RF Fundamentals

Signals

Outline

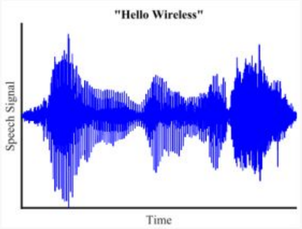
- Analog & Digital Signals
- Complex Numbers
- Frequency Domain Representation
- Fast Fourier Transform - FFT
- FFT Leakage
- GNU Radio Introduction
- Q&A

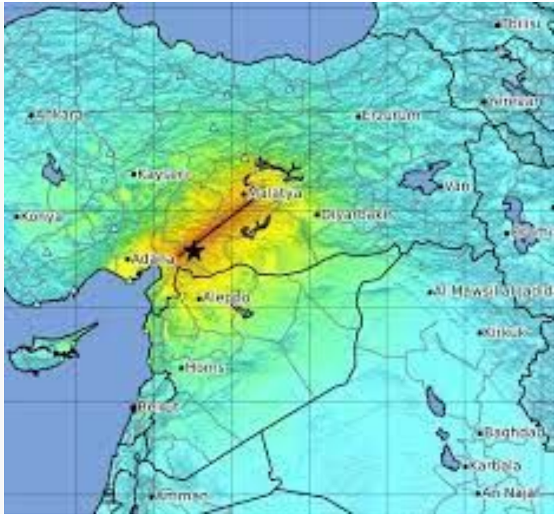
What is a signal?

A signal is any measurable quantity that varies with time

It carries or conveys information

- Speech
- GPS
- ECG
- Stock prices
- Earthquake

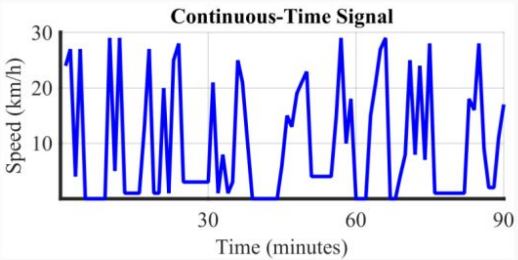




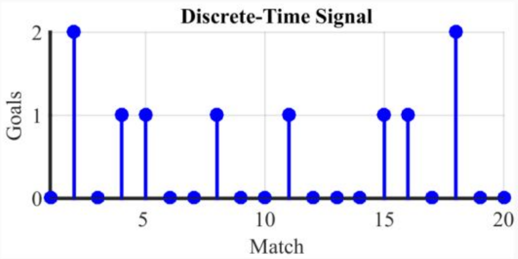
Continuous-Time vs Discrete-Time

- Continuous
  - Defined at every point
- Discrete
  - Only defined at discrete points in time

Player speed

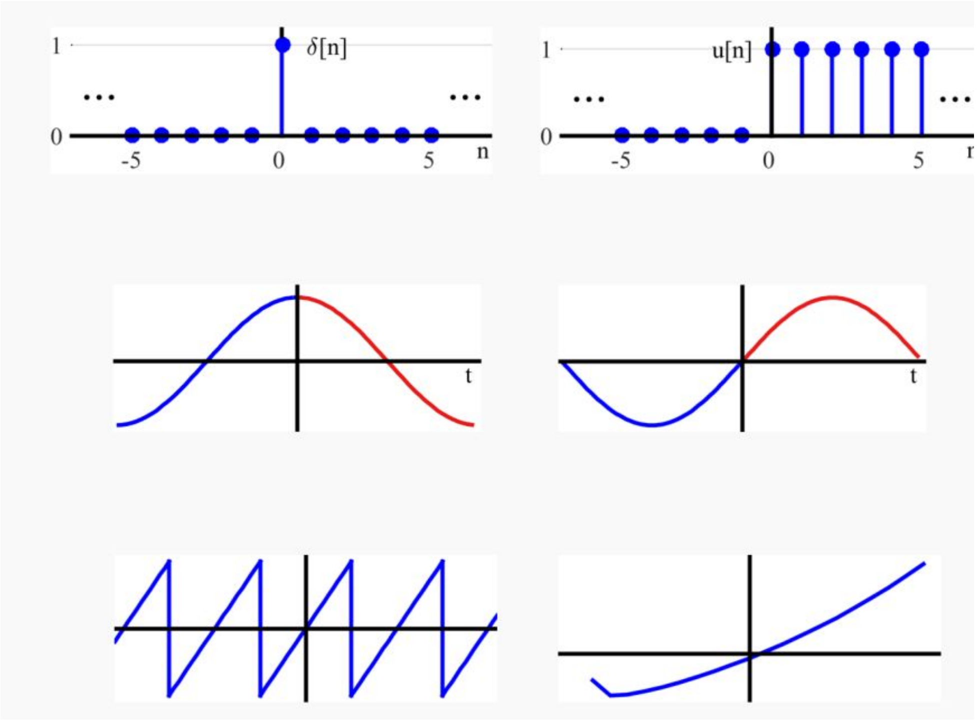


Player goals

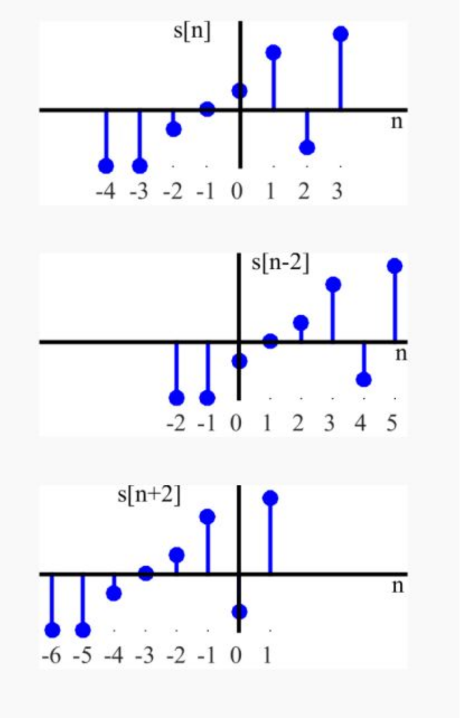


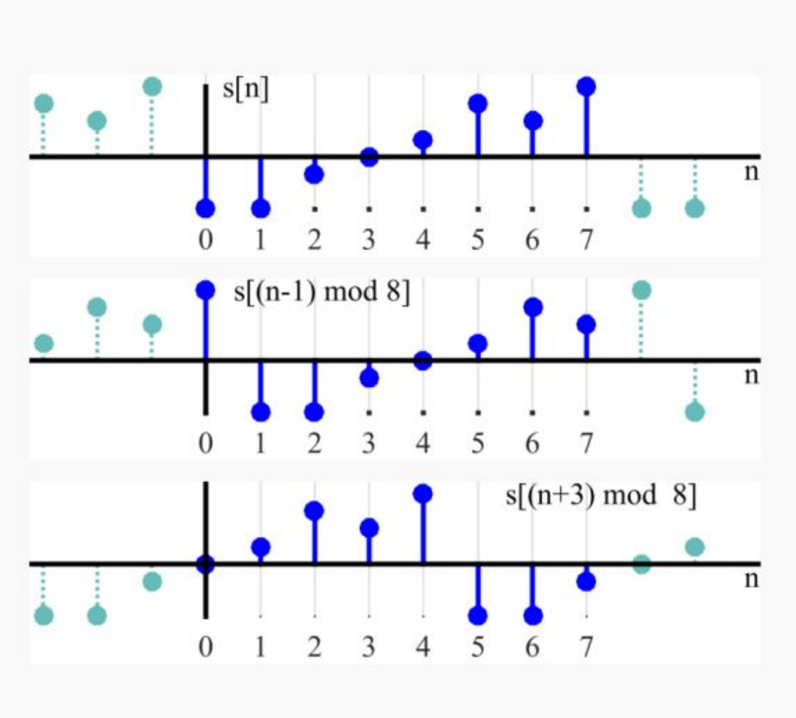
Basic Signals

- Unit impulse
- Unit step
- Even/Odd
- Periodic/Nonperiodic



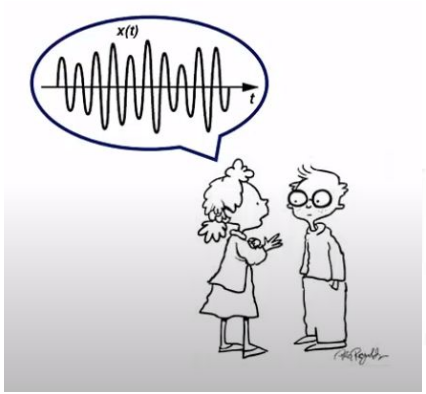
Shift in Time



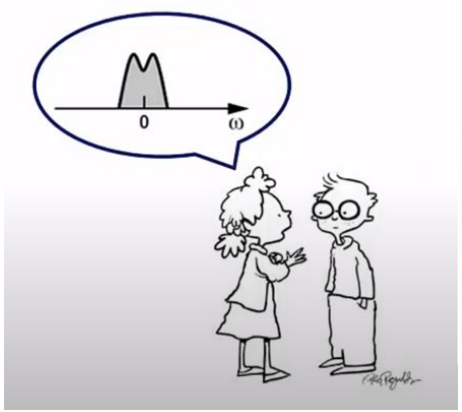


Time vs Frequency Domain

- The real world happens in the time domain

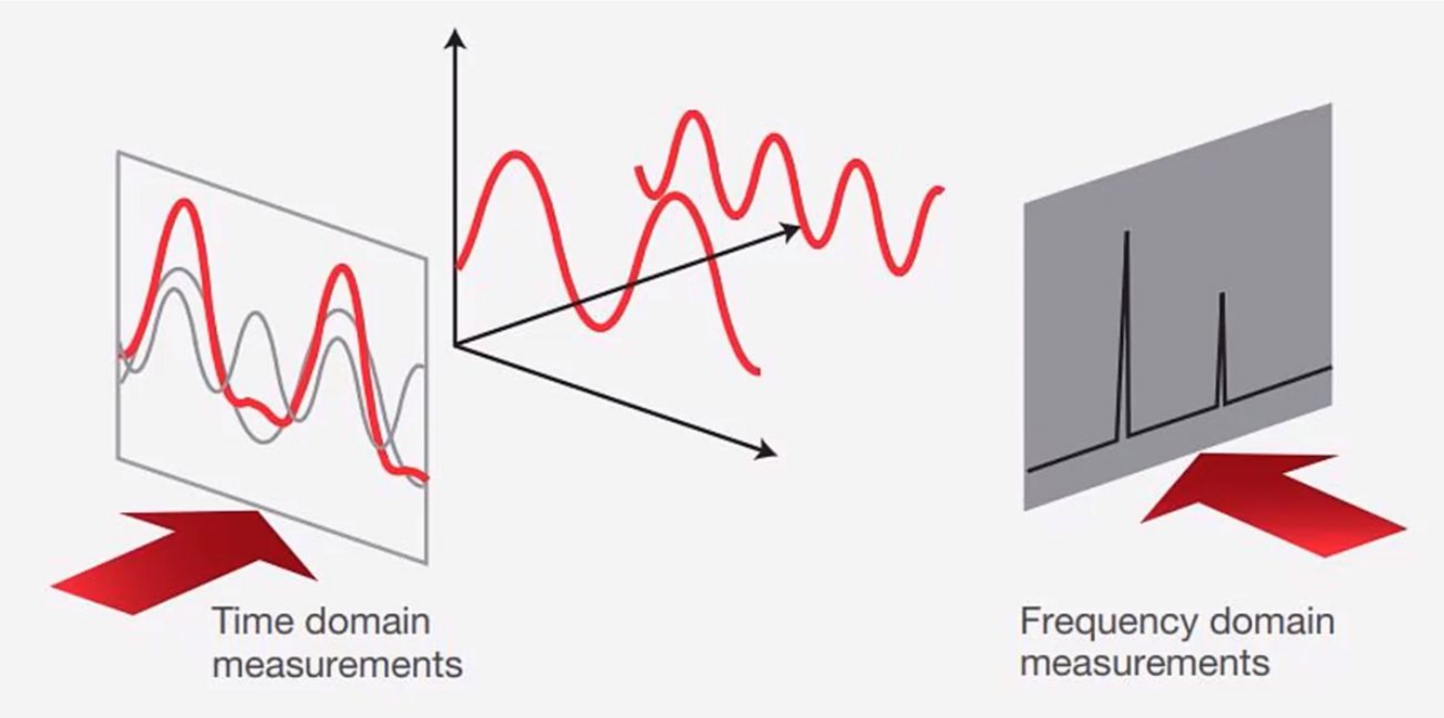


- Signals can be represented by frequency components



$$x(t) \xLeftrightarrow{\text{Fourier}} X(\omega)$$

Time vs Frequency Measurements

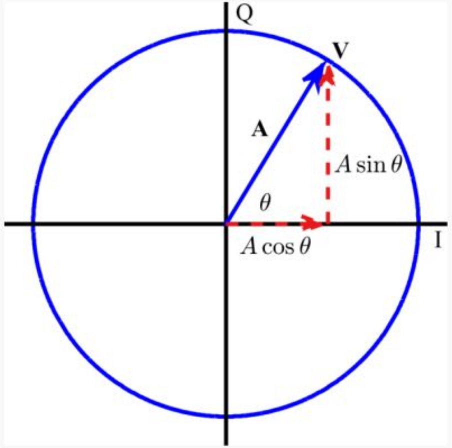


Complex Numbers

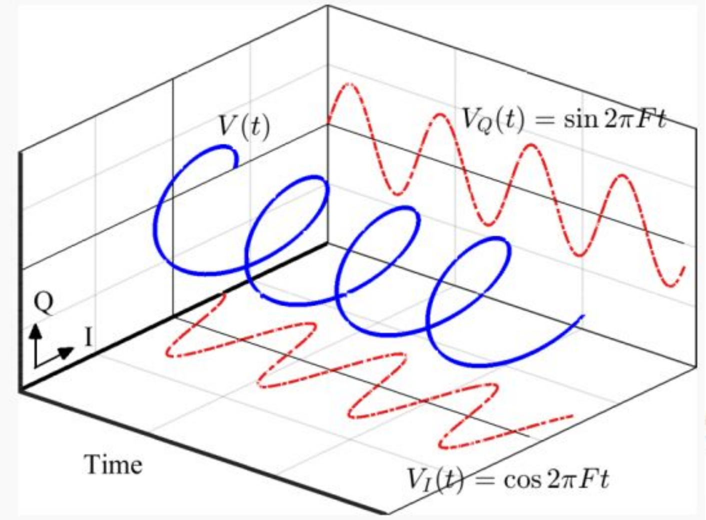
- Pair of real numbers
- I and Q parts
- Magnitude
- Phase

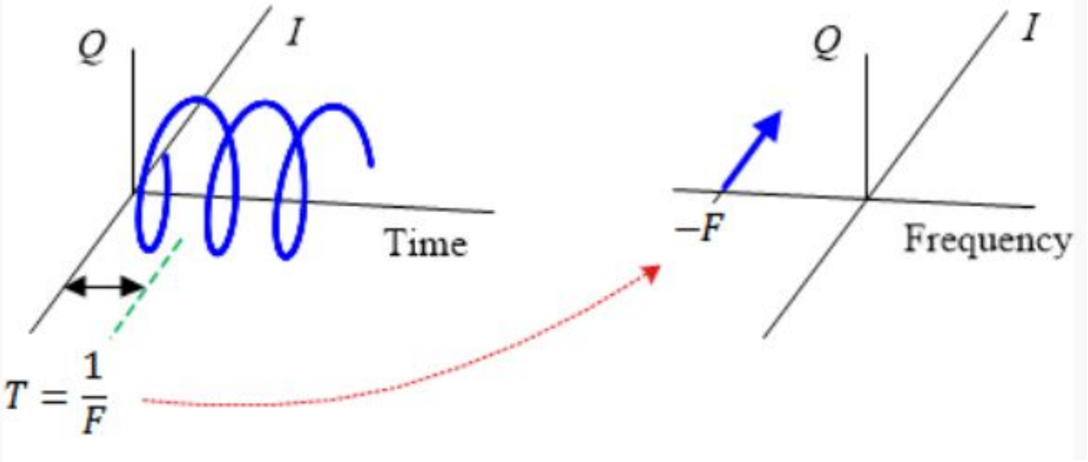
$$V_I = |V| \cos \measuredangle V$$
$$V_Q = |V| \sin \measuredangle V$$

$$|V| = \sqrt{V_I^2 + V_Q^2}$$

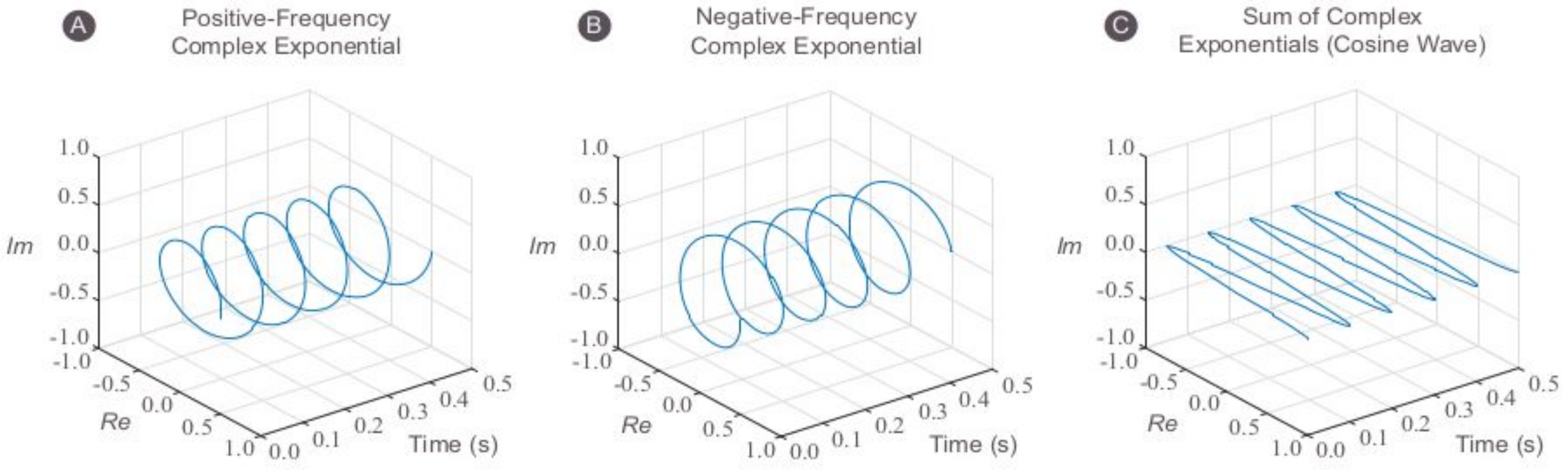


Complex Sinusoid



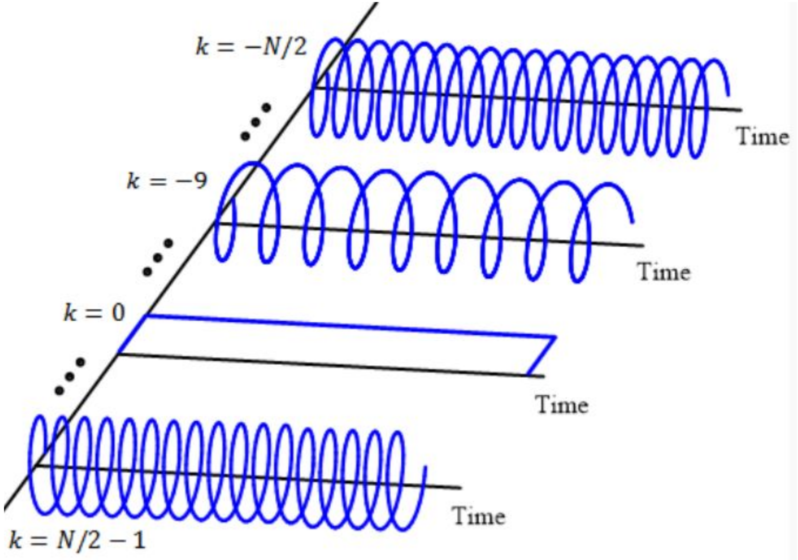


Positive/Negative Frequencies



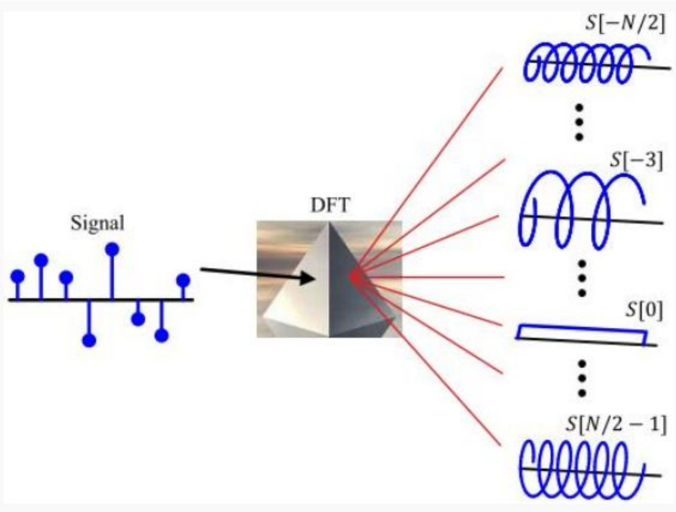
Set of Complex Sinusoids

k cycles per NTs seconds



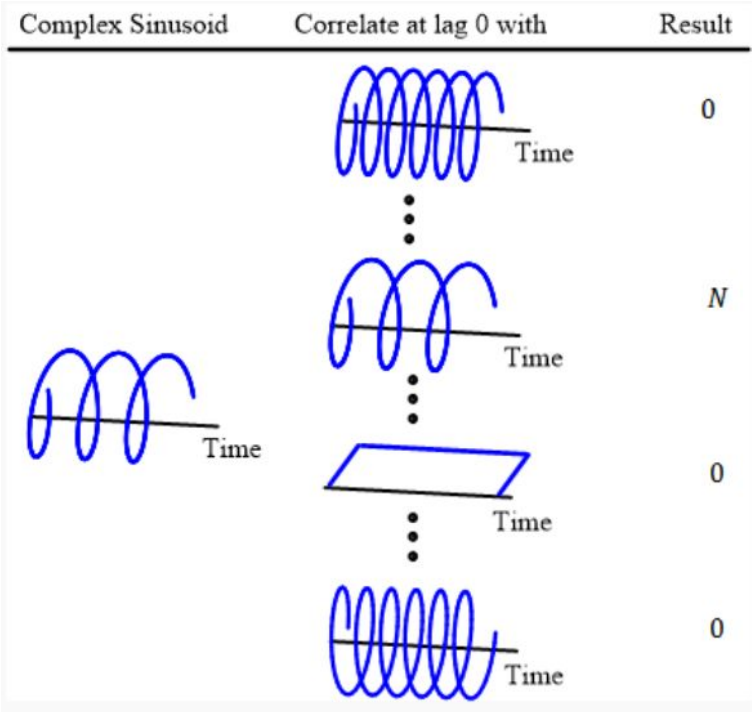
Discrete Fourier Transform

- **DFT** finds **amplitude** and **phase** contributions in a signal from each of the N discrete-time complex sinusoids
- These reference sinusoids are called **analysis frequencies**



Orthogonality

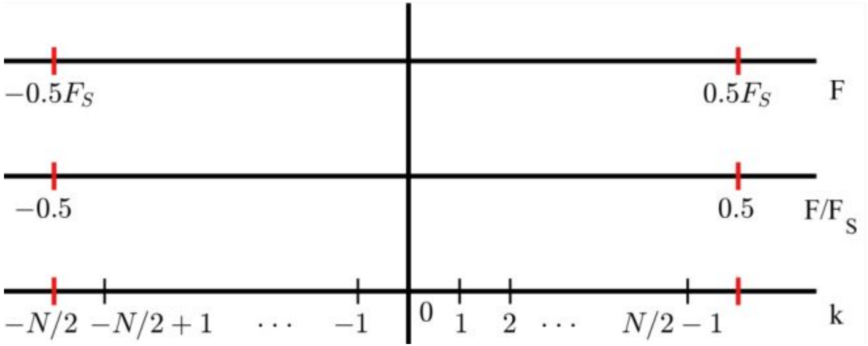
- Orthogonality is the basis for OFDM



Discrete Frequencies

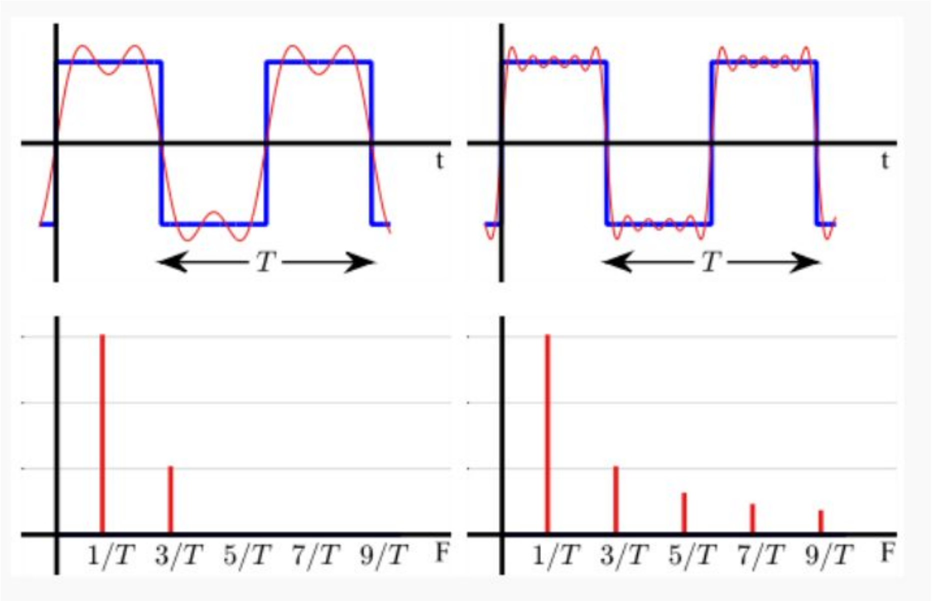
Suppose Fs = 100 and N = 10

- k=0 corresponds to 0Hz
- k=1 corresponds to 10Hz
- k=-2 corresponds to -20Hz



Making Up a Signal

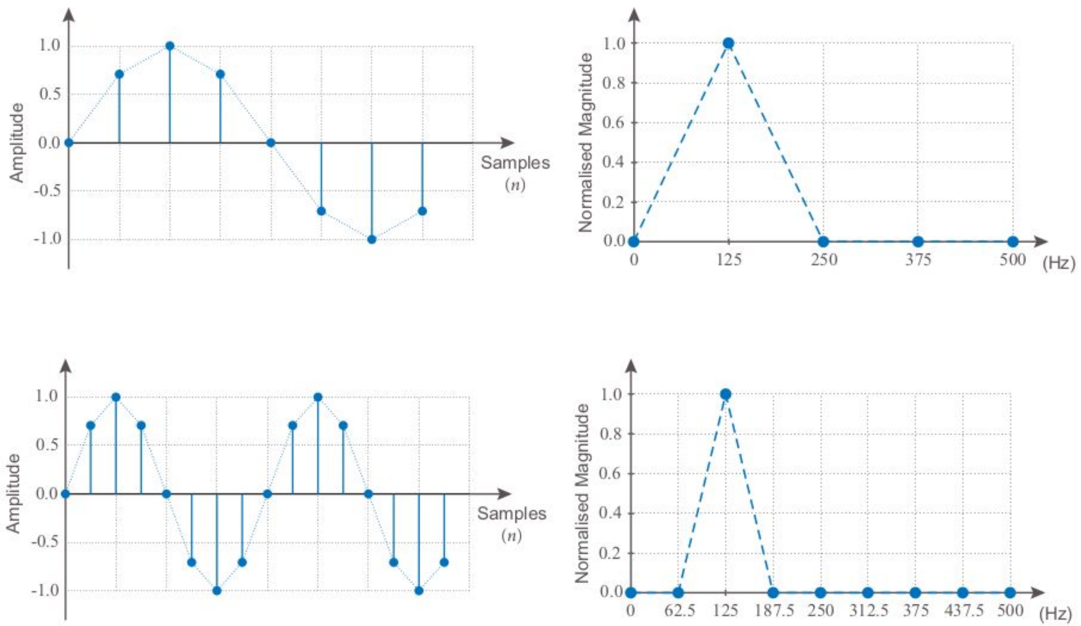
- Every signal is composed of sinusoids with different frequencies
- A better approximation is achieved with more sinusoids



Spectral Analysis

- **DFT** can be used to obtain the frequency representation of discrete-time waveforms
- **FFT** is not an approximation of the DFT; rather, it is the DFT and is effective when reducing computational complexity. We established that the FFT technique could only be used with DFT sizes that are a power of two.

125Hz sine wave sampled at 1kHz



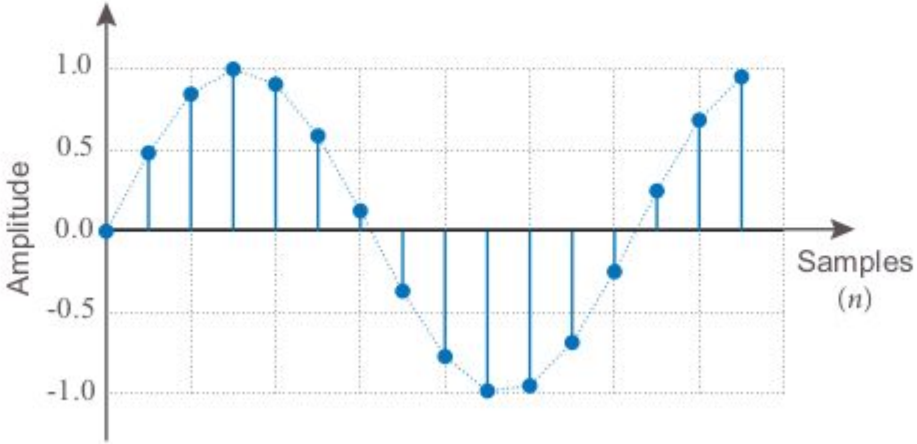
N = 8

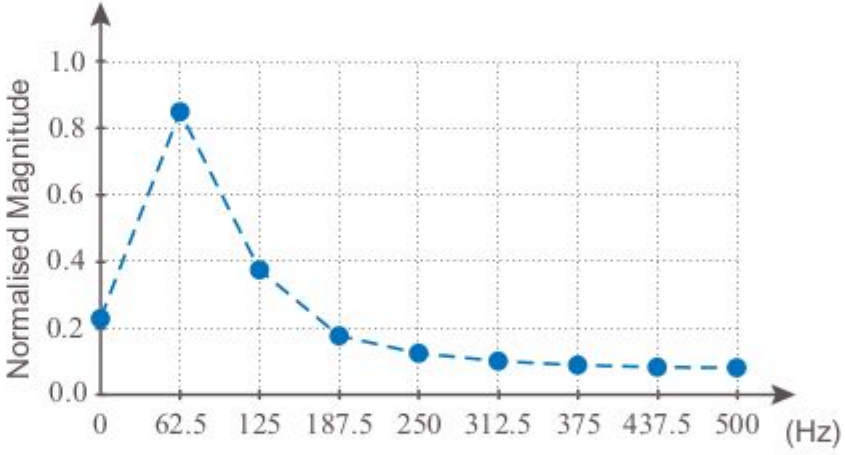
$$\Delta f = \frac{f_s}{N}$$

N = 16

Spectral Leakage

Now have a look at a discrete sine wave with a frequency of 80Hz sampled at 1kHz.

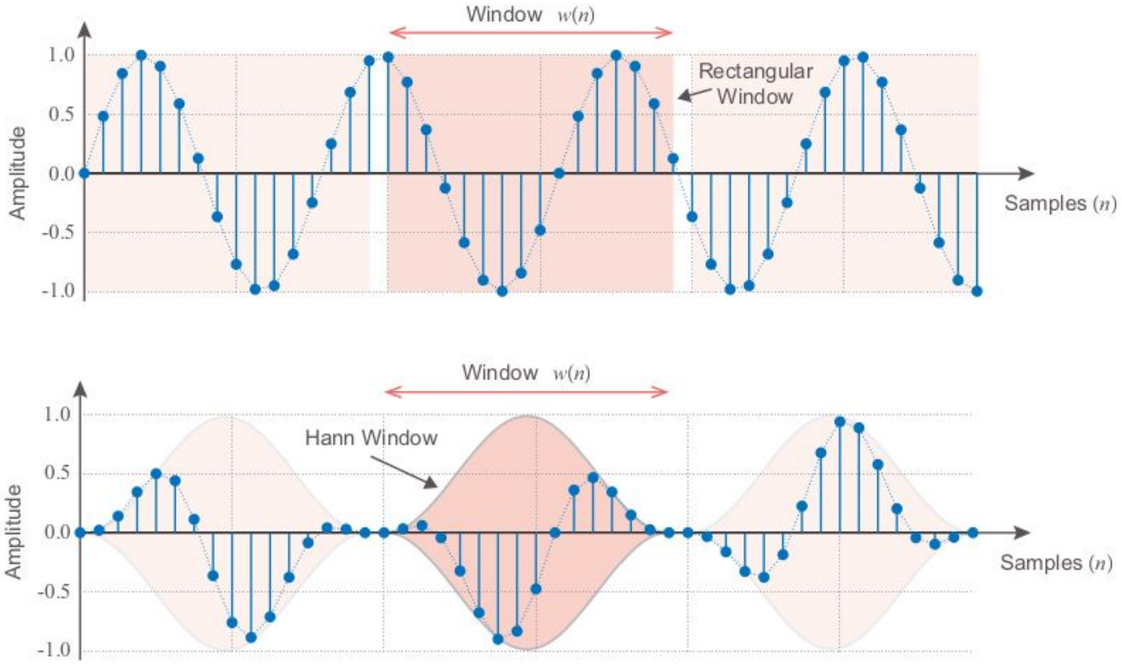




Windowing

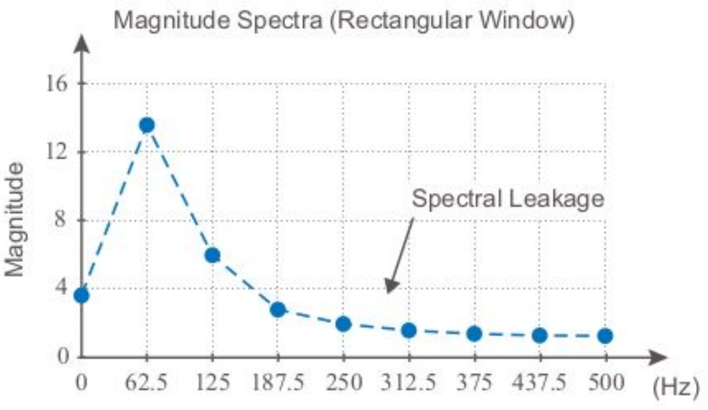
- We can reduce the effect of spectral leakage by applying particular windows to a discrete waveform before using the DFT
    - Hamming,
    - Hann,
    - Blackman-Harris and
    - Bartlett.

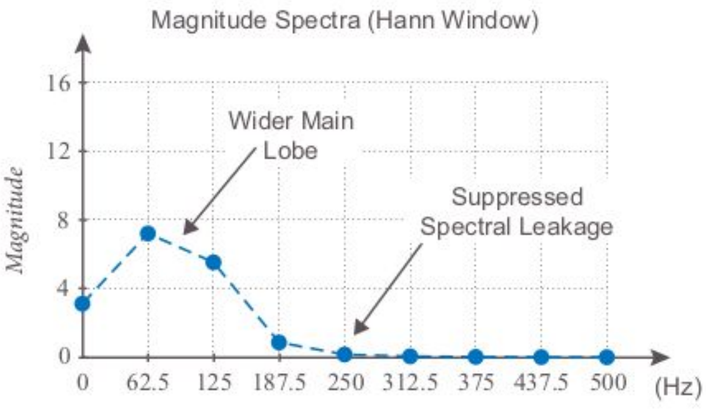
A Hann window applied to a discrete sine wave of 80Hz



A Hann window applied to a discrete sine wave of 80Hz

- Tapered windows can reduce spectral leakage in the DFT.
- However, there are some caveats.
    - Windowing has the effect of widening the main lobe of the peak frequency.
    - However, the side lobes that cause spectral leakage are reduced.

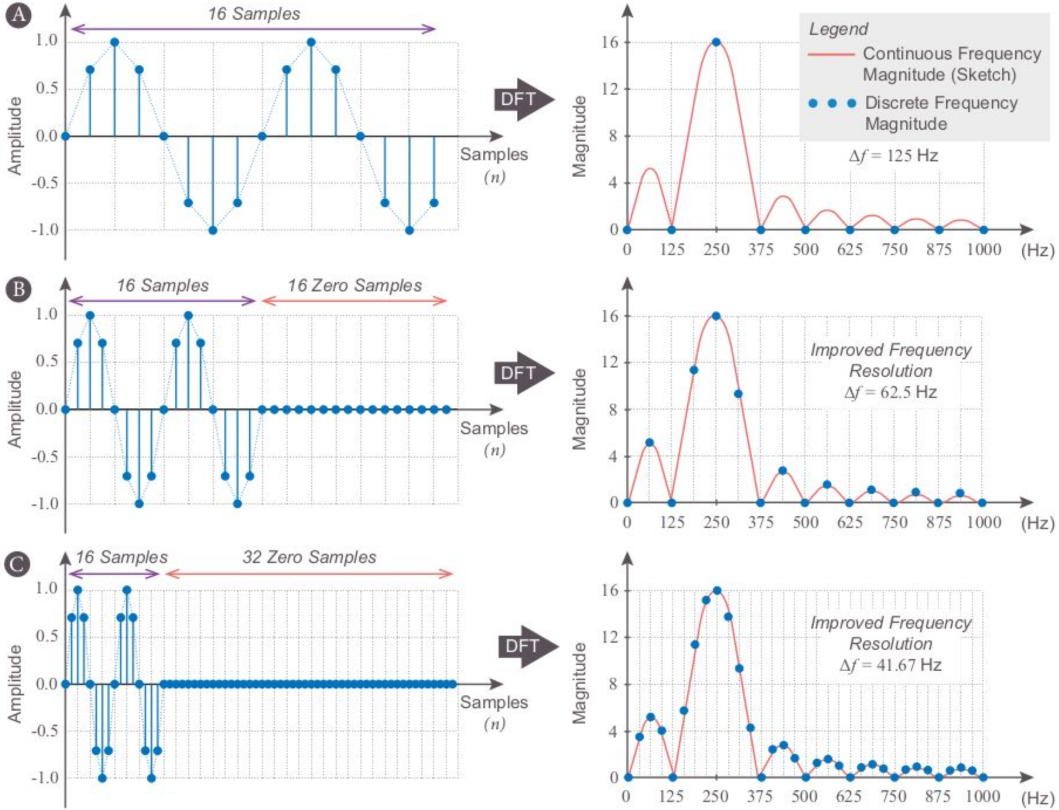




Zero Padding

- Zero padding is a technique that involves inserting zero-valued samples at the end of a discrete waveform to improve the frequency resolution of the DFT plot.
- The effect of zero padding is essentially an interpolation of the frequency sample points in the DFT and as such no extra ‘information’ is created on the signal

250 Hz Sine wave sampled at 2k Hz



Squarewave