23.04.2020 $\underline{13}$

№2.83

$\rho = 1$ г/л
$p = 101325$ Па
$\langle v \rangle$ - ? $v_{кв.}$ - ?

$$\langle v \rangle = \sqrt{\frac{8kT}{\pi m}} \quad v_{вер} = \sqrt{\frac{2kT}{m}} \quad v_{кв.} = \sqrt{\frac{3kT}{m}}$$

$$pV = \nu RT \quad pV = \frac{m}{M} RT = \frac{\rho V}{M} RT$$

$$p = \frac{\rho}{M} RT \quad \frac{RT}{M} = \frac{p}{\rho}$$

$$\langle v \rangle = \sqrt{\frac{8kT}{\pi m}} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8p}{\pi \rho}} = 2\sqrt{\frac{2p}{\pi \rho}} = 508 \text{ м/с}$$

$$v_{вер} = \sqrt{\frac{2kT}{m}} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2p}{\rho}} = 450 \text{ м/с}$$

$$v_{кв} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3p}{\rho}} = 551 \text{ м/с}$$

Ответ: $v_{вер} = 450$ м/с
$\langle v \rangle = 508$ м/с
$v_{кв} = 551$ м/с

№2.84

$\delta\eta = 1\%$
$F(v)$ - ?

$$F(u) = (4/\sqrt{\pi})\, u^2 \cdot e^{-u^2} \quad u = \frac{v}{v_{вер}}$$

Т.к. скорость отличается не более чем на 1% то наиболее вероятной, то:

a) $P\left(\frac{|v - v_{вер}|}{v_{вер}} < \delta\eta\right) = \int\limits_{1-\delta\eta}^{1+\delta\eta} dF(u) = \cancel{F(u)\Big|_{1-\delta\eta}^{1+\delta\eta}} =$

$|u-1| < \delta\eta$

$= \cancel{\frac{4}{\sqrt{\pi}} e^{-(1+\delta\eta)^2} \cdot (1+\delta\eta)^2 - \frac{4}{\sqrt{\pi}} e^{-(1-\delta\eta)^2} (1-\delta\eta)^2} =$

$u < 1+\delta\eta$

$u > 1-\delta\eta$

$= \int\limits_{1-\delta\eta}^{1+\delta\eta} \frac{4}{\sqrt{\pi}} u^2 \cdot e^{-u^2} du = \frac{4}{\sqrt{\pi}} \int\limits_{1-\delta\eta}^{1+\delta\eta} u^2 \cdot e^{-u^2} du =$

Теорема о среднем значении:

$$\int\limits_a^b f(x)dx = (b-a) f(c)$$

$1 + \delta\eta - 1 + \delta\eta = 2\delta\eta$

$c = 1 \qquad f(c) = 1^2 \cdot e^{-1} = e^{-1}$

$= \frac{4}{\sqrt{\pi}} \cdot 2\delta\eta \cdot e^{-1} = \frac{8\delta\eta}{\sqrt{\pi} \cdot e} = \frac{8 \cdot 0{,}01}{\sqrt{3{,}14} \cdot 2{,}7} = \frac{0{,}08}{4{,}78} = 0{,}0167$

б) $P\left(\frac{|v - v_{кв}|}{v_{кв}} < \delta\eta\right) = P(|v - v_{кв}| < \delta\eta \cdot v_{кв}) = P\left(\frac{|v - v_{кв}|}{v_{вер}} < \delta\eta \frac{v_{кв}}{v_{вер}}\right) =$

$= P\left(\left|\frac{v}{v_{вер}} - \frac{v_{кв}}{v_{вер}}\right| < \delta\eta \frac{v_{кв}}{v_{вер}}\right) = P\left(\left|u - \frac{v_{кв}}{v_{вер}}\right| < \delta\eta \frac{v_{кв}}{v_{вер}}\right) =$

$\frac{v_{кв}}{v_{вер}} = \frac{\sqrt{\frac{3RT}{M}}}{\sqrt{\frac{2RT}{M}}} = \sqrt{\frac{3}{2}}$

$= P\left(\left|u - \sqrt{\frac{3}{2}}\right| < \delta\eta \sqrt{\frac{3}{2}}\right) =$

$P\left(u < \sqrt{\frac{3}{2}}(1+\delta\eta)\right) \qquad P\left(u > \sqrt{\frac{3}{2}}(1-\delta\eta)\right)$

$= \int\limits_{\sqrt{\frac{3}{2}}(1-\delta\eta)}^{\sqrt{\frac{3}{2}}(1+\delta\eta)} dF(u) = \frac{4}{\sqrt{\pi}} \int\limits_{\sqrt{\frac{3}{2}}(1-\delta\eta)}^{\sqrt{\frac{3}{2}}(1+\delta\eta)} u^2 \cdot e^{-u^2} du =$

$\sqrt{\frac{3}{2}}(1+\delta\eta) - \sqrt{\frac{3}{2}}(1-\delta\eta) =$

$= \sqrt{\frac{3}{2}}(1+\delta\eta-1+\delta\eta) =$

$= \sqrt{\frac{3}{2}} \cdot 2\delta\eta = \sqrt{6}\,\delta\eta$

$c = \sqrt{\frac{3}{2}} \qquad f(c) = \left(\sqrt{\frac{3}{2}}\right)^2 \cdot e^{-\frac{3}{2}} = \frac{3e^{-3/2}}{2}$

$-$

$$= \frac{4}{\sqrt{\pi}} \sqrt{6}\,\delta y \cdot \frac{3}{2} e^{-3/2} = \frac{6\sqrt{6}}{\sqrt{\pi}}\,\delta y \cdot e^{-3/2} =$$

$$= \frac{6\sqrt{6}}{\sqrt{3,14}} \cdot 0,01 \cdot 2,7^{-3/2} = \frac{0,06\sqrt{6}}{1,77 \cdot \sqrt{2,7}} =$$

$$= \frac{0,0829}{4,4366} = 0,0187$$

Ответ: а) 0,0167 б) 0,0187

№285

T – ?
а) $v_{кв} = v_{вер} + 400 м/с$
б) $F(v)$ – max при $v = 420 м/с$

а) $$\sqrt{\frac{3RT}{M}} = \sqrt{\frac{2RT}{M}} + \Delta v$$

$$\sqrt{\frac{RT}{M}}\left(\sqrt{3} - \sqrt{2}\right) = \Delta v$$

$$\sqrt{T} = \frac{\Delta v \sqrt{M}}{\left(\sqrt{3} - \sqrt{2}\right)\sqrt{R}}$$

$$T = \frac{\Delta v^2 M}{\left(\sqrt{3} - \sqrt{2}\right)^2 R} = \frac{M \Delta v^2}{R\left(5 - 2\sqrt{6}\right)} = \frac{2 \cdot 400 \cdot 400 \cdot 10^{-3}}{8,314 \cdot 0,1} = \frac{320}{0,8314} = 384,89 =$$

$$= 385 \text{ (K)}$$

б) $$v = \sqrt{\frac{2RT}{M}} \qquad v^2 = \frac{2RT}{M} \qquad v_{вер} = v$$

$$T = \frac{Mv^2}{2R} = \frac{32 \cdot 10^{-3} \cdot 420 \cdot 420}{2 \cdot 8,314} = \frac{5644,8}{16,628} = 339,48 = 339 \text{ (K)}$$

Ответ: а) 385 K б) 339 K