretation

- even with constraint that all platforms must receive a positive (non-zero) allocation, optimal strategy is to give nearly entire budget to google.
- minimal allocation to F.B. and instagram.

x ————— x