PRESENTATION 6
JEE-OPT
Problem 16

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1. Problem
2. Solution
3. Plot

Problem Statement

Find the maximum value of the function

$$f(x) = 2x^3 - 15x^2 + 36x - 48 \tag{2.1}$$

on the set

$$A = \{x : x^2 + 20 \leq 9x\} \tag{2.2}$$

Solution

First let's find solutions of set A

$$x^2 - 9x + 20 \leq 0 \tag{3.1}$$

$$\Rightarrow (x-5)(x-4) \leq 0 \tag{3.2}$$

$$\Rightarrow 4 \leq x \leq 5 \tag{3.3}$$

Now we need to find maximum value of f(x) in A.
firstly lets find if there is any local maximum in A .
If f(x) has local maximum(minimum) at c in A
then max(f(x)) in A =max{f(4),f(c),f(5)}

If not f(x) is monotonic in A (since f is 3 degree polynomial function)
Then max(f(x))=max{f(4),f(5)}

$$f'(x) = 0 \tag{3.4}$$

when f(x) attains local maxima or minima,

$$f'(x) = 6x^2 - 30x + 36 \tag{3.5}$$

A numerical solution for (3.5) can be obtained as

$$x_{n+1} = x_n - \mu f'(x) \tag{3.6}$$

$$= x_n - \mu(6x^2 - 30x + 36) \tag{3.7}$$

where x_0 *is initial guess*.

The numerical solutions are

$$x_1 = 1.9998464849816984, \tag{3.8}$$

$$x_2 = 3.0001535150183014. \tag{3.9}$$

but $\mathrm{x}_1, x_2 \notin A$.
Therefore,
$max(f(x)) = max\{f(4), f(5)\}(3.10)$

$$f(4) = -16, \tag{3.11}$$

$$f(5) = 7. \tag{3.12}$$

The maximum value of the function

$$f(x) = 2x^3 - 15x^2 + 36x - 48 \tag{3.13}$$

on the set

$$A = \{x : x^2 + 20 \leq 9x\} \tag{3.14}$$

is f(5)$=$ 7

Plot

The code in
https://github.com/SRIJITH01/Srijith/blob/master/jeeopt.py
plots Fig. 2.

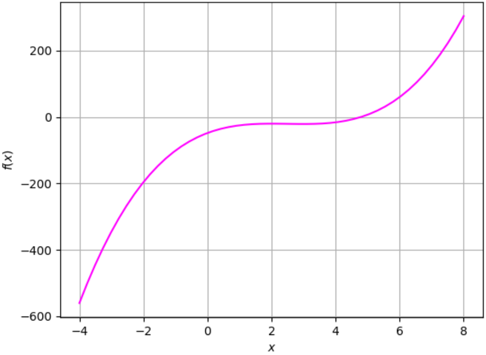

Figure: Graph of f(x) .

Plot

plots Fig. 2.

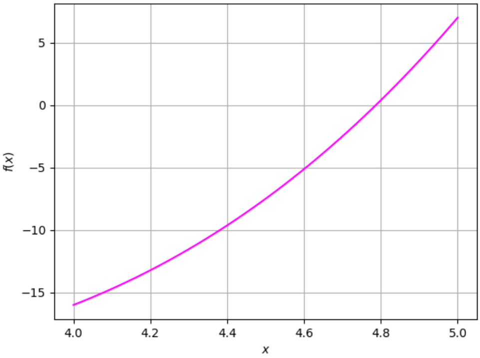

Figure: Graph of f(x).