34, 45, 47, 49, 50

ТЕРМОДИНАМИКА

$pV = \frac{m}{M}RT$

$p = p_0 e^{-Mgh/RT}$

$\left(p + \frac{a}{V_M^2}\right)(V_M - b) = RT$

$\gamma = 1 + \frac{R}{C_V} = \frac{C_P}{C_V}$

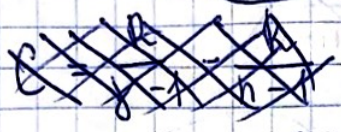
18.04.2020

№ 35

$C_V = ?$ $C_P = ?$

$m_1 = 7\,г$

$m_2 = 20\,г$



$C_V = \frac{R}{\gamma - 1}$

$U = U_1 + U_2$ $\quad (\nu_1 + \nu_2) C_V T = \nu_1 C_{V_1} T + \nu_2 C_{V_2} T$

$(\nu_1 + \nu_2) C_V = \nu_1 C_{V_1} + \nu_2 C_{V_2}$

$C_V = \frac{\nu_1 C_{V_1} + \nu_2 C_{V_2}}{\nu_1 + \nu_2}$

$C_V = \frac{\nu_1 \frac{R}{\gamma_1 - 1} + \nu_2 \frac{R}{\gamma_2 - 1}}{\nu_1 + \nu_2} =$

Табл. значения:

$\gamma_1 = 1{,}4$ $\quad \mu_1 = 28\,г/моль$

$\gamma_2 = 1{,}67$ $\quad \mu_2 = 40\,г/моль$

$= \frac{\mu_1 \mu_2 R \left(\frac{m_1}{\mu_1(\gamma_1 - 1)} + \frac{m_2}{\mu_2(\gamma_2 - 1)}\right)}{m_1 \mu_2 + m_2 \mu_1} =$

$= \frac{7 \cdot 40 \cdot 8{,}314 \left(\frac{7}{28(1{,}4-1)} + \frac{20}{40(1{,}67-1)}\right)}{7 \cdot 40 + 20 \cdot 28} =$

$= \frac{28 \cdot 40 \cdot 8{,}314 \left(\frac{7}{28 \cdot (1{,}4-1)} + \frac{20}{40(1{,}67-1)}\right)}{7 \cdot 40 + 20 \cdot 28} =$

$= \frac{16 \cdot 8{,}314 (0{,}625 + 0{,}746)}{12} = 15{,}2 \left(\frac{Дж}{моль \cdot К}\right)$

$C_P = \gamma = 1 + \frac{R}{C_V} = \frac{C_P}{C_V} \Rightarrow C_P = C_V + R$

$C_P = 23{,}514 \left(\frac{Дж}{моль \cdot К}\right)$

$\mu_{смеси} = \frac{m_1 + m_2}{\nu_1 + \nu_2} = \frac{m_1 + m_2}{\frac{m_1}{\mu_1} + \frac{m_2}{\mu_2}} = \frac{7 + 20}{\frac{7}{28} + \frac{20}{40}} = \frac{27}{0{,}25 + 0{,}5} = 36\,(г/моль)$

$C_V = \frac{C_V}{\mu} = \frac{15{,}2}{36} = 0{,}42 \left(\frac{Дж}{г \cdot К}\right)$ $\quad C_P = \frac{C_P}{\mu} = \frac{23{,}514}{36} = 0{,}65 \left(\frac{Дж}{г \cdot К}\right)$

№2.45

$pV^n = const$

γ, n - ?

$C < 0$

$A = Q - \Delta U = C_n \Delta T - C_V \Delta T = (C_n - C_V)\Delta T$

$C_n - C_V = -\frac{\nu R}{n-1}$

$A = \frac{\nu R (T_1 - T_2)}{n-1}$

Т.к. процесс политропический, то $A = -\Delta U$

$\Delta U = \frac{\nu R (T_2 - T_1)}{n-1}$

$pV^n = const$

Т.к. газ идеальный: $pV = \nu R T$ $\quad p = \frac{\nu R T}{V}$

$\frac{\nu R T}{V} V^n = const$ $\quad \nu R T V^{n-1} = const \Rightarrow$

$\Rightarrow T V^{n-1} = const$

Продиффер. TV^{n-1}:

$dT \cdot V^{n-1} + (n-1) V^{n-2} dV \cdot T = 0$

$\frac{dV}{dT} = \frac{V^{n-1}}{(1-n) V^{n-2} \cdot T} = \frac{V}{T(1-n)} =$

$= \frac{\nu R}{p(1-n)}$

Для $\nu = 1$ моль: $\frac{dV}{dT} = \frac{R}{p(1-n)}$

$C_n = C_V + p\frac{dV}{dT} = C_V + \frac{R}{1-n} = \frac{R}{\gamma - 1} - \frac{R}{n-1} \Rightarrow$

$\Rightarrow C_n = const$

$C_n = \frac{R(n-1-\gamma+1)}{(\gamma-1)(n-1)} = \frac{R(n-\gamma)}{(\gamma-1)(n-1)}$

$C < 0$ при $1 < n < \gamma$

№ 47

$\nu = 1$ моль; $M = 40$ г/моль

$\gamma = 1{,}67$; $n = 1{,}5$

$\Delta T = -26\,K = T_2 - T_1$

а) Q - ? б) A - ?

$$A = Q - \Delta U = C_n \Delta T - C_v \Delta T = (C_n - C_v)\Delta T$$

$$C_n - C_v = -\frac{\nu R}{n-1} \Rightarrow$$

$$\Rightarrow A = \frac{\nu R (T_1 - T_2)}{n-1} = \frac{\nu R \Delta T}{n-1} =$$

$$= \frac{1 \cdot 8{,}314 \cdot (-26)}{1{,}5 - 1} = 432{,}328\ (Дж)$$

$$\Delta U = \frac{\nu R \Delta T}{\gamma - 1} = \frac{1 \cdot 8{,}314 \cdot (-26)}{1{,}67 - 1} = -322{,}633\ (Дж)$$

$$Q = A + \Delta U = 432{,}326 \quad 322{,}633 = 109{,}693\ (Дж)$$

Ответ: а) 109, 693 Дж б) 432,328 Дж

№ 49

γ; $Q = -\Delta U$

а) C_n б) ур-е

в) $A\ (V_2 = \gamma V_1)$

$$Q = -\Delta U \Rightarrow \delta Q = -dU$$

а) $C = \frac{\delta Q}{dt}$ - молярная теплоемкость (для 1 моля)

$$C_n = \frac{-dU}{\nu dt} \qquad dU = \frac{\nu R}{\gamma - 1} dt$$

$$C_n = -\frac{\nu R dt}{(\gamma - 1)\nu dt} = \frac{R}{1 - \gamma}$$

б) $Q = \Delta U + A$

$\delta Q = dU + \delta A$

$-dU = dU + \delta A \qquad \delta A = -2dU \qquad pdV = -2\frac{\nu R}{\gamma - 1} dT$

Т.к. газ идеальный $p = \frac{\nu R T}{V}$

$$\frac{\nu R T dV}{V} = -2\frac{\nu R dT}{\gamma - 1}$$

$$\int \left(\frac{dV}{V} + \frac{2}{\gamma - 1}\frac{dT}{T}\right) = \int 0$$

$$\ln V + \frac{2}{\gamma - 1} \ln T = const \quad \Big| \cdot \frac{\gamma - 1}{2}$$

$$\frac{\gamma - 1}{2} \ln V + \ln T = const \qquad \ln V^{\frac{\gamma - 1}{2}} T = const$$

$$T V^{\frac{\gamma - 1}{2}} = const$$

б) Т.к. $TV^{\frac{\gamma-1}{2}} = const$, то $T_1 V_1^{\frac{\gamma-1}{2}} = T_2 V_2^{\frac{\gamma-1}{2}}$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\frac{\gamma-1}{2}} = \left(\frac{\eta V_1}{V_1}\right)^{\frac{\gamma-1}{2}} = \eta^{\frac{\gamma-1}{2}}$$

$$\Rightarrow T_2 = \frac{T_1}{\eta^{\frac{\gamma-1}{2}}} = T_1 \eta^{\frac{1-\gamma}{2}}$$

$$C_n = \frac{(n-\gamma)R}{(n-1)(\gamma-1)} = \frac{R}{(\gamma-1)}$$

$$\frac{n-\gamma}{n-1} = -1$$

$$n - \gamma = 1 - n \qquad n = \frac{1+\gamma}{2}$$

$$A = (C_n - C_V)\Delta T = \left(-\frac{R}{\gamma-1} - \frac{R}{\gamma-1}\right)\Delta T = -\frac{2R}{\gamma-1}\Delta T =$$

$$= \frac{-2R}{\gamma-1}(T_1 \eta^{\frac{1-\gamma}{2}} - T_1) = \boxed{\frac{2RT_1(1-\eta^{\frac{1-\gamma}{2}})}{\gamma-1}}$$

№50

$\nu = 1$ моль; γ

$p = aT^\alpha$, $a, \alpha = const$

а) $A - ?$ ΔT

б) $C_n - ?$ $C_n < 0$ $\alpha - ?$

$pV = \nu RT$ $\quad T = \frac{pV}{\nu R}$ $\quad \nu = 1$ н $\Rightarrow T = \frac{pV}{R}$

$p = aT^\alpha$ $\quad pT^{-\alpha} = a$

$$p\left(\frac{pV}{R}\right)^{-\alpha} = a \qquad p^{1-\alpha} V^{-\alpha} = aR$$

$$pV^{\frac{-\alpha}{1-\alpha}} = (aR^{\frac{1}{1-\alpha}})^{const}$$

$$pV^{\frac{\alpha}{\alpha-1}} = const \qquad n = \frac{\alpha}{\alpha-1}$$

$$A = Q - \Delta U = \Delta T C_n - C_V \Delta T = (C_n - C_V)\Delta T$$

$$C_n - C_V = -\frac{\nu R}{n-1} \Rightarrow A = \frac{\nu R \Delta T}{n-1} \quad \nu = 1 \text{ моль} \Rightarrow$$

$$\Rightarrow A = \frac{-R\Delta T}{n-1} = \frac{R\Delta T}{1-n} = \frac{R\Delta T}{\left(1-\frac{\alpha}{\alpha-1}\right)} = -R\Delta T(\alpha-1) =$$

$$= \boxed{R\Delta T(1-\alpha)}$$

б) $C_n = \frac{(n-\gamma)R}{(n-1)(\gamma-1)} = \frac{R}{\gamma-1} - \frac{R}{n-1} =$

$$= \frac{R}{\gamma-1} - \frac{R}{\left(\frac{\alpha}{\alpha-1}-1\right)} = \boxed{\frac{R}{\gamma-1} - R(\alpha-1)}$$