# Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with $\mu$ = 45 minutes and $\sigma$ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

   A. 0.3875
   B. 0.2676
   C. 0.5
   D. 0.6987

Ans:

Given the data,

$\mu$ = 45

$\sigma$ = 8

X=10 mins

Y=60 mins (1 hour)

Z_Score=(Y-X)- $\mu$ / $\sigma$

   =(60-10)-45/8

   =(50-45)/8

   =5/8

P(X>5/8) = 1-P(X<5/8)

   = 1-0.734

   =0.265

Option: B

2. -The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean $\mu$ = 38 and Standard deviation $\sigma$=6. For each statement below, please specify True/False. If false, briefly explain why.

   A. More employees at the processing center are older than 44 than between 38 and 44.
   B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans:

   A. Inorder to determine the truth of the given statement, we need to compare the proportions of employees those who are older than the 44 and those between 38 and 44.

Initially, we should calculate the z-score for 44 years:

Z_Score=(X)- $\mu$ / $\sigma$

  =44-38/6

  =1

P(Z>44)=1-P(Z<44)

  =1-0.841

  =0.159

Now, we should calculate the z-score for age between 38 to 44 years.

P(38<Z<44)= P(Z<44)-0.5

  =0.841-0.5

  =0.341

Since, the proportion of the age between the 38 to 44 years is having more probability compared to the proportion of the age more than the 44 years, hence the condition is false.


  B. To determine whether, we need to find the proportion of employees under the age of 30.

From scipy  import stats

Stats.norm.cdf(30,38,6)

0.0912

Total employees=400

Inorder to find the number of employees under 30 in a sample of 400 employees

We need to calculate the employees under 30 is

=0.0912*400

36.48

Therefore the statement represents the true, as there are 36 employees with an age of 30 years.

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between $2X_1$ and $X_1 + X_2$? Discuss both their distributions and parameters.

Ans: file attached.

4. Let $X \sim N(100, 20^2)$. Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

    A. 90.5, 105.9
    B. 80.2, 119.8
    C. 22, 78
    D. 48.5, 151.5
    E. 90.1, 109.9

Ans:

Inorder to find the value of a and b, probability of the random variable X

Mean=100

Variance=$20^2$=400

Taking a value between them=0.99

Z_Score=(X)-$\mu / \sigma$

As we need to find the z-scores of a and b which represents as $Z_a$ and $Z_b$

By using the standard normal distribution, we can find the z-scores corresponding to the cumulative probability for 0.005 and 0.995

    $Z_a$=-2.576
    $Z_b$=2.576

    Now by using the formula of X=$\mu$+Z$\sigma$

    Xa=100+(-2.576)(20)=48.48
    Xb=100+(2.576*20)=151.52

    Hence the values of a & b are 48.48,151.52

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit$_1 \sim N(5, 3^2)$ and Profit$_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
    A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
    B. Specify the 5th percentile of profit (in Rupees) for the company
    C. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: file attached