EE2101 Control Systems
Question 43

Srijith Reddy Pakala
EE19BTECH11041
Dept. of Electrical Engg.,
IIT Hyderabad.

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Problem Statement

The motor whose torque-speed characteristics are shown in Figure 1 drives the load shown in the diagram. Some of the gears have inertia. Find the transfer function,G(s)=$\theta_2(s)/E_a(s)$.

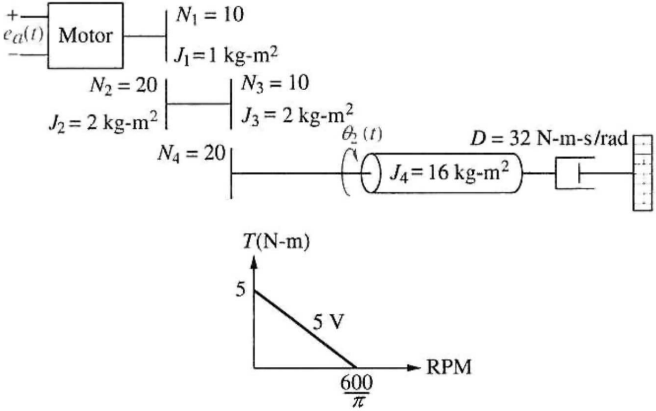


Figure 1

Solution

First we need to find the Mechanical constants J_m,D_m in the equation(3.1)

$$\frac{\theta_m(s)}{E_a} = \frac{K_t/R_aJ_m}{s[s+\frac{1}{J_m}(D_m+\frac{K_tK_b}{R_a})]} \tag{3.1}$$

The total inertia at the armature of the motor is

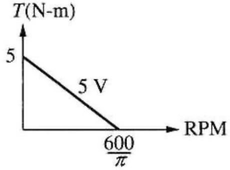
$$J_m = J_1 + (J_2 + J_3)\left(\frac{N_1}{N_2}\right)^2 + J_4\left(\frac{N_1N_3}{N_2N_4}\right)^2 \tag{3.2}$$

The total damping at the armature of the motor is

$$D_m = D\left(\frac{N_1N_3}{N_2N_4}\right)^2 \tag{3.3}$$

Solution

Now we will find the electrical constants, K_t/R_a and K_b.From the torque-speed curve of Figure 2.



$$T_{stall}=5$$
$$\omega_{no-load} = \frac{600}{\pi}$$
$$e_a=5$$

Hence the electrical constants are

$$\frac{K_t}{R_a} = \frac{T_{stall}}{e_a}=\frac{5}{5}=1$$

Solution

and

$$K_b = \frac{e_a}{\omega_{no-load}} = \frac{5}{\frac{600}{\pi}\frac{2\pi}{60}} = \frac{1}{4}$$

The above equations are derived in the text book "Norman.Nise control systems engineering chapter 2.8"
Given $N_1 = 10, N_2 = 20, N_3 = 10, N_4 = 20$
and $J_1 = 1kg - m^2, J_2 = 2kg - m^2, J_3 = 2kg - m^2$
$,J_4 = 16kg - m^2, and D = 32N - m - s/rad$

Solution

By substituting above values in equations (3.2) , (3.3) we get

$$J_m = 1 + (2+2)\left(\frac{1}{2}\right)^2 + 16\left(\frac{1}{4}\right)^2 = 3$$

$$D_m = 32\left(\frac{1}{4}\right)^2 = 2$$

Finally substituting all the variables in equation (3.1)

$$\frac{\theta_m(s)}{E_a} = \frac{\frac{1}{3}}{s\left[s+\frac{1}{3}\left(2+\frac{1}{4}\right)\right]} = \frac{1}{3s\left[s+0.75\right]}$$

Solution

Since

$$\theta_2(s) = \frac{1}{4}\theta_m(s)$$

Thus

$$\boxed{G(s) = \frac{\theta_2(s)}{E_a} = \frac{1}{12s\left[s + 0.75\right]}}$$

The code for the above calculations can be seen here


Thank you!