1. Movie Recommendation system

Matrix (R)

rows = Users
columns = movies
ratings = value

UserID \ MOVIEID	1	2	17	56
5	4	3	4	5
17	4	0	0	0
66	3	0	0	0

= R

- User 5 rated movie 1 with a 4
- User 17 rated movie 1 with a 4

similarly, the above matrix is explained

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$$R = \begin{bmatrix} 4 & 3 & 4 & 5 \\ 4 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 \end{bmatrix}$$

2. Initialize P and Q with small Random values

$$P = \begin{bmatrix} 0.5 \\ 0.3 \\ 0.2 \end{bmatrix}, \quad Q = \begin{bmatrix} 0.4 & 0.3 & 0.5 & 0.6 \end{bmatrix}$$

3. Compute predicted ratings

$$\hat{R} = P Q^T$$

$$\hat{R} = \begin{bmatrix} 0.5 \\ 0.3 \\ 0.2 \end{bmatrix} \begin{bmatrix} 0.4 & 0.3 & 0.5 & 0.6 \end{bmatrix}$$

$$\hat{R} = \begin{bmatrix} 0.2 & 0.15 & 0.25 & 0.3 \\ 0.12 & 0.09 & 0.15 & 0.18 \\ 0.08 & 0.06 & 0.1 & 0.12 \end{bmatrix}$$

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4. Compute Error matrix

$E = R - \hat{R}$

$E_{1,1} = 4 - 0.2 = 3.8$; $E_{1,2} = 3 - 0.15$, $E_{1,3} = 4 - 0.25 = 3.75$

$E_{1,4} = 4.7$, $E_{2,1} = 3.88$, $E_{3,1} = 2.92$

Error matrix (E) (observed ratings) $= \begin{bmatrix} 3.8 & 2.85 & 3.75 & 4.7 \\ 3.88 & - & - & - \\ 2.92 & - & - & - \end{bmatrix}$

5. Gradient Descent update rules

$P \leftarrow P + \alpha \cdot \sum E_{ij} Q_{jk}$

$Q \leftarrow Q + \alpha \sum e_{ij} P_{ik}$

$\alpha = 0.01$ (learning rate)

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6. Update P matrix

$P_1 \leftarrow P_1 + \alpha(e_{1,1}q_1 + e_{1,2}q_2 + e_{1,3}q_3 + e_{1,4}q_4)$

$P_1 \leftarrow 0.5 + 0.01\,(3.8 \times 0.4 + 2.85 \times 0.3 + 3.75 \times 0.5 + 4.7 \times 0.6)$

$P_1 \leftarrow 0.5 + 0.0707 = 0.5707$

$P_2 = 0.31552$

$P_3 = 0.21168$

7. Update Q Matrix

$Q_1 \leftarrow Q_1 + \alpha(e_{1,1}P_1 + e_{2,1}P_2 + e_{3,1}P_3)$

$Q_1 \leftarrow 0.43646$

$Q_2 = 0.31425$

Updated matrix after one iteration

$$P = \begin{bmatrix} 0.5707 \\ 0.31552 \\ 0.21168 \end{bmatrix}, \quad Q = \begin{bmatrix} 0.43648 & 0.31425 \\ 0.51435 & 0.61649 \end{bmatrix}$$

8. Predict $\hat{R}$

$$\hat{R} = P \times Q^T$$

$$= \begin{bmatrix} 0.2491 & 0.1793 & 0.2936 & 0.3518 \\ 0.1377 & 0.0992 & 0.1623 & 0.1945 \\ 0.0924 & 0.0665 & 0.1089 & 0.1305 \end{bmatrix}$$

Interpretation

- values are small due to one iteration
- with more iterations they will converge to actual values.

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Recommended movies for each user:

- User 1: Movie 4, 3 (highest predicted ratings: 0.3518, 0.2936)
- User 2: Movies 4, 3 (highest predicted ratings: 0.1945, 0.162)
- User 3: Movie 4, 3 (highest predicted ratings)

MAE:

User	Movie	R_{ij}	$\hat{R}_{ij}$	Absolute error	Squared error
1	1	4	0.2491	3.7509	14.0652
1	2	3	0.1793	2.8207	7.9563
1	3	4	0.2936	3.7064	13.7380
1	4	5	0.3518	4.6482	21.6137
2	1	4	0.3377	3.8623	14.9205
3	1	3	0.0924	2.9076	8.4559

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$$MAE = \frac{21.6961}{6} = 3.616$$

$$RMSE = \sqrt{\frac{80.7496}{6}} = \sqrt{13.4583} = 3.67$$

MAE & RMSE are quite high due to only one iteration.