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Homework 4

Q1) ⓐ $P(t) = -4x + y + 4 = 0 \rightarrow P(t) = -4x + y + 4h = 0$ in homog. coordinates

ⓑ homogeneous plane: $2x = 3h \rightarrow h = \frac{2}{3}x$

intersect. $-4x + y + 4h = 0$

$-4x + y + \frac{8}{3}x = 0 \rightarrow y = \frac{4}{3}x$. Choose $x = 1$: $\left[1, \frac{4}{3}, \frac{2}{3}\right]^T$ intersection

ⓒ Non-homogeneous intersection: $\left[1 / \frac{2}{3}, \frac{4}{3} / \frac{2}{3}\right] = \left[\frac{3}{2}, 2\right]$

Q2) ⓐ $(N\eta)^T MA = 0$ since MA is in the plane of the transformed triangle, so $(N\eta)^T MA = \eta^T A = 0$

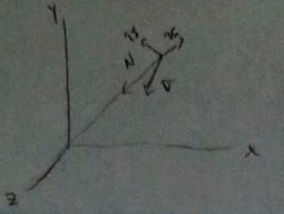
$\eta^T N^T MA = \eta^T A \rightarrow$ This means $N^T M = I_4$, so $N^T = M^{-1}$, and $N = (M^{-1})^T$

ⓑ $n_2 = (MA - MB) \times (MA - MC)$

ⓒ n_2 could not be computed via $n_2 = Mn_1$. If M is a shear transformation then Mn_1 may not necessarily be the correct normal vector for the newly transformed plane. For example, if a plane is just the xy plane with $n_1 = [0, 0, 1, 0]^T$ and $M = \begin{bmatrix} 1 & 0 & a & 0 \\ 0 & 1 & b & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, then $Mn_1 = [a, b, 1]^T$.

This is incorrect because the sheared xy plane still has a normal vector of $[0, 0, 1]^T$.

ⓓ If ΔABC's origin was the world origin and M is a rotation about the world's axes, then the normal vector would be $n_2 = Mn_1$



Q3) $P_{wv} = [10,10,10]^T$ $\qquad$ $\vec{V} = [0,0,1]^T,\ \vec{N} = [-1,-1,-1]^T$

$\vec{n} = \dfrac{-1[-1,-1,-1]}{\|[-1,-1,-1]\|} = \dfrac{\sqrt{3}}{3}[1,1,1]^T$ $\qquad$ $\vec{u} = \dfrac{-(\vec{V}\times\vec{N})}{\|\vec{V}\times\vec{N}\|} \to \det\begin{bmatrix} i & j & k \\ 0 & 0 & 1 \\ -1 & -1 & -1 \end{bmatrix} \to \dfrac{-[1,-1,0]}{\|[1,-1,0]\|} = \dfrac{\sqrt{2}}{2}[-1,1,0]^T$

$\vec{v} = \vec{n}\times\vec{u} = \dfrac{\sqrt{6}}{6}\det\begin{bmatrix} i & j & k \\ 1 & 1 & 1 \\ -1 & 1 & 0 \end{bmatrix} = \dfrac{\sqrt{6}}{6}[1,-1,2]^T$

$$M_{vw} = \begin{bmatrix} \vec{u}^T & -\vec{u}\cdot P_{wv} \\ \vec{v}^T & -\vec{v}\cdot P_{wv} \\ \vec{n}^T & -\vec{n}\cdot P_{wv} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -\sqrt{2}/2 & \sqrt{2}/2 & 0 & 0 \\ \sqrt{6}/6 & -\sqrt{6}/6 & \sqrt{6}/3 & -\frac{10}{3}\sqrt{6} \\ \sqrt{3}/3 & \sqrt{3}/3 & \sqrt{3}/3 & -10\sqrt{3} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$d = 6,\ n = 5,\ f = 25,\ (x_{min}, y_{min}) = (-4,-4),\ (x_{max}, y_{max}) = (4,4)$

$$M_{persp} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-(f+n)}{f-n} & \frac{-2fn}{(f-n)} \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \frac{2d}{x_{max}-x_{min}} & 0 & 0 & 0 \\ 0 & \frac{2d}{y_{max}-y_{min}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & \frac{x_{max}+x_{min}}{2d} & 0 \\ 0 & 1 & \frac{y_{max}+y_{min}}{2d} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -3/2 & -25/2 \\ 0 & 0 & -1 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} 3/2 & 0 & 0 & 0 \\ 0 & 3/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{Scaling}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{Shear}} = \begin{bmatrix} 3/2 & 0 & 0 & 0 \\ 0 & 3/2 & 0 & 0 \\ 0 & 0 & -3/2 & -25/2 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Viewport size: 200 x 200

$$M_{viewport} = \begin{bmatrix} 25 & 0 & 0 & 100 \\ 0 & -25 & 0 & 100 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad 25 = \frac{200}{4-(-4)} \text{ to get } (-4,4) \text{ to } (-100,100)$$

Scaling

Q4) $$\begin{bmatrix} \frac{1}{25} & 0 & 0 \\ 0 & -\frac{1}{25} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -100 \\ 0 & 1 & -100 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 20 \\ 50 \\ 1 \end{bmatrix} = \begin{bmatrix} (20-100)/25 \\ -(50-100)/25 \\ 1 \end{bmatrix} = \begin{bmatrix} -3.2 \\ 2 \\ 1 \end{bmatrix}$$