A Comparative Study of Eigenfaces and Fisherfaces

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***Abstract*—The PCA algorithm for face recognition as proposed by M.A Turk and Alex Pentland was efficient but also pointed out on the flaws of the algorithm under different lighting conditions. This paper discusses another developed algorithm that specifically takes care of the lighting scenario called Fisherfaces. A comparison between both the algorithms will also be done**

Keywords: PCA, LDA, Fisherfaces.

I. Introduction

The PCA finds a linear combination of features that maximizes the total variance in data. While this is clearly a powerful way to represent data, it doesn't consider any classes and so a lot of discriminative information may be lost when throwing components away. Imagine a situation where the variance is generated by an external source, let it be the light. The components identified by a PCA do not necessarily contain any discriminative information at all, so the projected samples are smeared together and a classification becomes impossible. In simpler terms, a variation in light could cause more discrimination over actual interpersonal differences, which is unfavourable. The Linear Discriminant Analysis or LDA was invented by the great statistician Sir R. A. Fisher, who successfully used it for classifying flowers in his 1936 paper *The use of multiple measurements in taxonomic problems* In order to find the combination of features that separates best between classes the Linear Discriminant Analysis maximizes the ratio of between-classes to within-classes scatter. The idea is simple: same classes should cluster tightly together, while different classes are as far away as possible from each other.

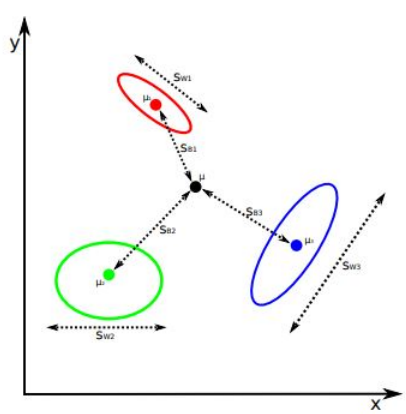

Fig. 1: This figure shows the scatter matrices S_B and S_W for a 3 class problem. μ represents the total mean and $[\mu_1, \mu_2, \mu_3]$ are the class means.

II. Algorithm

A. Dataset and Preprocessing

We used the yalefaces dataset consisting of 165 grayscale images of 15 subjects, each of resolution 243×320. Each subject had 11 images associated with it, consisting of the same subject in different lighting conditions and expressions First, we have split the dataset into training and testing images. Since it takes a significant amount of time to compute the eigenvectors of such a high resolution, we have reshaped each image into size 55×55 for faster computation. The image is then added to an array which will totally consist of N training images, with an array shape of $N \times 55 \times 55$, after which each image in the array is flattened, resulting in an array shape of $N \times 3025$. The next subsection will only discuss the algorithm for Fisherfaces. However, the important eigenfaces obtained through PCA will be used to compute Fisherfaces

B. Fisherfaces algorithm

- First, the training images are classified according to interpersonal changes. We have made use of all 15 subjects, along with 5 images per subject in the training set. Hence there are totally 15 classes with 5 samples in each class
- We then compute the weight coefficients obtained through the eigenfaces for each sample. Each sample is now expressed as a sum of these eigenfaces with the weights as the coefficients, and the weights(coefficients) are stored in a seperate vector
- Now, we compute the overall mean set of weights for the entire dataset and also, the mean weights of each class
- We can now calculate the between class scatter matrix S_B using the formula

$$S_B = \sum_{i=1}^{c} N_i(\mu_i - \mu)(\mu_i - \mu)^T \tag{1}$$

  where
  - c is the total number of classes
  - N_i is the total number of samples per class
  - μ_i is the mean weight of each class
  - μ is the overall mean weight
- The within class scatter matrix S_W can be computed using the formula

$$S_W = \sum_{i=1}^{c} \sum_{x_j \in X_i} (x_j - \mu_i)(x_j - \mu_i)^T \tag{2}$$

  where x_j is the weights of an image in class X_i

- Fisher's classic algorithm now looks for a projection *W*, that maximizes the class separability criterion:

$$W_{opt} = argmax_W \frac{|W^T S_B W|}{|W^T S_W W|} \tag{3}$$

- A solution for this eigenvalue problem is given by solving the general eigenvalue problem

$$S_B v_i = \lambda_i S_W v_i \tag{4}$$

$$S_W^{-1} S_B v_i = \lambda_i v_i \tag{5}$$

- The rank of S_W is at most $(N - c)$, with N samples and c classes. In pattern recognition problems the number of samples N is almost always smaller than the dimension of the input data (the number of pixels), so the scatter matrix S_W becomes singular.
- This was solved by performing a Principal Component Analysis on the data and projecting the samples into the $(N-c)$ dimensional space. A Linear Discriminant Analysis was then performed on the reduced data, because S_W isn't singular anymore.The optimization problem can be rewritten as:

$$W_{pca} = argmax_W |W^T S_T W| \tag{6}$$

$$W_{fld} = argmax_W \frac{|W^T W_{pca}^T S_B W_{pca} W|}{|W^T W_{pca}^T S_W W_{pca} W|} \tag{7}$$

- The transformation matrix W, that projects a sample into the (c-1) dimensional space is then given by:

$$W = W_{fld}^T W_{pca}^T \tag{8}$$

This matrix gives us the set of Fisherfaces

III. OBSERVATIONS AND RESULTS

First, the eigenfaces using LDA are plotted and then we plot the fisherfaces. In testing, the norm is calculated between the weights of the unknown vector and weights of the training set and the image is classified to the training image that corresponds to the minimum distance

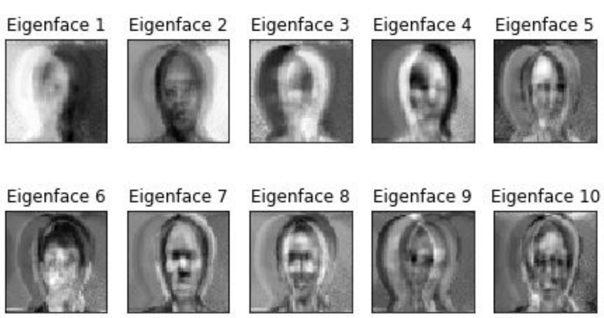

Fig. 2: Eigenfaces

On observation of the top 10 fisherfaces, we can see that on comparison with the eigenfaces, the variation in the components can be seen more in the face and structure of the individual, unlike eigenfaces where the first two eigenvectors mainly show the variance in lighting. From this we can infer

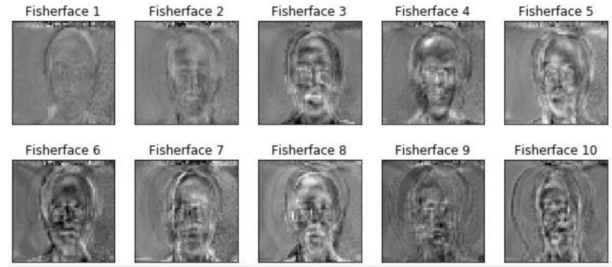

Fig. 3: Fisherfaces

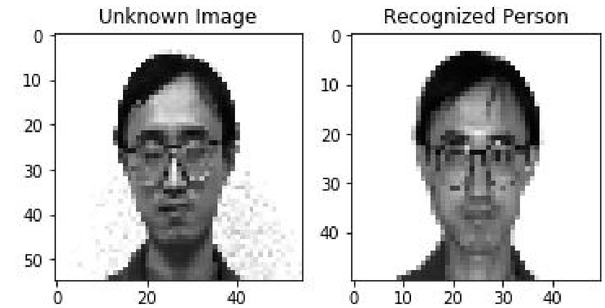

Fig. 4: Example of an unknown image classified using Fisherfaces

that the fisherfaces has dropped the principal components corresponding to lighting. On projecting a test image with the two fisherfaces, an accuracy of 57.1 percent was obtained, with five fisherfaces, an accuracy of 68.2 percent and 84 percent accuracy for 10 fisherfaces