Presentation2
PROJECT07
GROUP11

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Problem Statement

Let **A**,**B**,**C** be three unit vectors such that

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \frac{\sqrt{3}}{2}(\mathbf{B} + \mathbf{C}) \tag{2.1}$$

If **B** is not parallel to **C**,then find the angle between **A** and **B**.

Finding angle between **A** and **B**

Given that **A**,**B**,**C** are unit vectors and

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \frac{\sqrt{3}}{2}(\mathbf{B} + \mathbf{C}) \tag{3.1}$$

We know that

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A}.\mathbf{C})\mathbf{B} - (\mathbf{A}.\mathbf{B})\mathbf{C} \tag{3.2}$$

PROOF

Let

$$\mathbf{A} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} \begin{bmatrix} i & j & k \end{bmatrix} \tag{4.1}$$

$$\mathbf{B} = \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} \begin{bmatrix} i & j & k \end{bmatrix} \tag{4.2}$$

$$\mathbf{C} = \begin{bmatrix} c_x \\ c_y \\ c_z \end{bmatrix} \begin{bmatrix} i & j & k \end{bmatrix} \tag{4.3}$$

Now

$$\mathbf{B} \times \mathbf{C} = \begin{bmatrix} i & j & k \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{bmatrix} \tag{4.4}$$

$$\mathbf{B} \times \mathbf{C} = (b_y c_z - b_z c_y)i - (b_x c_z - b_z c_x)j + (b_x c_y - b_y c_x)k \tag{4.5}$$

Now

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \begin{bmatrix} i & j & k \\ a_x & a_y & a_z \\ b_y c_z - b_z c_y & -(b_x c_z - b_z c_x) & b_x c_y - b_y c_x \end{bmatrix} \tag{4.6}$$

Now expanding $\mathbf{A}\times(\mathbf{B} \times \mathbf{C})$ *and considering* $\mathbf{x}$ *component only*

$\implies \mathbf{i} : a_y(b_x c_y - b_y c_x) + a_z(b_x c_z - b_z c_x)$
$=(\mathrm{a}_y c_y + a_z c_z)b_x - (a_y b_y + a_z b_z)c_x$
$=(\mathrm{a}_y c_y + a_z c_z)b_x - (a_y b_y + a_z b_z)c_x + ((a_x c_x)b_x - (a_x b_x)c_x))$
$=(\mathrm{a}_x c_x + a_y c_y + a_z c_z)b_x - (a_x b_x + a_y b_y + a_z c_z)c_x$
$=\mathbf{i}((\mathrm{A.C})\mathrm{b}_x - (A.B)c_x)$
Similarly we can show for $\mathbf{j}$ and $\mathbf{k}$ components.
Hence

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A}.\mathbf{C})\mathbf{B} - (\mathbf{B}.\mathbf{C})\mathbf{A} \tag{4.7}$$

Now using equation (3.1) and equation(3.2)

$$(\mathbf{A}.\mathbf{C})\mathbf{B} - (\mathbf{A}.\mathbf{B})\mathbf{C} = \frac{\sqrt{3}}{2}(\mathbf{B} + \mathbf{C}) \tag{4.8}$$

Rearranging the terms we get the following equation

$$(\mathbf{A}.\mathbf{C} - \frac{\sqrt{3}}{2})\mathbf{B} = (\mathbf{A}.\mathbf{B} + \frac{\sqrt{3}}{2})\mathbf{C} \tag{4.9}$$

Applying cross product on both sides with **B** we get

$$(\mathbf{A}.\mathbf{C} - \frac{\sqrt{3}}{2})\mathbf{B} \times \mathbf{B} = (\mathbf{A}.\mathbf{B} + \frac{\sqrt{3}}{2})\mathbf{C} \times \mathbf{B} \tag{4.10}$$

Since

$$\mathbf{B} \times \mathbf{B} = 0 \tag{4.11}$$

therefore

$$(\mathbf{A}.\mathbf{B} + \frac{\sqrt{3}}{2})\mathbf{C} \times \mathbf{B} = 0 \tag{4.12}$$

Either

$$(\mathbf{A}.\mathbf{B} + \frac{\sqrt{3}}{2}) = 0 \tag{4.13}$$

or

$$\mathbf{C} \times \mathbf{B} = 0 \tag{4.14}$$

But since it is given that **B** is not parallel to **C**,

$$\mathbf{C} \times \mathbf{B} \neq 0 \tag{4.15}$$

So,

$$(\mathbf{A}.\mathbf{B} + \frac{\sqrt{3}}{2}) = 0 \tag{4.16}$$

implies

$$\mathbf{A}.\mathbf{B} = -\frac{\sqrt{3}}{2} \tag{4.17}$$

$$\mathbf{A}.\mathbf{B} = \|\mathbf{A}\|\|\mathbf{B}\|COS(\theta_1) \tag{4.18}$$

where θ_1 is angle between **A** and **B**. Since,

$$\|\mathbf{A}\| = \|\mathbf{B}\| = 1 \tag{4.19}$$

$$cos(\theta_1) = -\frac{\sqrt{3}}{2} \tag{4.20}$$

$$\theta_1 = \frac{5\pi}{6} \tag{4.21}$$

Similarly, we get equation(3.17)by substituting equation(3.8) in equation(3.4).

$$\mathbf{A}.\mathbf{C} = \frac{\sqrt{3}}{2} \tag{4.22}$$

$$\mathbf{A}.\mathbf{C} = \|\mathbf{A}\|\|\mathbf{C}\| COS(\theta_2) \tag{4.23}$$

where θ_2 is angle between **A** and **C**. Since,

$$\|\mathbf{A}\| = \|\mathbf{C}\| = 1 \tag{4.24}$$

$$cos(\theta_2) = \frac{\sqrt{3}}{2} \tag{4.25}$$

$$\theta_2 = \frac{\pi}{6} \tag{4.26}$$

Now we are going to plot one set of vectors satisfying given condition and

$$\mathbf{A} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{4.27}$$

$$\mathbf{B} = \begin{pmatrix} -\sqrt{3}/2 \\ 1/2 \\ 0 \end{pmatrix} \tag{4.28}$$

$$\mathbf{C} = \begin{pmatrix} \sqrt{3}/2 \\ 0 \\ 1/2 \end{pmatrix} \tag{4.29}$$

satisfy the given condition.

Plot

The code in

https://github.com/SRIJITH01/Srijith/blob/master/presentation2.py

plots Fig. 1.

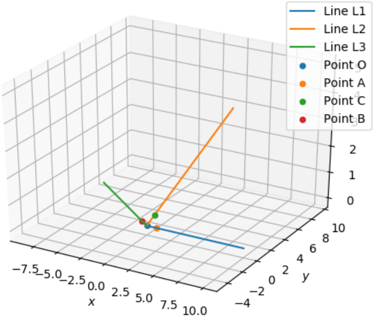

Figure: The above plot is set of vectors staisfying given condition