

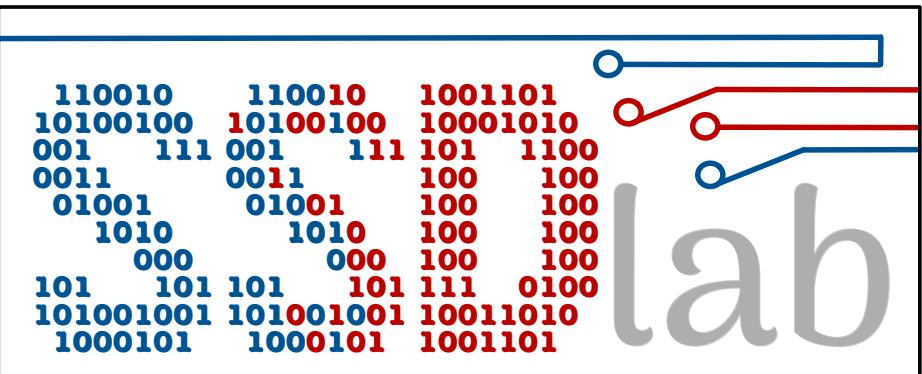
COSI 167A Advanced Data Systems

Class 6

Performance Analysis of LSM-Trees

Prof. Subhadeep Sarkar

<https://ssd-brandeis.github.io/COSI-167A/>



Class logistics

and administrivia

The third technical question is now available on the class website
(due before the class on **Tue, Sep 24**).

Deadline is at **12:45 PM** for all technical questions and reviews!

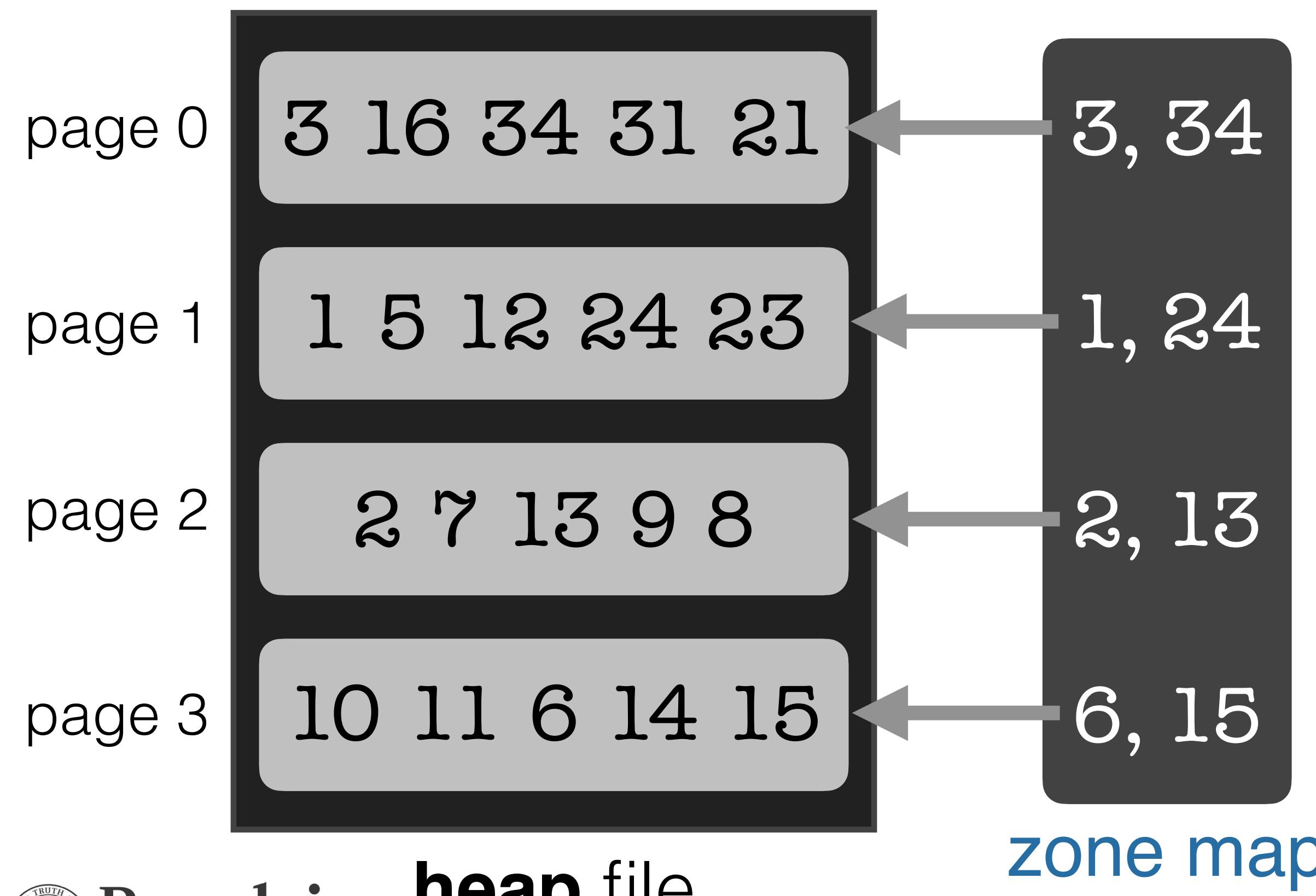
Project 1 is due in 3 days (**Fri, Sep 20**).

First guest lecture: next Tuesday (**Sep 24**).

Project 1

Testing the waters!

Implementing a Simple Zone Map



do not sort the data for
unsorted workloads

Requirement

`sort()`: may want to sort the
data **within a zone**
to facilitate binary search



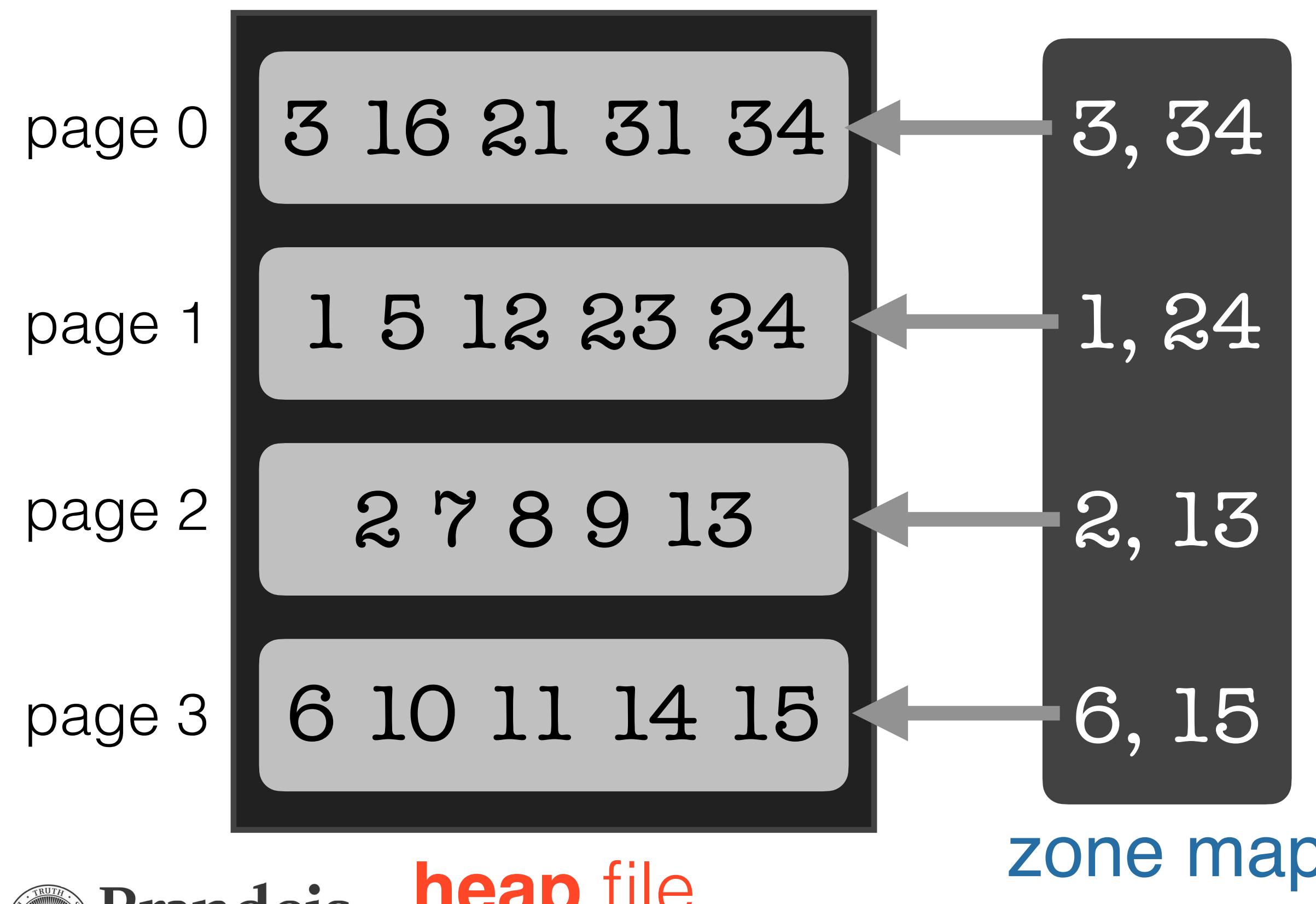
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heap file

Project 1

Testing the waters!

Implementing a Simple Zone Map



do not sort the data for
unsorted workloads

Requirement

`sort()`: may want to sort the
data **within a zone**
to facilitate binary search

Design optimization (optional)



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Today in COSI 167A

What's on the cards?

Queries in **LSM-trees**

Cost analysis

[P] ["Monkey: Optimal Navigable Key-Value Store", SIGMOD, 2017](#)

TECHNICAL QUESTION 2 [When does a tiered LSM-tree behave similarly to a leveled LSM-tree? What is the key contribution of the paper?](#)

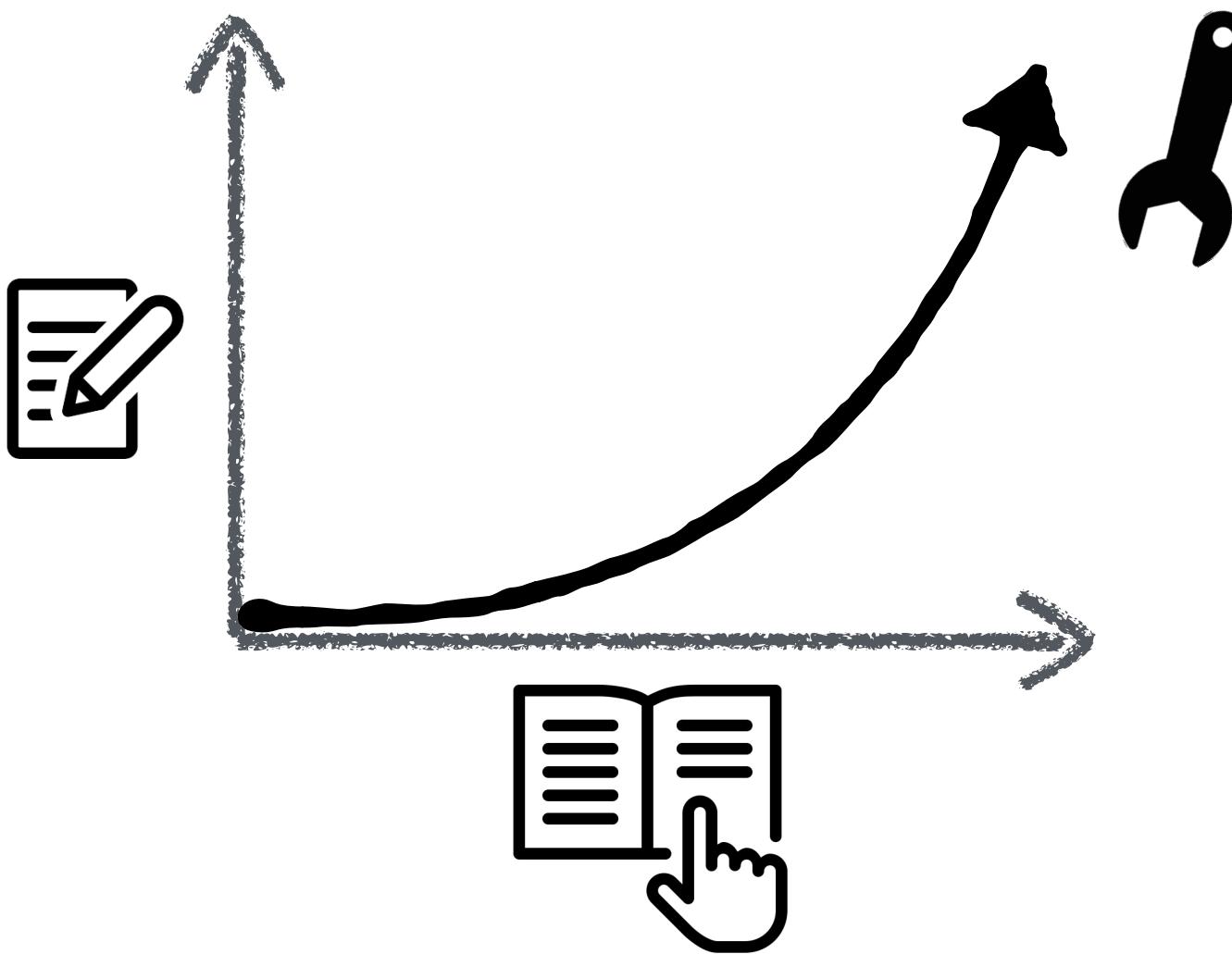
[B] ["The LSM Design Space and its Read Optimizations", ICDE, 2023](#)

Why LSM?

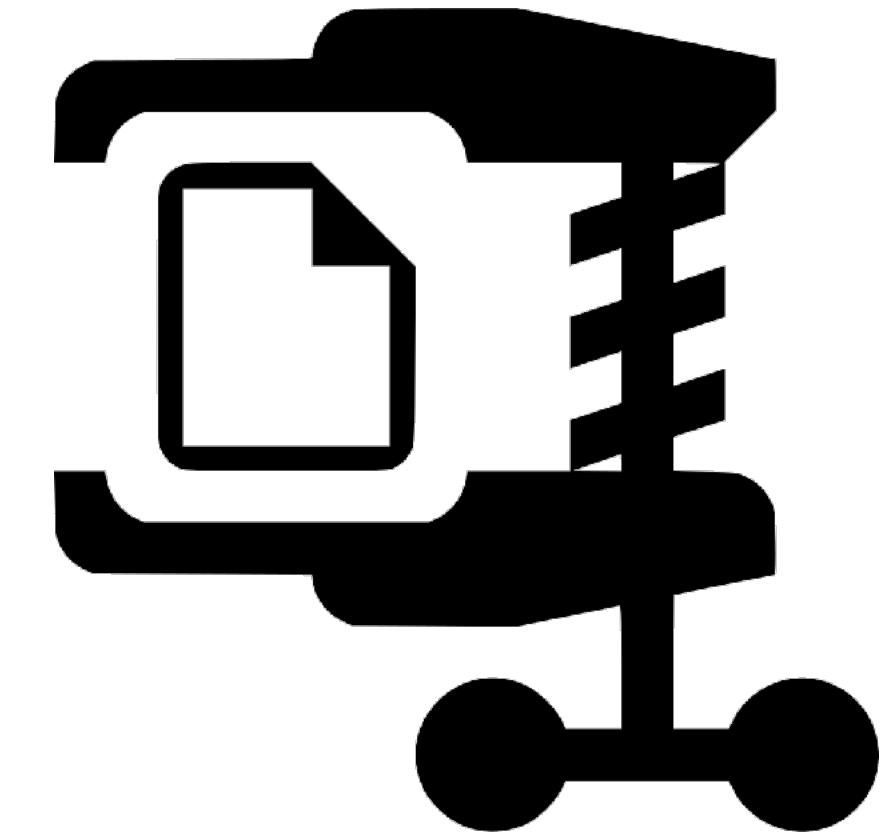
What's the hype all about?



fast writes



tunable read-write
performance



good space
utilization

LSM Operating Principles

Buffering ingestion

Immutable files on storage

Out-of-place updates & deletes

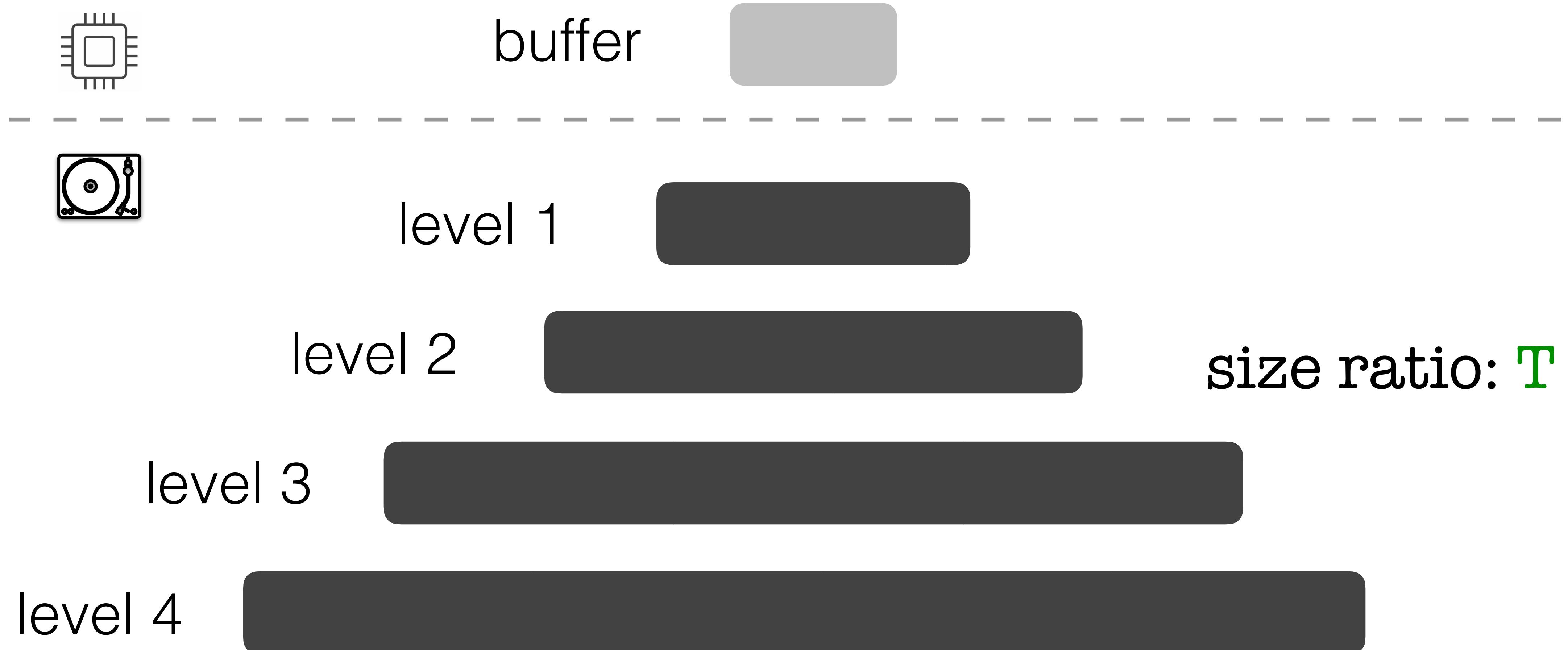
Periodic data layout reorganization

LSM basics

How do they look?

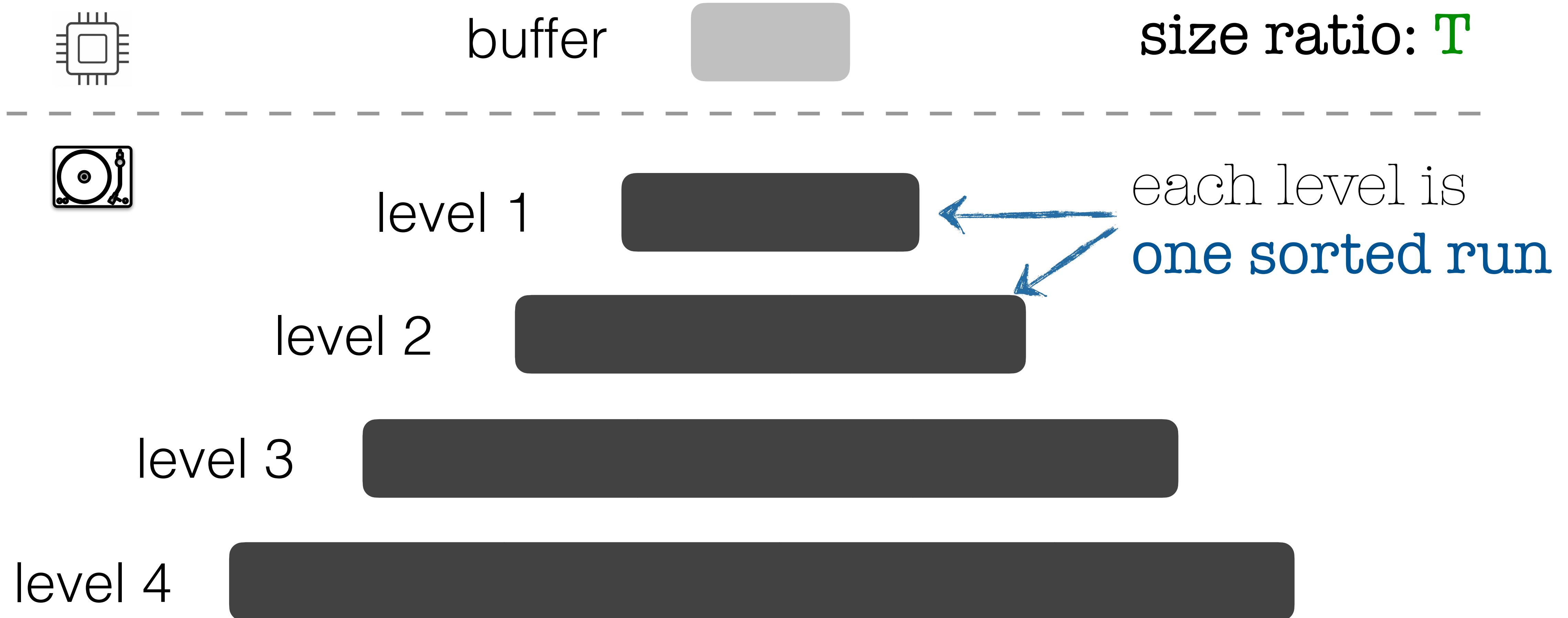
LSM basics

How do they look?



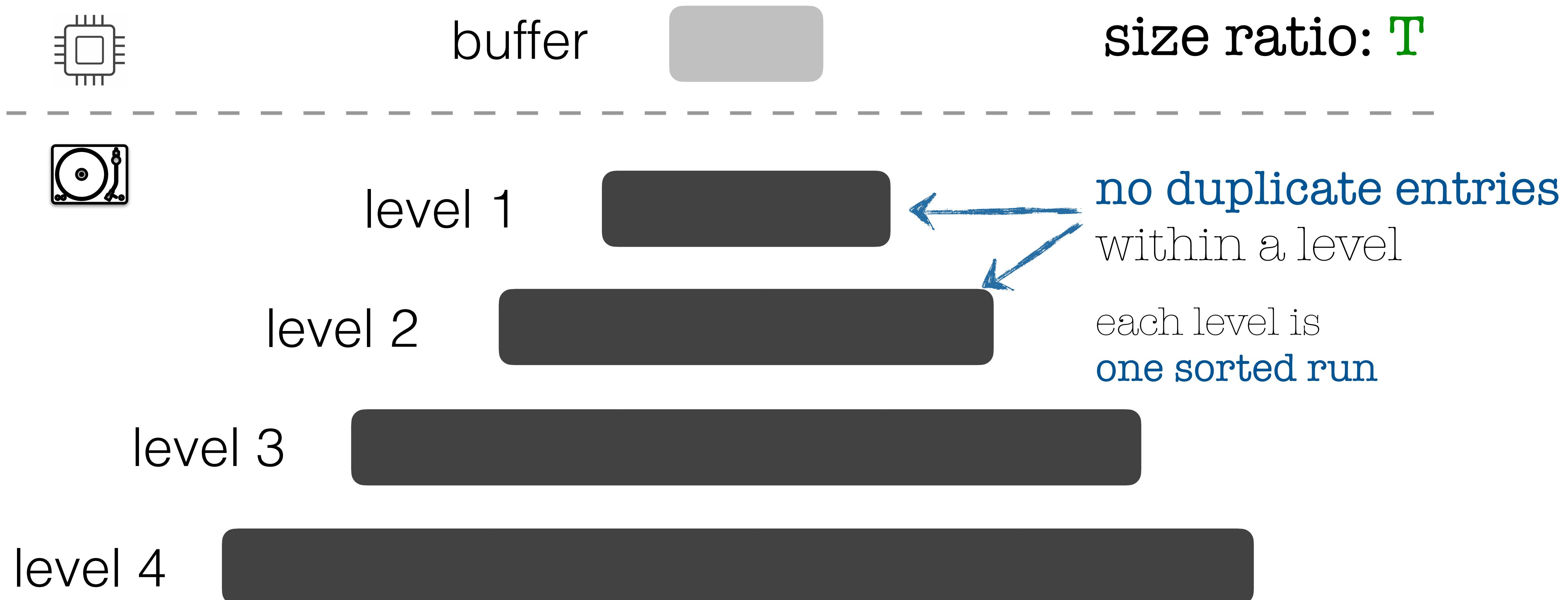
LSM basics

How do they look?

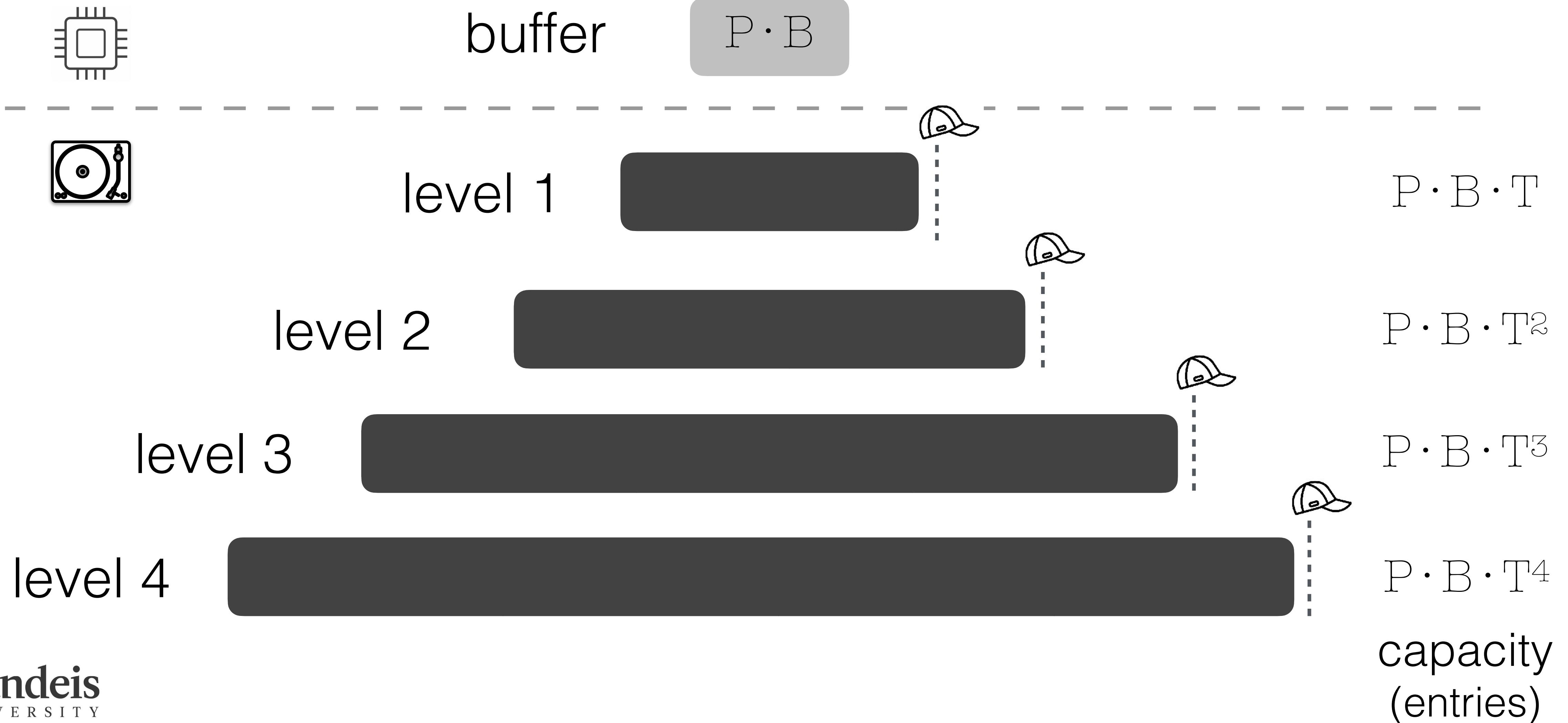


LSM basics

How do they look?



P : pages in buffer
 B : entries/page
 L : #levels
 T : size ratio

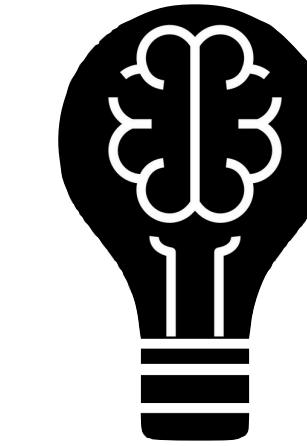
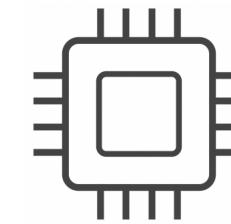


P : pages in buffer

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T : size ratio



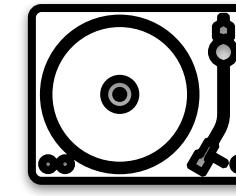
Thought Experiment 1



There are N entries in an LSM with size ratio T .

How many **levels** does the tree have?

$$P \cdot B$$



level 1

$$P \cdot B \cdot T$$

the total number of entries in the tree is N

level 2

$$P \cdot B \cdot T^2$$

level 3

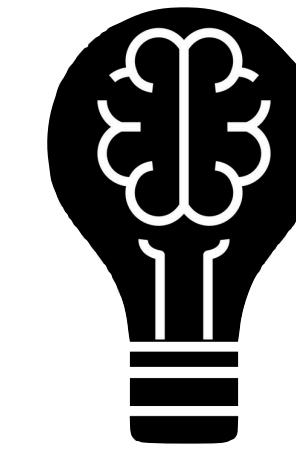
$$P \cdot B \cdot T^3$$

ooo

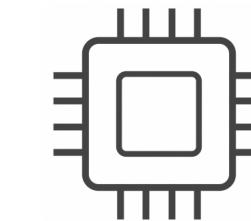
level L

$$P \cdot B \cdot T^L$$

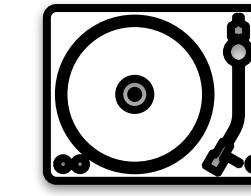
P : pages in buffer
 B : entries/page
 L : #levels
 T : size ratio



Thought Experiment 1



$$P \cdot B$$



level 1

$$P \cdot B \cdot T$$

the total number of entries in the tree is N

level 2

$$P \cdot B \cdot T^2$$

$$N = P \cdot B + P \cdot B \cdot T + P \cdot B \cdot T^2 + \dots + P \cdot B \cdot T^L$$

level 3

$$P \cdot B \cdot T^3$$

$$L = \log_T (N/(P \cdot B) \cdot ((T-1) / T))$$

ooo

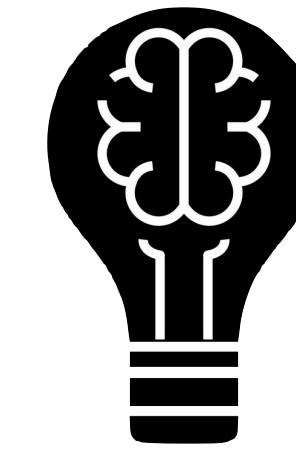
level L

$$P \cdot B \cdot T^L$$



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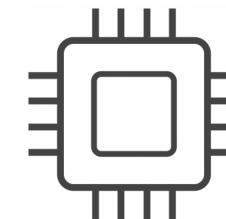
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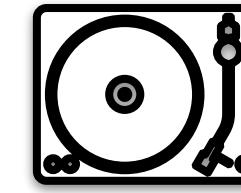
Thought Experiment 1



There are N entries in an LSM with size ratio T .
How many **levels** does the tree have?



$$P \cdot B$$



level 1

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$$N = P \cdot B + P \cdot B \cdot T + P \cdot B \cdot T^2 + \dots + P \cdot B \cdot T^L$$

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$$P \cdot B \cdot T^3$$

$$L = \log_T (N / (P \cdot B) \cdot ((T-1) / T))$$

ooo

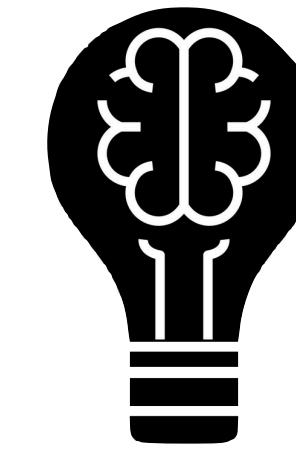
level L

$$P \cdot B \cdot T^L$$

behaves like a constant
for $2 < T < 20$



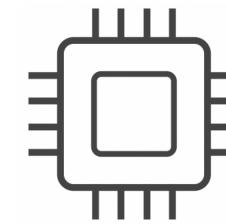
P : pages in buffer
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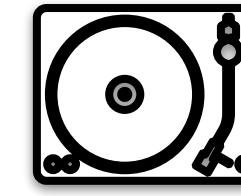
Thought Experiment 1



There are N entries in an LSM with size ratio T .
How many **levels** does the tree have?



$$P \cdot B$$



level 1

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the total number of entries in the tree is N

level 2

$$P \cdot B \cdot T^2$$

$$N = P \cdot B + P \cdot B \cdot T + P \cdot B \cdot T^2 + \dots + P \cdot B \cdot T^L$$

level 3

$$P \cdot B \cdot T^3$$

$$L = \log_T (N / (P \cdot B) \cdot ((T-1) / T))$$

ooo

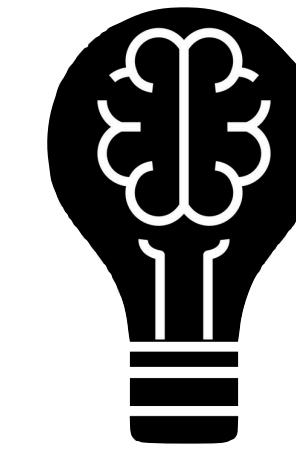
level L

$$P \cdot B \cdot T^L$$

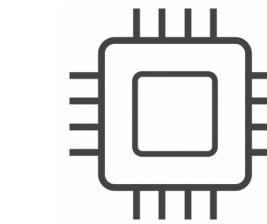
behaves like a constant
for $2 < T < 20$

$$L = \log_T (N / (P \cdot B))$$

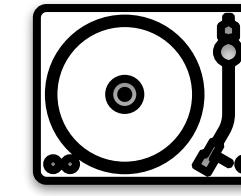
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Thought Experiment 1



$$P \cdot B$$



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$$N = P \cdot B + P \cdot B \cdot T + P \cdot B \cdot T^2 + \dots + P \cdot B \cdot T^L$$

level 3

$$P \cdot B \cdot T^3$$

$$L = \log_T (N / (P \cdot B) \cdot ((T-1) / T))$$



behaves like a constant
for $2 < T < 20$

$$L = \log_T (N / (P \cdot B))$$

$$L = O(\log_T (N))$$

Cost analysis

Counting all I/Os

Cost analysis

Counting all I/Os

data structure	ingestion cost	point lookup cost	range lookup cost
Leveled LSM-tree			
Tiered LSM-tree			
B+-tree			
Sorted array			
Log			

Cost analysis

Counting all I/Os

Ingestion cost

expected #I/Os performed to write a single entry to **disk**

note: if an entry is written **multiple times**, **count all I/Os**

Query cost

expected #I/Os performed to perform a single query

compare costs with and without **auxiliary (helper) data structures**

Ingestion cost

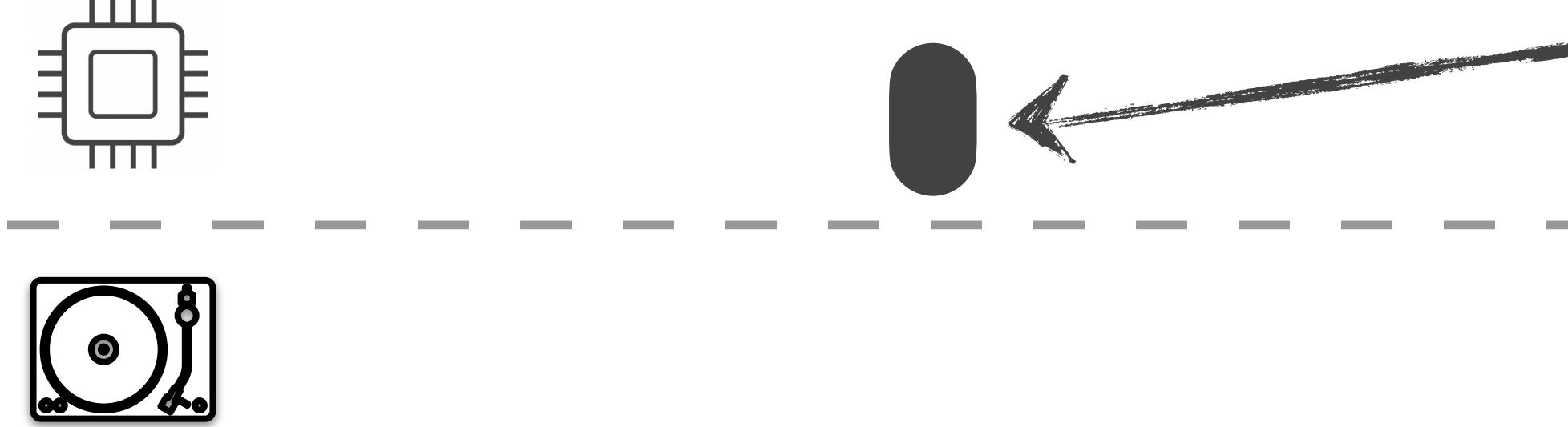
Inserts and updates

P : pages in buffer
 B : entries/page
 L : #levels
 T : size ratio

Ingestion cost

Inserts and updates

leveled LSM-tree



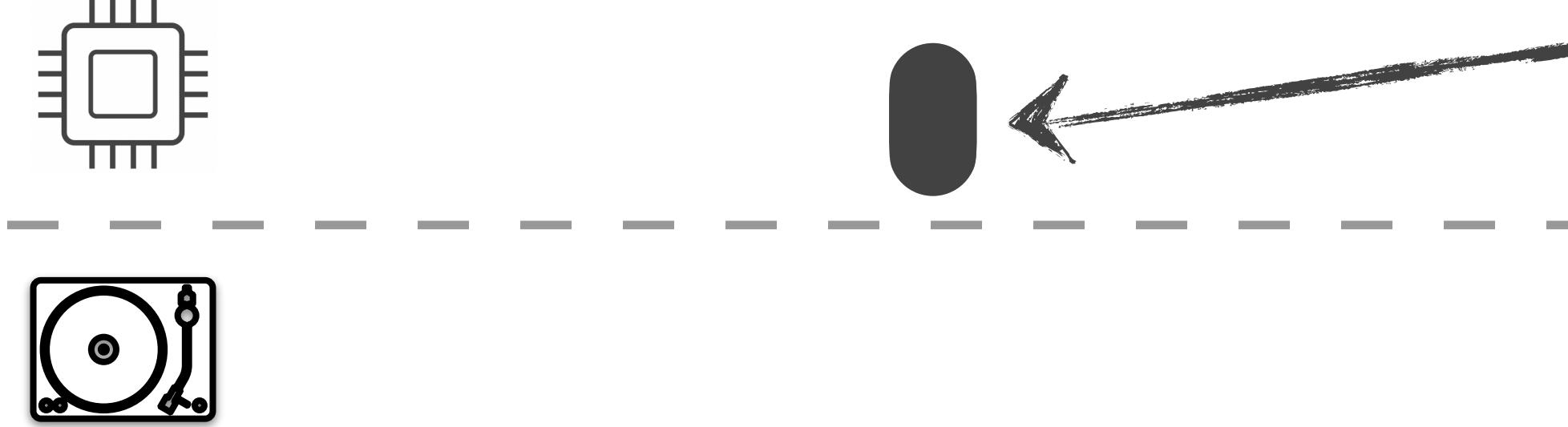
\mathbf{P} pages each with \mathbf{B} entries
total entries in buffer = $\mathbf{P} \cdot \mathbf{B}$

P : pages in buffer
 B : entries/page
 L : #levels
 T : size ratio

Ingestion cost

Inserts and updates

leveled LSM-tree



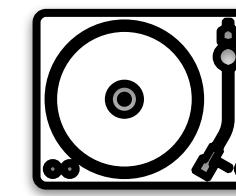
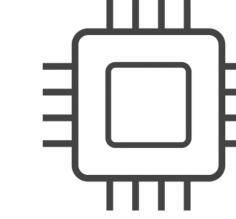
\mathbf{P} pages each with \mathbf{B} entries
total entries in buffer = $\mathbf{P} \cdot \mathbf{B}$

P : pages in buffer
 B : entries/page
 L : #levels
 T : size ratio

Ingestion cost

Inserts and updates

leveled LSM-tree



P pages each with B entries
total entries in buffer = $P \cdot B$

How many I/Os to flush the buffer?

P I/Os to flush the **P pages** in buffer



How many entries flushed per I/O?

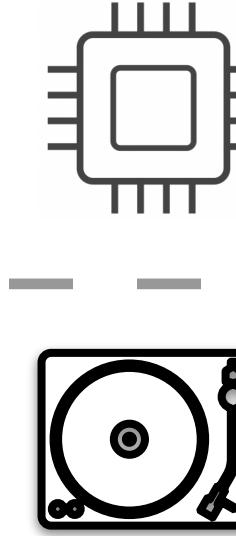
B entries per I/O

P : pages in buffer
 B : entries/page
 L : #levels
 T : size ratio

Ingestion cost

Inserts and updates

leveled LSM-tree



P pages each with B entries

total entries in buffer = $P \cdot B$

How many entries flushed per I/O?

B entries per I/O



How many I/Os to flush one entry?

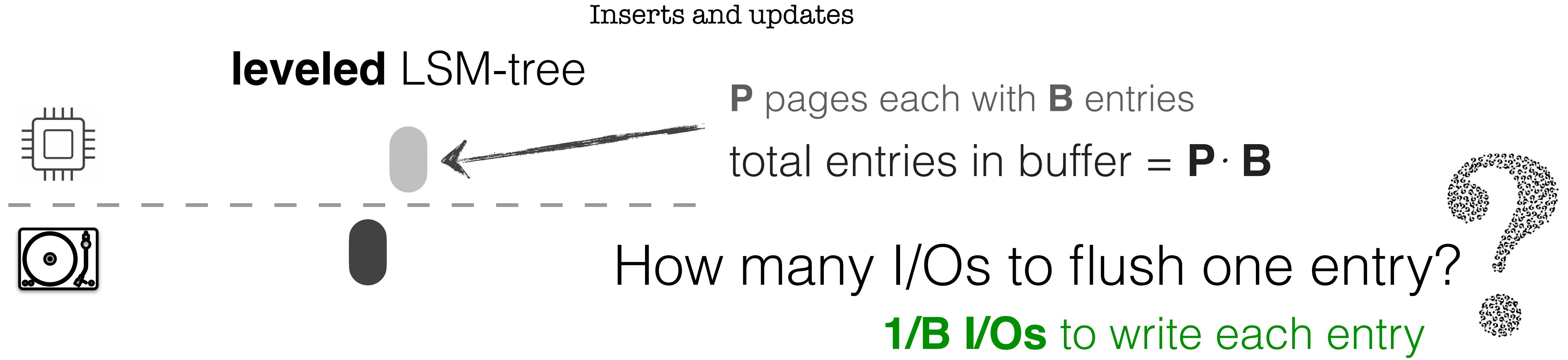
1/B I/Os to write each entry



ingestion cost in leveled LSM-trees?

P : pages in buffer
 B : entries/page
 L : #levels
 T : size ratio

Ingestion cost



Ingestion cost

expected #I/Os performed to write a single entry to **disk**

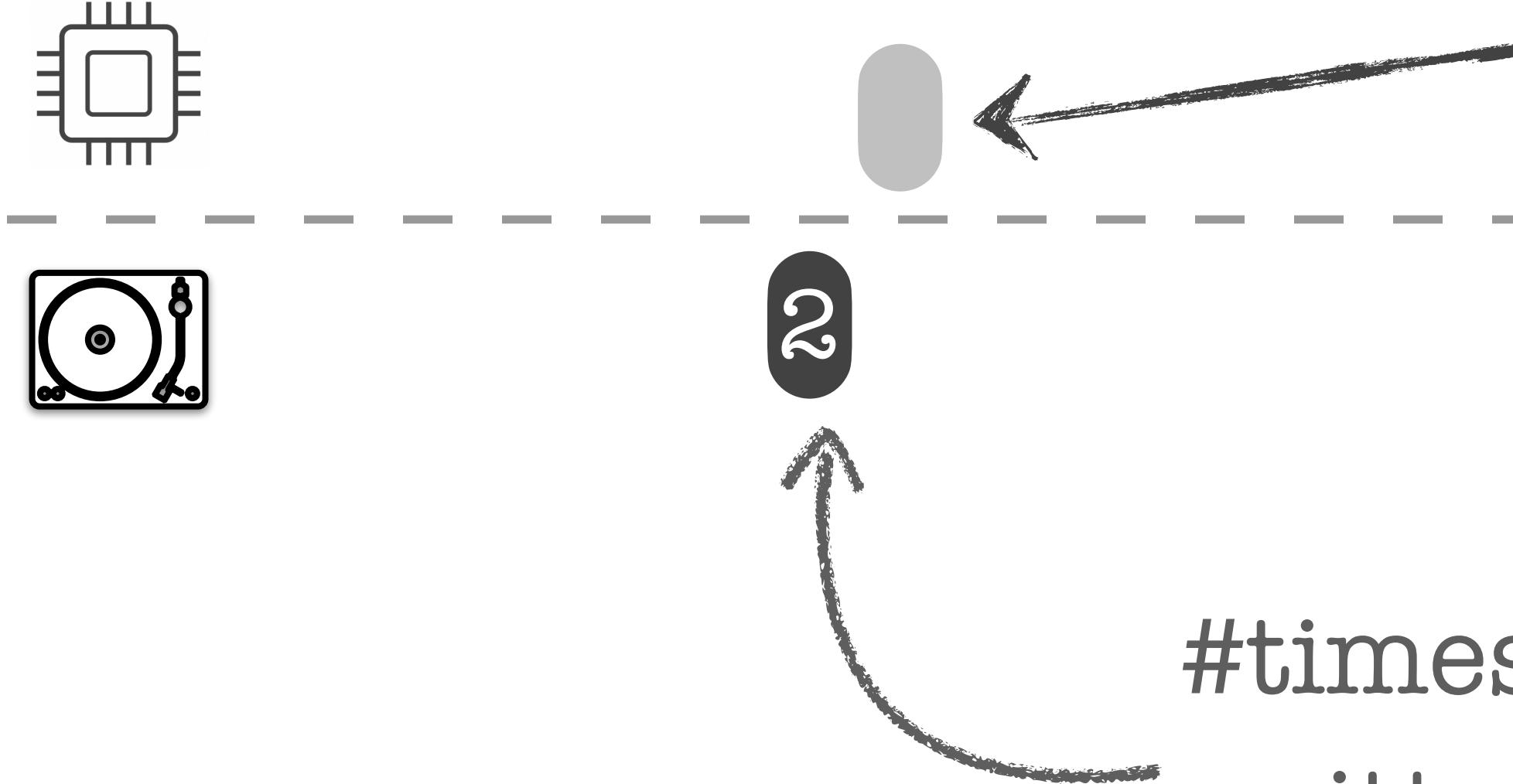
note: if an entry is written **multiple times**, **count all I/Os**

P : pages in buffer
 B : entries/page
 L : #levels
 T : size ratio

Ingestion cost

Inserts and updates

leveled LSM-tree



\mathbf{P} pages each with \mathbf{B} entries
total entries in buffer = $\mathbf{P} \cdot \mathbf{B}$

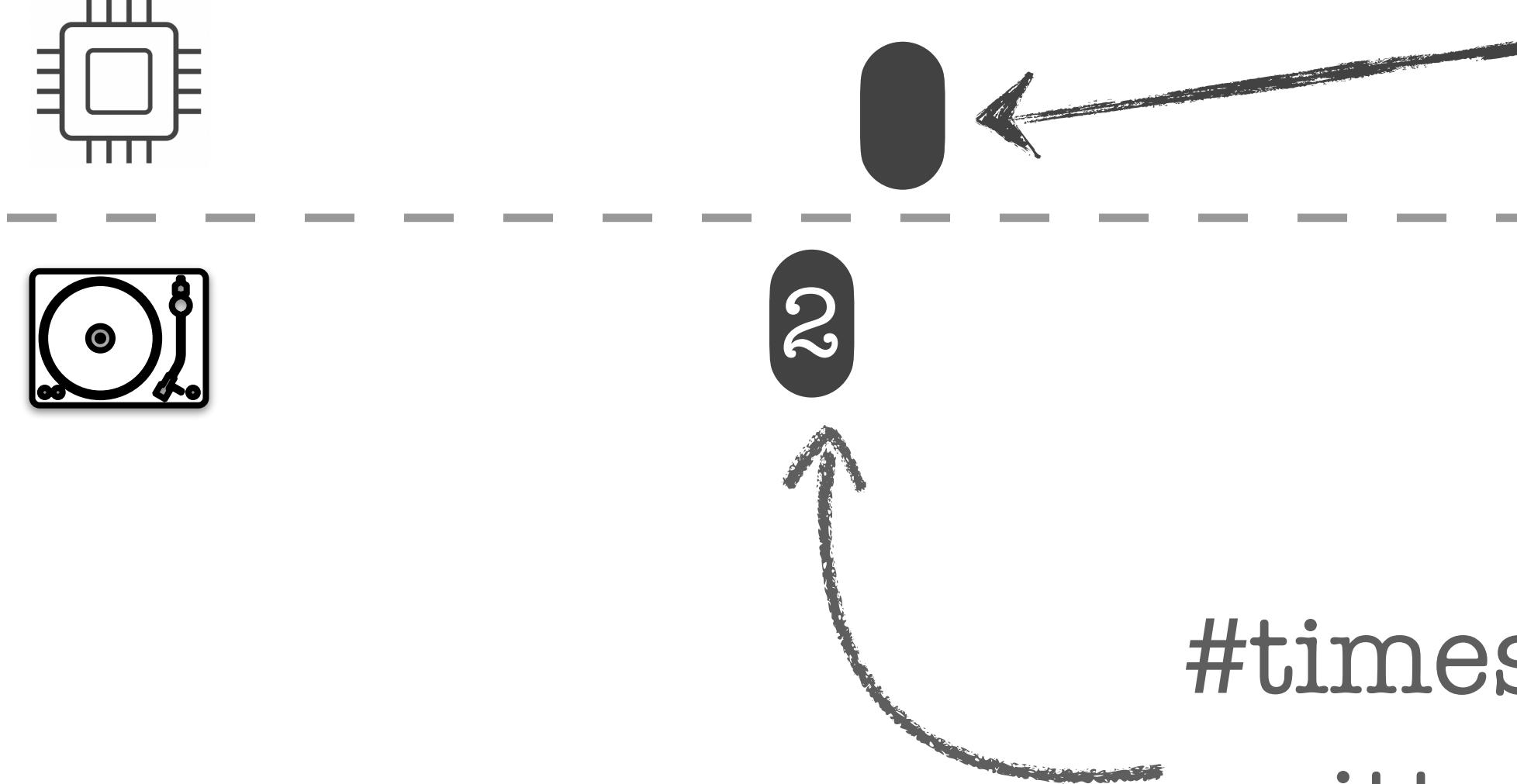
#times $\mathbf{2}$ has been
written to this level = $\mathbf{1}$

P : pages in buffer
 B : entries/page
 L : #levels
 T : size ratio

Ingestion cost

Inserts and updates

leveled LSM-tree



P pages each with B entries

total entries in buffer = $P \cdot B$

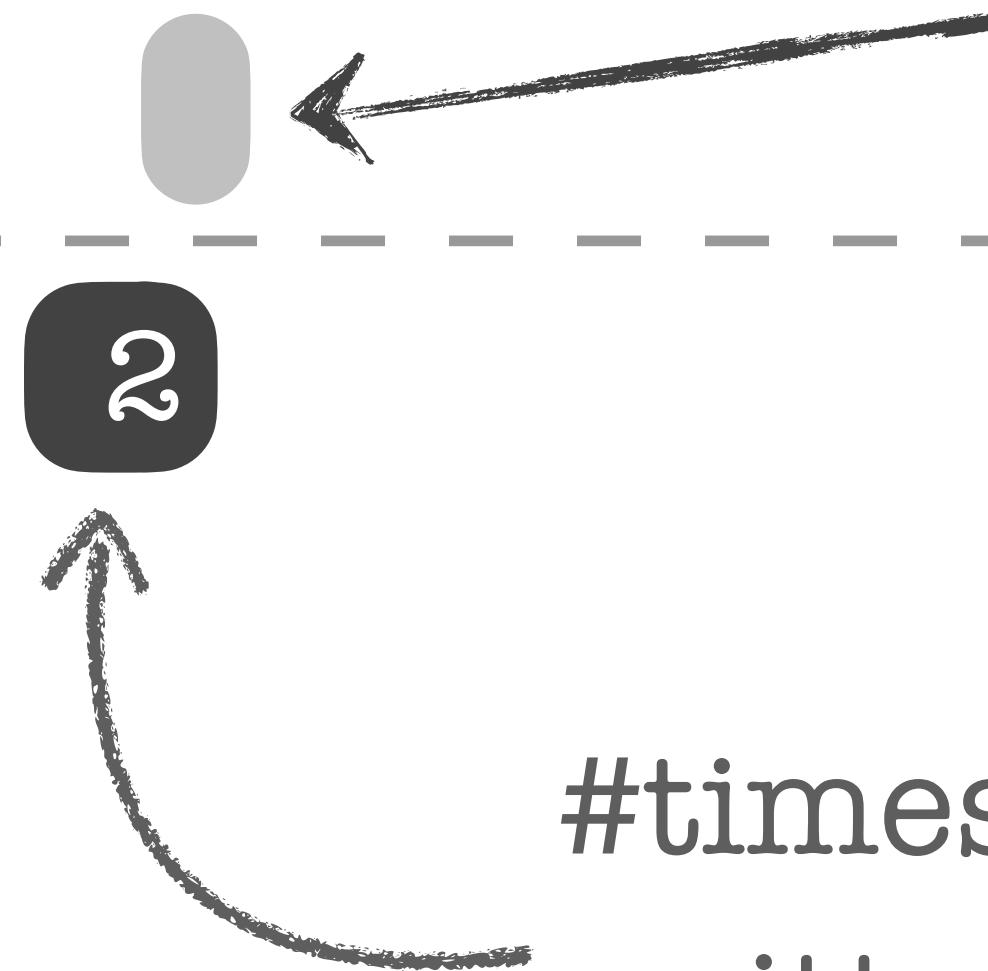
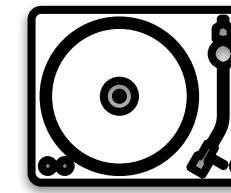
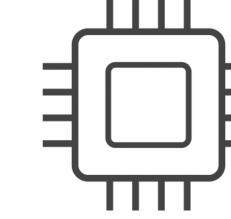
#times 2 has been
written to this level = 1

P : pages in buffer
 B : entries/page
 L : #levels
 T : size ratio

Ingestion cost

Inserts and updates

leveled LSM-tree



P pages each with B entries
total entries in buffer = $P \cdot B$

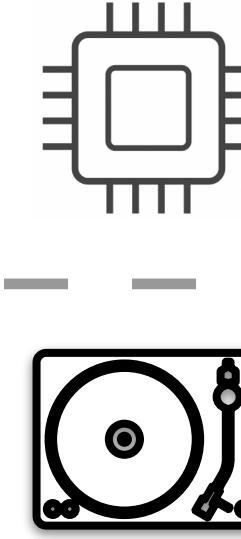
#times **2** has been
written to this level = **1**

P : pages in buffer
 B : entries/page
 L : #levels
 T : size ratio

Ingestion cost

Inserts and updates

leveled LSM-tree



P pages each with B entries
total entries in buffer = $P \cdot B$

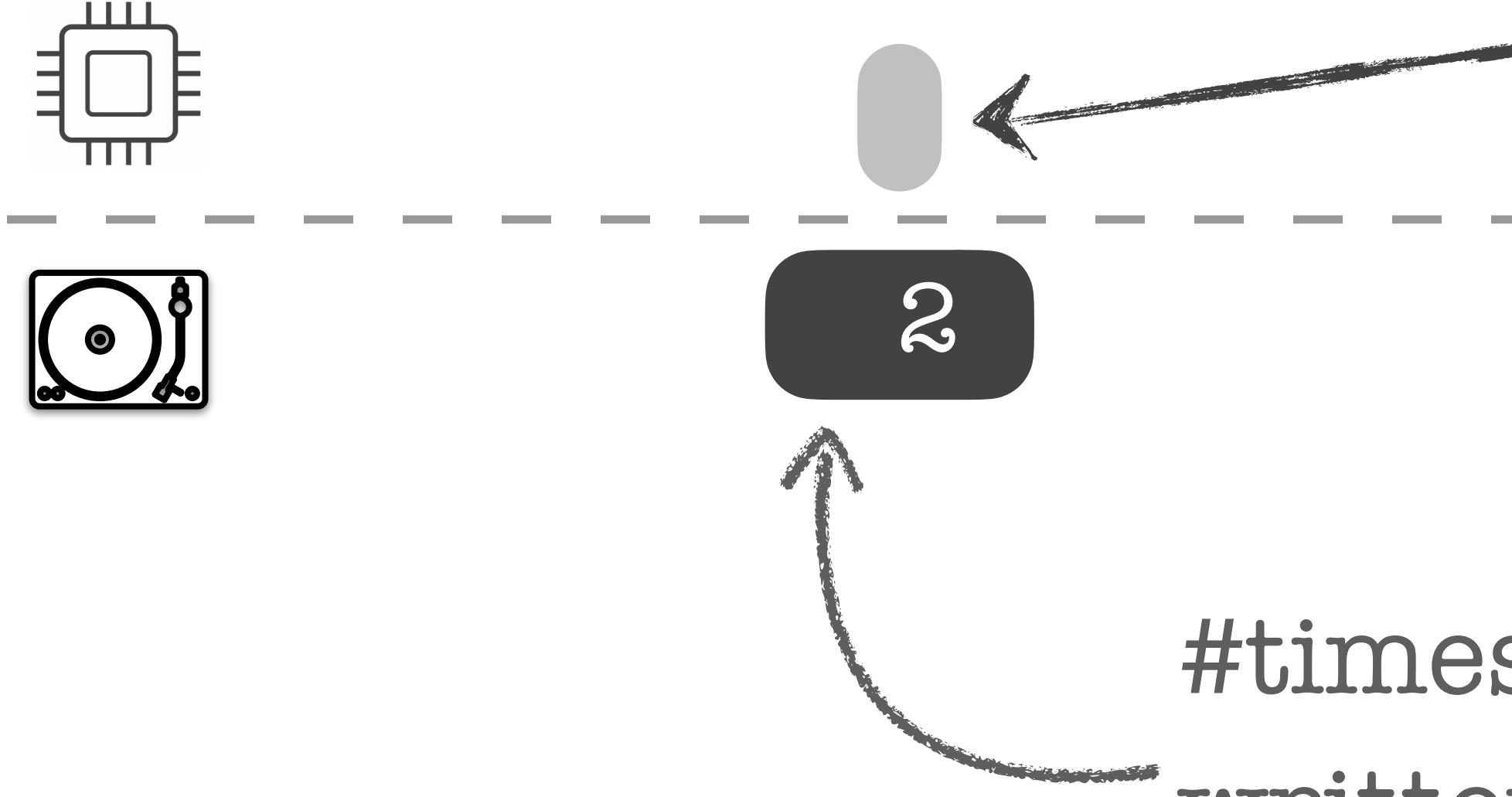
#times $\textcolor{blue}{2}$ has been
written to this level = $\textcolor{red}{2}$

P : pages in buffer
 B : entries/page
 L : #levels
 T : size ratio

Ingestion cost

Inserts and updates

leveled LSM-tree



\mathbf{P} pages each with \mathbf{B} entries
total entries in buffer = $\mathbf{P} \cdot \mathbf{B}$

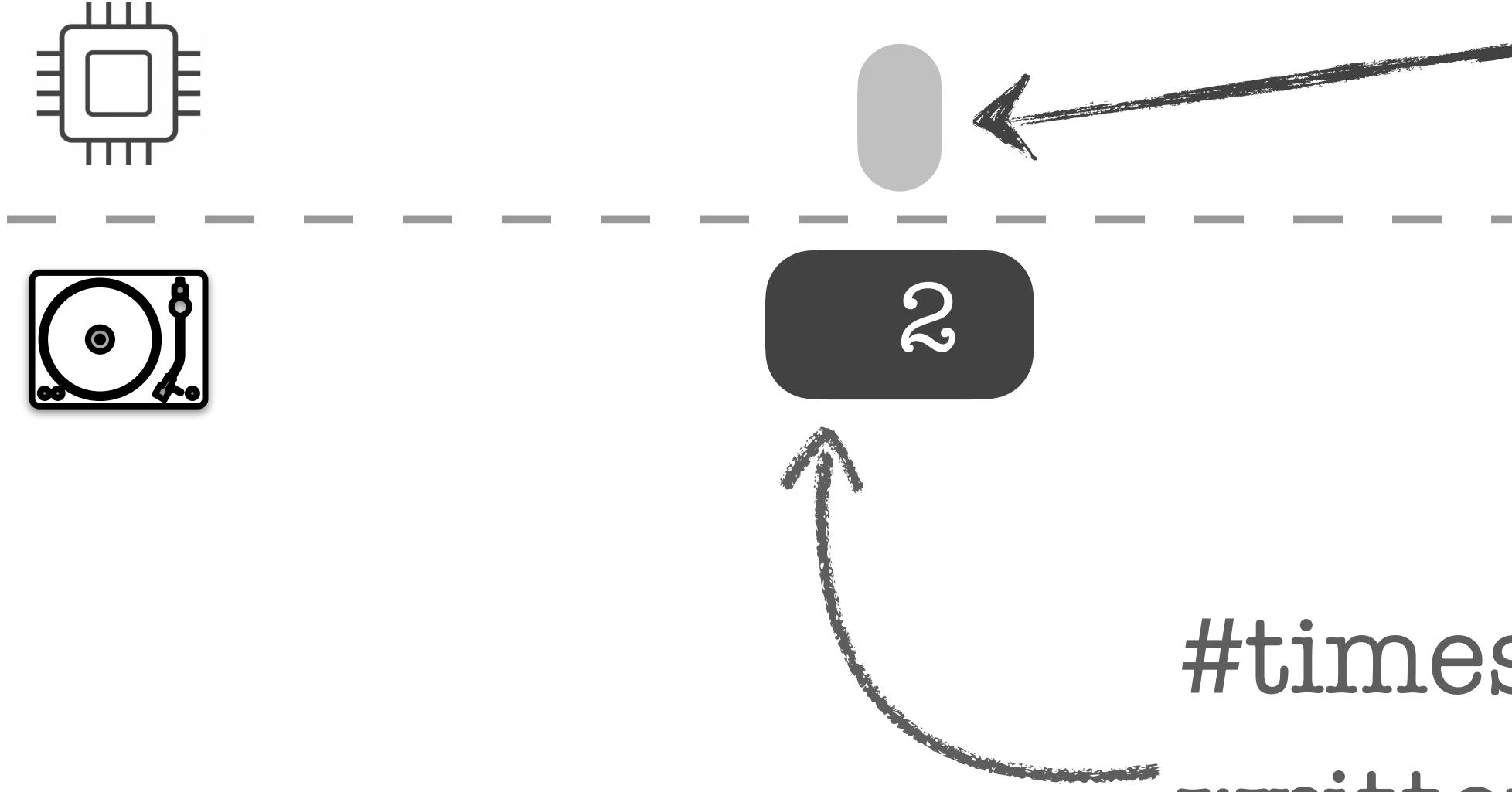
#times $\mathbf{2}$ has been
written to this level = $\mathbf{2}$

P : pages in buffer
 B : entries/page
 L : #levels
 T : size ratio

Ingestion cost

Inserts and updates

leveled LSM-tree



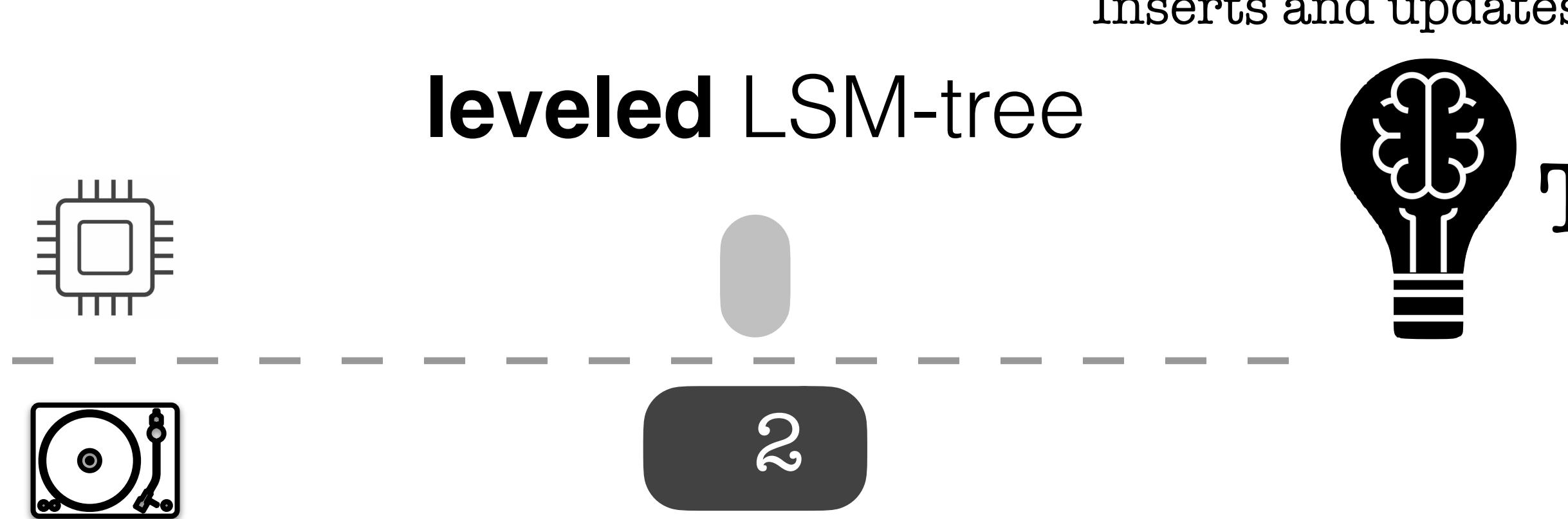
P pages each with B entries
total entries in buffer = $P \cdot B$

#times 2 has been
written to this level = 3

in general, #times an entry is written to a level = T

P : pages in buffer
 B : entries/page
 L : #levels
 T : size ratio

Ingestion cost



Thought Experiment 2

What is the ingestion cost
in leveled LSM-trees?

in general, #times an entry is written to a level = \mathbf{T}

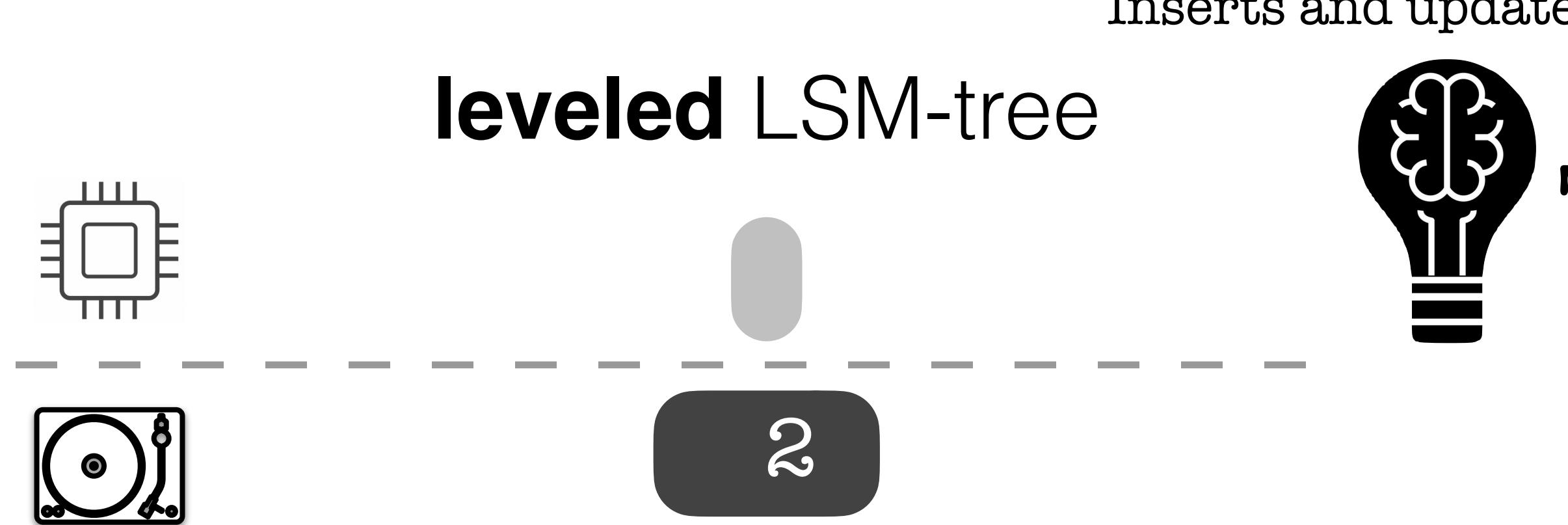
happens for all \mathbf{L} levels on disk

Total #times an entry is written in the tree = $\mathbf{L} \cdot \mathbf{T}$

B entries are written to disk per I/O

P : pages in buffer
 B : entries/page
 L : #levels
 T : size ratio

Ingestion cost



Thought Experiment 2

What is the ingestion cost
in leveled LSM-trees?

in general, #times an entry is written to a level = \mathbf{T}

happens for all \mathbf{L} levels on disk

Total #times an entry is written in the tree = $\mathbf{L} \cdot \mathbf{T}$

B entries are written to disk per I/O

Average number of I/Os to write a single entry = $\mathbf{L} \cdot \mathbf{T} / \mathbf{B}$

Cost analysis

Counting all I/Os

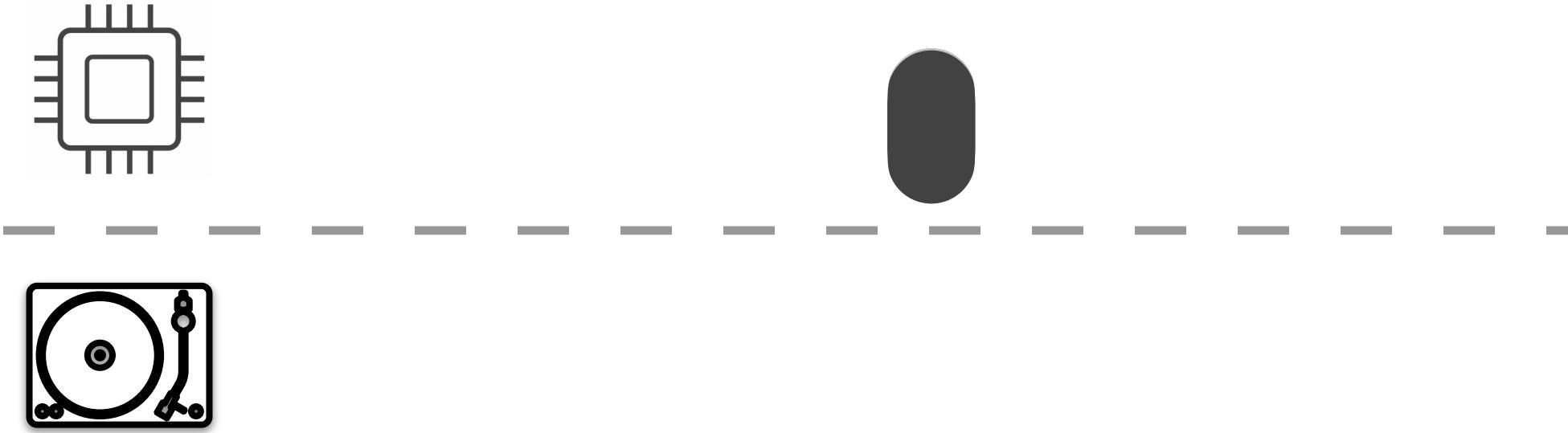
data structure	ingestion cost	point lookup cost	range lookup cost
Leveled LSM-tree	$O(L \cdot T / B)$		
Tiered LSM-tree			
B+-tree			
Sorted array			
Log			

P : pages in buffer
 B : entries/page
 L : #levels
 T : size ratio

Ingestion cost

Inserts and updates

tiered LSM-tree

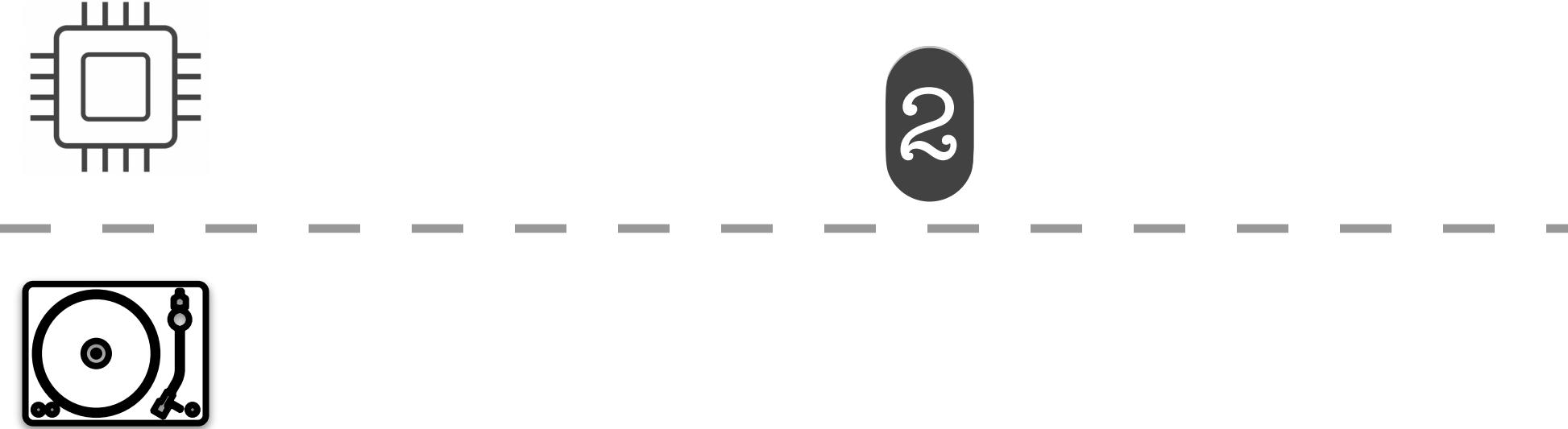


P : pages in buffer
 B : entries/page
 L : #levels
 T : size ratio

Ingestion cost

Inserts and updates

tiered LSM-tree

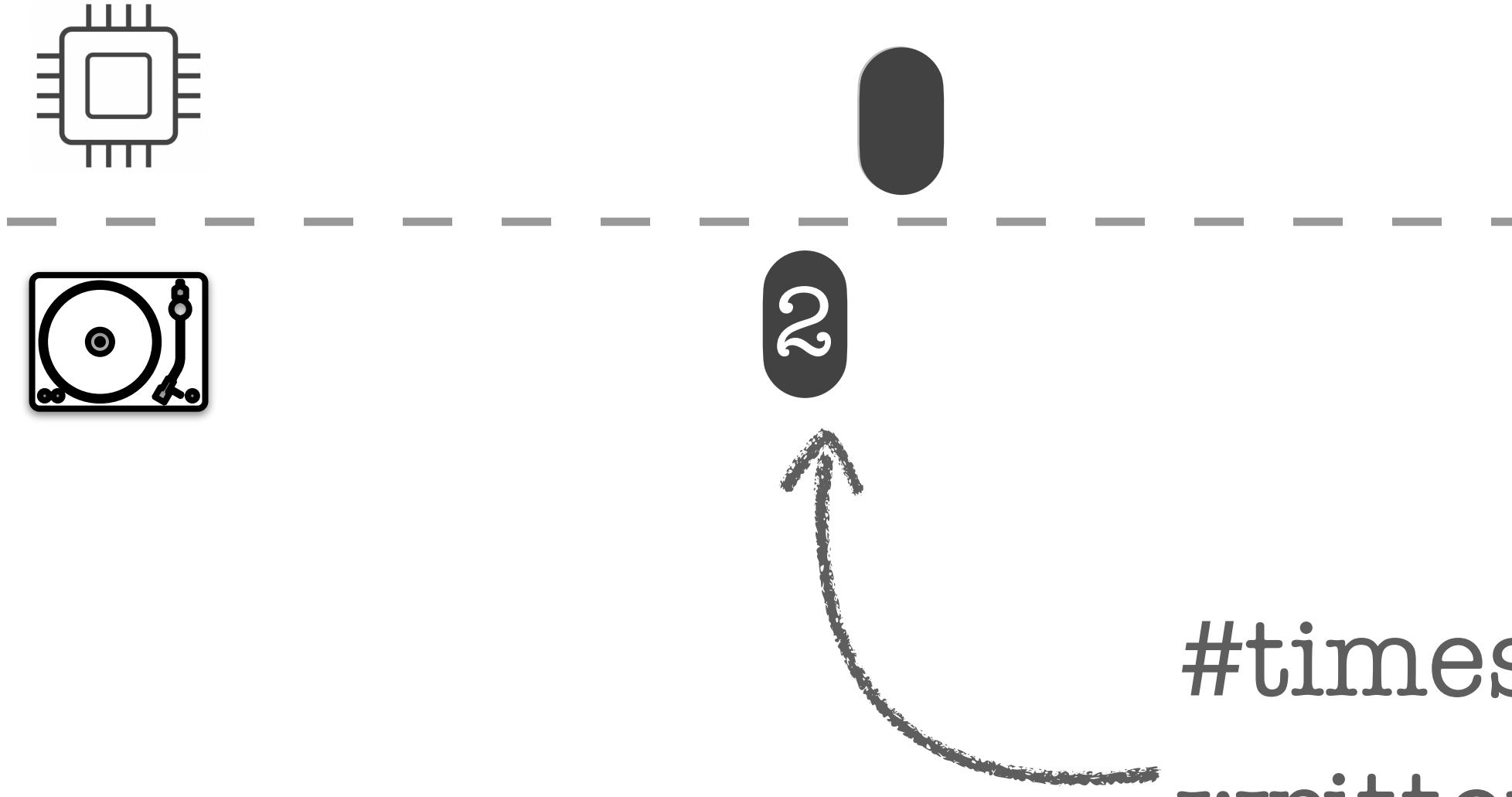


P : pages in buffer
 B : entries/page
 L : #levels
 T : size ratio

Ingestion cost

Inserts and updates

tiered LSM-tree



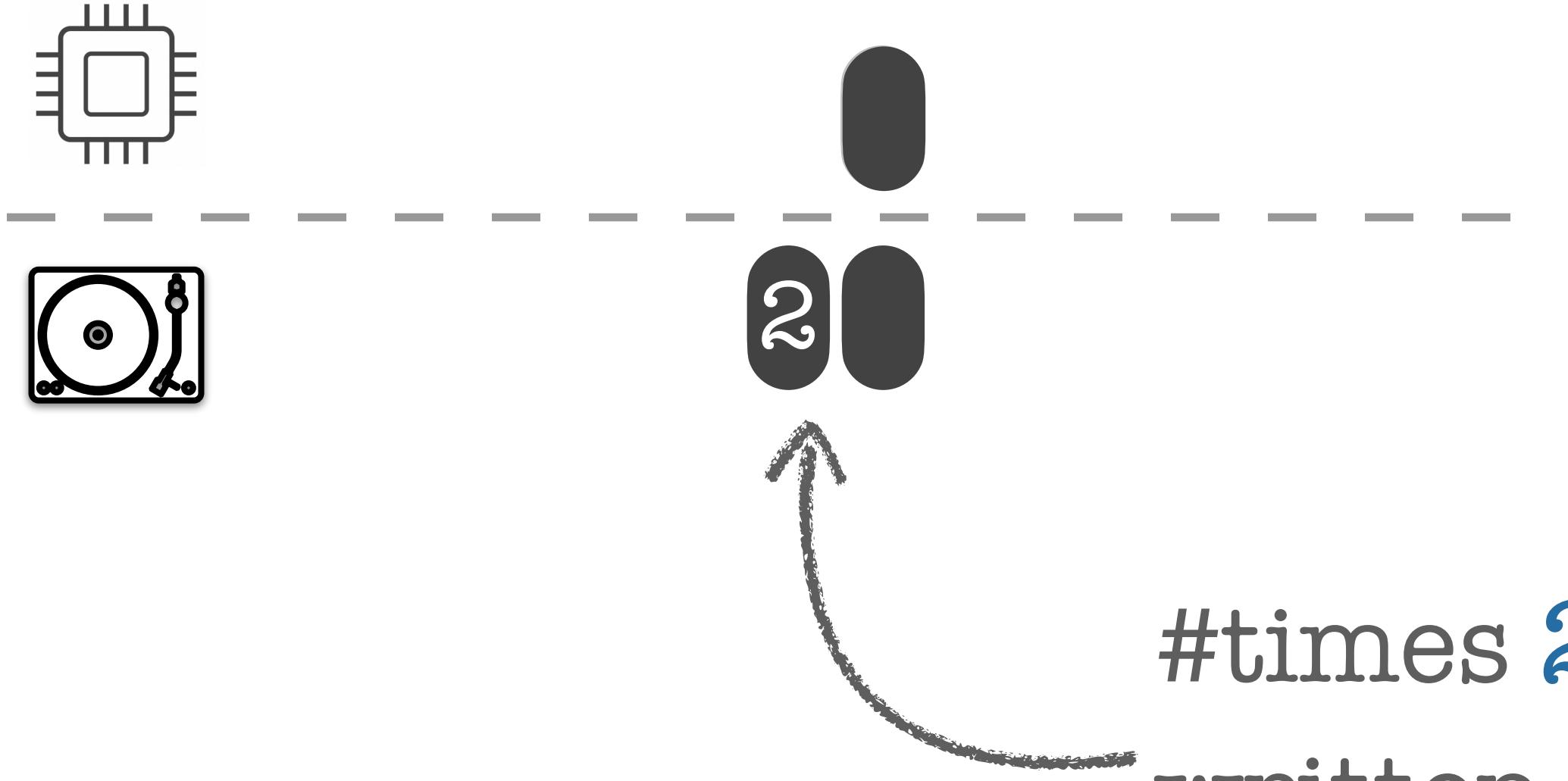
#times **2** has been
written to this level = **1**

P : pages in buffer
 B : entries/page
 L : #levels
 T : size ratio

Ingestion cost

Inserts and updates

tiered LSM-tree



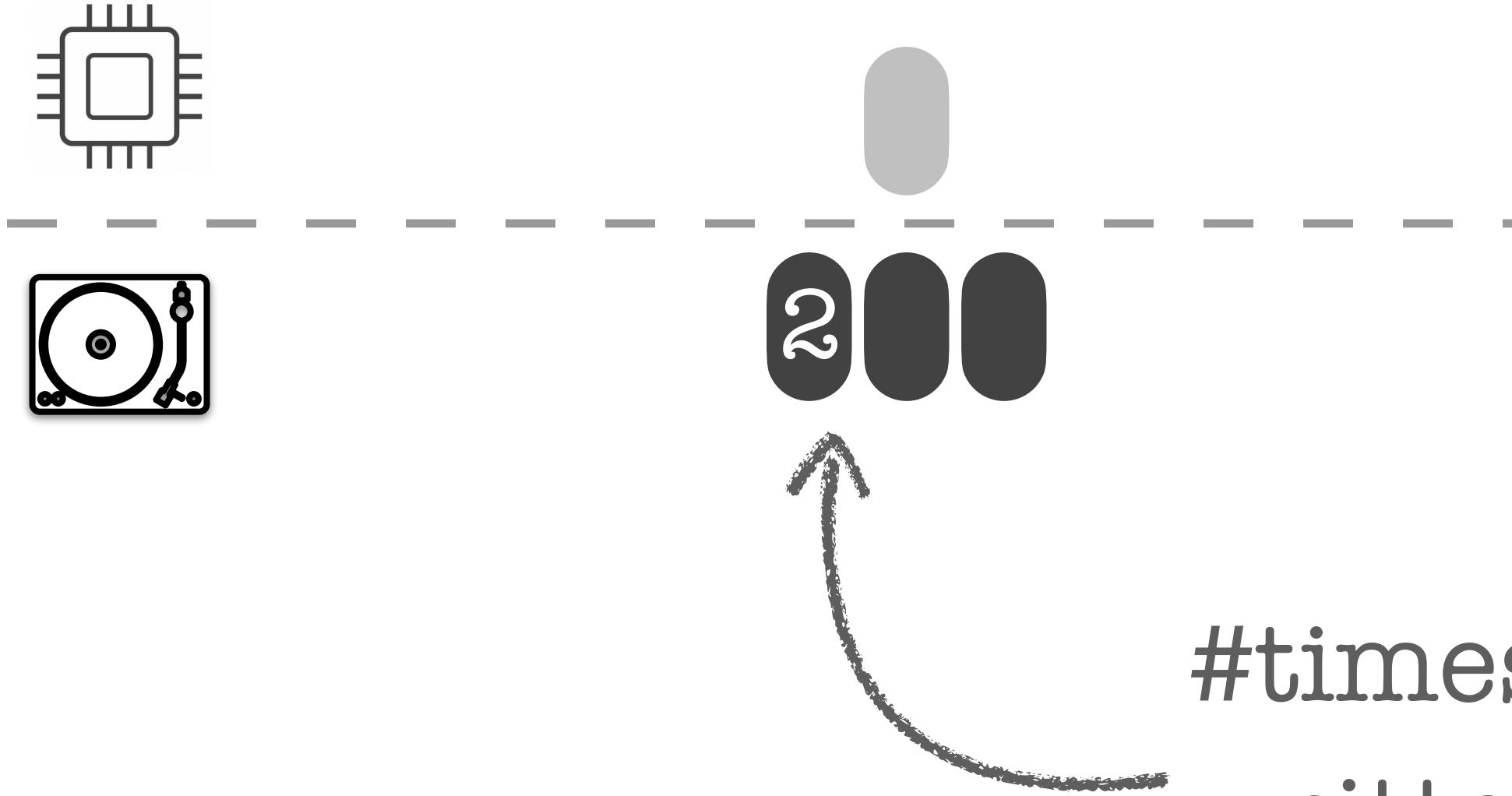
#times **2** has been
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 T : size ratio

Ingestion cost

Inserts and updates

tiered LSM-tree



#times 2 has been
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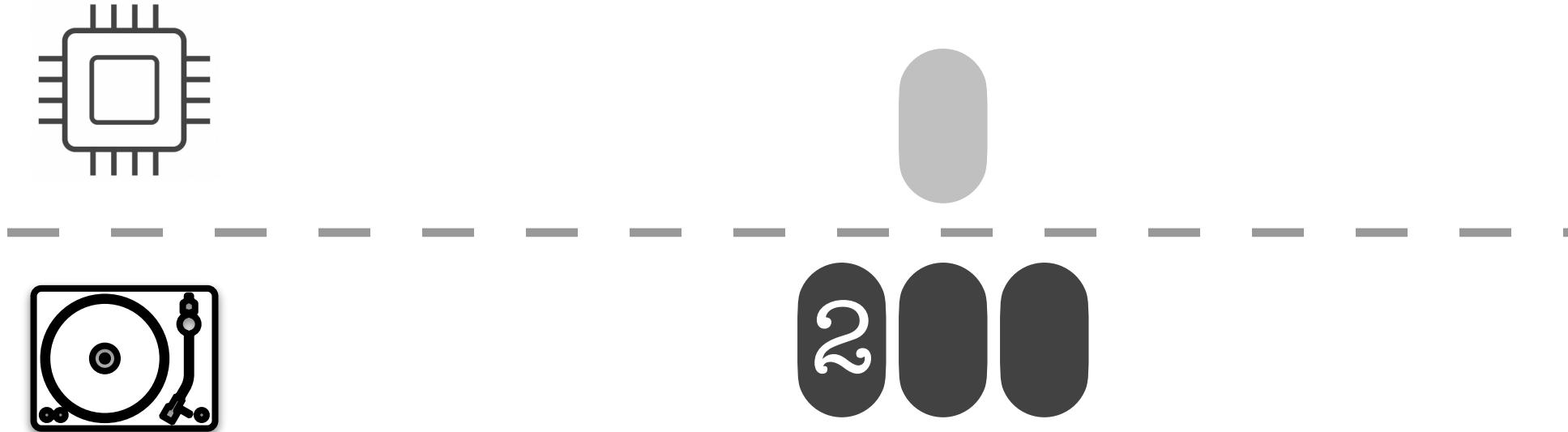
in general, #times an entry is written to a level = 1

P : pages in buffer
 B : entries/page
 L : #levels
 T : size ratio

Ingestion cost

Inserts and updates

tiered LSM-tree



in general, #times an entry is written to a level = **1**

↑ happens for all **L** levels on disk

Total #times an entry is written in the tree = **L**

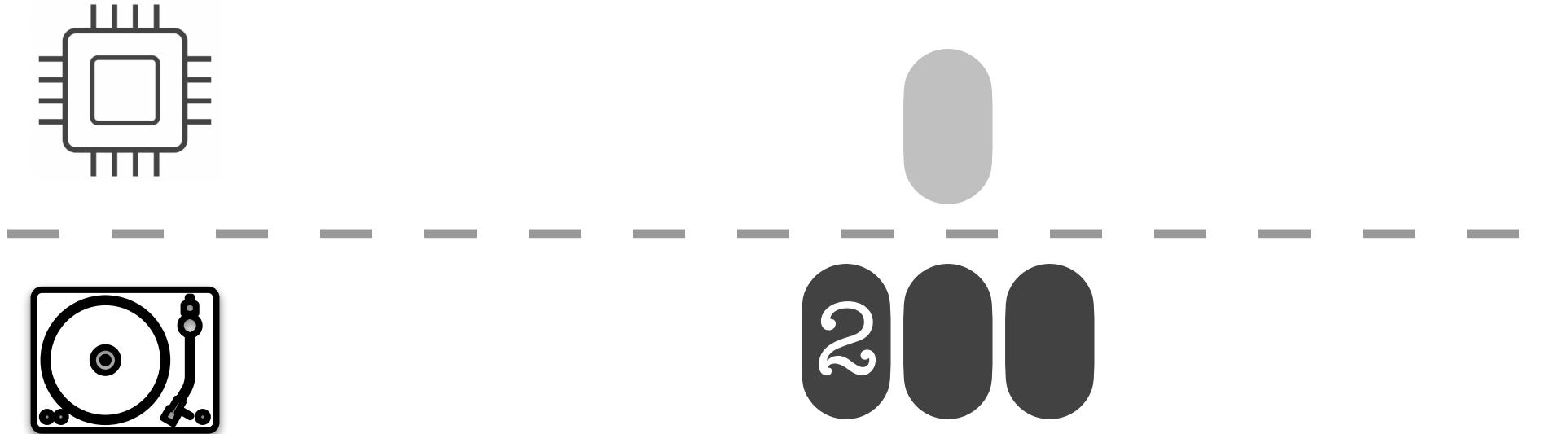
B entries are written to disk per I/O

P : pages in buffer
 B : entries/page
 L : #levels
 T : size ratio

Ingestion cost

Inserts and updates

tiered LSM-tree



in general, #times an entry is written to a level = **1**

happens for all **L** levels on disk

Total #times an entry is written in the tree = **L**

B entries are written to disk per I/O

Average number of I/Os to write a single entry = **L / B**

Cost analysis

Counting all I/Os

data structure	ingestion cost	point lookup cost	range lookup cost
Leveled LSM-tree	$O(L \cdot T / B)$		
Tiered LSM-tree	$O(L / B)$		
B+-tree			
Sorted array			
Log			

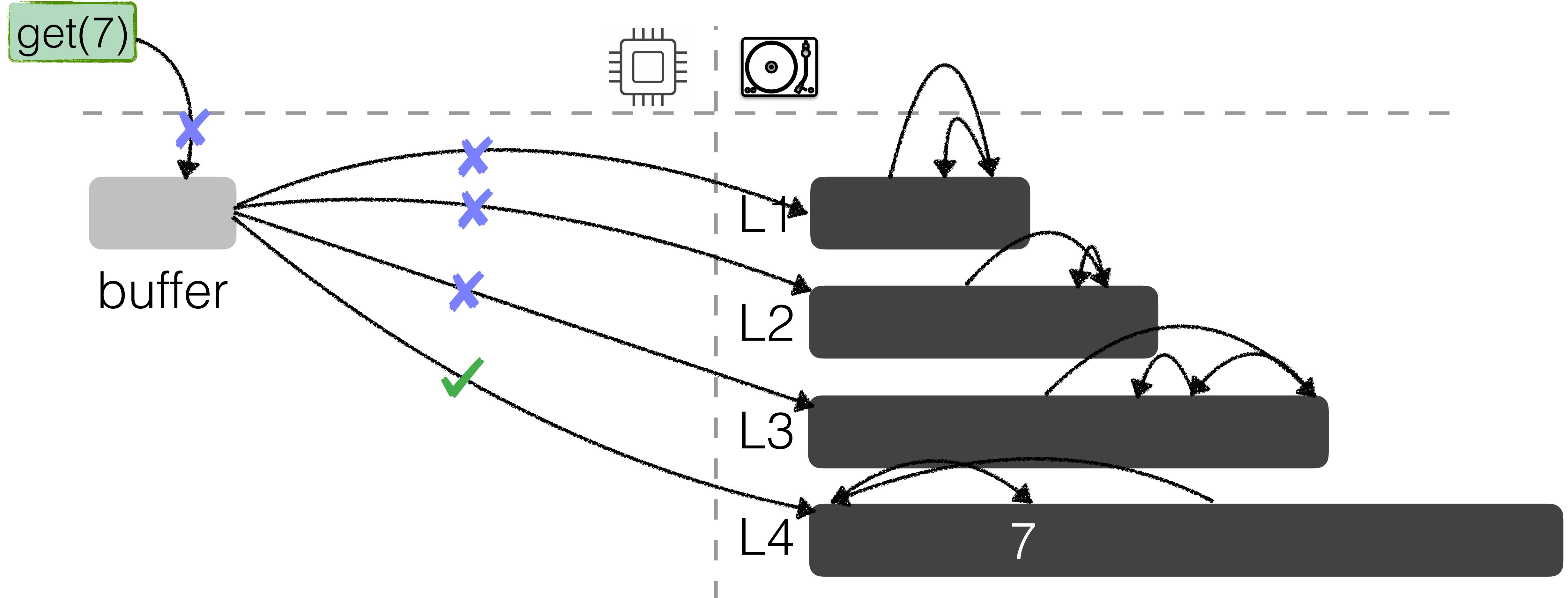
Point lookup cost

Looking for a specific key

P : pages in buffer
 B : entries/page
 L : #levels
 T : size ratio
 N : #entries

Point lookup cost

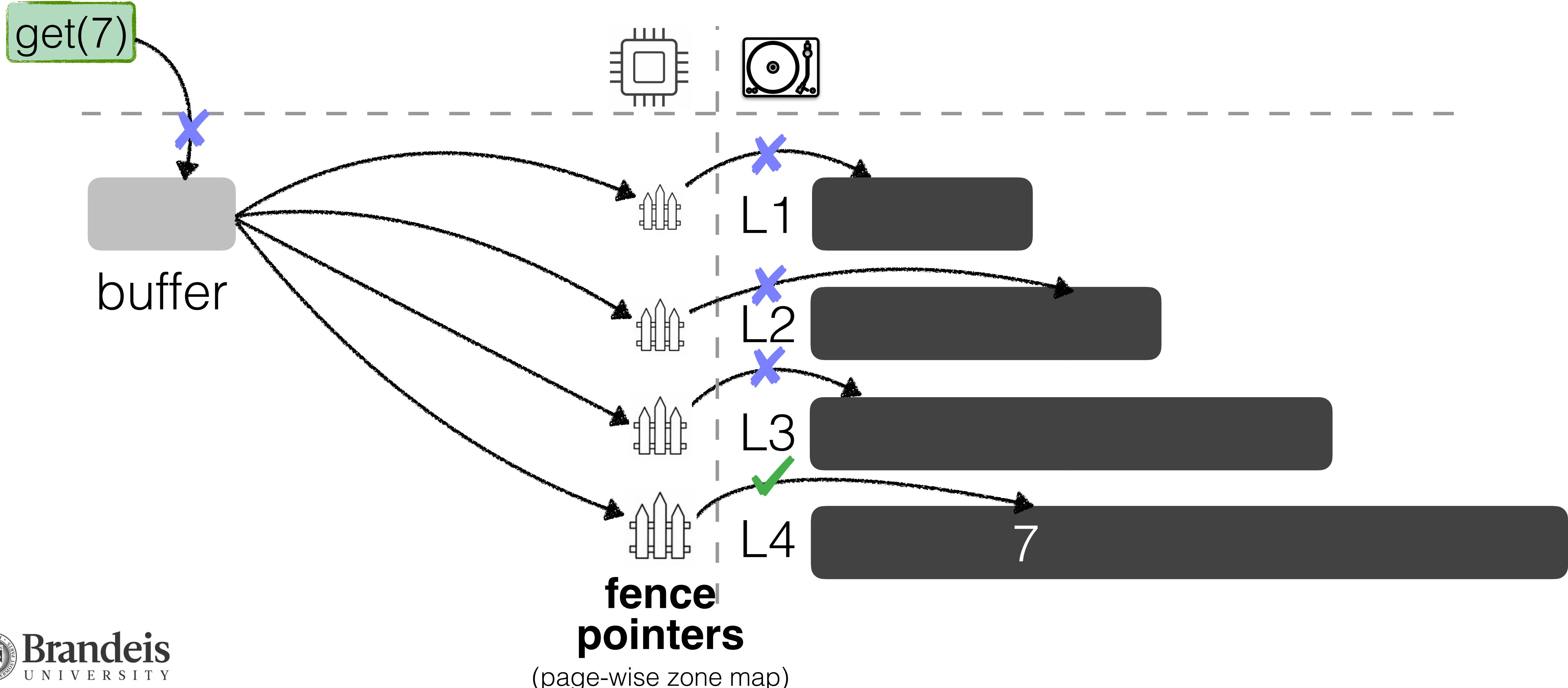
Looking for a specific key



P : pages in buffer
 B : entries/page
 L : #levels
 T : size ratio
 N : #entries

Point lookup cost

Looking for a specific key



Point lookup cost

Looking for a specific key



leveled LSM-tree

**fence
pointers**

(page-wise zone map)

Fence pointers

limit #I/Os per sorted run (level) = 1

Cost of a point lookup = L

Point lookup cost

Looking for a specific key

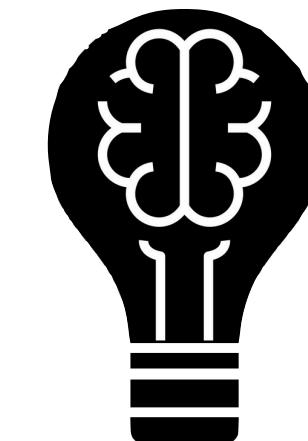


leveled LSM-tree

**fence
pointers**
(page-wise zone map)

limit #I/Os per sorted run (level) = 1

Cost of a point lookup = L



Thought Experiment 3

What is the point lookup cost
for entries **not in the tree**?



Point lookup cost

Looking for a specific key

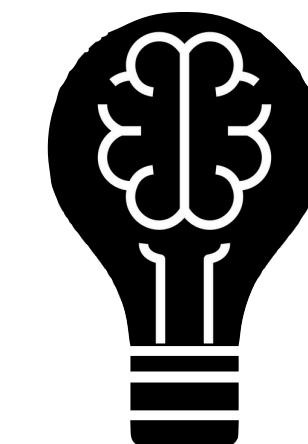


leveled LSM-tree

**fence
pointers**
(page-wise zone map)

limit #I/Os per sorted run (level) = 1

Cost of a point lookup = L



Thought Experiment 3

What is the point lookup cost
for entries **not in the tree**?

Same!



Cost analysis

Counting all I/Os

data structure	ingestion cost	point lookup cost	range lookup cost
Leveled LSM-tree	$O(L \cdot T / B)$	$O(L)$	
Tiered LSM-tree	$O(L / B)$		
B+-tree			
Sorted array			
Log			

Cost analysis

Counting all I/Os

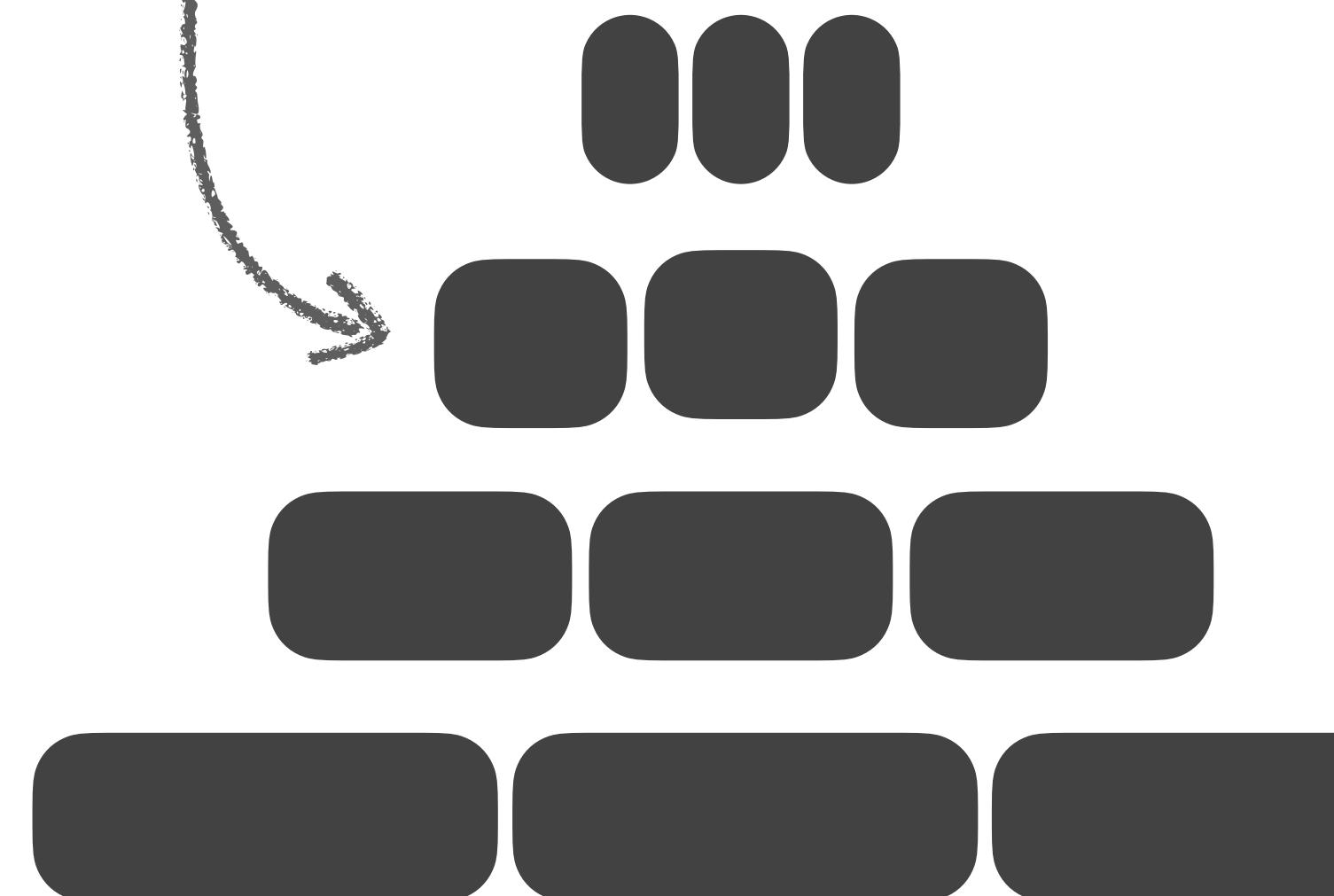
data structure	ingestion cost	point lookup cost*	range lookup cost
Leveled LSM-tree	$O(L \cdot T / B)$	$O(L)$	
Tiered LSM-tree	$O(L / B)$		
B+-tree			
Sorted array			
Log			

* with **fence pointers**

same cost for empty and non-empty lookups

Point lookup cost

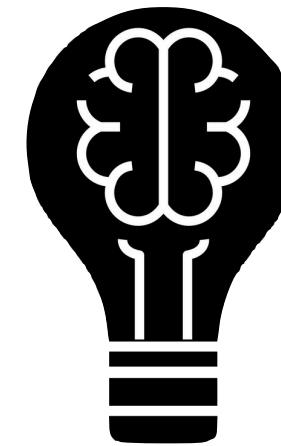
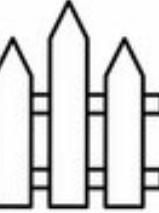
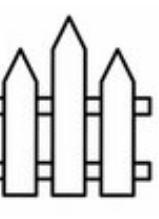
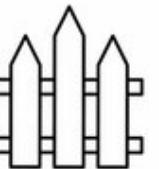
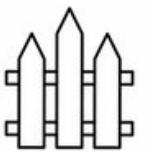
T sorted runs or tiers
per level



leveled LSM-tree

**fence
pointers**
(page-wise zone map)

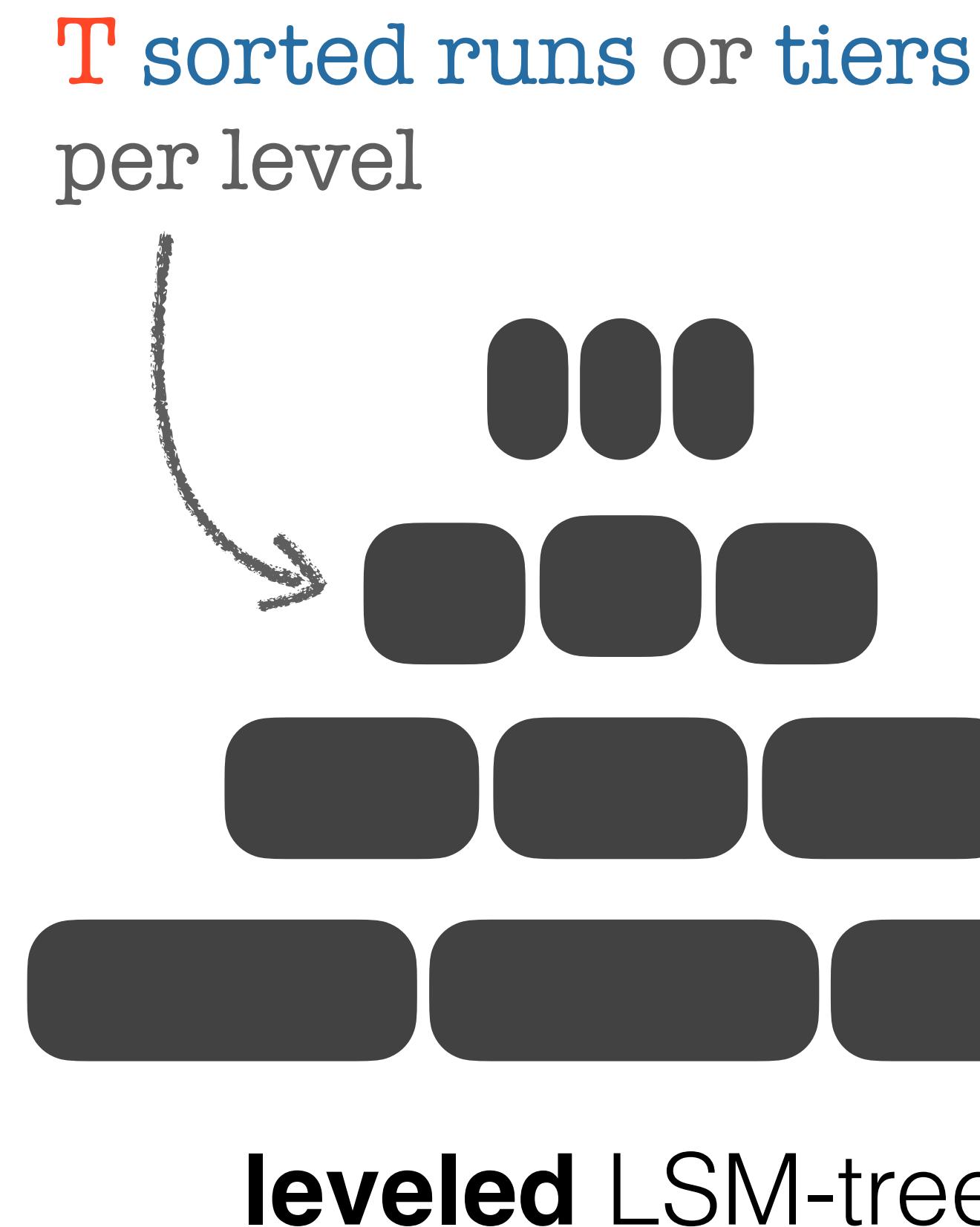
Looking for a specific key



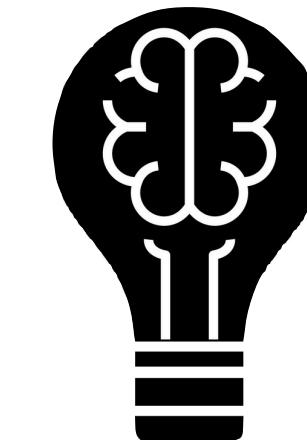
Thought Experiment 4
Point lookup cost in **tiered LSM**?



Point lookup cost



Looking for a specific key



Thought Experiment 4
Point lookup cost in **tiered LSM**?

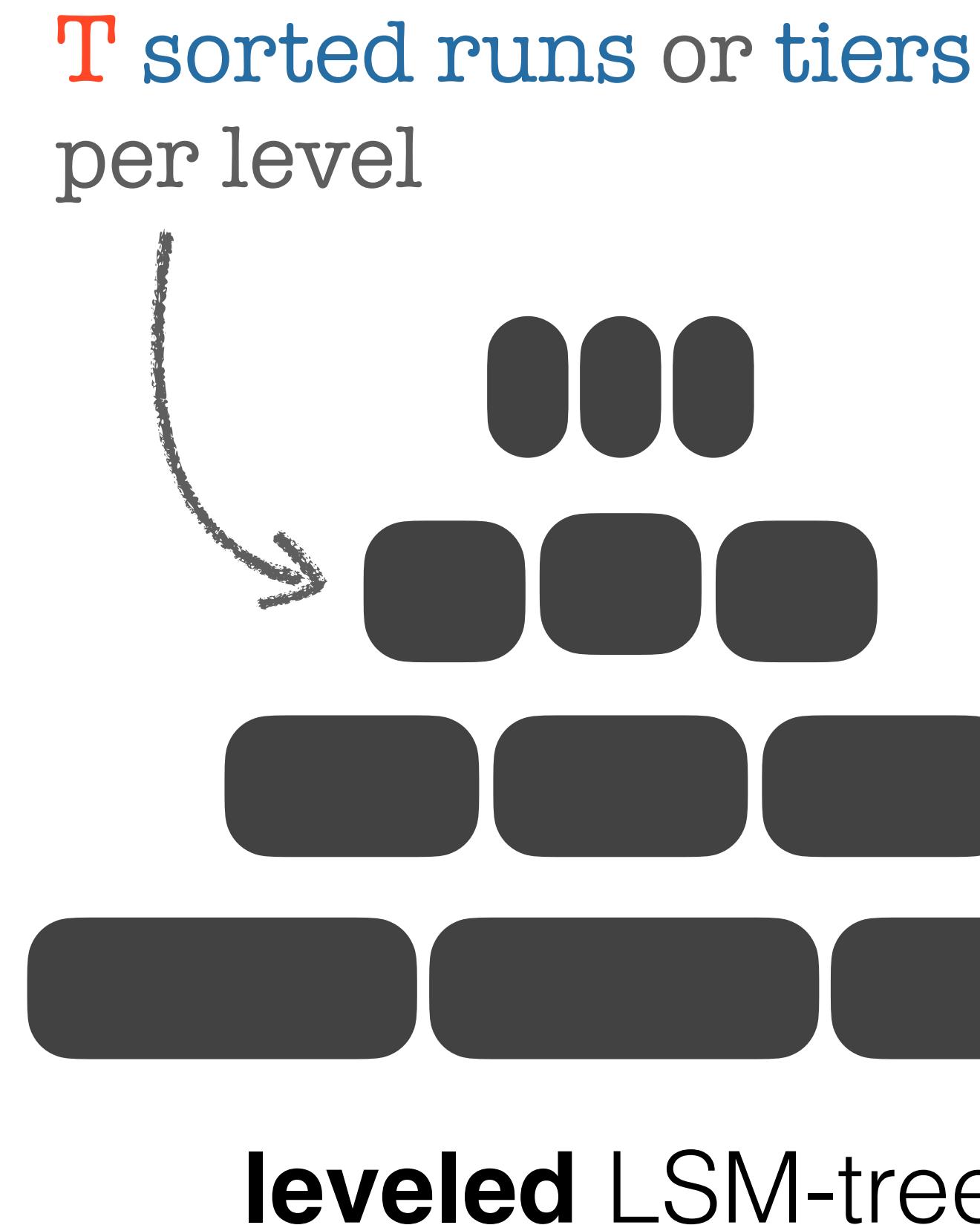
Fence pointers

limit #I/Os per sorted run = 1

T sorted runs per level

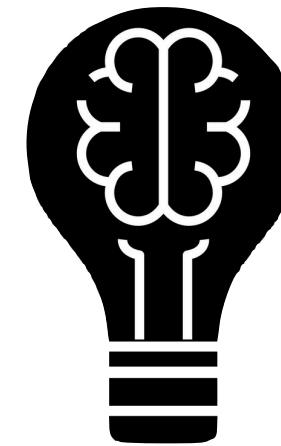
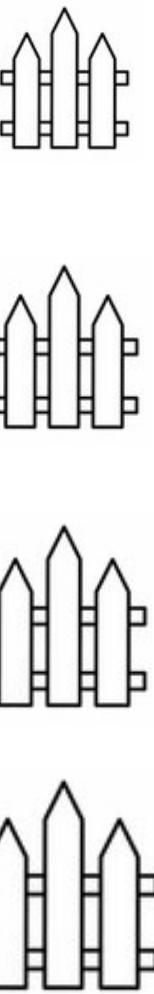


Point lookup cost



fence pointers
(page-wise zone map)

Looking for a specific key



Thought Experiment 4
Point lookup cost in **tiered LSM**?

Fence pointers

limit #I/Os per sorted run = 1

T sorted runs per level

Cost of a point lookup = **L • T**

Cost analysis

Counting all I/Os

data structure	ingestion cost	point lookup cost*	range lookup cost
Leveled LSM-tree	$O(L \cdot T / B)$	$O(\log_T(N))$	
Tiered LSM-tree	$O(L / B)$	$O(L \cdot T)$	
B+-tree			
Sorted array			
Log			

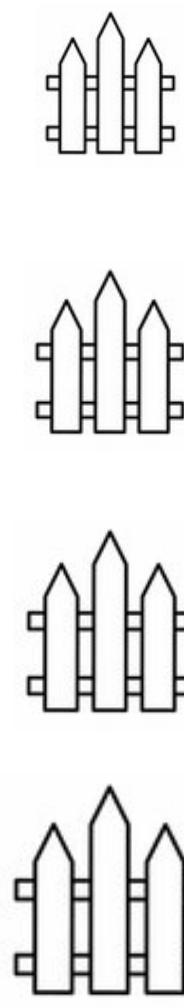
Lookups are still **very costly!** Can we do **better**?

Point lookup cost

Looking for a specific key



leveled LSM-tree



**fence
pointers**

(page-wise zone map)

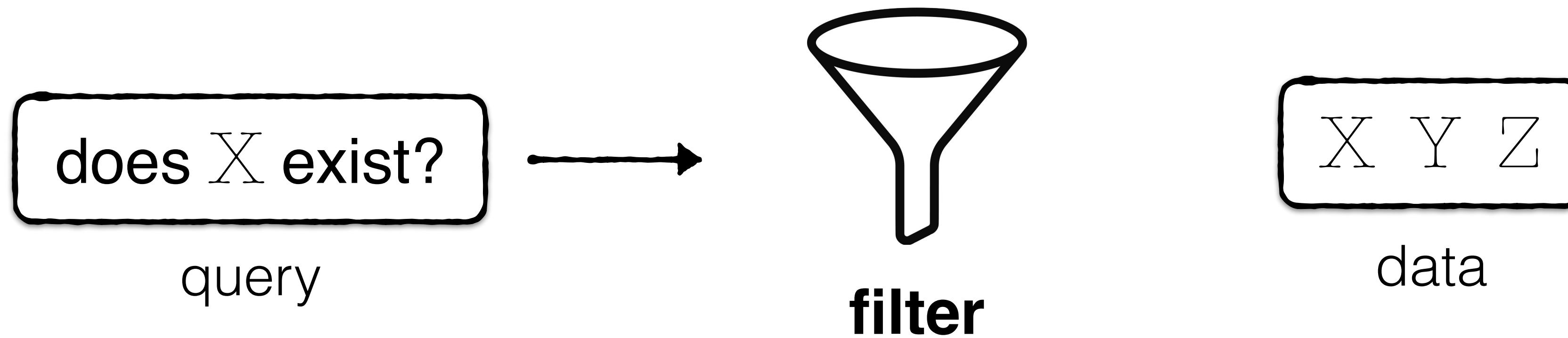


filter

What is a **filter**?

Answers membership queries

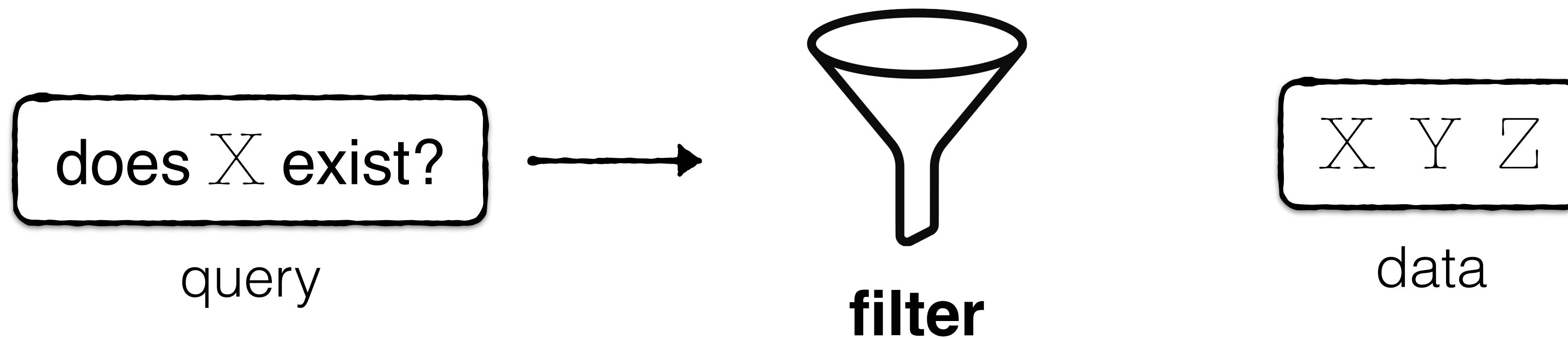
answers
membership queries



What is a **filter**?

Answers membership queries

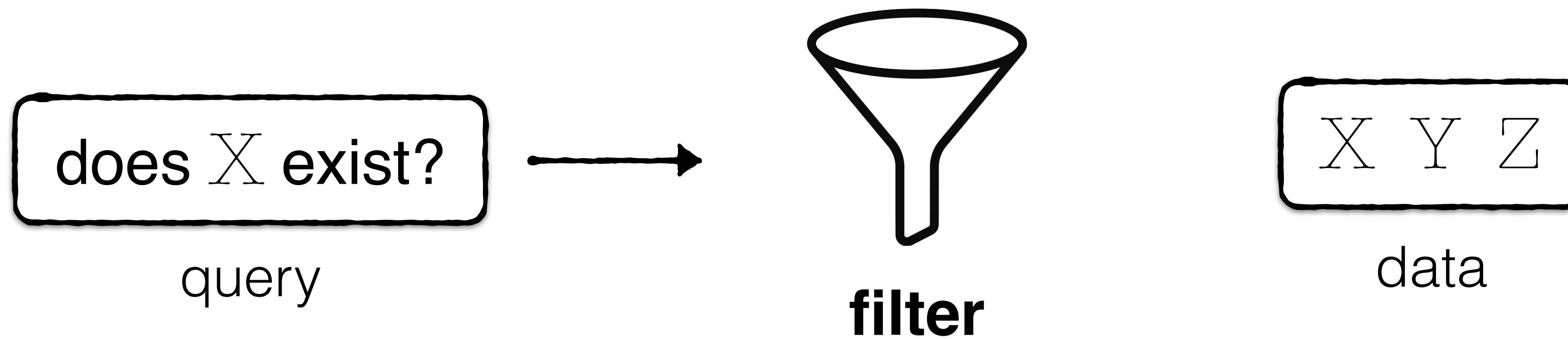
answers
membership queries



What is a **filter**?

Answers membership queries

answers
membership queries



**no false
negatives**

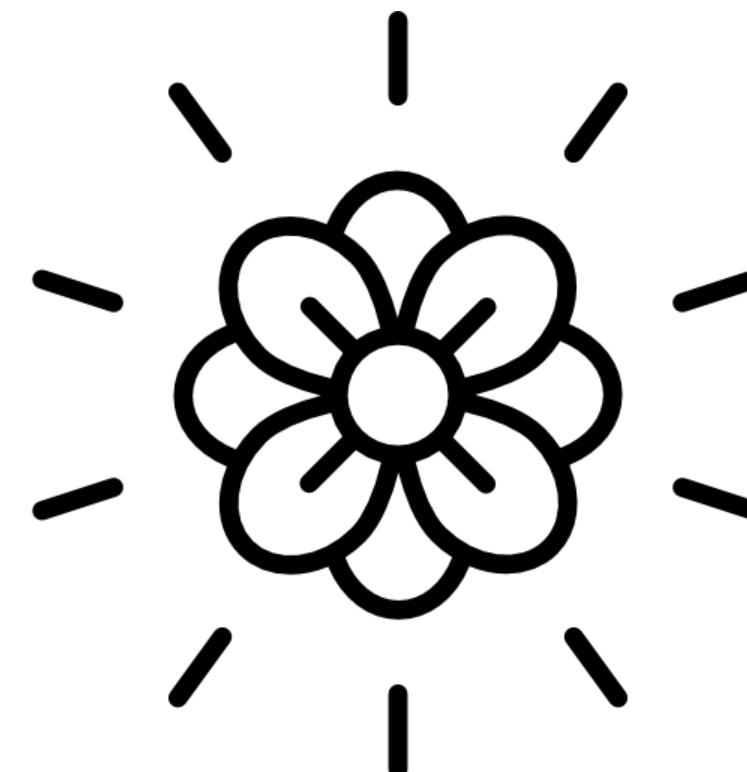


**false positives with
tunable probability**

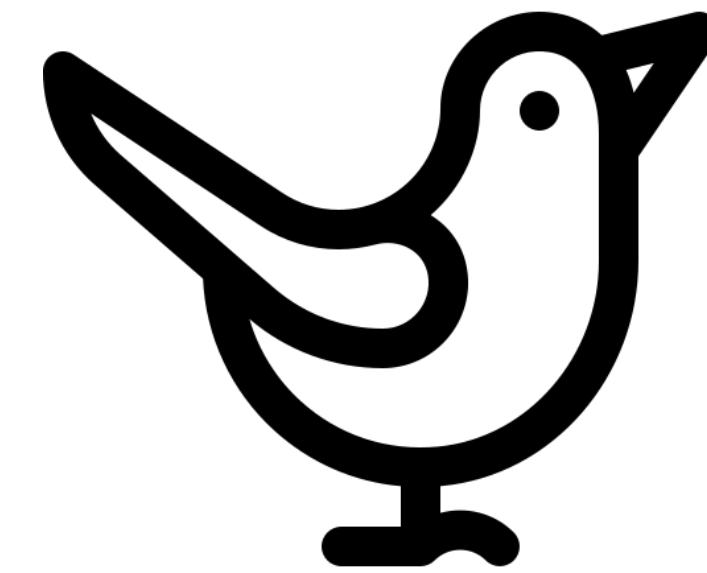


Types of filters?

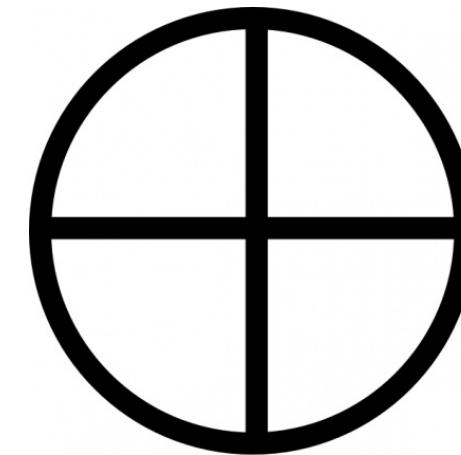
Point and range filters



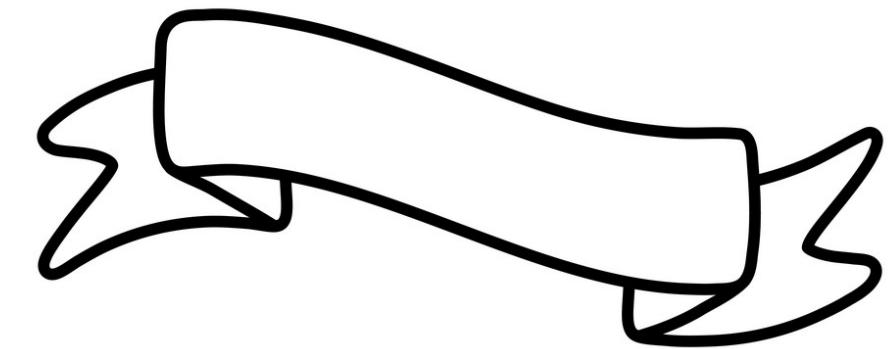
Bloom



Cuckoo



XOR



Ribbon

Bloom filter

Invented in 1970 by Burton Howard **Bloom**

Bloom filter

Invented in 1970 by Burton Howard **Bloom**

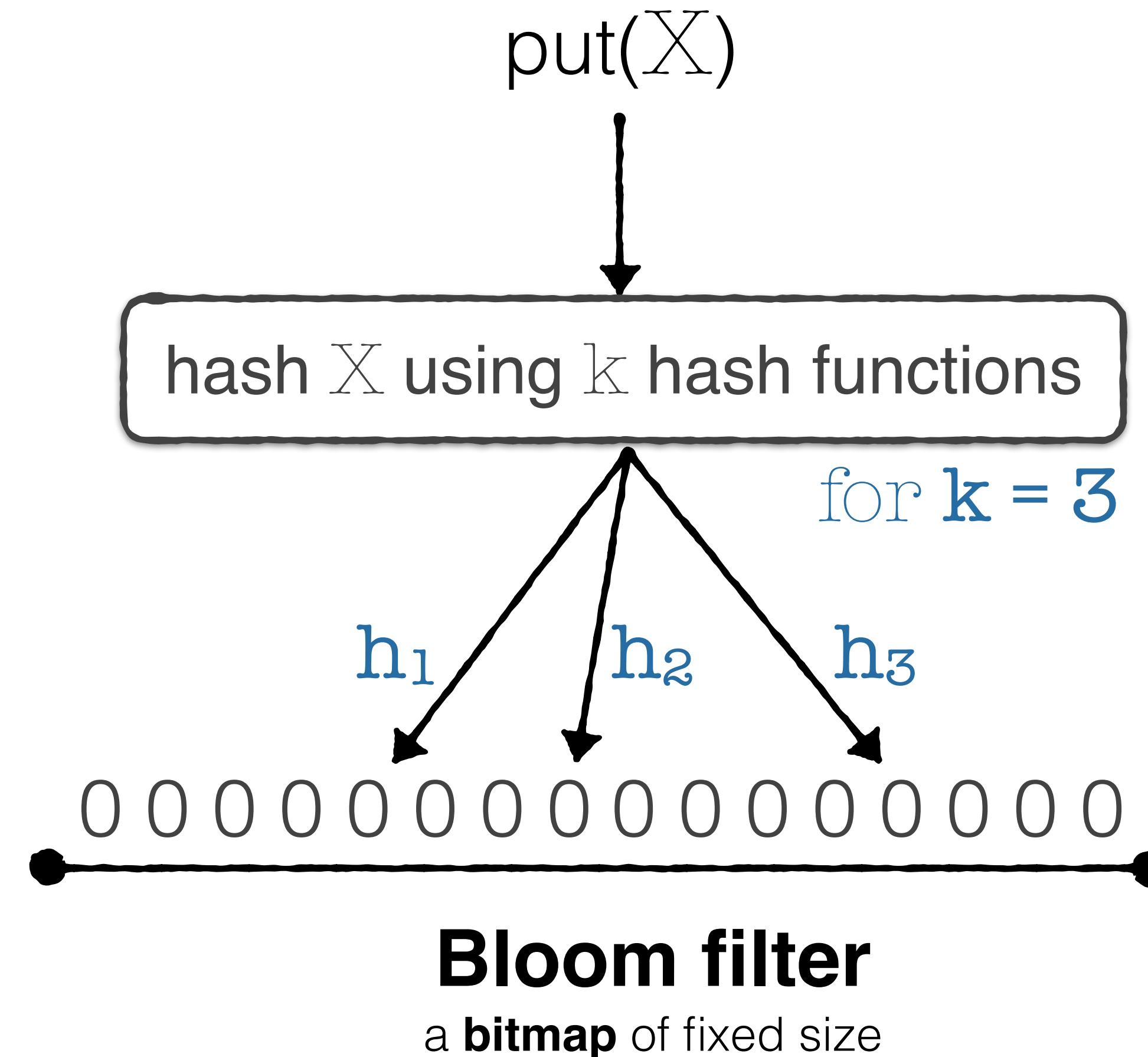


Bloom filter

a **bitmap** of fixed size

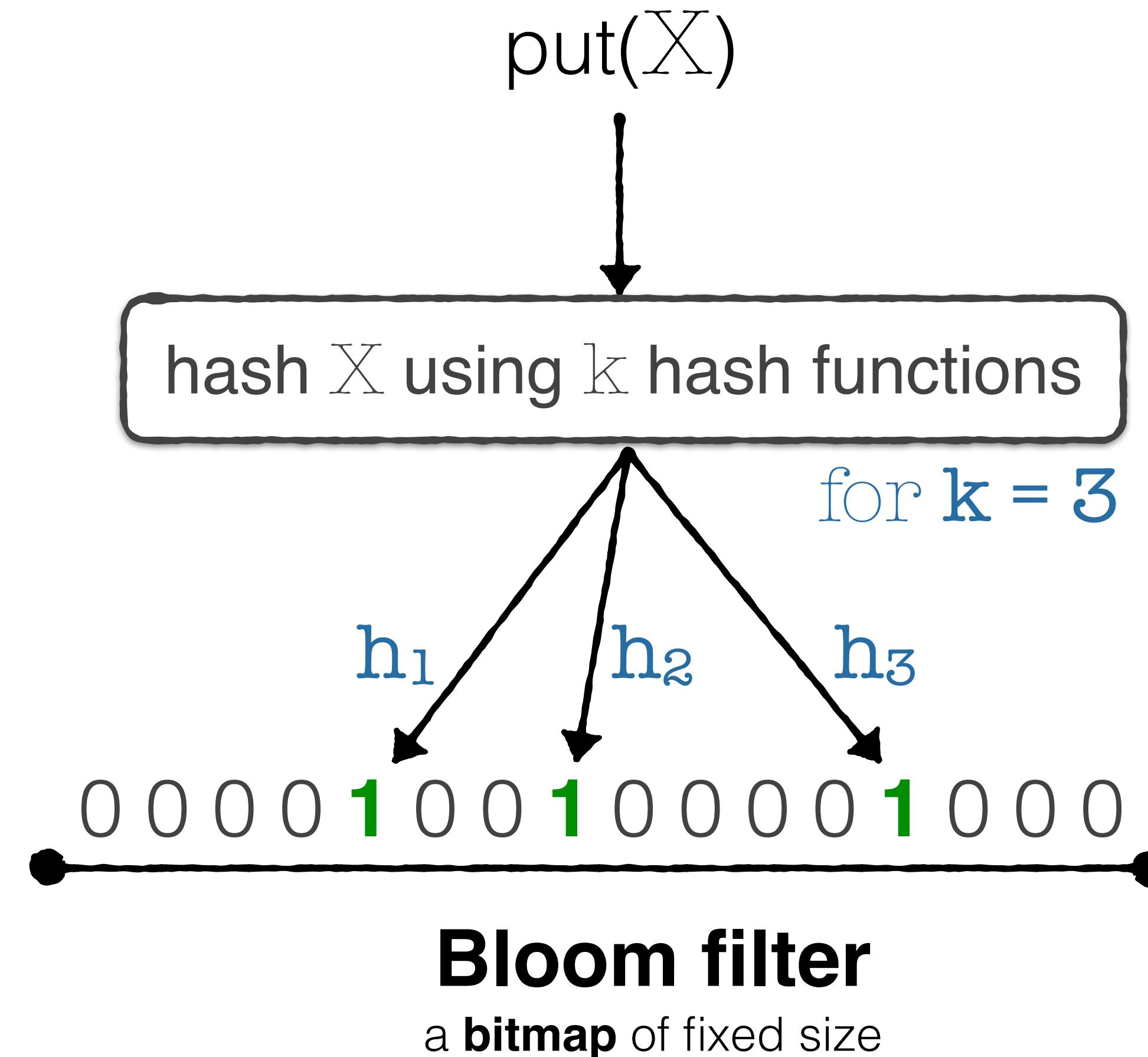
Bloom filter

Invented in 1970 by Burton Howard **Bloom**



Bloom filter

Invented in 1970 by Burton Howard **Bloom**



Bloom filter

Invented in 1970 by Burton Howard **Bloom**

get(X)



hash X using k hash functions

for $k = 3$

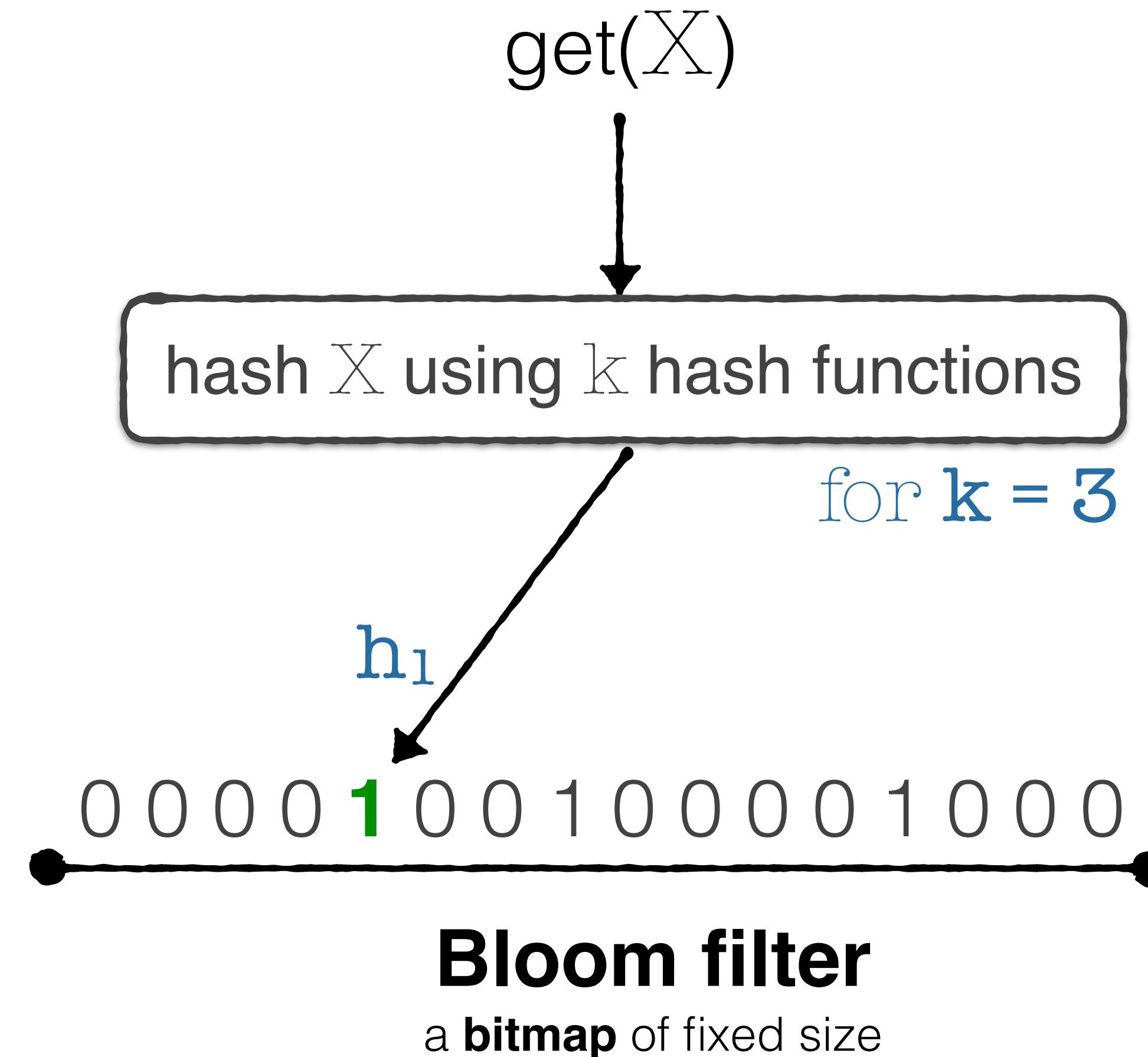
0 0 0 0 1 0 0 1 0 0 0 0 1 0 0 0

Bloom filter

a **bitmap** of fixed size

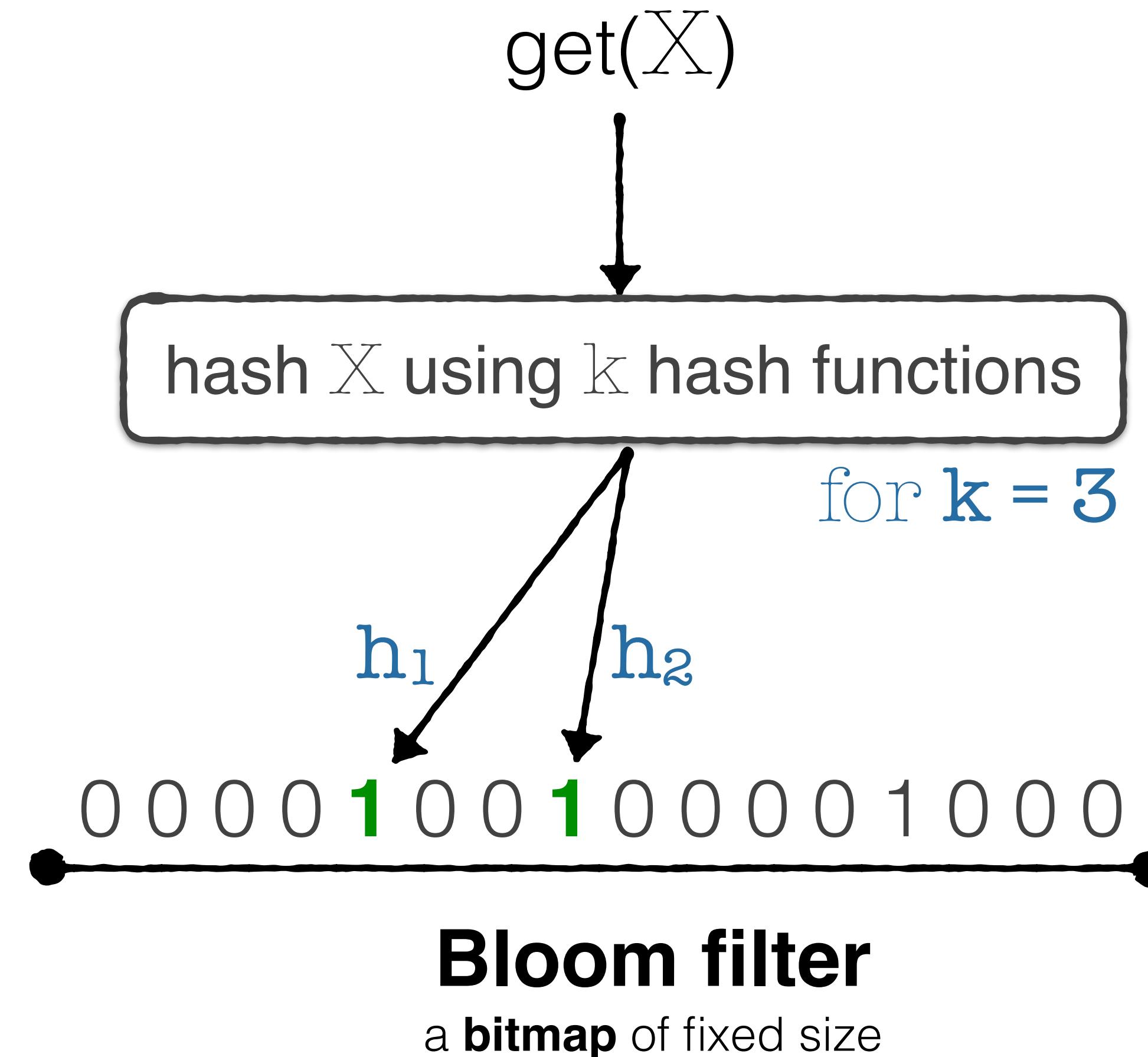
Bloom filter

Invented in 1970 by Burton Howard **Bloom**



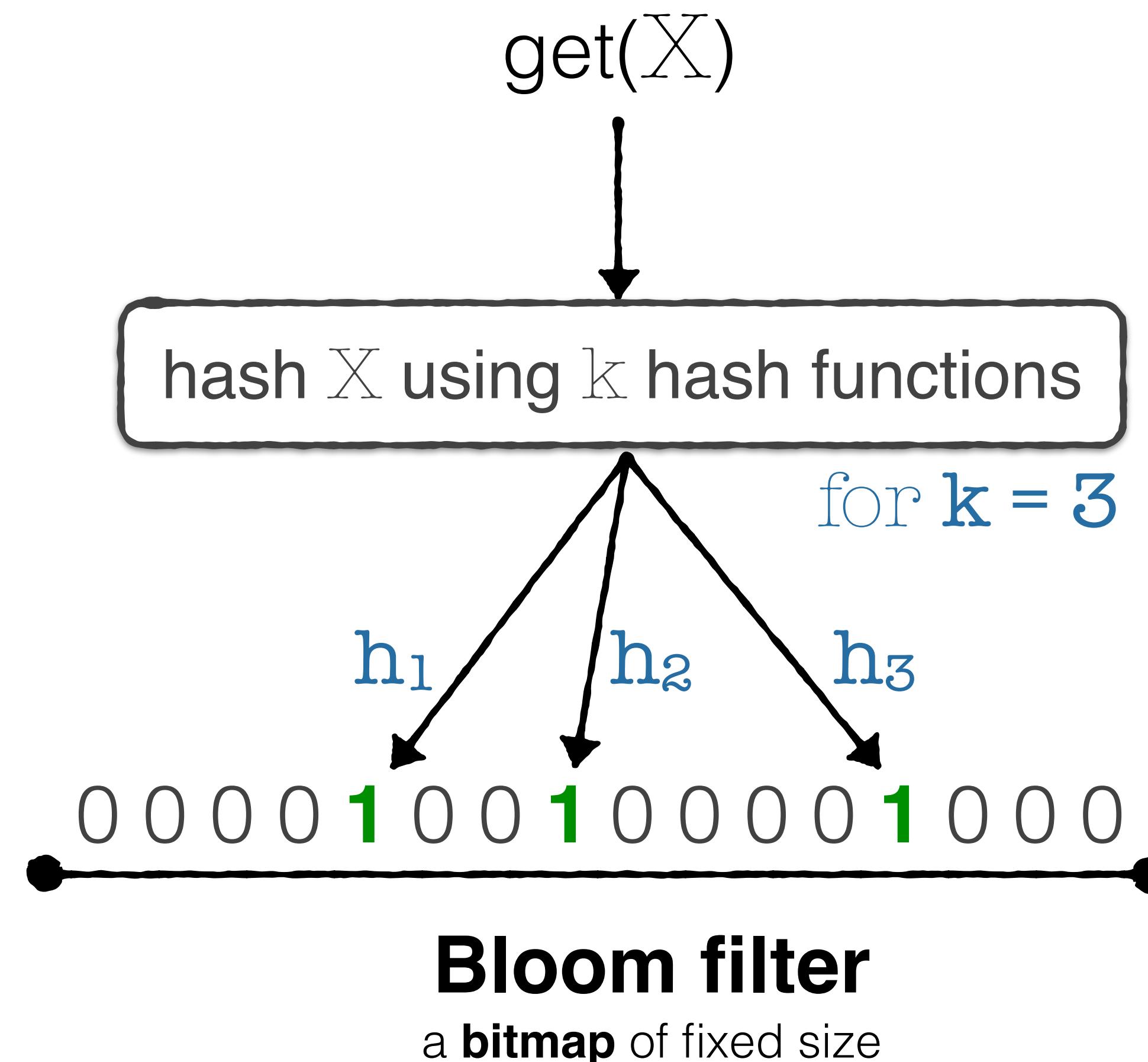
Bloom filter

Invented in 1970 by Burton Howard **Bloom**



Bloom filter

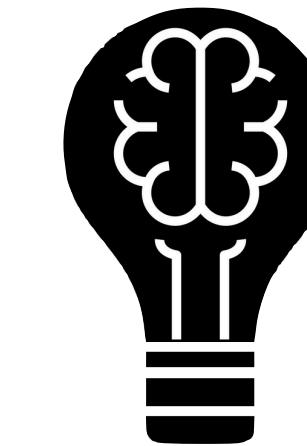
Invented in 1970 by Burton Howard **Bloom**



Gotcha

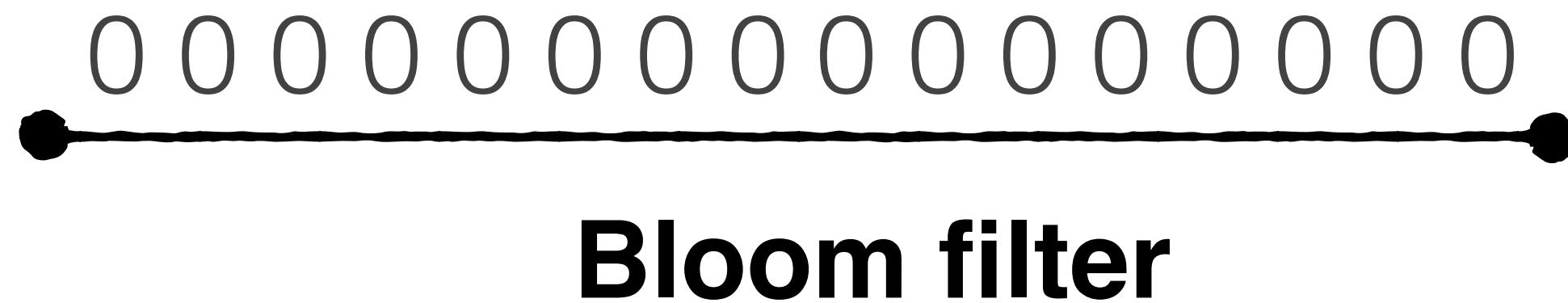
Bloom filter

Invented in 1970 by Burton Howard **Bloom**



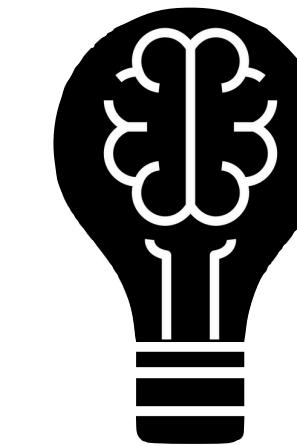
Thought Experiment 5

When does a Bloom filter return a **false positive** result?



Bloom filter

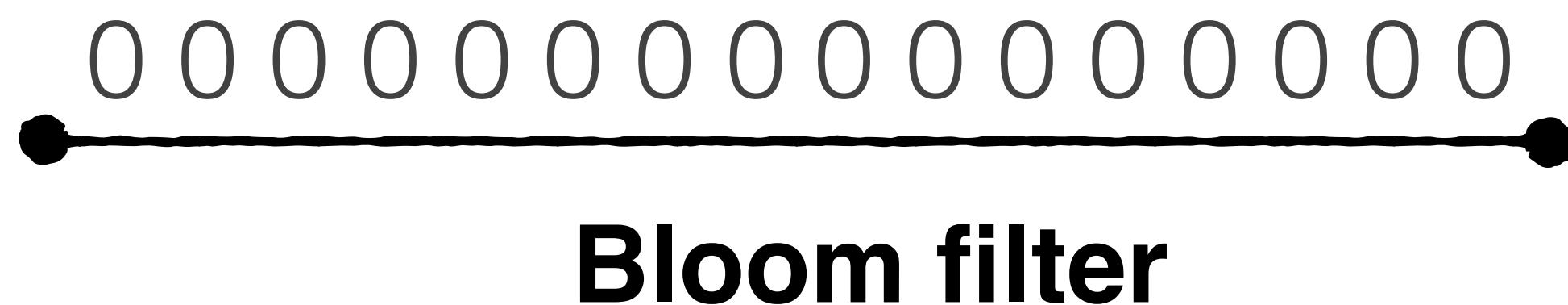
Invented in 1970 by Burton Howard **Bloom**



Thought Experiment 5

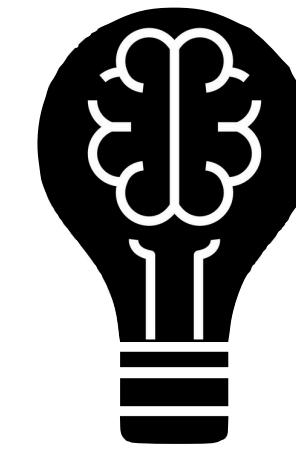
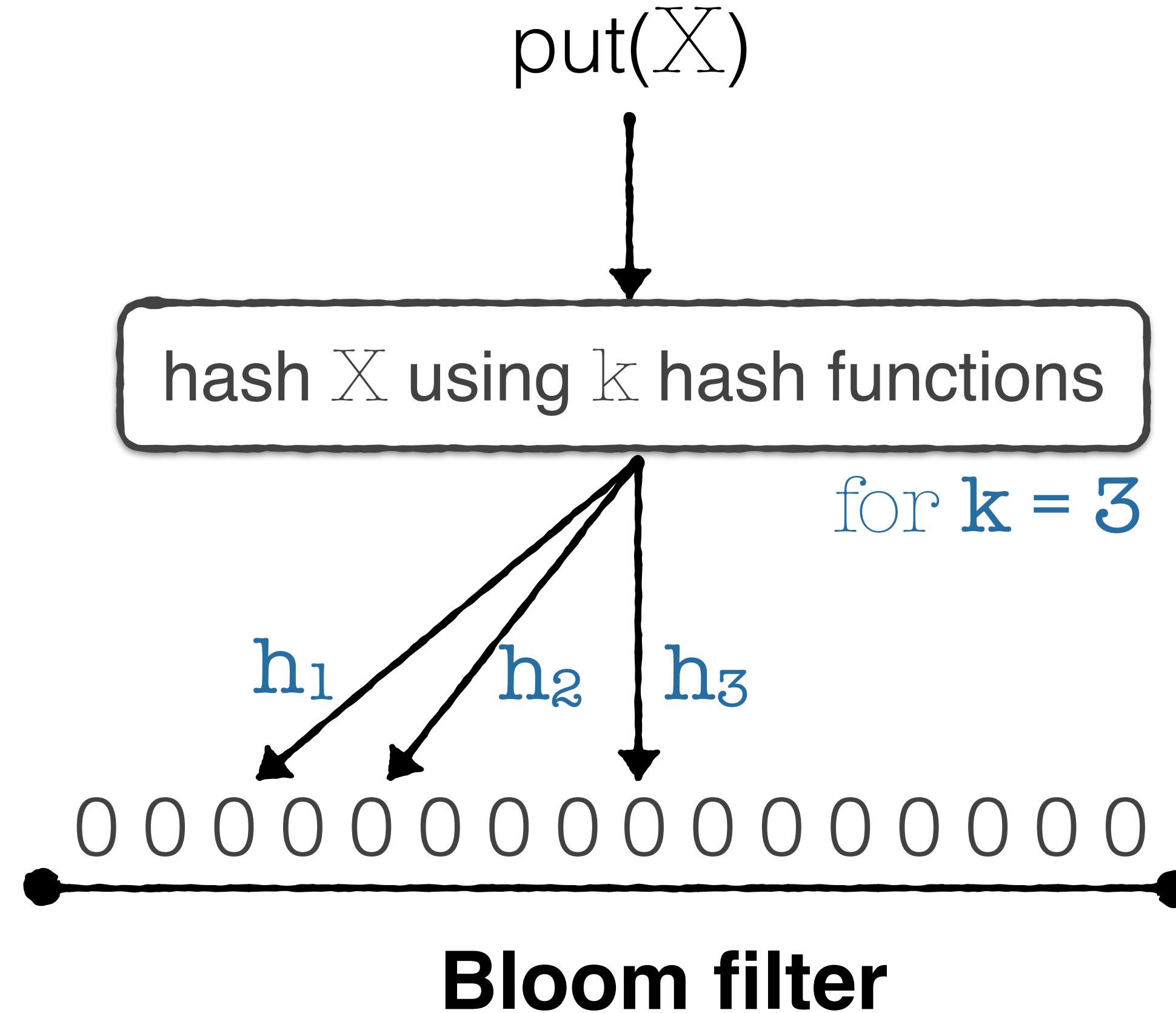
When does a Bloom filter return a **false positive** result?

$\text{put}(X) \longrightarrow \text{hashes bits } \{3,5,9\}$



Bloom filter

Invented in 1970 by Burton Howard **Bloom**



Thought Experiment 5

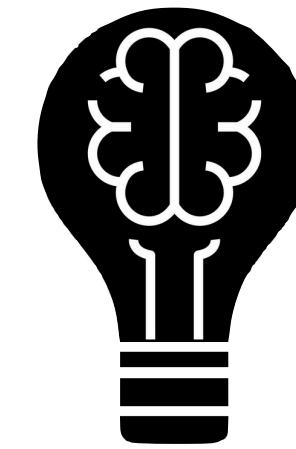
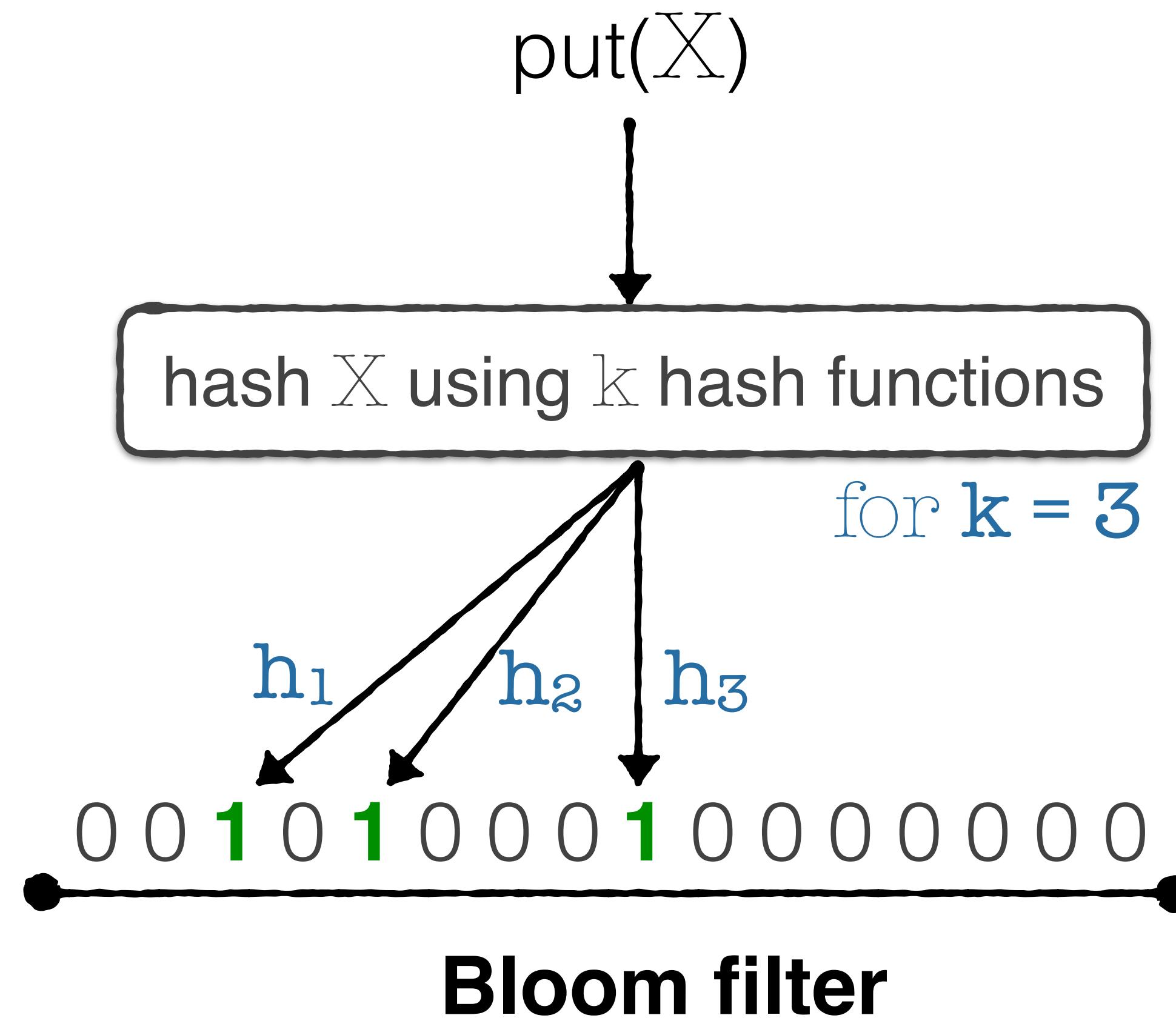
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Bloom filter

Invented in 1970 by Burton Howard **Bloom**



Thought Experiment 5

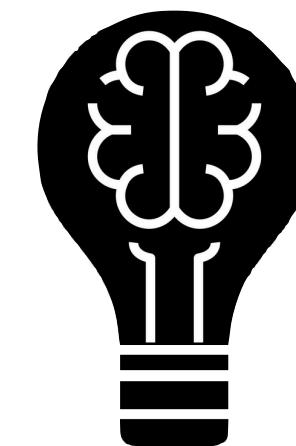
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Bloom filter

Invented in 1970 by Burton Howard **Bloom**

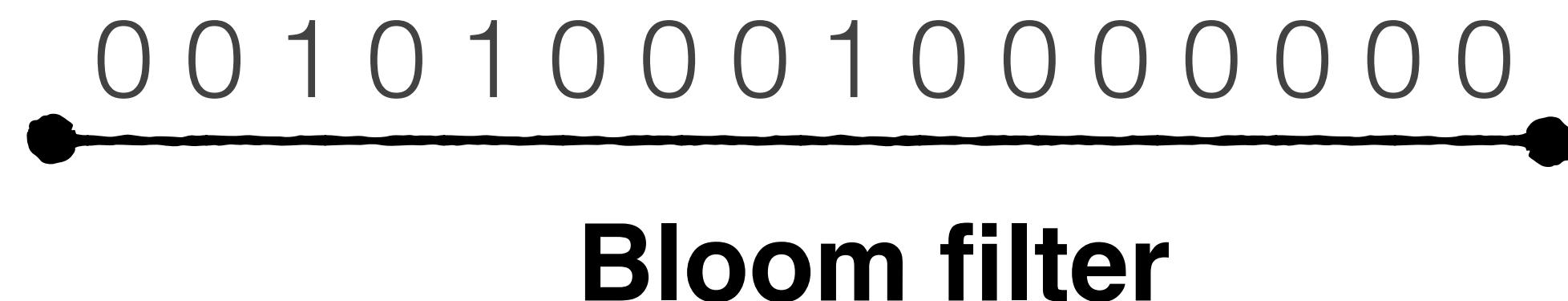


Thought Experiment 5

When does a Bloom filter return a **false positive** result?

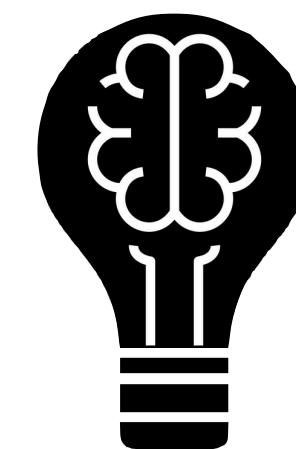
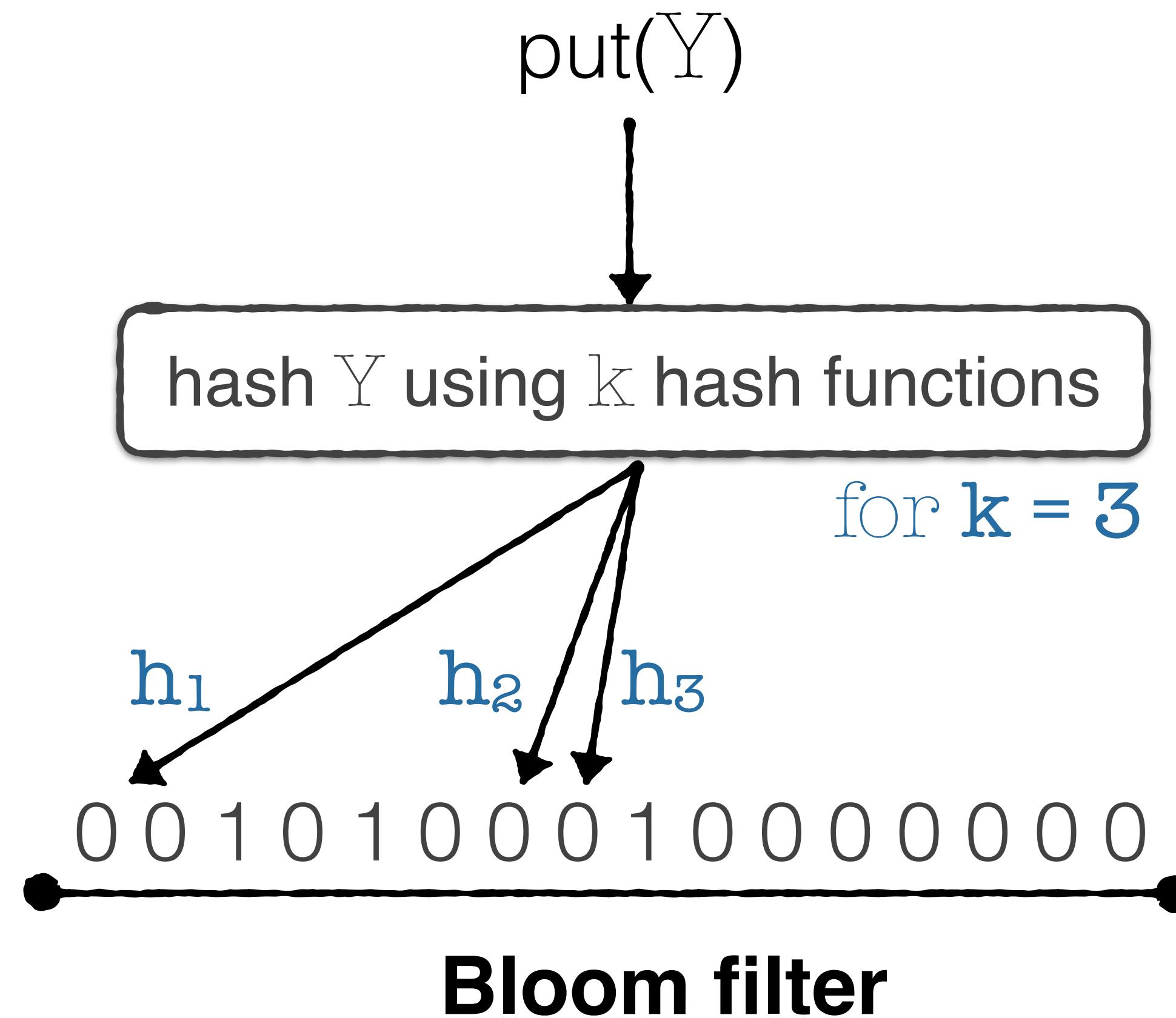
$\text{put}(X) \rightarrow \text{hashes bits } \{3,5,9\}$

$\text{put}(Y) \rightarrow \text{hashes bits } \{1,7,8\}$



Bloom filter

Invented in 1970 by Burton Howard **Bloom**



Thought Experiment 5

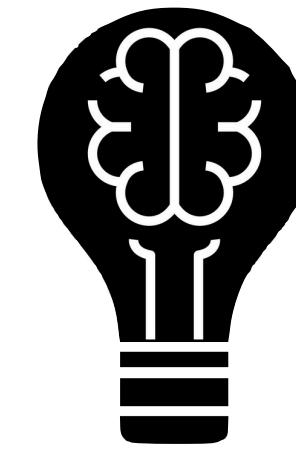
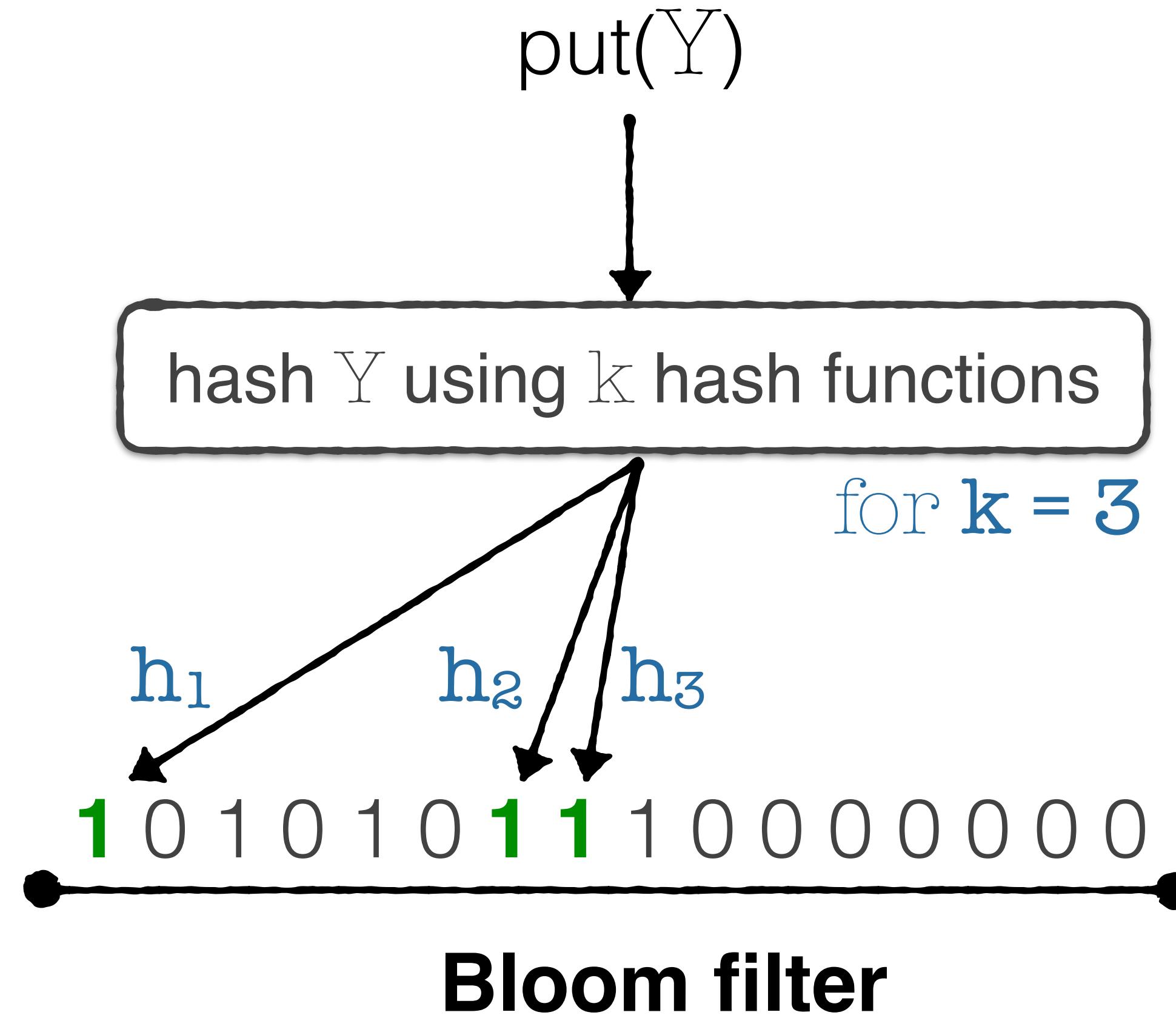
When does a Bloom filter return a **false positive** result?

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Bloom filter

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Thought Experiment 5

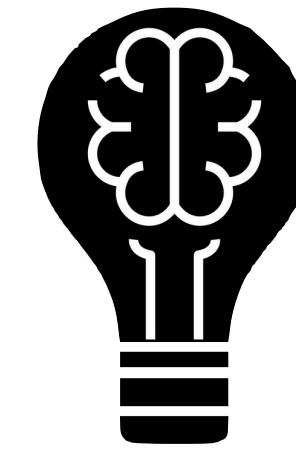
When does a Bloom filter return a **false positive** result?

$\text{put}(X) \rightarrow \text{hashes bits } \{3,5,9\}$

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Bloom filter

Invented in 1970 by Burton Howard **Bloom**



Thought Experiment 5

When does a Bloom filter return a **false positive** result?



Bloom filter

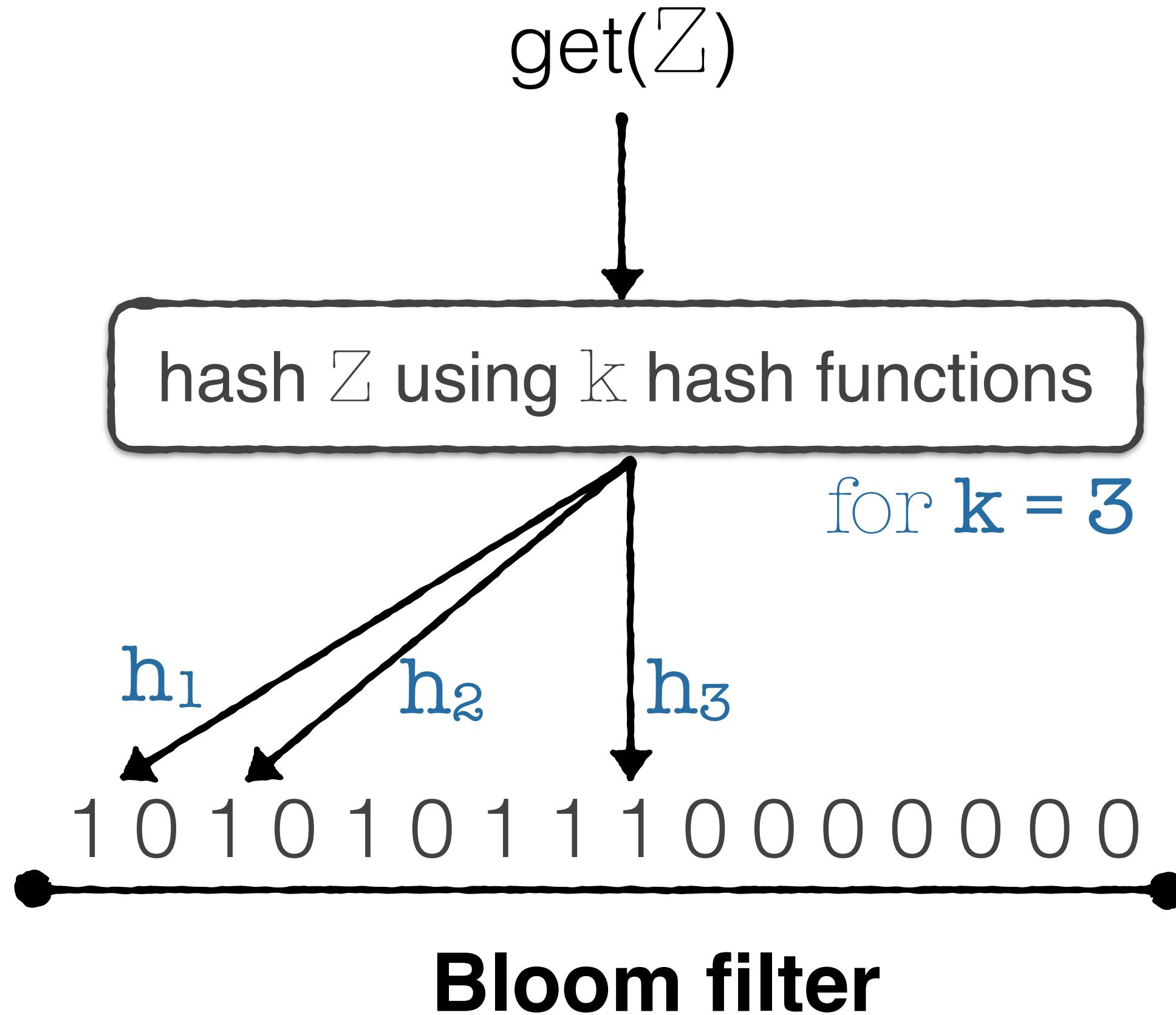
$\text{put}(X) \rightarrow \text{hashes bits } \{3,5,9\}$

$\text{put}(Y) \rightarrow \text{hashes bits } \{1,7,8\}$

$\text{get}(Z) \rightarrow \text{hashes bits } \{1,3,9\}$

Bloom filter

Invented in 1970 by Burton Howard **Bloom**



Thought Experiment 5

When does a Bloom filter return a **false positive** result?

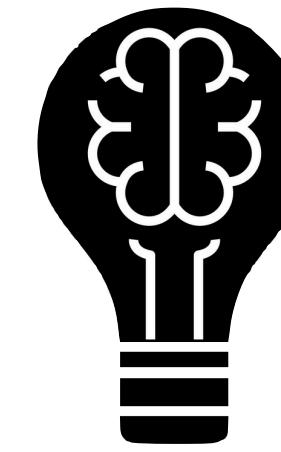
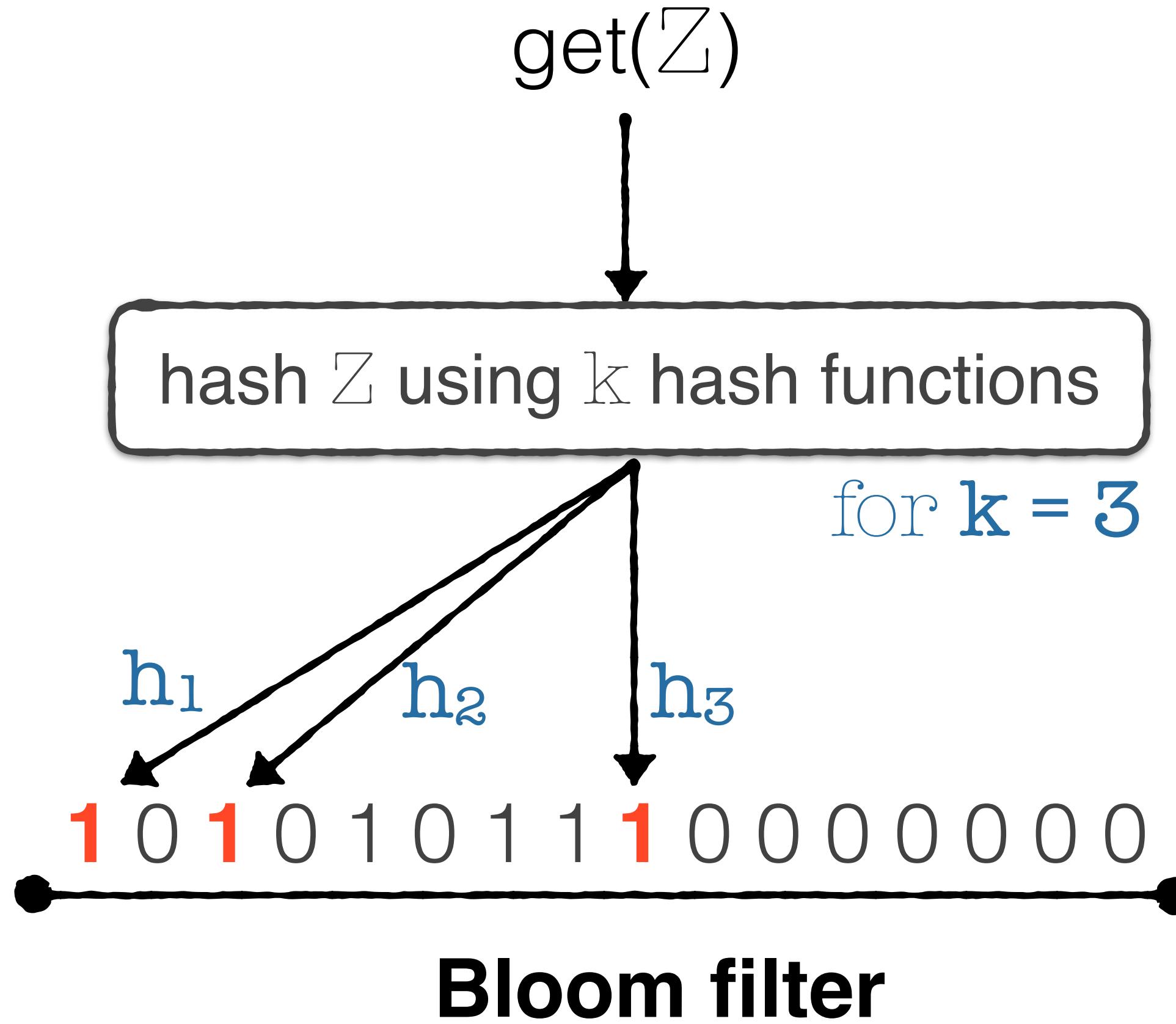
`put(X)` → hashes bits {3,5,9}

`put(Y)` → hashes bits {1,7,8}

`get(Z)` → hashes bits {1,3,9}

Bloom filter

Invented in 1970 by Burton Howard **Bloom**



Thought Experiment 5

When does a Bloom filter return a **false positive** result?

`put(X) → hashes bits {3,5,9}`

`put(Y) → hashes bits {1,7,8}`

`get(Z) → hashes bits {1,3,9}`



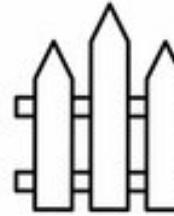
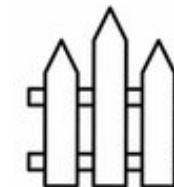
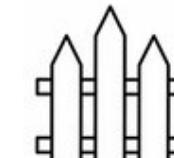
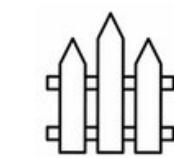
false positive

Point lookup cost

Looking for a specific key



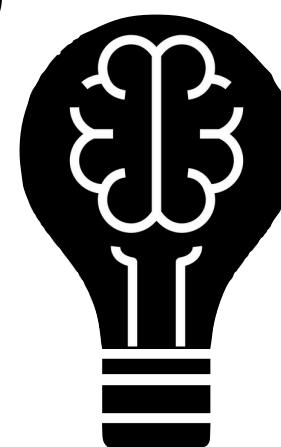
leveled LSM-tree



**fence
pointers**
(page-wise zone map)



filter
(one filter
per sorted run)



Thought Experiment 6

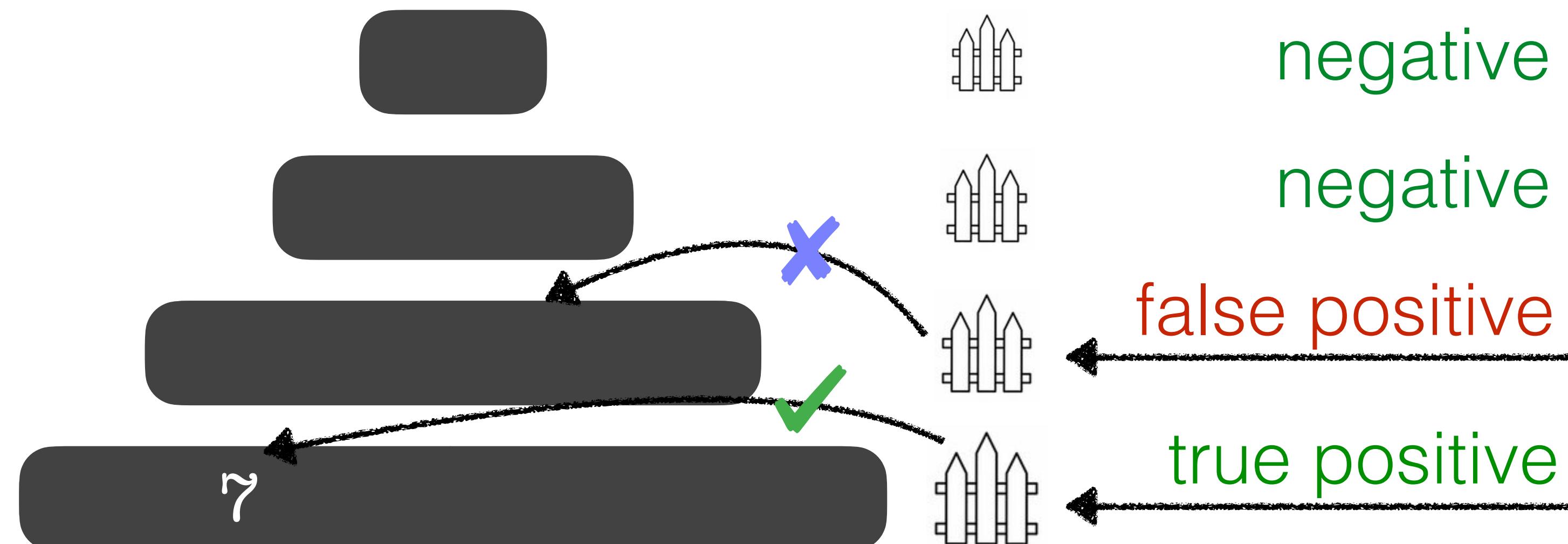
Cost of a **non-empty point lookup?**



Point lookup cost

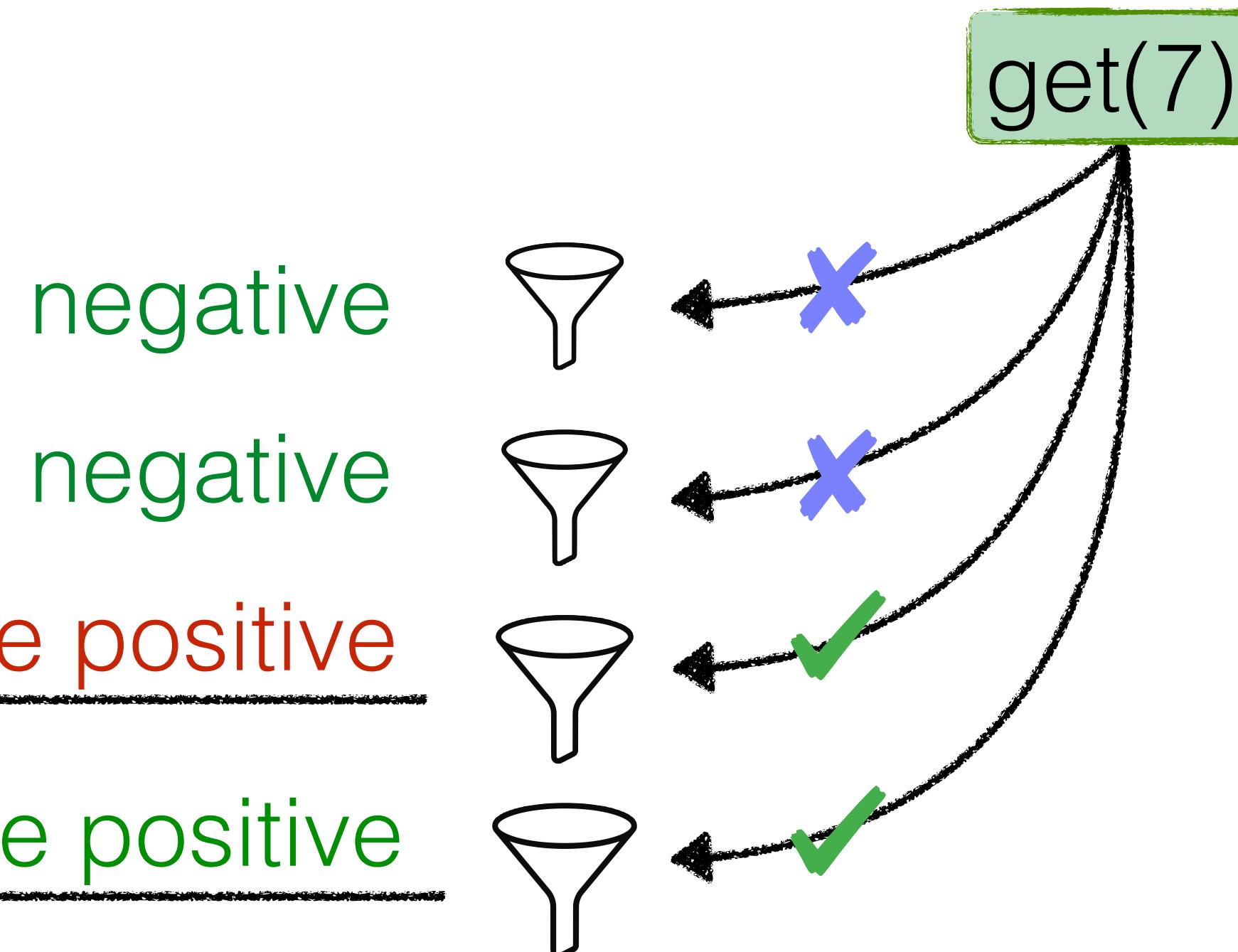
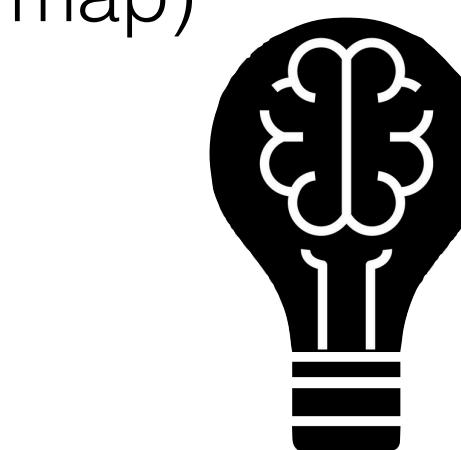
Looking for a specific key

ϕ = false positive rate of the filter



leveled LSM-tree

**fence
pointers**
(page-wise zone map)



filter
(one filter
per sorted run)

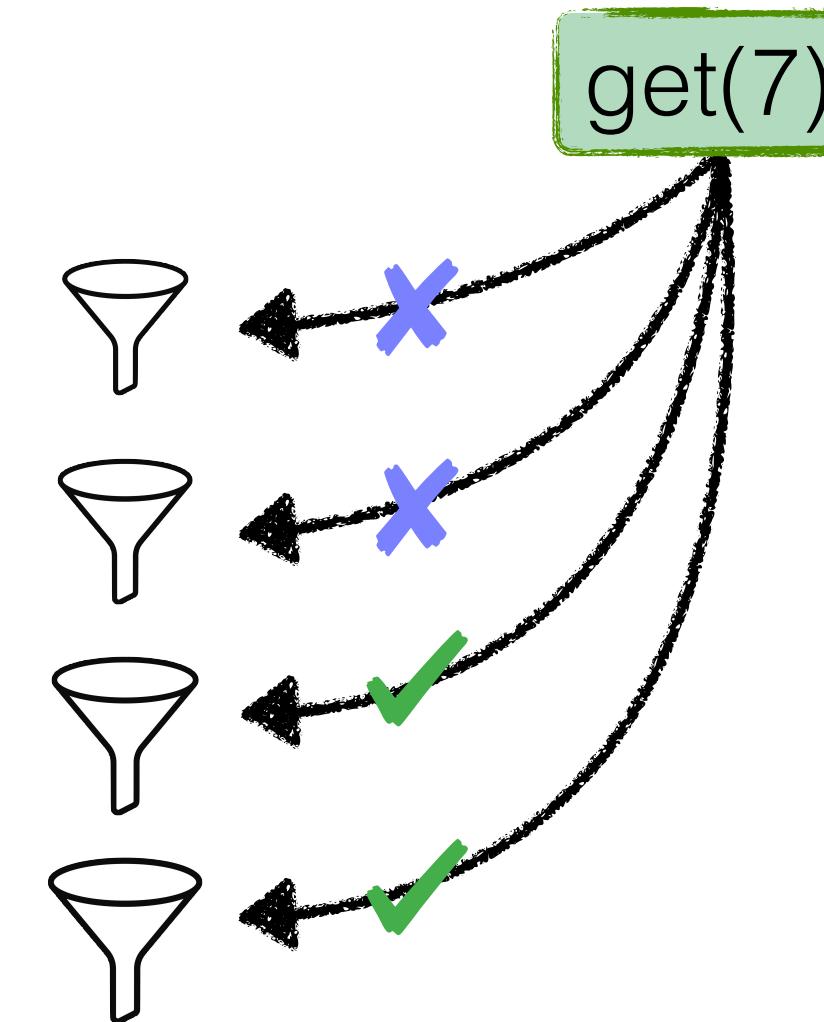
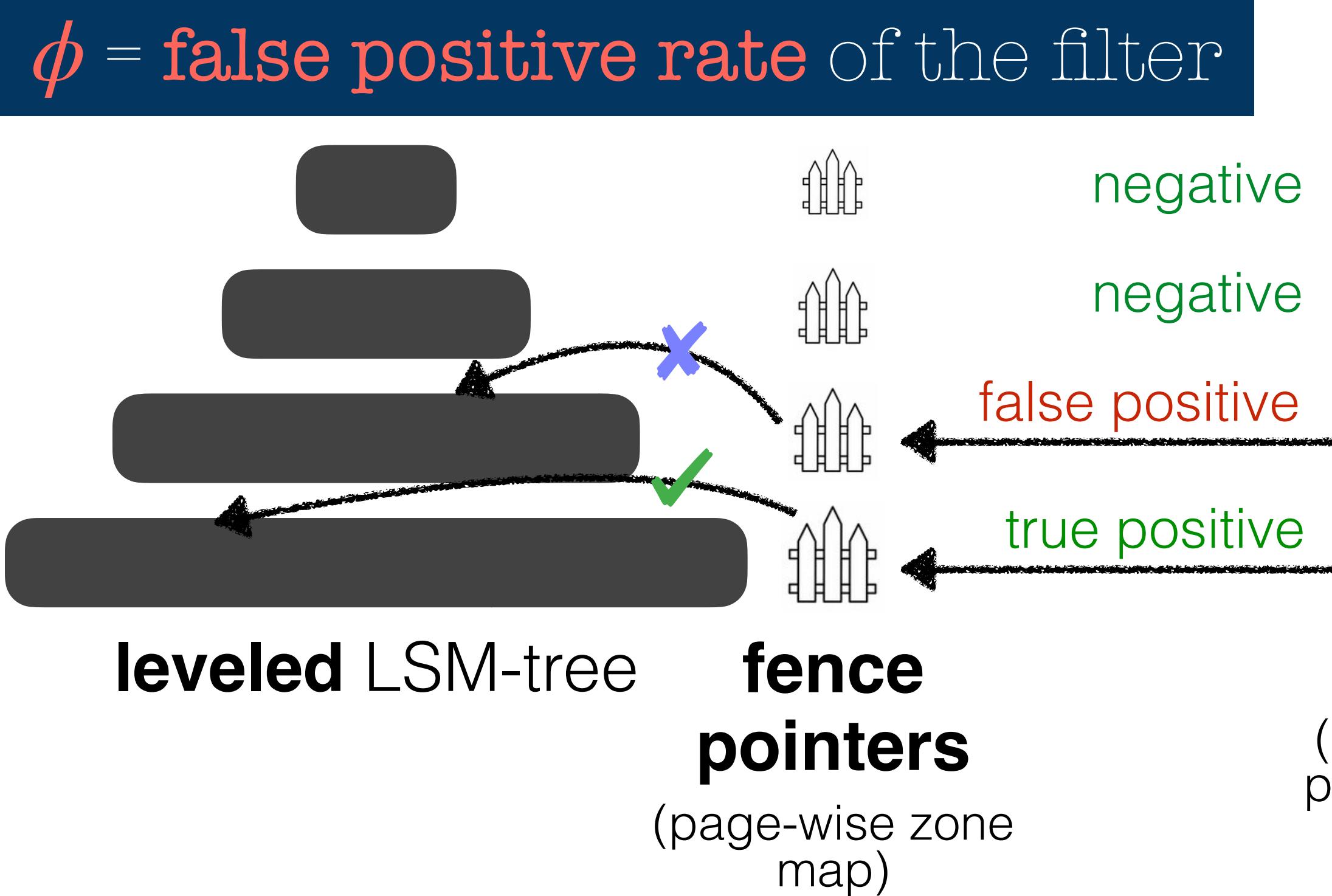
Thought Experiment 6

Cost of a **non-empty point lookup?**



Point lookup cost

Looking for a specific key

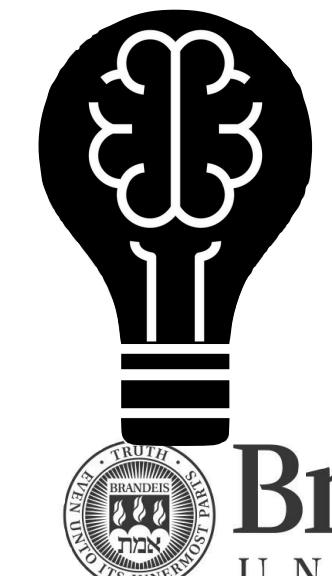


1 I/O for the sorted run (level) containing the data

+

1 I/O with probability ϕ for all other sorted runs

1 sorted runs per level



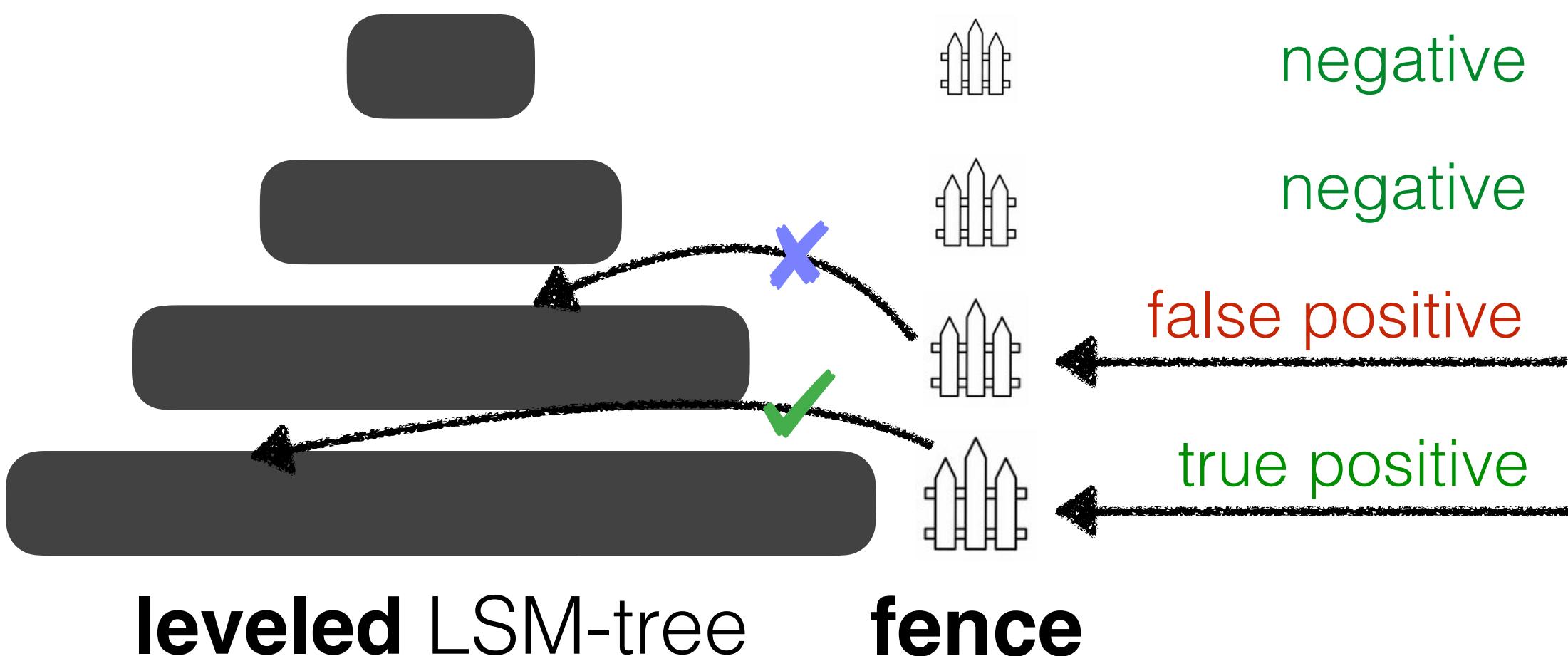
Thought Experiment 6
Cost of a **non-empty point lookup?**



Point lookup cost

Looking for a specific key

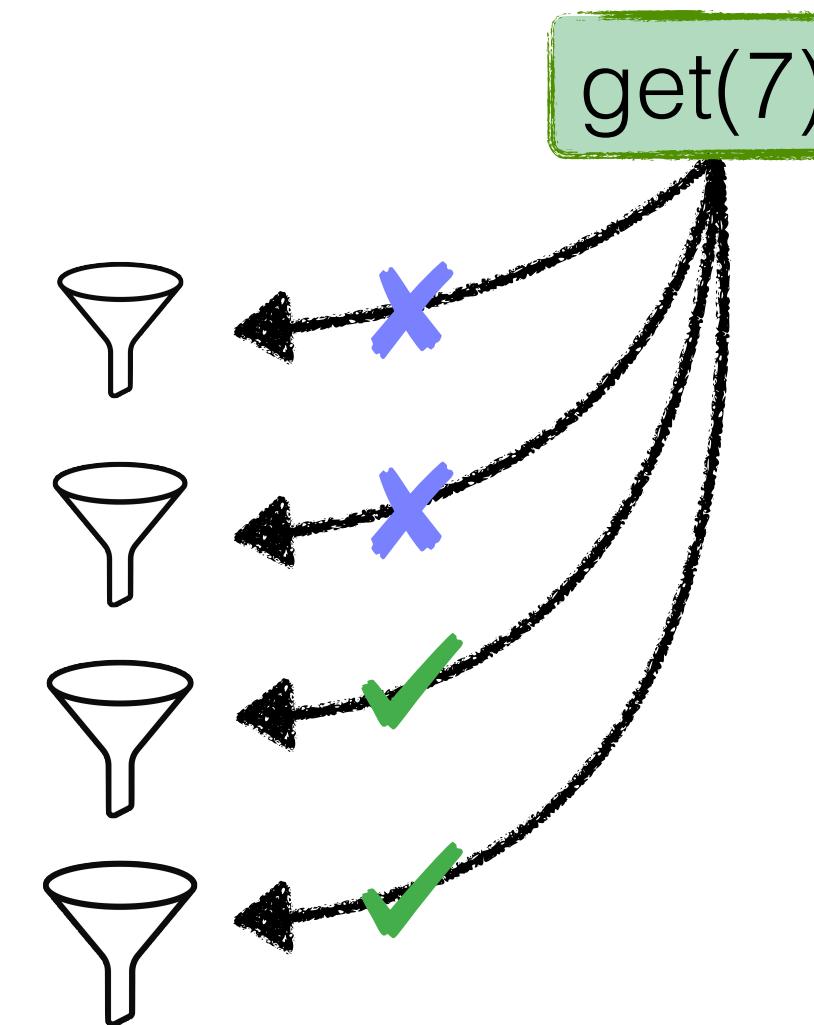
ϕ = false positive rate of the filter



leveled LSM-tree

**fence
pointers**
(page-wise zone
map)

filter
(one filter
per sorted
run)



1 I/O for the sorted run (level)
containing the data

+

1 I/O with probability ϕ for
all other sorted runs

1 sorted runs per level

Cost of non-empty point lookup

$$= 1 + \phi \cdot (L-1)$$

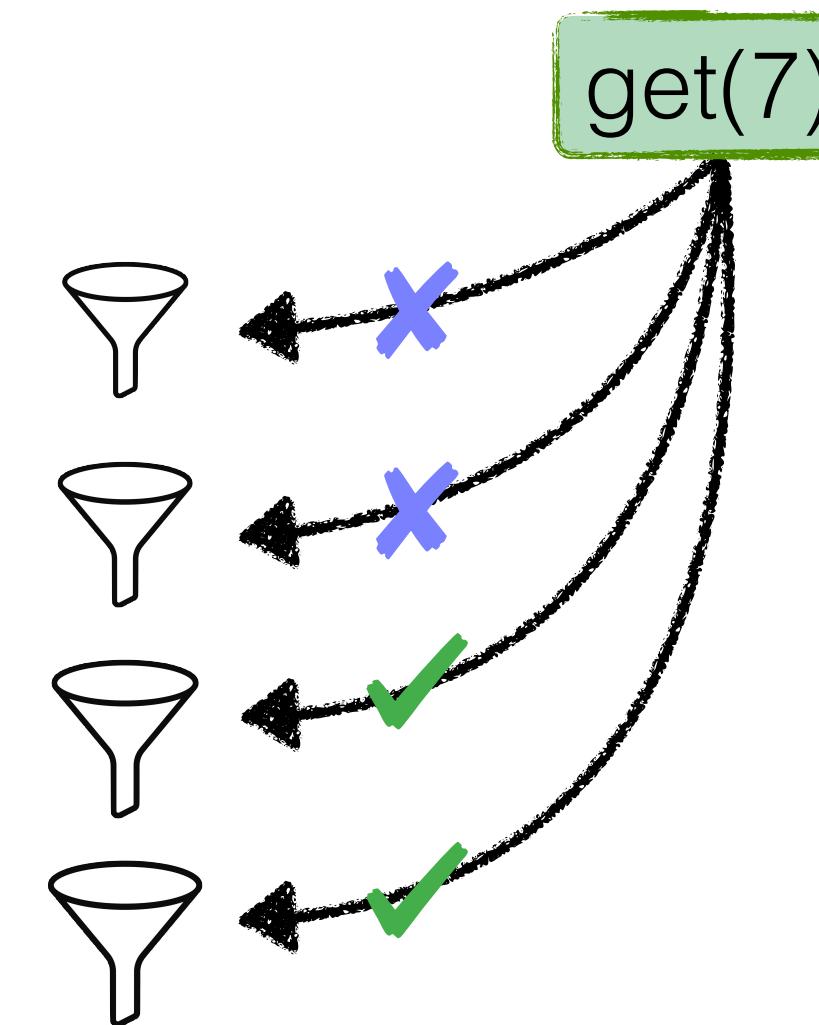
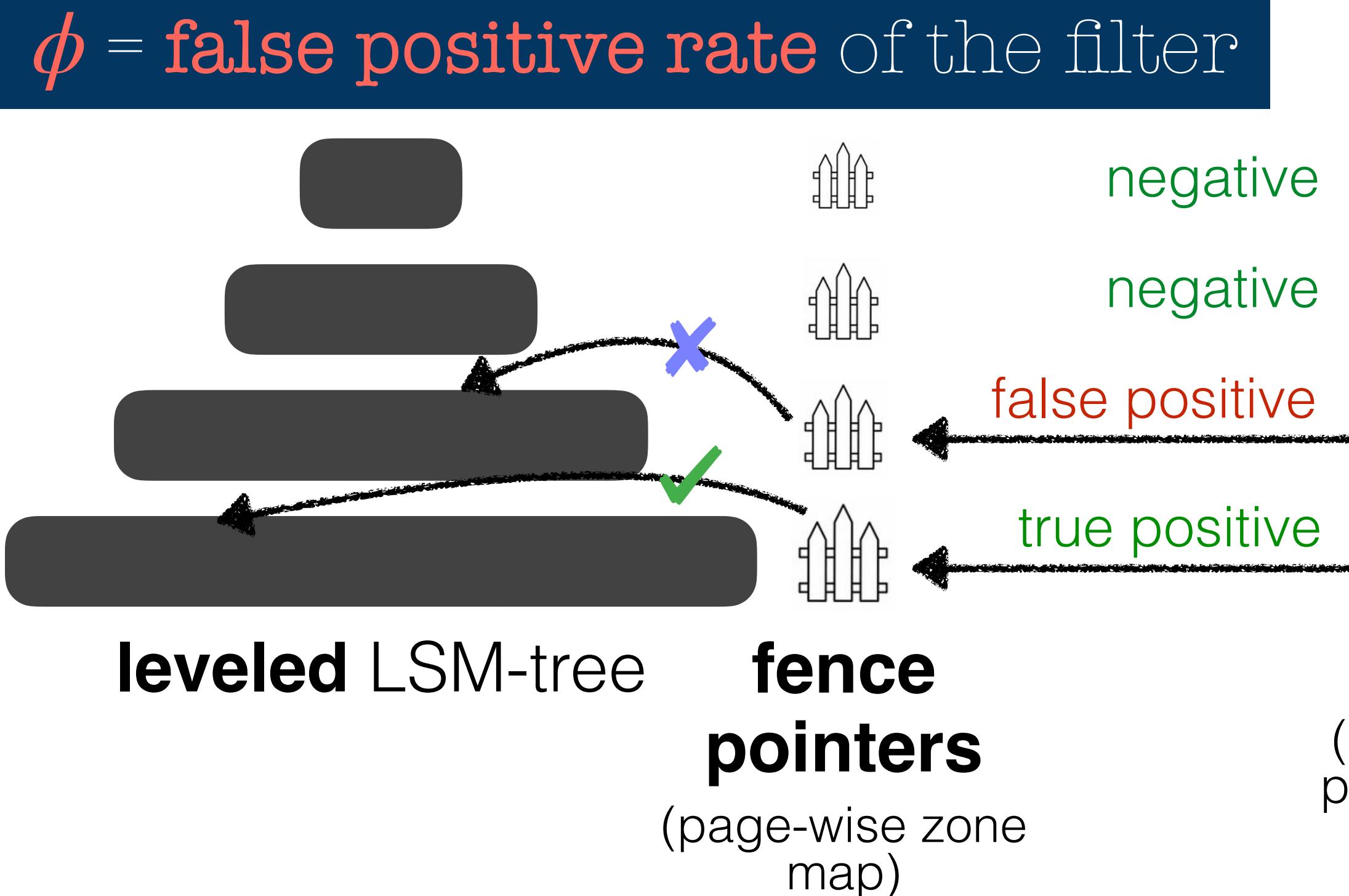
Thought Experiment 6

Cost of a **non-empty point lookup?**



Point lookup cost

Looking for a specific key



1 I/O for the sorted run (level) containing the data

+

1 I/O with probability ϕ for all other sorted runs

1 sorted runs per level

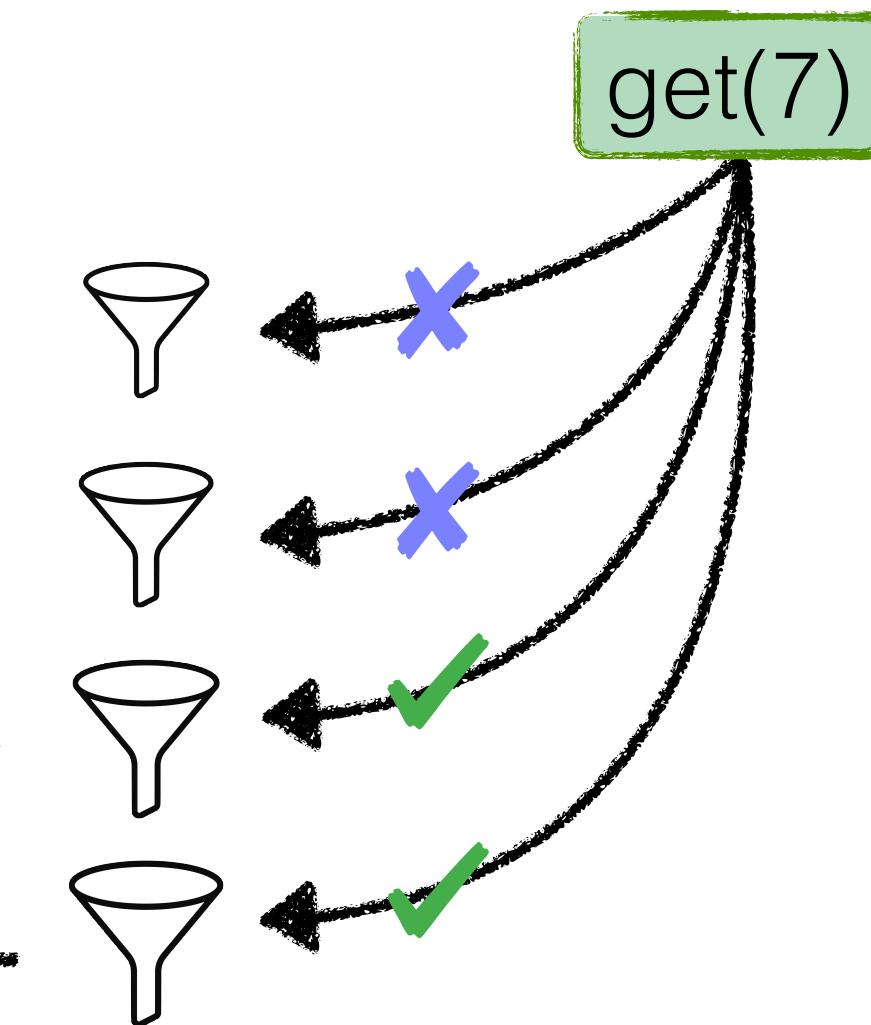
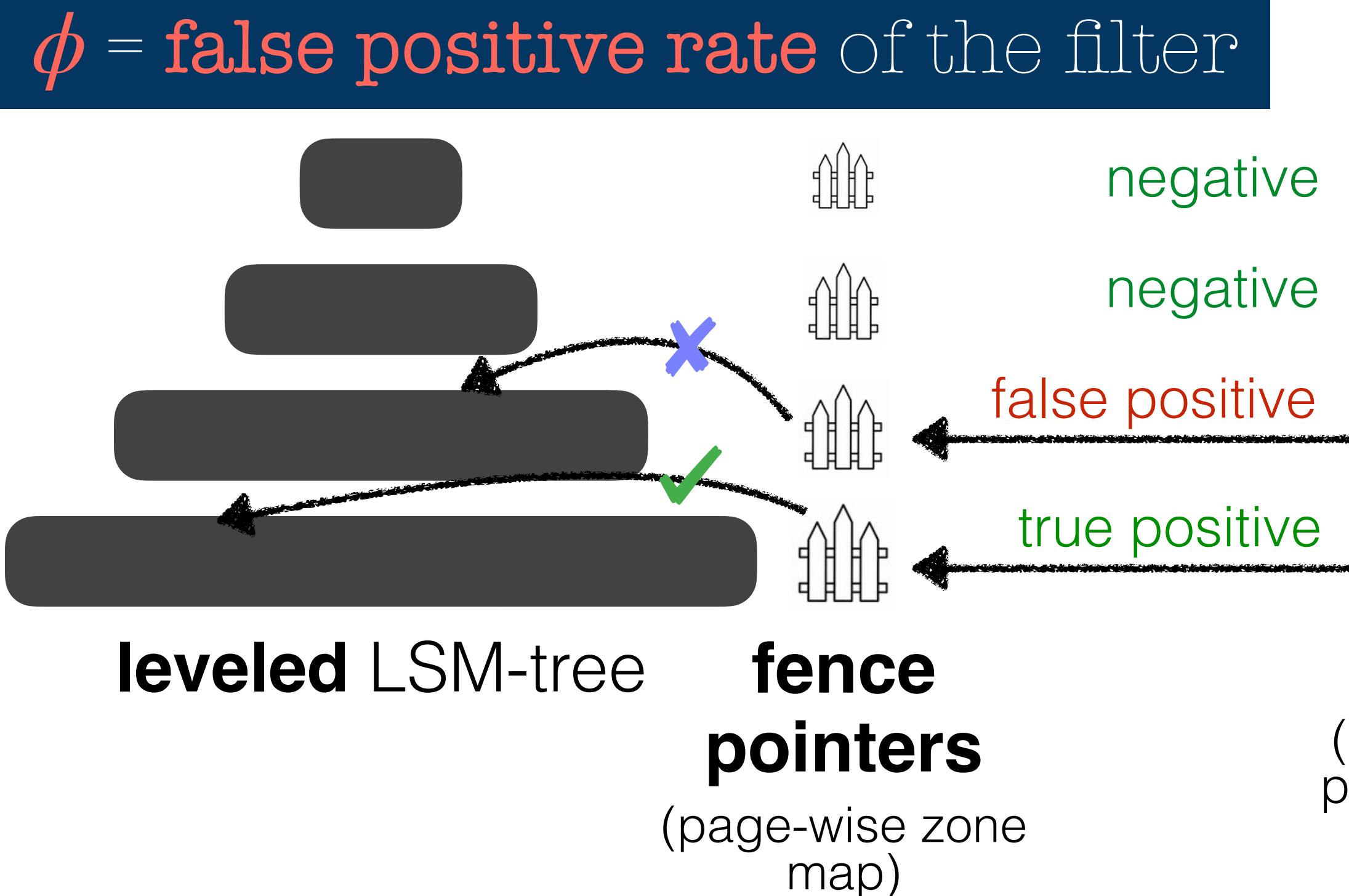
Cost of non-empty point lookup
= $1 + \phi \cdot (L-1)$

Is the lookup cost for entries
not in the tree the same?

No

Point lookup cost

Looking for a specific key



1 I/O for the sorted run (level) containing the data

+

1 I/O with probability ϕ for all other sorted runs

1 sorted runs per level

Cost of non-empty point lookup

$$= 1 + \phi \cdot (L-1)$$

Cost of empty point lookup = $\phi \cdot L$

Cost analysis

Counting all I/Os

data structure	ingestion cost	point lookup cost*	range lookup cost
Leveled LSM-tree	$O(L \cdot T / B)$	$O(1 + \phi \cdot L)$	
Tiered LSM-tree	$O(L / B)$	$O(L \cdot T)$	
B+-tree			
Sorted array			
Log			

* with **fence pointers & Bloom filter** with $FPR = \phi$

cost for **non-empty lookups** (remove “1 +” to get cost of empty lookups)

Cost analysis

Counting all I/Os

data structure	ingestion cost	point lookup cost*	range lookup cost
Leveled LSM-tree	$O(L \cdot T / B)$	$O(1 + \phi \cdot L)$	
Tiered LSM-tree	$O(L / B)$	$O(1 + \phi \cdot L \cdot T)$	
B+-tree			
Sorted array			
Log			

How do we compute ϕ ?

Computing FPR

Looking for a specific key

ϕ = false positive rate of the filter

depends on

memory available for filter (bits per entry)

#hash functions used

$$\text{optimal} = \ln(2) \cdot M$$

optimal FPR for Bloom filters = $2^{-M} \cdot \ln(2)$

Computing FPR

Looking for a specific key

ϕ = false positive rate of the filter

depends on

memory available for filter (bits per entry)

#hash functions used

$$\text{optimal} = \ln(2) \cdot M$$

optimal FPR for Bloom filters = $2^{-M} \cdot \ln(2)$

with 10 bits/entry: $\phi = 0.008$

Cost analysis

Counting all I/Os

$\phi = 0.008$ with 10 BPK

data structure	ingestion cost	point lookup cost*	range lookup cost
Leveled LSM-tree	$O(L \cdot T / B)$	$O(1 + \phi \cdot L)$	
Tiered LSM-tree	$O(L / B)$	$O(1 + \phi \cdot L \cdot T)$	
B+-tree			
Sorted array			
Log			

Monkey takes this **another step further!**



Monkey: Optimal Navigable Key-Value Store

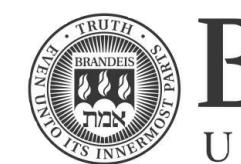
Niv Dayan

Manos Athanassoulis

Stratos Idreos

Harvard University

{dayan, manos, stratos}@seas.harvard.edu

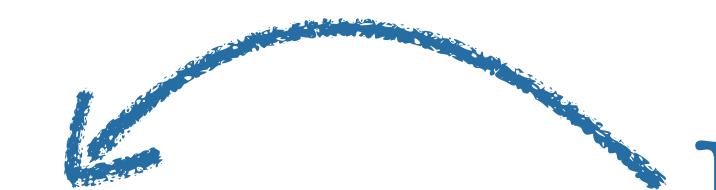
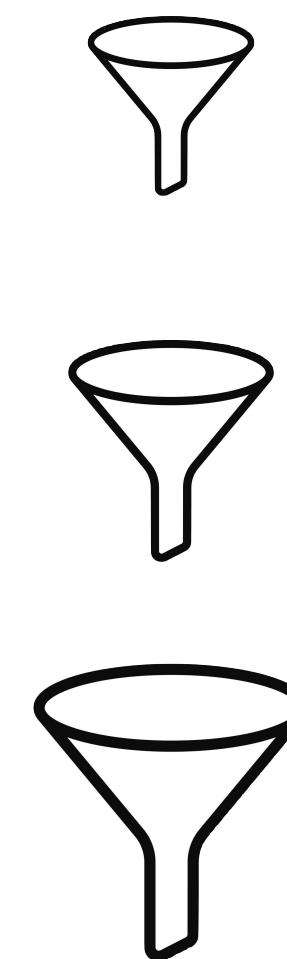
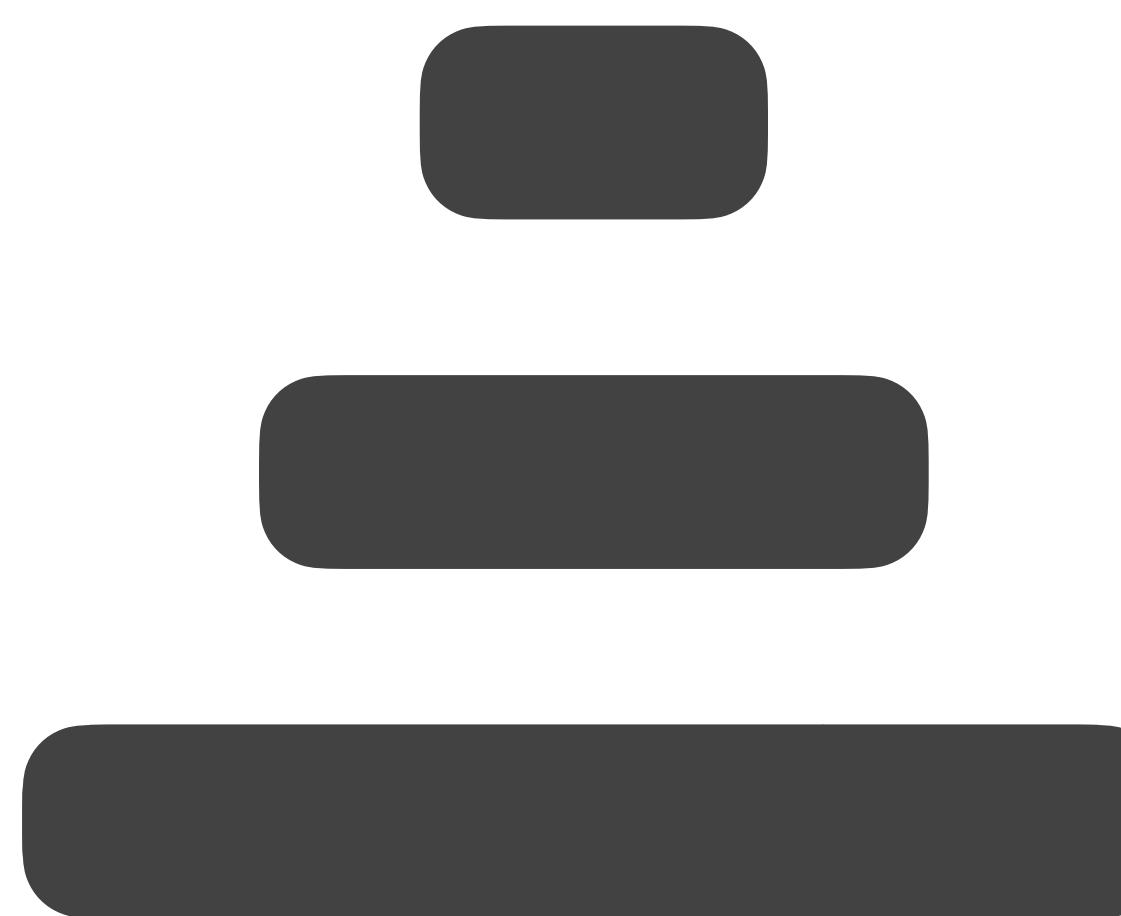


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Monkey

Optimal **N**avigable **K**ey-Value Store



Proportionally large filteres

filter

(one filter
per sorted run)

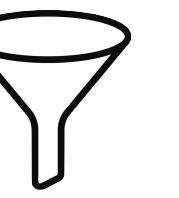


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U N I V E R S I T Y

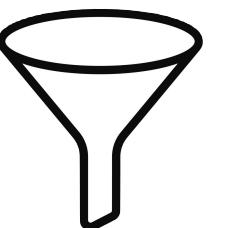


Monkey

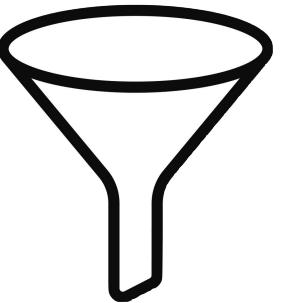
Optimal **N**avigable **K**ey-Value Store



M



M



M

filter

(one filter
per sorted run)

bits/entry



Monkey

Optimal **N**avigable **K**ey-Value Store



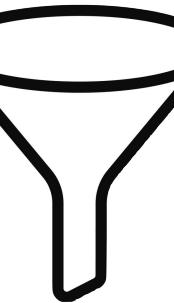
M

$2^{-M \cdot \ln(2)}$



M

$2^{-M \cdot \ln(2)}$



M

$2^{-M \cdot \ln(2)}$

filter

(one filter
per sorted run)

bits/entry

FPR

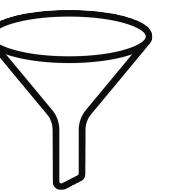


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U N I V E R S I T Y



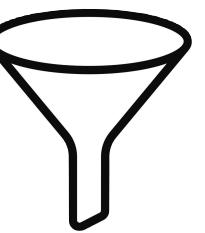
Monkey

Optimal **N**avigable **K**ey-Value Store



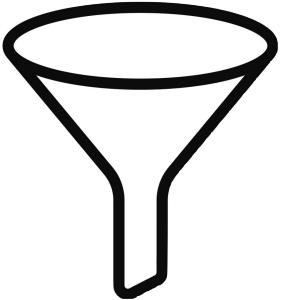
M

2^{-M}



M

2^{-M}



M

2^{-M}

filter

(one filter
per sorted run)

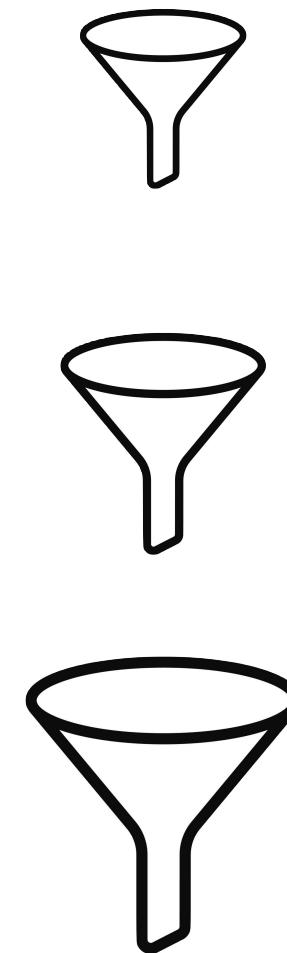
bits/entry

FPR



Monkey

Optimal **N**avigable **K**ey-Value Store



filter
(one filter
per sorted run)

M M M

2^{-M} 2^{-M} 2^{-M}

= $O(2^{-M} \cdot L)$



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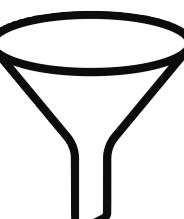
Monkey

Optimal **N**avigable **K**ey-Value Store



M

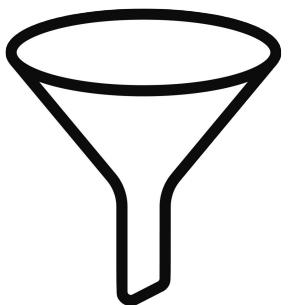
2^{-M}



M

2^{-M}

**most
memory**



M

2^{-M}

filter

(one filter
per sorted run)

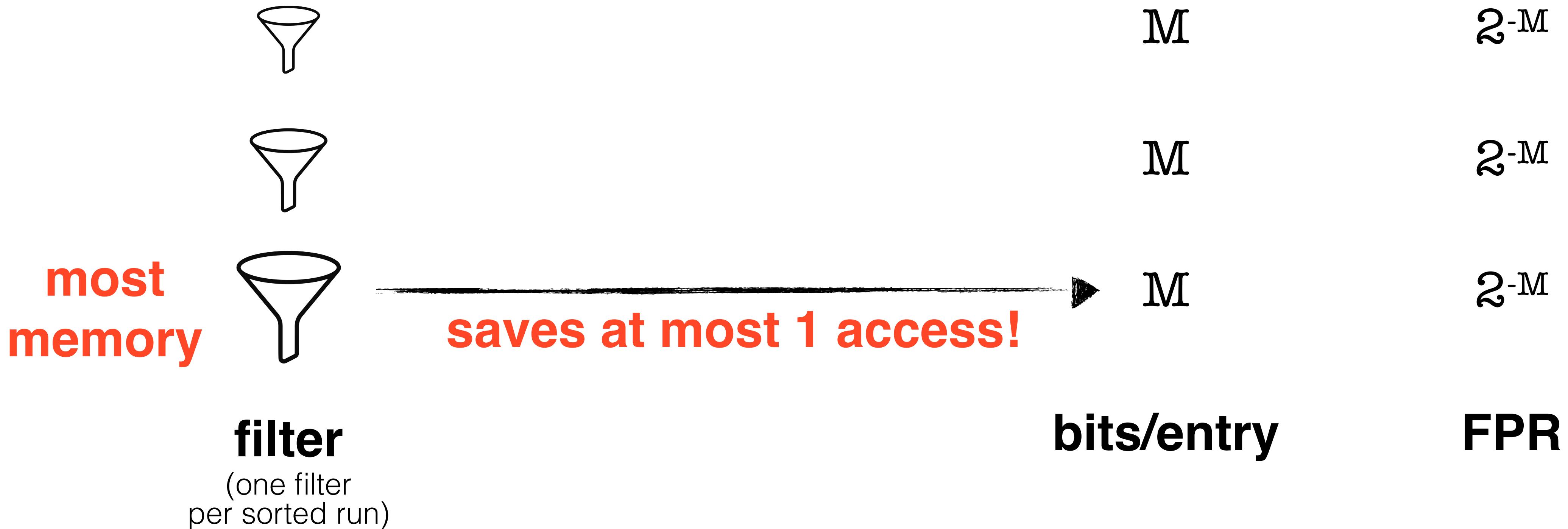
bits/entry

FPR



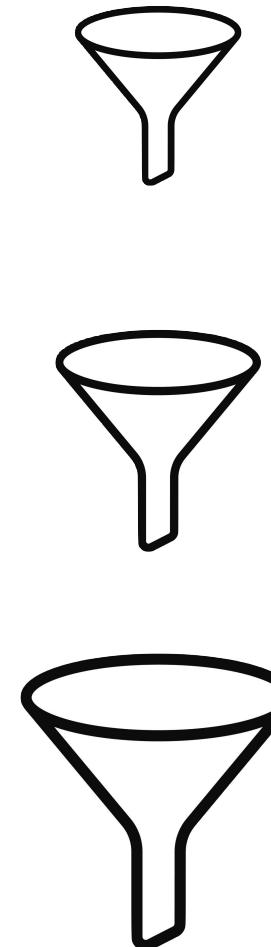
Monkey

Optimal **N**avigable **K**ey-Value Store

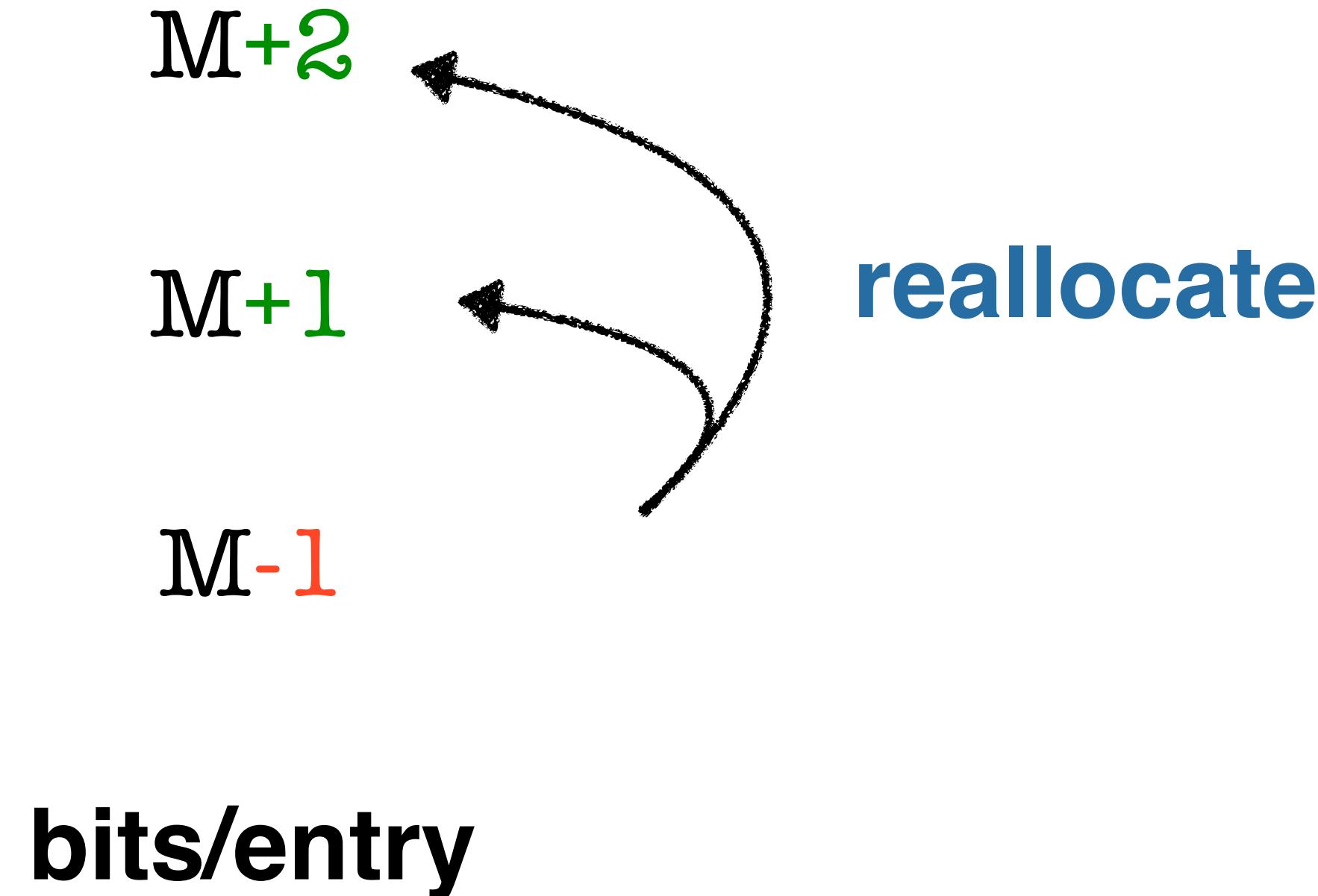


Monkey

Optimal **N**avigable **K**ey-Value Store

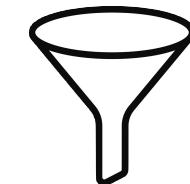


filter
(one filter
per sorted run)



Monkey

Optimal **N**avigable **K**ey-Value Store



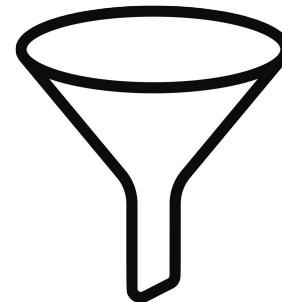
$M+2$

$2^{-(M+2)}$



$M+1$

$2^{-(M+1)}$



$M-1$

$2^{-(M-1)}$

filter
(one filter
per sorted run)

bits/entry

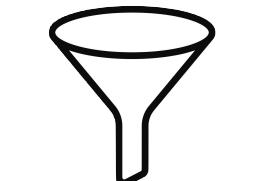
FPR



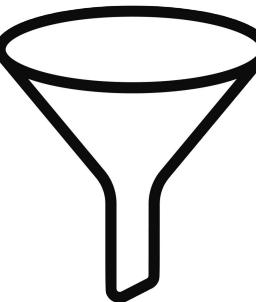
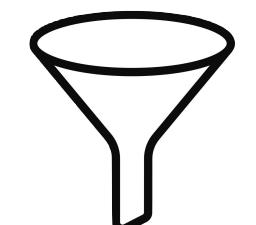
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Optimal **N**avigable **K**ey-Value Store



$$\begin{aligned} &\propto 2^{-M} / T^2 \\ &\propto 2^{-M} / T^1 \\ &\propto 2^{-M} / T^0 \end{aligned} \quad \left. \right\} = O(2^{-M})$$



filter

(one filter
per sorted run)

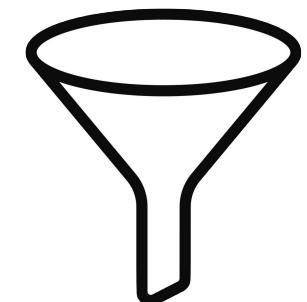
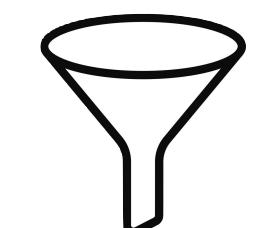
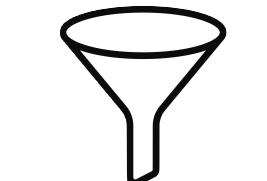
FPR



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Monkey

Optimal **N**avigable **K**ey-Value Store



filter

(one filter
per sorted run)

$$\propto 2^{-M} / T^2$$

$$\propto 2^{-M} / T^1$$

$$\propto 2^{-M} / T^0$$

}

$$= O(2^{-M}) < O(2^{-M} \cdot L)$$

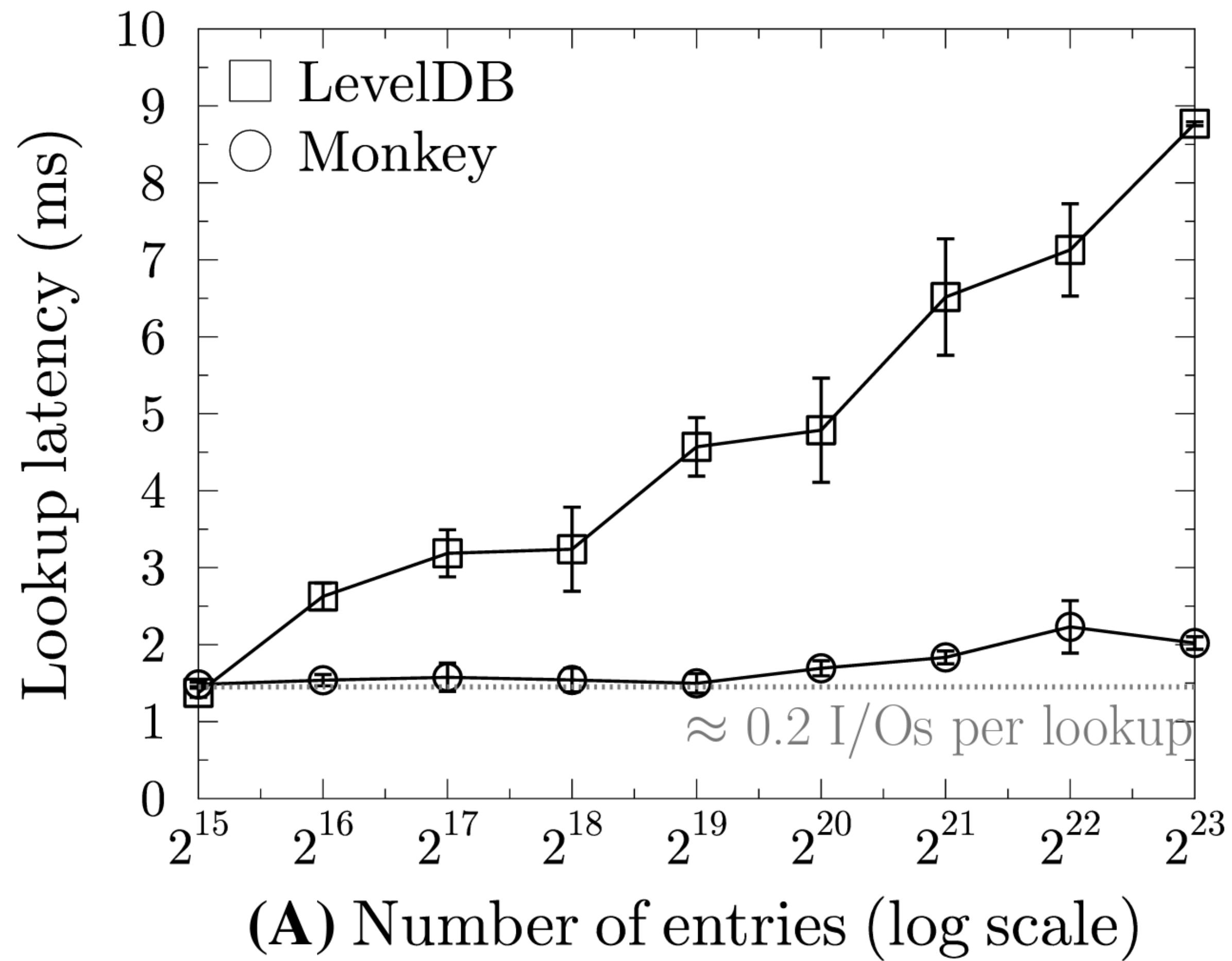
FPR



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U N I V E R S I T Y

Monkey

Optimal Navigable **Key**-Value Store



Cost analysis

Counting all I/Os

$\phi = 0.008$ with 10 BPK

data structure	ingestion cost	point lookup cost*	range lookup cost
Leveled LSM-tree	$O(L \cdot T / B)$	$O(1 + \phi \cdot L)$	
Tiered LSM-tree	$O(L / B)$	$O(1 + \phi \cdot L \cdot T)$	
B+-tree			
Sorted array			
Log			

* with **fence pointers & Bloom filter** with $FPR = \phi$

cost for **non-empty lookups** (remove “1 +” to get cost of empty lookups)

Cost analysis

Counting all I/Os

$\phi = 0.008$ with 10 BPK

data structure	ingestion cost	point lookup cost*	range lookup cost
Leveled LSM-tree	$O(L \cdot T / B)$	$O(1 + \phi \cdot L)^*$	
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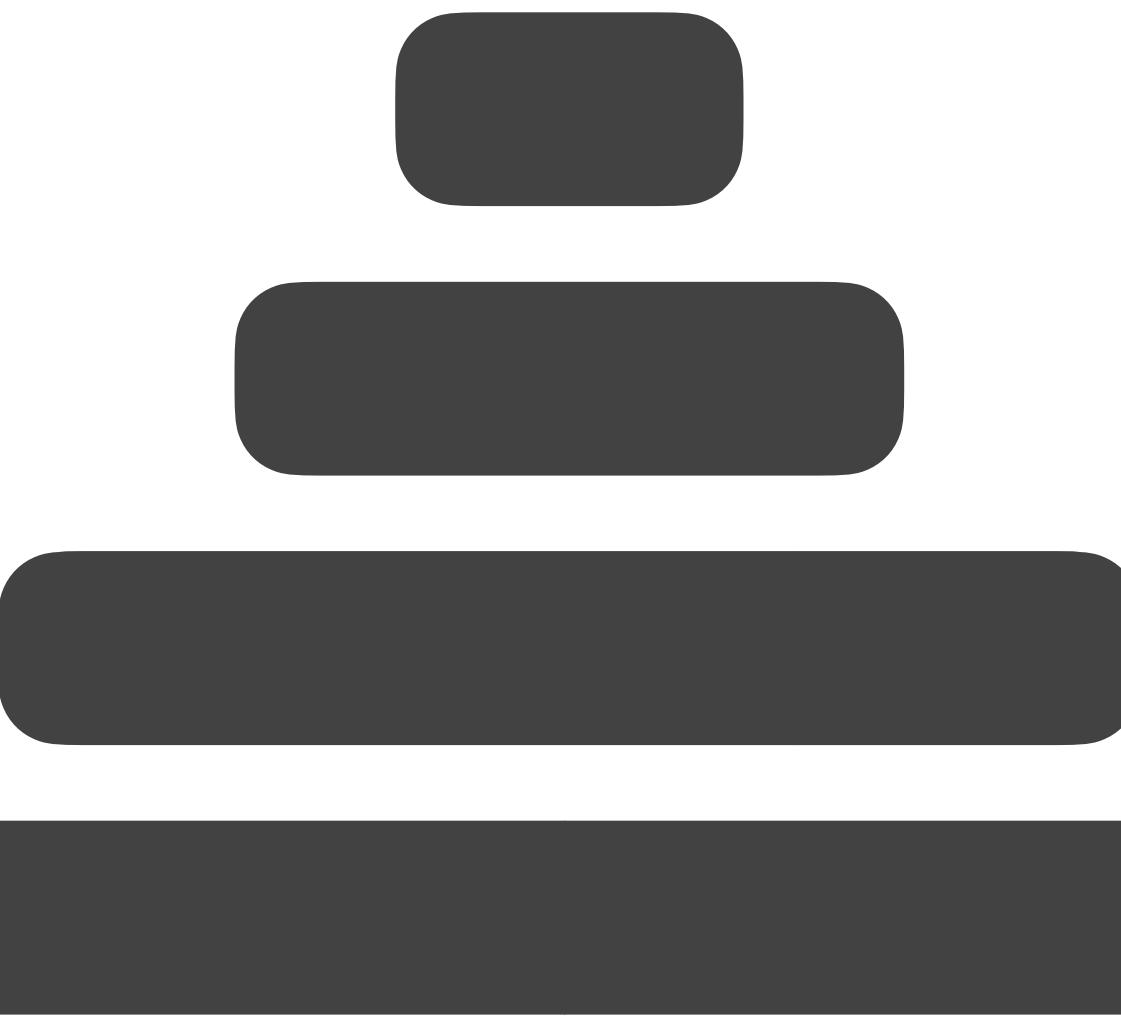
Monkey shaves off the “L” factor from the cost

Range lookup cost

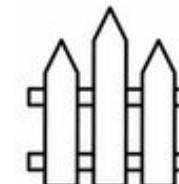
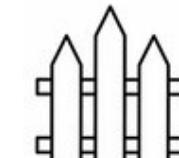
Looking for keys in a range

Range lookup cost

Looking for keys in a range

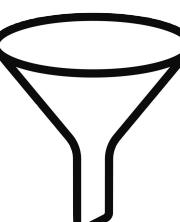
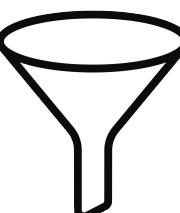


leveled LSM-tree



**fence
pointers**

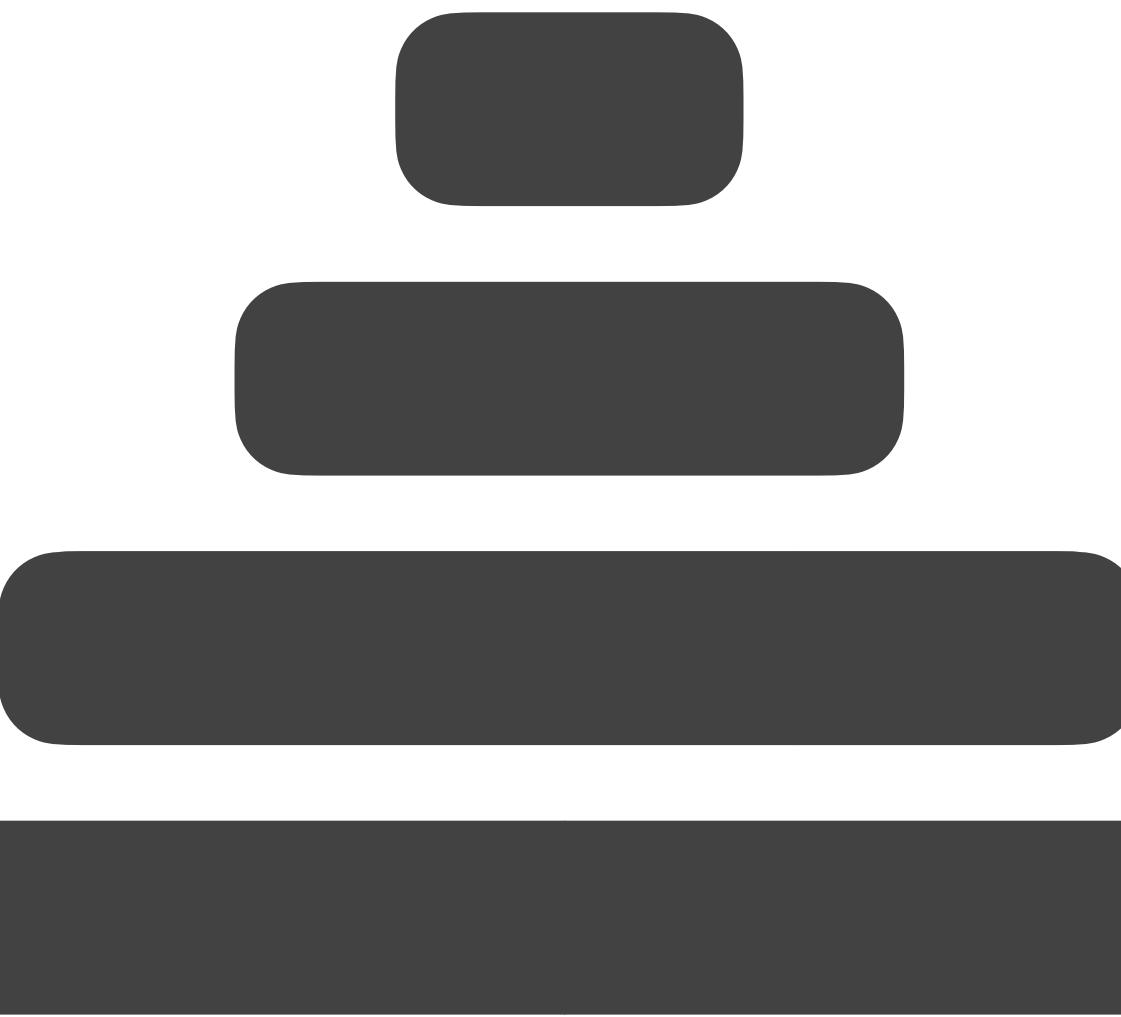
(page-wise zone map)



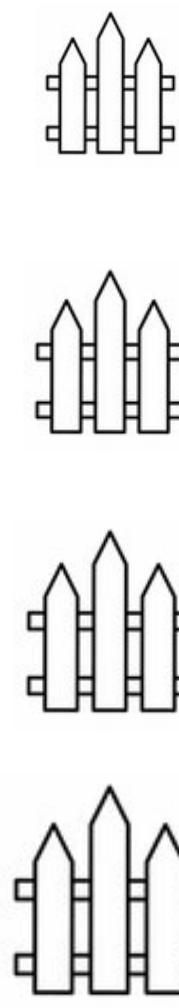
filter

Range lookup cost

Looking for keys in a range



leveled LSM-tree



**fence
pointers**

(page-wise zone map)

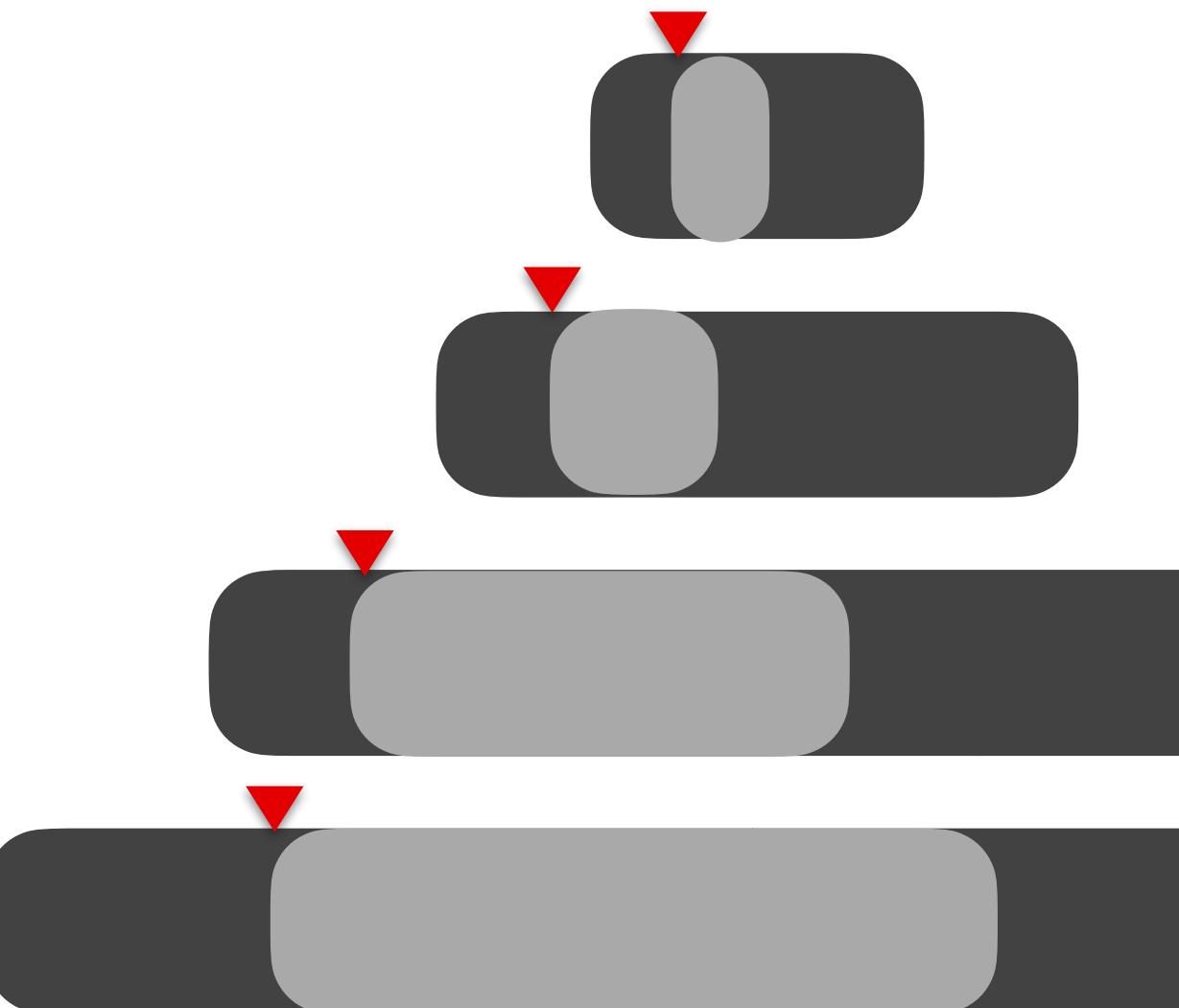


filter

Range lookup cost

Looking for keys in a range

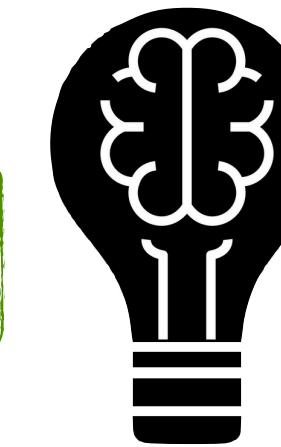
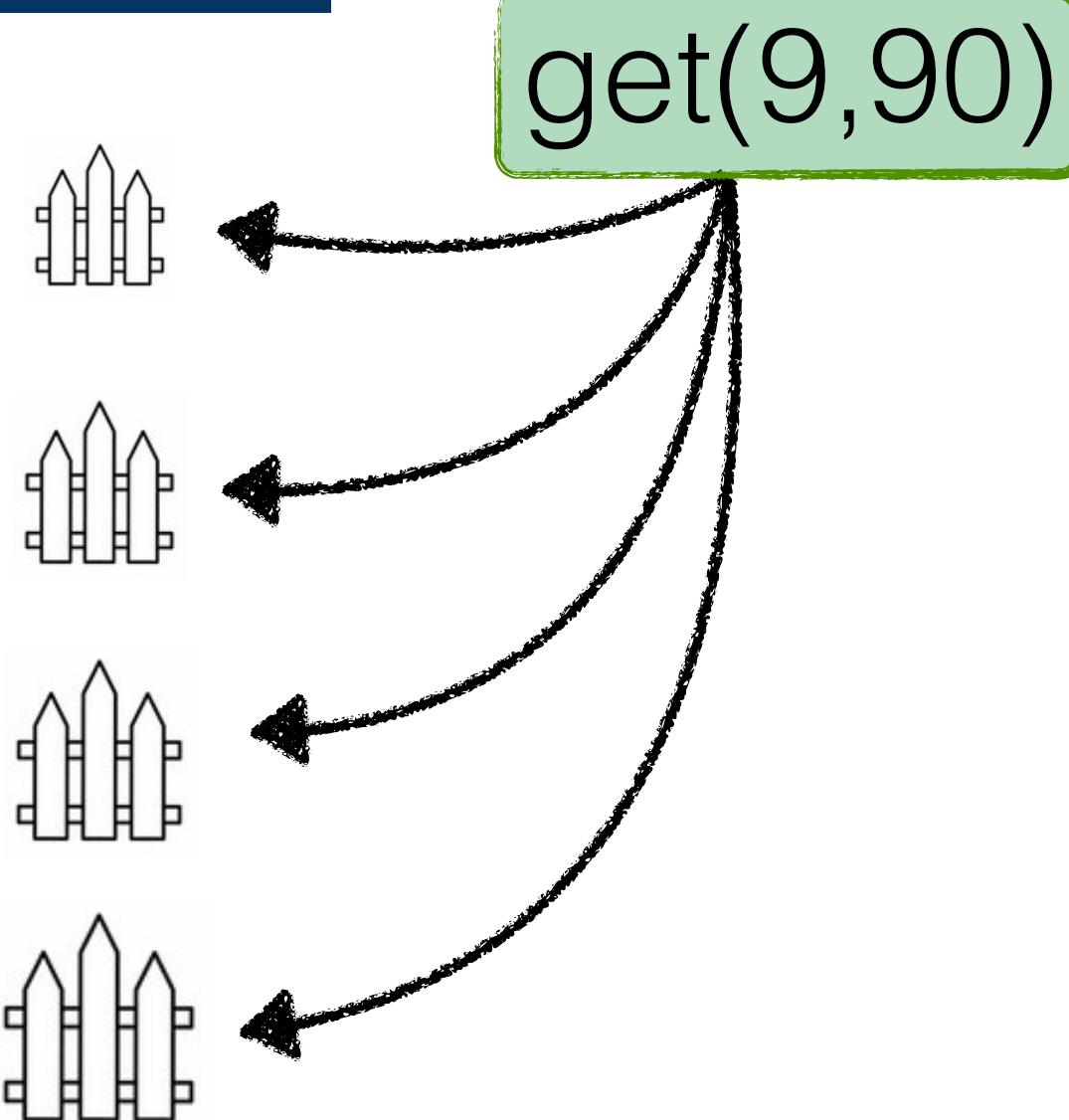
s = selectivity of the range query



leveled LSM-tree

**fence
pointers**

(page-wise zone map)



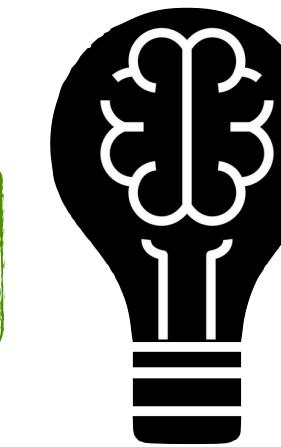
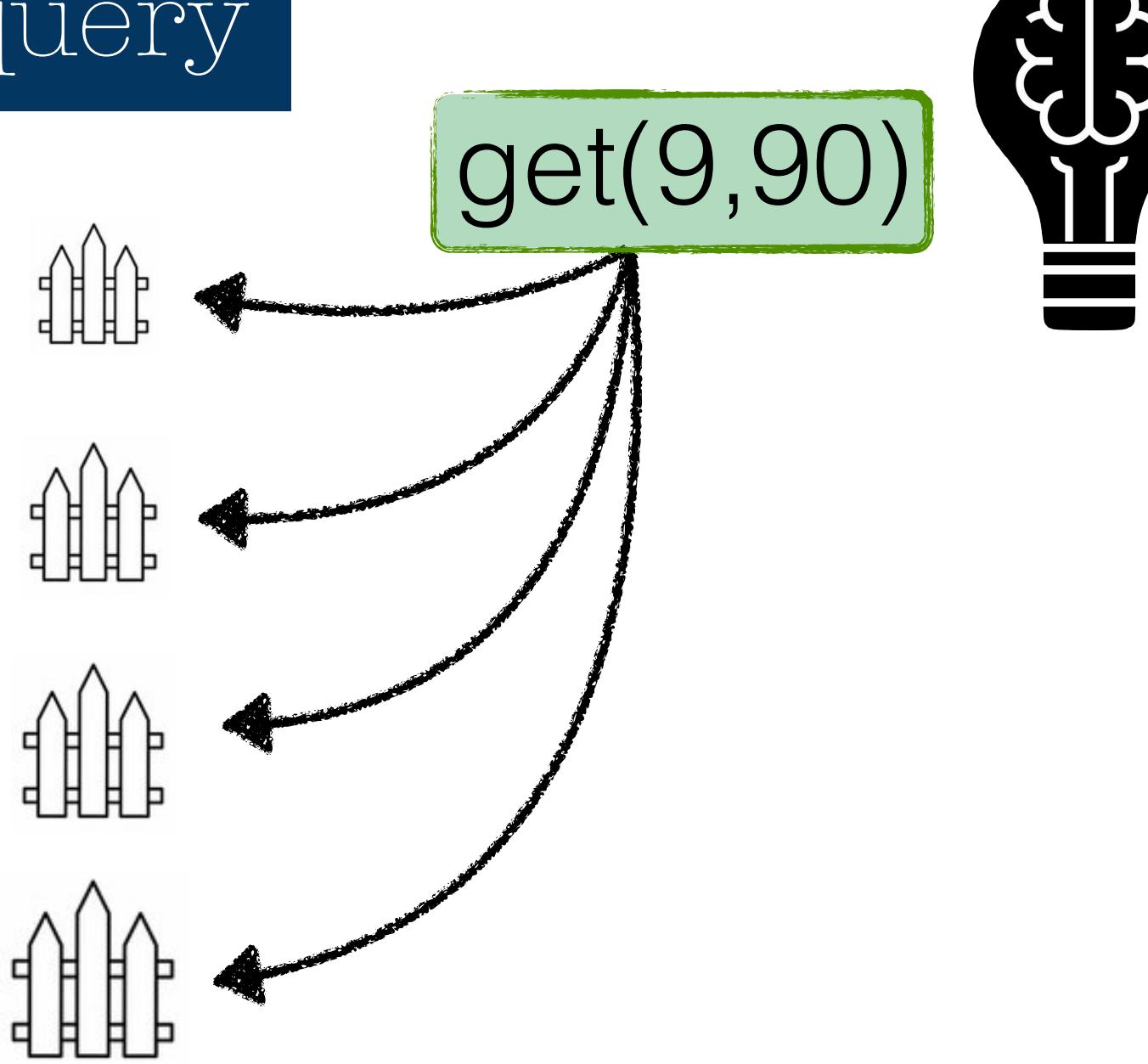
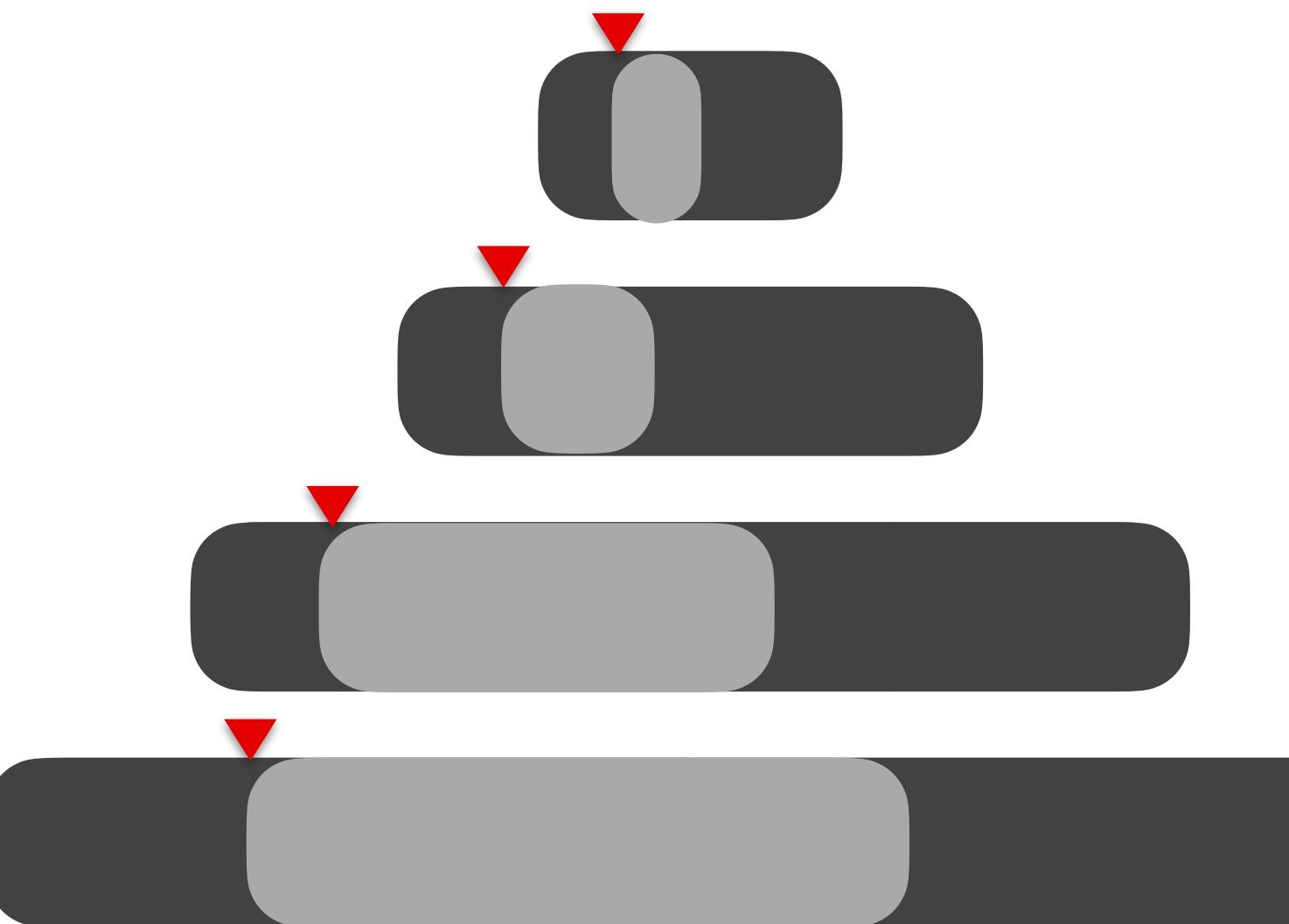
Thought Experiment ?
Cost of a **range query**?



Range lookup cost

Looking for keys in a range

$s = \text{selectivity}$ of the range query



Thought Experiment ?
Cost of a **range query**?

total entries in tree = N

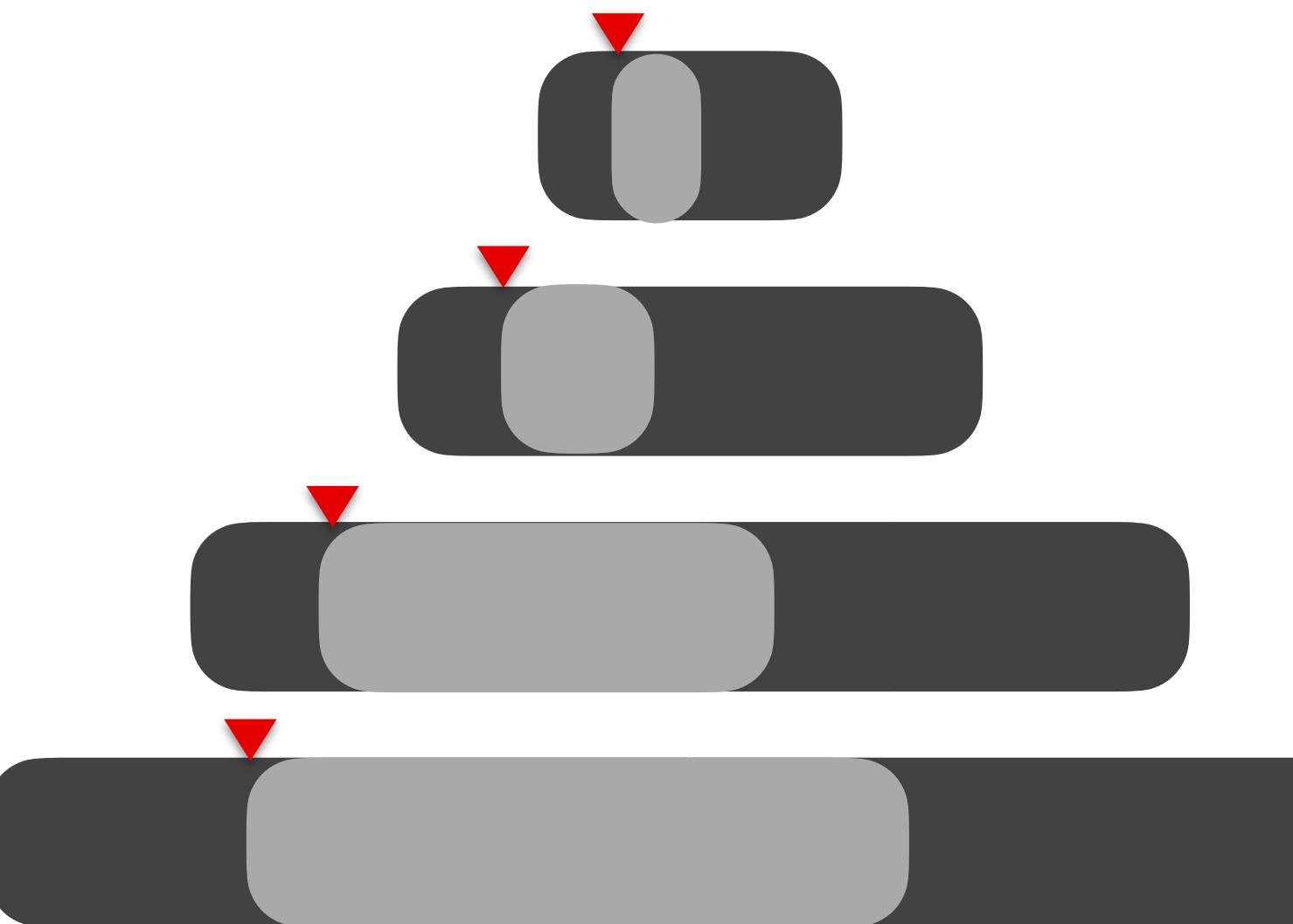
#entries per page = B

#pages in tree = N/B

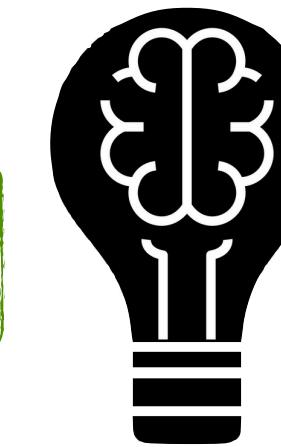
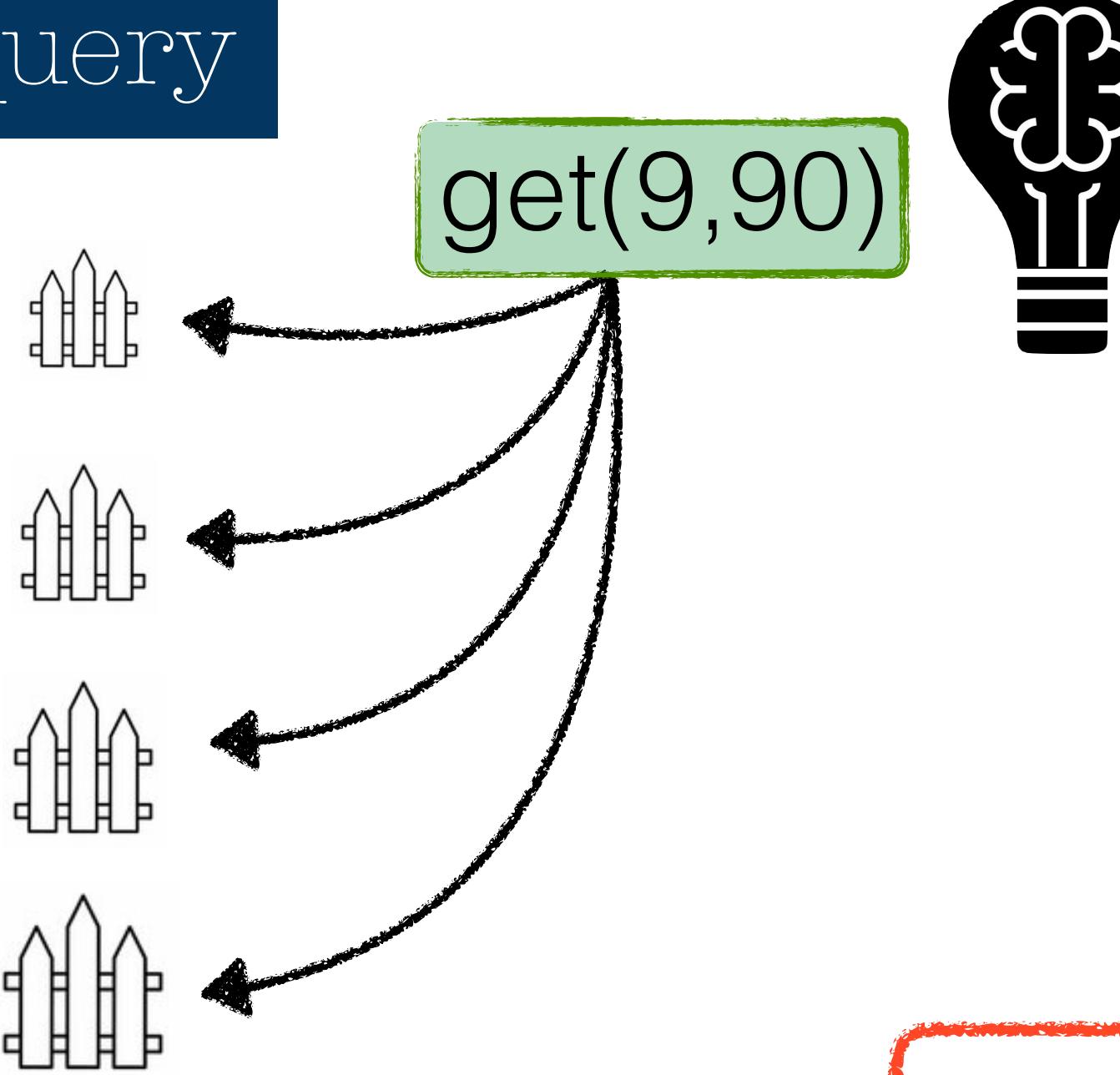
Range lookup cost

Looking for keys in a range

$s = \text{selectivity}$ of the range query



**fence
pointers**
(page-wise zone map)



Thought Experiment ?
Cost of a **range query**?

total entries in tree = N

#entries per page = B

#pages in tree = N/B

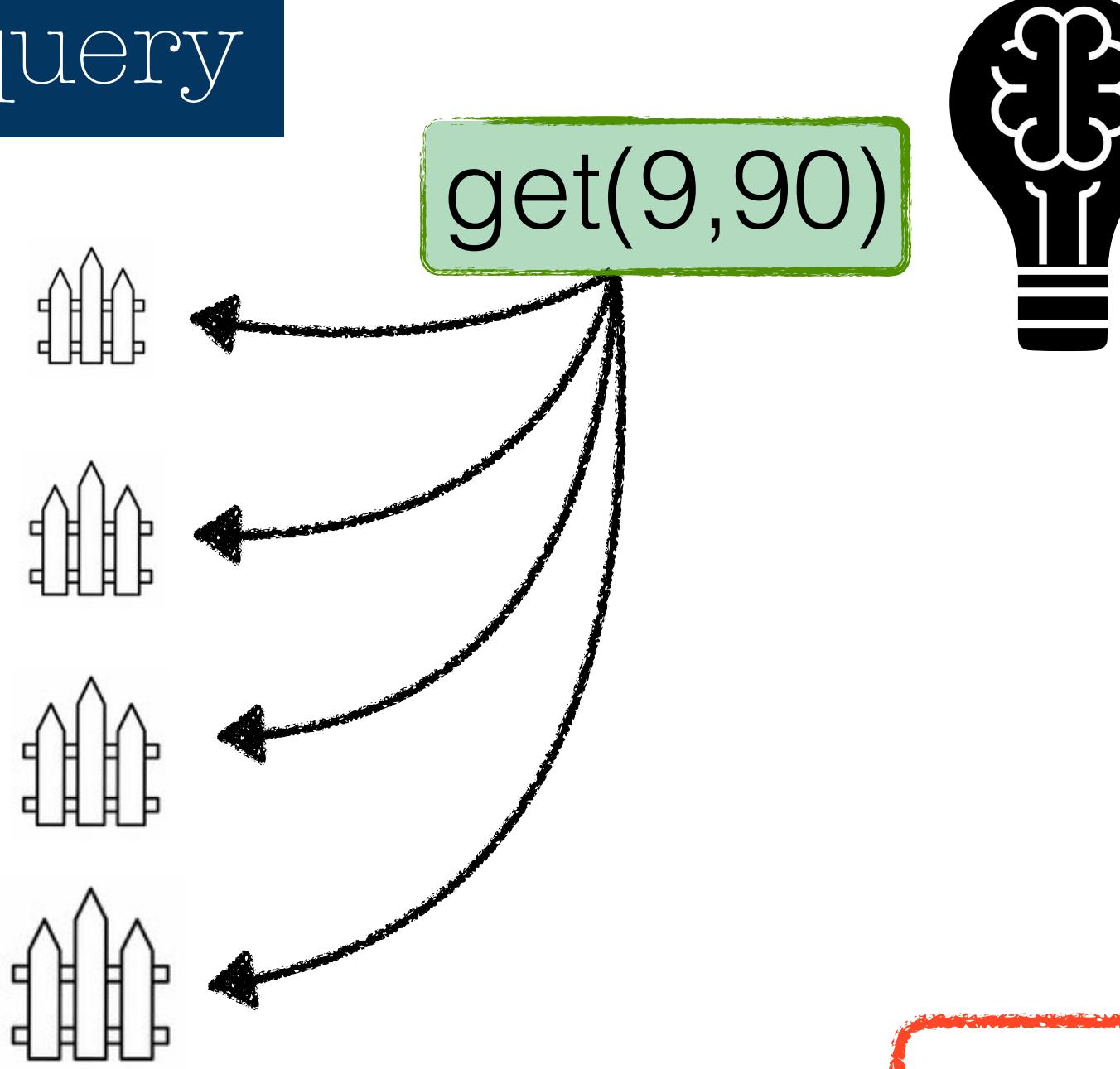
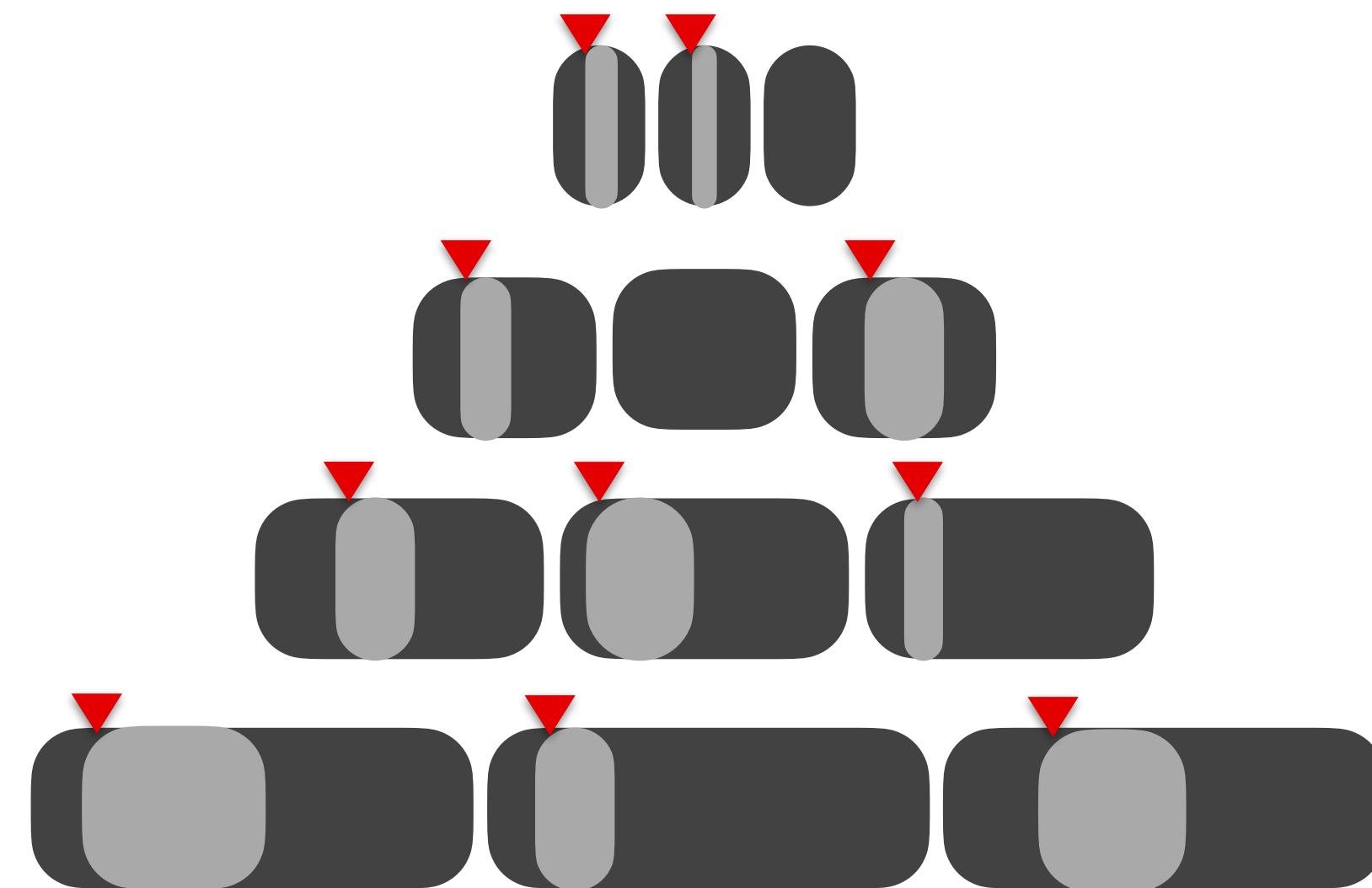
Cost of range lookup = $s \cdot N/B$

What about the **range query cost** in a **tiered LSM-tree**?

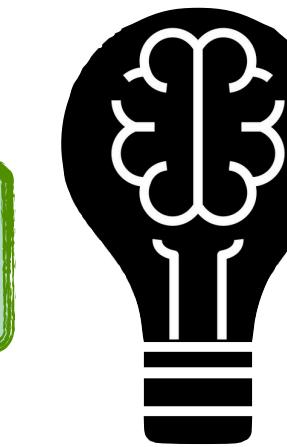
Range lookup cost

Looking for keys in a range

s = selectivity of the range query



same cost for
leveled & tiered LSM



Thought Experiment ?
Cost of a **range query**?

total entries in tree = N

#entries per page = B

#pages in tree = N/B

Cost of range lookup = $s \cdot N/B$

What about the **range query cost** in a **tiered LSM-tree**?

Cost analysis

Counting all I/Os

data structure	ingestion cost	point lookup cost [*]	range lookup cost
Leveled LSM-tree	$O(L \cdot T / B)$	$O(1 + \phi \cdot L)^*$	$O(s \cdot N / B)$
Tiered LSM-tree	$O(L / B)$	$O(1 + \phi \cdot L \cdot T)^*$	$O(s \cdot N / B)$
B+-tree			
Sorted array			
Log			

* with **fence pointers & Bloom filter** with $FPR = \phi$

cost for **non-empty lookups** (remove “ $1 +$ ” to get cost of empty lookups)



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Monkey shaves off the “L” factor from the cost

Cost analysis

Counting all I/Os

* long range lookups

data structure	ingestion cost	point lookup cost *	range lookup cost *
Leveled LSM-tree	$O(L \cdot T / B)$	$O(1 + \phi \cdot L)^*$	$O(s \cdot N / B)$
Tiered LSM-tree	$O(L / B)$	$O(1 + \phi \cdot L \cdot T)^*$	$O(s \cdot N / B)$
B+-tree			
Sorted array			
Log			

* with **fence pointers & Bloom filter** with $FPR = \phi$

cost for **non-empty lookups** (remove “ $1 +$ ” to get cost of empty lookups)

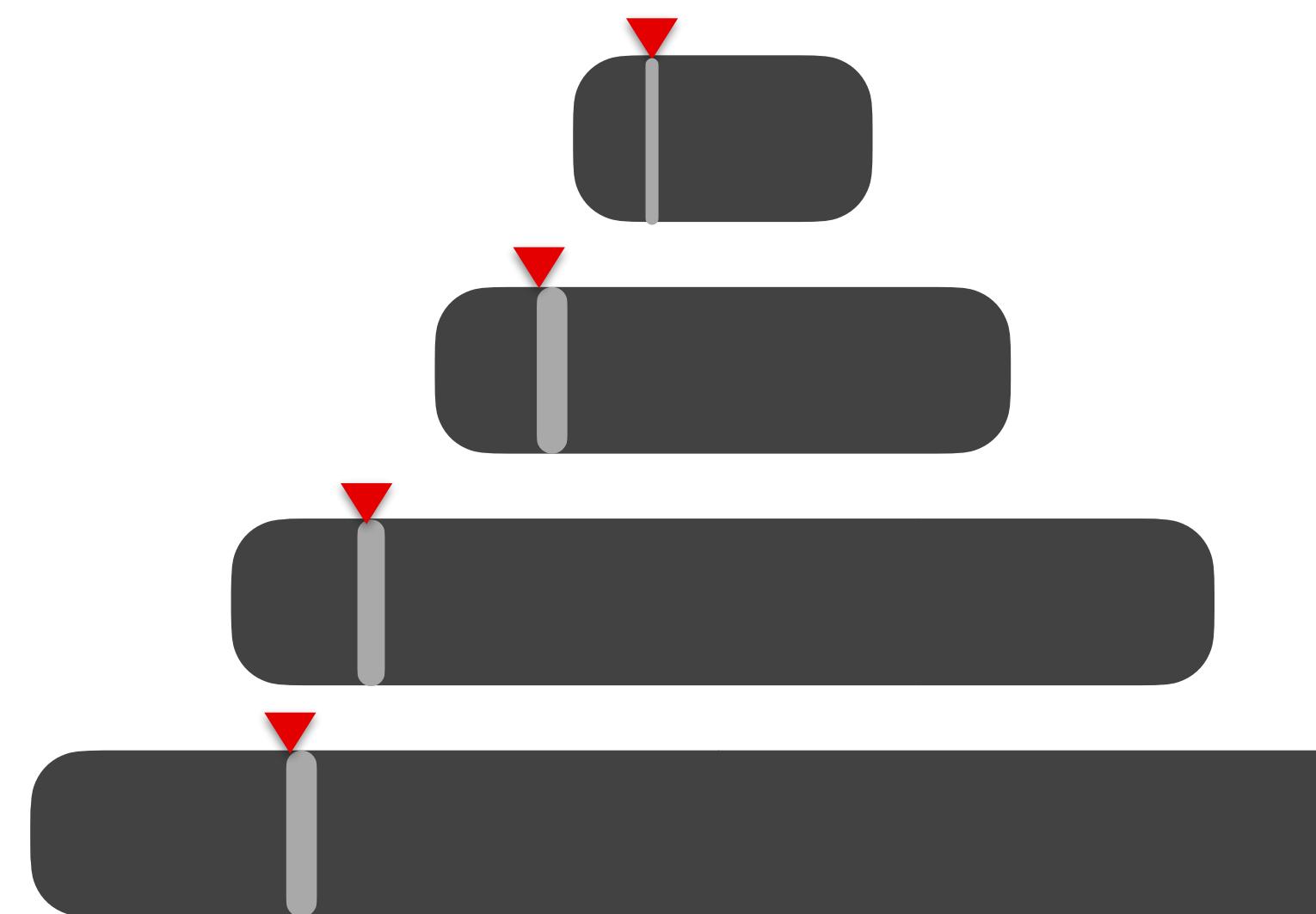


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Monkey shaves off the “L” factor from the cost

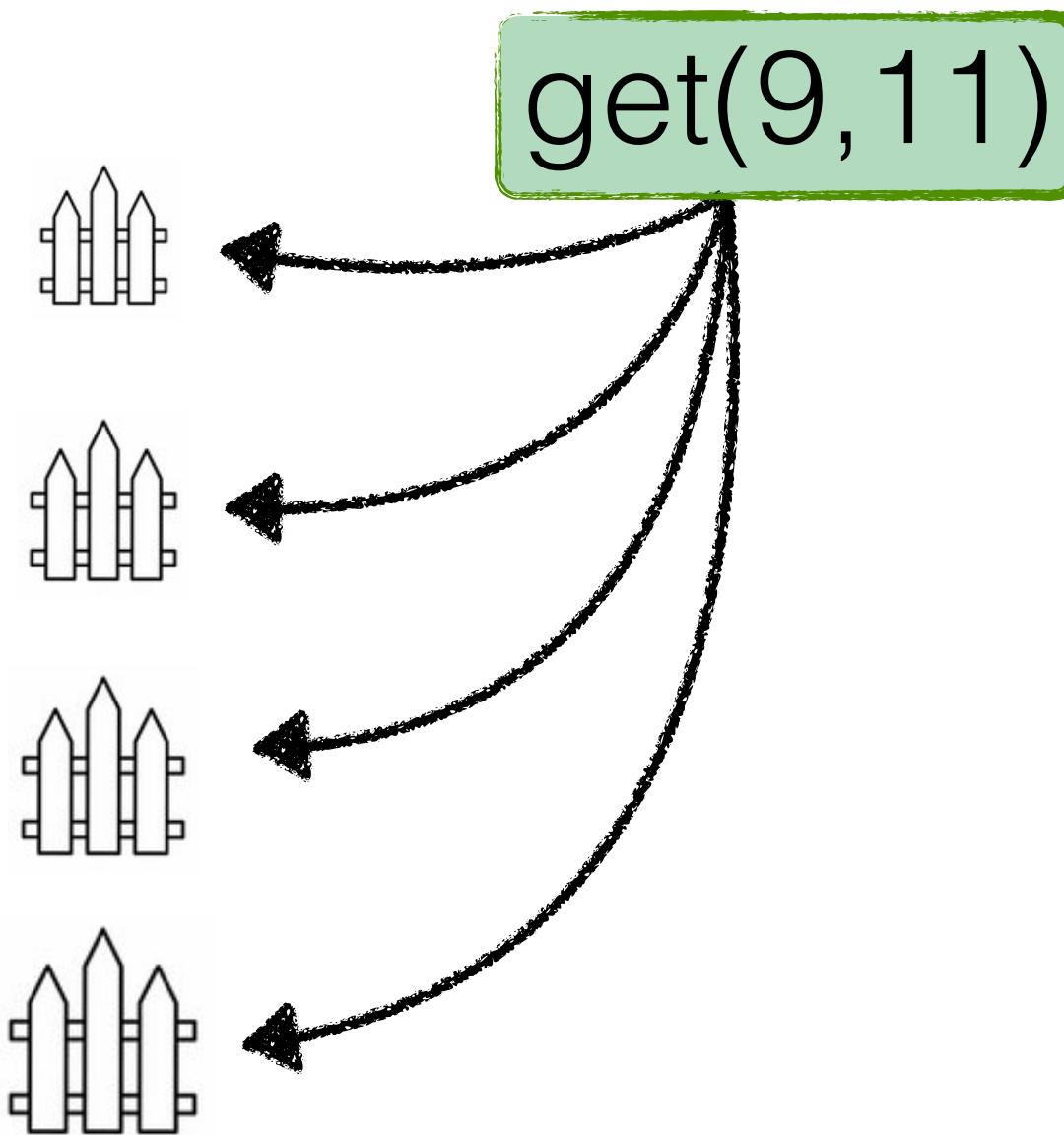
Range lookup cost

Looking for keys in a range



leveled LSM-tree

**fence
pointers**
(page-wise zone map)

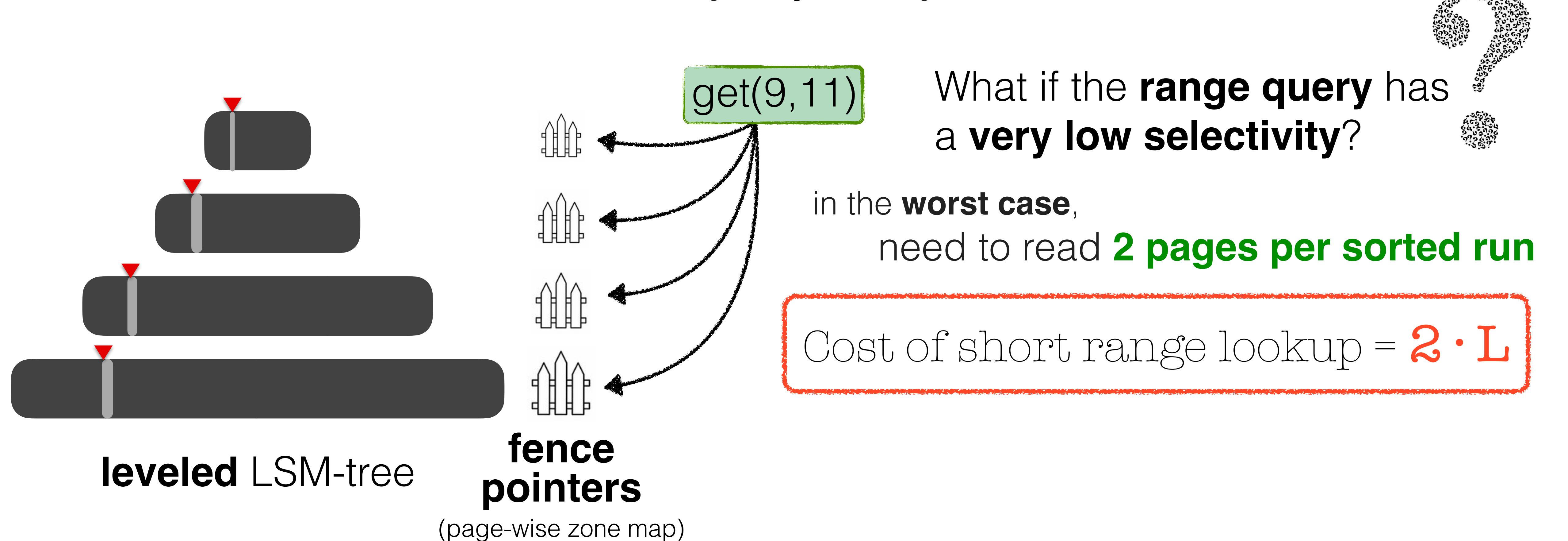


What if the **range query** has
a **very low selectivity**?



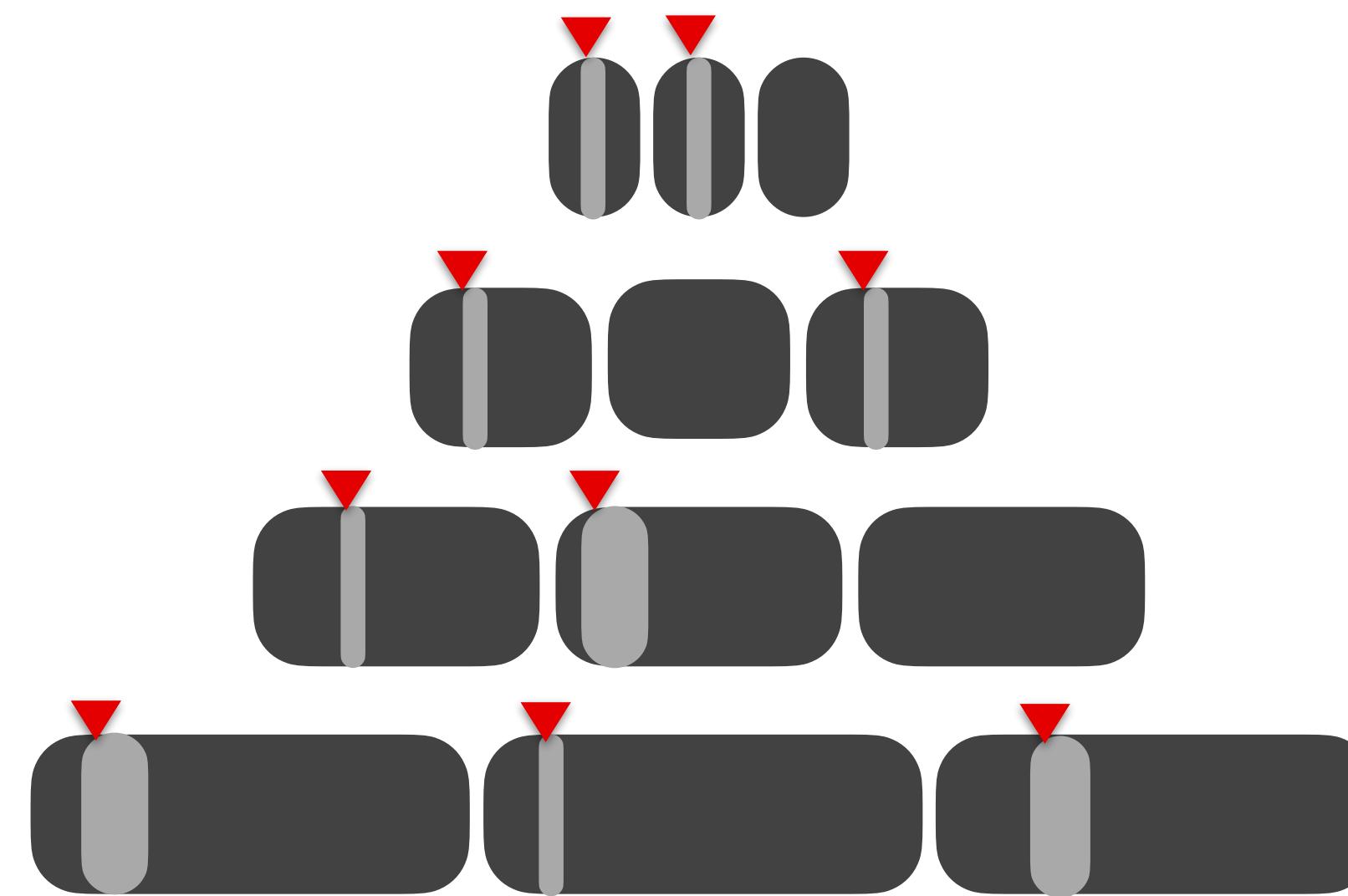
Range lookup cost

Looking for keys in a range



Range lookup cost

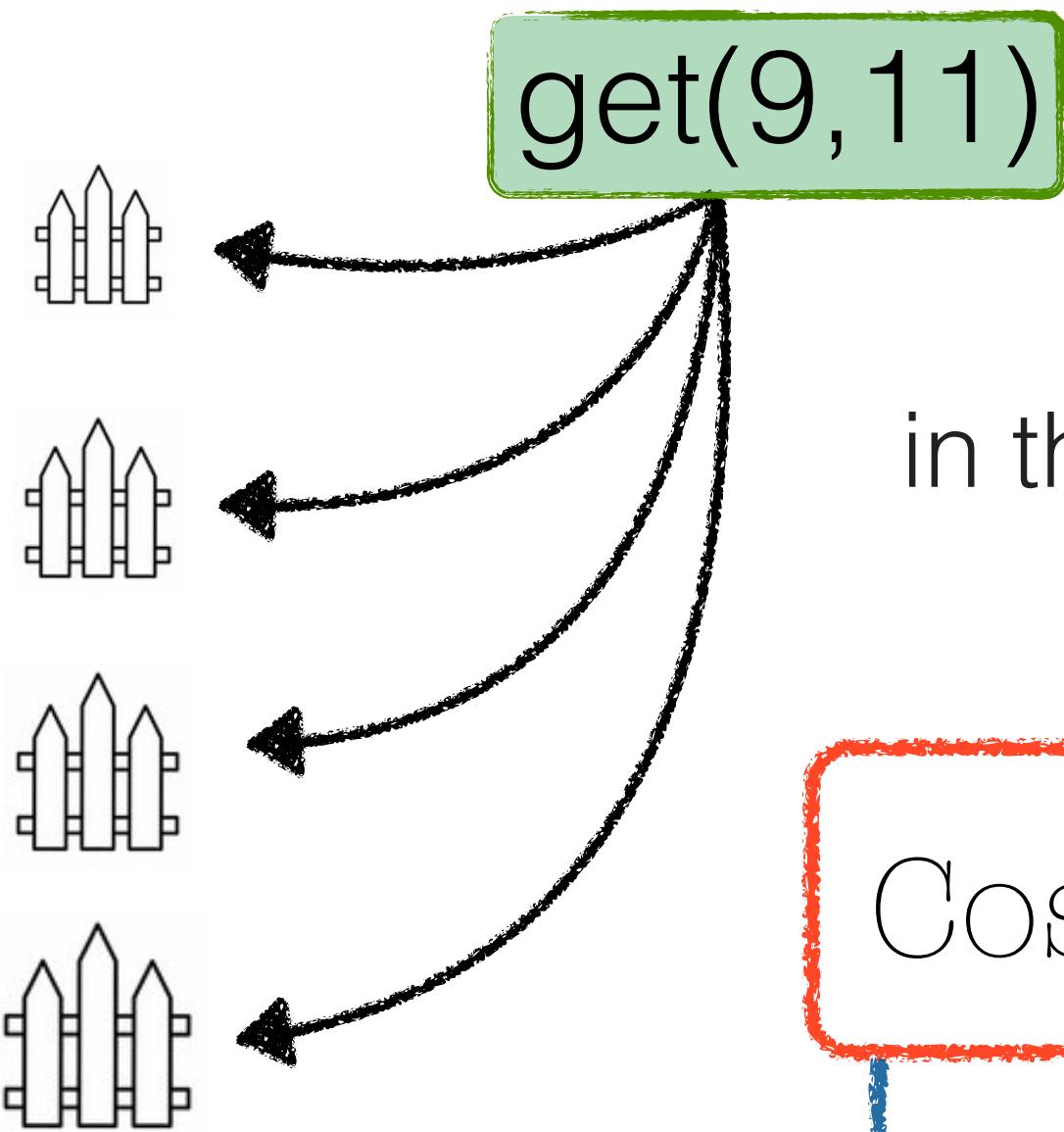
Looking for keys in a range



tiered LSM-tree

**fence
pointers**

(page-wise zone map)



What if the **range query** has
a **very low selectivity**?

in the **worst case**,
need to read **2 pages per sorted run**

Cost of short range lookup = $2 \cdot L$

leveling

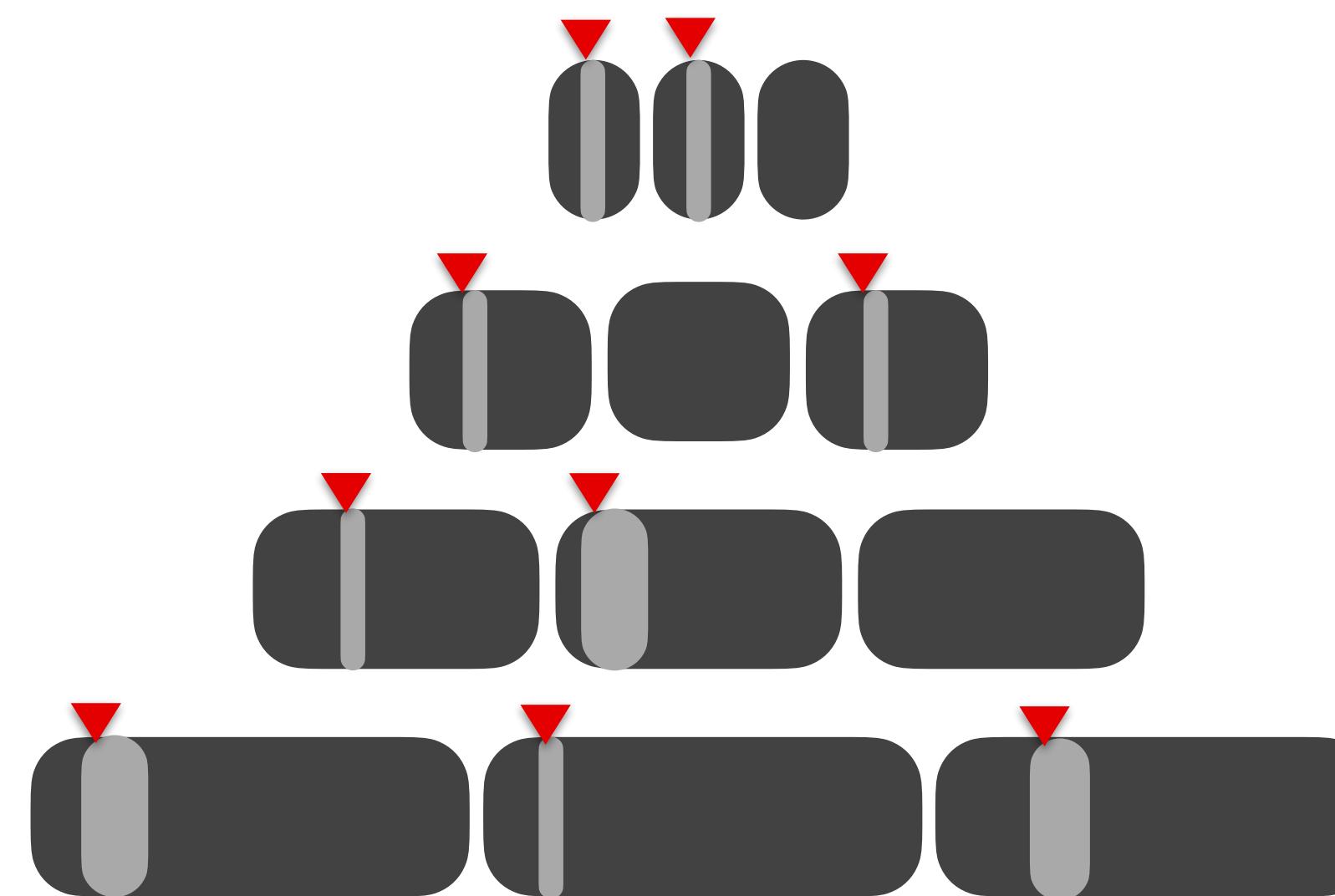
tiering

Cost of short range lookup = $2 \cdot L \cdot T$



Range lookup cost

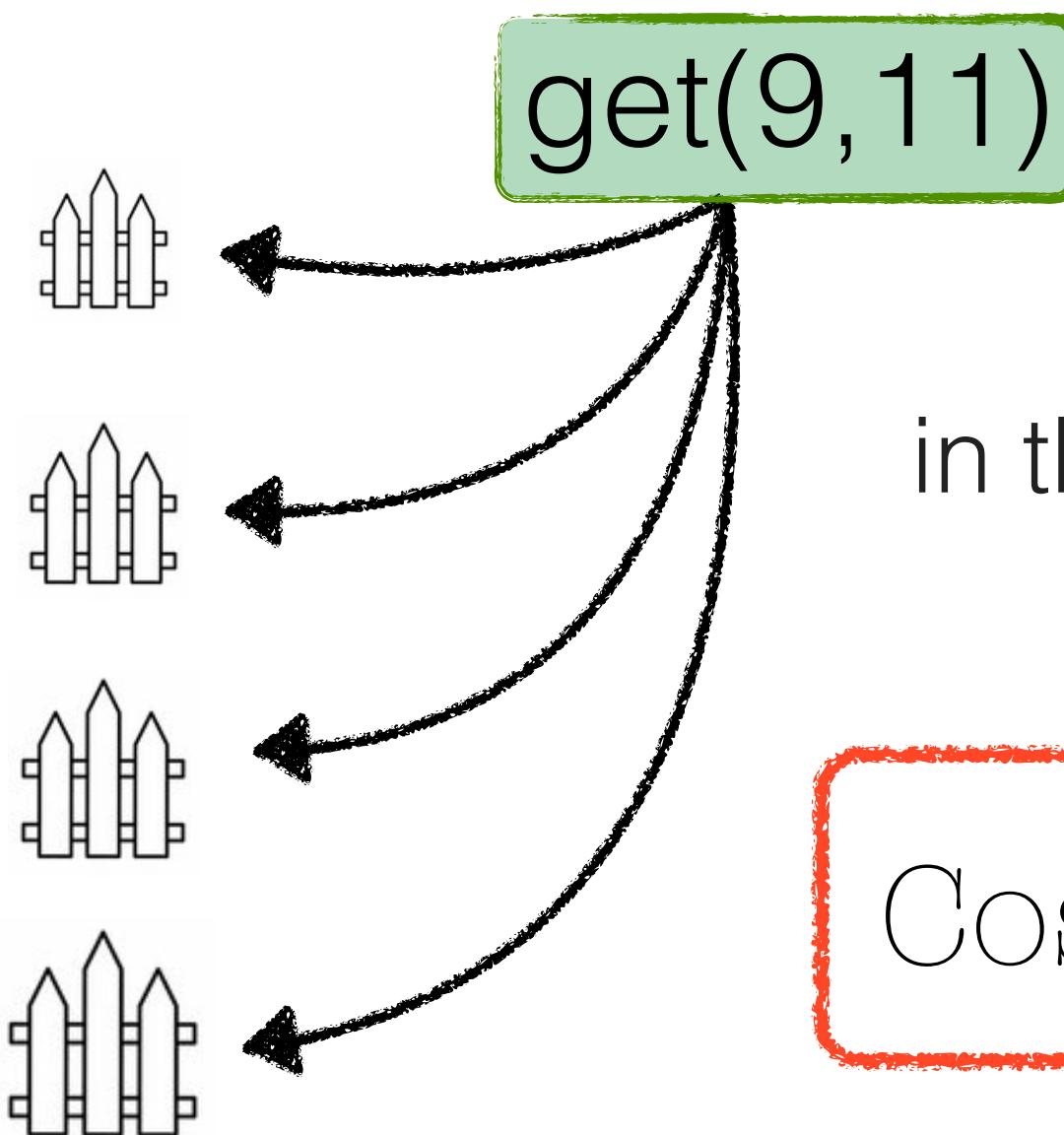
Looking for keys in a range



tiered LSM-tree

**fence
pointers**

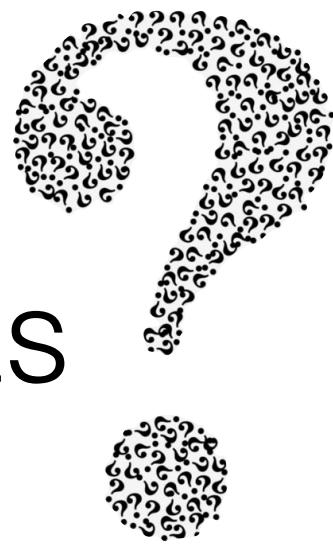
(page-wise zone map)



What if the **range query** has
a **very low selectivity**?

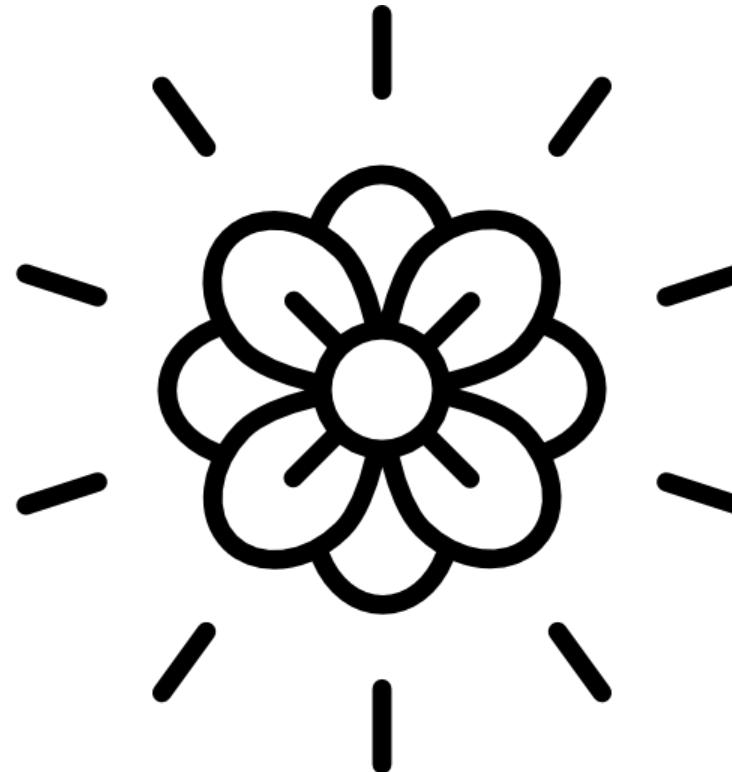
in the **worst case**,
need to read **2 pages per sorted run**

$$\text{Cost of short range lookup} = 2 \cdot L$$

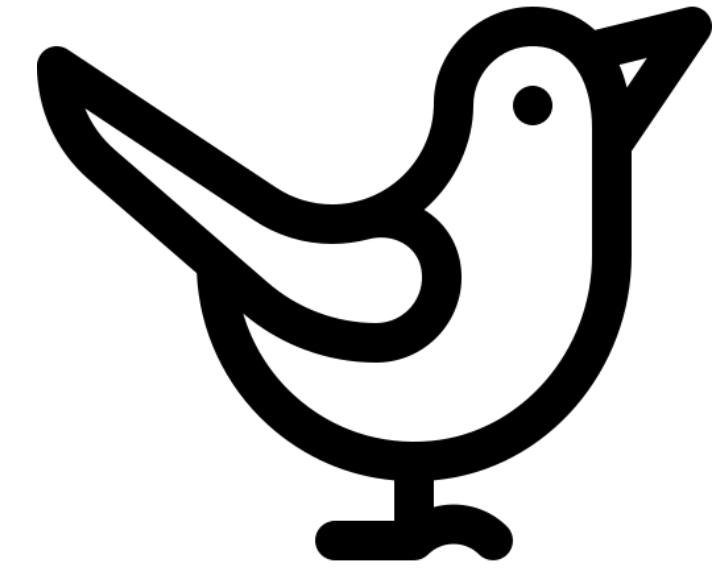


Filters?

Point and range filters



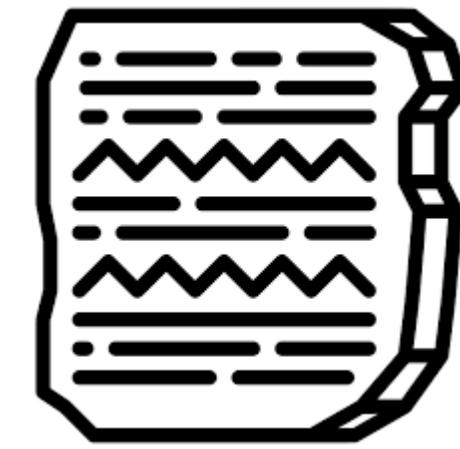
Bloom



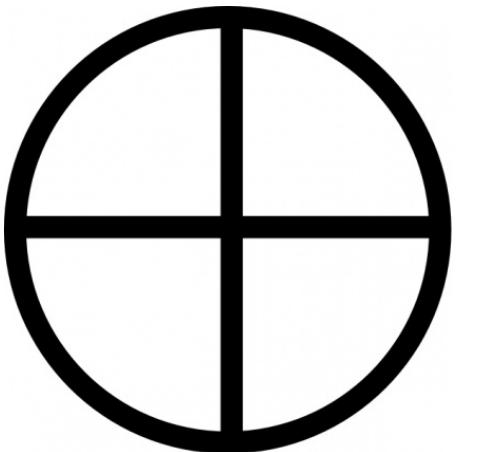
Cuckoo



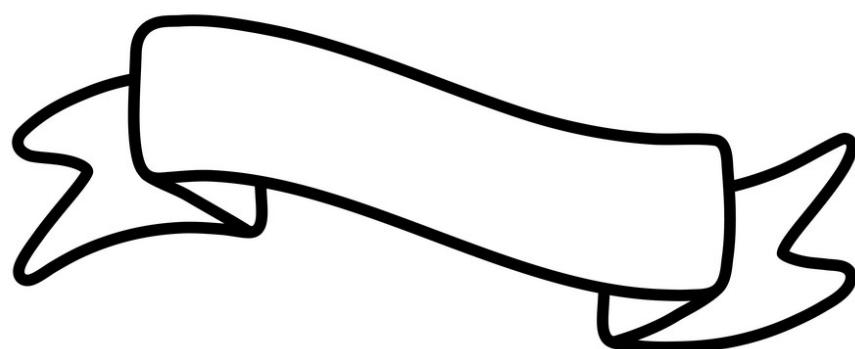
SuRF



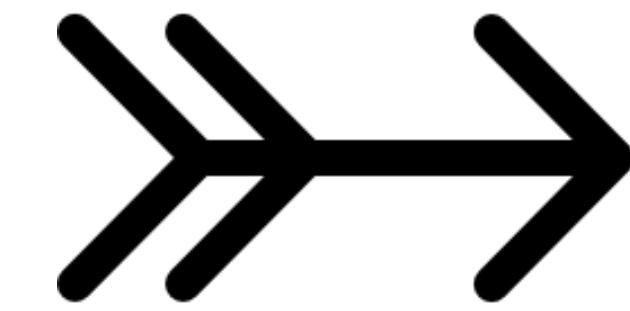
Rosetta



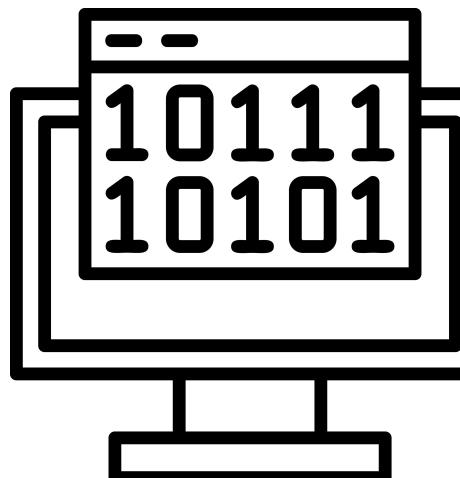
XOR



Ribbon



prefix



REncoder

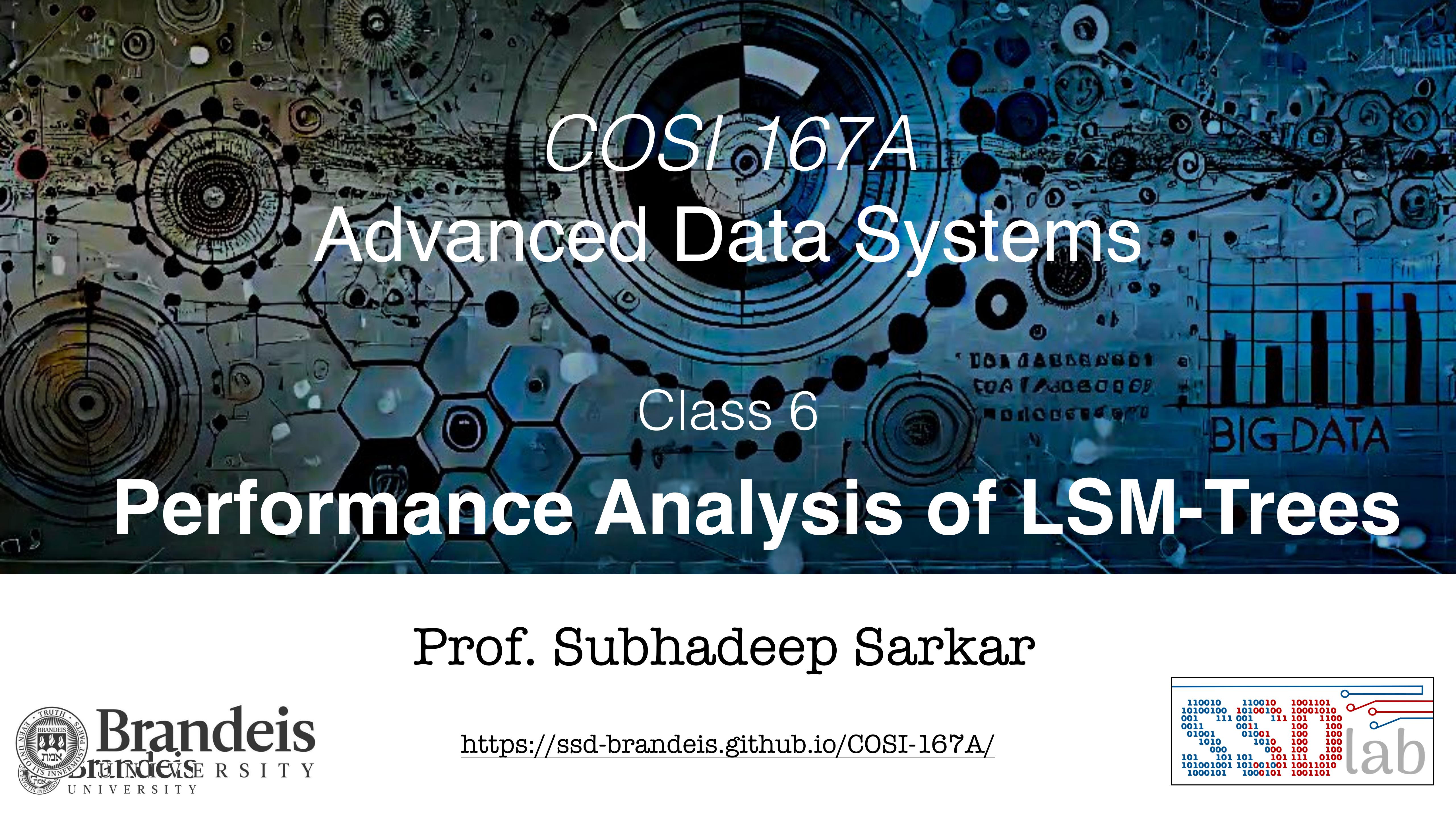


Next time in COSI 167A

More on LSMS

Cost analysis

Class Project discussion



COSI 167A Advanced Data Systems

Class 6

Performance Analysis of LSM-Trees

Prof. Subhadeep Sarkar

<https://ssd-brandeis.github.io/COSI-167A/>

