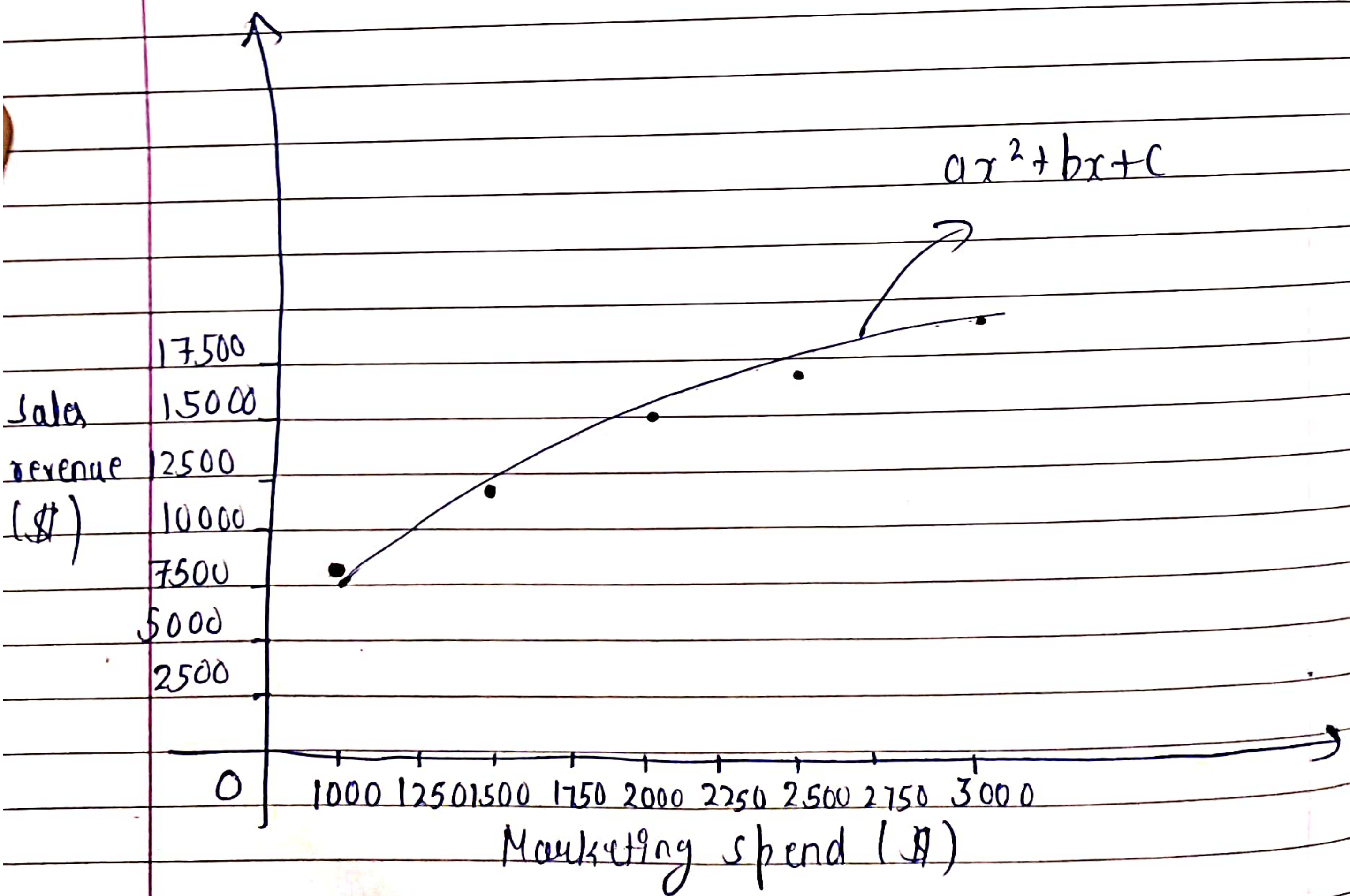
Case Study

1. Optimizing market spend

sample data:

Marketing Spend ($)	Salves Revenue ($)
1000	8000
1500	11500
2000	15000
2500	17000
3000	18500



· Fitting a Polynomial function

$f(x) = ax^2 + bx + c$

Polynomial regression:

Marketing spend (x): [1000, 1500, 2000, 2500, 3000]

Sales revenue (y): [8000, 11500, 15000, 17000, 18500]

$$y = X \cdot p$$

$$y = \begin{bmatrix} 8000 \\ 11500 \\ 15000 \\ 17000 \\ 18500 \end{bmatrix}, \quad X = \begin{bmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ x_3^2 & x_3 & 1 \\ x_4^2 & x_4 & 1 \\ x_5^2 & x_5 & 1 \end{bmatrix}$$

$$p = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

We solve for p (coefficients a, b and c) using normal equation:

$$p = (X^T X)^{-1} X^T y$$

① Design Matrix

- $$X = \begin{bmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ x_3^2 & x_3 & 1 \\ x_4^2 & x_4 & 1 \\ x_5 & x_5 & 1 \end{bmatrix} = \begin{bmatrix} 1000000 & 1000 & 1 \\ 2250000 & 1500 & 1 \\ 4000000 & 2000 & 1 \\ 6250000 & 2500 & 1 \\ 9000000 & 3000 & 1 \end{bmatrix}$$

Observation Vector (y):

$$y = \begin{bmatrix} 8000 \\ 11500 \\ 15000 \\ 17000 \\ 18500 \end{bmatrix}$$

Normal Eqn. Components:

- X^T

$$X^T = \begin{bmatrix} 1000000 & 2250000 & 4000000 & 6250000 & 9000000 \\ 1000 & 2500 & 2000 & 2500 & 3000 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

- $$X^TX = \begin{bmatrix} 1.925\times10^{14} & 1.25\times10^{10} & 15000000 \\ 1.25\times10^{10} & 11250000 & 150000 \\ 15000000 & 15000 & 5 \end{bmatrix}$$

- $$X^Ty = \begin{bmatrix} 7.5\times10^{10} \\ 45000000 \\ 82000 \end{bmatrix}$$

Page No.:
Date:

$$p = (X^T X)^{-1} X^T y$$

$$p = \begin{bmatrix} -0.001571 \\ 11.585714 \\ -2100 \end{bmatrix}$$

$$f(x) = -0.002x^2 + 12x - 2100$$

$$f'(x) = -0.004x + 12$$

$$f'(x) = 0$$

$$\Rightarrow -0.004x + 12 = 0$$
$$\Rightarrow -0.004x = -12$$
$$\Rightarrow x = \frac{12 \times 1000}{4} = 3000$$

- $f(1000) = -0.002(1000)^2 + 12(1000) - 2100$
  $= -2000 + 12000 - 2100$
  $= 7,900$

- $f(1500) = -0.002(1500)^2 + 12(1500) - 2100$
  $= -4500 + 18000 - 2100$
  $= 11,400$

- $f(3000) = -0.002(3000)^2 + 12(3000) - 2100$
  $= -18000 + 36000 - 2100$
  $= 15900$

- $f(4000) = -0.002(4000)^2 + 12(4000) - 2100$
  $= -32000 + 48000 - 2100$
  $= 13,900$ (Diminishing returns)

Page No.:

Date:

$\cdot f(3200) = -0.002(3200)^2 + 12(3200) - 2100$

$= -20{,}480 + 38{,}400 - 2100$

$= 15{,}820$ (still diminishing returns)

So, for marketing spend of 3000 (approx.) the sales revenue maximize. and even after spending more ~~than the actual~~ we would get diminishing returns.

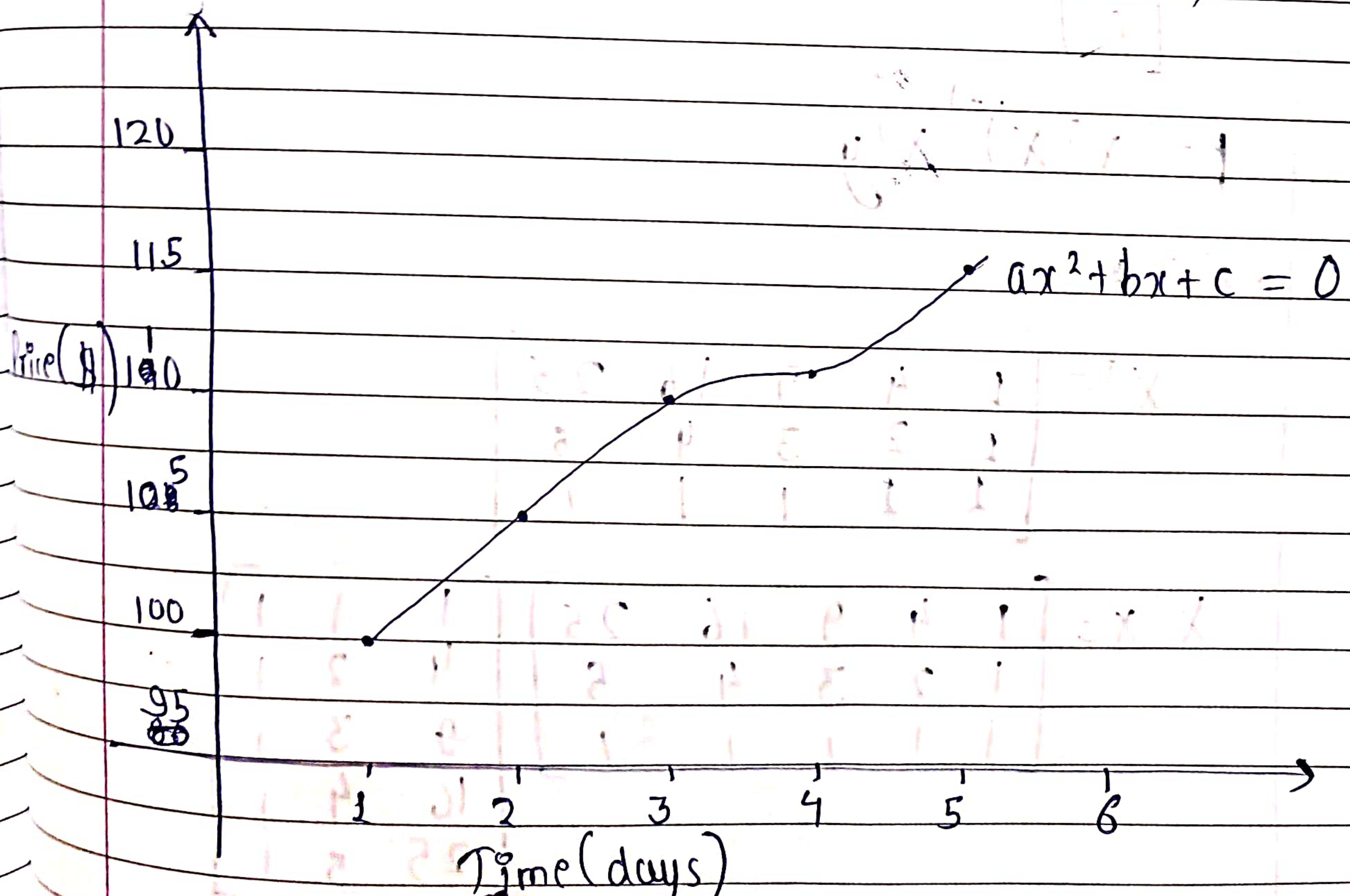
Date:

2. Predicting Stock Price Movements:

Context: hedge fund is interested in analyzing stock trends for informed trading decisions.

sample data:

Time (days)	Price ($)
1	100
2	105
3	110
4	112
5	118



Page No.:
Date:

- $X = \begin{bmatrix} t_1^2 & t_1 & 1 \\ t_2^2 & t_2 & 1 \\ t_3^2 & t_3 & 1 \\ t_4^2 & t_4 & 1 \\ t_5^2 & t_5 & 1 \end{bmatrix} = \begin{bmatrix} 1^2 & 1 & 1 \\ 2^2 & 2 & 1 \\ 3^2 & 3 & 1 \\ 4^2 & 4 & 1 \\ 5^2 & 5 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \\ 16 & 4 & 1 \\ 25 & 5 & 1 \end{bmatrix}$

- $y = \begin{bmatrix} 100 \\ 105 \\ 110 \\ 112 \\ 118 \end{bmatrix}$

- $p = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

$$p = (X^T X)^{-1} X^T y$$

- $X^T X$:

$$X^T = \begin{bmatrix} 1 & 4 & 9 & 16 & 25 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$X^T X = \begin{bmatrix} 1 & 4 & 9 & 16 & 25 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \\ 16 & 4 & 1 \\ 25 & 5 & 1 \end{bmatrix}$$

$$X^T X = \begin{bmatrix} 979 & 225 & 55 \\ 225 & 55 & 15 \\ 55 & 15 & 5 \end{bmatrix}$$

$$X^T y = \begin{bmatrix} 6252 \\ 1678 \\ 545 \end{bmatrix}$$

$$f(x) = ax^2 + bx + c$$

$$p = \bar{a} = \begin{bmatrix} -0.0714 \\ 4.7286 \\ 95.6 \end{bmatrix}$$

$$f(x) = -0.0714(x)^2 + 4.7286x + 95.6$$

- $f'(x) = (-0.0714)2x + 4.7286$

$$= -0.1428x + 4.7286$$

- $f''(x) = -0.1428$

3. Calculating (CLV) Customer Lifetime Value using Integration:

Monthly revenue: ($50)

Monthly Churn Rate: $f(t) = 0.05\,e^{-0.1t}$

$$CLV = \int_0^{12} 50 . e^{-0.1t}\,dt$$

$$\because \int e^{ax}\,dx = \frac{1}{a} e^{ax} + C$$

$$= \int e^{-0.1t}\,dt$$

$$= \frac{1}{-0.1} e^{-0.1t}$$

$$= -10e^{-0.1t}$$

$$CLV = 50 . \left[-10e^{-0.1t}\right]_0^{12}$$

$$= 50\left\{-10e^{-0.1\times 12} - \left[-10e^{-0.1(0)}\right]\right\}$$

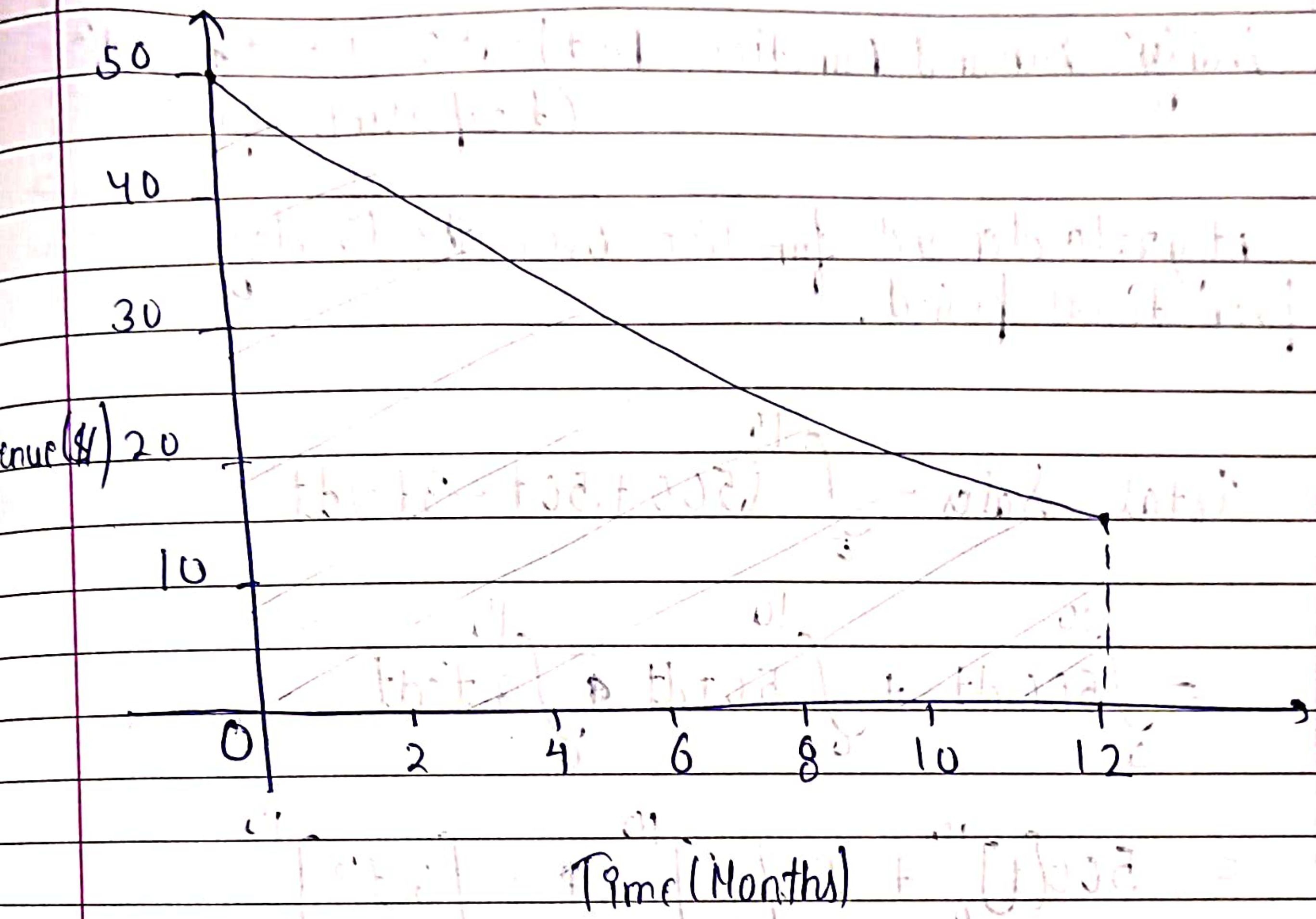
$$= 50\left\{-10e^{-1.2} - [-10]\right\}$$

$$= 50\left[-10e^{-1.2} + 10\right]$$

$$= 50 , 10\left[-e^{-1.2} + 1\right]$$

$$= 500\left[1 - e^{-1.2}\right]$$

$CLV = 500 \cdot (1 - 0.3012)$
$= 500 \times 0.6988$
≈ 349.40



curve represents decreasing revenue as churn affects the customer base.

The shaded region under the curve corresponds to the CLV, which is the total revenue accumulated over the 12-moth period, approx. $349.40

Page No.:
Date:

(4.) Demand Forecasting for E-commerce using Integration

Sample Data:

- Daily Demand Function: $D(t) = 500 + 50t - 3t^2$
  (t represents days)

- Integrate demand function over the 10-day promotional period.

$$\text{Total Sales} = \int_0^{10} (500 + 50t - 3t^2)\,dt$$

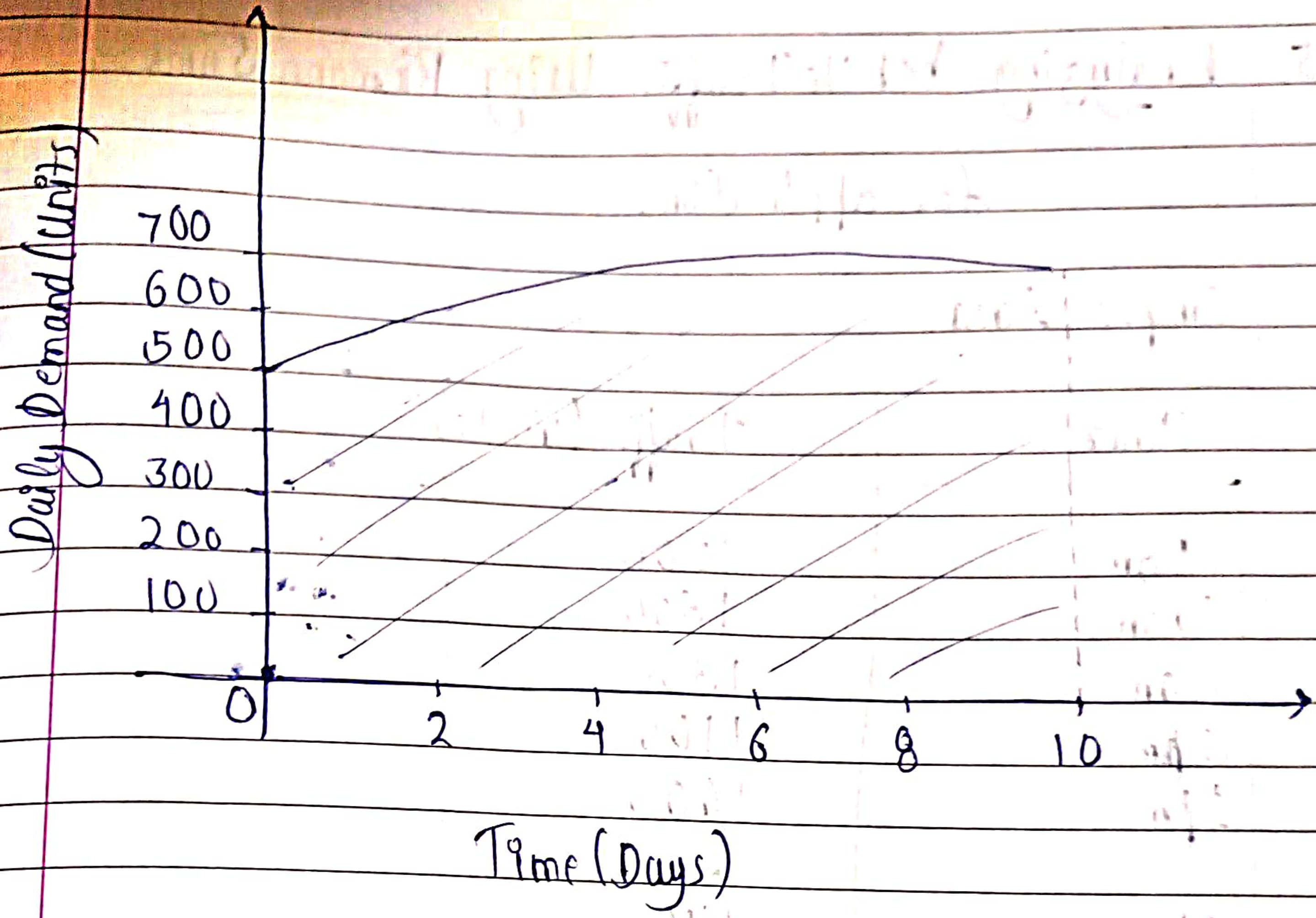
$$= \int_0^{10} 500\,dt + \int_0^{10} 50t\,dt - \int_0^{10} 3t^2\,dt$$

$$= 500[t]_0^{10} + 50\left[\frac{t^2}{2}\right]_0^{10} - \left[\frac{3t^3}{3}\right]_0^{10}$$

$$= 500[10 - 0] + 25[10^2 - 0^2] - [t^3]_0^{10}$$

$$= 5000 + 2500 - 1000$$
$$= 6500$$

total sales over the 10-day promotional period are 6,500 units.



shaded region : total sales over this period
approx. (6500 units)

6. Optimizing Inventory Using Differentiation:

$$C(x) = 200 + 20x + 0.5x^2$$

$C'(x) = 20 + 1.x$
$= 20 + x$

$C'(x) = 0$

$\Rightarrow 20 + x = 0 \Rightarrow x = -20$

$C''(x) = 1$

$\because C''(x) > 0$, function is concave up,

so, critical point is minimum.

But, $x = -20$ (does not make sense)

$\because$ negative months do not.

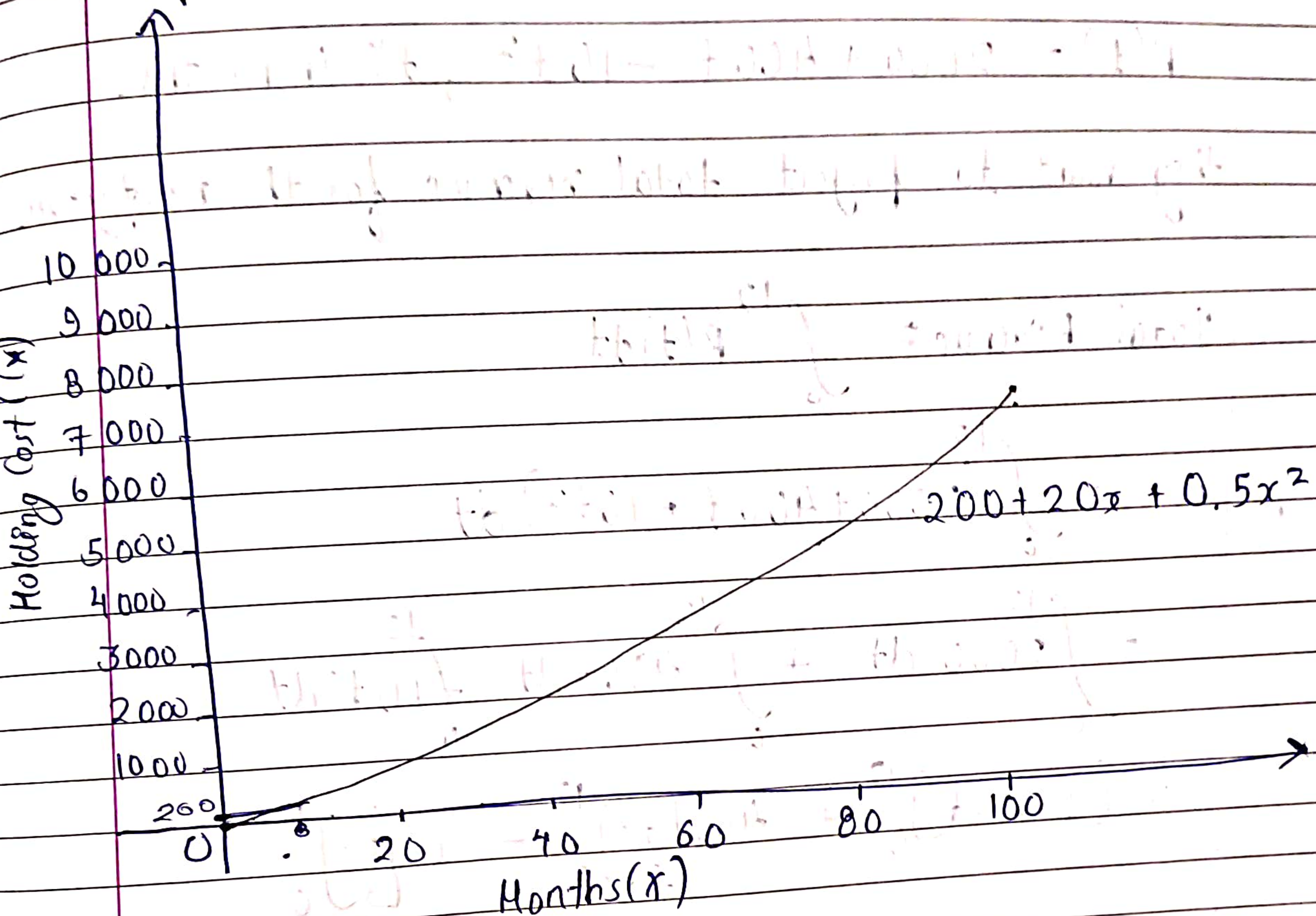
As the no. of months progress the holding cost will increase.

Conclusion:

* Either don't keep the product in inventory (not possible)
* there is no optimal no. of months to keep the product.

Page No.:
Date:

* the holding cost is increasing at a steady pace.



* the model is incorrect or it needs improvement.

7. Predicting Sales Growth using Integrals-

$R(t) = 2000 + 400t - 10t^2$, t is in months.

they want to project total revenue for the next year.

Total Revenue $= \int_0^{12} R(t)\,dt$

$$= \int_0^{12} (2000 + 400t - 10t^2)\,dt$$

$$= \int_0^{12} 2000\,dt + \int_0^{12} 400t\,dt \int_0^{12} 10t^2\,dt$$

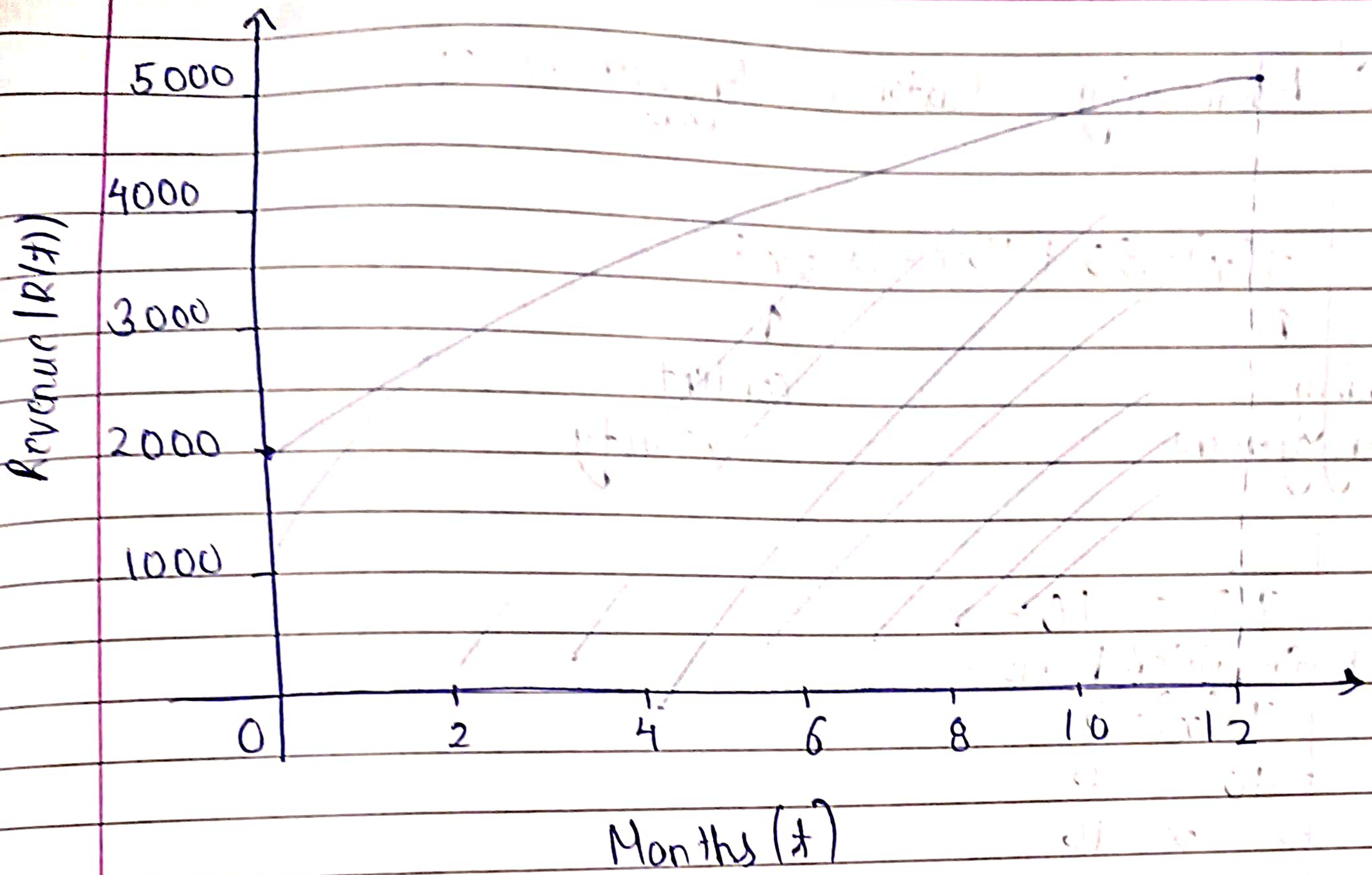
$$= \left[2000t\right]_0^{12} + 400\left[\frac{t^2}{2}\right]_0^{12} - 10\left[\frac{t^3}{3}\right]_0^{12}$$

$$= 2000[12-0] + 200\left[12^2 - 0^2\right] - \frac{10}{3}\left[(12)^3 - (0)^3\right]$$

$$= 24000 + 28,800 - \frac{10}{\cancel{3}_{1}} \times \cancel{12}^{4} \times 12 \times 12$$

$$= 24000 + 28,800 - 5,760$$

$$= 47,040$$



* the total projected revenue for next 12 months is 47,040

Page No.:
Date:

(8.) Maximizing Customer Engagement Time

$T(x) = 50 + 10x - 0.5x^2$

Daily engagement

content variety

$T'(x) = 10 - x$

* for critical points

$T'(x) = 0$

$\Rightarrow 10 - x = 0$

$\Rightarrow x = 10$

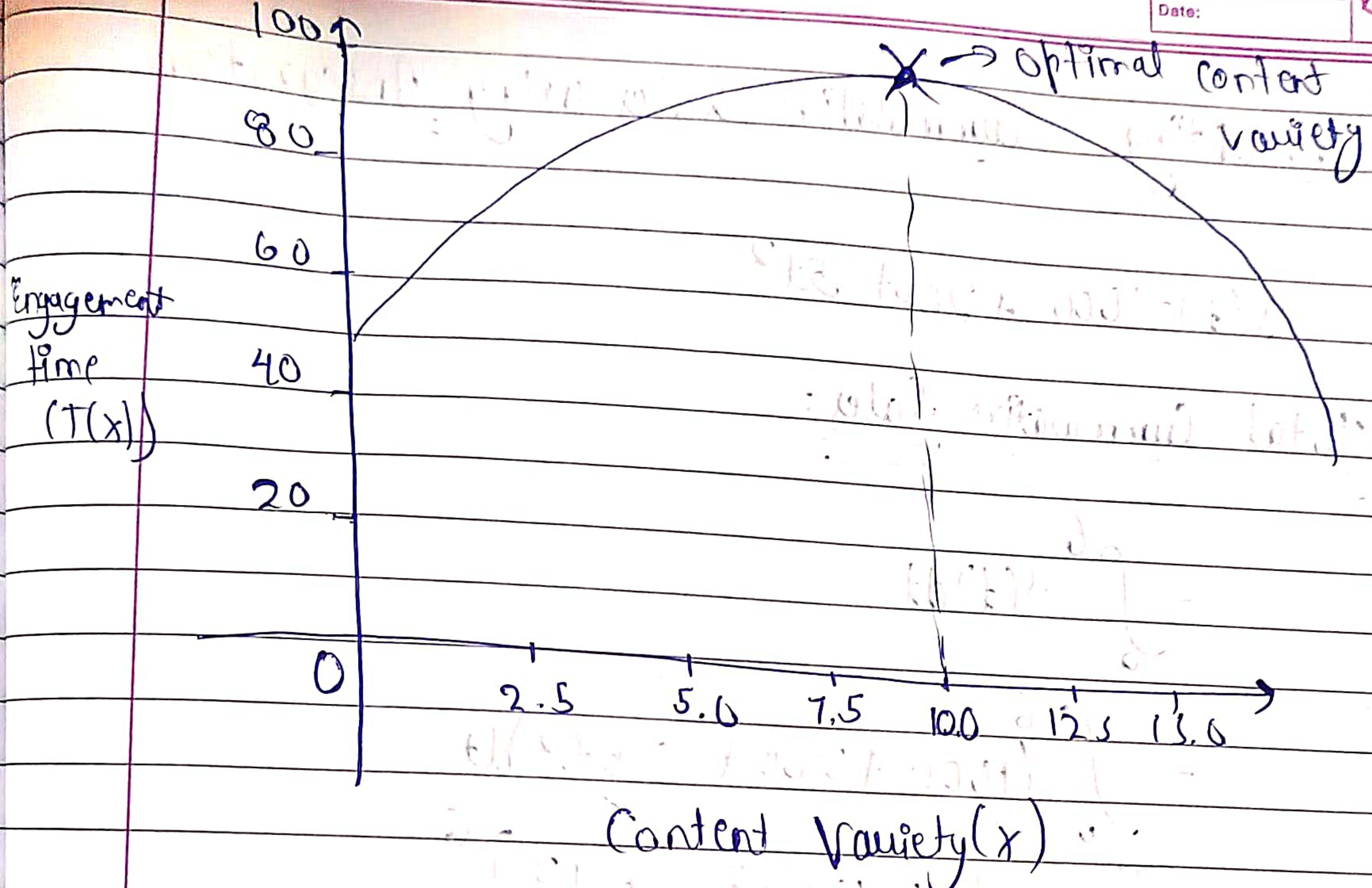
* $T''(x) = -1$

$\Rightarrow x = 10$ is a maximum, $\because T''(x) < 0$

$T(10) = 50 + 10(10) - 0.5(10)^2$

$= 50 + 100 - 50$

$= 100$



* So, for $x = 10$, engagement is maximized.

Page No.:
Date:

(9.) Predicting Cummulative Sales using Integration

$S(t) = 1000 + 200t - 5t^2$

Total Cummulative Sales:

$$= \int_0^6 S(t)\,dt$$

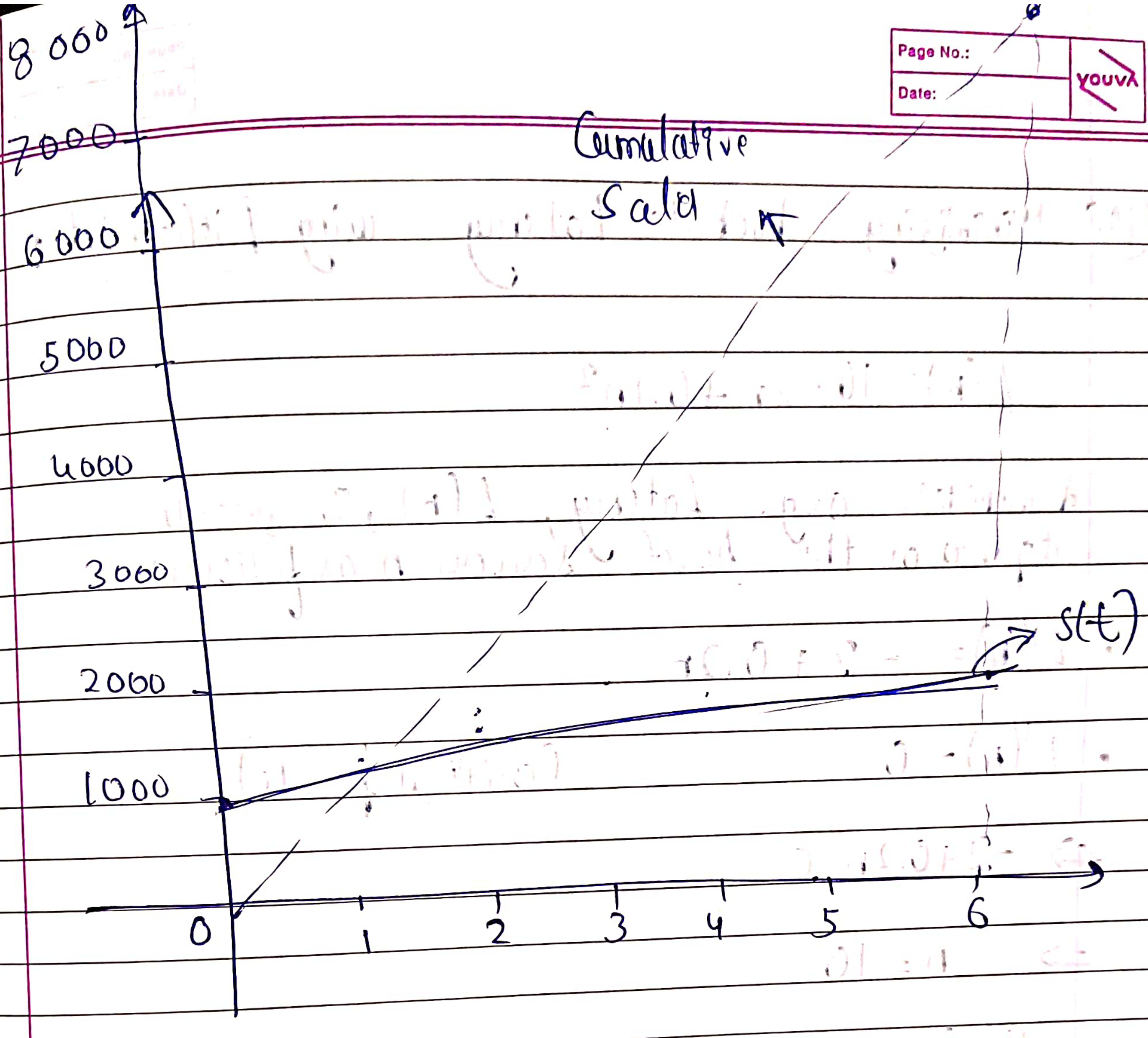
$$= \int_0^6 (1000 + 200t - 5t^2)\,dt$$

$$= \left[1000t + 100t^2 - \frac{5}{3}t^3\right]_0^6$$

$$= 1000 \times 6 + 100 \times 36 - \frac{5}{\cancel{3}_1} \times \cancel{6}^{2} \times 36$$

$$= 6000 + 3600 - 360$$

$$= 9240$$


Cumulative
Sala
8000
7000
6000
5000
4000
3000
2000
1000
0
1
2
3
4
5
6
S(t)

(10.) Minimizing Website Latency using Differentiation-

$$L(n) = 10 - 2n + 0.1n^2$$

A website's avg. Latency, $L(n)$, in seconds, depends on the no. of servers n as follows:

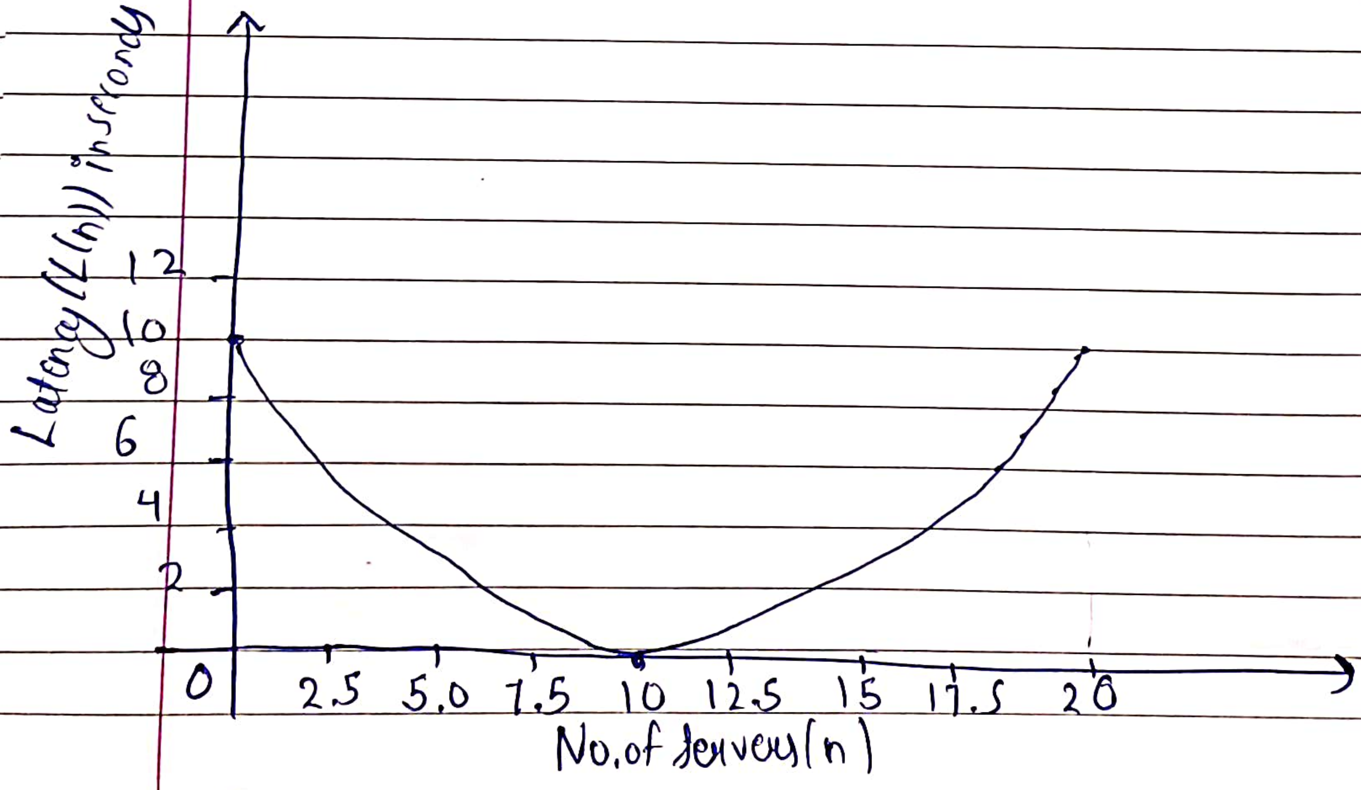
- $L'(n) = -2 + 0.2n$
- $L'(n) = 0$ (critical points).

$\Rightarrow -2 + 0.2n = 0$

$\Rightarrow n = 10$

- $L''(n) = 0.2$, $\because L''(n) > 0$, concave up (minimum)

$L(10) = 0$ seconds



at $n = 10$, the latency of server will be minimum.

Page No.:
Date:

(11.) Estimating Total Social Media Engagement using integration

$E(t) = 500 + 30t - t^2$, $t \to$ weeks

A company measures engagement growth over time in hours .

$$\text{Total Engagement} = \int_0^4 E(t)\,dt$$

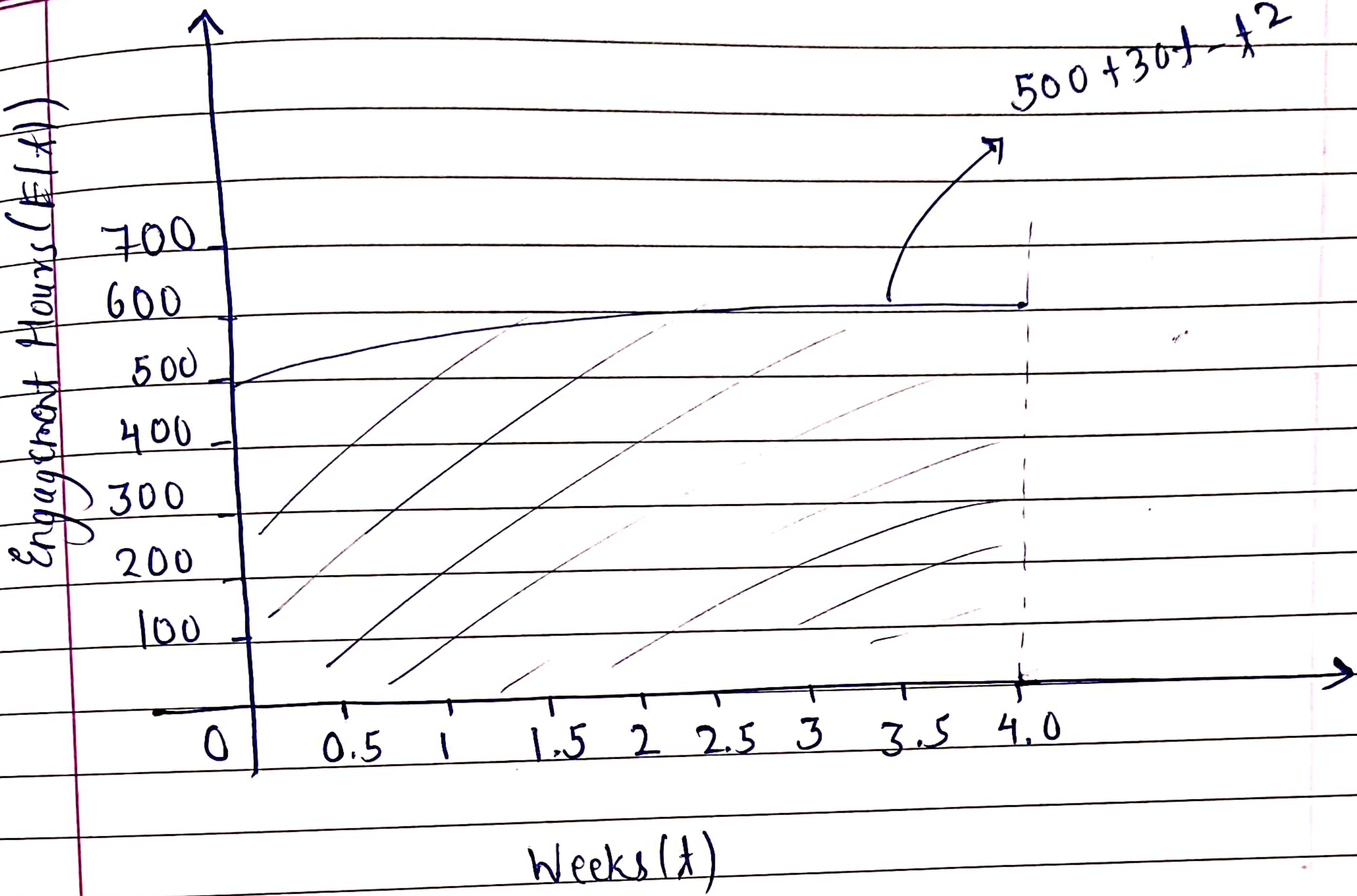
$$= \int_0^4 (500 + 30t - t^2)\,dt$$

$$= [500t]_0^4 + \frac{30}{2}[t^2]_0^4 - \left[\frac{t^3}{3}\right]_0^4$$

$$= 2000 + 15 \times 16 - \frac{4 \times 4 \times 4}{3}$$

$$= 2000 + 240 - 21.33$$

$$= 2218.67$$


$500+30t-t^2$
Engagement Hours (E(t))
700
600
500
400
300
200
100
0
0.5
1
1.5
2
2.5
3
3.5
4.0
Weeks (t)