Department of Computer Science and engg.- NITK Surathkal

CS-253 Design and Analysis of Algorithms

Assignment sheet- 3 Due Date: **14th April 2025**

Greedy algorithms and Dynamic Programming

1. **Dance partners:** You are pairing couples for a very conservative formal ball. There are n men and m women, and you know the height and gender of each person there. Each dancing couple must be a man and a woman, and the man must be at least as tall as, but no more than 3 inches taller than, his partner. You wish to maximize the number of dancing couples given this constraint.

2. Homework grade maximization: In a class, there are n assignments. You have H hours to spend on all assignments, and you cannot divide an hour between assignments, but must spend each hour entirely on a single assignment. The I'th hour you spend on assignment J will improve your grade on assignment J by B[I, J], where for each J, B[1, J] ≥ B[2, J] ≥ ... ≥ B[H, J] ≥ 0. In other words, if you spend h hours on assignment J, your grade will be Σi=1 B[I, J] and time spent on each project has diminishing returns, the next hour being worth less than the previous one. You want to divide your H hours between the assignments to maximize your total grade on all the assignments.
Give an efficient algorithm for this problem.

3. **An Activity-Selection Problem:** Given a set S = {1, 2, ..., n} of n proposed activities, with a start time s_i and a finish time f_i for each activity i, select a maximum-size set of mutually compatible activities.

4. Alice wants to throw a party and is deciding whom to call. She has n people to choose from, and she has made up a list of which pairs of these people know each other. She wants to pick as many people as possible, subject to two constraints: at the party, each person should have at least five other people whom they know and five other people whom they don't know.
Give an efficient algorithm that takes as input the list of n people and the list of pairs who know each other and outputs the best choice of party invitees. Give the running time in terms of n.

5. A contiguous subsequence of a list S is a subsequence made up of consecutive elements of S. For instance, if S is 5, 15, −30, 10, −5, 40, 10, then 15, −30, 10 is a contiguous subsequence but 5, 15, 40 is not. Give a linear-time algorithm for the following task:
Input: A list of numbers, a1 , a2 , . . . , an .
Output: The contiguous subsequence of maximum sum (a subsequence of length zero has sum zero).
For the preceding example, the answer would be 10, −5, 40, 10, with a sum of 55.
(Hint: For each j ∈ {1, 2, . . . , n}, consider contiguous subsequences ending exactly at position j.)

6. You are going on a long trip. You start on the road at mile post 0. Along the way there are n hotels, at mile posts $a_1 < a_2 < \cdots < a_n$, where each a_i is measured from the starting point. The only places you are allowed to stop are at these hotels, but you can choose which of the hotels you stop at. You must stop at the final hotel (at distance a_n), which is your destination.Algorithms You'd ideally like to travel 200 miles a day, but this may not be possible (depending on the spacing of the hotels). If you travel x miles during a day, the penalty for that day is $(200 - x)^2$. You want to plan your trip so as to minimize the total penalty—that is, the sum, over all travel days, of the daily penalties.
Give an efficient algorithm that determines the optimal sequence of hotels at which to stop.

7. A subsequence is palindromic if it is the same whether read left to right or right to left. For instance, the sequence A, C, G, T, G, T, C, A, A, A, A, T, C, G has many palindromic subsequences, including A, C, G, C, A and A, A, A, A (on the other hand, the subsequence A, C, T is not palindromic). Devise an algorithm that takes a sequence x[1 . . . n] and returns the (length of the) longest palindromic subsequence. Its running time should be O(n2).

8. Given two strings $x = x_1\ x_2 \cdot\cdot\cdot x_n$ and $y = y_1\ y_2 \cdot\cdot\cdot y_m$, we wish to find the length of their longest common substring, that is, the largest k for which there are indices i and j with $x_i\ x_{i+1} \cdot\cdot\cdot x_{i+k-1} = y_j\ y_{j+1} \cdot\cdot\cdot y_{j+k-1}$. Show how to do this in time O(mn).

9. Given an unlimited supply of coins of denominations $x_1\ x_2 \cdot\cdot\cdot x_n$, we wish to make change for a value v; that is, we wish to find a set of coins whose total value is v. This might not be possible: for instance, if the denominations are 5 and 10 then we can make change for 15 but not for 12. Give an O(nv) dynamic-programming algorithm for the following problem.
Input: $x_1\ x_2 \cdot\cdot\cdot x_n$; v.
Question: Is it possible to make change for v using coins of denominations $x_1\ x_2 \cdot\cdot\cdot x_n$?

10. Consider the following variation on the change-making problem (Exercise 11.): you are given denominations $x_1\ x_2 \cdot\cdot\cdot x_n$, and you want to make change for a value v, but you are allowed to use each denomination at most once. For instance, if the denominations are 1, 5, 10, 20, then you can make change for 16 = 1 + 15 and for 31 = 1 + 10 + 20 but not for 40 (because you can’t use 20 twice).
Input: Positive integers $x_1\ x_2 \cdot\cdot\cdot x_n$; another integer v.
Output: Can you make change for v, using each denomination x_i at most once?
Show how to solve this problem in time O(nv).

11. Given a convex polygon P = (v0, . . . , vn−1), and a weight function w on triangles, find a triangulation minimizing the total weight.

12. In the **art gallery gaurding** problem we are given a line L that represents a long hallway in an art gallery. We are also given a set $X= x_1\ x_2 \cdot\cdot\cdot x_{n-1}$ of real numbers that specify the positions of the paintings in this hallway. Suppose that a single gaurd can protect all the paintings within distance at most 1 of his or her position(on both sides). Design an algorithm for finding a placement of gaurds that uses the minimum number of gaurds to gaurd all the paintings with position in X .