Погрешность

Вариант 1

$\int_1^2 \sqrt{x}\cos(x^2)$ $n=2$, узлов $=3$, $p(x)=1$

$\approx (2-1)\left(A_1\sqrt{x}\cos(x_1^2) + A_2\sqrt{x}\cos(x_2^2) + A_3\cos x_3^2\right)$

$\omega(x) = (x-x_0)(x-x_1)(x-x_2) = x^3 + a_1x^2 + a_2x + a_3$

$$\int_1^2 p(x)\,(a_1x^2 + a_2x + a_3)x^m dx = -\int_1^2 p(x)\,x^{2+m+1}dx$$

$\forall m = \overline{0,2}$

$$\int_1^2 (a_1x^2 + a_2x + a_3)\,dx = -\int_1^2 x^3 dx = -\frac{15}{4}$$

$$\int_1^2 (a_1x^2 + a_2x + a_3)x\,dx = -\int_1^2 x^4 dx = -\frac{31}{5}$$

$$\int_1^2 (a_1x^2 + a_2x + a_3)x^2\,dx = -\int_1^2 x^5 dx = -\frac{21}{2}$$

$$\begin{cases} \frac{7a_1}{3} + \frac{3a_2}{2} + a_3 = -\frac{15}{4} \\ \frac{15a_1}{4} + \frac{7a_2}{3} + \frac{3a_3}{2} = -\frac{31}{5} \\ \frac{31a_1}{5} + \frac{15a_2}{4} + \frac{7a_3}{3} = -\frac{21}{2} \end{cases}$$

$a_1 = -\frac{9}{2}$ $a_2 = \frac{33}{5}$ $a_3 = -\frac{63}{20}$

$$\omega_3(x) = x^3 - \frac{9}{2}x^2 + \frac{33}{5}x - \frac{63}{20} = 0$$

$$x_0 = \frac{3}{2} \qquad x_1 = \frac{3}{2} - \frac{\sqrt{0,6}}{2} \qquad x_3 = \frac{3}{2} + \frac{\sqrt{0,6}}{2}$$

$$A_k = \int_1^2 p(x) \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{x - x_i}{x_k - x_i}\, dx \qquad k = \overline{0,2}$$

$$A_1 = \int_1^2 \frac{(x - x_1)}{(x_0 - x_1)} \cdot \left(\frac{x - x_2}{x_0 - x_2}\right) dx = \cancel{\int_1^2 \left(x - \frac{3}{2} + \frac{\sqrt{0,6}}{2}\right.}$$

$$= \int_1^2 \left(\frac{\left(x - \frac{3}{2} + \frac{\sqrt{0,6}}{2}\right)}{\frac{3}{2} - \frac{3}{2} + \frac{\sqrt{0,6}}{2}} \cdot \frac{x - \frac{3}{2} - \frac{\sqrt{0,6}}{2}}{\frac{3}{2} - \frac{3}{2} - \frac{\sqrt{0,6}}{2}}\right) dx = \frac{4}{9}$$

$$A_2 = \int_1^2 \frac{(x - x_0)}{x_1 - x_0} \cdot \frac{(x - x_2)}{x_1 - x_2}\, dx = \frac{5}{18}$$

$$A_3 = \int_1^2 \frac{(x - x_0)}{x_2 - x_0} \cdot \frac{(x - x_1)}{x_2 - x_1}\, dx = \frac{5}{18}$$

Получаем:

$$I = \frac{4}{9}\sqrt{\frac{3}{2}}\cos\left(\frac{3}{2}\right)^2 + \frac{5}{18}\sqrt{\frac{3}{2} - \frac{\sqrt{0,6}}{2}}\cos\left(\frac{3}{2} - \frac{\sqrt{0,6}}{2}\right)^2 +$$

$$+ \frac{5}{18}\sqrt{\frac{3}{2} + \frac{\sqrt{0,6}}{2}}\cos\left(\frac{3}{2} + \frac{\sqrt{0,6}}{2}\right)^2 = -0,59\,4635\ldots$$

Погрешность:

$$R \leq \frac{M_{2n+2}}{(2n+2)!}\int_1^2 p(x)\left(\omega_n(x)\right)^2 dx = \frac{M_6}{720}\int_1^2 p(x)\left(\omega_n(x)\right)^2$$

$$= \frac{M_6}{720} \int_1^2 \left(x^3 - \frac{9}{2}x^2 + \frac{33}{5}x - \frac{63}{20}\right)^2 dx =$$

$$M_6 = \max_{x=[1,2]} \left| \left(\sqrt{x} \cos x^2\right)^{(6)} \right| \approx \cancel{9{,}20} \approx 9000 \quad x = 2$$

$$= \frac{9000}{720} \cdot \int_1^2 \left(x^3 - \frac{9}{2}x^2 + \frac{33}{5}x - \frac{63}{20}\right)^2 dx =$$

$$= \frac{9000}{720 \cdot 2800} = 0{,}0044642\ldots$$

$$h = \frac{b-a}{n} = 0{,}1 = \frac{1}{n} \quad n = 10$$

$$h = 0{,}05 \Rightarrow n = 20$$

$$h = 0{,}025 \Rightarrow n = 40.$$

Формула средних прямоугольников

$$I(f) = \frac{b-a}{m}\left(f(x_{\frac{1}{2}}) + f(x_{\frac{3}{2}}) + \ldots + f(x_{\frac{2m-1}{2}})\right)$$

$$x_{i+\frac{1}{2}} = x_i + \frac{h}{2} \quad (i = \overline{0, m-1})$$

Формула Эйлера:

$$S[f|a,b] = \frac{h}{2}(f(a)+f(b)) + \frac{1}{12}h^2(f'(a)-f'(b))$$

$$I[f] = \int_a^b f(x)dx = \sum_{i=0}^{m-1}\int_{x_i}^{x_{i+1}} f(x)dx \approx \sum_{i=0}^{m-1} S[f|x_i, x_{i+1}] =$$

$$= \frac{h}{2}\left(f(x_0)+f(x_m) + 2\sum_{i=1}^{m-1} f(x_i)\right) + \frac{h^2}{12}(f'(x_0)-f'(x_m))$$

$$f'(x) = \frac{\cos x^2 - 4x^2 \sin x^2}{2\sqrt{x}}$$

$f'(1) \approx -1,146$ $f'(2) \approx -3,47$

Погрешность по Рунге:

Для формулы средних прямоугольников $k=2$.
(порядок точности)

$h=0,1$; $m=20$: $I[f] = -0,585581$

$h=0,05$; $m=40$: $I[f] = -0,591237$

$h=0,025$; $m=80$: $I[f] = -0,593851$

$$R_{2m} \approx \frac{S_{2m}-S_m}{2^{k-1}-1} = S_{2m}-S_m$$

или $$R_m \approx \frac{1}{1-\frac{1}{2^{k-1}}}(S_{2m}-S_m) = 2(S_{2m}-S_m)$$

$R_{20} \approx 2\cdot(S_{40}-S_{20}) = 0,\cancel{005656}$ $0,011312$

$R_{40} \approx S_{40}-S_{20}) = -0,005656$

$R_{80} = S_{80}-S_{40} = -0,002614$

Погрешность Рунге для Эйлера:

$h=0,1$, $m=20$: $I[f] = -0,5963185 = S_{20}$

$h=0,05$ $m=40$: $I[f] = -0,5963224 = S_{40}$

$h=0,025$ $m=80$: $I[f] = -0,5963227 = S_{80}$

порядок точности $k=5$

$$R_{2m} = \frac{S_{2m}-S_m}{2^{k-1}-1} = \frac{1}{15}(S_{2m}-S_m)$$

$$\cancel{R_{20}} = R_m = \frac{16}{15}(S_{2m}-S_m)$$

$$R_{20} = \frac{16}{15}(S_{40}-S_{20}) = 0,0000416$$

$$R_{40} = \frac{1}{15}(S_{40}-S_{20}) = 0,0000026$$

$$R_{80} = \frac{1}{15}(S_{80}-S_{40}) = 0,00000002$$