EECS 366 HW3

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You are given a scene with two objects and three coordinate frames: two object coordinate frames C_1 and C_2 attached to the objects, and a world coordinate frame W attached to world. The coordinate frames will be visually represented by three line segments starting from each origin, and extending unit length in the three principal directions of the corresponding frames.

The two object models are expresed in their respective model coordinates C_1 and C_2:
O_1: A tetrahedron with vertices at (0,0,0), (1,0,0), (0,1,0), and (0,0,1),
O_2: A cube with vertices at $(\pm1, \pm1, \pm1)$

1 Problem 1

O_1 initially at world coordinates $C_1 = (0, 10, 0)$ with principal axes aligned with world frame.
O_2 initially at world coordinates $C_2 = (10, 0, 20)$ with principal axes rotated 30° around y-axis (assuming object's y-axis here).

$$M_{W,C_1} = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$M_{W,C_2} = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 20 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \left[\begin{array}{ccc|c} cos\frac{\pi}{6} & 0 & sin\frac{\pi}{6} & 0 \\ 0 & 1 & 0 & 0 \\ -sin\frac{\pi}{6} & 0 & cos\frac{\pi}{6} & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} \frac{\sqrt{3}}{2} & 0 & \frac{1}{2} & 10 \\ 0 & 1 & 0 & 0 \\ -\frac{1}{2} & 0 & \frac{\sqrt{3}}{2} & 20 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

2 Problem 2

Coordinates of vertices of O_1 and C_1 in world frame:

$$C_1 = \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix}$$

$$O_{1,1} = M_{W,C_1} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix}$$

$$O_{1,2} = M_{W,C_1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ 0 \end{bmatrix}$$

$$O_{1,3} = M_{W,C_1} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 11 \\ 0 \end{bmatrix}$$

$$O_{1,4} = M_{W,C_1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ 1 \end{bmatrix}$$

3 Problem 3

Camera at world coordinates (-50,-50,-50) pointing at origin with $\overrightarrow{V} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$. What is the viewing transformation $M_{V,W}$ and what are the modelview transformations M_{V,C_1} and M_{V,C_2}?

$$\overrightarrow{N} = \begin{bmatrix} 50 \\ 50 \\ 50 \end{bmatrix}, \qquad \overrightarrow{n} = \frac{\overrightarrow{N}}{||\overrightarrow{N}||} = \frac{\sqrt{3}}{3}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},$$

$$P_{W,V} = \begin{bmatrix} 50 \\ 50 \\ 50 \end{bmatrix}, \qquad \overrightarrow{u} = \frac{\overrightarrow{V} \times \overrightarrow{N}}{||\overrightarrow{V} \times \overrightarrow{N}||} = \frac{\sqrt{2}}{2}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$\overrightarrow{v} = \overrightarrow{n} \times \overrightarrow{u} = \frac{\sqrt{6}}{6}\begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$$

Transformations:

$$M_{W,V} = \begin{bmatrix} [\ \overrightarrow{u} & \overrightarrow{v} & \overrightarrow{n}\ ] & P_{W,V} \\ & 0 & & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{6}}{6} & \frac{\sqrt{3}}{3} & -50 \\ 0 & \frac{\sqrt{6}}{3} & \frac{\sqrt{3}}{3} & -50 \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{6}}{6} & \frac{\sqrt{3}}{3} & -50 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M_{V,W} = M_{W,V}^{-1} = \begin{bmatrix} \overrightarrow{u}^T & -\overrightarrow{u} \cdot P_{W,V} \\ \overrightarrow{v}^T & -\overrightarrow{v} \cdot P_{W,V} \\ \overrightarrow{n}^T & -\overrightarrow{n} \cdot P_{W,V} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{3} & -\frac{\sqrt{6}}{6} & 0 \\ \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & 50\sqrt{3} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M_{V,C_1} = M_{V,W} M_{W,C_1} = \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{3} & -\frac{\sqrt{6}}{6} & \frac{10}{3}\sqrt{6} \\ \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & \frac{160}{3}\sqrt{3} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M_{V,C_2} = M_{V,W} M_{W,C_2} = \begin{bmatrix} 0.9659 & 0 & -0.2588 & -7.0711 \\ -0.1494 & 0.8165 & -0.5577 & -12.2474 \\ 0.2133 & 0.5774 & 0.7887 & 103.9230 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4 Problem 4

What are the coordinates of the vertices C_1 and W in the viewing frame?

$$W_V = M_{V,W} W = \begin{bmatrix} 0 \\ 0 \\ 50\sqrt{3} \end{bmatrix}$$

$$C_{1_V} = M_{V,W} C_{1_W} = \begin{bmatrix} 0 \\ \frac{10}{3}\sqrt{6} \\ \frac{160}{3}\sqrt{3} \end{bmatrix}$$

5 Problem 5

If O_1 is rotated around its x-axis 45°, what would M'_{W,C_1} and M'_{V,C_1} be?

$$M'_{W,C_1} = M_{W,C_1} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos\frac{\pi}{2} & -sin\frac{\pi}{2} & 0 \\ 0 & sin\frac{\pi}{2} & cos\frac{\pi}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.7071 & -0.5 & -0.5 & 0 \\ -0.4082 & 0.2887 & -0.8660 & 8.1650 \\ 0.5774 & 0.8165 & 0 & 92.3760 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M'_{V,C_1} = M_{V,W} M'_{W,C_1} = \begin{bmatrix} 0.7071 & -0.7071 & 0 & 0 \\ -0.4082 & -0.4082 & -0.8165 & 8.1650 \\ 0.5774 & 0.5774 & -0.5774 & 92.3760 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

6 Problem 6

If O_1 is subsequently rotated around its z-axis 60° followed by y-axis −30°, what would M''_{W,C_1} and M''_{V,C_1} be?

$$M''_{W,C_1} = M'_{W,C_1} \begin{bmatrix} cos\frac{\pi}{3} & -sin\frac{\pi}{3} & 0 & 0 \\ sin\frac{\pi}{3} & cos\frac{\pi}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos\frac{-\pi}{6} & 0 & sin\frac{-\pi}{6} & 0 \\ 0 & 1 & 0 & 0 \\ -sin\frac{-\pi}{6} & 0 & cos\frac{-\pi}{6} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.4330 & -0.8660 & -0.2500 & 0 \\ 0.1768 & 0.3536 & -0.9186 & 10 \\ 0.8839 & 0.3536 & 0.3062 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M''_{V,C_1} = M_{V,W} M''_{W,C_1} = \begin{bmatrix} -0.3188 & -0.8624 & -0.3933 & 0 \\ -0.3933 & 0.4979 & -0.7729 & 8.1650 \\ 0.8624 & -0.0918 & -0.4979 & 92.3760 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

7 Problem 7

If O_2 is rotated around the x-axis of the world frame for 30°, what would the be transformations M'_{W,C_2} and M'_{V,C_2}

$$M'_{W,C_2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos\frac{\pi}{6} & -sin\frac{\pi}{6} & 0 \\ 0 & sin\frac{\pi}{6} & cos\frac{\pi}{6} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} M_{W,C_2} = \begin{bmatrix} 0.866 & 0 & 0.5 & 10 \\ 0.25 & 0.8660 & -0.4330 & -10 \\ 0.4330 & 0.5 & 0.75 & 17.3205 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M'_{V,C_2} = M_{V,W} M_{W,C_2} = \begin{bmatrix} 0.9186 & -0.3536 & -0.1768 & -5.1764 \\ 0.0273 & 0.5030 & -0.8639 & -19.3185 \\ 0.3943 & 0.7887 & 0.4717 & 96.6025 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$