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2.

6.1.

a)

$$A = \left(\begin{array}{c|c|c} A_{00} & \alpha_{10} e_L & 0 \\ \hline \alpha_{10} e_L^T & \alpha_{11} & \alpha_{21} e_F^T \\ \hline 0 & \alpha_{21} e_F & A_{22} \end{array}\right), \quad L = \left(\begin{array}{c|c|c} L_{00} & 0 & 0 \\ \hline \lambda_{10} e_L^T & 1 & 0 \\ \hline 0 & \lambda_{21} e_F & L_{22} \end{array}\right).$$

$$D = \left(\begin{array}{c|c|c} D_{00} & 0 & 0 \\ \hline 0 & \delta_1 & 0 \\ \hline 0 & 0 & D_{22} \end{array}\right), \quad U = \left(\begin{array}{c|c|c} U_{00} & u_{01} e_L & 0 \\ \hline 0 & 1 & u_{12} e_F^T \\ \hline 0 & 0 & U_{22} \end{array}\right)$$

$$E = \left(\begin{array}{c|c|c} E_{00} & 0 & 0 \\ \hline 0 & \epsilon_1 & 0 \\ \hline 0 & 0 & E_{22} \end{array}\right).$$

$$\therefore LDL^T = \begin{pmatrix} L_{00} D_{00} L_{00}^T & L_{00} D_{00} \lambda_{10} e_L & 0 \\ \lambda_{10} e_L^T D_{00} L_{00}^T & \lambda_{10} e_L^T D_{00} \lambda_{10} e_L + \delta_1 & \delta_1 \lambda_{21} + e_F \\ 0 & \lambda_{21} e_F \delta_1 & \lambda_{21} e_F \delta_1 \lambda_{21} e_F + L_{22} D_{22} L_{22}^T \end{pmatrix}$$

$$UEU^T = \begin{pmatrix} U_{00} E_{00} U_{00}^T + u_{01} e_L \varepsilon_1 u_{01}^T e_L^T & u_{01} e_L \varepsilon_1 & 0 \\ \varepsilon_1 u_{01}^T e_L^T & \varepsilon_1 + u_{12} e_F E_{22} u_{12}^T e_F^T & u_{12} e_F^T E_{22} U_{22}^T \\ 0 & U_{22} E_{22} u_{12}^T e_F & U_{22} E_{22} U_{12}^T \end{pmatrix}$$

Given:

$$\begin{pmatrix} A_{00} & \alpha_{10} e_L & 0 \\ \alpha_{10} e_L^T & \alpha_{11} & \alpha_{21} e_F^T \\ 0 & \alpha_{21} e_F & A_{22} \end{pmatrix} = \begin{pmatrix} L_{00} & 0 & 0 \\ \lambda_{10} e_L^T & 1 & u_{12} e_F^T \\ 0 & 0 & U_{22} \end{pmatrix} \begin{pmatrix} D_{00} & 0 \\ 0 & \phi \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} L_{00} & 0 & 0 \\ \lambda_{10} e_L^T & 1 & u_{12} e_F^T \\ 0 & 0 & U_{22} \end{pmatrix}^T$$

$$= \begin{pmatrix} L_{00} D_{00} L_{00}^T & L_{00} D_{00} \lambda_{10} e_L^T & 0 \\ \lambda_{10} e_L^T D_{00} L_{00}^T & \lambda_{10} e_L^T D_{00} \lambda_{10} e_L + \phi + u_{12} e_F^T E_{22} U_{12}^T e_F^T & \\ 0 & U_{22} E_{22} u_{12}^T e_F & U_{22} E_{22} \end{pmatrix}$$

From above

$$\therefore \phi = \alpha_{11} - \lambda_{10} e_L^T D_{00} \lambda_{10} e_L - u_{12} e_F^T E_{22} u_{12}^T e_F \quad \text{— (1)}$$

we know

$$A = LDL^T$$

$$\Rightarrow \alpha_{11} = \lambda_{10} e_L^T D_{00} \lambda_{10} e_L + \delta_1$$

$$\therefore \lambda_{10} e_L^T D_{00} \lambda_{10} e_L = \alpha_{11} - \delta_1 \quad \text{— (2)}$$

$$A = UEU^T$$

$$\therefore \alpha_{11} = \varepsilon_1 + u_{12} e_F^T E_{22} u_{12}^T e_F$$

$$\therefore u_{12} e_F^T E_{22} u_{12}^T e_F^T = \alpha_{11} - \varepsilon_1 \quad \text{—}$$

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Substitute: (2) & (3) in (1).

$$\phi = \alpha_{11} - (\alpha_{11} - \delta_1) - (\alpha_{11} - \varepsilon_1)$$

$$= \delta_1 - \alpha_{11} + \varepsilon_1$$

$$= \delta_1 + \varepsilon_1 - \alpha_{11} \longrightarrow \text{Proved}$$

b) Cost of computing one twisted factorization is $O(n^3)$.

c. Cost of computing all twisted factorization is $O(n^4)$.

7.1: Computing the given $LDL^T x$:

$$\begin{pmatrix} L_{00} D_{00} L_{00}^T & L_{00} D_{00} \lambda_{10} e_L & 0 \\ \lambda_{10} e_L^T D_{00} L_{00}^T & \lambda_{10} e_L^T D_{00} \lambda_{10} e_L + \upsilon_{12} e_F^T E_{12} \upsilon_{12} e_F & \upsilon_{12} e_F^T E_{22} U_{22}^T \\ 0 & U_{22} E_{22} \upsilon_{12} e_F & U_{22} E_{22} U_{22}^T \end{pmatrix} * \begin{pmatrix} x_0 \\ \hline x_1 \\ \hline x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ \hline 0 \\ \hline 0 \end{pmatrix} \longrightarrow (1)$$