

В.	Image	(om	bruson	<u>w</u> i	ng -	SVP			
	Offine						,	1	
	A=	5 20 35	10 25 40	15 30 45		(mæhi in	c sepi	vents (0,:	pixel 255J)
					,				

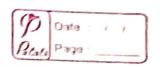
AT	transpose	of A)	, eq
AT	transpose	of A)	, dy

· Row 1 of A Tis column 1 of A	5	20	35
· Row 2 of AT 9s column 2 of A AT=	10	25	4.0
· Row 2 of AT is column 2 of A A'=	15	30	45
	_		

· Multiply ATA

ATA il a 3x3 matroc:

$$(A^{T}A)_{ij} = \sum_{i=1}^{3} (A^{T})_{i,k} A_{k,j}$$



$$C_{12} = 5.20+20.25+36935.40=100+500+1400=2000$$

$$C_{21} = 20.5 + 25.20 + 40.35 = 100 + 500 + 1400 = 2000$$

$$C_{22} = 20.20 + 25.25 + 40.40 = 2625$$

$$G_1 = 2256$$

&C32= 2850

· FFinal ATA matrix

(=	1650	2000	2250
	2000	2625	2850
	22.50	2856	3150
	_		_

1 650

AAT=

-		
•	Hultiply	AAT
	7	· · · · ·
	· (AAT)	= 650
	· (AAT), 2	= 1050

$$(AA^{T})_{12} = 1050$$

$$\cdot (AA^{T})_{13} = 1650$$

$$(AA^T)_{21} = 1056$$

$$\cdot (AA^T)_{2} = 1925$$

$$(AA^T)_3 = 3050$$

$$(AAT)_{3,2}^{3} = 1650$$

 $(AAT)_{3,2}^{3} = 3050$

$$(AAT)_0 = 4850$$

ATA-DI=	1650-7	2000	2250
	2000	2625-7	2850
	2250	2850	3150->
-1			_

calco Parie o take 25 common to make

$$H = 66 80 90$$
 $80 105 114$
 $90 114 126$

chanadmistic polynomial of M

· p(4)= det (4-4I

 $H-\mu I = 80$ 105- μ 114 90 114 126- μ

(66-4) (105-4) (1264) - 114x114] -80 (80(126-4) - 114-90)

+ 90 80x114-(105-4)90

 $\Rightarrow p(\mu) = \mu^3 - 297 \mu^2 + 980 \mu - 144 = 0$

· Numerical approximation of roots

 $\mu_1 = 0.154$ $\mu_2 = 3.18$ $\mu_3 \approx 293.66$

μ₁+μ₂+μ₃ = trace (H) = 66 + 105 + 126

H1 H2 H3 = det(H)= 144

· 4142 + 4143 + 42 H3 = 980

Eggen Values of ATA

· ATA = 25H, each eigen value is multiplied by 25

 $\lambda_1 \approx 25 \times 0.154 = 3.85$

 $\lambda_2 \approx 25 \times 3.18 = 79.5$ $\lambda_3 \approx 25 \times 293.66 = 7341.5$

Extract the Singular Values.

o, = √23 ≈ √7341.5 ≈ 85.68

 $G_2 = \sqrt{\lambda_2} = \sqrt{-9.5}$

 $G_3 = \sqrt{\lambda}_1 = \sqrt{3.85} \approx 1.96$

0,>62>03, singular values

Z matrix:

· langert Eigen Value:

13=7341.5

eigen-equation for Mi

(H-H3I)v=0-

(66-293.66) x + 84y + 90 z = 0

80x+(105-293.66)y+1144z=0

90x+114y+(126-293.66)z=0

-227-66x +80y+90z=0

80x - 188.66y + 114z = 0 90x + 114y - 167.66z = 0

we get, x=1, $y\approx 1.27$, $z\approx 1.40$

 $||v_1|| = \sqrt{1^2 + (1.2 +)^2 + (1.40)^2} \approx \sqrt{1 + 1.6129 + 1.96} \approx \sqrt{4.5729}$

 $\frac{1.27}{2.14} + \frac{140}{2.14} = (0.467, 0.593, 0.654)$

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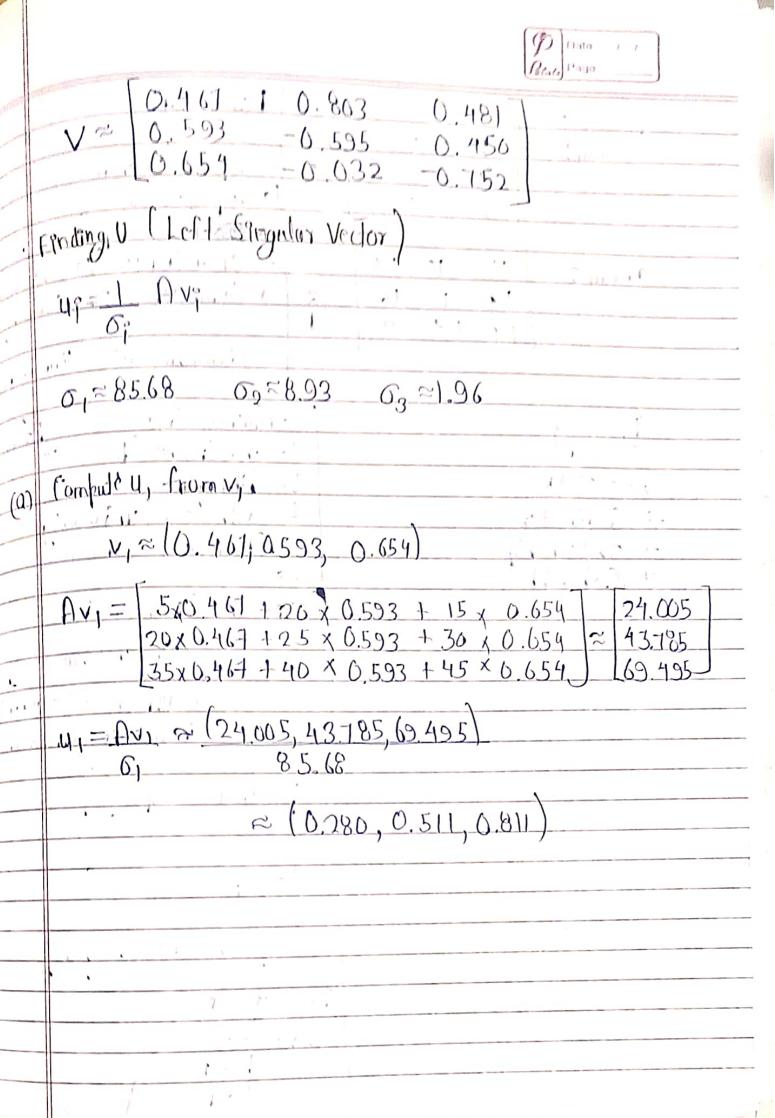
$$\begin{bmatrix} 66-3.18 & 80 & 90 \\ 80 & 105-3.18 & 114 & 9=0 \\ 90 & 114 & 126-3.18 \end{bmatrix}$$

$$v_2 \approx (1, -0.74), -0.04$$

$$||v_2|| \approx ||^{2} + (-0.74)|^{2} + (-0.04)^{2} \approx 1.245$$

$$v_2 \approx (0.803, -0.595, -0.032)$$

_				
	65,846	80	90	
	80	104.846	114	19 =
	96	(14	125.846	
	_		(~~~~	



Pape ____

(b) Compute uz from v2

$$v_2 \approx (0.803, -0.595, -0.032)$$

$$AV_2 \approx \begin{array}{l} 5 \times 0.803 + 20 \times (-0.595) + 15.(-0.032) \\ 20 \times 0.803 + 25 \times (-0.595) + 30 \times (-0.032) \\ 35 \times 0.803 + 40 \times (-0.595) + 45 \times (-0.032) \end{array}$$

$$y_2 = A v_2 \approx (-8.365, 0.225, 2.865)$$
 $\sigma_2 = 8.93$

$$\approx (-0.937, 0.025, 0.321)$$

$$V_3 \approx (0.48), 0.450, -0.752$$

$$Av_3 \approx \begin{bmatrix} 5 \times 0.481 + 20 \times 0.450 + 15 \times (-0.752) \\ 20 \times 0.481 + 25 \times 0.450 + 30 \times (-0.752) \\ 35 \times 0.481 + 40 \times 0.450 + 45 \times (-0.752) \end{bmatrix}$$

$$U_3 = AV_3 \approx (0.125, -1.69, 0.995) \approx (0.064, -0.862, 0.508)$$

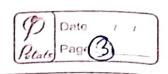
3 Final SVD components Right stegular vectors (V): $V = 0.461 \quad 0.803$ $0.593 \quad -0.595$ $0.659 \quad -0.032$ 0.481 0.4.50 -0.752· Siggular values (diagonal of E): ∑= 85.68 Left Singular v ectors (U): 0.064 -0.937 U= 6.780 0.51) -0.862 0.025 0.508 0.32 0.811

Interpretation:

-6, in significantly langer than 6, and 63.

"If we choose to keep only the langert singular value (k=1)

$$A_1 = 6_1 U_1 V_1 T_1$$



	we keep dominant features while ignoring fineraletails				
- 1	o, represents most importan				
	oz and oz add finer detai				
	Runk 1 - approximation	mutiply by 0, = 85.68			
	$A_1 = \sigma_1 u_1 v_1^T$	$A_1 = 11.20 14.23 15.69$ $20.42 25.96 28.65$			
0	G1 = 85.68	32.4) 41.19 45.40			
<u> </u>	$u_1 = 0.280$ 0.511	A respresents compressed			
	[0.81]	structure et contains dominant structure et A			
_	V ₁ = 0.467 0.593 0.654	-weachieve a significant			
	$(u_1v_1^7)_{ij}=(u_1)_j^3\times(v_1)_j^3$	compression. with using rank 1 compression.			
	$u_1v_1^T = 0.13076 0.16604 0.1831$ 6.23859 0.30292 0.3341				
	[0.37634 0.48052 0.5300]				
	,,				