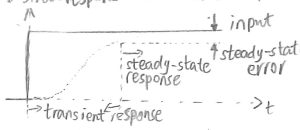
EIE3123 Dynamics Systems 控制理论 Cluesheet

```
>> pkg load control
>> roots([an, an-1, ..., a0])  # solve anx^n+...+a0=0
```

Simp Ctrl System

Input/stimulus → $\boxed{\text{Ctrl Sys}}$ → Output/response

Desired response → Actual response



Transfer Function

$X(s) \to \boxed{G(s)} \to Y(s)$ `>> G1 = tf([an_nume ... a0], [bn_denom, ..., b0])`

$b_n y^{(n)}(t) + \dots + b_0 y(t) = a_m x^{(m)}(t) + \dots + a_0 x(t)$

$\frac{C(s)}{R(s)}\left(\frac{Y(s)}{X(s)}\right) = G(s) = \frac{a_m s^m + \dots + a_0}{b_n s^n + \dots + b_0}$ (assume init = 0) $= K\frac{\prod_{i=1}^m (s-Z_i)}{\prod_{i=1}^n (s-P_i)}$ (normally $m<n$; $n-m$ zeros at inf)

Z_i: finite zeros, $H(s)|_{s=Z_i}=0$

P_i: finite poles, $H(s)|_{s=P_i}=\infty$

real pole: $\mathcal{L}^{-1}\left(\frac{A}{s-\sigma_i}\right) = Ae^{\sigma_i}u(t)$

cmplx pole: $\mathcal{L}^{-1}\left(\frac{As+B}{s^2-2\sigma_i s+(\sigma_i^2+w_i^2)}\right) = \mathcal{L}^{-1}\left(\frac{A(s-\sigma_i)+A\sigma_i+B}{(s-\sigma_i)^2+w_i^2}\right)$

$= u(t)e^{\sigma_i t}\left[A\cos(w_i t) + \left(\frac{A\sigma_i+B}{w_i}\right)\sin(w_i t)\right]$

```
>> G1 = zpk([z1, z2...], [P1, P2...], k)   # k: gain
>> step(G1)        >> t = 0:0.1:10
>> feedback(G1)    >> [y, t] = step(G1, t)
```

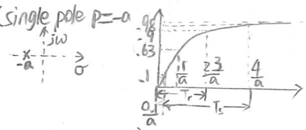
Laplace transform

Definition	$\mathcal{L}[f(t)] = F(s) = \int_{0^-}^{\infty} f(t)e^{-st}dt$
Linearity	$\mathcal{L}[k_1f_1(t)+k_2f_2(t)] = k_1F_1(s)+k_2F_2(s)$
Freq Shift	$\mathcal{L}[e^{\mp at}f(t)] = F(s\pm a)$
Time Shift	$\mathcal{L}[f(t-T)] = e^{-sT}F(s)$
Time Scaling	$\mathcal{L}[f(at)] = \frac{1}{a}F(\frac{s}{a})$
Differenciation	$\mathcal{L}[f^{(2)}(t)] = s^2F(s) - sf(0) - f^{(1)}(0^-)$
	$\mathcal{L}[f^{(n)}(t)] = s^nF(s) - \sum_{k=1}^{n} s^{n-k}f^{(k-1)}(0^-)$
Integration	$\mathcal{L}[\int_0^t f(\tau)d\tau] = \frac{F(s)}{s}$
Final/Initial Value	$f(\infty) = \lim_{s\to 0} sF(s)$ $f(0^+) = \lim_{s\to\infty} sF(s)$

$f(t) \leftrightarrow$	$F(s)$	$f(t) \leftrightarrow$	$F(s)$
$\delta(t)$	1	$e^{-at}u(t)$	$\frac{1}{s+a}$
$u(t)$	$1/s$	$\sin(wt)u(t)$	$\frac{w}{s^2+w^2}$
$t^n u(t)$	$n!/s^{n+1}$	$\cos(wt)u(t)$	$\frac{s}{s^2+w^2}$
$(t\sin(wt))u(t)$	$\frac{2ws}{(s^2+w^2)^2}$	$e^{-bt}\sin(wt)u(t)$	$\frac{w}{(s+b)^2+w^2}$
$(t\cos(wt))u(t)$	$\frac{s^2-w^2}{(s^2+w^2)^2}$	$e^{-bt}\cos(wt)u(t)$	$\frac{s+b}{(s+b)^2+w^2}$

1st order sys (single pole $p=-a$)

$R(s)\,(X(s)) \to \boxed{G(s) = \frac{a}{s+a}} \to C(s)\,(Y(s))$



$\tau = \frac{1}{a}$ = time constant

$a = \frac{1}{\tau}$ = exp freq

$T_r = 2.2/a$ = rise time

$T_s = 4/a$ = settling time

2nd order sys ($P_{1,2} = -\zeta w_n \pm w_n\sqrt{\zeta^2-1}$)

$G(s) = \frac{A}{s^2+2\zeta w_n s+w_n^2}$

w_n: natural freq without damping

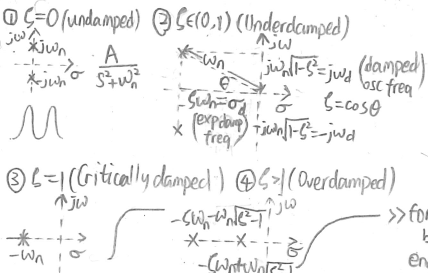
$\zeta = \frac{f_{exp\ decay}}{w_n} = \frac{T_{natural}}{2\pi a} \to a = \zeta w_n$

① $\zeta = 0$ (undamped) $\frac{A}{s^2+w_n^2}$

② $\zeta \in (0,1)$ (Underdamped): $jw_n\sqrt{1-\zeta^2} = jw_d$ (damped osc freq), $-\zeta w_n = \sigma_d$ (exp damp freq), $\zeta = \cos\theta$, $-jw_n\sqrt{1-\zeta^2} = -jw_d$

③ $\zeta = 1$ (Critically damped) $-w_n$

④ $\zeta > 1$ (Overdamped) $-\zeta w_n - w_n\sqrt{\zeta^2-1}$, $-\zeta w_n + w_n\sqrt{\zeta^2-1}$



$c(s) = \frac{w_n^2}{(s^2+2\zeta w_n s+w_n^2)s} = \frac{1}{s} - \frac{(s+\zeta w_n)+\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right)w_n\sqrt{1-\zeta^2}}{(s+\zeta w_n)^2+w_n^2(1-\zeta^2)}$

$c(t) = u(t)\left[1 - \frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta w_n t}\cos(w_n\sqrt{1-\zeta^2}\,t - \phi)\right]$

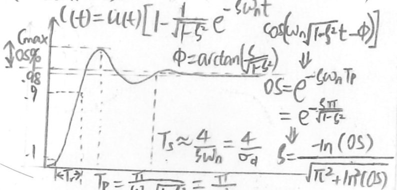
$\phi = \arctan\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right)$

$OS = e^{-\zeta w_n T_p} = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$

$\zeta = \frac{-\ln(OS)}{\sqrt{\pi^2+\ln^2(OS)}}$

$T_s \approx \frac{4}{\zeta w_n} = \frac{4}{\sigma_d}$

$T_p = \frac{\pi}{w_n\sqrt{1-\zeta^2}} = \frac{\pi}{w_d}$




```
>> for i=3:10     >> settle = .02*y(end)  >> [ymax, idx] = max(y)
   body           >> idx = find(abs(y-y(end))>settle) ... tp = t(idx)
   endfor            <(.02*y(end))) (1)
                  >> text(t(idx), y(idx), '')
```

$R(s) \to \bigoplus \xrightarrow{E(s)} \boxed{G(s)} \to C(s)$, feedback $\boxed{H(s)}$

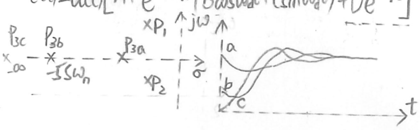
$\frac{C(s)}{R(s)} = \frac{G(s)}{1+H(s)G(s)}$

$\Phi_e = \frac{E(s)}{R(s)} = \frac{1}{1+H(s)G(s)}$

High order sys approx. (2nd order sys with additional poles)

3-pole sys with $P_3 = -\alpha_r$

$c(t) = u(t)\left[A + e^{-\zeta w_n t}(B\cos w_d t + C\sin w_d t) + De^{-\alpha_r t}\right]$



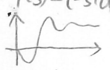
Effect of zero: amp of a response

LHP Z_i: $T(s) = (s+a)C(s) = sC(s) + aC(s)$

large a: $T(s) \approx aC(s)$ (scaled)

small a: $sC(s)$ introduce more OS

RHP Z_i: $T(s) = (-s+a)C(s) = -sC(s) + aC(s)$



initially moves to opposite direction

P-Z cancellation

Routh Stability Criterion

S^n	a_0	a_2	a_4
S^{n-1}	a_1	a_3	a_5
S^{n-2}	b_1	$b_2 = \frac{-\begin{vmatrix} a_0 & a_4 \\ a_1 & a_5\end{vmatrix}}{a_1}$	
...			

sign change: RHP pole

Sp. case I

s^5	1	2	6
s^4	3	6	9
s^3	0	3	
new s^3	$\varepsilon \to 0^+$	3	
...			

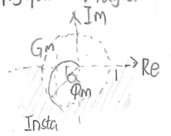
Sp. case II

s^5	1	3	2
s^4	2	6	4
s^3	0	0	
new s^3	8	12	
...			

$(2s^4+6s^2+4)' = 8s^3+12s$

?→A: ?→B: While 1: ?→C: ?→D: -(AD-BC)÷B◢ WhileEnd

Nyquist Diagram



$R(s) \to \bigoplus \to \boxed{G(s)} \to C(s)$

$G(s) = \frac{w_n^2}{s(s+2\zeta w_n)}$

$\Rightarrow T(s) = \frac{C(s)}{R(s)} = \frac{w_n^2}{s^2+2\zeta w_n s+w_n^2}$

let $|G(jw)| = 1$

$\Rightarrow \begin{cases} w_{0dB} = w_n\sqrt{-2\zeta^2+\sqrt{1+4\zeta^4}} \\ \angle G(jw_{0dB}) = -90 - \tan^{-1}\frac{w_{0dB}}{2\zeta w_n}\end{cases}$

$\Phi_m = 90 - \tan^{-1}\left(\frac{w_{0dB}}{2\zeta w_n}\right) = \tan^{-1}\left(\frac{2\zeta w_n}{w_{0dB}}\right)$

$= \tan^{-1}\left(\frac{2\zeta}{\sqrt{-2\zeta^2+\sqrt{1+4\zeta^4}}}\right)$

$\Rightarrow w_{BW} = w_n\sqrt{(1-2\zeta^2)+\sqrt{4\zeta^4-4\zeta^2+2}}$ (3dB) (feedback) $= \frac{4}{T_s\zeta}\sqrt{\sim} = \frac{\pi}{T_p\sqrt{1-\zeta^2}}\sqrt{\sim}$

$w_{0dB} \approx 0.5\,w_{BW}$

`>> margin(G1)` (open loop) # closed loop bode plot `>> bode(G1)`

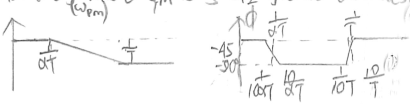
Lag Compensation

$G_c(s) = \frac{1}{\alpha}\left(\frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}\right)$ $(\alpha > 1)$

find the f ($w_{\Phi m}$) where ϕ_m is 5°~12° greater than desired

find $|M(w_{\Phi m})| = \alpha$

$w_{\Phi m} \approx 0.8\,w_{BW}$



Lead Compensation

$G_c(s) = \frac{1}{\beta}\left(\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}\right)$ $(\beta < 1)$ where $w_{max} = \frac{1}{T\sqrt{\beta}}$ $\phi_{max} = \sin^{-1}\left(\frac{1-\beta}{1+\beta}\right)$

closed loop BW → gain K → additional $\phi_{max\,lead}$ (+10°) (⇒β)

→ $|G_c(jw_{max})|$ → break freq, → reset K

$|G_c(jw_{max})| = \frac{1}{\sqrt{\beta}}$ $(\frac{1}{\beta} = \alpha = \gamma)$

combine (lead-lag): $G_c(s) = G_{lead}(s)\,G_{lag}(s) = \left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\gamma T_2}}\right)$

Digital Control & Z-transform

$G_{zoh}(s) = \frac{1}{s} - \frac{e^{-sT}}{s} = \frac{1-e^{-sT}}{s} \xrightarrow{Z} \frac{(1-z^{-1})}{z(s)}$

⎍ (zero order hold 零阶保持器)

Sampling $f^*(t) = \sum_{k=0}^{\infty} f(kT)\delta(t-kT)$

$\mathcal{L}\{f^*(t)\} = \sum_{k=0}^{\infty} f(kT)e^{-skT}$

$\xrightarrow{z=e^{sT}} Z\{f(t)\} = F(z) = \sum_{k=0}^{\infty} f(kT)z^{-k}$

```
>> Gz = c2d(G1, T, 'zoh')
>> rlocus(Gz)     >> hold # to plot
```

$Z(u(t)) = \frac{z}{z-1}$

• stability

$Z = e^{sT} = e^{(\sigma+jw)T} = e^{\sigma T}\angle wT$

• e_{ss} $\Phi_e = \frac{E(z)}{R(z)} = \frac{1}{1+G(z)H(z)}$

$e^*(\infty) = \lim_{z\to 1}(1-z^{-1})\frac{R(z)}{1+G(z)H(z)}$

• $T_s \approx \frac{4}{\zeta w_n} = \frac{4}{-\sigma}$ $(\Rightarrow \sigma = -\frac{4}{T_s})$

$z = e^{sT} = e^{(\sigma+jw)T} = e^{-\frac{4T}{T_s}}\angle wT$ → same T_s

• $T_P = \frac{\pi}{w}$ $(\Rightarrow w = \frac{\pi}{T_P})$

$Z = e^{sT} = e^{(\sigma+jw)T} = e^{\sigma T}\angle\left(\frac{\pi T}{T_P}\right)$ → same T_P

• ζ: $\frac{\sigma}{w} = \frac{-\zeta w_n}{w_n\sqrt{1-\zeta^2}} = -\frac{\zeta}{\sqrt{1-\zeta^2}}$

$\Rightarrow s = \sigma + jw = -w\frac{\zeta}{\sqrt{1-\zeta^2}} + jw$

$\Rightarrow z = e^{\frac{-wT\zeta}{\sqrt{1-\zeta^2}}}e^{jwT}$

```
>> theta = linspace(0, pi, 1000)
>> DF = ζ/sqrt(1-ζ^2)
>> x = exp(-theta*DF).*cos(theta)
>> y = exp(-theta*DF).*sin(theta)
>> plot(x, y)
```

Distrete Equiv

Tustin transformation

$s = \frac{2}{T}\left(\frac{z-1}{z+1}\right)$ (t)

`>> Gz = c2d(G, T, 'tustin')`

Matched P-Z (MPZ) ($z = e^{sT}$)

$\frac{U(s)}{E(s)} = K_c\frac{s+a}{s+b} \Rightarrow \frac{U(z)}{E(z)} = K_d\frac{z-e^{-aT}}{z-e^{-bT}}$

⇓ $s=0$ or $z=1$

$K_c\frac{a}{b} = K_d\frac{1-e^{-aT}}{1-e^{-bT}}$ $\left(K_d = K_c\frac{a}{b}\frac{(1-e^{-bT})}{1-e^{-aT}}\right)$

`>> Gz = c2d(G, T, 'matched')`