Social Network Analysis: Terms, Data, Theory & Vizualization

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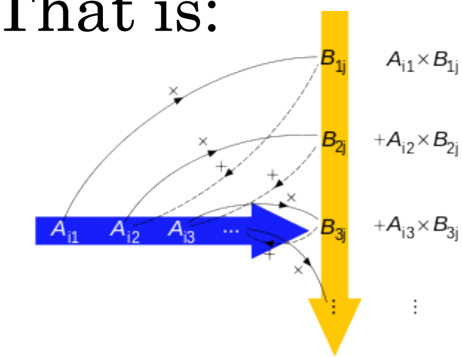
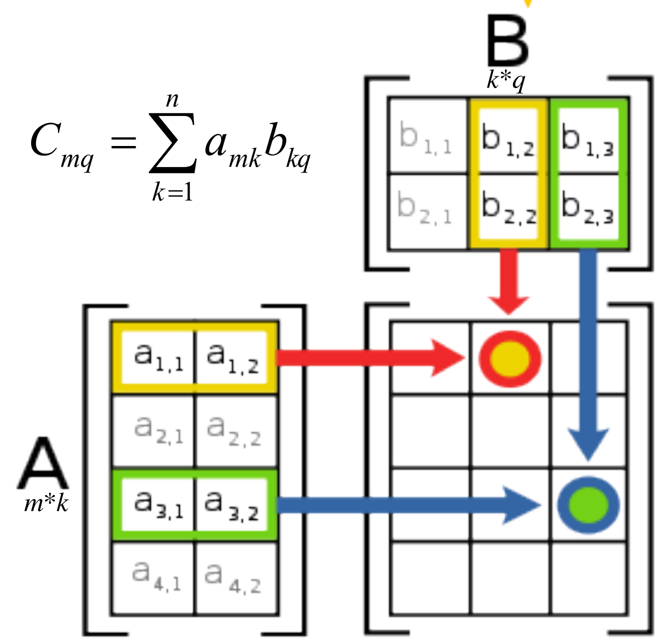
- For matrix multiplication order matters: AB ≠ BA.
- To be conformable, the number of rows in the 1st matrix must match the number of columns in the 2nd. That is:
  - A_{m*k} is conformable with B_{k*q}
  - Thus:
    - A_{4x2} x B_{2x3} = $C_{4 \text{ x } 3}$
      - but
    - A_{2x4} x B_{2x3} is not defined
- This is because in matrix multiplication, *elements* in the resulting matrix are:
  - the sum of
    - the element wise multiplication of the
    - corresponding:
      - rows from the 1st matrix, and
      - columns from the 2nd matrix:



$$C_{mq} = \sum_{k=1}^{n} a_{mk} b_{kq}$$



- Matrix operations require that the matrices are conformable (have compatible dimensions for the operation.
- For addition (+), subtraction (-), or element-wise multiplication (#), conformable matrices have the same number of rows ***and*** columns.
  - For these operations, the new cell values are the operation applied to the corresponding cell values in each initial matrix.

$$\mathbf{A} = \begin{matrix} a & d \\ b & e \\ c & f \end{matrix} \qquad \mathbf{B} = \begin{matrix} u & x \\ v & y \\ w & z \end{matrix} \qquad A+B = \begin{matrix} a+u & d+x \\ b+v & e+y \\ c+w & f+z \end{matrix} \qquad A-B = \begin{matrix} a-u & d-x \\ b-v & e-y \\ c-w & f-z \end{matrix}$$

Multiplication by a scalar: $3A = \begin{matrix} 3a & 3d \\ 3b & 3e \\ 3c & 3f \end{matrix}$ $\qquad A\#B = \begin{matrix} au & dx \\ bv & ey \\ cw & fz \end{matrix}$

Why Matrices?
Matrix Algebra & Graph Theory

Adjacency Matrix

0	0	1	0	0	0
0	0	1	0	0	0
1	1	0	1	0	0
0	0	1	0	1	1
0	0	0	1	0	1
0	0	0	1	1	0

$\times$

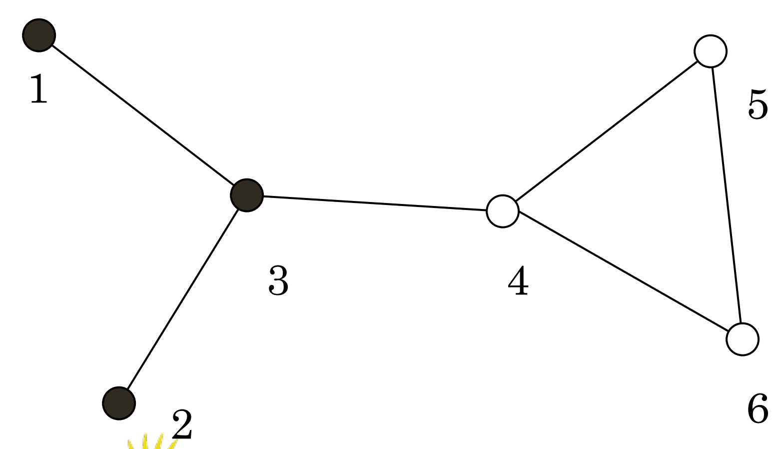
Attribute

1	0
1	0
1	0
0	1
0	1
0	1

$=$

Alter Attribute

1	0
1	0
2	1
1	2
0	2
0	2



Why Matrices?
Matrix Algebra & Graph Theory

0*1 + 0*1 + 1*1 + 0*0 + 0*0 + 0*0 = 1

1*1 + 1*1 + 0*1 + 1*0 + 0*0 + 0*0 = 2 …

0*0 + 0*0 + 1*0 + 0*1 + 0*1 + 0*1 = 0 …

Adjacency Matrix

0	0	1	0	0	0
0	0	1	0	0	0
1	1	0	1	0	0
0	0	1	0	1	1
0	0	0	1	0	1
0	0	0	1	1	0

×

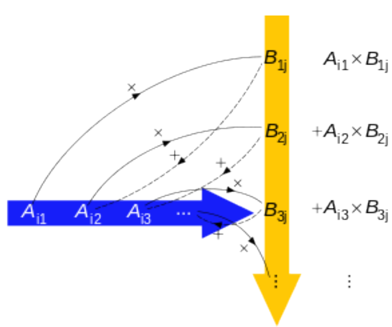
Attribute

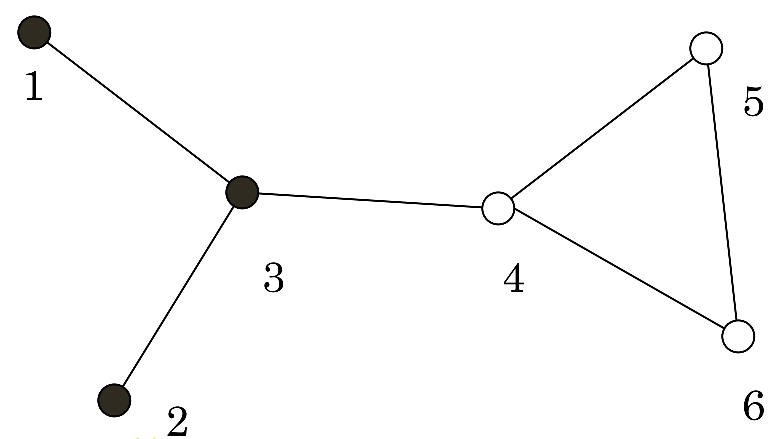
1	0
1	0
1	0
0	1
0	1
0	1

=

Alter Attribute

1	0
1	0
2	1
1	2
0	2
0	2





- The *transpose* (\` or ^{T}) of a matrix swaps the rows/columns:

$$\mathbf{A}^{t}_{ij} = \mathbf{A}_{ji}$$

- An M x N matrix becomes an N x M matrix.

$$\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}^{T} = \begin{bmatrix} a & c & e \\ b & d & f \end{bmatrix}$$

Why Matrices?
Matrix Algebra & Graph Theory

Transpose(Attribute)

1	**1**	**1**	**0**	**0**	**0**
0	0	0	1	1	1

×

Alter Attribute

1	0
1	0
2	1
1	2
0	2
0	2

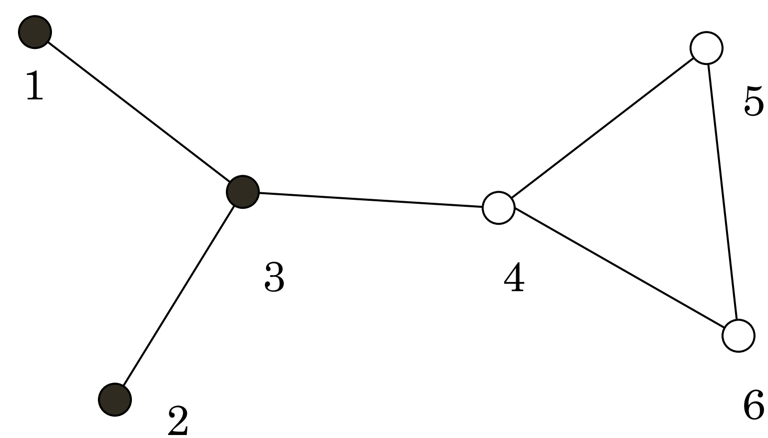
=

Attribute Mixing Matrix

4	**1**
1	**6**

Attribute

1	0
1	0
1	0
0	1
0	1
0	1



Why Matrices?
Matrix Algebra & Graph Theory

Adjacency Matrix

	1	2	3	4	5	6
1		0	1	0	0	0
2	0		0	0	0	0
3	0	1		1	0	0
4	0	0	1		1	1
5	0	0	0	0		1
6	0	0	0	0	1	

×

Attribute

1	0
1	0
0	1
0	1
1	0
0	1

=

Alter Attribute

0	1
0	0
1	1
1	2
0	1
1	0

Transpose(Attribute)

1	1	0	0	1	0
0	0	1	1	0	1

×

Alter Attribute

0	1
0	0
1	1
1	2
0	1
1	0

=

Attribute Mixing Matrix

	1	2
1	0	2
2	3	3