2.1 Integration

(1.) Basic integration rules.

1. Calculate indefinite integral

$\cdot \int (5x^3 - 4x^2 + 2)\,dx$

$= \int 5x^3\,dx - \int 4x^2\,dx + \int 2\,dx$

$= 5\left(\frac{x^4}{4}\right) - 4\left(\frac{x^3}{3}\right) + 2x + C_1 + C_2 + C_3$

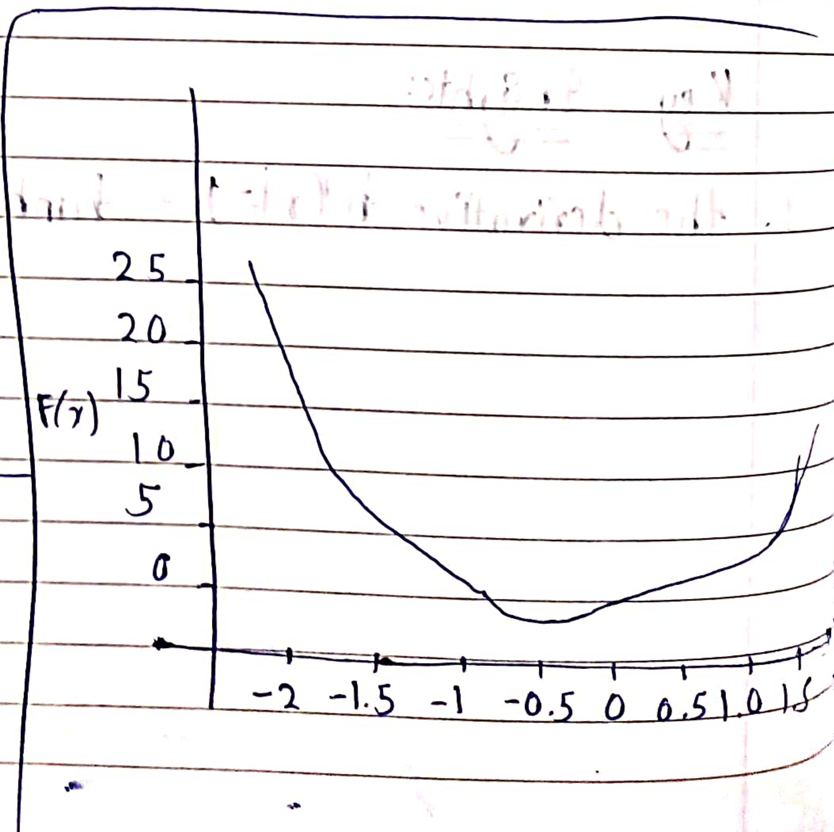
$= \frac{5x^4}{4} - \frac{4x^3}{3} + 2x + C$ $\quad [C = C_1 + C_2 + C_3$

2. $\int \frac{1}{x^2}\,dx$

$= \int x^{-2}\,dx$ $\quad [\because \int x^n = \frac{x^{n+1}}{n+1}$

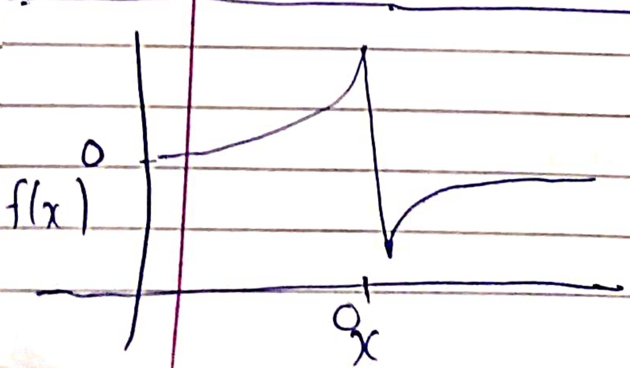
$= \frac{x^{-2+1}}{-2+1} = \frac{x^{-1}}{-1}$

$= \frac{-1}{x}$



3. $\int e^{3x}\,dx$

$= e^{3x} + C$



2. Application Problem:

$f(x) = 3x^2 + 2x$, rate at which data is received per hour.

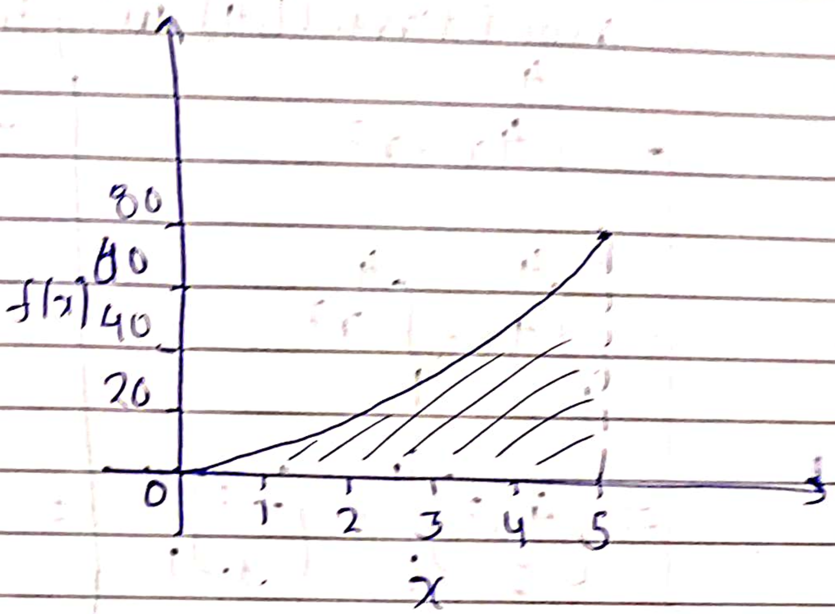
$$\int_0^5 (3x^2 + 2x)\,dx$$

$$= \left[\frac{3x^3}{3}\right]_0^5 + \left[\frac{2x^2}{2}\right]_0^5$$

$$= [x^3]_0^5 + [x^2]_0^5$$

$$= (125 - 0) + (25 - 0)$$

$$= 150$$



(2) Definite Integrals + A.U.C.

1. Definite integral

- $\displaystyle\int_1^3 (2x^2 + x)\,dx$

$$= \int_1^3 2x^2\,dx + \int_1^3 x\,dx$$

$$= \left[\frac{2x^3}{3}\right]_1^3 + \left[\frac{x^2}{2}\right]_1^3$$

$$= \left(\frac{2\times 27}{3} - \frac{2\times 1}{3}\right) + \frac{1}{2}(9 - 1)$$

$$= \frac{2}{3}[27 - 1] + \frac{1}{2}[8]$$

$$= \frac{2\times 26}{3} + 4 = \frac{52}{3} + 4 =$$

$17.33 + 4 = 21.33$

- $\displaystyle\int_0^{\pi} \sin(x)\,dx$

$$= [-\cos(x)]_0^{\pi}$$

$$= -[\cos(\pi) - \cos(0)]$$

$$= -[-1 - 1] \quad [\because \cos(\pi) = -1, \; \cos(0) = 1$$

$$= -[-2]$$

$$= 2$$

2. <u>Application problem:</u>

$$f(x) = 4x - x^2 ;$$

over interval $[1, 4]$ using integration.

$$= \int_1^4 4x - x^2$$

$$= \int_1^4 4x - \int_1^4 x^2$$

$$= \left[\frac{4x^2}{2}\right]_1^4 - \left[\frac{x^3}{3}\right]_1^4$$

$$= \left[\frac{4 \times 16}{2}\right] - \left[\frac{4 \times 1}{2}\right] - \left\{\frac{1}{3}\left[(4)^3 - (1)^3\right]\right\}$$

$$= 2 \times 15 - \left\{\frac{1}{3}(63)\right\}$$

$$= 30 - 21$$

$$= 9$$

3. <u>Probability Estimation</u>

1. <u>Integral for continuous probability:</u>

$$f(x) = \frac{1}{5} e^{-\frac{x}{5}}$$

$$\int_2^5 \frac{1}{5} e^{-x/5} \, dx$$

$$= \left[\frac{-1}{5} e^{-x/5}\right]_2^5$$

$$\cancel{= \frac{-1}{5}\left(e^{-\frac{5}{5}} - e^{-\frac{2}{5}}\right)}$$

$$\cancel{= \frac{-1}{5}\left[e^{-1} - e^{-\frac{2}{5}}\right]}$$

Page No.:
Date:

$$\left[e^{-x/5}\right]_2^5$$

• at $x = 5$

$$-e^{-5/5} = -e^{-1}$$

• at $x = 2$

$$-e^{-2/5}$$

$$P(2 \leq x \leq 5) = (-e^{-1}) - (-e^{-2/5}) = e^{-2/5} - e^{-1}$$

$$= 0.6703 - 0.3679$$

$$\approx 0.3024$$

Prob. that time b/w packets is b/w 2 and 5 minutes is approx. 0.3024

(30.24%)

Page No.:
Date:

4. Riemann Sums

1. Approximate the area:

$\int_1^3 x^2 dx$ by calc. each $f(x_i).\Delta x$, where $f(x)=x^2$

$$\text{Reimann Sum} = \sum_{i=1}^{n} f(x_i^*)\Delta x$$

$$\Delta x = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2} = 0.5$$

$$\left.\begin{array}{l} x_0 = 1 \\ \left.\begin{array}{l} x_1 = 1.5 \\ x_2 = 2 \\ x_3 = 2.5 \end{array}\right\} \end{array}\right\} \text{Left-end points}$$

$$\left.\begin{array}{l} x_1 = 1.5 \\ x_2 = 2 \\ x_3 = 2.5 \\ x_4 = 3 \end{array}\right\} \text{Right-end points}$$

- Using Left end point rule [$f(x)=x^2$]

$f(x_0)=(1)^2=1$

$f(x_1)=(1.5)^2=2.25$

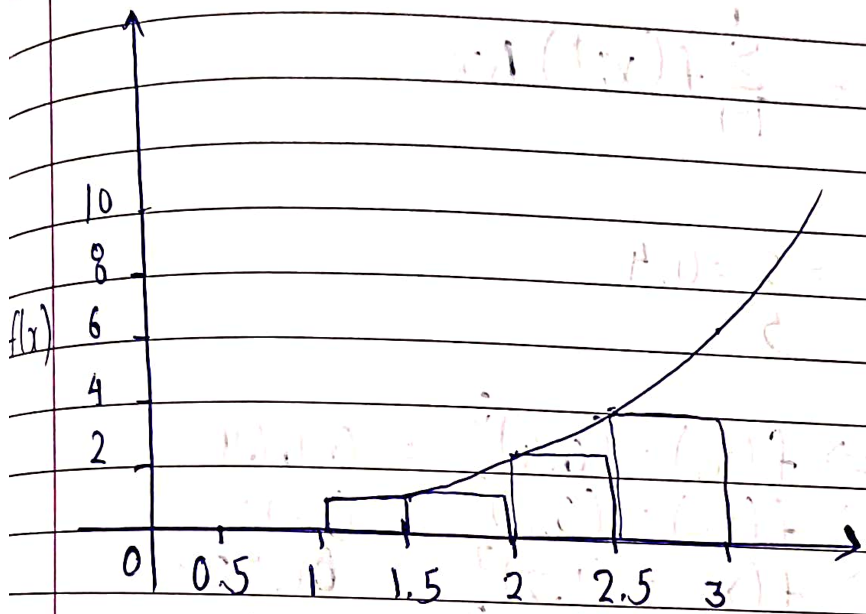
$f(x_2)=(2)^2=4$

$f(x_3)=(2.5)^2=6.25$

$$\text{Reimann Sum} = \Delta x \sum_{i=1}^{3} f(x_i)$$

$$\text{Sum} = 0.5[1+2.25+4+6.25]$$

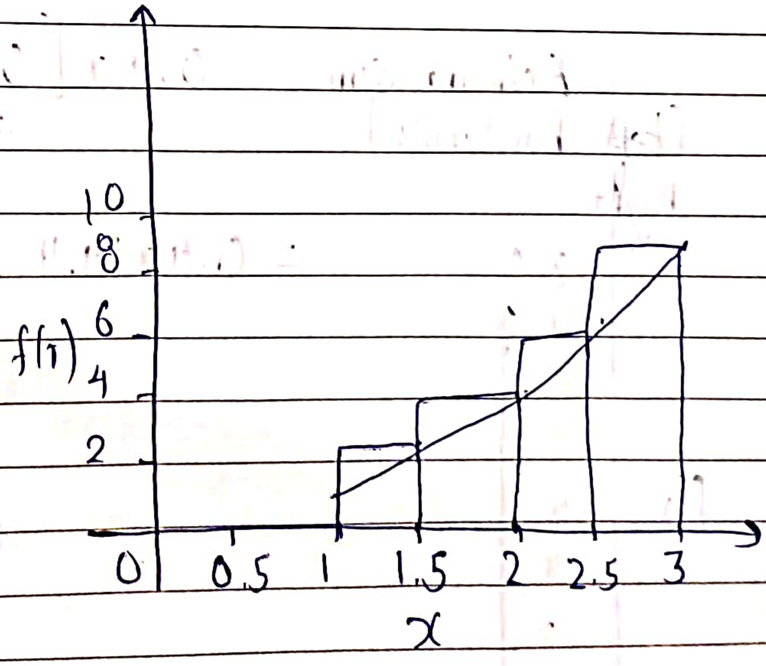
$$= 0.5 \times 13.5 = 6.75$$



- Using Right end point rule

$x_1 = 1.5$
$x_2 = 2$
$x_3 = 2.5$
$x_4 = 3$

$f(x_1) = (1.5)^2 = 2.25$
$f(x_2) = (2)^2 = 4$
$f(x_3) = (2.5)^2 = 6.25$
$f(x_4) = (3)^2 = 9$



$$\text{Reimann sum} = \Delta x \sum_{i=1}^{3} f(x_i)$$

$$= 0.5(2.25+4+6.25+9)$$

$$= 0.5 \times 21.5 = 10.75$$

Page No.:
Date:

2. Application Problems:

- $g(x) = x^3$, approximate the growth over interval $[0, 2]$ using Reimann sums with 5 rectangles.

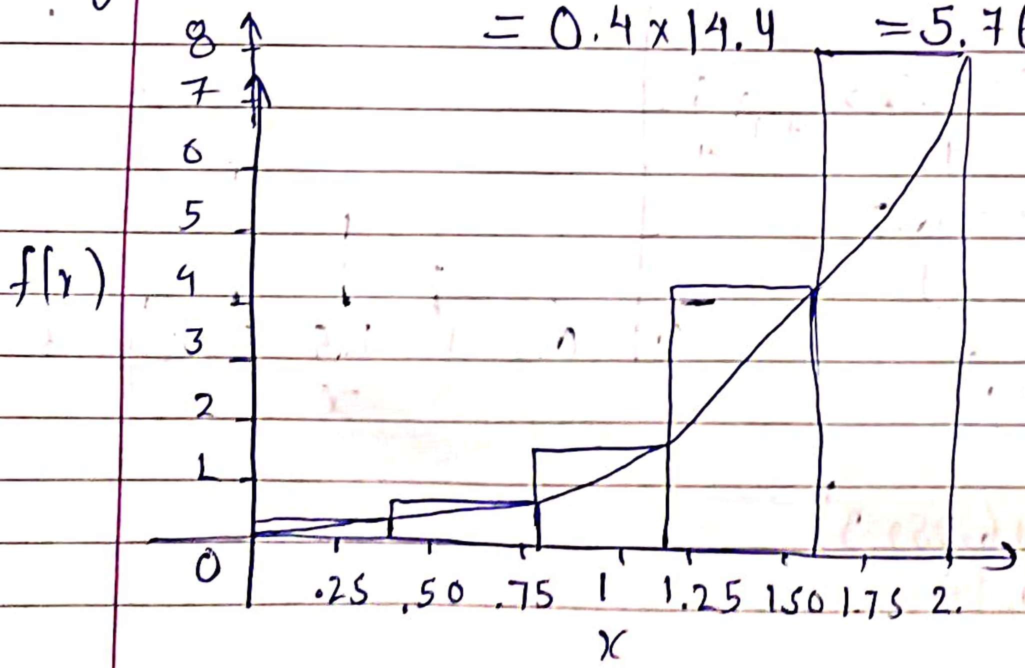
$$\text{Reimann sum} = \sum_{i=1}^{n} f(x_i^*)\,\Delta x$$

$$\Delta x = \frac{2-0}{5} = \frac{2}{5} = 0.4$$

$x_1 = 0.4 \Rightarrow f(x_1) = (0.4)^3 = 0.064$
$x_2 = 0.8 \Rightarrow f(x_2) = (0.8)^3 = 0.512$
$x_3 = 1.2 \Rightarrow f(x_3) = (1.2)^3 = 1.728$
$x_4 = 1.6 \Rightarrow f(x_4) = (1.6)^3 = 4.096$
$x_5 = 2 \Rightarrow f(x_5) = (2)^3 = 8$

Reimann sum (~~Left~~ Right hand rule) $= 0.4 \times [0.064 + 0.512 + 1.728 + 4.096 + 8]$

$= 0.4 \times 14.4 = 5.76$



5. Advanced Applications

1. Using Integrals for Data smoothing:

$f(x) = 2 + \cos(x)$, $[0, 2\pi]$

$$\text{Average value} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$= \frac{1}{2\pi - 0}\int_0^{2\pi} (2 + \cos(x))\,dx$$

$$= \frac{1}{2\pi}\int_0^{2\pi} (2 + \cos(x))\,dx$$

$$= 2[x]_0^{2\pi} + [\sin(x)]_0^{2\pi}$$

$$= 4\pi + \{\sin(2\pi) - \sin(0)\}$$

$$= 4\pi + \{0 - 0\}$$

$$= 4\pi$$

$$\text{Average Value} = \frac{1}{2\pi} \cdot 4\pi = 2$$