Presentation 3, 5.3

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September 3, 2019

Problem

Solution

Plot

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Problem Statement

Obtain a solution to the following

$$\max_{\mathbf{x}} 6x_1 + 5x_2 \tag{2.1}$$

with constraints

$$x_1 + x_2 \leq 5 \tag{2.2}$$

$$3x_1 + 2x_2 \leq 12 \tag{2.3}$$

$$where\ x_1, x_2 \geq 0 \tag{2.4}$$

using cvxpy.

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Solution

The given problem is expressed as follows

$$\max_{\mathbf{x}} \mathbf{cx} \text{ such that} \tag{3.1}$$

$$\mathbf{Ax} \leq \mathbf{b} \tag{3.2}$$

and

$$x_1, x_2 \geq 0 \tag{3.3}$$

where

$$\mathbf{c} = \begin{pmatrix} 6 & 5 \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 5 \\ 12 \end{pmatrix}, \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}. \tag{3.4}$$

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From eq(3.2)

$$\mathbf{x} \leq \mathbf{A}^{-1}\mathbf{b} \tag{3.5}$$

$$\Rightarrow \mathbf{x} \leq \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 5 \\ 12 \end{pmatrix} \tag{3.6}$$

$$\Rightarrow \mathbf{x} \leq \frac{\begin{pmatrix} 2 & -1 \\ -3 & 1 \end{pmatrix}}{(1 \times 2 - 3 \times 1)} \begin{pmatrix} 5 \\ 12 \end{pmatrix} \tag{3.7}$$

$$\Rightarrow \mathbf{x} \leq \begin{pmatrix} -2 & 1 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} 5 \\ 12 \end{pmatrix} \tag{3.8}$$

$$\Rightarrow \mathbf{x} \leq \begin{pmatrix} -2 \times 5 + 1 \times 12 \\ 3 \times 5 + (-1 \times 12) \end{pmatrix} \tag{3.9}$$

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$$\Rightarrow \mathbf{x} \leq \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{3.10}$$

Multiplying $\mathbf{c}$ on both sides of inequality

$$\mathbf{cx} \leq \mathbf{c} \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{3.11}$$

Therefore

$$\mathbf{cx} \leq \begin{pmatrix} 6 & 5 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{3.12}$$

$$\Rightarrow \mathbf{cx} \leq 27 \tag{3.13}$$

The maximum value of $\mathbf{cx}$ is 27 and is attained when

$$\mathbf{x} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{3.14}$$

i.e $x_1 = 2, x_2 = 3$
which also satisfies eq(3.3).

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Plot

The code in

https://github.com/SRIJITH01/Srijith/blob/master/presentation3.py

plots Fig. 1.

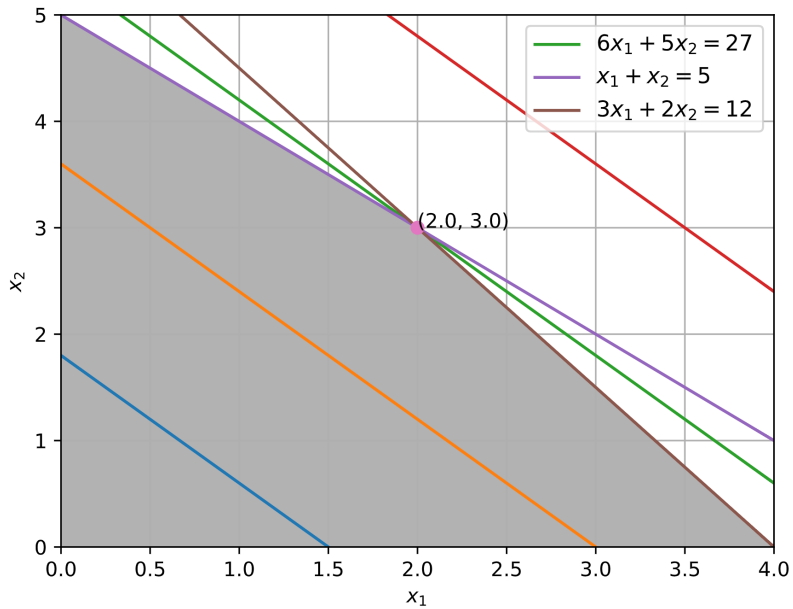

Figure: Graphical representation .

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