EIE3331 Communication Fundamentals Cluesheet

why modulation $\begin{cases} l_{antenna} > \frac{\lambda}{10} \\ \text{FDMA} \end{cases}$

Analog Modulation $\begin{cases} \text{AM} \\ \text{Angle M.} \begin{cases} \text{FM} \\ \text{PM} \end{cases} \end{cases}$

AWGN: $m(t) \rightarrow \oplus (n_w(t)) \rightarrow$ Low pass $[-W, W]$ $\rightarrow$ $S_o = S_r$, $N_o = \frac{\eta}{2} \times 2W = \eta W$

Fourier Transform

$\mathcal{F}$ / $\mathcal{F}^{-1}$ (inv.)

f: $G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi f t} dt$ $\quad g(t) = \int_{-\infty}^{\infty} G(f) e^{j2\pi f t} df$

ω: $G(\omega) = \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt$ $\quad g(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(\omega) e^{j\omega t} d\omega$

For: $x(t) = e^{j2\pi f_c t}$

$X(f) = \int_{-\infty}^{\infty} e^{j2\pi f_c t} e^{-j2\pi f t} dt = \int_{-\infty}^{\infty} e^{-j2\pi(f-f_c)} dt$

$= \delta(f - f_c)$ (unit pulse) (with only +ve real part)

For sine: $x(t) = \sin(\omega_c t) = \frac{e^{j\omega_c t} - e^{-j\omega_c t}}{2j}$

$X(f) = \frac{1}{2j}\delta(f-f_c) - \frac{1}{2j}\delta(f+f_c) = -\frac{j}{2}\delta(f-f_c) + \frac{j}{2}\delta(f+f_c)$

For cosine: $x(t) = \cos(\omega_c t) = \frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2}$

$X(f) = \frac{1}{2}\delta(f+f_c) + \frac{1}{2}\delta(f-f_c)$

$\xrightarrow{\text{conclude}}$ Freq. shifting (translation)

$x(t)e^{j2\pi f_0 t} \xrightarrow{\mathcal{F}} X(f-f_0)$, $\quad x(t)e^{j\omega_0 t} \xrightarrow{\mathcal{F}} X(\omega - \omega_0)$

$\Rightarrow \begin{cases} x(t)\sin\omega_0 t \leftrightarrow \frac{j}{2}(X(\omega+\omega_0) - X(\omega-\omega_0)) \\ x(t)\cos\omega_0 t \leftrightarrow \frac{1}{2}(X(\omega+\omega_0) + X(\omega-\omega_0)) \end{cases}$

$e^{j\beta\sin\omega_m t} \underset{(\mathcal{F}\text{series})}{=} \sum_{n=-\infty}^{n=\infty} C_n e^{jn\omega_m t} \rightarrow J_n(\beta)$, real num. $\begin{cases} \text{odd func. } (n = 2k+1) \\ \text{even func. } (n = 2k)\ k \in \mathbb{Z} \end{cases}$

Convolution Theorem: $f_1(x) \circledast f_2(x) = \int_{-\infty}^{\infty} f_1(x-\tau) f_2(\tau) d\tau = \int_{-\infty}^{\infty} f_1(\tau) f_2(x-\tau) d\tau$

$z(t) = x(t)y(t) \xrightarrow{\mathcal{F}} Z(f) = X(f) \circledast Y(f)$, $z(t) = x(t) \circledast y(t) \xrightarrow{\mathcal{F}} Z(f) = X(f)Y(f)$

Amplitude Modulation

= Amplitude deviation $(m(t))$ + Carrier. $\mu = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$ (<1 to avoid over modulation!)

① Conventional AM

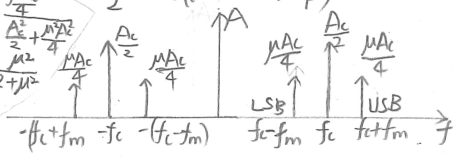
Single tone $[m(t) = A_m \cos \omega_m t]$: $\underbrace{kA_m}_{= k_a = \mu}$

$V(t) = A_c\cos(\omega_c t + \phi)[1 + kA_m \cos \omega_m t]$

$= A_c\cos(\omega_c t + \phi) + \frac{kA_mA_c}{2}\cos((\omega_c + \omega_m)t + \phi)$

$+ \frac{kA_mA_c}{2}\cos((\omega_c - \omega_m)t + \phi)$

$\eta_p = \frac{\frac{\mu^2 A_c^2}{4}}{\frac{A_c^2}{2} + \frac{\mu^2 A_c^2}{4}} = \frac{\mu^2}{2 + \mu^2}$



General:

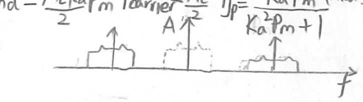
$V(t) = A_c(1 + k_a m(t))\cos(\omega_c t + \phi)$

$= A_c\cos(\omega_c t + \phi) + A_c k_a m(t)\cos(\omega_c t + \phi)$

$V(f) = \frac{A_c}{2}(\delta(f - f_c) + \delta(f + f_c)) +$

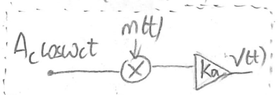
$\frac{A_c k_a}{2}(M(f + f_c) + M(f - f_c))$

$P_{sideband} = \frac{A_c^2 k_a^2}{2} P_m$ $\quad P_{carrier} = \frac{A_c^2}{2}$ $\quad \eta_p = \frac{k_a^2 P_m}{k_a^2 P_m + 1}$ (max $\frac{1}{3}$)



② Double Sideband-Suppressed Carrier

$V(t) = A_c\cos(\omega_c t) k_a m(t)$



$V(f) = \frac{A_c k_a}{2}(M(f + f_c) + M(f - f_c))$

$P_{sideband} = \frac{A_c^2 k_a^2}{2} P_m$

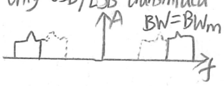
$\eta = 50\%$ LSB & USB carry info / 100%

③ Single Sideband

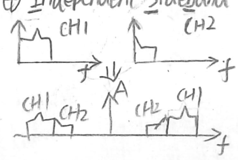
Based on DSB-SC

Only USB/LSB transmitted

BW = BW_m



④ Independent Sideband

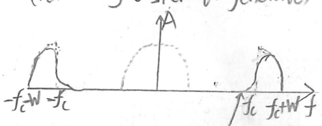


⑤ Vestigial Sideband 残留的

gradual transition from USB to LSB

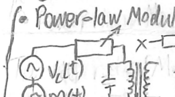
(process USB & LSB by a filter)

(relatively easier to generate)



To generate $V(t)$ (Modulator)

- Power-law Modulator (non-linear device) 幂律调制器



$x \rightarrow y = \alpha x + \beta x^2 + \gamma x^3 + \ldots \approx \alpha x + \beta x^2$

for $x = m(t) + V_c(t)$

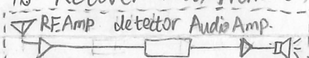
$y = \alpha(m(t) + V_c(t)) + \beta(m^2(t) + 2m(t)V_c(t) + V_c^2(t))$

$= \underbrace{\alpha m(t) + \beta m^2(t)}_{\text{low}} + \underbrace{2\beta m(t)V_c(t) + \alpha V_c t}_{\text{desired}} + \underbrace{\beta V_c^2(t)}_{\text{high}}$

- single-balanced modulator
- Ring modulator

} mainly for DSB-SC

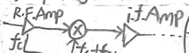
To Recover $m(t)$ From $V(t)$ (Demodulation)



$Q = \frac{f_0}{BW} \approx$ fixed const $\Rightarrow f_0 \propto BW$

(LC Resonant circuit / Acc. freq. Rej. noise) $\quad \Delta f_0 \rightarrow \Delta BW!$ $\begin{cases} \text{interference} \\ \text{noise} \end{cases}$

Sol.



(Superheterodyne Receiver 超外差式)

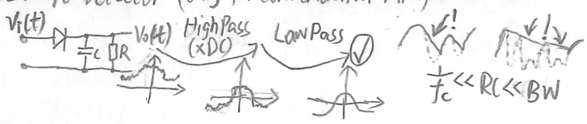
$A_c\cos(\omega_c t + \phi)[1 + m(t)]\cos(\omega_c + \omega_{if})t$

$= \frac{A_c}{2}[1 + m(t)][\underbrace{\cos((2\omega_c + \omega_{if})t + \phi)}_{\times \text{ low Pass}} + \cos(\omega_{if}t - \phi)]$

$\xrightarrow{\text{low pass}} \frac{A_c}{2}[1 + m(t)]\cos(\omega_{if}t - \phi)$ ✓

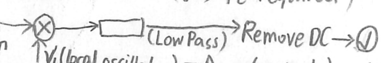
! Problem: image freq. at $f_c + 2f_{if}$

- Envelope Detector (only for conventional AM)



$\frac{1}{f_c} \ll RC \ll BW$

- Synchronous Detector (f_c, ϕ_c required)



V_l (local oscillator) $= A_l\cos(\omega_l t + \phi_l)$ with $\omega_l = \omega_c$, $\phi_l = \phi_c$

$A_c[1 + m(t)]\cos(\omega_c t + \phi_c) A_l\cos(\omega_l t + \phi_l)$

$= \frac{A_c A_l}{2}[1 + m(t)][\underbrace{\cos((\omega_c + \omega_l)t + \phi_c + \phi_l)}_{\times \text{ low Pass}} + \underbrace{\cos((\omega_c - \omega_l)t + \phi_c - \phi_l)}_{=1 \text{ if syn. else Phase/Freq error}}]$

Angle Modulation

= Carrier Signal + Angle (Phase) Deviation

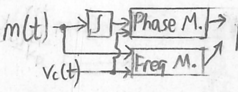
General: $V_o(t) = A_c\cos(\omega_c t + \phi_c + \psi(t))$

$= A_c \cdot \text{Re}[e^{j(\omega_c t + \phi_c)} e^{j\psi(t)}]$

Single tone: $V_o = A_c\cos(\omega_c t + \phi_c + k\cos(\omega_m t + \phi_m))$

$= A_c \cdot \text{Re}[e^{j(\omega_c t + \phi_c)} e^{jk\cos(\omega_m t + \phi_m)}]$

FM: $\psi(t) = k_f \int_{t_0}^{t} m(\tau) d\tau$, $\quad k_f = 2\pi f_d = \omega_d$



FM: $V_o(t) = A_c\cos(\omega_c t + \phi_c + 2\pi f_d \int_{t_0}^{t} m(\tau) d\tau)$

single tone: $V_o = A_c\cos(\omega_c t + \phi_c + 2\pi f_d \int_{t_0}^{t} A_m \cos(\omega_m \tau + \phi_m) d\tau)$

$= A_c\cos(\omega_c t + \phi_c + \underbrace{\frac{2\pi f_d A_m}{2\pi f_m}}_{\psi(t)_{max}} \sin(\omega_m t + \phi_m))$

$\psi(t)_{max} = D = \beta \rightarrow$ modulation index $\rightarrow$ deviation ratio

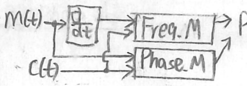
$\psi(t)_{max} = \left[\frac{2\pi f_d |m(t)|_{max}}{2\pi f_m(t)}\right]_{max}$

$= \frac{f_d |m(t)|_{max}}{BW} = \frac{\Delta f}{BW} = D = \beta$

PM: $\psi(t) = k_p m(t)$

$V_o(t) = A_c\cos(\omega_c t + \phi_c + k_p m(t))$

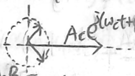
single tone: $V_o = A_c\cos(\omega_c t + \phi_c + k_p A_m \cos\omega_m t)$, $\beta = k_p A_m$



Narrowband Spectrum (when $|\psi(t)| \ll 1$) $\quad e^{j\psi(t)} \approx 1 + j\psi(t)$

$V_o(t) \approx A_c \text{Re}[e^{j(\omega_c t + \phi_c)}(1 + j\psi(t))] = A_c\cos(\omega_c t + \phi_c) - \psi(t)A_c\sin(\omega_c t + \phi_c)$

single tone: Phasor:

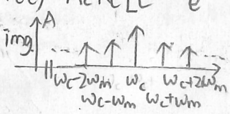


$A' \approx A_c$ $\quad \Delta\psi = \arcsin\left(\frac{A_c\beta\sin(\omega_m t + \phi_m)}{A'}\right) \approx \beta\sin(\omega_m t + \phi_m)$

$V_o(t) \approx A_c\cos(\omega_c t + \phi_c) + \frac{A_c\beta}{2}[\cos((\omega_c + \omega_m)t + \phi_c + \phi_m) - \cos((\omega_c - \omega_m)t + \phi_c - \phi_m)]$

Wideband Spectrum for single tone

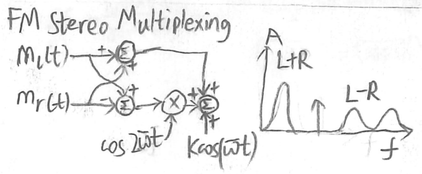
$V(t) = A_c \text{Re}[e^{j\omega_c t} e^{j\beta\sin(\omega_m t)}] = A_c \sum_{n=-\infty}^{n=\infty} J_n(\beta)\cos(\omega_c + n\omega_m)t$



$P_r = \frac{\frac{1}{2}A_c^2(J_0^2(\beta) + 2J_1^2(\beta) + 2\ldots)}{\frac{1}{2}A_c^2} = J_0^2(\beta) + 2\sum_{n=1}^{\infty} J_n^2(\beta)$ ($>98\%$)

Carson's Rule $k = \beta + 1$ (or $D + 1$)

FM Stereo Multiplexing

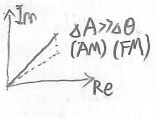


SNR AWGN

let $\left(\frac{S_o}{N_o}\right)_{Base} = \frac{S_r}{\eta W} = \gamma$

$\left(\frac{S_o}{N_o}\right)_{FM} = 3\beta^2\left(\frac{\overline{m^2(t)}}{m(t)^2_{max}}\right)\gamma$ $\quad \left(\frac{S_o}{N_o}\right)_{FM, st} = \frac{3\beta^2}{2}\gamma$

$\left(\frac{S_o}{N_o}\right)_{C\text{-}AM} = \eta_p\gamma$ $\quad \left(\frac{S_o}{N_o}\right)_{DSB\text{-}SC} = \gamma$

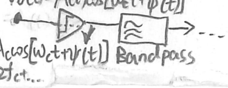


Distortion

Receiver $y(t) := \underset{!}{A(t)} 2\pi f_d m(t)$

Sol: Hard Limiter ($A_c(t)$ force to const, non-linear device)

$V_o(t) = A(t)\cos[\omega_c t + \psi(t)]$



To generate FM $v_0(t)$:

- Direct Generation: $v_0(t)=A_c\cos\left[2\pi f_c t + 2\pi f_d\int_{t_0}^{t} m(\tau)d\tau\right]$
  Voltage controlled Oscillator (VCO): $\Delta V_{in}\rightarrow\Delta f_0$ $\psi(t)$
- Indirect Generation:
  Generate narrowband $v(t)\approx A_c\cos(\omega_c t+\phi_c)-\psi(t)A_c\sin(\omega_c t+\phi_c)$
  Multiply freq. $(f_c\times N, f_d\times N)$ $2\psi(t)\rightarrow$ desired. But $f_c'>f_{c\,desired}$
  LO Adjust f_c $(f_c\pm f_{LO})$ minus f_c' by f_{LO} to $f_{c\,desired}$ (Low pass)

$m(t)\rightarrow\int\rightarrow 2\pi f_d$; $A_c\cos\omega_c t\rightarrow +90°\rightarrow\otimes\rightarrow\Sigma$ narrowband $\rightarrow$ freq. $\times N\rightarrow\otimes\leftarrow f_{LO}\rightarrow$ Low Pass
$f_0\pm f_{LO}$ $(f_0-f_{LO}=f_{c\,desired})$

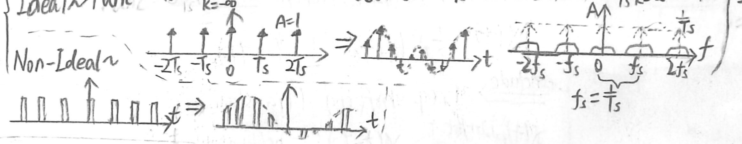
To Recover $m(t)$ From FM $v_0(t)$: $\left(\frac{d\psi(t)}{dt}\propto m(t)\right)$

- Frequency Discriminator: $y(t)=\frac{1}{2\pi}K_D\frac{d\psi(t)}{dt}\propto m(t)$ $[K_D]=V/Hz$
  $v_0(t)\rightarrow\boxed{\frac{d}{dt}}\rightarrow -A_c[\omega_c+\omega_d m(t)]\sin(\sim)\rightarrow$ Envelope Detector $\rightarrow$ Low Pass $\rightarrow -A_c\omega_d m(t)$
  should have $(\forall t)[\omega_c+\omega_d m(t)\geq 0]$ $(\Delta f<f_c)$
  $\boxed{\frac{d}{dt}}=\lim_{\tau\to 0}\frac{v_0(t)-v_0(t-\tau)}{\tau}$
  $v_0(t)\rightarrow\Sigma$, Delay τ $\approx\tau\frac{d\,v_0(t)}{dt}$
- RC Circuit/RLC Tuned Circuit + Envelope Detector
- Balanced Discriminator
- Phase-Locked Loop (PLL) let $\int_{t_0}^{t} m(\tau)d\tau=\theta_1$, $\int_{t_0}^{t} V(\tau)d\tau=\theta_2$
  $v_0(t)=\sin(\omega_c t+\theta_1)\rightarrow\otimes\rightarrow$ Low Pass $\rightarrow V(t)$; $r(t)$; $\cos(\omega_c t+\theta_2)$ VCO; $s(t)=v_0(t)r(t)$
  $\xRightarrow{\text{simplify}}$ $\theta_1(t)\rightarrow\otimes$, $s(t)=\sin(\theta_1-\theta_2)\approx\theta_1-\theta_2$, Low Pass $H(f)$, $\theta_2(t)$ VCO, $V(t)=\frac{d\theta_2}{dt}$ $\frac{d\theta_1}{dt}=m(t)$
  $[\theta_1(t)-\theta_2(t)]\circledast h(t)=\frac{d\theta_2}{dt}$ (?)
  $[\theta_1(f)-\theta_2(f)]H(f)=j2\pi f\theta_2(f)$
  $\theta_2(f)=\frac{H(f)}{H(f)+j2\pi f}\theta_1(f)\xrightarrow{H(f)\gg j2\pi f}\theta_1(f)$ (?)

ADC: Analog $\xrightarrow{\text{Sampling Quantize Encode}}$ Digital $\xrightarrow{\text{Modulation}}$

Sampling

- Ideal ~ Func: $\delta_{T_s}(t)=\sum_{k=-\infty}^{\infty}\delta(t-kT_s)$ $g_\delta(t)=g(t)\delta_{T_s}(t)$ $\mathcal{F}[\delta_{T_s}(t)]=\frac{1}{T_s}\sum_{k=-\infty}^{\infty}\delta(f-\frac{k}{T_s})$
- Non-Ideal ~



$\Rightarrow$ Sampling Theorem (by Nyquist): f_s should $\geq 2W_{signal}$, otherwise aliasing
Signal Recover From $g_s(t)$: Low Pass (Ideal vs. Practical) $\rightarrow$ over-sampling required!

Quantization

Func: $Q(x)=\hat{x}_i$ $(x\in R_i)$ (a Staircase Func)
Num of levels $L=2^R$ $\searrow$ Num of bits

- Non-Uniform: A-law $=\mathrm{sgn}(V_{in})\frac{A|V_{in}|}{1+\ln(A)}$, $0\leq V_{in}\leq\frac{1}{A}$; μ-law $=\mathrm{sgn}(V_{in})\frac{1+\ln(A|V_{in}|)}{1+\ln(A)}$, $\frac{1}{A}\leq V_{in}\leq 1$
- Uniform: step size $\Delta=\frac{2m_{max}}{2^R}=\frac{m_{max}}{2^{R-1}}$; $\hat{x}_i=\frac{1}{2}(m_i+m_{i+1})$

SQNR
Q: Quan. error $\sim U(-\frac{\Delta}{2},+\frac{\Delta}{2})$
$\sigma_Q^2=\frac{\Delta^2}{12}=\frac{m_{max}^2}{3\cdot 2^{2R}}$
$SNR_o=\frac{P}{\sigma_Q^2}=\frac{3P}{m_{max}^2}2^{2R}\Rightarrow SNR_o+6.02\text{dB}$ For each $R+1$

Coding: Natural Bin / Gray

Modulation

- Pulse Modulation: carry info based on pulse train
  - Pulse Code M: assign pulse shape
  - Pulse Amplitude M-M ($M=2^n$, or M-ary PAM) (Follow gray coding rule)
- Digital M.: carry info based on carrier wave
  - ASK A
  - FSK f
  - PSK ψ
  - QAM A,ψ

$BW=\frac{R f_s}{2}$ waveforms (line codes): 1 1 0 0 1
RZ: On-off RZ, Polar RZ, Bipolar RZ
NRZ: On-off NRZ, Polar NRZ

Pulse: basis of a symbol $p(t)$ overall signal $\sum_{n=-\infty}^{\infty}b_n p(t-nT_s)$

- Rectangular ~ ($-T_s/2$, $T_s/2$, V_m) $\xrightarrow{\mathcal{F}}$ ($\frac{1}{T_s}$, f_s) ! Inf. BW / Limit BW $\rightarrow$ Inter-Symbol Interference (original)
  Nyquist Criterion: No ISI at other sampling time
- Sinc ~ $p(t)=\frac{\sin(2B\pi t)}{2B\pi t}=\mathrm{Sinc}(2Bt)$ $T_s=\frac{1}{2B}$ ($-B$, B, f) $\therefore BW_{min}(2B\text{ symbol/s})=B$
  !Problem: inf time range, sensitive to jitter, decay slow (rate $=\frac{1}{t}$) sol. ⇓
- Raised cos ~ roll off factor $r=\frac{f_x}{D/2}=\frac{2f_x}{D}$ $B=\frac{D(1+r)}{2}$ $\rightarrow$ baud rate

Bin. Amplitude Shift Keying (On-Off K)

$m(t)=\sum_{n=-\infty}^{\infty}b_n p(t-nT_b)$ $v_0(t)=m(t)c(t)$
img. (f_c, f)

B Phase SK

simplify: BASK with $b_n=\pm 1$
$v_0(t)=A_c\cos(\omega_c t+\pi m(t))$
Demodulation (only syn.)
$v_0(t)\rightarrow\otimes\rightarrow$ Low Pass $\rightarrow m(t)A_c$; $A_c\cos(\omega_c t+\phi)$

Differential PSK

$v_0(t)=A_c\cos(\omega_c t+\phi(t))$
with $\phi(t)=\pi[m(t)\oplus\frac{\phi(t-T_b)}{\pi}]$
Data $\rightarrow\oplus$, Delay T_b
Recover $m(t)=m'(t)\oplus m'(t-1)$

M-ary

ASK $c_m(t)=A_m c(t)$
PSK $c_m(t)=A_c\cos(2\pi f_c t+\theta_m)$
FSK $c_m(t)=A_c\cos(2\pi f_{cm}t+\phi_{cm})$
General: $c_m(t)=\mathrm{Re}[\underbrace{A_m e^{j\theta_m}}_{S_m}A_c e^{j\omega_c t}]$ (PSK, ASK)

B Freq. SK

$c_0(t)=A_c\cos(2\pi f_{c0}t+\phi_{c0})$
$c_1(t)=A_c\cos(2\pi f_{c1}t+\phi_{c1})$
$m(t)=b(t)c_0(t)+\bar{b}(t)c_1(t)$
Phase discont.! $\xrightarrow{\text{sol}}$ Cont. Ph. FSK
img. (f_{c0} f_{c1}, f)
Carson's Rule: $B=2(\Delta f+W)$ $\Delta f=\frac{|f_0-f_1|}{2}$ $\rightarrow$ Large: big BW; Small: error detection

Demodulation
non-coherent: $v_0(t)\rightarrow H_0(f)\rightarrow$ Envelope, $H_1(f)\rightarrow$ Envelope $\rightarrow$ Compare (Discrete sample every T_b sec.)
coherent: $v_0(t)\rightarrow\otimes\, c_0(t)\rightarrow\int\rightarrow$ Low Pass; $\otimes\, c_1(t)\rightarrow\int\rightarrow$ Low Pass $\rightarrow$ Compare

$\int c_1(t)c_0(t)\approx 0$?
$\int c_0(t)c_0(t)\rightarrow\surd$
$\int c_1(t)c_1(t)\rightarrow\surd$
$\int c_0(t)c_1(t)\approx 0$?

c_0, c_1 should be orthogonal: $\int_0^{T_s}c_0(t)c_1(t)=0$
$\Rightarrow\frac{A_c^2}{2}\int_0^{T_s}[\underbrace{\cos(2\pi(f_0+f_1)t+\phi_0+\phi_1)}_{\times\text{ low pass}}+\cos(2\pi(f_0-f_1)t+\phi_0-\phi_1)]dt=0$
$\Rightarrow\frac{A_c^2\sin(2\pi(f_0-f_1)T_s+\phi_0-\phi_1)}{4\pi(f_0-f_1)}-\frac{A_c^2\sin(\phi_0-\phi_1)}{4\pi(f_0-f_1)}=0$

non-coherent: $\phi_0\neq\phi_1$
$\Rightarrow\sin(2\pi(f_0-f_1)T_s+\phi_0-\phi_1)=\sin(\phi_0-\phi_1)$ $(f_0\neq f_1)$
$\Rightarrow 2\pi(f_0-f_1)T_s=2\pi k$
$\Rightarrow f_0-f_1=\frac{k}{T_s}$

coherent: $\phi_0-\phi_1$ precompensate to 0
$\Rightarrow\sin(2\pi(f_0-f_1)T_s)=0$ $(f_0\neq f_1)$
$\Rightarrow 2\pi(f_0-f_1)T_s=\pi k$
$\Rightarrow f_0-f_1=\frac{k}{2T_s}$

Quaduature AM 正交调幅

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$S_m=A_m e^{j\theta_m}$
$\mathrm{Re}(S_m)\rightarrow\otimes$, $c(t)$, $-90°$, $\mathrm{Im}(S_m)\rightarrow\otimes\rightarrow\Sigma$ (Modulation)
$\rightarrow\otimes\rightarrow$ Low Pass $\rightarrow\frac{A_c}{2}\mathrm{Re}[S_m]$; $c(t)$, $+90°$; $\otimes\rightarrow$ Low Pass $\rightarrow\frac{A_c}{2}\mathrm{Im}[S_m]$ (Demodulation)

AWGN for digital transmission:

let $W\sim N(0,\sigma^2)$ $\sigma^2=N_0B$ $V_{th}=A$
$|W(f)|$, $\frac{N_0}{2}$, $-B$, B, f
$P_e=b_{1}P_1(V_{th})+b_0P_0(V_{th})\xrightarrow{\text{simplify}}P_0(V_{th})$
let $\mathrm{erf}(x)=\frac{2}{\sqrt{\pi}}\int_x^{\infty}e^{-x^2}dx$ $(\sim\mathrm{abs}[N(0,\frac{1}{\sqrt{2}})])$
$\mathrm{erfc}(x)=1-\mathrm{erf}(x)$
$P_e=\frac{1}{2}\mathrm{erfc}\left(\frac{A}{\sqrt{2}\sigma}\right)$ $\left(z=\frac{y+A}{\sqrt{2}\sigma}\right)$
$Q(x)=\frac{1}{2}\mathrm{erfc}\left(\frac{x}{\sqrt{2}}\right)$ $\left(x=\frac{A}{\sigma}\right)$ $\sqrt{SNR}$
$\therefore SNR\uparrow, Q(x)\downarrow, P_e\downarrow$

SNR at Receiver filter

$\eta_{(SNR)}=\frac{|x_o(T)|^2}{E[n^2(t)]}$ $|x_o(T)|^2=\left|\int_{-\infty}^{\infty}X(f)H(f)e^{j2\pi fT}df\right|^2$
$x(t)=bg(t)_{(p(t))}\xrightarrow{\mathcal{F}}X(f)=bG(f)$
$b=\pm 1\Rightarrow|x_o(T)|^2=\left|\int_{-\infty}^{\infty}G(f)H(f)e^{j2\pi fT}df\right|^2$
$\underset{\text{(Cauchy-Schwarz)}}{\leq}\left(\int_{-\infty}^{\infty}|H(f)|^2df\right)\left(\int_{-\infty}^{\infty}|G(f)|^2df\right)$ $H_{opt}(f)=kG^*(f)e^{-j2\pi fT}$
$E[n^2(t)]=\sigma_o^2=\int_{-\infty}^{\infty}|W(f)H(f)|^2df=\frac{N_0}{2}\int_{-\infty}^{\infty}|H(f)|^2df$
$\rightarrow\eta_{max}=\frac{\left(\int_{-\infty}^{\infty}|H(f)|^2df\right)\left(\int_{-\infty}^{\infty}|G(f)|^2df\right)}{\frac{N_0}{2}\left(\int_{-\infty}^{\infty}|H(f)|^2df\right)}=\frac{2\int_{-\infty}^{\infty}|G(f)|^2df}{N_0}$
condition: $H_{opt}(f)=kG^*(f)e^{-j2\pi fT}$

Matched Filter:

$\xrightarrow{\mathcal{F}^{-1}}h_{opt}(t)=k\int_{-\infty}^{\infty}G^*(f)e^{-j2\pi f(T-t)}df$, $G^*(f)\xlongequal{g(t)\text{ is real}}G(-f)$
$\searrow=kg(T-t)$ (matched with input pulse)
$E_b=\int_{-\infty}^{\infty}g^2(t)dt=\int_{-\infty}^{\infty}|G(f)|^2df$
$|x_o(T)|=|k|E_b$
$=\frac{|k|^2N_0}{2}\int_{-\infty}^{\infty}|G(f)|^2df=\frac{|k|^2N_0E_b}{2}\Rightarrow\eta_{max}=\frac{2(|k|E_b)^2}{|k|^2N_0E_b}=\frac{2E_b}{N_0}$
$\Rightarrow P_e^{opt}=Q(\sqrt{SNR})=Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$