1.4.6.1 *Terminal objects are unique up to isomorphism*

Proof. Assume $t1$ and $t2$ are terminal objects in a category $\mathcal{C}$. Since for every object $c \in \mathcal{C}$ there exists a unique arrow from c to each terminal object, there exist arrows $f : t1 \to t2$ and $g : t2 \to t1$. Thus $g \circ f = id_{t1}$ and $f \circ g = id_{t2}$, and f and g define our isomorphism. □

1.4.6.2 **Set** $\times$ **Set:**

Initial Object: $(\{\} \times \{\})$

Terminal Objects: $\{(\{A\} \times \{B\}) \mid A, B \in \textbf{Set}\}$

Set$^{\to}$**:**

Initial Objects: ?

Terminal Objects: ?

poset $\mathcal{P}$**:**

Initial Objects: The min objects $\{p \mid \forall p' \in \mathcal{P},\ p \leq p'\}$

Terminal Objects: The max objects $\{p \mid \forall p' \in \mathcal{P},\ p' \leq p\}$

1.4.6.3 A simple functional language with types UNIT and BOOL and arrows $true$: UNIT $\to$ BOOL and $false$: UNIT $\to$ BOOL can be formalized as a category with no initial objects. Initial objects must have one unique arrow to every other object in the category, and neither UNIT nor BOOL satisfy this criteria.

This category also has no terminal objects, for there are no arrows from BOOL to UNIT and two arrows from UNIT to BOOL.

The initial and terminal objects in a single-object category are the same. Likewise, the initial and terminal objects in any fully-connnected (clique) category are the same.