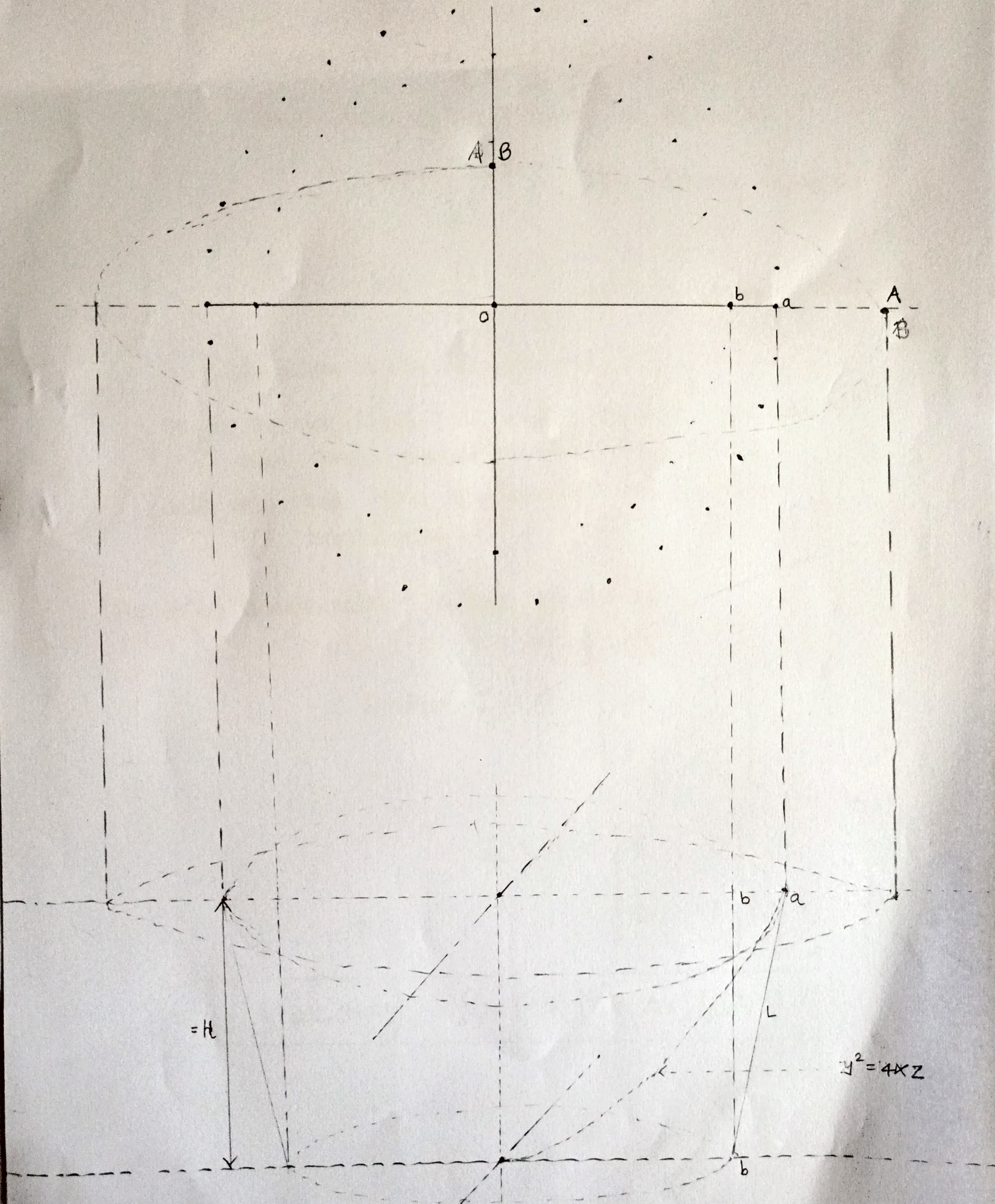
SMA-RING
WORKING TRANSFORMATION



mm

Define a transformation T: circular paraboloid $\longrightarrow$ elliptic mixed paraboloid. Hence

T: circle $\longrightarrow$ ellipse where T is the cross section component in T. (only the largest circle should be elliptic)

$T \in \mathbb{R}^3$ and $T \in \mathbb{R}^2$ in XY plane

T, T are non-linear transformations.

For T: Scaling matrices of transformation are M_x and M_y

i.e. $M_y = \begin{bmatrix} p & 0 \\ 0 & 1 \end{bmatrix}$ $p > 1$ (Assuming expansion)

$M_x = \begin{bmatrix} 1 & 0 \\ 0 & q \end{bmatrix}$ $q \in (0,1)$ (Assuming reduction)

In function algebra: $f(x,y) = (qx, py)$

To get one non-linear transformation equation, you will need complex numbers (or quarternions). So we will only show affine transformation and projective transformation.

For affine transformation: We have $u(x,y) = qx$

$v(x,y) = py$

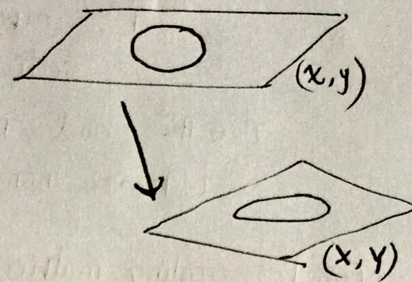
$\therefore$ Jacobian $J_f(x,y) = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} \end{bmatrix}$

$$J_f(x,y) = \begin{bmatrix} q & 0 \\ 0 & p \end{bmatrix}$$

$$\boxed{f(x+\Delta x, y+\Delta y) = f(x,y) + [\Delta x \;\; \Delta y] J_f(x,y)}$$

For projective transformation P :

$$P: \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \begin{bmatrix} C & D & 0 \\ E & F & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



$$X = Cx + Dy$$

$$Y = Ex + Fy$$

Area Analysis of elastic mesh on transformation T

Initially : Curved Surface Area (C_i) = CSA of frustum

$$= \pi L(a+b)$$

For our case, we have a parabola $y^2 = 4KZ$

$\therefore$ $\cancel{Z} = h = 125\text{mm} \Leftrightarrow y = 60\text{ mm}$

$\therefore$ $3600 = 4K(125)$

$$\boxed{K = 7.2}$$

We have : $a = 120$ mm

$b = 100$ mm

$\therefore$ $L^2 = (a-b)^2 + H^2$ $\quad (H = h - 83.33 = 41.67)$

$\therefore$ $L^2 = 400 + 1736.39$

$\therefore$ $\boxed{L = 42.15\text{ mm}}$

$\therefore$ $C_i = \pi(42.15)(220)$

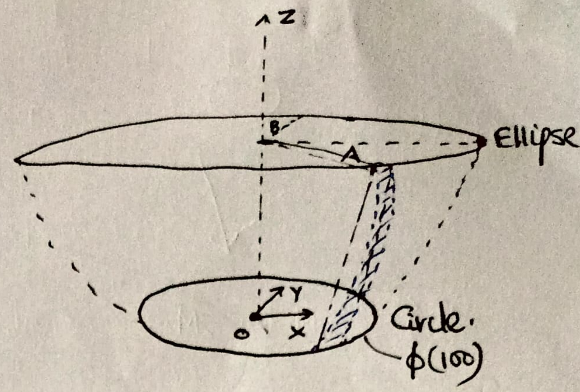
$\therefore$ $\boxed{C_i = 29126.49\text{ mm}^2}$

OR

$\boxed{C_i = 0.291\text{ m}^2}$

The elastic mesh will now be curved

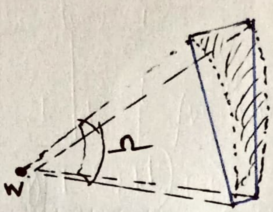
Take element of area as shown



Final Curved surface Area (C_f) is what we need.

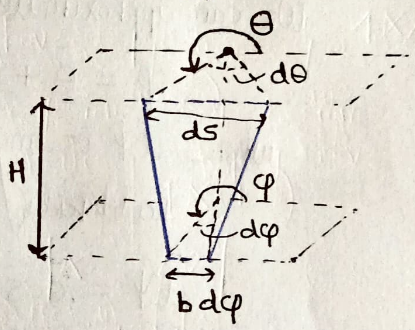
If the solid angle of this surface is Ω steradians,

$\boxed{dA_{proj} = \Omega\, dA}$ $\boxed{\Omega = \text{const.}}$



Take projection of elemental area taken

$\therefore dA = \frac{1}{\Omega}\, dA_{proj}$



Now, $ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

For ellipse ds :

$ds = A\sqrt{1 + e^2\sin^2\theta}\, d\theta$

where $e = \text{eccentricity} = \sqrt{1 - \frac{B^2}{A^2}}$

$\therefore dA_{proj} = \frac{1}{2}(ds + b\,d\varphi)H$ and $\frac{d\theta}{\theta} = \frac{d\varphi}{\varphi}$

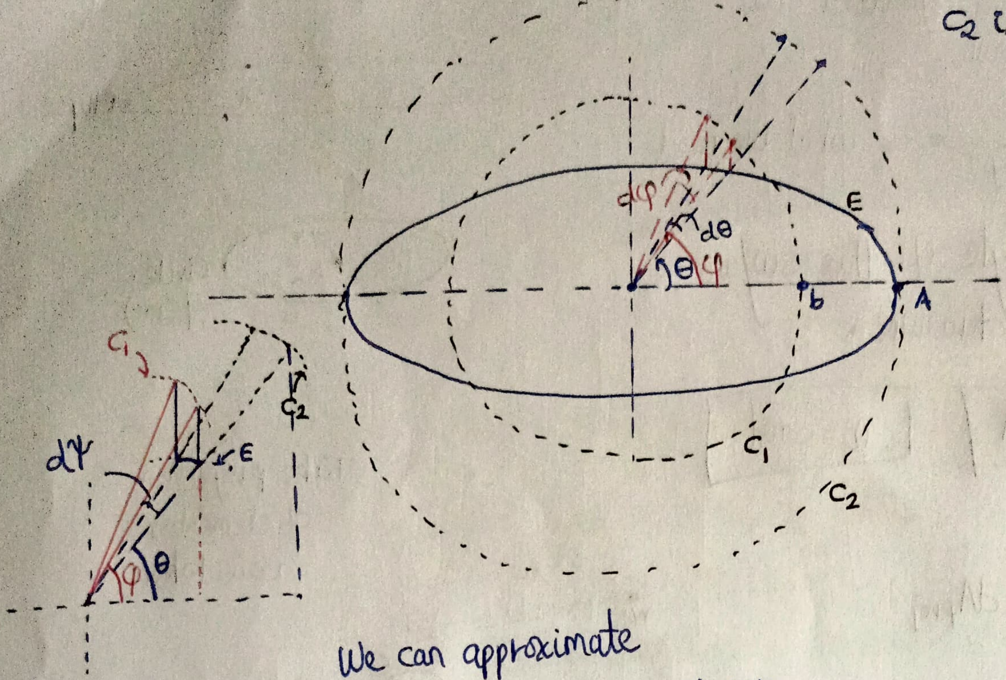
$= \frac{H}{2}\left[A\sqrt{1+e^2\sin^2\theta}\, d\theta + b\,d\varphi\right]$

$= \frac{H}{2}\left[\frac{A\theta}{\varphi}\sqrt{1+e^2\sin^2\theta} + b\right] d\varphi$

Now we need the relation b/w θ and φ.

$T: C_1 \rightarrow E$

C_2 is director circle



We can approximate

$$\varphi = \theta + \cancel{d\theta} + d\psi$$

where $d\psi$ is angle between unequal ~~radd~~ radii surface $r_1 = b$ and $r_2 = A$

$$d\psi = \cos^{-1}\left(\frac{-3b}{2A}\right) \quad \text{(By trigonometry)}$$

$$\varphi = \theta + \cos^{-1}\left(\frac{-3b}{2A}\right)$$

$$\text{i.e } \theta = \varphi - \cos^{-1}\left(\frac{-3b}{2A}\right)$$

$$\therefore \int_0^{A_{proj}} dA_{proj} = 4\int_0^{\pi/4} \frac{H}{2}\left[A\left(1 - \frac{\cos^{-1}\left(\frac{-3b}{2A}\right)}{\varphi}\right)\sqrt{1+e^2\sin^2\left(\varphi - \cos^{-1}\left(\frac{-3b}{2A}\right)\right)} + b\right] d\varphi$$

$$\therefore C_f = \frac{4}{\Omega}\int_0^{\pi/4} \frac{H}{2}\left[A\left(1 - \frac{\cos^{-1}\left(\frac{-3b}{2A}\right)}{\varphi}\right)\sqrt{1+e^2\sin^2\left(\varphi - \cos^{-1}\left(\frac{-3b}{2A}\right)\right)} + b\right] d\varphi$$

Assuming elastic mesh as a surface spring of constant K

Appx. $F = -K\Delta A$

$F = -K(C_f - 0.291)$ in our case.

Selecting appropriate SMA and elastic mesh can be done by this eq^n.

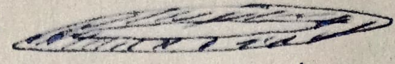
If we use SMA = Ni-Ti Alloy

Avg. Experimental max tensile strength = 700 MPa

Appx thickness of SMA ring (t) ≈ 4 mm

Area of shaded portion.

$$A_{sh} = \pi\left(120^2 - 116^2\right) \times 10^{-6}$$



Circle form

$$A_{sh} \simeq 2.97 \times 10^{-3}\ m^2$$

Max. tension ring can withstand before non-thermal deformation

$$T_{max} = 700 \times 10^{\cancel{6}\,3} \times 2.97 \times \cancel{10^{-3}}$$

$$\therefore T_{max} \simeq 2.08 \times 10^6\ N$$

$$T < T_{max}$$

$$K\left(\varsigma^{-0.291}\right) < 2 \times 10^6 \quad \text{for NiTi SMA}$$

$$\boxed{K < \frac{2 \times 10^6}{\varsigma^{-0.291}}}$$

Now we can choose appropriate elastic mesh according to above value range