$$\mathbf{P} = ()$$

Presentation PROJECT01

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Problem Statement

Draw the tangents through the point

$$\mathbf{P} = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$$

(2.0)

to the circle

$$C : x^T x = 9. \tag{2.0}$$

Finding the Centre and Radius

Let $\mathbf{O}$ be the centre of C. From the above equation of circle given in question,

$$\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{3.0}$$

also radius of circle c is r
r$=\sqrt{9} = 3$.

Finding the tangent points

let us name points on circle where tangent drawn from P meets the circle as Q1,Q2 whose solution is let's say Q.since PQ Is perpendicular to QO

$$(P - Q)^T (Q - O) = 0, \tag{3.0}$$

$$(P - Q) = k(omat.Q) \tag{3.0}$$

where

$$omat = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{3.0}$$

which implies

$$\|P - Q\|^2 = K^2 \|omat.Q\|^2 \tag{3.0}$$

since Q is a point on circle C,

$$\|Q\| = 3 \tag{3.0}$$

$$k^2 = \|P - Q\|^2 / 9 \tag{3.0}$$

$$\|P - Q\| = \sqrt{PO^2 - r^2} \tag{3.0}$$

$$k = +\sqrt{56}/3 \tag{3.0}$$

or

$$k = -\sqrt{56}/3 \tag{3.0}$$

$$P = (I + (k.omat)).Q \tag{3.0}$$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{3.0}$$

let's say (I+k.omat)=R then

$$Q = R^{-1}P \tag{3.0}$$

$$R = \begin{pmatrix} 1 & \sqrt{56}/3 \\ -\sqrt{56}/3 & 1 \end{pmatrix} \tag{3.0}$$

or

$$R = \begin{pmatrix} 1 & -\sqrt{56}/3 \\ \sqrt{56}/3 & 1 \end{pmatrix} \tag{3.0}$$

therefore

$$Q1 = \begin{pmatrix} 1 & -\sqrt{56}/3 \\ \sqrt{56}/3 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 7 \end{pmatrix} /(1 + 56/9)$$

(3.0)

$$Q1 = 9/65 \begin{pmatrix} 4 - 7\sqrt{56}/3 \\ 7 + 4\sqrt{56}/3 \end{pmatrix} \tag{3.0}$$

similarly,

$$Q2 = 9/65 \begin{pmatrix} 4 + 7\sqrt{56}/3 \\ 7 - 4\sqrt{56}/3 \end{pmatrix} \tag{3.0}$$

Finding the equation of tangents

The equation of tangent can be obtained from direction vector

$$m_1 = (P - Q1) \tag{3.0}$$

The normal vector n_1 *is*
defined as

$$n_1^T m_1 = 0 \tag{3.0}$$

and can be obtained as

$$n_1 = Q1 - O \tag{3.0}$$

$$n_1 = \begin{pmatrix} (392 - 12\sqrt{56})/65 \\ -(224 + 21\sqrt{56})/65 \end{pmatrix} \tag{3.0}$$

Therefore equation of tangent PQ1 is

$$n_1^T(x - P) = 0 \tag{3.0}$$

$$n_1^T x = n_1^T P \tag{3.0}$$

$$\begin{pmatrix} (392 - 12\sqrt{56})/65 \\ -(224 + 21\sqrt{56})/65 \end{pmatrix}^T x = \begin{pmatrix} (392 - 12\sqrt{56})/65 \\ -(224 + 21\sqrt{56})/65 \end{pmatrix}^T \begin{pmatrix} 4 \\ 7 \end{pmatrix} \tag{3.0}$$

$$\begin{pmatrix} (392 - 12\sqrt{56}) & -(224 + 21\sqrt{56}) \end{pmatrix} x \tag{3.0}$$

$$= \begin{pmatrix} (392 - 12\sqrt{56}) & -(224 + 21\sqrt{56}) \end{pmatrix} \begin{pmatrix} 4 \\ 7 \end{pmatrix} \tag{3.0}$$

The equaion of tangent PQ1 is

$$\begin{pmatrix} (392 - 12\sqrt{56}) & -(224 + 21\sqrt{56}) \end{pmatrix} x = -195\sqrt{56} \tag{3.0}$$

similarly,
The equation of tangent can be obtained from direction vector

$$m_2 = (P - Q2) \tag{3.0}$$

The normal vector $\mathrm{n}_2 is$
defined as

$$n_2^T m_2 = 0 \tag{3.0}$$

and can be obtained as

$$n_2 = Q2 - O \tag{3.0}$$

$$n_2 = \begin{pmatrix} (392 + 12\sqrt{56})/65 \\ -(224 - 21\sqrt{56})/65 \end{pmatrix} \tag{3.0}$$

Therefore equation of tangent PQ2 is

$$n_2^T(x - P) = 0 \tag{3.0}$$

$$n_2^T x = n_2^T P \tag{3.0}$$

$$\begin{pmatrix} (392 + 12\sqrt{56})/65 \\ -(224 - 21\sqrt{56})/65 \end{pmatrix}^T x = \begin{pmatrix} (392 + 12\sqrt{56})/65 \\ -(224 - 21\sqrt{56})/65 \end{pmatrix}^T \begin{pmatrix} 4 \\ 7 \end{pmatrix} \tag{3.0}$$

$$\begin{pmatrix} (392 + 12\sqrt{56}) & -(224 - 21\sqrt{56}) \end{pmatrix} x \tag{3.0}$$

$$= \begin{pmatrix} (392 + 12\sqrt{56}) & -(224 - 21\sqrt{56}) \end{pmatrix} \begin{pmatrix} 4 \\ 7 \end{pmatrix} \tag{3.0}$$

The equaion of tangent PQ2 is

$$\begin{pmatrix} (392 + 12\sqrt{56}) & -(224 - 21\sqrt{56}) \end{pmatrix} x = +195\sqrt{56} \tag{3.0}$$

Plot

https://github.com/SRIJITH01/Srijith/blob/master/srijith.py

plots Fig. 1.

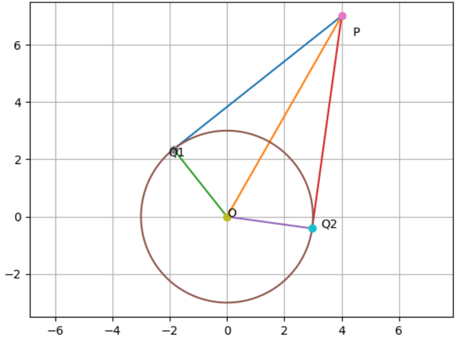

Figure: Tangents To circle.