Program Inversion
and Reversible Computation
2020

Reversible Computing:
Janus (2)
A Reversible Programming Language

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Today's Plan

- Programming techniques in a reversible language
- Reversible self-interpreter for Janus
  - Implementation of self-interpreter
  - Tower of self-interpreters
- An example of physical simulation in Janus
  - Discrete simulation of Schrödinger wave equation

Programming Techniques

- Each programming paradigm has its own programming techniques.
  - So do reversible languages

1. Zero-cleared copying, Zero-clearing by constant
2. Temporary stack
3. Code sharing by call and uncall
4. Call-uncall (Local Bennett's Method)

1. Zero-cleared Copying, Zero-clearing by Constant

- Zero-cleared copying:

```
{ x=0, … }
x ^= y
{ x=y, … }
```

Ex.

```
procedure main
  { n=0, … }
  n ^= 4
  { n=4, … }
  call fib
```

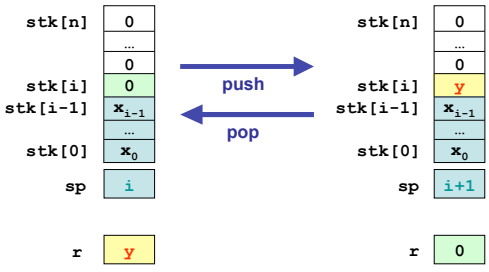
- Zero-clearing by constant:

```
{ x=y, … }
x ^= y
{ x=0, … }
```

Ex.

```
procedure main⁻¹
  uncall fib
  { n=4, … }
  n ^= 4
  { n=0, … }
```


2. Temporary Stack



2. Temporary Stack

- Array (initially zero-cleared): `tmp_stack[]`
- Stack pointer: `tmp_sp`
- Procedure (push, pop):

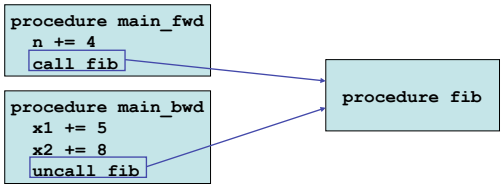
```
procedure alloc_tmp
  tmp_sp += 1
  tmp <=> tmp_stack[tmp_sp]
```

- Push: `call alloc_tmp` Save and clear `tmp`
- Pop: `uncall alloc_tmp` Restore cleared `tmp`

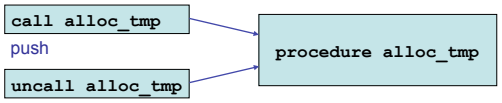
3. Code sharing by call and uncall

- The same procedure definition can be used with its inverse functionality by calling and uncalling it.

Ex.



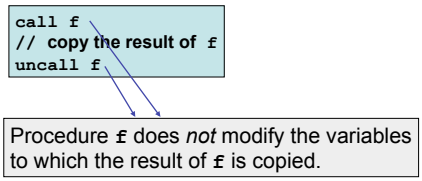
Ex.



4. Call-Uncall (Local Bennett's Method)

- We need the result of a procedure `f`.
- We want to undo all other side effects the computation has had on the store.

⇒ Use the "call-uncall" program pattern.



RTM-Interpreter in Janus (extended)

```
procedure main()
 ... RTM, tape and constants decl. and init. ...
from  q=QS    start state
do    call inst(q,left,s,right,q1,s1,s2,q2,pc)
      pc += 1                          } index of next rule
      if pc=PC_MAX then                }
         pc ^= PC_MAX                  }
      fi pc=0
until q=QF    final state                main loop
```

```
procedure pushtape(int s,stack stk)
 if empty(stk) && (s=BLANK) then
    s ^= BLANK  // zero-clear s
 else
    push(s,stk)
 fi empty(stk)
```



```
procedure inst(int q, stack left, int s, stack right,
               int q1, int s1, int s2, int q2, int pc)

if (q=q1[pc]) && (s=s1[pc]) then      // Symbol rule:
   q += q2[pc]-q1[pc]                 // set q to q2[pc]
   s += s2[pc]-s1[pc]                 // set s to s2[pc]
fi (q=q2[pc]) && (s=s2[pc])
if (q=q1[pc]) && (s1[pc]=SLASH) then // Move rule:
   q += q2[pc]-q1[pc]                 // set q to q2[pc]
   if s2[pc]=RIGHT then
      call   pushtape(s,left)         // push s on left
      uncall pushtape(s,right)        // pop right to s
   fi s2[pc]=RIGHT
   if s2[pc]=LEFT then
      call   pushtape(s,right)        // push s on right
      uncall pushtape(s,left)         // pop left to s
   fi s2[pc]=LEFT
fi (q=q2[pc]) && (s1[pc]=SLASH)
```


Review: Self-Interpreter

- A self-interpreter *sint* for *L* is an *L*-interpreter written in *L*:

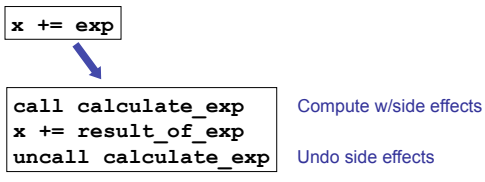
$$[\![sint]\!]_L[p, x] = [\![p]\!]_L\ x$$

- When *L* is a reversible language, the self-interpreter must be reversible.

Self-Interpreter for Janus

- Problem: evaluation of Janus expressions is backward nondeterministic!
- We need to implement those evaluation rules by reversible statements.

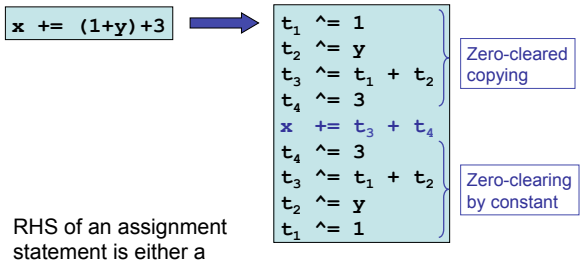
⇒ "Local Bennett's Method"



Outline of Solution

1. Simplified expressions (e.g., preprocessor)
   At most one operator in each expression.
2. Evaluation of simplified expressions.

1. Simplification of Janus Statements



RHS of an assignment statement is either a
- constant,
- variable, or
- binary expression with two variables.

Encoding of Janus Programs

- Since Janus has only numerical data, we encode Janus programs in two integer arrays:
  - **type[]** Type of syntactic construct (e.g., constant)
  - **para[]** Optional parameter (e.g., value of constant)
  - **pc** Program counter
- For example, assignment `x += 5` is encoded by

type	n_{aop}^{start}	n_{aop}	n_{con}	n_{con}	n_{aop}^{end}
para	4	n_{aop}^{plus}	271	5	4

Index of variable **x** in store

State of Self-Interpreter

- Syntactic types
  n_con, n_var, n_bop_start, …
- Syntactic parameters
  n_plus, n_minus, n_xor, …
- Store of interpreted program
  sigma[]
- Temporaries
  op, arg1, arg2, tmp, …
- Temporary stack
  tmp_sp, tmp_stack[]

Skipping over Syntactic Blocks

For example, assignment `x += 5` is encoded by

type	n_{aop}^{start}	n_{aop}	n_{con}	n_{con}	n_{aop}^{end}
para	4	n_{aop}^{plus}	271	5	4

Each statement has paired offsets.

```
procedure next
  next_tmp ^= para[pc]
  pc += next_tmp
  next_tmp ^= para[pc]
  pc += 1
```

Zero-cleared copying

set **next_tmp** to 0

Zero-clearing by constant

Remark: temporary `next_tmp` needed because `pc` may not occur on both sides of an assignment.

Skipping Forward and Backward

- Skipping over syntactic blocks in both directions:

type	n_{aop}^{start}	n_{aop}	n_{con}	n_{con}	n_{aop}^{end}
para	4	n_{aop}^{plus}	271	5	4

pc

call next

uncall next

type	n_{aop}^{start}	n_{aop}	n_{con}	n_{con}	n_{aop}^{end}
para	4	n_{aop}^{plus}	271	5	4

pc

Review: Syntax of Janus

$$
\begin{array}{rcl}
p & ::= & vdec^*\ \texttt{(procedure}\ id\ s\texttt{)}^+ \\
vdec & ::= & x \mid x\texttt{[}c\texttt{]} \\
s & ::= & x \oplus\texttt{= } e \mid x\texttt{[}e\texttt{]} \oplus\texttt{= } e \mid \\
 & & \texttt{if } e \texttt{ then } s \texttt{ else } s \texttt{ fi } e \mid \\
 & & \texttt{from } e \texttt{ do } s \texttt{ loop } s \texttt{ until } e \mid \\
 & & \texttt{call } id \mid \texttt{uncall } id \mid \texttt{skip} \mid s\ s \\
e & ::= & c \mid x \mid x\texttt{[}e\texttt{]} \mid \texttt{\~{}}e \mid e \odot e \\
c & ::= & 0 \mid 1 \mid \cdots \mid 4294967295 \\
\oplus & ::= & \texttt{+} \mid \texttt{-} \mid \texttt{\^{}} \\
\odot & ::= & \oplus \mid \texttt{*} \mid \texttt{/} \mid \texttt{\%} \mid \texttt{*/} \mid \texttt{\&} \mid \texttt{|} \mid \texttt{\&\&} \mid \texttt{||} \mid \\
 & & \texttt{<} \mid \texttt{>} \mid \texttt{=} \mid \texttt{!=} \mid \texttt{<=} \mid \texttt{>=}
\end{array}
$$

Assignment operations

Reversible Conditional

Reversible Loop

Procedure call/uncall

32-bits integers

Execution of Statements: Conditional

$$\frac{\sigma \vdash_{expr} e_1 \Rightarrow v_1 \quad \text{is-true?}(v_1) \qquad \sigma \vdash_{stmt} s_1 \Rightarrow \sigma' \qquad \sigma' \vdash_{expr} e_2 \Rightarrow v_2 \quad \text{is-true?}(v_2)}{\sigma \vdash_{stmt} \texttt{if } e_1 \texttt{ then } s_1 \texttt{ else } s_2 \texttt{ fi } e_2 \Rightarrow \sigma'}\ \text{IfTrue}$$

Test

Assertion

```
procedure exec_if
  pc += 1
  call eval
  if result
  then uncall eval
       pc -= 1
       call next // to then
       call exec
       call next // to fi
       pc += 1
       call eval
  fi result
  uncall eval
  pc -= 1
  call next // to end of stmt
  pc -= 1
```

Evaluation of the test **call-uncall**

Undoing the side effects of evaluating the test

```
else uncall eval
     pc -= 1
     call next // to then
     call next // to else
     call exec
     pc += 1
     call eval
```

Evaluation of the assertion

call-uncall

Undoing the side effects of evaluating the assertion

Execution of Statements: Loop

- Similar to conditional

Execution of Statements: Procedure Call

Execution of body of procedure *id*

$$\frac{\sigma \vdash_{stmt} \Gamma(id) \Rightarrow \sigma'}{\sigma \vdash_{stmt} \texttt{call } id \Rightarrow \sigma'}\ \text{Call}$$

```
procedure exec_call
  call exec_call_pc_swap
  call exec
  uncall next
  uncall exec_call_pc_swap
```

Save `pc` and set new `pc` to body of procedure.

call-uncall

Restore `pc`

```
procedure exec_call_pc_swap
  tmp ^= para[pc]
  pc <=> tmp
  call alloc_tmp
```

`pc:=para[pc]`

temporary stack

Execution of Statements: Procedure Uncall

$$\frac{\sigma \vdash_{stmt} \Gamma(id) \Rightarrow \sigma'}{\sigma \vdash_{stmt} \texttt{call } id \Rightarrow \sigma'}\ \text{Call}$$

$$\frac{\sigma' \vdash_{stmt} \Gamma(id) \Rightarrow \sigma}{\sigma \vdash_{stmt} \texttt{uncall } id \Rightarrow \sigma'}\ \text{Uncall}$$

- How to implement call and uncall in the interpreter?

```
procedure exec_uncall
        ?
```

Code-sharing by call-uncall

Review: Expression Evaluation is Irreversible

Judgment: $\sigma \vdash_{expr} e \Rightarrow v$ (Store, Exp, Val)

$$\frac{}{\sigma \vdash_{expr} c \Rightarrow [\![c]\!]}\ \text{Con} \qquad \frac{}{\sigma \vdash_{expr} x \Rightarrow \sigma(x)}\ \text{Var}$$

$$\frac{\sigma \vdash_{expr} e_1 \Rightarrow v_1 \quad \sigma \vdash_{expr} e_2 \Rightarrow v_2 \quad [\![\odot]\!](v_1, v_2) = v}{\sigma \vdash_{expr} e_1 \odot e_2 \Rightarrow v}\ \text{BinOp}$$

where $\odot \in \{ *, /, \&, <, >, \ldots \}$ are non-injective operators

Reversible Evaluation of Expressions

- Case: constants

Reversible update Evaluation of RHS (irreversible)

$$\frac{v' = [\![\oplus]\!](v, [\![c]\!])}{\sigma \uplus \{x \mapsto v\} \vdash_{stmt} x \;\; \oplus\!= c \Rightarrow \sigma \uplus \{x \mapsto v'\}}$$

- Case: variables

$$\frac{v' = [\![\oplus]\!](v, \sigma(y))}{\sigma \uplus \{x \mapsto v\} \vdash_{stmt} x \;\; \oplus\!= y \Rightarrow \sigma \uplus \{x \mapsto v'\}}$$

- Case: binary operators

$$\frac{v' = [\![\oplus]\!](v, [\![\odot]\!](\sigma(y_1), \sigma(y_2)))}{\sigma \uplus \{x \mapsto v\} \vdash_{stmt} x \;\; \oplus\!= y_1 \odot y_2 \Rightarrow \sigma \uplus \{x \mapsto v'\}}$$

Note: simplified expressions have at most one operator

Evaluation of Expressions: Dispatch

```
procedure eval
  if type[pc] = n_con
  then result ^= para[pc]
  else if type[pc] = n_var
       then result ^= sigma[para[pc]]
       else if type[pc] = n_bop_start
            then call eval_bop
                 call next
                 pc-=1
            else error
            fi type[pc] = n_bop_end
       fi type[pc] = n_var
  fi type[pc] = n_con
  pc+=1
```

Constant: value in `para[]`

Variable: look-up in `sigma[]`

Binary operator: call sub-procedure

Dispatch depends on `type[pc]`

Assertions on `type[pc]`

Evaluation of Expressions: Binary Operators

```
procedure eval_bop
  call eval_bop_args
  if op = n_plus
  then tmp ^= arg1 + arg2
  else if op = n_minus
       then tmp ^= arg1 - arg2
       else if op = n_xor
            then tmp ^= arg1 ^ arg2
            // ...
            fi op = n_xor
       fi op = n_minus
  fi op = n_plus
  uncall eval_bop_args
  result <=> tmp
```

Evaluate both arguments and return results in `arg1` and `arg2`

$v' = [\![\odot]\!](\sigma(y_1), \sigma(y_2))$ ($\sigma(y_1)$: arg1, $\sigma(y_2)$: arg2)

Side-effects are undone

call-uncall

The underlying operation is evaluated.

Evaluation of Expressions: Arguments

```
procedure eval_bop_args
  pc += 1
  op ^= para[pc]
  pc += 1
  call eval
  arg1 <=> result
  call eval
  arg2 <=> result
```

Determine the binary operator

Evaluate 1st argument

Evaluate 2nd argument

Execution of Statements: Reversible Update

$$\frac{v' = f(\sigma, \oplus, v, e)}{\sigma \uplus \{x \mapsto v\} \vdash_{stmt} x \;\; \oplus\!= e \Rightarrow \sigma \uplus \{x \mapsto v'\}}$$

```
procedure exec_aop
  call exec_aop_args
  call exec_aop_upd
  uncall exec_aop_args
  call next    // to end of stmt
  pc -= 1
```

Evaluation of RHS

Reversible Update

call-uncall

Execution of Statements: Dispatch

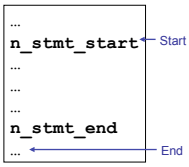
```
procedure exec1
  if type[pc] = n_aop_start
  then call exec_aop
  else if type[pc] = n_if_start
       then call exec_if
       else if type[pc] = n_from_start
            then call exec_from
            else if type[pc] = n_call
                 then call exec_call
                 else if type[pc] = n_uncall
                      then call exec_uncall
                      else if type[pc] = n_skip
                           else error
                           fi type[pc] = n_skip
                      fi type[pc] = n_uncall
                 fi type[pc] = n_call
            fi type[pc] = n_until_end
       fi type[pc] = n_fi_end
  fi type[pc] = n_aop_end
```

Dispatched depending on `type[pc]`

Execution of Statements: Top Level

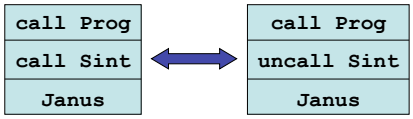
- Start from **n_stmt_start** and end with **n_stmt_end**
- In each iteration, a statement is executed by **exec1**

```
procedure exec
    from  type[pc] = n_stmt_start
    do    pc += 1
    loop  call exec1
    until type[pc] = n_stmt_end
    pc += 1
```



Short Summary: Self-Interpreter

- Implementation completed
- History-free
  - Uses only local Bennett's method
- Constant space
- Reversible self-interpreter



Experiments with Janus

- Many physical models describe reversible processes.
- Schrödinger Wave Equation
  - The fundamental equation of physics for describing quantum mechanical behavior.
  - F: $\mathcal{X}_{i,n+1} = \mathcal{X}_{i,n} + \alpha_i \mathcal{Y}_{i,n} - \epsilon(\mathcal{Y}_{i+1,n} + \mathcal{Y}_{i-1,n})$
    $\mathcal{Y}_{i,n+1} = \mathcal{Y}_{i,n} - \alpha_i \mathcal{X}_{i,n+1} + \epsilon(\mathcal{X}_{i+1,n+1} + \mathcal{X}_{i-1,n+1})$
  - C: $\mathcal{X}_{i,n} = \mathcal{X}_{i+128,n}$, $\mathcal{Y}_{i,n} = \mathcal{Y}_{i+128,n}$

† E. Fredkin. Feynman, Barton and the reversible Schrödinger difference equation. Feynman and computation: exploring the limits of computers, pages 337-348,1999.

M. P. Frank. Reversibility for Efficient Computing. PhD thesis, EECS Dept., MIT, 1999.

Simulation Program: Sch

$$\mathcal{X}_{i,n+1} = \mathcal{X}_{i,n} + \alpha_i \mathcal{Y}_{i,n} - \epsilon(\mathcal{Y}_{i+1,n} + \mathcal{Y}_{i-1,n})$$

```
procedure main
  ... // initialize arrays
  from  n=0
  loop  call step
        n += 1
  until n=maxn
```

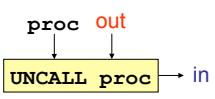
```
procedure step
  call stepX
  call stepY
```

```
procedure updateX
  X[i] += alpha[i] */ Y[i]
  X[i] -= epsilon */
          (Y[(i+1)%128] +
           Y[(i-1)%128])
```

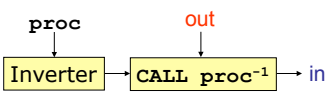
```
procedure stepX
  from  i=0
  loop  call updateX
        i += 1
  until i=128
  i -= 128
```


Review: Two Approaches to Inversion

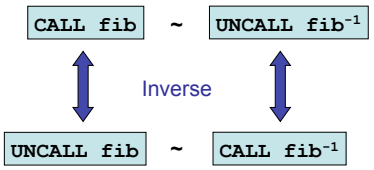
- Inverse interpretation of a procedure (one stage):

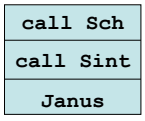


- Program inversion of a procedure (two stages):



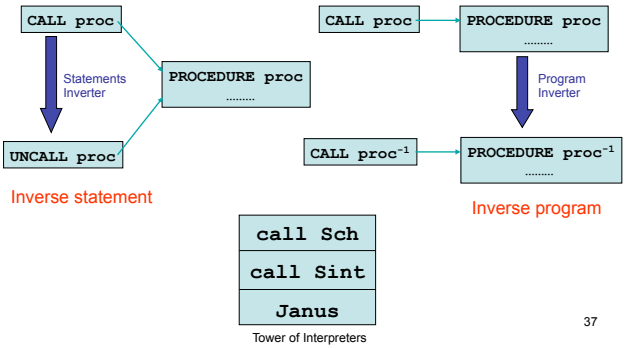
Review: Two ways of Invoking Procedures





Tower of Interpreters

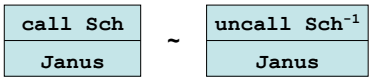
Review: Two Ways of Inversion



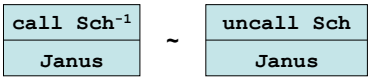
Tower of Interpreters

Inverse Call and Program Inversion

- Forward ($2^2 / 2 = 2$ possibilities)

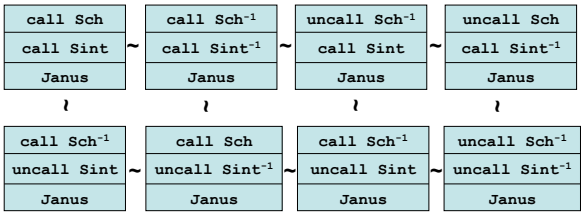


- Backward ($2^2 / 2 = 2$ possibilities)



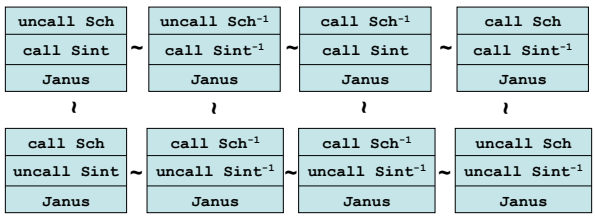
Inverse Call and Program Inversion

- Forward ($2^4 / 2 = 8$ possibilities)

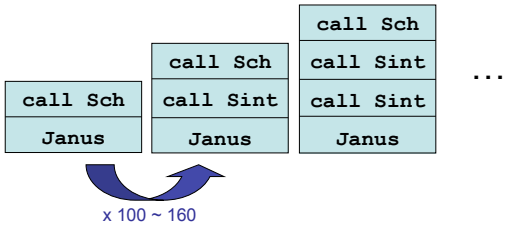


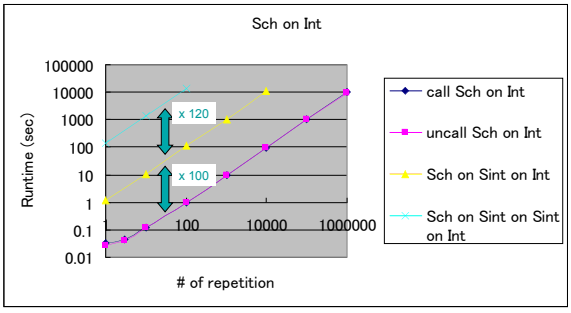
Inverse Call and Program Inversion

- Backward ($2^4 / 2 = 8$ possibilities)



Tower of Interpreters





Summary

- To implement the Janus self-interpreter, the operational semantics rules for expressions need to be implemented reversibly.
- Reversible programming paradigm has its own programming techniques.
- Janus self-interpreter realizes non-standard interpreter hierarchy.

Exercises: Free-Falling Object