Principles of Reversible Computation

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Computation is Physical

- **Computation is a physical process.**
  - paper and pencil
  - electronic device
  - biological system (such as the brain)
- **Any computation is subject to physical laws.**
  - Physical laws govern all information processing, *no matter how* it is accomplished.
- **Every logical state of a computing device must correspond to a physical state.**

Thermodynamical Minumum for Irreversible Computation

He then computed the thermodynamical minimum of energy per elementary act of information from the formula $kT \log_e N$ ergs, where k is Boltzmann's constant (1.4×10^{-16} ergs per degree), T is the temperature in absolute units, and N is the number of alternatives. For a binary act $N = 2$, and taking the temperature to be about 300 degrees absolute, he obtained 3×10^{-14} ergs for the thermodynamical minimum.

von Neumann 1949
in: Burks 1966, p.66

(1 erg = 10^{-7} joule)

Information Loss has a Physical Cost

- **Physical states cannot be destroyed,** so any bits that are **discarded logically turn into entropy.**
  - Consequence of basic thermodynam[...]
  - This irreversibility directly contribute[...] and therefore power consumption.

10^{-21} joules per erased bit at room temperature

Mismatch

- Our **best models of computation are irreversible**.
  - Carries over to actual implementations in computers.
- Good news: **Computation can be organized reversibly**.
  - Information destruction is *not* intrinsic to the concept of a computation process.

Reversible Computation (RC)

- Study of computing models deterministic in both the forward & backward directions.
- Adopts a physical principle of reality into a computing model without information loss.
- Sometimes RC a necessity, but also interesting in its own right – regardless of its physical motivation.

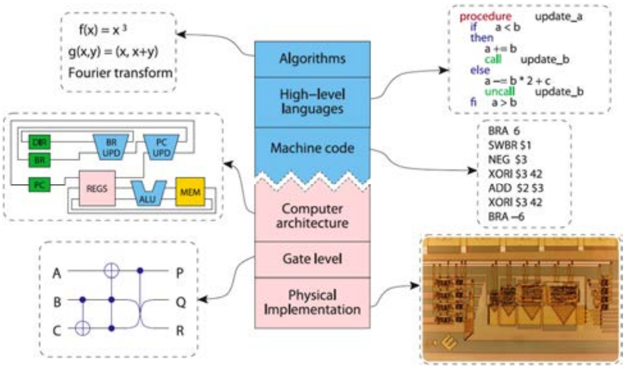
Reversible Languages

Languages:	fwd →	bwd ←	
• Logic languages	N	N	Prolog, ...
• Mainstream	D	N	C, ...
• Reversible languages	D	D	Janus, ...

State transitions:

- N ... non-deterministic
- D ... deterministic

Reversible Computing Systems

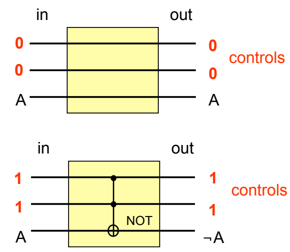


Reversible Logic Gates

Two essential properties:

1. The number of input lines is equal to the number of output lines (written $n \times n$).
2. Its Boolean function $B^n \rightarrow B^n$ is bijective.

Toffoli Gate [Toffoli '80]

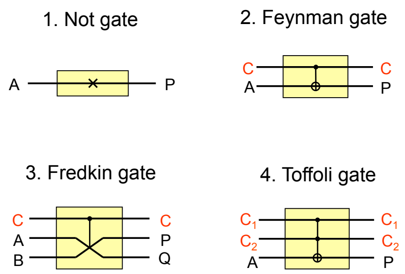


input			output		
C_1	C_2	A	C_1	C_2	P
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	1
1	1	1	1	1	0

Controlled-Controlled Not gate

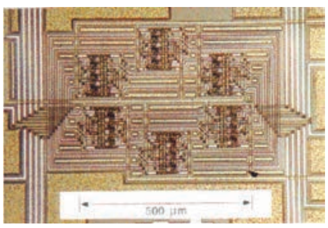
Universal: any rev. logic can be built by Toffoli gates

A Set of Reversible Logic Gates



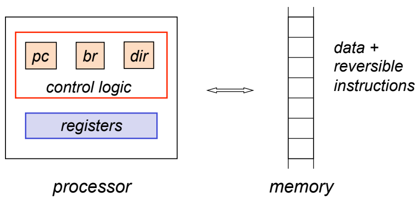
Realization of Reversible 4-Bit Adder

The circuit has *no power supply* and *no ground busbar*.
All energy provided to output pins comes from input pins.
Reversible gates used: Fredkin, Feynman, Toffoli.



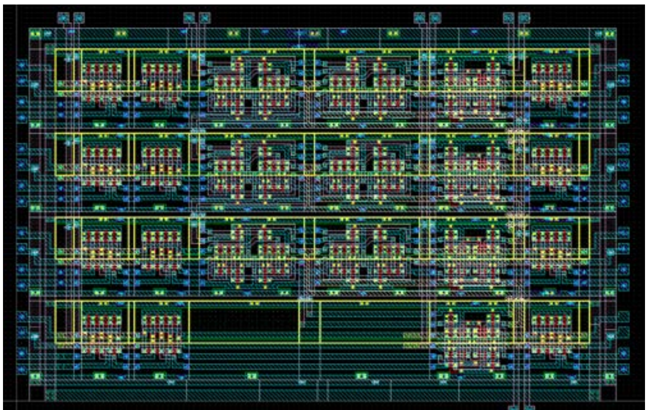
Dual-line pass-transistor logic [DeVos et al.]

Reversible Abstract Machine



Reversible von Neumann architecture

Shown reversible [AxelsenGlückYokoyama '07]



Fully Reversible Arithmetic-Logic Unit (ALU) [Thomsen et al.'10]

Gent's dual-line pass-transistors CMOS technology
256 transistors (0.35 μm), 117x77 μm size w/bonding pads

Computability

For all irreversible computing devices:

- Conventional (= irreversible) languages: equivalent to Turing-machine. [Turing 1936]

For all reversible computing devices:

- Reversible programming languages: equivalent to reversible Turing-machine. [Bennett 1974]
  = all computable <u>injective</u> functions

Two Main Steps – Example

1. **Specification**: **Injectivization** of non-injective **function**.

Fibonacci classic:	injectivize →	Fibonacci-pair injective:
fib(*n*)		[*fib*(*n*), *fib*(*n*+1)]

Recall: *fib*(1) = 1
fib(2) = 1

2. **Implementation**: in a **reversible programming language**.
   Algorithm design, programming, optimization, compilation.

[GlückKawabe'03]

Janus: a Reversible Language

- Imperative programming language
- Global store, no local store
- Scalar and array types, integer values
- Structured control-flow operators (IF, LOOP)
- Access to inverse semantics (UNCALL)
- Simple recursive procedures

[LutzDerby'82, YokoyamaGlück'07, YokoyamaAxelsenGlück'08]

Janus Example: Fibonacci-Pairs

```
n x1 x2
```

global store (integer variables)
initially zero

```
procedure fib
  if n=0 then x1 += 1
              x2 += 1
         else n -= 1
              call fib
              x1 += x2
              x1 <=> x2
  fi x1=x2
```

reversible procedure

test (n=0)

swap values of x1, x2

assertion (x1=x2)

Forward & Backward Computation

```
procedure main_fwd
  n += 4
  call fib
```

procedure forward

```
procedure main_bwd
  x1 += 5
  x2 += 8
  uncall fib
```

procedure backward

```
procedure fib
```

one procedure for both directions
⇒ reuse, code sharing

uncall: invokes inverse semantics

[ReillyFederighi65,LutzDerby82]

Program Inversion: Inverse Program

`n x1 x2` global store

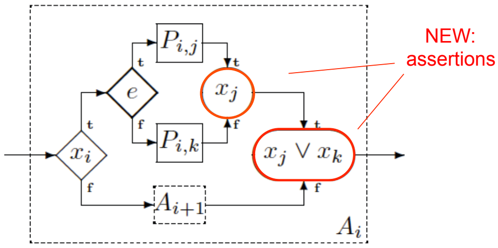
```
procedure fib
  if n=0 then x1+=1
              x2+=1
         else n-=1
              call fib
              x1+=x2
              x1<=>x2
  fi x1=x2
```

test (n=0); assertion (x1=x2)

⇒ fib^{-1} ?

Program Inversion $\mathcal{I}$

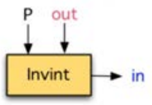
A Reversible Flowchart Diagram



[YokoyamaAxelsenGlück'08]

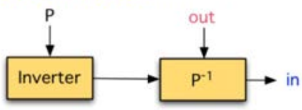
Two Approaches to Inversion of Program P in ⇒ out

- Inverse interpreter [McCarthy'56 ...]:



Applies to reversible & irreversible languages

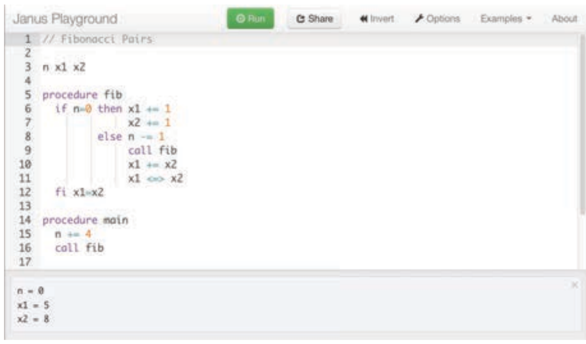
- Program inverter [Dijkstra'78, Gries'81...]:



- Related by Futamura projections [AbramovGlück'98]:

Janus Playground

http://topps.diku.dk/pirc/



Conclusion: Reversible Computing

Embraces the Theoretical, Practical, Technical, Applied.

- Computability, complexity, automata
- Abstract machines & computing models
- Languages design & implementation
  - Compilation, optimization, analysis, type systems
  - Program inversion, specialization, composition
  - Resource management: heap, parallelism
- Algorithms design
- Technical aspects
  - e.g. computer architectures
- Applied computing
  - e.g. robotics, rollback simulation, software engineering

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… and related references therein.